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Banks' net interest margin and changes in the term structure

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Non-technical summary

Research Question

When analysing the impact of interest rates onto banks' earnings, often only the interest level is investigated. Especially in the context of banks, however, it seems important to also consider the steepness of the term structure, as banks earn a substantial part of their interest income by granting long-term loans and financing these operations with short-term deposits. In this paper, we investigate the impact of the dynamics of the interest level and the steepness of the term structure onto banks' net interest income.

Contribution

We present an empirical model for the interest business of banks. This model is simplistic, but it is capable of reproducing empirical features of the interest business, for instance the short- and long-term impact of changes in the interest level and the earnings from term transformation. From the model, we derive implications, which we check with data from a quantitative survey among German banks.

Results

We show that our simplified model can replicate stylized features of different bank business models. Furthermore, the outcome of our parsimonious model for a bank's interest business is broadly in line with the results of the quantitative survey among German banks. Finally, our empirical analysis shows that above 10 per cent of the net interest income is due to term transformation.

Nichttechnische Zusammenfassung

Fragestellung

Häufig wird nur das Zinsniveau analysiert, wenn der Einfluss von Zinssätzen auf die Ertragslage von Banken untersucht wird. Besonders im Zusammenhang mit Banken ist aber auch die Steigung der Zinsstrukturkurve wichtig, denn Banken erzielen einen bedeutsamen Teil ihrer Einnahmen daraus, dass sie langfristige Kredite herausreichen und dies mit kurz laufenden Kundeneinlagen finanzieren. In dem Papier untersuchen wir, wie sich Änderungen im Zinsniveau und in der Steigung der Zinsstrukturkurve auf das Zinsergebnis der Banken auswirken.

Beitrag

Wir stellen ein empirisches Modell für das Zinsgeschäft der Banken vor. Das Modell ist stark vereinfacht, dafür kann es wichtige empirische Eigenschaften darstellen, zum Beispiel kurz- und langfristige Auswirkungen von Änderungen des Zinsniveaus und Erträge aus der Fristentransformation. Aus dem Modell leiten wir Implikationen ab, die wir mit Daten aus einer quantitativen Umfrage unter deutschen Banken überprüfen.

Ergebnisse

Wir können zeigen, dass unser vereinfachtes Modell bestimmte Eigenschaften verschiedener Geschäftsmodelle von Banken abbilden kann. Hinsichtlich der Implikationen aus dem vereinfachten Modell für das Zinsgeschäft der Banken zeigen wir, dass sie mit den Umfragedaten vereinbar sind. Schließlich zeigt unsere empirische Analyse, dass gut 10% der Nettozinseinnahmen der Banken auf Fristentransformation zurückgehen.

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Banks' Net Interest Margin and Changes in the Term Structure*

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Abstract

Understanding the impact of changing interest rates onto banks' net interest margin is of central importance for various stakeholders. The primary focus lies often on changes in the interest level. However, changes in the steepness are a second driver which also significantly impacts banks' interest business. We model the impact of an interest rate shock on a bank's net interest margin, where this shock consists not only of a level shift, but also of a change in the steepness of the term structure. Our simplified model can replicate stylized features of different bank business models. The outcome of our parsimonious model for a bank's interest business is broadly in line with the results of a quantitative survey among German small and medium-sized banks.

Keywords: Banks' net interest margin, Term Structure of Interest Rates

JEL classification: G21

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1 Introduction

The impact of changing interest rates on banks' interest business is of central importance for customers and investors on the one hand, and for policy makers and supervisors on the other hand. It is key to understand how a change in the term structure would affect bank rates and banks' net interest margin. This is particularly relevant in times where interest rates show significant changes.

When analysing interest rates, the focus often lies on the interest level, not on other characteristics of the term structure. As an approximation, this is empirically justified as yearly changes in the interest level account for about 90% of the variances of the changes in interest rates. Nevertheless, changes in the steepness of the term structure make up about 10 percent of the variance of interest rate changes.[1] Therefore, to gain a fairly complete picture of the impact of term structure movements, any model of the banks' interest business should preferably not only contain changes in the level, but also changes in the steepness of the term structure.

This seems especially important in the context of banks. A substantial part of their net interest income comes from making use of the usually positive steepness of the term structure. In other words, banks tend to grant long-term loans and finance these operations using short-term deposits, thereby benefiting from the usually higher interest rates for longer maturities.

In this paper, we set out to model the relationship between a bank's net interest income and the term structure of interest rates, and validate our model with empirical data. We look at the impact of changes in the level and in the steepness of the term structure on banks' interest margins. We do this with the help of passive investment strategies to mimic a bank's interest business, where these investment strategies consist in continuously investing in risk-free par-yield bonds of a certain maturity. We model these passive investment strategies such that we can incorporate changes in the level and in the steepness of the term structure.

We check the empirical fit of the modeling on the German banking sector, more precisely, with the results of a quantitative survey among small and medium-sized banks in Germany, known as the low-interest rate environment survey (LIRES). We think that the German banking sector is particularly relevant, as net interest income is by far the largest source of income for German banks. This is true not only for small and medium-sized banks, for instance saving banks and credit cooperatives, but also for large banks.[2] Therefore, net interest income and the corresponding term structure are important to assess German banks' profitability. However, as our model is quite general, it could also be applied to other jurisdictions.

In our analysis, we find that banks' interest business can be approximated by a portfolio of these passive investment strategies in bonds. We derive this conclusion from three observations: (i) A portfolio of these trading strategies is in line with the concept of a continuing banking business model, where maturing business is replaced by new business. (ii) We show empirically that a portfolio of these trading strategies explains more of the

[1] For further details, we refer to the mimeo Memmel and Heckmann-Draisbach (2022).

[2] In 2020, the share of net interest income with respect to German banks' operating profits was 67.3%; for savings banks and credit cooperatives, this share was 70.5% and 72.3%, for the large banks still 54.3%. See Deutsche Bundesbank (2021).

dynamics in banks' net interest margins than other sensible yet simple strategies. (iii) The results of the quantitative LIRE survey can be reasonably well explained by a portfolio of these trading strategies.

This paper's main contribution is the parsimonious modeling of banks' interest business, where we transfer a model for the interest business of banks that deals with parallel shifts to the more general case with additional changes in the steepness of the term structure. We show that despite the simplifications in the model, we can capture several features of the impact of changes in the term structure onto banks' net interest margin.

The paper is structured as follows. In Section 2, we give an overview over the related literature. Then, in Section 3, we explain the model setup for modeling banks' net interest margin. In Section 4, the empirical data used in the study is described and, in Section 5, we give the results. Section 6 concludes.

2 Related Literature

Our work is linked to several existing analyses. Memmel (2008) models the banks' interest business as a portfolio of many different bond portfolios, where for each balance sheet position, there is one bond portfolio. This makes it possible to closely model a bank's net interest income, at least regarding the risk-free interest rates. However, this model is entirely focused on the net interest income, not dealing with term transformation. By contrast, our approach takes account of the net interest margin and term transformation. In addition, it is parsimonious and closely related to regulatory figures.

Furthermore, our work is closely linked to Dräger, Heckmann-Draisbach, and Memmel (2021), where various aspects of risk management of German small and medium-sized banks are analyzed. The analysis there is based on the LIRES 2017 data and focuses on a positive parallel interest rate shock. In their model, the authors assume a stylized balance sheet with loans, bonds and deposits on the asset or the liability side, respectively. We build upon this work by assuming a similar model, which we extend by allowing shifts in the slope of the term structure (not only level shifts) and analysing various scenarios (not only positive parallel shifts) from the data.

It is empirically widely found that changes in the interest level are positively correlated with banks' net interest margins (see, for instance, Albertazzi and Gambacorta (2009), Oesterreichische Nationalbank (2013) and Claessens, Coleman, and Donnelly (2018)), i.e. an increase in the interest level is associated with higher net interest margins. This is especially true in a low-interest rate environment and in a long-term horizon. In the short-run, some authors find that banks may experience a negative effect from increasing interest rates, which is especially relevant for banks with a high amount of term transformation. This is found by Alessandri and Nelson (2015) for banks in the UK and by Busch and Memmel (2017) for banks in Germany. Our parsimonious model is able to reproduce these empirical features. The effects of the steepness of the term structure on banks are rarely investigated. One such study is carried out by English (2002) who analyzes the effect of the steepness of the term structure on banks' net interest margins, and he gets mixed results. We therefore extend the existing literature by contributing a dedicated study on the impact of changes in the steepness of the term structure onto banks' net interest margin.

Table 1: Studies on earnings from term transformation

Study	Share of NIM	Earnings [in bp per assets]	Sample
Memmel (2011)	12.3%	26.3	German banks, 2005-2009
Busch and Memmel (2016)	33.6%	73.3	German banks, 2012[3]
Chaudron et al. (2022)	8.3%[4]	11.4	Dutch banks, 2008Q1-2020Q4
Approximation in this paper[5]	-	50	German banks, 1975-2021
Study in this paper[6]	10.1%	18.7	German banks, 2014-2020

This table shows studies on earnings from term transformation. "NIM" stands for "net interest margin". "Assets" mean total assets, in the case of Chaudron et al. (2022), it means banking books assets. "bp" means basis points.

Here, it is important to note that we focus on the effect of a change in the term structure and not on the causes. That is, we do not explicitly model any circumstances why the term structure changes - these changes may be induced by monetary policy shocks, but one has to keep in mind that the response of the yield curve to monetary policy is nontrivial (see e.g. Tillmann (2020)and Katagiri (2022)) and other causes cannot be excluded.

Regarding other determinants of banks' net interest margins, such as credit risk, market power or interest rate risk, we find that in the literature (see, for instance, Ho and Saunders (1981), McShane and Sharpe (1985), Maudos and de Guevara (2004), Liebeg and Schwaiger (2006) and Heckmann-Draisbach and Moertel (2020)), their impact on the net interest margin is well documented. However, in our study, we assume that these determinants are time-constant, knowing that earnings from term transformation and earnings from other determinants may be interrelated (see Chaudron, de Haan, and Hoeberichts (2022)).

As to the contribution of term transformation to banks' earnings, it is known that German banks are much engaged in term transformation; Memmel (2011) and Busch and Memmel (2016) find that the contribution it makes to German banks' net interest income strongly depends on the time period under consideration and estimate that this contribution can account for up to around one-third of German banks' net interest income. We also make a contribution here and add a further estimate about the contribution from term transformation to banks' earnings. An overview over different studies can be found in Table 1. In this context, Chaudron et al. (2022) find that a bank's net interest margin NIM includes other time-varying components that increase when the earnings from term transformation decrease and vice versa. We will discuss this finding below.

3 Modeling Banks' Interest Business

3.1 Basic Model

The net interest income of banks is usually composed of the income and expenses of many interest-bearing instruments, such as loans and bonds on the asset side and deposits and issued bonds on the liability side. On average, the instruments on the asset side generate more interest income than the expenses from the instruments on the liability side. That's why, the net interest income, the difference between interest income and interest expenses, is usually positive. In this paper, we model the banks' interest business in a stylized manner, where we concentrate on the impact of changes in the term structure onto banks' net interest margin.[7] The model assumes one portfolio on the asset side and one portfolio on the liability side. The restriction to two portfolios makes the model more parsimonious, but still capable of reproducing some empirical features of banks' interest business, such as the incomplete pass-through to bank rates, term transformation and market power. Moreover, the restriction to two portfolios yields expressions for the change in the net interest margin that can be compared to supervisory reporting data and interpreted without much effort, thus yielding additional insights compared to more complex models.

We model the interest business of a bank as in Dräger et al. (2021): On the asset side, banks grant default-free loans (share: θ_A) in a revolving manner, i.e. whenever a loan matures, it is replaced by a new one which leads to a static balance sheet.[8] These loans have a maturity M_A and a coupon c equal to the then prevailing par-yield bond rate. The interest payments are taken out and constitute the interest income. In addition, banks hold cash (share: $1 - \theta_A$). On the liability side, there are default-free bonds (share: θ_L) with maturity M_L that the bank issues in a revolving manner; the rest of the liabilities consist of non-remunerated current accounts (share: $1 - \theta_L$). Note that the share θ_A can also be interpreted differently: Instead of the share of assets that have a pass-through of 100%, it can also be interpreted as the average pass-through on the asset side. The same reasoning applies to the liability side.[9]

The interest business of such a bank corresponds to a portfolio of passive trading strategies $S(m)$ that consist in investing in par-yield bonds of maturity m in a revolving manner. In our model, the portfolio of passive trading strategies is composed of a long position of θ_A in the strategy $S(M_A)$ and a short position of θ_L in the strategy $S(M_L)$. Suppose there is a parallel shift in the term structure by β_0 in time t_0, i.e. (if we denote the coupon of the par-yield bonds by $c(m)$):

[3] This study includes also the results for 2013, which are even higher as to term transformation.

[4] This share also includes interest on equity.

[5] See Appendix A.4

[6] See Section 5.3

[7] Banks' net interest margin is their net interest income divided by total assets. For other possible determinants of the net interest margin, see Section 2.

[8] Note that a static balance sheet is a common assumption in stress testing.

[9] In this model, a bank's market power can be measured by the share $1 - \theta_L$: if this bank is able to finance (large parts of) its operations with non-remunerated deposits, it has a strong market position, perhaps due to absent competitors or wide-spread branch-offices.

$$\triangle c(m) = \beta_0 \tag{1}$$

As existing contracts have to be fulfilled to the contractual "old" conditions, only new business (not existing business) is affected by the interest rate shock. Due to the revolving manner of the investment strategy, the share of new business (on the asset side) corresponds to T/M_A, capped at 100%. Among the new business, only the share θ_A counts, i.e. the fraction that is invested in the strategy with the par-yield bonds whose coupons have changed by β_0. The same is true of the liability side. Thus, after a time $T = t - t_0$ we expect a change in the bank's net interest margin $\triangle NIM$:

$$\begin{aligned} \triangle NIM(T) &= \beta_0 \cdot \theta_A \cdot min\left(\frac{T}{M_A}, 100\%\right) \\ &- \beta_0 \cdot \theta_L \cdot min\left(\frac{T}{M_L}, 100\%\right) \end{aligned} \tag{2}$$

This expressions describes two different effects: In the long run, the bank's net interest margin (NIM) changes by $\triangle NIM = \beta_0\,(\theta_A - \theta_L)$ because the shares θ_A (the loans on the asset side) and θ_L(the issued bonds on the liability side) are remunerated according to the market rate and the complements are not remunerated at all (cash and non-remunerated current accounts). In the short run, we look at the short time span T just after the shock in t_0 and observe that the share T/M_A of the loans have matured and are replaced by loans with a new coupon $c+\beta_0$. A similar reasoning applies to the liability side. Therefore, the short-term change in the net interest margin (NIM) is $\triangle NIM = T{\cdot}\beta_0{\cdot}(\theta_A/M_A - \theta_L/M_L)$.

An example may be helpful in understanding this point: Suppose Bank A grants loans with $M_A = 5$ years of maturity in a revolving manner. Further suppose that this business stands for $\theta_A = 90\%$ of the balance sheet (10% cash) and the liability side is composed of revolvingly issued bonds ($\theta_L = 70\%$, maturity $M_L = 2$) and of non-remunerated current accounts (30%). In this example, this bank has a long-run pass-through of 20% (=90%-70%), i.e. when the interest level goes up by 100 bp, its net interest margin will ultimately increase by 20 bp. In the short run, however, we will observe a drop in its net interest margin. More precisely, the initial decrease of the NIM has a slope of -17 bp/year (=90/5 - 70 /2) bp/year if the interest level goes up by 100 bp (see Figure 1, dotted line, the short-run effect corresponds to the slope at the beginning).

This model allows us to determine the consequences of a parallel interest rate shock for a bank:

- A bank benefits in the long run from an increase in the interest level (i.e. its net interest margin (NIM) increases) if the average pass-through on the asset side θ_A is larger than the one on the liability side θ_L. Empirically, this is often found (see, for instance, Albertazzi and Gambacorta (2009) and Claessens et al. (2018)).

- In the short run, it is possible that the net interest margin (NIM) becomes smaller as a consequence of a positive interest level shock, especially for banks that carry out a lot of term transformation. In case $\frac{\theta_A}{M_A} < \frac{\theta_L}{M_L}$, we have such a situation. This is found by Alessandri and Nelson (2015) for banks in the UK and by Busch and Memmel (2017) for banks in Germany.

3.2 Model Extension

So far, we have dealt only with parallel shifts of the term structure. In the following, we extend the model so that changes in the steepness of the term structure can be incorporated.

Again, we assume that an interest shock takes place at time $t = t_0$, having an impact on the interest level and on the steepness of the term structure, where the variable $T = t - t_0$ gives the period since the shock has happened. We use a linear model of the term structure[10] and relate – in an environment of low interest rates – small changes in the interest level (β_0) and in the steepness of the term structure (β_1) to the coupon of par-yield bonds ($c_t(m)$) of maturity m (details can be found in Appendix A.3, see especially Equation (28)). We thereby obtain:

$$\triangle c(m) = \beta_0 + \beta_1 \cdot m \tag{3}$$

Based on this expression, we can calculate the (instantaneous) deviation $C.NIM(T)$ of the net interest margin (from the baseline of no change) as a consequence of this interest shock where in the baseline, we assume that the term structure remains unaltered.[11] Equation (4) shows the deviation of the net interest margin from the baseline net interest margin.

$$\begin{aligned} C.NIM(T) &= \theta_A \cdot min\left(\frac{T}{M_A}, 100\%\right) \cdot (\beta_0 + \beta_1 M_A) \\ &- \theta_L \cdot min\left(\frac{T}{M_L}, 100\%\right) \cdot (\beta_0 + \beta_1 M_L) \end{aligned} \tag{4}$$

The reasoning is similar to the one in the previous chapter where we described the parallel shift. The example from above, this time with an increase of the slope by 10 bp/year, is shown in Figure 1 (solid line).

Without loss of generality, we assume $M_A > M_L > 0$, i.e. that the maturity on the asset-side is larger than the one of the liability side and that maturities are positive.[12] Concerning period T, i.e. the period since the interest shock has happened, we distinguish three cases:

- Case i): $T \leq M_L$

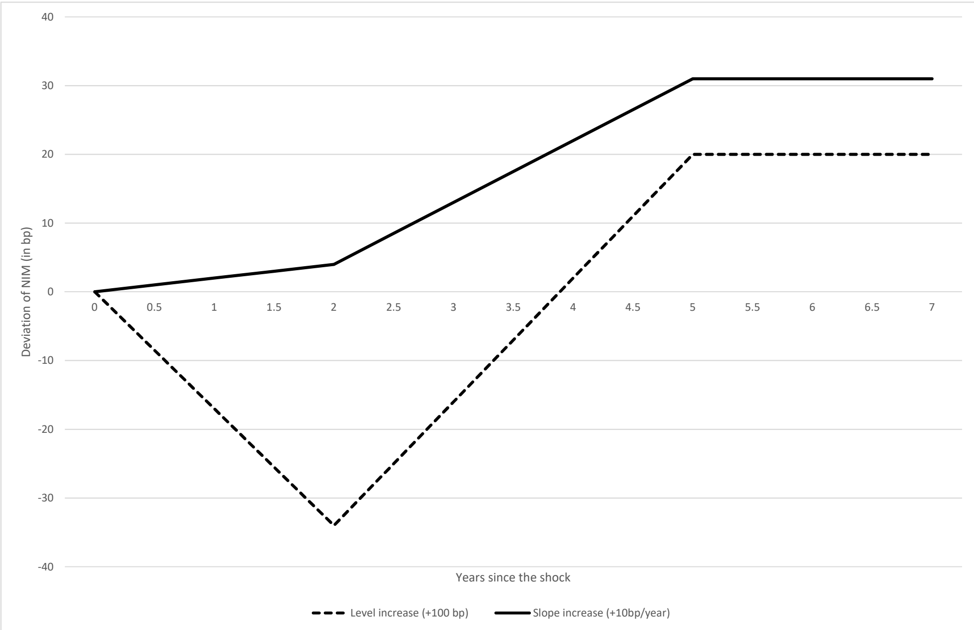
$$C.NIM(T) = T\left(\frac{\theta_A}{M_A} - \frac{\theta_L}{M_L}\right)\beta_0 + T\left(\theta_A - \theta_L\right)\beta_1 \tag{5}$$

[10] In a linear model, the yearly return increases linearly with the maturity $r_t(m) = \alpha_{0,t} + \alpha_{1,t} \cdot m$.

[11] To be in accordance with the data structure of the survey, we change the notation from $\triangle NIM$ to $C.NIM$, i.e. we do not report the change in the net interest margin from the starting point, but the deviation from the baseline of a constant term structure. In theory, the change $\triangle NIM$ is equal to the deviation $C.NIM$, but in practice, we observe that the net interest margin (NIM) is changing even in the baseline scenario of no change in the term structure; this is likely to earlier changes (in the past) in the term structure.

[12] As long as the maturities M_A and M_L are greater than zero, the effects can be computed in the Equations (5) and (6), only the conditions for the cases have to be altered if $M_A \leq M_L$. As to the simplified central bank in Table 5, the duration on the asset side, M_A, is zero, therefore, the short-term effect cannot be computed.

Figure 1: Deviations of the net interest margin ($C.NIM$) due to interest shocks



This figure shows the deviations of a bank's (instantaneous) net interest margin ($C.NIM$) as a consequence of two shocks to the term structure, namely a level shift by 100 bp (dotted line) and an increase in the slope by 10 bp/year (solid line).

- Case ii): $M_L < T \leq M_A$

$$C.NIM(T) = \left(T\frac{\theta_A}{M_A} - \theta_L\right)\beta_0 + (T\theta_A - M_L\theta_L)\,\beta_1 \tag{6}$$

- Case iii): $M_A < T$

$$C.NIM(T) = (\theta_A - \theta_L)\,\beta_0 + (M_A\theta_A - M_L\theta_L)\,\beta_1 \tag{7}$$

In the following, we mainly focus on Equation (7), for which we need two quantities for each bank, namely its long-run pass-through $\theta_A - \theta_L$ and its extent of term transformation $M_A\theta_A - M_L\theta_L$. The term $M_{A,i}\theta_{A,i} - M_{L,i}\theta_{L,i}$ is roughly proportional to the duration of the portfolio to mimic Bank i's interest business.[13] Under the assumption of a low interest level, we obtain for the euro duration of this bank's assets A_i minus the euro duration for its liabilities L_i:[14]

$$D_i^{€} := \frac{\partial A_i}{\partial \beta_0} - \frac{\partial L_i}{\partial \beta_0} = \frac{1}{2}\left(M_{A,i}\theta_{A,i} - M_{L,i}\theta_{L,i}\right)\cdot A_i \tag{8}$$

Banks in Germany have to report a similar value, namely for a shock of 200 bp, together with the Basel interest rate coefficient (see Equation (13)). The long-run pass-through $\theta_A - \theta_L$ can be determined by assigning to each balance sheet position (which is available for each bank and at monthly frequency) its pass-through as done in Dräger et al. (2021)). We discuss the relation to empirical quantities in more detail in Section (3.4). The following empirical equation derived from Equation (7) may then be estimated:

$$C.NIM_{i,k}(M_{A,i}) = \alpha + \beta_k \cdot (\theta_A - \theta_L)_i + \gamma_k \cdot (M_A\theta_A - M_L\theta_L)_i + \varepsilon_{i,k} \tag{9}$$

where $k = 1, ..., K$ stands for the relevant scenarios. This is the main equation of our analysis. The resulting estimates $\widehat{\beta}_k$ and $\widehat{\gamma}_k$ can be compared with the scenario parameters.

[13] In the following, we use two concepts of duration, namely the euro duration $D^{€}$ and the modified duration D_{mod}. The euro duration gives the euro amount of the change as a consequence of a small interest change and the modified duration is equal to the euro duration over the present value of the portfolio. As in our model a bank's equity is not explicitly accounted for, the bank's present value is zero, so that we cannot determine the modified duration of the bank, only its euro duration.

[14] To give an intuition for the duration formula: The modified duration of a par-yield bond corresponds approximately to its maturity, i.e. a par-yield bond with a maturity of five years loses approximately 5% of its value if the interest rate level rises by 1 percentage point (actually, this is only exact at an interest level of 0%). The passive investment strategy $S(m)$ consists in investing in par-yield bonds with maturity m so that this strategy consists at any time of bonds with a residual maturity equally distributed from zero maturity to maturity m, which leads to an average residual maturity of $m/2$, which is approximately equal to the strategy's modified duration (at an interest level of 2% and a maturity $m = 5$, the strategy's duration is $D_{mod} = 2.42$ (instead of 2.5). A portfolio consisting of a long position of $\theta_{A,i} \cdot A_i$ of the strategy $S(M_{A,i})$ and a short position of $\theta_{L,i} \cdot L_i$ of the strategy $S(M_{L,i})$ has the euro duration $D_i^{€} = \frac{1}{2}\left(M_{A,i}\theta_{A,i} - M_{L,i}\theta_{L,i}\right) \cdot A_i$. Note that, in the model, equity is not explicitly accounted for and that, therefore, the euro amount of a bank's assets is equal to its liabilities, i.e. $A_i = L_i$.

3.3 Checking Model Assumptions

In the model, there are several assumptions which we want to briefly motivate. First, we consider the assumption of loans being granted in a revolving manner. This assumption leads to a static balance sheet. It can be considered as agnostic about any management actions or changing conditions, by assuming that the initial portfolio mix is kept constant per bank. While this is a simplification, it is commonly considered as most appropriate assumption in the context of projections for stress tests, and thus quite undisputed in the field of stress testing.

Often, we make use of simplifications that are only exactly valid if the interest level is zero. This situation was nearly fulfilled in Germany during the low interest rate environment (2013-2021) and can thus be considered a reasonable assumption for the times of the LIRES.

One important assumption is that a bank's interest business can be modeled by a portfolio of passive trading strategies $S(m)$. We check one empirical implication of this assumption by running the following regression:

$$NIM_{t,i} = \alpha_i + \gamma_t + \beta \cdot \frac{F_{t,i}}{A_{t,i}} + \varepsilon_{t,i} \tag{10}$$

where α_i are bank fixed effects and γ_t are time fixed effects. $A_{t,i}$ are Bank i's total assets and $F_{t,i}$ are Bank i's earnings from term transformation in the period from $t-1$ to t under the assumption of a certain investment strategy, for instance a portfolio of the passive trading strategy (see also Equation (14)). If this assumption is valid, we expect the coefficient β to equal one. However, to note that Chaudron et al. (2022) find that Bank i's net interest margin NIM includes other time-varying components that increase when the earnings from term transformation decrease and vice versa, so that the coefficient β is less than one; in the case of their sample (Dutch banks), it is even close to zero.

3.4 Relating Model Parameters to Observable Quantities

In this section, we show how the model parameters can be related to observable quantities.

First, to obtain the two quantities that are necessary to estimate Equation (9), i.e. the long run pass-through $\theta_A - \theta_L$ and the exposure to term transformation $M_A\theta_A - M_L\theta_L$, we proceed as follows: balance sheet data of all German banks is used to determine bank-specific weights (w_{ij}) for the different balance sheet positions j. Let w_{ij} be the weight of balance sheet position j of bank i, then

$$\theta_{A,i} = \sum_{j=1}^{J} w_{ij} \cdot \theta_j^A \tag{11}$$

The same can be done on liability side. We use estimates of the long-run pass-throughs $\theta_{A,i}$ and $\theta_{L,i}$ from Memmel (2018) (details can be found in Table 1 in the cited article, sample period: January 2003 to March 2016) for 12 loan categories (that differ by initial maturity, by borrower or by kind of investment) and ten categories of deposits. Examples are the estimated pass-through for sight deposits of 0.38 and for housing loans of above 0.8. For some positions, the long-run pass-through is zero by definition, for instance for the position cash.

Now we turn to the quantity $M_A\theta_A - M_L\theta_L$. In principle, the maturity $M_{A,i}$ (and likewise the maturity $M_{L,i}$) could be estimated similar to $\theta_{A,i}$ (see Equation (11)). However, this estimate would not be as precise as that for $\theta_{A,i}$. For instance, off-balance sheet positions, mainly interest swaps, can be assumed to have a complete pass-through on the asset side and on the liability side, so that they do not alter a bank's net long-run pass-through. By contrast, they are likely to affect the term transformation. Therefore, we make use of the fact that the term $M_{A,i}\theta_{A,i} - M_{L,i}\theta_{L,i}$ is related to a bank's exposure to term transformation IRR_i, which is the the euro amount of the change in present value due to an interest rate shock of $\triangle r$. We approximately obtain:

$$D_i^{€} \approx -\frac{IRR_i}{\triangle r} \tag{12}$$

Together with Equation (8), we derive an expression for $M_{A,i}\theta_{A,i} - M_{L,i}\theta_{L,i}$, namely

$$M_{A,i}\theta_{A,i} - M_{L,i}\theta_{L,i} = -100 \cdot \frac{IRR_i}{A_i} \tag{13}$$

where we use the relationship $100 = 2/\triangle r$ for a positive shock of 200 bp.

Second, we explain how we calculate the earnings from term transformation $F_{t,i}$ in Equation (10). A bank's exposure to interest rate risk, the variable $IRR_{t,i}$, is taken from the banks' regular term transformation returns: Let IRR_i be the euro amount of the change in present value due to an interest rate shock of $\triangle r = 200bp$[15], which is reported quarterly. We use this information to scale the passive trading strategy: The risk from this strategy, measured by its duration $D_t^{€}(S(m_1)) - D_t^{€}(S(m_2))$, should be equal to the interest rate risk of the bank under question; the bank's hypothetical earnings from transformation correspond to the scaled earnings form the passive trading strategy $Re_t(S(m_1)) - Re_t(S(m_2))$. The formula is:

$$F_{t,i} = -50 \cdot \frac{Re_t(S(m_1)) - Re_t(S(m_2))}{D_t^{€}(S(m_1)) - D_t^{€}(S(m_2))} \cdot IRR_{t,i} \tag{14}$$

where $Re(\cdot)$ is the return of the term in brackets. Another modeling approach would be to redeploy the entire capital in each period. Then, in case of an investment in zero-bonds:

$$F_{t,i} = -50 \cdot \frac{r_t(m_1) - r_t(m_2)}{m_1 - m_2} \cdot IRR_{t,i} \tag{15}$$

with $r_t(m) = \alpha_{0,t} + \alpha_{1,t} \cdot m$. These expressions will be used to check different strategies in Equation (10).

4 Data

4.1 Reporting Data on Banks' Interest Business

We use several data points from regular reporting of banks, in particular balance sheet data and the quarterly reporting of banks' exposure to interest rate risk, both on a single-bank basis. Balance sheet data is taken for end-2016 and end-2018. As described in

[15]The "50" in the Equations (14) and (15) comes from the reciprocal value of $\triangle r = 200bp$.

Table 2: Summary statistics (bank level)

Variable	Year	Unit	Mean	SD	1st perc.	Median	99th perc.	Nobs
IRR	2016	-% per TA	1.96	1.03	-0.54	1.99	4.64	1419
IRR	2018	-% per TA	1.93	1.07	-0.77	1.96	4.76	1383
$\theta_A - \theta_L$	2016	% per TA	25.05	11.85	-11.86	26.43	49.13	1419
$\theta_A - \theta_L$	2018	% per TA	26.07	11.74	-12.67	27.71	48.71	1383

This table shows summary statistics at bank level. The data is from the banks' returns just before the wave of the quantitative survey took place, namely end-2016 figures for the wave 2017 and end-2018 figures for the wave 2019. IRR is a bank's exposure to interest rate risk and the difference $\theta_A - \theta_L$,, is its long-run net pass-through. "SD", "1st perc." and "99th perc." mean standard deviation, first percentile and 99th percentile.

section 3.4, the balance sheet data is used to estimate the bank-individual long run pass-through $\theta_A - \theta_L$, together with estimates of the long-run pass throughs per portfolio $\theta_{A,i}$ and $\theta_{L,i}$ from Memmel (2018).

For the regular reporting of banks' exposure to interest rate risk, we use the quarterly reported data of the euro amount of the change in present value due to an interest rate shock of $\triangle r = 200bp$. To get yearly data, we calculate (for Equations (14) and (15) in the last quarter of each year) the sum of the current quarter and of the 3 previous quarters, whereby we make use of the quarterly availability of the interest risk exposure data $IRR_{t,i}$. Again, we focus on end-2016 and end-2018.

In Table 2, we report summary statistics for banks' interest business. As can be seen, the mean and standard deviation of the quantities do not differ strongly between 2016 and 2018. The bank-individual lon-run pass-through shows a mean value of around 25, where the 99th percentile lies at nearly 50 (% per TA).

4.2 Low-Interest Rate Environment Survey

Every other year since 2013, German small and medium-sized banks have been subject to a quantitative survey, namely the low-interest rate environment survey (LIRES), which is conducted together with the LSI stress test.[16] Participation in the survey is compulsory. Based on the starting year, banks have to forecast their interest income and expenses (and other components of their profit and loss statement) for the following 5 years under a static balance sheet assumption, for different interest rate scenarios, i.e. for different assumptions on the term structure of interest rates. The data submitted by banks in this survey thus does not relate to actual or historical net interest income but to banks' own projections under given restrictions. This survey data is especially suitable for the purpose of this analysis, as it not only includes stress scenarios consisting of changes in the interest level, but also a scenario involving a change in steepness, more precisely a flattening of the term structure. Moreover, given that the only difference between the various scenarios are term structure changes, other effects that could have an impact on banks' net interest margin can be eliminated.

The empirical data we use in our study, i.e. the different waves of the LIRES, have

[16] In 2021, no survey wave took place as a consequence of the COVID-19 pandemic, but it was postponed to 2022.

Table 3: Scenarios in the LIRES waves

Number k	Scenario	Description	Change in the...	
			level	steepness
0	Baseline	Term structure remains constant	0	0
1	Turn	Term structure flattens	125	-11
2	Positive shift	All interest rates increase by 200 bp. The steepness does not change	200	0
3	Negative shift	All interest rates decrease by 100 bp. The steepness does not change	-100	0

This table shows descriptions of the scenarios. "bp" means basis points. All changes take place over night at the beginning of the five year horizon. Values in the two last columns are given in basis points.

already been used in several studies to learn about banks' interest business, see e.g. Busch, Drescher, and Memmel (2017), Heckmann-Draisbach and Moertel (2020), Dräger et al. (2021) and Busch, Littke, Memmel, and Niederauer (2021).

We focus on the data from the 2017 and 2019 waves of the survey, as these can be considered as established research data and the reporting was to some extent standardized between these surveys, thus providing comparability. For those two waves, we have 1419 (2017) and 1383 (2019) observations per data point. We use the reported net interest income for starting point and projection years as well as total assets at end-2016 and end-2018, respectively.

There are $K = 3$ scenarios that are relevant to us, namely two scenarios with a level shift and one scenario that includes a level shift and a flattening of the term structure. In addition, there is also the scenario of a time-constant term structure, which serves as reference point. We summarize these scenarios in Table 3. All of these scenarios assume a static balance sheet. Let $NIM(T)_{i,k}$ be the net interest margin of bank i in scenario k at time T, which we calculate as ratio between the reported net interest income at time T and total assets at the respective starting point. We calculate the deviation of bank i's net interest margin as

$$C.NIM_{i,k}(T) := NIM_{i,k}(T) - NIM_{i,0}(T) \tag{16}$$

where $k = 0$ is the baseline scenario of a time-constant term structure (see Table 3).

In Table 4, we show summary statistics of the deviation of banks' net interest margin in the positive shift, in the negative shift and in the turn scenario for the two different waves. As can be seen the mean short-run effect (one year) is negative, while the long-run effect (five years) is positive in the positive shift scenario. For the negative shift scenario, the results are almost mirrored. Regarding the turn scenario, the direction of the development of the net interest margin (NIM) is not straight-forward, as this scenario includes two opposing effects: a positive level shift and a reduction in the slope (see Table 3). Whereas the positive level shift leads to a long-run increase in the net interest margin (NIM), the reduction in the slope to a falling NIM. It seems as if the positive effect slightly dominates.

Table 4: Summary statistics (LIRES)

Variable	Scenario	Wave	Mean	SD	Share > 0	Nobs
$C.NIM(1)$	Pos. shift	2017	-10.10	27.93	25.65	1419
$C.NIM(1)$	Pos. shift	2019	-9.93	26.17	27.26	1383
$C.NIM(5)$	Pos. shift	2017	29.08	29.63	90.77	1419
$C.NIM(5)$	Pos. shift	2019	29.32	31.98	88.36	1383
$C.NIM(1)$	Neg. shift	2017	-0.33	19.23	41.51	1419
$C.NIM(1)$	Neg. shift	2019	-2.09	15.33	37.38	1383
$C.NIM(5)$	Neg. shift	2017	-19.76	23.43	14.45	1419
$C.NIM(5)$	Neg. shift	2019	-22.11	24.25	13.30	1383
$C.NIM(1)$	Turn	2017	-6.11	26.92	32.14	1419
$C.NIM(1)$	Turn	2019	-5.82	24.05	34.71	1383
$C.NIM(5)$	Turn	2017	-2.83	31.43	41.72	1419
$C.NIM(5)$	Turn	2019	-1.25	32.34	44.03	1383

This table shows summary statistics of the deviation of the NIM from the baseline scenario for the positive shift, negative shift and turn scenarios (see Table 3). "Share" is in per cent.

5 Results

5.1 Stylized Results from Parsimonious Model

In a first part, we evaluate the implications of the model equations for different time periods and show that the model can already capture various business models of banks.

The results are summed up in Table 5 for different idealized banks, where the short-term effect (the next to last column) is calculated from Equation (5) as Case i) and the long-term effect (last column) is taken from Equation (7) as Case iii). When we look at the short-term effects of an increase in the level of the term structure, we see that the deviation of the net interest margin is negative (for banks that carry out much term transformation). However, the long-run effects are often positive.

An upward-turning of the term structure is said to be beneficial for banks. In the linear term structure model $r_t(m) = \alpha_{0,t} + \alpha_{1,t} \cdot m$, this upward-turning is a combination of a decrease in the level, i.e. $\beta_0 < 0$, and an increase of the steepness, i.e. $\beta_1 > 0$. Even under the assumption of a positive net long-run pass-through $\theta_A - \theta_L$ and a negative short-run effect $\frac{\theta_A}{M_A} - \frac{\theta_l}{M_L}$, it is unclear whether the long-term effect is positive (see the cell in the last row and in the last column of Table 5). This is only the case if in addition

$$\frac{M_A\theta_A - M_L\theta_L}{\theta_A - \theta_L} > -\frac{\beta_0}{\beta_1} \tag{17}$$

Note that the expression $-\beta_0/\beta_1$ can be seen, according to the our linear term structure model $\triangle r_t(m) = \beta_0 + \beta_1 \cdot m$, as the pivotal point $m^*_{t_0}$ of a turning in the term structure (provided the two coefficients β_0 and β_1 have different signs, so that $m^* = -\beta_0/\beta_1$ is a positive maturity) and that, on the left-hand side of condition (17), there are bank characteristics and, on the right-hand side, there is a term structure characteristic. Regarding our sample of 564 observations for β_0 and β_1, we obtain for 384 points in time a positive value for $m^* = -\beta_0/\beta_1$and for 126 points in time Equation (17) is fulfilled for the average

Table 5: Impact on a bank's net interest margin (NIM)

<table>
<tr><th rowspan="2">No.</th><th colspan="4">Bank characteristic</th><th rowspan="2">Term structure</th><th colspan="2">$C.NIM$</th></tr>
<tr><th>$\theta_A - \theta_L$</th><th>$\frac{\theta_A}{M_A} - \frac{\theta_L}{M_L}$</th><th>$M_A\theta_A - M_L\theta_L$</th><th>Example</th><th>Short-term</th><th>Long-term</th></tr>
<tr><td>1</td><td>1</td><td>n.a.</td><td>0</td><td>Simplified central bank</td><td>pos. shift</td><td>n.a.</td><td>pos.</td></tr>
<tr><td rowspan="3">2</td><td rowspan="3">0</td><td rowspan="3">neg.</td><td rowspan="3">(pos.)</td><td rowspan="3">Commercial bank</td><td>pos. shift</td><td>neg.</td><td>0</td></tr>
<tr><td>pos. shift + inc. in steep.</td><td>neg.</td><td>pos.</td></tr>
<tr><td>neg. shift + inc. in steep.</td><td>pos.</td><td>pos.</td></tr>
<tr><td rowspan="3">3</td><td rowspan="3">pos.</td><td rowspan="3">neg.</td><td rowspan="3">(pos.)</td><td rowspan="3">Traditional bank</td><td>pos. shift</td><td>neg.</td><td>pos.</td></tr>
<tr><td>Pos. shift + inc. in steep.</td><td>?</td><td>pos.</td></tr>
<tr><td>Neg. shift + inc. in steep.</td><td>pos.</td><td>?</td></tr>
</table>

This table shows qualitatively the deviation of the net interest margin ($C.NIM$) for three idealized banks as a consequence of interest rate shocks. The simplified central bank has on its asset side loans to banks with a negligible maturity ($M_A = 0$) and on the liabilityside banknotes ($\theta_L = 0$). The commercial bank has on both sides loans and bonds with maturity $M_A > M_L$ and with complete pass-through. The traditional bank has on its liability side equity and deposits, so that $\theta_A > \theta_L$ and carries out term transformation, where we assume that the term transformation effect dominates (see the last row, second column with the entry "neg."). Entries in brackets "()" mean that they are derived from the assumptions in the two entries left to them. "?" means that the effect is indeterminate.

bank if mean values of 2016 are used (see Table 2).

Our model thus provides a condition for a long-term positive effect which sets the maturity and portfolio structure of a bank in relation to a observable change in the term structure.

5.2 Banks' Interest Business - Model Fit

We now turn to the question of how good our model fits to the survey data, for which we estimate Equation (9) for the three relevant scenarios (turn, negative shift and positive shift). The results in Table 6 show that modeling banks' interest business through the assumed bond portfolios captures several features. In this estimation, "theta" refers to the long-run pass-through $\theta_A - \theta_{L,}$ and "term" to the degree of term transformation $M_A\theta_A - M_L\theta_L$ in Equation (9). The following results can be highlighted:

- All estimates for the coefficient of "theta" have the right sign. However, in absolute terms, the theoretical values are significantly larger than the estimated values. This can be due to noisy values for the long-run pass-through "theta" (see Appendix A.5, Equation (41), something known as attenuation bias).

- For the turn scenario, it is noteworthy that in the specification including only a shift, the coefficient is insignificant and shows a very low R^2. This points to the importance of including changes in the steepness of the term structure when modelling the impact on banks' net interest margin.

- The coefficient γ related to the degree of term transformation "term" has the expected sign in the scenario "turn" of a flattening of the term structure. However, the estimates for "term" in the two shift scenarios are often significantly different from zero, the theoretical value in the case of a shift in the term structure (see Table 3). The cause for this may be that the new equilibrium (see Equation 7) is not reached after the maximal horizon of 5 years, which leads to a systematic bias. In Appendix A.5, it is shown that the coefficient $\tilde{\gamma}$ (for term) derived from the observed variables is equal to the sum of the true coefficient γ and a component depending on the correlation between the degree of term transformation "term" and the uncompleted change in the net interest margin (see Equations (37) and (42)).[17]

5.3 Banks' Interest Business - Earnings From Term Transformation and Assumption Check

We conduct an analysis on the banks' average earnings of term transformation, where we can make an educated guess, using the information from the summary statistics and Equation (35) in the Appendix A.4. This equation states that the contribution of term transformation is on average equal to the average steepness minus half of the trend in the interest level multiplied by banks' exposure to interest rate risk. According to Table 8, the average steepness is about 14 bp ("Mean", row 2) and the trend is about -22 bp ("Mean", row 7). According to Table 2, banks' average exposure to interest rate risk (measured as the change in present value due to an interest rate shock of +200 bp, divided by total assets, in percent) was close to 2 in 2016 and 2018; under the assumption that this exposure has been relatively constant through time, we set this exposure to 2. We obtain about 50 bp, which is within the range of the results of the studies named in Table 1. Note that the average earnings from this strategy are much higher than from the strategy of investing all funds in the then current zero bond and financing this operation by issuing short-term zero bonds. This strategy yields an average steepness of 14 bp. What is more, the risk is doubled, meaning that the average contribution is 14 bp if the risk of the bank

[17] *Uncompleted change* should be understood in the sense that after the survey horizon of $T_{max} = 5$ years, the change in the net interest margin $C.NIM(5)$ hat not yet attained the end point change $C.NIM(M_A)$. The following considerations may give rise to the belief that the correlation between the degree of term transformation and the uncompleted change is negative for the scenario of a positive shift. Assume that there are two sorts of banks (indexed by H and L) that differ only in the maximal maturity of the bonds on the asset side M_A. Table 9 in the appendix shows that the correlation between the variables "term" and ϕ is negative for the positive shift scenario. This may explain the significantly negative coefficients for "term" in the positive shift scenario in Table 6. For a negative shift, the correlation between "term" and ϕ is positive.

Table 6: Results at bank level

Scenario	Wave	theta		term		R^2	Nobs
		Theoretic	Empirical	Theoretic	Empirical		
Turn	2017	125	17.40***	-11	-8.46***	13.05	1351
Turn	2017	125	-4.88	-11		0.05	1351
Turn	2017	125		-11	-7.93***	12.41	1351
Pos. Shift	2017	200	85.53***	0	-7.12***	17.34	1350
Pos. Shift	2017	200	65.61***	0		9.00	1350
Pos. Shift	2017	200		0	-4.35***	3.43	1350
Neg. Shift	2017	-100	-22.84***	0	-1.85***	2.72	1346
Neg. Shift	2017	-100	-27.90***	0		2.02	1346
Neg. Shift	2017	-100		0	-2.57***	1.48	1346
Turn	2019	125	23.65***	-11	-10.95***	17.67	1318
Turn	2019	125	-8.43	-11		0.14	1318
Turn	2019	125		-11	-10.12***	16.68	1318
Pos. Shift	2019	200	112.27***	0	-8.99***	22.72	1317
Pos. Shift	2019	200	84.64***	0		12.25	1317
Pos. Shift	2019	200		0	-4.94***	3.56	1317
Neg. Shift	2019	-100	-36.91***	0	-1.10*	4.56	1312
Neg. Shift	2019	-100	-40.22***	0		4.32	1312
Neg. Shift	2019	-100		0	-2.42***	1.31	1312

This table shows the results of the regression (9), where the deviation in the net interest margin is taken in the fifth (and last) projection year of the corresponding survey wave (see Table 3). The column R^2 (in per cent) gives the coefficient of determination of Equation (9). "Nobs" gives the sample size. *, ** and *** mean significance at the 10%, 5% and 1% level. The columns "Theoretic" contain the values of Table 3.

Table 7: Modelling Term transformation

Strategy / Reporting	Duration (months)		R^2 (in %)	Coefficient
	Assets	Liabilities		
Passive trading strategy	42	6	64.82	0.66
Zero bonds	120	6	64.29	1.66
Structural contribution	-	-	64.39	5.32

This table showsthe duration on the asset side and on the liability that yields the highest explanatory power (R^2) for two strategies, namely for the passive trading strategy (where a constant fraction of the balance sheet is invested the then current par-yield bond) and for the strategy where the whole balance sheet is invested in the then current zero bond. For comparison, we also show the comparison with reporting figures on earnings from structural contribution.

is kept constant (compared to 50 bp). However, if we estimate the contribution according to Equation (10) and concentrate on the years where data is available, the contribution is much lower, namely 10.1% of NIM and 18.7 bp (see Table 1).

We now turn to the check of the assumptions, especially of the assumption of how to model the earnings from term transformation (here: Equation (12)), we deal with the question of how to best explain the share of the earnings from term transformation relative to the net interest income. From the banks' reporting, we have their net durations, but as Chaudron (2018) rightly states, the net duration does not give separately the durations on the asset or liability side, and hence not the maturities which we need for the passive trading strategies. We proceed as follows to obtain estimators for the durations on the asset and on the liability side: To find the combination of the trading strategies with the best fit, we try out all combinations of (m_1, m_2) of up to ten years in steps of six months, which yields 190 meaningful combinations[18], and compare the coefficient of determination R^2 of Equation (10). This done in Table 7.

We obtain the best fit (highest R^2 of Equation (10)) for a combination of the maturities (42, 6) for the passive trading strategy (see Equation (14)), yielding a coefficient of 0.66 (which is significantly smaller than the theoretical value of one, but within the expectations). The fit of this combination is better than the fit for all maturity combinations of the strategy of redeploying the whole capital in each period (see Equation (15)), which yields a maximal R^2 of 64.29%. We also challenged the fit against reporting figures of earnings from structural contributions, but here again, the model outperforms the data.[19] The R^2s seem to be in a tight range (64.82%, 64.29% and 64.39%). However, one has to take account of the finding that most of the explanation is due to the inclusion of time dummies in Equation (10). No testing seems possible as a consequence of trying out all combinations.

[18]The maturities m_1 and m_2 can each be equal to 20 different values, yielding 400 = 20 x 20 combinations. We subtract the 20 cases where both maturities are equal and exclude the 190 = 19 x 10 cases where the first maturity is smaller than the second maturity.

[19]Structural contribution may comprise more than just earnings from term transformation, e.g. earnings from own funds. To our knowledge, unfortunately, there is no isolated reporting of the earnings from term transformation, which is why we used this proxy for comparison.

5.4 Robustness Checks

As to the modeling of banks' interest business, we challenge our assumptions in Section 4 for deriving the relevant parameters and use other definitions of the long-run pass-through and the exposure to interest rate risk. More precisely, for the 2017 LIRES, we use alternative definitions of $\theta_{A,i}$ and $\theta_{L,i}$ which are based on the 2017 LIRES and were used similarly in Dräger et al. (2021). This leads to similar results as shown above.

Moreover, we check whether the investment strategy of redeploying the whole capital in each period yields a better fit for longer maturities (longer than 120 months = 10 years). Indeed, we find that the combination (210 = 17.5 years, 6) yields the best fit. However, this optimal fit for this strategy (measured by the R^2 of Equation (10)) is lower than the best fit of a portfolio using the passive trading strategy $S(m)$ in Subsection 4.1.

6 Conclusion

In our study, we model the impact of an interest rate shock on a bank's net interest margin with a parsimonious model that assumes a simplified portfolio of bonds. This model allows to analyse not only the impact of a change in interest rate level, but also a change in the steepness of the term structure.

We find that the portfolios of bonds can describe the interest business of banks well. The portfolios applying a passive trading strategy allow interest business to be modelled in a parsimonious way and at the same time allow empirical features of German banks' interest business, namely qualitatively different short-run and long-run net pass-through, and term transformation, to be reproduced. In addition, the model results fit the results of a quantitative survey and explain the dynamics of banks' net interest margin better than other plausible reference models. While our analysis focuses on the German banking sector, the model and setup could be easily transferred to other banking markets, which might be an interesting extension for future projects.

The modeling described in the paper may be used for stress testing banks' interest business with respect to changes in the term structure; e.g. it can be used to make coarse-grained predictions or to challenge the results in supervisory stress tests. This is especially relevant for banks with a significant exposure to interest rate risk, and can be particularly informative in times where significant changes in the term structure are expected.

A Appendix

A.1 Useful Formulae

For $\delta > 0$ and $m > 0$, we obtain:

$$\int_0^m exp(-\delta t)dt = \frac{1 - exp(-\delta m)}{\delta} \tag{18}$$

$$\int_0^m t \cdot exp(-\delta t)dt = \frac{1}{\delta^2}\left(1 - (1 + \delta m)\, exp(-\delta m)\right) \tag{19}$$

$$\int_0^m t^2 \cdot exp(-\delta t)dt = \frac{1}{\delta^3}\left(2 - \left(2 + 2\delta m + \delta^2 m^2\right) exp(-\delta m)\right) \tag{20}$$

For $N > 0$ as an integer, we obtain:

$$\sum_{i=1}^{N} i = \frac{N\,(N+1)}{2} \tag{21}$$

Assume that the vector θ (with dimension n) is multivariate normal $\theta \sim N(\mu; \Sigma)$ and that it is divided into two subvectors with the dimensions n_1 and n_2:

$$\begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} \sim N\left(\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}; \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}\right) \tag{22}$$

then

$$E\,(\theta_1|\theta_2 = x_2) = \mu_1 + \Sigma_{21}\Sigma_{22}^{-1}\,(x_2 - \mu_2) \tag{23}$$

A.2 Motivation for a Linear Model for Term Structure Changes

In this section, we briefly motivate why we chose to model term structure changes by a linear model. For this purpose, we investigate yearly changes of interest rates of different maturities of German government bonds with the help of a principal component analysis (PCA). In the following, we briefly describe the data used and the main results. For further details, we refer to the mimeo Memmel and Heckmann-Draisbach (2022).

A.2.1 Data on Term Structure

The interest rates are zero-bond rates, based on German government bonds and derived using the method according to Svensson (1994) with six parameters (see also Schich (1997) for the application to German data). Note that we are not dealing with single bonds, but with an already estimated term structure. The period covers nearly fifty years (monthly data, 1975-01 to 2021-12) and we use monthly data; in the paper, we have $dim(R_t) = 20$ maturities (maturities of up to 10 years in steps of 6 months) and $T_{Period} = 564$ monthly observations (47 years), yielding 11 280 observations. In our main analysis, we deal with the first 10 years of the term structure because this period seems to be the most relevant one for banks.

In Table 8, we report summary statistics. As to the average steepness, it is 14.09 bp per year (first column, second row), meaning that for each additional year of maturity, the return increases by around 14 bp. As a concrete example, a bond with 10 years of maturity yields on average 1.41% p.a. more than the short-term (0 year) interest rate, as the ten years of maturity contribute 10 times 14.09 bp of return. The 99th percentile of yearly changes is about 390 bp (fifth column, seventh row), significantly more than the 200 bp of a widespread regulatory shock (so-called Basel-Shock), which was informed by yearly changes. However, the interest rate changes tend to be larger for short maturities and when the interest level is higher, which was the case in the seventies and eighties of the last century.

Table 8: Summary statistics

Term structure	Model parameter	Unit	Mean	SD	1st perc.	Median	99th perc.
Level	Level	per cent	3.75	3.24	-0.99	3.82	11.65
	Steepness	bp per year	14.09	12.95	-21.39	15.83	39.08
Change (1 month)	Level	bp	-1.66	29.16	-91.49	-1.29	78.26
	Steepness	bp per year	-0.02	2.69	-6.57	-0.16	8.69
Change (3 months)	Level	bp	-5.12	61.88	-194.48	-2.77	172.11
	Steepness	bp per year	-0.07	5.12	-15.64	-0.31	15.04
Change (12 months)	Level	bp	-21.79	145.86	-389.77	-13.58	391.51
	Steepness	bp per year	-0.18	11.79	-32.5	-0.23	30.78

This table shows summary statistics for the level of and changes in the term structure (Period: 1975-01 to 2021-12). "SD", "bp", "1st perc." and "99th perc." mean standard deviation, basis points, first percentile and 99th percentile. The summary statistics are based on the model for the term structure $r_t(m) = \alpha_{0,t} + \alpha_{1,t} \cdot m$.

A.2.2 Results

The factor loadings of the three first components are displayed in Figure 2. The results are in line with the findings in the literature (see Litterman and Scheinkman (1991), Knez, Litterman, and Scheinkman (1994) and Bliss (1997)). The PCA is a completely statistical method, i.e. it is agnostic about possible structures (like level or steepness shifts) in the data. Yet the first component (i.e. the most important one) looks nearly like a parallel level shift (with longer maturities less affected) and the second component resembles a (concave) shift in the steepness. We furthermore check that the first two components nearly cover the whole variance of yearly interest rate changes (90.8% for the first component and 7.8% for the second component). With this background, and to have a simple model for the term structure, we opt for the linear two-factor term structure model $r_t(m) = \alpha_{0,t} + \alpha_{1,t} \cdot m$.

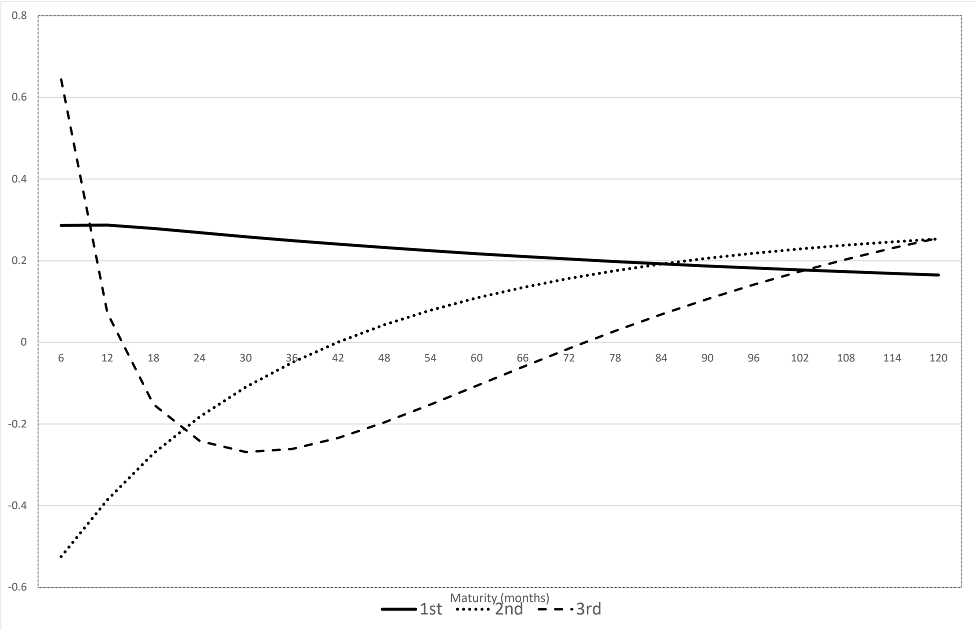
A.3 Coupon of a Par-Yield Bond

Using the definition that the present value of par-yield bonds is equal to one, we obtain

$$1 = c(m) \cdot \int_0^m exp(-r(t)t)dt + exp(-r(m)m) \tag{24}$$

where $r(m)$ is the spot rate, $c(m)$ is the coupon of the par-yield bond and m is its maturity. At a flat term structure, i.e. $r(m) = r \ \forall m$, and using the theorem about implicit functions, we obtain:

Figure 2: PCA: Factor loadings



This figure shows the factor loadings for the first three components of a principal component analysis (PCA) of yearly changes in interest rates of different maturities. German government bonds up to 120 months maturity in steps of 6 months. Monthly data; period: 1975-01 to 2021-12.

$$\frac{\partial c(m)}{\partial \alpha_i} = \frac{r^2 \int_0^m t \cdot \frac{\partial r}{\partial \alpha_i} \cdot exp(-rt)dt + rm \cdot \frac{\partial r}{\partial \alpha_i} \cdot exp(-rm)}{1 - exp(-rm)} \tag{25}$$

with $\frac{\partial r}{\partial \alpha_i} = f_i(\cdot)$ of the general additive model for the term structure $r(m) = \alpha_0 + \alpha_1 \cdot f_1(m) + \ldots + \alpha_n \cdot f_n(m)$ and r is the flat level of interest, i.e. $r(m) = r = \alpha_0$ meaning that the respective derivatives are determined at $\alpha_1, \ldots, \alpha_n = 0$, and $c(m) = r$. (For the numerator of Equation (25), we apply Equation (18) to Equation (24)). For instance, in the case of the linear term structure model $r(m) = \alpha_0 + \alpha_1 \cdot m$, we get $\frac{\partial r}{\partial \alpha_0} = 1$ and $\frac{\partial r}{\partial \alpha_1} = m$, yielding (applying Equation (19) to Equation (25)):

$$\frac{\partial c(m)}{\partial \alpha_0} = 1 \tag{26}$$

and (applying Equation (20) to Equation (25))

$$\frac{\partial c(m)}{\partial \alpha_1} = 2\frac{1 - (1 + r \cdot m)exp(-r \cdot m)}{r(1 - exp(-r \cdot m))}, \tag{27}$$

where the derivative is approximately equal to m, i.e. $lim_{r \to 0} \frac{\partial c(m)}{\partial \alpha_1} = m$. For the linear term structure model $r(m) = \alpha_0 + \alpha_1 \cdot m$, we obtain, as the limiting case for a small steepness:

$$c(m) \approx \alpha_0 + \alpha_1 m \tag{28}$$

A.4 Return of the Passive Trading Strategy $S(m)$

The return of the passive trading strategy $S(m)$ is the moving average of the current and past par-yield coupons (for the notation, see Section 3):

$$Re_t(S(m)) = \frac{1}{m} \sum_{i=1}^{m} c_{t-i+1} \tag{29}$$

For the linear term structure model $r(m) = \alpha_0 + \alpha_1 \cdot m$ and for a small steepness, we can express the par-yield coupon as in Equation (28) and obtain:

$$Re_t(S(m)) = \frac{1}{m} \sum_{i=1}^{m} \alpha_{0,t-i+1} + \sum_{i=1}^{m} \alpha_{1,t-i+1} \tag{30}$$

In the following, we model the interest level $\alpha_{0,t}$ as a constant μ and a time trend γ, blurred by a noise term $\eta_{0,t}$:

$$\alpha_{0,t} = \mu + \gamma \cdot t + \eta_{0,t} \tag{31}$$

and, for the steepness $\alpha_{1,t}$, we assume that it fluctuates around the average st:

$$\alpha_{1,t} = st + \eta_{1,t} \tag{32}$$

Using the modeling of Equations (31) and (32), we can rewrite Equation (30):

$$\begin{aligned} Re_t(S(m)) &= \mu + m \cdot st + \gamma \cdot \sum_{i=1}^{m}(t-i+1) + \varepsilon_t \\ &= \mu + m \cdot st + \gamma \cdot t - \gamma \cdot \frac{m-1}{2} + \varepsilon_t \end{aligned} \tag{33}$$

where we use Equation (21) to reformulate the sum expression and set $\varepsilon_t = \frac{1}{m}\sum_{i=1}^{m}\eta_{0,t-i+1} + \sum_{i=1}^{m}\eta_{1,t-i+1}$. Often, we look at the return difference of two trading strategies. Using (33), we obtain for the expectation of the return difference:

$$E\left(Re_t(S(M_A)) - Re_t(S(M_L))\right) = \left(st - \frac{\gamma}{2}\right) \cdot (M_A - M_L) \tag{34}$$

The risk of this return difference measured as the euro duration is for a small interest level and for a small steepness $D^{€} = \frac{1}{2}(M_A - M_L)$ and, for a Bank i, it is $D_i^{€} = -50 \cdot IRR_i$ (see Equation (12)). Therefore:

$$E\left(\frac{NII_i^{term}}{A_i}\right) = -100 \cdot \frac{IRR_i}{A_i} \cdot \left(st - \frac{\gamma}{2}\right) \tag{35}$$

i.e. the average contribution from term transformation, the term $E\left(\frac{NII_i^{term}}{A_i}\right)$ is equal to Bank i's standardized interest rate risk multiplied with the difference of the average steepness of the term structure and half of the time trend.

A.5 Biases in the Estimation

The true model is given by Equation (9), however we estimate (to keep the notation to a minimum, we drop the index k; we concentrate on the upward-shift scenario):

$$C.NIM(5)_i = \alpha + \tilde{\beta} \cdot theta_i + \tilde{\gamma} \cdot term_i + \tilde{\varepsilon}_i \tag{36}$$

We assume the following relationships between the theoretical values and their empirical counterparts.

$$C.NIM(5)_i = C.NIM(M_{A,i})_i + \phi_i \tag{37}$$

$$theta_i = (\theta_A - \theta_L)_i + \eta_i \tag{38}$$

$$term_i = (M_A\theta_A - M_L\theta_L)_i \tag{39}$$

To facilitate the calculation, the joint distribution is assumed to be normal, namely:

$$\begin{pmatrix} \theta_A - \theta_L \\ M_A\theta_A - M_L\theta_L \\ \varepsilon \\ \eta \\ \phi \end{pmatrix} \sim N\left(\begin{pmatrix} \mu_1 \\ \mu_2 \\ 0 \\ 0 \\ \delta \end{pmatrix}; \begin{pmatrix} \sigma_1^2 & 0 & 0 & 0 & 0 \\ 0 & \sigma_2^2 & 0 & 0 & \sigma_{2\theta} \\ 0 & 0 & \sigma_\varepsilon^2 & 0 & 0 \\ 0 & 0 & 0 & \sigma_\eta^2 & 0 \\ 0 & \sigma_{2\theta} & 0 & 0 & \sigma_\theta^2 \end{pmatrix}\right) \tag{40}$$

Table 9: Banks with differing maturities

Type of bank	H	L
$share$	$p > 0$	$1 - p > 0$
Maturity	$M_A^H > T_{max}$	$M_A^L < T_{max}$
$term$	$M_A^H \theta_A - M_L \theta_L$	$M_A^L \theta_A - M_L \theta_L$
$\phi = C.NIM(T_{max}) - C.NIM(M_A)$	$\phi^H < 0$	$\phi^L = 0$
$share \cdot \phi \cdot \left(term - \overline{term}\right)$	$p\phi^H \left(M_A^H - \overline{M}\right) \theta_A$	0

This table shows components of the covariance between the variables "term" and ϕ. In this example (which can be easily generalized), there are two sorts of banks that only differ in the maturity of the asset side (the assumptions of Section 3 are valid).

Using the conditional expectation in Equation (23), we obtain for the parameters:

$$\tilde{\beta} = \beta \cdot \frac{\sigma_1^2}{\sigma_1^2 + \sigma_\eta^2} \tag{41}$$

$$\tilde{\gamma} = \gamma + \frac{\sigma_{2\theta}}{\sigma_2^2} \tag{42}$$

With the assumptions laid down in Subsection 5.3, one can calculate the covariance between "term" and ϕ (see Table 9).

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