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Optimal timing of policy interventions in troubled banks

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Non-technical summary

Research Question

A key problem faced by policymakers is deciding whether and when to optimally resolve a distressed bank whose solvency is uncertain. If it was straightforward to distinguish whether a distressed bank is fundamentally insolvent or just illiquid, the optimal policy would be equally straightforward: support illiquid and resolve insolvent banks. In practice, such a distinction is seldom possible without conducting a time-consuming financial audit. Delaying intervention, however, gives uninsured creditors time to withdraw their claims, which raises the cost of bailing out insured depositors if the bank is ultimately resolved.

Contribution

The present paper uses a dynamic banking model to analyze a policy authority (PA)'s decision to optimally resolve a distressed bank. The bank's solvency becomes uncertain due to a shock and its junior wholesale creditors begin withdrawing their funds. The bank repays withdrawals by liquidating its assets. Wholesale withdrawals therefore drain resources from the bank and dilute the bank's insured depositors' claims. Over time, news may reveal the bank's true solvency state. The PA maximizes aggregate output and decides when to resolve or to support the bank with liquidity or equity injections.

Results

For the PA, delaying resolution is valuable because the PA can make a more efficient decision (avoiding the resolution of a solvent bank) if news may arrive. Delaying resolution is also costly because withdrawals raise the PA's cost of covering deposit insurance. The optimal resolution date trades off these costs. Through liquidity support, the PA always avoids the inefficient resolution of a solvent bank but exposes itself to counterparty risk. Thus, it is sometimes optimal to delay support to lower the PA's counterpary risk exposure. Outright equity injections are preferred to liquidity support if the bank's debt maturity structure is short or if it holds only a small amount of insured deposits.

Nichttechnische Zusammenfassung

Fragestellung

Besteht Unsicherheit über die Solvenz einer finanziell angeschlagenen Bank, muss die Aufsichtsbehörde entscheiden, ob und wann diese abgewickelt werden soll. Dieses Problem ließe sich einfach lösen, wenn eindeutig entschieden werden könnte, ob das Institut bereits insolvent oder lediglich in Liquiditätsnot geraten ist: die insolvente Bank sollte abgewickelt, die illiquide unterstützt werden. Diese Unterscheidung ist ohne eine zeitaufwändige Buchprüfung kaum zu treffen. Währenddessen können nachrangige Gläubiger jedoch ihre Gelder abziehen. Diese Rückzahlungen verringern die Ressourcen, die im Abwicklungsfall zur Deckung versicherter Einlagen zur Verfügung stehen.

Beitrag

Das Papier untersucht mittels eines dynamischen Modells einer finanziell angeschlagenen Bank die Entscheidung einer Aufsichtsbehörde über den optimalen Abwicklungszeitpunkt. Aufgrund von Unsicherheit über die Solvenz der Bank ziehen unbesicherte (nachrangige) Gläubiger ihre Gelder ab. Um deren Forderungen zu bedienen, veräußert die Bank Teile ihrer Aktiva. Dadurch stehen der Einlagensicherung weniger Vermögenswerte zur Deckung versicherter Einleger zur Verfügung. In jedem Zeitpunkt besteht die Möglichkeit, dass öffentliche Informationen über die tatsächliche Solvenz der Bank bekannt werden. Die Aufsichtsbehörde maximiert den gesamtwirtschaftlichen Output und entscheidet, ob und wann sie die Bank abwickelt oder sie mit Liquidität oder Eigenkapital unterstützt.

Ergebnisse

Für die Behörde ist es vorteilhaft, die Abwicklung der Bank zu verzögern, um etwaige Informationen über ihre tatsächliche Solvenz zu nutzen und die Abwicklung einer eigentlich solventen Bank zu vermeiden. Dadurch erhalten unbesicherte Gläubiger jedoch Zeit, ihre Gelder abzuziehen, wodurch sich die Kosten der Einlagensicherung erhöhen. Im optimalen Interventionszeitpunkt müssen sich Grenzvorteile und -kosten gerade entsprechen. Durch Gewährung von Liquiditätshilfen kann die Behörde es immer vermeiden, fälschlicherweise eine solvente Bank abzuwickeln, geht dabei jedoch ein Gegenparteirisiko gegenüber der Bank ein. Daher kann es vorteilhaft sein, Liquiditätshilfen nur mit Verzögerung zu gewähren, um die Risikobelastung der Behörde zu minimieren. Die Behörde bevorzugt öffentlich finanzierte Rekapitalisierungen gegenüber Liquiditätshilfen, wenn die Bankverbindlichkeiten eine kürzere durchschnittliche Laufzeit aufweisen oder die Bank sich nur mit einem geringen Anteil versicherter Einlagen finanziert.

Deutsche Bundesbank Discussion Paper No 10/2022

Optimal Timing of Policy Interventions in Troubled Banks*

Philipp J. König† Paul Mayer‡ David Pothier§

Abstract

We analyze the problem of a policy authority (PA) that must decide when to resolve a troubled bank whose underlying solvency is uncertain. Delaying resolution increases the chance that information arrives that reveals the bank's true solvency state. However, delaying resolution also gives uninsured creditors the opportunity to withdraw, which raises the cost of bailing out insured depositors. The optimal resolution date trades off these costs with the option value of making a more efficient resolution decision following the arrival of information. Providing the bank with liquidity support buys the PA time to wait for information, but increases the PA's losses if the bank is insolvent. The PA may therefore optimally choose to delay the provision of liquidity support in order to minimize its losses.

Keywords: Bank Resolution, Lender of Last Resort, Banking Crises

JEL Classifications: G01, G21, G28

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"[T]he failure of [a] bank to adjust its liquidity on the open market means that there is at least a whiff of suspicion of insolvency." — Goodhart (1999) —

1 Introduction

A key challenge for policymakers is judging whether a bank facing liquidity problems is fundamentally insolvent (a *gone concern*), or whether it is just illiquid and could avoid insolvency if given adequate policy support (a *going concern*). If this distinction was straightforward, the optimal policy for dealing with troubled banks would be equally straightforward: support illiquid banks and resolve insolvent ones. In practice, such a clear distinction is seldom possible without a detailed examination of a bank's books. Conducting a financial audit, however, takes time and gives uninsured creditors the chance to withdraw their claims before policymakers intervene, potentially increasing the cost of bailing out insured depositors if the bank is ultimately resolved. Thus, a key problem of resolution policy is deciding *when* to optimally resolve a troubled bank whose solvency state is uncertain.

While resolution policies differ in their details across jurisdictions, a common feature is that resolution authorities enjoy considerable discretion in determining whether a financial institution has reached a point of non-viability (PONV) and should be resolved. For example, the Federal Deposit Insurance Corporation (FDIC) explicitly states that "*under certain circumstances, [it] may delay resolution of a critically undercapitalized insured depository institution if a determination is made that it is in the best interest of the deposit insurance fund*" (FDIC, 2019).[1] Policymakers' flexibility regarding the timing of resolution raises a number of important questions. First, when (and under what conditions) will a policymaker optimally resolve a troubled bank facing liquidity problems?

[1] In Europe, the European Banking Authority's (EBA) guidelines for declaring a financial institution non-viable explicitly state that they "[...] do not purport to constrain the ultimate discretion of the competent authority and of the resolution authority in making the determination that an institution is failing or likely to fail" (EBA, 2015).

Second, how does the optimal resolution date (i.e., the optimal PONV) depend on the bank's balance sheet characteristics and broader economic conditions? Third, how is the optimal timing of resolution affected by the availability of other commonly used policy tools such as liquidity support and equity injections?

To answer these questions, we develop a stylized dynamic banking model in continuous time. There is a representative bank, wholesale creditors, retail depositors and a policy authority (PA). The bank enters the economy with a legacy asset financed by insured retail deposits and overlapping issues of uninsured wholesale debt. At some interim date, a verifiable shock realizes that may impair the bank's asset. Whether the bank's asset is impaired by the shock is not immediately observable, which creates uncertainty about the bank's underlying solvency. Given this uncertainty, wholesale creditors become unwilling to roll over their claims when they come due. To obtain the liquidity needed to repay withdrawing wholesale creditors, the bank must sell parts of its asset.

The PA's primary role is to choose whether and when to resolve the bank by enforcing a complete write-down of its outstanding wholesale debt and liquidating its remaining assets. The PA's objective is to maximize expected aggregate output.[2] The optimal intervention decision is affected by two key frictions. First, liquidating unimpaired assets is costly; e.g., because transferring ownership of these assets to agents other than the bank destroys value. Second, there is a social cost to public funds; e.g., due to the distortionary effects of taxation. Because of this social cost, bailing out insured depositors creates a deadweight loss.

As time passes, information may arrive that reveals the bank's solvency state. Delaying intervention and waiting for information has value since it may avoid the inefficient resolution of a solvent bank. Delaying intervention, however, also gives uninsured creditors time to withdraw maturing debt, which drains resources from the bank and increases

[2] We are agnostic about the exact institutional identity of the policy authority in our model. It could be a central bank, a deposit insurance fund, a chartering authority, or a combination of various different institutions. By considering a single policy authority, we purposefully abstract from potential conflicts-of-interest between various policy-making institutions. Doing so allows us to focus on the dynamic trade-offs inherent to resolution policy. See Repullo (2000) and Kahn and Santos (2005) for papers studying the optimal allocation of regulatory authority across institutions with conflicting mandates.

the cost of bailing out insured depositors if the bank is resolved. Because debt withdrawals circumvent the *de jure* seniority of insured deposits, we refer to this increase in the cost of bailing out insured depositors as the "dilution costs" of delaying resolution.

The failure of IndyMac in July 2008 provides an illustrative example of how uncertainty about a bank's solvency can lead to costly delays in the resolution process. Very active in the non-prime mortgage market in the years leading up to the 2007/08 financial crisis, IndyMac's financial conditions began to deteriorate after US house prices started to decline. Concerned about the bank's solvency, the Office of Thrift Supervsion (OTS) and the FDIC decided to examine IndyMac's books ahead of their on-site inspection schedule, which led its CAMELS rating to be downgraded from 2 to 5 (the worst possible rating).[3] Once IndyMac's financial problems became publicly known, uninsured depositors began to withdraw *en masse* and the bank's liquidity position rapidly deteriorated. Only afterwards did the FDIC place the bank under receivership. The IndyMac resolution proved to be the most expensive in FDIC history, costing about 12 billion USD (FDIC, 2017).[4]

Just like in the Indy Mac case, the PA in our model must choose whether and when to resolve a troubled bank. We show that this policy problem can be framed in terms of minimizing a weighted sum of *type-I* errors (forcing a solvent bank into resolution) and *type-II* errors (allowing an insolvent bank to continue operating), with the weights depending on the social costs of public funds. If the social costs are sufficiently large, the PA resolves the bank immediately after wholesale creditors begin to withdraw. The reason is that the costs of a *type-II* error are so large that the PA never chooses to wait for information. For smaller social costs, the relative weight of avoiding a *type-I* error increases and the PA becomes willing to delay resolution in the hope that new information reveals the bank to be solvent. Because the bank must meet debt withdrawals by fire-selling

[3]The CAMELS (Capital, Asset Quality, Management, Earnings, Sensitivity to Market Risk) rating system is an internal supervisory tool first adopted by US regulatory institutions in 1979.

[4]Besides IndyMac, other prominent examples of bank failures during the 2007/08 financial crisis include Washington Mutual in September 2008 and Wachovia in December 2008. Two common features of these cases were the large losses the banks incurred on their (non-prime) mortgage loan portfolios and the subsequent run-offs by uninsured depositors that were sparked by the uncertainty about the banks' solvency once information about these losses became public (FDIC, 2017).

assets, the value of information – i.e., the gain from avoiding a *type-I* error – decreases over time. The PA therefore optimally resolves the bank once the dilution costs from allowing wholesale debt withdrawals to continue exceed the value of information. The optimal resolution date can be viewed as the PA's assessment that the bank's point of non-viability (PONV) has been reached.

Our baseline model focuses on the optimal timing of resolution. In practice, policy authorities have access to other policy tools that can be employed before resolution measures are invoked; e.g., central bank emergency facilities or treasury recapitalization programs.[5] We extend the analysis by considering how the optimal timing of resolution is affected if the PA can provide the bank with liquidity support (i.e., act as a lender of last resort) or inject equity capital into the bank.

Enlarging the PA's policy options to include the provision of liquidity support changes the trade-offs faced by the PA in two important ways. First, liquidity support preserves the value of information over time by allowing the bank to meet debt withdrawals without fire-selling assets. Second, liquidity support exposes the PA to counterparty risk. The reason is that by providing liquidity support the PA obtains a claim against the bank, implying that the PA bears an additional loss if the bank's asset turns out to be impaired.

Which of these two effects dominates depends on the social cost of public funds. If there is no cost of public funds, the PA's optimal policy consists of supporting the bank immediately after wholesale creditors start withdrawing, and maintaining support until information arrives. This policy eliminates the risk of making a *type-I* error: if the bank is revealed to be solvent, wholesale creditors stop withdrawing and liquidity support is terminated; if, instead, the bank is revealed to be insolvent, the PA resolves the bank and makes insured retail depositors whole. *Type-II* errors have no effect on aggregate output in this case since transfers from the PA do not create a deadweight loss. If the disbursement of public funds involves a social cost, *type-II* errors matter for aggregate output. The reason is that a marginal increase in the PA's loss reduces aggregate output

[5] See Laeven and Valencia (2010) for an overview of policy interventions in distressed banks during the 2007/08 financial crisis, including liquidity support, government guarantees and public recapitalizations.

by more than a marginal decrease in bank profits. As a result, the PA may choose to delay the provision of liquidity support in order to reduce its counterparty risk exposure. Such a policy may be optimal even though it forces the bank to engage in costly fire-sales before liquidity support is granted.[6]

Instead of providing liquidity support, we also consider the effect of allowing the PA to inject equity capital into the bank. A sufficiently large equity injection allows the bank to refinance maturing wholesale debt by issuing new, long-term risky debt. Contrary to liquidity support, it is never optimal to delay equity injections. The reason is that (risky) debt refinancing allows to reallocate cash flows from the bank to the PA without requiring the bank to engage in costly fire-sales. We show that the PA prefers equity injections over liquidity support whenever the minimum equity injection required to enable the bank to refinance its wholesale debt stock is smaller than the PA's expected loss from providing liquidity support.

Related Literature. Our paper builds on several strands of the literature. First, our model is related to the literature on bank resolution. An early contribution by Mailath and Mester (1994) studies a regulator's incentive to shut down distressed banks when banks can engage in inefficient risk-shifting and their closure involves opportunity costs. Acharya and Yorulmazer (2007) extend their analysis to a setting with multiple banks and systemic shocks. Other, more recent, papers study the design of resolution policies in different contexts.[7] Schilling (2019) studies how regulatory forbearance affects creditors' withdrawal incentives. Walther and White (2020) characterize optimal bail-in rules when regulatory interventions may signal negative information to a bank's creditors. All of these papers emphasize the strategic interaction between regulatory authorities and banks' creditors. Our model, which abstracts from such strategic considerations, complements

[6] We show in Appendix A3 that our results are robust if the PA can charge a penalty rate above the risk-free rate, or if the bank has cash balances that it can draw down to meet debt withdrawals.

[7] There also exists a recent literature studying the design of bank resolution policies in a multinational setting, including Calzolari and Loranth (2011), Bolton and Oehmke (2019), and Segura and Vicente (2019).

these papers by studying the optimal timing of resolution.

Our model shares important similarities with the "real options" literature, including McDonald and Siegel (1986) and Ingersoll and Ross (1992). These papers study the optimal timing of a firm's decision to invest in an irreversible project when the underlying benefits of the project are uncertain. The problem of the PA in our model is conceptually closely related, as it involves the PA choosing the timing of an irreversible resolution decision when the underlying financial condition of a distressed bank is uncertain.

Our paper also contributes to the literature on the design and effects of lender of last resort (LLR) interventions. Thornton (1802) and Bagehot (1873) were the first to study under what conditions a LLR should lend to banks in a liquidity crisis. Rochet and Vives (2004) provide a theoretical foundation to Bagehot's doctrine that a LLR should lend to solvent but illiquid banks, while Freixas, Rochet, and Parigi (2004) emphasize the difficulty of basing policy decisions on a clear-cut distinction between insolvency and illiquidity – a criticism previously raised by Goodhart (1999). Our model also emphasizes the uncertainty involved in distinguishing insolvent from illiquid banks. However, unlike most of the extant literature, we focus on the dynamic trade-offs of LLR interventions.

In this regard, our model is most closely related to Santos and Suarez (2019). The dynamic banking model in our paper is largely inspired by theirs, but the focus is different. They show how regulatory liquidity requirements improve the efficiency of LLR interventions by buying policymakers time before making an intervention decision. Two key differences between their framework and ours that deserve to be highlighted are: (i) we assume that bailing out insured deposits is costly, and (ii) we allow for the partial liquidation of the bank's assets. Even though these differences may seem innocuous, they fundamentally change the trade-offs affecting the PA's intervention decision. In particular, in Santos and Suarez (2019), the policy trade-off is essentially static since it only depends on how the expected value of the bank's asset compares to its liquidation value. Hence, in their model, it is always optimal to delay the decision of whether to resolve or support the bank until the bank has depleted its cash balances. In our model, in contrast, the policy problem is inherently dynamic, with the PA optimally trading off the value of information against the dilution costs of delaying intervention over time.

Our model also shares common features with the dynamic banking models of He and Xiong (2012) and He and Manela (2016). In contrast to these papers, we abstract from coordination problems among wholesale creditors and their information acquisition incentives and focus instead on the optimal timing of policy interventions.

The bank in our model is passive and its balance sheet is exogenously given. We deliberately abstract from *ex ante* moral hazard concerns in order to highlight the *ex post* trade-offs faced by policymakers.[8] In particular, we show that providing liquidity support to a troubled bank need not always be *ex post* optimal since it increases the cost borne by policy authorities if the bank is ultimately revealed to be insolvent.[9] Other papers studying *ex post* policy interventions in troubled banks include Philippon and Skreta (2012), Bruche and Llobet (2014) and Segura and Suarez (2020). These papers adopt a mechanism design approach, and characterize the optimal design of bank bailouts when banks are subject to a debt overhang problem and are privately informed about the quality of their assets. Rather than studying the static implementation of recapitalization programs under asymmetric information, our dynamic model focuses on the timing of policy interventions under uncertainty.

2 The Model

We consider a model in continuous time with a representative bank, a continuum of investors and a Policy Authority (PA). Time is denoted by $t \in \mathbb{R}$. All agents are risk neutral and there is no discounting.

[8] There exists an extensive literature studying the *ex ante* incentive effects of LLR interventions: e.g., Repullo (2005), Ratnovski (2009), and Jeanne and Korinek (2020)

[9] The negative effect of LLR support on the cost of bailing out depositors has been previously recognized by economic historians, including Bordo (1989), but seems to have been largely ignored by more recent theoretical contributions. Choi, Santos, and Yorulmazer (2021) point to another unintended consequence of LLR operations; namely, the adverse effect on the quality of collateral in private funding markets.

2.1 The Bank

The bank enters the economy before date 0 with a legacy asset (e.g., a loan portfolio), financed by a mix of insured deposits (δ) and uninsured wholesale debt (D_0). We normalize the bank's balance sheet size to 1, so that the bank's equity is equal to $E_0 \equiv 1 - \delta - D_0$. The asset is perfectly divisible. To simplify the exposition, we assume that the asset matures at some date $T \rightarrow \infty$.[10]

At date 0, a verifiable shock realizes that may adversely affect the quality of the bank's asset (e.g., increase the fraction of non-performing loans in its portfolio). With probability $\mu \in (0,1)$, the bank's asset is unaffected by the shock and generates a cash flow of R_g at maturity. With converse probability $1-\mu$, the bank's asset is impaired by the shock and its cash flow at maturity is reduced to $R_b < R_g$. We refer to a bank with an unimpaired asset as a *good bank*, and a bank with an impaired asset as a *bad bank*. The asset generates no cash flow prior to maturity, but can be liquidated at any date $t \geq 0$ for a value of $\ell_i \leq R_i$, where $i \in \{g, b\}$ denotes the bank's type.[11]

Whether or not the bank's asset is impaired by the shock is initially unobservable: i.e., the bank's type is unknown at date 0. However, the probability μ is common knowledge, which allows to calculate the asset's expected cash flow at maturity, $R \equiv \mu R_g + (1-\mu) R_b$, and the asset's expected liquidation value, $\ell \equiv \mu \ell_g + (1-\mu)\ell_b$.

Public information (news) about the quality of the bank's asset randomly arrives over time.[12] For simplicity, we assume that if news arrives, it perfectly reveals the bank's type to all agents in the economy. News is assumed to follow a Poisson process with intensity $\lambda > 0$. Given this Poisson process, the probability that news arrives before some date t

[10] Our analysis would be essentially unchanged if, instead of having infinite maturity, the asset matured according to a Poisson process with a sufficiently small intensity, so that any other process with positive intensity (e.g., the maturing of the bank's debt claims) arrives earlier almost surely.

[11] The assumption that liquidation may destroy value is meant to reflect frictions that reduce the asset's liquidation value below its value in best use (Shleifer and Vishny, 1992). For example, such costs could stem from asset buyers being less efficient at monitoring borrowers than the bank, which reduces the value of loans if they are sold.

[12] We do not explicitly model where information comes from, and take the information process as exogenous. In practice, public information about the quality of a bank's assets comes from various sources, including bank supervisors, credit rating agencies, financial markets, etc.

follows an exponential distribution and is given by:

$$p(t) = 1 - e^{-\lambda t}.$$

2.2 Investors

The bank's debt is held by the investors. An amount D_0 of this debt is uninsured and is uniformly held across a subset of investors, which we refer to as uninsured wholesale creditors. The remaining debt δ is insured and is held by the remaining investors, which we refer to as insured retail depositors.

Following Santos and Suarez (2019), we assume that uninsured creditors are given the option to "put their debt back" to the bank in exchange for a fixed repayment $D > D_0$ at some random exercise date.[13] This assumption is akin to assuming that wholesale debt consists of overlapping issues of zero-coupon debt with fixed maturity. Creditors' option to put their debt arrives according to an independent Poisson process with intensity $\gamma > 0$. Given this Poisson process, the fraction of uninsured creditors who receive the option to put their debt before some date t is equal to:

$$1 - n(t) = 1 - e^{-\gamma t}.$$

At any date t, uninsured creditors with the option to put their debt back to the bank must decide whether to withdraw or roll over, given all the available information.[14] Uninsured creditors make their withdraw/roll over decision in order to maximize their expected payoff. We assume that uninsured creditors choose to stay invested in the bank if they are indifferent between withdrawing and rolling over (e.g., due to small transaction costs that must be incurred if they withdraw).

[13] The repayment D is negotiated before date 0 and is therefore determined outside the model. We assume that the probability that the bank is hit by the shock is sufficiently small such that it was feasible for the bank to raise financing before date 0.

[14] We assume that the dispersed ownership of wholesale debt, and the resulting hold up problem among uninsured creditors, rules out the possibility of privately restructuring the bank's wholesale debt.

Insured deposits are a stable source of financing and are never withdrawn before the asset matures.[15] Insured depositors are senior to uninsured creditors at maturity, or following a regulatory intervention that leads to the resolution of the bank prior to maturity. In case the bank is unable to meet insured depositors' claims in full, they are made whole by a deposit insurance fund (financed by the PA). There is a social cost to public funds $\phi > 0$ that is incurred per unit of funds spent by the PA to make insured depositors whole.[16]

Assumption 1. *Asset cash flows and debt face values satisfy:*

$$\mu(R_g - \delta) < D < R_g - \delta \quad \textit{and} \quad R_b = \ell_b < \ell_g < \min\{\delta, D\}$$

The assumption $D < R_g - \delta$ implies that a good bank is solvent in the absence of asset sales. Conditional on being hit by the shock, however, the bank cannot issue new claims since $\mu(R_g - \delta) < D$ and $R_b < \delta$. Consequently, the bank must meet debt withdrawals by partially liquidating its asset.

The bank's asset can be sold at any date t to deep-pocketed asset buyers at a per unit price equal to its liquidation value given all the available information at the time of sale. Thus, the asset can be sold for ℓ if liquidation takes place before news arrives, or for ℓ_i if liquidation takes place after news arrives, depending on whether the bank's type $i \in \{g, b\}$ is revealed to be good or bad. To simplify the exposition, we assume $\ell_g < D$ so that even a good bank eventually defaults if sufficiently many uninsured creditors withdraw, and that $\ell_g < \delta$ so that the PA must always bail out insured depositors if the bank's asset is liquidated before it matures. Finally, we also assume $\ell_b = R_b$ so that the liquidation of an impaired asset does not destroy value. These last three assumptions can be relaxed without changing the qualitative nature of our results.[17]

[15] Recent empirical evidence, including Chen, Goldstein, Huang, and Vashishtha (2020), show that insured deposits are "sticky", in the sense that they are withdrawn slower and in smaller quantities than uninsured deposits.

[16] The social cost of public funds could reflect, for example, distortions caused by taxation if additional revenue has to be raised to pay back insured deposits (Dahlby, 2008).

[17] What is important is that the asset's expected liquidation value before information arrives (ℓ) is less

2.3 The Policy Authority

The Policy Authority (PA) enters the economy at date 0 and decides whether to intervene at some date $t \geq 0$. Its objective is to maximize expected aggregate output, defined as:

$$\mathbb{E}[\text{Total Cash Flows} + \phi(\text{Loss of the PA})].$$

Aggregate output equals total cash flows (i.e., bank profits plus investor income minus any transfer from the PA) plus any loss incurred by the PA scaled by the social cost of public funds, ϕ. If $\phi = 0$, aggregate output is unaffected by how cash flows are allocated between the bank, investors and the PA. In this case, bailing out insured depositors does not involve a deadweight loss. If $\phi > 0$, the allocation of cash flows matters for aggregate output. In particular, increasing the PA's loss by one unit reduces aggregate output by more than reducing bank profits or investor income by one unit.

Section 3 considers the baseline model where the PA decides whether to resolve the bank at some date $t \geq 0$. If the bank is resolved, the PA enforces a complete write-down of the bank's outstanding uninsured debt and liquidates its remaining assets.[18] Any loss to the PA from bailing out insured depositors following resolution is scaled by the social cost of public funds. Section 4 extends the analysis by allowing the PA to act as a lender of last resort (LLR) and to lend funds to the bank at the (zero) risk-free rate. If the bank turns out to be insolvent, any additional loss incurred by the PA from the provision of liquidity support is also scaled by the social cost of public funds. Section 5 provides a discussion of two key aspects of the model, namely: (i) how the availability of liquidity support affects

than its expected cash flow at maturity (R), but exceeds the cash flow of an impaired asset at maturity (R_b). Similarly, what is important is that the face value of debt (D) and the face value of insured deposits (δ) exceed the asset's liquidation value in the absence of information (ℓ).

[18] Resolution in our model reflects current regulatory practices. For example, in the United States, the FDIC is responsible for taking a critically undercapitalized bank into receivership. The FDIC then sells the franchise of the failing institution to another financial institution or retains the bank's assets and pays insured depositors directly. Similarly, in Europe, the Bank Recovery and Resolution Directive (BRRD) enables resolution authorities to dispose of an institution's franchise, temporarily transferring it to a publicly owned entity, and/or writing down bail-inable debt. We abstract from differences in resolution tools by assuming that the PA always obtains a payoff equal to the liquidation value of the bank's remaining assets following resolution.

the optimal timing of resolution; and (ii) the conditions under which liquidity support is preferred to publicly-financed equity injections. Section 6 concludes.

3 Optimal Timing of Resolution

Before characterizing the optimal timing of the PA's resolution decision, we first analyze uninsured creditors' incentives to roll over or withdraw their claims.

3.1 Creditors' withdrawal decision

It is strictly dominant for uninsured creditors to start exercising their option to put their debt back to the bank as soon as the opportunity arises. By withdrawing, uninsured creditors secure repayment from the liquidation proceeds of the bank's asset and circumvent the seniority of insured depositors in resolution. If they were to roll over, uninsured creditors would run the risk of not being repaid, either because the bank's asset turns out to be impaired or because the PA chooses to resolve the bank before its asset matures.

The bank cannot issue new claims in the absence of information (*cf.,* Assumption 1), and must finance withdrawals by partially selling its asset. If withdrawals last until date t and no news arrives in the meantime, the share of assets the bank has to sell in order to meet withdrawals is equal to:

$$1 - z(t) = \frac{(1 - n(t))D}{\ell}.$$

The selling of assets progressively erodes a good bank's equity value. Hence, there exists a critical date after which even a good bank becomes insolvent. This critical date τ is determined by the following condition:

$$z(\tau)R_g - n(\tau)D - \delta = 0,$$

which, given the definitions of $z(t)$ and $n(t)$, can be solved for τ:

$$\tau = \frac{1}{\gamma} \ln \left(\frac{\left(\frac{R_g}{\ell} - 1\right) D}{\delta + \left(\frac{D}{\ell} - 1\right) R_g} \right) > 0,$$

where the inequality follows from Assumption 1.

The bank does not immediately default at date τ because it still has assets that can be liquidated. However, since insured depositors are senior at maturity, it is strictly dominant for uninsured creditors to continue withdrawing for all $t > \tau$ even if the bank is revealed to be good. Similarly, it is strictly dominant for uninsured creditors to continue withdrawing if the bank is revealed to be bad since a bad bank is always insolvent.

In the absence of intervention by the PA, wholesale debt withdrawals only stop if good news arrives before date τ. Since a good bank is still solvent at date $t \leq \tau$, uninsured creditors that roll over their claims in this case are guaranteed full repayment at maturity providing that all other creditors do the same. We assume that uninsured creditors coordinate on the equilibrium in which they all roll over.[19]

Lemma 1. *It is strictly dominant for uninsured creditors to start withdrawing at date* 0*. Given withdrawals that last t periods, it is strictly dominant for uninsured creditors to continue withdrawing after the arrival of bad news or if $t > \tau$. It is weakly dominant for uninsured creditors to stop withdrawing if good news arrives before τ.*

3.2 Optimal intervention: waiting versus resolution

The PA has the option to enforce a complete write-down of the bank's outstanding uninsured debt and liquidate its remaining assets at any date $t \geq 0$. The PA's problem consists of deciding when to intervene and resolve the bank (if ever).

[19]This assumption can be justified by the fact that the PA would optimally bail out the bank at no cost if uninsured creditors were to continue withdrawing following the arrival of good news before τ.

The value of resolving the bank at date t is equal to:

$$L(t) = \ell + \phi(z(t)\ell - \delta). \tag{1}$$

The first term of Equation (1) equals the total proceeds from liquidation: i.e., the liquidation value of the bank's remaining assets, $z(t)\ell$, plus the cash flow from selling assets to cover debt withdrawals before date t, $(1 - z(t))\ell$. The second term equals the cost of bailing out insured depositors following resolution: i.e., the difference between the liquidation value of the bank's remaining assets less the face value of insured deposits scaled by the social cost of public funds, ϕ. Since $z'(t) < 0$, the value from resolving the bank strictly decreases in t: i.e., $L'(t) = \phi n'(t)D < 0$. The reason is that delaying resolution gives uninsured creditors time to extract more resources from the bank, which increases the cost of bailing out insured depositors following resolution. We refer to this effect as the *dilution costs* of delaying resolution.

Instead of resolving the bank at date t, the PA has the option to delay its resolution decision and wait for news. The benefit of waiting for news stems from avoiding the inefficient resolution of a good bank, and depends on whether news arrives before or after date τ. If good news arrives before τ, uninsured creditors stop withdrawing and intervention becomes unnecessary. If bad news arrives before τ, the bank is revealed to be insolvent and the PA optimally resolves the bank. Consequently, expected aggregate output conditional on news arriving at date $t' \leq \tau$ is:

$$V(t') = z(t')R + (1 - z(t'))\ell + (1 - \mu)\phi(z(t')R_b - \delta), \quad \forall t' \leq \tau. \tag{2}$$

The first term of Equation (2) equals the expected value of the bank's remaining assets. The second term equals the liquidation proceeds used to pay back debt withdrawn before date t'. The last term equals the PA's expected loss from bailing out insured depositors if the bank is revealed to be bad (insured depositors do not have to be bailed out if the bank is revealed to be good since a good bank is still solvent at date $t' \leq \tau$).

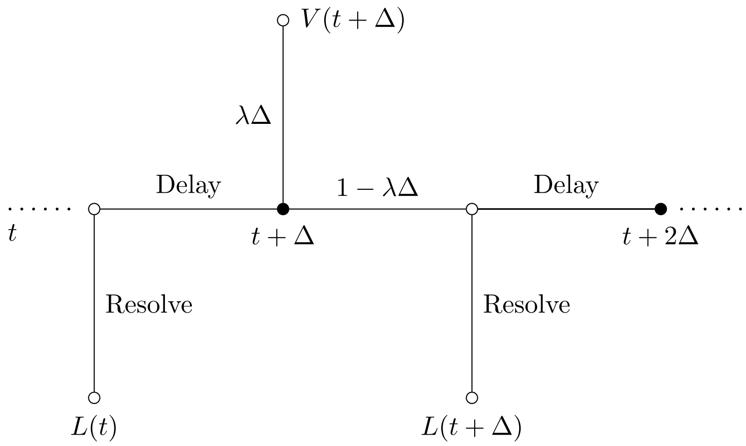


Figure 1: Timing of the PA's intervention decision.

If news arrives after date τ, wholesale creditors continue withdrawing regardless of whether the bank is revealed to be good or bad (*cf.*, Lemma 1). Since the arrival of good news does not stop the costly fire-selling assets to meet debt withdrawals, the PA optimally resolves the bank regardless of its type. Expected aggregate output conditional on information arriving at date $t' > \tau$ is therefore $V(t') = L(t')$ for all $t' > \tau$.

Figure 1 summarizes the timing of the PA's intervention decision. At every date t, the PA must choose whether to resolve the bank or to delay resolution for an arbitrarily small interval of time Δ. Given the Poisson process driving information arrival, the PA expects news to arrive with probability $\lambda\Delta$ if it delays resolution. If news arrives, expected aggregate output is equal to $V(t+\Delta)$. With complementary probability $1-\lambda\Delta$ news does not arrive, in which case the PA must again choose at date $t+\Delta$ whether to resolve the bank for a value of $L(t+\Delta)$ or to continue waiting for news.

An optimal intervention policy consists of a threshold date t_r such that the PA resolves the bank at date t_r if news does not arrive before. Given this policy rule, the value of delaying resolution at date t as $\Delta \to 0$ is equal to:[20]

$$W(t;t_r) = \int_0^{t_r-t} V(t+k)dp(k) + (1-p(t_r-t))L(t_r). \tag{3}$$

[20]See Lemma A1 in the Appendix for a formal derivation of the PA's value function.

Differentiating Equation (3) with respect to t_r, it follows that any interior optimal resolution date $t_r^* \in (0, \tau)$ must satisfy the following first-order condition:

$$\lambda\big(V(t_r^*) - L(t_r^*)\big) + L'(t_r^*) = 0. \tag{4}$$

The difference $V(t_r^*) - L(t_r^*)$ corresponds to the *value of information* at the optimal resolution date. This difference measures the net gain from choosing the optimal policy action following the arrival of information compared to resolving the bank. The first term of Equation (4), which equals the value of information multiplied by the arrival rate of news λ, captures the benefit of delaying resolution for an arbitrarily short period of time at date t_r^*. The second term, $L'(t_r^*)$, equals the dilution costs from allowing wholesale debt withdrawals to continue for a similarly short period of time at date t_r^*.

Before date τ, the value of information can be positive or negative, depending on the value of ϕ. After date τ, in contrast, the value of information is always zero since $V(t') = L(t')$ for all $t' > \tau$. Thus, it is never optimal for the PA to delay resolution to after date τ: i.e., $t_r^* \leq \tau$.[21] In order for the PA to sometimes choose an interior optimal resolution date, $t_r^* \in (0, \tau)$, we impose the following assumption.[22]

Assumption 2.

$$\mu\lambda\delta < \gamma(D - \ell). \tag{5}$$

Assumption 2 implies that marginally delaying resolution increases the PA's expected loss. The left-hand side of condition (5) equals the reduction in the PA's expected loss from marginally delaying resolution: i.e., the instantaneous probability that good news arrives multiplied by the face value of insured deposits. The right-hand side equals the increase in dilution costs from marginally delaying resolution: i.e., the fraction of debt withdrawals per unit of time multiplied by the reduction in the bank's equity value due to asset sales.

[21] See the proof of Proposition 1 in Appendix A1 for a formal proof of this claim.

[22] In Appendix A2, we discuss what happens if Assumption 2 is violated and show that the optimal resolution date in this case is either $t_r^* = 0$ or $t_r^* = \tau$, depending on the value of ϕ.

Proposition 1. *Given Assumption 2, there exist threshold values, $\underline{\phi}$ and $\overline{\phi}$, such that the PA's optimal intervention decision in the absence of news satisfies:*

- *If $\phi < \underline{\phi}$: The PA resolves the bank at date $t_r^* = \tau$.*
- *If $\phi \in [\underline{\phi}, \overline{\phi})$: The PA resolves the bank at date $t_r^* \in (0, \tau)$.*
- *If $\phi \geq \overline{\phi}$: The PA resolves the bank at date $t_r^* = 0$.*

If bad news arrives before t_r^*, the bank is immediately resolved. If good news arrives before t_r^*, uninsured creditors stop withdrawing and the PA never intervenes.

When deciding whether or not to intervene, the PA trades off the value of information with the dilution costs of delaying resolution. On the one hand, delaying resolution reduces the likelihood of a *type-I* error: i.e., the risk that the PA mistakenly chooses to resolve a solvent bank. On the other hand, delaying resolution allows wholesale debt withdrawals to continue and reduces the resources the PA can seize after resolution. Not liquidating the bank today therefore increases the cost of a *type-II* error: i.e., the PA's loss from bailing out insured depositors in case the bank is ultimately resolved.

The timing of the PA's resolution decision critically depends on the social costs of public funds. If there are no social costs ($\phi = 0$), the dilution costs are absent. While uninsured debt withdrawals reduce the resources the PA can seize after resolution, they do not create a deadweight loss since they just transfer resources from the PA to uninsured creditors. In this case, it is always optimal to delay resolution until date τ. Such a policy maximizes the likelihood that uninsured creditors stop withdrawing following the arrival of good news, thereby minimizing the risk that the PA inefficiently resolves a solvent bank. By continuity, it follows that if the social costs are sufficiently small, the PA never liquidates the bank before date τ (unless bad news arrives before).

If the social costs are large ($\phi \geq \overline{\phi}$), the PA resolves the bank immediately after wholesale creditors start withdrawing: i.e., at date 0. In this case, the dilution costs from allowing wholesale debt withdrawals to continue always exceed the value of information. Waiting for good news simply doesn't pay.

For intermediate values $\phi \in [\underline{\phi}, \overline{\phi})$, the PA optimally delays resolution to after date 0 but chooses to resolve the bank at some date $t_r^* < \tau$ if news does not arrive before. Rather than resolving the bank immediately after withdrawals start, the PA is willing to wait in the hope that good news arrives tomorrow. As time passes, however, the dilution costs from allowing wholesale debt withdrawals to continue become too large compared to the value from avoiding the inefficient resolution of a solvent bank. The reason is that the bank must meet debt withdrawals by fire-selling assets, which leads the value of information to decrease over time. The PA therefore optimally resolves the bank strictly before date τ.

3.3 Implications for the optimal timing of resolution

In this subsection, we show how the optimal resolution date is affected by changes in broader economic conditions (the arrival rate of information, λ, and the marketability of the bank's asset, ℓ) and the bank's liability structure (the share of insured deposits, δ, and the intensity of debt withdrawals, γ).

Corollary 1 (Broader Economic and Regulatory Environment)**.** *For $\phi \in (\underline{\phi}, \overline{\phi})$, the optimal resolution date t_r^*:*

1. *strictly increases in the the arrival rate of information, $\frac{\partial t_r^*}{\partial \lambda} > 0$;*
2. *can increase or decrease in the marketability of bank assets, $\frac{\partial t_r^*}{\partial \ell} \gtrless 0$.*

Arrival rate of information. A higher arrival rate of information, λ, increases the value of information but has no effect on the dilution costs from delaying resolution. Hence, increasing λ leads the PA to optimally delay its resolution decision. Figure 2a provides a numerical example of the effect of an increase in λ on the optimal resolution date. The figure plots the value of information (black lines) and the dilution costs (red line) as functions of time. The optimal resolution date, t_r^*, lies at the intersection of the two curves. A higher value of λ leads to a rightward shift in the value of information (black dashed line), while leaving the dilution costs unchanged. As a consequence, the optimal resolution date increases from t_r^* to $t_r^{*\prime}$.

A higher value of λ can be interpreted as reflecting more efficient information collection; e.g., due to improvements in institutional processes or technological developments. Examples of the former include the harmonization of recovery and resolution frameworks in the European Union with the goal to increase the speed of resolution activities (EU, 2014), or the adoption of common principles that improve coordinated resolution of multinational banks (FSB, 2014). A recent example of technological developments that facilitate regulatory information collection is the development and implementation of a global legal identifier system that enhances regulators' surveillance powers (FSB, 2019).

Asset marketability. Changes in the liquidation value, ℓ, have an ambiguous effect on the optimal resolution date. The ambiguity stems from two countervailing effects of ℓ on the value of information (as with changes in λ, changes in ℓ do not affect the dilution costs from delaying resolution). First, an increase in asset marketability implies that the bank needs to sell less assets in order to meet a given amount of debt withdrawals. Larger values of ℓ therefore increase expected aggregate output conditional on information, $V(t)$, which raises the value of information. Second, an increase in asset marketability increases the value from resolution, $L(t)$, because the PA obtains a higher price from liquidating the bank's assets. This effect lowers the value of information. Hence, depending on which of these two effect dominates, the optimal resolution date can either increase or decrease if the marketability of the bank's asset improves.

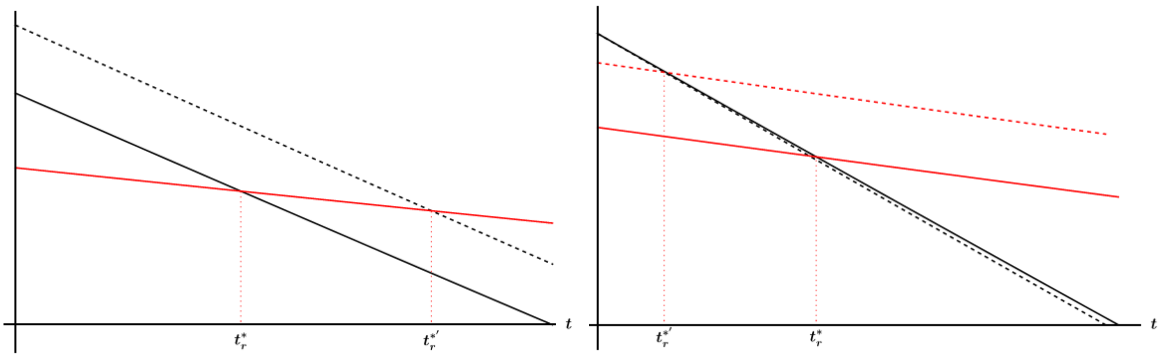
Corollary 2 (Bank Liability Structure)**.** *For $\phi \in (\underline{\phi}, \overline{\phi})$, the optimal resolution date t_r^*:*

1. *strictly increases in the the share of insured deposits, $\frac{\partial t_r^*}{\partial \delta} > 0$;*
2. *can increase or decrease in the intensity of debt withdrawals, $\frac{\partial t_r^*}{\partial \gamma} \gtrless 0$.*

Share of insured deposits. An increase in the amount of insured deposits, δ, leads the PA to resolve the bank later. The reason is that a higher share of insured deposits

Figure 2: Comparative statics of γ and λ.

The figure shows the expected value of information (black lines), $\lambda(V(t_r^*) - L(t_r^*))$, and the dilution costs (red lines), $L'(t_r^*)$. The optimal resolution date t_r^* is at the intersection of the curves. Baseline parameter values: $\lambda = 0.05$, $\gamma = 0.99$, $\mu = 0.25$, $\phi = 0.01$, $D = 0.75$, $\delta = 0.25$, $R_g = 1.1$, $\ell_g = 0.2$, $R_b = \ell_b = 0.18$.



(a) Increase in λ: An increase in the arrival rate of information, λ, from 0.05 to 0.0516 increases the optimal resolution date.

(b) Increase in γ: An increase in the intensity of debt withdrawals, γ, from 0.999 to 1.024 reduces the optimal resolution date.

increases the benefit of avoiding a *type-I* error since more insured deposits have to be bailed out if a solvent bank is inefficiently resolved. Graphically, the effect of an increase in insured deposits (and a corresponding decrease in initial equity) is akin to the effect of a higher arrival rate of information: i.e., the value of information shifts to the right, while the dilution costs remain unaffected (see Figure 2a).[23] This result suggests that regulatory authorities will be more inclined to resolve troubled banks later if these are financed by a higher share of insured deposits, compared to otherwise similar banks with fewer insured deposits.

Intensity of debt withdrawals. The ratio $1/\gamma$ can be interpreted as the average maturity of the bank's wholesale debt, with a higher value of γ corresponding to a shorter debt maturity structure. Increasing γ has an ambiguous effect on the PA's optimal resolution date. First, a shorter debt maturity structure reduces the value of information because fewer assets are left on the bank's balance sheet once news arrives. Taken on its

[23] Qualitatively, the result that an increase in insured deposits increases the optimal resolution date is independent of whether the increase in δ is matched by a reduction in bank equity or in the amount of wholesale debt. However, if the increase in insured deposits is matched by a reduction in wholesale debt, the dilution costs also decrease, which leads the PA to delay resolution even more compared to the case where higher insured deposits are matched by a reduction in bank equity.

own, this effect incentivizes the PA to resolve the bank earlier. Second, a shorter debt maturity structure has two opposing effects on the dilution costs. On the one hand, the dilution costs increase because a larger fraction of outstanding debt is expected to be withdrawn if intervention is marginally delayed. This *flow effect* reinforces the effect of γ on the value of information. On the other hand, the bank's stock of outstanding debt at any given date declines if γ increases. This *stock effect* reduces the dilution costs and incentivizes the PA to delay resolution.

The numerical example in Figure 2b illustrates a case where the flow effect outweighs the stock effect, so that a shorter debt maturity structure leads to a rightward shift in dilution costs (red curves) and an earlier optimal resolution date. This numerical result suggests that regulatory measures that incentivize banks to use more stable sources of funding (e.g., Basel III's net stable funding ratio) optimally lead regulatory authorities to delay the resolution of troubled banks.

4 Optimal Intervention with Liquidity Support

In this section, we analyze how the optimal timing of the PA's resolution decision is affected if the PA can also act as a lender of last resort (LLR) by lending funds to the bank at the (zero) risk-free rate.

4.1 Value from liquidity support

By lending funds to the bank, the PA allows the bank to cover debt withdrawals without having to sell assets. Liquidity support, however, affects neither the assets' cash flow nor the face value of the bank's debt liabilities. Consequently, liquidity support does not change wholesale creditors' withdrawal incentives and Lemma 1 continues to apply.

In exchange for the funds supplied by the PA, the bank provides the PA with (senior) claims against its future cash flows.[24] Liquidity support therefore exposes the PA

[24] We can interpret the seniority of the PA's claim as akin to a collateral requirement in an emergency lending operation. As in practice, if the central bank obtains a senior (or collateralized) claim, it reduces

to counterparty risk: if the bank turns out to be bad, the PA must not only bail out insured depositors but also makes an additional loss on any unpaid claims it holds against the bank. Thus, compared to delaying resolution and waiting for news to arrive, liquidity support preserves the value of information over time by avoiding the liquidation of unimpaired assets but also increases the PA's loss conditional on the bank being insolvent.

Consider a policy whereby the PA initiates liquidity support at date $t_s \geq 0$ and continuously maintains support until news arrives at some date $t' > t_s$. The value of such a policy depends on whether liquidity support is initiated before or after date τ. We show in the Appendix that the PA will never choose to initiate liquidity support after date τ.[25] Hence, in what follows, we focus on the case where $t_s \leq \tau$.

If the PA initiates support before date τ and the bank is revealed to be good, withdrawals stop and the PA does not incur any loss. In contrast, if the bank is revealed to be bad, the PA resolves the bank and incurs a loss from bailing out insured depositors and from any unpaid claims it holds against the bank. Expected aggregate output conditional on information arriving at date $t' > t_s$ is therefore:

$$V_s(t'; t_s) = V(t_s) - (1-\mu)\phi(n(t_s) - n(t'))D, \quad \forall t_s \leq \tau. \tag{6}$$

Since the provision of liquidity support allows the bank to pay back withdrawing creditors without selling assets, the bank's total cash flow and the cost of bailing out insured depositors of a bad bank remain unchanged between date t_s (when support is initiated) and date t' (when news arrives). The second term of Equation (6) equals the PA's additional loss from granting liquidity support if the bank turns out to be bad and reflects the claims the PA holds against the bank by the time news arrives, which amount to $(n(t_s) - n(t'))D$. The following assumption imposes an upper bound on the PA's additional loss from providing liquidity support.[26]

the resources that are left to cover insured deposits in case the bank defaults. In our model, since all losses are ultimately borne by the PA, it is irrelevant whether losses originate from liquidity provision or from covering insured deposits.

[25] See Lemma A6 in Appendix A1 for a formal proof of this claim.

[26] Assumption 3 implies an upper bound $\bar{\ell} < \delta$. We discuss what happens if Assumption 1 is violated

Assumption 3.

$$(1-\mu)\frac{\gamma}{\lambda+\gamma}D < \mu(\delta-\ell). \tag{7}$$

The left-hand side of condition (7) equals the expected value of wholesale debt withdrawals from a bad bank before news arrives, which corresponds to the PA's expected counterparty risk exposure from providing liquidity support at date 0. Assumption 3 states that this additional liability from providing liquidity support is strictly less than the benefit from not having to bail out insured depositors of a good bank conditional on liquidity support being maintained until news arrives.

4.2 Optimal intervention policy

Optimal intervention policies with liquidity support can be summarized by two thresholds, t_s and $t_r \geq t_s$, where t_s denotes the date at which the PA initiates liquidity support (if ever) and t_r denotes the date at which the PA resolves the bank (if ever).[27] Given this policy rule, the PA's value function (3) becomes:

$$S(t;t_s,t_r) = \int_0^{t_s-t} V(t+k)dp(k) + \int_{t_s-t}^{t_r-t} V_s(t+k;t_s)dp(k) + (1-p(t_r-t))L(t_r). \tag{8}$$

The first term of Equation (8) equals expected aggregate output if the PA delays intervention after date 0 and news arrives before liquidity support is initiated or the bank is resolved. The second term equals expected aggregate output if news arrives after liquidity support is initiated at date t_s. The last term equals aggregate output if the bank is resolved at date t_r and news does not arrive before. Maximizing Equation (8) with respect to t_s and t_r yields the PA's optimal intervention policy.

Proposition 2. *Given Assumption 3, there exist threshold values, ϕ^* and ϕ^{**}, such that the PA's optimal intervention decision in the absence of news satisfies:*

in Appendix A2 and show that the PA in this case will never choose to delay liquidity support.

[27]We show in the proof of Proposition 2 that restricting attention to monotone policies is without loss of generality: i.e., the PA will never choose to terminate liquidity support without resolving the bank.

- *If $\phi < \phi^*$: The PA provides liquidity support immediately at date 0 and maintains support until news arrives: i.e., $t_s^* = 0$ and $t_r^* = \infty$.*

- *If $\phi \in [\phi^*, \phi^{**})$: The PA delays liquidity support until date τ and then maintains support until news arrives: i.e., $t_s^* = \tau$ and $t_r^* = \infty$.*

- *If $\phi \geq \phi^{**}$: The PA resolves the bank at date 0: i.e., $t_s^* = t_r^* = 0$.*

If the PA supports the bank and good news arrives, wholesale debt withdrawals stop and the PA ends its liquidity support. If bad news arrives, the bank is immediately resolved.

Figure 3 provides a graphical illustration of Proposition 2. As in Section 3, it is useful to consider the benchmark case where there are no social costs to public funds. If $\phi = 0$, the optimal intervention policy consists of providing liquidity support at date 0 and maintaining support until news arrives. Granting immediate liquidity support ensures that unimpaired assets are never sold and eliminates the risk of a *type-I* error. Without the social costs, there is no deadweight loss from bailing out insured deposits of a bad bank, and hence no costs associated with making a *type-II* error.

When $\phi > 0$, the optimal policy depends on the liquidation value of the bank's asset. For small values of ℓ, the PA never resolves the bank in the absence of news, regardless of the value of ϕ. The only decision the PA has to make is whether to support the bank at date 0 (if $\phi < \phi^*$) or to delay the provision of liquidity support to date τ (if $\phi \geq \phi^*$). Liquidity support in this case not only avoids costly fire-sales, it also lowers the PA's expected loss compared to resolution. Consequently, the PA always prefers to support the bank.

Delaying liquidity support has two opposing effects on aggregate output when $t_s \leq \tau$. On the one hand, delaying support forces the bank to cover debt withdrawals by fire-selling assets, which lowers the bank's profits if it turns out to be good. On the other hand, delaying support reduces the PA's expected loss. The reason is that delaying support lowers the PA's loss from unpaid claims it holds against the bank in case the bank turns out to be bad. When $\phi \geq \phi^*$, the gain from reducing the PA's counterparty risk exposure

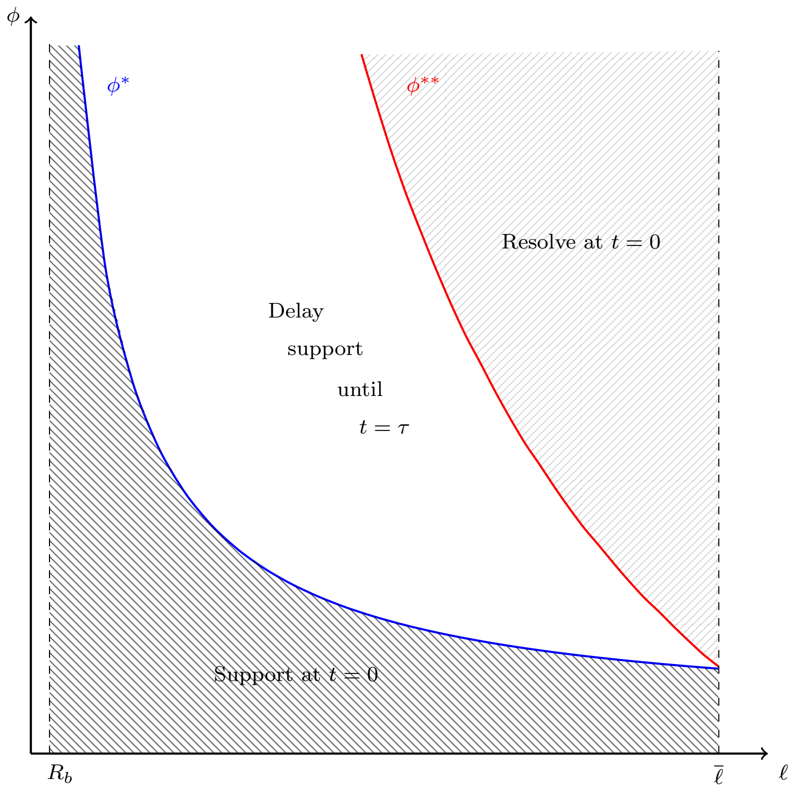


Figure 3: Intervention regimes in $\phi - \ell$ space.
The graph plots the two thresholds, $\phi^*(\ell)$ and $\phi^{**}(\ell)$, as functions of ℓ and shows the different policy regions in $\phi - \ell$ space. The dark-shaded region corresponds to parameter values where providing liquidity support at date 0 is optimal. The unshaded region corresponds to parameter values where delaying liquidity support until date τ is optimal. The light shaded region corresponds to parameter values where resolving the bank at date 0 is optimal.

exceeds the loss from fire-selling assets for all $t_s \leq \tau$. Hence, the PA optimally delays liquidity support to date τ. Further delaying the provision of liquidity support is never optimal because a good bank has negative equity if it covers debt withdrawals by selling assets beyond date τ. As a result, any reduction in the bank's cash flow from selling assets after date τ is ultimately borne by the PA.

For large values of ℓ, resolving the bank immediately at date 0 may be optimal. In this case, the provision of liquidity support entails a trade-off: i.e., it avoids costly fire-sales but increases the PA's expected loss compared to immediate resolution. If $\phi > \phi^{**}$, the PA's additional loss from providing liquidity support is too great compared to the value of avoiding a *type-I* error. Thus, the PA optimally resolves the bank at date 0. Contrary to the case without liquidity support, resolving the bank after date 0 in the absence of news is never optimal. The reason is that liquidity support preserves the value of information over time, implying that liquidity support always dominates delaying resolution.

4.3 Interpreting the frictions

The parameters ϕ and ℓ parameterize the two key frictions in our model: i.e., the social cost of public funds and the deadweight loss from liquidating unimpaired assets. Although these frictions enter our model in reduced form, variations in these parameters provide insights as to how systemic externalities or changes in macroeconomic conditions affect the PA's optimal intervention decision.

First, the social cost of public funds may be lower for systemically important banks (e.g., due to spillovers that the bailout of depositors of such banks imposes on other financial institutions). The failure of systemically important banks may also generate large fire-sale externalities due to the size of their balance sheets. Hence, banks can be ordered according to their systemic relevance along a positively sloped ray in Figure 3, with more systemically relevant banks at the lower end of the ray. Our model implies that regulatory authorities will optimally provide immediate liquidity support to systemically important banks and choose to resolve less systemically relevant ones.

Second, the social cost of public funds may be larger during economic recessions when the opportunity cost of public funds is particularly high (e.g., due to other government interventions such as unemployment programs or fiscal support to non-financial corporations). Market liquidity may also be impaired during recessions because of widespread asset sales and weakened balance sheets. The macroeconomic environment can therefore be mapped along a negatively sloped ray in Figure 3, with recessions characterized by small values of ℓ and large values of ϕ. Our model implies that regulatory authorities will optimally delay the provision of liquidity support during bad economic times and make more speedy resolution and support decisions during booms.

5 Discussion

This section provides a discussion of two key aspects of the model. First, we discuss how the presence of liquidity support affects the optimal timing of resolution. Second, we analyze the conditions under which liquidity support is preferred to policies whereby the PA directly injects equity capital into the bank.

5.1 Liquidity support and the timing of resolution

Comparing the optimal intervention decision with and without liquidity support (i.e., Propositions 1 and 2) shows how liquidity support affects the optimal timing of resolution.

Proposition 3. *The presence of liquidity support implies that:*

- *If $\phi < \phi^{**}$, the PA never resolves the bank unless it is revealed to be insolvent.*
- *The PA becomes less inclined to resolve the bank at date 0: i.e., $\bar{\phi} < \phi^{**}$.*

Without liquidity support, the PA chooses to resolve the bank at or before date τ, unless good news arrives before. Delaying resolution beyond date τ is never optimal since wholesale debt withdrawals do not stop following the arrival of good news after that date. With liquidity support, in contrast, the PA only resolves the bank if news arrives and reveals the bank's asset to be impaired. The reason is that liquidity support prevents debt withdrawals from eroding a good bank's equity value. The PA can therefore avoid the insolvency of a good bank by providing liquidity support before date τ and maintaining support until news arrives. In other words, liquidity support buys the PA time before having to make an irreversible resolution decision, thereby eliminating the risk that the PA mistakenly chooses to resolve a solvent bank.

If the social cost of public funds is sufficiently high, the PA resolves the bank at date 0, regardless of whether liquidity support is available. The critical cost threshold that triggers the resolution of the bank, however, is strictly larger when liquidity support is available: i.e., $\phi^{**} > \overline{\phi}$. The reason is that, with liquidity support, the value of information remains constant over time since the bank does not have to sell assets to meet debt withdrawals. As a consequence, the cost threshold that makes the PA just indifferent between resolving the bank at date 0 and delaying intervention to a later date is strictly larger when liquidity support is available.

5.2 Liquidity support vs. equity injections

The key benefit of liquidity support is that it preserves the value of information over time by avoiding the fire-selling of unimpaired assets. An alternative policy that avoids costly asset sales is to directly inject equity into the bank so as to incentivize wholesale creditors to refinance their claims.

This subsection analyzes the effect of such equity injections. We consider a policy whereby the PA injects equity capital E into the bank at some date $t_e \geq 0$ in exchange for (preferred) stock that is junior to wholesale debt but senior to the bank's inside equity. Given this equity injection, the bank seeks to refinance all of its outstanding wholesale debt with new long-term debt maturing at $T \to \infty$ with face value F. We assume that, following the equity injection, the PA can force the bank to refinance its wholesale debt as long as it leaves the bank with non-negative profits. Thus, in order for debt refinancing to be feasible, the face value F must satisfy the bank's limited liability condition:

$$\mu\big(z(t_e)R_g - \delta - n(t_e)F\big) \geq 0, \tag{9}$$

and wholesale creditors' break-even condition:

$$\mu \min\left\{F, \frac{z(t_e)R_g - \delta + E}{n(t_e)}\right\} + (1-\mu)\min\left\{F, \frac{(z(t_e)R_b - \delta + E)_+}{n(t_e)}\right\} \geq D, \tag{10}$$

where $(\cdot)_+ \equiv \max\{\cdot, 0\}$. The PA's optimal equity injection maximizes expected aggregate output at date t_e:

$$E^*(t_e) = \arg\max_E \left\{ z(t_e)R + (1 - z(t_e))\ell - (1-\mu)\phi \max\{E, \delta - z(t_e)R_b\} \right\},$$

subject to the constraints (9) and (10).

Lemma 2. *The optimal equity injection at date t_e is:*

$$E^*(t_e) = \frac{\delta + n(t_e)D - z(t_e)R}{(1-\mu)}.$$

Expected aggregate output at any date t following an equity injection of $E^*(t_e)$ at date t_e equals:

$$V_b(t_e) = V(t_e) - \phi\Big(n(t_e)D - \mu(z(t_e)R_g - \delta)\Big). \tag{11}$$

Just like the case with liquidity support, the bank's total cash flow and the cost of bailing out insured depositors of a bad bank remain fixed at their date t_e-level if the PA injects equity. The reason is that equity injections allow the bank to avoid costly asset sales by eliminating the maturity mistmatch on the bank's balance sheet. The second term of Equation (11) equals the PA's additional loss from equity injections: i.e., the difference between the value of outstanding wholesale debt at date t_e and the bank's remaining cash flow net of insured deposits.

Proposition 4. *It is never optimal to delay equity injections to after date 0: i.e., $t_e^* = 0$. The PA strictly prefers to provide liquidity support compared to injecting an amount of equity $E^*(0)$ at date 0 if and only if:*

$$(1-\mu)\frac{\gamma}{\lambda+\gamma}D < D - \mu(R_g - \delta). \tag{12}$$

Contrary to liquidity support, it is never optimal to delay equity injections. The value from delaying liquidity support stems from the fact that it lowers the PA's loss from unpaid claims it holds against the bank in case the bank turns out to be bad. Reducing the PA's counterparty risk exposure, however, comes at the cost of requiring the bank to meet debt withdrawals by fire-selling assets before liquidity support is granted, which lowers the profits of a good bank. Equity injections allow to implement this reallocation of cash flows from the bank to the PA without incurring the deadweight loss from fire-selling assets. In particular, the optimal equity injection $E^*(0)$ reduces the bank's profits to zero by forcing the bank to refinance its outstanding wholesale debt with face value D with (risky) long-term debt with face value $F = R_g - \delta$. Such a policy minimizes the amount of equity the PA must inject in order for debt refinancing to be feasible, and thereby minimizes the PA's additional loss in case the bank defaults.

Whether equity injections are preferred to liquidity support depends on the amount of equity the PA must inject in excess of the amount needed to bail out insured depositors of a bad bank: i.e., $(1-\mu)(E^*(0) - (\delta - R_b)) = D - \mu(R_g - \delta)$. This transfer corresponds to the additional loss the PA must incur in order for wholesale creditors to be willing to refinance their claims. If this transfer is greater than the PA's counterparty risk exposure from providing liquidity support at date 0 – the left-hand side of condition (12) – the PA always prefers liquidity support over equity injections. Otherwise, if condition (12) fails to hold, equity injections dominate liquidity support at date 0.

Corollary 3. *The PA is more inclined to provide the bank with liquidity support rather than injecting equity at date* 0 *when: (i) the intensity of debt withdrawals* (γ) *is low; (ii) the arrival rate of information* (λ) *is high; (iii) the share of insured deposits* (δ) *is high.*

Corollary 3 follows directly from condition (12). The PA tends to prefer liquidity support over equity injections if the intensity of debt withdrawals is low, or if the arrival rate of news is high, because these factors reduce the PA's counterparty risk exposure from providing liquidity support. More specifically, a low value of γ and high value of λ imply that only few wholesale creditors withdraw their claims before information arrives. The PA also prefers liquidity support over equity injections if the share of insured deposits is high. The reason is that a higher value of δ increases the minimum amount of equity the PA has to inject in order for the bank to be able to refinance its wholesale debt.

6 Conclusion

Our paper proposes a positive theory of the optimal timing of policy interventions in troubled banks, emphasizing the role of two key frictions: (i) the costly fire-selling of bank assets, and (ii) the social cost of public funds. The policy trade-off in our model can be framed in terms of minimizing a weighted sum of *type-I* errors (resolving a solvent bank) and *type-II* errors (keeping an insolvent bank afloat). Delaying resolution creates value by lowering the likelihood of inefficiently resolving a solvent bank but also gives uninsured creditors time to withdraw maturing debt, which increases the cost of bailing out insured depositors if the bank is ultimately resolved.

Our results provide insights into the effects and timing of LLR interventions. We show that liquidity support, by preventing costly asset sales, buys the policy authority time to make a more efficient resolution decision. The reduction in the risk of making a *type-I* error, however, comes at the expense of increasing the cost of a *type-II* error because liquidity support raises the policy authority's loss if the bank turns out to be insolvent. As a consequence, the policy authority may sometimes optimally delay the provision of liquidity support. We show that liquidity support is preferred to outright equity injections if the bank has a relatively long debt maturity structure, or if a significant share of the bank's debt liabilities consists of insured deposits.

Our model was motivated by the practical difficulties involved in distinguishing insolvent from illiquid banks, but it may also be applied to regulatory authorities' decision to forbear during banking crises. More specifically, delaying intervention and waiting for good news to arrive can be interpreted as a regulator "gambling" for the resurrection of a distressed bank in order to avoid expending public funds in resolving or supporting it. We show that such gambling behavior can be optimal, even if the face value of a bank's debt liabilities is known to exceed the expected cash flow of its assets.

Appendix

A1 Proofs

Proof of Proposition 1. The proof is based on a number of auxiliary lemmas.

Lemma A1. *Consider a policy whereby the PA waits until date t_r and then, absent news, resolves the bank. At date $t \le t_r$, the value of this policy is*

$$W(t;t_r) \;=\; \int_0^{t_r-t} X(t+k)dp(k) \;+\; \int_{t_r-t}^{\infty} L(t_r)dp(k),$$

where $L(t)$ is defined in Equation (1) and

$$X(t+k) \;=\; \begin{cases} V(t+k) & \text{if } t \le \tau, \\ L(t+k) & \text{if } t > \tau. \end{cases}$$

Proof of Lemma A1. To derive $W(t;t_r)$, note that, conditional on news arriving before date τ, the PA optimally resolves the bank if news is bad, and does not intervene if news is good (because withdrawals stop in this case). If information arrives after date τ, the PA resolves the bank regardless of whether news is good or bad. Thus, expected output conditional on news arriving at date $t+k$ is $X(t+k)$.

If news has not arrived before date $t < t_r$ and the PA waits for a small time interval Δ, then news arrives with probability $\lambda\Delta$ in which case aggregate output is $X(t+\Delta)$. With converse probability, $1-\lambda\Delta$, no news arrives and the PA continues to wait. If news does not arrive before date t_r, the bank is resolved and information becomes irrelevant: i.e., expected output conditional on information arriving equals $L(t_r)$ for all $t > t_r$. Thus, the value from waiting at date t is:

$$W(t;t_r) \;=\; \lambda\Delta K(t+\Delta) + (1-\lambda\Delta)W(t+\Delta;t_r),$$

where

$$K(t) \;=\; \begin{cases} X(t) & \text{if } t < t_r, \\ L(t_r) & \text{if } t \ge t_r. \end{cases}$$

Re-arranging the previous equation yields:

$$\frac{W(t+\Delta;t_r) - W(t;t_r)}{\Delta} + \lambda K(t+\Delta) - \lambda W(t+\Delta;t_r) \;=\; 0.$$

Taking the limit $\Delta \to 0$, it follows that the value of the policy to wait until t_r and then to resolve the bank absent news is determined by the solution to the first-order non-homogeneous differential equation:

$$W'(t) - \lambda W(t) + \lambda K(t) \;=\; 0.$$

Its stable (forward) solution is given by (see Takayama (1985) or Arnold (1992))

$$W(t) \;=\; \lambda \int_t^{\infty} e^{(t-k)\lambda} K(k)\, dk \;+\; Ce^{\lambda t}. \tag{A1}$$

We set the constant $C = 0$ in order to guarantee convergence of the solution since $\lambda > 0$. Using the piecewise function $K(t)$, one obtains the value function Equation (3) in the text. □

For any t, the optimal resolution date t_r^* maximizes $W(t; t_r^*)$. We first show that $t_r^* \leq \tau$. To see this, suppose that $t_r > \tau$. Using the definition of $X(t)$, it follows that

$$W(t;t_r) = \int_0^{\tau - t} V(t+k) dp(k) \;+\; \int_{\tau - t}^{t_r - t} L(t+k) dp(k) \;+\; \int_{t_r - t}^{\infty} L(t_r) dp(k).$$

Differentiating $W(t; t_r)$ with respect to t_r yields:

$$\frac{\partial W(t;t_r)}{\partial t_r} \;=\; (1 - p(t_r - t))L'(t_r) \;<\; 0, \quad \forall t_r > \tau.$$

Since the value from marginally delaying resolution after date τ is negative, the PA always resolves the bank at or before date τ absent news. We can therefore restrict attention to resolution dates $t_r \in [0, \tau]$.

Differentiating $W(t; t_r)$ for $t_r \in [0, \tau]$ yields

$$\frac{\partial W(t;t_r)}{\partial t_r} \;\equiv\; F(t_r) \;=\; (1 - p(t_r - t))\lambda\,(V(t_r) - L(t_r)) + (1 - p(t_r - t))L'(t_r).$$

Lemma A2. *Define* $\hat{\phi} \equiv \frac{\frac{\lambda}{\lambda+\gamma}(R-\ell)}{\ell - (1-\mu)\frac{\lambda}{\lambda+\gamma}R_b}$ *and let* t_r^* *denote the value of* t_r *such that* $F(t_r) = 0$. *Any interior solution* t_r^* *maximizes* $W(t; t_r)$ *for all* $t \in [0, \tau]$ *if and only if* $\phi < \hat{\phi}$.

Proof of Lemma A2. Consider t_r^* such that $F(t_r^*) = 0$. In order for t_r^* to constitute a maximum we must have $F'(t_r^*) < 0$. Differentiating $F(t_r)$ yields:

$$\begin{aligned} F'(t_r) \;&=\; -\,\lambda F(t_r) \;+\; (1 - p(t_r - t))\,(\lambda(V'(t_r) - L'(t_r)) + L''(t_r)) \\ &=\; -\,\lambda F(t_r) \;-\; (1 - p(t_r - t))\,(\lambda + \gamma) n'(t_r) \frac{D}{\ell} \left(\ell - (1-\mu)\frac{\lambda}{\lambda+\gamma} R_b \right) \left(\phi - \hat{\phi} \right). \end{aligned}$$

Evaluating the latter at t_r^* yields:

$$F'(t_r^*) \;=\; -(1-p(t_r^*))\,(\lambda+\gamma)n'(t_r^*)\frac{D}{\ell}\left(\ell-(1-\mu)\frac{\lambda}{\lambda+\gamma}R_b\right)\left(\phi-\hat{\phi}\right) \;<\;0 \quad\Leftrightarrow\quad \phi<\hat{\phi},$$

since $n'(t) < 0$ and $\ell \geq R_b$. □

Lemma A3. *Given Assumption 2, there exist threshold values $\underline{\phi}$ and $\overline{\phi}$, with $\underline{\phi} < \overline{\phi}$, such that $W(t; t_r)$ admits an interior maximum $t_r^* \in (0, \tau)$ if and only if $\phi \in (\underline{\phi}, \overline{\phi})$.*

Proof of Lemma A3. Define $\Phi(t_r)$ as the value of ϕ such that $F(t_r) = 0$. Explicitly:

$$\Phi(t_r) \;=\; \frac{z(t_r)(R-\ell)}{z(t_r)\ell-\mu\delta-(1-\mu)z(t_r)R_b+\frac{\gamma}{\lambda}n(t_r)D}.$$

By definition of $\Phi(t_r)$, we have that $F(t_r) < 0$ if and only if $\phi > \Phi(t_r)$. Note that

$$\begin{aligned}\Phi'(t_r) &\propto z'(t_r)\left(z(t_r)\ell-\mu\delta-(1-\mu)z(t_r)R_b+\frac{\gamma}{\lambda}n(t_r)D\right)-z(t_r)\left(z'(t_r)(\ell-(1-\mu)R_b)+\frac{\gamma}{\lambda}n'(t_r)D\right)\\ &= -\frac{n'(t_r)D}{\ell}\left(\mu\delta-\frac{\gamma}{\lambda}(D-\ell)\right) \;<\;0 \quad\Leftrightarrow\quad \lambda\mu\delta\;<\;\gamma(D-\ell),\end{aligned}$$

where the last inequality holds due to Assumption 2.

Define $\underline{\phi} \equiv \Phi(\tau)$ and $\overline{\phi} \equiv \Phi(0)$ and note that $(\underline{\phi}, \overline{\phi}) \neq \emptyset$ because $\Phi'(t_r) < 0$. Suppose $\phi \in (\underline{\phi}, \overline{\phi})$, implying that $F(0) > 0$ and $F(\tau) < 0$. Since $F(t_r)$ is continuous, by the intermediate value theorem there exists a t_r^* such that $F(t_r^*) = 0$. To show that t_r^* constitutes a maximum it suffices to show that $\overline{\phi} < \hat{\phi}$ (*viz.* Lemma A2), which is equivalent to:

$$\ell-(1-\mu)\frac{\lambda}{\lambda+\gamma}R_b \;<\; \frac{\lambda}{\lambda+\gamma}\left(\ell-\mu\delta-(1-\mu)R_b+\frac{\gamma}{\lambda}D\right) \quad\Leftrightarrow\quad \lambda\mu\delta\;<\;\gamma(D-\ell),$$

which, again, is always satisfied due to Assumption 2. □

Lemma A4. *Suppose that $\phi < \underline{\phi}$, then $W(t; t_r)$ is maximized at $t_r^* = \tau$ for all t. Suppose that $\overline{\phi} < \phi$, then $W(t; t_r)$ is maximized at $t_r^* = 0$ for all t.*

Proof of Lemma A4. Suppose that $\phi < \underline{\phi}$. Because $\underline{\phi} < \Phi(t_r)$ for all $t_r < \tau$, it follows that $\phi < \Phi(t_r)$ for all $t_r \leq \tau$, implying that $F(t_r) > 0$ for all $t_r \leq \tau$. Hence, the marginal value from delaying intervention is strictly positive for all $t_r \leq \tau$. As a consequence, the PA optimally resolves the bank at date $t_r^* = \tau$. A similar argument shows that the PA optimally resolves the bank at $t_r^* = 0$ if $\phi > \overline{\phi}$. □

The proof of Proposition 1 follows by combining Lemmas A2-A4. □

Proof of Corollaries 1 and 2. The proof follows by applying the implicit function theorem to the first-order condition $F(t_r^*) = 0$. Since $F'(t_r^*) < 0$, the sign of the respective derivatives is given by the sign of the derivatives of $F(t_r^*)$ with respect to the different parameters. We can decompose the effect of the parameters on $F(t_r^*)$ into the effects on the value of information, $\lambda(V(t_r^*) - L(t_r^*))$, and the dilution costs, $L'(t_r^*)$. Thus, for λ, ℓ and δ:

$$\frac{\partial t_r^*}{\partial \lambda} \propto \frac{\partial(\lambda(V(t_r^*) - L(t_r^*))}{\partial \lambda} = V(t_r^*) - L(t_r^*) > 0,$$
$$\frac{\partial t_r^*}{\partial \ell} \propto \frac{\partial(\lambda(V(t_r^*) - L(t_r^*))}{\partial \ell} = \underbrace{\frac{\partial z(t_r^*)}{\partial \ell}(R - \ell + \phi((1-\mu)R_b - \ell))}_{>0 \text{ since } \phi < \hat{\phi}} - \underbrace{\phi z(t_r^*)}_{>0} \gtrless 0,$$
$$\frac{\partial t_r^*}{\partial \delta} \propto \frac{\partial(\lambda(V(t_r^*) - L(t_r^*))}{\partial \delta} = \mu\phi > 0,$$

and for γ:

$$\begin{aligned}\frac{\partial t_r^*}{\partial \gamma} &\propto \frac{\partial(\lambda(V(t_r^*) - L(t_r^*))}{\partial \gamma} + \frac{\partial L'(t_r^*)}{\partial \gamma} \\ &= \underbrace{\frac{\partial z(t_r^*)}{\partial \gamma}(R - \ell + \phi((1-\mu)R_b - \ell))}_{<0 \text{ since } \phi < \hat{\phi}} + \underbrace{\phi\gamma n(t_r^*)D\left(t_r^* - \frac{1}{\gamma}\right)}_{\gtrless 0} \gtrless 0.\end{aligned}$$

□

Proof of Proposition 2. The proof is based on a number of auxiliary lemmas. We begin by specifying the PA's optimal action conditional on news arriving.

Lemma A5. *Suppose the PA prefers to initiate liquidity support at date t_s in the absence of news. Then:*

1. *If bad news arrives at date $t' \gtrless t_s$, the PA resolves the bank.*
2. *If good news arrives at date $t' \geq t_s$, the PA either stops support if $t_s \leq \tau$ or maintains support until all wholesale debt is withdrawn if $t_s > \tau$.*
3. *If good news arrives at date $t' < t_s$, withdrawals either stop if $t' \leq \tau$ or the PA initiates support and maintains support until all wholesale debt is withdrawn if $t' > \tau$.*

Proof of Lemma A5.

1. The claim follows immediately from the fact that liquidating bad assets is costless: i.e., $\ell_b = R_b$.
2. If good news arrives at $t' \geq t_s$ and $t_s \leq \tau$, debt withdrawals stop since a good bank is solvent. Hence, the PA terminates liquidity support. If $t_s > \tau$, the run does not stop because a good bank is insolvent. To prove the claim that the PA optimally maintains support until all wholesale

debt is withdrawn, suppose towards a contradiction that the PA instead resolves the bank after the arrival of good news. Expected aggregate output conditional on news arriving in this case would equal $L(t')$. Since $L'(t) < 0$, this implies that the PA is better off resolving the bank at date t_s rather than initiating support, contradicting the assumption that the PA prefers to initiate liquidity support at date t_s in the absence of news.

3. If good news arrives at $t' < t_s$ and $t' \leq \tau$, debt withdrawals stop since a good bank is solvent. Hence, the PA never initiates liquidity support. From above, we know that the PA prefers maintaining support to resolving the bank if good news arrives at $t' \geq t_s$ whenever $t_s > \tau$, which implies the following inequality:

$$\begin{aligned} & z(t_s)R_g + \phi(z(t_s)R_g - \delta - n(t_s)D) \geq z(t_s)\ell_g + \phi(z(t_s)\ell_g - \delta - (n(t_s) - n(t'))D) \\ \Leftrightarrow \quad & (1+\phi)(R_g - \ell_g) \geq \phi \frac{n(t')}{z(t_s)} D, \quad \forall\, t' \geq t_s. \end{aligned} \tag{A2}$$

For the PA to prefer initiating liquidity support if good news arrives at date $t' < t_s$, we must have:

$$\begin{aligned} & z(t')R_g + \phi(z(t')R_g - \delta - n(t')D) \geq z(t')\ell_g + \phi(z(t')\ell_g - \delta) \\ \Leftrightarrow \quad & (1+\phi)(R_g - \ell_g) \geq \phi \frac{n(t')}{z(t')} D, \quad \forall\, t' < t_s. \end{aligned} \tag{A3}$$

The claim follows because condition (A2) implies condition (A3). To see this, note that since condition (A2) must hold for all $t' \geq t_s$, it must hold in particular at $t' = t_s$ when $n(t')$ takes on its largest value. Hence, it is sufficient to show that:

$$\frac{n(t_s)}{z(t_s)} > \frac{n(t')}{z(t')} \Leftrightarrow n(t')(D - \ell) > n(t_s)(D - \ell), \quad \forall t' < t_s,$$

where the inequality follows from $D > \ell$ by Assumption 1 and $n(t') > n(t_s)$ for all $t' < t_s$.

□

We use Lemma A5 to derive the PA's value function with liquidity support. Consider the policy of delaying intervention until date t_s and then, absent news, initiating liquidity support until date t_r and then, again absent news, resolving the bank. We show below that restricting attention to this policy space is without loss of generality: i.e., the PA will never stop liquidity support in the absence of news.

Using the PA's optimal actions conditional on news as specified in Lemma A5, expected aggregate output conditional on news arriving at date $t' < t_s$ is:

$$V_w(t') = \begin{cases} V(t') & \text{if } t' \leq \tau \\ V(t') + \mu\phi(z(t')R_g - \delta - n(t')D) & \text{if } t' > \tau \end{cases}$$

and expected aggregate output conditional on news arriving at date $t' \geq t_s$ is:

$$V_s(t'; t_s) = \begin{cases} V(t_s) - (1-\mu)\phi(n(t_s) - n(t'))D & \text{if } t_s \leq \tau \\ V(t_s) - (1-\mu)\phi(n(t_s) - n(t'))D + \mu\phi(z(t_s)R_g - \delta - n(t_s)D) & \text{if } t_s > \tau \end{cases}$$

The PA's value function can then be written as:

$$S(t; t_s, t_r) = \int_0^{t_s - t} V_w(t+k)dp(k) + \int_{t_s-t}^{t_r-t} V_s(t+k; t_s)dp(k) + \int_{t_r-t}^{\infty} L(t_r)dp(k) \tag{A4}$$

Lemma A6. *The PA never delays liquidity support to after date τ: i.e., $t_s^* \leq \tau$.*

Proof of Lemma A6. To prove the claim, it suffices to show that $S(t; t_s, t_r)$ is decreasing in t_s for $t_s > \tau$. Differentiating Equation (A4) with respect to t_s for $t_s > \tau$ yields:

$$\begin{aligned} \frac{\partial S(t; t_s, t_r)}{\partial t_s} &= p'(t_s - t)\underbrace{(V_w(t_s) - V_s(t_s; t_s))}_{=0} + \int_{t_s-t}^{\infty} \frac{\partial V_s(t+k; t_s)}{\partial t_s} dp(k) \\ &= (1 - p(t_s - t))(1+\phi)(R - \ell)\frac{n'(t_s)D}{\ell} < 0, \quad \forall t_s > \tau, \end{aligned}$$

where the inequality follows from $n'(t_s) < 0$. Hence, we must have $t_s^* \leq \tau$. □

Lemma A7. *The PA either resolves the bank at date 0 or never resolves the bank in the absence of information: i.e., $t_r^* \in \{0, \infty\}$.*

Proof Lemma A7. To show that the PA never resolves the bank after granting liquidity support in the absence of information, fix $t_s \geq 0$ and assume towards a contradiction that the PA optimally resolves the bank at some date $t_r \in (t_s, \infty)$ in the absence of information. This optimal resolution date must satisfy the following first-order condition:

$$\left.\frac{\partial S(t; t_s, t_r)}{\partial t_r}\right|_{t_r = t_r^*} = (1 - p(t_r^* - t))\Big(\lambda(V_s(t_r^*; t_s) - L(t_r^*)) + L'(t_r^*)\Big) = 0. \tag{A5}$$

The claim follows from the fact that the value function $S(t; t_s, t_r)$ is convex in t_r at any interior solution to (A5). To see this, differentiate (A5) with respect to t_r and evaluate the derivative at t_r^*:

$$\left.\frac{\partial^2 S(t;t_s,t_r)}{\partial t_r^2}\right|_{t_r=t_r^*} = (1-p(t_r^*-t))\left(\lambda\left(\left.\frac{\partial V_s(t_r;t_s)}{\partial t_r}\right|_{t_r=t_r^*} - L'(t_r^*)\right) + L''(t_r^*)\right) > 0.$$

The inequality follows from the fact that $\frac{\partial V_s(t;t_s)}{\partial t} - L'(t) = -\mu\phi n'(t)D > 0$ and that $L''(t) = \phi n''(t)D > 0$. Hence, any interior solution to condition (A5) must be a minimum, and we must have $t_r^* \in \{t_s, \infty\}$.

To show that the PA never resolves the bank after date 0 in the absence of information, notice that since $S(t;t_s,t_r)$ is convex in t_r, a sufficient condition for the PA to find it optimal to initiate liquidity support rather than resolving the bank at date t_s is:

$$\left.\frac{\partial S(t;t_s,t_r)}{\partial t_r}\right|_{t_r=t_s} > 0 \quad \Leftrightarrow \quad \phi < \Phi(t_s),$$

From the proof of Proposition 1, we know that $W(t;t_s) > 0$ for all $\phi < \Phi(t_s)$. To prove the claim, suppose the PA prefers to resolve the bank at some date $t_s > 0$ in the absence of information. In order for the PA to prefer waiting until date t_s instead of resolving the bank before, we must have $\phi < \Phi(t_s)$. But then, it must be that the PA prefers granting liquidity support rather than resolving the bank at date t_s, contradicting the supposition. Hence, we must have $t_r^* \in \{0, \infty\}$. □

Lemma A8. *Define* $\phi^*(\ell) \equiv \frac{R-\ell}{(1-\mu)(\ell-R_b)}$.

1. *If* $\phi < \phi^*(\ell)$*, it is never optimal to delay liquidity support. Otherwise, delaying liquidity support until date* τ *is always preferred to initiating liquidity support before date* τ*: i.e.,* $t_s^* \in \{0, \tau\}$.
2. *If the PA initiates liquidity support at date* $t_s^* \leq \tau$*, the PA never stops liquidity support in the absence of news.*
3. $\phi^*(\ell)$ *is strictly decreasing and convex in* ℓ.

Proof of Lemma A8.

1. From Lemma A6, we know that it is never optimal for the PA to delay liquidity support to after date τ. The claim then follows because, for $t_s \leq \tau$, $S(t;t_s,t_r)$ is strictly decreasing in t_s if and only if $\phi < \phi^*$. To show this, differentiate Equation (A4) with respect to t_s for $t_s \leq \tau$:

   $$\frac{\partial S(t;t_s,t_r)}{\partial t_s} = (1-p(t_s-t))z'(t_s)\Big((R-\ell) - (1-\mu)\phi(\ell-R_b)\Big) < 0 \quad \Leftrightarrow \quad \phi < \phi^*.$$

   where the inequality follows from $z'(t_s) < 0$. Hence, we must have $t_s^* \in \{0, \tau\}$.
2. From Lemma A7, we know that if the PA introduces liquidity support it never resolves the bank in the absence of information. We also know from Proposition 1 that the PA always prefers resolution

to waiting at date τ, implying that the PA will never stop liquidity support initiated at date τ in the absence of news. To show that the PA never stops liquidity support initiated before date τ in the absence of news, consider the following alternative policy: suppose the PA delays intervention until date $t_s < \tau$ and then, absent news, initiates liquidity support at date t_s until date t_1 and then, again absent news, stops liquidity support without resolving the bank. If the PA continues to wait for news at date t_1, expected aggregate output conditional on news arriving at date t' is:

$$\widehat{V}(t;t_s,t_1) = \hat{z}(t;t_s,t_1)R + (1 - z(t;t_s,t_1))\ell + (1-\mu)\phi\left(\hat{z}(t;t_s,t_1)R_b - \delta + (n(t_1) - n(t_s))D\right),$$

where:

$$\hat{z}(t;t_s,t_1) = \left(z(t_s) - \frac{(n(t_1) - n(t))D}{\ell}\right).$$

The claim follows because the derivative of the PA's value function with respect to t_1 is strictly increasing if and only if $\phi < \phi^*(\ell)$:

$$\begin{aligned}\frac{\partial \widehat{S}(t;t_s,t_1)}{\partial t_1} &= p'(t_1 - t)\underbrace{\left(V_s(t_1;t_s) - \widehat{V}(t_1;t_s;t_1)\right)}_{=0} + \int_{t_1-t}^{\infty} \frac{\partial \widehat{V}(t;t_s,t_1)}{\partial t_1} dp(k) \\ &= -(1 - p(t_1 - t)(1-\mu)(\ell - R_b)(\phi^*(\ell) - \phi)\frac{n'(t_1)D}{\ell} > 0,\end{aligned}$$

where the inequality follows because $n'(t_1) < 0$. Hence, it is never optimal to stop liquidity support that is initiated before date τ.

3. Differentiating $\phi^*(\ell)$ with respect to ℓ yields:

$$\frac{d\phi^*}{d\ell} = -\frac{(R - R_b)}{(1-\mu)(\ell - R_b)^2} < 0, \quad \frac{d^2\phi^*}{d\ell^2} = \frac{2(R - R_b)}{(1-\mu)(\ell - R_b)^3} > 0.$$

□

Lemmas A7 to A8 jointly characterize the set of candidate policy solutions. In particular, they imply that we can restrict attention to three alternative policies: (i) resolve the bank at date 0 ($t_s^* = t_r^* = 0$); (ii) initiate liquidity support at date 0 and maintain liquidity support until news arrives ($t_s^* = 0, t_r^* = \infty$); (iii) or delay intervention until date τ and then provide liquidity support until news arrives ($t_s^* = \tau, t_r^* = \infty$).

Lemma A9. *Define* $\underline{\ell} \equiv \mu\delta + (1-\mu)R_b - (1-\mu)\frac{\gamma}{\gamma+\lambda}D$ *and* $\phi_0(\ell) \equiv \frac{R-\ell}{\ell-\underline{\ell}}$*:*

1. *If* $\ell \le \underline{\ell}$*, the PA prefers liquidity support over resolution at date* 0 *for all values of* ϕ*.*

2. *If* $\ell > \underline{\ell}$*, the PA prefers liquidity support over resolution at date* 0 *if and only if* $\phi \le \phi_0$*.*

3. $\phi_0(\ell)$ *is strictly decreasing and convex in* ℓ*.*

Proof of Lemma A9.

1. The difference between initiating liquidity support at date 0 (and then maintaining liquidity support until news arrives) and resolving the bank at date 0 is:

$$
\begin{aligned}
S(0;0,\infty) - S(0;0,0) = & \int_0^\infty V_s(k;0)dp(k) - L(0) \\
= & R - \ell - \phi\left(\mu(\ell-\delta) + (1-\mu)\left(\ell - R_b + \frac{\gamma}{\lambda+\gamma}D\right)\right) \\
= & R - \ell - \phi(\ell - \underline{\ell}).
\end{aligned}
$$

Since $R > \ell$, it follows that $S(0;0,\infty) > S(0;0,0)$ for all values of ϕ whenever $\ell < \underline{\ell}$.

2. If $\ell > \underline{\ell}$, then it follows from above that $S(0;0,\infty) > S(0;0,0)$ if and only if:

$$
\phi < \phi_0(\ell) \equiv \frac{R-\ell}{\ell-\underline{\ell}} = \frac{R-\ell}{\ell - \mu\delta - (1-\mu)R_b + (1-\mu)\frac{\gamma}{\lambda+\gamma}D}.
$$

3. The properties of $\phi_0(\ell)$ are:

$$
\frac{d\phi_0}{d\ell} = -\frac{R - \left(\mu\delta + (1-\mu)R_b - (1-\mu)\frac{\gamma}{\lambda+\gamma}D\right)}{\left(\ell - \left(\mu\delta + (1-\mu)R_b - (1-\mu)\frac{\gamma}{\lambda+\gamma}D\right)\right)^2} < 0,
$$

$$
\frac{d^2\phi_0}{d\ell^2} = \frac{2\left(R - \left(\mu\delta + (1-\mu)R_b - (1-\mu)\frac{\gamma}{\lambda+\gamma}D\right)\right)}{\left(\ell - \left(\mu\delta + (1-\mu)R_b - (1-\mu)\frac{\gamma}{\lambda+\gamma}D\right)\right)^3} > 0.
$$

□

Lemma A10. *Assumption 3 implies an upper bound for ℓ, given by $\bar{\ell} \equiv \delta - \frac{(1-\mu)}{\mu}\frac{\gamma}{\lambda+\gamma}D$:*

1. *The interval $(\underline{\ell}, \bar{\ell})$ is non-epmty.*

2. *There exists $\hat{\ell} \in (\underline{\ell}, \bar{\ell})$ such that:*

   (a) *If $\ell < \hat{\ell}$, the PA prefers delaying intervention until date τ and then, absent news, initiating liquidity support rather than resolving the bank at date 0 for all values of ϕ.*

   (b) *If $\ell \geq \hat{\ell}$, the PA prefers delaying intervention until date τ and then, absent news, initiating liquidity support rather than resolving the bank at date 0 if and only if*

$$
\phi < \phi_1 \equiv \frac{\left(\int_0^\tau z(k)dp(k) + \int_\tau^\infty z(\tau)dp(k)\right)(R-\ell)}{\left(\ell - \mu\delta - (1-\mu)\left(\int_0^\tau z(k)dp(k) + \int_\tau^\infty z(\tau)dp(k)\right)R_b + (1-\mu)\int_\tau^\infty (n(k)-n(\tau))Ddp(k)\right)}.
$$

3. $\phi^* < \phi_0 < \phi_1$ *if and only if* $\ell < \bar{\ell}$.

Proof of Lemma A10.

1. $\underline{\ell} < \bar{\ell}$ is equivalent to

$$\mu\delta + (1-\mu)R_b - (1-\mu)\frac{\gamma}{\lambda+\gamma}D \ < \ \delta - \frac{(1-\mu)}{\mu}\frac{\gamma}{(\gamma+\lambda)}D \ \Leftrightarrow\ \mu(\delta - R_b) > (1-\mu)\frac{\gamma}{(\gamma+\lambda)}D,$$

which is satisfied by Assumption 3.

2. The PA prefers delaying intervention until date τ and then initiating liquidity support compared to resolving the bank at date 0 if and only if:

$$S(0;\tau,\infty) - S(0;0,0) \ = \ \int_0^\tau V(k)dp(k) \ + \ \int_\tau^\infty V_s(k;\tau)\,dp(k) - L(0) > 0$$

$$\Leftrightarrow \quad \phi < \phi_1 \equiv \frac{\left(\int_0^\tau z(k)dp(k) \ + \ \int_\tau^\infty z(\tau)dp(k)\right)(R-\ell)}{\left(\ell - \mu\delta - (1-\mu)\left(\int_0^\tau z(k)dp(k) \ + \ \int_\tau^\infty z(\tau)dp(k)\right)R_b + (1-\mu)\int_\tau^\infty (n(k)-n(\tau))Ddp(k)\right)}.$$

Using the definitions of ϕ_0 and ϕ_1, it follows that $\phi_0 < \phi_1$ is equivalent to:

$$\ell - \mu\delta + (1-\mu)(1-p(\tau))n(\tau)\frac{\gamma}{\lambda+\gamma}D < (p(\tau)\xi(\tau) + (1-p(\tau))z(\tau))\left(\ell - \mu\delta + (1-\mu)\frac{\gamma}{\lambda+\gamma}D\right),$$

where $\xi(\tau) \equiv \int_0^\tau z(k)dp(k)$. Note that:

$$p(\tau)\xi(\tau) + (1-p(\tau))z(\tau) = 1 - (1-(1-p(\tau))n(\tau))\frac{\gamma}{\gamma+\lambda}\frac{D}{\ell}.$$

Rewriting the inequality $\phi_0 < \phi_1$ accordingly, it follows that:

$$\phi_0 < \phi_1 \ \Leftrightarrow \ell \ < \bar{\ell} \equiv \delta - \frac{(1-\mu)}{\mu}\frac{\gamma}{\lambda+\gamma}D.$$

Since $\lim_{\ell \downarrow \underline{\ell}} \phi_0 = \infty$ and $\phi_1 > \phi_0$ for $\ell \in (\underline{\ell}, \bar{\ell})$, it follows by continuity of ϕ_0 and ϕ_1 that there exists an $\hat{\ell} \in (\underline{\ell}, \bar{\ell})$ such that $\lim_{\ell \downarrow \hat{\ell}} \phi_1 = \infty$. For $\ell < \hat{\ell}$, $S(0;\tau,\infty) > S(0;0,0)$ for all values of ϕ, while for $\ell \geq \hat{\ell}$ we have that $S(0;\tau,\infty) > S(0;0,0)$ if and only if $\phi < \phi_1$.

3. From above it follows that $\phi_0 < \phi_1$ if and only if $\ell < \bar{\ell}$. Note further that $\phi^* < \phi_0$ is equivalent to

$$\ell - \mu\delta - (1-\mu)R_b + (1-\mu)\frac{\gamma}{\gamma+\lambda}D \ < \ (1-\mu)(\ell - R_b) \Leftrightarrow \ \ell < \bar{\ell} \equiv \delta - \frac{(1-\mu)}{\mu}\frac{\gamma}{\gamma+\lambda}D.$$

Hence, $\phi^* < \phi_0 < \phi_1$ if and only if $\ell < \bar{\ell}$.

□

To complete the proof, we distinguish between two different cases, depending on the value of ℓ:

1. $\ell \leq \hat{\ell}$: For $\phi < \phi^*$, Lemmas A8 and A9 imply $S(0;0,0) < S(0;\tau,\infty) < S(0;0,\infty)$. Thus, the PA supports the bank at date 0 and maintains support until news arrives. If $\phi \geq \phi^*$, Lemmas A8 and A9 imply $S(0;0,0) < S(0;0,\infty) < S(0;\tau,\infty)$. Hence, the PA prefers to delay liquidity support until date τ and then maintains support until news arrives.

2. $\ell \in (\hat{\ell}, \overline{\ell}]$: If $\phi < \phi^*$, the PA supports the bank at date 0 and maintains support until news arrives. If $\phi \in [\phi^*, \phi_1)$, the PA delays liquidity support until date τ and maintains support until news arrives. Finally, if $\phi \geq \phi_1$, then Lemma A10 implies $S(0;0,\infty) < S(0;\tau,\infty) < S(0;0,0)$. Hence, the PA resolves the bank at date 0.

To finish, we set $\phi^{**} \equiv \phi_1$. □

Proof of Proposition 3. The first part of the proposition follows from Proposition 2. To prove the second part, note that:

$$\overline{\phi} < \phi_0 \Leftrightarrow (1-\mu)\frac{\gamma}{\lambda+\gamma} < 1.$$

From Lemma A10, $\phi_0 < \phi_1 \equiv \phi^{**}$ and therefore $\overline{\phi} < \phi^{**}$. Hence, for $\phi \in (\overline{\phi}, \phi^{**})$, the PA resolves the bank at date 0 if liquidity support is not available, but does not if liquidity support is available. □

Proof of Lemma 2. Notice that aggregate output with equity injections is strictly decreasing in E. Thus, the optimal equity injection is the minimum value of E that simultaneously satisfies conditions (9) and (10). There are two cases to consider, depending on whether $E \gtrless \delta - z(t_e)R_b$.

Consider first the case where $E \leq \delta - z(t_e)R_b$. In this case, in order for wholesale creditors to accept to refinance, the face value F must be such that $F \geq D/\mu$. However, since $D > \mu(R_g - \delta)$ and $\ell < D$ by Assumption 1, this violates the bank's limited liability constraint (9). To see this, notice that:

$$z(t_e)R_g - \delta - n(t_e)\frac{D}{\mu} = n(t_e)\left(R_g - \delta - \frac{D}{\mu}\right) + (1-n(t_e))\left(R_g\left(1-\frac{D}{\ell}\right) - \delta\right) < 0.$$

Consider next the case where $E > \delta - z(t_e)R_b$. In this case, in order for wholesale creditors to accept to refinance, the face value F must be such that:

$$n(t_e)F \geq \min\left\{n(t_e)D, \frac{n(t_e)D - (1-\mu)(z(t_e)R_b - \delta + E)}{\mu}\right\},$$

which is decreasing in E. From the bank's limited liability constraint (9), it follows that we must also have $n(t_e)F \leq z(t_e)R_g - \delta$. Hence, the minimum value of E that simultaneously satisfies conditions (9) and (10) is E^*. To complete the proof, notice that $E^* > \delta - z(t_e)R_b$ is equivalent to $n(t_e)D > \mu(z(t_e)R_g - \delta)$, which always holds by Assumption 1. □

Proof of Proposition 4. The optimal timing of equity injections maximizes the following value function:

$$B(t; t_e) = \int_0^{t_e - t} V_w(t+k)dp(k) + \int_{t_e - t}^{\infty} V_b(t_0)dp(k). \tag{A6}$$

Without loss of generality, we restrict attention to equity injections at dates $t_e \leq \tau$. The reason is that after date τ, the PA strictly prefers liquidity support to injecting equity (see below). Differentiating Equation (A6) with respect to $t_e \leq \tau$ yields:

$$\frac{\partial B(t; t_e)}{\partial t_e} = (1 - p(t_e - t))\Big(\lambda\phi(n(t_e)D - \mu(z(t_e)R_g - \delta)) + (1+\phi)(R - \ell)n'(t_e)\frac{D}{\ell}\Big). \tag{A7}$$

To prove that the PA never delays equity injections after date 0, we show that the value function (A6) is convex in t_e at any interior solution to (A7). To see this, let $t_e^* \in (0, \tau)$ denote the critical point such that $\frac{\partial B(t; t_e^*)}{\partial t_e} = 0$. Differentiating (A7) with respect to t_e and evaluating the derivative at $t_e^* \in (0, \tau)$:

$$\frac{\partial^2 B(t; t_e)}{\partial t_e^2}\Big|_{t_e^* \in (0,\tau)} = (1 - p(t_e^* - t))\Big(\lambda\phi(\ell - \mu R_g)n'(t_e^*)\frac{D}{\ell} - \gamma(1+\phi)(R-\ell)n'(t_e^*)\frac{D}{\ell}\Big).$$

Using the first-order condition (A7), it follows that:

$$\begin{aligned}\frac{\partial^2 B(t; t_e)}{\partial t_e^2}\Big|_{t_e^* \in (0,\tau)} &= (1 - p(t_e^* - t))\lambda\gamma\phi\left((\mu R_g - \ell)n(t_e^*)\frac{D}{\ell} + (n(t_e^*)D - \mu(z(t_e^*)R_g - \delta)\right) \\ &= -(1 - p(t_e^* - t))\lambda\gamma\phi\mu\left(R_g\left(1 - \frac{D}{\ell}\right) - \delta\right) > 0.\end{aligned}$$

where the inequality follows because $D > \ell$ by Assumption 1. Hence, any interior solution to (A7) must be a minimum, implying that $t_e^* \in \{0, \tau\}$.

Note further that the PA strictly prefers liquidity support over equity injections at dates $t > \tau$. To see this, suppose that the PA has waited until date $t_e > \tau$. The value from injecting equity at this date is strictly smaller than the value from initiating liquidity support:

$$B(t_e; t_e) = V(t_e) - \phi\Big(\mu(n(t_e)D + \delta - z(t_e)R_g) + (1-\mu)n(t_e)D\Big) < S(t_e; t_e, t_e) = V(t_e) - \phi(1-\mu)\frac{\gamma}{\lambda+\gamma}n(t_e)D,$$

where the inequality follows from the fact that $\mu(n(t_e)D + \delta - z(t_e)R_g) < 0$ for all $t_e > \tau$ by the definition of τ. Thus, the optimal date for injecting equity must be $t_e^* = 0$.

The PA prefers equity injections at date 0 to providing immediate liquidity support whenever:

$$B(0;0) - S(0;0,\infty) > 0 \quad \Leftrightarrow \quad D - \mu(R_g - \delta) < (1-\mu)\int_0^{\infty}(1-n(k))Ddp(k).$$

Solving the integral yields condition (12). □

A2 Violation of Assumptions 2 and 3

A2.1 Violation of Assumption 2

The following proposition shows the consequences of violating Assumption 2 for the optimal timing of resolution. Suppose instead that:

$$\lambda\mu\delta > \gamma(D - \ell). \tag{A8}$$

Proposition A1. *Suppose condition (A8) holds. The PA optimally resolves the bank at date $t_r^* \in \{0, \tau\}$. Specifically, there exists $\overline{\overline{\phi}}$ such that $t_r^* = 0$ if and only if $\phi < \overline{\overline{\phi}}$ and $t_r^* = \tau$ otherwise.*

Proof of Proposition A1. We can still restrict attention to resolution dates $t_r \in [0, \tau]$ because, for any $t_r > \tau$, the marginal value from delaying resolution is strictly negative (*viz.* proof of Proposition 1). Given condition (A8), it follows from the proof of Proposition 1 that $\Phi(t_r)$ is strictly increasing in t_r and $\hat{\phi} < \overline{\phi}$. Thus, $\hat{\phi} < \overline{\phi} < \underline{\phi}$. As a consequence, whenever $\phi < \overline{\phi}$, the marginal value from delaying resolution is positive: i.e., $F(t_r) > 0$ for all $t_r \in [0, \tau]$. Thus, the PA resolves the bank at date τ. Conversely, if $\phi > \underline{\phi}$, $F(t_r) < 0$ for all $t_r \in [0, \tau]$ and the PA resolves the bank at date 0. Consider $\phi \in (\overline{\phi}, \underline{\phi})$, which also implies $\phi > \hat{\phi}$. In this case, any interior solution $F(t_r) = 0$ minimizes $W(t; t_r)$. Thus, depending on whether $W(0; 0) \gtrless W(0; \tau)$, the PA either resolves the bank at date 0 or date τ. The value $\overline{\overline{\phi}}$ is such that $W(0; 0) = W(0; \tau)$. □

A2.2 Violation of Assumption 3

The following proposition shows the consequences of violating Assumption 3 for the optimal intervention policy with liquidity support. Suppose that Assumption 3 fails to hold, that is:

$$\ell > \overline{\ell} \equiv \delta - \frac{(1-\mu)}{\mu} \frac{\gamma}{(\gamma + \lambda)} D. \tag{A9}$$

Proposition A2. *Suppose condition (A9) holds. The PA never delays liquidity support to after date 0. The PA provides liquidity support at date 0 if and only if $\phi < \phi_0$. Otherwise, if $\phi \geq \phi_0$, the PA resolves the bank at date 0.*

Proof of Proposition A2. From Lemma A10, it follows that if $\ell > \overline{\ell}$, then $\phi_1 < \phi_0 < \phi^*$. Lemmas A8, A9 and A10 then imply that if $\phi < \phi_0$ we must have $\max\{S(0; 0, 0), S(0; \tau, \infty)\} < S(0; 0, \infty)$. If $\phi \geq \phi_0$, then $S(0; 0, 0) > \max\{S(0; 0, \infty), S(0; \tau, \infty)\}$. Hence, if $\phi < \phi_0$, the PA supports the bank at date 0 and maintains support until news arrives. Conversely, if $\phi \geq \phi_0$, the PA resolves the bank at date 0. □

Figure A1 illustrates Proposition A2.

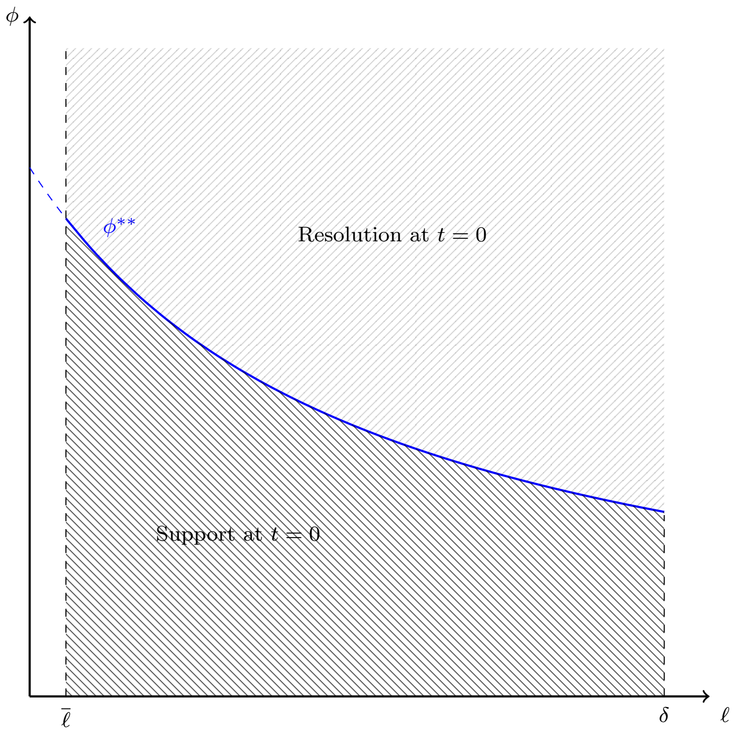


Figure A1: Intervention regimes in $\phi - \ell$ space.
If Assumption 3 is violated, the only relevant threshold is $\phi^0 \equiv \phi^{**}$ which divides the areas into support at date 0 and resolution at date 0.

A3 Extensions

In this Appendix, we discuss two extensions of the model with liquidity support. First, we allow the PA to provide liquidity support at a mark-up over the risk-free rate (a so-called "penalty rate"). Second, we consider how the PA's optimal policy changes if the bank holds liquid assets that can be used to cover debt repayments without fire-selling assets. We show that our main results and the policy trade-offs are robust to these extensions of the model.

A3.1 Penalty rates

Granting liquidity support at date 0 implies that the bank benefits from a subsidy since the PA offers funding at a time when a good bank still has positive profits. The PA can reduce this subsidy by charging a mark-up over the risk-free rate, or a "penalty rate", on funds drawn down by the bank. Charging a penalty rate, however, does not affect total cash flows nor does it lower the loss incurred by the PA if the bank turns out to be bad since the penalty rate is only paid if the bank is good. Thus, offering liquidity support at a penalty rate has no effect on expected aggregate output and, consequently, does not affect the PA's optimal intervention decision.

Proposition A3. *The charging of a penalty rate does not affect the PA's decision to grant liquidity support, nor its decision to delay the provision of liquidity support.*

A3.2 Liquidity holdings

Suppose that, in addition to its long-term asset, the bank holds liquid assets such as cash or central bank reserves. Given the normalization of the bank's balance sheet to unity, we assume that the bank holds a share $m \in (0, \overline{m})$ of its total assets in cash and the remaining share $1 - m$ in the long-term asset.[28]

Without liquidity support, the bank covers debt withdrawals by first depleting its cash holdings before fire-selling assets. In the absence of news, the bank's cash balances are depleted at date:

$$\tau_m = \frac{1}{\gamma} \ln \left(\frac{D}{D - m} \right).$$

The bank begins to sell its long-term asset if withdrawals continue after date τ_m and, in the absence of either news or liquidity support, a good bank becomes insolvent at some date $\hat{\tau}$ (which depends on m).

Similar to the provision of liquidity support, using cash holdings to cover debt withdrwawals prevents the deadweight loss from fire sales. Running down cash balances, however, does not avoid the dilution costs from delaying resolution since the repayment of wholesale creditors reduces the resources the PA can seize if it resolves the bank.

Proposition A4. *The PA is indifferent between providing liquidity support or delaying support and letting the bank cover debt repayments by running down its cash balances.*

Proof of Proposition A4. Expected aggregate output when the PA lets the bank run down its cash balances and then provides liquidity support at date τ_m is:

$$\begin{aligned} &\int_0^{\tau_m} \Big(R(1-m) + m + (1-\mu)\phi \left((1-m)R_b + m - (1-n(k))D - \delta \right) \Big) dp(k) \\ &+ \int_{\tau_m}^{\infty} \left(R(1-m) + m + (1-\mu)\phi \left((1-m)R_b - \left(1 - \frac{m}{D} - n(k) \right) D - \delta \right) \right) dp(k) \\ &= \int_0^{\infty} \Big(R(1-m) + m + (1-\mu)\phi \left((1-m)R_b + m - (1-n(k))D - \delta \right) \Big) dp(k) \end{aligned}$$

where the first line is expected aggregate output from waiting until the bank's cash balances are exhausted (at date τ_m), and the second line is expected aggregate output when liquidity support is introduced at date τ_m and the bank's outstanding wholesale debt is $n(\tau_m)D = (1 - m/D)D = D - m$. It is immediate to see that the third line equals expected aggregate output under a policy of immediate liquidity support

[28] The upper bound $\bar{m}$ follows by maintaining an assumption equivalent to Assumption 1. That is: $R_g(1-m) + m > D + \delta$, which implies $m < \frac{R_g - D - \delta}{R_g - 1}$ and thus $\bar{m} = \min \left\{ 1, \frac{R_g - D - \delta}{R_g - 1} \right\}$.

when the bank holds a share $(1 - m)$ of its balance sheet in long-term assets and m in cash balances (*viz.* proof of Proposition 2). □

Aside from letting the bank meet debt withdrawals before date τ_m by running down its cash balances, the PA's optimal intervention decision is qualitatively the same as in Proposition 2. For sufficiently large social costs above some threshold ϕ_m^{**}, the PA liquidates the bank at date 0. For sufficiently small social costs below some other threshold ϕ_m^*, the PA is indifferent between offering immediate support at date 0, or delaying liquidity support until date τ_m and then providing support until news arrives. For intermediate social costs between ϕ_m^* and ϕ_m^{**}, the bank draws down its cash holdings and (provided that news does not arrive before) starts to fire-sell assets at date τ_m, while the PA delays the provision of liquidity support until the good bank has zero equity at some date $\hat{\tau}$.

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