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On the macroeconomic effects of reinvestments in asset purchase programmes

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Non-technical summary

Research question

Central banks have resorted to asset purchase programmes to stimulate the economy in light of a binding effective lower bound (ELB) on short-term nominal interest rates. A feature of recent programmes is the announcement of a reinvestment policy: the central bank keeps the overall volume of assets, i.e. the sum of net asset purchases, on the balance sheet constant for some time. What are the qualitative and quantitative effects of such reinvestment policies?

Contribution

We present a systematic model-based analysis of the macroeconomic effects of reinvestment policies. We quantify the macroeconomic effects of reinvestments by mapping key features of the Eurosystem's pandemic emergency purchase programme (PEPP) into our model – a programme that explicitly contained a reinvestment policy. Additionally, we use a piecewise-linear approach to perform stochastic simulations with a state-dependent asset purchase programme. This allows analysing how reinvestment policies can mitigate the constraints of upper purchase limits.

Results

We obtain four main results. First, omitting reinvestments in an asset purchase programme that embeds key features of the PEPP reduces the peak effect on inflation by roughly one third. Second, monetary policy can achieve a given macroeconomic stimulus by substituting a higher overall volume of assets on the central bank's balance sheet (more net purchases) with longer reinvestments. Based on the same programme as above, we show that monetary policy can decrease the overall volume by €400bn (or 30%) if it extends the reinvestment period from six to twelve quarters. If monetary policy completely abstains from reinvestments, it has to increase the overall volume by €1000bn (or 70%). Third, our stochastic simulations reveal that reinvestments can undo the detrimental impact of upper purchase limits on the inflation bias. For an upper purchase limit of 25% (33%; 50%), monetary policy can prolong the reinvestment period by five (four; two) quarters to reach the same inflation bias as in the case without an upper limit. Fourth, the quantitative impact of reinvestments depends on how agents form expectations. When they are boundedly rational, the macroeconomic impact of asset purchases in general as well as the marginal benefit of reinvestments are lower.

Nichttechnische Zusammenfassung

Fragestellung

Angesichts einer bindenden effektiven Zinsuntergrenze haben Zentralbanken auf Anleihekäufe zurückgegriffen, um die gesamtwirtschaftliche Entwicklung zu stabilisieren. Ein Merkmal der jüngsten Programme ist die Ankündigung einer Reinvestitionspolitik: Die Zentralbank hält das Gesamtvolumen der Aktiva, d. h. die Summe der Nettokäufe, für einige Zeit auf der Bilanz konstant. Welche qualitativen und quantitativen Auswirkungen hat eine solche Reinvestitionspolitik?

Beitrag

Wir stellen eine systematische modellbasierte Analyse der makroökonomischen Auswirkungen der Reinvestitionspolitik vor. Wir quantifizieren die makroökonomischen Effekte von Reinvestitionen, indem wir wesentliche Merkmale des Pandemie Notfallankaufprogramms (PEPP) des Eurosystems in unserem Modell abbilden – ein Programm, das explizit eine Reinvestitionspolitik enthielt. Darüber hinaus verwenden wir einen stückweise linearen (piecewise linear) Ansatz, um stochastische Simulationen mit einem zustandsabhängigen Ankaufprogramm durchzuführen. Auf diese Weise analysieren wir, wie Reinvestitionspolitiken die Beschränkungen etwaiger Kaufobergrenzen abmildern können.

Ergebnisse

Wir ermitteln vier wesentliche Ergebnisse. Erstens verringert der Wegfall von Reinvestitionen in einem Anleihekaufprogramm, das wesentliche Merkmale des PEPP enthält, den maximalen Inflationseffekt um rund ein Drittel. Zweitens kann die Geldpolitik einen ähnlichen großen makroökonomischen Impuls erzielen, wenn sie ein höheres Gesamtvolumen der Aktiva in der Zentralbankbilanz (mehr Nettoankäufe) durch längere Reinvestitionen ersetzt. Auf der Grundlage des obigen Programms zeigen wir, dass die Geldpolitik das Gesamtvolumen um 400 Mrd. € (oder 30%) verringern kann, wenn sie den Reinvestitionszeitraum von sechs auf zwölf Quartale verlängert. Verzichtet die Geldpolitik demgegenüber vollständig auf Reinvestitionen, muss sie das Gesamtvolumen um 1000 Mrd. € (oder 70%) erhöhen. Drittens zeigen unsere stochastischen Simulationen, dass Reinvestitionen die dämpfenden Auswirkungen von Kaufobergrenzen auf die Inflationsverzerrung aufheben können. Bei einer Kaufobergrenze von 25% (33%; 50%) kann die Geldpolitik den Reinvestitionszeitraum um fünf (vier; zwei) Quartale verlängern, um die gleiche Inflationsrate zu erreichen wie im Fall ohne Kaufobergrenze. Viertens hängen die quantitativen Auswirkungen von Reinvestitionen entscheidend davon ab, wie die Akteure ihre Erwartungen bilden. Wenn sie begrenzt rational sind, verringern sich die makroökonomischen Auswirkungen von Anleihekäufen im Allgemeinen sowie von Reinvestitionen im Besonderen.

Deutsche Bundesbank Discussion Paper No 47/2022

On the Macroeconomic Effects of Reinvestments in Asset Purchase Programmes*

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Abstract

A feature of recent monetary policy asset purchase programmes is the reinvestment policy: the central bank announces to keep the overall volume of assets on its balance sheet constant for some time. In this paper, we systematically assess the macroeconomic effects of such reinvestment policies. Conceptually, monetary policy can achieve a given macroeconomic stimulus by substituting higher overall volumes (more net purchases) with longer reinvestments. Quantitatively, we find that omitting reinvestments in a programme that embeds key features of the Eurosystem's pandemic emergency purchase programme reduces the effect on inflation by roughly one third. Stochastic simulations reveal that reinvestment policies can be applied to mitigate the constraints of upper purchase limits. Introducing bounded rationality attenuates the effects of reinvestment policies.

Keywords: Reinvestment, Stock effect, State-dependent asset purchases, Cognitive discounting, Bayesian estimation

JEL Classification: D78, E31, E44, E52, E58

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1 Introduction

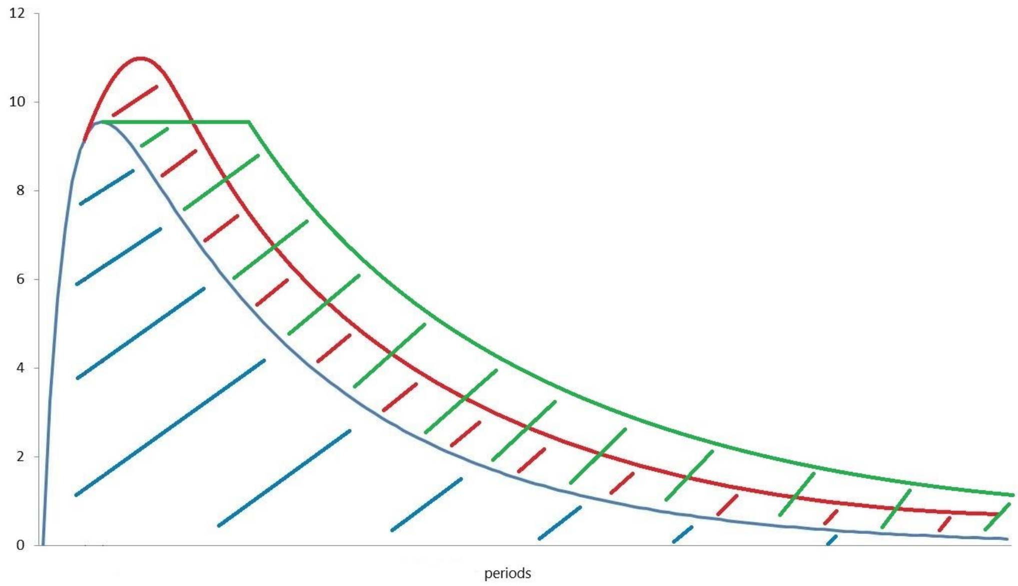
In the wake of the Great Recession of 2007/08 and the COVID-19 pandemic, central banks in advanced economies resorted to asset purchase programmes to stimulate the economy. A feature of recent programmes is the announcement of a reinvestment period. This is the period the central bank is going to hold the overall volume of assets, i.e. the sum of all net purchases, constant on the balance sheet. For example, the European Central Bank (ECB) announced on 4th June 2020: *"The maturing principal payments from securities purchased under the pandemic emergency purchase programme will be reinvested until at least the end of 2022"* (ECB, 2020). The length and the size of reinvestments also play a role in the recent normalisation of the Fed's balance sheet (Federal Reserve, 2022).

This paper presents a systematic analysis of reinvestment policies. Specifically, we assess their qualitative and quantitative effects within a dynamic stochastic general equilibrium (DSGE) model. In this framework, primarily the stock effect determines the macroeconomic effect of asset purchase programmes. The stock effect is an announcement effect. It implies that financial market participants immediately factor in the central bank's credible announcement of how the stock of assets on its balance sheet will evolve over time.[1] This evolution includes how long the overall volume that the central bank ultimately holds stays constant on its balance sheet, i.e., how long the central bank reinvests any maturing assets. As a result, it is the announcement of the whole evolution of the stock of assets on the central bank's balance sheet that affects financial markets and ultimately the economy at large.

Within our framework, we show that an additional reinvestment period enhances the macroeconomic stimulus of an asset purchase programme for a given overall volume, i.e. without the need of more net purchases. The reason is that the announcement of a longer period of reinvesting maturing assets constitutes an additional stock effect. Figure 1 shows this in a stylised way. The blue curve illustrates an expected evolution of the stock of assets on the central bank's balance sheet over time. The blue dashed area

[1] In contrast, the flow effect describes the impact of ongoing asset purchases in each period.

Figure 1: Illustration of stock effect – shifting the area beneath the curve



Notes: The figure depicts the asset holdings on the central bank's balance sheet (x-axis; in % of outstanding debt) over time (y-axis). The blue curve represents a baseline programme. The red curve illustrates an increase in net purchases and thus larger overall volume (red shaded area) compared to the blue curve. The green curve shows a programme with the same overall volume as the blue curve, but where reinvestments of maturing assets keep the balance sheet constant for some time. The stock effect implies that if the areas beneath the red and green curve are similar, so are the macroeconomic effect of each programme.

beneath the curve captures the stock effect for this programme. This area becomes larger when net purchases are higher and therefore the overall volume larger (red curve). As a result, there is an associated additional stock effect (red dashed area) and, correspondingly, a larger expansionary stimulus (all else being equal). Alternatively, one can generate an additional stock effect by reinvesting maturing assets for some time (green curve and associated green area). This keeps the balance sheet and therefore the overall volume of the programme constant for some time. In case the area beneath the red curve is similar as beneath the green one, the stock effect implies that the macroeconomic stimulus of both purchase programmes should be similar.[2]

In essence, this reasoning implies that the central bank can substitute higher overall volumes (more net purchases) with longer reinvestments (longer constant balance sheet size). Thus, the central bank has effectively two margins of adjustment to alter the

[2]See the discussion in Section 3 and 5 why the macroeconomic effects will be not exactly the same.

impact of an asset purchase programme. Accordingly, reinvestments are a feature of a purchase programme that can achieve two things. First, it can increase the stock effect and thus the intended macroeconomic stimulus – the comparison of the areas beneath the blue and the green curve. Second, it can shift area from the present towards the future to keep the same stimulus – the comparison of the areas beneath the red and the green curve. This allows the central bank to deploy reinvestment policies as an integral part of its monetary stance.

In order to quantify the impact of reinvestments, we employ a medium-scale two-agent New Keynesian model with a financial sector. The model features financial frictions (specifically a loan-in-advance constraint and limits to arbitrage) so that government bond purchases affect inflation and output. The existence of a second household that is "hand-to-mouth" and that receives countercyclical transfers mitigates the expansionary effects of future policies (to tame the forward guidance puzzle that is pertinent in this class of models, see e.g. McKay, Nakamura and Steinsson, 2016). The model framework is close to Gerke, Giesen and Scheer (2020a), which builds on Carlstrom, Fuerst and Paustian (2017). We estimate the model from 1999Q1 to 2014Q4 on eight euro area time series (GDP, inflation, investment, hours worked, short- and long-term interest rates, wage growth and net worth of financial intermediaries).

We quantify the impact of the reinvestment policy in a purchase programme that resembles the Eurosystem's pandemic emergency purchase programme (PEPP). We implement key features of the PEPP as communicated in June 2020, like the overall volume of €1350bn and six quarters of reinvestments in a baseline simulation.[3] To isolate the impact of the reinvestment policy, we run a counterfactual simulation without reinvestment. Thus, net purchases and the overall volume remain the same, but the periods of unwinding start directly afterwards. The omission of reinvestment reduces the macroeconomic impact in general, but leaves the dynamics largely unchanged. Overall, it reduces the peak effect of inflation by roughly one third.

[3] We regard all purchases under the PEPP as government bond purchases. According to the data from "History of monthly net purchase under the PEPP last update: 07 June 2022", at least 80% (and up to 97%) of purchases were alloted to public sector securities.

We then illustrate the substitutability between higher overall volumes and longer reinvestments when the macroeconomic effect should be the same as in the baseline simulation. We find that the central bank would have to raise the overall volume by €1000bn (≈ 70%) if there was no reinvestment period. In contrast, the central bank could have reduced its overall volume by €400bn (≈ 30%) if it doubled the reinvestment period to 12 quarters. Thus, our simulations indicate a quantitatively considerable role of reinvestments in stimulating the economy.

Monetary policy can use reinvestments in practice to mitigate the restrictive effects of self-imposed and/or legal upper purchase limits. For example, such limits are publicly announced in the case of the Eurosystem's public sector purchase programme (PSPP), the Federal Reserve's secondary market purchases of Treasury securities or the Bank of England asset purchase facility.[4] To soften or mitigate the impact of limits, the central bank can resort to reinvestments. This captures and applies the above-mentioned logic that monetary policy can substitute present net purchases with future reinvestments: while monetary policy cannot increase net purchases above the limit in the present, it can promise to keep the balance sheet constant via reinvestments for some time. This mirrors to some extent a lower-for-longer approach with respect to the interest rate at the effective lower bound (Eggertsson and Woodford, 2003): While the central bank cannot lower its policy rate below the effective lower bound (ELB) in the present, it promises to keep the interest rate lower for longer in the future.[5]

In order to demonstrate how reinvestment can mitigate upper purchase limits, we perform stochastic simulations. For this purpose, we extend our model framework in two dimensions. First, we add an ELB on nominal interest rates as a constraint. Second, we embed a state-dependent public asset purchase programme with limits and reinvestment into this framework. The extension reflects that net purchases only take

[4]For the Eurosystem, see Decision (EU) 2015/774 of the European Central Bank. For the Federal Reserve, see the FAQs on Treasury Purchases on `https://www.newyorkfed.org/markets/treasury-reinvestments-purchases-faq`. For the Bank of England see the Consolidated Market Notice: Asset Purchase Facility: Gilt Purchases - Market Notice published on 11 June 2019.

[5]Gerke, Kienzler and Scheer (2021) illustrate how make-up strategies, which embed a lower-for-longer approach, might help to mitigate the consequences of upper purchase limits. Make-up strategies would reduce the severity of the ELB and thus the need for asset purchase programmes at the ELB.

place when interest rates are at the ELB and that the size of net purchases depends on the inflation shortfall from its target. It thereby captures key features of the public sector purchase programme (PSPP) of the Eurosystem in the past. We estimate the strength of the state dependency with data on inflation and PSPP purchases. To simulate the model with an ELB and a state-dependent, non-linear purchase programme, we employ an extension of the piecewise-linear solution approach of Kulish and Pagan (2017).

We obtain four main results. First, the ELB causes a significant deterioration in economic performance when the central bank has only the short-term nominal interest rate at its disposal. This is reflected by a negative inflation bias of about 50bp – i.e. the average inflation rate would be about 1.5% instead of 2% – and an increase in macroeconomic volatility.[6] Second, if the central bank does resort to asset purchases without an upper limit or reinvestment, it can lower the inflation bias to 20bps. Hence, asset purchases are effective but might not necessarily be sufficient on their own to reach the inflation target.[7] Third, if upper limits of either 25%, 33% and 50% of outstanding debt restrict the possible overall volume of the purchase programme, the inflation bias increases between 10bps (50% upper limit) and 25bps (25% upper limit) relative to the case of unlimited purchases. The lower the limit, the lower the potential stimulus and thus the larger the inflation bias and the macroeconomic volatility. Fourth, longer reinvestment periods help to mitigate the impact of upper limits. In particular, if the central bank prolongs reinvestment periods by two to five quarters (depneding on the level of the limit) it reaches roughly the same inflation bias as in the case without an upper limit. The lower the limit, the longer the necessary reinvestment.

Our stochastic simulations illustrate a notable impact of reinvestment policies. As explained above, this stems from the stock effect, which is an announcement effect.

[6] Both effects are consistent with the macroeconomic literature on the ELB. See, for example, Kiley and Roberts (2017) and Bernanke (2020) for the US, and Andrade, Galí, Le Bihan and Matheron (2021) and Coenen, Montes-Galdón and Schmidt (2021) for the euro area.

[7] While we focus on asset purchases to enhance monetary stimulus, the central bank can also follow other alternative unconventional policies. Examples include the use of a negative interest rate policy (e.g. Ulate, 2021; Gerke, Giesen and Scheer, 2021), forward guidance (e.g. Campell, Evans, Fisher and Justiniano, 2012) or a mix of unconventional policies (e.g. Sims and Wu, 2021).

Hence, the impact of reinvestment depends on the agents being rational and forward-looking. However, recent research on expectation formation has increasingly documented substantial deviations from full information rational expectations (Coibion, Gorodnichenko and Kamdar, 2018; Afrouzi, Kwon, Landier, Ma and Thesmar, 2021). As a cross-check, we therefore assess how effective asset purchases and reinvestments remain when expectations deviate from rational expectations. Specifically, we assume that agents have boundedly rational expectations in the spirit of Gabaix (2020). This assumption renders agents partially myopic in the sense that they additionally discount the future. We find that asset purchases in general are then less effective in stimulating the economy. With respect to reinvestments, we find that the marginal benefit of an additional period of reinvestment is lower when agents are boundedly rational. Both results appear intuitive: as agents are not perfectly forward-looking, the stock effect is discounted. Essentially, the area under the curve becomes de facto smaller when agents are boundedly rational than if they are fully rational. As a result, the announcement of asset purchase programmes in general and reinvestments in particular are less effective. We leave a full-fledged quantitative analysis of reinvestment policies when agents are boundedly rational for future research.

Related Literature

Our paper is related to the literature on the effect of asset purchases and the effective lower bound (ELB) and it touches on the literature on the implications of deviations from rational expectations. Although a relatively large volume of literature on the macroeconomic effects of asset purchases has evolved over the last two decades, we are, to the best of our knowledge, the first to assess systematically the effects of reinvestment policies.

The main mechanism of the paper relies on the predominance of the stock effect to determine the macroeconomic impact of asset purchases and reinvestments. This is in line with empirical evidence (e.g. D'Amico and King, 2013; Sudo and Tanaka, 2021) and also holds in other models (e.g. Chen, Cúrdia and Ferrero, 2012; Gertler and Karadi, 2013).

There is an extensive literature studying the impact of asset purchases on yields, output, inflation expectations and inflation rates. In general, the empirical evidence points to significant effects on financing conditions, output and inflation.[8] We add to the literature by explicitly allowing for a reinvestment policy. This not only renders an asset purchase programme empirically more realistic, but also emphasises the two margins that the central bank can adjust: the size of its programme and the time that the assets stay constant on its balance sheet.

We implement an endogenous asset purchase programme in our simulations. Kiley (2018) and Bernanke (2020) tie the amount of fixed purchases to a threshold value for the output gap. Our specification is closely related to Burlon, Notarpietro and Pisani (2019). They condition the amount of net purchases to the expected inflation shortfall from target. However, they estimate the strength of the state dependency based on five data points (the changes in fixed monthly purchases that occured). We extend their approach by using monthly data on realised inflation and net purchases from March 2015 to December 2018. Coenen, Montes-Galdón and Smets (2020) condition the purchased volume on a (model-implied) shadow interest rate that would have prevailed in the absence of the ELB. The advantage of our modelling approach is that the central bank does not have to rely on a latent variable, which is not observable in real time.

Lastly, we allow for deviations from rational expectations in a medium-scale model. We follow Erceg, Jakab and Lindé (2021) and Ilabaca, Meggiorini and Milani (2020) and add a behavioral element to the model by introducing "cognitive discounting" as in Gabaix (2020).

The rest of the paper is organised as follows: Section 2 lays out the model framework in detail as well as the estimation. Section 3 illustrates the substitutability of higher net purchases and longer reinvestments with reference to the PEPP. Section 4 quantifies the impact of limits and the amount of reinvestment needed to mitigate the limits. Section 5 shows how deviations from rational expectations can worsen the substitutability

[8] For a summary and references to various papers in this area, see Bhattarai and Neely (2022) or chapter 2.5 in Work stream on the price stability objective (2021).

between net purchase and reinvestment. Finally, Section 6 concludes.

2 Framework and estimation

This section lays out in detail the framework and the parametrisation/estimation of the model. We use the model of Gerke, Giesen and Scheer (2020a), which is a quantitative two-agent New Keynesian model that builds on the work of Carlstrom et al. (2017), Galí, López-Salido and Vallés (2007) and Bilbiie (2008). The economy consists of households, three types of firms (final goods firm, intermediate goods firm and capital goods firm), a government, financial intermediaries and segmented financial markets. The latter allow us to analyse the effects of asset purchases.

2.1 Households

The economy is populated by two types of households: measure $1-\lambda$ of households has complete access to financial markets and can smooth consumption through short-term deposits and the accumulation of real capital – we call them Ricardian or optimising households. The remaining fraction λ has no access to financial markets (it can neither borrow nor save) and in every period consumes its labour income and transfers entirely – we call them hand-to-mouth households.

Each optimising household (denoted by the superscript o) maximises expected lifetime utility

$$E_t \sum_{s=0}^{\infty} \beta^s d_{t+s} \left\{ \ln\left(C^o_{t+s} - hC^o_{t+s-1}\right) - B\frac{H^{1+\eta}_{t+s}}{1+\eta} \right\}, \tag{1}$$

where C^o_t denotes private consumption, h degree of habit, H_t the (individual) labour input (scaled by B to normalise labour input in the steady state) and d_t a demand shock. The latter is given by:

$$\ln(d_t) = (1-\rho_d)\ln(d) + \rho_d \ln(d_{t-1}) + \varepsilon_{d,t}. \tag{2}$$

The budget constraint of the optimising household is given by

$$C_t^o + P_t^k I_t^o + \frac{D_t}{P_t} + (1 + \kappa Q_t)\frac{F_{t-1}}{P_t} = w_t H_t + R_t^k K_t + \frac{D_{t-1}}{P_t} R_{t-1}^d + div_t - T_t^o + \frac{Q_t F_t}{P_t} \quad (3)$$

Households invest in real capital I_t at the price P_t^k, save deposits D_t and repay their outstanding debt including a coupon payment of 1, $(1 + \kappa Q_t)\frac{F_{t-1}}{P_t}$ (see below for more details). They earn labour income $w_t H_t$ (to be specified below), a return on capital $R_t^k K_t$ and deposits $R_{t-1}^d \frac{D_{t-1}}{P_t}$ and dividends div_t net of taxes T_t^o (which consist of a lump-sum part and a re-distributive part; see subsection 2.4 for details). div_t comprises dividends from the financial intermediaries (div_t^{FI}), a capital goods producer (div_t^{CP}) and an intermediate goods producer (div_t^{IG}).

There is a need for intermediation through the financial system since all of the household's investment purchases must be financed beforehand by issuing new investment bonds (hence, there is a loan-in-advance constraint). The price of such bonds is denoted by Q_t and offers the payment stream $1, \kappa, \kappa^2, \ldots$, following Woodford (2001). CI_t denotes the number of new perpetuities issued in t, the household's stock of nominal liabilities F_t is then given by

$$F_t = \kappa F_{t-1} + CI_t \;\; \Leftrightarrow CI_t = F_t - \kappa F_{t-1}. \quad (4)$$

The loan-in-advance constraint is then given by:

$$P_t^k I_t \leq \frac{Q_t CI_t}{P_t} \quad (5)$$

The law of motion for capital follows:

$$K_t = (1 - \delta) K_{t-1} + I_t. \quad (6)$$

The Ricardian household maximises utility (1) subject to the budget constraint (3), the loan-in-advance constraint (5) and the law of motion for capital (6). The first-order conditions are given by:

$$\Lambda_t^o = \frac{d_t}{C_t^o - hC_{t-1}^o} - E_t \frac{\beta h d_{t+1}}{C_{t+1}^o - hC_t^o} \tag{7}$$

$$\Lambda_t^o = E_t \beta \frac{\Lambda_{t+1}^o}{\Pi_{t+1}} R_t^d \quad \text{with} \quad \Pi_{t+1} = \frac{P_{t+1}}{P_t} \tag{8}$$

$$\Lambda_t^o M_t Q_t = E_t \frac{\beta \Lambda_{t+1}^o \left(1 + \kappa Q_{t+1} M_{t+1}\right)}{\Pi_{t+1}} \tag{9}$$

$$\Lambda_t^o M_t P_t^k = E_t \beta \Lambda_{t+1}^o \left[R_{t+1}^k + M_{t+1} P_{t+1}^k (1 - \delta)\right] \tag{10}$$

with $M_t = 1 + \frac{\vartheta_t}{\Lambda_t}$ or $\Lambda_t M_t = \Lambda_t^o + \vartheta_t$. The first two equations comprise the typical Euler equation for deposits, and the third the one for investment bonds. The fourth, equation (10), describes the demand for capital. It is distorted by the time-varying wedge M_t which depends on the multiplier of the loan-in-advance constraint (5). As discussed in great detail in Carlstrom et al. (2017), this distortion acts as a mark-up on the price of new capital, giving rise to a term premium. Such a term premium exists due to the segmented markets and the leverage constraint of the banks that limit the arbitrage across the term structure (see next subsection).

The budget constraint of hand-to-mouth households is much simpler as they neither borrow nor save and only consume their labour income less taxes:

$$C_t^h = w_t H_t - T_t^h, \tag{11}$$

where their consumption is C_t^h, their labour income is $w_t H_t$ (see below) and T_t^h are taxes that hand-to-mouth households have to pay. Overall taxes are given by a time-invariant component T^h and a countercyclical transfer scheme:

$$T_t^h = \frac{\tau}{\lambda} (Y_t - Y) + T^h. \tag{12}$$

$\tau \geq 0$ captures the degree of countercyclical transfers which rebates income whenever aggregate output is different from the steady state $(Y_t - Y)$. Although this transfer scheme is stylised, it captures parsimoniously automatic stabilisers that are found in more complex settings (see, for instance, McKay and Reis, 2016; Leeper, Plante and Traum, 2010). Additionally, it is the most direct way to introduce redistribution within the two types of households.

2.1.1 Labour agencies

Each household supplies a specialised type of labour H_t^j, irrespective of whether it is a Ricardian or a hand-to-mouth household (in the spirit of Erceg, Henderson and Levin, 2000). Since firms do not differentiate between the two household types when hiring labour for a specialised type j, the supply of hours and the wage rate are the same for both groups. The labour agencies bundle the specialised labour inputs into a homogeneous labour output that they sell to an intermediate good firm according to

$$H_t = \left[\int_0^1 \left(H_t^j\right)^{1/\left(1+\lambda_{w,t}\right)} dj\right]^{1+\lambda_{w,t}} \tag{13}$$

where $\lambda_{w,t}$ is the wage mark-up, following the log-normal process

$$\ln\left(\lambda_{w,t}\right) = (1-\rho_{\lambda^w})\ln(\lambda_w) + \rho_{\lambda^w}\ln\left(\lambda_{w,t-1}\right) + \varepsilon_{\lambda_{w,t}}. \tag{14}$$

The demand for the different types of labour inputs is given by

$$H_t^j = \left(\frac{W_t^j}{W_t}\right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} H_t \tag{15}$$

In each period, the probability of resetting the wage is $(1-\theta_w)$, while with the complementary probability (θ_w) the wage is automatically increased following the indexation rule:

$$W_t^j = \Pi_{t-1}^{\iota_w} W_{t-1}^j$$

The maximisation problem of a given union for the specialised labour input j is given by:

$$\max_{\tilde{W}_t} E_t \sum_{s=0}^{\infty} (\beta\theta_w)^s \left\{ (1-\lambda)\, u\left(C^o_{t+s}\right) + \lambda u\left(C^h_{t+s}\right) - d_{t+s}\Lambda^a_{t+s} B \frac{H^{1+\eta}_{t+s}}{1+\eta} \right\}$$

s.t. the budget constraints (3), (11) and labour demand (15) and with $\Lambda^a_{t+s} = (1-\lambda)\,\Lambda^o_{t+s} + \lambda\Lambda^h_{t+s}$ (similar to Colciago, 2011).[9]

2.2 Financial intermediaries

The financial intermediaries (FIs) in the model use accumulated net worth N_t and short-term deposits D_t to finance investment bonds F_t and long-term government bonds B_t. Their balance sheet is given by:

$$\underbrace{Q_t \frac{B_t}{P_t}}_{\bar{B}_t} + \underbrace{Q_t \frac{F_t}{P_t}}_{\bar{F}_t} = N_t + \frac{D_t}{P_t} = L_t N_t, \tag{16}$$

where L_t denotes leverage, $\bar{B}_t$ and $\bar{F}_t$ the real market value of government and investment bonds. Note that investment and government bonds are perfect substitutes since they offer the same payment streams and thus are valued at the same price Q_t. Define the return on those bonds as R^L_t:

$$R^L_t \equiv \frac{1+\kappa Q_t}{Q_{t-1}}. \tag{17}$$

Every period an FI receives the coupon payment of 1 from its old assets in $t-1$. Its income is thus $(1+\kappa Q_t)\left(\frac{B_{t-1}}{P_t} + \frac{F_{t-1}}{P_t}\right)$. It purchases new assets at price Q_t, such that the real value of these purchases is $Q_t\left(\frac{F_t}{P_t} + \frac{B_t}{P_t}\right)$. It further collects new deposits D_t and has to pay out interest rate expenses on the deposits of the previous period $R^d_{t-1}\frac{D_{t-1}}{P_t}$. Any deviation of net worth from the steady state will be costly: $f(N_t)N_t$, with $f(N_t) = \frac{\Psi_n}{2}\left(\frac{N_t - N}{N}\right)^2$. Thus, the remaining dividend payments are given by interest

[9] We define $\Lambda^h_{t+s} = d_{t+s}\frac{1}{c^h_{t+s}}$, i.e. without habit.

income less the expenditures:

$$div_t^{FI} = (1 + \kappa Q_t)\left(\frac{B_{t-1}}{P_t} + \frac{F_{t-1}}{P_t}\right) + \frac{D_t}{P_t} - Q_t\left(\frac{F_t}{P_t} + \frac{B_t}{P_t}\right) - R_{t-1}^d \frac{D_{t-1}}{P_t} - f(N_t)N_t$$

$$\Leftrightarrow div_t^{FI} + (1 + N_t)\, f(N_t) = \underbrace{\frac{P_{t-1}}{P_t}\left(\left(R_t^L - R_{t-1}^d\right)L_{t-1} + R_{t-1}^d\right)N_{t-1}}_{\text{profits}}, \tag{18}$$

where the definition of the return R_t^L and the banks' balance sheet (16) were substituted. This equation shows that profits will be partly paid out as dividends div_t^{FI} to the (Ricardian) households while the rest is retained as net worth for subsequent activity. The FI discounts dividend flows using the (Ricardian) household's pricing kernel augmented with additional impatience $\zeta < 1$. The latter is needed to restrict accumulation of net worth and growth out of the constraint. Ultimately, this allows for a positive excess return of long-term debt over deposits also in the steady state.

The FI then chooses dividends div_t^{FI} and net worth N_t to maximise expected dividend payments

$$V_t = E_t \sum_{s=0}^{\infty} (\beta\zeta)^s \Lambda_{t+s} div_{t+s}^{FI} \tag{19}$$

subject to (18). This yields the following first-order condition:

$$\Lambda_t \left[1 + f(N_t) + N_t f'(N_t)\right] = E_t \Lambda_{t+1} \beta\zeta \frac{P_t}{P_{t+1}} \left[\left(R_{t+1}^L - R_t^d\right)L_t + R_t^d\right]. \tag{20}$$

The FIs are subject to a simple hold-up problem which limits their ability to attract deposits (similar in spirit to Gertler and Karadi, 2013). We follow the approach by Carlstrom et al. (2017) and arrive at the following expression for the leverage constraint L_t:

$$L_t = \frac{1}{\left[1 + (\Phi_t - 1)\, E_t \frac{R_{t+1}^L}{R_t^d}\right]}, \tag{21}$$

where Φ_t measures exogenous changes in the financial friction:

$$\ln(\Phi_t) = (1 - \rho_\Phi)\ln(\Phi) + \rho_\Phi \ln(\Phi_{t-1}) + \varepsilon_{\Phi,t}. \tag{22}$$

2.3 Goods market

Perfectly competitive final goods producers combine differentiated intermediate goods $Y_t(i)$ into a homogeneous good Y_t according to the technology:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{1+\lambda_{p,t}}} di \right]^{1+\lambda_{p,t}}$$

where $\lambda_{p,t}$ is the time-varying price mark-up that evolves according to

$$\ln\left(\lambda_{p,t}\right) = (1 - \rho_{\lambda^p}) \ln(\lambda_p) + \rho_{\lambda^p} \ln\left(\lambda_{p,t-1}\right) + \varepsilon_{\lambda_{p,t}}. \tag{23}$$

Profit maximisation leads to the following demand function:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} Y_t, \tag{24}$$

with

$$P_t = \left[\int_0^1 P_t(i)^{-\frac{1}{\lambda_{w,t}}} di \right]^{-\lambda_{w,t}}. \tag{25}$$

A continuum of monopolistic competitive firms combines capital K_{t-1} and labour H_t to produce intermediate goods according to a standard Cobb-Douglas technology. The production function is given by:

$$Y_t(i) = A_t K_{t-1}(i)^{\alpha} H_t(i)^{1-\alpha} \tag{26}$$

with

$$\ln(A_t) = (1 - \rho_A) \ln(A) + \rho_A \ln(A_{t-1}) + \varepsilon_{A,t}. \tag{27}$$

The intermediate goods producers set prices based on Calvo contracts. In each period firms adjust their prices with probability $\left(1 - \theta_p\right)$ independently of previous adjustments. Those firms that cannot adjust their prices in a given period re-set their prices according to the following indexation rule:

$$P_t(i) = \Pi_{t-1}^{\iota_p} P_{t-1}(i).$$

Firms that can adjust their prices face the following problem:

$$\max_{P_t^*} E_t \sum_{s=0}^{\infty} \theta_p^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_t} \left[\frac{P_t^* \left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_p} \right)}{P_{t+s}} Y_{t+s}(i) - \frac{W_{t+s}}{P_{t+s}} H_{t+s}(i) - R_{t+s}^k K_{t-1+s}(i) \right],$$

subject to labour demand (15) and $Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon_{p,t}} Y_t$. It holds that dividends are given by $div_t^{IG} = Y_t - w_t H_t - R_t^k K_{t-1}$.

The capital goods producers take final investment output I_t and sell it with a mark-up to the households, subject to adjustment costs. Therefore, dividends $div_t^{CP} = P_t^k I_t^n - I_t = P_t^k \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t - I_t$, where the investment-specific technology shock follows an AR(1) process:

$$\ln(\mu_t) = (1 - \rho_\mu) \ln(\mu) + \rho_\mu \ln(\mu_{t-1}) + \varepsilon_{\mu,t}. \tag{28}$$

The profit maximisation is then described by

$$\max_{I_t} E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t+s} \left[P_{t+s}^k \mu_{t+s} \left[1 - S\left(\frac{I_{t+s}}{I_{t+s-1}}\right)\right] I_{t+s}) - I_{t+s} \right]. \tag{29}$$

2.4 Government policies

The government consists of two authorities. First, fiscal policy focuses on the redistribution between the two types of households. Second, a central bank sets the interest rate (and later also resorts to asset purchases).

2.4.1 Fiscal policy

The government collects taxes T_t in a lump-sum fashion and issues government bonds $\frac{Q_t B_t^g}{P_t}$ to finance its outstanding debt including coupon payments $(1+\kappa Q_t)\frac{B_{t-1}^g}{P_t}$. Note, B_t^g denotes the amount of government bonds that the government issued in period t. B_t is the amount of government bonds that is held by the financial intermediaries. It holds that

$$B_t^g = B_t. \tag{30}$$

This equilibrium condition will be different in Sections 3 to 5 where we allow for central bank holdings of government debt, B_t^{CB} – see Section.

The government budget constraint is given by

$$\frac{Q_t B_t^g}{P_t} + T_t = (1+\kappa Q_t)\frac{B_{t-1}^g}{P_t}. \tag{31}$$

Note that tax-income $T_t = \lambda T_t^h + (1-\lambda) T_t^o$ is net of the countercyclical transfers paid to hand-to-mouth households. Implicitly, there is a redistribution of countercyclical transfers $\tau(Y_t - Y)$ from optimising to hand-to-mouth households (via the government). The respective tax rules for both agents are given by the following two equations:

$$T_t^o = \frac{1}{1-\lambda}\left(T^t + T^o - \tau(Y_t - Y)\right) \tag{32}$$

$$T_t^h = T^h + \frac{\tau}{\lambda}(Y_t - Y). \tag{33}$$

For simplicity, only the Ricardian households finance the government. Additionally, they are involved in the countercyclical transfer system in which the hand-to-mouth households participate as well. The degree of countercyclicality is given by τ. T^o and T^h are chosen such that consumption of hand-to-mouth and Ricardian households coincides in the steady state.

2.4.2 Central bank

The central bank follows a Taylor rule when setting its short-term policy rate R_t (since short-term government debt and bank deposits are perfect substitutes, it holds that $R_t^d = R_t$):

$$\ln(R_t) = (1-\rho)\ln(R) + \rho\ln(R_{t-1}) + (1-\rho)\left(\tau_\pi(\pi_t - \pi) + \tau_y(y_t - y_{t-1})\right) + \ln\left(R_t^\epsilon\right), \quad (34)$$

with

$$\ln\left(R_t^\epsilon\right) = (1-\rho_m)\ln(R^\epsilon) + \rho_m \ln\left(R_{t-1}^\epsilon\right) + \varepsilon_{R,t}.$$

2.5 Aggregation

Using the household and the government budget constraint, as well as all dividend payments, one arrives at the aggregate resource constraint

$$Y_t = C_t + I_t + f(N_t)N_t, \quad (35)$$

where aggregate consumption and investment are given by a weighted average of the respective variables for optimising and hand-to-mouth households:

$$C_t = (1-\lambda)\,C_t^o + \lambda C_t^h \quad (36)$$

and

$$I_t = (1-\lambda)\,I_t^o. \quad (37)$$

Similarly, the aggregate capital stock is given by

$$K_t = (1-\lambda)\,K_t^o. \quad (38)$$

2.6 Estimation

After linearising the model around the deterministic steady state, we estimate it with Bayesian methods. We use eight quarterly euro area time series for the sample period 1999Q1 to 2014Q4.[10]

2.6.1 Data

We use a total of eight observables for the euro area: real per capita GDP, real investment, gross inflation, employment growth, real wage growth, the short-term interest rate, the long-term interest rate and real bank net worth growth. The time series on bank net worth is taken from the European Central Bank's MFI Balance Sheet Items Statistics. All the other variables are taken from the Area-Wide Model database of the ECB (see the online data appendix). Since we have only seven structural shocks in the model, we add a measurement error to the observation equation for bank net worth in order to avoid stochastic singularity.

Per capita output and investment are obtained by dividing real GDP (YER) and investment (ITR) by the labour force (LFN). Growth rates are log-differences. Inflation is measured as the growth rate of the Harmonised Index of Consumer Prices (HICPSA). Employment growth is the log-difference of total employment (LNN). For the real wage series we first divide the nominal wage rate per capita (WRN) by the HICPSA and then take the log-difference. Our short-term nominal interest rate is the 3-month Euribor rate (STN) and our long-term nominal interest rate is the euro area 10-year government benchmark bond yield (LTN). Real bank net worth is obtained by dividing the nominal capital and reserves of euro area monetary financial institutions (NWB) by HICPSA and taking the log-difference. All growth series are de-meaned with their respective sample mean. The following table summarises the observation equations, where a hat denotes the log-deviation from the steady state, i.e. $\hat{y}_t = \ln(Y_t) - \ln(Y)$.

[10] The estimation is closely related to previous work in Gerke, Giesen and Scheer (2020b). We use the Dynare software package for the estimation, see Adjemian, Bastani, Juillard, Karamé, Maih, Mihoubi, Mutschler, Perendia, Pfeifer, Ratto and Villemot (2011) for details. We end in 2014 because of the period of binding effective lower bounds from 2015 onwards.

$$\begin{bmatrix} \text{dlGDP}_t \\ \text{dlInvestment}_t \\ \text{dlHICPSA}_t \\ \text{ShortInterestRate}_t \\ \text{LongInterestRate}_t \\ \text{dlHours}_t \\ \text{dlWages}_t \\ \text{dlNetworth}_t \end{bmatrix} = 100 \cdot \begin{bmatrix} 0 \\ 0 \\ \log(\Pi) \\ \log(\Pi/\beta) \\ \log(\Pi/\beta) + 0.01/4 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \hat{y}_t - \hat{y}_{t-1} \\ \hat{x}_t - \hat{x}_{t-1} \\ \hat{\pi}_t \\ \hat{r}_t \\ \hat{r}_t^{L,10} \\ \hat{h}_t - \hat{h}_{t-1} \\ \hat{w}_t - \hat{w}_{t-1} \\ \hat{n}_t - \hat{n}_{t-1} + \varepsilon_{n,t} \end{bmatrix}.$$

2.6.2 Calibration and prior distributions

As is common in the literature, we calibrate a subset of the structural parameters. We mostly follow the calibration of Carlstrom et al. (2017) and Coenen, Karadi, Schmidt and Warne (2018). The time preference β is set to 0.99, yielding a steady-state annual real interest rate of roughly 4% (for the simulations below, we also calibrate this parameter such that the steady-state annual real interest rate is at 0.5%, see Section 3). The labour income share α is set to 0.33 and the depreciation rate to $\delta = 0.025$, which implies a 10% annual depreciation of the capital stock. The steady-state mark-ups of prices and wages are set to 20%, i.e. $\lambda_w = \lambda_p = 0.2$. The leverage ratio is set to 6 which implies $\zeta = 0.9854$. We impose the condition that in the steady state the annual long-term rate R^L is one percentage point above the short-term rate, i.e. $R^L = R^{L,10} = R + 0.01/4$ (in line with the data). In order to estimate the model with a 10-year government bond (similar to its empirical counterpart) we set $\kappa = 0.975$. It was not possible to identify the share of hand-to-mouth households λ and the redistribution coefficient τ simultaneously in the data. Therefore, we calibrate the share of hand-to-mouth households to 30% according to empirical evidence (e.g. Dolls, Fuest and Peichl, 2012; Bilbiie and Straub, 2013; Fève and Sahuc, 2017).[11] However, the share of hand-to-mouth households does not effect the aggregate impact of asset purchases

[11] As a cross-check, we estimated the model with the calibrated redistribution τ (at the posterior mean of Table 1) and found a share of hand-to-mouth households of around 35%.

Table 1: Prior and posterior distribution of estimated parameters

		Prior			Posterior			
		Dist	Mean	SE	Mode	Mean	5 percent	95 percent
Utility & technology								
h	Habit	B	0.500	0.2000	0.7921	0.7897	0.7211	0.8576
η	Inverse Frisch	G	2.000	0.5000	1.5801	1.7460	1.0043	2.4766
ψ_I	Investment adj.	G	10.000	1.0000	14.1247	14.2112	12.3537	16.0382
ψ_N	Net worth adj.	G	3.000	1.0000	6.2004	6.5485	4.6520	8.3655
Stickiness								
ι_p	Price indexation	B	0.600	0.1000	0.4872	0.5187	0.3465	0.6859
ι_w	Wage indexation	B	0.600	0.1000	0.3189	0.3443	0.2252	0.4591
θ_p	Price stickiness	B	0.700	0.1000	0.8015	0.8127	0.7573	0.8692
θ_w	Wage stickiness	B	0.700	0.1000	0.8646	0.8581	0.8131	0.9036
Government policy								
ρ_m	MP smoothing	B	0.750	0.1000	0.7679	0.7655	0.7174	0.8148
τ_π	MP on inflation	N	1.500	0.1000	1.5969	1.6280	1.4818	1.7696
τ_y	MP on output	N	0.500	0.1000	0.6146	0.6217	0.4652	0.7758
τ	Redistribution	B	0.100	0.0500	0.1666	0.1756	0.0642	0.2855
AR(1) shocks								
ρ_a	TFP	B	0.600	0.2000	0.9846	0.9800	0.9637	0.9979
ρ_ϕ	Financial friction	B	0.600	0.2000	0.7491	0.7366	0.6742	0.8010
ρ_μ	Investment	B	0.600	0.2000	0.9316	0.9246	0.8849	0.9644
ρ_{λ_w}	Wage mark-up	B	0.600	0.2000	0.2405	0.2760	0.0731	0.4565
ρ_{λ_p}	Price mark-up	B	0.600	0.2000	0.5130	0.4569	0.2393	0.6647
ρ_d	Demand	B	0.600	0.2000	0.5868	0.5874	0.4328	0.7493
ρ_{res}	Monetary policy	B	0.600	0.2000	0.4843	0.4730	0.3328	0.6176
std shocks								
ε_R	Monetary policy	IG	0.010	1.0000	0.0030	0.0031	0.0027	0.0035
ε_A	TFP	IG	0.010	1.0000	0.0058	0.0059	0.0050	0.0067
ε_ϕ	Financial friction	IG	0.050	1.0000	0.1623	0.1736	0.1290	0.2145
ε_μ	Investment	IG	0.500	1.0000	0.0985	0.1007	0.0833	0.1179
ε_{λ_w}	Wage mark-up	IG	0.100	1.0000	0.8377	0.9434	0.2504	1.6418
ε_{λ_p}	Price mark-up	IG	0.100	1.0000	0.0438	0.0591	0.0242	0.0967
ε_D	Demand	IG	0.100	1.0000	0.0346	0.0375	0.0258	0.0492
ε_N^{ME}	ME on net worth	IG	0.001	1.0000	0.0119	0.0122	0.0104	0.0139

Notes: B stands for the beta, G for the gamma, IG for the inverted gamma and N for the normal distribution.

quantitatively strong, see Appendix C.

The choices for the prior distributions are summarised in columns 2 to 4 of Table 1. The first block of parameters determine the shape of the utility and cost functions. For the level of consumption habit h, we use a beta distribution with a mean of 0.5 and a standard deviation of 0.2. The inverse Frisch elasticity η has a relatively flat prior centred around 2. The prior mean and standard deviation for the investment adjustment costs Ψ_I are taken from the posterior mode of Coenen et al. (2018).

For the degree of indexation and stickiness, we use a beta distribution centred around 0.6 and 0.7, respectively, with a standard deviation of 0.1 for all four parameters, which is slightly below the values in Coenen et al. (2018).

The prior for the persistence of monetary policy is a beta distribution with mean 0.75 and standard deviation of 0.1. The two Taylor coefficients on inflation and output growth both follow a normal distribution centred around 1.5 and 0.5 respectively. For the size of redistribution we take a relatively flat prior around 0.1, which is the posterior mean for a US estimate (Leeper et al., 2010).

We set the prior distribution for all autocorrelations of the exogenous shock processes as a beta distribution which is centred around 0.6 with a standard deviation of 0.2. For better identification of the autocorrelation of the monetary policy shock and the persistence in the Taylor rule, we use a slightly tighter prior for the latter. All priors for the standard deviations of shocks follow a relatively flat inverse gamma distribution with a standard deviation of 1. The prior of the demand shock as well as the wage and price markup are centred around 0.1. For TFP and monetary policy we use a lower value of 0.01. For the financial friction and the investment-specific technology shock, we use a larger value of 0.5. The mean for the measurement error on net worth is set to 10% of the standard deviation of the underlying data sample.

2.6.3 Posterior distribution

With the prior distributions specified above, we draw from the posterior distributions using the Metropolis-Hastings algorithm with two chains, each with 2,000,000 draws. In order to assess the convergence of the chains, we compute several measures following Brooks and Gelman (1998). The interval of the posterior distribution which is covered by the chains, as well as the second moment of the posterior distribution, seem to be stable for most parameters after approximately 1,000,000 draws. To ensure that results are reported based on parameter draws that have converged, we report results based on the last 100,000 draws of each chain.

The last columns of Table 1 report the posterior mode, the posterior mean, and the lower and upper bounds of the 90% posterior density interval of the estimated parameters obtained by the Metropolis-Hastings algorithm. Most of our estimates are in line with similar estimates for the euro area (e.g. Smets and Wouters, 2003; Coenen et al., 2018). In the online appendix we plot the prior and posterior distribution of each parameter.

Compared to the above two studies, our data points to a slightly higher value of habit persistence and wage stickiness as well as a much lower persistence of monetary policy (around 0.77 compared to over 0.9 in the other two studies). However, note that the monetary policy shock is relatively persistent. We estimate a degree of redistribution $\tau \sim 0.17$.[12] This is relatively close to Leeper et al. (2010), who find τ in the range of 0.05 to 0.25 with a mean of 0.13 for a similar transfer rule in a representative agent model.

3 Reinvestment policies: the case of the ECB's pandemic emergency purchase programme

In this section, we illustrate two aspects. First, the quantitative effects of a reinvestment policy. Second, the substitutability of a larger overall volume (more net purchases) vs. a longer reinvestment period (longer constant balance sheet size). Throughout our simulations, we consider asset purchases as government bond purchases. Most of the purchases under the Eurosystem's purchase programmes are indeed government bonds.

In order to quantify the macroeconomic impact of a reinvestment policy, we use our estimated model. As a baseline scenario, we implement an asset purchase programme that resembles the PEPP as of June 2020 with respect to the announced overall volume, the periods of net purchases and the periods of reinvestment.[13] In particular, we assume that the central bank conducts net purchases for five consecutive quarters (2020Q2-

[12]In Gerke, Giesen and Scheer (2020a), the transfer coefficient $\bar{\tau}$ is around 0.5 with the transfer rule $T_t^h = \bar{\tau}(Y_t - Y) + T^h$. Here, we use the transfer rule $T_t^h = \frac{\tau}{\lambda}(Y_t - Y) + T^h$. Hence our value of τ is, in principle, a scaled version from Gerke, Giesen and Scheer (2020a), as the following holds: $\bar{\tau} = \frac{\tau}{\lambda}$.

[13]We do not attempt to quantify the impact of the actual PEPP since we abstract from some issues that are important for assessing its macroeconomic effects properly. For instance, we do not capture the pandemic itself (supply/demand shock). Nor do we capture potentially smaller effects of monetary policy during a pandemic.

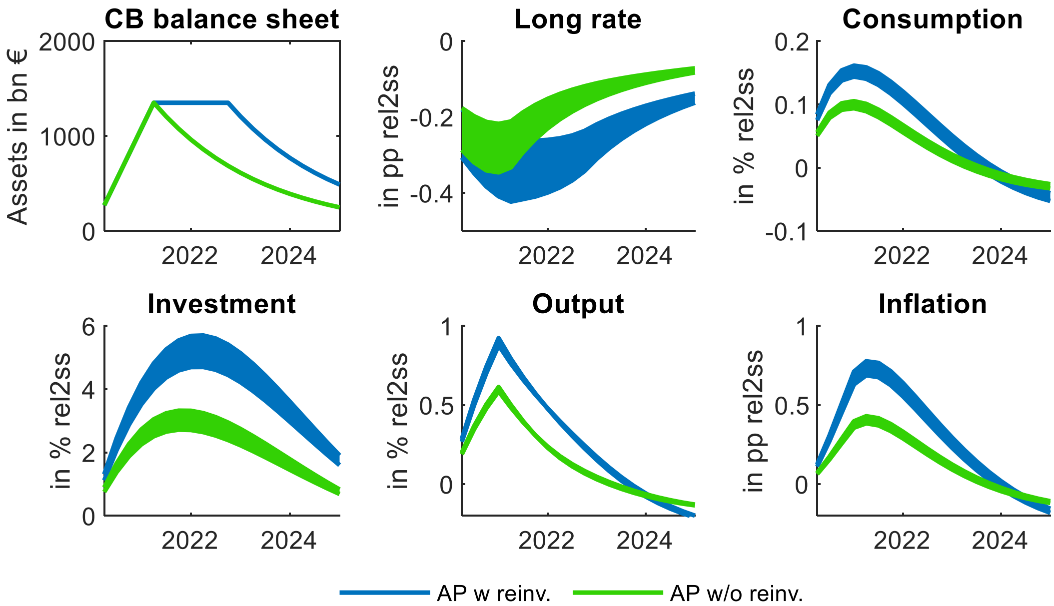
2021Q2) until the overall volume of €1350bn (≈ 11% of GDP) is reached. Following the periods of net purchases, the balance sheet stays constant for six quarters (2021Q3-2022Q4) where the central bank reinvests all maturing assets. Subsequently, the assets on the central bank's balance sheet are reduced gradually, which we model via an AR(1) process with persistence of 0.9.[14]

To implement such an asset purchase path we assume perfect foresight. Hence, the agents know the complete evolution of the stock of assets on the central bank's balance sheet over time. This reflects our assumption that the central bank has announced a credible purchase path. There are thus no further surprises beyond the first period (we relax this assumption in Section 4). Additionally, we allow for different time-preference rates β. While we assume a value that implies a natural real interest rate of 4% during the estimation (1999-2014), there is empirical evidence that the natural rate has declined notably during the last decades. In addition to results generated with a natural rate of 4%, we therefore calibrate β to match a natural rate of 0.5%, in line with empirical evidence for the euro area (Brand, Bielecki and Penalver, 2018). In Figure 2, we thus show ranges whose boundaries correspond to the two different parameterisations of β.

The blue range in Figure 2 depicts our baseline scenario, i.e. a purchase programme that resembles the PEPP in its key features (upper left panel) as well as its associated macroeconomic impact (all other panels). The credible announcement of such a bond purchase programme stimulates the economy. Due to limits to arbitrage, asset purchases by the central bank increase bond prices and lower long-term government bond yields (upper middle panel). Lower yields in turn raise consumption (upper right panel). The purchase of government bonds induces a portfolio rebalancing on the asset side of financial intermediaries towards investment bonds. This raises the price of investment bonds (which are perfect substitutes for government bonds) and lowers their yields (again due to limits to arbitrage). Higher bond prices relax the loan-in-

[14] The average maturity of government bonds is 10 years in the model. Hence, an AR(1) of 0.9 implies that the central bank actively sells bonds (negative net purchases) on top of not reinvesting maturing bonds.

Figure 2: Impact of reinvestment policy



Notes: The figure shows the impact of an asset purchase programme that resembles key features of the PEPP on macroeconomic variables (blue area, "AP w reinv" = asset purchases with reinvestment policy). All results are shown relative to steady state. Output and inflation are annualised. The green area isolates the impact of the overall volume when there is no reinvestment period. The range of results is due to different natural rates of 0.5% and 4%.

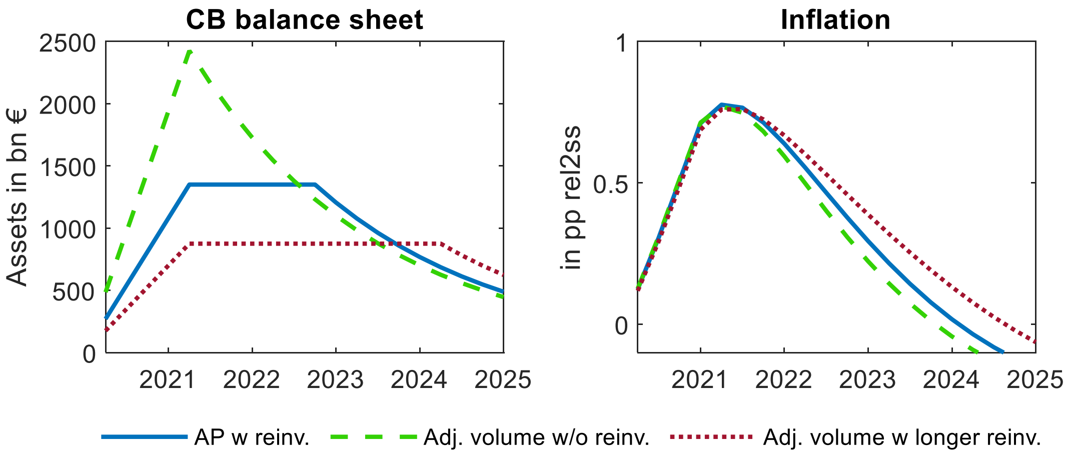
advance constraint, which incentivises investment (bottom left panel). Higher demand raises output (bottom middle panel) and ultimately increases inflation (bottom right panel).[15]

Overall, our baseline results indicate that such a purchase programme results in a peak annualised inflation effect between 0.6pp and 0.7pp, and annual inflation of roughly 0.5pp in 2022. This is in line with estimates of the PEPP (see, for instance, ECB, 2020). Note that, although the purchase programme path continues after 2025, the aggregate effects become negligible from 2025 onwards. This illustrates that the announcement of the programme is the predominant driver of the macroeconomic effects.[16] Put

[15]For the sake of illustration, we left the short-term policy rate unconstrained. This helps us to isolate the macroeconomic impact of asset purchases. As a result of the stimulus, the policy rate increases. In practice, asset purchase programmes have usually been conducted during times when the policy rate was constrained, that is, when it remained at the ELB. In this case, the macroeconomic impact of asset purchases would be larger and reflect the interplay of a constrained policy rate with asset purchases (Gerke, Giesen and Scheer, 2020a; Sahuc, 2016). We introduce the ELB in our model in Section 4, when we analyse the role of reinvestment policies in mitigating the restrictive effects of upper purchase limits.

[16]The reason why the macroeconomic effects nevertheless play out over several quarters are the

Figure 3: Substitutability of overall volume and reinvestment



Notes: The figure illustrates the substitutability of overall volumes (green dashed line; adjusted volume without reinvestment) and longer reinvestments (blue dotted line; adjusted volume with longer reinvestment) such that the overall macroeconomic impact on inflation is similar to the baseline (black solid line, asset purchases with reinvestment). Output growth and inflation are annualised values. The annual natural rate is calibrated to 0.5%.

differently, the stock effect is present in the model.

To isolate the impact of reinvestments, we simulate a counterfactual purchase programme without reinvestment (green area in Figure 2, again with a natural rate of 0.5% and 4%). Thus, net purchases remain the same, but the periods of unwinding start directly after the end of net purchases. As expected, the omission of reinvestment reduces the macroeconomic impact in general, but leaves the qualitative dynamics largely unchanged. For the case at hand, it reduces the peak effect of annualised inflation by roughly one third to about 0.4pp. The magnitude of the reduction depends on the area under the curve for the stock of assets on the central bank's balance sheet over time that is lost in case of no reinvestment. In the present simulation, it is the difference between the black solid line and the red-dashed line in the left panel. The longer the reinvestment period in case of the black solid line the larger this area becomes.

Due to reinvestment, the central bank effectively has two margins for adjusting its monetary stimulus. Put differently, there is a potential substitutability of higher overall volumes and longer reinvestments. We illustrate this in Figure 3. Suppose the central

propagation mechanisms of the model, not the ongoing purchases per se.

bank intends to change the number of periods of reinvestment without altering the overall macroeconomic effect. In other words, the peak annualised effect on the inflation rate should remain around 0.6pp, as in the baseline simulation. First, abstract from any reinvestment (green dashed line). This implies that the central bank would then have to raise the overall volume by €1000bn – roughly 70% – to €2350bn (= €1350bn + €1000bn). The amount of government debt on the central bank's balance sheet would increase accordingly from 11% of GDP in the baseline to 21% of GDP. Second, we double the reinvestment period to a total of 12 quarters (blue dotted line). In this case, the central bank could have reduced its overall volume by €400bn – roughly 30% – to €950bn. This implies that the amount of government debt on the central bank's balance sheet would decrease to 7% relative to GDP.

For each alternative purchase programme (green and blue line), we chose the overall volume such that the peak inflation effect (right panel) is the same as in the baseline scenario. The dynamics of inflation remain similar: According to the logic of the stock effect, if the areas below the respective curves (left panel) are similar, the macroeconomic response should be too. However, the dynamics after the peak are slightly different across the scenarios mainly due to discounting effects and endogenous propagation.[17]

A reinvestment policy therefore allows the central bank to substitute net purchases today with reinvestment purchases in the future while maintaining a given macroeconmic stimulus. In the above simulations, the central bank is able to reduce its holdings of government bonds from 21% of GDP to 7% of GDP without reducing its expansionary stance. We now explore how monetary policy can apply this substitutability to cope with upper purchase limits.

[17] Discounting implies that future purchases are in general less stimulative than present purchases – see Section 5 for a related point. Endogenous propagation, like a build-up via habit or capital accumulation, implies that future purchases might be actually more stimulative than present purchases.

4 Reinvestments as a means to mitigate limits of asset purchases

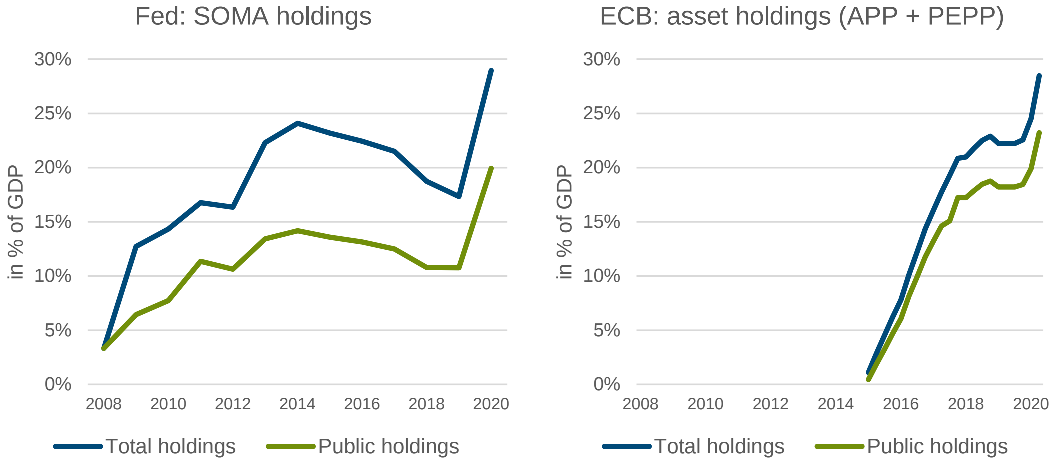
In the previous section, we illustrated how reinvestment policies can enhance a monetary stimulus without changing the overall volume of a purchase programme. This is an important insight since, in practice, asset purchase programmes are subject to self-imposed and/or legal upper purchase limits. To evaluate the extent to which reinvestment policies can mitigate the restrictive effects of upper purchase limits, we proceed in two steps. We first assess how much upper purchase limits constrain the effectiveness of asset purchase programmes. We then quantify whether and how much reinvestments can mitigate the impact of limits.

Upper purchase limits were publicly announced in the case of the Federal Reserve's secondary market purchases of Treasury securities and the Eurosystem's public sector purchase programme (PSPP). One reason for such limits lies in the risk of large purchase programmes having undesirable side effects. As an example, the ECB states that limits are necessary *"... to safeguard market functioning and price formation as well as to mitigate the risk of the ECB becoming a dominant creditor of euro area governments"*.[18] As Figure 4 reveals, public debt holdings represent by far the largest share of the Federal Reserve's or the Eurosystem's asset purchase programmes.

If upper purchase limits restrict net purchases and thus the overall volume of asset purchases, they reduce the degree of monetary policy expansion and accordingly their effectiveness. However, the results of the previous section implies, that the central bank can then resort to reinvestments to soften or mitigate the impact of limits on net purchases: While monetary policy cannot extend net purchases above the limit in the present, it can promise to keep the balance sheet constant via reinvestments for some time in the future. This intertemporal substitutability mirrors, to some extent, the lower-for-longer logic with respect to the interest rate at the effective lower bound (Eggertsson and Woodford, 2003): Although the central bank cannot lower its policy

[18]See ECB Q&A on the PSPP: `https://www.ecb.europa.eu/mopo/implement/app/html/pspp-qa.en.html`.

Figure 4: Central bank balance sheets



Sources: Historical data for the 'System Open Market Account (SOMA) Holdings of Domestic Securities', provided by the NY Fed; 'History of monthly net purchase under the PEPP' and 'History of cumulative net purchase under the APP', provided by the ECB; Data for inflation and GDP are retrieved from the BEA and the Statistical Data Warehouse.

rate below the ELB in the present, it can promise to keep the interest rate lower for longer in the future.

4.1 Augmenting the model with an ELB and a state-dependent asset purchase programme

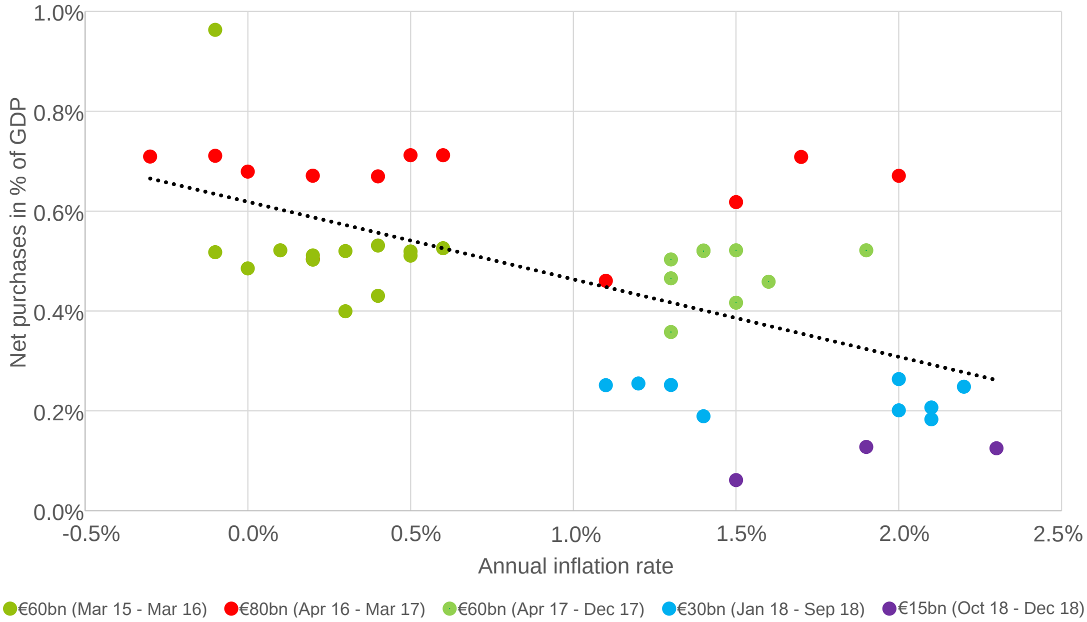
We now quantify the impact of limits and reinvestment within our model. Specifically, we simulate selected aggregate statistics like the average inflation rate and output growth and their respective variances. To do so, we extend our model along two dimensions. First, we add an effective lower bound (ELB) on nominal interest rates as a constraint, such that the monetary policy rule (in linearised form) reads

$$\ln(R_t) = \max\left\{\rho \ln(R_{t-1}) + (1-\rho)\left(\ln(R) + \tau_\pi(\pi_t - \pi) + \tau_y(y_t - y_{t-1})\right) + R_t^\epsilon; \ln(R^{ELB})\right\}, \tag{39}$$

where R^{ELB} is the gross interest rate at the ELB (possibly implying a negative net interest rate), R is the gross interest rate in the steady state, $\pi_t = \ln(\Pi_t)$ the net inflation rate, π the steady state net inflation rate and $y_t = \ln(Y_t)$.

Second, we embed a state-dependent public asset purchase programme which takes into

Figure 5: Relationship between net purchases in % of GDP and inflation



Notes: The colours denote different fixed monthly PSPP purchases during the periods 2015-2018. PSPP purchases are retrieved from the 'History of cumulative net purchases under the APP', which is available on the ECB's website; data for inflation and GDP are retrieved from the Statistical Data Warehouse. See the online data appendix for details.

account both limits and reinvestments. The state dependence reflects the observation that in practice net purchases have taken place only when nominal interest rate is at the ELB and that the size of net purchases depends on the inflation shortfall from its target. The latter is motivated by two reasons. For one, most central banks of advanced economies focus primarily on price stability. It is therefore natural to assume a relationship between the size of net purchases and the inflation rate. For another, as illustrated in Figure 5, we find empirical evidence for a negative relationship between the amount of net purchases under the Eurosystem's public sector purchase programme (PSPP) and the inflation rate in the euro area.[19]

In order to estimate the strength of such a state dependency, we run an OLS regression:

[19] Even though the Eurosystem purchased a fixed amount each month, it changed those monthly volumes with respect to the underlying economic situation. For instance, the Eurosystem announced in January 2015 that it would purchase €60bn monthly under its asset purchase programme (the PSPP is part of the APP). As the inflation rate did not revert to levels consistent with the Eurosystem's mandate, it increased the volume to €80bn. Then, starting in April 2017, it gradually reduced the volume of net purchases again in light of improved inflation rates. The Eurosystem then stopped the programme (initially) in December 2018, before restarting net purchases in November 2019.

$NetPurchases_t = A + \phi_b \pi_t^a + \varepsilon_t$. Specifically, we regress net purchases from April 2015 to December 2019 on the monthly year-over-year inflation rate π_t^a – for details on the data, see appendix A. We estimate a parameter of $\phi_b = -0.12$ and $A \approx 0$. This implies that if the inflation rate falls by 1 percentage point, net purchases increase by 0.12% relative to GDP, roughly €149bn, per month. In quarterly values, $\phi_b = -0.36$ and net purchases increase by about €450bn per quarter for each 1pp fall of the inflation rate.

This estimate is in line with alternative estimates like in Burlon et al. (2019), which find asset purchases to increase by €118bn per month if the inflation rate decreases by 1 percentage point. Specifically, they condition the amount of net purchases to the expected inflation shortfall from target (in the "medium term", i.e., at the end of the respective projection horizon). However, they estimate the strength of the state dependency based on only five data points, namely the changes in fixed monthly purchases that occurred during their sample. In this sense, we complement their approach by using monthly data on realised inflation and net purchases from March 2015 to December 2018. Yet, the overall magnitude of state dependency is relatively similar (€149bn vs. €118bn).

We activate the following asset purchase rule whenever the ELB binds:

$$\hat{b}_t^{CB} = \hat{b}_{t-1}^{CB} + \phi_b \hat{\pi}_t^a,$$

where we substituted

$$NetPurchases_t = b_t^{CB} - b_{t-1}^{CB} = \hat{b}_t^{CB} - \hat{b}_{t-1}^{CB},\ \hat{b}_t^{CB} = b_t^{CB} - b^{CB}$$
$$\text{and } \hat{\pi}_t^a = \hat{\pi}_t + \hat{\pi}_{t-1} + \hat{\pi}_{t-2} + \hat{\pi}_{t-3},\ \hat{\pi}_t = \pi_t - \pi.$$

In case no limit was binding, the central bank reduces its balance sheet once the ELB stops binding: $\hat{b}_t^{CB} = \rho_b \hat{b}_{t-1}^{CB}$. We set $\rho_b = 0.99$ to capture a gradual unwinding of the balance sheet. If the limit was not binding, no reinvestment was started.[20]

[20] In principle, we could also model that a reinvestment period follows any net purchase period. However, since we are interested in reinvestments as a means to mitigate the restrictive effects of limits, it seems natural and more transparent to implement reinvestments only when a limit was binding.

To introduce upper purchase limits, we restrict the overall volume on the central bank's balance sheet to 25%, 33% or 50% of all outstanding public debt. These limits were specifically relevant for the Eurosystem's PSPP. The Federal Reserve had a larger limit of 70%, but restricted the size of net purchases when the overall volume was above 35% (see references in footnote 4).

As Figure 4 illustrates, limits of 50% or higher are not yet a binding constraint. However, once one takes the legacy of previous purchases into account, even larger limits might become a binding constraint in the future. For instance, the Federal Reserve only managed to reduce the size of its balance sheet by roughly one third after 2014 and due to the pandemic, the Fed increased net purchases dramatically. With upper purchase limits, the net asset purchase rule at the ELB extends to $\hat{b}_t^{CB} = \min\left\{\hat{b}_{t-1}^{CB} - \phi_b \hat{\pi}_t^a ; \hat{b}^*\right\}$, $\hat{b}^* = \{0.25, 0.33, 0.5 \text{ or } 1\}$. The latter limit rules out "unlimited" asset purchases. As Table 2 reveals, this will not be a restrictive assumption (see 2nd column, last row).

If the overall volume reaches a limit, net purchases are zero and the balance sheet size remains at the respective limit: $\hat{b}_t^{CB} = \hat{b}^*$, $\hat{b}^* = \{0.25, 0.33 \text{ or } 0.5\}$. Importantly, this is not considered a reinvestment period, although the balance sheet is constant. It is rather that the central bank would like to increase its balance sheet (during the ELB period), but the limit prevents this endeavour. Consequently, reinvestment policies according to our definition only start when the ELB ceases to bind. In this case, the central bank resorts to reinvestments for $\bar{T}$ periods:

$$\hat{b}_t^{CB} = \hat{b}_{t-1}^{CB}, \text{ if } R_t > R^{ELB},\ t \in T^{reinv} \text{ and } \hat{b}_{t-1}^{CB} = \hat{b}^*$$

T^{reinv} consists of the periods $t^*, t^* + 1, \ldots, t^* + \bar{T}$, where t^* is the last period in which the ELB was binding.

We choose the length of the reinvestment periods $\bar{T}$ such that the simulated average inflation rate in the case of binding limits with reinvestments is roughly the same as in the case without limits and no reinvestment (see further below for details). The timing regarding the evolution of the central bank balance sheet is thus as follows: Whenever the ELB binds, the central bank resorts to net purchases (subject to a limit). After the ELB stops binding, i.e. when the policy rate lifts off, the central bank unwinds its balance sheet when no limit was reached. When a limit was reached, it keeps the balance sheet constant for another $\bar{T}$ periods.

Taken together, the general form of the rule that governs the evolution of the central bank's balance sheet in the model is as follows ($\hat{b}^* = \{0.25, 0.33 \text{ or } 0.5\}$):

$$\hat{b}_t^{CB} = \begin{cases} \min\left\{\hat{b}_{t-1}^{CB} - \phi_b \hat{\pi}_t^a ; \hat{b}^*\right\} & \text{if } R_t = R^{ELB} \\ \hat{b}_{t-1}^{CB} & \text{if } R_t > R^{ELB},\ t \in T^{reinv},\ \hat{b}_{t-1}^{CB} = \hat{b}^* \\ \rho_b \hat{b}_{t-1}^{CB} & \text{else} \end{cases} \tag{40}$$

To solve the model with an ELB and such a state-dependent, non-linear purchase programme, we extend the piecewise-linear approach for structural changes developed by Kulish and Pagan (2017) – for details, see appendix B in the appendix.[21] We translate the above non-linearities and state dependence of the rules (monetary policy rule and the net purchase rule) into different regimes. In particular, we assume that, at any point in time, one of four regimes describes the economy: an unconstrained regime (M1) and three constrained regimes (M2a), (M2b), and (M2c), which can be regarded as sub-regimes of an overall constrained regime (M2). Once the timing of each regime is determined endogenously, the algorithm solves for the policy function via backward iteration and determines the model dynamics. This continues until expectations of each regime are consistent with the model dynamics.

In the unconstrained regime (M1), the economy is given as laid out in Section 2 with the addition that $\hat{b}_t^{CB} = \rho_b \hat{b}_{t-1}^{CB}$. Therefore, the central bank sets its policy rate according to its Taylor rule and does not resort to any net asset purchases (it only reduces its balance sheet gradually, in case it resorted to net purchases in the past). In the

[21] Thanks to Carlos Montes-Galdón for providing us with a first replication kit of this approach.

constrained regime (M2), one of the following non-linearities applies.

(M2a) The ELB binds ($R_t = R^{ELB}$), but there are no asset purchases ($\hat{b}_t^{CB} = 0$). This regime merely serves as a benchmark to quantify the impact of the ELB.

(M2b) The ELB binds ($R_t = R^{ELB}$), the central bank resorts to asset purchases at the ELB, with/without upper purchase limits ($\hat{b}^*$), according to the rule $\hat{b}_t^{CB} = \min\left\{\hat{b}_{t-1}^{CB} - \phi_b \hat{\pi}_t^a; \hat{b}^*\right\}$. This allows us to quantify the macroeconomic impact of asset purchases and limits. Note that this formulation implies another non-linearity due to the min operator.

(M2c) After the ELB ceases to bind ($R_t > R^{ELB}$) and if the limit was binding in the past ($\hat{b}_{t-1}^{CB} = \hat{b}^*$), the central bank reinvests all maturing assets to keep the balance sheet constant for some time: $\hat{b}_t^{CB} = \hat{b}_{t-1}^{CB}$ for $t \in T^{reinv}$. This allows us to quantify the impact of reinvestments to mitigate the limits.

Each of the above non-linearities / sub-regimes represents in itself a new set of equations that describe the economy. Importantly, the algorithm is flexible enough to capture very elaborate cases like an expected double-dip recession, in which there is a mix of constrained and unconstrained regimes (with limits and reinvestments). As already pointed out, the agents anticipate the whole evolution of the purchase programme. That is, whenever an ELB is binding, the agents expect net purchases to start. They also anticipate that if a limit is going to bind in the future, net purchases are constrained. In case the central bank implements a reinvestment period, agents take this also into account. Put shortly, there is no uncertainty with respect to the asset purchase programme.

4.2 Quantitative results

For the simulations, we draw exogenous shocks from the estimated distributions.[22] Based on these shocks, we generate 2500 simulations with a length of 200 periods each and discard the first 100 periods for initialisation. In order to roughly map current features in the euro area, we assume a symmetric inflation target of 2%, a long-term equilibrium real interest rate of 0.5% and an ELB of -0.5% (lowest value of the deposit facility rate).[23]

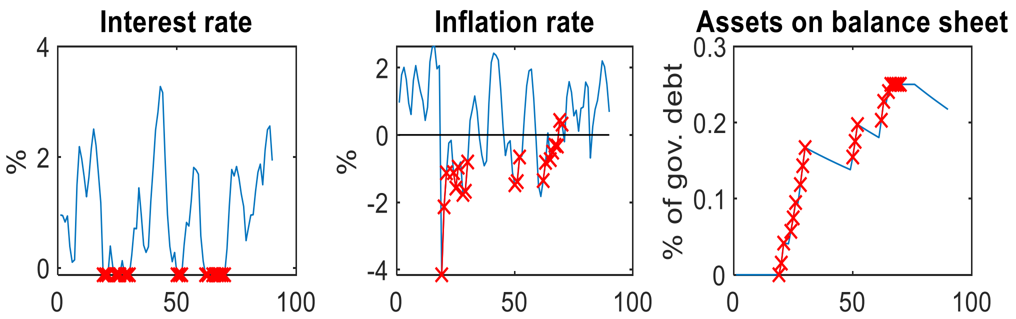
For illustration, Figure 6 depicts part of a single simulation as an example. In this case, the limit was at 25% of outstanding debt and there was a reinvestment of 6Q after the ELB ceases to bind. A red cross denotes a binding ELB period. Once the ELB binds (left panel), the central bank resorts to net purchases and the assets on the balance sheet increase (right panel). The greater the inflation shortfall (middle panel), the larger the size of net purchases. Around period 70, the upper purchase limit is reached so the balance sheet stays constant at that level. After the ELB ceases to bind, the central bank reinvests the maturing assets such that the balance sheet stays constant for 6Q (illustrated by the plateau without the red crosses).

We now assess the quantitative implications of purchase programmes with and without upper purchase limits. This is followed by an analysis of the quantitative relevance of reinvestment policies. Specifically, we contrast three scenarios in which the ELB is a binding constraint. In the first scenario, the central bank adjusts only its short-term policy rate to stabilise the economy. It does not resort to asset purchases. In the second scenario, whenever the ELB is binding the central bank resorts to (unlimited) asset

[22] We use all shocks except for the investment-specific shock $\varepsilon_{\mu,t}$. This shock induced a high incidence of double-, triple- or quadruple-dip recessions. This reduced computing speed dramatically. Additionally, the investment-specific shock gives rise to reversal puzzles (Carlstrom, Fuerst and Paustian, 2015; Gerke, Giesen and Kienzler, 2020) already for a relatively short ELB spell. However, without the investment-specific shock, the ELB frequency dropped to around 10 to 15% if the central bank only has the short-term policy rate at its disposal. We thus scale the other shocks in the simulations by a factor of 1.5 in order to generate a binding ELB frequency in the first scenario of around 30% – which is in line with the frequency when all shocks were included.

[23] Note that possible side effects are not part of the model analysis. Considering these is important in order to better understand the central bank's options for fulfilling its mandate. A complete analysis of those costs is beyond the scope of this paper (for a discussion of side effects, see, for instance, Altavilla, Lemke, Linzert, Tapking and von Landesberger, 2021).

Figure 6: Example of one particular stochastic simulation



Note: The figure shows the interest rate (left panel), inflation rate (middle panel) and the assets on the central bank's balance sheet (right panel) of a particular stoachstic simulation. The model in Section 2 is hit with stochastic shocks based on the estimated shock processes. The annual inflation target is 2%, the long-run level of the annual real rate is 0.5% and the ELB is -0.5% (see the main text for further details). A red cross denotes a binding ELB period. Whenever the ELB binds, state-dependent net purchases start (right panel) until a limit of 25% is reached. After the ELB ceases to bind thereafter, the balance sheet stays constant due to a reinvestment period of six quarters.

purchases using its state-dependent purchase rule. In the third scenario, the central bank initiates asset purchases at the ELB, but obeys an upper purchase limit of 25%, 33% or 50%, respectively. In all scenarios, there is no reinvestment.

The simulation results for the respective average inflation rates are depicted in Figure 7. We note three main findings. First, the ELB causes a sizable negative inflation bias if the central bank can only adjust the short-term nominal interest rate. Second, asset purchases that capture key features of past purchase programmes reduce the negative inflation bias, but do not completely eliminate it. Third, purchase limits reduce the effectiveness of asset purchase programmes.

The left marker of Figure 7 illustrates the first main result. It shows an average inflation rate of around 1.5% in the baseline scenario, i.e. without an asset purchase programme. In other words, the effective lower bound causes an average inflation rate that is around 50 basis points below the inflation target of 2%.

If the central bank does resort to unconstrained asset purchases at the effective lower bound (scenario 2), the average inflation rate increases towards the inflation target (middle marker). The simulations underscore why such programmes have established

Figure 7: Average inflation rate for different policy scenarios

Note: Average annual inflation rates (in %) based on stochastic simulations that take the ELB into account. Annual inflation target is 2%, long-run level of the annual real rate is 0.5%, ELB is at -0.5%. Left marker: central bank has only interest rate at its disposal. Middle marker: central bank resorts to state-dependent asset purchases at the ELB without a limit. Right marker: central bank resorts to state-dependent asset purchases at the ELB with an upper purchase limit of 25% (lower end), 33% and 50% (upper end).

themselves as part of unconventional monetary policy measures at the ELB. Nevertheless, the average inflation rate of just over 1.8% remains below the targeted rate of 2%.[24]

If monetary policy is subject to upper limits on its asset purchases, it will be more difficult to reach the inflation target compared to the unconstrained case. The right marker in Figure 7 shows the extent to which an upper limit of 25%, 33% or 50% on the purchase volume reduces the effectiveness of asset purchase programmes. Depending on the limit, the average inflation rate drops by around 10 to 25 basis points compared to a programme without an upper limit. Hence, in comparison to a case without asset purchases (scenario 1), the inflation rate is closer to its target. Nevertheless, with an average inflation rate of below 1.8%, the central bank misses its target by a greater

[24] One particular reason is the parametrisation of the state-dependent purchase rule. As described above, the strength of the state dependence was estimated based on the Eurosystem's PSPP. Of course, if we assume a stronger state dependency, asset purchases can raise the average inflation rate to the inflation target.

margin than in the scenario without limits (scenario 2).

When central banks can resort to asset purchases with and without limits, it not only affects the average inflation rate but, of course, the economy at large, too: Table 2 provides selected summary statistics of the above simulations. If the central bank can purchase more assets at the ELB (25% limit to 50% limit to unlimited purchases), there is a corresponding increase of stimulus. This results in a lower ELB frequency and duration (first two rows), a reduced volatility of inflation (fourth row) and output (fifth row), and larger holdings of government debt on the balance sheet (sixth and seventh row; row seven depicts the average holding only during ELB periods, i.e., when there are net purchases). The last row in Table 2 shows how often the respective limit was binding during the periods of net purchases, that is, when the ELB was binding. For instance, in roughly 30% of time, the central bank was restricted by an upper purchase limit of 25% (last column).

Table 2: Summary statistics: stochastic simulations, asset purchases with limits

	No asset purchases	Asset purchases	Asset purchases with limits		
Summary statistics	0%/0Q	100%/0Q	50%/0Q	33%/0Q	25%/0Q
Frequency ELB	27.8	21.7	22.0	22.2	22.8
Avg. duration ELB in Q	4.0	3.4	3.5	3.5	3.6
Mean inflation (a)	1.55	1.83	1.73	1.62	1.56
Std inflation (a)	7.6	4.6	7.1	8.1	8.1
Std output growth	13.6	5.1	7.5	11.1	12.7
Size CB holdings (% of gov. bonds)	0.0	23.6	21.9	18.4	15.5
at ELB (% of gov. bonds)	0.0	28.3	26.1	21.4	17.8
Limit binding (% of time at ELB)	100	0.04	6.5	19.4	29.5

Notes: Summary statistics based on stochastic simulations for the baseline scenario (no asset purchases) and alternative asset purchase scenarios with/without limit, which are described in the main text of Section 4. Annual inflation target is 2%, long-run level of the annual real rate is 0.5%, ELB is at -0.5%. The inflation rate is annualised.

The central bank can mitigate the impact of such limits via reinvestment. As we have illustrated in Section 3, a reinvestment policy allows the central bank to substitute net purchases today with reinvestment purchases in the future. Hence, in our simulations we now allow the central bank to reinvest maturing assets for some time after the ELB ceases to bind. Importantly, as described above, the central bank only resorts to reinvestments if an upper purchase limit has been reached. We calibrate the length of reinvestment periods such that the average inflation rate is as close as possible to the average inflation rate in the case of unlimited asset purchases.[25]

We summarise the aggregate results in Table 3. According to our simulations, the central bank would have to reinvest maturing assets for two quarters (50% limit), four quarters (33% limit) or five quarters (25% limit) to reach the same inflation rate as in the case without an upper limit. Therefore, as a rule of thumb, one quarter of reinvestment increases the average inflation rate by roughly 5 basis points. Furthermore, the macroeconomic volatility is also lower than without reinvestments (comparison of Table 2 and Table 3). Nevertheless, the volatility is still higher than with unlimited asset purchases. This implies that reinvestments do not perfectly compensate for possible upper purchase limits. They seem to undo the negative inflation bias but neither the additional volatility of inflation nor output.

Table 3: Summary statistics: stochastic simulations, asset purchases with limits and reinvestments

	Asset purchases	Asset purchases with limits and reinvestment		
Summary statistics	100%/0Q	50%/2Q	33%/4Q	25%/5Q
Frequency ELB	21.7	22.1	22.7	23.4
Avg. duration ELB in Q	3.4	3.4	3.5	3.6
Mean inflation (a)	1.83	1.83	1.83	1.82
Std inflation (a)	4.6	4.9	5.5	5.8
Std output growth	5.1	5.7	7.2	8.3
Size CB holdings (% of gov. bonds)	23.6	22.2	19.0	16.3
at ELB (% of gov. bonds)	28.3	26.7	22.7	19.2
Limit binding (% of time at ELB)	0.04	7.6	22.9	35.2

Notes: Summary statistics based on stochastic simulations for the asset purchases scenario without limit, and alternative reinvestment scenarios, which are described in the main text of Section 4. Annual inflation target is 2%, long-run level of the annual real rate is 0.5%, ELB is at -0.5%. The inflation rate is annualised.

To sum up, the stochastic simulations illustrate four main results. First, the effective lower bound on interest rates constrains the central bank from reaching its inflation target. Second, asset purchases in isolation are not necessarily sufficient to reach the

[25] It is not always possible to match the average inflation rate exactly (see the last column for the row "Mean inflation" in Table 3). The reason is that the periods in the model are in discrete time, while a numerically identical result would require continuous periods, e.g. 5.2 quarters.

inflation target. Third, when upper limits restrict the possible overall volume of an asset purchase programme, the average inflation rate declines compared to the case without limits (while still being higher than in the case without asset purchases). Fourth, reinvestments help to mitigate the impact of upper limits.

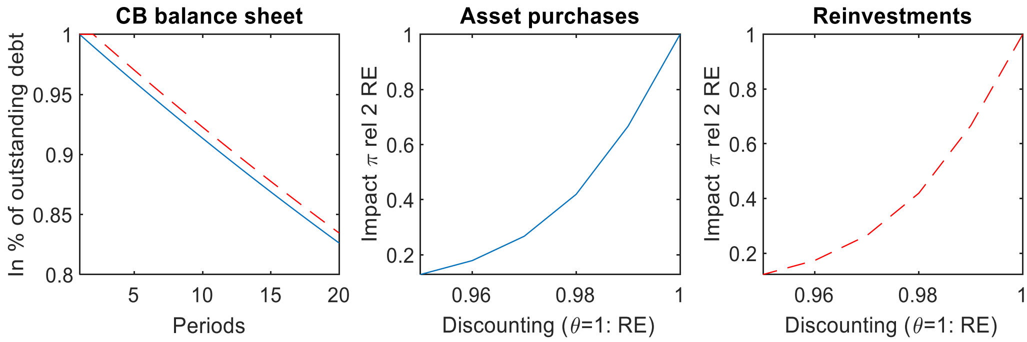
5 Robustness with respect to expectation formation

The previous sections illustrated how reinvestment policies can be applied to enhance the macroeconomic stimulus of asset purchases in light of binding limits. If the central bank substitutes net purchases today with reinvestment purchases in the future, the rational agents in the economy expect a similar stimulus today due to the stock effect. This mechanism depends crucially on the agents being rational and forward-looking and the announcement being credible. With rational expectations, future changes in the economic environment play a decisive role in determining present dynamics, e.g. via intertemporal substitution. However, recent survey-based and experimental microevidence of expectation formation has increasingly documented substantial deviations from full information rational expectations (Coibion et al., 2018; Afrouzi et al., 2021).

In this section, we therefore gauge the effectiveness of reinvestments when expectations deviate from full information rational expectations. We follow Erceg et al. (2021) and Kolasa, Ravgotra and Zabczyk (2022) and add an ad-hoc behavioural element to the model. Specifically, we introduce "cognitive discounting" following Gabaix (2020). This deviation from rational expectations renders agents partially myopic as they discount future variables. In this sense, the forward-looking agents in the economy have boundedly rational expectations.

We implement bounded rationality by replacing each forward-looking variable $x_{t+1|t} = E_t x_{t+1}$ in the linearised equilibrium conditions of the rational expectations version of the model with $\theta x_{t+1|t} = \theta E_t x_{t+1}$. The cognitive discount parameter θ satisfies $0 < \theta \leq 1$, where $\theta = 1$ nests the rational expectations case. While Gabaix (2020) derives this cognitive discount parameter from first principles, we introduce it ad hoc.

Figure 8: Asset purchases and reinvestments with boundedly rational expectations



Note: The left panel depicts two asset purchase programmes that are evaluated under different degrees of bounded rationality (θ). The middle panel compares the average inflation rate of the blue solid asset purchase programme relative to rational expectations. The right panel contrasts the marginal impact ont he average inflation rate of a reinvestment period with rational expectations.

Doing so allows us to capture in a stylised way elements of the aforementioned evidence implying that informational frictions are likely to dampen the role of expectations in determining current macroeconomic dynamics. For the subsequent simulations, we vary the cognitive discount parameter but leave all the other parameters in line with our estimation.

The middle panel of Figure 8 assesses the relative impact of asset purchases in general (i.e. without reinvestment) on the average inflation rate under boundedly rational expectations compared to the impact under rational expectations. For the sake of illustration, we assume a net purchase of 1% of outstanding debt in the first quarter, which is followed by a gradual unwinding (blue solid line, left panel). We then reduce the discounting parameter from $\theta = 1$ (rational expectations) to $\theta = 0.95$ (the estimated value of Erceg et al. (2021) for the euro area); we keep the same purchase path. Under boundedly rational expectations, asset purchases become less effective (middle panel). Intuitively, as agents are not perfectly forward-looking anymore, they discount the stock effect accordingly. Figuratively speaking, the area under the curve of an asset purchase path becomes de facto smaller when agents are only boundedly rational (i.e. in contrast to Figure 1 in Section 1).

In order to make up for this lost efficiency, monetary policy can enlarge its asset purchase programmes – all else being equal. Larger purchase programmes, however, hit more frequently binding limits. Accordingly, the central bank would be forced more often to resort to reinvestments when expectations are boundedly rational.

Yet, our simulations reveal that the marginal benefit in terms of inflation of an additional period of reinvestment is lower if agents discount future variables, see the right panel of Figure 8. We again implement a net purchase of 1% in the first quarter, but keep the balance sheet constant at 1% for one quarter.[26] The right panel then depicts the marginal increase in the average inflation rate for different degrees of bounded rationality relative to the marginal increase under rational expectations. For a discounting value of $\theta = 0.99$, the marginal effectiveness of reinvestments is already visibly smaller than with rational expectations. The higher the discounting, the lower the marginal impact.

One final comment is in order. In the above simulations, we have only introduced and varied the discounting parameter (keeping all other parameters unchanged). This might distort the impact of discounting. For example, it might be the case that the "endogenous" persistence due to discounting is connected with the "exogenous" persistences (habit, financial friction, net worth adjustment costs etc.). If discounting is high, the "exogenous" persistence might change, which would affect the dynamics of asset purchases. A proper assessment of the effectiveness of reinvestments when expectations are boundedly rational is, however, beyond the scope of this paper. It would require a fully-fledged estimation with cognitive discounting as a prerequisite. We leave this, and the question of how much additional reinvestment would be needed in case of boundedly rational agents compared to our findings, for future research.

[26]In general, the marginal increase in reinvestment is not constant over the periods of reinvestments. Quantitatively it turns out that it is relatively flat for a wide range of reinvestments up to 20 quarters. For transparency, we therefore show only the marginal increase of the first reinvestment period. Hence, the right panel shows the difference of the average inflation rate for the red dashed and the blue solid asset purchase path. This is the marginal increase in inflation due to one period of reinvestment.

6 Conclusion

A notable feature of recent asset purchase programmes is the announcement of how long the central bank is going to hold the overall volume, i.e. the sum of net purchases, constant on its balance sheet. This is the reinvestment period. In this paper, we systematically assess the qualitative and quantitative effects of such reinvestment policies. Our model-based analysis illustrates that an additional period of reinvestment enhances the macroeconomic stimulus of an asset purchase programme for a given overall volume, i.e. without increasing net purchases.

The economic reason is straightforward: In our model, primarily the stock effect determines the macroeconomic effect of asset purchase programmes. It implies that financial market participants immediately factor in the central bank's announcement of how the stock of assets on its balance sheet will evolve over time. If the announcement of purchases is credible, the stock effect allows the central bank to substitute higher overall volumes with longer reinvestments to obtain the same macroeconomic stimulus.

We obtain four main results. First, omitting reinvestments in an asset purchase programme that embeds key features of the PEPP reduces the peak effect on inflation by one third. Second, monetary policy can achieve a given macroeconomic stimulus by substituting a higher overall volume of assets on the central bank's balance sheet (more net purchases) with longer reinvestments. Based on the same programme as above, we show that monetary policy can decrease the overall volume by €400bn (or 30%) if it extends the reinvestment period from six to twelve quarters. If monetary policy completely abstains from reinvestments, it has to increase the overall volume by €1000bn (or 70%). Third, our stochastic simulations reveal that reinvestments can undo the dampening impact of upper purchase limits on the inflation bias. For an upper purchase limit of 25% (33%; 50%), monetary policy can prolong the reinvestment period by five (four; two) quarters to reach the same inflation bias as in the case without an upper limit. Fourth, the quantitative impact of reinvestments depends on how agents form expectations. If agents are boundedly rational, the macroeconomic impact of asset purchases in general as well as the marginal benefit of reinvestments are lower.

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A Data

This section describes the data used for estimation. There is a link after each source (within the online pdf).

Definition of observables

Real per capita output growth: $\Delta Y_t^{dat} = \frac{(YER_t/LFN_t)-(YER_{t-1}/LFN_{t-1})}{(YER_{t-1}/LFN_{t-1})}$

Real per capita investment growth: $\Delta I_t^{dat} = \frac{(ITR_t/LFN_t)-(ITR_{t-1}/LFN_{t-1})}{(ITR_{t-1}/LFN_{t-1})}$

Gross inflation: $\Pi_t^{dat} = 1 + \frac{HICPYSA_t - HICPYSA_{t-1}}{HICPYSA_{t-1}}$

Real wage growth: $\Delta w_t^{dat} = \frac{(WRN_t/HICPYSA_t)-(WRN_{t-1}/HICPYSA_{t-1})}{(WRN_{t-1}/HICPYSA_{t-1})}$

Short-term interest rates: $R_t^{dat} = \frac{STN_t}{4*100}$

Long-term interest rates: $R_t^{L,dat} = \frac{LTN_t}{4*100}$

Real bank net worth growth: $\Delta N_t^{dat} = \frac{(NWB_t/HICPYSA_t)-(NWB_{t-1}/HICPYSA_{t-1})}{(NWB_{t-1}/HICPYSA_{t-1})}$

Data description

All data are seasonally adjusted. Except for those series, which are based upon data in monthly frequency, the reference area is always equal to 'Euro Area 19 (fixed composition)' . The former - more precisely raw data for HICP, interest rates and net worth of financial intermediaries - are only availbale for reference area 'Euro Area (Changing composition)'.[27]

YER: Gross domestic product at market prices, Million Euros, Chain linked volume (rebased), Reference year 2015. Source: ECB Statistical Data Warehouse.

LFN: Labor force (Thousands of persons), Based on definition of AWM database: LFN = LNN(= Total employment)/(1− URX(= Unemployment rate)). Source: ECB SDW; LNN, URX.

ITR: Gross fixed capital formation, Million Euros, Chain linked volume (rebased), Reference year 2015. Source: ECB SDW.

HICPYSA: HICP - Overall index, Monthly Index, Base year 2015=100. Source: ECB SDW.

LNN: Total employment (Thousands of persons). Source: ECB SDW.

WRN: Nominal wage rate per head, bases on defintion of AWM database: WRN = WIN(=Compensation of employees(D1))/LNN. Source: Eurostat, WIN; ECB SDW, LNN.

STN: Nominal short-term interest rate, Euribor 3-months, Percent per annum. Source: ECB SDW.

[27] Time series of interest rates and banks' net worth are taken from the 'FM - Financial markets data' respectively 'BSI- Balance Sheet Items' datasets. Both do not provide any data for reference area 'Euro Area 19 (Fixed composition)'. For inflation or rather the HICP, Eurostat makes seasonally unadjusted data for both 'Euro Area (Changing composition)' as well as 'Euro Area 19 (Fixed composition)' available. In this case both series are the same, hence we stick with the seasonally adjusted date from the Statistical data warehouse and a changing composition of euro area member states.

NWB: Capital and reserves (net) of Monetary and Financial Institutions (MFIs) reporting sector, Outstanding amounts at the end of the period (stocks), Millions of Euro. Source: ECB SDW.

B Solution method

To solve the model with an ELB and a state-dependent, non-linear purchase programme, we employ the piecewise-linear approach for structural changes developed by Kulish and Pagan (2017). The approach is similar in spirit to Guerrieri and Iacoviello (2015). The difference between these approaches is the context in which they developed their respective piecewise-linear approach. Kulish and Pagan (2017) simulate a model with a structural break that is determined by a policymaker. Guerrieri and Iacoviello (2015) model a structural change that was driven by an exogenous shock only. If the (expected) duration of the regime is the same for both approaches, the reduced-form matrices associated with that duration are exactly the same.

We translate the non-linearities and state dependence of the monetary policy rule (34) and the net purchase rule (40) into regimes. In particular, we assume that, at any point in time, one of four regimes describes the economy: an unconstrained regime (M1) and three constrained regimes (M2a), (M2b), and (M2c), which can be regarded as sub-regimes of an overall constrained regime (M2). Once the timing of each regime is determined endogenously, the algorithm solves for the policy function via backward iteration and determines the model dynamics. This continues until expectations of each regime are consistent with the model dynamics.

In the unconstrained regime (M1), the economy is given as laid out in Section 2 with the addition that $\hat{b}_t^{CB} = \rho_b \hat{b}_{t-1}^{CB}$. Therefore, the central bank sets its policy rate according to its Taylor rule and does not resort to any net asset purchases (it only reduces its balance sheet gradually, in case it resorted to net purchases in the past). In the constrained regime (M2), one of the following non-linearities applies.

(M2a) The ELB binds ($R_t = R^{ELB}$), but there are no asset purchases ($\hat{b}_t^{CB} = 0$). This regime merely serves as a benchmark to quantify the impact of the ELB.

(M2b) The ELB binds ($R_t = R^{ELB}$), the central bank resorts to asset purchases at the ELB, with/without upper purchase limits ($\hat{b}^*$),4 according to the rule $\hat{b}_t^{CB} = \min\left\{\hat{b}_{t-1}^{CB} - \phi_b \hat{\pi}_t^a; \hat{b}^*\right\}$. This allows us to quantify the macroeconomic impact of asset purchases and limits. Note that this formulation implies another non-linearity due to the min operator.

(M2c) After the ELB ceases to bind ($R_t > R^{ELB}$) and if the limit was binding in the past ($\hat{b}_{t-1}^{CB} = \hat{b}^*$), the central bank reinvests all maturing assets to keep the balance sheet constant for some time: $\hat{b}_t^{CB} = \hat{b}_{t-1}^{CB}$ for $t \in T^{reinv}$. This allows us to quantify the impact of reinvestments to mitigate the limits.

We approximate the dynamic equations of the respective regimes up to first order, such that they have the following representations.

Unconstrained regime:

$$Ax_t = C + Bx_{t-1} + DE_t x_{t+1} + F\varepsilon_t \tag{M1}$$

Constrained regime:

$$A_t^* x_t = C_t^* + B_t^* x_{t-1} + D_t^* E_t x_{t+1} + F_t^* \varepsilon_t \tag{M2}$$

In both regimes, E_t denotes the expectations operator, x_t the vector of endogenous variables and ε_t the vector of exogenous variables. The matrices A, B, C, D, F and $A_t^*, B_t^*, C_t^*, D_t^*, F_t^*$ are of conformable dimensions that capture the structural parameters of the economic system. To keep the notation short, we bundle all regimes (M2a to M2c) into a time-varying regime (M2).[28]

To fix ideas, think of (M2) first as only representing the case when the ELB binds and the central bank resorts to asset purchases without a limit, i.e. $\hat{b}_t^{CB} = \hat{b}_{t-1}^{CB} - \phi_b \hat{\pi}_t^a$. The agents in the economy expect this regime to be in place for some time T. Now consider the extension when an upper purchase limit binds after some periods with net purchases. In this case, the agents first expect the same regime to hold as before, i.e. $\hat{b}_t^{CB} = \hat{b}_{t-1}^{CB} - \phi_b \hat{\pi}_t^a$, until some period T^1. After T^1, the net purchase rule changes to $\hat{b}_t^{CB} = \hat{b}^*$. The general form of (M2) captures both regimes. Lastly, suppose that there is a reinvestment period after the ELB ceases to bind. In this case, the agents in the economy anticipate first a regime with net purchases, then a regime where asset purchases are at the limit and then a regime in which the central bank keeps the balance sheet constant: $\hat{b}_t^{CB} = \hat{b}_{t-1}^{CB}$ for $t \in T^{reinv}$. The algorithm is general enough to capture such cases and more elaborate ones like an expected double-dip recession, in which there is a mix of constrained and unconstrained regimes (with limits and reinvestments).

Without loss of generality, assume the following timing:

- The constrained regime is in place in periods 1 until T^{constr}: M2 applies.

[28] Technically, we implemented the scenario in Section 3 with the same algorithm. We defined $\hat{b}_t^{CB}$ as an exogenous variable $\hat{b}_t^{CB} = \bar{b}_t$, where the latter variable takes the values as explained in that section. After the end of net purchases or reinvestments (in which case $\hat{b}_t^{CB} = \hat{b}_{t-1}^{CB}$ holds), the assets on the balance sheet are endogenously reduced according to the AR(1) process. This procedure is the same as a perfect foresight simulation, as long as the path of the endogenous variable $\hat{b}_t^{CB}$ is the same as $\bar{b}_t$.

- The unconstrained regime is in place from $T^{constr} + 1$ onwards: M1 applies.

The solution to such an economic system gives us a time-varying (non-linear) policy function (Guerrieri and Iacoviello, 2015). We obtain it by an iterative procedure, where the underlying assumption is that after period $T^{constr} + 1$, the unconstrained regime stays in place forever. In other words, once either regime applies, agents fully incorporate the change in the structure of the economy. However, they do not anticipate the initiation of a regime change. As a result, precautionary effects are not incorporated. Therefore, the solution after period T^{constr} is obtained via standard perturbation. It is given by

$$x_t = J + Qx_{t-1} + G\varepsilon_t, \quad \forall t > T^{constr} \tag{41}$$

Given this solution, one can substitute for the expectation in T^{constr}, i.e. regime (M1), and solve the model for T^{constr}. This continues until period 1. For a detailed description, see Kulish and Pagan (2017) or Guerrieri and Iacoviello (2015).

Ultimately, the policy function is given by

$$x_t = J_t + Q_t x_{t-1} + G_t \varepsilon_t \tag{42}$$

with

$$J_t = J,\ Q_t = Q,\ G_t = G \ \ \forall t > T^{constr}, \text{ i.e. the time-invariant policy function of (41)}$$

and

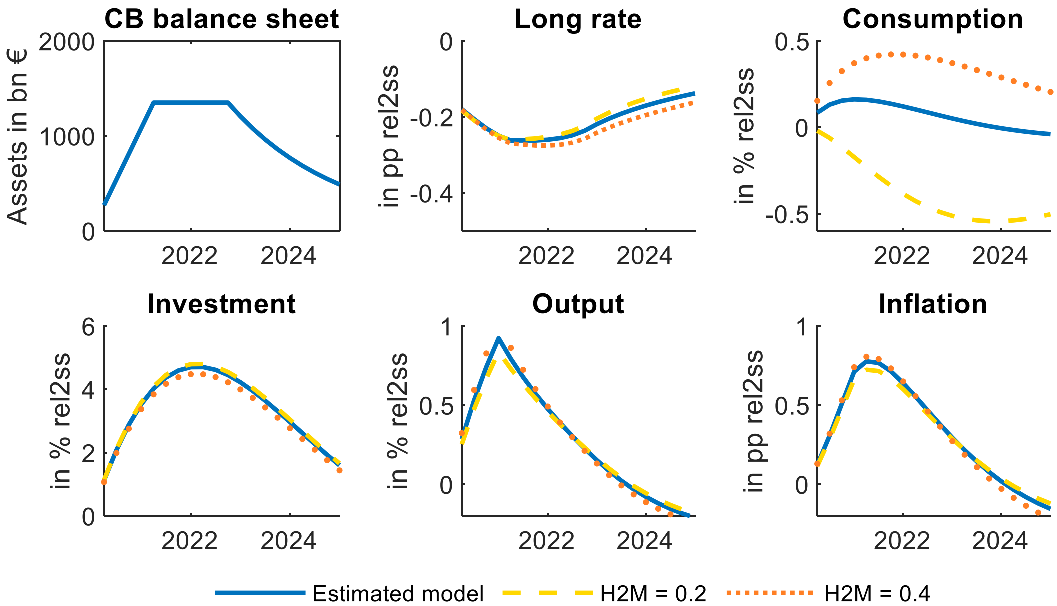
$$\Xi_t = \left(A_t^* - D_t^* Q_{t+1}\right)^{-1}, J_t = \Xi_t \left(C_t^* + D_t^* J_{t+1}\right), Q_t = \Xi_t B_t^*, G_t = \Xi_t F_t^* \text{ for } 1 \leq t \leq T^{constr}.$$

C Impact of hand-to-mouth share

As explained in Section 2.6, we calibrated the share of hand-to-mouth households in order to estimate the strength of the countercyclical transfer scheme. In order to assess how sensitive our results are with respect to this parameter, we repeat the simulation in Section 3 with different hand-to-mouth shares.

Figure 9 depicts the underlying asset purchase path as well as some selected macroeconomic variables (as in Figure 2). The blue line serves as the baseline; again, we assume that the natural rate of interest is 0.5%. The orange-dashed line illustrates the impact of a higher hand-to-mouth share, the yellow-dotted line illustrates a lower share. The most visible difference is the impact on consumption, which is smaller the lower the share. However, this is compensated by a relative larger investment response (note the different scales on the vertical axis for consumption and investment). Hence,

Figure 9: Impact of reinvestment policy



Notes: The figure shows the impact of an asset purchase programme that resembles key features of the PEPP on macroeconomic variables (blue area, "AP w reinv" = asset purchases with reinvestment policy). All results are shown relative to steady state. Output and inflation are annualised. The green area isolates the impact of the overall volume when there is no reinvestment period. The range of results is due to different natural rates of 0.5% and 4%.

the aggregate effects of inflation and output are by and large similar. We therefore conclude that key results are not affected by the share of hand-to-mouth households.