SUPPLEMENTARY RESULTS

Shadow-Rate VARs*

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*The views expressed herein are solely those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of Cleveland, the Federal Reserve System, the Eurosystem or the Deutsche Bundesbank. Replication codes are available at https://github.com/elmarmertens/CCMMshadowrateVAR-code. Corresponding author: Todd E. Clark, todd.clark@researchfed.org.

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I (a) 6m and 1y shadow rates

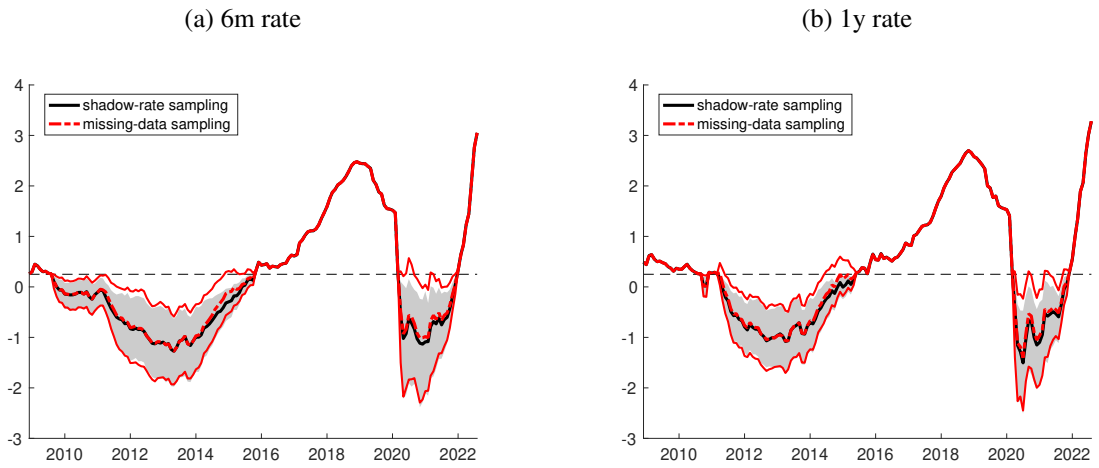
Figure S.1 provides full-sample and quasi-real-time estimates of shadow rates for the 6-month and 1-year Treasury maturities, obtained with the simple shadow-rate VAR including a range of interest rates. The contours of these estimates (both full sample and quasi-real time) follow those shown in the paper for the federal funds rate.

I (b) Shadow rates from hybrid shadow-rate VAR

Figure S.2 reports various shadow rate estimates obtained from the hybrid shadow-rate VAR. Panel (a) compares the smoothed shadow-rate estimates of the federal funds rate against updated estimates obtained from Krippner (2013, 2015) and Wu and Xia (2016). Panel (b) compares smoothed and quasi-real time shadow-rate estimates of the federal funds rate. Panels (c) and (d) show full-sample and quasi-real-time estimates of shadow rates for the 6-month and 1-year Treasury maturities.

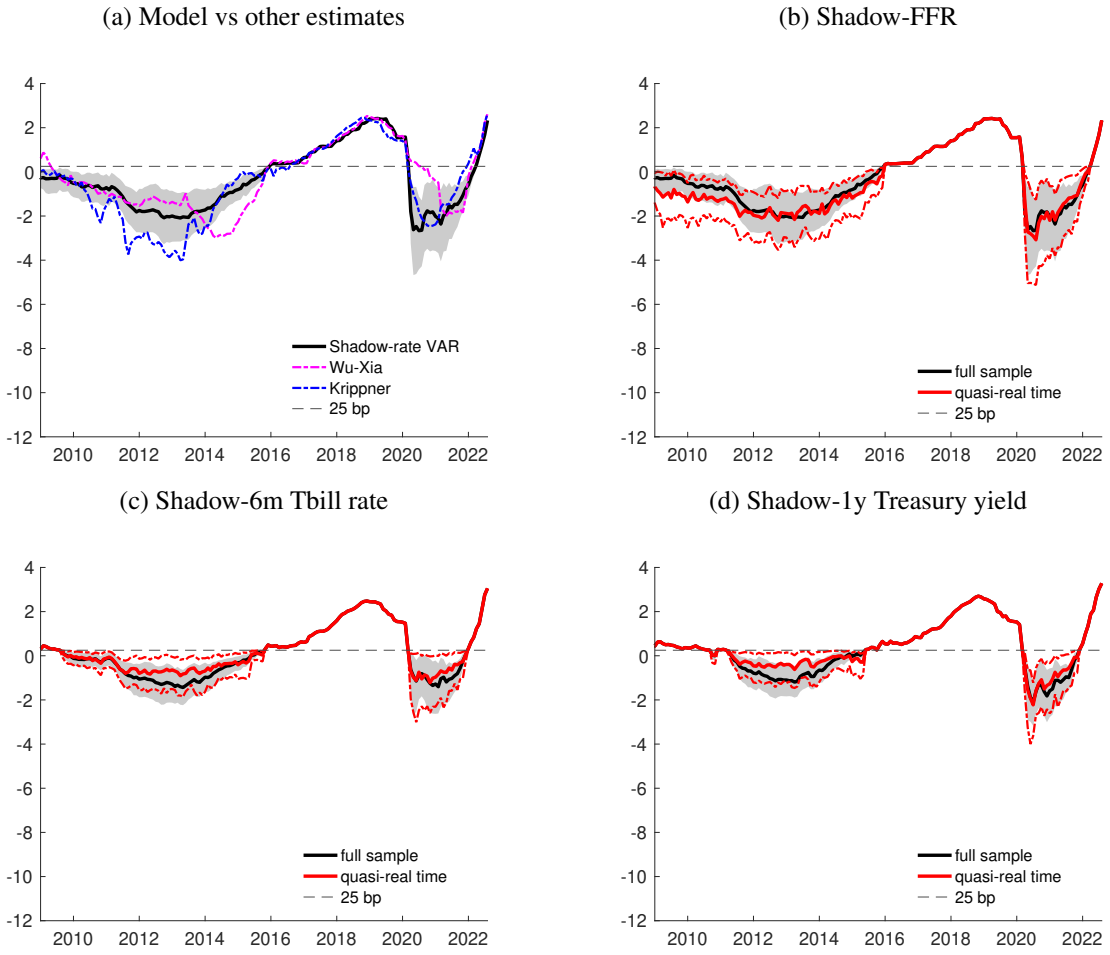
Figure S.3 reports differences in parameter and SV estimates between the missing-data approach (that does not impose the ELB) and shadow-rate sampling (which enforces the ELB). We find that shadow-rate sampling (as opposed to missing-data sampling) increases the odds of drawing shadow-rate values below the ELB by adjusting the SV path for shocks to the shadow rate equation, with only minor effects on the VARs' transition coefficients, except for some shifts in intercepts of medium- to longer-term interest-rate equations. (In results not shown, we find only minor effects on SV paths for other variables.)

Figure S.1: Shadow-rate estimates for 6m and 1y rates (simple shadow-rate VAR model)



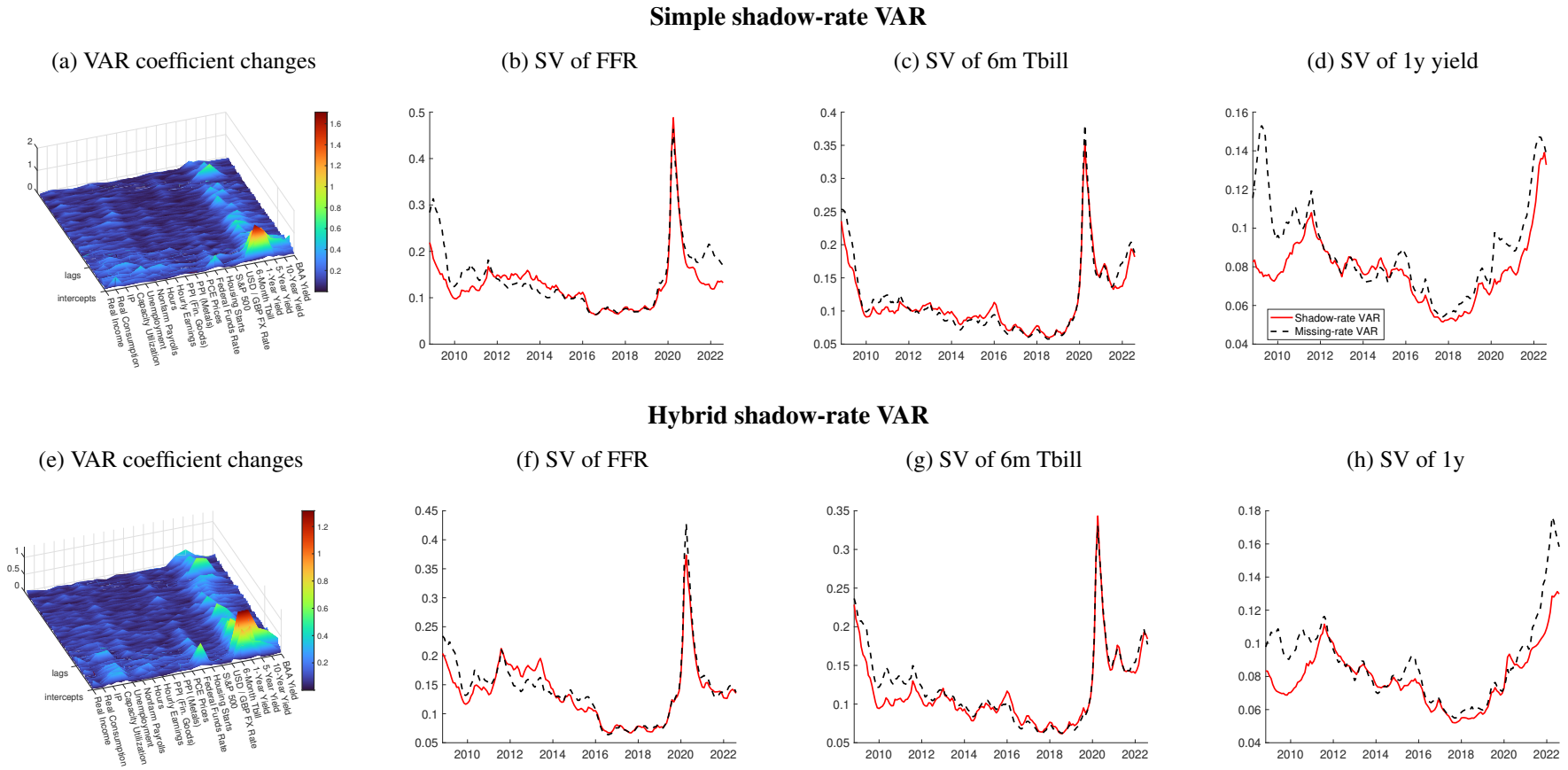
Note: Panels (a) and (b) report smoothed and quasi-real time shadow-rate estimates for 6m and 1y interest rates from our simple shadow-rate VAR. The quasi-real-time estimates are the end-of-sample estimates produced by recursive estimation of the model starting in January 2009. Each estimation conditions on available data since 1959:03, but the figure omits the period prior to 2008 during which the ELB did not bind. Posterior medians are shown as thick lines; grey shaded areas and thin lines depict 90 percent uncertainty bands.

Figure S.2: Shadow-rate estimates from hybrid shadow-rate VAR



Note: Shadow-rate estimates from the hybrid version of our shadow-rate VAR, estimated using the full set of our 20 variables as listed in Table 1 of the paper. Panel (a) compares the smoothed shadow-rate estimates against updated estimates obtained from Krippner (2013, 2015) and Wu and Xia (2016). Panel (b) compares smoothed and quasi-real time shadow-rate estimates of the federal funds rate. The quasi-real-time estimates are the end-of-sample estimates produced by recursive estimation of the model starting in January 2009. Each estimation conditions on available data since 1959:03, but the figure omits the period prior to 2008 during which the ELB did not bind. Posterior medians are shown as thick lines; grey shaded areas and thin lines depict 90 percent uncertainty bands. Panels (c) and (d) show full-sample and quasi-real-time estimates of shadow rates for the 6-month and 1-year Treasury maturities.

Figure S.3: Effects of shadow-rate sampling on parameters and SV



Note: Differences in estimates for parameters and selected SV paths when comparing the shadow-rate VAR against a missing-data approach that treats observations of nominal interest rates at the ELB as missing. Panels (a)–(d) report results based on the simple shadow-rate VAR, and Panels (e)–(h) display corresponding results from the hybrid shadow-rate VAR. Panels (a) and (e) show absolute values of standardized differences in mean estimates of VAR coefficients; the estimates are standardized based on the posterior standard deviation of each coefficient generated by the missing-data approach. The SV paths shown in the other panels reflect posterior medians. All estimates are estimated from the full data sample, from 1959:03 through 2022:08.

Figure S.4 provides comparisons to assess the effects of shadow-rate modeling and enforcement of the ELB in model estimation, using the hybrid shadow-rate VAR. These results are similar to those reported in the paper for the simple shadow-rate VAR. Focusing on the federal funds rate and its shadow rate, Panel (a) compares shadow-rate (black) and missing-data (red) draws for the shadow rate s_t obtained from the posterior of our hybrid shadow-rate VAR. Shadow-rate draws are obtained from the truncated posterior for s_t that satisfies the ELB, and described by the problem of drawing from $\boldsymbol{S}|\boldsymbol{Y}$. Missing-data draws are obtained from the underlying (and untruncated) posterior of the missing data problem, which ignores the ELB, and correspond to draws from $\boldsymbol{S}|\bar{\boldsymbol{Y}}$. As the chart comparison indicates, the posteriors obtained from these alternatives are very similar. Panel (b) compares missing-data posteriors obtained from two sets of estimates obtained from hybrid shadow-rate VARs: In the baseline (red), parameter and SV draws reflect shadow-rate sampling (as also shown in Panel (a)). In the alternative version (blue), parameters and SV are drawn while treating the policy rate at the ELB as missing data and without requiring that missing data draws lie below the ELB. The comparison highlights the non-negligible effects of shadow-rate sampling, which takes into account observations of interest rates at the ELB, on model estimates of parameters and SV. Without forcing the draws of missing interest rate observations to lie at or below the ELB, the upper bound of the posterior credible set rises above the ELB by early 2015 (but largely not during the more recent episode), which contradicts observations of the federal funds rate that were at the ELB during those times. In contrast, the use of shadow-rate sampling, as opposed to a missing-data approach, leads to estimates of parameters and SV that increase the odds of obtaining missing-data draws for the shadow rate that lie below the ELB (for observations when the ELB binds).

I (c) Shadow-rate estimates from ex yields data

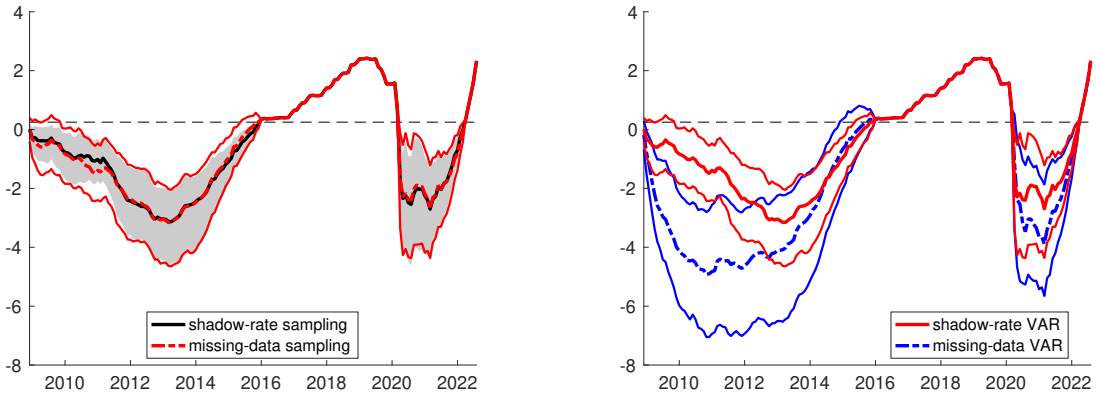
A similar pattern is observed when the estimation excludes all data on yields (except for the federal funds rate), as shown in Panels (c) and (d) of Figure S.4. With the federal funds rate the only interest rate in the model, when the draws of missing interest rate observations are not forced to lie at or below the ELB, the upper bound of the posterior credible set shows an even longer period above the ELB, from about 2012 through 2015 and again in 2020. Similar to Figure S.3, Figure S.5 shows that, also in the ex-yields case, shadow-rate sampling (as opposed to missing-data sampling) increases the odds of drawing shadow-rate values below the ELB by adjusting the SV path for shocks to the shadow rate equation, with relatively minor effects on the VARs' transition coefficients. Compared to the case where yields are included in the estimation, Figure S.5 displays stronger changes in intercepts of some macro equations, notably industrial production and capacity utilization.

Figure S.4: Effect of imposing ELB on shadow-rate estimates in hybrid model

Hybrid shadow-rate VAR estimates from data including yields

(a) Missing-data and shadow-rate draws

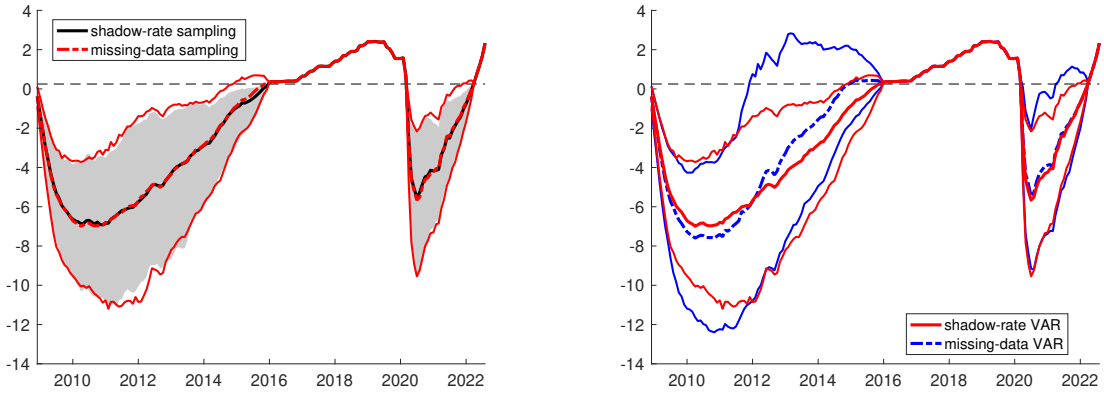
(b) Missing-data draws w/and w/o shadow-rate sampling



Estimates from data excluding yields

(c) Missing-data and shadow-rate draws

(d) Missing-data draws w/and w/o shadow-rate sampling

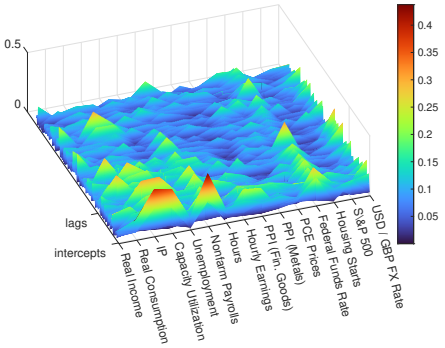


Note: Panel (a) compares shadow-rate (black) and missing-data (red) draws for s_t obtained from the posterior of our hybrid shadow-rate VAR. Shadow-rate draws are obtained from the truncated posterior for s_t that satisfies the ELB. Missing-data draws are obtained from the underlying (and untruncated) posterior of the missing data problem that ignores the ELB. Panel (b) displays missing-data posteriors obtained from two sets of hybrid shadow-rate VAR estimates: In the baseline (red), parameter and SV draws reflect shadow-rate sampling. In the alternative version (blue), parameters and SV are drawn while treating the policy rate at the ELB as missing data and without requiring that missing data draws lie below the ELB. In this panel, medians are reported as thick lines and 90 percent uncertainty bands are reported with the grey shaded area or thin lines. Panels (c) and (b) provide corresponding estimates from a VAR that omits all interest rate data except for the federal funds rate.

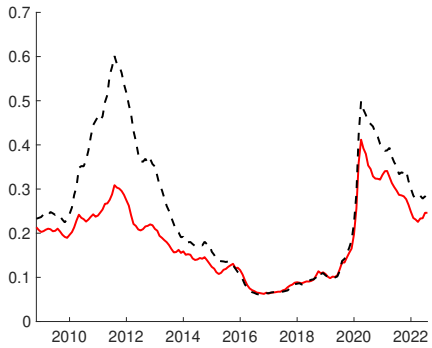
Figure S.5: Effects of shadow-rate sampling on parameters and SV (ex yields)

Simple shadow-rate VAR

(a) VAR coefficient changes

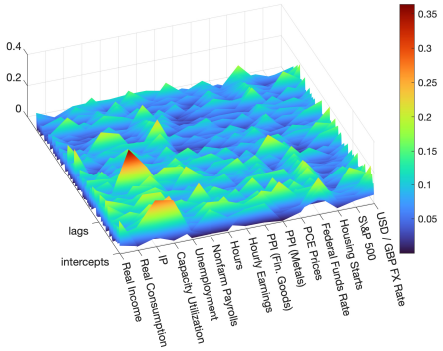


(c) SV of FFR

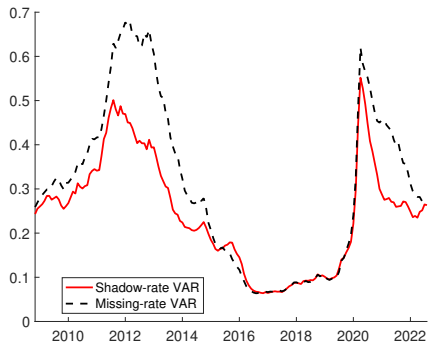


Hybrid shadow-rate VAR

(b) VAR coefficient changes



(d) SV of FFR

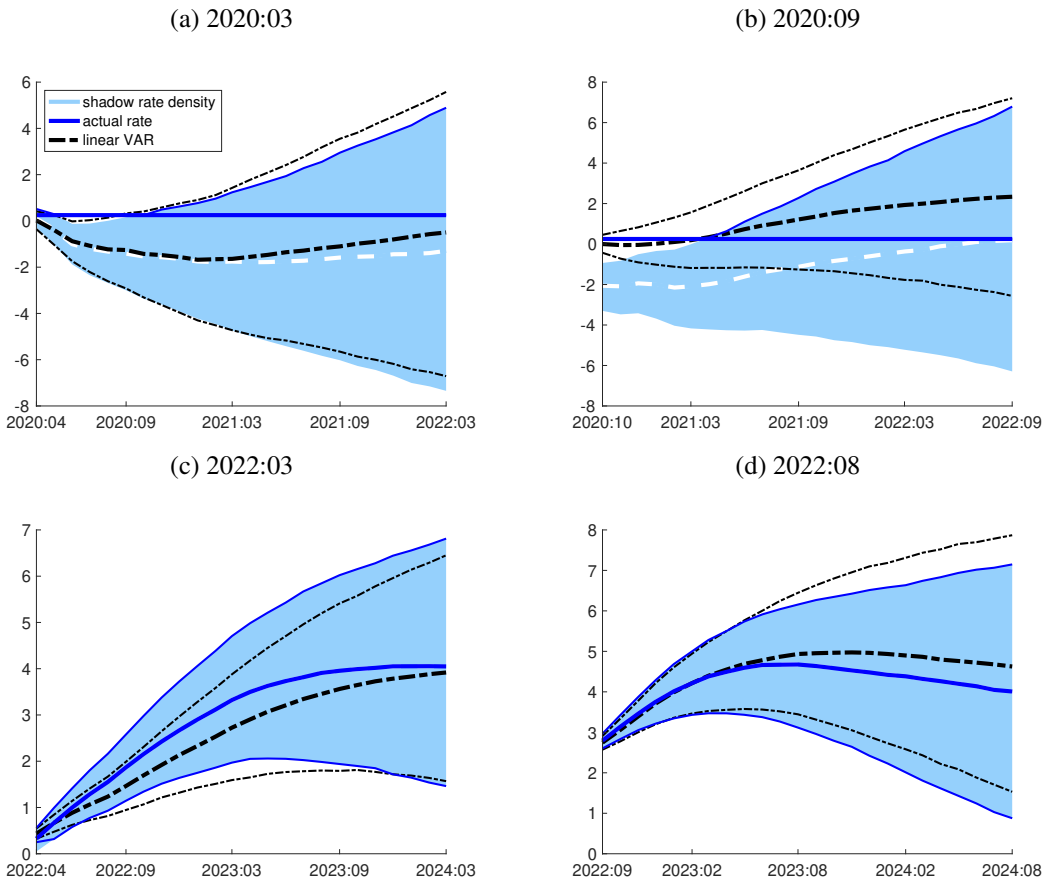


Note: Differences in estimates for parameters and selected SV paths when comparing the shadow-rate VAR against a missing-data approach that treats observations of nominal interest rates at the ELB as missing. Panels (a) and (c) report results based on the simple shadow-rate VAR, and Panels (b) and (d) display corresponding results from the hybrid shadow-rate VAR. Panels (a) and (b) show absolute values of standardized differences in mean estimates of VAR coefficients; the estimates are standardized based on the posterior standard deviation of each coefficient generated by the missing-data approach. The SV paths shown in the other panels reflect posterior medians. All estimates are estimated from the full data sample, but excluding yield measures other than the federal funds rate, from 1959:03 through 2022:08.

II Hybrid vs. simple shadow-rate VAR: additional forecast comparisons

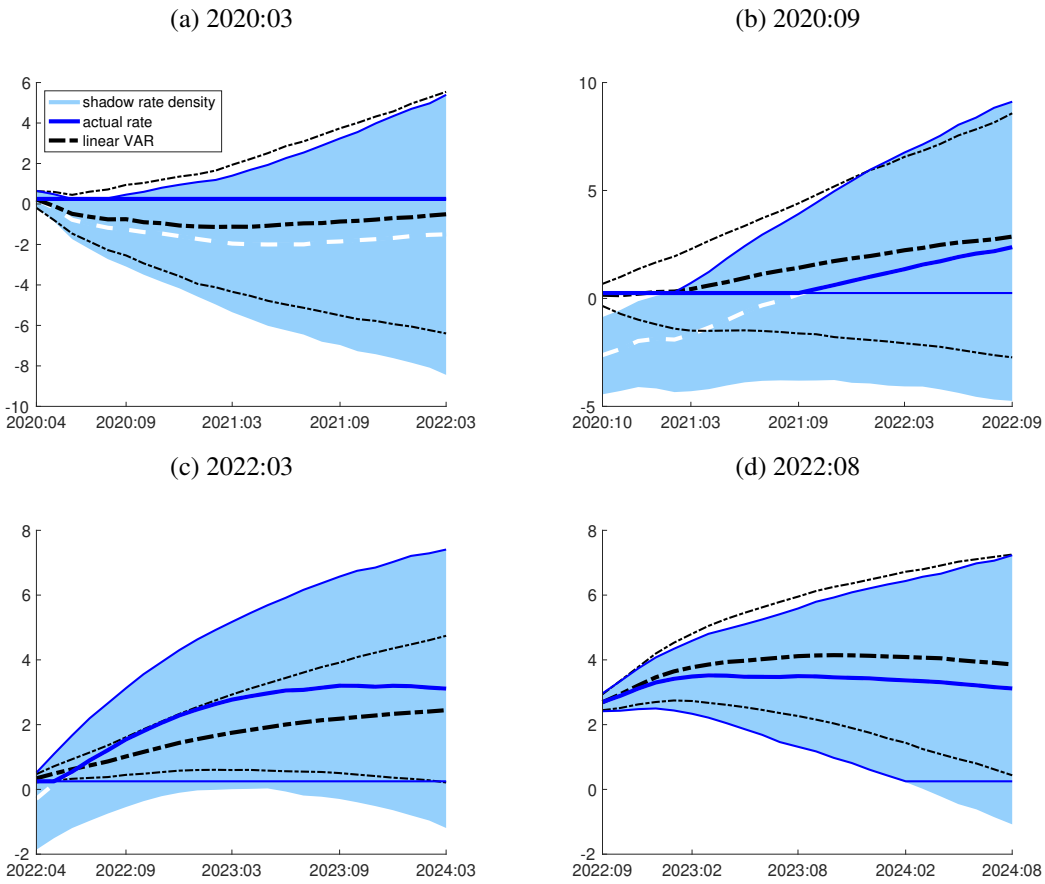
Section II (a) provides additional figures of predictive densities of shadow and actual interest rats in 2020-2022 to complement those shown from the simple shadow-rate VAR in the paper. The densities shown here are generated from hybrid shadow-rate VAR, as well as hybrid and simple shadow-rate VARs estimated without yields data (except for the federal funds rate), and are qualitatively similar to those reported in the paper. Section II (b) directly compares the forecast accuracy of simple and hybrid shadow-rate VARs (as opposed to comparing each of these models against the linear VAR as shown in the paper). Section II (c) describes a variant of the hybrid shadow-rate VAR that allows the matrix of shock impacts, A, to switch at the ELB, and reports on the model's forecast accuracy.

Figure S.6: Predictive densities of the federal funds rate (hybrid shadow-rate VAR)



Note: Predictive density for the actual and shadow values of the federal funds rate, simulated out of sample at different jump-off dates. Dashed-dotted (black) lines depict the predictive density for the actual rate as generated from the standard VAR. The shaded (light blue) area with dashed (white) lines represent the hybrid shadow-rate VAR's predictive density of the shadow rate, while solid lines (dark blue) reflect the corresponding censored density for the actual interest rate. Posterior medians and 68 percent bands.

Figure S.7: Predictive densities of the federal funds rate (simple shadow-rate VAR, ex yields)

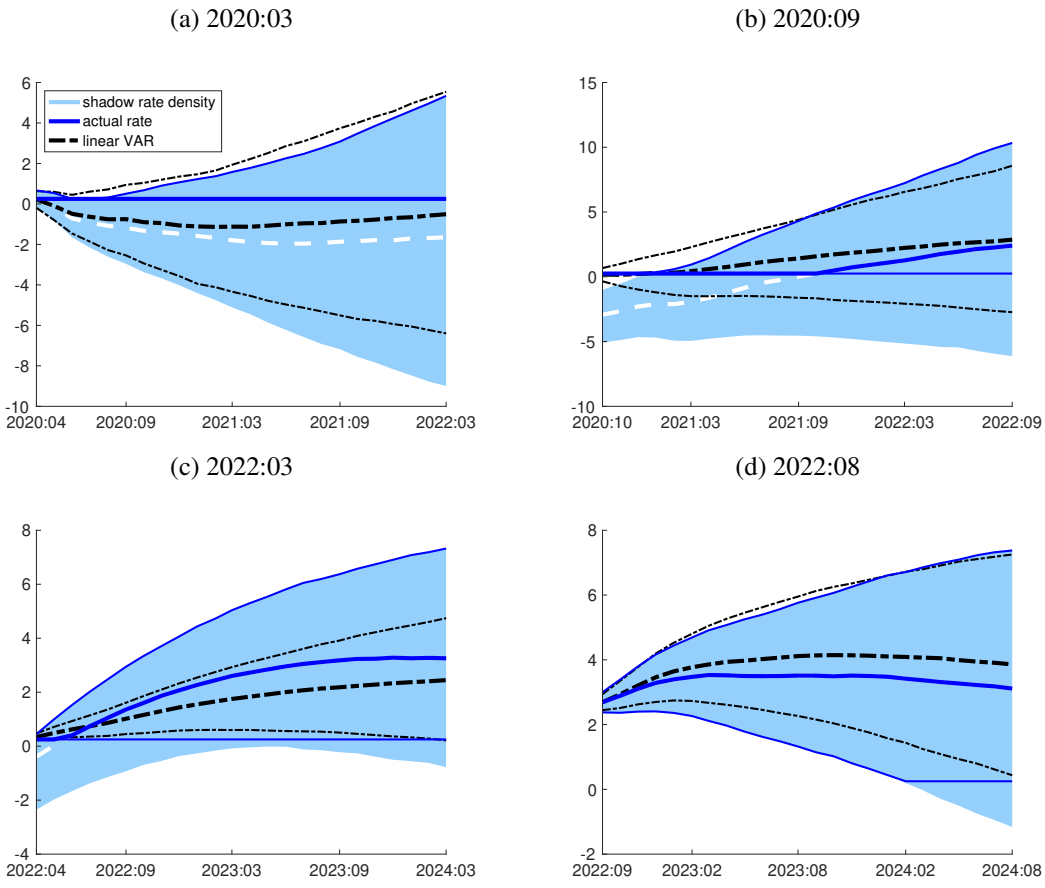


Note: Predictive density for the actual and shadow values of the federal funds rate, simulated out of sample at different jump-off dates. Dashed-dotted (black) lines depict the predictive density for the actual rate as generated from the linear VAR. The shaded (light blue) area with dashed (white) lines represent the simple shadow-rate VAR's predictive density of the shadow rate, while solid lines (dark blue) reflect the corresponding censored density for the actual interest rate. Posterior medians and 68 percent bands.

II (a) Predictive densities for the federal funds rate in 2020-2022

Figure S.6 reports predictive densities for actual and shadow values of the federal funds rate from the standard linear and hybrid shadow-rate VAR for selected origins during the Covid period. Results are broadly comparable to those reported in the main text. In addition, Figures S.7 and S.8 provide similar results obtained from simple and hybrid shadow-rate VARs using data excluding yields (except for the federal funds rate).

Figure S.8: Predictive densities of the federal funds rate (hybrid shadow-rate VAR, ex yields)



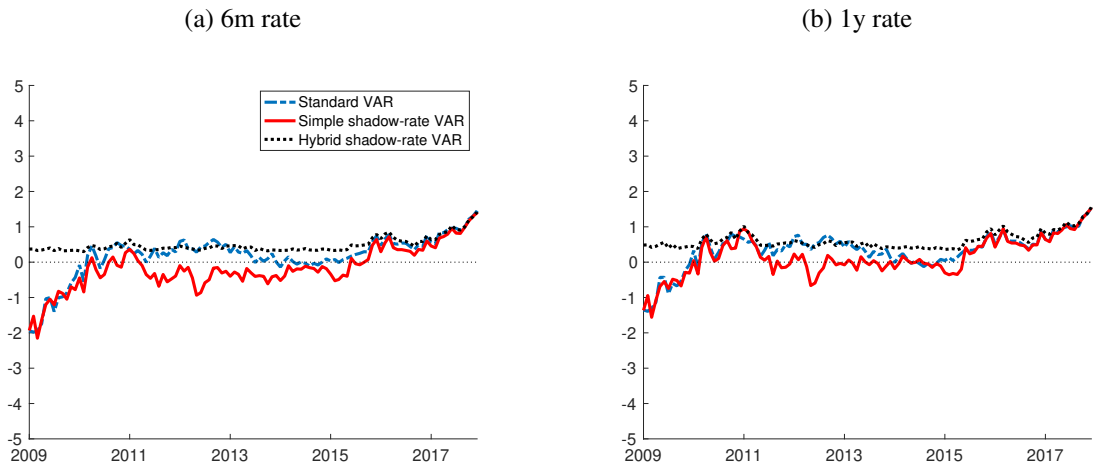
Note: Predictive density for the actual and shadow values of the federal funds rate, simulated out of sample at different jump-off dates. Dashed-dotted (black) lines depict the predictive density for the actual rate as generated from the linear VAR. The shaded (light blue) area with dashed (white) lines represent the hybrid shadow-rate VAR's predictive density of the shadow rate, while solid lines (dark blue) reflect the corresponding censored density for the actual interest rate. Posterior medians and 68 percent bands.

II (b) Direct forecast comparison: simple vs hybrid shadow-rate VAR

Tables S.1 through S.3 compare the forecast accuracy of the hybrid shadow-rate VAR to that of the simple shadow-rate VAR. We provide the comparisons for the benchmark model with a range of interest rates and smaller models in which the federal funds rate is the only interest rate, over samples that end in December 2017 as well as August 2022. In these results directly comparing the hybrid and shadow-rate VAR forecasts, the hybrid forecasts are modestly better for most economic activity indicators (e.g., IP, capacity utilization, and unemployment) at longer horizons and modestly worse for only a few indicators (e.g., hourly earnings).

To illustrate salient differences in the forecast-relevant policy rates that underlay predictions of macroeconomic (and other non-interest-rate) variables, Figure 4 of the paper compares 12-month ahead averages of the forecast-relevant policy rates expected by each model out of sample. Figure S.9 provides corresponding 12-month ahead averages of forecast-relevant 6m and 1y rates.

Figure S.9: Expected averages of forecast-relevant 6m and 1y rates (12-months ahead)



Note: 12-month ahead expected averages of 6m and 1y rates (actual or shadow), generated out-of-sample at forecast origins 2009:01 – 2017:12. For each VAR model, the interest rate measures shown correspond to the forecast-relevant policy rate variables used in the model equations for macroeconomic variables, x_t: For the simple shadow-rate VAR, shadow rate expectations are shown. For the hybrid shadow-rate VAR, predictions of actual rates are reported, and for the standard VAR, which ignores the ELB, (uncensored) federal funds rate predictions are shown.

Table S.1: Forecast performance of simple and hybrid shadow-rate VARs

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	1.00	0.99	1.01	**1.14****	1.01	1.01	1.01	0.98	1.01**	1.02**	1.00	0.99
Real Consumption	1.00	1.01	0.98	**0.92**	1.00	1.01	1.00	**0.90**	1.00	1.00	0.99	0.97**
IP	0.99	0.98*	0.96	0.98	0.99	0.98	**0.95****	**0.95**	0.99	0.98*	0.97*	0.99
Capacity Utilization	0.97	**0.94***	**0.91***	**0.84****	**0.94*****	**0.90*****	**0.86*****	**0.84****	0.97**	**0.94****	**0.91****	**0.88*****
Unemployment	1.01	1.00	0.99	**0.91**	1.00	1.01	1.03	**0.95**	1.01	1.00	0.98	**0.94**
Nonfarm Payrolls	1.00	1.01	1.00	**0.94**	0.98	0.97	**0.93**	**0.90***	0.98	0.98	0.96	**0.95***
Hours	1.01	1.02	1.02	1.01	1.00	1.01	1.01	0.99	1.00	1.01	1.01	1.00
Hourly Earnings	1.00	1.01	1.01	1.04***	1.00	1.00	1.01	1.02	1.00	1.01	1.01	1.02*
PPI (Fin. Goods)	1.00	0.99	0.97	1.01	1.01	1.00	0.98	1.00	1.01	1.00	0.99	1.00
PPI (Metals)	1.00	1.00	0.98**	0.99	1.00	0.99	0.99	0.99*	1.00	0.99	0.99***	0.99*
PCE Prices	1.00	0.98	0.97	**1.05****	0.99	0.98	0.98	**1.05*****	1.00	0.99	0.99	1.04***
Federal Funds Rate	1.02	0.98	0.96	**0.93****	**1.05**	**0.94**	**0.95***	0.99	0.99	0.96	**0.93**	**0.90****
Housing Starts	0.99	0.99	**0.95**	1.00	1.00	0.99	**0.93**	0.98	0.99	1.00	0.98	1.02
S&P 500	1.01	1.00	1.01	1.01	1.02	1.01	1.01*	1.01	1.00	1.00	1.01**	1.00
USD / GBP FX Rate	1.02*	1.01	1.01	1.03	1.03***	1.02*	1.03	**1.08**	1.02**	1.01	1.01	1.02
6-Month Tbill	0.99	0.96	**0.94*****	**0.90*****	0.98	**0.95**	**0.90****	**0.86*****	0.98	**0.94***	**0.90*****	**0.86*****
1-Year Yield	0.97*	**0.95*****	**0.93*****	**0.90*****	0.99	**0.93****	**0.85*****	**0.84****	0.98	**0.94*****	**0.90*****	**0.87*****
5-Year Yield	1.02	1.00	**0.95**	**0.89*****	1.02	1.03	1.00	**0.95**	1.01	1.00	0.97	**0.94***
10-Year Yield	1.02	1.01	0.98	**0.94***	1.02	1.02	0.99	1.00	1.01	1.01	0.99	0.99
BAA Yield	1.00	0.98	0.99	**1.05**	0.99	0.98	1.00	**1.08****	0.99	0.99	1.00	1.03

Note: Comparison of "simple shadow-rate VAR" (baseline, in denominator) against "hybrid shadow-rate VAR." Values below 1 indicate improvement over baseline. Evaluation window with forecast origins from 2009:01 through 2017:12 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

Table S.2: Forecast performance of simple and hybrid shadow-rate VARs (ex yields)

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	1.00	0.99	1.00	0.99	0.99	0.99	0.97	**0.91**	1.00	0.99	0.98	0.96
Real Consumption	1.00	0.98	**0.94**	**0.91**	0.97	0.98	**0.91**	**0.94**	1.00	0.99	0.97	0.97
IP	0.98	0.96	**0.88**	**0.81**	0.96*	**0.95**	**0.88***	**0.83**	0.98	0.96	**0.91**	**0.89**
Capacity Utilization	**0.94**	**0.89**	**0.78****	**0.66**	**0.89****	**0.87***	**0.77****	**0.66**	**0.94****	**0.88****	**0.80****	**0.71**
Unemployment	0.98	**0.93****	**0.80****	**0.67**	0.97	**0.94**	**0.78****	**0.61**	0.98	**0.95****	**0.84*****	**0.71**
Nonfarm Payrolls	1.02	1.04	**0.89**	**0.71**	1.02	0.98	**0.79***	**0.68**	1.00	0.98	**0.86****	**0.78**
Hours	0.98	0.99	1.00	**0.87**	0.96*	0.98	0.96	**0.80**	0.98	0.98	0.97	**0.89**
Hourly Earnings	1.01*	1.01	1.03	**1.06**	1.01	1.01	1.03**	**1.07**	1.01**	1.01**	1.02**	1.04*
PPI (Fin. Goods)	0.99	0.99	0.99	**1.05**	1.00	0.99	1.00	1.02	1.00	1.00	0.99	1.01
PPI (Metals)	1.00	0.99	1.00	1.00	0.99	0.99	1.00	0.98	0.99	0.99	1.00	1.00
PCE Prices	**0.95***	**0.93**	**0.89**	**0.91**	**0.95***	**0.94**	**0.92**	**0.91**	0.96*	0.96	**0.93**	**0.93**
Federal Funds Rate	**3.94**	**3.36**	**2.33**	**1.71**	**1.55**	1.02	**0.89**	1.00	**1.61**	**1.28**	0.98	**0.91**
Housing Starts	**0.86****	**0.79**	**0.70**	**0.73**	**0.88****	**0.83***	**0.77**	**0.81**	**0.90****	**0.85***	**0.78**	**0.81**
S&P 500	1.02	1.01	1.00	0.99	1.01	1.01	0.99	0.98*	1.01	1.01	1.00	0.99*
USD / GBP FX Rate	1.02	0.99	1.00	1.01	1.03	1.01	1.02	1.02	1.02	1.00	1.01	1.02*

Note: Comparison of "simple shadow-rate VAR" (baseline, in denominator) against "hybrid shadow-rate VAR." Values below 1 indicate improvement over baseline. Evaluation window with forecast origins from 2009:01 through 2017:12 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

Table S.3: Forecast performance of simple and hybrid shadow-rate VARs (incl. COVID-19)

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	1.00	1.00	0.96	**1.05**	1.00	1.00	1.00	0.99	1.00	1.00	1.00	0.99
Real Consumption	1.00	1.00	1.00	1.01	1.00	1.00	1.00	0.97	1.00	1.00	1.00	0.99**
IP	0.99*	1.00*	0.99**	1.00	0.99	0.98	0.97**	0.98	0.99	0.99**	0.98**	0.99
Capacity Utilization	1.00	0.98***	**0.94***	**0.89****	0.97**	**0.93*****	**0.89*****	**0.86****	0.98***	**0.95*****	**0.93*****	**0.90*****
Unemployment	1.00	1.01*	1.03	**1.07**	1.01	1.02	1.04	1.01	1.01	1.01	1.01	0.99
Nonfarm Payrolls	1.00	1.00	1.00	1.00	1.00	1.00	0.98	0.96**	1.00	1.00	0.99	0.99*
Hours	1.00	1.01	1.03	1.00	1.01	1.01	1.01	0.98	1.00	1.00	1.00	1.00
Hourly Earnings	1.01	1.00	1.02**	**1.05****	1.00	1.00	1.00	1.01	1.00	1.00	1.00	1.01
PPI (Fin. Goods)	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.01	1.00	1.00	1.00	1.00
PPI (Metals)	0.99*	1.00	0.98**	0.98	1.00	1.00	1.00	0.99	1.00	1.00	0.99**	0.99**
PCE Prices	0.99	0.99	0.98	1.02	0.99	0.98	1.00	1.04***	1.00	0.99	1.00	1.03***
Federal Funds Rate	0.98	**0.92****	**0.90**	**0.88**	1.00	**0.94****	0.96	1.04	0.98	**0.93****	**0.91***	**0.92****
Housing Starts	1.03	**1.05**	1.04	**1.17**	1.03	1.04	0.99	1.02	1.01	1.01	0.99	1.02
S&P 500	1.00	0.99	1.00	**1.07**	1.00	0.99	1.01	1.01*	1.00	1.00	1.01***	1.00
USD / GBP FX Rate	1.02*	1.01*	1.00	1.01	1.02***	1.02**	1.02	1.04	1.01**	1.01	1.00	1.01
6-Month Tbill	**0.94****	**0.91****	**0.90***	**0.87**	**0.94***	**0.92***	**0.92****	**0.94**	**0.94****	**0.91*****	**0.91*****	**0.90*****
1-Year Yield	**0.92****	**0.90****	**0.90***	**0.89**	**0.93****	**0.92****	**0.88*****	**0.92**	**0.93****	**0.91*****	**0.92*****	**0.90*****
5-Year Yield	1.00	0.98	**0.95***	**0.89**	1.02	1.02	0.98	1.00	1.00	1.00	0.98	0.96
10-Year Yield	1.02	1.00	0.98	**0.93**	1.02*	1.02	0.98	1.01	1.01	1.00	0.99	0.99
BAA Yield	0.99	0.98**	0.99	1.04	1.00	0.98*	0.99	**1.08****	0.99	0.99	1.00	1.02*

Note: Comparison of "simple shadow-rate VAR" (baseline, in denominator) against "hybrid shadow-rate VAR." Values below 1 indicate improvement over baseline. Evaluation window with forecast origins from 2009:01 through 2022:08 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Due to the close behavior of some of the models compared, and rounding of the reported values, one of the comparisons shows a significant ratio of 1.00. This case arises from persistent differences in performance that are, however, too small to be relevant after rounding. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

Table S.4: Forecast performance of simple and hybrid shadow-rate VARs (ex yields, incl. COVID-19)

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	1.00	1.00	0.98	**1.15**	0.99*	0.99*	0.99	0.97	1.00	1.00	1.00	0.99
Real Consumption	1.00	1.00	1.00	1.00	0.99	1.00	0.97	0.98	1.00	1.00	0.99	0.99
IP	0.99	0.99	0.98*	0.97	0.98**	0.97*	**0.93****	**0.90**	0.99	0.99	0.96	**0.95**
Capacity Utilization	0.97	**0.95****	**0.88**	**0.70**	**0.94*****	**0.90****	**0.82****	**0.71**	0.96***	**0.93*****	**0.86****	**0.78**
Unemployment	1.00	1.00	1.01	0.97	1.00	1.00	**0.94**	**0.79**	1.00	0.99	0.96*	**0.85**
Nonfarm Payrolls	1.00	1.00	1.00	1.00	1.00	1.00	**0.94***	**0.88**	1.00	1.00	0.97*	**0.94**
Hours	0.99	1.00	1.03	**0.91**	0.98	1.00	0.99	**0.84***	0.99	0.99	0.99	**0.93**
Hourly Earnings	1.01**	1.01	1.02**	1.04	1.01	1.00	1.03**	1.04	1.00	1.01	1.01*	1.02
PPI (Fin. Goods)	1.00	1.00	1.00	1.02	1.01	1.00	1.00	1.01	1.01	1.01	1.00	1.01
PPI (Metals)	0.99	0.99	1.00	0.98	0.99	0.99	1.00	0.98	0.99	0.99	1.00	1.00
PCE Prices	0.96*	0.96	**0.91***	0.98	0.97	0.96	**0.94**	**0.93***	0.98	0.98	**0.95**	**0.95***
Federal Funds Rate	**2.14**	**1.71**	**1.34**	**1.16**	**1.19**	**1.05**	**0.94***	1.02	**1.18**	**1.09**	0.99	**0.94**
Housing Starts	**0.93**	**0.84**	**0.72**	**0.79**	**0.95**	**0.88***	**0.79***	**0.86**	**0.94****	**0.89****	**0.83**	**0.86**
S&P 500	1.00	1.00	1.00	0.99	1.00	1.00	0.99*	0.99	1.00	1.00	1.00	0.99
USD / GBP FX Rate	1.02	0.99	1.00	1.00	1.02	1.01	1.01	1.01	1.01	1.01	1.01	1.01

Note: Comparison of "simple shadow-rate VAR" (baseline, in denominator) against "hybrid shadow-rate VAR." Values below 1 indicate improvement over baseline. Evaluation window with forecast origins from 2009:01 through 2022:08 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

II (c) Hybrid model with impact matrix switching at ELB

In our forecasting and IRF results, to entertain ELB-driven parameter change while constraining its extent to make estimation and forecasting feasible, we consider a version of the hybrid VAR extended to allow the impact matrix A to change at the ELB, and denote the regime switching impact matrix $A\left(\mathbf{1}_{(i_{t-1}\leq ELB)}\right)$. In this case, the model of equation (6) in the paper is extended so that the innovation vector takes the form:

$$v_t = A\left(\mathbf{1}_{(i_{t-1}\leq ELB)}\right)^{-1} \Lambda_t^{0.5}\varepsilon_t, \tag{S.1}$$

$$A\left(\mathbf{1}_{(i_{t-1}\leq ELB)}\right) = A + \mathbf{1}_{(i_{t-1}\leq ELB)} \cdot A_{\text{elb}} \tag{S.2}$$

where A_{elb} represents the change in the impact matrix that occurs when the policy rate (lagged one period to avoid endogeneity issues) is at or below the ELB. We estimate the model with an informative prior on A_{elb}, and retain an informative prior on A (as in our baseline specification).

The prior on A_{elb} is specified as a multivariate normal with mean zero, and diagonal variance-covariance matrix, with each diagonal element set to a variance of 1/1000. Despite such tightness in the prior, the model without switching shock impacts (i.e. $A_{\text{elb}} = 0$) performs much better. Tables S.5 and S.6 compare the forecast accuracy (with and without inclusion of COVID-19 data) of the model with switching A to that of the same hybrid shadow-rate VAR with fixed A (as considered in the paper).

Table S.5: Forecast performance of hybrid shadow-rate VAR w/o and w/switching impact matrix

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	1.00	1.00	0.99	**0.95**	1.00	1.00	1.00	1.01	1.00	1.01	1.02*	**1.05****
Real Consumption	1.02	1.02	1.03	**1.05**	1.00	1.02	1.00	1.01	1.02	1.02	**1.06**	**1.06**
IP	1.03	1.00	**1.05***	**1.07**	1.03*	1.00	1.04	**1.09****	1.03*	1.03*	**1.08****	**1.12*****
Capacity Utilization	1.01	0.97	0.97	**1.09**	0.99	**0.93****	0.99	1.03	1.00	0.99	1.01	**1.08**
Unemployment	1.02	1.01	**0.95***	**1.12**	1.00	0.98	**0.93****	**0.92****	1.02	**1.05**	**1.07**	**1.14**
Nonfarm Payrolls	0.97	**0.95**	1.04	**1.50*****	0.99	0.97	1.01	**1.08*****	0.99	1.00	**1.06***	**1.17****
Hours	1.01	0.98	**0.95**	**1.08**	1.01	0.97	**0.95**	0.98	1.02	1.03	1.03	**1.11****
Hourly Earnings	1.00	1.02	1.01	1.01	0.99	1.00	1.02	1.02	1.00	1.02	**1.05**	**1.08***
PPI (Fin. Goods)	1.02*	1.03*	**1.07*****	1.04	1.03*	1.04*	**1.09****	**1.07**	1.03**	1.03**	**1.08****	**1.08***
PPI (Metals)	1.00	1.04**	1.04	**1.11*****	1.03	1.03*	**1.07**	**1.06****	1.01	1.04***	**1.05****	**1.09*****
PCE Prices	**1.05*****	**1.07*****	**1.14*****	**1.06***	**1.06*****	**1.10*****	**1.21*****	**1.17*****	**1.07*****	**1.09*****	**1.17*****	**1.16*****
Federal Funds Rate	**3.13**	**2.71**	**2.55****	**3.01****	**1.82**	**1.34**	**0.91**	**0.94**	**1.75**	**1.48**	**1.22**	**1.29**
Housing Starts	0.97	0.97	**0.95**	**1.09**	0.98	0.98	0.97	1.00	**1.16***	**1.19**	**1.17**	**1.19**
S&P 500	**1.07**	1.04*	1.04*	**1.12**	1.04	1.03	1.03*	1.00	1.03*	1.03*	**1.05****	**1.06*****
USD / GBP FX Rate	0.99	0.98	1.01	0.98	1.01	1.01	1.03	0.97	0.99	1.01	1.03	1.03
6-Month Tbill	**3.13**	**2.27**	**1.82**	**2.03**	**0.92**	**0.89**	**0.88***	1.01	**1.32**	**1.18**	**1.08**	**1.16**
1-Year Yield	**3.13**	**2.31**	**1.85**	**2.07**	**0.89***	**0.84****	**0.93**	1.02	**1.25**	**1.16**	**1.09**	**1.19**
5-Year Yield	**1.57**	**1.44**	**1.69**	**2.41**	1.02	1.04	**1.10**	**1.17**	**1.22***	**1.17**	**1.21***	**1.25****
10-Year Yield	**1.48**	**1.46**	**1.74**	**2.73**	1.00	1.03	**1.18**	**1.23**	**1.24***	**1.24**	**1.35***	**1.34*****
BAA Yield	**1.23***	**1.64**	**2.07**	**2.83**	**1.07**	**1.23****	**1.31***	**1.44****	**1.35****	**1.39****	**1.51****	**1.53*****

Note: Comparison of "hybrid shadow-rate VAR" (baseline, in denominator) against "hybrid shadow-rate VAR (A switching)." Values below 1 indicate improvement over baseline. Evaluation window with forecast origins from 2009:01 through 2017:12 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h+1$ lags. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

Table S.6: Forecast performance of hybrid shadow-rate VAR w/o and w/switching impact matrix (incl. COVID-19)

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	1.00	1.00	1.02	**1.08**	1.00	1.00	0.99	0.99	1.01*	1.01	1.01*	1.04
Real Consumption	1.00	1.00	1.00*	1.04	1.01	1.00	0.99	1.02	1.02	1.01	1.03*	**1.05**
IP	1.00	1.00	1.00	**1.09**	1.02*	1.00	1.00	1.03	1.02**	1.02**	1.04**	**1.08****
Capacity Utilization	0.98	1.00	**1.09**	**1.83**	0.99	0.98	1.03	**1.05**	0.99	1.00	1.03	**1.10**
Unemployment	0.99	0.99	1.03	**1.72**	0.99	0.99	0.99	**1.05**	1.00	1.01	1.04*	**1.17***
Nonfarm Payrolls	1.00	1.00	1.00	**1.08**	1.01	1.00	1.03	**1.07**	1.01	1.01	1.02	**1.09***
Hours	0.99	1.00	**1.06**	**1.81**	0.99	0.97*	0.97	1.02	1.03	1.04	**1.07**	**1.18***
Hourly Earnings	1.00	1.00	0.96	**1.09**	1.00	1.01	1.01	1.02	1.01	1.02	**1.05***	**1.10****
PPI (Fin. Goods)	1.00	1.02*	1.03*	**1.08**	1.02	1.03*	**1.07*****	**1.06***	1.02*	1.03**	**1.06*****	**1.08****
PPI (Metals)	1.00	1.04**	**1.07***	**1.19**	1.02	1.03**	**1.07****	**1.06****	1.00	1.04***	**1.06****	**1.11*****
PCE Prices	**1.08**	**1.05*****	**1.05**	**1.08****	**1.06****	**1.08*****	**1.13*****	**1.12*****	**1.07****	**1.06*****	**1.11*****	**1.14*****
Federal Funds Rate	**14.67**	**4.65**	**5.88**	**8.51**	**4.58**	**1.25**	**0.93**	**1.06**	**4.73**	**1.56***	**1.40**	**1.84**
Housing Starts	1.00	**1.08**	**1.46**	**3.18**	0.99	1.01	1.03	**1.15**	**1.14****	**1.18***	**1.21***	**1.34***
S&P 500	**1.11**	1.01	**1.20**	**2.09**	**1.06**	1.00	**1.07***	0.99	**1.09***	**1.05***	**1.09***	**1.14**
USD / GBP FX Rate	1.00	1.00	1.04	**1.15**	1.01	1.01	1.04	0.96*	1.00	1.02	**1.05****	**1.07**
6-Month Tbill	**1.76**	**1.76***	**1.67**	**1.79**	**0.94**	1.03	0.97	1.04	**1.13**	**1.25**	**1.13**	**1.15**
1-Year Yield	**2.02**	**1.77***	**1.66**	**1.72**	**1.09**	1.04	1.02	**1.05**	**1.29***	**1.19***	**1.12**	**1.17**
5-Year Yield	**1.40**	**1.33**	**1.46**	**1.81**	**1.08**	**1.11**	**1.17**	**1.17***	**1.18****	**1.16****	**1.19****	**1.20****
10-Year Yield	**1.43**	**1.41**	**1.54**	**2.02**	**1.12**	**1.15**	**1.25***	**1.25**	**1.25****	**1.26****	**1.30****	**1.27*****
BAA Yield	**1.20***	**1.46**	**1.79**	**2.37**	**1.06**	**1.19****	**1.20***	**1.29****	**1.28****	**1.29****	**1.38****	**1.40*****

Note: Comparison of "hybrid shadow-rate VAR" (baseline, in denominator) against "hybrid shadow-rate VAR (A switching)." Values below 1 indicate improvement over baseline. Evaluation window with forecast origins from 2009:01 through 2022:08 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Due to the close behavior of some of the models compared, and rounding of the reported values, one of the comparisons shows a significant ratio of 1.00. This case arises from persistent differences in performance that are, however, too small to be relevant after rounding. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

III Generalized impulse responses: additional results

This part of the appendix provides additional estimates of generalized impulse response functions (GIRFs). Section III (a)) reports GIRFs for shocks to the excess bond premium (EBP) and Section III (b) reports estimates for shocks to financial uncertainty as measured by the VXO measure of stock market volatility. Finally, Section III (c) reports GIRFs from a small-size VAR, consisting of the EBP shock, unemployment rate, PCE inflation, federal funds rate (and in that order).

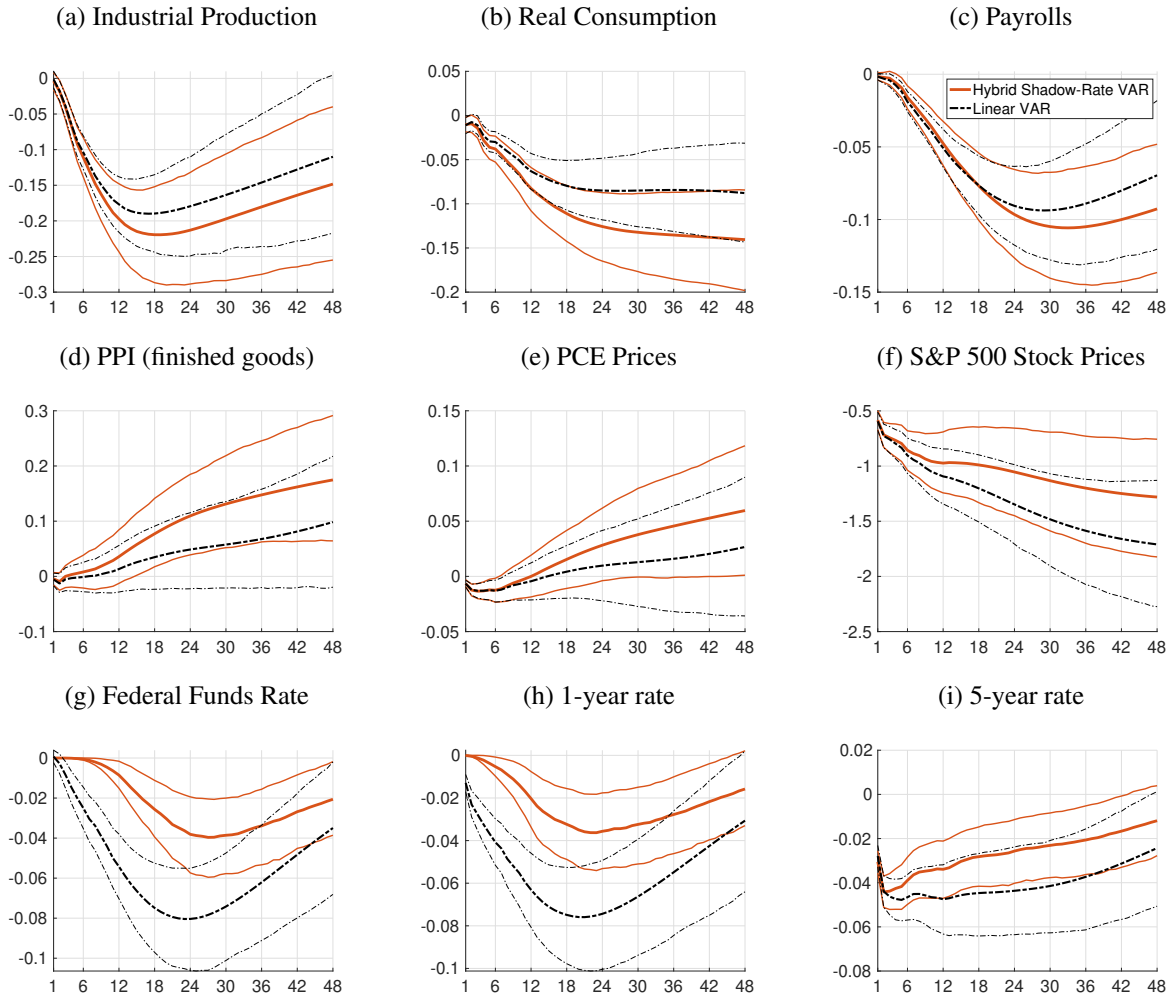
III (a) GIRFs to EBP shocks

Figure S.10 compares GIRFs obtained from the hybrid shadow-rate VAR (as shown in the paper) against those obtained from a linear VAR (in both cases, the GIRFs pertain to an EBP shock and originate in December 2012). For the linear VAR, the GIRFs are, of course, independent of the shock origin, as the ELB is ignored. As a consequence, estimated responses from the linear VAR differ from those obtained from our hybrid shadow-rate VAR for December 2012 (when the ELB was binding). They differ in ways that are quite similar to the comparison shown in the paper between GIRFs from the hybrid VAR that originate in January 2007 (when the ELB has not been binding) as opposed to December 2012. Figure S.11 presents GIRFs originating in December 2012 to an EBP shock when excluding yields from the model, with a comparison against the linear VAR shown in Figure S.12.

Overall, results from the hybrid shadow-rate VAR are qualitatively similar to those with yields included, as in the paper's presented results. However, when yields are excluded from the estimation, the difference between responses from linear and shadow-rate VAR are nearly muted, which speaks to the importance of incorporating multiple interest rates (instead of only the federal funds rate) in the analysis.

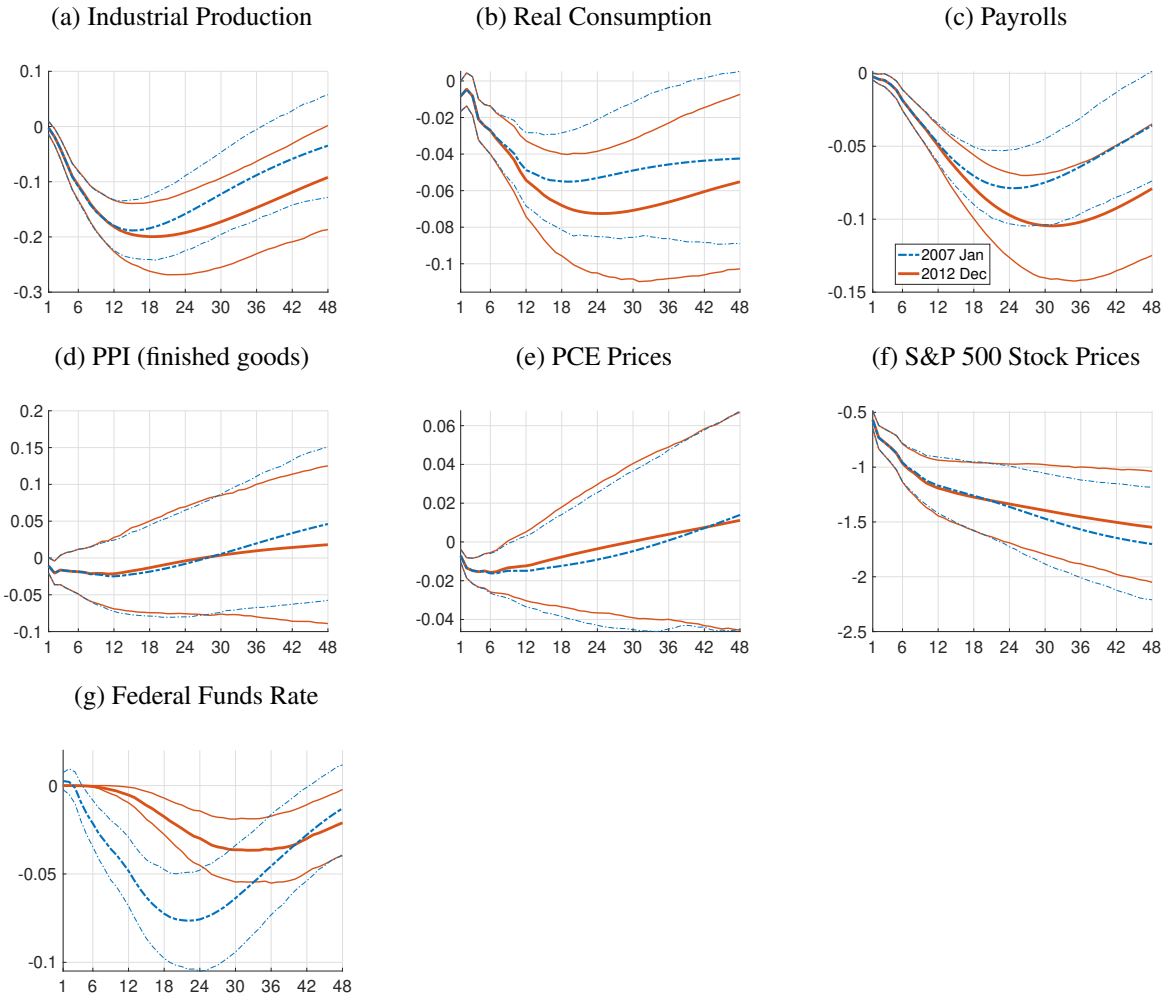
Using again the full data set (including yields), Figures S.13 through S.15 present GIRFs for different sizes and signs of the shock at different dates. For a given GIRF origin, there are very limited sign and size effects (except for short-term interest rates), whereas the size of responses differs across dates at or away from the ELB, as discussed in the paper.

Figure S.10: Generalized Impulse Responses: Hybrid vs Linear VAR (EBP Shock in 2012)



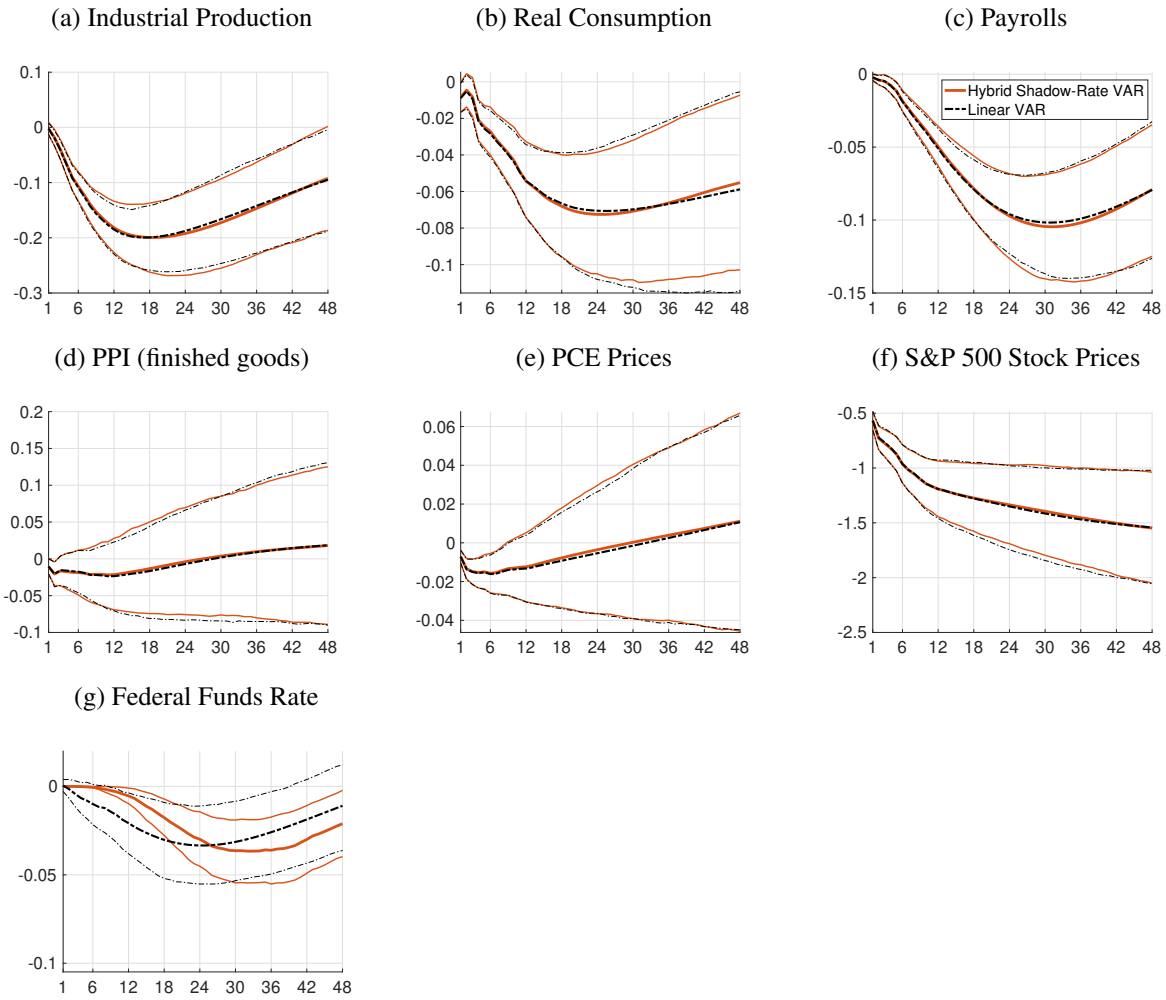
Note: Estimates obtained from full-sample posterior of linear and hybrid shadow-rate VARs, respectively, for GIRF in December 2012. Size of EBP shock has been set to average SV level of .11 for the years 2005-2006 (as obtained from the hybrid shadow-rate VAR, and with details described in Section 3.6 of the paper). Horizon of 48 months. Posterior means 68 percent credible sets. Interest rates in levels, and cumulated log levels (times 100) for all other variables.

Figure S.11: Generalized Impulse Responses to an EBP Shock in 2007 and 2012 (ex Yields)



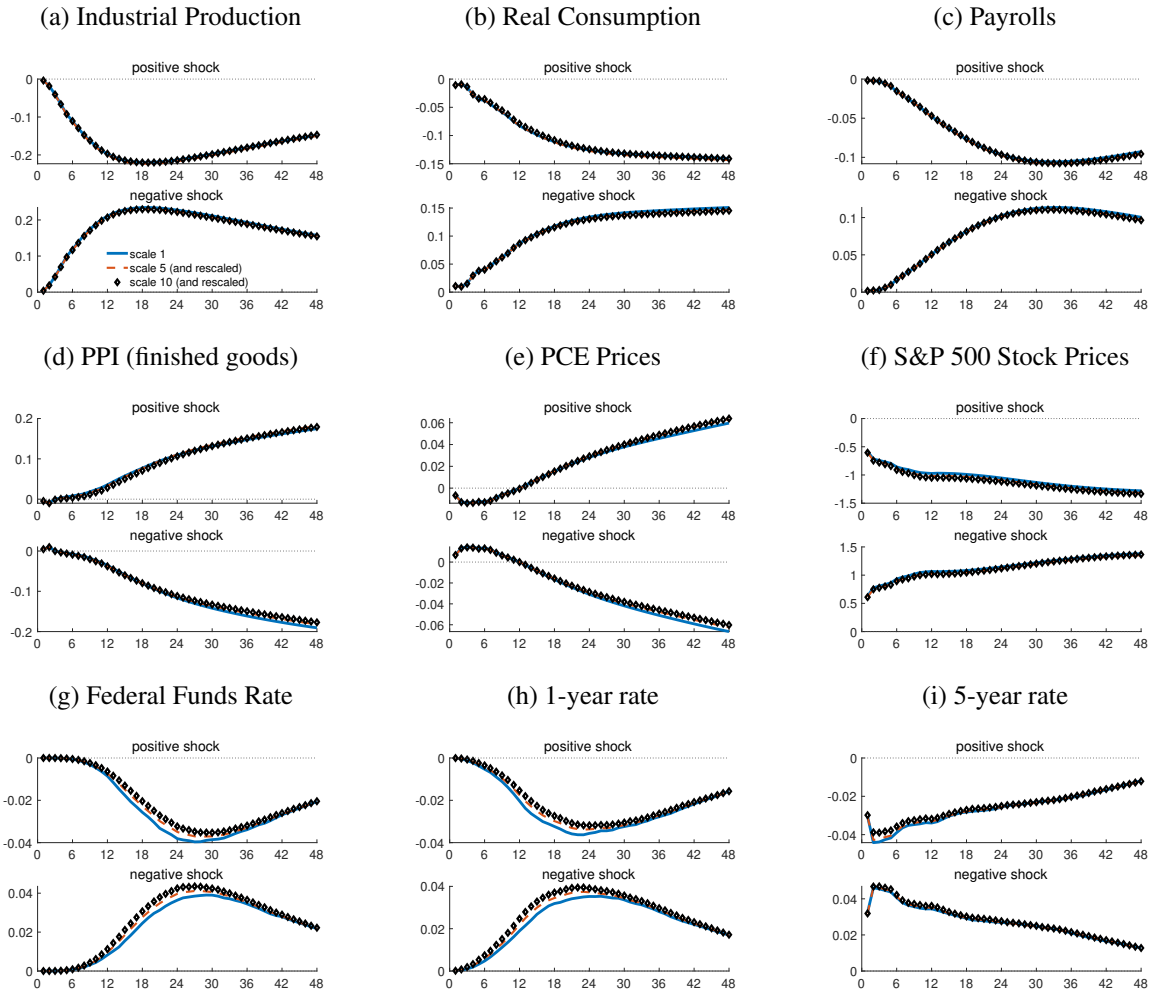
Note: Estimates obtained from full-sample posterior (conditioned on "ex-yields" data) of hybrid shadow-rate VAR for two different GIRF origins. Size of EBP shock has been set to average SV level of 0.11 for the years 2005-2006 (obtained from model with yields data as described in Section 3.6 of the paper). Horizon of 48 months. Posterior means 68 percent credible sets. Interest rates in levels, and cumulated log levels (times 100) for all other variables.

Figure S.12: Generalized Impulse Responses: Hybrid vs Linear VAR (EBP Shock in 2012, ex Yields)



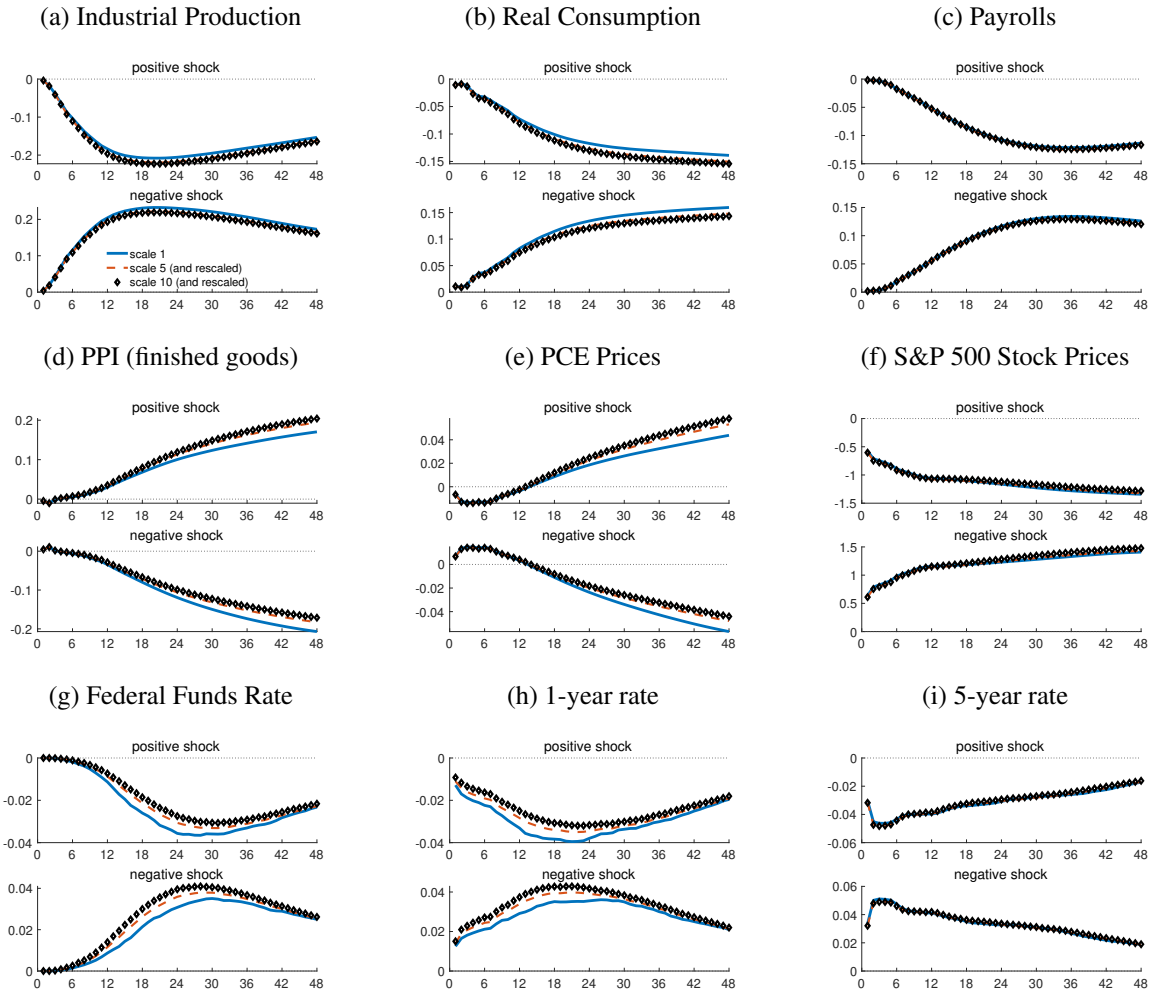
Note: Estimates obtained from full-sample posterior (conditioned on "ex-yields" data) of linear and hybrid shadow-rate VARs, respectively, for GIRF in December 2012. Size of EBP shock has been set to average SV level of .11 for the years 2005-2006 (as obtained from the hybrid shadow-rate VAR with all yields, and with details described in Section 3.6 of the paper). Horizon of 48 months. Posterior means 68 percent credible sets. Interest rates in levels, and cumulated log levels (times 100) for all other variables.

Figure S.13: GIRF to EBP shocks of different sizes and signs in December 2012



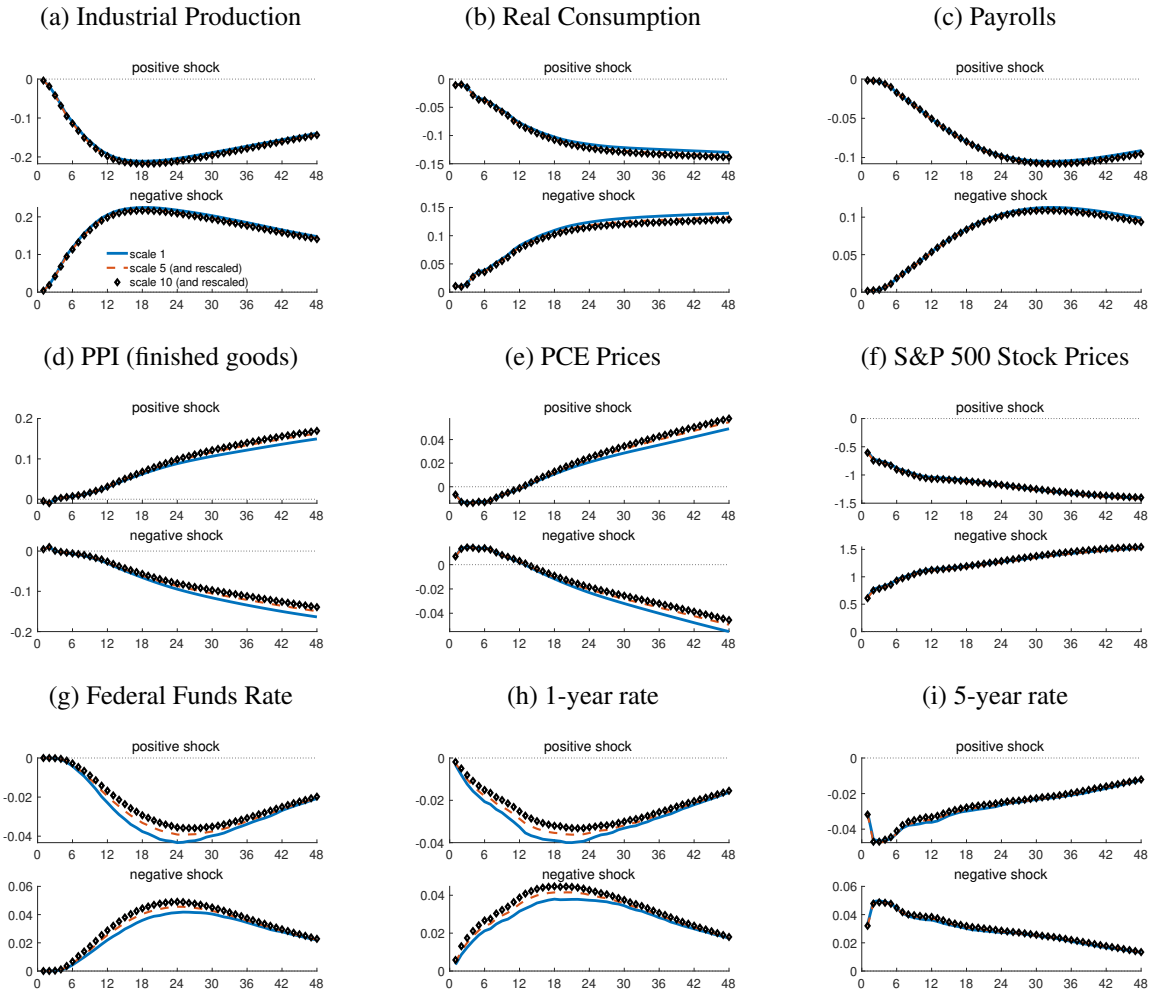
Note: GIRFs to positively and negatively signed EBP shocks of different sizes and with origin of the baseline forecast in December 2012. The shock sizes considered are $k \cdot \sigma$ for $k = 1, 5, 10$ and $\sigma = 0.11$ (reflecting the average SV level of EBP shocks for the years 2005-2006 as described in Section 3.6 of the paper). Posterior mean estimates simulated from full-sample posterior of hybrid shadow-rate VAR. Interest rates in levels, and cumulated log levels (times 100) for all other variables.

Figure S.14: GIRF to EBP shocks of different sizes and signs in December 2010



Note: GIRFs to positively and negatively signed EBP shocks of different sizes and with origin of the baseline forecast in December 2010. The shock sizes considered are $k \cdot \sigma$ for $k = 1, 5, 10$ and $\sigma = 0.11$ (reflecting the average SV level of EBP shocks for the years 2005-2006 as described in Section 3.6 of the paper). Posterior mean estimates simulated from full-sample posterior of hybrid shadow-rate VAR. Interest rates in levels, and cumulated log levels (times 100) for all other variables.

Figure S.15: GIRF to EBP shocks of different sizes and signs in December 2014



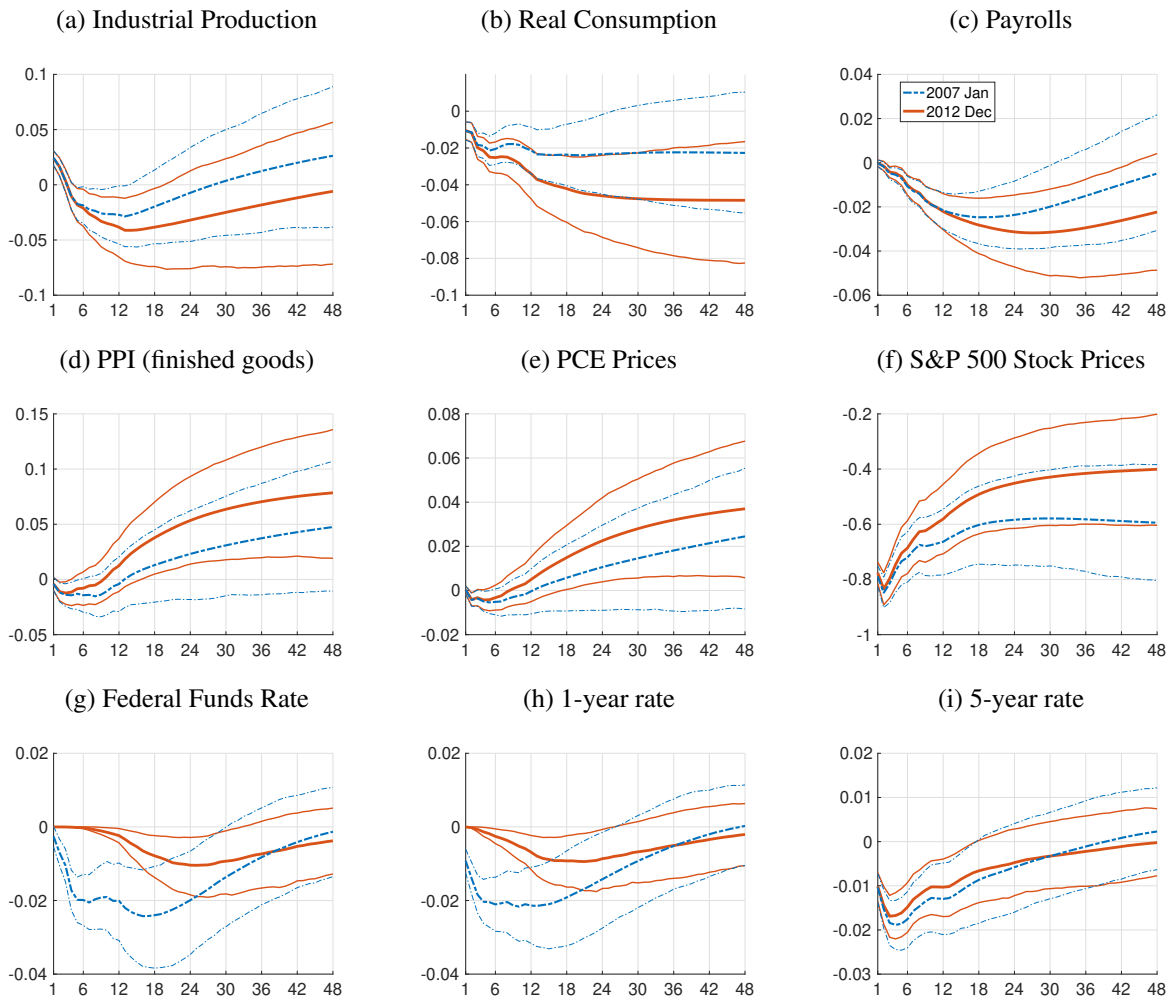
Note: GIRFs to positively and negatively signed EBP shocks of different sizes and with origin of the baseline forecast in December 2014. The shock sizes considered are $k \cdot \sigma$ for $k = 1, 5, 10$ and $\sigma = 0.11$ (reflecting the average SV level of EBP shocks for the years 2005-2006 as described in Section 3.6 of the paper). Posterior mean estimates simulated from full-sample posterior of hybrid shadow-rate VAR. Interest rates in levels, and cumulated log levels (times 100) for all other variables.

III (b) GIRFs to VXO shocks

As further robustness check to the GIRF estimates based on EBP shocks reported in the paper, we also consider shocks to financial uncertainty, as measured by the extended VXO index provided in FRED-MD. The extended series splices historical data on options-implied volatility to realized volatility measures for earlier dates.[1] The first observation of the series is available for July 1962 and the estimation sample has been adjusted accordingly in this case. Figures S.16 and S.18 presents GIRFs to a VXO shock (obtained from hybrid shadow-rate and linear VARs), and the effects are qualitatively similar to those of an EBP shock as shown in Figure 6 of the paper. Figures S.17 and S.19 presents the corresponding GIRFs that obtain when yields data is excluded from the model. Overall, as for the EBP shock, results from the hybrid shadow-rate VAR are qualitatively similar to those with yields included (whereas differences with the linear VAR are again muted when yields are dropped from the data). Figures S.20 through S.22 present GIRFs for different sizes and signs of the VXO shock, and for different dates. Again as for the EBP shock, there are very limited sign and size effects (except for short-term interest rates) for a given GIRF origin.

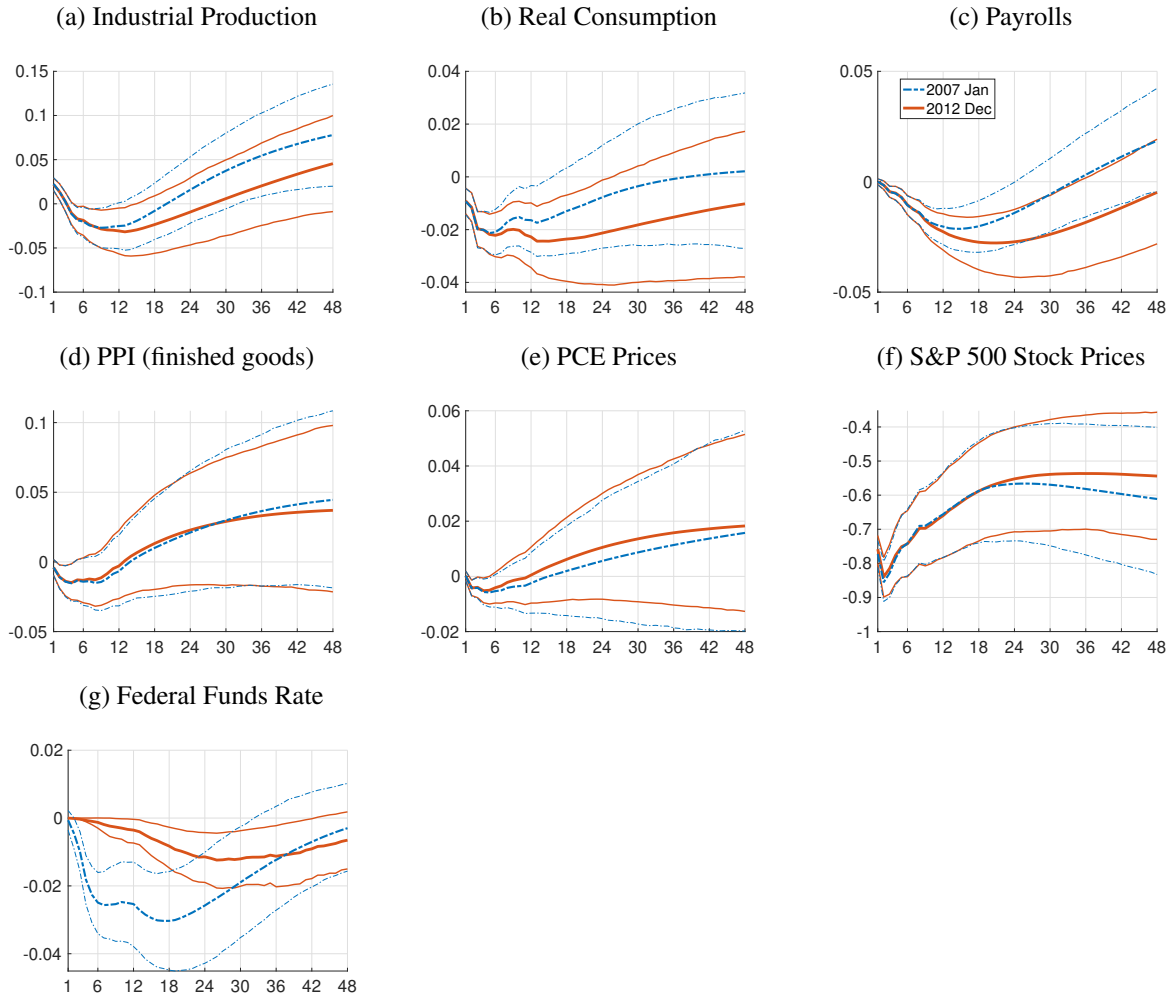
[1]The extended VXO series is denoted "VXOCLSx" in FRED-MD and has been added to FRED-MD in 2015-09 as documented at https://files.stlouisfed.org/files/htdocs/uploads/fredmdchanges.pdf.

Figure S.16: Generalized Impulse Responses to a VXO Shock in 2007 and 2012



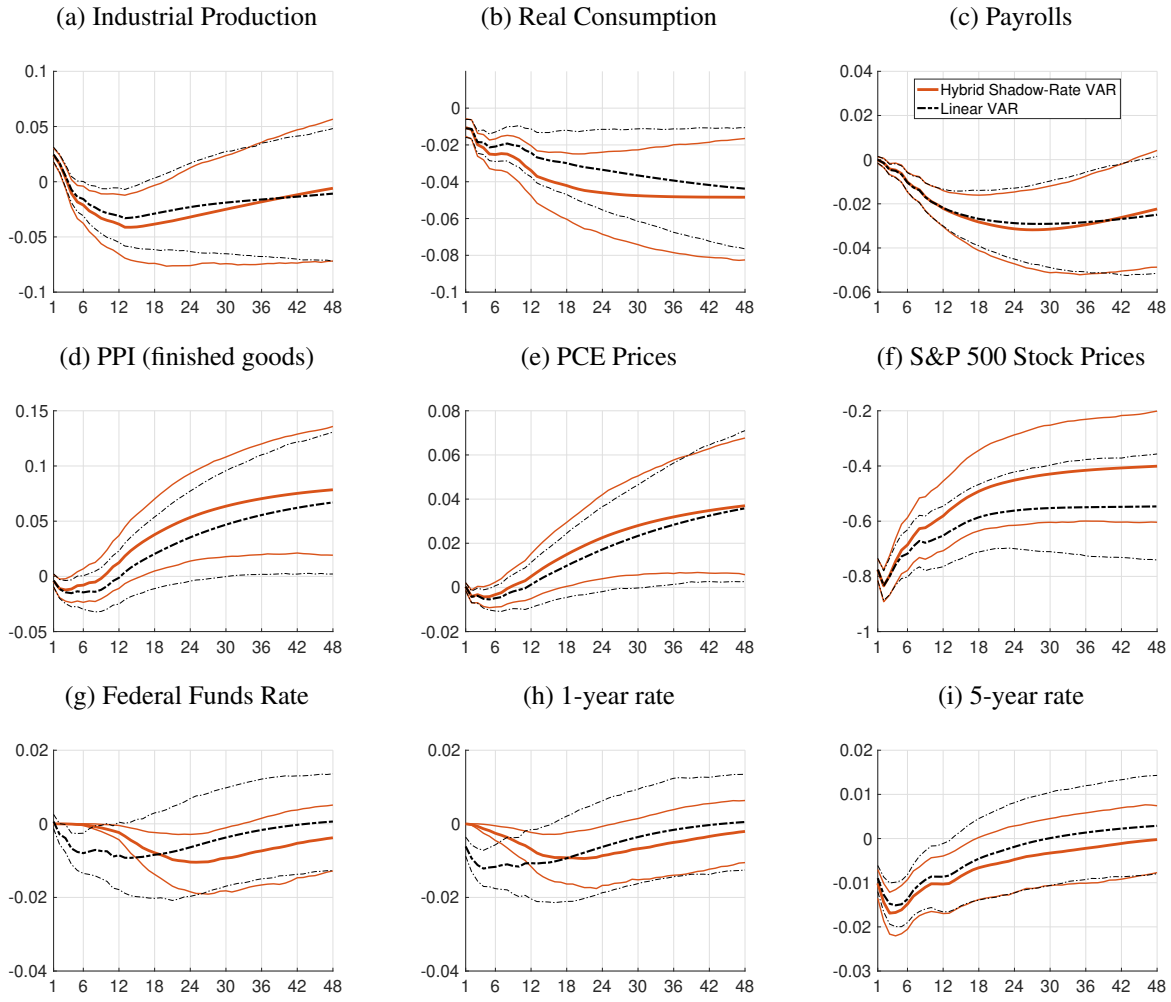
Note: Estimates obtained from full-sample posterior of hybrid shadow-rate VAR for two different GIRF origins. Size of VXO shock has been set to average SV level of 1.27 for the years 2005-2006 (with details described in Section 3.6 of the paper). Horizon of 48 months. Posterior means and 68 percent credible sets. Interest rates in levels, and cumulated log levels (times 100) for all other variables.

Figure S.17: Generalized Impulse Responses to a VXO Shock in 2007 and 2012 (ex Yields)



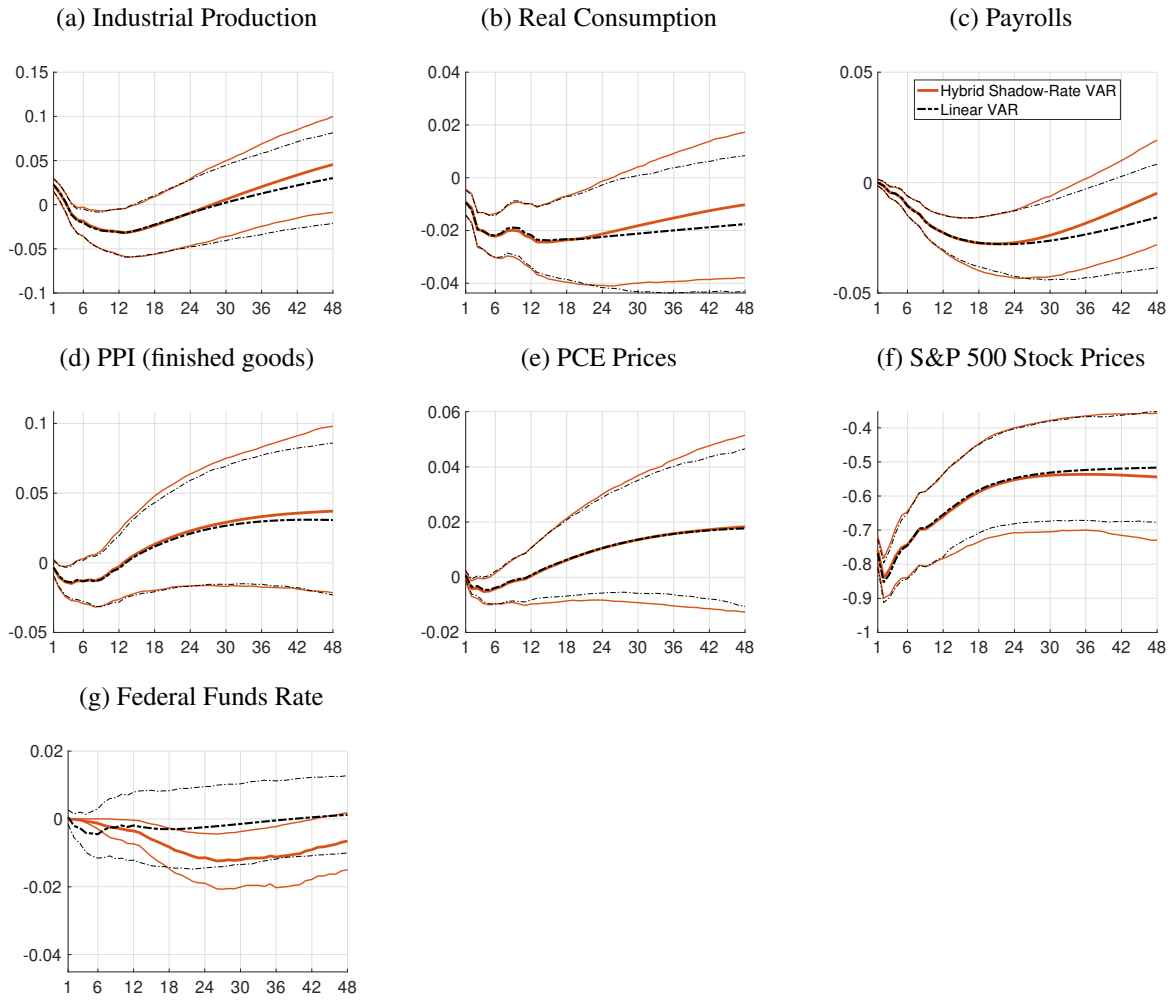
Note: Estimates obtained from full-sample posterior (conditioned on "ex-yields" data) of hybrid shadow-rate VAR for two different GIRF origins. Size of VXO shock has been set to average SV level of 1.27 for the years 2005-2006 (obtained from model with yields data as described in Section 3.6 of the paper). Horizon of 48 months. Posterior means 68 percent credible sets. Interest rates in levels, and cumulated log levels (times 100) for all other variables.

Figure S.18: Generalized Impulse Responses: Hybrid vs Linear VAR (VXO Shock in 2012)



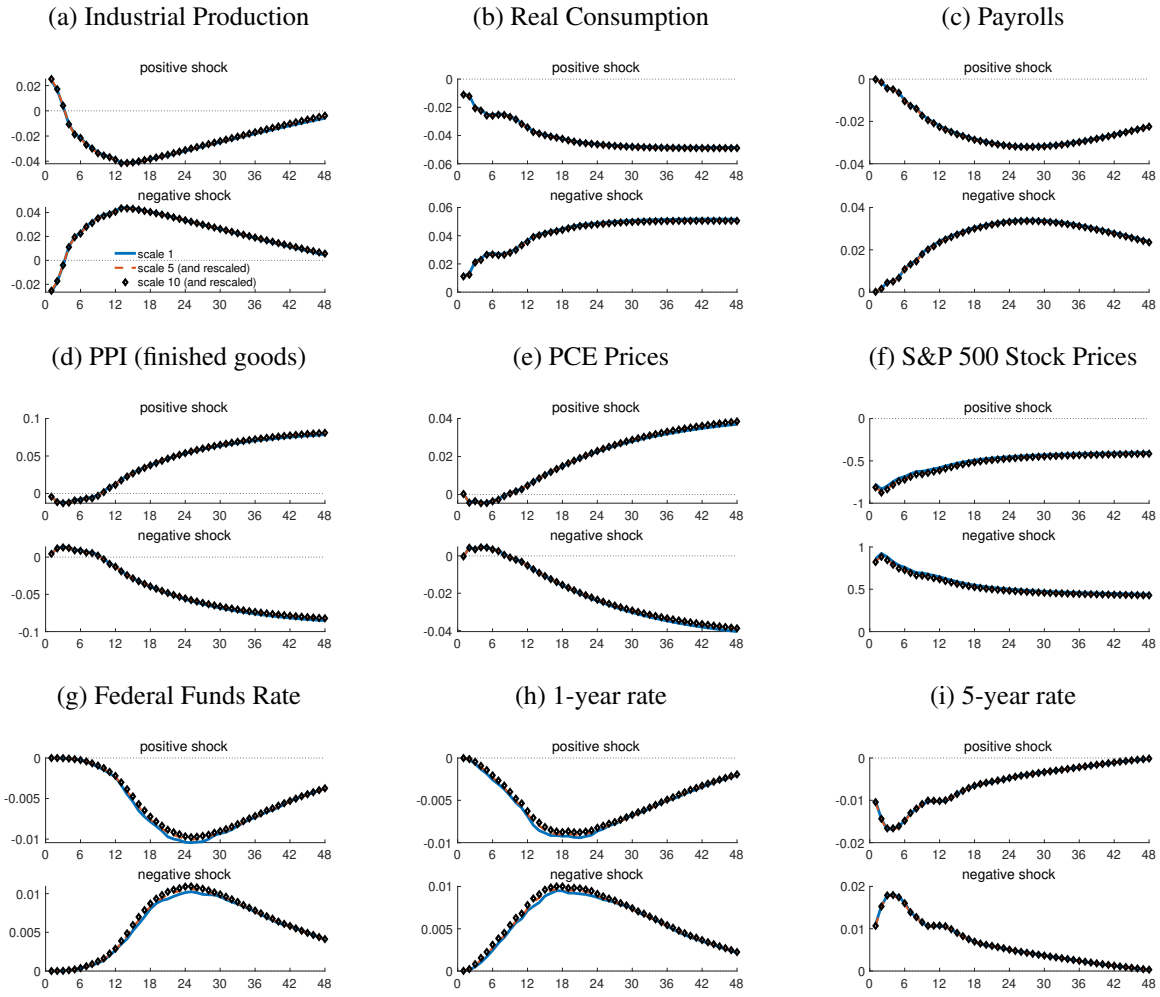
Note: Estimates obtained from full-sample posterior of linear and hybrid shadow-rate VARs, respectively, for GIRF in December 2012. Size of VXO shock has been set to average SV level of 1.27 for the years 2005-2006 (as obtained from the hybrid shadow-rate VAR, and with details described in Section 3.6 of the paper). Horizon of 48 months. Posterior means 68 percent credible sets. Interest rates in levels, and cumulated log levels (times 100) for all other variables.

Figure S.19: Generalized Impulse Responses: Hybrid vs Linear VAR (VXO Shock in 2012, ex Yields)



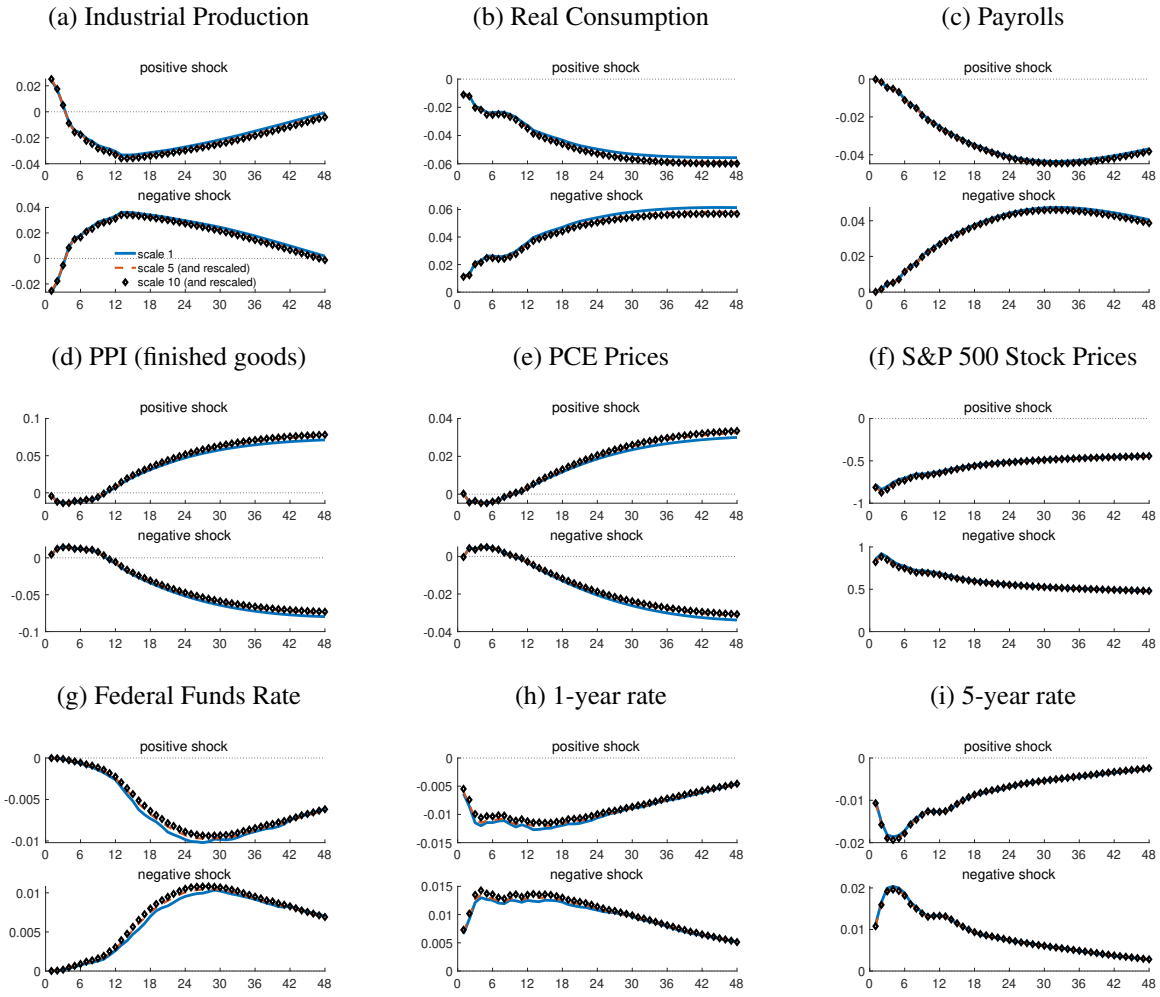
Note: Estimates obtained from full-sample posterior (conditioned on "ex-yields" data) of linear and hybrid shadow-rate VARs, respectively, for GIRF in December 2012. Size of VXO shock has been set to average SV level of 1.27 for the years 2005-2006 (as obtained from the hybrid shadow-rate VAR with all yields, and with details described in Section 3.6 of the paper). Horizon of 48 months. Posterior means 68 percent credible sets. Interest rates in levels, and cumulated log levels (times 100) for all other variables.

Figure S.20: GIRF to VXO shocks of different sizes and signs in December 2012



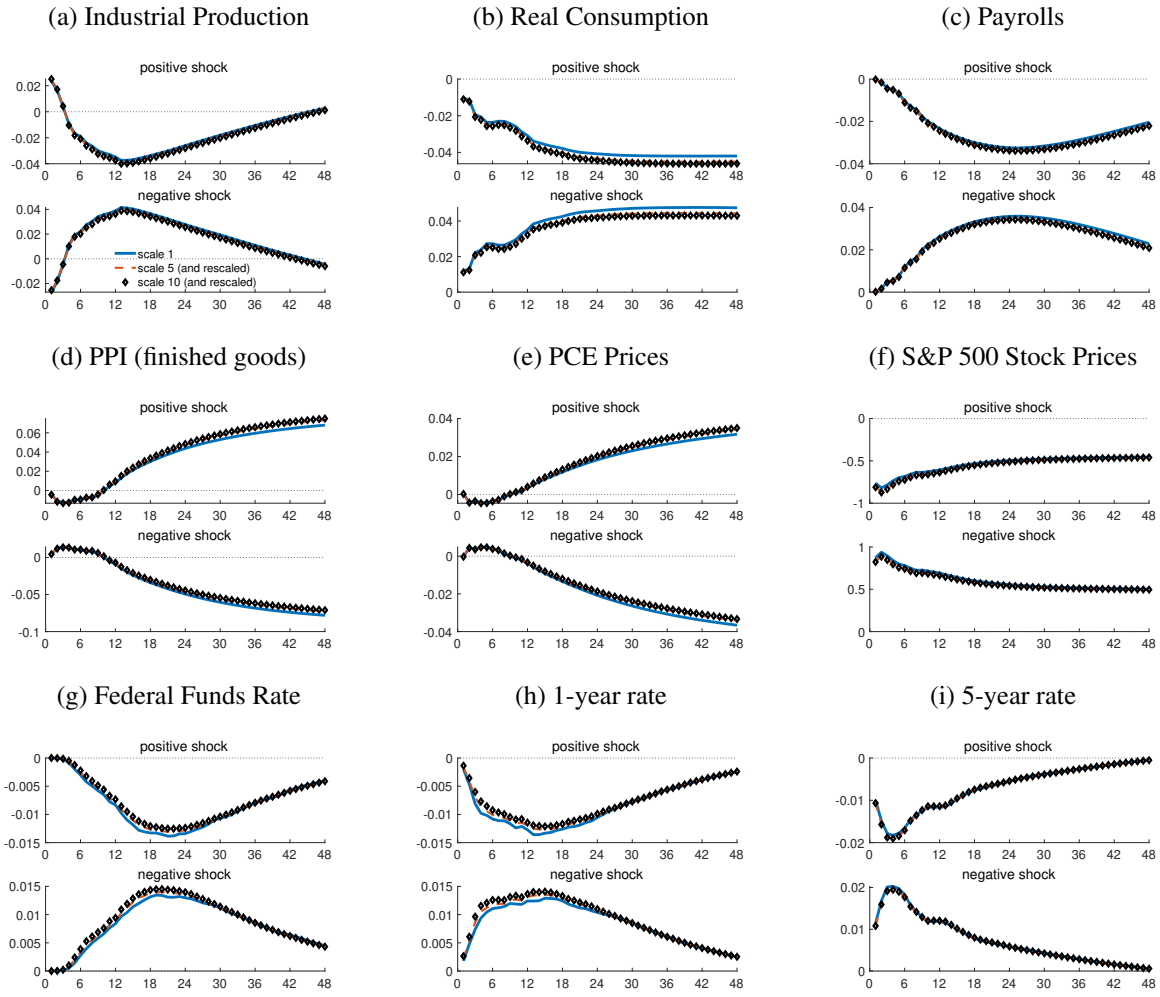
Note: GIRFs to positively and negatively signed VXO shocks of different sizes and with origin of the baseline forecast in December 2012. The shock sizes considered are $k \cdot \sigma$ for $k = 1, 5, 10$ and $\sigma = 1.27$ (reflecting the average SV level of VXO shocks for the years 2005-2006 as described in Section 3.6 of the paper). Posterior mean estimates simulated from full-sample posterior of hybrid shadow-rate VAR. Interest rates in levels, and cumulated log levels (times 100) for all other variables.

Figure S.21: GIRF to VXO shocks of different sizes and signs in December 2010



Note: GIRFs to positively and negatively signed VXO shocks of different sizes and with origin of the baseline forecast in December 2010. The shock sizes considered are $k \cdot \sigma$ for $k = 1, 5, 10$ and $\sigma = 1.27$ (reflecting the average SV level of VXO shocks for the years 2005-2006 as described in Section 3.6 of the paper). Posterior mean estimates simulated from full-sample posterior of hybrid shadow-rate VAR. Interest rates in levels, and cumulated log levels (times 100) for all other variables.

Figure S.22: GIRF to VXO shocks of different sizes and signs in December 2014



Note: GIRFs to positively and negatively signed VXO shocks of different sizes and with origin of the baseline forecast in December 2014. The shock sizes considered are $k \cdot \sigma$ for $k = 1, 5, 10$ and $\sigma = 1.27$ (reflecting the average SV level of VXO shocks for the years 2005-2006 as described in Section 3.6 of the paper). Posterior mean estimates simulated from full-sample posterior of hybrid shadow-rate VAR. Interest rates in levels, and cumulated log levels (times 100) for all other variables.

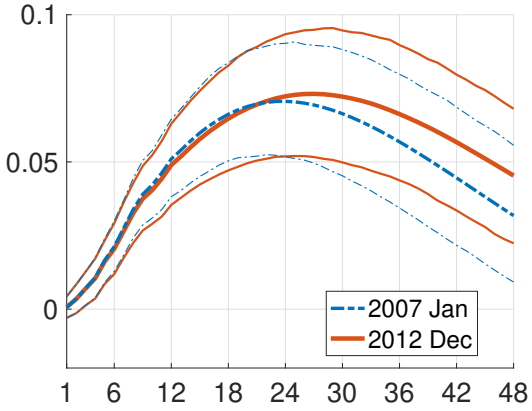
III (c) GIRFs from small-scale VAR

Here we reports GIRFs from a small-size VAR, consisting of the EBP shock, unemployment rate, PCE inflation, federal funds rate (and in that order). Figure S.23 provides GIRFs estimated from the hybrid shadow-rate VAR for origins in January 2007 (when the economy was well away from the ELB) and December 2012 (when the ELB was binding.) Figure S.24 compares GIRFs from a linear VAR and the hybrid shadow-rate VAR for December 2012. (For the linear VAR, the GIRFs are, of course, independent of the shock origin.) Figures S.25 through S.27 present GIRFs for different sizes and signs of the shock, and for different origin dates.

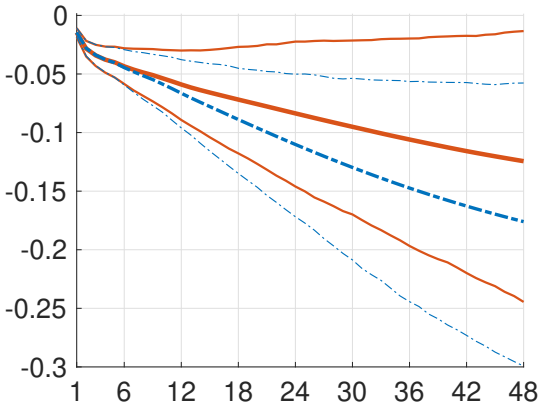
The smaller model generates responses that are qualitatively similar to those for our medium-size model, in the sense that at the ELB the interest rate decreases less so that unemployment increases more and inflation decreases less. Yet, the differences emerge more slowly than in the larger model, which provides a more granular representation of the economy that better captures the shock transmission mechanism. These results underscore the importance of working with such larger shadow-rate models, and use of our shadow-rate sampling methods assures their computational feasibility.

Figure S.23: Generalized Impulse Responses to an EBP Shock in 2007 and 2012 (small VAR)

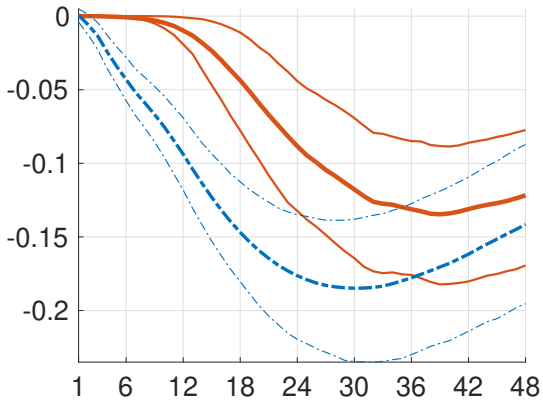
(a) Unemployment Rate



(b) PCE Prices



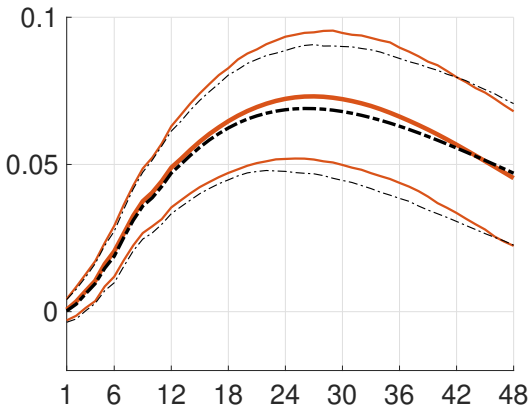
(c) Federal Funds Rate



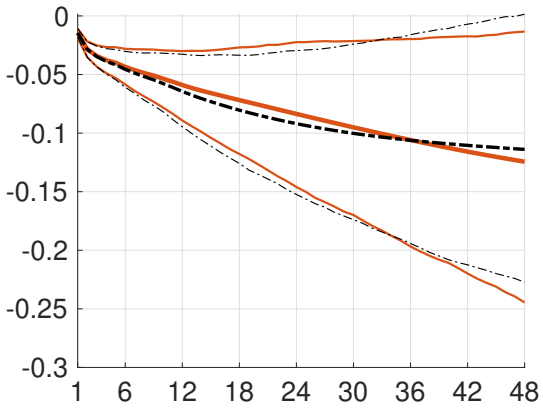
Note: Estimates obtained from full-sample posterior of hybrid shadow-rate VAR for two different GIRF origins. Small-size VAR using only EBP, unemployment rate, PCE inflation and the federal funds rate. Size of EBP shock has been set to average SV level of 0.11 for the years 2005-2006 (with details described in Section 3.6 of the paper). Horizon of 48 months. Posterior means and 68 percent credible sets. Interest rates in levels, and cumulated log levels (times 100) for all other variables.

Figure S.24: Generalized Impulse Responses: Hybrid vs Linear VAR (small VAR)

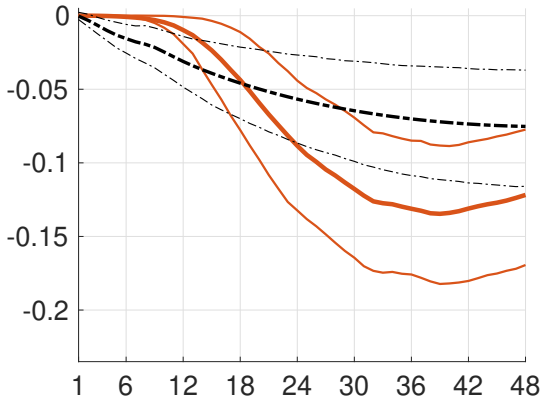
(a) Unemployment Rate



(b) PCE Prices

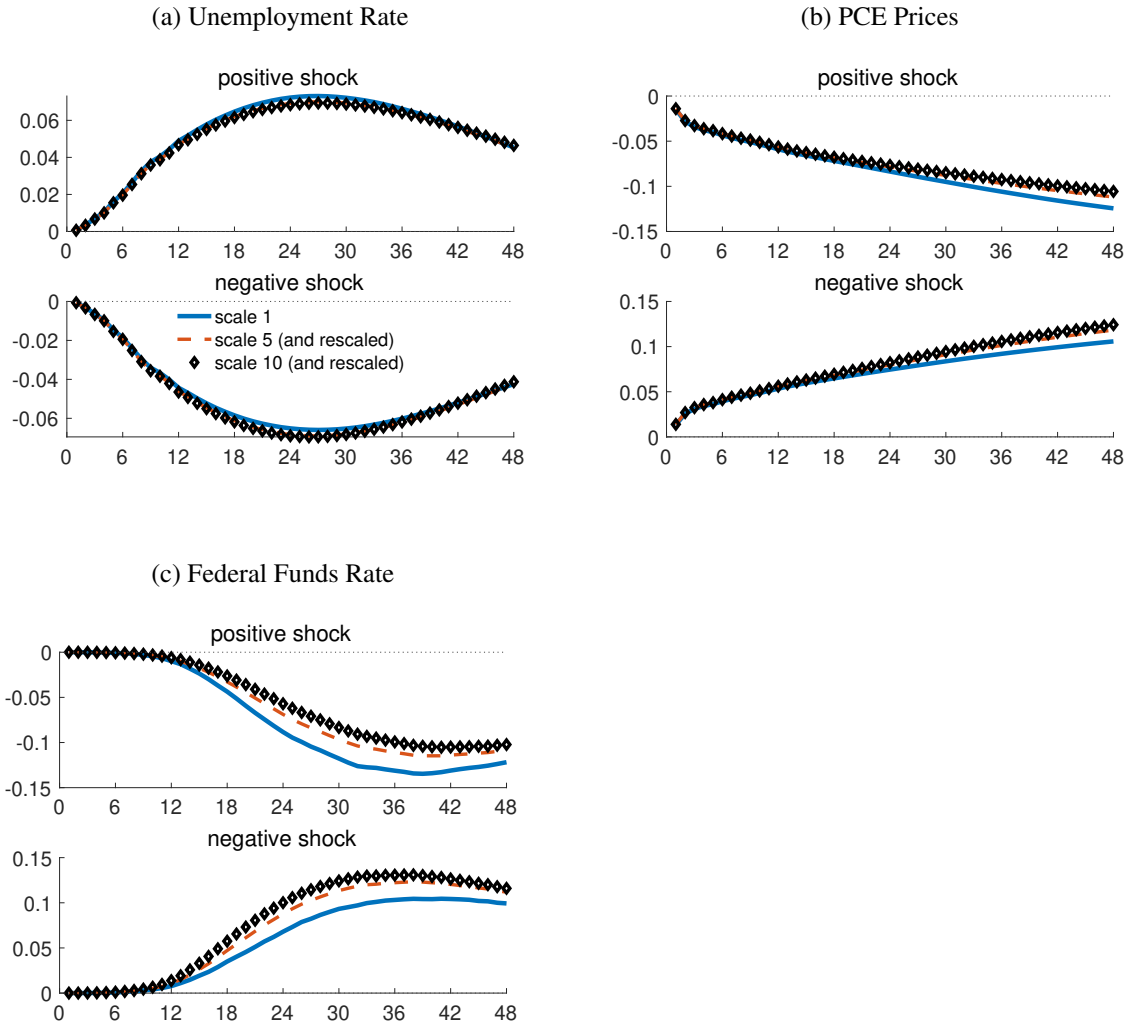


(c) Federal Funds Rate



Note: Estimates obtained from full-sample posterior of linear and hybrid shadow-rate VARs, respectively, for GIRF in December 2012. Small-size VARs using only EBP, unemployment rate, PCE inflation and the federal funds rate. Size of EBP shock has been set to average SV level of 0.11 for the years 2005-2006 (as obtained from the hybrid shadow-rate VAR, and with details described in Section 3.6 of the paper). Horizon of 48 months. Posterior means 68 percent credible sets. Interest rates in levels, and cumulated log levels (times 100) for all other variables.

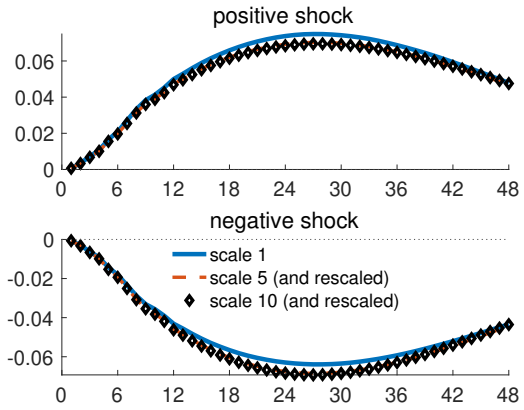
Figure S.25: GIRF to EBP shocks of different sizes and signs in small VAR in December 2012



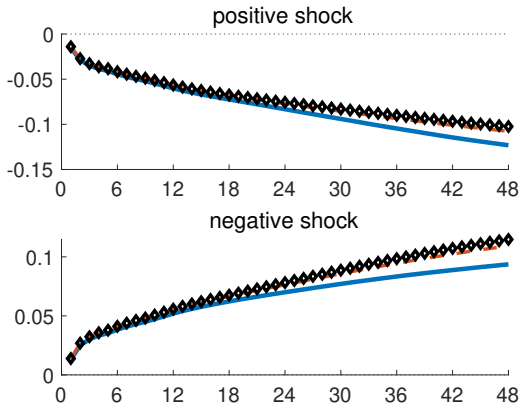
Note: GIRFs to positively and negatively signed EBP shocks of different sizes and with origin of the baseline forecast in December 2012. The shock sizes considered are $k \cdot \sigma$ for $k = 1, 5, 10$ and $\sigma = 0.11$ (reflecting the average SV level of EBP shocks for the years 2005-2006 as described in Section 3.6 of the paper). Posterior mean estimates simulated from full-sample posterior of hybrid shadow-rate VAR using only EBP, unemployment rate, PCE inflation and the federal funds rate. Interest rates in levels, and cumulated log levels (times 100) for all other variables.

Figure S.26: GIRF to EBP shocks of different sizes and signs in small VAR in December 2010

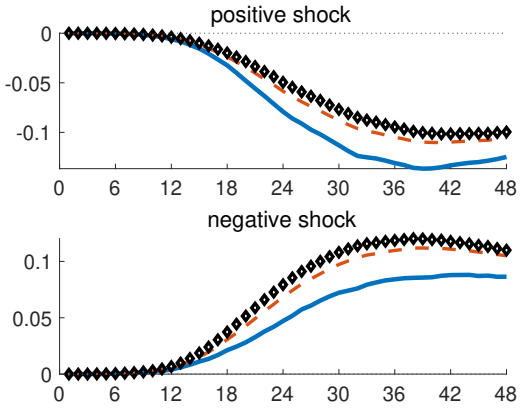
(a) Unemployment Rate



(b) PCE Prices



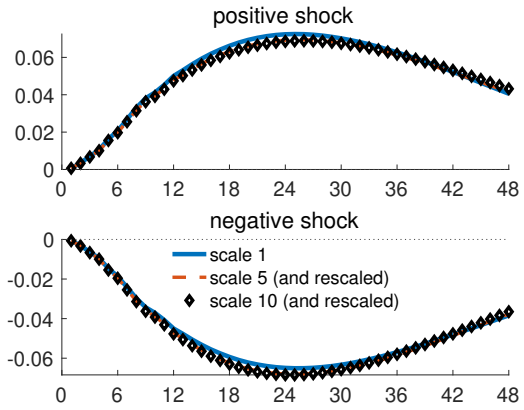
(c) Federal Funds Rate



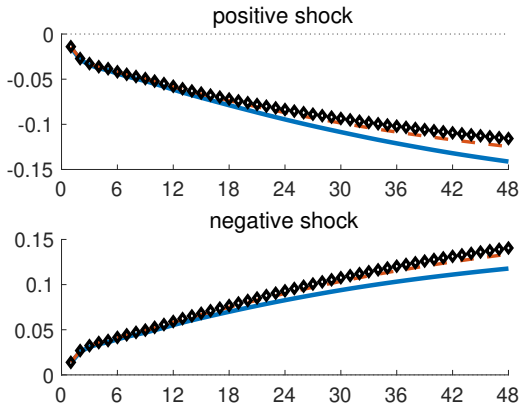
Note: GIRFs to positively and negatively signed EBP shocks of different sizes and with origin of the baseline forecast in December 2010. The shock sizes considered are $k \cdot \sigma$ for $k = 1, 5, 10$ and $\sigma = 0.11$ (reflecting the average SV level of EBP shocks for the years 2005-2006 as described in Section 3.6 of the paper). Posterior mean estimates simulated from full-sample posterior of hybrid shadow-rate VAR using only EBP, unemployment rate, PCE inflation and the federal funds rate. Interest rates in levels, and cumulated log levels (times 100) for all other variables.

Figure S.27: GIRF to EBP shocks of different sizes and signs in small VAR in December 2014

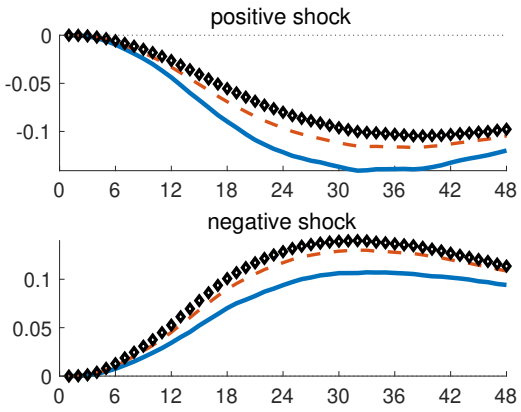
(a) Unemployment Rate



(b) PCE Prices



(c) Federal Funds Rate



Note: GIRFs to positively and negatively signed EBP shocks of different sizes and with origin of the baseline forecast in December 2014. The shock sizes considered are $k \cdot \sigma$ for $k = 1, 5, 10$ and $\sigma = 0.11$ (reflecting the average SV level of EBP shocks for the years 2005-2006 as described in Section 3.6 of the paper). Posterior mean estimates simulated from full-sample posterior of hybrid shadow-rate VAR using only EBP, unemployment rate, PCE inflation and the federal funds rate. Interest rates in levels, and cumulated log levels (times 100) for all other variables.

IV Forecast performance of shadow-rate vs linear VARs: additional results

IV (a) Forecast performance of baseline models incl. COVID-19

Tables S.7 and S.8 repeat the forecast evaluations provided in the paper's tables, but with the end point of the evaluation sample extended from December 2019 to August 2022. The results in the longer sample align with the results described in the paper for the sample ending with the forecast origin of December 2017, for which the end point of the evaluation sample is December 2019.

Table S.7: Forecast performance of standard vs. simple shadow-rate VAR (incl. COVID-19)

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	1.00	1.00	0.97	**0.89**	1.01	1.00	1.00	1.00	1.01	1.01	1.01	1.01
Real Consumption	1.00	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.01*	1.00
IP	1.01*	1.00	1.00	1.00	1.02**	1.02*	1.03**	1.01*	1.01**	1.01*	1.02***	1.01
Capacity Utilization	1.01*	1.01	**1.05****	**1.09*****	1.04***	**1.06*****	**1.12*****	**1.12****	1.03***	1.04***	**1.08*****	**1.08*****
Unemployment	1.00	1.00	0.98	**0.95**	0.99	0.98	0.96	**0.95**	0.99	0.99	0.99	0.98
Nonfarm Payrolls	1.00	1.00	1.00	1.00	1.00	1.01	1.02	1.02	1.00	1.01	1.02**	1.01*
Hours	1.00	1.00	1.00	1.03*	0.99	1.00	1.02	**1.05**	1.00	1.01	1.01	1.02
Hourly Earnings	1.00	1.00	1.01	1.01	1.00	1.01	1.00	1.01	1.00	1.01	1.00	1.00
PPI (Fin. Goods)	1.00	1.00	1.01	0.99*	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
PPI (Metals)	1.00	1.00	1.00	1.02	1.00	1.00	1.00	1.01**	1.00	1.00	1.00	1.01*
PCE Prices	1.01	1.01	1.02	0.96	1.01	1.01	1.02	1.01	1.00	1.01	1.01	1.01
Federal Funds Rate	**0.44****	**0.48***	**0.64**	**1.22**	**0.25*****	**0.35*****	**0.41****	**0.47****	**0.29*****	**0.35*****	**0.42****	**0.50*****
Housing Starts	0.97	**0.94**	**0.92**	**0.87**	0.97	**0.94**	**0.95**	**0.94**	0.99	0.98	0.97	**0.94**
S&P 500	1.00	1.01	1.01*	0.97	0.99	1.01	0.99	0.99	1.00	1.01	1.00	1.01*
USD / GBP FX Rate	0.99**	0.99**	0.99*	0.98	0.98**	0.99**	0.99	0.98	0.99**	0.99**	1.00	0.99
6-Month Tbill	**0.70****	**0.73**	**0.85**	**1.32**	**0.43*****	**0.49*****	**0.53****	**0.63****	**0.46*****	**0.51*****	**0.58****	**0.63*****
1-Year Yield	**0.79**	**0.86**	**0.93**	**1.32**	**0.61*****	**0.60*****	**0.62****	**0.73****	**0.63*****	**0.64*****	**0.67*****	**0.68*****
5-Year Yield	0.98	0.98	0.97	**1.19**	**0.92****	**0.91****	**0.91*****	**0.93*****	**0.95**	**0.93****	**0.91*****	**0.87*****
10-Year Yield	0.99	0.98	0.96	**1.18**	0.97	0.96	**0.91*****	**0.92***	0.98	0.96	**0.94***	**0.94****
BAA Yield	0.97	0.99	1.01	**1.08**	0.98	0.96	0.99	1.02	0.99	1.00	1.01	1.03

Note: Comparison of "standard linear VAR" (baseline, in denominator) against "simple shadow-rate VAR." Values below 1 indicate improvement over baseline. Evaluation window with forecast origins from 2009:01 through 2022:08 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

Table S.8: Forecast performance of standard vs. hybrid shadow-rate VAR (incl. COVID-19)

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	1.00	1.00	**0.93**	**0.93**	1.01	1.00	1.01	0.99	1.01	1.00	1.01	1.00
Real Consumption	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.97	1.00	1.00	1.00	0.99**
IP	1.00	1.00	0.99	1.00	1.01	1.01	1.00	0.99	1.01*	1.00	1.00	1.00
Capacity Utilization	1.01	0.99	0.99	0.97	1.01	0.99	1.00	0.97	1.01	0.99	1.00	0.98
Unemployment	1.00	1.01	1.01	1.01	1.00	1.00	1.00	**0.95****	1.00	1.00	1.00	0.97***
Nonfarm Payrolls	1.00	1.00	1.00	1.00	0.99*	1.00	1.00	0.98	1.00	1.01	1.01	1.00
Hours	1.00	1.01**	1.03	1.03	1.00	1.01	1.03	1.03	1.01*	1.01**	1.01	1.01
Hourly Earnings	1.00	1.01	1.02*	**1.06****	0.99	1.00	1.00	1.02**	1.00	1.00	1.00	1.01
PPI (Fin. Goods)	1.00	1.00	1.00	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
PPI (Metals)	0.99**	1.00	0.99*	1.00	1.00	0.99	0.99	1.00	1.00	0.99*	0.99	1.00
PCE Prices	1.00	1.00	1.01	0.97	1.00	0.99	1.02	**1.06*****	1.00	1.00	1.01	1.04***
Federal Funds Rate	**0.43****	**0.44***	**0.58**	**1.07**	**0.26*****	**0.33*****	**0.40****	**0.49****	**0.29*****	**0.33*****	**0.39****	**0.46*****
Housing Starts	1.00	0.99	0.96	1.02	1.00	0.98	**0.94**	**0.95**	1.00	0.99	0.96	0.96
S&P 500	0.99	1.00	1.01	1.04	0.99	0.99	1.00	1.00	1.00	1.01	1.01**	1.00*
USD / GBP FX Rate	1.01	1.00	0.99	0.99	1.01	1.01	1.01	1.02	1.00	1.00	1.00	1.00
6-Month Tbill	**0.66****	**0.66**	**0.77**	**1.15**	**0.40*****	**0.44*****	**0.49****	**0.59****	**0.43*****	**0.46*****	**0.53*****	**0.57*****
1-Year Yield	**0.73****	**0.77***	**0.84**	**1.18**	**0.57*****	**0.54*****	**0.55*****	**0.67****	**0.58*****	**0.59*****	**0.62*****	**0.62*****
5-Year Yield	0.98	0.96	**0.93**	**1.06**	**0.93**	**0.93**	**0.90***	**0.92**	**0.95**	**0.93***	**0.90*****	**0.84*****
10-Year Yield	1.00	0.98	**0.94**	**1.10**	0.99	0.97	**0.89****	**0.93**	0.99	0.97	**0.94**	**0.93***
BAA Yield	0.96	0.97	1.00	**1.13**	0.98	**0.94**	0.98	**1.10**	0.99	0.99	1.01	**1.05**

Note: Comparison of "standard linear VAR" (baseline, in denominator) against "hybrid shadow-rate VAR." Values below 1 indicate improvement over baseline. Evaluation window with forecast origins from 2009:01 through 2022:08 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Due to the close behavior of some of the models compared, and rounding of the reported values, one of the comparisons shows a significant ratio of 1.00. This case arises from persistent differences in performance that are, however, too small to be relevant after rounding. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

IV (b) Forecast performance of shadow-rate vs linear VARs with ex-yields data

Tables S.9 through S.12 evaluate forecast accuracy in smaller VAR specifications in which the only interest rate is the federal funds rate. The first two tables evaluate the forecast accuracy of a simple shadow-rate VAR and a hybrid shadow-rate VAR, relative to a standard VAR, for the paper's evaluation sample that ends in December 2017. In these results, without term structure data included in the models, a simple shadow-rate VAR performs worse in average forecasting than a standard linear VAR, whereas macro forecasts from the hybrid shadow-rate VAR are on par with a linear VAR (and thus more accurate than those from the simple shadow-rate VAR). The following two tables provide similar results for the sample through August 2022.

Table S.9: Forecast performance of standard vs. simple shadow-rate VAR (ex yields)

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	1.00	1.00	1.01	**0.94**	1.01	1.01	1.03	**1.06**	1.01	1.00	1.02	1.04**
Real Consumption	1.00	1.03	**1.05**	1.04	1.02	1.02	**1.06**	1.03	1.00	1.02**	1.04*	1.03**
IP	1.01	1.04	**1.15***	**1.16**	1.03	**1.06**	**1.13***	**1.15**	1.02	1.04	**1.11***	**1.10***
Capacity Utilization	**1.06**	**1.12**	**1.35****	**1.55***	**1.10****	**1.14***	**1.36****	**1.46****	**1.06****	**1.12****	**1.30*****	**1.45****
Unemployment	1.02	**1.08****	**1.29*****	**1.54***	1.03*	**1.07**	**1.31****	**1.53****	1.02	**1.06****	**1.22*****	**1.44****
Nonfarm Payrolls	0.97	1.01	**1.31****	**1.47**	0.98	**1.06**	**1.35*****	**1.42****	1.00	**1.06**	**1.24*****	**1.32****
Hours	1.01	1.01	**1.07**	**1.29****	1.03	1.03	**1.09**	**1.38****	1.01	1.02	**1.07**	**1.20*****
Hourly Earnings	1.00	1.00	0.99	**0.95***	0.99	1.00	0.98*	**0.95**	1.00	1.00	0.99	0.98
PPI (Fin. Goods)	1.01	1.00	1.00	0.98**	1.00	0.99	1.00	0.98*	1.00	1.00	1.00	0.99
PPI (Metals)	1.01	1.01	0.99	1.02	1.01	1.01	1.00	1.02	1.00	1.00	0.99	1.01**
PCE Prices	1.04	**1.05**	**1.08**	**1.06**	1.04	1.03	**1.05**	**1.05**	1.03	1.03	**1.05**	**1.05**
Federal Funds Rate	**0.34**	**0.25**	**0.29**	**0.58**	**0.11*****	**0.10****	**0.23***	**0.38****	**0.15*****	**0.14****	**0.21***	**0.38****
Housing Starts	**1.15****	**1.24***	**1.22**	**1.12**	**1.13****	**1.18****	**1.18****	**1.12***	**1.10*****	**1.16****	**1.17****	**1.13****
S&P 500	0.99	1.01	1.00	1.00	0.99	1.01	1.02	1.01	0.99	1.00	1.01	1.02**
USD / GBP FX Rate	0.99	1.00	0.99	1.00	0.98	0.99	0.98	0.99	0.99	0.99	0.99	0.99

Note: Comparison of "standard linear VAR" (baseline, in denominator) against "simple shadow-rate VAR." Values below 1 indicate improvement over baseline. Evaluation window with forecast origins from 2009:01 through 2017:12 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

Table S.10: Forecast performance of standard vs. hybrid shadow-rate VAR (ex yields)

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	1.00	0.99	1.01	**0.93****	1.00	1.00	1.00	0.97	1.00	1.00	1.00	1.00
Real Consumption	1.00	1.01	0.99	**0.95**	1.00	1.01	0.96	0.96	1.00	1.01	1.00	1.00
IP	0.99	1.01	1.01*	**0.94**	0.99	1.01	0.99	**0.95**	0.99	1.00	1.01	0.98
Capacity Utilization	0.99	0.99	**1.06***	1.03	0.98	1.00	**1.05***	0.97	0.99	0.99	1.03**	1.02
Unemployment	1.00	1.01	1.03	1.02	1.00	1.01	1.03*	**0.94**	1.00	1.01	1.03**	1.02
Nonfarm Payrolls	0.99	**1.05****	**1.17***	**1.05**	1.00	1.04**	**1.07***	0.96	1.00	1.03**	**1.07****	1.03
Hours	0.99**	1.01	**1.07***	**1.13**	0.99**	1.01	1.04	**1.10****	0.99***	1.01	1.04**	**1.06****
Hourly Earnings	1.01*	1.01	1.01	1.00	1.00	1.01	1.01*	1.02	1.01	1.01*	1.01***	1.02**
PPI (Fin. Goods)	1.00	0.99**	0.99	1.03	1.00	0.99**	0.99	1.00	1.00	0.99	1.00	1.01
PPI (Metals)	1.00	1.00	0.99*	1.02	1.00	0.99	0.99	1.00	1.00	1.00	1.00	1.01***
PCE Prices	0.98**	0.97	0.96	0.97	0.99**	0.97*	0.97	0.96*	0.99*	0.98	0.98	0.98
Federal Funds Rate	**1.35**	**0.84**	**0.68**	0.99	**0.18****	**0.11****	**0.20***	**0.38****	**0.24****	**0.18***	**0.21***	**0.35****
Housing Starts	0.99	0.98	**0.85**	**0.83**	1.00	0.97	**0.90**	**0.91**	1.00	0.99	**0.92**	**0.92**
S&P 500	1.01	1.02*	1.00	0.99	1.01	1.02	1.00	0.99	1.00	1.01	1.00	1.01
USD / GBP FX Rate	1.01	0.99	1.00	1.01	1.01	1.00	1.00	1.01	1.00	0.99	1.00	1.01

Note: Comparison of "standard linear VAR" (baseline, in denominator) against "hybrid shadow-rate VAR." Values below 1 indicate improvement over baseline. Evaluation window with forecast origins from 2009:01 through 2017:12 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

Table S.11: Forecast performance of standard vs. simple shadow-rate VAR (ex yields, incl. COVID-19)

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	1.00	1.00	1.01	0.99	1.01**	1.01	1.01	1.02	1.00	1.00	1.00	1.01
Real Consumption	1.00	1.00**	1.00*	0.99	1.01*	1.01	1.02	1.01	1.00	1.01**	1.01*	1.01***
IP	1.00	1.01*	1.02**	1.02	1.01	1.03*	**1.07****	**1.08**	1.01	1.02	1.04*	**1.05**
Capacity Utilization	1.03	**1.05****	**1.21****	**1.45***	**1.06*****	**1.10****	**1.26****	**1.38****	1.04***	**1.07*****	**1.19*****	**1.32****
Unemployment	1.00	1.00	1.00	**1.08**	1.00	1.00	**1.07**	**1.24**	1.00	1.01	**1.05****	**1.19***
Nonfarm Payrolls	1.00	1.00	1.00**	1.01	0.99	1.01	**1.10*****	**1.13****	1.00	1.01	**1.05*****	**1.08****
Hours	1.01	1.00	1.01	**1.16***	1.01	1.01	1.04	**1.28****	1.01	1.01	1.04	**1.12****
Hourly Earnings	1.00	1.01	0.98	0.96*	0.99*	1.01	0.99	0.97	1.00	1.00	0.99	0.99
PPI (Fin. Goods)	1.00	0.99	0.99	1.00	0.99	0.99	0.99	0.99	0.99	0.99	1.00	1.00
PPI (Metals)	1.01	1.01	1.00	1.03*	1.01	1.01	1.00	1.02*	1.00	1.00	1.00	1.01**
PCE Prices	1.02	1.02	1.03	1.04	1.02	1.02	1.04	1.04	1.01	1.01	1.03	1.03
Federal Funds Rate	**0.51***	**0.49**	**0.61**	**1.27**	**0.28*****	**0.31****	**0.38****	**0.41****	**0.34*****	**0.35****	**0.39****	**0.45*****
Housing Starts	**1.07***	**1.17***	**1.21***	**1.10**	**1.05**	**1.12****	**1.16****	**1.10**	**1.06****	**1.11****	**1.13****	**1.09***
S&P 500	1.00	1.01	1.00	0.99	1.00	1.01	1.01	1.01	1.00	1.00	1.01	1.01**
USD / GBP FX Rate	0.98	1.00	0.99	1.00	0.98	0.99	0.98	1.00	0.99*	0.99	0.99	1.00

Note: Comparison of "standard linear VAR" (baseline, in denominator) against "simple shadow-rate VAR." Values below 1 indicate improvement over baseline. Evaluation window with forecast origins from 2009:01 through 2022:08 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Due to the close behavior of some of the models compared, and rounding of the reported values, a few comparisons show significant ratios of 1.00. These cases arise from persistent differences in performance that are, however, too small to be relevant after rounding. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

Table S.12: Forecast performance of standard vs. hybrid shadow-rate VAR (ex yields, incl. COVID-19)

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	1.00	1.00	0.99	**1.14**	1.00	1.00	1.00	0.99	1.00	1.00	1.00	1.00
Real Consumption	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.99	1.00	1.00*	1.00	1.00
IP	1.00	1.00	1.00**	0.99	0.99	1.00	0.99	0.97	1.00	1.00	1.00	1.00
Capacity Utilization	1.00	1.00	**1.06****	1.02	0.99	0.99	1.04*	0.98	1.00	1.00	1.02**	1.02
Unemployment	1.00	1.00	1.01	**1.05**	1.00	1.00	1.01	0.98	1.00	1.00	1.01	1.02
Nonfarm Payrolls	1.00	1.00	1.00***	1.00	0.99*	1.01	1.03**	0.99	1.00	1.01***	1.02***	1.01
Hours	1.00	1.00	1.04**	**1.05**	0.99	1.00	1.03	**1.07****	1.00	1.01	1.03**	1.04**
Hourly Earnings	1.01	1.01*	1.00	1.00	1.00	1.01*	1.01**	1.01	1.00	1.01	1.01	1.01
PPI (Fin. Goods)	1.00	0.99**	0.99	1.02	1.00	0.99**	1.00	1.00	1.00	1.00	1.00	1.00
PPI (Metals)	1.00	1.00*	1.00	1.00	1.00	1.00	0.99	1.00	1.00*	1.00**	1.00	1.01**
PCE Prices	0.99**	0.98*	**0.95****	1.02	0.99**	0.98*	0.97	0.97**	0.99*	0.99*	0.98*	0.98*
Federal Funds Rate	**1.08**	**0.84**	**0.82**	**1.47**	**0.33*****	**0.32****	**0.35****	**0.42****	**0.40*****	**0.38****	**0.39****	**0.43*****
Housing Starts	1.00	0.98	**0.87**	**0.87**	1.00	0.98	**0.91**	**0.94**	1.00	0.99	**0.94**	**0.94**
S&P 500	1.00	1.01	1.00	0.98	1.00	1.01	1.00	1.00	1.00	1.00	1.00	1.01
USD / GBP FX Rate	1.00	0.99	0.99	1.00	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.01

Note: Comparison of "standard linear VAR" (baseline, in denominator) against "hybrid shadow-rate VAR." Values below 1 indicate improvement over baseline. Evaluation window with forecast origins from 2009:01 through 2022:08 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Due to the close behavior of some of the models compared, and rounding of the reported values, a few comparisons show significant ratios of 1.00. These cases arise from persistent differences in performance that are, however, too small to be relevant after rounding. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

IV (c) Forecast performance of plug-in VARs

Tables S.13 through S.20 provide forecast accuracy comparisons of the simple shadow-rate and hybrid shadow-rate VARs to plug-in VARs that use updated estimates from Wu-Xia or Krippner as shadow-rate "data," first for an evaluation sample ending with the forecast origin of December 2017, for which the end point of the evaluation sample is December 2019, then for an evaluation sample with an end point of August 2022. The first two tables compare simple shadow-rate VAR forecast accuracy to plug-in VARs using the Wu-Xia and Krippner rates. The next two compare hybrid shadow-rate VAR forecast accuracy to plug-in VARs using the Wu-Xia and Krippner rates. We find notable benefits for point and density forecasting with our shadow-rate and hybrid VARs as compared to a plug-in model.

Table S.13: Comparison of simple shadow-rate VAR against plug-in VAR (Wu-Xia)

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	1.00	1.00	0.99	**0.85***	0.97	0.96*	**0.95**	**0.91****	0.98	0.98*	0.98	0.96***
Real Consumption	0.99	0.99	0.97**	**0.91*****	0.99	0.98	0.97	**0.90****	0.99	0.99	0.98**	**0.95*****
IP	1.04**	1.02	1.04**	**0.86**	1.04**	1.02	**1.05****	0.99	1.03**	1.02	1.04**	0.99
Capacity Utilization	1.03	**1.05**	**1.07**	0.96	**1.06****	**1.08***	**1.10***	**0.95**	1.02	1.04	**1.05**	0.96
Unemployment	0.99	1.00	1.00	**0.91**	1.01	1.01	0.98	**0.86**	0.98	0.99	0.99	**0.91**
Nonfarm Payrolls	0.99	1.00	1.01	**0.88**	1.02	1.02	**1.05**	**0.89**	1.01	1.02	1.02	**0.94**
Hours	0.98	0.97	0.97	**0.92***	0.97	**0.95**	0.98	**0.91**	0.98	0.96	0.97	**0.92****
Hourly Earnings	0.99	0.99	0.97	1.00	0.98	1.00	0.98	1.00	0.99	0.99	0.97	0.99
PPI (Fin. Goods)	1.00	0.98	0.98	**0.90****	0.99	0.97*	0.98	**0.94**	0.99	0.97*	0.97*	**0.95***
PPI (Metals)	0.98*	0.97**	0.97**	0.97	0.98*	0.97**	0.96**	0.96	0.98**	0.97***	0.97**	0.98
PCE Prices	0.98	0.97	0.97	0.98	0.99	0.97	0.99	**0.94**	0.98	0.96	0.96	0.96
Policy Rate	**0.57***	**0.74**	1.02	**1.20*****	**0.51***	**0.83**	**1.17**	**1.10**	**0.60***	**0.83**	**1.14**	**1.22***
Housing Starts	1.00	0.97	0.96	0.98	0.99	0.97	**0.94**	**0.89*****	0.99	0.96	**0.94**	**0.93****
S&P 500	0.96*	0.98*	0.99	**1.11**	0.97	0.98	0.98	0.99**	0.98*	0.99	0.99	1.01
USD / GBP FX Rate	1.00	1.00	1.00	0.96**	1.00	1.00	1.01	0.98	0.99	1.00	1.01	0.99
6-Month Tbill	**0.88**	0.96	**1.07**	**1.10**	**0.95**	0.99	1.04	**1.09**	**0.87**	0.96	**1.11**	**1.16**
1-Year Yield	1.04	**1.11**	**1.17**	**1.15**	**1.10**	**1.13**	**1.19**	**1.19**	1.01	**1.11**	**1.20**	**1.18**
5-Year Yield	**0.94**	0.98	**1.12**	**1.23***	**0.87**	**0.86**	0.98	**1.13**	**0.91**	**0.91**	0.98	**1.09**
10-Year Yield	0.96	**0.95**	1.00	**1.10**	**0.91**	**0.89**	**0.93**	1.01	**0.94**	**0.92**	**0.93**	0.99
BAA Yield	**0.91**	**0.86***	**0.78***	**0.76****	**0.93**	**0.88***	**0.82****	**0.75****	**0.93****	**0.88****	**0.83****	**0.83*****

Note: Comparison of "Plug-in VAR (WuXia)" (baseline, in denominator) against "simple shadow-rate VAR." Values below 1 indicate improvement over baseline. Evaluation window with forecast origins from 2009:01 through 2017:12 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

Table S.14: Comparison of simple shadow-rate VAR against plug-in VAR (Krippner)

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	0.99	1.01	0.99	**0.84***	0.96**	0.99	0.97	**0.94****	0.97**	0.99	0.99	0.96***
Real Consumption	1.00	0.99	0.99	**0.95****	1.00	0.99	0.98	**0.94****	0.99	0.99*	0.98***	0.96***
IP	1.01	0.99	1.01	**0.94**	1.03	1.00	1.02	0.97	1.01	1.00	0.99	0.96**
Capacity Utilization	**1.05***	**1.07**	**1.08***	0.96	**1.07**	**1.11***	**1.13****	**0.95**	1.02	**1.05**	**1.06**	**0.95**
Unemployment	0.96	0.96	1.03	1.02	0.98	0.99	1.00	1.03	0.97	0.98	1.01	0.99
Nonfarm Payrolls	1.00	1.02	**1.05**	**0.93**	1.02	1.04	**1.07**	**0.93**	1.01	1.03	1.03	**0.94**
Hours	0.97	0.98	1.01	**0.94**	0.96	0.97	1.02	**0.94**	0.97	0.97	0.99	**0.92**
Hourly Earnings	1.00	1.01	1.00	1.01	0.99	1.01	1.00	1.04***	0.99	1.00	0.99	1.00
PPI (Fin. Goods)	0.99	0.99	1.00	**0.95**	0.99	0.97	1.00	0.97	0.99	0.98	0.99	0.96
PPI (Metals)	0.97	0.96*	**0.95***	**0.95**	0.98	0.96*	**0.93****	**0.94**	0.96*	0.96**	0.96**	0.96*
PCE Prices	0.98	0.98	0.99	0.98	1.00	0.97	0.99	0.97	0.98	0.97	0.97	**0.95**
Policy Rate	**0.89**	**0.88**	**0.88**	1.03	1.03	**1.10**	1.01	1.02	**1.06**	1.00	0.98	**1.14**
Housing Starts	1.01	1.02	**1.06**	**1.08**	0.99	1.03	**1.07**	**1.05**	1.00	1.01	1.01	1.01
S&P 500	0.96*	0.98	0.99	**1.06**	1.00	1.00	0.99	0.97*	0.97**	0.98	0.98**	0.98
USD / GBP FX Rate	1.01	1.01	1.00	0.97	1.00	1.01	1.01	0.99	1.00	1.00	1.00	0.99
6-Month Tbill	**0.56*****	**0.68*****	**0.80***	**0.85**	**1.11**	**1.08**	1.00	1.02	**0.62*****	**0.73****	**0.89**	0.98
1-Year Yield	**0.70*****	**0.82***	**0.92**	**0.93**	**1.15**	**1.20**	**1.18**	**1.18**	**0.79****	**0.91**	1.04	**1.05**
5-Year Yield	1.01	**1.05**	**1.13**	**1.13**	**0.94**	**0.95**	1.02	**0.93**	**0.95**	0.97	0.99	1.00
10-Year Yield	0.98	0.99	1.04	**1.09**	0.97	0.97	1.01	**0.91**	0.96	0.96	**0.95**	**0.93**
BAA Yield	0.96	**0.95**	**0.91**	**0.85*****	0.99	0.97	0.96	**0.79*****	0.97	0.96	**0.91*****	**0.84*****

Note: Comparison of "Plug-in VAR (Krippner)" (baseline, in denominator) against "simple shadow-rate VAR." Values below 1 indicate improvement over baseline. Evaluation window with forecast origins from 2009:01 through 2017:12 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

Table S.15: Comparison of hybrid shadow-rate VAR against plug-in VAR (Wu-Xia)

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	0.99	1.00	0.99	0.97	0.98	0.97	0.96*	**0.89****	0.99	0.99	0.98	0.96**
Real Consumption	0.99	1.00	**0.95****	**0.83****	0.99	0.99	0.97	**0.82*****	0.99	0.99	0.96***	**0.92*****
IP	1.03**	0.99	1.00	**0.85**	1.03**	0.99	1.00	**0.94**	1.03*	1.00	1.00	0.98
Capacity Utilization	1.00	0.98	0.97	**0.81****	1.00	0.98	**0.94**	**0.80****	1.00	0.98	0.96*	**0.84*****
Unemployment	0.99	1.00	0.98	**0.82*****	1.01	1.02	1.01	**0.82*****	0.99	0.99	0.97	**0.86*****
Nonfarm Payrolls	0.99	1.00	1.01	**0.83***	1.00	0.99	0.97	**0.80****	0.99	1.00	0.98	**0.89*****
Hours	0.99	0.98	0.99	**0.92***	0.97	0.97	0.99	**0.90**	0.98	0.97	0.97	**0.92****
Hourly Earnings	0.99	0.99	0.98	1.04	0.98	1.00	0.99	1.02	0.99	0.99	0.98	1.00
PPI (Fin. Goods)	1.00	0.97***	**0.94****	**0.91***	1.00	0.97**	0.96**	**0.94**	1.00	0.97***	0.96***	**0.95**
PPI (Metals)	0.97*	0.96*	0.96***	0.96	0.98	0.96**	**0.94*****	**0.95**	0.98**	0.96**	0.96***	0.98
PCE Prices	0.98	**0.95**	**0.94****	1.03	0.99	**0.95**	0.97	0.99	0.98	0.96	**0.95***	0.99
Policy Rate	**0.58***	**0.73**	0.98	**1.12**	**0.53***	**0.78**	**1.11**	**1.09**	**0.60****	**0.80**	**1.06**	**1.10**
Housing Starts	0.99	0.96	**0.91**	0.99	0.99	0.96	**0.88**	**0.87**	0.98	0.96	**0.92**	**0.95**
S&P 500	0.98*	0.98	1.00	**1.11**	0.99	0.99	1.00	1.00	0.99	1.00	1.00	1.01
USD / GBP FX Rate	1.02	1.02	1.01	1.00	1.03*	1.03	1.04	**1.06**	1.01	1.01	1.02	1.01
6-Month Tbill	**0.87**	**0.93**	1.00	0.99	**0.93**	**0.94**	**0.93**	**0.93**	**0.85**	**0.90**	1.00	1.00
1-Year Yield	1.01	**1.05**	**1.08**	1.04	**1.09**	1.04	1.01	1.00	0.99	1.04	**1.08**	1.03
5-Year Yield	**0.95**	0.98	**1.07**	**1.10**	**0.89**	**0.88**	0.98	**1.08**	**0.91**	**0.91**	0.96	1.03
10-Year Yield	0.97	0.96	0.98	1.03	**0.93**	**0.91**	**0.91**	1.01	**0.95**	**0.93**	**0.92**	0.98
BAA Yield	**0.91**	**0.85***	**0.77***	**0.80***	**0.93**	**0.86***	**0.82***	**0.81**	**0.92****	**0.87****	**0.83***	**0.86*****

Note: Comparison of "Plug-in VAR (WuXia)" (baseline, in denominator) against "hybrid shadow-rate VAR." Values below 1 indicate improvement over baseline. Evaluation window with forecast origins from 2009:01 through 2017:12 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

Table S.16: Comparison of hybrid shadow-rate VAR against plug-in VAR (Krippner)

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	0.99*	1.00	1.00	**0.95**	0.97	1.00	0.98	**0.92****	0.99	1.01	0.99	**0.95*****
Real Consumption	1.00	1.00	0.97	**0.87***	0.99	1.00	0.98	**0.85***	0.99	0.99	0.97**	**0.93*****
IP	1.01	0.97	0.97	**0.92**	1.03	0.98	0.97	**0.92**	1.00	0.98	0.96**	**0.95**
Capacity Utilization	1.02	1.00	0.98	**0.81****	1.01	1.00	0.97	**0.80***	0.99	0.98	0.96*	**0.83*****
Unemployment	0.96	0.96	1.01	**0.93**	0.98	1.00	1.03	0.97	0.97	0.97	0.99	**0.93**
Nonfarm Payrolls	1.00	1.02	1.04	**0.88**	1.00	1.01	0.99	**0.83**	1.00	1.01	0.99	**0.89*****
Hours	0.98	1.00	1.03	**0.94**	0.96	0.99	1.03	**0.94**	0.97	0.97	1.00	**0.92**
Hourly Earnings	1.00	1.02	1.01	**1.05*****	0.99	1.01	1.01	**1.05*****	1.00	1.01	1.00	1.01
PPI (Fin. Goods)	0.99	0.98*	0.97	**0.95**	1.00	0.97*	0.98	0.97	1.00	0.98*	0.98	0.96
PPI (Metals)	0.97	0.96*	**0.94****	**0.94**	0.98	**0.95***	**0.92****	**0.93**	0.96*	**0.95****	**0.95*****	**0.95***
PCE Prices	0.98	0.96	**0.95**	1.03	0.99	**0.95****	0.97	1.03	0.99	0.97	0.96*	0.99
Policy Rate	**0.90**	**0.86**	**0.84***	0.96	**1.07**	1.03	0.96	1.02	**1.05**	0.96	**0.91**	1.03
Housing Starts	1.00	1.01	1.00	**1.08**	0.99	1.02	0.99	1.03	1.00	1.00	0.99	1.02
S&P 500	0.97	0.98	0.99	**1.06**	1.01	1.01	1.00	0.97	0.97	0.98	0.99	0.98
USD / GBP FX Rate	1.03	1.02	1.01	1.01	1.03*	1.03	1.04	**1.06**	1.02	1.01	1.01	1.01
6-Month Tbill	**0.55*****	**0.65*****	**0.75****	**0.76***	**1.09**	1.03	**0.90**	**0.87**	**0.61*****	**0.69****	**0.80***	**0.84***
1-Year Yield	**0.68*****	**0.77****	**0.86**	**0.84***	**1.14**	**1.11**	1.00	0.99	**0.78****	**0.85**	**0.93**	**0.91**
5-Year Yield	1.02	**1.05**	**1.08**	1.01	0.96	0.98	1.02	**0.89**	0.96	0.97	0.97	**0.94**
10-Year Yield	1.00	1.00	1.02	1.03	0.98	0.99	0.99	**0.91**	0.97	0.96	**0.94**	**0.92**
BAA Yield	0.96	**0.93**	**0.90**	**0.90****	0.98	0.96	0.97	**0.85****	0.96	**0.95**	**0.91****	**0.86*****

Note: Comparison of "Plug-in VAR (Krippner)" (baseline, in denominator) against "hybrid shadow-rate VAR." Values below 1 indicate improvement over baseline. Evaluation window with forecast origins from 2009:01 through 2017:12 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

Table S.17: Comparison of simple shadow-rate VAR against plug-in VAR (Wu-Xia, incl. COVID-19)

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	1.00	1.00	0.99	**0.91**	1.00	1.00	0.99	0.97*	1.00	1.00	0.99	1.00
Real Consumption	1.00	1.00	1.00	0.99*	0.99	0.99	0.98*	0.97*	1.00	1.00	0.99**	0.98**
IP	1.01*	1.00	1.01**	0.98	1.02**	1.02	1.03**	0.99	1.02**	1.01	1.02***	1.00
Capacity Utilization	0.99	0.98	1.02	0.97	1.03*	**1.05***	**1.07***	0.97	1.01	1.02	1.04	0.97
Unemployment	0.99	0.98	0.97	**0.95**	0.98	0.97	**0.95**	**0.90**	0.98**	0.98	0.97	**0.95**
Nonfarm Payrolls	1.00	1.00	1.00	0.99	1.00	0.99	1.01	0.97	0.99	0.99	1.00	0.98*
Hours	0.99	0.98*	0.99	**0.95****	0.97**	0.96*	1.00	**0.94**	0.98*	0.97**	0.98	**0.94****
Hourly Earnings	0.99	1.00	0.99	0.99	0.99	1.00	0.99	1.00	1.00	1.00	0.98	0.99
PPI (Fin. Goods)	1.00	0.99	0.99	**0.91****	1.00	0.98	0.99	0.96	1.00	0.99	0.98	0.96*
PPI (Metals)	0.99	0.98	0.98**	0.98	0.98*	0.97**	0.96***	0.97	0.99**	0.98**	0.98**	0.99
PCE Prices	0.99	0.98	0.99	**0.92**	1.00	0.98	0.99	**0.95**	0.99	0.98	0.97	0.96
Policy Rate	**0.81***	**0.86**	**1.05**	**1.34**	**0.66*****	**0.81**	**0.92**	0.96	**0.73*****	**0.83**	0.98	**1.11**
Housing Starts	0.97	**0.92**	**0.92**	0.97	0.97	**0.93**	**0.91***	**0.89*****	0.98	**0.95***	**0.94****	**0.94*****
S&P 500	0.99	1.00	1.01	1.02	0.99	1.00	0.99	0.99**	0.99	1.00	0.99	1.00
USD / GBP FX Rate	1.00	1.00	1.00	0.97**	1.00	0.99	1.01	0.99	0.99	0.99	1.00	0.99
6-Month Tbill	**0.87****	**0.93**	0.99	**1.15**	**0.87***	**0.88**	**0.92**	1.04	**0.85*****	**0.91**	0.99	**1.09**
1-Year Yield	**0.93**	0.97	1.02	**1.17**	**0.95**	**0.94**	1.00	**1.11**	**0.93**	0.98	1.04	**1.11**
5-Year Yield	0.96	0.99	**1.06**	**1.21**	**0.90**	**0.88**	0.97	**1.07**	**0.94**	**0.93**	0.98	**1.06**
10-Year Yield	0.98	0.97	1.00	**1.13**	**0.95**	**0.92**	**0.94**	1.01	0.97	**0.95**	**0.95**	0.99
BAA Yield	**0.93***	**0.91***	**0.84****	**0.87****	**0.95***	**0.90****	**0.86****	**0.83***	**0.95***	**0.93***	**0.88***	**0.88****

Note: Comparison of "Plug-in VAR (WuXia)" (baseline, in denominator) against "simple shadow-rate VAR." Values below 1 indicate improvement over baseline. Evaluation window with forecast origins from 2009:01 through 2022:08 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

Table S.18: Comparison of simple shadow-rate VAR against plug-in VAR (Krippner, incl. COVID-19)

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	1.00	1.00	0.98	**0.79**	1.00	1.00	1.00	0.98	1.00	1.00	1.00	0.99
Real Consumption	1.00	1.00*	1.00	0.98	1.00	0.99	0.99**	0.98	1.00	1.00	1.00	0.99
IP	1.00	1.00	1.00	0.99	1.02	1.01	1.01	0.98	1.01	1.00	1.00	0.98*
Capacity Utilization	1.01**	1.03**	**1.07****	0.96	1.03*	**1.07****	**1.11*****	0.97	1.01	1.03*	1.04*	0.96
Unemployment	0.99	0.99	0.98	0.97	0.98	0.98	0.97	1.00	0.98*	0.98	0.99	0.99
Nonfarm Payrolls	1.00	1.00	1.00	1.00	0.99	1.00	1.02	0.97	1.00	1.00	1.01	0.98
Hours	1.00	1.00	1.03	**0.93**	0.97	0.99	1.03	0.96	0.99	0.99	1.00	**0.95**
Hourly Earnings	1.01	1.01**	1.02	1.00	1.00	1.01	1.01	1.03***	1.00	1.01	1.00	1.00
PPI (Fin. Goods)	1.00	1.00	1.01	**0.95***	0.99	0.99	1.01	0.98	1.00	0.99	1.00	0.98
PPI (Metals)	0.99	0.98	0.97*	0.98	0.98	0.97*	**0.95****	0.96	0.98	0.97**	0.97**	0.97
PCE Prices	0.99	0.99	1.00	**0.94**	1.00	0.99	1.01	0.98	0.99	0.99	0.99	0.97
Policy Rate	**0.80****	**0.88**	1.00	**1.08**	**0.73**	0.96	0.98	**0.93**	**0.78***	**0.94**	0.99	**1.07**
Housing Starts	0.98	0.98	1.00	1.02	0.98	1.00	1.02	1.02	1.00	1.00	1.00	1.00
S&P 500	0.99	1.01	1.00	1.02	1.00	1.02	0.99	0.98	0.99	0.99	0.98***	0.98*
USD / GBP FX Rate	1.01	1.00	1.00	0.98	1.01	1.00	1.00	0.99	1.01	1.00	1.00	0.99
6-Month Tbill	**0.77*****	**0.80*****	**0.86****	**0.93**	**0.91**	**0.93**	0.96	1.02	**0.76*****	**0.82*****	**0.92**	0.99
1-Year Yield	**0.82*****	**0.86****	**0.91**	**0.95**	0.97	1.00	**1.07**	**1.14**	**0.84*****	**0.91**	1.00	1.04
5-Year Yield	1.01	1.03	**1.05**	**1.06**	0.96	0.97	1.02	**0.95**	0.97	0.99	1.00	1.00
10-Year Yield	0.98	0.99	1.01	1.04	0.99	0.97	1.01	**0.93**	0.97	0.97	0.97	**0.95**
BAA Yield	**0.95**	**0.94***	**0.91****	**0.87****	0.97	**0.94**	**0.94**	**0.85****	0.97*	0.96**	**0.93*****	**0.88*****

Note: Comparison of "Plug-in VAR (Krippner)" (baseline, in denominator) against "simple shadow-rate VAR." Values below 1 indicate improvement over baseline. Evaluation window with forecast origins from 2009:01 through 2022:08 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Due to the close behavior of some of the models compared, and rounding of the reported values, one of the comparisons shows a significant ratio of 1.00. This case arises from persistent differences in performance that are, however, too small to be relevant after rounding. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

Table S.19: Comparison of hybrid shadow-rate VAR against plug-in VAR (Wu-Xia, incl. COVID-19)

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	1.00	1.00	**0.94**	**0.95**	0.99	0.99	0.99	0.97**	1.00	1.00	0.99*	0.98**
Real Consumption	1.00	1.00	1.00	1.00	1.00	0.99	0.99	**0.94****	0.99**	0.99**	0.99***	0.97***
IP	1.00	1.00	1.00	0.98	1.01	1.00	1.00	0.97	1.01	1.00	1.00	0.99
Capacity Utilization	0.99	0.96	0.97	**0.86***	1.00	0.97	**0.95***	**0.84****	0.99	0.97***	0.97**	**0.88*****
Unemployment	0.99	0.99	1.00	1.01	0.99	1.00	0.99	**0.91****	0.99	0.99	0.99	**0.94****
Nonfarm Payrolls	1.00	1.00	1.00	1.00	0.99	0.99	0.99	**0.93****	0.99	0.99	0.99	0.97***
Hours	0.99	0.99	1.03	**0.95****	0.99	0.97	1.01	**0.93**	0.99	0.98	0.98	**0.94****
Hourly Earnings	1.00	1.00	1.00	1.04	0.99	1.00	0.99	1.01	1.00	1.00	0.98	1.00
PPI (Fin. Goods)	1.00	0.99	0.99	**0.91***	1.00	0.98*	0.99	0.97	1.00	0.99*	0.98	0.97
PPI (Metals)	0.98**	0.98*	0.96***	0.96*	0.98*	0.97*	0.96***	0.97	0.98**	0.98**	0.97***	0.98
PCE Prices	0.98	0.97	0.97	**0.94**	0.99	0.97*	0.99	0.99	0.98	0.97	0.97	0.99
Policy Rate	**0.80***	**0.79***	**0.95**	**1.19**	**0.67*****	**0.77***	**0.88**	1.00	**0.72*****	**0.77****	**0.89**	1.02
Housing Starts	1.00	0.97	**0.95**	**1.14**	0.99	0.97	**0.90**	**0.91**	0.99	0.97	**0.93**	**0.95**
S&P 500	0.98	0.99	1.01	**1.10**	0.99	0.99	1.00	1.00	0.99	1.00	1.00	1.00
USD / GBP FX Rate	1.02	1.01	1.00	0.98	1.02*	1.02	1.03	1.04	1.01	1.00	1.00	1.00
6-Month Tbill	**0.82*****	**0.84***	**0.89**	1.01	**0.82****	**0.81***	**0.84**	0.98	**0.79*****	**0.83****	**0.90**	0.98
1-Year Yield	**0.86***	**0.88**	**0.92**	1.04	**0.89**	**0.86**	**0.89**	1.02	**0.87***	**0.89**	**0.95**	1.00
5-Year Yield	0.97	0.97	1.01	**1.08**	**0.92**	**0.90**	**0.95**	**1.07**	**0.94**	**0.93**	0.96	1.02
10-Year Yield	1.00	0.97	0.98	**1.05**	0.97	**0.93**	**0.93**	1.02	0.98	**0.95**	**0.94**	0.99
BAA Yield	**0.93***	**0.89****	**0.83****	**0.91**	**0.94***	**0.88****	**0.85****	**0.89**	**0.95***	**0.92****	**0.88***	**0.90****

Note: Comparison of "Plug-in VAR (WuXia)" (baseline, in denominator) against "hybrid shadow-rate VAR." Values below 1 indicate improvement over baseline. Evaluation window with forecast origins from 2009:01 through 2022:08 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

Table S.20: Comparison of hybrid shadow-rate VAR against plug-in VAR (Krippner, incl. COVID-19)

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	1.00	1.00	**0.94**	**0.83**	1.00	1.00	1.00	0.98*	0.99	1.00	1.00	0.98***
Real Consumption	1.00	1.00	1.00	0.99	1.00	0.99	0.99	**0.95***	1.00	0.99**	0.99**	0.98***
IP	1.00	1.00	0.99	0.99	1.01	1.00	0.98*	0.96	1.00	0.99	0.98**	0.97
Capacity Utilization	1.01	1.01	1.01	**0.86***	1.01	0.99	0.99	**0.83***	0.99	0.98**	0.97*	**0.87*****
Unemployment	0.99	0.99	1.00	1.04	0.99	1.00	1.01	1.01	0.99	0.99	1.00	0.98
Nonfarm Payrolls	1.00	1.00	1.00	1.00	0.98*	0.99	1.00	**0.94***	1.00	1.00	1.00	0.97***
Hours	1.00	1.01	**1.06***	**0.93**	0.98	1.00	**1.05**	**0.94**	0.99	0.99	1.01	**0.95**
Hourly Earnings	1.01	1.02*	1.03	**1.05*****	0.99	1.01	1.01	1.04***	1.00	1.01	1.00	1.01
PPI (Fin. Goods)	1.00	1.00	1.01	**0.95**	1.00	0.99	1.00	0.99	1.00	0.99	1.00	0.98
PPI (Metals)	0.98	0.97	**0.95****	0.96	0.98	0.96*	**0.94****	**0.95**	0.98*	0.97**	0.96**	0.96*
PCE Prices	0.98	0.98	0.99	**0.95**	0.99	0.98	1.00	1.03	0.99	0.99	0.99	1.00
Policy Rate	**0.79****	**0.81****	**0.90***	**0.95**	**0.74**	**0.90**	**0.94**	0.97	**0.77***	**0.87**	**0.91****	0.98
Housing Starts	1.01	1.03	1.04	**1.20**	1.01	**1.05**	1.01	1.04	1.00	1.01	1.00	1.02
S&P 500	0.99	1.00	1.00	**1.10**	1.00	1.01	1.00	0.98	0.99	0.99	0.99	0.98*
USD / GBP FX Rate	1.03*	1.01	1.00	0.99	1.03**	1.02	1.02	1.03	1.02*	1.01	1.00	1.00
6-Month Tbill	**0.72*****	**0.73*****	**0.78****	**0.81***	**0.85***	**0.85**	**0.88**	0.96	**0.71*****	**0.74*****	**0.84****	**0.89***
1-Year Yield	**0.75*****	**0.78****	**0.82***	**0.85***	**0.90**	**0.92**	**0.95**	1.04	**0.78*****	**0.83****	**0.91**	**0.94**
5-Year Yield	1.01	1.01	1.00	**0.95**	0.98	0.99	1.01	**0.95**	0.98	0.98	0.98	0.97
10-Year Yield	1.00	0.99	0.99	0.97	1.01	0.98	0.99	**0.94**	0.98	0.97	0.96	**0.95**
BAA Yield	**0.94***	**0.92****	**0.90***	**0.91****	0.96	**0.92***	**0.93**	**0.92**	0.96**	**0.95****	**0.93*****	**0.90*****

Note: Comparison of "Plug-in VAR (Krippner)" (baseline, in denominator) against "hybrid shadow-rate VAR." Values below 1 indicate improvement over baseline. Evaluation window with forecast origins from 2009:01 through 2022:08 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

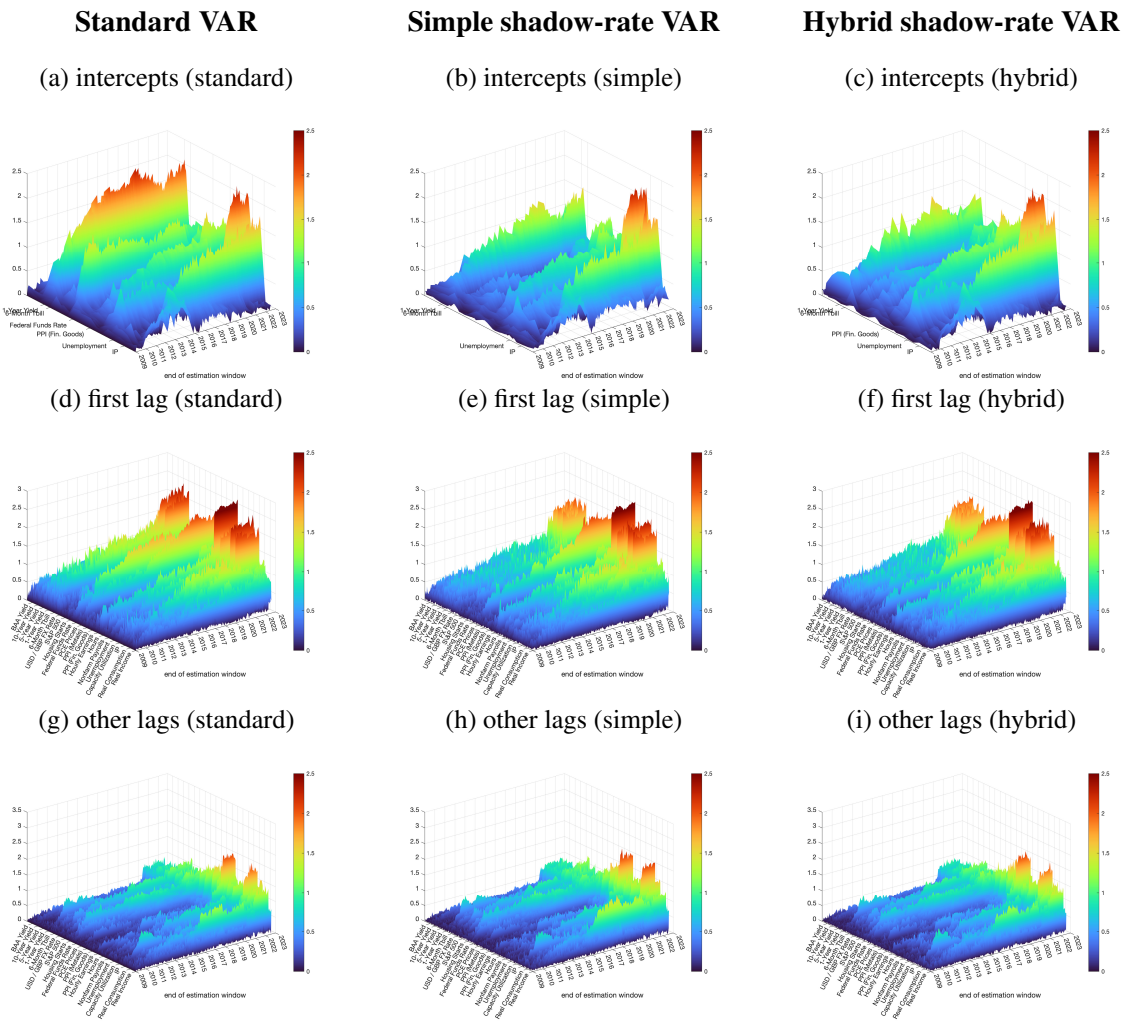
V Parameter stability over time

Our various VAR models (linear VAR, simple/hybrid shadow-rate VAR) treat the parameters, C_j, of the VAR's transition equations as time-invariant. As a simple check we examine the stability of recursive estimates of the VAR from January 2009 through the end of our sample in August 2022, by comparing differences in the posterior means for each coefficient relative to the linear VAR estimates obtained over a data sample ending in 2009:01 (our first observation of a binding ELB). All estimation samples use data available since 1959:03. For sake of comparability, we standardize the change in the posterior means obtained by different models and at subsequent forecast origins, by dividing the changes in posterior means by the posterior standard deviation of the linear VAR estimated through 2009:01.

Figure S.28 provides charts of the standardized differences in coefficient estimates (relative to the linear VAR per 2009:01) over time, for each variable and month since 2009:01, for the intercepts and lag coefficients generated from our 20-variable data set. Figure S.29 provides analogous charts obtained when yield data other than the federal funds rate are omitted from the estimation. Each figure shows estimates obtained from the linear VAR, as well as the simple and hybrid shadow-rate VARs.

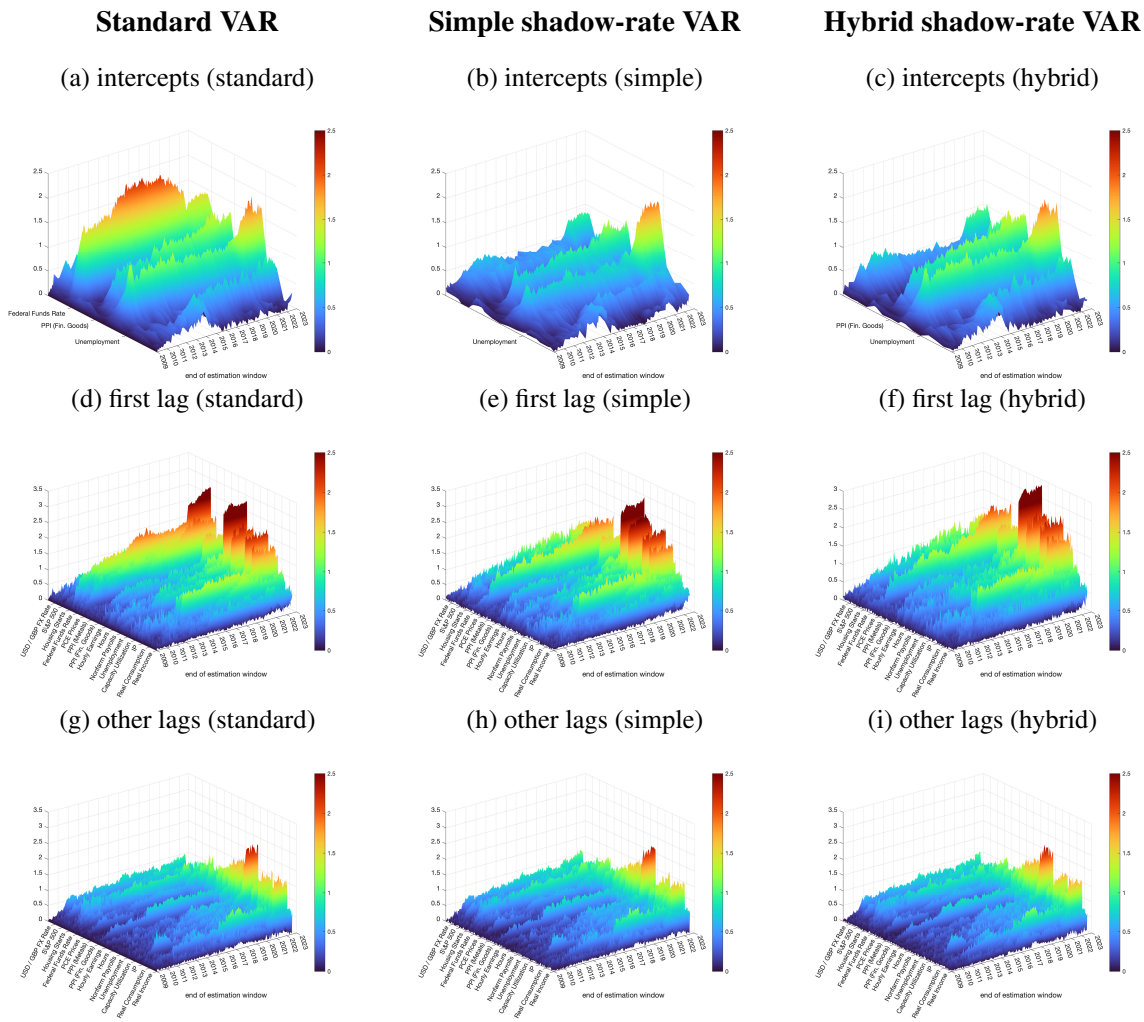
In Figure S.28, the most notable parameter changes occur for intercepts related to interest rates in the linear VAR. As visible in Panel (a) of Figure S.28, intercepts in the linear VAR's equations for several interest rates, but also PPI inflation and the unemployment rate, changed very soon after the onset of the first ELB episode. In contrast, intercept changes of the simple and hybrid shadow-rate VARs are almost negligible (or limited to the COVID-19 period) as shown in Panels (b) and (c) of the figure. In all three specifications, first-lag coefficients of a few equations, in particular for interest rates, display some further changes, in particular since late 2014 (and then again since COVID-19). As shown in Figure S.29, when the VARs are estimated while excluding yield data (other than the federal funds rate), parameter changes in all coefficients are most pronounced for the linear VAR (or limited to COVID-19).

Figure S.28: Parameter changes over time



Note: Absolute values of standardized differences in mean estimates of VAR coefficients; the estimates are standardized based on the posterior standard deviation of each coefficient generated by the linear VAR using data through 2009:01. All estimation windows use available data since 1959:03.

Figure S.29: Parameter changes over time (ex yields)



Note: Absolute values of standardized differences in mean estimates of VAR coefficients; the estimates are standardized based on the posterior standard deviation of each coefficient generated by the linear VAR using data through 2009:01. All estimation windows use available data since 1959:03.

VI The role of different yields for shadow-rates and forecast performance

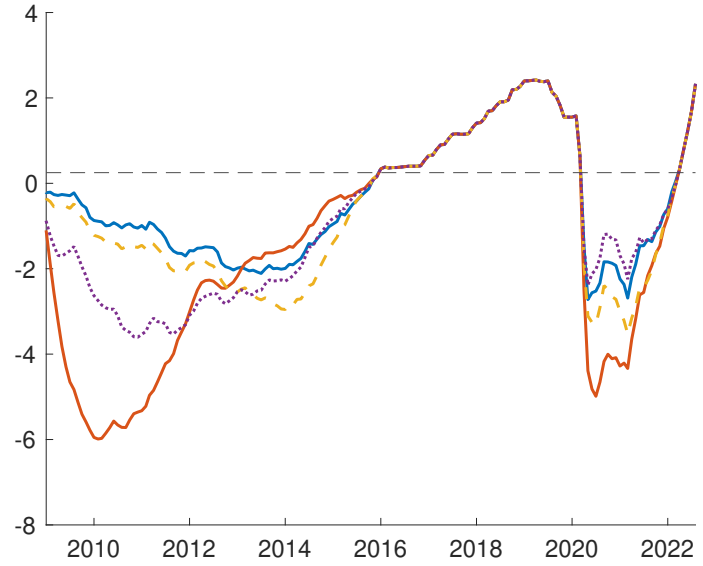
This part of the appendix considers the effects on shadow-rate estimates and forecast accuracy from separately adding yields of different maturities to the "ex yields" data set. The "ex yields" data set includes only the federal funds rate as interest rate measures, alongside the other 14 macroeconomic and financial variables listed in Table 1 of the paper. The main focus of our comparisons is to add all short-term or all long-term rates to the ex-yields data. The short-term rates added comprise the 6-month and 1-year Treasury rates (on top of the federal funds rate), whereas the set of long-term interest rates consists of 5- and 10-year Treasury rates and the 20-year BAA corporate yield. Section VI (a) provides shadow rate estimates, and Section VI (b) reports forecast accuracy tables. The results are not sensitive to the inclusion of data from the COVID-19 pandemic and results for sub-samples are omitted for the sake of brevity.

VI (a) Shadow-rate estimates when selected yields are added to the federal funds rate

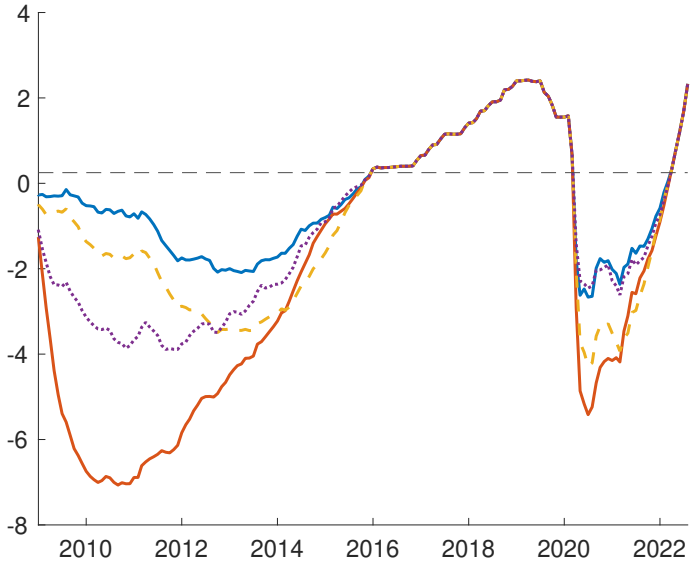
Figure S.30 present shadow rate estimates from models that add either short- or long-term rates to the federal funds rate in estimation. Figure S.31 provides further details by adding selected interest rates individually. The main message is that including only the federal funds rate in the model leads to shadow rate estimates that are much more negative than when other yields are included. Among the various maturities, adding the one-year rate leads to estimates that are closest to those obtained with all the yields included.

Figure S.30: Shadow rates estimates obtained when different groups of yields are included in the VAR

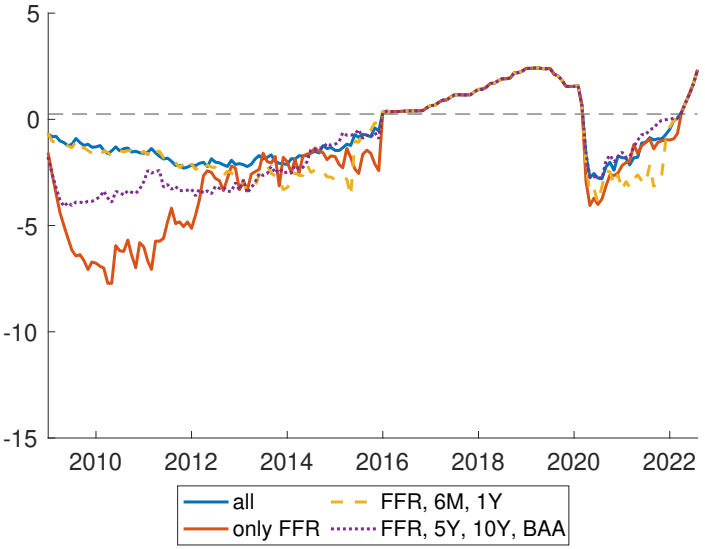
(a) Final estimates from simple shadow-rate VAR



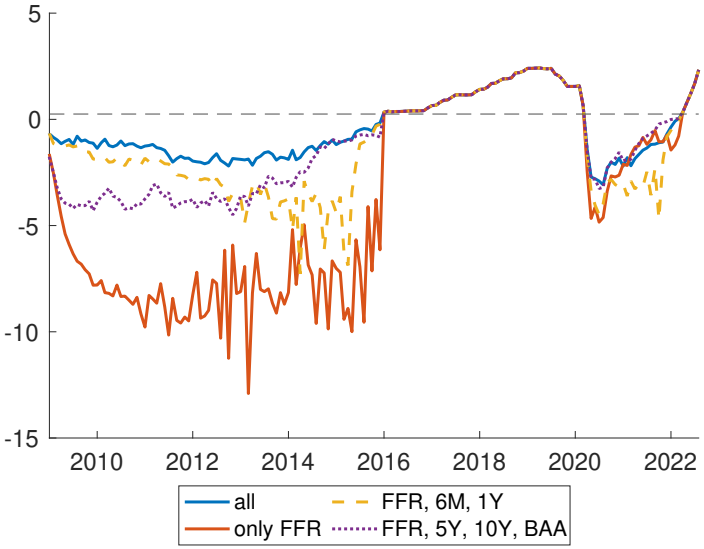
(b) Final estimates from hybrid shadow-rate VAR



(c) QRT estimates from simple shadow-rate VAR



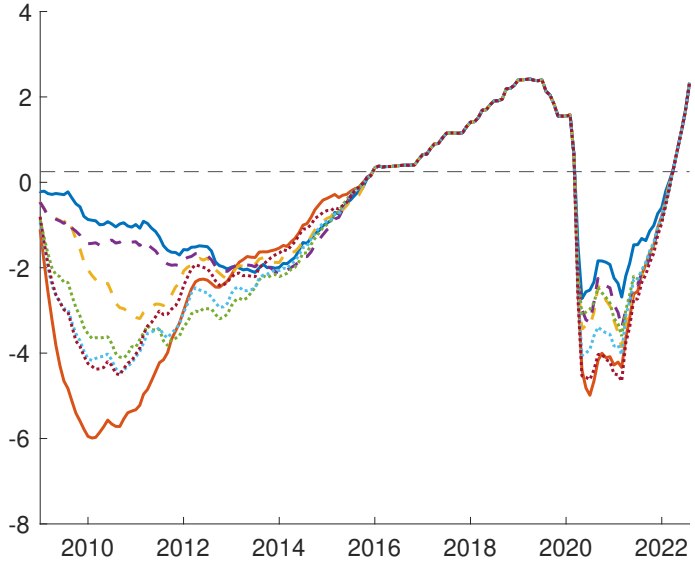
(d) QRT estimates from hybrid shadow-rate VAR



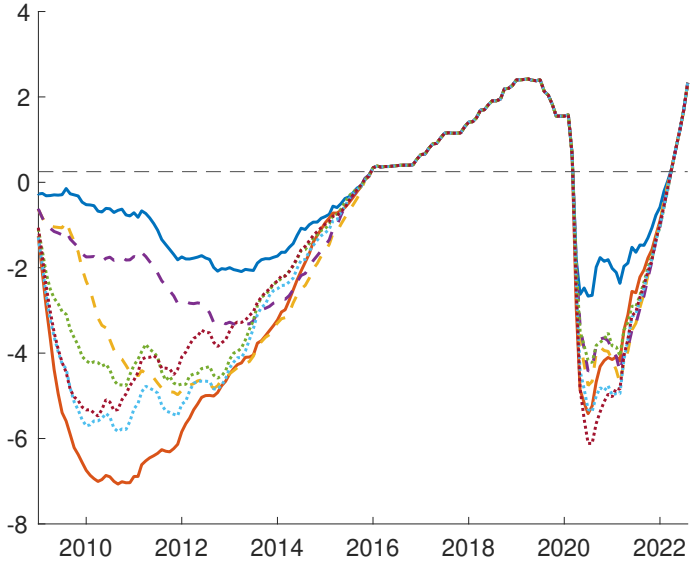
Note: Shadow-rate estimates obtained from models conditioned on different sets of interest rates (and all other 14 macroeconomic and financial variables listed in Table tab:datalist of the paper).

Figure S.31: Shadow rates estimates obtained when different yields are included in the VAR

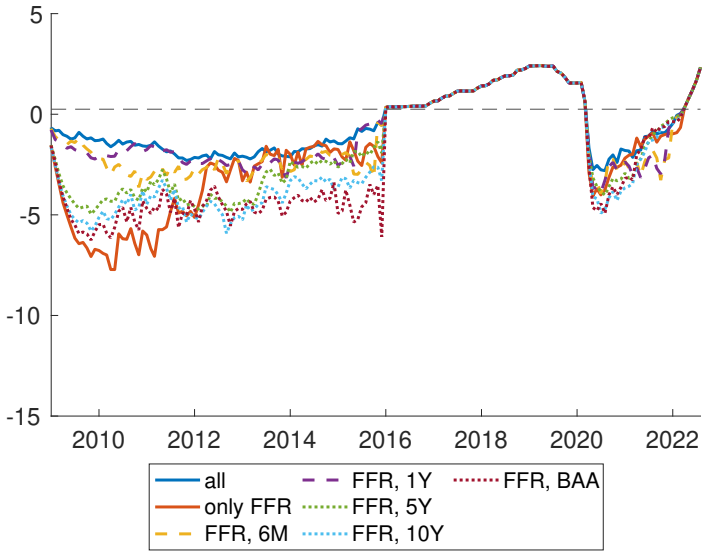
(a) Final estimates from simple shadow-rate VAR



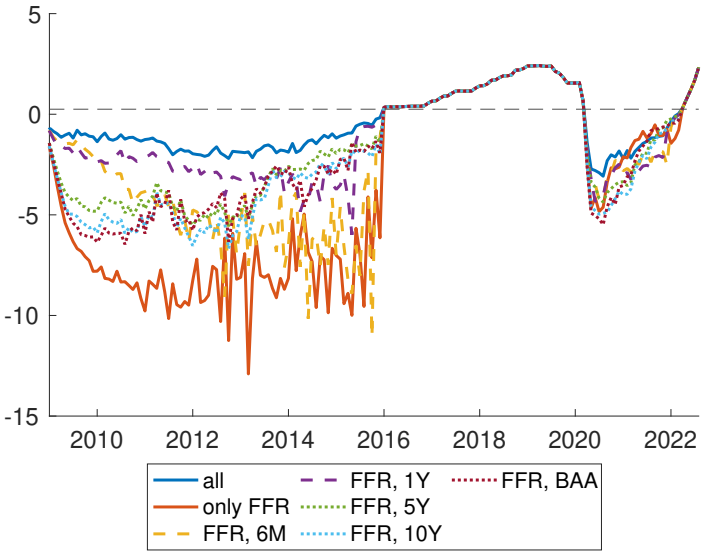
(b) Final estimates from hybrid shadow-rate VAR



(c) QRT estimates from simple shadow-rate VAR



(d) QRT estimates from hybrid shadow-rate VAR



Note: Shadow-rate estimates obtained from models conditioned on different sets of interest rates (and all other 14 macroeconomic and financial variables listed in Table tab:datalist of the paper).

VI (b) Forecast performance when adding only shorter- or longer-term yields to the federal funds rate

Tables S.21 through S.26 assess the forecast performance of linear, simple, and hybrid shadow rate models when including only the federal funds rate and either long maturities or short ones. It turns out that adding long maturities often improves the forecasting performance, in particular for variables such as capacity utilization, hourly earnings, PPI, PCE prices, and the federal funds rate (but not for capacity utilization in the case of the linear and hybrid VARs). Adding instead short-term yields also improves the forecasts but in general less than when adding long-term rates, except for the federal funds rate, where gains are often larger. These differences can be partly attributed to the different estimates of the shadow rates mentioned earlier.

Table S.21: Forecast performance of linear VAR w/o and w/longer-term yields

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	1.00	1.00	0.99	**1.15**	1.00	1.00	0.99	1.00	1.00	1.00	0.99	1.00
Real Consumption	1.01	1.00	1.00	1.00	1.00	0.98**	0.96**	0.97	1.00	1.00	1.00	1.01
IP	1.00	1.00	1.01	1.00	1.00	1.01	0.98	**0.95****	0.99*	0.99	0.98*	0.98*
Capacity Utilization	0.96	0.96	**0.95**	**0.87**	**0.95****	**0.93****	**0.89***	**0.85**	**0.95*****	**0.95****	**0.93***	**0.90**
Unemployment	0.99	0.99	1.01	**1.08**	0.98	0.98	1.01	**1.12**	0.99*	0.99	1.00	**1.08**
Nonfarm Payrolls	1.00	1.00	1.00	1.01	0.99	0.98	0.99	0.96	1.00	0.99	1.00	0.98
Hours	0.96	0.99	1.02	1.03	0.98	1.00	1.02	1.03	0.98	0.99	1.01	1.02
Hourly Earnings	0.97	**0.93**	**0.91**	1.00	0.98	**0.95**	**0.91**	0.99	0.98	0.96	0.97	1.02
PPI (Fin. Goods)	**0.95****	**0.95*****	**0.94****	1.03	**0.95****	**0.95****	**0.94***	1.01	0.96**	0.96**	0.96*	1.02
PPI (Metals)	0.98	0.98	1.00	1.01	0.98	0.99	1.01	1.02	0.99	0.99	1.00	1.02***
PCE Prices	**0.94**	**0.93***	**0.94**	**1.10**	0.96	**0.94**	0.96	**1.07**	**0.95**	**0.95**	0.97	**1.07***
Federal Funds Rate	**0.95**	0.96	1.01	**1.07**	**0.95**	0.96	1.01	0.99	0.96	0.97	1.03	**1.05**
Housing Starts	1.04	**1.07**	**1.10**	**1.18**	1.03	**1.09**	**1.21**	**1.37**	1.01	1.03	**1.08**	**1.15**
S&P 500	1.02	1.01	1.00	0.99	1.02	1.02	1.01***	1.01	1.02**	1.01*	1.01	1.02*
USD / GBP FX Rate	1.00	1.00	1.00	1.00	1.01	1.00	1.00	1.01	1.00	1.00	1.01	1.02**

Note: Comparison of "all variables ex yields" (baseline, in denominator) against "FFR plus 5Y, 10Y, and BAA (and 14 others)." Values below 1 indicate improvement over baseline. Forecasts generated by standard linear VAR. Evaluation window with forecast origins from 2009:01 through 2022:08 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. The variables listed reflect those common to both datasets. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

Table S.22: Forecast performance of simple shadow-rate VAR w/o and w/longer-term yields

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	1.00	1.00	1.00	1.00	0.99	0.99	0.99	0.98	0.99*	0.99	1.00	1.00
Real Consumption	1.00	1.00	1.00	1.00	0.99	0.99*	0.97**	0.97	1.00	0.99	1.00	1.00
IP	1.00	1.00	0.99	0.99	0.98	0.99	**0.95****	**0.94***	0.98**	0.98**	0.97**	0.96*
Capacity Utilization	**0.95**	**0.94**	**0.87***	**0.77**	**0.94****	**0.92**	**0.87**	**0.80**	**0.95*****	**0.93****	**0.88****	**0.81***
Unemployment	1.00	1.00	1.01	0.99	0.99	0.98	0.96	**0.95**	0.99	0.99	0.98	**0.94**
Nonfarm Payrolls	1.00	1.00	1.00	0.99	1.00	0.99	**0.94****	**0.91**	1.00	0.99	0.97**	**0.94***
Hours	0.96	0.97	1.01	**0.93**	0.97	0.99	1.01	**0.88**	0.97*	0.98	0.99	**0.94***
Hourly Earnings	0.96	**0.94**	**0.94**	1.02	0.99	0.96	**0.95**	1.02	0.99	0.98	0.99	1.03**
PPI (Fin. Goods)	0.98	0.98	0.98**	1.01	0.98	0.99	0.98	1.02	0.98	0.99	0.98**	1.02
PPI (Metals)	0.98	0.99	1.01	1.00	0.98	0.99	1.01	1.01	0.99	1.00	1.01	1.02**
PCE Prices	0.97	0.96	0.97	1.01	0.98	0.97	0.98	**1.05**	0.98	0.97	0.99	**1.05**
Federal Funds Rate	**0.78***	**0.77****	**0.84***	**0.89**	**0.93**	0.97	0.96	**1.11**	**0.87**	**0.86**	**0.92**	0.98
Housing Starts	1.00	0.97	0.96	0.97	1.01	1.02	**1.07**	**1.14**	0.98	0.97	1.00	1.00
S&P 500	1.02	1.01	1.01*	1.00	1.01	1.01	1.01	0.99**	1.02*	1.01	1.01	1.01
USD / GBP FX Rate	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.01**

Note: Comparison of "all variables ex yields" (baseline, in denominator) against "FFR plus 5Y, 10Y, and BAA (and 14 others)." Values below 1 indicate improvement over baseline. Forecasts generated by simple shadow-rate VAR. Evaluation window with forecast origins from 2009:01 through 2022:08 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. The variables listed reflect those common to both datasets. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

Table S.23: Forecast performance of hybrid shadow-rate VAR w/o and w/longer-term yields

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	1.00	1.00	1.02	**0.92**	1.00	1.01	1.01	1.00	1.00	1.00	1.01	1.00
Real Consumption	1.00	1.00	1.00	1.00	1.00	0.99	0.97	**0.94***	1.00	1.00	0.99	0.99
IP	1.00	1.00	1.00	1.00	0.99	1.01	1.00	0.98	0.98*	0.99	0.99	0.99
Capacity Utilization	0.96	0.98	0.96	**0.83****	0.96	**0.95**	**0.90**	**0.84**	0.96***	0.96	**0.94**	**0.86*****
Unemployment	1.00	1.00	1.01	1.04	0.99	0.98	1.00	**1.09**	0.99	0.99	1.00	1.04
Nonfarm Payrolls	1.00	1.00	1.00	0.99	1.00	0.99	0.98	**0.95**	1.00	0.99	0.99	0.97**
Hours	**0.94**	0.97	1.02	1.00	0.99	1.00	**1.05**	1.00	0.98	0.99	1.01	0.99
Hourly Earnings	**0.95***	**0.93***	**0.90**	0.98	0.97	**0.94***	**0.90***	0.97	0.98	0.96*	0.96	1.01
PPI (Fin. Goods)	**0.95****	**0.95****	**0.95***	0.98	**0.95****	0.96	**0.94**	1.02	**0.95****	0.96*	0.96	1.02
PPI (Metals)	0.98*	0.98	1.00	1.04	0.98	0.99	1.01	1.02	0.99	0.99	1.00	1.02**
PCE Prices	**0.95**	**0.94**	1.00	**1.05**	0.96	**0.94**	0.98	**1.18*****	0.96	0.96	1.00	**1.13*****
Federal Funds Rate	**0.37**	**0.46**	**0.61**	**0.69****	**0.80**	**0.92**	0.99	**1.08**	**0.74**	**0.79***	**0.88**	**0.94**
Housing Starts	**1.05**	**1.08**	**1.17**	**1.30**	1.04	**1.09**	**1.20**	**1.30**	1.01	1.02	**1.07**	**1.13**
S&P 500	1.02	1.01	1.01**	1.00	1.02	1.01	1.02**	1.02	1.02*	1.01	1.01	1.01
USD / GBP FX Rate	1.01	1.02	1.01	1.04	1.01	1.02	1.03	**1.05**	1.00	1.01	1.01	1.02

Note: Comparison of "all variables ex yields" (baseline, in denominator) against "FFR plus 5Y, 10Y, and BAA (and 14 others)." Values below 1 indicate improvement over baseline. Forecasts generated by hybrid shadow-rate VAR. Evaluation window with forecast origins from 2009:01 through 2022:08 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. The variables listed reflect those common to both datasets. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

Table S.24: Forecast performance of linear VAR w/o and w/short-term yields

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	0.99	1.00	0.98	1.00	1.00	1.00	0.99	0.99	0.99	0.99	0.99	1.00
Real Consumption	1.00	1.00	1.00	1.00	1.00	0.99	0.98	0.98	1.00	1.00	0.99	1.00
IP	1.00	1.00	1.00	1.00	0.99*	0.99	0.98***	0.96**	0.99**	0.99**	0.99***	0.99
Capacity Utilization	0.98	0.97**	0.96**	**0.89****	0.96***	**0.94*****	**0.91*****	**0.87****	0.97***	0.96***	**0.95*****	**0.92*****
Unemployment	1.00	1.00	1.00	1.03	1.00	1.01	1.02	1.02	1.00	1.00	1.01	1.02
Nonfarm Payrolls	1.00	1.00	1.00	1.00	1.00	1.00	1.00	**0.94**	1.00	1.00	1.00	0.98
Hours	1.00	1.00	1.02	1.00	1.00	1.01	1.02	0.98	1.00	1.00	1.01	1.00
Hourly Earnings	1.01	1.00	0.98*	0.97**	0.98***	0.99	0.98	0.98	0.99	1.00	0.99	1.00
PPI (Fin. Goods)	0.98***	0.98***	0.98**	1.02	0.98***	0.98***	0.97**	1.00	0.98**	0.98**	0.99	1.00
PPI (Metals)	0.99	0.99	1.00	1.01	1.00	1.00	1.00	1.00	0.99	1.00	1.00	1.01***
PCE Prices	0.99*	0.98***	0.97***	1.03	0.99*	0.98**	0.98**	0.99	0.98**	0.98***	0.98**	1.00
Federal Funds Rate	**0.76****	**0.71***	**0.73**	**0.82**	**0.76*****	**0.69*****	**0.73*****	**0.86**	**0.78*****	**0.75*****	**0.77*****	**0.87***
Housing Starts	0.99	**0.95**	**0.87**	**0.87**	1.00	0.96	**0.95**	1.00	0.98	0.96	**0.93**	**0.94**
S&P 500	1.01	1.02	1.00	1.00	1.02*	1.02	1.01	1.01	1.01	1.01	1.01	1.01
USD / GBP FX Rate	1.01	1.01***	1.01	1.02***	1.01*	1.01**	1.01*	1.02**	1.01	1.01	1.01*	1.01***

Note: Comparison of "all variables ex yields" (baseline, in denominator) against "FFR plus 6M and 1Y (and 14 others)." Values below 1 indicate improvement over baseline. Forecasts generated by standard linear VAR. Evaluation window with forecast origins from 2009:01 through 2022:08 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. The variables listed reflect those common to both datasets. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

Table S.25: Forecast performance of simple shadow-rate VAR w/o and w/short-term yields

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	1.00	1.00	1.00	1.00	0.99	1.00	0.99	0.98	1.00	1.00	1.00	1.01
Real Consumption	1.00	1.00	1.00	1.00	0.99*	1.00	0.97*	0.97	1.00	1.00	1.00	0.99
IP	1.00	1.00	0.99	0.98	1.00	0.98	**0.94***	**0.93**	1.00	0.99	0.98	0.97
Capacity Utilization	0.98	0.96*	**0.89**	**0.73**	0.96**	**0.94***	**0.87**	**0.78**	0.98**	0.96*	**0.90**	**0.81**
Unemployment	1.00	1.00	0.99	**0.91**	1.00	0.99	**0.93***	**0.81**	1.00	0.99*	0.96**	**0.86**
Nonfarm Payrolls	1.00	1.00	1.00	0.99	1.00	0.98	**0.94***	**0.88**	1.00	0.99	0.97*	**0.94**
Hours	0.99	1.01	1.02	**0.88**	1.00	1.03	1.02	**0.82**	1.00	1.01	1.00	**0.92**
Hourly Earnings	1.01	1.00	1.00	1.02	1.00	1.00	1.01	1.02	1.00	1.00	1.01	1.02**
PPI (Fin. Goods)	0.99*	0.99	0.99	1.01	0.99**	0.99	0.99*	1.01*	0.99	1.00	1.00	1.01**
PPI (Metals)	0.99	0.99	1.01	1.00	0.99*	0.99	1.00	0.99	0.99	1.00	1.01*	1.01
PCE Prices	0.98*	0.97	0.96	0.98	0.98*	0.98	0.97	0.98	0.98*	0.98	0.98	0.99
Federal Funds Rate	**0.67****	**0.62****	**0.73***	**0.80**	**0.73****	**0.77**	**0.94**	**1.16**	**0.69*****	**0.70****	**0.86**	**1.06**
Housing Starts	**0.94****	**0.86***	**0.81**	**0.81**	**0.95**	**0.90****	**0.89**	**0.92**	**0.95****	**0.91***	**0.90**	**0.90**
S&P 500	1.01	1.00	1.00	0.99**	1.01	1.01	1.00	0.99	1.01**	1.00	1.00	1.00
USD / GBP FX Rate	1.01	1.00	1.01	1.01**	1.01	1.01	1.01	1.01*	1.01	1.01	1.01	1.01**

Note: Comparison of "all variables ex yields" (baseline, in denominator) against "FFR plus 6M and 1Y (and 14 others)." Values below 1 indicate improvement over baseline. Forecasts generated by simple shadow-rate VAR. Evaluation window with forecast origins from 2009:01 through 2022:08 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. The variables listed reflect those common to both datasets. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

Table S.26: Forecast performance of hybrid shadow-rate VAR w/o and w/short-term yields

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	0.99	1.00	**1.05**	**0.88**	0.99	1.00	1.00	1.00	0.99	1.00	1.00	1.01
Real Consumption	1.00	1.00	1.00	0.99	1.00	1.00	0.98*	0.96*	1.00	1.00	0.99	0.99
IP	1.00	1.00	1.00	1.00	1.00	0.99	0.98***	0.97**	0.99	0.99***	0.98***	0.99**
Capacity Utilization	0.97	0.97**	**0.92*****	**0.81*****	0.97**	**0.94*****	**0.88*****	**0.81*****	0.98**	0.96***	**0.92*****	**0.87*****
Unemployment	1.00	1.00	1.01	0.99	1.01	1.01	1.02	1.02	1.00	1.00	1.01	0.99
Nonfarm Payrolls	1.00	1.00	1.00	0.99	1.00	0.99	0.98	**0.94****	1.00	0.99**	0.98**	0.97***
Hours	0.98	1.00	1.00	0.97	1.00	1.01	1.00	0.99	1.00	1.00	1.00	0.99
Hourly Earnings	1.00	0.99	0.97	1.02	0.98***	0.98	0.98	1.02	0.99	0.99	0.99	1.03***
PPI (Fin. Goods)	0.98***	0.98**	0.99*	1.02	0.97***	0.98**	0.98*	1.01	0.98***	0.98**	0.99	1.01
PPI (Metals)	0.99	1.00	1.01	1.02	0.99	1.00	1.00	1.01	1.00	1.00	1.00	1.01**
PCE Prices	0.99	0.98**	1.02	1.01	0.99	0.98*	1.00	1.04**	0.99*	0.98**	1.00	1.04**
Federal Funds Rate	**0.33**	**0.38**	**0.55**	**0.69**	**0.62****	**0.71***	0.96	**1.17**	**0.60****	**0.64****	**0.83**	**1.06**
Housing Starts	0.99	**0.95**	**0.90**	**0.90**	1.00	0.96	0.97	0.97	0.98	0.96	**0.95**	**0.95**
S&P 500	1.02*	1.01	1.01	1.01	1.03*	1.02	1.03*	1.01	1.02**	1.01*	1.01*	1.01*
USD / GBP FX Rate	1.02**	1.02**	1.01	1.00	1.01*	1.02**	1.02**	1.02	1.01**	1.01	1.01*	1.01

Note: Comparison of "all variables ex yields" (baseline, in denominator) against "FFR plus 6M and 1Y (and 14 others)." Values below 1 indicate improvement over baseline. Forecasts generated by hybrid shadow-rate VAR. Evaluation window with forecast origins from 2009:01 through 2022:08 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. The variables listed reflect those common to both datasets. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

VII Forecast performance compared to a linear VAR that omits short-term interest rates

It has been suggested in the literature, e.g. Debortoli, Gali, and Gambetti (2019), to only include longer-term rates in VARs to avoid issues related to the ELB. Therefore, in Tables S.27 through S.32 we assess the relative performance of VARs that only include longer-term rates with respect to that of the linear, simple shadow, and hybrid shadow-rate VARs with all rates, excluding or including the Covid observations in the evaluation. It turns out that the VAR with long-term rates only is generally worse than that with all rates for most real variables, but better for the 5- and 10-year yields. The same holds for the simple and hybrid shadow-rate VARs, but the latter in particular is better also for the 5- and 10-year yields (though not when the Covid period is included in the evaluation). Overall, it seems that omitting short-term rates from the VAR is not a good idea as in general it leads to a deterioration in the forecasting performance.

Table S.27: Forecast performance of linear VAR without short-term interest rates

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	1.00	0.99	1.01	**1.16**	1.02	1.02*	1.01	1.01	1.00	1.00	0.99	1.00
Real Consumption	1.00	0.97*	**0.94**	**0.95**	0.99	0.96	**0.91**	0.98	0.99	0.97**	0.96	0.99
IP	0.99	0.98	**0.92****	**0.89****	0.97*	0.97	**0.91****	**0.88****	0.98	0.97	**0.93****	**0.93****
Capacity Utilization	**0.95***	**0.93**	**0.88***	**0.79**	**0.92*****	**0.89****	**0.81****	**0.77**	**0.95*****	**0.92****	**0.87****	**0.83**
Unemployment	1.03**	1.04	1.02	1.00	1.01	**1.06**	1.01	1.03	1.03***	1.03	1.00	1.01
Nonfarm Payrolls	1.02	1.02	0.96	**0.84**	0.99	0.97	**0.88****	**0.89**	1.00	0.99	**0.94***	**0.94**
Hours	1.00	1.01	1.04	0.96	1.00	1.02	1.01	**0.94**	1.01	1.01	1.02	0.99
Hourly Earnings	1.01	1.00	1.00	0.99*	1.00	0.99	0.99	0.98	1.00	1.00	1.01	1.00
PPI (Fin. Goods)	**0.95****	0.97	0.97	0.98	**0.95****	0.98	0.97*	0.97	0.96*	0.97	0.98	0.99
PPI (Metals)	0.99	0.99	1.01	1.00	0.99	0.99	1.01	1.00	0.99	1.00	1.01	1.01*
PCE Prices	**0.94****	**0.93***	**0.93***	**0.95***	**0.94****	**0.93****	**0.92****	**0.92*****	**0.95****	**0.94***	**0.94****	**0.95****
Housing Starts	0.96	**0.95**	1.00	**1.17**	0.97	0.97	1.02	**1.27**	0.97	0.97	1.01	**1.17**
S&P 500	0.96	0.97	0.98***	0.96	**0.95****	0.96	0.96***	0.99	0.98*	0.99	0.99	1.02***
USD / GBP FX Rate	1.03*	1.02***	1.01***	1.03**	1.04**	1.02**	1.02**	1.04	1.03**	1.02***	1.01***	1.04***
5-Year Yield	1.04	**1.07**	**1.18****	**1.18***	**1.06**	**1.10**	**1.23***	**1.22**	**1.05**	**1.09**	**1.14****	**1.10***
10-Year Yield	**1.05**	**1.06**	**1.15**	**1.23**	**1.05**	1.04	**1.15**	**1.20**	**1.05**	**1.06**	**1.11**	**1.11***
BAA Yield	**0.94**	**0.91**	**0.88**	**0.94**	0.96	**0.93**	**0.87**	1.01	0.98	**0.94**	**0.93**	0.96

Note: Comparison of "Linear VAR without short-term yields" (baseline, in denominator) against "all variables." Values below 1 indicate improvement over baseline. Baseline estimated from linear VAR, alternative forecasts generated by standard linear VAR. Evaluation window with forecast origins from 2009:01 through 2017:12 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

Table S.28: Forecast performance of linear VAR without short-term interest rates vs all data in simple shadow-rate VAR

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	1.00	1.00	1.01	1.02	1.00	1.00	1.01	0.99	0.99	0.99	1.00	1.00
Real Consumption	1.00	0.97***	0.97	**0.93**	0.98*	0.96**	**0.93***	0.98	0.99*	0.97***	0.97	0.99
IP	1.00	0.99	0.96	**0.89****	1.00	0.99	0.96	**0.89****	0.99	0.99	0.97	**0.94****
Capacity Utilization	0.98	0.98	0.96	**0.87**	0.98	0.96	**0.92**	**0.86**	0.97*	0.97	**0.95**	**0.90**
Unemployment	1.03	1.04	1.02	1.00	1.01	1.04	0.97	0.99	1.02*	1.03	1.01	1.01
Nonfarm Payrolls	1.00	1.01	1.00	**0.88**	0.99	0.99	**0.95**	**0.90**	1.00	1.01	0.99	0.96
Hours	1.01	1.02*	1.04*	0.99	1.01	1.02**	1.02	0.98	1.01	1.02*	1.03*	1.01
Hourly Earnings	1.01*	1.01	1.00	0.99	1.01	1.00	0.98	0.99	1.01*	1.00	1.00	1.00
PPI (Fin. Goods)	**0.95****	0.96*	0.97	0.97*	**0.95*****	0.96*	0.97*	0.98	**0.95****	0.96*	0.97	0.99
PPI (Metals)	0.98	0.98	1.01	1.02*	0.98	0.99	1.01	1.01	0.99	1.00	1.01*	1.02***
PCE Prices	**0.94****	**0.93****	**0.94***	0.96	**0.94*****	**0.93*****	**0.93****	**0.94***	**0.94*****	**0.94****	**0.94****	0.96*
Housing Starts	0.97	**0.94**	**0.95**	**1.06**	0.97	**0.95**	1.00	**1.17**	0.98	0.96	0.97	**1.07**
S&P 500	**0.95**	0.98	0.98*	1.00	**0.93****	**0.95**	**0.95****	0.99	0.97	0.99	0.99*	1.03***
USD / GBP FX Rate	1.02	1.01**	1.00	1.00	1.02	1.01	1.00	1.00	1.02	1.01*	1.01*	1.02**
5-Year Yield	0.97	0.96	1.00	1.00	0.99	1.00	**1.10**	**1.10**	1.02	1.02	1.03	**0.95**
10-Year Yield	1.00	0.99	1.01	1.01	0.99	1.00	1.02	**1.09**	1.02	1.01	1.03	**1.05**
BAA Yield	**0.92**	**0.90**	**0.89**	**0.87**	0.96	**0.92**	**0.89**	0.99	0.97	**0.95**	**0.95**	0.99

Note: Comparison of "Linear VAR without short-term yields" (baseline, in denominator) against "all variables." Values below 1 indicate improvement over baseline. Baseline estimated from linear VAR, alternative forecasts generated by simple shadow-rate VAR. Evaluation window with forecast origins from 2009:01 through 2017:12 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

Table S.29: Forecast performance of linear VAR without short-term interest rates vs all data in hybrid shadow-rate VAR

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	0.99	0.99	1.01	**1.16****	1.01	1.02	1.02	0.97	1.00	1.01	1.00	0.99
Real Consumption	1.00	0.98	**0.94**	**0.85**	0.98	0.97	**0.93**	**0.88**	0.99	0.97*	0.96	0.96
IP	1.00	0.97	**0.92***	**0.87***	0.99	0.97	**0.91***	**0.85***	0.99	0.97	**0.93***	**0.93***
Capacity Utilization	**0.95**	**0.92***	**0.87***	**0.73***	**0.92****	**0.87****	**0.79*****	**0.72***	**0.95****	**0.91*****	**0.87*****	**0.80***
Unemployment	1.04*	1.04	1.01	**0.91**	1.02	**1.05**	1.00	**0.94**	1.03**	1.02	1.00	**0.95**
Nonfarm Payrolls	1.00	1.02	1.00	**0.83**	0.97	0.96	**0.88****	**0.81**	0.98	0.98	**0.95**	**0.92**
Hours	1.02	1.04	**1.06**	0.99	1.01	1.04	1.04	0.97	1.02	1.02	1.04	1.00
Hourly Earnings	1.01	1.01	1.01	1.03*	1.00	1.00	0.99	1.01	1.01	1.01	1.01	1.02*
PPI (Fin. Goods)	**0.95****	**0.95**	**0.94**	0.97	**0.95****	0.96	**0.95**	0.98	0.96*	0.96	0.96	0.99
PPI (Metals)	0.98	0.98	0.99	1.01	0.98	0.99	1.00	1.00	0.99	0.99	1.00	1.01***
PCE Prices	**0.94****	**0.91****	**0.91***	1.01	**0.94****	**0.91****	**0.91****	0.99	**0.95****	**0.93****	**0.93***	1.00
Housing Starts	0.96	**0.93**	**0.90**	**1.06**	0.97	**0.94**	**0.93**	**1.15**	0.97	0.96	**0.95**	**1.09**
S&P 500	0.96*	0.98	0.99	1.01	**0.95****	0.96	0.97	0.99	0.98**	0.99	1.00	1.03***
USD / GBP FX Rate	1.04	1.02**	1.02	1.03	**1.05****	1.03**	1.03*	**1.08**	1.04*	1.02**	1.02*	**1.05***
5-Year Yield	0.99	0.96	**0.95**	**0.89**	1.02	1.03	**1.10**	**1.05**	1.03	1.03	1.00	**0.90****
10-Year Yield	1.02	1.00	1.00	**0.95**	1.01	1.01	1.01	**1.09**	1.03	1.02	1.02	1.04
BAA Yield	**0.92**	**0.89**	**0.88**	**0.91**	**0.95**	**0.90**	**0.90**	**1.07**	0.96	**0.94**	**0.95**	1.02

Note: Comparison of "Linear VAR without short-term yields" (baseline, in denominator) against "all variables." Values below 1 indicate improvement over baseline. Baseline estimated from linear VAR, alternative forecasts generated by hybrid shadow-rate VAR. Evaluation window with forecast origins from 2009:01 through 2017:12 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

Table S.30: Forecast performance of linear VAR without short-term interest rates (incl. COVID-19)

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	1.00	1.00	1.01	0.96	1.00	1.01	1.00	1.00	1.00	1.00	1.00	1.00
Real Consumption	1.00	1.00	1.00	1.01	1.00	1.00	0.97	0.98	1.00	1.00	0.99	1.00
IP	1.00	1.00	1.00	0.97*	0.98*	0.98	0.96**	**0.93****	0.99*	0.99	0.97*	0.97*
Capacity Utilization	0.99*	1.00	**0.93**	**0.81**	**0.95*****	**0.92*****	**0.83*****	**0.79**	0.96*****	**0.94*****	**0.90*****	**0.87**
Unemployment	1.00	0.99	1.00	**1.08**	1.00	1.02	1.00	1.04	1.00	1.00	0.99	1.02
Nonfarm Payrolls	1.00	1.00	1.00	1.00	1.00	0.98	0.97*	**0.95**	0.99	0.99	0.98**	0.98
Hours	1.01	1.02	1.02	0.96	1.01	1.03*	1.01	**0.94**	1.01	1.01	1.01	0.99
Hourly Earnings	1.01	1.01	0.99	0.98**	1.00	0.99	0.99	0.99	1.00	1.00	1.00	1.00
PPI (Fin. Goods)	0.98	0.98*	0.97**	**0.93**	0.98	0.98	0.97**	0.98	0.98	0.98	0.98	0.99
PPI (Metals)	0.99	0.99	1.00	0.96	0.99	1.00	1.01	1.00	0.99	1.00	1.01	1.01**
PCE Prices	0.96**	**0.95****	**0.94****	**0.93***	0.96**	**0.94*****	**0.93*****	**0.94*****	0.97*	0.96**	**0.95****	0.96*****
Housing Starts	1.01	1.03	**1.06**	**1.25**	1.02	1.04	**1.07**	**1.29**	1.00	1.01	1.03	**1.16**
S&P 500	0.99	0.99	0.99**	1.01	0.97*	0.98	0.98*	1.00	0.99	0.99	1.00	1.03*****
USD / GBP FX Rate	1.04**	1.02*****	1.01**	0.98	1.03*****	1.03*****	1.01	1.03	1.03*****	1.02*****	1.01*****	1.03*****
5-Year Yield	1.02	**1.08****	**1.16*****	**1.14***	**1.05**	**1.11****	**1.23*****	**1.14**	1.04	**1.09****	**1.13*****	**1.10****
10-Year Yield	1.04	**1.07***	**1.16****	**1.18***	1.04	**1.06**	**1.19****	**1.17**	1.04	**1.06**	**1.12***	**1.11****
BAA Yield	0.99	0.98	0.98	**0.93**	0.99	1.00	0.98	0.96	1.00	0.99	0.98	0.97

Note: Comparison of "Linear VAR without short-term yields" (baseline, in denominator) against "all variables." Values below 1 indicate improvement over baseline. Baseline estimated from linear VAR, alternative forecasts generated by standard linear VAR. Evaluation window with forecast origins from 2009:01 through 2022:08 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

Table S.31: Forecast performance of linear VAR without short-term interest rates vs all data in simple shadow-rate VAR (incl. COVID-19)

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	1.00	1.00	0.99	**0.85**	1.00	1.01	1.00	0.99	1.01	1.00	1.00	1.01
Real Consumption	1.00	1.00	1.00	0.99	1.00	1.00	0.97*	0.99	1.00	1.00	1.00	1.01
IP	1.00	1.00	1.00	0.97*	1.00	1.01	0.98	**0.94****	1.00	1.00	0.99	0.98
Capacity Utilization	1.01	1.01	0.98	**0.88**	0.98	0.97	**0.94***	**0.89**	0.99	0.98	0.97	**0.94**
Unemployment	0.99	0.99	0.98	1.02	0.99	1.00	0.96*	0.98	1.00	0.99	0.98	1.00
Nonfarm Payrolls	1.00	1.00	1.00	1.00	0.99	0.99	0.99	0.97	1.00	0.99	1.00	0.99
Hours	1.01	1.02**	1.02*	0.98	1.00	1.03***	1.03	0.99	1.01*	1.01***	1.02**	1.01
Hourly Earnings	1.00	1.01*	1.00	0.98	1.00	1.00	0.99	1.00	1.01	1.01	1.00	1.01
PPI (Fin. Goods)	0.98	0.98	0.99	**0.92**	0.97**	0.97*	0.97**	0.98	0.98*	0.98	0.98*	0.99
PPI (Metals)	0.99	0.99	1.01	0.99	0.99	0.99	1.00	1.01	0.99	1.00	1.01	1.02***
PCE Prices	0.97**	0.96**	0.96**	**0.89**	0.97**	**0.95*****	**0.94*****	**0.95****	0.97**	0.96**	0.96**	0.97**
Housing Starts	0.98	0.97	0.98	**1.09**	0.99	0.98	1.02	**1.21***	0.99	0.99	1.00	**1.09**
S&P 500	0.98	1.00	1.00	0.98	0.96	0.98	0.97**	0.99	0.99	1.00	1.00	1.03***
USD / GBP FX Rate	1.02*	1.01**	1.00	0.96	1.02	1.01*	1.00	1.00	1.02**	1.01**	1.01**	1.02***
5-Year Yield	1.00	**1.06**	**1.13**	**1.35**	0.97	1.01	**1.12***	**1.06**	0.99	1.02	1.03	0.96
10-Year Yield	1.03	**1.05**	**1.12**	**1.40**	1.01	1.01	**1.08**	**1.08**	1.02	1.02	**1.05**	1.04
BAA Yield	0.96	0.97	0.99	1.00	0.97	**0.95**	0.97	0.98	0.99	0.99	0.99	0.99

Note: Comparison of "Linear VAR without short-term yields" (baseline, in denominator) against "all variables." Values below 1 indicate improvement over baseline. Baseline estimated from linear VAR, alternative forecasts generated by simple shadow-rate VAR. Evaluation window with forecast origins from 2009:01 through 2022:08 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

Table S.32: Forecast performance of linear VAR without short-term interest rates vs all data in hybrid shadow-rate VAR (incl. COVID-19)

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	1.00	1.00	**0.95**	**0.89**	1.00	1.00	1.01	0.99	1.00	1.00	1.00	1.00
Real Consumption	1.00	1.00	1.00	1.00	1.00	1.00	0.98	0.96	1.00	1.00	1.00	0.99
IP	1.00	1.00	0.99**	0.97	0.99	0.99	0.96*	**0.92***	0.99	0.99	0.98	0.97
Capacity Utilization	1.00	0.99	**0.93**	**0.78***	0.96***	**0.91*****	**0.83*****	**0.77***	0.97***	**0.94*****	**0.90*****	**0.85***
Unemployment	1.00	1.00	1.00	**1.09**	1.00	1.02	1.00	0.99	1.01	1.00	1.00	0.99
Nonfarm Payrolls	1.00	1.00	1.00	1.00	0.99	0.98	0.97*	**0.93**	0.99	0.99	0.99	0.98
Hours	1.01**	1.03**	**1.05**	0.98	1.01	1.03**	1.04	0.97	1.01*	1.02*	1.02	1.00
Hourly Earnings	1.01	1.01	1.02	1.04*	1.00	1.00	0.99	1.01	1.01	1.01	1.00	1.01
PPI (Fin. Goods)	0.98*	0.98	0.98	**0.92**	0.97	0.98	0.97	0.98	0.98	0.98	0.98	0.99
PPI (Metals)	0.98	0.99	0.99	0.97	0.99	0.99	1.00	1.00	0.99	0.99	1.00	1.01***
PCE Prices	0.96**	**0.95***	**0.94***	**0.90**	0.96**	**0.93*****	**0.94****	0.99	0.97*	0.96**	0.96*	1.00
Housing Starts	1.01	1.02	1.01	**1.28**	1.02	1.02	1.01	**1.22**	1.00	1.00	0.99	**1.11**
S&P 500	0.98	0.99	1.00	**1.06**	0.96**	0.97*	0.98	1.00	0.99	1.00	1.01	1.03***
USD / GBP FX Rate	1.04*	1.02***	1.01	0.97	1.04***	1.04***	1.02	**1.05**	1.03**	1.02***	1.01	1.03
5-Year Yield	1.00	1.04	**1.07**	**1.21**	0.98	1.03	**1.10***	**1.05**	0.99	1.01	1.01	**0.93***
10-Year Yield	**1.05**	**1.05**	**1.09**	**1.30**	1.03	1.03	**1.06**	**1.09**	1.03	1.03	1.04	1.03
BAA Yield	**0.95**	**0.95**	0.97	**1.05**	0.97	**0.93**	0.96	**1.06**	0.98	0.98	0.99	1.01

Note: Comparison of "Linear VAR without short-term yields" (baseline, in denominator) against "all variables." Values below 1 indicate improvement over baseline. Baseline estimated from linear VAR, alternative forecasts generated by hybrid shadow-rate VAR. Evaluation window with forecast origins from 2009:01 through 2022:08 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

VIII Shadow rates and forecasts when assuming other ELB values

The baseline results reported thus far, are all conditioned on an ELB value of 25 basis points. Section VIII (a) describes forecast accuracy and shadow-rate estimates obtained when assuming an ELB value of 12.5 basis points, and Section VIII (b) provides corresponding material for the case of when the ELB value is set to 50 basis points. As in our main results, and for sake of comparability, forecasts from linear and shadow-rate VARs are compared against realized interest-rate values that are censored at the ELB; in case of the robustness checks reported below, the alternative ELB values of 12.5 and 50 basis points are applied for this purpose as well.

VIII (a) ELB=12.5 basis points

Tables S.33 through S.36 provide forecast accuracy comparisons of the simple shadow-rate and hybrid shadow-rate VARs to standard VARs that assume a value of 12.5 basis points for the ELB. The value of 12.5 basis points corresponds to the middle of the FOMC's target range for the federal funds rate during each of the two ELB episodes covered in our data. Specifically, Tables S.33 and Tables S.34 are counterparts to Tables 2 and 3 in the paper, and Tables S.35 and S.36 mirror Tables S.7 and S.8 in this appendix (except for the choice of ELB value). In all cases, results are broadly similar, indicating robustness to the alternative choice in ELB value. More specifically, the advantages of the shadow-rate VAR over the standard linear VAR are preserved when we lower the ELB value from the baseline 25 basis points to 12.5 basis points. However, with the lower setting of the ELB value, the relative performance of the hybrid specification deteriorates some (at longer forecast horizons) relative to the paper's results with a setting of 25 basis points; with the lower setting, the hybrid specification's forecast accuracy falls a little short of the shadow-rate VAR. This modest deterioration likely reflects the fact that, with a lower ELB value barely above the minimum actual funds rate ever observed, the shadow rate more strongly resembles the actual funds rate in the period of the Great Recession and ensuing slow recovery.

Figures S.32 and S.33 report shadow-rate estimates for our baseline data set, and generated with the assumed value of ELB=12.5bp from simple and hybrid shadow-rate VARs, respectively. Figure S.34 provides corresponding estimates when yields data (other than the federal funds rate) are excluded. As noted above, with the lower ELB value that corresponds to the 0 to 25 basis point target range used by the FOMC and is close to the observed funds rate for much of the 2009-2014 period, by construction the shadow rate is equal to the actual funds rate for more of the sample than is the case with our baseline estimates using an ELB value of 25 basis points. In turn, our estimated shadow rate based on an ELB value of 12.5 basis points is less similar to the term structure model-based estimates of Krippner and Wu-Xia, commonly exceeding these estimates for much of the 2009-2014 period.

Table S.33: ELB=12.5bp: Forecast performance of standard vs. simple shadow-rate VAR

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	1.00	1.00	0.99**	**0.88**	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.01
Real Consumption	0.99	1.00	1.00	1.00	0.99	1.00	1.01	1.00	0.99	1.00	1.00	1.00**
IP	1.01*	1.01**	1.01*	0.97**	1.01	1.02***	1.02	0.99**	1.00	1.01**	1.01**	1.00
Capacity Utilization	1.00	1.00	1.00	1.01	0.99	0.99	0.99	1.02	1.00	1.00	1.00	1.00
Unemployment	1.00	0.98	0.98	0.98	1.00	0.98	0.96	0.96*	1.00	0.98	0.98	0.98
Nonfarm Payrolls	0.98	1.00	1.00	1.00	0.98*	1.00	1.01	0.99	0.99	1.00	1.00	1.00
Hours	1.00	1.01	1.00	1.00	1.00	1.00	1.00	1.01	1.00	1.00	1.00	1.00
Hourly Earnings	1.01**	1.01**	1.00	1.00	1.01	1.01*	1.00	1.01	1.01**	1.01	1.00	1.00
PPI (Fin. Goods)	1.00	0.99***	1.00	0.99	1.00	0.99***	1.00	1.00	1.00	1.00	1.00	1.00
PPI (Metals)	0.99**	0.99	1.00	1.00	1.00	1.00	1.00	1.01**	1.00	1.00	1.00	1.00
PCE Prices	1.00	0.99*	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Federal Funds Rate	**0.34**	**0.34**	**0.42**	**0.70**	**0.30*****	**0.32***	**0.39**	**0.59**	**0.33*****	**0.33****	**0.41***	**0.55****
Housing Starts	0.99	0.99	0.97	0.98	0.98	0.98	0.98	0.97	0.99	1.00	0.98	0.98
S&P 500	1.00	1.01*	1.01	**1.05**	1.00	1.02**	1.01	1.00*	1.00	1.01*	1.00	1.00
USD / GBP FX Rate	1.00	1.00	1.00	0.97***	0.99	1.00	1.00	0.99*	0.99*	1.00	1.00	1.00***
6-Month Tbill	**0.50**	**0.52**	**0.62**	**0.80**	**0.50****	**0.50***	**0.57**	**0.68***	**0.51*****	**0.51***	**0.57***	**0.63****
1-Year Yield	**0.77**	**0.73**	**0.75**	**0.85**	**0.81****	**0.71***	**0.70**	**0.74***	**0.79*****	**0.72****	**0.70***	**0.67*****
5-Year Yield	1.00	0.98	**0.94**	**0.88**	1.02	1.01	0.99	0.97*	1.01	1.00	0.97*	**0.90*****
10-Year Yield	1.00	0.99	0.97	**0.93**	1.01	1.01	0.98	0.98	1.00	1.00	0.99	0.97*
BAA Yield	1.01	1.02	1.02	0.97	1.02*	1.02	1.01	1.03*	1.02*	1.02	1.01	1.03***

Note: Comparison of "standard linear VAR" (baseline, in denominator) against "simple shadow-rate VAR," and assuming an ELB of 12.5 basis points. Values below 1 indicate improvement over baseline. Evaluation window with forecast origins from 2009:01 through 2017:12 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Due to the close behavior of some of the models compared, and rounding of the reported values, a few comparisons show significant ratios of 1.00. These cases arise from persistent differences in performance that are, however, too small to be relevant after rounding. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

Table S.34: ELB=12.5bp: Forecast performance of standard vs. hybrid shadow-rate VAR

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	1.00	1.00	0.99	**0.94**	1.00	1.00	1.01	0.98	1.00	1.00	1.01**	1.00
Real Consumption	1.00	1.01	1.00	1.01	1.00	1.01	1.03	**0.92**	1.00	1.01*	1.00	0.98
IP	1.01*	1.01	1.01	**1.06**	1.01***	1.01	1.01	**0.92**	1.01	1.00	1.00	0.97
Capacity Utilization	0.99**	0.99*	1.02	0.96	0.98*	0.99	1.00	0.98	0.99*	0.99*	1.01	0.98
Unemployment	1.00	0.99	1.00	0.98	1.00	0.99	1.00	0.99	1.00	0.99	1.00	0.99
Nonfarm Payrolls	0.98**	1.01	**1.07**	**1.13**	0.99	1.00	1.04	1.03	0.99*	1.00	1.03	1.01
Hours	1.00	1.01	1.03	1.04	1.00	1.00	1.02	**1.05**	1.00	1.00	1.02	1.02
Hourly Earnings	1.01	1.02**	1.02*	1.01	1.00	1.01	1.01	1.01	1.01*	1.01*	1.01	1.00
PPI (Fin. Goods)	1.00	0.99	0.97	1.01	1.00	0.99	0.98	1.00	1.00	0.99	0.99	1.00
PPI (Metals)	1.00	1.00	0.99	1.02	0.99**	1.00	0.99	1.01*	1.00*	1.00	0.99	1.00
PCE Prices	0.99	0.98	**0.95**	1.02	0.99**	0.98	**0.95**	1.03***	0.99*	0.99	0.97	1.02***
Federal Funds Rate	**0.34**	**0.34**	**0.42**	**0.68**	**0.31*****	**0.33***	**0.40**	**0.58**	**0.33*****	**0.33****	**0.40***	**0.54****
Housing Starts	0.99	0.99	**0.93**	0.97	0.99	0.97	**0.93**	0.98	1.00	0.99	0.96	0.99
S&P 500	1.01	1.00	1.01**	0.97	1.02**	1.00	1.02**	1.00	1.01	1.00	1.01**	1.00
USD / GBP FX Rate	1.01	1.01	1.00	1.00	1.01	1.01*	1.02	1.04	1.00	1.01*	1.00	1.01
6-Month Tbill	**0.51**	**0.52**	**0.62**	**0.78**	**0.51****	**0.51***	**0.57**	**0.67***	**0.51*****	**0.51***	**0.57***	**0.61****
1-Year Yield	**0.78**	**0.74**	**0.75**	**0.82**	**0.82****	**0.72***	**0.71**	**0.73***	**0.80****	**0.73***	**0.69***	**0.65*****
5-Year Yield	1.00	0.99	**0.94**	**0.86**	1.01	1.01	1.00	0.98	1.01	1.01	0.98	**0.89*****
10-Year Yield	1.00	1.00	0.98	**0.92**	0.99	1.01	0.99	1.01	1.00	1.00	0.99	0.98**
BAA Yield	1.01	1.02	1.02	0.99	1.01	1.01	1.01	**1.09****	1.01*	1.02	1.01	1.04***

Note: Comparison of "standard linear VAR" (baseline, in denominator) against "hybrid shadow-rate VAR," and assuming an ELB of 12.5 basis points. Values below 1 indicate improvement over baseline. Evaluation window with forecast origins from 2009:01 through 2017:12 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Due to the close behavior of some of the models compared, and rounding of the reported values, one of the comparisons shows a significant ratio of 1.00. This case arises from persistent differences in performance that are, however, too small to be relevant after rounding. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

Table S.35: ELB=12.5bp: Forecast performance of standard vs. simple shadow-rate VAR (incl. COVID-19)

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	1.00	1.00	1.00	**0.92**	1.01*	1.00	1.00	1.00	1.00	1.00	1.01	1.00
Real Consumption	1.00	1.00	1.00	0.99	0.99	1.00	1.01*	1.00	0.99*	1.00	1.00	1.00
IP	1.00*	1.00	1.00	1.00	1.00	1.01*	1.01*	1.00	1.00	1.01*	1.01**	1.00
Capacity Utilization	0.99	0.99	1.00	1.01	1.00	1.00	1.00	1.02	1.00	1.00	1.01	1.01
Unemployment	1.00	0.99	0.98	0.97	0.99	0.96**	**0.95***	0.96**	0.99	0.98**	0.98*	0.98**
Nonfarm Payrolls	1.00	1.00	1.00	1.00	0.99*	0.99	1.00	1.00	0.99*	1.00	1.00	1.00
Hours	1.00	0.99	1.01	1.00	1.00	1.00	1.01	1.01	1.00	1.00	1.01	1.00
Hourly Earnings	1.01**	1.01**	1.01	**0.95**	1.01	1.01**	1.01	1.00	1.01**	1.01**	1.00	1.00
PPI (Fin. Goods)	1.00*	1.00	1.00	0.97	1.01	0.99*	1.00	1.00	1.00	1.00	1.00	1.00
PPI (Metals)	1.00	1.00	1.00	1.02	1.00	1.00	1.00	1.01***	1.00	1.00	1.00	1.00
PCE Prices	1.00	1.00	1.01	**0.95**	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Federal Funds Rate	**0.51***	**0.57**	**0.77**	**1.33**	**0.38*****	**0.45****	**0.55***	**0.65**	**0.42*****	**0.46****	**0.55***	**0.61****
Housing Starts	0.97*	0.96	**0.95**	0.96	0.97**	**0.95****	0.96	0.97	0.98*	0.98	0.97	0.98
S&P 500	1.00	1.01*	1.00	1.00	1.00	1.01***	1.01	1.01**	1.00	1.01**	1.00	1.00
USD / GBP FX Rate	0.99*	1.00	1.00	0.98**	0.99**	1.00	1.00	1.00	0.99**	1.00	1.00	1.00**
6-Month Tbill	**0.79**	**0.81**	**0.93**	**1.32**	**0.60*****	**0.63****	**0.70**	**0.74***	**0.63*****	**0.64****	**0.69***	**0.68*****
1-Year Yield	**0.83**	**0.90**	0.99	**1.28**	**0.77*****	**0.76****	**0.79**	**0.79***	**0.77*****	**0.77*****	**0.77****	**0.71*****
5-Year Yield	0.98	1.00	1.01	**1.17**	0.97	0.99	0.99	0.97**	0.97	0.97	0.97**	**0.90*****
10-Year Yield	1.00	1.01	1.02	**1.19**	1.00	1.00	0.99	0.98	0.99	0.99	0.99	0.96*
BAA Yield	0.99	1.01	1.03	**1.09**	1.00	0.99	1.00	1.04***	1.00	1.01	1.01	1.01

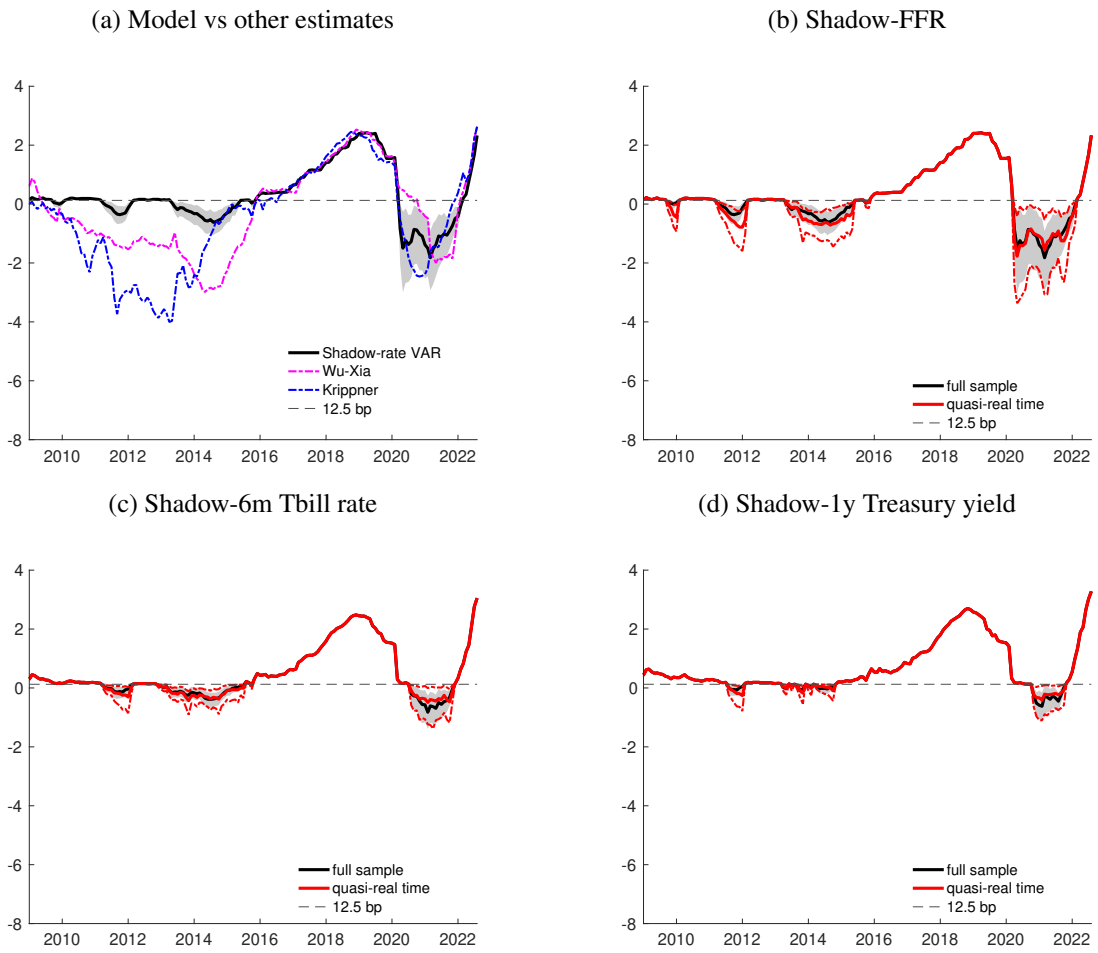
Note: Comparison of "standard linear VAR" (baseline, in denominator) against "simple shadow-rate VAR," and assuming an ELB of 12.5 basis points. Values below 1 indicate improvement over baseline. Evaluation window with forecast origins from 2009:01 through 2022:08 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Due to the close behavior of some of the models compared, and rounding of the reported values, a few comparisons show significant ratios of 1.00. These cases arise from persistent differences in performance that are, however, too small to be relevant after rounding. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

Table S.36: ELB=12.5bp: Forecast performance of standard vs. hybrid shadow-rate VAR (incl. COVID-19)

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	1.00	1.00	1.02	0.96	1.00	1.00	1.01**	0.99	1.00	1.00	1.00	1.00
Real Consumption	1.00	1.00	1.00	1.00	1.00	1.00	1.01	0.97	1.00	1.00	1.00	0.99
IP	1.00	1.00	1.00	1.02	1.01	1.01	1.00	0.96	1.00	1.00	1.00	0.98
Capacity Utilization	0.98	0.96	0.99	0.99	0.98	0.98	1.00	0.99	0.99	0.99	1.00	0.98
Unemployment	1.00	0.99	0.99	1.01	1.00	0.99**	0.99	1.01	1.00*	0.99**	0.99	1.00
Nonfarm Payrolls	1.00	1.00	1.00	1.00	0.99	0.99	1.01	1.01	1.00	1.00	1.01	1.00
Hours	1.00	0.99	1.02	1.02	1.00	1.00	1.03	1.03	1.00	1.00	1.02	1.01
Hourly Earnings	1.00	1.00	0.99	**0.94**	1.00	1.00	1.00	1.00	1.00	1.01*	1.01	1.00
PPI (Fin. Goods)	1.00	1.00	0.99	0.98	1.00	1.00	0.99	1.00	1.00	1.00	0.99	1.00
PPI (Metals)	1.00	1.00*	1.00	1.04	0.99**	1.00	0.99	1.01*	1.00*	1.00	0.99	1.00
PCE Prices	0.98*	1.00	1.00	0.98	0.99**	0.99	0.98	1.02***	0.99**	0.99	0.99	1.01***
Federal Funds Rate	**0.51***	**0.56***	**0.76**	**1.24**	**0.38*****	**0.43****	**0.55***	**0.64***	**0.42*****	**0.45*****	**0.54***	**0.60****
Housing Starts	1.01	1.01	0.99	**1.08**	1.00	0.99	0.98	1.03	1.00	1.00	0.98	1.01
S&P 500	1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.01	1.00	1.00	1.01	1.00
USD / GBP FX Rate	1.01	1.00	1.00	1.00	1.01*	1.01*	1.01	1.02	1.00	1.00	1.00	1.00
6-Month Tbill	**0.79**	**0.80**	**0.92**	**1.20**	**0.61*****	**0.63****	**0.70**	**0.73***	**0.62*****	**0.63****	**0.69****	**0.67*****
1-Year Yield	**0.82**	**0.89**	0.98	**1.18**	**0.76*****	**0.76****	**0.79**	**0.78***	**0.76*****	**0.76*****	**0.77****	**0.70*****
5-Year Yield	0.98	1.01	1.01	**1.11**	0.97	0.98	1.00	0.99	0.97	0.98	0.97	**0.90*****
10-Year Yield	1.00	1.01	1.02	**1.17**	0.99	1.00	1.00	1.00	0.99	0.99	0.99	0.97
BAA Yield	0.99	1.01	1.02	**1.11**	0.99	0.98	1.00	**1.08*****	1.00	1.01	1.01	1.02

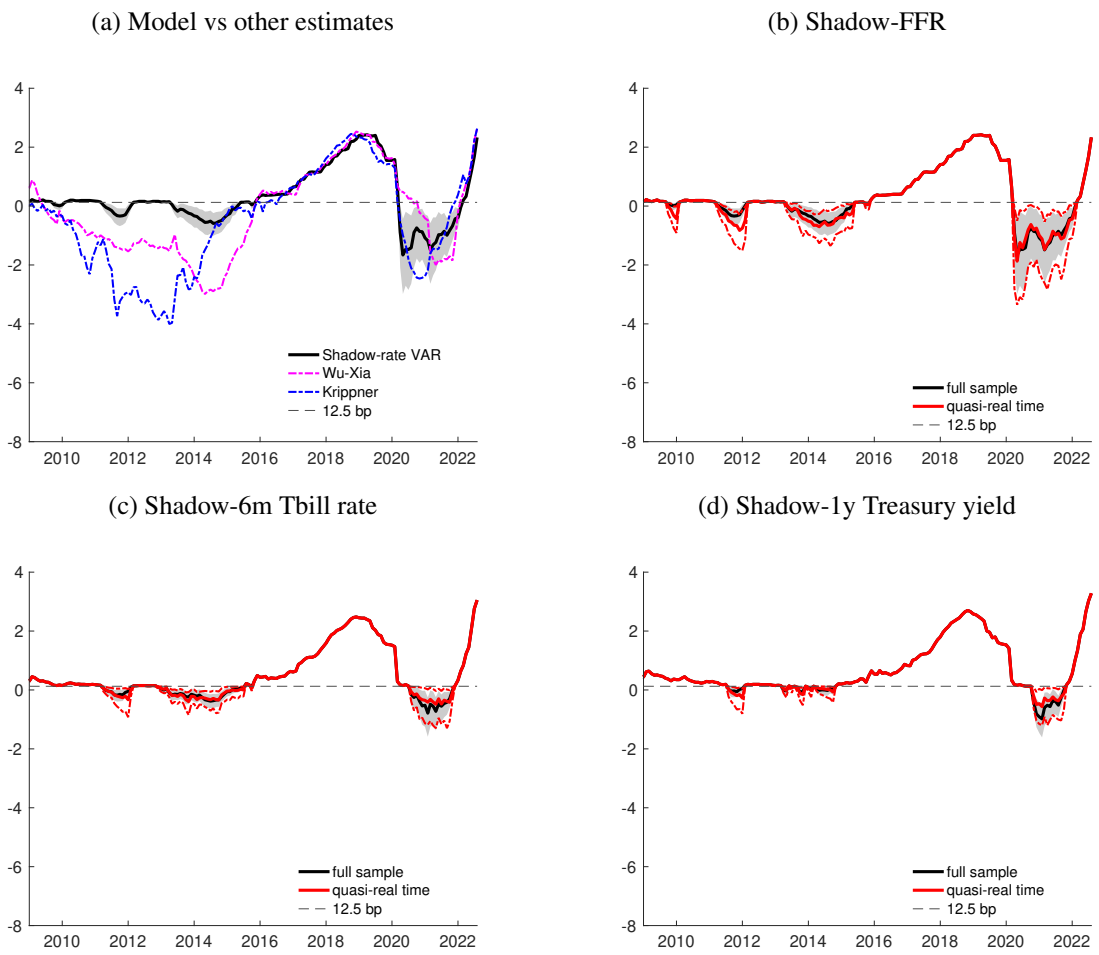
Note: Comparison of "standard linear VAR" (baseline, in denominator) against "hybrid shadow-rate VAR," and assuming an ELB of 12.5 basis points. Values below 1 indicate improvement over baseline. Evaluation window with forecast origins from 2009:01 through 2022:08 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Due to the close behavior of some of the models compared, and rounding of the reported values, a few comparisons show significant ratios of 1.00. These cases arise from persistent differences in performance that are, however, too small to be relevant after rounding. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

Figure S.32: ELB=12.5bp: Shadow-rate estimates from simple shadow-rate VAR



Note: Shadow-rate estimates from the simple version of our shadow-rate VAR, estimated using the full set of our 20 variables as listed in Table 1 of the paper, and when the value of ELB is set to 12.5 basis points (bp). Panel (a) compares the smoothed shadow-rate estimates against updated estimates obtained from Krippner (2013, 2015) and Wu and Xia (2016). Panel (b) compares smoothed and quasi-real time shadow-rate estimates of the federal funds rate. The quasi-real-time estimates are the end-of-sample estimates produced by recursive estimation of the model starting in January 2009. Each estimation conditions on available data since 1959:03, but the figure omits the period prior to 2008 during which the ELB did not bind. Posterior medians are shown as thick lines; grey shaded areas and thin lines depict 90 percent uncertainty bands. Panels (c) and (d) show full-sample and quasi-real-time estimates of shadow rates for the 6-month and 1-year Treasury maturities.

Figure S.33: ELB=12.5bp: Shadow-rate estimates from hybrid shadow-rate VAR

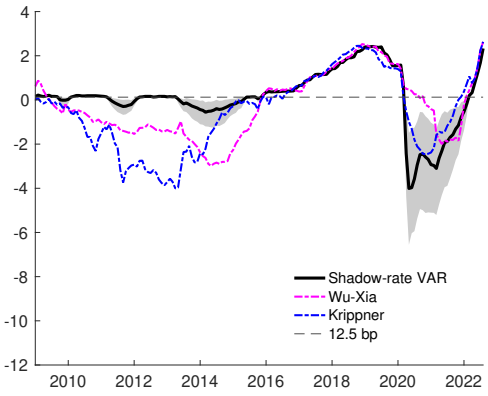


Note: Shadow-rate estimates from the hybrid version of our shadow-rate VAR, estimated using the full set of our 20 variables as listed in Table 1 of the paper, and when the value of ELB is set to 12.5 basis points (bp). Panel (a) compares the smoothed shadow-rate estimates against updated estimates obtained from Krippner (2013, 2015) and Wu and Xia (2016). Panel (b) compares smoothed and quasi-real time shadow-rate estimates of the federal funds rate. The quasi-real-time estimates are the end-of-sample estimates produced by recursive estimation of the model starting in January 2009. Each estimation conditions on available data since 1959:03, but the figure omits the period prior to 2008 during which the ELB did not bind. Posterior medians are shown as thick lines; grey shaded areas and thin lines depict 90 percent uncertainty bands. Panels (c) and (d) show full-sample and quasi-real-time estimates of shadow rates for the 6-month and 1-year Treasury maturities.

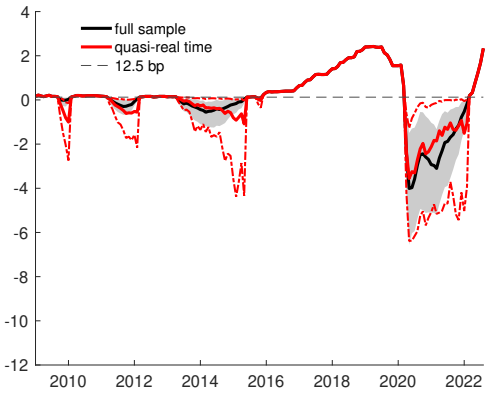
Figure S.34: ELB=12.5bp: Shadow-rate estimates generated from ex-yields data

Simple shadow-rate VAR

(a) Model vs other estimates

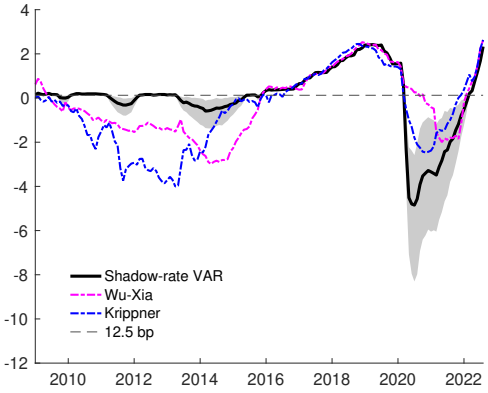


(b) Shadow-FFR

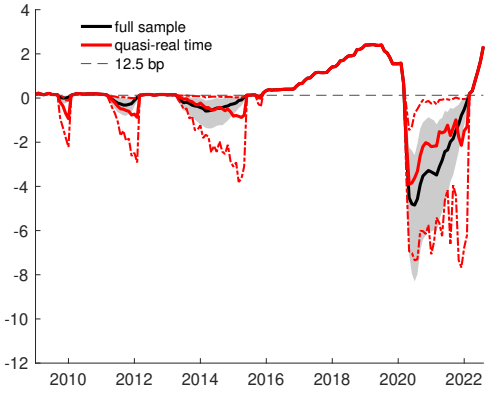


Hybrid shadow-rate VAR

(c) Model vs other estimates



(d) Shadow-FFR



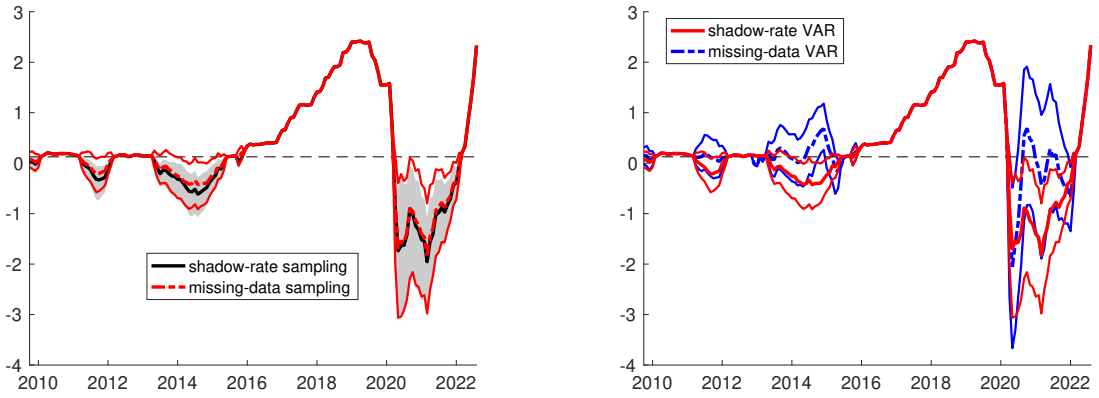
Note: Shadow-rate estimates from simple and hybrid shadow-rate VARs, estimated when excluding yields data (other than the federal funds rate) from our data set as listed in Table 1 of the paper, and when the value of ELB is set to 12.5 basis points (bp). Panel (c) compares the smoothed shadow-rate estimates against updated estimates obtained from Krippner (2013, 2015) and Wu and Xia (2016). Panel (d) compares smoothed and quasi-real time shadow-rate estimates of the federal funds rate. The quasi-real-time estimates are the end-of-sample estimates produced by recursive estimation of the model starting in January 2009. Each estimation conditions on available data since 1959:03, but the figure omits the period prior to 2008 during which the ELB did not bind. Posterior medians are shown as thick lines; grey shaded areas and thin lines depict 90 percent uncertainty bands.

Figure S.35: Effect of imposing ELB=12.5bp on shadow-rate estimates

Simple shadow-rate VAR estimates from data including yields

(a) Missing-data and shadow-rate draws

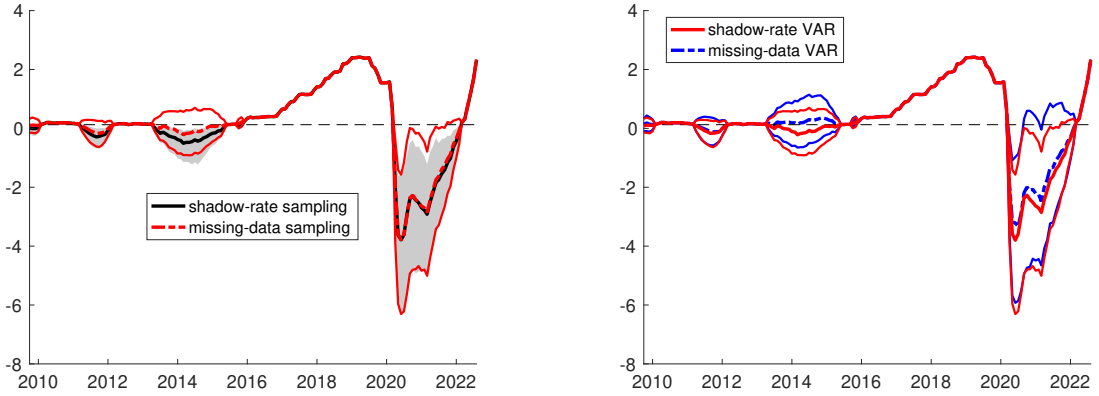
(b) Missing-data draws w/and w/o shadow-rate sampling



Estimates from data excluding yields

(c) Missing-data and shadow-rate draws

(d) Missing-data draws w/and w/o shadow-rate sampling



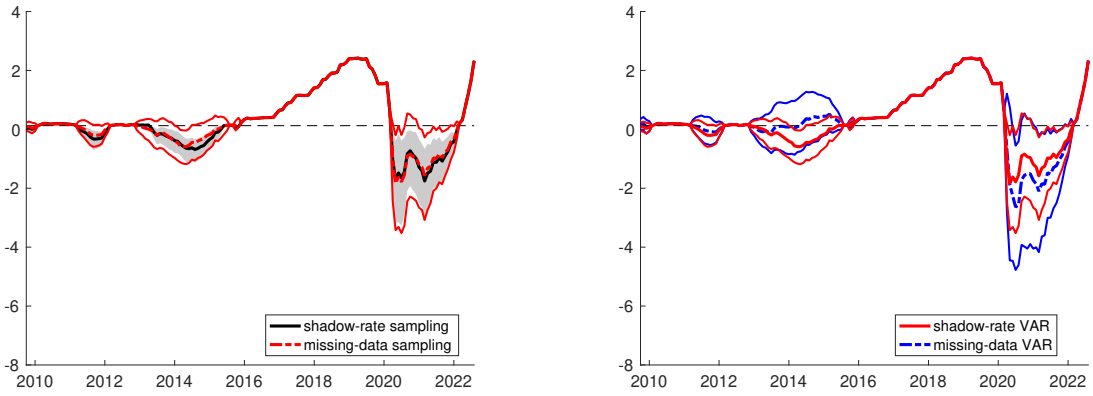
Note: Panel ((a)) compares shadow-rate (black) and missing-data (red) draws for s_t obtained from the posterior of our simple shadow-rate VAR. Shadow-rate draws are obtained from the truncated posterior for s_t that satisfies the ELB. Missing-data draws are obtained from the underlying (and untruncated) posterior of the missing data problem that ignores the ELB. Panel ((b)) displays missing-data posteriors obtained from two sets of simple shadow-rate VAR estimates: In the baseline (red), parameter and SV draws reflect shadow-rate sampling. In the alternative version (blue), parameters and SV are drawn while treating the policy rate at the ELB as missing data and without requiring that missing data draws lie below the ELB. Panels ((c)) and ((b)) provide corresponding estimates from a VAR that omits all interest rate data except for the federal funds rate. Each panel shows reports medians (thick lines) and 90 percent uncertainty bands (grey shaded areas or thin lines).

Figure S.36: Effect of imposing ELB=12.5bp on shadow-rate estimates in hybrid model

Hybrid shadow-rate VAR estimates from data including yields

(a) Missing-data and shadow-rate draws

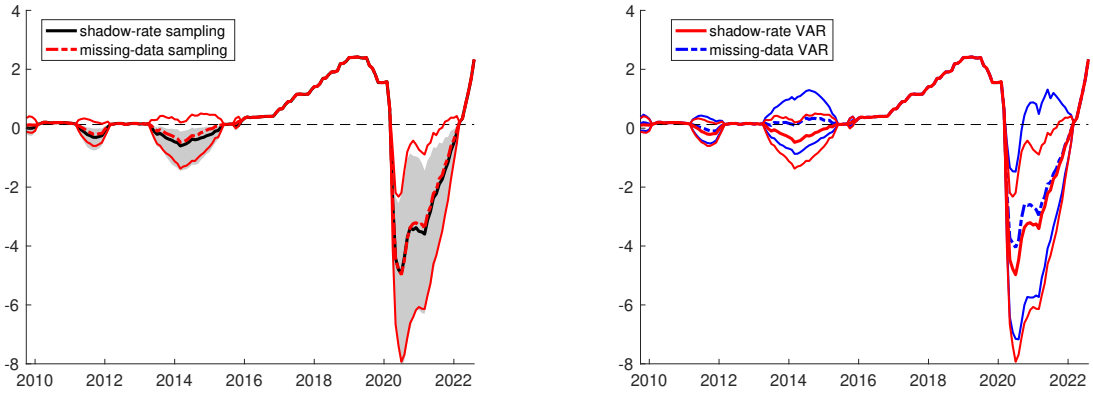
(b) Missing-data draws w/and w/o shadow-rate sampling



Estimates from data excluding yields

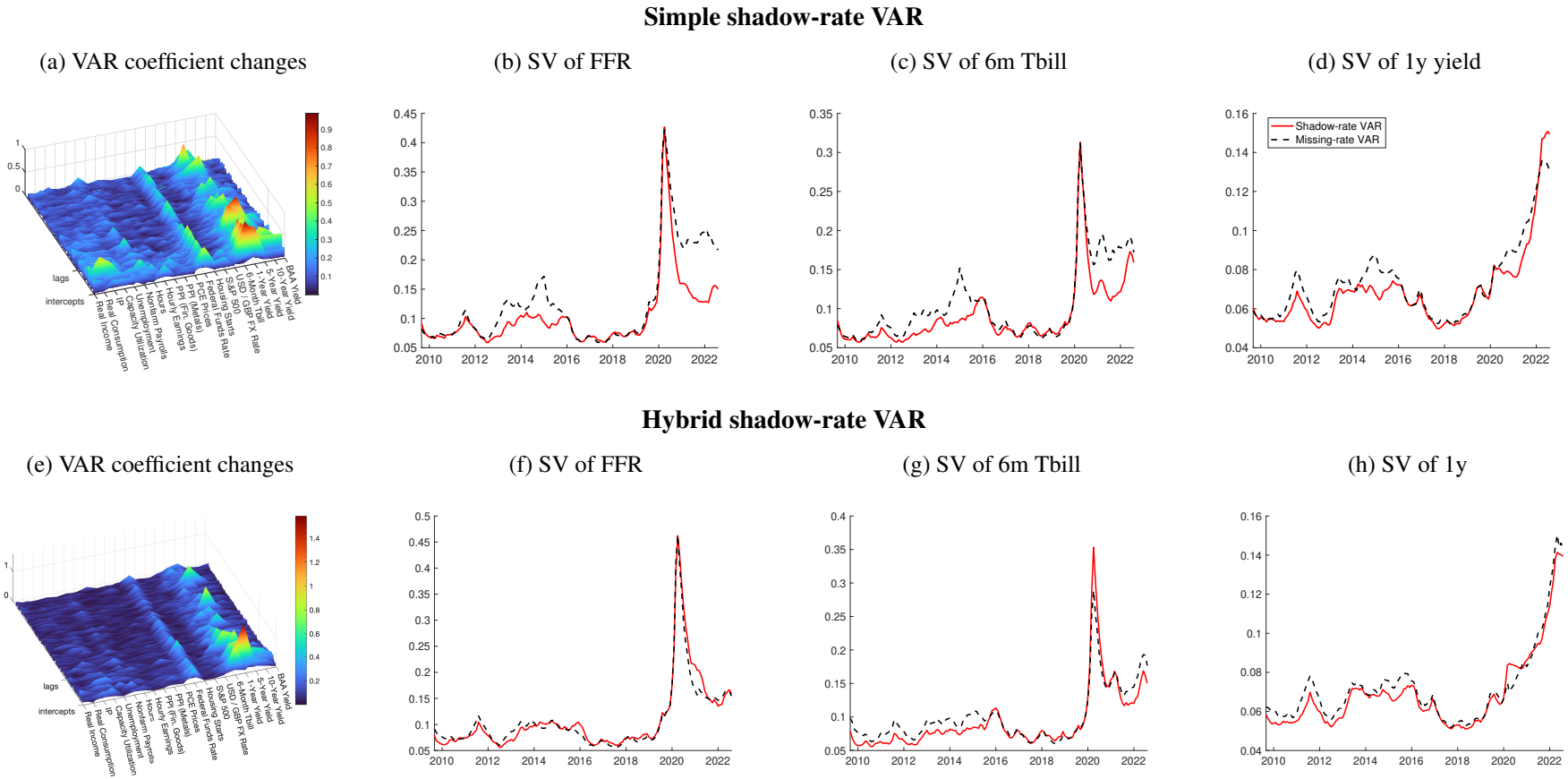
(c) Missing-data and shadow-rate draws

(d) Missing-data draws w/and w/o shadow-rate sampling



Note: Panel (a) compares shadow-rate (black) and missing-data (red) draws for s_t obtained from the posterior of our hybrid shadow-rate VAR. Shadow-rate draws are obtained from the truncated posterior for s_t that satisfies the ELB. Missing-data draws are obtained from the underlying (and untruncated) posterior of the missing data problem that ignores the ELB. Panel (b) displays missing-data posteriors obtained from two sets of hybrid shadow-rate VAR estimates: In the baseline (red), parameter and SV draws reflect shadow-rate sampling. In the alternative version (blue), parameters and SV are drawn while treating the policy rate at the ELB as missing data and without requiring that missing data draws lie below the ELB. In this panel, medians are reported as thick lines and 90 percent uncertainty bands are reported with the grey shaded area or thin lines. Panels (c) and (b) provide corresponding estimates from a VAR that omits all interest rate data except for the federal funds rate.

Figure S.37: ELB=12.5bp: Effects of shadow-rate sampling on parameters and SV

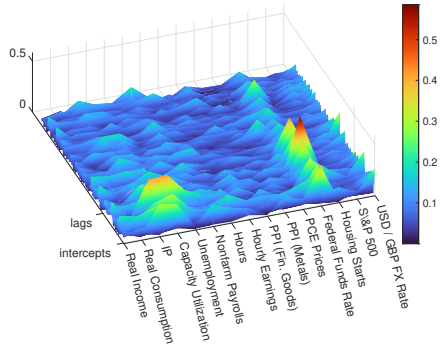


Note: Differences in estimates for parameters and selected SV paths when comparing the shadow-rate VAR against a missing-data approach that treats observations of nominal interest rates at the ELB as missing. Panels (a)–(d) report results based on the simple shadow-rate VAR, and Panels (e)–(h) display corresponding results from the hybrid shadow-rate VAR. Panels (a) and (e) show absolute values of standardized differences in mean estimates of VAR coefficients; the estimates are standardized based on the posterior standard deviation of each coefficient generated by the missing-data approach. The SV paths shown in the other panels reflect posterior medians. All estimates are estimated from the full data sample, from 1959:03 through 2022:08.

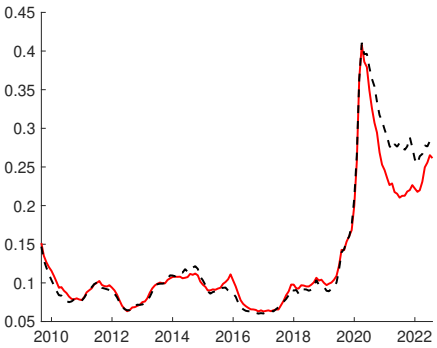
Figure S.38: ELB=12.5bp: Effects of shadow-rate sampling on parameters and SV (ex yields)

Simple shadow-rate VAR

(a) VAR coefficient changes

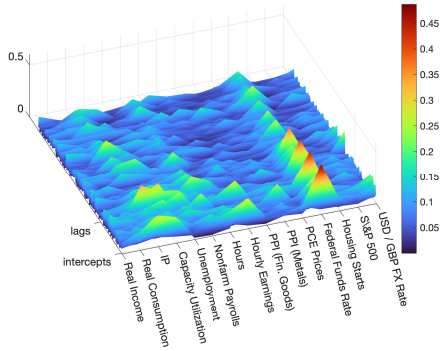


(c) SV of FFR

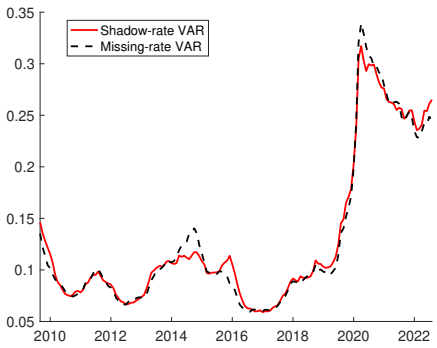


Hybrid shadow-rate VAR

(b) VAR coefficient changes



(d) SV of FFR



Note: Differences in estimates for parameters and selected SV paths when comparing the shadow-rate VAR against a missing-data approach that treats observations of nominal interest rates at the ELB as missing. Panels (a) and (c) report results based on the simple shadow-rate VAR, and Panels (b) and (d) display corresponding results from the hybrid shadow-rate VAR. Panels (a) and (b) show absolute values of standardized differences in mean estimates of VAR coefficients; the estimates are standardized based on the posterior standard deviation of each coefficient generated by the missing-data approach. The SV paths shown in the other panels reflect posterior medians. All estimates are estimated from the full data sample, but excluding yield measures other than the federal funds rate, from 1959:03 through 2022:08.

VIII (b) ELB=50 basis points

Tables S.37 through S.40 provide forecast accuracy comparisons of the simple shadow-rate and hybrid shadow-rate VARs to standard VARs that assume a value of 50 basis points for the ELB. Specifically, Tables S.37 and Tables S.38 are counterparts to Tables 2 and 3 in the paper, and Tables S.39 and S.40 mirror Tables S.7 and S.8 in this appendix (except for the choice of ELB value).

Overall, contours of the estimated shadow rates, and relative forecast performance of the simple shadow-rate VAR are similar to our baseline results, indicating robustness to the alternative choice in ELB value. More specifically, with the higher ELB value, the simple shadow-rate VAR continues to beat the forecast accuracy of a standard VAR for interest rates — by a modestly larger margin with the 50 basis points ELB setting than the 25 basis points ELB setting — although, at longer forecast horizons, it falls short of the standard VAR's accuracy for several macroeconomic variables.

Figures S.39 and S.40 report shadow-rate estimates for our baseline data set, and generated with the assumed value of ELB = 50 basis points from simple and hybrid shadow-rate VARs, respectively. Figure S.41 provides corresponding estimates when yields data (other than the federal funds rate) are excluded. The shadow rate estimates based on the higher ELB value are qualitatively similar to the baseline estimates with the 25 basis points ELB, although following the Great Recession, the alternative estimates turn more negative than in the baseline case. With the higher ELB setting, the shadow rate estimates come to even more strongly resemble Krippner's estimates. Mechanically, with the higher ELB value, more observations of the federal funds rate in the historical sample are censored, and the model infers shadow rate values even more negative than in the paper's baseline.

Not surprisingly, however, the choice of a higher value for the censoring constraint on actual rates affects forecast simulations from the hybrid VAR more noticeably, since forecasts of macroeconomic variables are now conditioned on actual rates that are assumed to not fall below 50 basis points (instead of 25 basis points). For good parts of our evaluation window, this choice conditions macroeconomic forecasts on policy rate projections that are about 25 basis points higher than they otherwise would have been, with negative impacts for longer-horizon forecasts.

Table S.37: ELB=50bp: Forecast performance of standard vs. simple shadow-rate VAR

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	1.00	1.00	0.99	**0.88**	0.98*	0.98	0.99	0.98	0.99	1.00	1.02	1.01
Real Consumption	1.00	1.01	**1.05**	1.01	1.00	1.02	**1.09***	1.01	1.00	1.02*	1.04**	1.02
IP	1.03*	1.01	**1.06***	1.02	1.04**	1.02	**1.07****	**1.05**	1.02*	1.02	**1.05***	1.04**
Capacity Utilization	**1.06*****	**1.07****	**1.15*****	**1.23****	**1.08*****	**1.09****	**1.22*****	**1.27*****	**1.05*****	**1.07*****	**1.15*****	**1.19*****
Unemployment	1.00	1.00	1.04	**1.10**	1.00	0.98	1.02	**1.09**	1.00	1.00	1.04	**1.08**
Nonfarm Payrolls	0.97	0.97	**1.07**	**1.19***	0.99	1.02	**1.14****	**1.18**	1.00	1.01	**1.09****	**1.10*****
Hours	1.01	1.01	1.01	**1.05***	1.01	1.00	1.02	**1.08***	1.01	1.00	1.02	1.04**
Hourly Earnings	1.00	1.01	0.99	0.97*	1.01	1.02	1.01	0.98	1.00	1.01	1.00	0.99
PPI (Fin. Goods)	1.03*	1.01	1.01	0.97**	1.02	1.00	1.00	0.98	1.02	1.01	1.01	1.00
PPI (Metals)	1.00	1.00	0.99	1.01	1.00	1.01	1.00	1.01	1.00	1.01	1.00	1.01***
PCE Prices	1.03	1.03	1.02	0.99	1.03	1.02	1.01	1.01	1.02	1.01	1.01	1.01
Federal Funds Rate	**0.13*****	**0.17***	**0.25**	**0.46**	**0.06*****	**0.12*****	**0.22****	**0.32****	**0.06*****	**0.12*****	**0.21****	**0.35*****
Housing Starts	**1.05**	**1.08**	**1.06**	**0.92**	1.04	**1.07**	**1.07**	0.96	1.04	**1.06**	1.04	**0.94**
S&P 500	1.00	1.01	1.01	**1.07**	0.98	0.99	1.00	1.01	1.00	1.01	1.01	1.01**
USD / GBP FX Rate	0.98	0.99	0.99**	**0.95*****	0.97*	0.98	0.97**	0.96**	0.98*	0.99	0.98***	0.99*
6-Month Tbill	**0.19*****	**0.29***	**0.41**	**0.62**	**0.09*****	**0.19*****	**0.30****	**0.37*****	**0.09*****	**0.18*****	**0.31****	**0.44*****
1-Year Yield	**0.42*****	**0.51***	**0.58**	**0.74**	**0.18*****	**0.29*****	**0.36****	**0.44*****	**0.22*****	**0.32*****	**0.42*****	**0.50*****
5-Year Yield	**0.93**	**0.87****	**0.75*****	**0.74***	**0.93**	**0.91**	**0.83***	**0.77****	0.96	**0.91**	**0.82*****	**0.75*****
10-Year Yield	**0.95**	**0.93**	**0.81****	**0.67*****	**0.95**	**0.95**	**0.86**	**0.79**	0.97	**0.95**	**0.89**	**0.88***
BAA Yield	1.00	1.04	**1.13**	**1.06**	**1.05**	**1.05**	**1.09**	**1.11**	1.02	**1.05**	**1.08**	**1.11**

Note: Comparison of "standard linear VAR" (baseline, in denominator) against "simple shadow-rate VAR," and assuming an ELB of 50 basis points. Values below 1 indicate improvement over baseline. Evaluation window with forecast origins from 2009:01 through 2017:12 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h+1$ lags. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

Table S.38: ELB=50bp: Forecast performance of standard vs. hybrid shadow-rate VAR

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	1.00	1.00	1.04***	**5.42**	0.99	0.99	**1.13*****	**1.20**	0.99	0.99	**1.11*****	**2.07***
Real Consumption	1.01	1.02*	**1.11***	**23.29**	1.01	**1.05*****	**1.09****	**1.11**	1.01	1.04***	**1.12*****	**2.37***
IP	1.03***	1.04	**1.25*****	**22.57**	1.04***	1.04	**1.24*****	**1.41*****	1.02***	1.03	**1.24*****	**2.61****
Capacity Utilization	0.99	0.96	1.00	**2.81****	0.98	**0.94**	**0.94**	**1.06**	0.99	0.96	1.02	**1.68***
Unemployment	1.01	0.97*	**0.87****	**2.74***	1.00	0.97	**0.84****	**0.67*****	1.00	0.99	0.96	**1.44**
Nonfarm Payrolls	0.97**	0.99	**1.09**	**7.46**	0.97***	0.96	0.99	**0.84**	0.98**	0.99	**1.10****	**2.13****
Hours	1.00	1.01	**1.09**	**12.37**	1.00	1.01	**1.05**	**1.35****	1.00	1.01	**1.11****	**2.26****
Hourly Earnings	1.00	1.03**	**1.14****	**10.47**	0.99	1.02*	**1.07*****	**1.26****	1.00	1.02***	**1.13*****	**2.23****
PPI (Fin. Goods)	0.99	0.97	0.99	**5.42**	0.99	0.98	1.01	**1.15**	0.99	0.98	1.03	**1.99***
PPI (Metals)	0.99***	0.99	1.04	**14.24**	0.99***	0.99	1.00	1.04	0.99***	1.00	1.03**	**2.00***
PCE Prices	0.99	0.98	**1.17****	**5.32**	0.99	0.97	**1.20****	**1.71*****	0.99	0.99	**1.19*****	**2.37****
Federal Funds Rate	**0.13*****	**0.17***	**0.24**	**0.37***	**0.07*****	**0.13*****	**0.22****	**0.32****	**0.06*****	**0.12*****	**0.21****	**0.34****
Housing Starts	1.01	1.00	**0.95**	**6.79**	1.02**	0.99	**0.94**	**0.95**	1.01*	1.02	1.01	**1.68**
S&P 500	1.02	1.03	**1.28***	**44.16**	1.01	1.01	**1.07**	**1.27****	1.01	1.04**	**1.14*****	**3.13***
USD / GBP FX Rate	1.01	1.01	**1.05****	**4.65**	1.01	1.02	1.04	**1.10****	1.00	1.01	**1.05*****	**1.72****
6-Month Tbill	**0.27*****	**0.47**	**1.11**	**16.16**	**0.08*****	**0.17*****	**0.29****	**0.39*****	**0.11*****	**0.23*****	**0.42****	**1.21**
1-Year Yield	**0.90**	**1.51**	**4.73**	**165.08**	**0.23*****	**0.40****	**0.68**	**3.09**	**0.40*****	**0.66**	**1.21**	**9.94**
5-Year Yield	1.01	**1.06**	**2.73****	**76.20**	**1.08**	**1.07**	**1.26**	**2.74**	**1.11**	**1.13**	**1.38**	**5.98***
10-Year Yield	**1.14**	**1.23**	**2.85****	**82.50**	**1.18**	**1.32***	**1.70***	**4.15**	**1.19****	**1.32****	**1.82****	**7.10****
BAA Yield	**1.29****	**1.75*****	**3.48****	**99.17**	**1.31*****	**1.64*****	**2.61*****	**5.29*****	**1.29*****	**1.64*****	**2.54*****	**7.48****

Note: Comparison of "standard linear VAR" (baseline, in denominator) against "hybrid shadow-rate VAR," and assuming an ELB of 50 basis points. Values below 1 indicate improvement over baseline. Evaluation window with forecast origins from 2009:01 through 2017:12 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

Table S.39: ELB=50bp: Forecast performance of standard vs. simple shadow-rate VAR (incl. COVID-19)

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	1.00	1.00	1.01	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Real Consumption	1.00	1.00	1.00**	1.00	0.99	1.00	1.03*	1.01	0.99	1.00	1.01	1.00
IP	1.01**	1.00	1.00	1.00	1.02**	1.02	1.04**	1.03**	1.01**	1.01	1.02*	1.02**
Capacity Utilization	1.02**	1.03**	**1.09*****	**1.19****	**1.06*****	**1.09*****	**1.21*****	**1.25*****	1.04***	**1.06*****	**1.11*****	**1.14*****
Unemployment	0.99	0.99	0.97	**0.95**	0.99*	0.97*	0.97	1.00	0.99**	0.99	0.99	1.01
Nonfarm Payrolls	1.00	1.00	1.00	1.00*	0.99	1.00	1.03	**1.07****	1.00	1.00	1.02***	1.03***
Hours	1.01	0.98	1.02	1.04*	1.00	1.00	1.03	**1.08****	1.01	1.00	1.02	1.03*
Hourly Earnings	1.00	1.01	1.00	0.97	1.00	1.01	1.01	0.98	1.00	1.01	1.00	0.99*
PPI (Fin. Goods)	1.01	1.00	1.00	0.98**	1.01	0.99	1.00	0.99	1.01	1.00	1.00	1.00
PPI (Metals)	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.01**	1.00	1.00	1.00	1.01***
PCE Prices	1.01	1.00	1.01	0.98	1.01	1.01	1.01	1.01	1.00	1.00	1.01	1.01
Federal Funds Rate	**0.31*****	**0.36***	**0.54**	**1.11**	**0.15*****	**0.25*****	**0.33****	**0.38*****	**0.16*****	**0.25*****	**0.34****	**0.42*****
Housing Starts	1.00	1.00	1.00	**0.91**	1.00	1.01	1.02	0.96	1.01	1.03	1.02	0.96
S&P 500	1.00	1.01	1.00	1.02	1.00	1.01	1.00	1.01	1.00	1.01	1.01	1.01
USD / GBP FX Rate	0.99*	0.99*	0.99	0.96***	0.98**	0.98*	0.98	0.97*	0.99*	0.99*	0.99*	0.99
6-Month Tbill	**0.47*****	**0.55***	**0.70**	**1.15**	**0.24*****	**0.36*****	**0.41*****	**0.47*****	**0.25*****	**0.35*****	**0.44*****	**0.51*****
1-Year Yield	**0.63****	**0.73***	**0.82**	**1.21**	**0.38*****	**0.47*****	**0.47*****	**0.53*****	**0.40*****	**0.48*****	**0.52*****	**0.56*****
5-Year Yield	**0.95**	**0.94**	**0.92**	**1.16**	**0.90**	**0.90**	**0.85****	**0.83***	**0.91**	**0.90****	**0.85*****	**0.79*****
10-Year Yield	1.00	1.00	**0.95**	**1.16**	0.96	0.96	**0.90**	**0.84**	0.97	0.96	**0.93**	**0.90****
BAA Yield	0.98	1.01	**1.07**	**1.20**	1.01	0.99	1.03	**1.13**	1.00	1.02	**1.05**	**1.09**

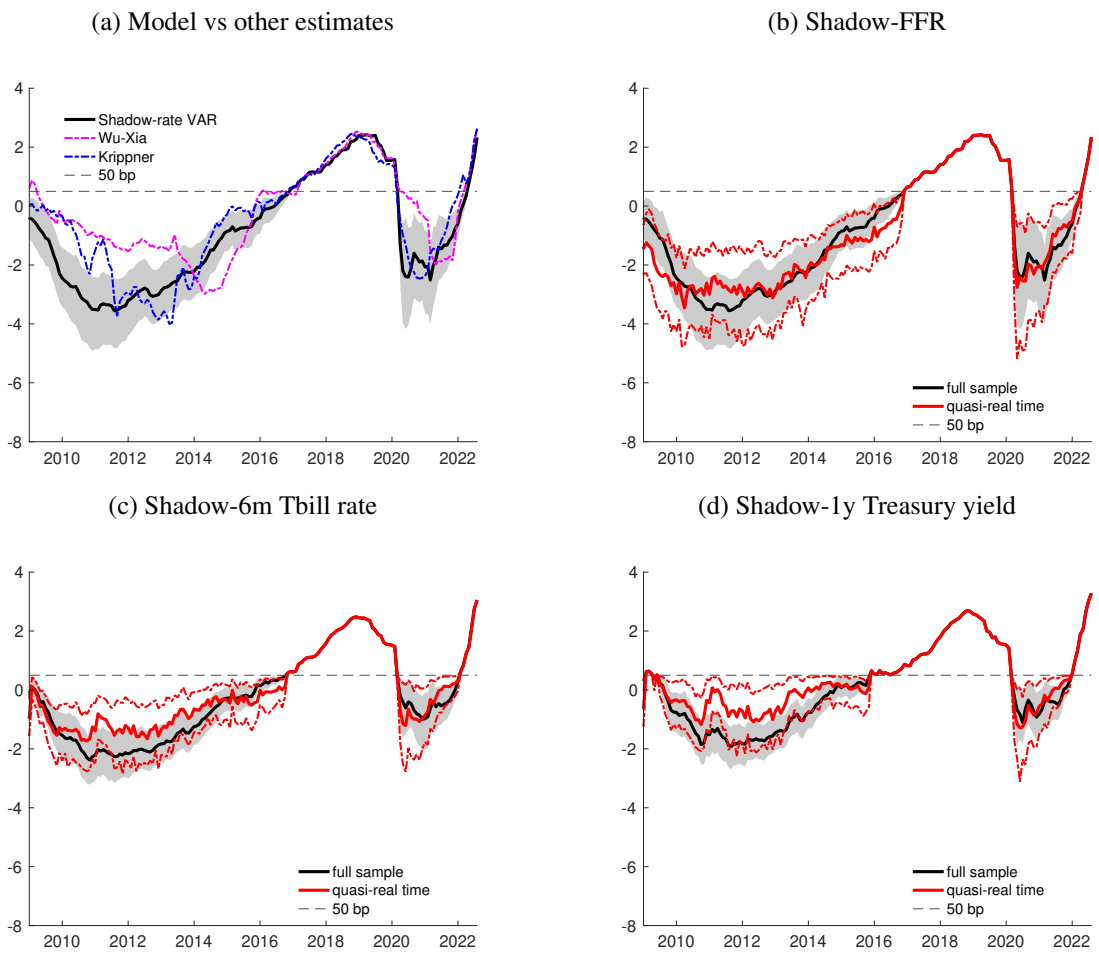
Note: Comparison of "standard linear VAR" (baseline, in denominator) against "simple shadow-rate VAR," and assuming an ELB of 50 basis points. Values below 1 indicate improvement over baseline. Evaluation window with forecast origins from 2009:01 through 2022:08 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Due to the close behavior of some of the models compared, and rounding of the reported values, a few comparisons show significant ratios of 1.00. These cases arise from persistent differences in performance that are, however, too small to be relevant after rounding. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

Table S.40: ELB=50bp: Forecast performance of standard vs. hybrid shadow-rate VAR (incl. COVID-19)

	RMSE				MAE				CRPS			
Variable / Horizon	3	6	12	24	3	6	12	24	3	6	12	24
Real Income	1.00	1.00	1.01	**1.67**	1.00	1.00	1.04**	**1.05**	1.00	0.99	1.02*	**1.29***
Real Consumption	1.00	1.00	1.01**	**3.59**	1.00	1.01	1.03**	1.04	1.00	1.00	1.03**	**1.36**
IP	0.99	1.01*	1.02**	**7.08**	1.01	1.02*	**1.10****	**1.21****	1.00	1.01	**1.09****	**1.69***
Capacity Utilization	1.00	1.00	1.01	**2.47****	0.99	0.97	0.97	**1.06**	0.99*	0.98	1.01	**1.47**
Unemployment	1.00	1.00	1.00	**1.43**	1.00	0.99	**0.95***	**0.84****	1.00	1.00	0.99	**1.21**
Nonfarm Payrolls	1.00	1.00	1.00	**1.17**	0.99***	0.98	1.00	0.96	0.99***	1.00	1.02**	**1.26***
Hours	1.00	1.00	**1.06***	**8.89**	1.00	1.01	**1.05**	**1.25***	1.01	1.01	**1.07****	**1.81***
Hourly Earnings	1.00	1.02**	**1.08****	**8.25**	0.99	1.01	**1.05****	**1.18****	0.99	1.01	**1.07*****	**1.81***
PPI (Fin. Goods)	1.00	0.99*	1.00	**3.58**	0.99	0.99	1.01	**1.09**	1.00	0.99	1.02	**1.60***
PPI (Metals)	0.99***	0.99	1.03	**10.49**	0.99***	0.99	1.00	1.03*	0.99**	1.00	1.02*	**1.71***
PCE Prices	0.98**	0.99	**1.11****	**3.91**	0.98*	0.98	**1.13****	**1.47*****	0.99**	0.99	**1.12****	**1.87****
Federal Funds Rate	**0.34*****	**0.36****	**0.49**	0.97	**0.19*****	**0.27*****	**0.32****	**0.39****	**0.19*****	**0.26*****	**0.33****	**0.40*****
Housing Starts	1.02*	1.03	0.99	**6.13**	1.02***	1.01	0.97	0.98	1.01**	1.01	1.01	**1.52**
S&P 500	1.00	1.01	**1.15***	**30.11**	1.00	1.01	**1.05**	**1.19****	1.01	1.02***	**1.09*****	**2.39***
USD / GBP FX Rate	1.00	1.01	1.03	**4.16**	1.01	1.01	1.03	**1.06**	1.00	1.00	1.03**	**1.54***
6-Month Tbill	**0.55*****	**0.68**	**1.15**	**15.05**	**0.26*****	**0.36*****	**0.41*****	**0.50*****	**0.29*****	**0.40*****	**0.52****	**1.09**
1-Year Yield	**0.83**	**1.28**	**3.97**	**151.39**	**0.41*****	**0.53*****	**0.68**	**2.71**	**0.51*****	**0.67****	1.04	**7.70**
5-Year Yield	0.99	**1.05**	**2.29***	**65.88**	1.00	1.01	**1.14**	**2.36**	1.00	1.03	**1.22**	**4.57**
10-Year Yield	**1.11***	**1.18***	**2.36****	**70.02**	**1.11**	**1.21***	**1.45**	**3.31**	**1.12***	**1.21****	**1.53***	**5.33***
BAA Yield	**1.19****	**1.49*****	**2.73****	**78.24**	**1.19****	**1.39*****	**2.01****	**4.05****	**1.18*****	**1.40*****	**1.99*****	**5.47****

Note: Comparison of "standard linear VAR" (baseline, in denominator) against "hybrid shadow-rate VAR," and assuming an ELB of 50 basis points. Values below 1 indicate improvement over baseline. Evaluation window with forecast origins from 2009:01 through 2022:08 (and outcome data as far as available). Significance assessed by Diebold-Mariano-West test using Newey-West standard errors with $h + 1$ lags. Performance differences of 5 percent and more (relative to baseline) are indicated by bold face numbers.

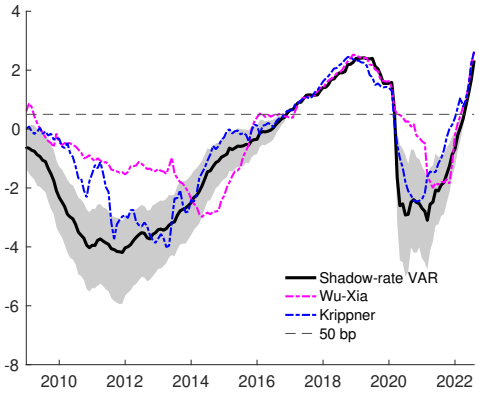
Figure S.39: ELB=50bp: Shadow-rate estimates from simple shadow-rate VAR



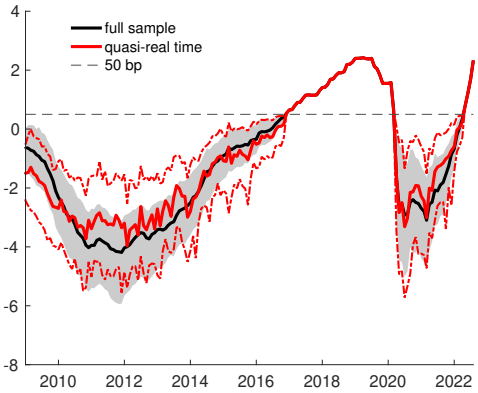
Note: Shadow-rate estimates from the simple version of our shadow-rate VAR, estimated using the full set of our 20 variables as listed in Table 1 of the paper, and when the value of ELB is set to 12.5 basis points (bp). Panel (a) compares the smoothed shadow-rate estimates against updated estimates obtained from Krippner (2013, 2015) and Wu and Xia (2016). Panel (b) compares smoothed and quasi-real time shadow-rate estimates of the federal funds rate. The quasi-real-time estimates are the end-of-sample estimates produced by recursive estimation of the model starting in January 2009. Each estimation conditions on available data since 1959:03, but the figure omits the period prior to 2008 during which the ELB did not bind. Posterior medians are shown as thick lines; grey shaded areas and thin lines depict 90 percent uncertainty bands. Panels (c) and (d) show full-sample and quasi-real-time estimates of shadow rates for the 6-month and 1-year Treasury maturities.

Figure S.40: ELB=50bp: Shadow-rate estimates from hybrid shadow-rate VAR

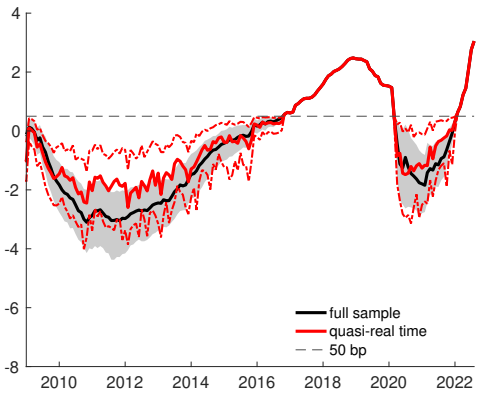
(a) Model vs other estimates



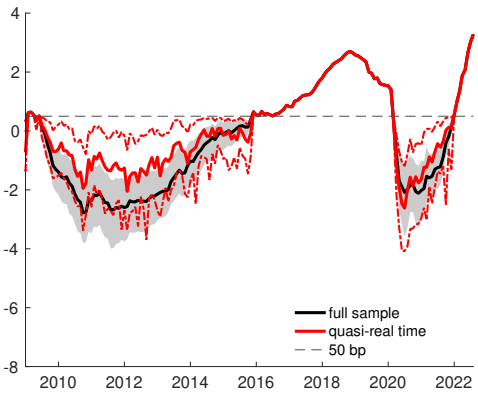
(b) Shadow-FFR



(c) Shadow-6m Tbill rate



(d) Shadow-1y Treasury yield

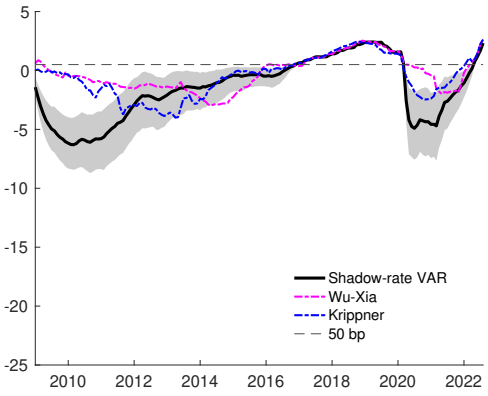


Note: Shadow-rate estimates from the hybrid version of our shadow-rate VAR, estimated using the full set of our 20 variables as listed in Table 1 of the paper, and when the value of ELB is set to 50 basis points (bp). Panel (a) compares the smoothed shadow-rate estimates against updated estimates obtained from Krippner (2013, 2015) and Wu and Xia (2016). Panel (b) compares smoothed and quasi-real time shadow-rate estimates of the federal funds rate. The quasi-real-time estimates are the end-of-sample estimates produced by recursive estimation of the model starting in January 2009. Each estimation conditions on available data since 1959:03, but the figure omits the period prior to 2008 during which the ELB did not bind. Posterior medians are shown as thick lines; grey shaded areas and thin lines depict 90 percent uncertainty bands. Panels (c) and (d) show full-sample and quasi-real-time estimates of shadow rates for the 6-month and 1-year Treasury maturities.

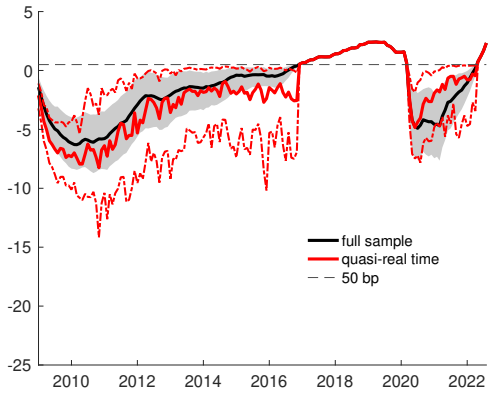
Figure S.41: ELB=50bp: Shadow-rate estimates generated from ex-yields data

Simple shadow-rate VAR

(a) Model vs other estimates

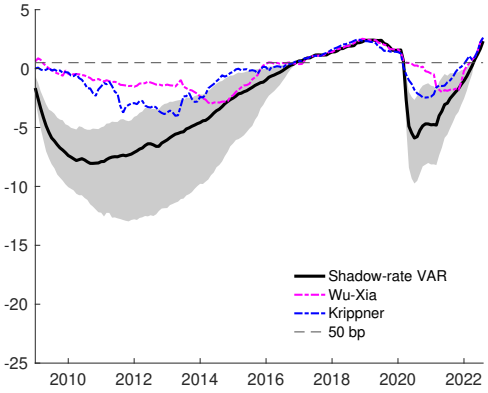


(b) Shadow-FFR

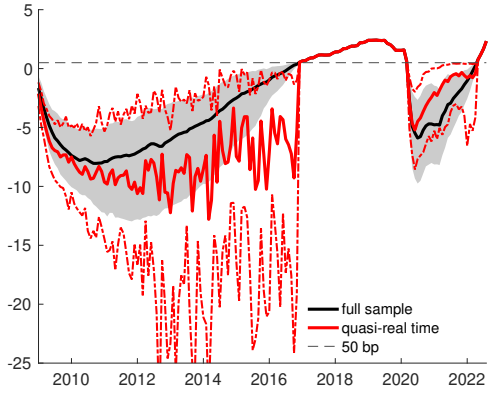


Hybrid shadow-rate VAR

(c) Model vs other estimates



(d) Shadow-FFR



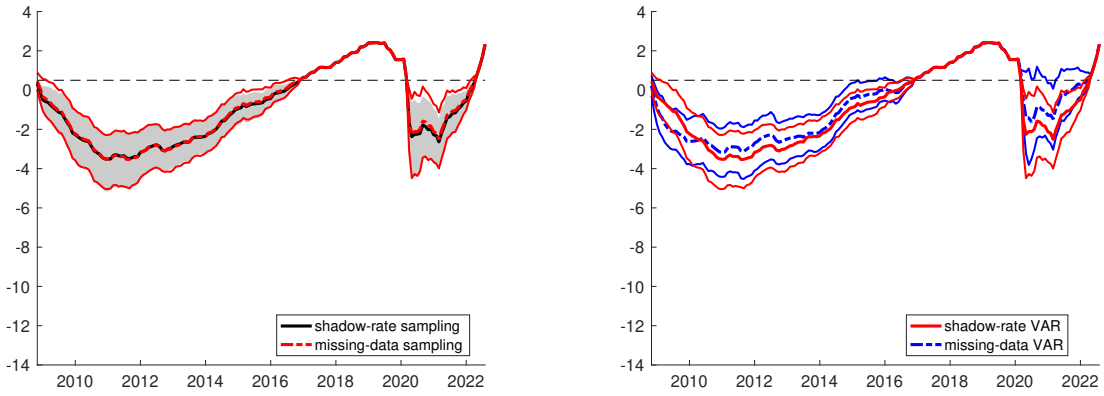
Note: Shadow-rate estimates from simple and hybrid shadow-rate VARs, estimated when excluding yields data (other than the federal funds rate) from our data set as listed in Table 1 of the paper, and when the value of ELB is set to 50 basis points (bp). Panel (c) compares the smoothed shadow-rate estimates against updated estimates obtained from Krippner (2013, 2015) and Wu and Xia (2016). Panel (d) compares smoothed and quasi-real time shadow-rate estimates of the federal funds rate. The quasi-real-time estimates are the end-of-sample estimates produced by recursive estimation of the model starting in January 2009. Each estimation conditions on available data since 1959:03, but the figure omits the period prior to 2008 during which the ELB did not bind. Posterior medians are shown as thick lines; grey shaded areas and thin lines depict 90 percent uncertainty bands.

Figure S.42: Effect of imposing ELB=50bp on shadow-rate estimates

Simple shadow-rate VAR estimates from data including yields

(a) Missing-data and shadow-rate draws

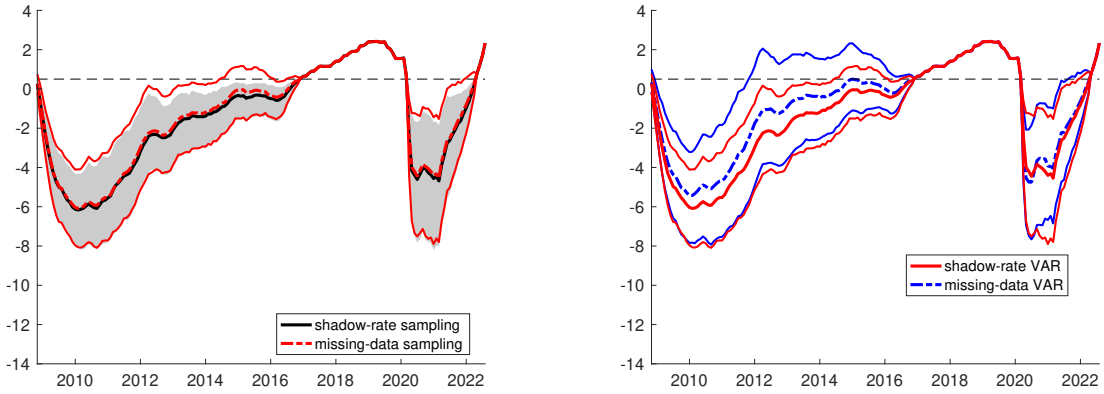
(b) Missing-data draws w/and w/o shadow-rate sampling



Estimates from data excluding yields

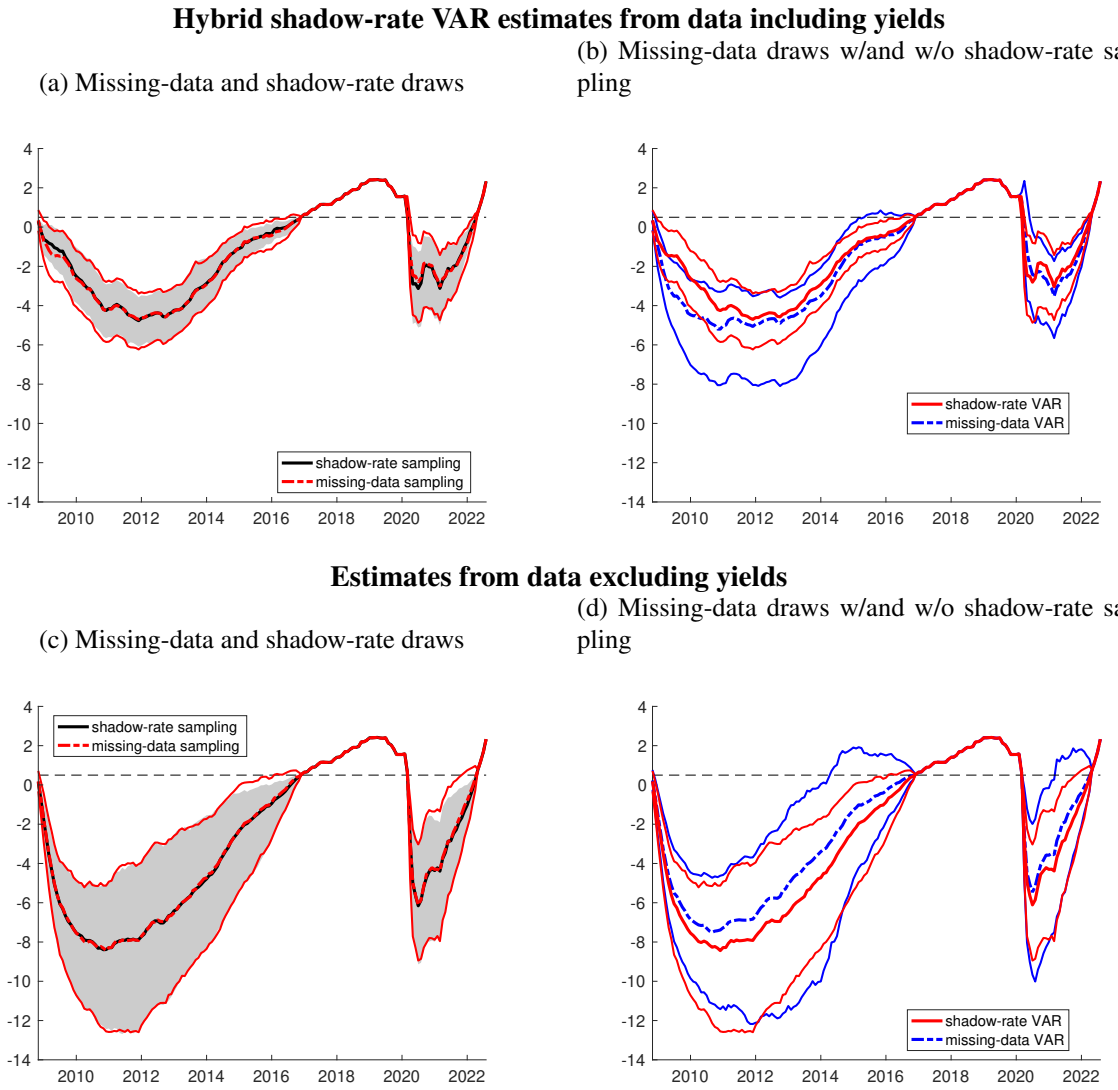
(c) Missing-data and shadow-rate draws

(d) Missing-data draws w/and w/o shadow-rate sampling



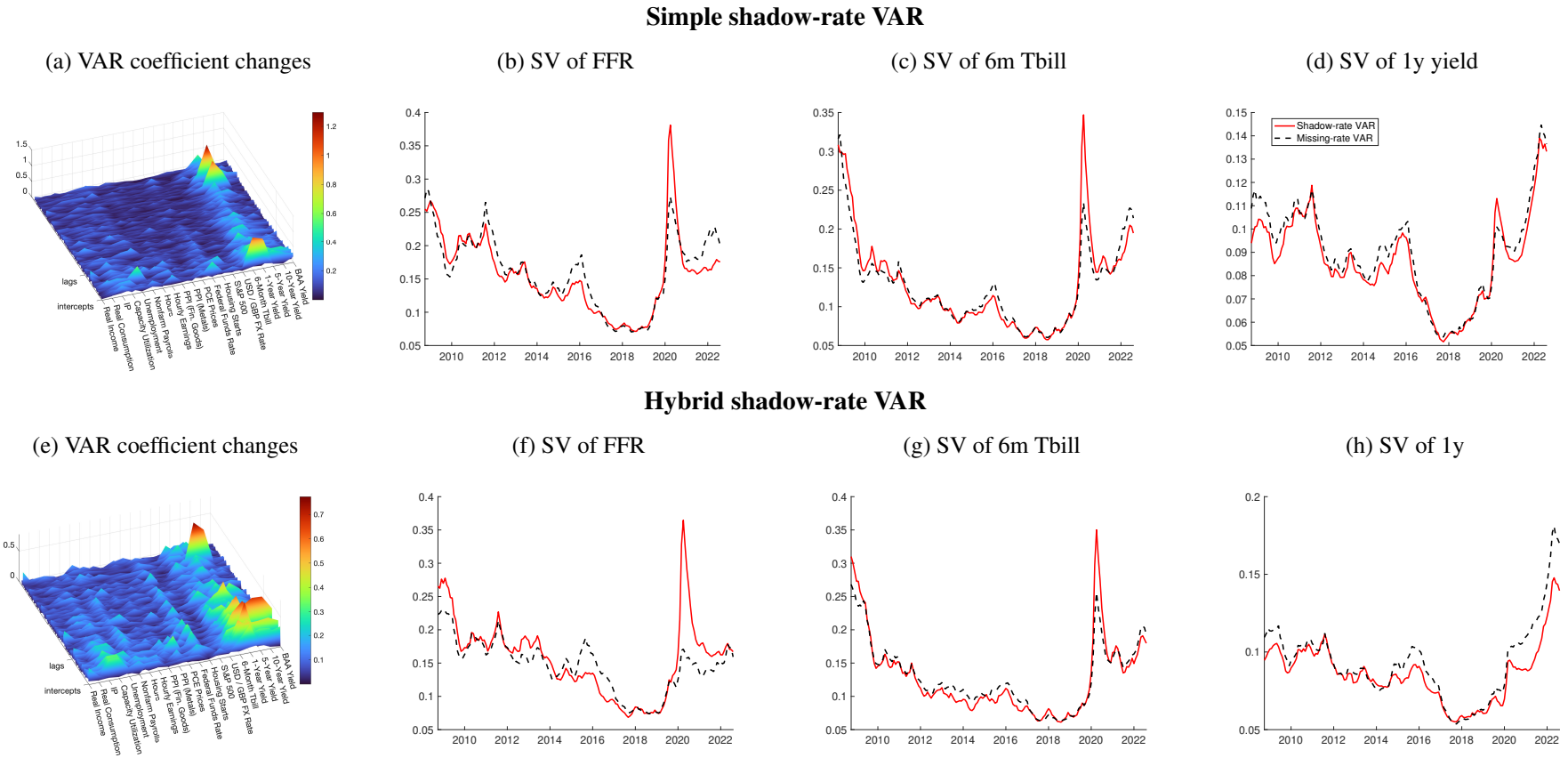
Note: Panel ((a)) compares shadow-rate (black) and missing-data (red) draws for s_t obtained from the posterior of our simple shadow-rate VAR. Shadow-rate draws are obtained from the truncated posterior for s_t that satisfies the ELB. Missing-data draws are obtained from the underlying (and untruncated) posterior of the missing data problem that ignores the ELB. Panel ((b)) displays missing-data posteriors obtained from two sets of simple shadow-rate VAR estimates: In the baseline (red), parameter and SV draws reflect shadow-rate sampling. In the alternative version (blue), parameters and SV are drawn while treating the policy rate at the ELB as missing data and without requiring that missing data draws lie below the ELB. Panels ((c)) and ((b)) provide corresponding estimates from a VAR that omits all interest rate data except for the federal funds rate. Each panel shows reports medians (thick lines) and 90 percent uncertainty bands (grey shaded areas or thin lines).

Figure S.43: Effect of imposing ELB=50bp on shadow-rate estimates in hybrid model



Note: Panel (a) compares shadow-rate (black) and missing-data (red) draws for s_t obtained from the posterior of our hybrid shadow-rate VAR. Shadow-rate draws are obtained from the truncated posterior for s_t that satisfies the ELB. Missing-data draws are obtained from the underlying (and untruncated) posterior of the missing data problem that ignores the ELB. Panel (b) displays missing-data posteriors obtained from two sets of hybrid shadow-rate VAR estimates: In the baseline (red), parameter and SV draws reflect shadow-rate sampling. In the alternative version (blue), parameters and SV are drawn while treating the policy rate at the ELB as missing data and without requiring that missing data draws lie below the ELB. In this panel, medians are reported as thick lines and 90 percent uncertainty bands are reported with the grey shaded area or thin lines. Panels (c) and (b) provide corresponding estimates from a VAR that omits all interest rate data except for the federal funds rate.

Figure S.44: ELB=50bp: Effects of shadow-rate sampling on parameters and SV

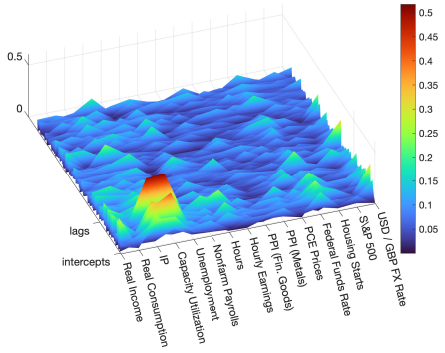


Note: Differences in estimates for parameters and selected SV paths when comparing the shadow-rate VAR against a missing-data approach that treats observations of nominal interest rates at the ELB as missing. Panels (a)–(d) report results based on the simple shadow-rate VAR, and Panels (e)–(h) display corresponding results from the hybrid shadow-rate VAR. Panels (a) and (e) show absolute values of standardized differences in mean estimates of VAR coefficients; the estimates are standardized based on the posterior standard deviation of each coefficient generated by the missing-data approach. The SV paths shown in the other panels reflect posterior medians. All estimates are estimated from the full data sample, from 1959:03 through 2022:08.

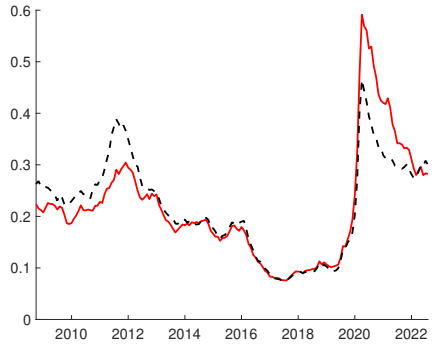
Figure S.45: ELB=50bp: Effects of shadow-rate sampling on parameters and SV (ex yields)

Simple shadow-rate VAR

(a) VAR coefficient changes

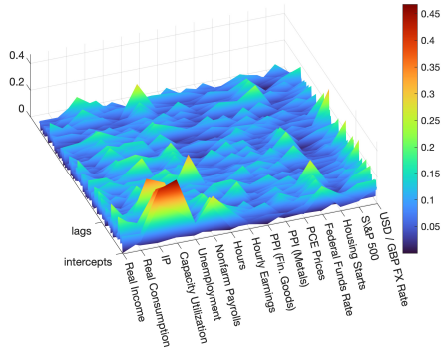


(c) SV of FFR

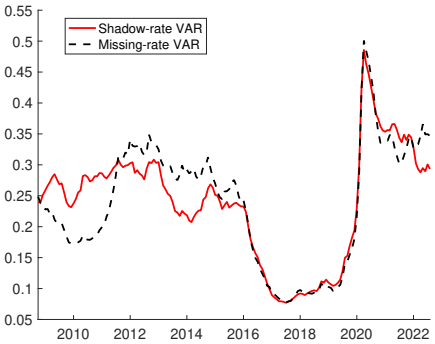


Hybrid shadow-rate VAR

(b) VAR coefficient changes



(d) SV of FFR



Note: Differences in estimates for parameters and selected SV paths when comparing the shadow-rate VAR against a missing-data approach that treats observations of nominal interest rates at the ELB as missing. Panels (a) and (c) report results based on the simple shadow-rate VAR, and Panels (b) and (d) display corresponding results from the hybrid shadow-rate VAR. Panels (a) and (b) show absolute values of standardized differences in mean estimates of VAR coefficients; the estimates are standardized based on the posterior standard deviation of each coefficient generated by the missing-data approach. The SV paths shown in the other panels reflect posterior medians. All estimates are estimated from the full data sample, but excluding yield measures other than the federal funds rate, from 1959:03 through 2022:08.

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[2] Krippner, Leo (2013), "Measuring the stance of monetary policy in zero lower bound environments," *Economics Letters*, 118, 135–138.

[3] ——— (2015), *Zero Lower Bound Term Structure Modeling: A Practitioner's Guide*: Palgrave Macmillan.

[4] Wu, Jing Cynthia, and Fan Dora Xia (2016), "Measuring the macroeconomic impact of monetary policy at the zero lower bound," *Journal of Money, Credit and Banking*, 48, 253–291.