DEUTSCHE BUNDESBANK
EUROSYSTEM

Discussion Paper

Deutsche Bundesbank
No 28/2024

Digital transformation and its impact on labour productivity – A multi-sector perspective

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Internet http://www.bundesbank.de

ISBN 978–3–98848–003–3
ISSN 2941–7503

Non-technical summary

Research Question

Since the mid-20th century, a fast-growing phenomenon has been transforming our economy: digitalisation. At the same time, aggregate labour productivity growth has declined in a number of advanced economies. This coincidence has fuelled an ongoing debate about the importance of digitalisation for labour productivity growth. Against this background, we ask how important efficiency gains in the digital sectors are for labour productivity in Germany, France and the United States and what role input-output linkages play in transmitting these efficiency gains to the aggregate economy.

Contribution

The data show that a significant part of the output of digital goods producing sectors serves as an intermediate input. While traditional approaches have mainly focused on the role of investment in digital capital as a key transmission channel, we study the impact of efficiency gains in the digital goods producing sectors on aggregate labour productivity through the lens of a multi-sector dynamic general equilibrium model with detailed input-output linkages.

Results

Without the total factor productivity (TFP) gains in the digital sectors, aggregate labour productivity growth would have been about half as high in Germany, France and the United States between 1996 and 2020. Input-output linkages are a key transmission mechanism. If production linkages are disregarded, the productivity impact of the digital sectors is considerably lower.

Nichttechnische Zusammenfassung

Fragestellung

Seit Mitte des 20. Jahrhunderts hat ein sich rasch entwickelndes Phänomen unsere Wirtschaft verändert: die Digitalisierung. Gleichzeitig ist das Wachstum der gesamtwirtschaftlichen Arbeitsproduktivität in einer Reihe von Industrieländern zurückgegangen. Diese Koinzidenz hat eine anhaltende Debatte über die Bedeutung der Digitalisierung für das Arbeitsproduktivitätswachstum entfacht. Vor diesem Hintergrund untersuchen wir, wie wichtig Effizienzsteigerungen in den digitalen Sektoren für die Arbeitsproduktivität in Deutschland, Frankreich und den USA sind und welche Rolle Input-Output-Verknüpfungen bei der Übertragung dieser Effizienzsteigerungen auf die Gesamtwirtschaft spielen.

Beitrag

Die Daten zeigen, dass ein erheblicher Teil der Produktion von Digitalgüter produzierenden Wirtschaftssektoren in Form von Vorleistungen genutzt werden. Während sich traditionelle Ansätze auf die Rolle von Investitionen in den digitalen Kapitalstock als zentralen Übertragungskanal fokussierten, untersuchen wir die Auswirkungen von Effizienzsteigerungen in den Digitalsektoren auf die gesamtwirtschaftliche Arbeitsproduktivität anhand eines dynamischen allgemeinen Gleichgewichtsmodells mit detaillierten Input-Output-Verknüpfungen.

Ergebnisse

Ohne die TFP-Zuwächse in den Digitalsektoren wäre das Wachstum der Arbeitsproduktivität in Deutschland, Frankreich und den USA zwischen 1996 und 2020 nur etwa halb so hoch ausgefallen. Werden sektorale Verknüpfungen über Vorleistungen außer Acht gelassen, fällt der Bedeutung der Digitalsektoren für die aggregierte Arbeitsproduktivität erheblich niedriger aus. Input-Output-Verknüpfungen sind demnach ein zentraler Übertragungsmechanismus.

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Digital transformation and its impact on labour productivity - A multi-sector perspective*

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Abstract

In recent years, there has been a controversial debate on how the rapid diffusion of digital technologies affects labour productivity in advanced economies. Using a multi-sector dynamic general equilibrium model, we show that cumulative labour productivity growth in the United States, Germany and France over the period from 1996 to 2020 would have been about half as high without the efficiency gains from the sectors producing digital goods – despite their relatively small size in terms of gross value added. This is not only because TFP growth in the digital sectors is exceptionally high, but also because other sectors benefit from these efficiency improvements via production linkages.

Keywords: dynamic general equilibrium model, sectoral linkages, production network, digitalisation.

JEL classification: E17, E23, E24, O33, O41, O47.

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1 Introduction

Since the mid-20th century, a fast-growing phenomenon has been transforming our economy: digitalisation. Yet, its effect on overall labour productivity growth in advanced economies remains a contentious topic (see, e.g., Gordon, 2016; Byrne, Oliner, and Sichel, 2017; Brynjolfsson, Rock, and Syverson, 2019).

Against this background, our paper analyses the impact of efficiency gains in digital goods producing sectors on aggregate labour productivity in three advanced economies – Germany, France and the United States – with a particular focus on a specific transmission channel: the role of digital intermediate inputs. Table 1 indicates that input-output linkages might be a relevant channel through which digitalisation can have a productivity-enhancing effect: In the United States, Germany and France, half or more of the output produced by digital sectors is used as intermediate inputs.[1]

To examine the role of intermediates as a transmission channel of sectoral efficiency gains, we construct a multi-sector dynamic general equilibrium model with a production network. This framework enables an in-depth investigation of how efficiency improvements in the production of digital goods have affected labour productivity growth through production linkages. Our study thereby adds to the growing body of research emphasising the significance of sector-specific developments in shaping macroeconomic patterns (see Foerster, Hornstein, Sarte, and Watson, 2022; Gaggl, Gorry, and vom Lehn, 2023).

Table 1: Usage of digital goods in percent

	Germany	France	United States
Consumption	31	26	24
Investment	16	17	25
Intermediate inputs	53	57	51

Notes: Share of gross output of the digital goods producing sectors in the year 2000 used as consumption goods, investment goods or intermediate inputs. Digital sectors consist of divisions C26 *Manufacture of computer, electronic and optical products* and C27 *Manufacture of electrical equipment* of the NACE Rev. 2 classification as well as section J *Information and communication*. Source: World Input Output Database.

According to the model simulations, cumulative labour productivity growth in the United States, Germany and France would have been about half as high between 1996 and 2020 without total factor productivity (TFP) growth in the digital sectors, compared to the benchmark simulation. Quantitatively, this amounts to a difference of 25 percentage points in the United States, and disparities of 15 and 11 percentage points in Germany and France, respectively. Production networks play a key role in transmitting TFP growth from the digital sectors into the broader economy. In a second set of simulations, we

[1] Digital sectors are defined on the basis of the current version of the statistical classification of economic activities in the European Community (NACE Rev. 2) as the *Manufacture of computer, electronic and optical products* and *Manufacture of electrical equipment* (divisions C26 and C27) as well as the IT services sector *Information and communication* (section J).

show that disregarding network connections substantially reduces the influence of the digital sectors on the simulated trajectories. When digital intermediates are not taken into account, simulated labour productivity growth is 10, 7 and 5 percentage points lower than in the baseline scenario for the United States, Germany and France, respectively.

Our modelling framework is a closed-economy, flexible-price dynamic general equilibrium model with multiple interrelated production sectors. Goods can be consumed or invested. Compared to the conventional one-sector model, however, sectoral output also serves as an intermediate input for production, thereby affecting aggregate labour productivity growth through production networks. In terms of parameterisation, the sectors deviate from each other with respect to their factor intensities and relevance for consumption, investment and intermediate input demand. The benchmark model covers eight sectors that are calibrated using the latest version of the World Input-Output Database (WIOD, see Timmer, Dietzenbacher, Los, Stehrer, and De Vries, 2015). We set up separate versions of the model for Germany, France and the U.S. respectively, and solve them non-linearly.

Sector-specific shocks to TFP growth are the only exogenous driver in the model. They are derived from empirical measures of sectoral TFP growth, which are corrected for changes in capacity utilisation. The estimates show that TFP growth in the digital sectors was significantly higher than in the remaining sectors. For instance, TFP in the United States' digital sectors tripled from 1996 to 2020, whereas it increased by a mere 10 percent in the remaining sectors.

To examine the role of digital transformation in labour productivity growth, we construct two counterfactual scenarios. In the first, we simulate the path of aggregate labour productivity under the assumption that there is no growth of TFP in the digital sectors. This allows us to evaluate the importance of efficiency gains in the digital sectors as a whole. In the second, we analyse how labour productivity would have evolved if digital products had not served as inputs in other sectors of the economy. This exercise sheds light on the role of the production network. The counterfactual scenarios show that TFP growth in the digital sectors is a major driver of aggregate productivity growth. For the United States, the model predicts that aggregate labour productivity would have stagnated since the mid-2000s without the efficiency gains from the digital sectors. Significant productivity contributions from the digital sectors are also present in the other countries studied. Moreover, the simulations emphasise the importance of input-output linkages as a transmission channel. When digital output is used exclusively for consumption or investment purposes, labour productivity growth between 1996 and 2020 falls considerably short compared to the baseline model in all of the countries considered.

Looking at the cross-country differences in the production structure more broadly, we find that the higher labour productivity growth in the United States is not only due to the marked efficiency improvements in the digital sectors, but also promoted by an overall favourable production structure. This conclusion is drawn from replicating the benchmark analyses for the United States, while replacing the input-output matrices with those corresponding to Germany and France.

Related Literature Our paper is related particularly to analyses integrating production networks into dynamic general equilibrium models as well as work centering on the impact of digitalisation on productivity.

The studies that come closest to ours are those that use dynamic multi-sector flexible-price general equilibrium models to highlight the role of sectoral developments for the evolution of key macroeconomic variables, such as Foerster et al. (2022), Gaggl et al. (2023), and vom Lehn and Winberry (2022). The latter focuses on the effects of TFP shocks and their amplification through investment networks over the business cycle frequency. Foerster et al. (2022) and Gaggl et al. (2023), in turn, examine the contribution of sectoral developments to aggregate growth.

Both Foerster et al. (2022) and Gaggl et al. (2023) characterise an aggregate balanced growth path and decompose aggregate GDP growth into its sectoral contributions. Specifically, Foerster et al. (2022) use Cobb-Douglas preferences and a production structure with heterogeneous sectoral parameters to derive sector-specific multipliers that can be decomposed into a direct and an indirect (network) effect. Their framework, however, does not allow for production networks to change endogenously over time as pointed out by Gaggl et al. (2023), who study structural change by specifying production and preference functions with constant elasticity of substitution (CES). They find that inputs produced by different sectors are substitutes in the production of investment goods and complements in the production of intermediates and consumption goods, implying that structural change is mainly driven by investment. However, to characterise the balanced growth path, the authors assume that the sectoral production parameters are identical across sectors.

While our study also highlights the importance of sectoral developments for macroeconomic trends, it conceptually complements the works of Foerster et al. (2022) and Gaggl et al. (2023) in that we directly capture the impact of sectoral TFP changes on aggregate variables by constructing counterfactuals rather than decomposing sectoral contributions along an aggregate balanced growth path. This allows us to combine CES-type preferences and production structure with sector-specific production parameters. Moreover, since we solve the model nonlinearly, the effects of sectoral TFP shocks are state-dependent in our setting.

Other papers using multi-sector dynamic general equilibrium models but with nominal rigidities are those of Bouakez, Cardia, and Ruge-Murcia (2011), Pasten, Schoenle, and Weber (2020), Bouakez, Rachedi, and Santoro (2021), and Peri, Rachedi, and Varotto (ming). While the former uses a multi-sector framework to assess the impact of intersectoral linkages on the government expenditure multiplier in the United States, the latter examines the effects of government investment. Bouakez et al. (2021), in turn, introduces durable goods into a multi-sector framework to examine the sectoral and aggregate effects of monetary policy shocks. Pasten et al. (2020) analyse the role of heterogeneous price rigidities for the responses of sectoral output and inflation to a monetary policy shock in the United States (see also Bouakez, Cardia, and Ruge-Murcia, 2014). In addition to analysing prototypical macro policies, multi-sector dynamic general equilibrium models have recently also increasingly been used in an environmental context (see, e.g., Hinterlang, Martin, Röhe, Stähler, and Strobel, 2022; Ernst, Hinterlang, Mahle, and Stähler, 2023).

Further studies that investigate the effects of sectoral TFP shocks in a static but rich multi-sector environment include, inter alia, those of Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012), Baqaee and Farhi (2019, 2020), and Bigio and LaO (2020).

Interrelation of digitalisation and productivity has been the focus of a vast strand of literature (see, e.g., Gordon, 2000; Oliner, Sichel, and Stroh, 2007; Jorgenson, Ho, and Stiroh, 2008; Byrne, Fernald, and Reinsdorf, 2017; Byrne et al., 2017; Goldfarb and Tucker, 2019; Byrne, 2022; Acemoglu, Autor, and Patterson, 2023). The coincidence of a rapid spread of digital technologies and declining labour productivity growth – often termed a modern productivity paradox – has fuelled a controversial debate about the impact of digitalisation on aggregate labour productivity growth in advanced economies (see, e.g., Gordon, 2016; van Ark, 2016; Brynjolfsson et al., 2019). Previous work on this topic has focused, in particular, on investment as the central transmission channel (see, e.g., Greenwood, Hercowitz, and Krusell, 1997; Jorgenson and Stiroh, 2000; Oliner and Sichel, 2000; Basu, Fernald, Oulton, and Srinivasan, 2003; Brynjolfsson and Hitt, 2003; Cette, Clerc, and Bresson, 2015; Bergeaud, Cette, and Lecat, 2017). Digital products, however, enter production processes not only as capital goods but also as intermediate inputs. Against this background, we explicitly account for the role of input-output linkages in the diffusion of efficiency gains.

The rest of the paper is organised as follows. Section 2 describes the construction of sectoral TFP shocks. The theoretical model is introduced in Section 3, its calibration in Section 4. Simulation design and results are described in Section 5. Section 6 discusses the results and provides several robustness checks and Section 7 concludes.

2 Constructing time series of sectoral TFP shocks

In order to examine the impact of efficiency gains in digital sectors on aggregate productivity growth, we calculate TFP growth rates at the sectoral level and feed them into our theoretical model. A key challenge in estimating TFP lies in the measurement of the production factors used. For example, the calculated TFP growth series may be distorted by idle assets or under-utilisation of labour.

We use a two-step procedure to account for the degree of capacity utilisation in the measurement of TFP growth rates, similar to Basu, Fernald, and Kimball (2006) and Comin, Quintana, Schmitz, and Trigari (2020).[2] Specifically, we first derive Solow residuals using annual sectoral data from the 2023 version of the EU KLEMS data for Germany, France and the United States. The growth accounting exercise builds on a standard Cobb-Douglas production function and spans the time period from 1997 to 2020.[3] The sample comprises 21 sectors that cover the non-farm, non-mining private market economy (see Table 2). In a second step, we adjust the Solow residuals for changes in the utilisation of production factors within a panel model. Specifically, we regress the sector-specific Solow residuals on measures for sectoral capacity utilisation and sector-specific fixed effects. As the level of capacity utilisation is, in general, unobserved, we apply two different proxies to measure it. For the European countries, we adapt the approach of Comin et al. (2020) and use survey answers about the level of factor utilisation from the European Commission's business and consumer surveys. For the United States, we follow Basu et al. (2006) in using changes in average weekly hours worked.[4] Our final sectoral TFP series are given

[2] Details for the estimation can be found in the Appendix.

[3] The respective sectoral labour and capital income shares are permitted to be time-varying. The income shares show only little variation over the sample period; see supplementary Appendix.

[4] As in Basu et al. (2006) and Comin et al. (2020), the impact of utilisation on TFP growth is estimated with a instrumental variable approach (see Appendix for details).

by the difference between the sector-specific Solow residual and the estimated impact of capacity utilisation.

Table 2: Sectors

Manufacturing	Non-manufacturing
Food, beverages, tobacco products [C10-C12]	Construction [F]
Textiles, wearing apparel, leather [C13-C15]	Trade; repair of motor vehicles [G]
Wood, paper, printing [C16-C18]	Transportation and storage [H]
Chemicals, basic pharmaceutical products [C20-C21]	Accommodation and food service activities [I]
Rubber, plastic, non-metallic mineral products [C22-C23]	**Information and communication [J]**
Metal products [C24-C25]	Financial and insurance activities [K]
Computer, electronic, optical products [C26-C27]	Professional, scientific and technical activities [M]
Machinery and equipment n.e.c. [C28]	Administrative and support service activities [N]
Motor vehicles, trailers, other transport equipment [C29-C30]	Arts, entertainment, recreation ; other services [R-S]
Furniture, jewellery, musical instruments, toys [C31-C33]	
Electricity, gas, steam, air conditioning supply [D]	
Water supply, sewerage, waste [E]	

Notes: List of sectors included in the estimations with NACE codes in parentheses. Digital sectors are highlighted in bold. For the United States, sectors D-E enter the model as an aggregate and C20-C21 is replaced by C20 owing to missing data.

Figure 1 shows the path of utilisation-adjusted TFP in digital and non-digital sectors between 1996 and 2020.[5] In all countries, TFP growth in the digital sectors was substantially larger than in the remaining sectors. In the United States and Germany the TFP level in the digital branches more than doubled in the respective years. The growth rate in France is a bit lower but still considerably higher than for non-digital branches. TFP in non-digital sectors increased only little over the same period in all countries.

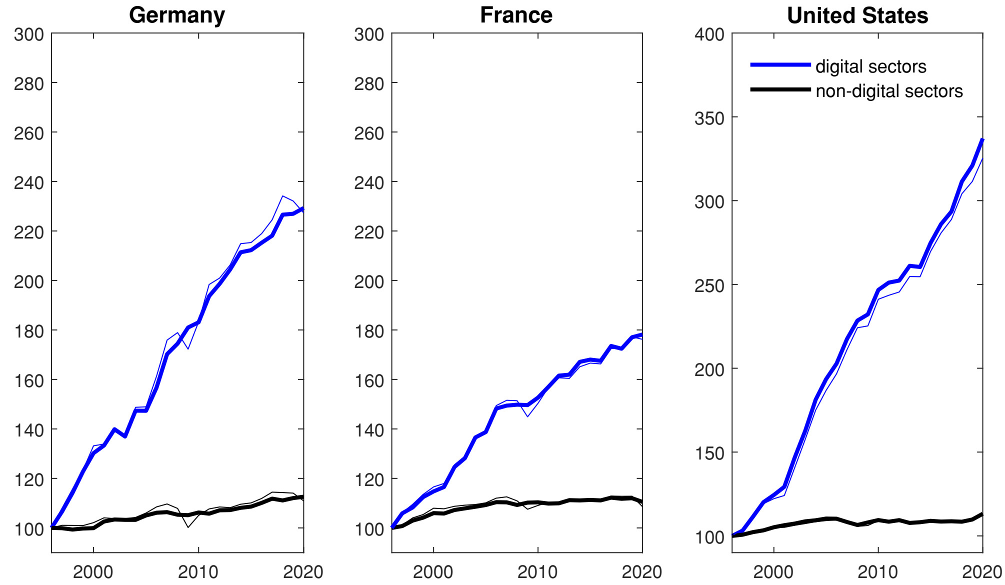
3 The model

Our analysis is based on a flexible-price model that includes a representative household, a set of $\mathcal{S} = \{1, 2, ..., S\}$ production sectors each containing a perfectly competitive representative firm, and perfectly competitive consumption, investment and intermediate goods retailers.[6]

[5]To compute TFP measures for digital and non-digital sectors, we aggregate sectoral TFP growth rates, using their value-added contributions as weights.

[6]A detailed derivation of the model is presented in the Appendix.

Figure 1: TFP in digital and non-digital sectors



Notes: Figure plots indices of aggregate TFP in the digital (NACE C26-C27 and NACE J) and non-digital sectors (rest) from 1996 to 2020 (1996=100). Thin lines show TFP without adjustment for capacity utilisation.

3.1 Representative household

The representative household maximises the stream of expected utility,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \kappa_N \frac{N_t^{1+\zeta}}{1+\zeta} \right\} \tag{1}$$

by choosing a sequence of consumption C_t, labour supply N_t and physical capital investment I_t, where $0 < \beta < 1$ is the subjective discount factor, σ denotes the inverse of the elasticity of intertemporal substitution for consumption, κ_N measures the disutility of labour and ζ represents the inverse of the Frisch labour supply elasticity. $\mathbb{E}_0$ is the expectations operator at time $t = 0$. Given the consumer price index (CPI) P_t^C, the representative household's optimisation problem is subject to the followed budget constraint, which is cast in real terms as

$$C_t + P_t^I I_t = w_t N_t + r_t^k K_{t-1}. \tag{2}$$

The variable P_t^I is the CPI-deflated real price of investment goods, w_t is the real wage and r_t^k is the real rental rate of capital K_t. The capital accumulation process is represented by the following law of motion

$$K_t = (1-\delta)K_{t-1} + I_t, \tag{3}$$

with δ denoting the rate of phyiscal capital depreciation.

We follow Bouakez, Rachedi, and Santoro (2023) in assuming that the household's preferences for consumption and investment goods spanning all sectors are represented by a function with constant elasticity of substitution (CES):

$$X_t = \left[\sum_{s=1}^{S} \psi_{X,s}^{1-\sigma_X} X_{s,t}^{\sigma_X} \right]^{\frac{1}{\sigma_X}}, \quad \sum_{s=1}^{S} \psi_{X,s} = 1, \quad \psi_{X,s} \in [0,1], \quad X \in \{C, I\}$$

where X represents either consumption (C) or investment (I). The term $\psi_{X,s}$ represents the relative preference or weight assigned to goods from sector s within the consumption or investment bundle. The elasticity of substitution among these goods is controlled by $\sigma_X \in (-\infty, 1)$, with the corresponding elasticity being $1/(1-\sigma_X)$. Demand for goods from sector s as a function of the relative prices and the aggregate bundle X_t is given by the first-order condition

$$X_{s,t} = \psi_{X,s} \left(\frac{P_{s,t}}{P_t^X} \right)^{-\left(\frac{1}{1-\sigma_X}\right)} X_t. \tag{4}$$

The aggregate price index for consumption or investment goods bundles is given by

$$P_t^X = \left[\sum_{s=1}^{S} \psi_{X,s} \left(P_{s,t}\right)^{-\frac{\sigma_X}{(1-\sigma_X)}} \right]^{-\frac{(1-\sigma_X)}{\sigma_X}}. \tag{5}$$

3.2 Labour and capital

The allocation of labour and capital to the various sectors $s \in \mathcal{S}$ within the economy is based on the assumption of representative labour and capital agencies operating in a perfectly competitive market. These agencies employ the aggregate labour supply N_t at the prevailing real wage rate w_t and rent out the aggregate capital stock K_t at the real rental rate r_t^k. They then supply these factors to producers of intermediate goods across S distinct sectors.

We follow Bouakez et al. (2023) in assuming that the total amount of labour provided by the household is a CES function of the labour supplied to each sector, that is

$$N_t = \left(\sum_{s=1}^{S} \omega_{N,s}^{1-\nu_N} N_{s,t}^{\nu_N} \right)^{\frac{1}{\nu_N}},$$

where $\omega_{N,s}$ is the weight attached to labour provided to sector $s \in \mathcal{S}$. The parameter ν_N determines the elasticity of substitution of labour across sectors $1/(1-\nu_N)$, capturing the degree of labour mobility. Similarly, aggregate capital, K_t bundles sectoral capital flows, $K_{s,t}$ with an elasticity of substitution $1/(1-\nu_K)$. In the limiting case where $\nu_N, \nu_K \to 1$, we observe perfect mobility of labour and capital, leading to uniformity in nominal wages and capital returns across all sectors. Conversely, when $\nu_N, \nu_K > 1$, capital and labour are imperfectly mobile, which allows for disparities in wages and capital returns among sectors. The parameters $\nu_N, \nu_K > 1$ are thus instrumental in succinctly capturing the gradual reallocation of labour and capital in response to shocks. The first-order conditions for sectoral demand for labour and capital are, respectively, given by

$$N_{s,t} = \omega_{N,s} \left(\frac{w_{s,t}}{w_t} \right)^{-\frac{1}{(1-\nu_N)}} N_t \tag{6}$$

and

$$K_{s,t} = \omega_{K,s} \left(\frac{r^K_{s,t+1}}{r^K_{t+1}} \right)^{-\left(\frac{1}{1-\nu_K}\right)} K_t. \tag{7}$$

3.3 Sectoral production and intermediate inputs

Within each sector $s \in \mathcal{S}$, we consider a representative firm that engages in the production of sector-specific output, $y_{s,t}$, by combining labour, $N_{s,t}$, capital, $K_{s,t-1}$, and a composite of intermediate inputs, $H_{s,t}$. The firms operate under a Cobb-Douglas production function[7]

$$y_{s,t} = \left(\varepsilon^{VA}_{s,t} K^{1-\alpha_{N,s}}_{s,t-1} N^{\alpha_{N,s}}_{s,t} \right)^{\alpha_{H,s}} (H_{s,t})^{1-\alpha_{H,s}}, \tag{8}$$

where $\varepsilon^{VA}_{s,t}$ represents sectoral TFP. The parameters $\alpha_{N,s}, \alpha_{H,s} \in (0,1)$ determine the sector-specific output elasticity with respect to capital, labour, and intermediate inputs. We assume that TFP follows a random walk process with innovations $e_{s,t}$,

$$\frac{\varepsilon^{VA}_{s,t}}{\varepsilon^{VA}_{s,t-1}} = 1 + e_{s,t}. \tag{9}$$

The intermediate input bundle, $H_{s,t}$, is produced by a perfectly competitive intermediate-goods retailer that operates a CES production technology

$$H_{s,t} = \left(\sum_{j=1}^{S} \psi^{1-\sigma_{H,s}}_{H,s,j} H^{\sigma_{H,s}}_{s,j,t} \right)^{\frac{1}{\sigma_{H,s}}}, \tag{10}$$

where $\psi_{H,s,j}$ determines the weight of the input from sector j in the production of the intermediate-goods bundle for sector s, and $\sigma_{H,s}$ is the parameter that dictates the elasticity of substitution among these inputs. The optimal demand for intermediate goods originating from sector j and used in sector s is derived from the first-order condition

$$H_{s,j,t} = \psi_{H,s,j} \left(\frac{P_{j,t}}{P^H_{s,t}} \right)^{\left(-\frac{1}{1-\sigma_{H,s}}\right)} H_{s,t}. \tag{11}$$

[7]The choice of a unitary substitution elasticity between value added and intermediate inputs (Cobb-Douglas production function) is motivated by the estimate of Atalay (2017) and implies constant shares of labour and intermediate inputs. Appendix A.2 presents time series of $\alpha_{N,s}$ and $\alpha_{H,s}$, supporting the assumption's validity for the examined time period.

3.4 Market clearing and aggregation

In each sector s, product market clearing implies

$$y_{s,t} = C_{s,t} + I_{s,t} + \sum_{j=1}^{S} H_{j,s,t}. \tag{12}$$

At the aggregate level, CPI-deflated sectoral value added is defined as

$$Y_t^{va} = C_t + P_t^I I_t. \tag{13}$$

4 Calibration and solution method

There are two subsets of model parameters. The first comprises general parameters related to the aggregate economy, which are based on values from the literature (see Table 3 for an overview). The second set of parameters captures the heterogeneity on the production side. These are calibrated using data from input-output tables. The model is specified for three different countries: Germany, France, and the United States.

Table 3: Baseline calibration of general parameters

Variable/Parameter	Symbol	Value
Discount factor	β	0.97
Elasticity of intertemporal substitution	σ	1.25
Inverse Frisch elasticity of labour supply	ζ	1.00
Labour disutility scaling parameter	κ_N	19.94
Capital depreciation rate	δ^k	0.10
Parameters governing the substitution elasticities:		
Consumption	σ_C	1-1/1.01
Investment	σ_I	1-1/1.01
Labour	ν_N	2.00
Capital	ν_K	2.00
Intermediate inputs	$\sigma_{H,s}$	1-1/0.20

Notes: Parameterised for the annual frequency. The table shows calibrated values for general parameters as described in the main text.

General parameters The model is calibrated to the annual frequency. We set the discount factor to $\beta = 0.968$, which implies an annual interest rate of 3.3% in each country. The intertemporal elasticity of substitution is fixed at a value of $\sigma_c = 1.25$. The Frisch elasticity of labour supply is set to unity (i.e. $\zeta = 1$). The relative weight of the disutility of labour is calibrated to match a targeted aggregate labour supply of $\bar{N} = 0.33$. We assume a capital depreciation rate of 10%, which is a standard choice in the literature.

Substitution elasticities for consumption and investment goods are set to unity, as in Foerster et al. (2022) and vom Lehn and Winberry (2022). For consumption goods, this value is close to the estimate for the EU of Hobijn and Nechio (2019), who estimate a substitution elasticity of 1.01 in a ten-sector specification of their multi-sector model. Regarding the parameters governing the substitution elasticities of labour and capital ν_N and ν_K, we follow Bouakez et al. (2021) and choose a value of two. In the sectoral production functions, we assume a unitary substitution elasticity between value added and intermediate inputs in the sectoral production function, i.e., the Cobb-Douglas production function, and a relatively low substitution elasticity of intermediate inputs of 0.2. These values are specified based on the estimates of Atalay (2017) for the United States.[8] We provide further discussion and sensitivity analysis of these key parameters in Section 6.

Sector-specific production parameters For each country, we calibrate the sector-specific output elasticities, $\alpha_{N,s}$ and $\alpha_{H,s}$, the respective relative weights assigned to consumption, investment and intermediate goods, $\psi_{H,s,j}$, $\psi_{C,s}$, and $\psi_{I,s}$, and the weights attached to labour and capital provided by sector s, $\omega_{N,s}$, $\omega_{K,s}$, based on data for 2000 (which is the earliest available year) of the latest vintage of the World Input-Output Database (WIOD).[9] Further information regarding the computation of sectoral parameters and their values is provided in Appendix 7. The benchmark economy in each country consists of $S = 8$ sectors, relying on the NACE Rev. 2 classification.[10,11] The digital goods producing sectors consists of NACE section J *Information and Communication* as well as NACE divisions C26 *Manufacture of computer, electronic and optical products* and C27 *Manufacture of electrical equipment*. The effects of different sectoral aggregations are discussed in Section 6. An overview encompassing all sectors included in the benchmark economy and the respective sector's share in real value added is provided in Table A.1.

[8]Appendix D in Atalay (2017) reports estimates of the elasticity of substitution within the basket of intermediate inputs of 0.2 and between the bundle of intermediate inputs and the capital-labour composite of one, based on a dataset that includes nine sectors.

[9]Figure C.8 shows the very similar simulation results for model variants with 19 and 20 sectors, which represents the greatest possible granularity. For further information regarding the WIOD, see also Timmer et al., 2015. We employ the calibration toolkit introduced in Hinterlang, Martin, Röhe, Stähler, and Strobel (2023).

[10]NACE is a derived classification of the International Standard Industrial Classification of All Economic Activities (ISIC). The first level and the second level of ISIC Rev. 4 (sections and divisions) are identical to sections and divisions of NACE Rev. 2. This allows us to merge information from the WIOD (classified according to ISIC Rev. 4) and EU KLEMS database (classified according to NACE Rev. 2)

[11]Overall, TFP data for the three euro area countries cover 21 sections/divisions. We merge, however, the manufacturing divisions C10 – C18, C20 – C25, and C28 – 33. Furthermore, we merge the digital divisions *Manufacture of computer, electronic and optical products* and *Manufacture of electrical equipment* (divisions C26 and C27) as well as *Information and communication* (section J); the utilities sectors *Electricity* (section D) and *Water supply and waste management* (section E); *Wholesale and retail trade, repair of motor vehicles and motorcycles* (section G) and *Transportation and storage* (section H); *Accomodation and food service activities* (I), *Professional scientific and technical activities* (section M), and *Administrative and support service activities* (section N); as well as *Arts, entertainment and recreation* (section R) and *Other service activities* (section S).

Solution method We compute forecasts of the model variables conditional on sectoral TFP shocks. The model is solved nonlinearly using the extended path method (see, e.g., Fair and Taylor, 1983; Adjemian and Juillard, 2010).[12] The method indirectly characterises the decision rules by generating time series for the endogenous variables. Specifically, in each period, the model is simulated under perfect foresight conditional on previous period's outcomes (or initial conditions) and a vector of shocks that occur only in the current period. The simulation results of each period are then concatenated.

The solution algorithm accounts for the full nonlinearities of the model but requires a stance on the agents' expectation horizon. That is, in every period, agents assume that beside the TFP shocks materialising in the current period, there will be no further shocks. Since we are not trying to capture the risk associated with TFP shocks, we consider this adequate as it allows us to capture the nonlinearities of the models.[13]

5 Results

Before examining the role of efficiency gains in the digital sectors and the transmission channels for aggregate labour productivity growth, the following section briefly outlines the propagation of a TFP shock in the model and compares the model fit to the data.

5.1 Propagation of a TFP shock

The effect of an unexpected change in sectoral TFP is standard. A shock to digital sector TFP, for instance, lowers marginal costs and the price of digital output falls. This stimulates demand for these goods, both for consumption and investment purposes and as intermediate inputs. As far as possible, products from other sectors are replaced by comparatively cheaper digital goods. Due to limited substitutability, however, the demand for other goods rises as well.

5.2 Benchmark simulation

Figure 2 compares the simulated paths of aggregate labour productivity for the three countries under consideration to their unconditional empirical counterparts.[14] TFP changes are the only exogenous driver of long-run growth. According to the data, empirical labour productivity grew by about 27 percent in Germany and France and by about twice as much in the United States over the period from 1996 to 2020. The model's predictions of labour productivity growth for each country are close to the data, with the model predicting cumulative growth of 28 percent and 26 percent for Germany and France, and 44

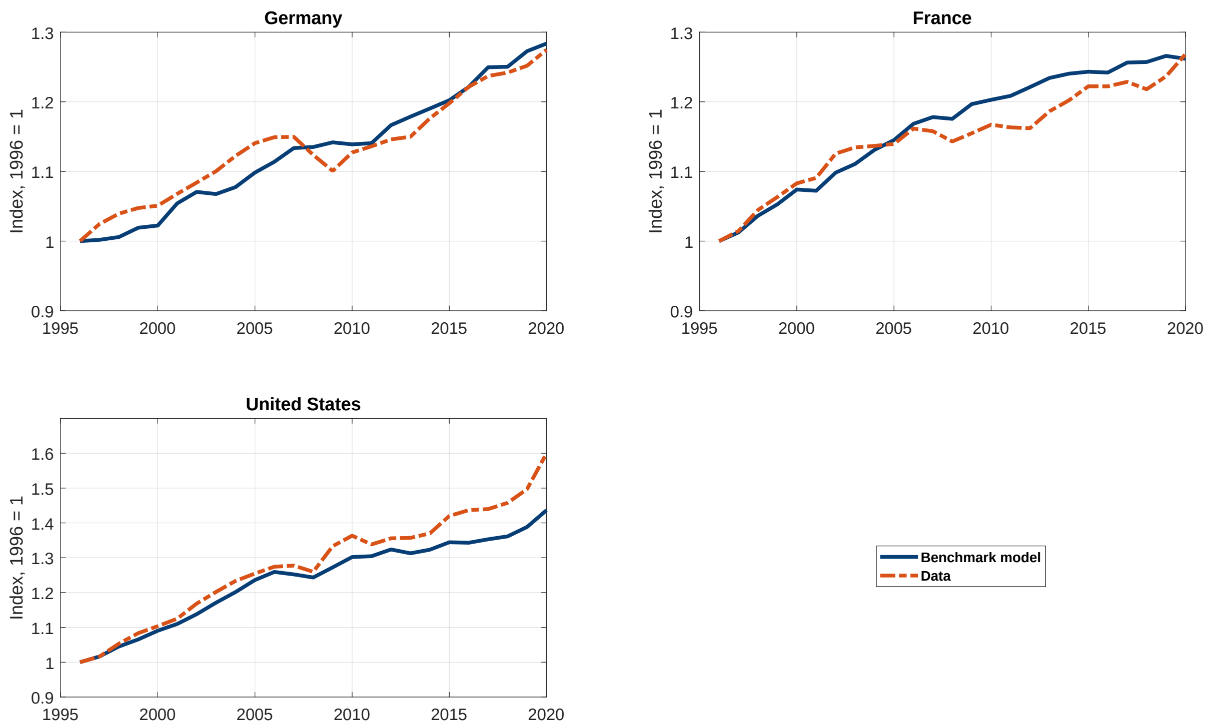
[12]We implement the solution algorithm following Gadatsch, Stähler, and Weigert (2016).

[13]Figure 1 reveals large differences in the sectoral TFP series. Hence, a first-order approximation of the model is likely to be inaccurate since the decision rules of the agents would remain unchanged regardless of the distance from the initial steady state. Another option would be to simulate the model under perfect foresight. However, this assumes that the model agents know the full TFP path – an assumption we consider too strong.

[14]The empirical measure of aggregate labour productivity used for this comparison covers the same sectors as the model. The respective paths of real gross value added and labour input are displayed in Figure C.2 in the Appendix.

percent for the United States. Because the TFP shocks fed to the model are utilisation-adjusted, the conditional forecasts do not show the decline in labour productivity during the Great Recession period.

Figure 2: Model-implied and empirical (unconditional) aggregate labour productivity.



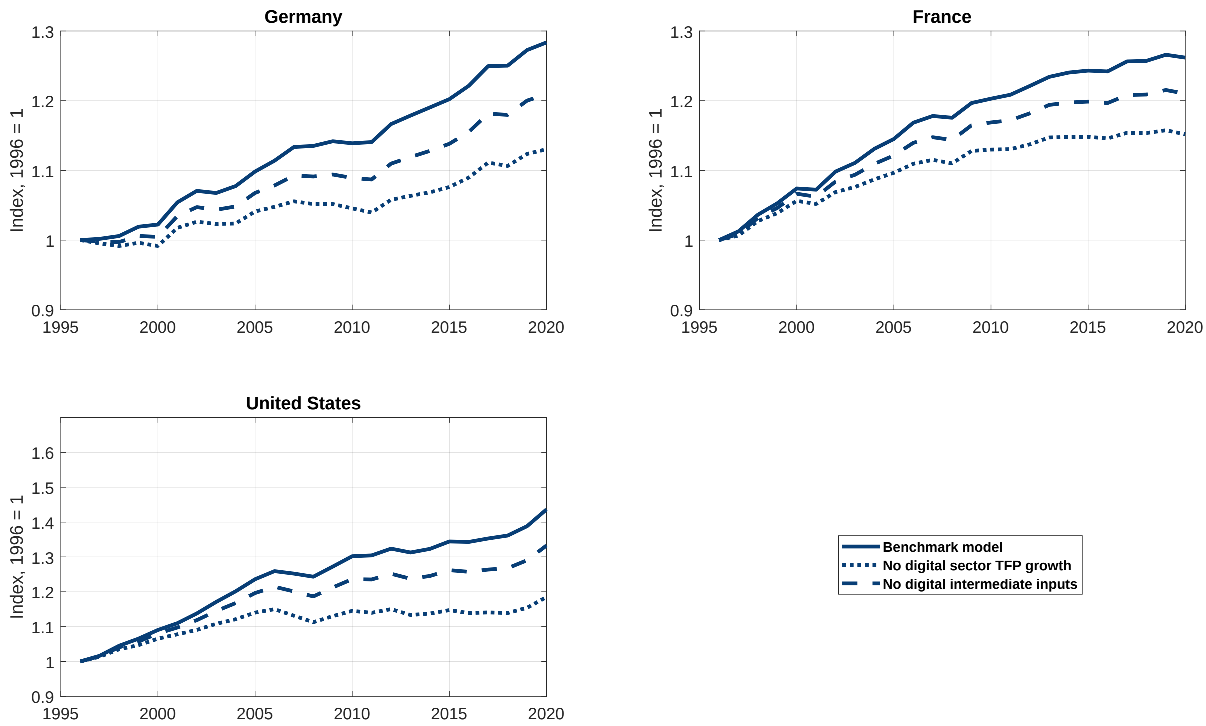
Notes: The figure plots model-implied aggregate labour productivity (blue straight line) and its empirical counterpart (orange dashed-dotted line) for the three countries considered. Model-implied labour productivity is calculated as the ratio of aggregate value added to aggregate labour input. The empirical counterpart is constructed as the value-weighted sum of sectoral labour productivity over the same sectors included in our model. Data range from 1996 to 2020.

5.3 The impact of the digital transformation on aggregate labour productivity

To assess the role of efficiency gains in digital sectors and their propagation for aggregate labour productivity, we perform two counterfactual analyses. In a first exercise, we focus on the impact of TFP growth in the digital sectors. Our reference point is the benchmark simulation. We contrast the latter with a counterfactual scenario in which TFP growth in the digital sectors is assumed to be constant over the simulation horizon.

Figure 3 shows that labour productivity would have been significantly lower in all countries if there had been no TFP growth in the digital sectors. Hence, despite their comparatively small share in total gross value added, the substantial increase in digi-

Figure 3: Model-implied and counterfactual aggregate labour productivity.



Notes: The figure plots model-implied aggregate labour productivity (blue straight line) and two counterfactuals. One completely abstracts from TFP growth in digital sectors (blue dotted line). In the other counterfactual, digital output cannot be used as an intermediate input (blue dashed line). Data range from 1996 to 2020.

tal sector TFP has a considerable impact on aggregate productivity.[15] In the United States, labour productivity would have grown only by about 19 percent, i.e. cumulative growth would have been 25 percentage points lower than in the benchmark simulation. In the counterfactual simulations for Germany and France, cumulative productivity growth would have been 15 and 11 percentage points lower, respectively, compared to the benchmark scenario.

In the second counterfactual analysis, we explicitly focus on the role of intermediates as a transmission mechanism of digital efficiency gains. Here, we do not restrict TFP growth in the digital sector, but assume that digital goods are used exclusively for consumption or investment purposes and not as intermediate inputs. To do so, we set the respective elements of the input-output matrix to zero and rescale the remaining values so that the share of non-digital intermediate inputs in each sector sums up to one.

The dashed line in Figure 2 shows the corresponding simulation results. Compared to the first counterfactual scenario, the simulation results are closer to the benchmark. However, aggregate labour productivity growth is significantly lower when digital goods do not serve as intermediate inputs. Hence, digital inputs play an important role in transmitting the efficiency gains in the digital sector to the overall economy. Neglecting this transmission channel may therefore lead to a considerable underestimation of the effects of the digital transformation.

A rising relative importance of digital inputs in all intermediate goods suggests that the indirect transmission channel has gained in importance over time. To illustrate this, it is useful to analyse the value and share of digital intermediate goods in all digital intermediate goods over time. To this end, we compute the quantity and value shares of digital intermediate goods in all intermediate goods as

$$s_t^q = \frac{\sum_{s=1}^{S} H_{s,Digi,t}}{\sum_{j=1}^{S}\sum_{s=1}^{S} H_{s,j,t}}, \tag{14}$$

and

$$s_t^v = \frac{\sum_{s=1}^{S} P_{Digi,t} H_{s,Digi,t}}{\sum_{j=1}^{S}\sum_{s=1}^{S} P_{j,t} H_{s,j,t}}, \tag{15}$$

respectively. Table 4 shows the respective values in 1996, 2020 and the change between the two time periods. The initial share of digital intermediate goods in all intermediate goods is about 14 percent in France, 13 percent in Germany and 13.5 percent in the United States.[16] s_t^q increases over time in all countries, with the largest absolute increase observed in the US, by about 7 percentage points. In Germany and France, s_t^q increases by about 3 percentage points and 4 percentage points, respectively. The stronger rise in the digital intermediate share in the United States partly reflects the higher growth rate of digital sector TFP. By contrast, the expenditure shares s_t^v decreased in all countries due to the change in the relative price of digital goods over time, as shown in Figure C.3.

[15] Table A.1 shows the real value added share of the sectors in the initial period, which is about 11 percent in the United States, 9 percent in France and 10 percent Germany.

[16] The values of s_t^q and s_t^v coincide in the first period because we set all relative prices equal to one in the initial steady state.

Table 4: Share of digital goods in all intermediate inputs in percent

	Germany			France			United States		
	1996	2020	$\Delta_{1996\to2020}$	1996	2020	$\Delta_{1996\to2020}$	1996	2020	$\Delta_{1996\to2020}$
s_t^v	13.50	11.34	−2.16	14.04	13.47	−0.57	13.11	11.39	−1.72
s_t^q	13.50	17.34	3.84	14.04	17.39	3.35	13.11	20.11	7.00

Notes: The table shows the quantity and value share of digital intermediate inputs in all intermediate inputs, s_t^v and s_t^q, respectively, in 1996 and 2020 in percent as well as change between the two dates in percentage points.

6 Discussion

6.1 Centrality and sectoral TFP growth

Our simulation exercise indicates that production networks play a central role in the transmission of the efficiency gains in digital branches. To illustrate the relevance of its design, we repeat the benchmark simulation for the United States presented in Section 5 but change the values of the input-output matrix Ψ_H to those of Germany and France, respectively. If the US production network would have been equal to the German or French one, aggregate productivity growth in the United States over the period from 1996 to 2020 would have been 2 to 3 percentage points lower; see also Figure C.10. Hence, the higher labour productivity growth in the United States is not only driven by relatively stronger TFP growth but also by the favourable structure of the production network.

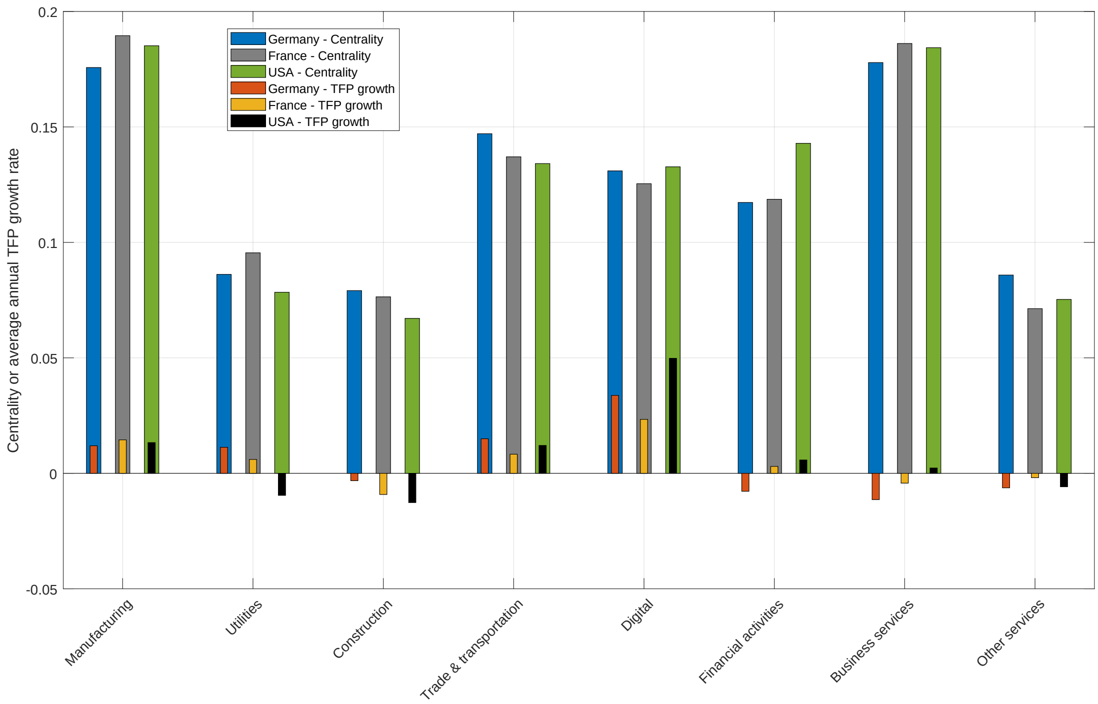
The role of the production network can be further investigated by assessing each sector's relevance using a (Bonacich) centrality measure (Bonacich, 1972; Carvalho, 2014).[17] As outlined in Carvalho and Tahbaz-Salehi (2019), the more important an industry is as an input supplier to other central industries, the more central it is to the production network.

Figure 4 shows the centrality measures together with the average annual sectoral TFP growth rates over the period from 1996 to 2020. The sectors in the model economy can be broadly classified into three tiers. The industries with the highest centrality score are manufacturing and business services, which include professional, scientific and technical services, as well as administrative and support services. These are followed by slightly less upstream sectors – trade and transportation, financial and insurance activities, and the digital sector. The least upstream sectors are construction and other services, which encompass arts, entertainment and recreation, and other services.

The most central sectors are also receiving the highest share of digital intermediate inputs in all three economies. Specifically, the share of digital intermediate goods used

[17]Following Carvalho (2014), the vector of centrality measures for the included sectors is computed as $(1-\alpha_H)/S(I-\lambda\Psi_H')^{-1}$, with λ= 0.5 and α_H, Ψ_H and I corresponding to the vector of stacked sectoral intermediate input shares, to the input-output matrix and to the identity matrix, respectively. The sectoral parameters are based on WIOD data for the year 2000 (see Section 4). For a more detailed view, Figure C.9 in the Appendix provides the graph for the 20-sector variant, the highest level of granularity our data can offer.

Figure 4: Centrality and average sectoral TFP growth across countries



Notes: The figure shows the (Bonacich) centrality measure for the initial period of benchmark model as well as the average sectoral TFP growth rate over the period from 1996 to 2020. The computation of the centrality measure follows Carvalho (2014).

in a given sector relative to the total volume of intermediate inputs produced by the digital sector, computed as $s^q_{Digi} = H_{s,Digi} \div \sum_{s=1}^{S} H_{s,Digi}$, are the highest in the most important nodes in the production network: manufacturing and business services (see Table 5). However, while the United States has, on average, witnessed positive TFP growth in the business services sector between 1996 and 2020, the average growth rates in Germany and France in the sector were negative over this time period. Therefore, in contrast to the two European countries considered, positive TFP growth in the United States business services sector raised the demand for and hence the diffusion of digital goods, thereby further amplifying the impact of the digital sectors' substantial efficiency gains on aggregate productivity.

Table 5: Distribution of intermediate inputs from the digital sector across all sectors

	Manuf.	Constr.	Utilities	Tr. & Tr.	Digi.	Fin. act.	Busi. serv.	Other serv.
$s^q_{DE,Digi}$	19.22	2.95	6.21	8.16	43.53	3.66	14.37	1.89
$s^q_{FR,Digi}$	20.85	1.27	5.33	12.57	31.64	10.75	15.74	1.85
$s^q_{US,Digi}$	16.97	1.63	4.07	9.18	46.10	6.90	13.57	1.58

Notes: The table illustrates the distribution of intermediate inputs from the digital sector across all sectors included in the model, expressed as a percentage of the total volume of intermediate inputs originating from the digital sector.

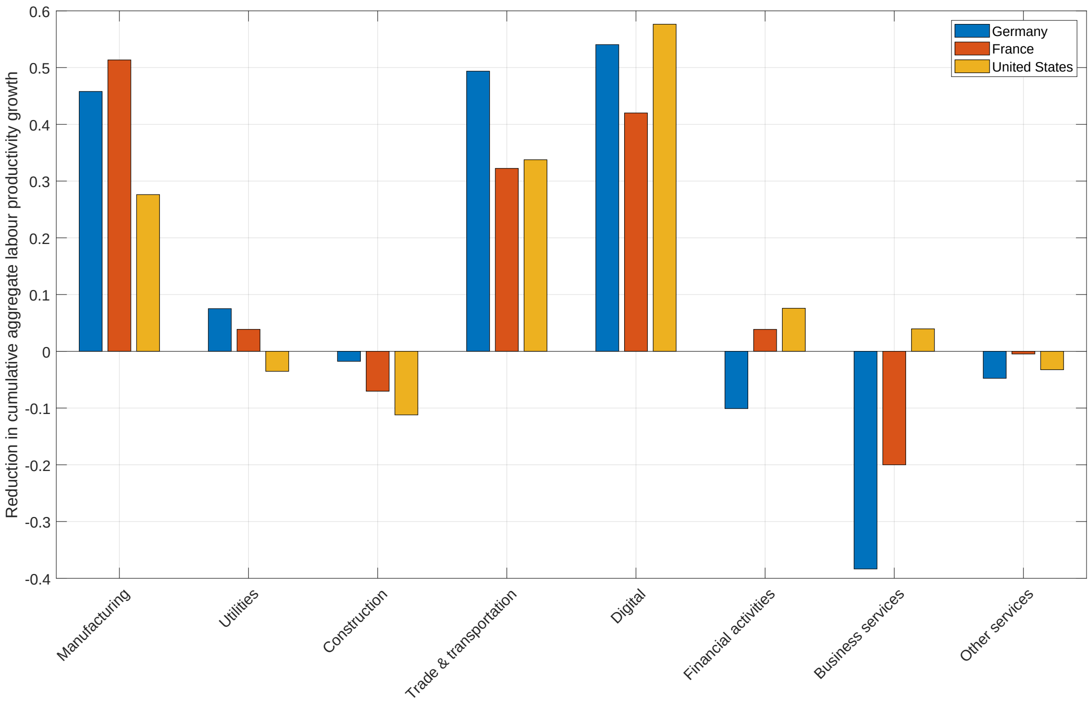
6.2 How important are non-digital sectors for labour productivity growth?

In order to gauge the relative importance of the digital sector in propelling aggregate labour productivity, we compare its contribution to that of the other, non-digital sectors. To do so, we proceed as in the first counterfactual scenario and simulate the path of aggregate labour productivity, assuming a zero growth rate for each non-digital sector at a time. Figure 5 shows that the absence of TFP growth in the manufacturing sector would have resulted in a cumulative aggregate labour productivity growth from 1996 to 2020 that would have been by about 45 percent lower in Germany, 50 percent lower in France, and 30 percent lower in the United States relative to the benchmark simulation.

More broadly, the exercise reveals somewhat similar effects across countries but very heterogeneous effects across sectors. Manufacturing, trade and transportation, and the digital sector have been instrumental in driving labour productivity growth across all countries under consideration.[18] Several sectors have a negative impact on overall productivity, which is not unexpected given their negative average TFP growth rates as illustrated in Figure 5 and the fact that the extent to which various goods are substitutable is limited.

[18] It is important to note that Wholesale of information and communication equipment and Retail sale of information and communication equipment are components of the trade and transportation sector, which could potentially lead to an underestimation of the digital transformation in our model. However, due to the constraints of the available data, we are unable to segregate the TFP series of these two subsectors from those of other subsectors within the trade and transportation sector.

Figure 5: Reduction in aggregate labour productivity growth when not taking into account sectoral TFP growth of a specific sector



Notes: The figure shows the loss in aggregate cumulative labour productivity growth over the period from 1996 to 2020 that would have occurred without TFP growth for a given sector.

6.3 Varying the elasticity of substitution

The substitution elasticities are among the key parameters in the model. Empirical evidence on their magnitude, especially for European countries, is scarce. In our baseline simulations, we follow the studies of vom Lehn and Winberry (2022) and Foerster et al. (2022) for the United States in specifying elasticities of substitution of (almost) unity in the consumption and investment goods.[19] Gaggl et al. (2023), however, estimate a long-run elasticity of 2.36 for the investment goods bundle in the United States, and values of 0 and about 0.5 for consumption and intermediate goods, respectively. Regarding intermediate inputs vom Lehn and Winberry (2022) and Foerster et al. (2022) both choose an elasticity of one.

To assess the robustness of our benchmark results, we run two different scenarios. In the first, we set the elasticity of substitution of investment goods to a relatively high value of 2.36, based on the estimate in Gaggl et al. (2023). In the second, we specify a value of 1.01 for the substitution elasticity of intermediate inputs, consumption and investment goods, closely following vom Lehn and Winberry (2022) and Foerster et al. (2022). Table 6 summarises the results of the two counterfactual scenarios presented in section 5 for different parameterisation. Specifically, the table shows the loss in cumulative labour productivity growth in the two scenarios over the period from 1996 to 2020 without TFP growth in the digital sectors (scenario I) and excluding digital intermediate inputs (scenario II) for the different values of the elasticity of substitution.

Table 6: Loss in cumulative labour productivity growth for different values of the elasticity of substitution in percentage points

Substitution elasticities	Loss in labour productivity growth		
	Germany	France	United States
	Counterfactual I		
Benchmark model	15.33	11.00	25.15
$\sigma_I = \sigma_H = \sigma_C = 1.01$	14.41	9.88	24.43
$\sigma_I = 2.36, \sigma_H = 0.20, \sigma_C = 1.01$	15.04	10.61	24.28
	Counterfactual II		
Benchmark model	7.42	5.17	10.36
$\sigma_I = \sigma_H = \sigma_C = 1.01$	6.29	3.76	9.41
$\sigma_I = 2.36, \sigma_H = 0.20, \sigma_C = 1.01$	6.41	4.37	7.48

Notes: The table shows the percentage point difference in cumulative labour productivity growth to the benchmark simulation between 1996 and 2020 for different values of the elasticity of substitution in counterfactuals I and II relative to the benchmark scenario. Counterfactual I refers to the simulation in which TFP growth in the digital sector is neglected. Counterfactual II refers to the scenario in which output produced by the digital sector can be used only for consumption and investment purposes but not as intermediate input.

[19] Hobijn and Nechio (2019) find a value of one for consumption goods in the ten-sector estimation and a value that is even higher if more sectors are included.

The findings are quite intuitive. As shown in Figures C.5 to C.7 in the Appendix, the higher the elasticity, the greater the cumulative growth in labour productivity over time. The reason is that goods produced more efficiently (and thus sold at a lower price) can more easily substituted for other goods in the respective bundle. Quantitatively, however, we find the effects of changes in the substitution elasticity to be small, which applies to both counterfactual analyses.

7 Conclusion

This study explores the impact of the significant efficiency gains in digital sectors on labour productivity in the Germany, France, and the United States using a multi-sector dynamic general equilibrium model with a production network. Despite the digital sector's relatively small size in terms of gross value added, aggregate productivity growth in these economies would have been considerably lower without the efficiency improvements in this sector. This is not only due to the exceptionally high TFP growth in the digital sectors, but also to an amplification through the production network: Input-output linkages serve as an important channel for spreading these efficiency gains throughout the economy. Neglecting the role of production networks may thus lead to a significant underestimation of the efficiency gains linked to digital advancement.

Furthermore, our study uncovers notable differences in production networks between countries. Specifically, we show that the production network of the United States is more conducive to leveraging the efficiency gains of the digital sectors. While investigating the underlying causes of these disparities is beyond the scope of this paper, the findings suggest an interesting avenue for future research.

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Appendix A: Calibration details

A.1 Sectoral parameterisation

Table A.1 shows the sectors included in the benchmark model as well as their respective shares in real gross value added.

Table A.1: Overview over sectors

Sector	NACE code	Sectoral share in real value added		
		Germany	France	United States
Manufacturing	C10–C18, C20–C25, C28–C30	0.20	0.16	0.17
Construction	F	0.05	0.05	0.03
Utilities	D–E	0.07	0.07	0.06
Trade and transportation	G–H	0.25	0.26	0.26
Digital	J, C26–C27	0.10	0.09	0.11
Financial and insurance activities	K	0.08	0.09	0.11
Business services	I, M–N	0.20	0.25	0.20
Other services	R–S	0.06	0.03	0.05

Notes: The shares in real gross value added may not sum up to one due to rounding. Numbers reported refer to the initial period.

The production technology of intermediate goods producers differs across sectors as we allow for sector-specific factor intensities of labour, capital and intermediate inputs. Moreover, all sectors contribute differently to final demand. The parameterisation for each sector s is derived using data from the most recent release of the WIOD for the year 2000. It includes data on socioeconomic accounts as well as input-output tables for 56 sectors and 43 countries. We build datasets for the two European countries and for the United States. The socioeconomic accounts help us to pin down $\omega_{N,s}$, $\omega_{K,s}$, $\alpha_{N,s}$ and $\alpha_{H,s}$, and we can use the provided input-output tables to match inter-sectoral trade shares, $\psi_{H,s,j}$, as well as the sectoral shares in the consumption and investment good bundles, $\psi_{C,s}$ and $\psi_{I,s}$, respectively (see Tables A.2 to A.7). In order to determine sector-specific labour and capital supply, we first sum up the number of persons engaged and the nominal capital stock over all sectors, and then compute the respective shares $\omega_{N,s}$ and $\omega_{K,s}$. Dividing the amount of intermediate inputs by gross output per industry yields the factor intensities for intermediate inputs, $1 - \alpha_{H,s}$. In combination with the share of gross output that flows into labour compensation, we can fix the values for $\alpha_{N,s}$.

Parameters $\psi_{H,s,j}$ describe the share of intermediate inputs consumed by sector s that are produced by sector j. To obtain these, we first compute the total sum of intermediate inputs for each sector and then the respective shares of the producing sectors, using the input-output tables. Relying on WIOD's national accounts data, the distribution of final consumption expenditure by households and gross fixed capital formation across sectors can be derived, giving us the CES bundle shares $\psi_{C,s}$ and $\psi_{I,s}$. To facilitate calculations, we normalise relative prices to one in the initial steady state.

Germany

Table A.2: Baseline calibration of sector-specific parameters

	$\alpha_{N,s}$	$\alpha_{H,s}$	$\omega_{N,s}$	$\omega_{K,s}$	$\psi_{C,s}$	$\psi_{I,s}$
Manufacturing	0.735	0.346	0.241	0.242	0.321	0.363
Utilities	0.472	0.499	0.018	0.186	0.062	0.008
Construction	0.917	0.429	0.101	0.024	0.007	0.359
Trade and transportation	0.748	0.533	0.276	0.192	0.255	0.053
Digital	0.550	0.476	0.069	0.088	0.104	0.161
Fin. activities	0.772	0.472	0.045	0.061	0.086	0.001
Business services	0.512	0.597	0.183	0.135	0.102	0.052
Other services	0.652	0.665	0.067	0.072	0.063	0.003

Notes: The table shows calibrated values for sector-specific parameters as described in the main text. The values were computed using year 2000 data from the latest vintage of the WIOD.

Table A.3: Input-Output Matrix

	$\psi_{H,1,j}$	$\psi_{H,2,j}$	$\psi_{H,3,j}$	$\psi_{H,4,j}$	$\psi_{H,5,j}$	$\psi_{H,6,j}$	$\psi_{H,7,j}$	$\psi_{H,8,j}$
$\psi_{H,s,1}$	0.61	0.137	0.435	0.117	0.192	0.019	0.115	0.093
$\psi_{H,s,2}$	0.041	0.297	0.010	0.026	0.013	0.009	0.022	0.035
$\psi_{H,s,3}$	0.009	0.091	0.133	0.022	0.013	0.010	0.024	0.033
$\psi_{H,s,4}$	0.152	0.123	0.140	0.520	0.142	0.026	0.085	0.087
$\psi_{H,s,5}$	0.055	0.084	0.119	0.055	0.443	0.063	0.157	0.107
$\psi_{H,s,6}$	0.017	0.041	0.040	0.056	0.021	0.615	0.048	0.098
$\psi_{H,s,7}$	0.112	0.210	0.116	0.192	0.151	0.243	0.518	0.175
$\psi_{H,s,8}$	0.005	0.016	0.006	0.011	0.026	0.014	0.030	0.372

Notes: The table shows calibrated values for sector-specific parameters as described in the main text. The second entry in the first row, for example, shows that 4.1 percent of the intermediate inputs used in sector 1 were produced in sector 2. The digital sector corresponds to $s = 5$. The values were computed using year 2000 data from the latest vintage of the WIOD.

France

Table A.4: Baseline calibration of sector-specific parameters

	$\alpha_{N,s}$	$\alpha_{H,s}$	$\omega_{N,s}$	$\omega_{K,s}$	$\psi_{C,s}$	$\psi_{I,s}$
Manufacturing	0.613	0.317	0.192	0.244	0.304	0.225
Utilities	0.387	0.489	0.015	0.127	0.043	0.001
Construction	0.767	0.382	0.089	0.038	0.012	0.464
Trade and transportation	0.695	0.523	0.279	0.182	0.293	0.068
Digital	0.534	0.480	0.060	0.119	0.089	0.153
Fin. activities	0.614	0.406	0.041	0.048	0.092	0.001
Business services	0.692	0.552	0.251	0.183	0.127	0.078
Other services	0.749	0.611	0.073	0.060	0.040	0.011

Notes: The table shows calibrated values for sector-specific parameters as described in the main text. The values were computed using year 2000 data from the latest vintage of the WIOD.

Table A.5: Input-Output Matrix

	$\psi_{H,1,j}$	$\psi_{H,2,j}$	$\psi_{H,3,j}$	$\psi_{H,4,j}$	$\psi_{H,5,j}$	$\psi_{H,6,j}$	$\psi_{H,7,j}$	$\psi_{H,8,j}$
$\psi_{H,s,1}$	0.563	0.199	0.387	0.126	0.241	0.023	0.183	0.207
$\psi_{H,s,2}$	0.038	0.45	0.012	0.023	0.026	0.006	0.019	0.056
$\psi_{H,s,3}$	0.007	0.038	0.196	0.006	0.013	0.008	0.013	0.032
$\psi_{H,s,4}$	0.147	0.074	0.119	0.412	0.110	0.031	0.114	0.142
$\psi_{H,s,5}$	0.062	0.042	0.074	0.083	0.347	0.139	0.122	0.146
$\psi_{H,s,6}$	0.021	0.027	0.043	0.099	0.034	0.566	0.070	0.063
$\psi_{H,s,7}$	0.156	0.162	0.164	0.239	0.223	0.219	0.465	0.229
$\psi_{H,s,8}$	0.007	0.009	0.005	0.012	0.008	0.007	0.014	0.126

Notes: The table shows calibrated values for sector-specific parameters as described in the main text. The second entry in the first row, for example, shows that 3.8 percent of the intermediate inputs used in sector 1 were produced in sector 2. The digital sector corresponds to $s = 5$. The values were computed using year 2000 data from the latest vintage of the WIOD.

United States

Table A.6: Baseline calibration of sector-specific parameters

	$\alpha_{N,s}$	$\alpha_{H,s}$	$\omega_{N,s}$	$\omega_{K,s}$	$\psi_{C,s}$	$\psi_{I,s}$
Manufacturing	0.598	0.37	0.147	0.198	0.229	0.224
Utilities	0.317	0.500	0.009	0.124	0.046	0.001
Construction	0.833	0.507	0.082	0.019	0.001	0.350
Trade and transportation	0.610	0.659	0.268	0.213	0.307	0.081
Digital	0.639	0.461	0.077	0.184	0.087	0.231
Fin. activities	0.586	0.517	0.058	0.086	0.131	0.006
Business services	0.693	0.612	0.300	0.131	0.131	0.106
Other services	0.714	0.661	0.060	0.045	0.069	0.002

Notes: The table shows calibrated values for sector-specific parameters as described in the main text. The values were computed using year 2000 data from the latest vintage of the WIOD.

Table A.7: Input-Output Matrix

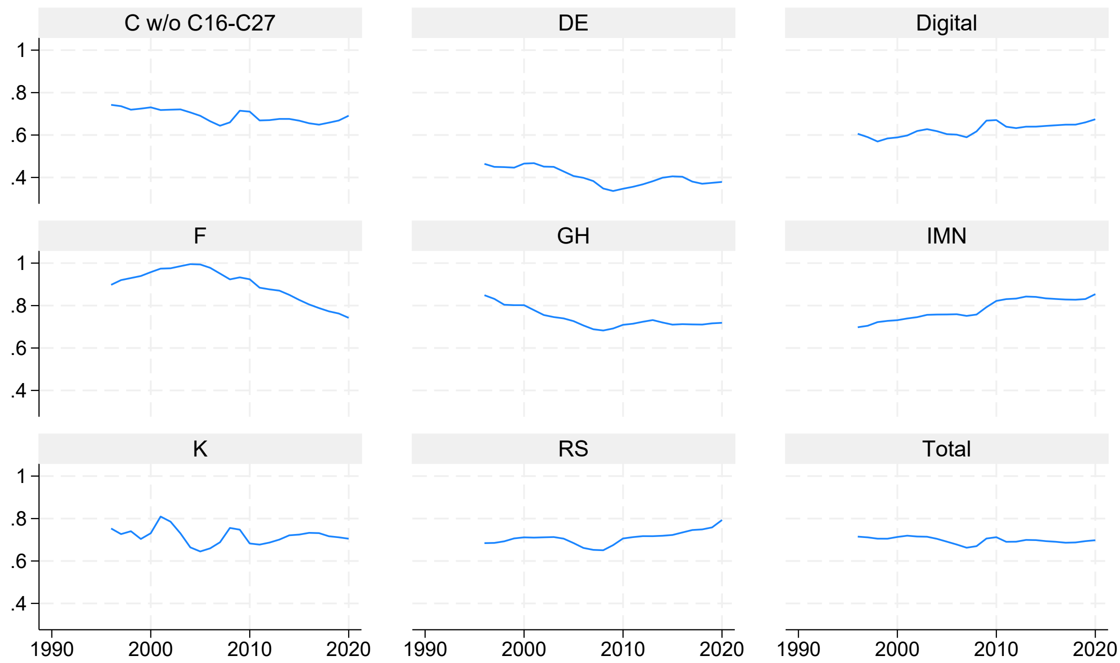
	$\psi_{H,1,j}$	$\psi_{H,2,j}$	$\psi_{H,3,j}$	$\psi_{H,4,j}$	$\psi_{H,5,j}$	$\psi_{H,6,j}$	$\psi_{H,7,j}$	$\psi_{H,8,j}$
$\psi_{H,s,1}$	0.611	0.064	0.48	0.156	0.15	0.03	0.186	0.204
$\psi_{H,s,2}$	0.037	0.076	0.017	0.041	0.016	0.007	0.032	0.043
$\psi_{H,s,3}$	0.006	0.029	0.001	0.009	0.006	0.007	0.009	0.015
$\psi_{H,s,4}$	0.136	0.246	0.244	0.245	0.096	0.039	0.113	0.139
$\psi_{H,s,5}$	0.066	0.069	0.104	0.094	0.435	0.089	0.141	0.090
$\psi_{H,s,6}$	0.024	0.202	0.027	0.115	0.040	0.587	0.106	0.19
$\psi_{H,s,7}$	0.113	0.290	0.110	0.318	0.234	0.226	0.381	0.216
$\psi_{H,s,8}$	0.007	0.024	0.017	0.021	0.023	0.015	0.031	0.102

Notes: The table shows calibrated values for sector-specific parameters as described in the main text. The second entry in the first row, for example, shows that 3.7 percent of the intermediate inputs used in sector 1 were produced in sector 2. The digital sector corresponds to $s = 5$. The values were computed using year 2000 data from the latest vintage of the WIOD.

A.2 Descriptive evidence on the labour income share

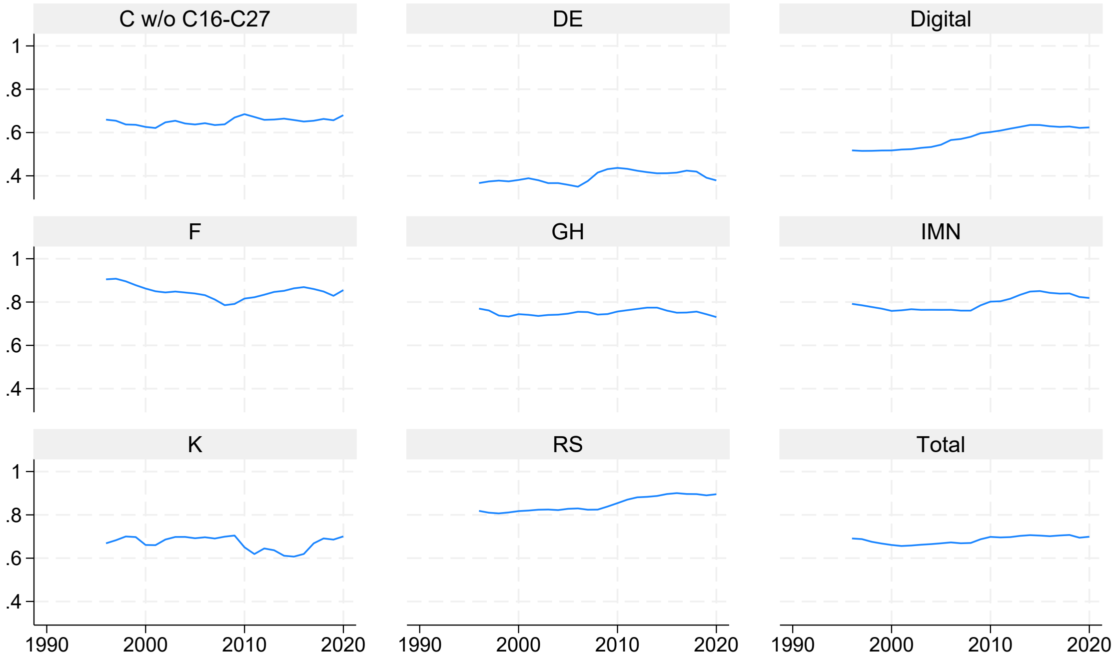
Atalay (2017) estimates the elasticity of substitution within the intermediate goods basket and between the bundles of intermediate goods and gross value added. The preferred specification of the author yields a relatively low substitution elasticity within the intermediate goods bundle. This elasticity is contingent on the number of sectors incorporated in the estimation but is consistently estimated to be quite low. Furthermore, a unitary substitution elasticity between the bundles of intermediate goods and value added is indicated as the preferred estimate. This specification implies that the ratios of labour income and (nominal) intermediate inputs to gross value added are constant. Figures A.3 – A.5 below show that this is indeed in line with data from both EU KLEMS and WIOD over the time period considered in our analysis.

Figure A.1: Ratio of labour income and gross value added for Germany



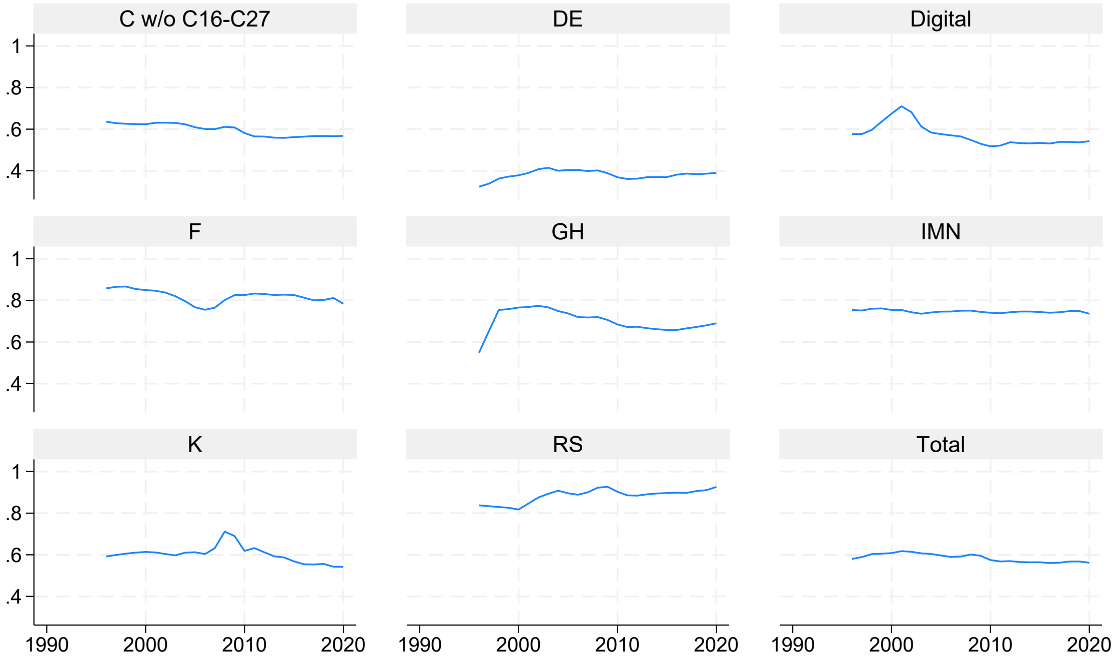
Notes: The figure plots the ratio of nominal labour remuneration to nominal gross value added, using EU KLEMS data for Germany for the period from 1996 to 2020.

Figure A.2: Ratio of labour income and gross value added for France



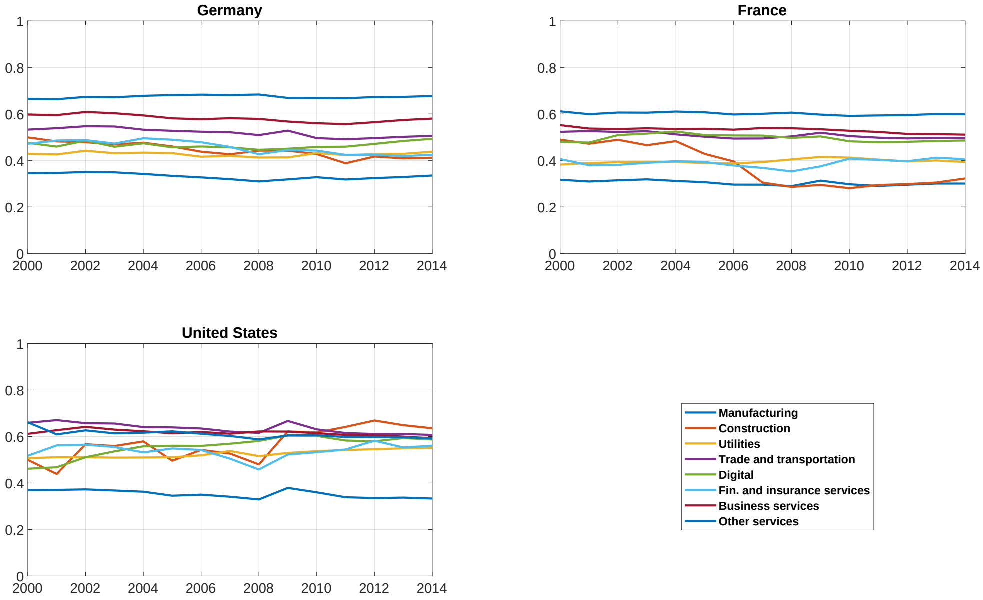
Notes: The figure plots the ratio of nominal labour remuneration to nominal gross value added, using EU KLEMS data for France for the period from 1996 to 2020.

Figure A.3: Ratio of labour income and gross value added for the United States



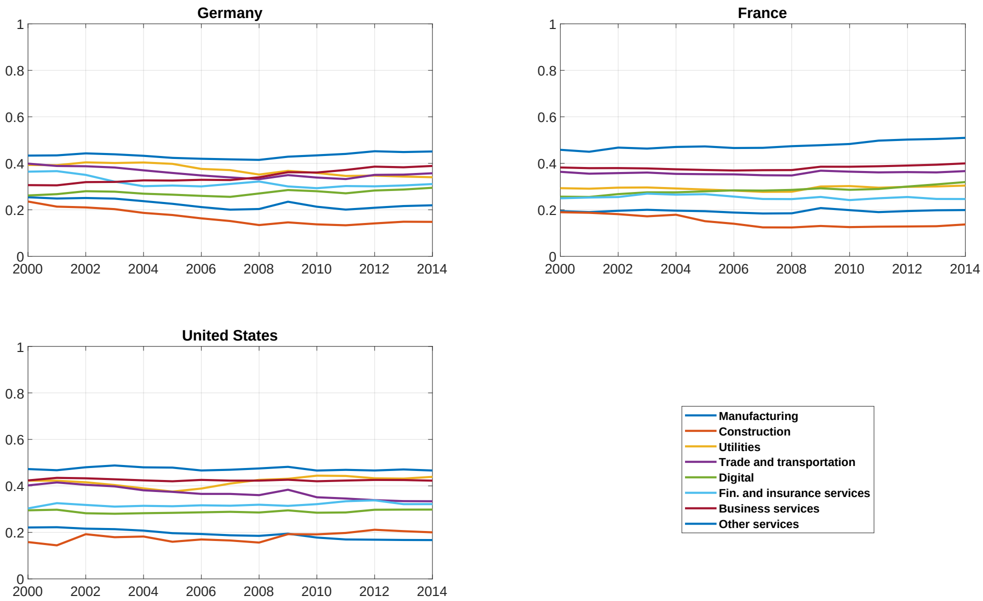
Notes: The figure plots the ratio of nominal labour remuneration to nominal gross value added, using EU KLEMS data for the United States for the period from 1996 to 2020.

Figure A.4: Ratio of intermediate inputs share of gross output



Notes: The figure plots the ratio of nominal intermediate inputs and nominal gross output using WIOD data for the period from 2000 to 2014.

Figure A.5: Ratio of labour income and gross output



Notes: The figure plots the ratio of nominal labour income and nominal gross output using WIOD data for the period from 2000 to 2014.

Appendix B: Details on the construction of TFP shocks

In this Appendix, we provide additional information for the derivation of country-specific TFP shocks.

B.1 Estimation

The utilisation-adjusted TFP series are estimated in a two-step approach. In the first step, we calculate yearly sectoral Solow residuals ($s_{j,t}$) by applying a standard growth accounting technique (Solow, 1957):

$$s_{j,t} = dy_{j,t} - \alpha_{j,t} dl_{j,t} - (1 - \alpha_{j,t}) dk_{j,t}, \tag{B.1}$$

assuming that sectoral production, $y_{j,t}$, can be captured by a standard Cobb-Douglas production function with constant returns to scale and production factors labour ($l_{j,t}$) and capital ($k_{j,t}$). The labour share $\alpha_{j,t}$ is calculated as the average labour remuneration over the last and current year divided by total production (Tornqvist, 1936). The capital share is given by $1 - \alpha_{j,t}$.

The growth accounting exercise is based on several assumptions. Equation (B.1) follows from a Cobb-Douglas production function with constant returns to scale. Moreover, we assume perfect competition on the factor markets when deriving the factor shares. Hence, prices equal marginal costs and factor weights correspond to their respective output shares. Introducing imperfect competition in a growth accounting exercise is not trivial and might be a source of additional biases (see Hulten, 2010).[20] Furthermore, we use value added growth instead of gross output growth in the growth accounting exercise.[21] Further underlying assumptions are a technology level that is Hick's-neutral and a stable functional relationship between inputs and output (see Hulten, 2010).

In order to extract TFP growth, the calculated Solow residuals are corrected for changes in capacity utilisation in a second step. Here, we regress the Solow residual of sector j on a proxy for the change in unobserved capacity utilisation $d\widetilde{u}_{j,t}$;

$$ds_{j,t} = c_j + \beta_z d\widetilde{u}_{j,t} + \nu_{j,t}, \tag{B.2}$$

where c_j is a sector fixed effect and $\nu_{j,t}$ a residual. To account for differences in the effect of capacity utilisation across different areas of the economy, the economic sectors are divided into two subgroups: manufacturing and non-manufacturing sectors. The panel estimations are conducted separately for each group in each country. The coefficient β_z captures the impact of utilisation changes for sectors in group z. Thus, the estimated utilisation-adjusted TFP growth for sector j in subgroup z, $d\hat{a}_{j,t}$, is given by the sum of c_j and $\nu_{j,t}$.

Since changes in capacity utilisation can also be driven by exogenous changes in TFP, an instrumental variables approach is needed for the estimation of equation (B.2) (Basu

[20] Comin et al. (2020) estimate TFP growth rates allowing for positive profits. For most countries in their sample the estimated impact of profit shares is small.

[21] While using gross output would reduce our sample as the necessary data are missing for some sectors it would, however, hardly affect our results (see Section B.5).

et al., 2006). We use three structural shock series: an oil price shock, an international financial market shock and a macroeconomic uncertainty shock.

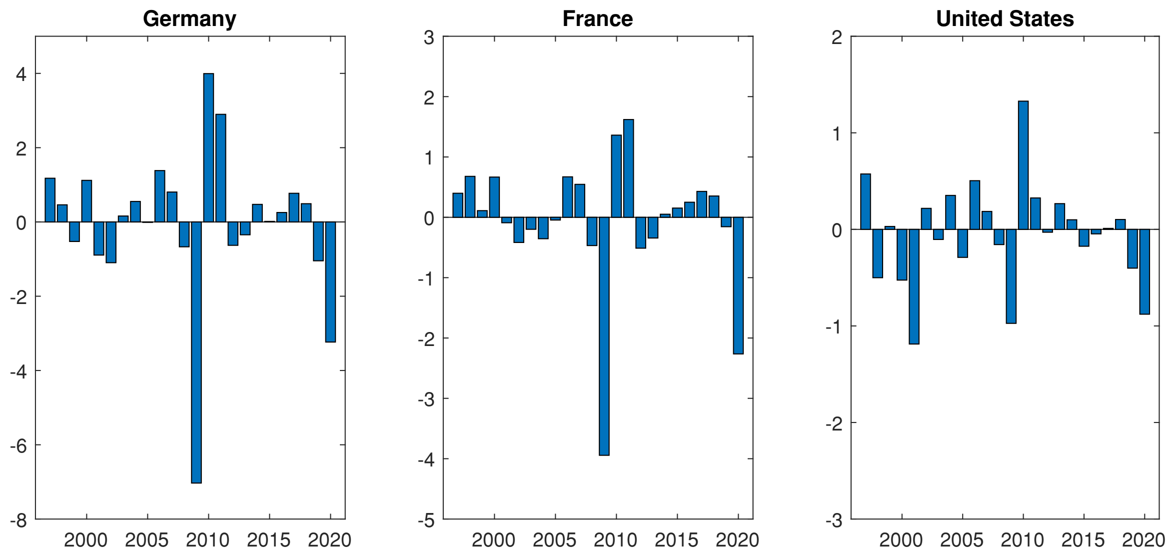
B.2 Data and proxies for sectoral capacity utilisation

The growth accounting exercise uses data from the 2023 version of EU KLEMS, which covers the period from 1997 to 2020. The sample comprises 21 sectors and covers the non-farm, non-mining private market economy. The included sectors are displayed in Table B.1 and Table B.2. To account for differences in the effect of capacity utilisation across different areas of the economy, the economic sectors were divided into two groups: manufacturing and non-manufacturing sectors. For the United States, Basu et al. (2006) additionally separate durable and non-durable manufacturing. Because the European data cover a shorter time period, we expand the panel dimension by considering only two groups but each with more sectors. This increases the precision of the estimated coefficients and, additionally, raises the validity of the instruments used. The panel estimations are conducted separately for each group in each country. For the United States the sector structure slightly differs from Table 2 owing to missing data: the estimations include only C20 rather than C20-21, and sectors D and E are combined.

We apply two different proxies to measure capacity utilisation in the sectors. For the European countries we use survey answers about the level of factor utilisation from the European Commission's business and consumer surveys. In the survey, manufacturing firms are asked "*at what capacity is your company currently operating (as a percentage of full capacity)?*", while service providers are asked "*If the demand addressed to your firm expanded, could you increase your volume of activity with your present resources? If so, by how much?*". The data are available for manufacturing sectors at the NACE two-digit level since 1980, for service sectors since 2011. As in Comin et al. (2020), we prolong the latter series until 1997 using the growth rate of average capacity utilisation in the manufacturing sectors. For some sectors, the survey does not provide data or the data is not available for the majority of years. In these cases, we use the series for average utilisation in manufacturing and services instead. Details are provided in Table B.1 and Table B.2. The survey does not provide data for electricity (NACE D), water supply and waste management (NACE E), trade (NACE G) or construction (NACE F). For the German and French construction sector we use additional survey data on capacity utilisation from the ifo institute and Insee. For the rest, changes in utilisation are proxied by changes in average capacity utilisation in the domestic manufacturing sector. Moreover, since data for the financial (NACE K) and recreational sectors (NACE R-S) are scarce on the two-digit level, we use average utilisation in the services sectors in these branches. The capacity series are aggregated at the NACE one-digit level and for sector groups using value added shares. We use average value added shares over the years 2005-2020 (manufacturing) and 2011-2020 (services) to aggregate the utilisation series. The time horizons vary owing to the availability of data on value added at the two-digit NACE level. Manufacturing sectoral data for gross value added start in 2005. For the United States, we follow Basu et al. (2006) and use the log change in average weekly hours per worker as proxy for changes in utilisation, as provided by EU KLEMS. Figure B.1 shows the estimated impact of utilisation changes on TFP growth for Germany, France and the

United States. A comparison between the unadjusted and adjusted measures reveals that capacity utilisation has an effect on the measurement of TFP mainly at business cycle frequency. This is especially true for the years during the global financial and economic crisis. A similar pattern can be seen for the start of the COVID-19 pandemic in 2020. Temporary reductions in unadjusted TFP indicators can therefore often be explained by changes in capacity utilisation and, consequently, do not necessarily represent efficiency declines. The overall trend of TFP, however, is less affected by changes in utilisation as can be seen from Figure 1 in the main text.

Figure B.1: Estimated impact of utilisation changes on Solow residuals



Notes: The estimated aggregate impact of utilisation is derived as the difference between the unadjusted Solow residuals and the utilisation-adjusted series of TFP growth, both aggregated across sectors using value added shares. See equation (B.2).

B.3 Shock series

We use three different shock series as instruments in the panel regressions: oil price shocks, international financial market shocks, and macroeconomic uncertainty shocks. The oil price shocks are taken from Känzig (2021). They rely on high-frequency data. Specifically, the series is based on changes in oil future prices in a tight window around OPEC production announcements. Financial market shocks reflect the unexpected part of United States corporate credit risk premia and are provided by Gilchrist and Zakrajsek (2012). The calculation of macroeconomic uncertainty shocks draws on work of Jurado, Ludvigson, and Ng (2015) and Meinen and Röhe (2017). The shocks are generated in a Bayesian VAR model for each country, identified by recursive ordering, with macroeconomic uncertainty entering as the first variable. The uncertainty indicators are derived from the conditional volatility of the unforecastable component of a broad set of macroeconomic variables. For the United States, we use macroeconomic uncertainty as generated by Jurado et al. (2015). The European series are an updated version of the ones in Meinen

Table B.1: Utilisation proxies used for European countries: Sectors C-E

Sector	Utilisation proxy
Food, beverages, tobacco products (C10-C12)	Utilisation surveys of the European commission for C10, C11 and C12, aggregated for C10-C12 using value added shares (average for years 2005-2018). For Germany data for C12 is missing in most years and we use C10-C11 instead.
Textiles, wearing apparel, leather (C13-C15)	Utilisation surveys of the European commission for C13, C14 and C15, aggregated for C13-C15 using value added shares (average for years 2005-2018).
Wood, paper, printing (C16-C18)	Utilisation surveys of the European commission for C16, C17 and C18, aggregated for C16-C18 using value added shares (average for years 2005-2018).
Chemicals, basic pharmaceutical products (C20-C21)	Utilisation surveys of the European commission for C20 and C21, aggregated for C20-C21 using value added shares (average for years 2005-2018). For the US only data for C20 are available.
Rubber, plastic, non-metallic mineral products (C22-C23)	Utilisation surveys of the European commission for C22 and C23, aggregated for C22-C23 using value added shares (average for years 2005-2018).
Metal products (C24-C25)	Utilisation surveys of the European commission for C24 and C25, aggregated for C24-C25 using value added shares (average for years 2005-2018).
Computer, electronic, optical products (C26-C27)	Utilisation surveys of the European commission for C26 and C27, aggregated for C26-C27 using value added shares (average for years 2005-2018).
Machinery and equipment n.e.c. (C28)	Utilisation surveys of the European commission for C28
Motor vehicles, trailers, other transport equipment (C29-C30)	Utilisation surveys of the European commission for C29 and C30, aggregated for C29-C30 using value added shares (average for years 2005-2018).
Furniture, jewellery, musical instruments, toys (C31-C33)	Utilisation surveys of the European commission for C31, C32 and C33, aggregated for C31-C33 using value added shares (average for years 2005-2018).
Electricity, gas, steam, air conditioning supply (D)	Utilisation surveys of the European commission for aggregate manufacturing. For the US, D and E are combined in one sector.
Water supply, sewerage, waste (E)	Utilisation surveys of the European commission for aggregate manufacturing. For the US, D and E are combined in one sector.

Notes: List of sectors included in the estimations with NACE codes in parentheses and the proxy used for production factor utilisation in the estimations. Survey data on utilisation stem from the business and consumer and surveys of the European Commission. There are no data for sectors D and E and we use the average for the manufacturing sector instead. For the United States we use changes in average weekly hours worked, provided by EU KLEMS, as a proxy for changes in capacity utilisation, following Basu et al. (2006).

and Röhe (2017). Besides macroeconomic uncertainty the VAR models include a stock price index,[22] the shadow short rate of Krippner (2013), the CPI index, the unemployment rate and industrial production as dependent variables. Unless specified otherwise,, the data are taken from Haver Analytics. The estimation frequency is monthly, with 12

[22]Here we choose the CDAX for Germany, the CAC for France, and the S&P 500 for the United States.

Table B.2: Utilisation proxies for the included sectors: Sectors F-S

Sector	Utilisation proxy
Construction (F)	Utilisation surveys of the European commission for aggregate manufacturing. For Germany and France separate data for utilisation in the construction sector, taken from ifo and Insee.
Trade; repair of motor vehicles (G)	Utilisation surveys of the European commission for aggregate manufacturing
Transportation and storage (H)	Utilisation surveys of the European commission for H49, H50, H51, H52, and H53, aggregated to H using value added shares (average for years 2011-2018), (before 2011: aggregate manufacturing). For Germany, data for H50, H51, H52, for France data for H50 and H51 are missing.
Accommodation and food service activities (I)	Utilisation surveys of the European commission for I55 and I56, aggregated to I using value added shares (average for years 2011-2018), (before 2011: aggregate manufacturing).
Information and communication (J)	Utilisation surveys of the European commission for J58, J59, J60, J61, J62 and J63, aggregated to J using value added shares (average for years 2011-2018), (before 2011: aggregate manufacturing). For Germany, data for J58-J61 and J63 are missing.
Financial and insurance activities (K)	Utilisation surveys of the European commission for the aggregate service sector (before 2011: aggregate manufacturing)
Professional, scientific and technical activities (M)	Utilisation surveys of the European commission for M69, M70, M71, M72, M73, and M74, aggregated to M using value added shares (average for years 2011-2018), (before 2011: aggregate manufacturing). Data for M75 are missing.
Administrative and support service activities (N)	Utilisation surveys of the European commission for N77, N78, N79, N80, N81 and N82, aggregated to N using value added shares (average for years 2011-2018), (before 2011: aggregate manufacturing). Data for N80 are missing in Germany.
Arts, entertainment, recreation; other services (R-S)	Utilisation surveys of the European commission for the aggregate service sector (before 2011: aggregate manufacturing)

Notes: List of sectors included in the estimations with NACE codes in parentheses and the proxy used for capacity utilisation in the estimations. Survey data about utilisation stem from the business and consumer and surveys of the European Commission. Utilisation data for service sectors are available starting from 2011. Before 2011, the utilisation is calculated backward until 1997 using the growth rate of capacity utilisation in the aggregate manufacturing sector. For sectors K and R-S, individual utilisation data is missing for most countries in the majority of years. We use the series for average utilisation in the service sectors instead. Moreover, the European Commission's survey does not provide data for construction (F) and trade (G) and we use the manufacturing sector's aggregate series.

lags included, using a Minnesota normal Wishard prior. The uncertainty shock reflects the median of the shock distribution (100,000 draws from posterior).

Tests for weak instruments confirm that the chosen instruments are sufficiently correlated with changes in capacity utilisation. We report F-statistics (Montiel Olea and Pflueger, 2013) in Table B.3. TFP growth is only adjusted for changes in utilisation if β is statistically significant at the 10% level at least. The regression results are presented in Table B.3. The coefficient is significant for all cases except for the non-manufacturing sectors in the United States, which is also the only regression in which statistical tests reject the validity of the instruments.

Basu et al. (2006) use oil price shocks, government spending shocks and a monetary policy shock series in their estimation for U.S. TFP growth. The oil price shocks reflect lagged changes in the Brent oil price (calculated as the log difference between the current quarters real price of oil minus the maximum price in the last 4 quarters, lagged by one period and averaged to yearly means). The instruments in Comin et al. (2020) are an oil price shock series (following the one in Basu et al. (2006)), an economic policy uncertainty shock, financial shocks (GZ spread) and a monetary policy shock series. We also tested alternative series. However, a measure of unexpected changes in government spending as well as monetary policy shocks did not meet the criteria for valid instruments and were excluded from the estimations. Using the oil price shock by Basu et al. (2006) instead of the series by Känzig (2021) hardly changes the results. However, the relevance of instruments is higher when we use the measure of Känzig (2021).

B.4 Estimation results

Table B.3 shows the estimation results of the country-specific panel regressions of the Solow residuals on changes in capacity utilisation, together with F-statistics for testing for potential weak instruments. The panel estimations are conducted by country for two groups of sectors: manufacturing sectors and non-manufacturing sectors.

Table B.3: Regression results

	DE		**FR**		**US**	
Utilisation proxy:	Surveys		Surveys		Average hours worked	
	Manuf.	Other	Manuf.	Other	Manuf.	Other
Coefficient	0.68	0.26	0.46	0.27	2.59	0.62
Std.	0.14	0.06	0.12	0.09	0.60	1.13
P-val.	0.00	0.00	0.00	0.00	0.00	0.59
Obs.	288	216	288	216	253	207
F-KP	22.0	28.0	15.4	15.0	20.4	10.6
F-MP	28.5	37.4	19.0	25.8	23.9	15.1
Crit-MP5	27.7	26.7	24.8	26.0	17.6	23.3
Crit-MP10	17.0	16.4	15.0	16.0	11.1	14.4

Notes: Estimation results of regressing sectoral Solow residuals on a proxy for changes in sectoral capacity utilisation (survey data for European countries; changes in average weekly hours worked for the United States) and sector-specific fixed effects. Estimation is conducted for two subgroups (see Section 2 of the main text), manufacturing sectors (comprising NACE C sub-sectors, NACE D and E) and others (comprising NACE F to K, M, N, R-S), by two-stage-least squares with three instruments: an oil price shock, a macroeconomic uncertainty shock, and a financial shock series (see main text). Standard errors are robust. *F-KP* presents the first stage F-statistic of Kleibergen and Paap (2006). *F-MP* shows the effective F-statistic by Montiel Olea and Pflueger (2013), which is robust for heteroskedasticity, autocorrelation and clustering, with *Crit-MP5*, *Crit-MP10* being the critical values at the 5% and 10% level, respectively.

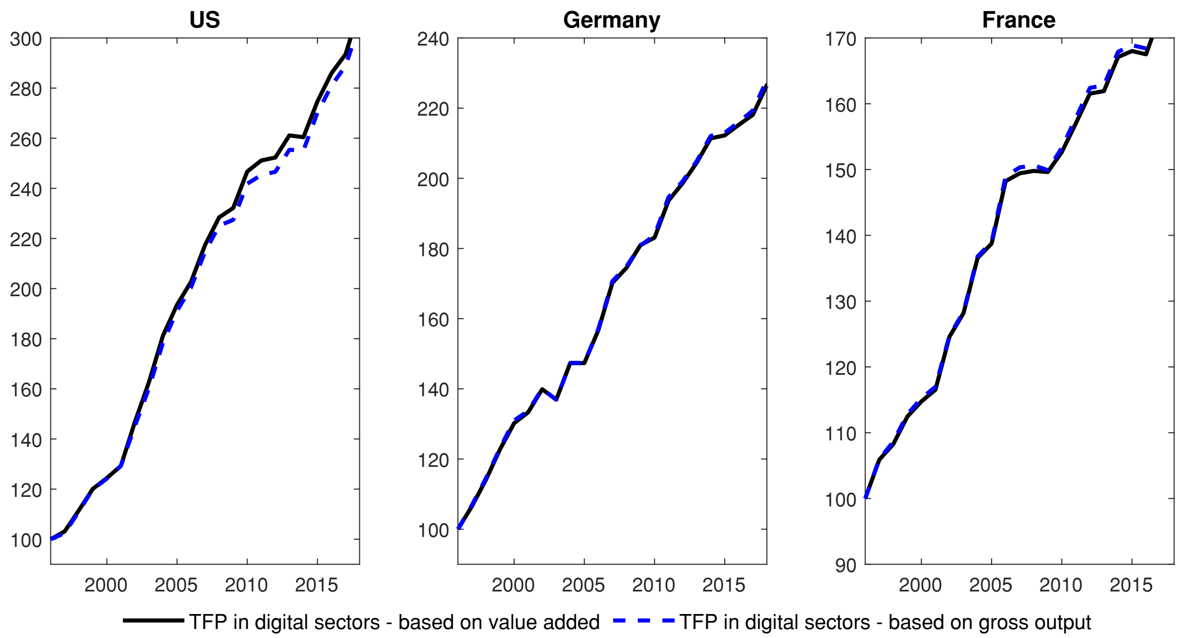
B.5 Robustness: Alternative growth accounting

Figure B.2 contrasts our baseline measure of TFP, which is derived from real gross value added growth, with a TFP measure that is calculated on basis of gross output growth. For the latter, the Solow residual $s^g_{j,t}$ is calculated as follows:

$$s^g_{j,t} = dy^g_{j,t} - \alpha^l_{j,t} dl_{j,t} - (\alpha^k_{j,t}) dk_{j,t} - (1 - \alpha^k_{j,t} - \alpha^l_{j,t}) dm_{j,t}, \tag{B.3}$$

where $m_{j,t}$ are real intermediate inputs in sector j in year t and $\alpha^l_{j,t}$ ($\alpha^k_{j,t}$) are calculated by dividing the sectoral nominal compensation of labour (capital) by sectoral nominal gross output. The utilisation adjustment is conducted in the same way as before. In case of gross output, the sector TFP growth series are aggregated with Domar weights, calculated as sectoral gross output over sectoral value added, whereas the TFP growth series based on value added are aggregated with value added weights (see Hulten, 2010).

Figure B.2: TFP in digital sectors using gross output growth



Notes: Figure contrasts indices for utilisation-adjusted TFP in the digital sectors (NACE C26-C27 and NACE J) from 1996 to 2020 (1996=100) based on gross output growth with those derived from gross value added growth.

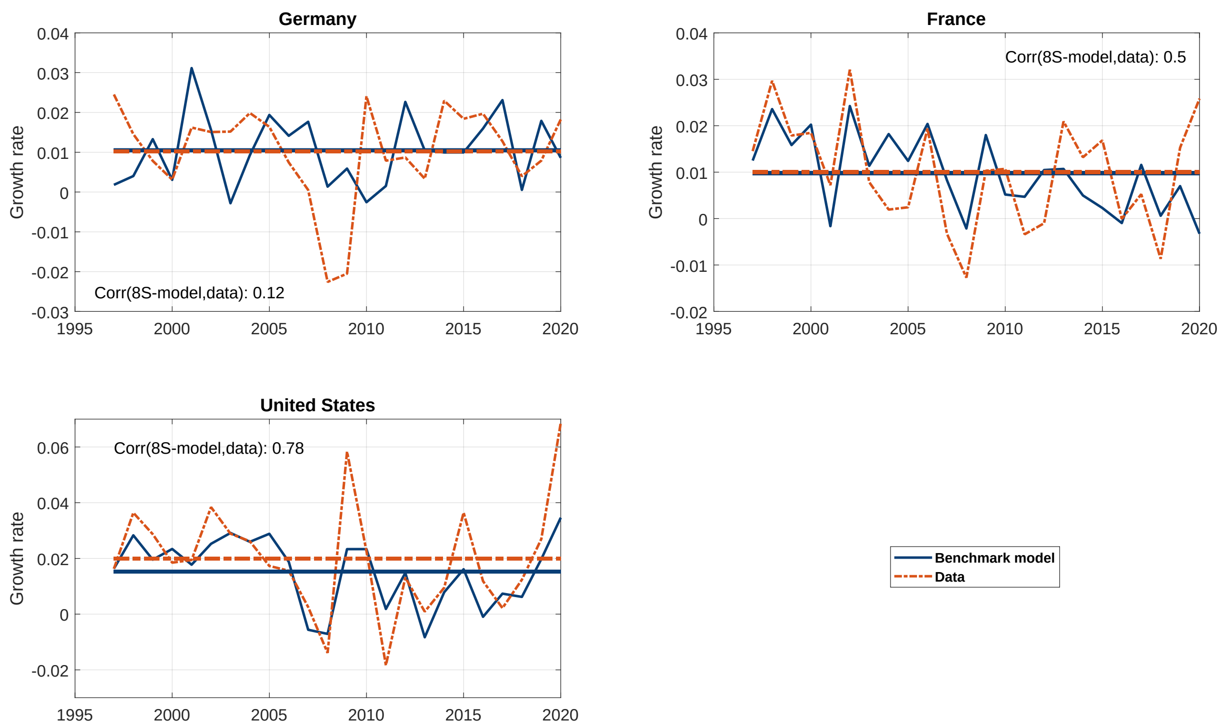
As shown in Figure B.2 TFP in the digital sectors is hardly affected by the choice of the output measure. In this paper, we use the TFP measure derived from data on gross value added as it is available for a larger set of sectors. In particular, price indices and hence price-adjusted data for intermediate goods are missing for some sectors in the United States.

Appendix C: Robustness and additional results

C.1 Labour productivity growth: Model vs. data

We also explore the model's ability to capture fluctuations in labour productivity around the trend. Figure C.1 shows labour productivity growth rates simulated by the benchmark model, their empirical counterparts, and average annual growth rate in each country (horizontal lines).

Figure C.1: Model-implied and actual labour productivity growth



Notes: The figure plots the model-implied aggregate labour productivity growth (straight blue line) and its empirical counterpart (dashed orange line). The horizontal lines indicate the average annual growth rate over the period from 1996 to 2020 for Germany, France, and the United States.

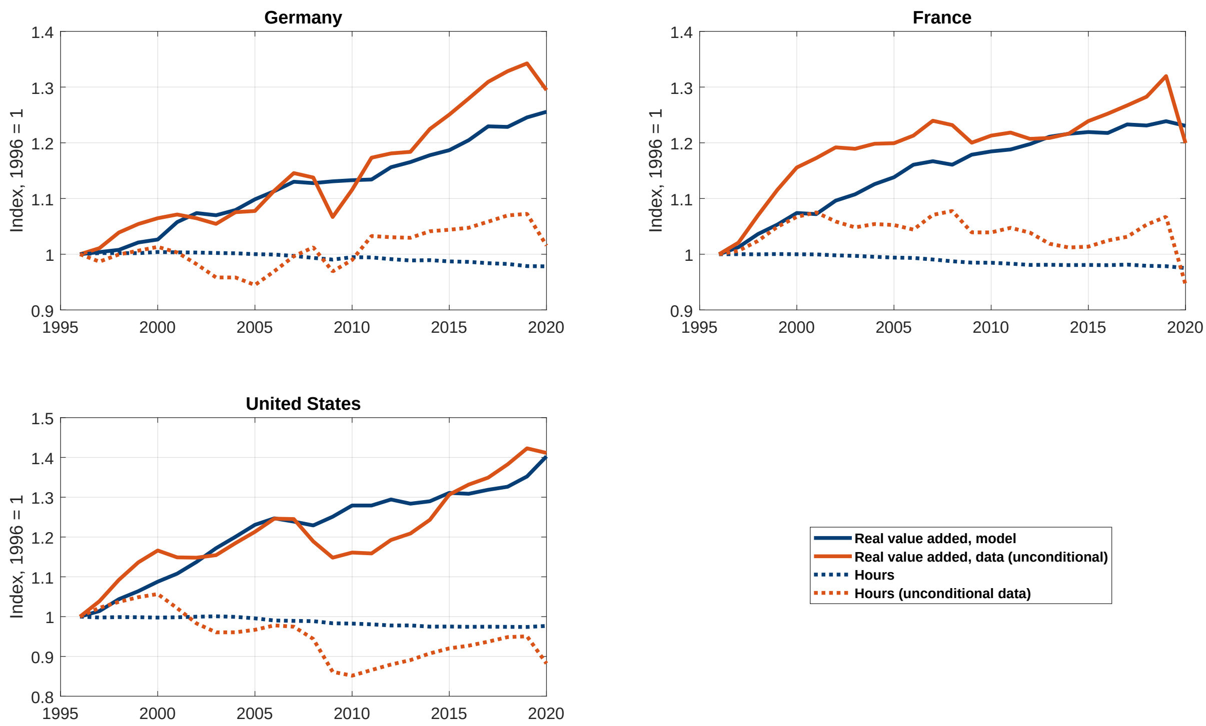
As expected from the results presented in section 5, the average annual growth rates of the benchmark simulation and the empirical counterparts are relatively close, with the model simulations slightly underestimating the average growth rates in the data.

Interestingly, the model captures the variation in labour productivity around the trend relatively well in the United States and in France, as reflected by correlation coefficients of 0.78 and 0.5, respectively. In Germany, by contrast, the correlation is considerably lower at a value of 0.05. Further analyses (which are available upon request) indicate that this low correlation is driven by the utilisation adjustment. Using unadjusted TFP data results in a correlation coefficient of 0.5 as well.

C.2 Gross value added and hours worked: Model vs. data

To further dissect the development of labour productivity, Figure C.2 shows the model-implied paths for real gross value added and hours worked in Germany, France, and the United States in period from 1996 to 2020 period and compares them to their empirical counterparts. Overall, the model fits the data reasonably well, especially in the United States.

Figure C.2: Model-implied and empirical components of labour productivity, real gross value added and labour



Notes: The figure plots model-implied real gross value added and and hours worked as well as its empirical counterparts. Data span the period from 1996 to 2020 for Germany, France, and the United States.

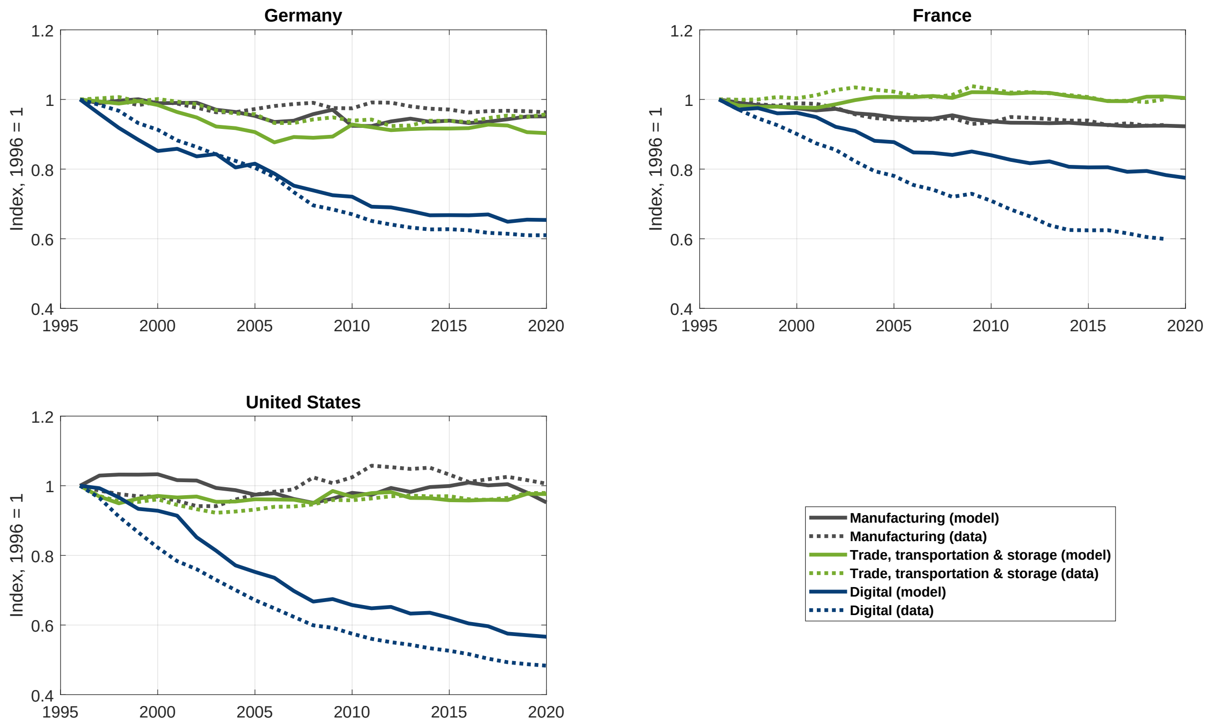
C.3 Relative prices

In the literature, the inverse of the relative price of investment goods has frequently been used as measure of (investment-specific) technological progress.[23] Since our benchmark

[23]See, e.g., Justiniano, Primiceri, and Tambalotti (2011).

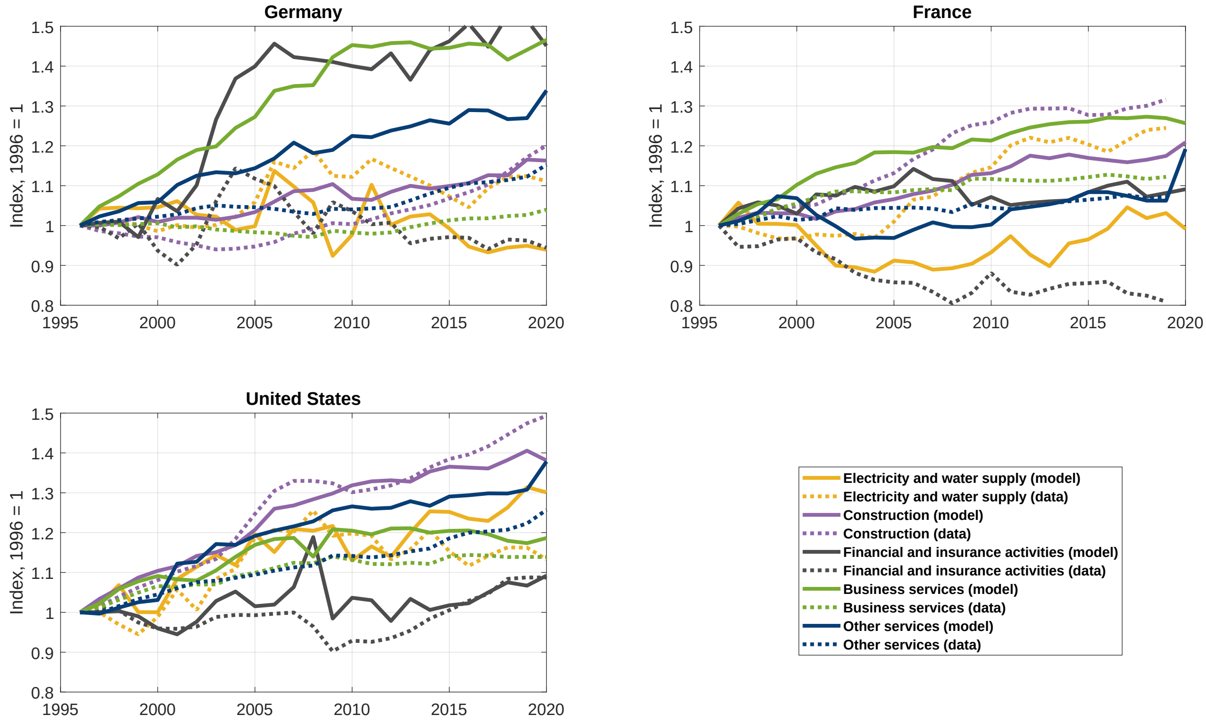
results are conditional on TFP sequences for the complete set of sectors, we take the converse route and compare the model-implied relative gross output prices with its empirical counterpart (see Figures C.3 and C.4). According to Figure C.3, the model captures the relative prices over time reasonably well, although the relative price decline of digital goods are slightly underestimated, especially in France. In all countries, the largest relative price decreases are observed in the digital sector, while prices in the other two sectors decrease only moderately.

Figure C.3: Model-implied and empirical relative gross output prices for the manufacturing sector, the trade and transport sector, and the digital sector.



Notes: The empirical gross output prices are computed by generating deflators for each sector and dividing them by the PCE deflator.

Figure C.4: Model-implied and empirical relative gross output prices.



Notes: The empirical gross output prices are computed by generating deflators for each sector and dividing them by the PCE deflator.

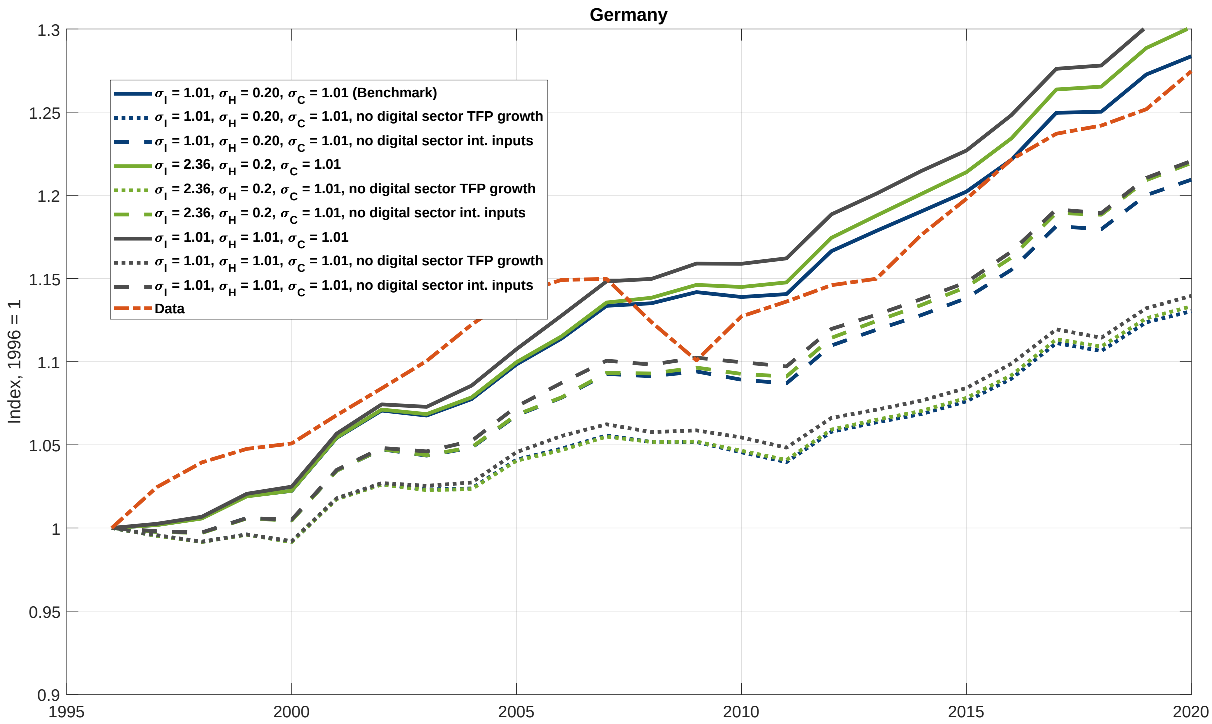
C.4 Alternative parameterisations of the elasticity of substitution

Figures C.5 to C.7 show the sensitivity of our benchmark simulation to the use of different values of the elasticity of substitution. For the two selected scenarios, we repeat the two counterfactual simulations from the main text. The results appear quite intuitive. The higher the elasticity of substitution, the higher the cumulative growth in labour productivity over time, since goods produced more efficiently (and thus sold at a lower price) can more easily substitute for consumption, investment and intermediate goods from other sectors.

Figure C.5: Robustness analysis of the elasticity of substitution - Germany

Figure C.6: Robustness analysis of the elasticity of substitution - France

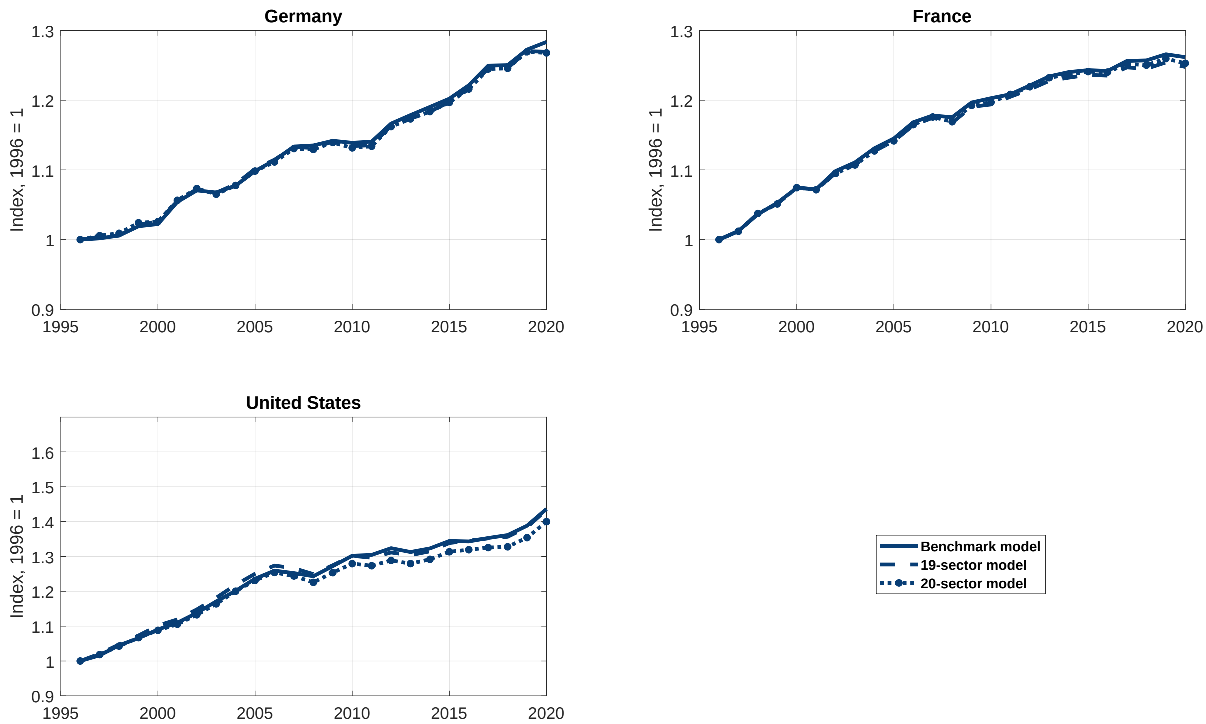
Figure C.7: Robustness analysis of the elasticity of substitution - United States



C.5 Sectoral aggregation

Figure C.8 shows the sensitivity of the benchmark simulations to different specifications of the number of economic sectors. Specifically, the figure contrasts the evolution of aggregate labour productivity of the eight-sector benchmark model with a 19 and 20-sector model. The 20-sector model is the most granular disaggregation level possible given the availability of TFP data. The difference between the 19 and 20-sector models is that the NACE divisions C26-C27 and section J are merged in the former variant, but not in the latter. In all countries, the simulated labour productivity tends to increase as the number of sectors decreases, although this effect is relatively small. The reason for this is that the elasticities of substitution in the more disaggregated model versions are set to the same values as in the benchmark model, but the heterogeneity in TFP development tends to increase as more sectors are specified.

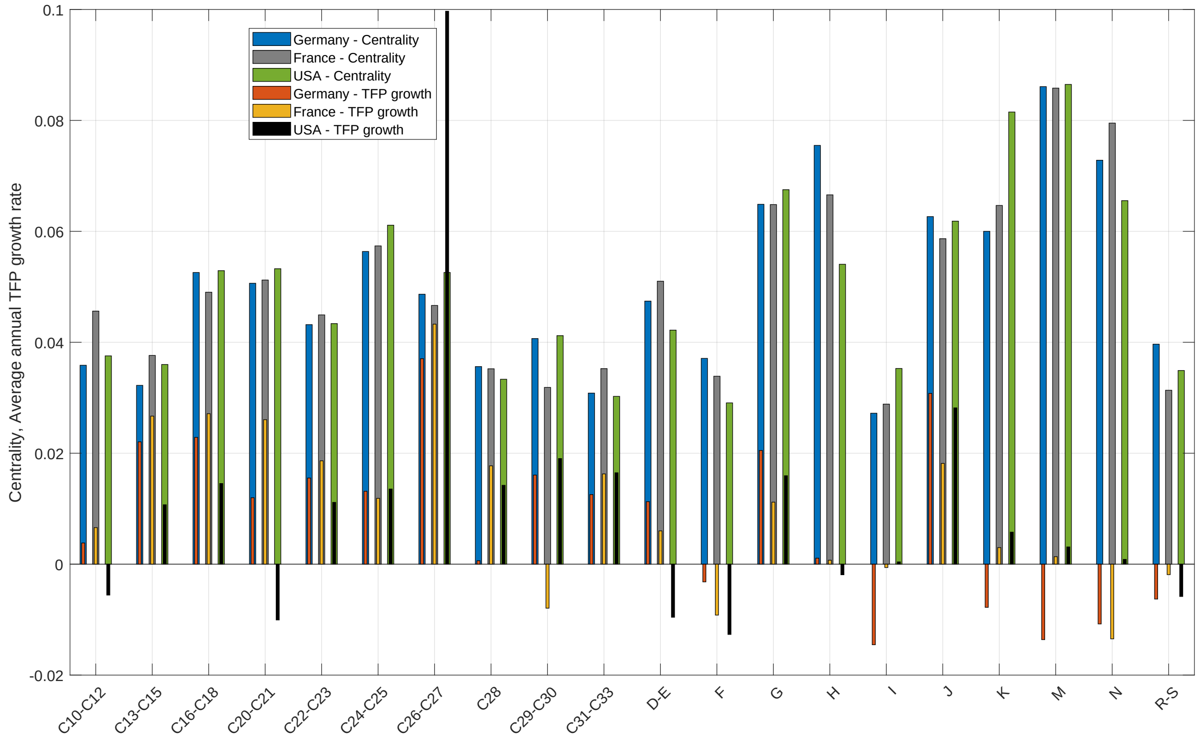
Figure C.8: Simulations of labour productivity using different numbers of sectors



Notes: The figure plots model-implied labour productivity for different specifications of the sector number.

C.6 Centrality in the 20-sector model

Figure C.9: Centrality and average sectoral TFP growth across countries



Notes: The (Bonacich) centrality measure for the initial period of the 20-sector model as well as the average sectoral TFP growth rate over the period from 1996 to 2020. The computation of the centrality measure follows Carvalho (2014).

C.7 Isolating the role of the input-output matrix

Figure C.10: Effects of replacing the United States input-output matrix

Notes: The figure shows how labour productivity growth changes in simulations when using Germany's or France's input-output matrix instead of that of the United States.

Appendix D: Model setup and equilibrium conditions

This Appendix contains further details on the baseline model. Section D.1 provides the derivation of the model equations. Section D.2 summarises the nonlinear equilibrium conditions. Section A.2 depicts the evolution of the labour share of value added from EU KLEMS as well as the labour and intermediate input share from WIOD over time.

D.1 Model derivation

- Households:
  The representative household chooses $\{C_t, I_t, K_t, N_t\}_{t=0}^{\infty}$ to maximise utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \kappa_N \frac{N_t^{1+\zeta}}{1+\zeta} \right)$$

subject to the budget constraint

$$C_t + P_t^I I_t = w_t N_t + r_t^k K_{t-1}$$

and the law of motion for capital

$$K_t = (1-\delta)K_{t-1} + I_t.$$

Hence, the Lagrangian can be written as

$$\begin{aligned} \Lambda = \mathbb{E}_0 \sum_{t=0}^{\infty} \Bigg\{ & \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \kappa_N \frac{N_t^{1+\zeta}}{1+\zeta} \right) \\ & - \beta^t \lambda_t \Bigg[C_t + P_t^I \left[K_t - (1-\delta)K_{t-1} \right] \\ & - w_t N_t - r_t^k K_{t-1} \Bigg] \Bigg\}. \end{aligned}$$

The first-order conditions corresponding to this problem are

$$\Lambda_C = C_t^{-\sigma} - \lambda_t = 0,$$

$$\Lambda_N = \kappa_N N_t^{\zeta} - \lambda_t w_t = 0,$$

$$\Lambda_K = \lambda_t - \beta \mathbb{E}_t \left[\lambda_{t+1} \frac{r_{t+1}^k + (1-\delta) P_{t+1}^I}{P_t^I} \right] = 0$$

and

$$\Lambda_\lambda = C_t + P_t^I \left[K_t - (1-\delta)K_{t-1}\right] \\ - w_t N_t - r_t^k K_{t-1} = 0.$$

Finally, we impose a standard transversality condition to guarantee that capital does not grow too quickly:

$$\lim_{t\to\infty} \beta^t \lambda_t K_t = 0.$$

- Consumption-goods, investment-goods and intermediate-goods retailers:

The representative consumption-goods retailer optimisation problem is given by

$$\max_{C_{s,t}} C_t - \sum_{s=1}^{S} P_{s,t} C_{s,t},$$

subject to the technology constraint

$$C_t = \left(\sum_{s=1}^{S} \psi_{C,s}^{1-\sigma_C} C_{s,t}^{\sigma_C}\right)^{\frac{1}{\sigma_C}}.$$

Therefore, the retailer's optimization problem can be written as

$$\max_{C_{s,t}} \left(\sum_{s=1}^{S} \psi_{C,s}^{1-\sigma_C} C_{s,t}^{\sigma_C}\right)^{\frac{1}{\sigma_C}} - \sum_{s=1}^{S} P_{s,t} C_{s,t},$$

which leads to the following first-order condition characterising the demand for consumption goods:

$$C_{s,t} = \psi_{C,s} P_{s,t}^{-\frac{1}{(1-\sigma_C)}} C_t.$$

By plugging this expression into the constant elasticity of substitution aggregator of consumption goods it can be shown that

$$P_t^C = \left[\sum_{s=1}^{S} \psi_{C,s} \tilde{P}_{s,t}^{-\frac{\sigma_C}{(1-\sigma_C)}}\right]^{-\frac{(1-\sigma_C)}{\sigma_C}},$$

with $P_{s,t} = \tilde{P}_{s,t}/P_t^C$.

Hence, CPI inflation, $\pi_t^{CPI} = P_t^C / P_{t-1}^C$, can be expressed as

$$\pi_t^{CPI} = \left[\sum_{s=1}^{S} \psi_{C,s} (\pi_{s,t}^{PPI} P_{s,t-1})^{-\frac{(\sigma_C)}{1-\sigma_C}}\right]^{-\frac{(1-\sigma_C)}{\sigma_C}},$$

with $\pi_{s,t}^{PPI} = \tilde{P}_{s,t}/\tilde{P}_{s,t-1}$.
Analogously, the optimisation problem of the representative investment goods retailer is given by

$$\max_{I_{s,t}} P_t^I I_t - \sum_{s=1}^{S} P_{s,t} I_{s,t},$$

subject to the technology constraint

$$I_t = \left(\sum_{s=1}^{S} \psi_{I,s}^{1-\sigma_I} I_{s,t}^{\sigma_I} \right)^{\frac{1}{\sigma_I}}.$$

Therefore, the retailer's optimisation problem can be written as

$$\max_{I_{s,t}} P_t^I \left(\sum_{s=1}^{S} \psi_{I,s}^{1-\sigma_I} I_{s,t}^{\sigma_I} \right)^{\frac{1}{\sigma_I}} - \sum_{s=1}^{S} P_{s,t} I_{s,t}.$$

The first-order conditions corresponding to this problem is

$$I_{s,t} = \psi_{I,s} \left[\frac{P_{s,t}}{P_t^I} \right]^{-\left(\frac{1}{1-\sigma_I}\right)} I_t,$$

while the price index is given by

$$P_t^I = \left[\sum_{s=1}^{S} \psi_{I,s} (P_{s,t})^{-\frac{\sigma_I}{(1-\sigma_I)}} \right]^{-\frac{(1-\sigma_I)}{\sigma_I}}.$$

Finally, the optimisation problem of the representative intermediate goods retailer can be expressed as

$$\max_{H_{s,j,t}} P_{s,t}^H H_{s,t} - \sum_{j=1}^{S} P_{j,t} H_{s,j,t},$$

subject to the technology constraint

$$H_{s,t} = \left[\sum_{j=1}^{S} \psi_{H,s,j}^{1-\sigma_{H,s}} H_{s,j,t}^{\sigma_{H,s}} \right]^{\frac{1}{\sigma_{H,s}}}.$$

Hence, the retailer solves

$$\max_{H_{s,j,t}} P_{s,t}^H \left[\sum_{j=1}^{S} \psi_{H,s,j}^{1-\sigma_{H,s}} H_{s,j,t}^{\sigma_{H,s}} \right]^{\frac{1}{\sigma_{H,s}}} - \sum_{j=1}^{S} P_{j,t} H_{s,j,t},$$

which leads to the first-order condition

$$H_{s,j,t} = \psi_{H,s,j} \left(\frac{P_{j,t}}{P^H_{s,t}} \right)^{-\frac{1}{\left(1-\sigma_{H,s}\right)}} H_{s,t}$$

while the price index is given by

$$P^H_{s,t} = \left[\sum_{j=1}^{S} \psi_{H,s,j} \left(P_{j,t}\right)^{-\frac{\sigma_{H,s}}{\left(1-\sigma_{H,s}\right)}} \right]^{-\frac{\left(1-\sigma_{H,s}\right)}{\sigma_{H,s}}} .$$

- Labour and capital supply

  The optimisation problem with respect to labour supply can be written as

$$\max_{N_{s,t}} w_{s,t} N_{s,t} - w_t N_t,$$

  subject to the technology constraint

$$N_t = \left(\sum_{s=1}^{S} \omega_{N,s}^{1-\nu_N} N_{s,t}^{\nu_N} \right)^{\frac{1}{\nu_N}}$$

  or more succinctly

$$\max_{N_{s,t}} w_{s,t} N_{s,t} - w_t \left(\sum_{s=1}^{S} \omega_{N,s}^{1-\nu_N} N_{s,t}^{\nu_N} \right)^{\frac{1}{\nu_N}} ,$$

  which leads to the following first-order condition characterising the sector-specific demand for labour types:

$$N_{s,t} = \omega_{N,s} \left(\frac{w_{s,t}}{w_t} \right)^{-\frac{1}{(1-\nu_N)}} N_t \quad \forall s \in \mathcal{S}.$$

  By plugging this expression into the CES aggregator of labour goods, we get the aggregate wage index:

$$w_t = \left[\sum_{s=1}^{S} \omega_{N,s} w_{s,t}^{-\frac{\nu_N}{(1-\nu_N)}} \right]^{-\frac{(1-\nu_N)}{\nu_N}} .$$

  Similarly, the optimisation problem related to the supply of capital can be written as

$$\max_{K_{s,t-1}} r^K_{s,t} K_{s,t-1} - r^K_t K_{t-1},$$

subject to the technology constraint

$$K_t = \left(\sum_{s=1}^{S} \omega_{K,s}^{1-\nu_K} K_{s,t}^{\nu_K} \right)^{\frac{1}{\nu_K}}.$$

Hence, the agency's optimisation problem can be expressed as

$$\max_{K_{s,t-1}} r_{s,t}^K K_{s,t-1} - r_t^K \left(\sum_{s=1}^{S} \omega_{K,s}^{1-\nu_K} K_{s,t-1}^{\nu_K} \right)^{\frac{1}{\nu_K}},$$

which leads to the following first-order condition characterising the demand for capital:

$$K_{s,t} = \omega_{K,s} \left(\frac{r_{s,t+1}^K}{r_{t+1}^K} \right)^{-\frac{1}{(1-\nu_K)}} K_t \quad \forall s \in \mathcal{S}.$$

By plugging this expression into the CES aggregator of capital goods, we get:

$$r_t^K = \left[\sum_{s=1}^{S} \omega_{K,s} (r_{s,t}^K)^{-\frac{\nu_K}{(1-\nu_K)}} \right]^{-\frac{(1-\nu_K)}{\nu_K}}.$$

- Production

  First, the representative firm in each sector s minimises its costs

$$w_{s,t} N_{s,t} + r_{s,t}^k K_{s,t-1} + P_{s,t}^H H_{s,t}$$

subject to the constant returns to scale production technology

$$y_{s,t} = \left(\varepsilon_{s,t}^{VA} K_{s,t-1}^{1-\alpha_{N,s}} N_{s,t}^{\alpha_{N,s}} \right)^{\alpha_{H,s}} (H_{s,t})^{1-\alpha_{H,s}}.$$

Therefore, the optimisation problem can be written as

$$\min_{N_{s,t}, K_{s,t-1}, H_{s,t}} w_{s,t} N_{s,t} + r_{s,t}^k K_{s,t-1} + P_{s,t}^H H_{s,t} + mc_{s,t} \left[y_{s,t} - \left(\varepsilon_{s,t}^{VA} K_{s,t-1}^{1-\alpha_{N,s}} N_{s,t}^{\alpha_{N,s}} \right)^{\alpha_{H,s}} (H_{s,t})^{1-\alpha_{H,s}} \right].$$

The first-order conditions corresponding to this problem are:

$$\begin{aligned} w_{s,t} &= \alpha_{H,s} \alpha_{N,s} mc_{s,t} \frac{y_{s,t}}{N_{s,t}}, \\ r_{s,t}^k &= \alpha_{H,s} (1 - \alpha_{N,s}) mc_{s,t} \frac{y_{s,t}}{K_{s,t-1}}, \\ P_{s,t}^H &= (1 - \alpha_{H,s}) mc_{s,t} \frac{y_{s,t}}{Hs,t}. \end{aligned}$$

Second, the representative firm chooses $y_{s,t}$ to maximise its profits

$$\max_{y_{st}} \Pi_{s,t} = P_{s,t} y_{s,t} - mc_{s,t} y_{s,t}$$

The first-order condition of the problem is

$$P_{s,t} = mc_{s,t}.$$

- Market clearing

  In each sector s product market clearing implies

$$y_{s,t} = C_{s,t} + I_{s,t} + \sum_{j=1}^{S} H_{j,s,t}.$$

  At the aggregate level, CPI-deflated sectoral value added is defined as

$$Y_t^{va} = C_t + P_t^I I_t.$$

D.2 Representing the equilibrium

The baseline model is characterised by the following nonlinear difference equations:

$$\lambda_t = C_t^{-\sigma}, \tag{D.1}$$

$$\kappa_N N_t^{\zeta} = \lambda_t w_t \tag{D.2}$$

$$\lambda_t = \beta \mathbb{E}_t \left[\lambda_{t+1} \frac{r_{t+1}^k + (1-\delta) P_{t+1}^I}{P_t^I} \right], \tag{D.3}$$

$$C_t + P_t^I I_t = w_t N_t + r_t^k K_{t-1}, \tag{D.4}$$

$$K_t = (1-\delta) K_{t-1} + I_t, \tag{D.5}$$

$$C_{s,t} = \psi_{C,s} P_{s,t}^{-\frac{1}{(1-\sigma_C)}} C_t, \tag{D.6}$$

$$1 = \left[\sum_{s=1}^{S} \psi_{C,s} P_{s,t}^{-\frac{\sigma_C}{(1-\sigma_C)}} \right]^{-\frac{(1-\sigma_C)}{\sigma_C}}, \tag{D.7}$$

$$I_{s,t} = \psi_{I,s} \left(\frac{P_{s,t}}{P_t^I} \right)^{-\frac{1}{(1-\sigma_I)}} I_t, \tag{D.8}$$

$$P_t^I = \left[\sum_{s=1}^{S} \psi_{I,s}(P_{s,t})^{-\frac{\sigma_I}{(1-\sigma_I)}}\right]^{-\frac{(1-\sigma_I)}{\sigma_I}}, \tag{D.9}$$

$$H_{s,j,t} = \psi_{H,s,j} \left(\frac{P_{j,t}}{P_{s,t}^H}\right)^{-\frac{1}{\left(1-\sigma_{H,s}\right)}} H_{s,t}, \tag{D.10}$$

$$P_{s,t}^H = \left[\sum_{j=1}^{S} \psi_{H,s,j} \left(P_{j,t}\right)^{-\frac{\sigma_{H,s}}{\left(1-\sigma_{H,s}\right)}}\right]^{-\frac{\left(1-\sigma_{H,s}\right)}{\sigma_{H,s}}}, \tag{D.11}$$

$$N_{s,t} = \omega_{N,s} \left(\frac{w_{s,t}}{w_t}\right)^{-\frac{1}{(1-\nu_N)}} N_t, \tag{D.12}$$

$$w_t = \left[\sum_{s=1}^{S} \omega_{N,s} w_{s,t}^{-\frac{\nu_N}{(1-\nu_N)}}\right]^{-\frac{(1-\nu_N)}{\nu_N}}, \tag{D.13}$$

$$K_{s,t} = \omega_{K,s} \left(\frac{r_{s,t+1}^K}{r_{t+1}^K}\right)^{-\frac{1}{(1-\nu_K)}} K_t, \tag{D.14}$$

$$r_t^K = \left[\sum_{s=1}^{S} \omega_{K,s}(r_{s,t}^K)^{-\frac{\nu_K}{(1-\nu_K)}}\right]^{-\frac{(1-\nu_K)}{\nu_K}}, \tag{D.15}$$

$$y_{s,t} = \left(\varepsilon_{s,t}^{VA} K_{s,t-1}^{1-\alpha_{N,s}} N_{s,t}^{\alpha_{N,s}}\right)^{\alpha_{H,s}} (H_{s,t})^{1-\alpha_{H,s}}, \tag{D.16}$$

$$w_{s,t} = \alpha_{H,s}\alpha_{N,s} mc_{s,t} \frac{y_{s,t}}{N_{s,t}}, \tag{D.17}$$

$$r_{s,t}^k = \alpha_{H,s}(1-\alpha_{N,s}) mc_{s,t} \frac{y_{s,t}}{K_{s,t-1}}, \tag{D.18}$$

$$P_{s,t}^H = (1-\alpha_{H,s}) mc_{s,t} \frac{y_{s,t}}{Hs,t}, \tag{D.19}$$

$$P_{s,t} = \tilde{m}c_{s,t}, \tag{D.20}$$

$$y_{s,t} = C_{s,t} + I_{s,t} + \sum_{j=1}^{S} H_{j,s,t} \tag{D.21}$$

and

$$Y_t^{va} = C_t + P_t^I I_t. \tag{D.22}$$