DEUTSCHE
BUNDESBANK
EUROSYSTEM

Discussion Paper

Deutsche Bundesbank
No 18/2023

Monetary policy rules under bounded rationality

Michael Dobrew
(European Central Bank)

Rafael Gerke
(Deutsche Bundesbank)

Daniel Kienzler
(Deutsche Bundesbank)

Alexander Schwemmer
(German Federal Ministry of Finance)

Deutsche Bundesbank, Wilhelm-Epstein-Straße 14, 60431 Frankfurt am Main,
Postfach 10 06 02, 60006 Frankfurt am Main

Tel +49 69 9566-0

Please address all orders in writing to: Deutsche Bundesbank,
Press and Public Relations Division, at the above address or via fax +49 69 9566-3077

Internet http://www.bundesbank.de

ISBN 978–3–95729–949–9
ISSN 2749–2958

Non-technical summary

Research question

How does bounded rationality affect the benefits of history-dependent monetary policy strategies such as price level targeting (PLT) or average inflation targeting (AIT)? These strategies take into account past deviations of prices or inflation from target. Via the expectations channel they can exhibit powerful stabilisation properties when monetary policy faces a trade-off between the stabilisation of inflation and real activity as well as during effective lower bound (ELB) periods. Consequently, the efficacy of history-dependent strategies strongly depends on agents being rational and forward-looking. However, recent research on expectation formation has increasingly documented considerable deviations from full information rational expectations.

Contribution

We employ a New Keynesian model with sticky prices and sticky wages, an effective lower bound on interest rates and boundedly rational agents. In this framework, monetary policy faces two difficulties in stabilising the economy. First, supply shocks induce a trade-off between the stabilisation of inflation and real activity. Second, demand shocks are difficult to accommodate when interest rates hit the ELB. Bounded rationality reduces the strength of the expectations channel. This reduces the effectiveness of history-dependent strategies in mitigating the trade-off arising from supply shocks and stabilising the economy at the ELB. We compare a range of optimised history- and non-history-dependent interest-rate rules according to their model-consistent welfare performance. To do so, we run stochastic simulations across different degrees of bounded rationality.

Results

Our stochastic simulations show that for demand shocks history-dependent interest-rate rules lose their advantage vis-à-vis the non-history dependent inflation-targeting (IT) rule as the degree of bounded rationality increases. For trade-off inducing supply shocks, the IT rule even outperforms history-dependent rules for a sufficiently high degree of bounded rationality. If the degree of bounded rationality is low, the expectations channel is not crucially weakened and history-dependent rules retain their advantages over the IT rule. Furthermore, we show that an interest-rate rule, which responds to an exponential - instead of an arithmetic - average of inflation rates performs remarkably well independently of the degree of bounded rationality. Such a rule unifies the advantages of history-dependent rules for low degrees of bounded rationality and the advantages of non-history dependent rules for high degrees of bounded rationality.

Nichttechnische Zusammenfassung

Forschungsfrage

Wie wirkt sich beschränkte Rationalität auf die Vorteile vergangenheitsabhängiger geldpolitischer Strategien – wie beispielsweise Preisniveausteuerung (PLT) oder die Steuerung der durchschnittlichen Inflationsrate (AIT) – aus? Solche Strategien berücksichtigen vergangene Abweichungen der Preise oder der Inflation vom Zielwert. Mittels des Erwartungskanals können sie starke Stabilisierungswirkungen aufweisen, wenn sich die Geldpolitik einem Tradeoff zwischen der Stabilisierung der Inflationsrate und der realen Wirtschaftsaktivität gegenübersieht und während Phasen an der effektiven Zinsuntergrenze. Folglich hängt die Wirksamkeit vergangenheitsabhängiger Strategien stark davon ab, ob die Akteure rational und zukunftsorientiert sind. Neuere Untersuchungen zur Erwartungsbildung dokumentieren jedoch zunehmend beträchtliche Abweichungen von vollständig rationalen Erwartungen.

Beitrag

Wir verwenden ein Neukeynesianisches Modell mit rigiden Preisen und Löhnen, einer effektiven Zinsuntergrenze sowie beschränkt rationalen Akteuren. In diesem Rahmen sieht sich die Geldpolitik zwei Herausforderungen gegenüber, die wirtschaftliche Entwicklung zu stabilisieren. Erstens führen Angebotsschocks zu einem Zielkonflikt (Trade-off) zwischen der Stabilisierung der Inflationsrate und der realen Wirtschaftsaktivität. Zweitens sind Nachfrageschocks schwer zu bewältigen, wenn die Zinsen an die effektive Zinsuntergrenze stoßen. Beschränkte Rationalität verringert die Stärke des Erwartungskanals. Entsprechend verringert sich die Effektivität vergangenheitsabhängiger Strategien bei aus Angebotsschocks resultierenden Zielkonflikten und bei der Stabilisierung der wirtschaftlichen Entwicklung an der effektiven Zinsuntergrenze. Wir vergleichen eine Reihe von optimierten vergangenheits- und nicht vergangenheitsabhängigen Zinsregeln anhand eines modellkonsistenten Wohlfahrtskriteriums, indem wir stochastische Simulationen über verschiedene Grade beschränkter Rationalität durchführen.

Ergebnisse

Unsere stochastischen Simulationen zeigen, dass mit zunehmendem Grad der beschränkten Rationalität vergangenheitsabhängige Zinsregeln bei Nachfrageschocks ihren Vorteil gegenüber einer nicht-vergangenheitsabhängigen Regel für Inflationssteuerung (IT) verlieren. Bei Angebotsschocks, die einen Trade-off induzieren, übertrifft die IT-Regel sogar die vergangenheitsabhängigen Regeln für einen hinreichend hohen Grad an begrenzter Rationalität. Ist der Grad der begrenzten Rationalität dagegen gering, wird der

Erwartungskanal nicht entscheidend geschwächt, und vergangenheitsabhängige Regeln behalten ihre Vorteile gegenüber der IT-Regel. Darüber hinaus zeigen wir, dass eine Regel, die auf einen exponentiellen, statt auf einen arithmetischen, Durchschnitt der Inflationsrate reagiert, unabhängig vom Grad der beschränkten Rationalität bemerkenswert gut funktioniert. Eine solche Regel vereint die Vorteile vergangenheitsabhängiger Regeln für niedrige Grade an beschränkter Rationalität und die Vorteile nicht vergangenheitsabhängiger Regeln für hohe Grade an beschränkter Rationalität.

Deutsche Bundesbank Discussion Paper No 18/2023

Monetary policy rules under bounded rationality*

Michael Dobrew† Rafael Gerke‡ Daniel Kienzler§
Alexander Schwemmer¶

June 22, 2023

Abstract

We study the welfare performance of various simple monetary policy rules under bounded rationality (BR) along the lines of Gabaix (2020) in a New Keynesian model with sticky wages and an effective lower bound (ELB) on interest rates. Policy strategies with a strong history dependence lose their advantage over inflation targeting in mitigating a demand-driven recessions when interest rates are constrained by the ELB. For supply shocks, inflation targeting outperforms history-dependent rules for a sufficiently high degree of BR. An exponential average inflation targeting rule, which features a variable degree of history dependence, performs remarkably well, independent of the degree of BR.

Keywords: Bounded Rationality, Sticky Wages, Monetary Policy Strategies, Zero Lower Bound

JEL Codes: E20, E24, E31, E32, E52

*We thank Michał Brzoza-Brzezina, Sebastian Giesen, Tom Holden, Gerhard Illing, Jochen Mankart, Emanuel Mönch, Assaf Razin, Joost Röttger, Harald Uhlig, Johannes Wacks, Henning Weber, Mirko Wiederholt, Andreas Worms and Peter Zorn for helpful comments and suggestions. The views expressed in this paper are those of the authors and do not necessarily represent those of the ECB, the Bundesbank, the Eurosystem, or the German Federal Ministry of Finance.

†European Central Bank, michael.dobrew@ecb.europa.eu
‡Deutsche Bundesbank, rafael.gerke@bundesbank.de
§Deutsche Bundesbank, daniel.kienzler@bundesbank.de
¶German Federal Ministry of Finance, alexander.schwemmer@bmf.bund.de

1 Introduction

The long-term decline in the natural real rate[1] together with an effective lower bound (ELB) on nominal interest rates limited monetary policy's leeway to provide stimulus when inflation rates were too low not long ago. Consequently, major central banks analysed several alternative interest-rate rules within their reviews of their monetary policy strategy[2] to evaluate possibilities of providing policy stimulus at the ELB. More recently, monetary policy has been challenged by severe supply shocks that have induced a trade-off between stabilizing high inflation rates and decreasing real activity.

In principle, history-dependent strategies like average-inflation targeting (AIT) or price-level targeting (PLT) can help with both challenges via the expectations channel. History-dependent strategies promise to "make up" past and current deviations of prices or inflation from target in the future. For example, when inflation is below target during an ELB episode, a history-dependent strategy keeps the policy rate low even after the ELB ceases to bind in order to make up for this inflation shortfall through a future expansion. If agents rationally expect higher future inflation this lowers real rates already today, stabilising contemporaneous inflation.[3] In case of a supply shock that increases inflation and decreases economic activity, a history-dependent strategy can mitigate the trade-off by promising to compensate the high inflation rates by inflation rates below target in the future: Lower expected inflation rates imply a higher real rate and hence a lower inflation rate today without having to slow down real activity today.

Hence, the efficacy of history-dependent strategies depends heavily on agents' expectations being rational and forward-looking, both for supply shocks that induce a trade-off and demand shocks that drive the policy rate to the ELB. However, recent research on expectation formation has increasingly documented substantial deviations from full information rational expectations.[4] Thus, a key question for monetary policy makers is whether history-dependent strategies still perform well when expectations deviate from full-information rational expectations.

In this paper, we analyse the performance of different optimised history-dependent interest-rate rules compared to an optimised non-history dependent inflation targeting (IT) rule in an ELB-constrained economy with boundedly rational agents and a role for supply shocks. To that end, we employ a New Keynesian model with sticky prices and sticky wages as in Erceg et al. (2000). In this set-up the divine coincidence breaks down for technology shocks. Instead, they induce a trade-off between stabilising inflation and real activity. In the spirit of Gabaix (2020), we assume that the agents are partially myopic in the sense that they discount expectations of future variables. This weakens the stabilising effects of the expectations channel, particularly at the ELB, and the mitigation of the trade-off for supply

[1] See e.g. Brand et al. (2018) or Holston et al. (2017).

[2] E.g. the Federal Reserve concluded its strategy review in 2019, the Eurosystem in 2021, the Bank of Canada engages in a 5-year returning review process and the Bank of Japan has conducted a smaller policy review in the beginning of 2021.

[3] Other possibilities to mitigate the ELB constraint in the realm of monetary policy include negative interest rates, forward guidance, and asset purchase programmes. Wo do not consider these tools in this paper.

[4] For survey evidence, see e.g. Coibion et al. (2018). For experimental evidence, see e.g. Afrouzi et al. (2021).

shocks.

Our results show that for demand shocks history-dependent interest-rate rules lose their advantage vis-à-vis the IT rule as the degree of myopia increases and the strength of the expectations channel fades. Since there is no trade-off for demand shocks (only the ELB fricition), there is virtually no difference anymore between the optimised history-dependent rules and the optimised IT rule for high degrees of myopia. For trade-off inducing supply shocks, the IT rule even outperforms history-dependent rules for a sufficiently high degree of myopia. This is because without an effective expectations channel, history-dependent rules induce a lot of volatility in real activity in order to stabilise inflation. If the degree of myopia is low, the expectations channel is not crucially weakened and history-dependent rules retain their advantages over the non-history dependent IT rule, as in a rational expectations setting.

In practice, monetary policy makers face considerable uncertainty about the degree of myopia. Therefore, a robust interest-rate rule should perform reasonably well across different degrees of myopia, and across different shock constellations. Essentially, on the one hand, such a rule should exhibit some form of history-dependence in order to reap the benefits of this feature in case the degree of myopia is low. On the other hand, it should resemble an IT rule when the degree of myopia is high. In principle, the AIT rule can fulfill these requirements as it performs in between the IT and the PLT rule for both low and high degrees of myopia. However, in particular for trade-off inducing technology shocks, conventional AIT that features an arithmetic, or simple, moving average of inflation rates exhibits an inherent volatility-inducing character. The reason is that any past deviation of inflation from its target directly affects the average for a given time frame, calling for a compensation by monetary policy in order to achieve its average inflation target. But once the initial deviation drops out of the averaging window, monetary policy is required to compensate the previous compensation to achieve its average inflation target, and so on. Conventional AIT thus has the disadvantage that it requires periodic fluctuations in the inflation rate to achieve its average inflation target.

A possibility to circumvent this disadvantage while still retaining the advantages of the conventional AIT rule is to employ an exponential AIT (eAIT) rule. Under an eAIT rule, monetary policy responds to an exponential moving average of the current and all past inflation deviations instead of an arithmetic, or simple, moving average with a fixed averaging window as in conventional AIT rules. This has two crucial consequences, one that is general and one that is specific to our BR setting. First, under the eAIT rule past inflation rates never actually drop out of the average, which generally avoids the inherent volatility-inducing character of the AIT rule. Second, under eAIT, past inflation rates in the average receive exponentially decaying weights over time. That is, past inflation deviations are assigned a higher weight the closer they are to the present period. This is in contrast with the conventional AIT rule where all (past) inflation deviations entering the average receive equal weights. Thus, in comparison to a conventional AIT rule, the higher weights on inflation rates closer to the present "tilt" the character of the eAIT rule towards an IT rule. The effect of this tilting is more pronounced for higher degrees of bounded rationality, as this further reduces the influence that inflation rates further in the past have on expectations about future inflation rates. Consequently, the eAIT rule performs remarkably well independently of the degree of myopia. On the one hand, it preserves the history-dependent character of the conventional AIT rule and thus approximates the good welfare performance of the PLT rule for low degrees of myopia. On the other hand, the eAIT rule shifts weights in the targeted average to the present and is thus able to approximate the good performance of the IT rule when the degree of myopia is high. Moreover, across all degrees of myopia, the eAIT rule avoids inherently inducing volatility like the AIT rule.

To compare the performance of the different monetary policy strategies under different degrees of myopia whilst taking into account the ELB, we proceed as follows: For each instrument rule and each degree of myopia, we span a wide grid over the rules' parameters. On each grid node, we run a stochastic simulation within which we solve the model with the extended path algorithm as described in Fair and Taylor (1983).[5] These simulations give us frequency distributions of the model variables. An advantage of our relatively simple model structure is that we can use the model-consistent welfare loss function to evaluate the performance of each rule. Thus, for each degree of myopia we can compare the welfare performance of optimised interest-rate rules conditional on the type of shock.[6]

The remainder of the paper is structured as follows. Section 2 gives an overview of the related literature. Section 3 presents the model, its calibration, and the numerical methods used to solve the model and conduct the grid search for finding the optimal coefficients of the rules. In section 4 we present the welfare comparison of the various optimised interest-rate rules for demand and supply shocks. Section 5 concludes.

2 Literature

The paper draws on several branches of the literature. The way we model agents' myopia is inspired by Gabaix (2020). He takes into account discounting of future deviations from steady state, which he introduces via expectations regarding future variables that are biased towards the steady state. Similar to Erceg et al. (2021), we do not micro-found myopia but use a shortcut to capture the idea that expectational effects are attenuated on an aggregate level. This idea has been formalised in different ways. For example, García-Schmidt and Woodford (2019) and Farhi and Werning (2019) introduce different variants of level-k thinking, an approach to bounded rationality that involves a finite number of updating rounds when deducing the behaviour of other agents in the future. Woodford (2019) introduces agents with a finite planning horizon, and Angeletos and Lian (2018) relax the assumption of common knowledge, resulting in a form of myopia on the aggregate level.[7]

[5]We implement the computations with the software package Dynare, see Adjemian et al. (2021).

[6]We consider a demand shock in the form of a discount factor shock and a supply shock in the form of a technology shock.

[7]Other attempts to remedy the implausibly strong forward-lookingness of the standard New Keynesian model include Bilbiie (2021) and Michaillat and Saez (2021). The former considers a simple (analytical) two-agent incomplete-markets formulation of the New Keynesian model, in which agents self-insure against idiosyncratic transitions into worse states. The paper shows that the empirically relevant case of procyclical

Gabaix (2020) shows that in the New Keynesian model with bounded rationality, strict price-level targeting is not the optimal policy anymore in the case of a cost-push shock. This relates to our results concerning a technology shock in the sticky-price-sticky-wage economy. Benchimol and Bounader (2021) study optimal monetary policy under bounded rationality. They show that PLT remains the optimal policy as long as agents continue to form expectations about inflation rationally, even if they are myopic with respect to the rest of the economy. In addition, we consider an economy subject to the ELB which is the prime motivation to consider history-dependent policy strategies in the first place. In that regard, we share a part of the setting of Nakata et al. (2019). They analyse optimal monetary policy at the effective lower bound in a New Keynesian model with discounting in the Euler equation as well as the Phillips curve. However, unlike the present paper, they abstract from wage rigidities and only focus on demand shocks. In contrast to Benchimol and Bounader (2021) and Nakata et al. (2019), we consider simple instrument rules that played a prominent role in the context of recent strategy reviews of leading central banks.

Our paper also relates to other analyses that study history-dependent monetary policies with simple rules, taking into account the ELB. Reifschneider and Williams (2000) track the sum of past deviations of the policy rate from the desired rate due to the ELB and propose to compensate this shortfall in stimulus by keeping the policy rate lower-for-longer. Nakov (2008) studies AIT and PLT rules, among others, in the light of the ELB. In contrast to these papers, we add boundedly rational agents and study how this feature affects the performance of history-dependent monetary policies. Bernanke et al. (2019) analyse history-dependent policy rules in a large model for the US and consider reduced effectiveness of history-dependent policies due to imperfect credibility of the monetary policy framework. Similarly, Coenen et al. (2021) use a large model for the euro area to compare history-dependent monetary policy rules.[8] Erceg et al. (2021) compare history-dependent policy rule in a medium-scale model for the euro area and additionally consider boundedly rational agents as in Gabaix (2020) as we do. In contrast to these studies, we use a smaller model, which allows us to use a model-consistent welfare function in order to optimise the coefficients of our policy rules and produce welfare rankings of the different rules. Wagner et al. (2022) use the standard New Keynesian model with bounded rationality as in Gabaix (2020) and an ELB, and compare the welfare performance of different (history-dependent) monetary policy rules. They also find that the relative performance of history-dependent rules vis-a-vis an IT rule diminishes with a greater degree of bounded rationality. In contrast to them, our model features wage rigidities, which reinforces the weakening of the expectations channel due to bounded rationality. Moreover, our analysis features an eAIT rule that performs well across different degrees of myopia.

income risk in fact reinforces forward-lookingness of the model; unrealistic countercyclical income risk mitigates this. Michaillat and Saez (2021) study how wealth preferences (in terms of bond holdings) affect the dynamic properties of the New Keynesian model. They find that wealth preferences can act similar to a discounting term in the Euler equation. Also, they show that this has important implications concerning the dynamics at the effective lower bound.

[8]Their model principally also allows for limited credibility. However, our understanding is that they consider limited credibility only for their analysis of forward guidance, not for their analysis of history-dependent policy rules.

Two other papers that we know of also study AIT with an exponential moving average of the inflation rate. Nakata et al. (2020) analyse the welfare properties of eAIT under rational and boundedly rational expectations, and determine the optimal smoothing parameter of the exponential moving average in both cases.[9] Their motivation for using an exponential moving average is a technical one. Using an arithmetic moving average involves incorporating a possibly large number of endogenous state variables (the lags of the inflation rate) into the model. This would imply prohibitively large computational costs given their global solution method that fully takes into account uncertainty. In contrast, we use the extended path algorithm to solve the model. This choice allows for considering arithmetic as well as exponential moving averages in the interest-rate rule while taking into account the effective lower bound. While Nakata et al. (2020) argue that eAIT can be considered a crude proxy for AIT, our results below for technology shocks rather suggest that the eAIT rule can lead to quite different results than the regular AIT rule. Honkapohja and McClung (2021) analyse the stability properties of various AIT rules, among them a rule involving the exponential moving average of the inflation rate, under adaptive learning. In contrast, our focus is on the welfare properties of different monetary policy strategies in a setting with myopic agents.

3 Model

In this section we augment a New Keynesian model with sticky prices and sticky wages as in Erceg et al. (2000) with bounded rationality in the spirit of Gabaix (2020) and an ELB on the policy rate. We present the model's log-linearised equilibrium conditions and delegate the derivation of the micro-founded non-linear equilibrium conditions to Appendix A.[10]

3.1 Private Sector

Time is discrete and the private sector of the economy features four agents: households, monopolistically competitive labour unions setting wages and two types of firms. Intermediate goods firms engage in monopolistic competition producing output which in turn is used by a competitive final goods firm to produce the final consumption good. Agents' expectations are crucial in determining equilibrium outcomes and in our model they generally need not be rational. Each sector might form expectations differently and we therefore denote expectations of each sector $s \in \{H, F, U\}$ in period t regarding some variable x_τ in period $\tau > t$ as $\mathcal{E}_t^s[x_\tau]$.

Households' optimisation problem gives rise to a dynamic IS curve relating period-t the (log-linearised) deviation of output y_t from steady state to the same output deviation in the

[9]Their analysis is in the spirit of the policy delegation literature, i.e., the monetary authority derives its decision rule from maximising under discretion an objective function that involves average inflation.

[10]While we could in principle conduct our simulations presented in later sections with the non-linear equilibrium conditions, we found that linearising them is conducive to the convergence of the extended path algorithm and thus speeds up the simulations. In any case, the key non-linearity in the model, namely the ELB, is fully taken into account in our simulation exercises.

next period, the contemporaneous real interest rate r_t and a demand shock z_t

$$y_t = \mathcal{E}_t^H[y_{t+1}] + \frac{\rho + z_t - r_t}{\sigma}, \tag{1}$$

where $1/\sigma$ denotes the elasticity of intertemporal substitution of consumption and ρ is both the rate of time preference and the steady-state real interest rate.

The ex-ante real rate is related to the nominal interest rate i_t and inflation expectations of households π_t^e via the Fisher equation

$$r_t = i_t - \pi_t^e. \tag{2}$$

We follow Gabaix (2020) and assume that inflation expectations in the Fisher equation are actually rational, i.e. $\pi_t^e = \mathbb{E}_t[\pi_{t+1}]$, where $\mathbb{E}_t[\cdot]$ is the expectations operator, conditional on information available in period t. [11]

Households supply labour to unions which set wages according to Calvo (1983). Only a fraction $1 - \theta_w$ of wages are optimised in each period. We assume that the remainder of unions have their wages indexed to the steady-state inflation rate π^*. Standard derivations then yield the New Keynesian wage Phillips curve

$$\pi_{w,t} - \pi^* = \kappa_w x_t - \lambda_w \hat{w}_t + \beta \mathcal{E}_t^U[\pi_{w,t+1} - \pi^*], \tag{3}$$

relating wage inflation $\pi_{w,t}$ to its expected future value, the current (log-linear) output gap $x_t = y_t - y_{n,t}$, and the real wage gap $\hat{w}_t = w_t - w_{n,t}$. $y_{n,t}$ and $w_{n,t}$ are the natural output and the natural wage, respectively, and will be defined below. The slope parameters κ_w and λ_w are composite parameters depending amongst other things on θ_w and are fully described in appendix A.

We assume sticky prices on the firm side similarly to sticky wages on the union side. Intermediate goods firms are monopolistically competitive and each period only a fraction θ_p can reset their prices. Prices that are not reset are indexed to inflation. This setup gives rise to the New Keynesian price Phillips curve

$$\pi_t - \pi^* = \kappa_p x_t + \lambda_p \hat{w}_t + \beta \mathcal{E}_t^F[\pi_{t+1} - \pi^*], \tag{4}$$

relating the current inflation rate π_t to future inflation, the output gap and the wage gap. κ_p and λ_p again are composite parameters, which depend among other things on θ_p. $\beta = 1/(1+\rho)$ is the steady-state real discount factor.

Stickiness in both nominal wages and prices implies that the real wage adjusts only sluggishly and the evolution of the real wage is governed by

$$w_t = w_{t-1} + \pi_{w,t} - \pi_t. \tag{5}$$

Introducing sticky wages in conjunction with sticky nominal prices implies that the real

[11] It would be straightforward to allow for non-rational expectations in the Fisher equation as well. However, the effect is minuscule compared to behavioural expectations in other equations.

wage can no longer adjust freely in response to shocks. Accordingly, monetary policy needs to decide on the relative roles of price and wage inflation in bringing about real wage adjustment. In such a setup, the divine coincidence disappears for technology shocks as these shocks directly affect the desired real wage. As a result, technology shocks have an effect on economic welfare under alternative policy frameworks. Hence, taking into account sticky wages and technology shocks gives a more complete picture of the challenges monetary policy faces in bringing about welfare-optimal outcomes: monetary policy does not only have to cope with demand shocks driving the economy to the ELB but also with the trade-offs induced by supply shocks.

3.1.1 Natural Variables

Natural output and the natural real wage are a function of possibly time-varying total factor productivity a_t and structural parameters. In the linearised models, we can write

$$y_{n,t} = \psi_{y,n} a_t, \quad \text{and} \tag{6}$$

$$w_{n,t} = \psi_{w,n} a_t, \tag{7}$$

where $\psi_{y,n}, \psi_{w,n}$ are composite parameters explained in appendix A.

Absent price and wage rigidities, a version of the IS curve (1) can be used to define the *natural rate of interest* r_t^* as the interest that satisfies

$$y_{n,t} = \mathcal{E}_t^H[y_{n,t+1}] + \frac{\rho + z_t - r_t^*}{\sigma}. \tag{8}$$

Note that one can also combine (1) and (8) to obtain an IS curve in terms of the output gap x_t as

$$x_t = \mathcal{E}_t^H[x_{t+1}] + \frac{r_t^* - r_t}{\sigma}. \tag{9}$$

3.1.2 Shock Processes

Our model features two exogenous variables, a demand shock z_t and a technology shock a_t. Each exogenous variable follows its own AR(1) processes with mean $\bar{x}$, persistence ρ_x, and innovation $\varepsilon_{x,t}$ for $x \in \{z, a\}$ such that

$$z_t = (1 - \rho_z)\bar{z} + \rho_z z_{t-1} + \varepsilon_{z,t}, \tag{10}$$

$$a_t = (1 - \rho_a)\bar{a} + \rho_a a_{t-1} + \varepsilon_{a,t}, \tag{11}$$

where we normalise $\bar{a} = \bar{z} = 0$. Each innovation $\varepsilon_{x,t}$ is drawn from an i.i.d. normal distribution with mean 0 and variance $\sigma_x^2 \geq 0$:

$$\varepsilon_{x,t} \sim \mathcal{N}(0, \sigma_x^2), \quad x \in \{z, a\}, \tag{12}$$

where the realisation of the innovation in period t is not known or anticipated until the start of the period, i.e.

$$\mathbb{E}_t[\varepsilon_{x,t}] = \mathcal{E}_t^j[\varepsilon_{x,t}] = 0, \quad x \in \{z, a\}, j = \in \{H, F, U\}$$

3.2 Expectations Formation under Bounded Rationality

By now it is well known that New Keynesian models with rational expectations, i.e. $\mathcal{E}_t^j[\cdot] = \mathbb{E}_t[\cdot]$ for $j \in \{H, F, U\}$, give rise to various paradoxes and puzzles. Among these, the forward-guidance puzzle (Del Negro et al., 2023) stands out as the most prominent one. Events like interest rate changes that take place far in the future, but are already known today, have quantitatively large effects on contemporaneous inflation and output. This counter-intuitive result and other puzzles are a direct effect of the very strong expectations channel in the workhorse model. Households under rational expectations are extremely forward-looking and even events occurring in the very distant future substantially matter for their contemporary decision making. This particularly matters for monetary policy strategies that rely on policy reactions that will take place in the future but are already announced and therefore known today.

For this reason we employ boundedly rational expectations in the spirit of Gabaix (2020).[12] In this approach, agents do not fully understand the world, in particular as regards events that are happening far in the future. Instead, future events get discounted relative to the rational benchmark which leads to myopia regarding future variations around the steady state. In particular, agents form expectations according to

$$\mathcal{E}_t^j[x_{t+1}] = M_j \mathbb{E}_t[x_{t+1}] + (1 - M_j)\bar{x} \tag{13}$$

for $j \in \{H, F, U\}$, i.e. they perceive future (state) variables x_{t+1} to be biased towards their corresponding steady state $\bar{x}$.[13]

This modifies the IS curve and the price and wage Phillips curves according to

$$x_t = M_H \mathbb{E}_t[x_{t+1}] + \frac{r_t^* - r_t}{\sigma}, \tag{14}$$

$$\pi_t - \pi^* = \kappa_p x_t + \lambda_p \hat{w}_t + \beta M_F \mathbb{E}_t[\pi_{t+1} - \pi^*], \tag{15}$$

$$\pi_{w,t} - \pi^* = \kappa_w x_t - \lambda_w \hat{w}_t + \beta M_U \mathbb{E}_t[\pi_{w,t+1} - \pi^*]. \tag{16}$$

Boundedly rational expectations imply discounting in the dynamic IS curve as well as in the Phillips curves, which mitigates the expectations channel. In particular, expectations about future variables are less sensitive than in the benchmark rational expectations case whenever $M_j < 1$.

[12] Other recent papers in a similar vein are Benchimol and Bounader (2021), Nakata et al. (2019) and Erceg et al. (2021). As recently discussed by an Angeletos et al. (2020) or Angeletos and Lian (2022), this in general leads to an under-reaction of expectations and real variables.

[13] Note that for the deviations of the various variables from steady-state, we directly have that they are biased toward 0.

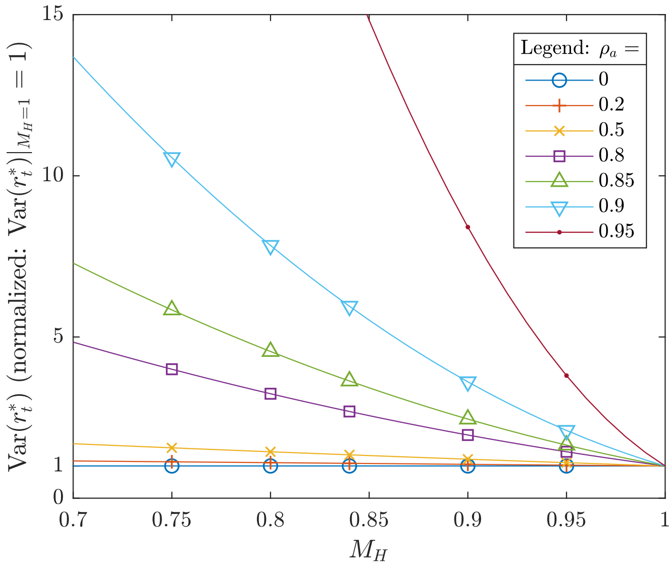

Figure 1: Effects of myopia on the variance of the natural interest rate for technology shocks

Concerning the natural (flexible-price) version of the economy, first note that natural output and wage are not forward-looking variables and as such, they are not affected by this myopia. Likewise, absent price and wage stickiness, the expectational parameters on the expectations of price- and wage-setters are irrelevant, i.e., M_F, M_U do not affect the natural variables at all. However, the natural rate of interest r_t^* is affected by the discounting in the IS curve. Combining (6) and (8) yields the evolution of the natural rate

$$r_t^* = \rho + z_t + \sigma(M_H \rho_a - 1)\psi_{y,n} a_t. \tag{17}$$

The variance of the natural rate is then given by

$$\text{Var}(r_t^*) = \text{Var}(z_t) + \sigma^2 \psi_{y,n}^2 (M_H \rho_a - 1)^2 \text{Var}(a_t). \tag{18}$$

In particular, for given variances of the shock processes for a_t and z_t, as long as ρ_z and ρ_a are strictly positive (and less than one), the variance of the natural interest rate is strictly decreasing in M_H for $M_H \in [0, 1]$. In other words, with persistent shocks, ceteris paribus, more discounting in the IS equation increases the variance of the natural rate. This increase is stronger the more persistent the shocks are and the lower M_H becomes (the relationship is quadratic in M_H). Figure 1 illustrates this by plotting the increase in volatility of the natural interest rate relative to the rational-expectations benchmark across values for M_H and the shock persistence.

Equations (14)-(16) together with the shock processes fully describe the private sector of the economy. To close the model, we specify how monetary policy is conducted.

3.3 Prototypical interest-rate rules

We compare history-dependent and non-history-dependent monetary policy strategies and operationalise the strategies via simple interest-rate rules.[14] Under non-history-dependent interest-rate rules, the monetary-policy maker only seeks to stabilise current inflation and possibly real economic activity, but fully disregards past actions and economic developments when setting the current policy rate. The most prominent example of such a strategy is IT.

In contrast, under history-dependent rules, inflation developments in the past have to be compensated in order to reach the target variable and thus play a prominent role in current policy setting. The most prominent examples of such a strategy are PLT, where the target variable is the price level, and AIT, where the target variable is some average of current and past inflation rates.

We analyse IT in the form of a standard Taylor-type rule. That is, we assume that the monetary policy maker does not know the natural variables and thus bases its interest rate decisions on a simple rule that depends on inflation, the deviation of inflation from target, a constant intercept term, and the deviation of output from its steady state. Thus, the rule reads

$$i_t^* = \rho + \pi_t + \phi_\pi(\pi_t - \pi^*) + \phi_y y_t, \tag{19}$$

where i_t^* is the desired nominal rate (which will not be constrained by the ELB), and $\phi_\pi > 0$ and $\phi_y \geq 0$ are coefficients controlling the monetary policy response to deviations of inflation from target and the deviation of output from steady state, respectively. We contrast the IT rule to several history-dependent strategies. First, we analyse a (flexible) PLT rule of the form

$$i_t^* = \rho + \pi_t + \phi_\pi \hat{p}_t + \phi_y y_t, \tag{20}$$

where the log deviation $\hat{p}_t$ of the (log-linearised) price level $p_t = p_{t-1} + \pi_t$ from its target path $p_t^* = p_0^* + t\pi^*$ evolves according to

$$\hat{p}_t := p_t - p_t^* = (p_{t-1} + \pi_t) - (p_{t-1}^* + \pi^*) = \hat{p}_{t-1} + \pi_t - \pi^*. \tag{21}$$

Second, we analyse an AIT rule with the general form given by

$$i_t^* = \rho + \pi_t + \phi_\pi \tilde{\pi}_t + \phi_y y_t, \tag{22}$$

where $\tilde{\pi}_t$ is the deviation of average inflation from target. Average inflation is measured by

[14] Work related to recent monetary policy strategy reviews by major central banks also chose to operationalise the different kinds of monetary policy strategies with simple interest-rate rules, see, e.g., Cecioni et al. (2021). Interest-rate rules thus play an important role in monetary policy practice.

a simple moving average (SMA) of the form

$$\bar{\pi}_t|_{SMA,T} = \frac{1}{T}\sum_{\ell=0}^{T-1}(\pi_{t-\ell} - \pi^*), \tag{23}$$

which is widely studied in the literature and where T denotes the length of the averaging window. To keep the rules comparable across time windows, we re-normalise the averages such that the response coefficient of inflation on nominal interest rate setting is fixed at ϕ_π. This means that for the simple-moving average, we have

$$\tilde{\pi}_t := T\bar{\pi}_t|_{SMA,T} = \sum_{\ell=0}^{T-1}(\pi_{t-\ell} - \pi^*) \tag{24}$$

Note that with this specification, in the limit we get inflation targeting with $T \to 1$ and price-level targeting with $T \to \infty$.

For all rules that we study we assume that there is an ELB $i_{ELB} = 0$ on the nominal interest rate, which constrains monetary policy to set the nominal interest rate according to[15]

$$i_t = \max\{i_{ELB}, i_t^*\}. \tag{25}$$

3.4 Welfare

The welfare criterion we use to compare the different interest-rate rules is the expected utility of the representative agent under the rational expectations operator, $W = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t, N_t)$. This follows the behavioural economics literature which views agents as using heuristics in their behaviour but experiencing utility from the actual consumption and leisure stream (Gabaix, 2020).

The welfare loss function can then be written as a linear combination of the expected quadratic deviations of the output gap, price inflation and wage inflation from their respective targets and is given by

$$\mathbb{L} = \frac{1}{2}\left\{ \left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right)\mathbb{E}\left[x_t^2\right] + \frac{\epsilon_p}{\lambda_p}\mathbb{E}\left[(\pi_t - \pi^*)^2\right] + \frac{\epsilon_w(1-\alpha)}{\lambda_w}\mathbb{E}\left[(\pi_{w,t} - \pi^*)^2\right] \right\} \tag{26}$$

where the weights are functions of the underlying structural parameters. Note that these weights are not impacted by myopia.[16]

[15] Note that we do not consider interest rate smoothing for our welfare simulations further below in order to focus on the inherent history-dependent characteristic of the PLT and AIT rules.

[16] To formally derive this loss function as a second order approximation to households' discounted utility, we also implicitly assume that the steady state has zero inflation and is efficient, i.e. that there are subsidies in place that undo the distortions arising from market power in the labour and goods market.

Table 1: Calibration of parameters

Parameter	Value	Description
β	0.9975	Preference Parameter
σ	1.00	Risk Aversion
φ	5.00	Inverse Frisch elasticity
α	0.25	Production Returns to Scale
θ_p	0.75	Prob. of price resetting
θ_w	0.75	Prob. of wage resetting
ϵ_p	9.00	Elasticity of substitution btw. goods
ϵ_w	4.50	Elasticity of substitution btw. workers
ρ_z	0.85	Persistence demand shock
ρ_a	0.85	Persistence technology shock
σ_a	0.0199	Standard deviation technology shock
σ_z	0.0357	Standard deviation demand shock
π^*	0.005	Annual inflation: 2%

3.5 Calibration

Our baseline calibration mostly follows Galí (2015).[17] Each period in the model corresponds to a quarter. The preference parameter β is calibrated so that the annualised steady-state real rate is 1%. We assume log utility ($\sigma = 1$) and target a Frisch elasticity of labour supply of 0.2 yielding $\varphi = 5$. We set $\alpha = 0.25$ implying decreasing returns to scale. Setting $\epsilon_p = 9$ implies a steady state price markup of 12.5%. The elasticity of substitution between differentiated types of labour ϵ_w is set to a lower value of 4.5 implying an average wage markup of around 30%.[18] The reset probability of prices and wages is set to 0.75. This yields an average price and wage duration of one year consistent with much of the empirical evidence. The inflation target π^* is set so that annual inflation equals 1.9%, consistent with the former target of the ECB.[19]

For the persistence of technology and discount factor shocks we use standard values of 0.85. The standard deviations are calibrated to yield an ELB incidence of 20% under rational expectations in the case of inflation targeting.[20] The ELB is set to 0%. Throughout most of the analysis below, we implement for simplicity $M_H = M_F = M_U$. In these cases, we only write M and vary its value in a range of 0.5 to 1. This allows us to analyse the impact of varying degrees of myopia on New Keynesian puzzles as well as its effects on monetary policy strategies.

[17] We also investigated a calibration as in Gabaix (2020) where the Phillips curves and the IS curve are flatter. The main insights do not change much under this alternative calibration.

[18] In a model featuring unemployment this would imply an average unemployment rate of around 5% which is a standard value in the literature.

[19] Note that since we adjust the shock size below so as to achieve a certain ELB frequency under rational expectations, the choice of π^* does not affect our results in any way: As long as we assume that non-adjusting firms update their prices by steady-state inflation, the choice of π^* in steady state only determines the distance between the steady-state nominal interest rate and its effective lower bound.

[20] In particular, we calibrate shocks such that the ELB frequency under the IT rule is 20% for $\phi_\pi = 0.5$ and $\phi_y = \frac{0.5}{4}$.

3.6 Numerical method

We need to solve our model under various monetary policy rules given an occasionally binding constraint due to the zero lower bound and the real wage as a state variable. To do so we rely on the deterministic extended path algorithm first proposed by Fair and Taylor (1983). The extended path algorithm combines the accuracy of deterministic perfect foresight solutions with the ability to provide an accurate account of non-linearities. This crucially relies on the assumption that agents react to random current period shocks but assume that no shocks will occur in future periods, i.e. that the economy asymptotically returns to equilibrium after the current period shocks. Therefore, while in our numerical simulations the economy is hit by shocks every period, agents only act to contemporary ones, i.e. we assume certainty equivalence.

We optimise the response coefficients of the different monetary policy rules for each degree of myopia and each type of shock separately. This ensures a fair comparison in which all the rules perform as best as they can – that is, without implicitly biasing the comparison by assigning sub-optimal parameters to a rule – conditional on the type of shocks and the degree of myopia. We numerically optimise parameters by conducting a grid search for each respective rule and calculate welfare from equation (26) using the variances arising from stochastic simulations at each grid point.[21]

4 The welfare performance of different monetary policy rules under bounded rationality

In this section, we compare the performance of interest-rate rules described in Section 3.3 across different degrees of myopia using stochastic simulations.[22] For each degree of myopia, we re-optimise the interest-rate rule coefficients so that each rule can perform in the best way possible for a given environment. To gain insights into the driving mechanisms for the results, we consider demand and supply shocks separately.[23] Throughout the analysis, we assume that all agents in the economy are myopic to the same degree.

4.1 Demand shocks

Figure 2 shows the welfare comparison of different policy rules for discount factor shocks taking into account the ELB. Under mild forms of myopia, the typical results from the New Keynesian model carry over. History-dependent rules outperform the IT rule due to their

[21] The grid in general covers parameter values in the range from 0 to 10 for both parameters (regular grid), where $\phi_\pi \geq 0.5$. For the demand shock, we also admit values up to 1000 (20,50,100,200,500,1000) for both parameters. For the technology shock, we also add gridpoints between 0 and 0.5 and between 10 and 20 for ϕ_π. Note that for the technology shock, the constraint $\phi_y \geq 0$ will be binding, see the discussion below. Hence, in the appendix, we also add grid points with a negative ϕ_y.

[22] We introduce exponential AIT in Section 4.4.

[23] Appendix C presents an estimation of the model and a subsequent welfare analysis with joint shocks.

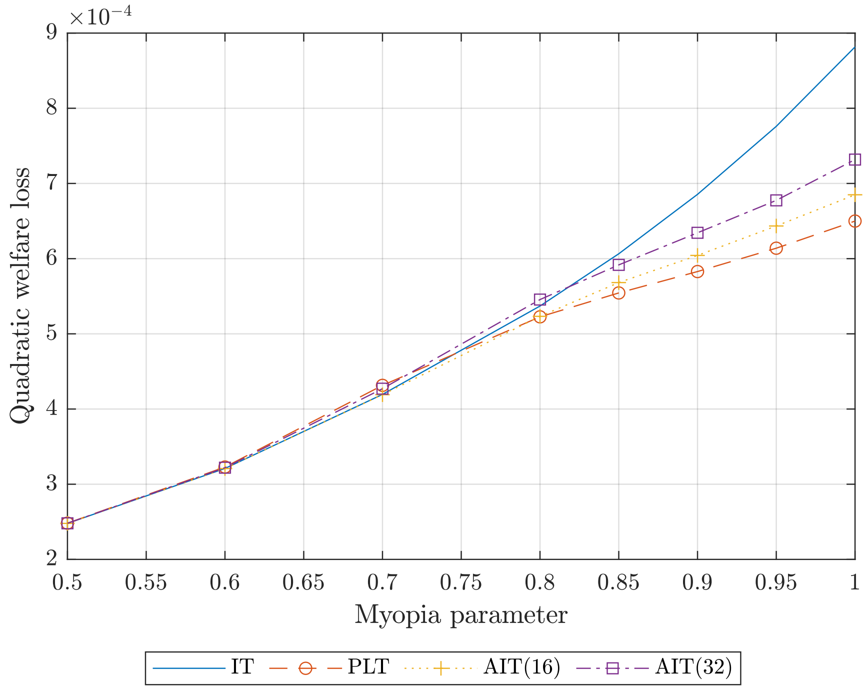

Figure 2: Welfare comparison for demand shocks

inherent lower-for-longer component. The PLT rule, which fully reaps the benefits from the expectations channel, is the best performing rule as long as myopia is not too severe.

For larger degrees of myopia, the power of the expectations channel decreases. Consequently, history-dependent rules gradually lose their advantage over the non-history-dependent IT rule for increasing degrees of myopia. For sufficiently low values of M, the different optimised rules become almost identical in terms of their performance.

The optimised parameters shown in Figure 3 exhibit a similar patterns across strategies for demand shocks. For low degrees of myopia, close to rational expectations, it is optimal under all rules to react as strongly as possible to inflation or price gaps and with smaller reaction coefficients to output deviations. For history dependent strategies, this reverses under high degrees of myopia. In order to provide additional stimulus at the ELB and to counter the ensuing recessions, it is optimal to strongly react to the output gap and almost entirely neglect inflation gaps – this also avoids generating additional volatility. Meanwhile, for the IT rule, myopia mostly implies that monetary policy should *also* put a greater weight on the output gap, whereas it is still optimal to react as strongly as possible to inflation deviations (coefficient value of $\phi_\pi = 1000$, upper grid boundary).

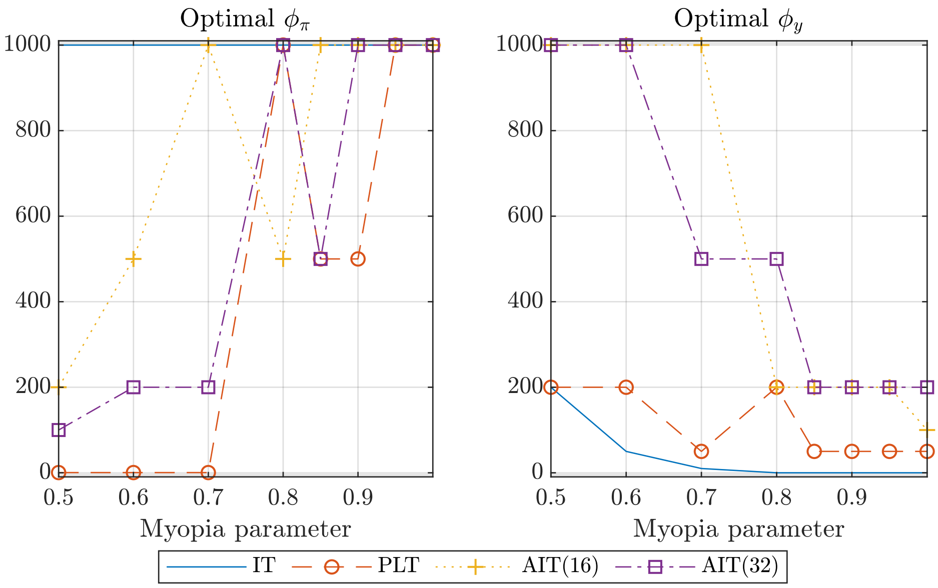

Figure 3: Optimal policy rule parameters for demand shocks

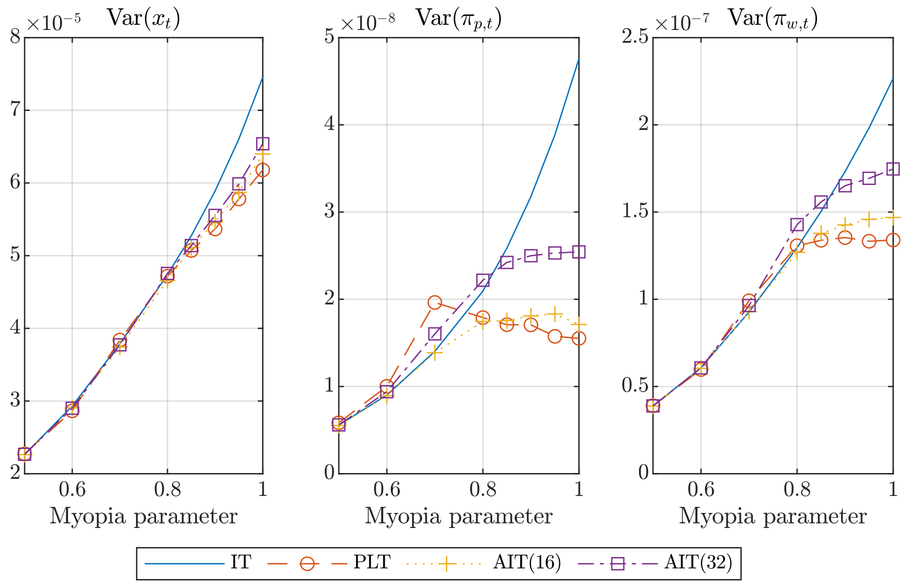

Figure 4: Variances for demand shocks

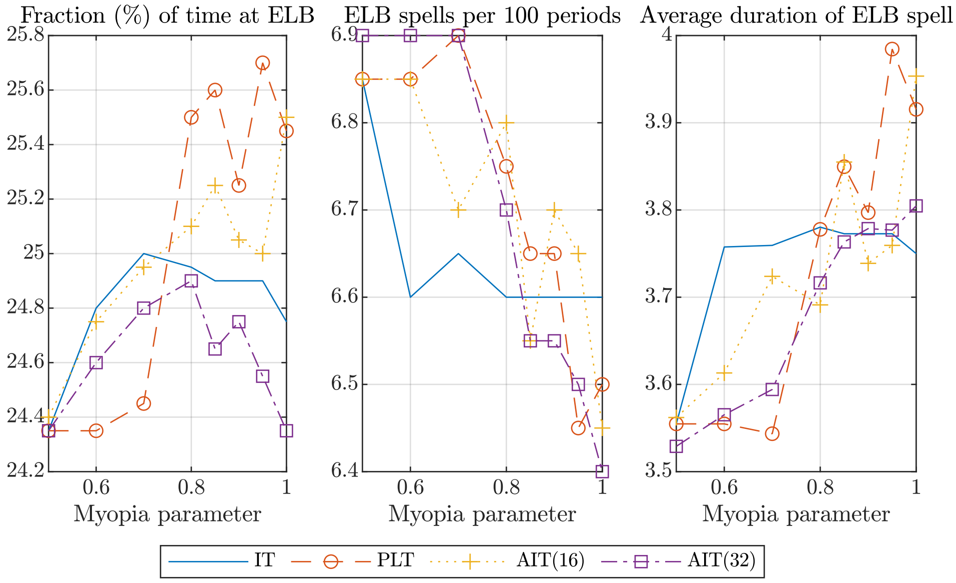

Figure 5: ELB statistics for demand shocks

Likewise, the patterns for the variances underlying the welfare losses shown in Figure 4 are broadly similar across the different interest-rate rules. Notably, inflation targeting 'catches up' in terms of each variance. Also, as shown in Figure 5, the ELB statistics in the simulations are broadly similar (watch for the scaling on the y-axis). In this regard, average inflation targeting with a short horizon appears to be a relatively attractive policy here. The most remarkable element of the figure is that with an increasing degree of myopia the various policies hit the ELB more often (in terms of number of ELB spells per 100 periods), but the average spell becomes shorter.

4.2 Technology shocks

Figure (6) shows the welfare comparison of different policy strategies under technology shocks. Under rational expectations and under mild myopia, the expectations channel remains sufficiently strong so that the PLT rule yields the lowest welfare losses, while the IT rule yields the highest welfare losses.

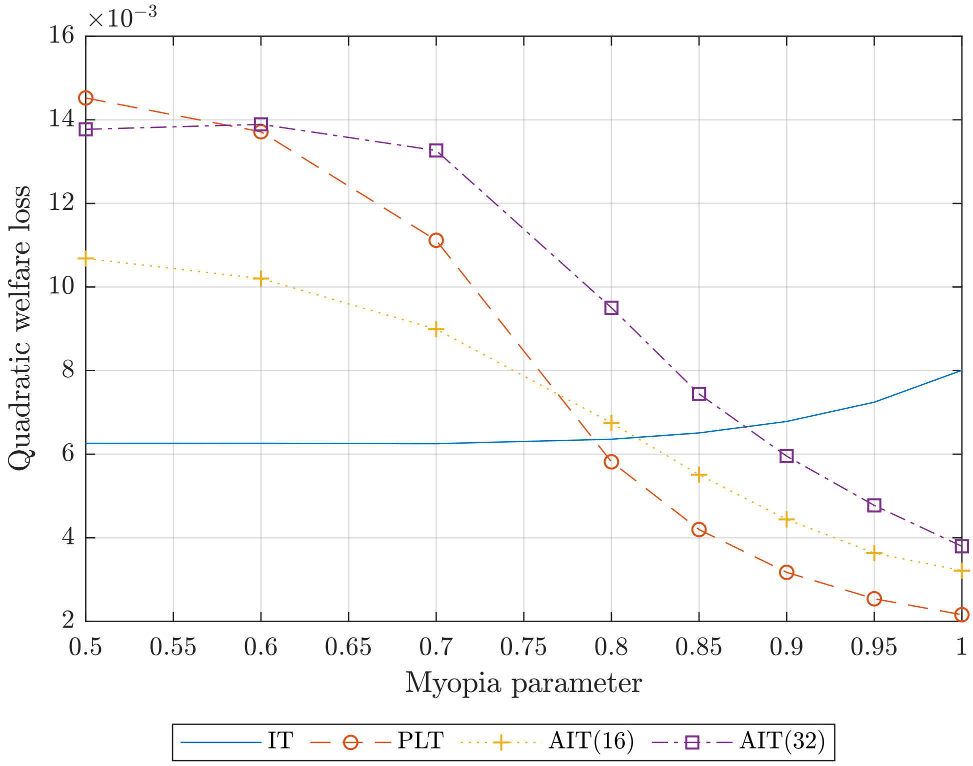

Figure 6: Welfare comparison for technology shocks

Under higher degrees of bounded rationality ($M < 0.8$), the advantage of the PLT rule and other history-dependent rules turns into a disadvantage. Rankings completely reverse and the IT rule turns out to yield the lowest welfare losses. This is mostly due to the IT rule yielding remarkably stable welfare losses across different degrees of myopia whereas losses under the history-dependent rules gradually rise for higher degrees of myopia.

Figure 7 shows the optimal policy coefficients for each degree of myopia. The grey shaded areas show the bounds of the grid used for the grid search. Under technology shocks, it is never optimal to react to deviations of output from steady state irrespective of the specific interest-rate rule.[24] In contrast, the optimal response coefficients for the nominal variable differ markedly between the history-dependent rules and the non-history-dependent IT rule. Under the IT rule, it is optimal to react much stronger to inflation as to

[24] This is due to the fact that we restrict the response parameter to be positive. In fact, since the Taylor rule is not specified in terms of the output gap, this is of little surprise since with technology shocks, output under-reacts, giving an output gap of the opposite sign relative to the deviation of output from steady state. If we allow the central bank to observe natural variables and set $i_t = r_t^* + \pi_t + \phi_\pi(\pi_t - \pi^*) + \phi_y x_t$ (or a similar variant for PLT and AIT), the performance of the various monetary policy rules becomes nearly identical. In any case, in these scenarios, a TFP shock actually calls for (in absolute terms) large coefficients on the output gap and small ones on the measure of inflation. Similarly, if we stick to the assumption that the central bank does not include natural variables in its Taylor rule, it would be optimal to set a negative coefficient on the output deviation from steady state. This optimal coefficient becomes more negative for smaller values of the myopia parameters (remember, the natural rate reacts more strongly with higher degrees of myopia relative to natural output). I.e., with technology shocks in the EHL framework, output-gap targeting becomes a key ingredient of optimal monetary policy, relating our results to those of Garín et al. (2016). Figure 26 in Appendix B shows the welfare comparison if we allow for negative reaction parameters and also refine the grid for ϕ_π. Figure 27 Appendix B depicts the resulting optimal parameters. It should be noted that our main insights are not affected, but it becomes clear that the importance of the output gap increases in the degree of myopia.

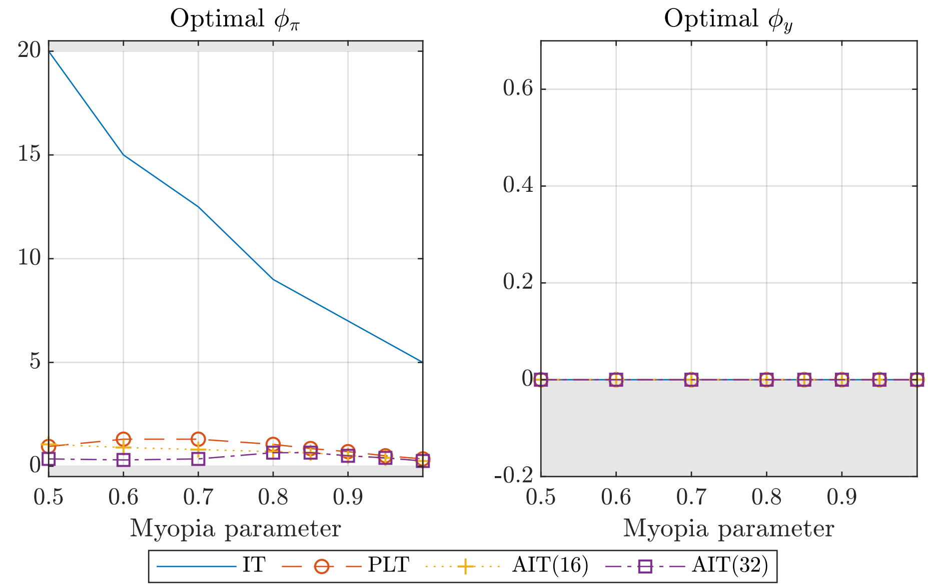

Figure 7: Optimal policy rule parameters for technology shocks

average inflation under the AIT rule and to the price level under the PLT rule. The reason can clearly be seen in Figure 8, which shows the variances underlying the welfare rankings. The flat welfare line across degrees of myopia under IT is the result of a monotonic decline of volatility of inflation and wage inflation (whereas the volatility of output monotonically increases). This, in turn, is a result of the declining severity of ELB recessions. For the history-dependent rules, the picture is more nuanced. While dealing very well with the ELB under rational expectations, mild degrees of myopia lead to increasing volatility of all variables in the loss function. On the one hand, the effectiveness of the history-dependent component which originally leads to low economic volatility is lowered. On the other hand, myopia also increases the volatility of the natural rate. Pure history-dependent strategies ($\phi_y = 0$) immediately push the economy to the ELB for a larger fraction of time, leading to fewer, but longer spells at the ELB. In addition, this makes the cost-push component of technology shocks relatively more important, raising the costs associated with history-dependent strategies.[25]

[25]This result is muted if monetary policy reacts to natural variables or if we allow for a negative ϕ_y in the present model. In any case, with technology shocks, as myopia increases, the optimal response shifts from stabilising inflation to stabilising the output gap. See Figure 27, which depicts the optimal coefficients for the case where we also allow for $\phi_y < 0$.

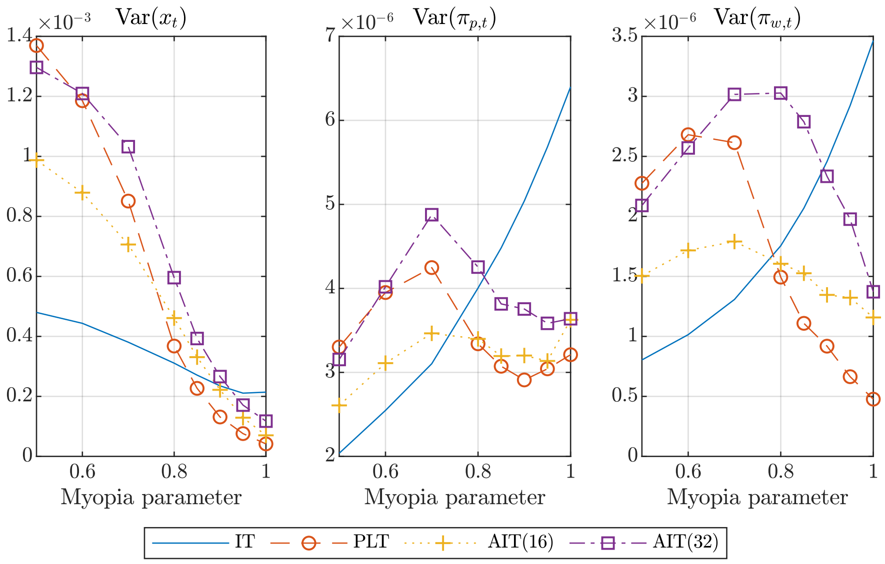

Figure 8: Variances for technology shocks

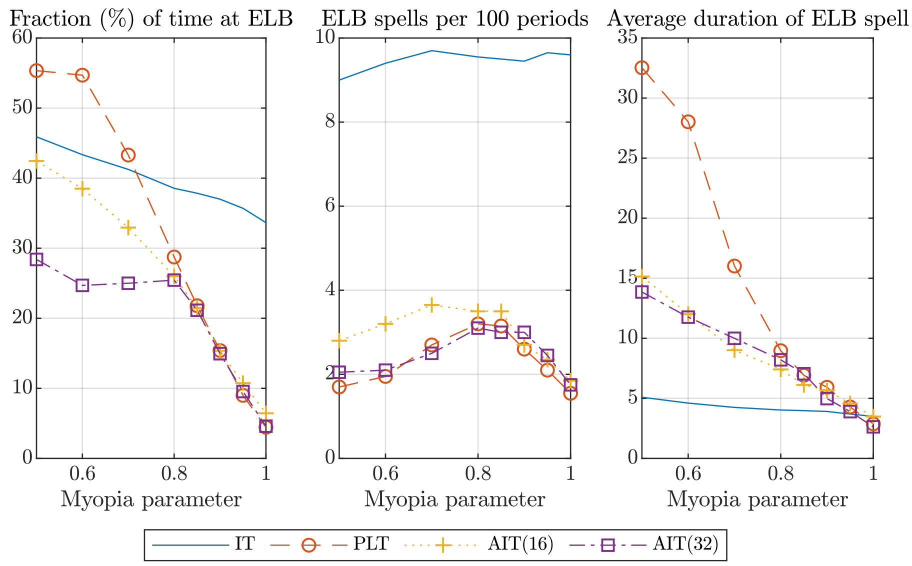

Figure 9: ELB statistics for technology shocks

Figure 9 shows the corresponding ELB statistics. Larger degrees of myopia lead to fewer spells at the ELB across all policy rules. This is because myopia increases the volatility of the natural rate subsequently leading to fewer ELB incidences. At the same time the average duration and time spend at the ELB increases for all history-dependent strategies. While ELB periods are less severe under bounded rationality, the effectiveness of the expectations channel is also reduced. History-dependent strategies therefore need to keep nominal rates lower for even longer in order to achieve their target and compensate the reduced effectiveness.[26]

[26] This is in line with the results obtained by Nakata et al. (2019).

4.3 Inspecting the mechanism

As already known from the discussion in Gabaix (2020), with myopic agents, a stationary price level is no longer the optimal policy in a simple New Keynesian model for shocks that induce a trade-off. Thus, it may come as no surprise that under the supply shock the performance of the PLT and AIT rules deteriorates more and more relative to the IT rule as the degree of myopia increases. Under demand shocks, which do not induce a trade-off, the PLT and AIT rules lose their advantage compared to the IT rule for increasing degrees of myopia. The underlying reason in both cases is that the power of the history-dependent rules is intimately related to the effectiveness of the expectations channel,[27] which operates at maximum strength with rational expectations and becomes weaker with increasing degrees of myopia.

Hence, under rational expectations and mild degrees of myopia, the history-dependent rules can deal better than IT with the constraints posed by the ELB and trade-offs between inflation and real activity. In ELB episodes, agents expect monetary policy to make up past negative deviations from the inflation target in the future, lowering real rates and stabilising inflation today. In case of a trade-off between stabilising high inflation and low real activity, the history-dependent rules can harness the expectations channel in that expected low inflation in the future lowers inflation today without having to decrease real activity further.

For higher degrees of myopia and hence a weaker expectations channel, these advantages of history-dependent rules fade or even turn into a disadvantage. For demand shocks, high degrees of myopia imply that inflation expectations cannot act anymore in a stabilising manner as under low degrees of myopia or rational expectations. Since there is no trade-off involved, all rules virtually yield the same welfare loss. For trade-off inducing supply shocks, the weaker expectations channel cannot act anymore as a means to lower inflation without necessarily having to lower real activity. Instead, in order to bring down inflation, monetary policy has to decrease economic activity, inducing much volatility into the economy. This effect becomes stronger the higher the degree of myopia. This is why history-dependent rules perform worse than the IT rule for high degrees of myopia. Since it exhibits the strongest degree of history dependence the volatility-increasing effect is strongest for the PLT rule.

Hence, AIT – which includes only a limited number of past inflation rates in a simple moving average and hence exhibits a weaker degree of history dependence – in principle should act as a good compromise across different degrees of myopia. First, past deviations from the average inflation target are corrected for in the following periods to some extent, so AIT can harness the expectations channel for low degrees of myopia, although not as strongly as PLT. Therefore, increasing myopia should affect AIT less than PLT. Second, the memory of AIT is limited: After a certain number of periods, past deviations drop out of the averaging window, so the potentially volatility-increasing effects of past deviations for high degrees of myopia are mitigated. Taken together, the AIT rule, while harnessing the

[27] To enhance the argument that the expectations channel and its attenuation through myopia is driving our results, Appendix E shows that the solution or mitigation of several New Keynesian paradoxes that are known to rely on the expectations channel is reinforced by the interaction between sticky wages and myopia.

expectations channel to some extent for low degrees of myopia, should become more powerful compared to the PLT rule as myopia increases. Indeed, our results in section 4 show that this is actually the case.

What contributes to the impression that the AIT rule might be an obvious compromise is that its performance lies in between the performance of the PLT and the IT rule for both low and high degrees of myopia and for both demand and supply shocks. In the case of demand shocks, the AIT rule even comes very close to the best performing rule for low and high degrees of myopia (the PLT rule and the IT rule, respectively; see Figure 2). This may come at little surprise since monetary policy does not have to confront a trade-off in the case of demand shocks and we compare optimised rules. However, in the case of trade-off-inducing technology shocks, while the AIT rule again represents middle ground, its welfare losses are markedly higher than those of the best performing rules, especially for high degrees of myopia (see Figure 6). This casts doubt on whether the AIT rule would actually be a desirable compromise candidate.

Moreover, a fact that usually receives little attention is that simple moving averages may actually introduce volatility on their own when used as a guide for monetary policy. The reason is that any past deviation of inflation from its target directly affects the average for a given time frame, but once it drops out of the averaging window this effect reverses. Thus, if monetary policy uses the simple moving average as guidance, that benchmark may well switch signs suddenly.

To illustrate this, consider a monetary policy maker that pursues an average inflation target with an averaging window of T periods. Assume that in period t_0, starting from a steady state with $\pi_t = \pi^*$ for $t \leq t_0$, inflation π_{t_0} exogenously deviates from its target π^* by an amount $\Delta\pi$. From period $t_0 + 1$ onwards monetary policy has perfect control over the current inflation rate and sets inflation such that the average $\bar{\pi}_{t,T} := \frac{1}{T}\sum_{s=0}^{T-1} \pi_{t-s}$ satisfies $\bar{\pi}_{t,T} = \pi^*$ again. For the sake of illustration, assume that monetary policy sets $\pi_{t_0+1} = \pi^* - \Delta\pi$ in period $t_0 + 1$, fulfilling its target in that period, and over the course of the next periods it keeps $\pi_t = \pi^*$. However, in period $t_0 + T$, period t_0 drops out of the averaging window. Thus, monetary policy would set $\pi_{t_0+T} = \pi^* + \Delta\pi$. In the next period, period $t_0 + 1$ drops out of the averaging window, requiring again a negative deviation from target. This would repeat every T periods, as depicted in Figure 10 a). For comparison, we show what would happen under PLT (panel b)) and under IT (panel c)). Under PLT, the increase in period t_0 would be countered with a negative deviation in the following period, but since no period would ever drop out of the averaging window, no further deviation from the target would be required in the absence of an exogenous shock. Under IT, the opposite holds: since there is no averaging window, deviations immediately 'drop out' of memory, i.e., from $t_0 + 1$ onwards, π_t could be set to the target value.

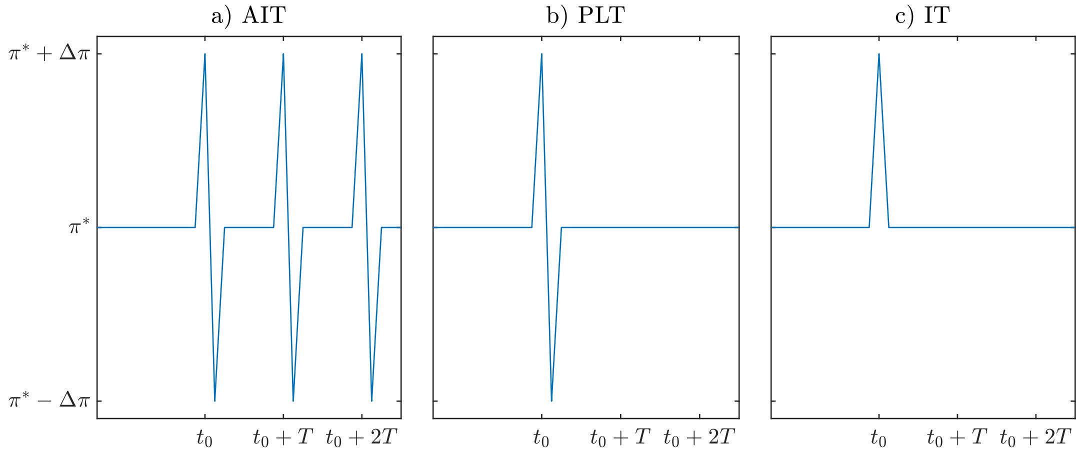

Figure 10: Illustration: Cycling behaviour of average-inflation targeting

Hence, in this stylised framework, targeting a simple moving average would actually introduce an undamped cycle. Any shock that leads to a deviation of inflation from target would require periodically repeating deviations of inflation from its target. This, by itself, would increase welfare-detrimental variations in inflation. In a fully specified model, and especially if monetary policy only has indirect control of inflation via nominal interest rate setting, these cycles would be mitigated and smoother than depicted in the Figure. However, the underlying mechanism still remains. Note that the described inherent volatility-inducing characteristic of AIT is particularly relevant for shocks that induce a trade-off for monetary policy since in the case of demand shocks, monetary policy can (under ideal circumstances) immediately undo shocks that make inflation deviate from its target and, hence, the periodically repeating pattern of deviations does not occur.

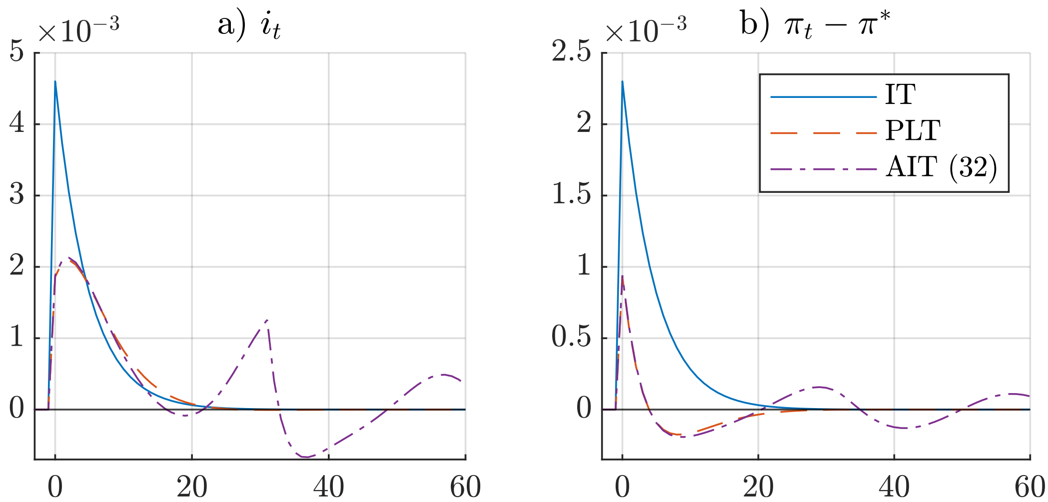

Figure 11: Illustration: Cycling behaviour of average-inflation targeting in the model
Note: The figure shows the impulse response functions of the nominal interest rate i_t and the inflation rate π_t to a inflationary supply shock for $\rho_a = 0.85$, $\phi_\pi = 1$, $\phi_y = 0$, with rational expectations.

Figure 11 shows that this cycling behaviour is also present in our model. It depicts the impulse responses of nominal interest rates and inflation to an inflationary shock for a baseline parameterisation and with rational expectations. To illustrate the effect clearly, we set

$\phi_y = 0$ and $\phi_\pi = 1$. Similar to what we have seen before, (pure) AIT features oscillating spikes in the nominal interest rate as past periods with high (or low) inflation drop out of the averaging window.[28] This induces volatility in the inflation rate.

To sum up the aforementioned observations, the AIT rule exhibits some desirable features when it comes to the performance across different degrees of myopia. These include history dependence which is beneficial for low degrees of myopia, and a limited memory compared to the PLT rule which is beneficial when high degrees of myopia weaken the expectations channel substantially. In this sense, the AIT rule represents a compromise between the PLT and the IT rule. However, for trade-off-inducing technology shocks the welfare losses under the AIT rule are markedly higher than those of the best performing rules for the different degrees of myopia, which the inherent volatility-inducing characteristic of AIT is crucially contributing to. In this sense, the AIT rule is not an attractive compromise. As a consequence, a strategy that preserves the aforementioned benefits and avoids the disadvantage of inherently generating volatility has the potential to perform better than conventional AIT across different degrees of myopia and across different types of shocks. In the following section, we present exponential Average-Inflation Targeting (eAIT) as a promising candidate for such a strategy.

4.4 Exponential AIT

4.4.1 Characteristics of the exponential moving average

In contrast to the AIT rule, where a simple (or arithmetic) moving average of the inflation rate enters the interest-rate rule, the argument that enters the eAIT rule is an exponential moving average of the inflation rate. The exponential moving average is an infinite impulse response filter, i.e., the average is applied to all past observations according to

$$\bar{\pi}_{t|EMA,T_{eAIT}} = \sum_{s=0}^{\infty} \eta_s \pi_{t-s} \quad \text{with } \eta_0 \geq 0 \text{ and } \eta_s = \rho_{eAIT}\eta_{s-1} = \rho_{eAIT}^s \eta_0 \text{ for } s \geq 1. \tag{27}$$

Here, $\rho_{eAIT} \in [0, 1)$ is a smoothing parameter. If the weights are normalised such that

$$\sum_{s=0}^{\infty} \eta_s = 1,$$

ρ_{eAIT} and η_0 are linked according to $\eta_0 = 1 - \rho_{eAIT}$. Moreover, equation (27) can be restated recursively as

$$\bar{\pi}_{t|EMA,T_{eAIT}} = \eta_0 \pi_t + \rho_{eAIT} \bar{\pi}_{t-1|EMA,T_{eAIT}}, \tag{28}$$

[28] Allowing for $\phi_y \neq 0$ ameliorates the cycling behaviour of AIT strategies to some extent, because it increases the weight on current deviations. However, the general tendency of cyclicality remains.

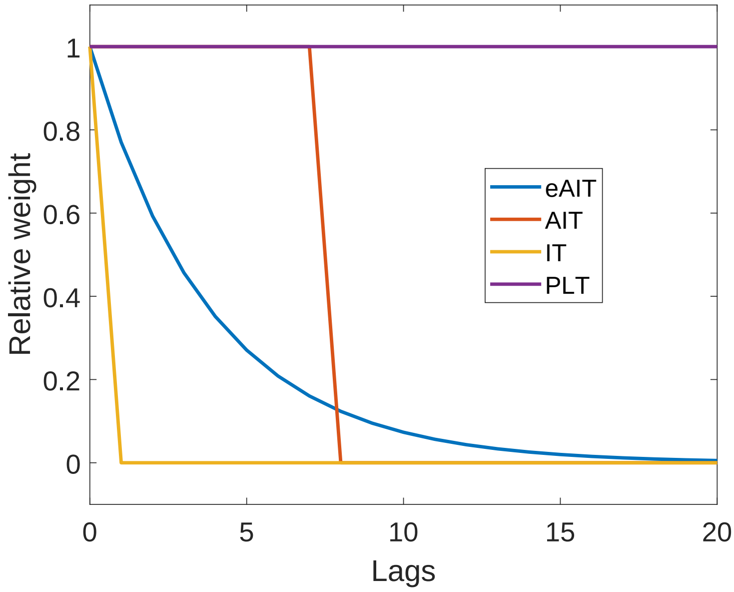

Figure 12: Weights on different lags of inflation in different monetary policy rules ($T = 8$ in the AIT rule, $T_{eAIT} = 8$ in the eAIT rule). Weights normalised to 1 for lag 0.

which with $\eta_0 = 1 - \rho_{eAIT}$ gives

$$\bar{\pi}_{t|EMA,T_{eAIT}} = (1 - \rho_{eAIT})\pi_t + \rho_{eAIT}\bar{\pi}_{t-1|EMA,T_{eAIT}}. \tag{29}$$

Equation (29) with appropriately normalised weights nests both a single-period inflation rate for $\rho_{eAIT} = 0$ and the (linearised) law of motion of the price level for $\rho_{eAIT} = 1$. Entering into an interest-rate rule, these two special cases would correspond to the IT rule and the PLT rule, respectively.

Note that with the exponential moving average being an infinite impulse response filter, there is actually not a direct equivalent for the averaging window in a simple moving average. Nevertheless, the smoothing parameter in the exponential moving average is sometimes expressed as

$$\rho_{eAIT} = \frac{T_{eAIT} - 1}{T_{eAIT} + 1} \tag{30}$$

to define a T_{eAIT}-period exponential moving average.

Figure 12 shows the relative weights that AIT and eAIT assign to different lags of inflation, exemplified for $T = T_{eAIT} = 8$. For reference, we also include the relative weights for IT and PLT in the graph. The relative weights are normalised such that the relative weight on current inflation is one. The relative weight on all lagged values is zero for inflation targeting and one for price-level targeting. A simple moving average of inflation as in the AIT rule has relative weights one for lags up to $T - 1$ and zero afterwards. The relative weights of the exponential moving average in (29) with $T_{eAIT} = 8$ in $\rho_{eAIT} = \frac{T_{eAIT}-1}{T_{eAIT}+1}$ follow an exponentially declining function across lags. That is, for $0 < \rho_{eAIT} < 1$, past inflation deviations are assigned a higher weight the closer they are to the present period. The relative weight

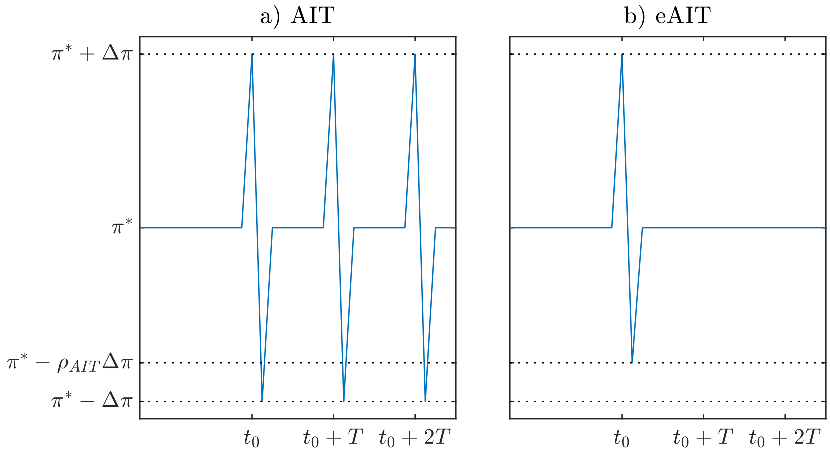

Figure 13: Illustration: cycling behaviour of AIT and eAIT

of two adjacent periods in the exponential moving average is always given by

$$\frac{\eta_s}{\eta_{s-1}} = \frac{\rho_{eAIT}^s \eta_0}{\rho_{eAIT}^{s-1} \eta_0} = \rho_{eAIT}.$$

This implies that if a deviation of inflation from target is cancelled out subsequently, no additional adjustments will be necessary to reach the targeted exponential moving average of the inflation rate in the absence of additional shocks.[29]

To illustrate this graphically, consider Figure 13. Compared to the simple moving average used in the AIT rule in panel a) (which is the same graph as panel a) in Figure 10), the exponential moving average used in the eAIT rule in panel b) does not feature the periodically repeating pattern of inflation deviations. Also, it becomes evident that the response of the central bank targeting an exponential moving average will respond similar to one that stabilises the price level (see panel b) in Figure 10). The major difference is that the response in the period after the shock is smaller in magnitude than the shock itself, which is due to the exponential decay in weights. After this initial response, absent further shocks, the central bank could just set the desired inflation rate forever.

4.4.2 The welfare performance of the eAIT rule

In the following, we analyse the welfare performance of an interest-rate rule that takes an exponential moving average of inflation as an argument. The rule is given by equation (22), where average inflation is measured by the exponential moving average in equation (29), and the corresponding smoothing parameter is determined as in equation (30).[30]

[29]The underlying reason is that the past error never actually drops out of an averaging window, and thus never shows up with its sign reversed.

[30]To be precise, we operationalise this as

$$\tilde{\pi}_t := \hat{\pi}_t + \frac{T-1}{T+1}\tilde{\pi}_{t-1}$$

in equation (22). Note that our specification of eAIT is closely related to interest-rate smoothing (where past shadow rates are used to smooth current ones).

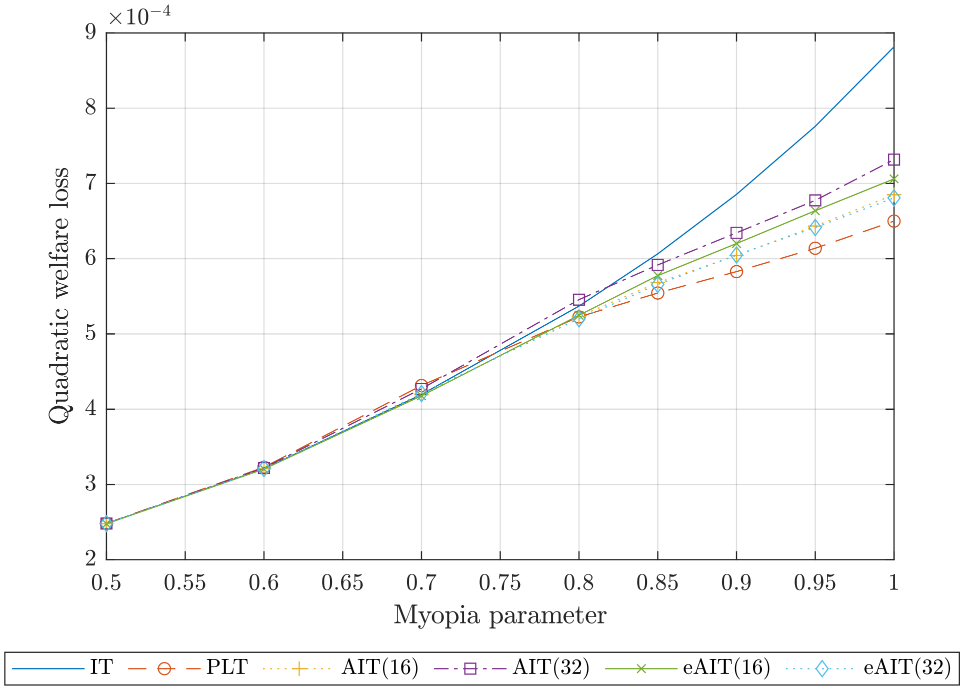

Figure 14: Welfare comparison of all monetary policy rules, demand shock

As in previous sections, there is an ELB on the policy rate as in equation (25). For a given T_{eAIT}, we optimise the coefficients in the eAIT rule for each degree of myopia and each shock according to the welfare criterion in equation (26).

Figure 14 shows welfare losses for demand shocks as in Figure 2, where now two lines for eAIT are added: one for eAIT with $T_{eAIT} = 16$ (green line with crosses) and one for eAIT with $T_{eAIT} = 32$ (light blue line with diamonds). As described above, the demand shock does not imply a trade-off for monetary policy, and the inherent volatility-inducing characteristic of AIT does not take effect for this type of shock. Hence, the conventional AIT rule already performs very well. In fact, for nearly rational expectations, eAIT with a lower autoregressive coefficient performs slightly worse than conventional AIT with a smaller averaging window.

In contrast, for trade-off-inducing technology shocks, Figure 15 shows that the eAIT rules can result in a marked improvement in terms of welfare losses compared to conventional AIT. For low degrees of myopia, the performance of the eAIT rules is very close to the best performing rule, the PLT rule, and markedly better than for the conventional AIT rules. As the degree of myopia increases, the deterioration in terms of welfare losses is much smaller under the eAIT rules than under the PLT rule. Consequently, the eAIT rules exhibit lower welfare losses than the PLT rule for a value of M below 0.9. In our simulations, eAIT with $T_{eAIT} = 32$ is the best performing rule for values of the myopia parameter between 0.85 and 0.9. However, the differences are relatively small and could be due to numerical imprecision. For even more myopic agents, the welfare losses of this eAIT rule increase and approach those of the IT rule for $M < 0.7$. However, for $M < 0.8$, eAIT with $T_{eAIT} = 16$ clearly exhibits the lowest welfare losses. In any case, the welfare losses under the eAIT rule with $T_{eAIT} = 16$ are markedly lower than the ones under the conventional AIT rules for the whole

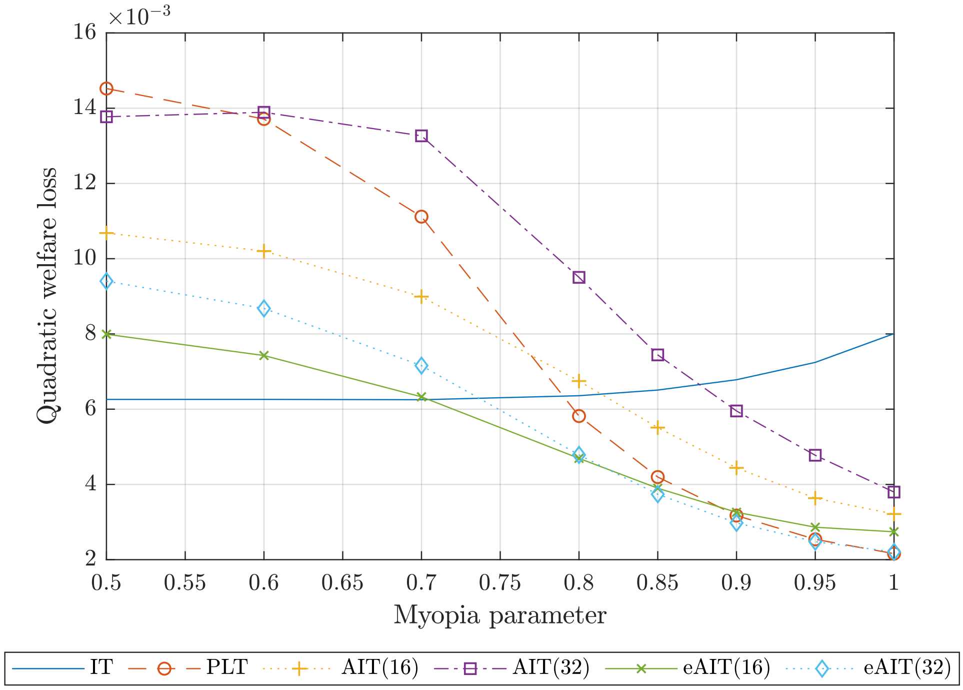

Figure 15: Welfare comparison of all monetary policy rules, technology shock

spectrum of degrees of myopia.

Given these observations, eAIT can be regarded as a robust strategy: Welfare losses are minimal or close to minimal among the considered rules across different degrees of myopia and different types of shocks. For demand shocks, eAIT performs almost as well as the best performing rules for low and high degrees of myopia. For technology shocks, eAIT can significantly improve on conventional AIT and its performance is very close or even somewhat better than that of the top performing rules for low and high degrees of myopia. This has three main reasons. First, in contrast to the conventional AIT rule, shock-induced deviations of the inflation rate from target do not trigger periodic policy-induced fluctuations of inflation under the eAIT rule, as described above. This benefits the eAIT rule's performance across the whole spectrum of degrees of myopia – especially for technology shocks, where smoothing out inflation is not optimal. Second, just like the conventional AIT rule and the PLT rule, the eAIT rule exhibits history dependence. This is beneficial when the expectations channel is powerful, i.e., for low degrees of myopia. Note that the history-dependent character of the eAIT rule is more akin to the PLT rule than to the conventional AIT rule since the exponential moving average entering the eAIT rule is an infinite impulse-response filter and, hence, all past inflation deviations are relevant for current and future monetary policy. This is why the eAIT rules come much closer to the PLT rule than the conventional AIT rule for low degrees of myopia in the case of technology shocks (see Figure 15). Third, the exponentially decaying weights of the exponential moving average in the eAIT rule imply that inflation deviations are assigned a higher weight, the closer they are to the present period. This 'tilts' the character of the eAIT rule – regarding the performance with myopic agents – towards an IT rule (in which the weight is one on the present inflation rate and zero on past inflation rates). The effect of this tilting is more pronounced for high degrees of myopia as this further reduces the influence that inflation deviations further in the past have on expectations about future inflation rates. In this way, the eAIT rule is able to approximate the IT rule when the degree of myopia is high.

4.5 Communicating an exponential AIT rule

While exponential AIT proves to be a very robust rule to both supply and demand shocks the natural question arises how such a rule could be communicated to the public. AIT itself already seems difficult to communicate due to the multitude of parameters relevant to such a rule. Those include the length of the averaging window, the strength of the reaction to past deviations, the amount of time to correct deviations from the inflation target and perhaps even an asymmetric element as in the Federal Reserves new monetary policy strategy.

While seemingly a daunting task, Hoffmann et al. (2022) have documented that reaping the benefits of AIT is possible despite providing very little details on how the strategy would exactly work. In a randomized control trial they show that communicating target inflation on average leads to higher inflation expectations in the control group without providing participants any details about the averaging window or the strength of the reaction to deviations from the target.[31] Simply knowing that policy makers aim for an average inflation rate of 2 % is sufficient.

In light of this positive evidence, exponential AIT seems even easier and more credible to communicate compared to a seemingly standard AIT rule. The biggest difference concerns averaging inflation rates exponentially instead of arithmetically. Yet, this only means that more recent inflation outcomes matter more to the policy maker than outcomes further in the past. Given that more recent inflation developments influence current and future inflation outcomes more strongly and therefore are more relevant for policy, it seems only natural for policy makers to communicate this orientation towards the recent past. Furthermore, as discussed before arithmetic AIT implies a volatile adjustment path due to the mechanic nature of past inflation readings dropping out of the averaging window. An exponential rule can instead actively communicate it's intend against this mechanical update of an average number and instead credibly emphasize the smoothness of the future interest rate path it intends to implement.

5 Conclusion

In a New Keynesian model with sticky prices, sticky wages, an ELB on the policy rate and bounded rationality, we have studied the welfare performance of various interest-rate rules. History-dependent rules perform better than an IT rule for demand and supply shocks when agents have rational expectations or are mildly myopic. For stronger degrees of myopia the advantages of history-dependent strategies fade or turn into a disadvantage and IT becomes the best performing rule. However, an interest-rate rule responding to an exponential mov-

[31] At the time of the randomized control trial the inflation rate was considerably below target.

ing average of current and past inflation rates is extraordinarily robust by performing well across different shocks and degrees of myopia.

References

ADJEMIAN, S., H. BASTANI, M. JUILLARD, F. KARAMÉ, J. MAIH, F. MIHOUBI, W. MUTSCHLER, G. PERENDIA, J. PFEIFER, M. RATTO, AND S. VILLEMOT (2021): "Dynare: Reference Manual Version 4," Dynare Working Papers 1, CEPREMAP.

AFROUZI, H., S. Y. KWON, A. LANDIER, Y. MA, AND D. THESMAR (2021): "Overreaction in Expectations: Evidence and Theory," Working paper.

ANGELETOS, G.-M., Z. HUO, AND K. A. SASTRY (2020): *Imperfect Macroeconomic Expectations: Evidence and Theory*, University of Chicago Press.

ANGELETOS, G.-M. AND C. LIAN (2018): "Forward Guidance without Common Knowledge," *American Economic Review*, 108, 2477–2512.

——— (2022): "Dampening General Equilibrium: Incomplete Information and Bounded Rationality," Working Paper 29776, National Bureau of Economic Research.

ASCARI, G. AND A. M. SBORDONE (2014): "The Macroeconomics of Trend Inflation," *Journal of Economic Literature*, 52, 679–739.

BENCHIMOL, J. AND L. BOUNADER (2021): "Optimal Monetary Policy Under Bounded Rationality," Dynare Working Papers 67, CEPREMAP.

BERNANKE, B. S., M. T. KILEY, AND J. M. ROBERTS (2019): "Monetary Policy Strategies for a Low-Rate Environment," *AEA Papers and Proceedings*, 109, 421–426.

BILBIIE, F. O. (2021): "Monetary Policy and Heterogeneity: An Analytical Framework," Tech. rep.

BRAND, C., M. BIELECKI, AND A. PENALVER (2018): "The Natural Rate of Interest: Estimates, Drivers, and Challenges to Monetary Policy," Occasional Paper Series 217, European Central Bank.

BRZOZA-BRZEZINA, M., P. GALIŃSKI, AND K. MAKARSKI (2022): "Monetary policy in a two-country model with behavioral expectations," NBP Working Papers 353, Narodowy Bank Polski, Economic Research Department.

CALVO, G. A. (1983): "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics*, 12, 383–398.

CECIONI, M., G. COENEN, R. GERKE, H. LE BIHAN, AND R. MOTTO (2021): "The ECB's Price Stability Framework: Past Experience, and Current and Future Challenges," Occasional Paper Series 269, European Central Bank.

COENEN, G., C. MONTES-GALDÓN, AND S. SCHMIDT (2021): "Macroeconomic Stabilisation and Monetary Policy Effectiveness in a Low-Interest-Rate Environment," *Journal of Economic Dynamics and Control*, 132.

COIBION, O., Y. GORODNICHENKO, AND R. KAMDAR (2018): "The Formation of Expectations, Inflation, and the Phillips Curve," *Journal of Economic Literature*, 56, 1447–1491.

DEL NEGRO, M., M. GIANNONI, AND C. PATTERSON (2023): "The Forward Guidance Puzzle," *Journal of Political Economy Macroeconomics*, 1, 43–79.

EGGERTSSON, G. B. (2010): "The Paradox of Toil," *FRB of New York Staff Report*.

EGGERTSSON, G. B. AND P. KRUGMAN (2012): "Debt, Deleveraging, and the Liquidity Trap: A Fisher-Minsky-Koo Approach," *The Quarterly Journal of Economics*, 127, 1469–1513.

EGGERTSSON, G. B. AND M. WOODFORD (2003): "The Zero Bound on Interest Rates and Optimal Monetary Policy," *Brookings Papers on Economic Activity*, 2003, 139–211.

ERCEG, C. J., D. W. HENDERSON, AND A. T. LEVIN (2000): "Optimal Monetary Policy with Staggered Wage and Price Contracts," *Journal of Monetary Economics*, 46, 281–313.

ERCEG, C. J., Z. JAKAB, AND J. LINDÉ (2021): "Monetary Policy Strategies for the European Central Bank," *Journal of Economic Dynamics and Control*, 132, 104211.

FAIR, R. AND J. TAYLOR (1983): "Solution and Maximum Likelihood Estimation of Dynamic Nonlinear Rational Expectations Models," *Econometrica*, 51, 1169–85.

FARHI, E. AND I. WERNING (2019): "Monetary Policy, Bounded Rationality, and Incomplete Markets," *American Economic Review*, 109, 3887–3928.

GABAIX, X. (2020): "A Behavioral New Keynesian Model," *American Economic Review*, 110, 2271–2327.

GALÍ, J. (2015): *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework and Its Applications Second Edition*, no. 10495 in Economics Books, Princeton University Press.

GARCÍA-SCHMIDT, M. AND M. WOODFORD (2019): "Are Low Interest Rates Deflationary? A Paradox of Perfect-Foresight Analysis," *American Economic Review*, 109, 86–120.

GARÍN, J., R. LESTER, AND E. SIMS (2016): "On the desirability of nominal GDP targeting," *Journal of Economic Dynamics and Control*, 69, 21–44.

GEIGER, F. AND F. SCHUPP (2018): "With a little help from my friends: Survey-based derivation of euro area short rate expectations at the effective lower bound," Discussion Papers 27/2018, Deutsche Bundesbank.

GÓMEZ, A. A. J. E. G. R. J. E. S., J. E. GALLEGOS, R. JAIMES, AND E. S. GÓMEZ (2022): "Reconciling Empirics and Theory: The Behavioral Hybrid New Keynesian Model," Vniversitas Económica 018560, Universidad Javeriana - Bogotá.

HOFFMANN, M., E. MOENCH, L. PAVLOVA, AND G. SCHULTEFRANKENFELD (2022): "Would households understand average inflation targeting?" *Journal of Monetary Economics*, 129, S52–S66.

HOLSTON, K., T. LAUBACH, AND J. C. WILLIAMS (2017): "Measuring the Natural Rate of Interest: International Trends and Determinants," *Journal of International Economics*, 108, 59–75.

HONKAPOHJA, S. AND N. MCCLUNG (2021): "On Robustness of Average Inflation Targeting," Research Discussion Papers 6/2021, Bank of Finland.

ILABACA, F., G. MEGGIORINI, AND F. MILANI (2020): "Bounded rationality, monetary policy, and macroeconomic stability," *Economics Letters*, 186, 1–5.

MEGGIORINI, G. AND F. MILANI (2021): "Behavioral New Keynesian Models: Learning vs. Cognitive Discounting," CESifo Working Paper Series 9039, CESifo.

MICHAILLAT, P. AND E. SAEZ (2021): "Resolving New Keynesian Anomalies with Wealth in the Utility Function," *The Review of Economics and Statistics*, 1–19.

NAKATA, T., R. OGAKI, S. SCHMIDT, AND P. YOO (2019): "Attenuating the Forward Guidance Puzzle: Implications for Optimal Monetary Policy," *Journal of Economic Dynamics and Control*, 105, 90–106.

NAKATA, T., S. SCHMIDT, AND F. BUDIANTO (2020): "Average Inflation Targeting and the Interest Rate Lower Bound," Working Paper Series 2394, European Central Bank.

NAKOV, A. (2008): "Optimal and Simple Monetary Policy Rules with Zero Floor on the Nominal Interest Rate," *International Journal of Central Banking*, 4, 73–127.

REIFSCHNEIDER, D. AND J. C. WILLIAMS (2000): "Three Lessons for Monetary Policy in a Low-Inflation Era," *Journal of Money, Credit, and Banking*, 32, 936–966.

SMETS, F. AND R. WOUTERS (2007): "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, 97, 586–606.

WAGNER, J., T. SCHLANGER, AND Y. ZHANG (2022): "A Horse Race of Alternative Monetary Policy Regimes Under Bounded Rationality," Discussion Papers 2022-4, Bank of Canada.

WERNING, I. (2011): "Managing a Liquidity Trap: Monetary and Fiscal Policy," NBER Working Papers 17344, National Bureau of Economic Research, Inc.

WOODFORD, M. (2019): "Monetary Policy Analysis when Planning Horizons Are Finite," *NBER Macroeconomics Annual*, 33, 1–50.

A Derivation of the Full Model

A.1 Household Behaviour

There is a continuum of households $h \in [0,1]$ with identical preferences

$$U_{h,t_0} = \mathcal{E}^h_{t_0}\left[\sum_{t\geq t_0} \beta^t d_t \left(u(c_{h,t}) - \frac{\nu}{1+\varphi} n_{h,t}^{1+\varphi}\right)\right], \tag{31}$$

where $c_{h,t}$ is h's consumption and $n_{h,t}$ is h's labour supply, φ is the household's inverse Frisch elasticity, d_t is a discount factor shock, and the function governing period utility flow from consumption $u(c)$ is given by

$$u(c) = \begin{cases} \ln(c) & \text{if} \quad \sigma = 1, \\ \frac{c^{1-\sigma}-1}{1-\sigma} & \text{otherwise.} \end{cases}$$

Here σ is both the inverse of the elasticity of intertemporal substitution as well as the coefficient of relative risk aversion.

Denote h's period-t wage by $w_{h,t}$, the wage-tax (or subsidy) by τ_w and the price level by P_t. Each household obtains nominal wage income $(1-\tau_w)W_{h,t}n_{h,t}$, real profits Π^F_t and transfers $P_t\tau_{ht}$ and spends it on final-goods consumption $P_t c_{ht}$ and liquid bonds $P_t b_{ht}$, these liquid bonds then yield a nominal return of $(1+i_t)P_t b_{ht}$ in the next period.

That is, the households's nominal period-budget constraint is given by

$$P_t(c_{ht} + b_t) = (1-\tau_w)W_{h,t}n_{h,t} + P_t(\Pi^F_t + \tau_{ht}) + (1+i_{t-1})P_{t-1}b_{h,t-1} \tag{32}$$

and the equivalent in real terms is

$$(c_{ht} + b_t) = (1-\tau_w)\frac{W_{h,t}}{P_t}n_{h,t} + \Pi^F_t + \tau_{ht} + R_t b_{h,t-1} \tag{33}$$

where

$$R_t = \frac{1+i_{t-1}}{1+\pi_t}$$

is the real interest factor between periods $t-1$ and t.

We assume that the household acts as a price-taker on all markets except for the labour market. On that market, we assume that households form unions U, which have market power. In particular, each household delegates its wage bargaining to the union, which then also determines the labour supply n_{ht}.

I.e., the household's optimisation problem in period t_0 is given by

$$\max_{(c_{ht},b_{ht})_{t\geq t_0}} \quad U_{h,t_0} \quad \text{s.t.} \quad (33) \text{ holds } \forall t$$

Denoting as λ_{ht} h's Lagrange multiplier on (33) in t, we obtain the first order conditions:

$$u'(c_{ht}) = \lambda_{ht} \tag{34}$$
$$\lambda_{ht} d_t = \beta \mathcal{E}_t^h [d_{t+1} R_{t+1} \lambda_{h,t+1}], \tag{35}$$

which with $u'(c) = c^{-\sigma}$ delivers the standard Euler equation

$$d_t c_{ht}^{-\sigma} = \beta \mathcal{E}_t^h \left[d_{t+1} R_{t+1} c_{h,t+1}^{-\sigma} \right]$$

Following Erceg et al. (2000), we assume that $(\tau_{ht})_{h \in [0,1], t \geq t_0}$ is such that in each period, all households choose the same level of consumption $c_{ht} = C_t^H \forall t$. Aggregate consumption is then given by

$$C_t = \int_{h \in [0,1]} c_{ht} dh = \int_{h \in [0,1]} C_t^H dh = C_t^H.$$

Also, we assume that all households form the same expectations regarding the future $\mathcal{E}_t^h[\cdot] = \mathcal{E}_t^H[\cdot]$. Then, there is an aggregate Euler equation

$$d_t C_t^{-\sigma} = \beta \mathcal{E}_t^H \left[d_{t+1} R_{t+1} C_{t+1}^{-\sigma} \right] \tag{36}$$

and an aggregate stochastic discount factor

$$\mathcal{Q}_{t|t+1} = \beta d_{t+1} C_t^{\sigma} C_{t+1}^{-\sigma} / d_t. \tag{37}$$

which satisfies

$$1 = \mathcal{E}_t^H \left[R_{t+1} \mathcal{Q}_{t|t+1} \right] \tag{38}$$

A.2 Final-Goods Firm

The final good Y_t is produced by a competitive firm, using intermediate inputs $y_{ft}, f \in \mathcal{F}$ according to a production function, where each input in $\mathcal{F}$ with $|\mathcal{F}| = 1$ is produced by one intermediate firm, indexed also with f, according to

$$Y_t = \left(\int_{f \in \mathcal{F}} y_{ft}^{\frac{\epsilon_p - 1}{\epsilon_p}} df \right)^{\frac{\epsilon_p}{\epsilon_p - 1}} \tag{39}$$

where $\epsilon_p > 1$ is the elasticity of substitution across varieties. With the price of the final good being given by P_t and intermediate input prices given by p_{ft}, profits of the final-goods firm are given by

$$\Pi_t^{\text{final}} := P_t Y_t - \int_{f \in \mathcal{F}} p_{f,t} y_{ft} df \tag{40}$$

As is usual with this production function, profit maximisation and a zero-profit condition give demand for each intermediate good as

$$y_{ft} = \left(\frac{p_{ft}}{P_t}\right)^{-\epsilon_p} Y_t \tag{41}$$

and an equation for the price level

$$P_t = \left(\int_{f \in \mathcal{F}} p_{ft}^{1-\epsilon_p} df\right)^{\frac{1}{1-\epsilon_p}}. \tag{42}$$

A.3 Intermediate-Goods Producers

Each intermediate goods firm f uses an aggregate labour input $N_{f,t}$ to produce its intermediate good according to a production function

$$q_{ft} = A_t N_{f,t}^{1-\alpha}, \tag{43}$$

where α measures decreasing returns to scale and A_t is time-varying productivity.

The aggregate labour input is priced at the real wage w_t and there is a production subsidy τ_p such that nominal profits of the firm are given by

$$P_t \Pi_{f,t} = p_{ft} q_{ft}(1+\tau_p) - P_t w_t N_{ft} \tag{44}$$

Each firm can only re-optimise its price p_{ft} in a given period with a probability $1-\theta_p$. In case the firm cannot reoptimise its price, the price is updated according to $p_{ft} = p_{f,t-1}(1+\pi^*)$. Firms also engage in monopolistic competition, taking into account the demand curves $q_{ft} = y_{ft}$ as in (41). I.e., their maximisation problem in period t_0 yields the recursive formulation

$$\Gamma_{f,t_0} := \max_{(p_{ft_0})} \left\{ \begin{array}{l} \mathcal{E}_{t_0}^F \left[\sum_{t \geq t_0} \theta^{t-t_0} \mathcal{Q}_{t_0|t} \left(\frac{p_{ft|t_0}}{P_t} q_{ft|t_0}(1+\tau_p) - w_t N_{ft|t_0} \right) + (1-\theta_p) \mathcal{Q}_{t_0|t_0+1} \Gamma_{f,t_0+1} \right] \\ \qquad \text{s.t } t \geq t_0 : \\ \qquad\qquad p_{ft|t_0} = \begin{cases} (1+\pi^*) p_{f,t-1|t_0}, & \text{if} \quad t > t_0, \\ p_{ft_0}, & \text{if} \quad t = t_0, \end{cases} \\ \qquad\qquad q_{ft|t_0} = A_t N_{f,t|t_0}^{1-\alpha}, \\ \qquad\qquad q_{ft|t_0} = \left(\frac{p_{ft|t_0}}{P_t}\right)^{-\epsilon_p} Y_t, \\ \qquad\qquad \mathcal{E}_{t_0}^F[(A_t, Y_t, P_t, w_t)_{t \geq t_0}] \text{ given} \end{array} \right\}$$

for the value of an optimising firm Γ_{f,t_0} at the start of a period t_0. Here, the stochastic discount factor $\mathcal{Q}_{t_0|t}$ between non-adjacent periods t_0, t is given by

$$\mathcal{Q}_{t_0|t} = \prod_{s=t_0}^{t-1} \mathcal{Q}_{s|s+1} \quad \text{for } t \geq t_0, t \neq t_0 + 1$$

This gives rise to an optimal pricing decision given by

$$p^*_{ft} = p^*_t = P_t \left(\frac{\epsilon_p}{(\epsilon_p - 1)(1+\tau_p)(1-\alpha)} \frac{\Xi^1_t}{\Xi^2_t} \right)^{\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}}, \quad \text{where} \tag{45}$$

$$\Xi^1_t = w_t \left(\frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} + \theta \mathcal{E}^F_t \left[\left(\frac{1+\pi_{t+1}}{1+\pi^*} \right)^{\frac{\epsilon_p}{1-\alpha}} \mathcal{Q}_{t|t+1} \Xi^1_{t+1} \right] \quad \text{and} \tag{46}$$

$$\Xi^2_t = Y_t + \theta \mathcal{E}^F_t \left[\left(\frac{1+\pi_{t+1}}{1+\pi^*} \right)^{\epsilon_p - 1} \mathcal{Q}_{t|t+1} \Xi^2_{t+1} \right], \tag{47}$$

which is the same for all firms optimising in t.

Each intermediate-goods producer also generates real profits

$$\pi_{f_t} = (1+\tau_p) \left(\frac{p_{ft}}{P_t} \right)^{1-\epsilon_p} Y_t - w_t \left(\frac{p_{ft}}{P_t} \right)^{\frac{-\epsilon_p}{1-\alpha}} \left(\frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \tag{48}$$

and has labour demand

$$N_{ft} = \left(\frac{p_{ft}}{P_t} \right)^{\frac{-\epsilon_p}{1-\alpha}} \left(\frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \tag{49}$$

The price-level evolves according to

$$P_t^{1-\epsilon_p} = (1-\theta_p) \left(p^*_t \right)^{1-\epsilon_p} + \theta_p (1+\pi^*)^{1-\epsilon_p} P_{t-1}^{1-\epsilon_p}$$

or, in terms of the inflation rate

$$1 = (1-\theta_p) \left(\frac{p^*_t}{P_t} \right)^{1-\epsilon_p} + \theta_p \left(\frac{1+\pi^*}{1+\pi_t} \right)^{1-\epsilon_p} \tag{50}$$

A.4 Labour Packers

We assume that the labour input used by intermediate goods firms is provided by a competitive labour packer who buys up the labour supplied by the various unions and aggregates it up according to the "production function"

$$N_t = \left(\int_{u \in \mathcal{U}} n_{ut}^{\frac{\epsilon_w - 1}{\epsilon_w}} du \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}, \tag{51}$$

where ϵ_w is the elasticity of substitution across different union's labour input.

With the real price of the aggregate labour output given by the real wage w_t and individual unions' wage rates $w_{u,t}$, we obtain profits

$$\Pi_t^N := w_t N_t - \int_{u \in \mathcal{U}} n_{ut} w_{ut} du. \tag{52}$$

Maximising this subject to the production function and zero profits, we obtain a labour demand for individual union's labour supply

$$n_{ut} = \left(\frac{w_{ut}}{w_t}\right)^{-\epsilon_w} N_t \tag{53}$$

and an equation for the real wage (index)

$$w_t = \left(\int_{u \in \mathcal{U}} w_{ut}^{1-\epsilon_w} du\right)^{\frac{1}{1-\epsilon_w}}, \tag{54}$$

Equations (53) and (54) can also be expressed in nominal terms as

$$n_{ut} = \left(\frac{W_{ut}}{W_t}\right)^{-\epsilon_w} N_t \quad \text{and} \tag{55}$$

$$W_t = \left(\int_{u \in \mathcal{U}} W_{ut}^{1-\epsilon_w} du\right)^{\frac{1}{1-\epsilon_w}}, \tag{56}$$

$$\text{where } W_t = P_t w_t \quad \text{and} \tag{57}$$

$$W_{ut} = P_t w_{ut}. \tag{58}$$

A.5 Unions / Wage Setters

Each household is assigned to a union, whose objective it is to maximise its member's utility, taken as given the member's consumption decision. As mentioned before, households' consumption is perfectly insured, however, their labour supply is subject to market clearing at a given wage rate and as such each union takes (55) into account. In period t_0, each union discounts a real income stream in period t by $\mathcal{Q}_{t_0|t}$, the disutility of the union stems from the utility function (31). Similar to intermediate-goods producers, unions can only reoptimise in any given period with probability $1 - \theta_w$, with a probabilty of θ_w their nominal wage is simply adjusted for steady-state inflation π^*.

The dynamic programming problem of an optimising firm in period t_0 can thus be written in recursive form as

$$V_t^u = \max_{(W_{ut_0}^*)} \left\{ \begin{array}{l} \mathcal{E}_{t_0}^u \left[\sum_{t \geq t_0} \theta_w^{t-t_0} \mathcal{Q}_{t_0|t} \left((1-\tau_w) \frac{W_{ut|t_0}}{P_t} n_{ut|t_0} - \frac{\nu}{1+\varphi} C_t^\sigma n_{ut|t_0}^{1+\varphi} \right) \right. \\ \qquad \left. + (1-\theta_w) \mathcal{Q}_{t_0|t_0+1} V_{t_0+1}^u \right] \\ \quad \text{s.t } t \geq t_0 : \\ \qquad W_{ut|t_0} = \begin{cases} (1+\pi^*) W_{u,t-1|t_0}, & \text{if} \quad t > t_0, \\ W_{ut_0}^*, & \text{if} \quad t = t_0, \end{cases} \\ \qquad n_{ut|t_0} = \left(\frac{W_{ut|t_0}}{W_t} \right)^{-\epsilon_w} N_t, \\ \qquad\qquad \mathcal{E}_{t_0}^F [(C_t, W_t, N_t, W_t, P_t, \mathcal{Q}_{t_0|t})_{t \geq t_0}] \text{ given} \end{array} \right\}.$$

This gives rise to optimal wage setting behaviour

$$(W_{ut}^*)^{1+\epsilon_w \varphi} = P_t W_t^{\epsilon_w \varphi} \frac{\nu \epsilon_w}{(\epsilon_w - 1)(1-\tau_w)} \frac{X_{1t}}{X_{2t}}, \tag{59}$$

$$X_{1t} = C_t^\sigma N_t^{1+\varphi} + \theta_w \mathcal{E}_t^u \left[\left(\frac{1+\pi_{w,t+1}}{1+\pi^*} \right)^{\epsilon_w(1+\varphi)} \mathcal{Q}_{t|t+1} X_{1,t+1} \right], \tag{60}$$

$$X_{2t} = N_t + \theta_w \mathcal{E}_t^u \left[\frac{(1+\pi_{w,t+1})^{\epsilon_w}}{(1+\pi^*)^{\epsilon_w - 1}(1+\pi_{t+1})} \mathcal{Q}_{t|t+1} X_{2,t+1} \right], \tag{61}$$

where

$$(1+\pi_{w,t}) = \frac{W_t}{W_{t-1}} \tag{62}$$

The nominal wage index thus evolves according to

$$W_t^{1-\epsilon_w} = (1-\theta_w)(W_t^*)^{1-\epsilon_w} + \theta_w [(1+\pi^*) W_{t-1}]^{1-\epsilon_w}, \tag{63}$$

which we can reformulate as

$$1 = (1-\theta_w) \left(W_t^* / W_t \right)^{1-\epsilon_w} + \theta_w \left[\frac{1+\pi^*}{1+\pi_{w,t}} \right]^{1-\epsilon_w}, \tag{64}$$

We can express (59) in real terms as

$$w_{ut}^* = \frac{W_{ut}^*}{P_t} = \left[w_t^{\epsilon_w \varphi} \frac{\nu \epsilon_w}{(\epsilon_w - 1)(1-\tau_w)} \frac{X_{1t}}{X_{2t}} \right]^{\frac{1}{1+\epsilon_w \varphi}}, \tag{65}$$

where the real wage evolves according to

$$w_t = (1+\pi_{wt}) w_{t-1} / (1+\pi_t) \tag{66}$$

and (64) can be written as

$$1 = (1-\theta_w)\left(w_t^*/w_t\right)^{1-\epsilon_w} + \theta_w \left[\frac{1+\pi^*}{1+\pi_{w,t}}\right]^{1-\epsilon_w}, \tag{67}$$

A.6 Fiscal Policy, Market Clearing

We assume that liquid bonds are in zero-net supply, i.e.

$$\int_{h\in[0,1]} b_{h,t} dh = 0 \tag{68}$$

Also, the government purchases goods $G_t = g_t Y_t$ and runs a balanced budget. As such, the budget constraint of the government reads as

$$\int_{h\in[0,1]} \tau_{ht} dh + \tau_w \int_{h\in[0,1]} w_{ht} n_{ht} dh = \sum_{f\in\mathcal{F}} \tau_p y_{ft} df = G_t \tag{69}$$

Market clearing in the final-goods sector then implies

$$C_t + G_t = Y_t = \int_{f\in\mathcal{F}} y_{ft} df \tag{70}$$

or

$$C_t = Y_t(1-g_t) \tag{71}$$

We still need to aggregate up firms' profits from (48), taking into account that the final-goods firms and the labour packer make zero profits:

$$\begin{aligned}
\Pi_t^F &= \Pi_t^{final} + \Pi_t^N + \int_{f\in\mathcal{F}} \pi_{f_t} df \\
&= 0 + 0 + \pi_{f_t} df \\
&= \int_{f\in\mathcal{F}} \left((1+\tau_p)\left(\frac{p_{ft}}{P_t}\right)^{1-\epsilon_p} Y_t - \left(\frac{p_{ft}}{P_t}\right)^{\frac{-\epsilon_p}{1-\alpha}} \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \right) df \\
&= (1+\tau_p) \int_{f\in\mathcal{F}} \left(\left(\frac{p_{ft}}{P_t}\right)^{1-\epsilon_p} \right) Y_t - \int_{f\in\mathcal{F}} \left(\left(\frac{p_{ft}}{P_t}\right)^{\frac{-\epsilon_p}{1-\alpha}} \right) \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} df
\end{aligned} \tag{72}$$

Similarly, aggregate labour demand is given by

$$N_t = \int_{f\in\mathcal{F}} N_{ft} df = \int_{f\in\mathcal{F}} \left(\frac{p_{ft}}{P_t}\right)^{\frac{-\epsilon_p}{1-\alpha}} \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} df = \int_{f\in\mathcal{F}} \left(\frac{p_{ft}}{P_t}\right)^{\frac{-\epsilon_p}{1-\alpha}} df \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \tag{73}$$

Note that for any $x \in \mathbb{R}$, we have

$$s_t^p(x) := \int_{f\in\mathcal{F}} \left(\frac{p_{ft}}{P_t}\right)^x df \tag{74}$$

$$=(1-\theta_p)\left(\frac{p_t^*}{P_t}\right)^x + \theta_p \int_{f\in\mathcal{F}} \left((1+\pi*)\frac{p_{f,t-1}}{P_t}\right)^x df \tag{75}$$

$$=(1-\theta_p)\left(\frac{p_t^*}{P_t}\right)^x + \theta_p \left(\frac{1+\pi^*}{1+\pi_t}\right)^x \int_{f\in\mathcal{F}} \left(\frac{p_{f,t-1}}{P_{t-1}}\right)^x df \tag{76}$$

$$=(1-\theta_p)\left(\frac{p_t^*}{P_t}\right)^x + \theta_p \left(\frac{1+\pi^*}{1+\pi_t}\right)^x s_{t-1}^p(x), \tag{77}$$

where, note we have

$$s_t^p(1) = s_t^p(0-\epsilon_p) = 1 \tag{78}$$

This allows us to obtain

$$\Pi_t^F = (1+\tau_p)Y_t - s_t\left(\frac{-\epsilon_p}{1-\alpha}\right)\left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \tag{79}$$

Similarly, aggregate labour demand is given by

$$N_t = -s_t\left(\frac{-\epsilon_p}{1-\alpha}\right)\left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \tag{80}$$

An equilibrium in this economy, given structural parameters $(\sigma, \nu, \rho, \beta, \alpha, \epsilon_p, \epsilon_w, \theta_p, \theta_w)$, policy parameters $(\tau_w, \tau_p, \phi_\pi, \phi_y, \pi^*)$, state variables $(w_{t-1}.W_{t-1}, P_{t-1})$ is a sequence of state-contingent aggregate variabels $(C_t, Y_t, G_t, R_t, \pi_t, \pi_{w,t}, w_t, i_{t-1}, \Xi_{1,t}, \Xi_{2,t}, X_{1,t}, X_{2,t})$

B Additional Results

B.1 Additional Figures for Section 4

Simulated Means

Below, a couple of additional results are presented for section 4, starting with the mean values of the simulated variables under TFP and demand shocks in figures 16 and 17, respectively. In this section, we always show all policy rules from the main text.

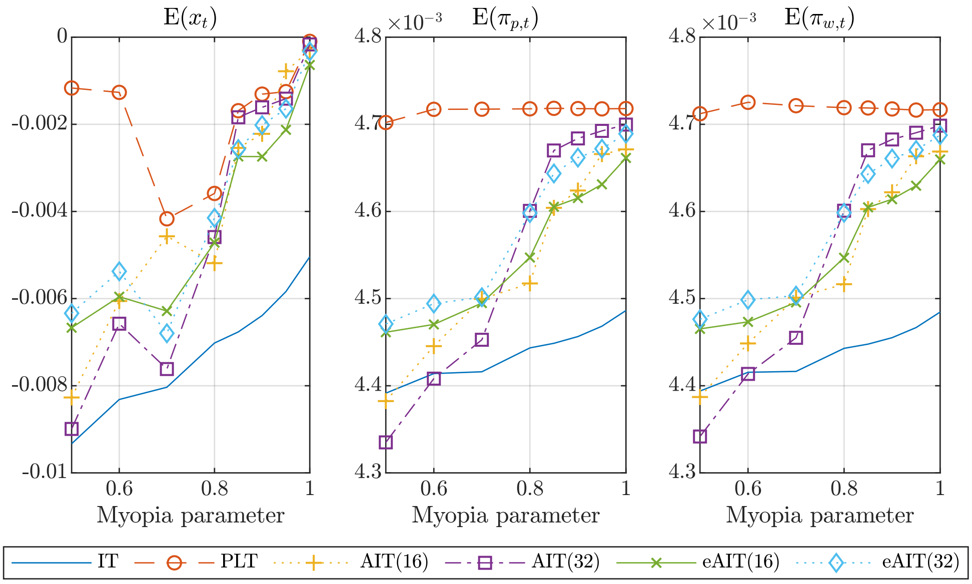

Figure 16: Means of selected simulated variables, TFP shock

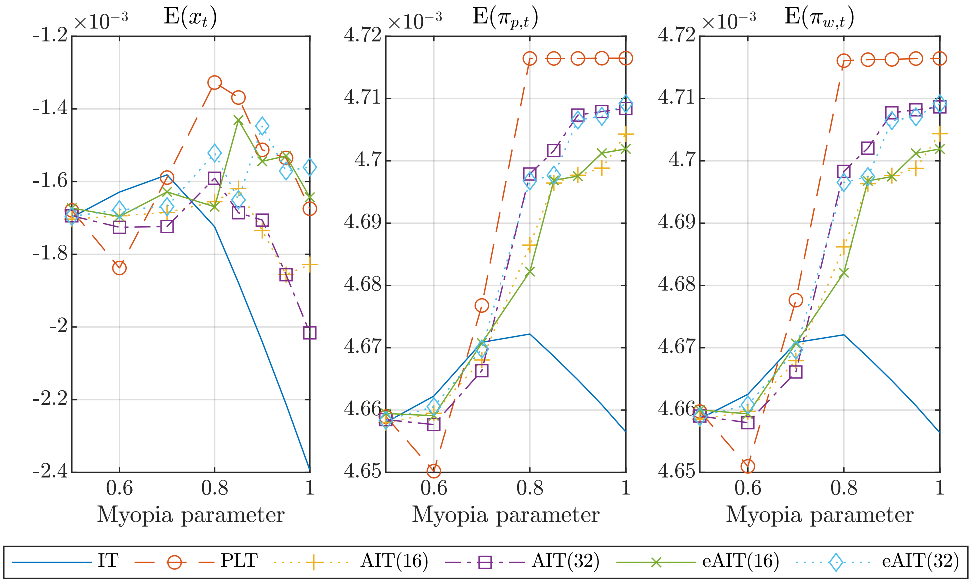

Figure 17: Means of selected simulated variables, demand shock

It is noteworthy that the presence of the effective lower bound implies that the mean output gap is negative for all simulations. Also, the increased volatility in the natural rate makes the mean output gap larger for TFP shocks for the various AIT or eAIT rules. For IT, this wedge declines with less forward-looking agents, and for PLT there is a non-monotonous behaviour. Notably, it becomes clear that as the TFP shock leads to a lot more time spent at the ELB with history dependent policies, those, in particular PLT, have less effective time (i.e., unconstrained time) to stabilise (average) inflation. Consequently, mean inflation rates start to differ from the target rate as the economy becomes more myopic.

Welfare Purely Based on Variances

Instead of using equation (26) to measure welfare losses, often a simple variance-based approximation is used:

$$\mathbb{L}_2 = \frac{1}{2}\left\{ \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) \text{Var}\left(x_t\right) + \frac{\epsilon_p}{\lambda_p}\text{Var}\left(\pi_t^p\right) + \frac{\epsilon_w(1-\alpha)}{\lambda_w}\text{Var}\left(\pi_t^w\right) \right\} \tag{81}$$

Figures 18 and (19) present the resulting welfare approximations for our experiments conducted in the main text. Note that the overall pattern is unaffected: inflation targeting becomes more, price-level targeting less attractive as myopia increases. However, for technology shocks note that eAIT with $T_{eAIT} = 32$ in this case exactly approaches IT as we make agents more myopic. Moreover, in Figure 19, it is easier to see that inflation targeting overtakes price-level targeting, also it overtakes it a bit earlier. This is due to the fact that history dependent strategies at the ELB counteract deflationary biases etc., making the mean deviation relevant from a welfare perspective. This, however, is neglected under equation (81).

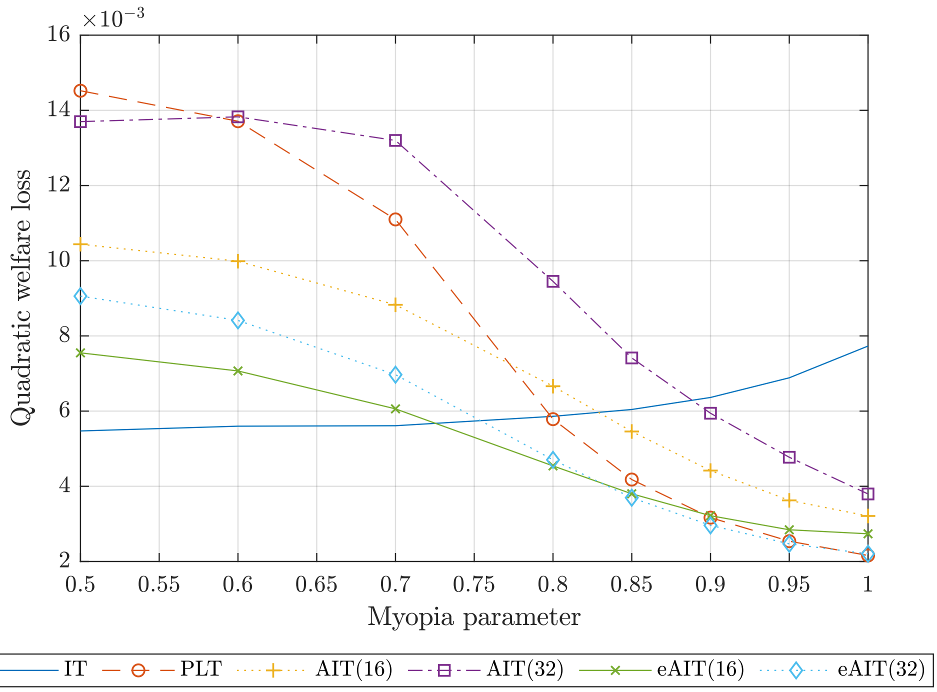

Figure 18: Welfare comparison for technology shocks, purely variance-based

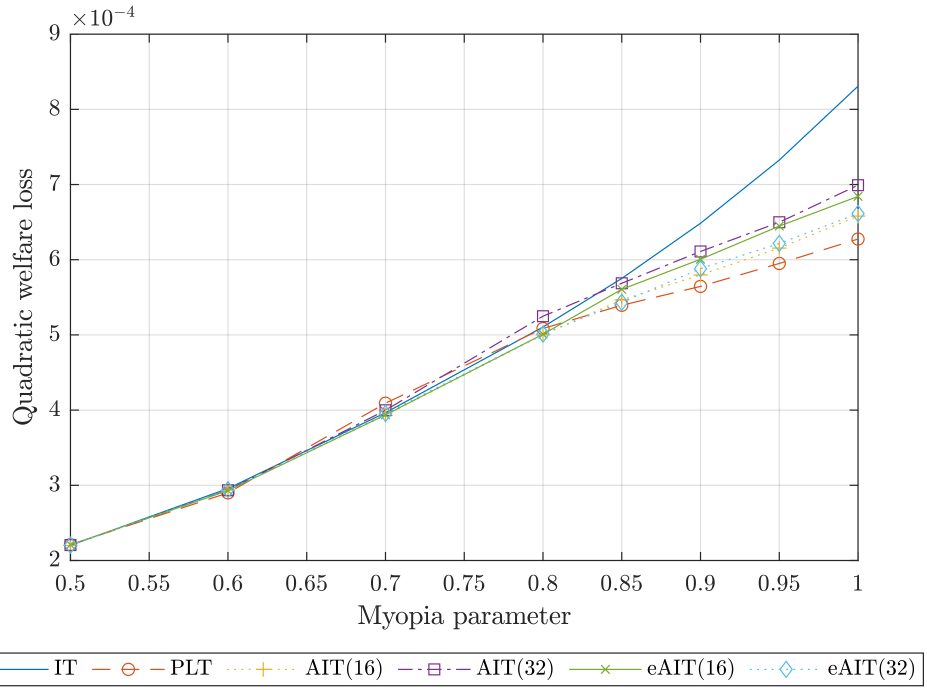

Figure 19: Welfare comparison for demand shocks, purely variance-based

Optimised Parameters, Variances and ELB Statistics with all Monetary Policy Rules

Now, we present the results for the optimised parameters (Figures 20 and 21), the variance plots (Figures 22 and 23) and the plots depicting the ELB statistics for the analysis with the eAIT rules included.

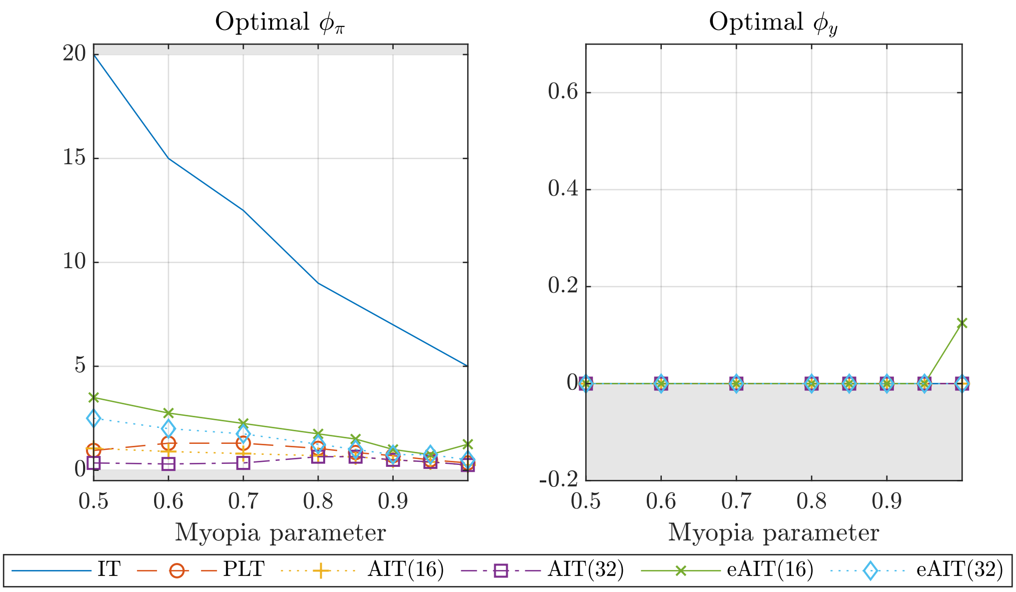

Figure 20: Optimised parameters, all monetary policy rules, TFP shock

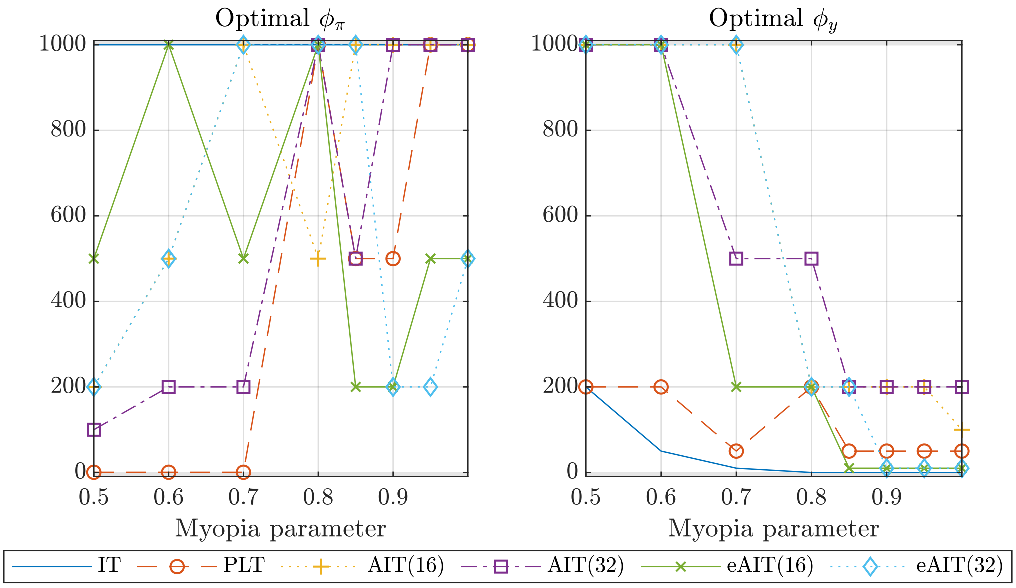

Figure 21: Optimised parameters, all monetary policy rules, demand shock

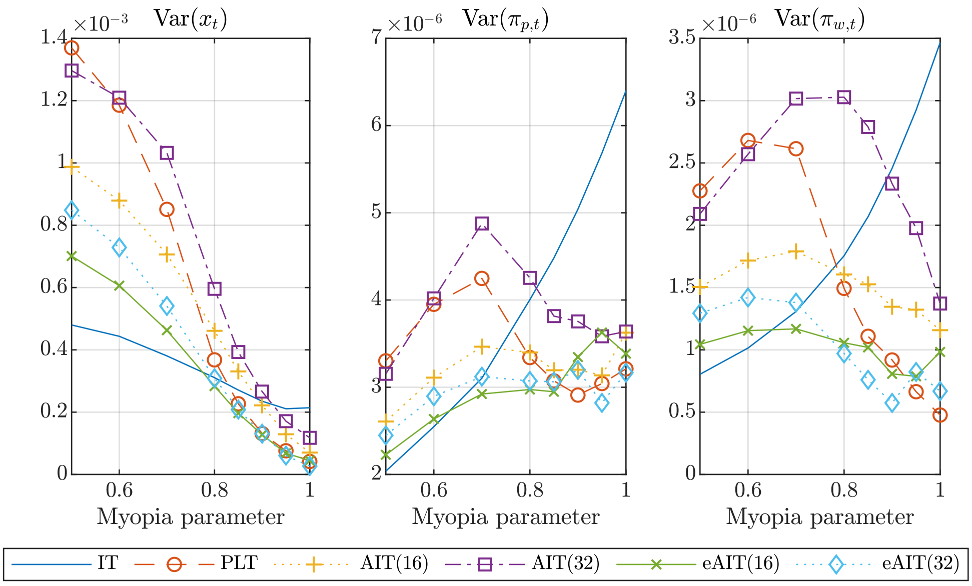

Figure 22: Variances, all monetary policy rules, TFP shock

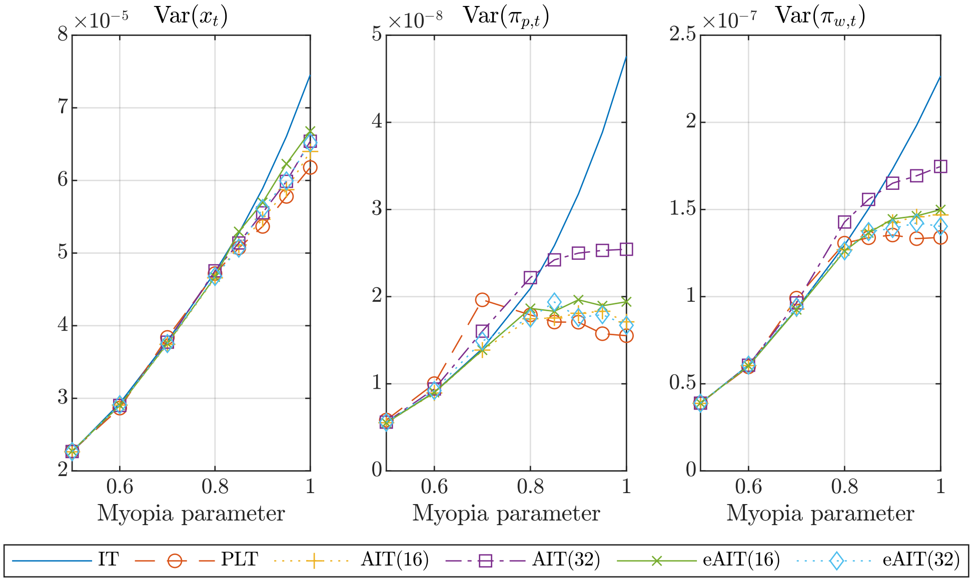

Figure 23: Variances, all monetary policy rules, demand shock

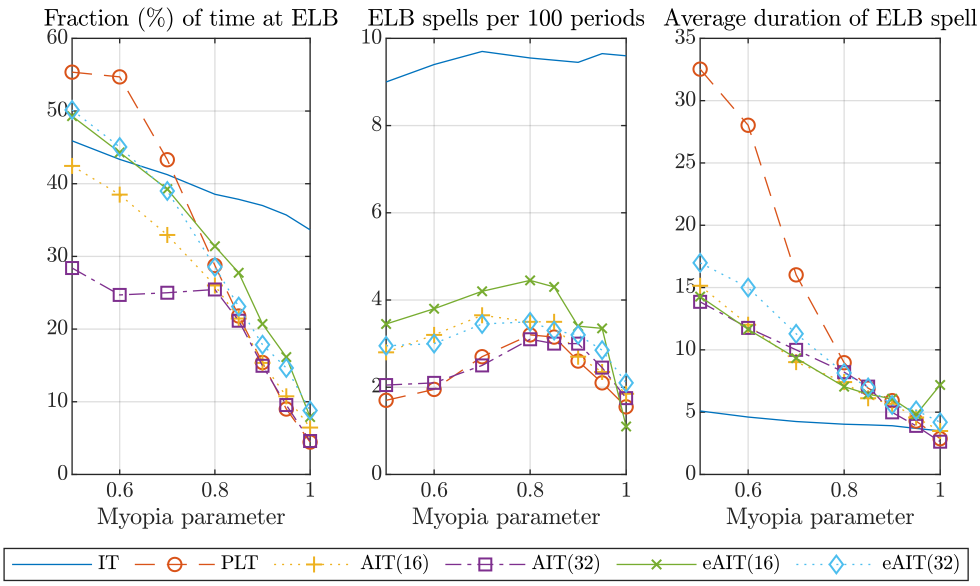

Figure 24: ELB statistics, all monetary policy rules, TFP shock

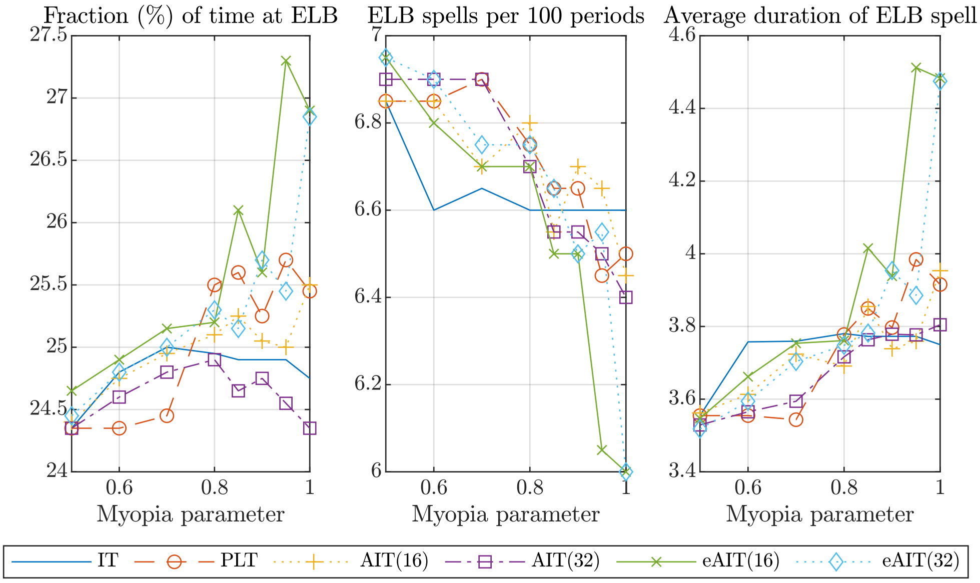

Figure 25: ELB statistics, all monetary policy rules, demand shock

TFP Shock, Allowing for a Negative Reaction Parameter on Output

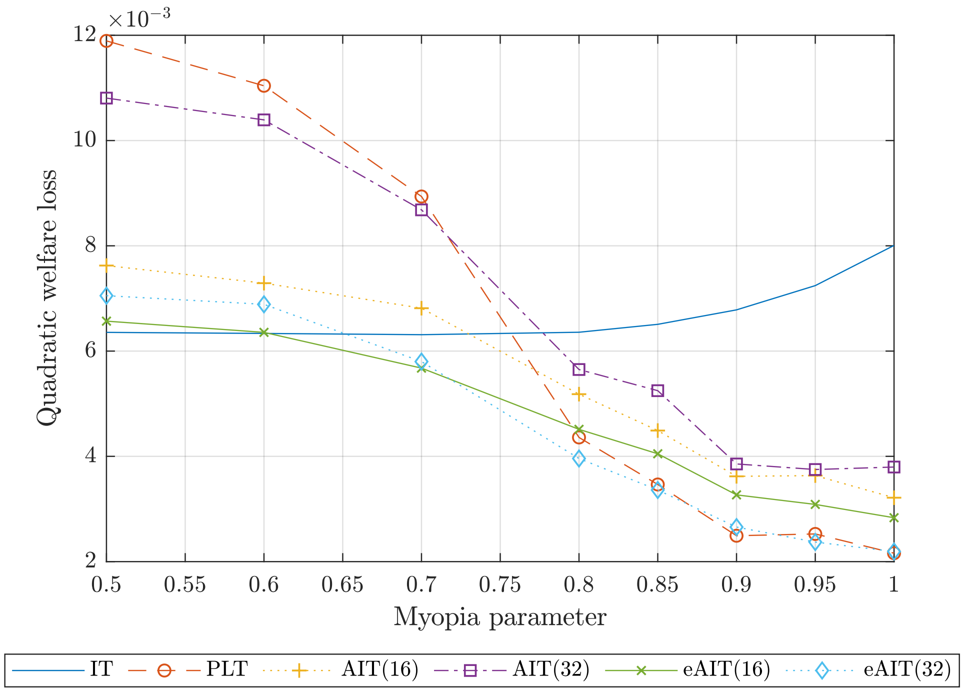

Figure 26: Welfare comparison for technology shocks, with negative output coefficients allowed

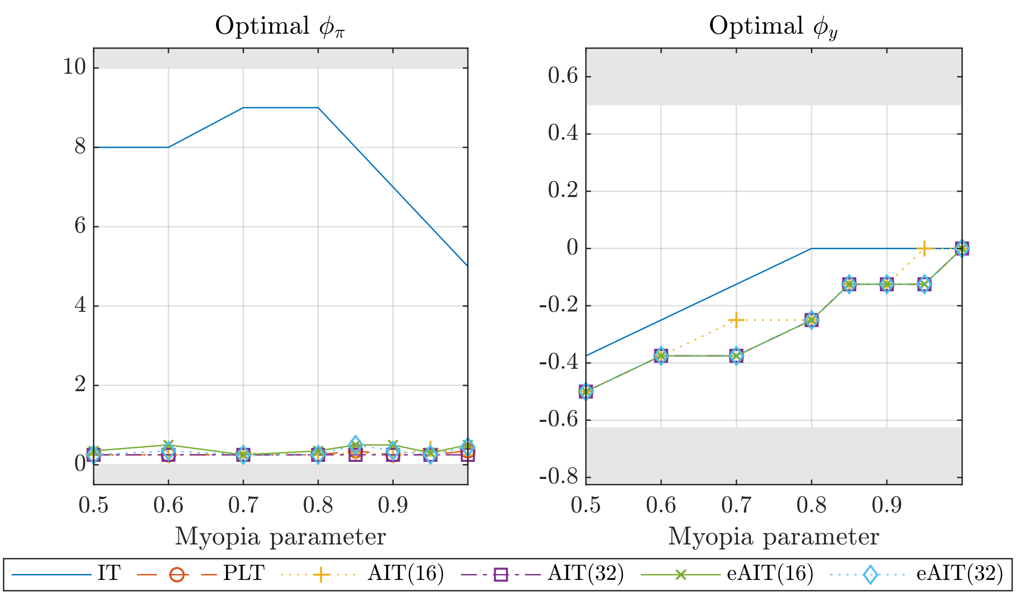

Figure 27: Optimal reaction parameters, TFP shock, with negative output coefficients allowed

TFP Shock, Keeping Volatility of Natural Rate Constant

In Figure 28 we provide the welfare analysis for the TFP shock when we rescale the variance of the TFP shock so as to keep the variance of the natural rate constant (and allow for negative output coefficients). From the results in the main text, it is evident that this rescaling scales down the variance of natural output by quite a lot. As a result, all variances decrease and the economy hits the ELB less often (for $M < 0.8$, it only hits it with IT or PLT, not with any form of AIT or eAIT). This affects the welfare ranking, making IT and PLT perform relatively worse, even though on average, all rules now become better for lower myopia. However, we still observe that IT 'catches up' with history-dependent strategies. Also, the eAIT rules still perform relatively good.

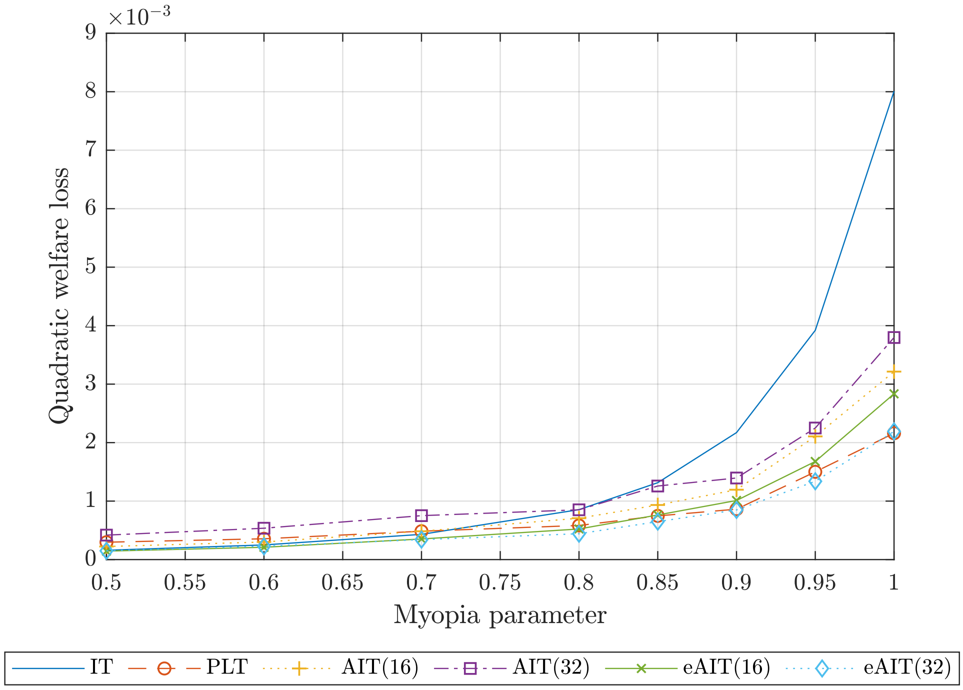

Figure 28: Welfare comparison for technology shocks, variance of natural rate being kept constant

C Joint shocks

In the main text we evaluated various interest rate rules separately for demand and supply shocks. This enabled us not only to clearly study the economic mechanisms underlying each shock but also their relative importance. Especially supply shocks trigger large welfare differences between different rules and are responsible for the volatility-inducing character of AIT.

However, in real time policy makers rarely face demand and supply shocks separately but have to react to a mix of both. In consequence, an empirically realistic mix of shocks becomes relevant. Therefore, we estimate the model with Bayesian techniques and are hence able to extract, among others, the relative shock strength for the demand and supply shock as well as the myopia parameter M. Given the estimated parameters (except the ones for the interest-rate rule), we then re-optimise the parameters in the different interest-rate rules and evaluate the rules' relative welfare performances for joint demand and supply shocks.

C.1 Estimation

To estimate our model we use quarterly European data from 1999Q1 to 2021Q3. The observables are output growth, inflation, wage growth and the short-term interest rate. We use the shadow short rate as computed by the term-structure model in Geiger and Schupp (2018) as a measure for the short-term interest rate, as our data sample includes periods where the policy rate remained at the effective lower bound with unconventional policy measures in place.[32]

We make several adaptations to our model to achieve a better fit of the data and obtain convergence in the parameter estimates. First, we include a smoothing term in the interest rate rule to capture the persistence inherent in data on interest rates. Second, we use output growth in the interest rate rule instead of the output gap. Third, we add further shocks to the model in order to avoid stochastic singularity. As our model contains two shocks but uses four observables for estimation, we add a monetary policy shock ν_t and a measurement error on wage growth m_t. [33]

Priors

We split parameters into two sets. We calibrate the first set of parameters as we have a rather small model and therefore a parsimonious set of data series. These parameters can not be well identified through estimation. This regards the discount factor β, risk aversion σ, the inverse Frisch elasticity φ, the production returns to scale α, the elasticity of substitution between goods ε_p, and the elasticity of substitution between workers ε_w. The calibrated values correspond to table 1, except for the discount factor, which we set to 0.99 to account for the higher real interest rate in our data.

We estimate parameters in the second set. The priors we use are shown in table 2 and mostly follow Smets and Wouters (2007). Otherwise, we choose the prior of the persistence parameter in the monetary policy shock process ρ_ν to be the same as for the other persistence parameters. We harmonise the prior means for the standard deviations in the monetary policy innovation and the measurement error so that they both approximately equal 10% of the standard deviation in the shadow rate time series and the real wage growth time series, respectively. The prior standard deviation for those shock innovations equals the other prior

[32] For details about the data and our observation equations, see appendix D

[33] The interest rate rule that we use for estimating the model thus reads

$$i_t^* = \rho_{i^*} i_{t-1}^* + (1-\rho_{i^*})\left(\rho + \pi_t + \phi_\pi(\pi_t - \pi^*) + \phi_y y_t\right) + \nu_t, \tag{82}$$

where $\nu_t = (1-\rho_\nu)\bar{\nu} + \rho_\nu \nu_{t-1} + \varepsilon_{\nu,t}$, with $\varepsilon_{\nu,t} \sim \mathcal{N}(0, \sigma_\nu^2)$. Nevertheless, in the welfare analysis of the subsequent section we use the rules as defined in section 3.3 with re-optimised parameters. The rule presented here is adapted merely for estimation purposes.

Table 2: Prior distribution of estimated parameters

			Prior	
		Dist	Mean	SE
Stickiness				
θ_p	Price stickiness	B	0.500	0.100
θ_w	Wage stickiness	B	0.500	0.100
Interest-rate rule				
ρ_{i^*}	MP smoothing	B	0.750	0.100
ϕ_π	MP on inflation	N	1.500	0.250
ϕ_y	MP on output	N	0.125	0.050
AR(1) shocks				
ρ_a	Supply	B	0.500	0.200
ρ_z	Demand	B	0.500	0.200
ρ_v	Monetary policy	B	0.500	0.200
Std shocks				
σ_a	Supply	IG	0.100	2.000
σ_z	Demand	IG	0.100	2.000
σ_v	Monetary policy	IG	0.001	2.000
σ_m	Measurement error	IG	0.001	2.000
M	Myopia	B	varied	varied

Notes: B abbreviates beta, G gamma, IG inverted gamma and N normal distribution.

standard deviations of the shock innovations. Hence, the prior for these parameters is an inverse gamma distribution with mean 0.001 and standard deviation 2.

So far, there have been only few attempts to structurally estimate the myopia parameter M in the literature and prior specifications and posterior moments differ widely. Papers that allow for a flexible, more agnostic prior typically estimate a high degree of myopia. Ilabaca et al. (2020) estimate a posterior mean between 0.41 and 0.60 for firms and between 0.71 and 0.85 for households assuming a beta prior with mean 0.8 and standard deviation 0.15. Meggiorini and Milani (2021) choose the same prior but arrive at a lower posterior mean of 0.42.[34] Brzoza-Brzezina et al. (2022) choose a beta prior with mean 0.85 and standard deviation 0.05 and estimate a posterior mean of 0.71. Gómez et al. (2022) choose a lower prior mean of 0.75 and standard deviation 0.15 and arrive at a posterior median between 0.36 and 0.39. By contrast, Erceg et al. (2021) select a very tight prior assuming close to rational expectations. Their beta prior with mean 0.975 and standard deviation 0.0125 features almost no probability mass below 0.92 and yields a posterior mode of 0.95.[35]

Instead of choosing a specific prior for the degree of myopia, we opt to remain agnostic and run multiple estimations with different prior means and standard deviations. For this purpose we take Ilabaca et al. (2020) and Erceg et al. (2021) as polar cases and estimate our

[34]This is the estimated posterior mean of their expectational assumption that is comparable to ours. They show that the posterior mean of the myopia parameter varies quite a lot with the underlying expectational assumption. For example, if they assume subjective expectations with Euler equation learning, they arrive at a posterior mean of 0.94.

[35]In their baseline specification, they assume that only the households are myopic. In a robustness exercise they also assume that firms are myopic and choose the same prior distribution for the firms' myopia parameter as for the one of households. In this case, they arrive at a posterior median of 0.94 for the households' myopia parameter and 0.95 for the firms' myopia parameter.

Table 3: Prior and posterior distributions of myopia parameter M

	Dist	Prior Mean	Prior SE	Posterior Mean	Posterior 5 %	Posterior 95 %	LML	$\frac{\sigma_a}{\sigma_z}$
Ilabaca et al. (2020)	B	0.800	0.150	0.449	0.256	0.652	1421.751	0.428
Erceg et al. (2021)	B	0.975	0.0125	0.951	0.912	0.991	1403.970	1.277
Intermediate 1	B	0.850	0.100	0.558	0.401	0.712	1419.920	0.479
Intermediate 2	B	0.950	0.030	0.798	0.717	0.878	1410.962	0.867
Intermediate 3	B	0.900	0.050	0.718	0.621	0.815	1415.701	0.688

Notes: B abbreviates beta distribution. LML is log marginal likelihood, computed using the modified harmonic mean approximation.

model additionally with "intermediate" priors. The specifications are shown in columns two to four in table 3 and refer to both polar cases as well as priors close to Ilabaca et al. (2020) ("Intermediate 1"), close to Erceg et al. (2021) ("Intermediate 2") or approximately between both cases ("Intermediate 3").

Estimation results

Our estimation results are shown in columns five to eight of table 3.[36]

Three results are apparent. First, we underscore that the prior distribution of the myopia parameter has a substantial influence on its posterior distribution. A wide prior as e.g. in Ilabaca et al. (2020) results in a low posterior mean of 0.449 implying high degrees of boundedness of expectations. Instead, a tight prior as in Erceg et al. (2021) results in a high posterior mean of 0.951 implying close to rational expectations.

Second, the estimation favors a model with a high degree of myopia. The log marginal likelihood is highest for the model imposing the prior in Ilabaca et al. (2020) and lowest for the model imposing the prior in Erceg et al. (2021). Generally, a lower prior mean and thus a lower posterior mean results in a higher log marginal likelihood.[37]

Third, the importance of the supply shock relative to the demand shock is highly dependent on the degree of myopia. Supply shocks are more important and their size relative to demand shocks highest when myopia is estimated to be low as with the Erceg et al. (2021) prior. Besides this polar case however, we estimate demand shocks to be considerably more important especially for high degree of myopia, which is the case under all other priors. Generally, the higher the degree of myopia, the higher the importance of demand shocks relative to supply shocks.

[36] We draw from the posterior distribution using the Metropolis-Hastings algorithm with one chain and 300,000 draws. We discard the first 50% of the draws as burn-in and validate convergence by means of the test proposed in Geweke (1992). We use Dynare to perform the estimation, see Adjemian et al. (2021).

[37] This does not hold generally. For example, if we decrease the prior mean to 0.5 (and keep the prior standard deviation at 0.15 as in Ilabaca et al. (2020)), the log marginal likelihood is much lower, namely 1123.166 (and the posterior mean of the myopia parameter is 0.340).

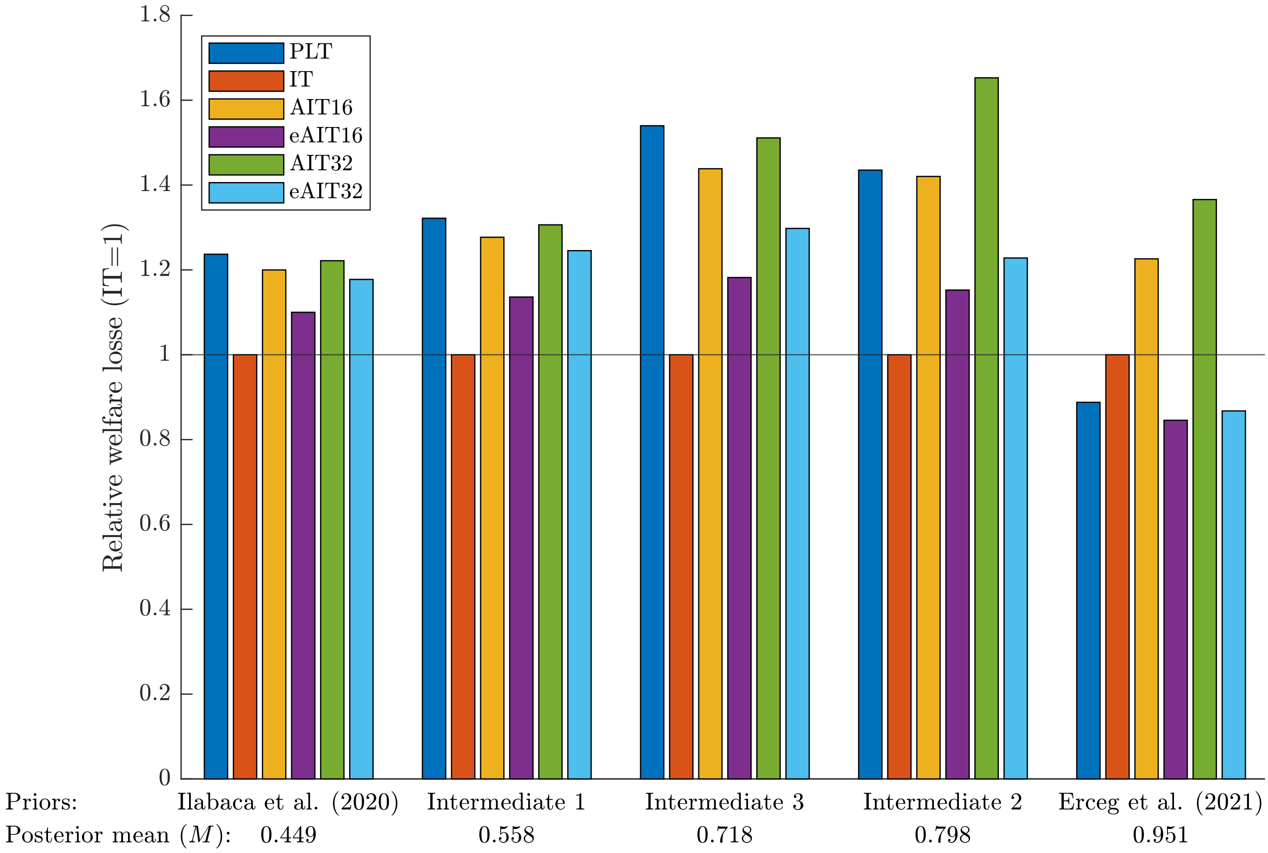

Figure 29: Welfare comparison of all monetary policy rules, joint shocks

C.2 Welfare rankings with estimated parameters and joint shocks

Given our estimation results we compare welfare across interest-rate rules when the economy is subject to both supply and demand shocks. For this purpose we re-optimise our policy rules given the estimated values for the persistence and relative standard deviations of the shocks and the degree of price and wage stickiness θ_p and θ_w. [38]

Figure 29 shows our welfare comparison. Each set of bars corresponds to an estimation resulting from a different prior for the degree of myopia. On the left we show the wide prior as in Ilabaca et al. (2020) and on the right the tight prior of Erceg et al. (2021). The bars are normalised so that the welfare loss is shown relative to that of the IT rule.

Overall, the insights of our single shock analysis carry over to the analysis with joint shocks. First, as the degree of myopia increases PLT looses the advantage of its history-dependency and the IT rule increasingly performs better. Second, the eAIT rule is the only strategy that can keep up with PLT when expectations are close to rational as shown in the far right set of bars in figure 29. Third, eAIT is the most robust history-dependent strategy across high degrees of myopia. As discussed above, this is because in the eAIT rule, past inflation deviations are assigned a higher weight the closer they are to the present period and thus "tilt" the character of the eAIT rule towards the IT rule for high degrees of myopia.

[38] To be consistent with the welfare calculation in the case of single shocks, we calibrate the absolute values of the standard deviations to target an ELB frequency of 20%.

Thus, the welfare performance of the eAIT rule manages to stay close to that of the IT rule, which is the best performing rule for high degrees of myopia.

D Estimation details

D.1 Data

We use quarterly European data from 1999Q1 to 2021Q3 and as observables output growth, inflation, wage growth and the short-term interest rate.

Definition of observables

Real per capita output growth: $\Delta Y_t^{dat} = log\left(\frac{YER_t}{LFN_t}\right) - log\left(\frac{YER_{t-1}}{LFN_{t-1}}\right) - \text{mean}(\Delta Y^{dat})$

Net inflation rate: $\Pi_t^{dat} = log(HICP_t^Q) - log(HICP_{t-1}^Q)$

Shadow short rate: $SSR_t^{dat} = SSR_t^Q/400$

Real wage growth: $\Delta w_t^{dat} = log\left(\frac{WRN_t}{HICP_t^Q}\right) - log\left(\frac{WRN_{t-1}}{HICP_{t-1}^Q}\right) - \text{mean}(\Delta w^{dat})$

Observation equations

$$\begin{aligned}
\Delta Y_t^{dat} &= y_t - y_{t-1} \\
\Pi_t^{dat} &= \pi_t \\
SSR_t^{dat} &= i_t^* \\
\Delta w_t^{dat} &= w_t - w_{t-1} + m_t,
\end{aligned}$$

where $m_t \sim \mathcal{N}(0, \sigma_m^2)$ is a measurement error on real wage growth.

Data description

YER: Gross domestic product at market prices - Euro area 19 (fixed composition) - Domestic (home or reference area), Total economy, Euro, Chain linked volume (rebased), Non transformed data, Calendar and seasonally adjusted data. Source: Eurostat.[39]

LFN: Labor force calculated as $LFN_t = \frac{LNN_t}{1 - \frac{URX_t}{100}}$.

LNN: Total employment (in thousands of persons) - Euro area 19 (fixed composition) as of 1 January 2015 - Domestic (home or reference area), Total economy, Total - All activities, Persons, Not applicable, Non transformed data, Calendar and seasonally adjusted data. Source: Eurostat.[40]

[39] Series key: MNA.Q.Y.I8.W2.S1.S1.B.B1GQ._Z._Z._Z.EUR.LR.N
[40] Series key: ENA.Q.Y.I8.W2.S1.S1._Z.EMP._Z._T._Z.PS._Z.N

URX: Unemployment rate - Euro area 19 (fixed composition) as of 1 January 2015; European Labour Force Survey; Total; Age 15 to 74; Total; Seasonally adjusted, not working day adjusted. Source: Eurostat.[41]

HICP^Q: Quarterly harmonised index of consumer inflation calculated from monthly data as $HICP_t^Q = \frac{1}{3}\sum_{i=t-2}^{t} HICP_i^M$.

HICP^M: Monthly harmonised index of consumer prices - Euro area (changing composition) - Overall index, Working day and seasonally adjusted. Source: European Commission (Eurostat) and European Central Bank calculations based on Eurostat data.[42]

SSR^Q: Quarterly shadow short rate calculated from monthly data as $SSR_t^Q = \frac{1}{3}\sum_{i=t-2}^{t} SSR_i^M$.

SSR^M: Monthly shadow short rate as calculated in Geiger and Schupp (2018).

WRN: Quarterly nominal wage per employee calculated as $WRN_t = \frac{WIN_t}{LNN_t}$.

WIN: Quarterly compensation of employees - Euro area 19 (fixed composition) as of 1 January 2015 - Current prices, million euro - sesonally and calender adjusted. Source: Eurostat.[43]

[41] Series key: LFSI.Q.I8.S.UNEHRT.TOTAL0.15_74.T
[42] Series key: ICP.M.U2.Y.000000.3.INX
[43] Series key: NAMQ_10_GDP

D.2 Priors and posteriors

Table 4: Prior and posterior distribution of estimated parameters, prior distribution of M as in Ilabaca et al. (2020)

			Prior		Posterior			
		Dist	Mean	SE	Mode	Mean	5 percent	95 percent
Stickiness								
θ_p	Price stickiness	B	0.5000	0.1000	0.6160	0.6167	0.5575	0.6739
θ_w	Wage stickiness	B	0.5000	0.1000	0.6828	0.6801	0.6196	0.7412
Interest-rate rule								
ρ_{i^*}	MP smoothing	B	0.7500	0.1000	0.9331	0.9320	0.9059	0.9582
ϕ_π	MP on inflation	N	1.5000	0.2500	1.6256	1.6160	1.2298	1.9842
ϕ_y	MP on output	N	0.1250	0.0500	0.1131	0.1130	0.0396	0.1849
AR(1) shocks								
ρ_a	Supply	B	0.5000	0.2000	0.3015	0.3179	0.1582	0.4656
ρ_z	Demand	B	0.5000	0.2000	0.8846	0.8833	0.8278	0.9400
ρ_v	Monetary policy	B	0.5000	0.2000	0.4828	0.4851	0.3367	0.6438
std shocks								
σ_a	Supply	IG	0.1000	2.0000	0.0347	0.0367	0.0239	0.0489
σ_z	Demand	IG	0.1000	2.0000	0.0752	0.0858	0.0497	0.1214
σ_v	Monetary policy	IG	0.0010	2.0000	0.0009	0.0010	0.0008	0.0011
σ_m	Measurement error	IG	0.0010	2.0000	0.0082	0.008	0.0072	0.0092
M	Myopia	B	0.8000	0.1500	0.4909	0.4492	0.2560	0.6517

Notes: B stands for the beta, G for the gamma, IG for the inverted gamma and N for the normal distribution.

Table 5: Prior and posterior distribution of estimated parameters, prior distribution of M as in Erceg et al. (2021)

			Prior		Posterior			
		Dist	Mean	SE	Mode	Mean	5 percent	95 percent
Stickiness								
θ_p	Price stickiness	B	0.5000	0.1000	0.6674	0.6698	0.6105	0.7278
θ_w	Wage stickiness	B	0.5000	0.1000	0.7160	0.7331	0.6419	0.8288
Interest-rate rule								
ρ_{i^*}	MP smoothing	B	0.7500	0.1000	0.9076	0.9052	0.8839	0.9270
ϕ_π	MP on inflation	N	1.5000	0.2500	2.0510	1.9969	1.6327	2.3640
ϕ_y	MP on output	N	0.1250	0.0500	0.1579	0.1609	0.0946	0.2291
AR(1) shocks								
ρ_a	Supply	B	0.5000	0.2000	0.4765	0.4588	0.2417	0.6568
ρ_z	Demand	B	0.5000	0.2000	0.9019	0.9079	0.8708	0.9413
ρ_v	Monetary policy	B	0.5000	0.2000	0.3775	0.3826	0.2596	0.5099
std shocks								
σ_a	Supply	IG	0.1000	2.0000	0.0382	0.0424	0.0231	0.0618
σ_z	Demand	IG	0.1000	2.0000	0.0302	0.0332	0.0256	0.0400
σ_v	Monetary policy	IG	0.0010	2.0000	0.0010	0.0011	0.0009	0.0012
σ_m	Measurement error	IG	0.0010	2.0000	0.0079	0.0080	0.0070	0.0090
M	Myopia	B	0.9750	0.0125	0.9676	0.9506	0.9116	0.9905

Notes: B stands for the beta, G for the gamma, IG for the inverted gamma and N for the normal distribution.

Table 6: Prior and posterior distribution of estimated parameters, prior distribution of M as in "Intermediate 1" specification in Table 3

		Prior			Posterior			
		Dist	Mean	SE	Mode	Mean	5 percent	95 percent
Stickiness								
θ_p	Price stickiness	B	0.5000	0.1000	0.6304	0.6320	0.5744	0.6912
θ_w	Wage stickiness	B	0.5000	0.1000	0.7056	0.7023	0.6446	0.7600
Interest-rate rule								
ρ_{i^*}	MP smoothing	B	0.7500	0.1000	0.9298	0.9292	0.9029	0.9551
ϕ_π	MP on inflation	N	1.5000	0.2500	1.6321	1.6215	1.2306	1.9936
ϕ_y	MP on output	N	0.1250	0.0500	0.1200	0.1191	0.0488	0.1926
AR(1) shocks								
ρ_a	Supply	B	0.5000	0.2000	0.3052	0.3143	0.1627	0.4636
ρ_z	Demand	B	0.5000	0.2000	0.8966	0.8979	0.8438	0.9495
ρ_v	Monetary policy	B	0.5000	0.2000	0.4862	0.4864	0.3313	0.6388
std shocks								
σ_a	Supply	IG	0.1000	2.0000	0.0371	0.0397	0.0252	0.0536
σ_z	Demand	IG	0.1000	2.0000	0.0687	0.0829	0.0452	0.1178
σ_v	Monetary policy	IG	0.0010	2.0000	0.0009	0.0010	0.0009	0.0011
σ_m	Measurement error	IG	0.0010	2.0000	0.0080	0.0082	0.0072	0.0092
M	Myopia	B	0.8500	0.1000	0.5948	0.5580	0.4009	0.7117

Notes: B stands for the beta, G for the gamma, IG for the inverted gamma and N for the normal distribution.

Table 7: Prior and posterior distribution of estimated parameters, prior distribution of M as in "Intermediate 2" specification in Table 3

		Prior			Posterior			
		Dist	Mean	SE	Mode	Mean	5 percent	95 percent
Stickiness								
θ_p	Price stickiness	B	0.5000	0.1000	0.6640	0.6702	0.6073	0.7321
θ_w	Wage stickiness	B	0.5000	0.1000	0.7681	0.7626	0.7074	0.8175
Interest-rate rule								
ρ_{i^*}	MP smoothing	B	0.7500	0.1000	0.9167	0.9154	0.8912	0.9420
ϕ_π	MP on inflation	N	1.5000	0.2500	1.7012	1.6944	1.3230	2.0790
ϕ_y	MP on output	N	0.1250	0.0500	0.1470	0.1446	0.0733	0.2113
AR(1) shocks								
ρ_a	Supply	B	0.5000	0.2000	0.3370	0.3424	0.1816	0.5040
ρ_z	Demand	B	0.5000	0.2000	0.9200	0.9204	0.8813	0.9570
ρ_v	Monetary policy	B	0.5000	0.2000	0.4619	0.4614	0.3115	0.6001
std shocks								
σ_a	Supply	IG	0.1000	2.0000	0.0440	0.0488	0.0274	0.0698
σ_z	Demand	IG	0.1000	2.0000	0.0494	0.0563	0.0338	0.0769
σ_v	Monetary policy	IG	0.0010	2.0000	0.0010	0.0010	0.0009	0.0011
σ_m	Measurement error	IG	0.0010	2.0000	0.0080	0.0082	0.0072	0.0092
M	Myopia	B	0.9500	0.0300	0.8098	0.7979	0.7165	0.8783

Notes: B stands for the beta, G for the gamma, IG for the inverted gamma and N for the normal distribution.

Table 8: Prior and posterior distribution of estimated parameters, prior distribution of M as in "Intermediate 3" specification in Table 3

		Prior			Posterior			
		Dist	Mean	SE	Mode	Mean	5 percent	95 percent
Stickiness								
θ_p	Price stickiness	B	0.5000	0.1000	0.6531	0.6564	0.5971	0.7161
θ_w	Wage stickiness	B	0.5000	0.1000	0.7450	0.7405	0.6854	0.7938
Interest-rate rule								
ρ_{i^*}	MP smoothing	B	0.7500	0.1000	0.9221	0.9215	0.8963	0.9482
ϕ_π	MP on inflation	N	1.5000	0.2500	1.6607	1.6496	1.2872	2.0344
ϕ_y	MP on output	N	0.1250	0.0500	0.1362	0.1360	0.0650	0.2059
AR(1) shocks								
ρ_a	Supply	B	0.5000	0.2000	0.3207	0.3246	0.1660	0.4751
ρ_z	Demand	B	0.5000	0.2000	0.9137	0.9136	0.8711	0.9550
ρ_v	Monetary policy	B	0.5000	0.2000	0.4799	0.4792	0.3305	0.6268
std shocks								
σ_a	Supply	IG	0.1000	2.0000	0.0417	0.0451	0.0275	0.0623
σ_z	Demand	IG	0.1000	2.0000	0.0572	0.0656	0.0390	0.0905
σ_v	Monetary policy	IG	0.0010	2.0000	0.0010	0.0010	0.0009	0.0011
σ_m	Measurement error	IG	0.0010	2.0000	0.0080	0.0082	0.0072	0.0092
M	Myopia	B	0.9000	0.0500	0.7381	0.7183	0.6207	0.8153

Notes: B stands for the beta, G for the gamma, IG for the inverted gamma and N for the normal distribution.

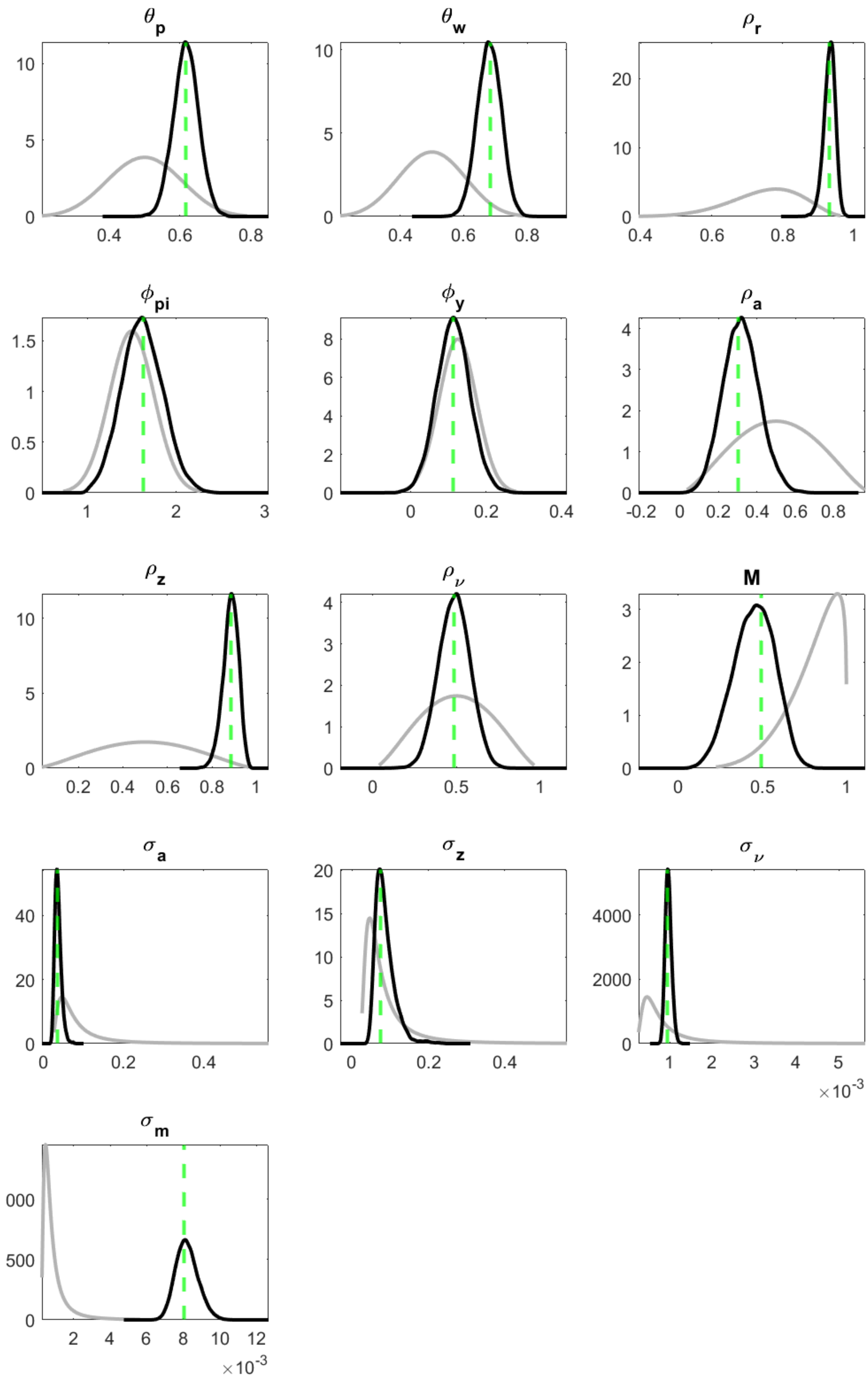

Figure 30: Prior and posterior plots of estimated parameters, prior distribution of M as in Ilabaca et al. (2020)

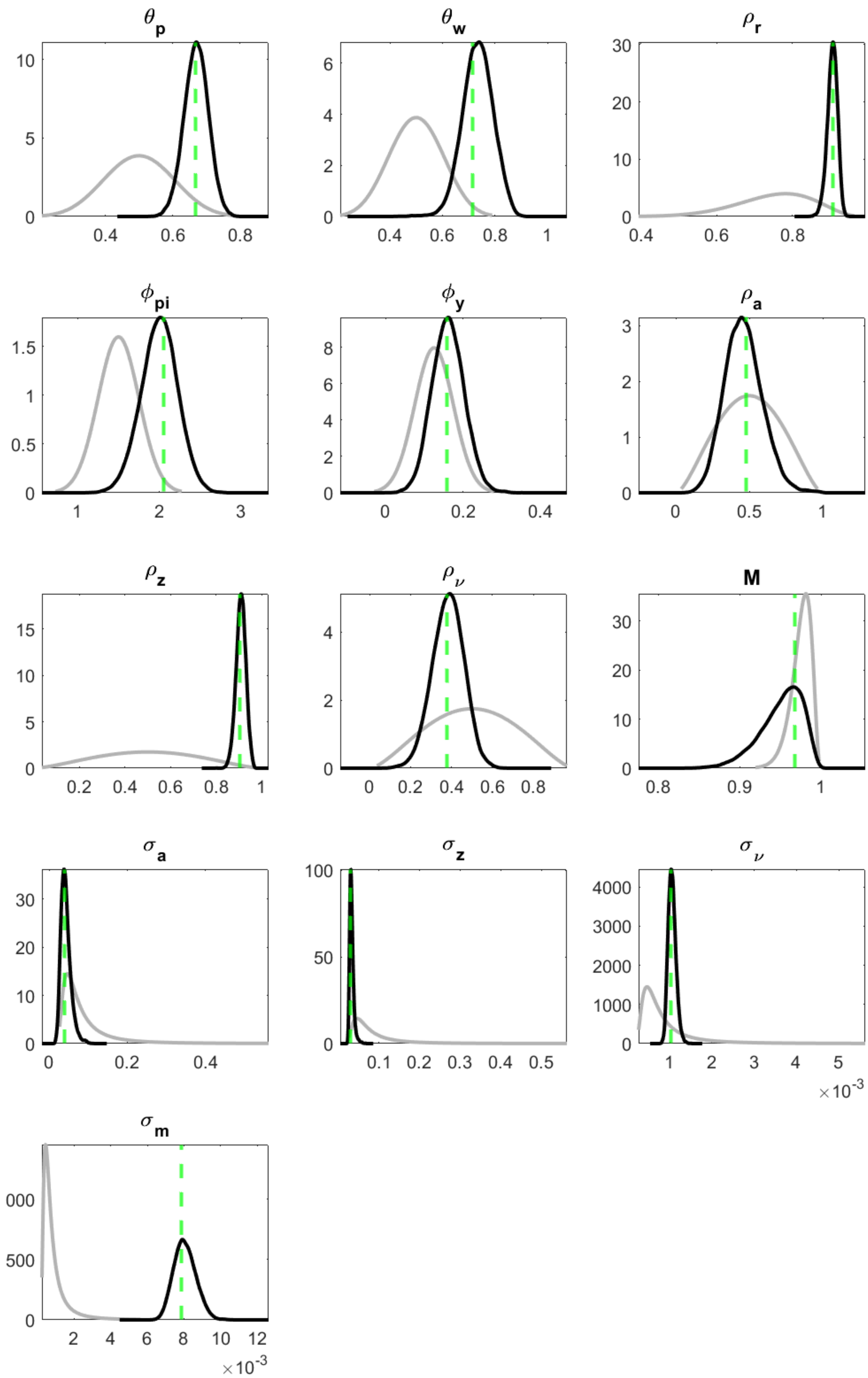

Figure 31: Prior and posterior plots of estimated parameters, prior distribution of M as in Erceg et al. (2021)

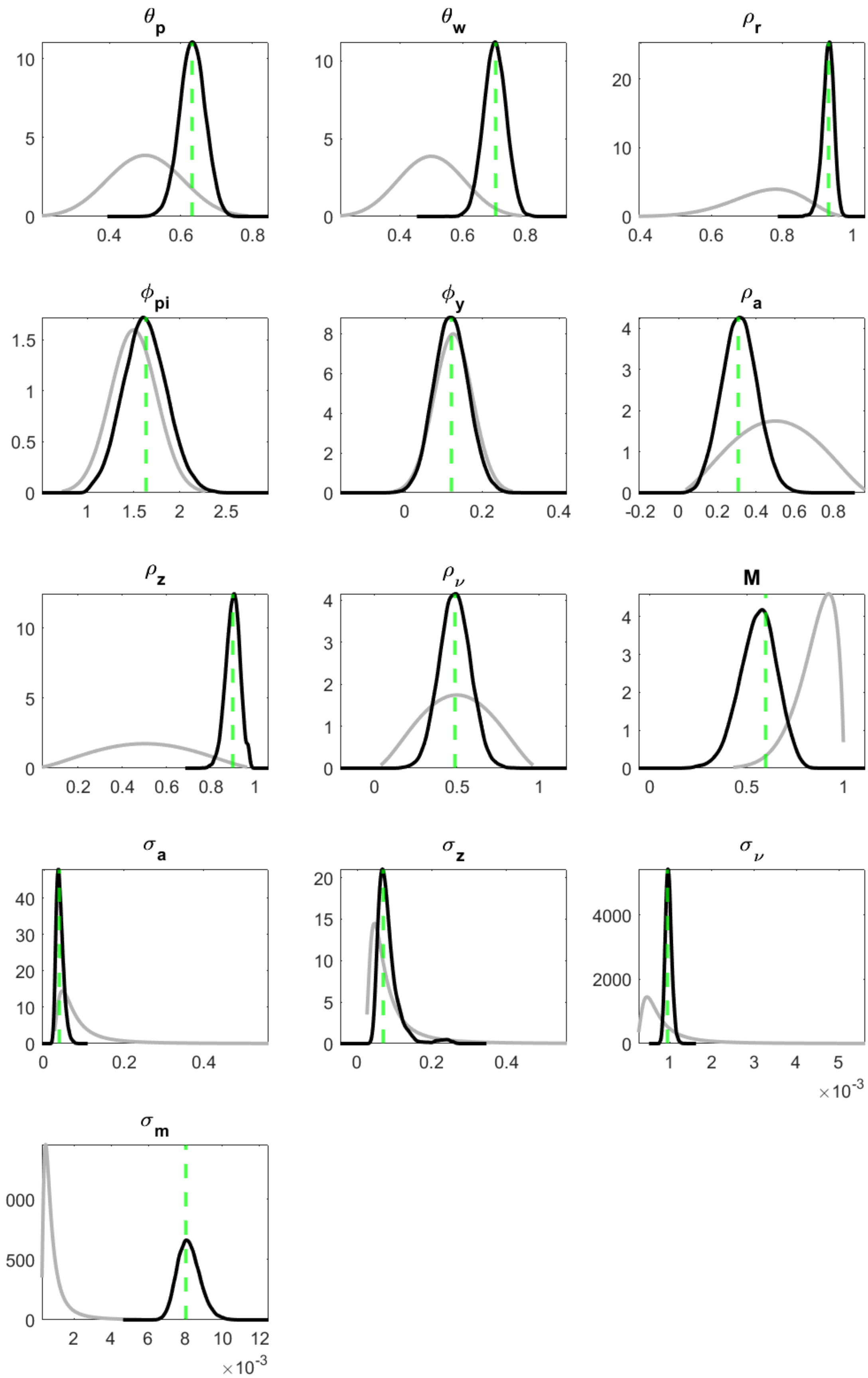

Figure 32: Prior and posterior plots of estimated parameters, prior distribution of M as in "Intermediate 1" specification in Table 3

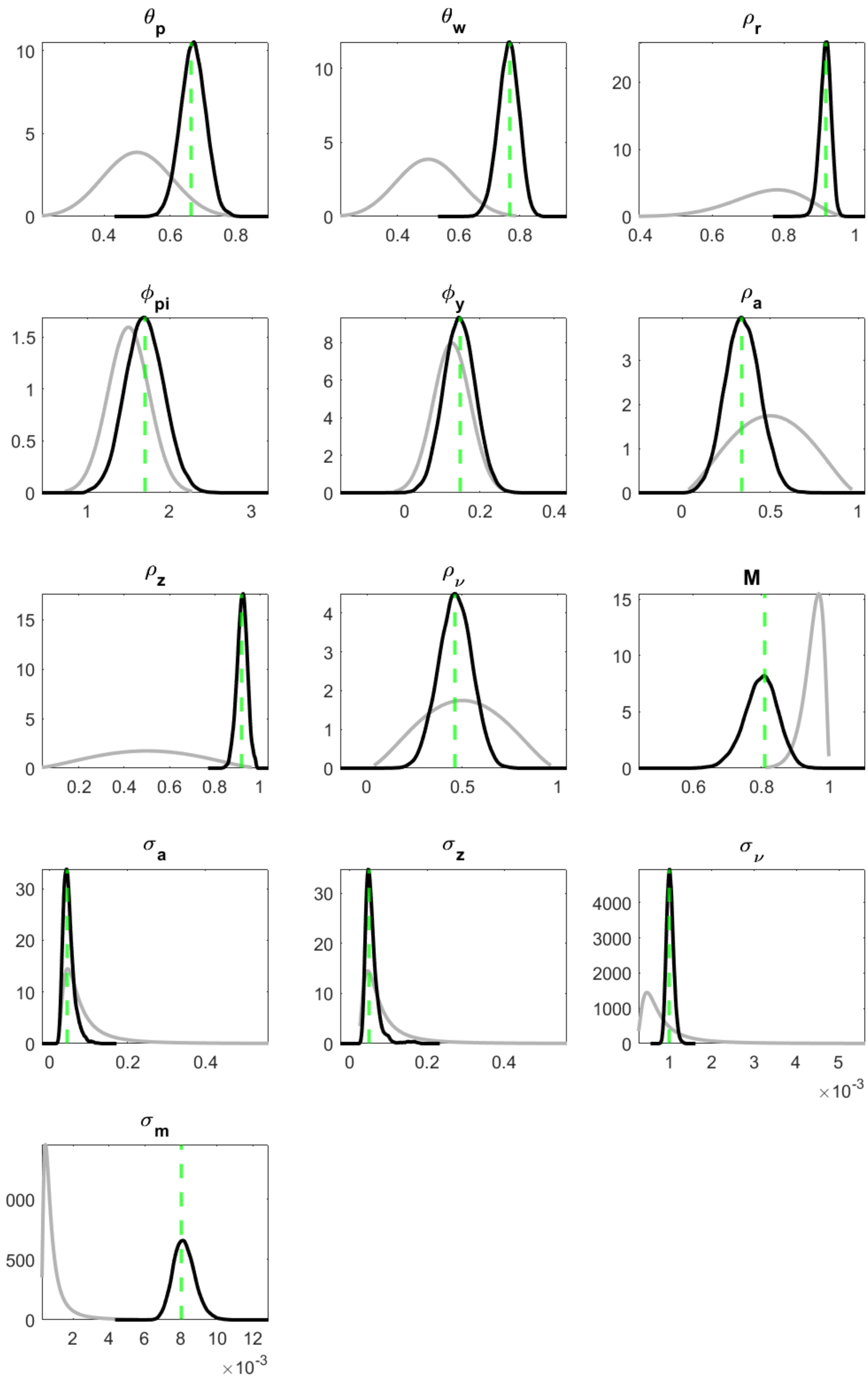

Figure 33: Prior and posterior plots of estimated parameters, prior distribution of M as in "Intermediate 2" specification in Table 3

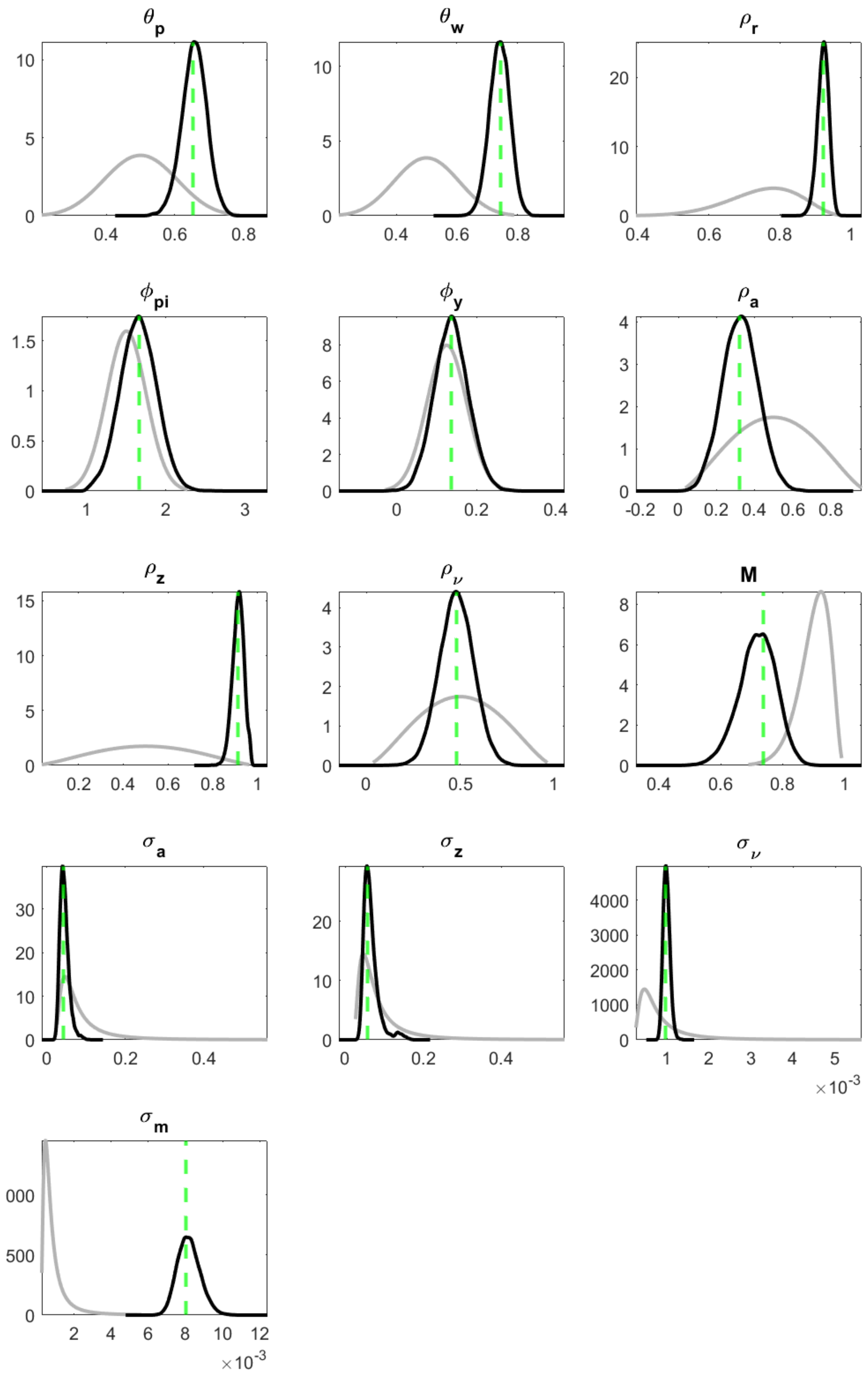

Figure 34: Prior and posterior plots of estimated parameters, prior distribution of M as in "Intermediate 3" specification in Table 3

E Implications of sticky wages under bounded rationality

In this appendix we analyse the interactions between sticky wages and boundedly rational expectations. As shown in Gabaix (2020), bounded rationality in itself suffices to solve a number of paradoxes arising in New Keynesian models. We extend this insight and show that wages stickiness in addition to price stickiness considerably enhances bounded rationality as a solution to these paradoxes. We first focus on determinacy properties, the severity of zero lower bound episodes and the forward guidance puzzle. The channel that is responsible for the paradoxes is the large impact of expectations of future variables on current economic outcomes. The strong performance of history-dependent policy rules in the traditional New Keynesian model depends on the same channel. Thus, mitigating the paradoxes by weakening this channel also reduces the power of history-dependent policy rules. Subsequently, we additionally analyse the influence of boundedly rational expectations and sticky wages on the Neo-Fisherian paradox, the paradox of toil, and the paradox of flexibility.

E.1 Enhanced determinacy

The traditional New Keynesian model suffers from the existence of a continuum of equilibria if monetary policy is sufficiently passive. This occurs when the Taylor principle is violated, i.e. when monetary policy acts according to an IT rule but reacts to inflation deviations less than one-for-one (in our setup: $\phi_\pi < 0$). This exposes the economy to the possibility of self-fulfilling sunspot fluctuations therefore implying indeterminacy. Panel a) of Figure 35 shows such determinacy regions in the ϕ_y-ϕ_π-space when all agents have either rational (solid lines) or boundedly rational (dashed lines) expectations contrasting the case when only prices (red lines) or also wages (blue lines) are sticky.

Under rational expectations, obeying the Taylor principle is crucial to guarantee existence of a unique equilibrium if monetary policy does not react to output deviations ($\phi_y = 0$).[44] This holds irrespective of the degree of stickiness of prices and wages. In the standard model, sticky wages enlarges the determinacy issues but only if monetary policy also responds to output deviations ($\phi_y > 0$).

Under bounded rationality and in combination with sticky wages, a number of new results emerge. First, even if monetary policy does not respond to output, the Taylor principle need not be satisfied. Crucially however, the degree to which monetary policy can deviate from the Taylor principle depends on the degree of myopia as well as the degree of wage stickiness. If only prices are sticky and expectations are modestly bounded ($M = 0.9$ as in Figure 35), only small deviations from the Taylor principle are possible. In contrast, if wages are also sticky, substantial deviations from the Taylor principle are possible. Second, this implies that under sticky wages a smaller degree of myopia is necessary to guarantee

[44] This holds for a target inflation rate of zero. If the target inflation rate is above zero, the Taylor principle is – in general – not sufficient to guarantee the existence of a unique equilibrium for $\phi_y = 0$, see Ascari and Sbordone (2014). But with our assumption of perfect indexation for non-optimising firms, this issue becomes irrelevant.

uniqueness of equilibria.[45] Third, with an increasingly stronger reaction to output, the reaction to inflation required to achieve determinacy is increasingly reduced with sticky wages compared to sticky prices only. This holds for rational expectations as well, but is substantially stronger under myopia. If expectations are bounded, sticky wages cause a particularly pronounced enlargement of the determinacy region.

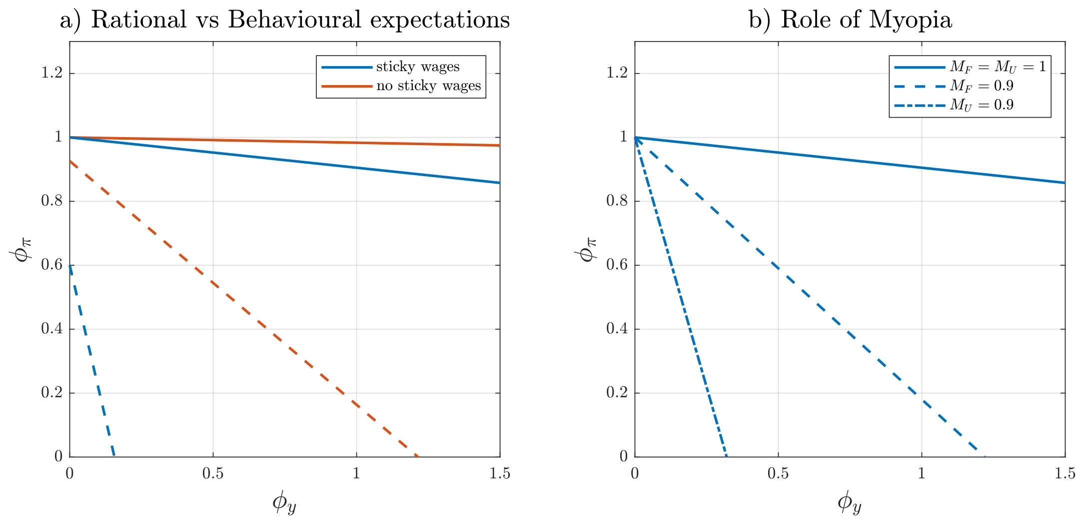

Figure 35: Determinacy

Note: The left-hand panel shows determinacy regions as a function of monetary policy parameters comparing the case when only prices or also wages are sticky. Solid lines indicate rational expectations, dashed lines indicate behavioural expectations with $M = 0.9$. Areas below the lines are indeterminacy regions for the respective case. The right-hand panel shows the influence of partial myopia under sticky wages.

Panel b) of Figure 35 breaks down relative contributions of myopia to the improved determinacy properties under sticky prices and wages. We set either $M_F = 0.9$ or $M_U = 0.9$, while keeping the other myopia factors at 1. Myopic firms or unions, i.e., myopia in just the price or the wage Phillips curve, tilts the boundary of indeterminacy without relaxing the Taylor principle if the central bank does not react to output gap deviations. However, myopia in the wage Phillips curve requires only little additional responsiveness to output gap deviations ($\phi_y > 0.3$) to guarantee uniqueness compared to myopia in the price Phillips curve ($\phi_y > 0.9$). This implies that myopia on the side of wage unions helps more to eliminate multiple equilibria.[46]

E.2 Decreased severity of the ELB

To study the implications of the ELB under bounded rationality with sticky wages, we conduct the following experiment:[47] Let monetary policy follow the IT rule (19) with $\phi_\pi = 0.5$

[45] Gabaix(2020) terms this the "strong bounded rationality principle", i.e. the degree of myopia necessary to guarantee determinacy. Under sticky prices only, this principle requires $M = 0.8$, whereas under sticky wages, $M = 0.85$ is sufficient.

[46] In line with the previous footnote, myopia in the IS curve only does not alter the determinacy properties.

[47] This experiment is based on the one reported by (Gabaix, 2020, p. 19 et seq.), which in turn is based on ideas put forth by Werning (2011) and Eggertsson and Woodford (2003). The main difference is that we

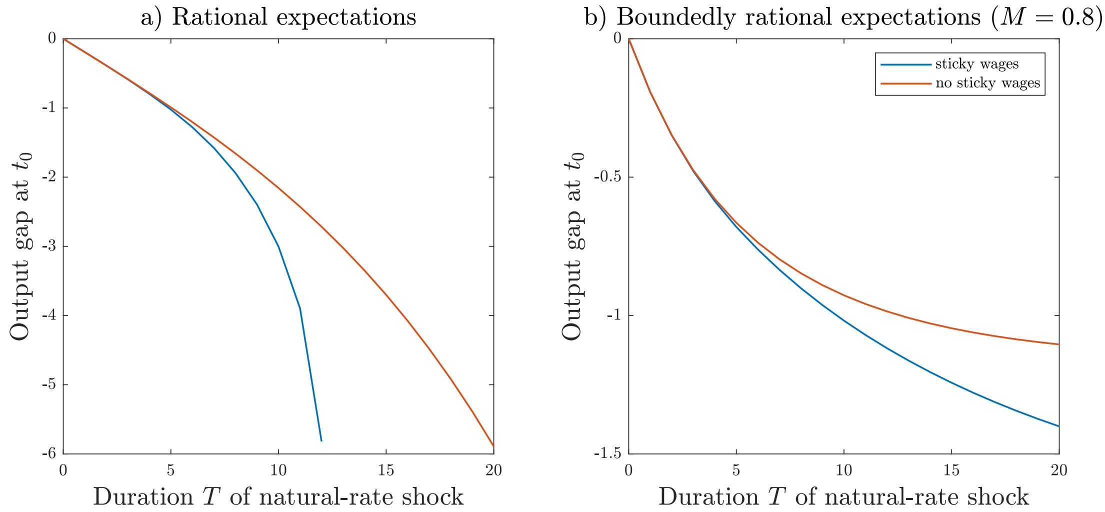

Figure 36: Severity of ELB

Note: This figure compares the severity of recessions induced by a constant decline in r_t^* to -0.20 for T quarters under rational (lhs) and boundedly rational (rhs) expectations.

and $\phi_y = 0.125$. At t_0, the natural real interest rate r_t^* drops to -0.20 for exactly $T \geq 0$ periods, which drives the economy to the effective lower bound. After T periods, r_t^* reverts to its steady state $1/\beta - 1$. Figure 36 depicts the severity of such an ELB recession (measured as the output gap x_{t_0} in the first period t_0) as a function of the duration of the shock to the natural rate. Panel a) depicts the situation with rational expectations (i.e. $M = 1$), panel b) the corresponding one in a model with boundedly rational expectations and $M = 0.8$. In each panel, we compare the severity of the ELB with and without sticky wages. In all cases depicted, the output gap decreases with the duration of the shock to r_t^*. Under rational expectations, both with and without sticky wages the output gap in the initial period at the ELB decreases without bounds as T increases.[48] Note that, under rational expectations, the unbounded decrease is much stronger with sticky wages as T increases. In contrast, under boundedly rational expectations, the decrease in the output gap is bounded both with and without sticky wages. However, comparing both cases with their corresponding rational expectations counterpart in panel a), it is evident that the decrease in the severity of the ELB is much more pronounced under sticky wages. In fact, the decrease in the output gap is almost as attenuated as in the case without sticky wages (note the difference in the scale of the vertical axis between panels a) and b)) despite the much stronger decrease under rational expectations. It is in this sense that the combination of sticky wages and bounded rationality enhances the resolution of the paradox of excessively large recessions at the ELB compared to a case without sticky wages.

The reason why the same shock to the natural rate is more severe in absolute terms in the model with sticky wages for approximately $T > 5$ is that by adding sticky wages, we add an endogenous state variable, the real wage w_t, which only adjusts sluggishly. At the ELB, as

consider a model with sticky wages. Also, compared to the before-mentioned sources we add $\phi_y > 0$, which is, however, inconsequential.

[48] In fact, for the experiment considered, with sticky wages and sticky prices, the numeric solver was not able to solve the model anymore for $T > 13$. Note that with a higher ϕ_π, the cut-off level of T increases.

output contracts, the real wage decreases. Given the deflationary pressure on prices, wage inflation has to be even lower than price inflation. Conversely, when exiting the ELB, the real wage has to increase again, which requires $\pi_{w,t} > \pi_t$ for some time. Under our calibration, the price Phillips curve is steeper than the wage Phillips curve, implying that the majority of this adjustment has to occur via lower price inflation. That is, with wage rigidities, there is an extra, endogenous disinflationary force keeping inflation low. If the shock to r_t^* is severe enough and causes a long spell at the ELB, this disinflationary force can endogenously cause this spell to be substantially longer.[49]

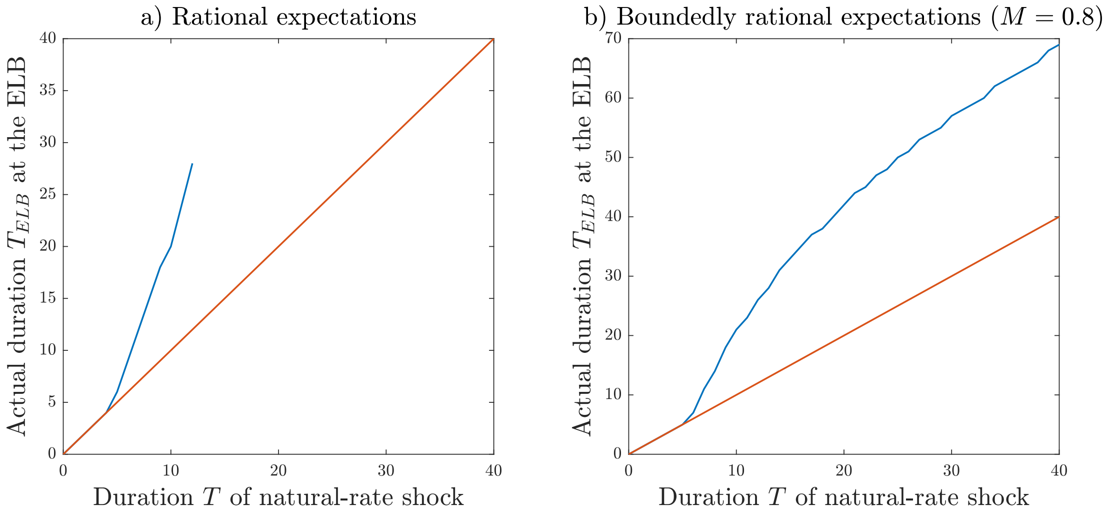

Figure 37: Severity of ELB: Mapping the natural-rate shock to ELB durations
Note: This figure compares the actual duration of the ELB (T_{ELB}), induced by a constant decline in r_t^* to -0.20 for T quarters under rational (lhs) and boundedly rational (rhs) expectations.

[49]See Figures 37 and 38 for more details on this experiment. In particular, Figure 37 maps the natural-rate shocks depicted in Figure 36 to actual durations at the effective lower bound. Note that for both rational and boundedly-rational expectations, with sticky wages, the ELB duration generally starts to be longer than the pure shock to the natural rate. Note however, with rational expectations, this increase is stronger before the line ends. This, in turn is due to the real wage, which declines more because of the larger output gap. From the right-hand panel, it becomes clear that with boundedly rational agents, the duration at the ELB does not grow as fast – which is mostly caused by a smaller (in absolute terms) output gap. For sake of comparison, the boundedly rational New Keynesian model with only sticky prices does not feature endogenous prolonging of the ELB spells. As such, the duration of the ELB spell coincides with the duration of the shock to the natural rate. Figure 38 finally compares the actual duration at the ELB (from Figure 37) and the initial output gap (from Figure 36), where the layout of the figure is the same as in 36. Obviously, with boundedly rational agents and sticky wages, the output loss in the first period at the ELB is bounded even for very long episodes at the ELB – contrary to rational expectations.

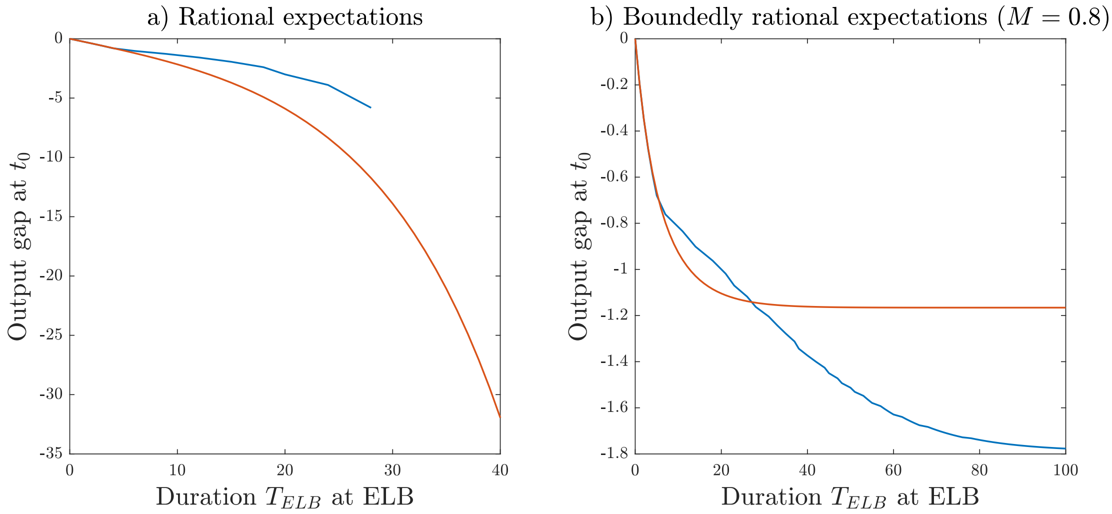

Figure 38: Severity of ELB: relationship between actual spells at the ELB and initial output gap.
Note: This figure compares the severity of recessions (measured as x_{t_0}) due to a spell at the effective lower bound from t_0 for T_{ELB} periods, induced by a constant decline in r_t^* to -0.20 for T quarters (not reported, see Figure 37 for the mapping) under rational (lhs) and boundedly rational (rhs) expectations.

E.3 Reduced power of forward guidance

The canonical New Keynesian model is subject to the well-known forward guidance puzzle (Del Negro et al., 2023). Reductions in the interest rate that take place further in the future have a stronger contemporaneous effect on output and inflation. Panel a) of Figure 39 illustrates this. Both the versions with sticky prices only and sticky wages in addition are subject to the puzzle, although the magnitude is lower for the case of sticky wages.

Bounded rationality in itself already provides a solution to the forward guidance puzzle. However, the middle panel of figure 39 shows that sticky wages act as a complement and further help reduce the puzzle. Crucial to resolving the puzzle is myopic behaviour of households that leads to discounting in the IS curve (panel b)). Nevertheless, sticky wages act as an additional anchor because households and firms - even if myopic - understand that future wages cannot be freely set but are linked to past wages through wage rigidity. Additionally, myopia on the firm and union side further reduces the magnitude of contemporaneous changes in inflation albeit with nuanced differences. Myopia on the firm side reduces forward-guidance effects in the short and medium term compared to myopia on the union side. The latter however is more important to the quantitative resolution of the puzzle in the long term.

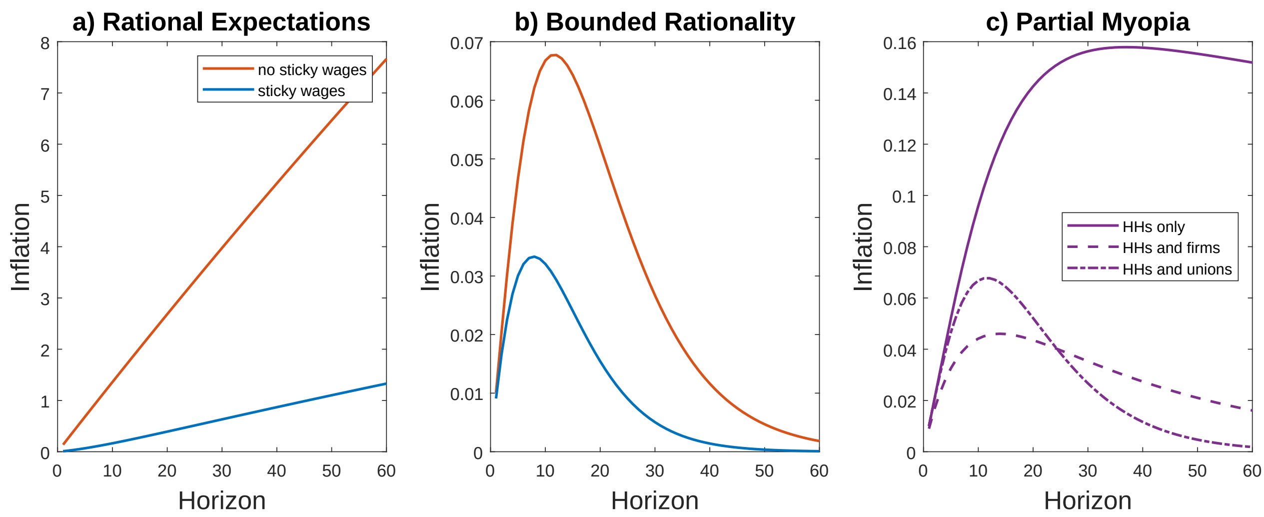

Figure 39: Strength of Forward Guidance
The left figure compares the strength of forward guidance in the New Keynesian model with sticky prices only against sticky wages in addition under rational expectations. The middle graph shows the case of sticky wages in addition to sticky prices when all agents are myopic and $M = 0.9$. The right hand graph compares cases of partial myopia.

E.4 Other New Keynesian Paradoxes

Before, we focused on those paradoxes of the standard New Keynesian model under rational expectations that were crucial for the comparison of different monetary policy strategies. For completeness, in the followng subsections we also discuss three other puzzles of the New Keynesian model: the paradox of flexibility, the paradox of toil and the Neo-Fisherian puzzle.

Paradox of Flexibility

The paradox of flexibility (Eggertsson and Krugman, 2012) states that with sticky prices (and sticky wages), the ELB becomes more harmful as prices (or wages) become more flexible, i.e. as θ_p, θ_w decrease. The underlying factor is that the slope of the Phillips curves increases with rising flexibility. I.e., a negative output gap requires more deflation, which at the effective lower bound implies a bigger gap between realised real rate and natural rate. This paradox is not resolved by reducing forward-lookingness of agents (see also Gabaix, 2020). However, by breaking the strong link of output gap and inflation rates across periods that one can obtain under rational expectations, this effect is somewhat muted for longer spells at the effective lower bound.

Paradox of Toil

The paradox of toil (Eggertsson, 2010) states that, at the effective lower bound, expansionary supply-side shocks can be contractionary. The reason is that positive TFP shocks are deflationary and are accompanied by a decrease in the natural rate (see equation (8)). However, with the economy already at the lower bound, nominal rates cannot drop any further, and the deflationary pressure actually raises the real interest rate. As is evident from (8), with bounded rationality, a given TFP shock causes bigger adjustments of the natural rate, this actually makes the paradox of toil more severe. At the same time, since the Phillips curves become less forward-looking, the deflationary effects of persistent TFP shocks are somewhat muted. I.e., overall, the paradox of toil could become more or less severe; however, in any case, it would remain effective.

Neo-Fisherian Paradox

The Fisher equation describes the long-run relationship between nominal and natural interest rates and expected inflation. Permanent increases in the nominal rate imply higher long-run inflation given that the natural rate is exogenously given. The Neo-Fisherian paradox posits that such permanent increases also lead to higher inflation rates in the short-run. To analyse how Neo-Fisherian the model under bounded rationality and sticky wages is, we follow the experiment by (Gabaix, 2020, p. 38 et seq.). To forecast inflation firms and unions forecast (the same) default inflation $\pi_t^d = (1-\xi)\bar{\pi}_t + \xi\bar{\pi}_t^{CB}$ which is a function of $\bar{\pi}_t$ and $\bar{\pi}_t^{CB}$, i.e. moving averages of past inflation and inflation guidance with $\xi \in [0,1]$. Those averages in turn are given by $\bar{\pi}_t = (1-\eta)\bar{\pi}_{t-1} + \eta\pi_{t-1}$ and $\bar{\pi}_t^{CB} = (1-\eta_{CB})\bar{\pi}_{t-1}^{CB} + \eta_{CB}\pi_{t-1}^{CB}$.

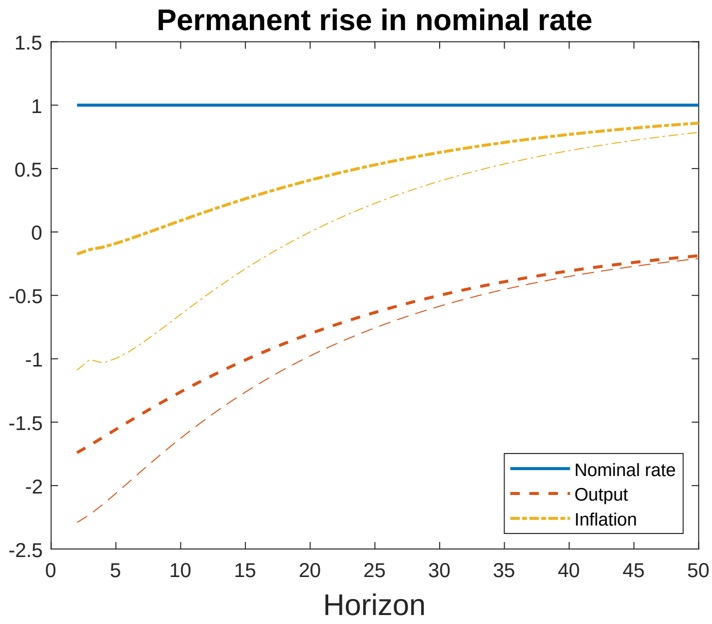

Figure 40: Neo-Fisherian puzzle

Note: The figure shows the impulse responses to an unexpected permanent 1% increase in the nominal interest rate under sticky wages. Lighter lines indicate the case with sticky prices only.

At time 0 the central bank announces an immediate, permanent, unexpected rise in the nominal rate of 1% and its corresponding target inflation $\pi_t^{CB} = 1\%$.[50]
Figure 40 shows the result of a permanent increase in the nominal rate. The dashed line shows that in the short-run inflation (yellow) reacts negatively to the permanent increase. However, the initial magnitude is quite small especially in comparison to the sticky price model (light dashed). While bounded rationality thus solves the Neo-Fisherian puzzle, the effects are weaker with sticky wages. In the sticky wage model, convergence to the long-run level is faster.

[50] In addition to the calibration in table 1 we use $\xi = 0.8$, $\eta = 0.5$ and $\eta_{CB} = 0.05$.