DEUTSCHE BUNDESBANK
EUROSYSTEM

Discussion Paper

Deutsche Bundesbank
No 06/2022

Banks' strategic interaction, adverse price dynamics and systemic liquidity risk

Ulrich Krüger
Christoph Roling
Leonid Silbermann
Lui-Hsian Wong

Deutsche Bundesbank, Wilhelm-Epstein-Straße 14, 60431 Frankfurt am Main,
Postfach 10 06 02, 60006 Frankfurt am Main

Tel +49 69 9566-0

Please address all orders in writing to: Deutsche Bundesbank,
Press and Public Relations Division, at the above address or via fax +49 69 9566-3077

Internet http://www.bundesbank.de

ISBN 978–3–95729–874–4
ISSN 2749–2958

Non-technical summary

Research Question

Systemic liquidity risk depends on the interaction of market participants. Thus, coordination failures between market participants can lead to a liquidity crisis. This paper addresses the question of how to measure short-term systemic liquidity risk when focusing on banks' strategic interaction via adverse price dynamics in distressed asset sales.

Contribution

To our knowledge, our theoretical analysis is the first to model strategic interaction between banks that aim to remain liquid while minimising the loss in market value that they incur in a distress sale triggered by a system-wide funding shock. Our empirical analysis makes use of granular regulatory data for German banks which make possible to pinpoint the resilience of the German banking system to a short-term liquidity shock. We propose an indicator, called the Systemic Liquidity Buffer (SLB). We benchmark the SLB with microprudential indicators on liquidity risk (e.g. Liquidity Coverage Ratio). In additional analyses, we investigate the liquidity risks from banks' US dollar business and we evaluate the impact of suddenly rising interest rates on liquidity in the banking system.

Results

In the empirical analyses on banks in Germany we find that our indicator turns out to be substantially lower than aggregate microprudential indicators on liquidity risk, as latter indicators do not take the impact of distress sales into account. However, measured by the SLB the banking system remains resilient to a systemic liquidity shock and even increased its resilience during the recent COVID-19 crisis.

Nichttechnische Zusammenfassung

Fragestellung

Systemische Liquiditätsrisiken hängen von strategischen Interaktionen der Marktteilnehmer ab. So können Koordinationsprobleme zwischen Marktteilnehmern zu einer systemischen Liquiditätskrise führen. Das Papier befasst sich mit der Frage, wie kurzfristige systemische Liquiditätsrisiken gemessen werden können, wenn strategische Interaktion von Banken bei Notverkäufen von Vermögenswerten berücksichtigt werden.

Beitrag

Nach unserem Wissen ist unsere theoretische Analyse die erste, die die strategische Interaktion zwischen Banken bei einem Notverkauf modelliert, der durch einen systemweiten Finanzierungsschock ausgelöst wird. Hierbei zielen Banken darauf ab, jederzeit liquide zu bleiben und gleichzeitig Marktwertverluste zu minimieren, die sie während des Notverkaufs erleiden. Unsere empirische Analyse verwendet granulare regulatorische Datenmeldungen für deutsche Banken, die es ermöglichen, die Widerstandsfähigkeit des deutschen Bankensystems gegenüber kurzfristigen Liquiditätsrisiken zu bestimmen. Wir definieren einen Indikator für den systemischen Liquiditätspuffer. Zudem vergleichen wir unseren Indikator mit mikroprudenziellen Kennzahlen des Liquiditätsrisikos (z.B. 'Liquidity Coverage Ratio'). Außerdem untersuchen wir die Liquiditätsrisiken aus dem US-Dollar-Geschäft von Banken und bewerten die Auswirkungen plötzlich steigender Zinsen auf die Liquidität im Bankensystem.

Ergebnisse

In der empirischen Analyse für Banken in Deutschland finden wir, dass unser Indikator erheblich niedriger ist als aufsichtsrechtliche Liquiditätkennzahlen, da letztgenannte die Auswirkungen von Notverkäufen nicht berücksichtigen. Gemessen anhand des systemischen Liquiditätspuffers bleibt das Bankensystem jedoch gegenüber einem systemischen Liquiditätsschock widerstandsfähig und erhöhte sogar seine Resilienz während der jüngsten COVID-19-Krise.

Bundesbank Discussion Paper No 06/2022

Banks' Strategic Interaction, Adverse Price Dynamics and Systemic Liquidity Risk*

Ulrich Krüger
Deutsche Bundesbank

Christoph Roling
Deutsche Bundesbank

Leonid Silbermann
Deutsche Bundesbank

Lui-Hsian Wong
Deutsche Bundesbank

Abstract

In this paper we introduce two measures, the Systemic Liquidity Buffer (SLB) and the Systemic Liquidity Shortfall (SLS) to assess liquidity in the banking system. The SLB takes an aggregated perspective on liquidity risks in the banking system. In contrast, the SLS focusses on the problematic banks which suffer a liquidity shortfall. These measures provide an add-on to regulatory liquidity measures such as the LCR because they better incorporate a systemic perspective: (1) They model the impact of a funding shock by valuing assets at depressed market prices, (2) Doing so, they explicitly incorporate banks' strategic responses to a market undergoing sharp price declines. We test our approach using several applications capturing both a short (5 days) and a medium-term (30 days) stress scenario, a sudden rise in interest rates, the impact of banks' US dollar business and the recent COVID-19 crisis.

Keywords: Systemic liquidity risk, market liquidity, funding liquidity, contagion, fire sales

JEL classification: C63, G01, G17, G21, G28.

---

*Corresponding author: Lui-Hsian Wong Deutsche Bundesbank, Financial Stability Department, Wilhelm-Epstein-Str. 14, 60431 Frankfurt am Main. Phone: +49 69 9566 7412. E-mail: lui-hsian.wong@bundesbank.de. The authors thank Sabrina Baier, Markus Brunnermeier, Marvin Borsch, Alexandros Gilch, Alexander Schmidt, Gerardo Ferrara, Natalie Kessler, Sébastien Dereeper, Henry Penikas, participants of the 10th International Conference "Financial Engineering and Banking Society" in 2021, organized by the Université de Lille, participants of the 37th Symposium on Money, Banking and Finance in 2021 organized by the Banque de France, participants of the Joint CNB/ECB/ESRB Workshop "Sources of structural systemic risk in the financial system: identification and measurement" 2019 in Prague, participants of the CCBS Workshop "Managing liquidity and funding risk" 2019 at the Bank of England in London, participants of the CCBS Workshop "Systemic risk assessment: identification and monitoring" 2019 at the Bank of England in London, and seminar participants at the Bundesbank for helpful comments.

1 Introduction

Financial crises have gone hand in hand with liquidity crises, in particular with a drying-up of liquidity in the banking system.[1] As a consequence of the financial crisis of 2007/2008, policy-makers have introduced new measures of liquidity risk for individual financial institutions, which regulate the level and composition of liquid assets and financial liabilities.[2] Beyond liquidity risk at a single entity, academic research has focused on economic mechanisms that have the potential to destabilise the whole financial system (Morris and Shin, 2004; Brunnermeier and Pedersen, 2009; Allen and Gale, 2010; Krishnamurty, 2010). These models provide insight into a possible source of endogenous liquidity risk: An initial negative shock to asset prices and a run on short-term liabilities may force some financial institutions to reduce their balance sheet. As these institutions shed some of their assets to raise cash and to repay debt, they exert further downward pressure on market prices. Other institutions see a decline in the market value of their assets and may find it difficult to meet their short-term obligations, inducing them to sell assets as well. When institutions' funding suddenly evaporates and the system goes through a self-reinforcing cycle of price declines, systemic liquidity risk materialises.[3]

In this paper, we measure systemic liquidity risk by focusing on banks' strategic interaction via adverse price dynamics. The model tests the resilience of the banking system to an exogenous funding shock. Specifically, we take a widespread bank run over a short time horizon (e.g. 5 days) as given, in which institutions must repay debt fully and imme-

[1]Examples include the liquidity crisis associated with Long-Term Capital Management (LTCM) in 1998 (Gatev, Schuermann, and Strahan, 2007), the Great Financial Crisis in 2007/2008 (Brunnermeier, 2009) or the European debt crisis in the autumn of 2011 (Correa, Sapriza, and Zlate, 2016).

[2]For an overview of the framework of liquidity risk management put forward by the Basel Committee on Banking Supervision (BCBS), see Basel Committee on Banking Supervision (2010). For details on the main metrics in this framework, the Liquidity Coverage Ratio (LCR) and the Net Stable Funding Ratio (NSFR), see Basel Committee on Banking Supervision (2013) and Basel Committee on Banking Supervision (2014).

[3]To the best of our knowledge, there is no commonly used definition of systemic liquidity risk. The International Monetary Fund (2011) defines it as "the risk that multiple institutions may face simultaneous difficulties in rolling over their short-term debts or in obtaining new short-term funding through widespread dislocations of money and capital markets". Our notion above emphasises strains in funding markets *and* institutions' reactions to price changes and is similar to Krishnamurty (2010) and Shin (2010).

diately according to contractual maturities. From this, we derive an endogenous shock to market liquidity of banks' securities. The projected cash outflows force banks to offload securities if their initial cash reserves are insufficient to service their liabilities. Banks' decision-making is characterised by two objectives. First, they aim to stay liquid at any point in time by generating enough cash through sales of securities. Second, they aim to minimise losses in the securities' market value by taking into consideration the price impact of their own strategy and strategies of other banks. This gives rise to a coordination problem in terms of the timing and volume of sale. On the one hand, if banks expect the market price of these securities to fall (because other banks will sell securities to bridge their funding bottlenecks), it may be rational for them to dispose them as early as possible to minimise their market value losses. On the other hand, if a bank has a large portfolio and large expected outflows, its selling strategies alone can depress prices. Consequently, it will rather first use existing cash reserves to service the outflows, and try to divide the sale of securities up into small portions and to extend it over a longer period of time so as not to single-handedly accelerate the price drop. Banks with a large portfolio will therefore tend to act more cautiously than banks that have little influence over the market price.

Our model makes three specific assumptions. First we disregard the role of the central bank as a lender of last resort, as we want to test the resilience of the banking system to a widespread funding shock without assistance from the central bank. Consequently, we assume banks do not have access to central bank funding, in particular repo transactions vis-à-vis the central bank are excluded.[4] This assumption is guided by the macroprudential, i.e. preventive focus of our liquidity metrics. They are supposed to pick up liquidity risks that do not anticipate lender of last resort activities in the spirit of Bagehot (1873) as addressing those liquidity risks in a timely manner would ideally make central bank intervention less likely and necessary.[5] Second, we assume that banks cannot rely on

[4]In Section 4.2.2 we consider a model variant which permits central bank funding and compare the results with those of the basic model.

[5]Additionally, distinguishing between liquidity and solvency problems might be challenging in a crisis (see, for example, Thakor (2015)).

interbank credit as a potential funding source during episodes of stress. In particular, banks cannot offset the cash outflows through the interbank repo market. In such a scenario banks depend on outright asset sales as a short-term funding source to service their liabilities. Third, we assume banks restrict asset sales to securities designated as liquid assets according to the Capital Requirements Regulation (CRR), i.e. high-quality liquid assets (HQLA) which are eligible for the short-term Liquidity Coverage Requirement (LCR). We are thus following the underlying concept of the LCR where banks should build up a buffer of HQLA that can be used in times of stress. Adopting such a regulatory perspective allows us to compare readily the results produced by our model with the LCR.

The model provides a macroprudential perspective as it reflects banks' endogenous response function to a funding shock. This response function can intensify the liquidity crisis via distress sales of securities. It produces distress prices which can be interpreted as liquidity weights, are assigned to a bank's securities portfolio and depend on system level factors. In particular, these liquidity weights depend on the overall level of short-term funding in the banking system. The higher the reliance on short-term funding in the banking system (cash outflows in a widespread bank run over a short time horizon respectively), the higher banks' selling volume and the higher the drop in securities' market prices during the distress sale ceteris paribus. Second, the model also links liquidity weights of a bank's securities portfolio with the commonality of banks' securities portfolio composition. The higher the commonality between these portfolios, the higher the drop in securities' market prices during the distress sale ceteris paribus. Importantly, the liquidity weights produced by the model vary over time, depending on the current macro-development of short-term funding and the current composition of the securities portfolio in the banking system. Assigning time-variant liquidity weights is one advantage of our model compared to microprudential liquidity measures, such as the LCR which assigns fix liquidity weights for assets.

For the further analysis, we ask two questions: How much liquidity is in the banking system once the assumed scenario of a widespread banking run and resulting distress sales of securities unfolds? Moreover, if some institutions become illiquid, how severe is the liquidity shortage in the system? The Systemic Liquidity Buffer (SLB) presents an answer to the first question. It calculates the available net liquidity after the funding shock based on two components: (1) liquid assets valued at simulated distress prices, minus (2) net outflows from on-balance sheet and off-balance sheet transactions. The level of this buffer measures how vulnerable the system is to roll-over risk: Low or even negative values indicate that the system as a whole does not have enough liquid assets to withstand a system-wide bank run. The buffer aggregates the amount of liquid assets individual institutions have once roll-over risk materialises and market prices may be depressed. In this scenario, some institutions in the system may have sufficient liquidity, while other institutions may not be able to meet their financial obligations. By restricting attention to institutions that actually become illiquid, we arrive at an answer to the second question. The Systemic Liquidity Shortfall (SLS) is the amount of liquidity that the system would need to ensure that all institutions withstand the funding shock. Taken together, the SLB and the SLS are meant to support policy analysis to assess the stability of the banking system by measuring the resilience of the banking system to liquidity risk.

Aggregate risk measures have received attention in recent years. Brunnermeier, Gorton, and Krishnamurthy (2011) and Brunnermeier, Gorton, and Krishnamurthy (2014) provide a conceptual framework for measuring liquidity risk in the financial system. They propose the Liquidity Mismatch Index (LMI). This index is designed to quantify the amount of liquidity that an institution has considering its ability to turn its asset into cash and the maturity profile of its debt. The latter information indicates the urgency with which its debt has to be repaid. Building on this framework, Bai, Krishnamurty, and Weymuller (2018) motivate liquidity weights theoretically and estimate them empirically. These weights are attached to each item on the asset and liability side of a bank's balance sheet and reflect the tension between the market liquidity of assets and the funding liquidity of liabilities. The difference between weighted assets and weighted liabilities yields the LMI.

Our approach adds theoretical and empirical results to the framework of Brunnermeier et al. (2014). It differs from the implementation of the LMI in Bai et al. (2018) in the way the impact of sudden liquidity outflows and the deterioration of prices of securities are assessed. Our theoretical analysis builds on a model in which banks face an exogenous liquidity outflows in two periods. In reaction to these outflows they may choose which share of a certain type of asset they sell in each period in order to raise cash. We obtain four optimal selling strategies, which are driven by the underlying trade-off of early selling at favorable prices or shifting sales to later periods to optimise a bank's own impact on market prices. The respective analysis formalises the best course for action (Brunnermeier et al., 2014) of an institution in response to the funding shock. We also show that in a model with two banks, equilibria exist in the sense that there are pairs of selling strategies which are mutually compatible for each bank to maintain liquidity during the stress period.

Empirically, we observe contractual *flows* from banks' balance sheets, i.e. for given maturity buckets (overnight, up to two days, up to three days etc.), we can directly obtain the net liquidity outflow that each bank faces. Therefore, a calibration exercise to derive liability-side weights from *stocks* of liabilities as in Bai et al. (2018) is not necessary in our approach. Turning to the asset side of the balance sheet, we assign banks' liquid assets to several classes, such as government bonds, corporate bonds or common stocks. We then assess the market liquidity of banks' assets by simulating a sequence of distress sales (Greenwood, Landier, and Thesmar, 2015). In response to the funding shock, each bank sells some of its liquid assets according to the selling strategies implied by our theoretical model. The resulting sales volumes induce a price decline, which we derive from an empirical price impact measure in the spirit of Amihud (2002). Accordingly, the stressed market prices yield the cash-equivalent value of banks' assets (Brunnermeier et al., 2014). Continuing this selling process for a given number of periods, say 5 days, we can simulate the evolution of the market value of banks' liquid assets (SLB) and the liquidity shortfall (SLS). Note that our primary focus is the impact of distress sales on liquidity in the banking system, rather than the leverage ratio. In this respect, our analysis differs from Greenwood et al. (2015). Although leverage played a major role in the build-up of risk in the banking system before the Great Financial Crisis (GFC) in 2007/2008 (Geanakoplos, 2009; Adrian and Shin, 2010), once investors lose confidence in institutions and refuse to roll-over debt, maintaining liquidity becomes the primary objective of banks in the very short run, which we focus on in this paper.

We study systemic liquidity risk in the German banking system in four empirical applications. First, we examine the distribution of liquidity risk in the cross section. In a run on the banking system that takes 5 days, the total liquidity shortfall SLS is about EUR 18 bn, of which one third is attributable to systemically important banks. If we analyze the SLS by business model, the entire shortfall is concentrated on commercial banks, while savings or cooperative banks face no shortage. These findings highlight the potential of the SLS to spot vulnerabilities in the banking system. Moreover, the SLB illustrates the potentially severe economic consequences of a distress sale spiral: comparing the stock of liquid assets valued at the end of the stress episode at possibly depressed market prices relative to the value of the liquid assets before the run on the banking system started reveals an overall loss in market value of EUR 52 bn. While this loss is 3% of liquid assets, it accounts for 10% of banks' aggregate Tier 1 capital.[6]

Second, we examine the evolution of systemic liquidity risk over time. To this end, we aggregate individual excess liquidity according to bank-specific (or microprudential) regulatory measures and compare this indicator of liquidity risk with the SLB. Before the most intense period of the GFC in September 2008, bank-specific excess liquidity aggregated over all banks was positive and rising steadily, showing no sign of possible liquidity risks in the banking system. In contrast, the SLB reached its lowest and negative level in mid-2007, pointing to a severe vulnerability of the system to liquidity risk, which was prevalent at that time. The reason for the divergence between the two measures before the crisis is the strong impact of distress sales on security prices due to a sharp increase

[6] This implicitly assumes fair-value accounting and abstracts from the complexities that the security valuation method has on recognising banks' losses (see Schmidt, Noth, and Tonzer (2021)).

in short-term funding in the banking system in the run-up to the crisis from June 2003 to June 2007: When aggregating individual excess liquidity, we implicitly assume that each bank can sell a particular security at the current market price. But this assumption may underestimate downward price pressure exerted by banks collectively in a liquidity crisis, especially when banks simultaneously face funding bottlenecks due to excessive short-term refinancing. This latter effect is captured in the SLB, making it a more appropriate measure of aggregate risk.

In the framework of Brunnermeier et al. (2014), aggregate liquidity risk is evaluated in different states of the world and for several stress scenarios. We provide two examples of relevant scenarios that extend our baseline results. While the baseline specification assumes a run on banks' total debt, we also assess the impact of a run on US dollar denominated debt. The US dollar is the most important foreign currency for internationally active German banks. Funding in this currency tended to be vulnerable in times of market-wide stress as in the autumn 2011 (Ivashina, Scharfstein, and Stein, 2015; Correa et al., 2016). We therefore study the impact of roll-over risk in US dollar on liquidity in the banking system as our third application. The fourth application deals with the interaction of interest rate risk and liquidity risk. An upward shift in the yield curve increases repricing risk for banks that fund fixed-rate loans with variable-rate deposits (English, van den Heuvel, and Zakrajsek, 2018). Here, we are concerned with a repricing of banks' liquid securities due to an upward shift in the yield curve immediately before they face a run on their debt.

The plan of the paper is as follows. In Section 2, we present our methodology to measure systemic liquidity risk. Section 3 discusses our theoretical model with two banks, two time periods and one asset with the intent to draw conclusions regarding the strategic interactions of banks. Section 4 presents an empirical version of the model and discusses the four policy applications for the German banking system. Section 5 concludes.

2 The Systemic Liquidity Buffer

In this section, we suggest a measure of systemic liquidity risk and lay out the building blocks of our model. Section 2.1 introduces the systemic liquidity buffer, and Section 2.2 outlines the model we use to derive this measure of systemic liquidity risk.

2.1 Overview

Declining asset prices lower the market value of banks' portfolios when they reevaluate their assets in a marked-to-market environment. In a financial crisis, some institutions may be forced to sell some of their assets to raise cash. If demand for these assets is not perfectly inelastic, a falling price affects other institutions in the system through this asset valuation channel (Cifuentes, Ferrucci, and Shin, 2005). It is the main goal of the paper to model this type of connectedness between banks, both from a theoretical and an empirical point of view. To this end, we introduce a measure of systemic liquidity risk, which we call the systemic liquidity buffer.

In our model, each bank faces liquidity outflows for a given period of time, say 5 days or 30 days. This period of liquidity stress applies to all banks at the same time. Banks draw on their initial cash reserve to meet these obligations. They can also raise new cash by selling securities from their portfolio. We assume banks restrict asset sales to securities designated as liquid assets according to the CRR, i.e. high-quality liquid assets (HQLA) for the LCR. Thus, for bank i, cash evolves according to

$$c_{i,t+1} = c_{i,t} + v_{i,t} - l_{i,t} \tag{1}$$

for $t = 1, 2, \ldots, T$, where T is the period in which the stress event ends, and $c_{i,1}$ is given as the initial cash position. Here, $l_{i,t}$ denotes outflows from deposits and other forms of debt. Banks have a cash amount of $c_{i,t}$ at the beginning of the period and possibly raise new cash $v_{i,t}$ in this period by selling securities from their portfolio. This portfolio can be broken down into several asset classes (government bonds, corporate bonds, stocks etc.). For notational ease, we bundle these sales into a single term $v_{i,t}$.

In each asset class, sales take place at current market prices prevailing at that time. If several banks sell securities simultaneously, prices may fall, and all banks update the market value of their existing security holdings accordingly.

At the end of the stress period, we take stock of a bank's remaining liquid funds and the market value of the remaining security portfolio. We define the systemic liquidity buffer (SLB) for bank i as

$$SLB_{i,T+1} = c_{i,T+1} + a_{i,T+1} = c_{i,1} + \sum_{t=1}^{T} (v_{i,t} - l_{i,t}) + a_{i,T+1}, \tag{2}$$

where $a_{i,T+1}$ denotes the market value of the security portfolio, evaluated at prices prevailing after the stress period has ended. While $c_{i,T+1}$ includes actual cash flows in terms of outflows and asset sales, this second term $a_{i,T+1}$ is a hypothetical cash flow. Here, we ask how much cash the bank could generate if it sold the entire remaining portfolio.

If a given bank has large initial cash holdings, sees only little outflows, or if market prices do not change much, this liquidity buffer is large and the bank is resilient to a liquidity shock of this kind. By contrast, if the bank has only little cash to begin with, deals with large outflows, and is not able to raise enough cash under stress, the buffer may be small or even negative, indicating that the bank is prone to liquidity risk.

By aggregating the SLB across all banks, we obtain the liquidity buffer of the entire banking system,

$$SLB_{T+1} = \sum_{i=1}^{N} SLB_{i,T+1} \tag{3}$$

where N denotes the total number of banks in the system. We also introduce the systemic liquidity shortfall (SLS),

$$SLS_{T+1} = \sum_{i=1}^{N} \min \{SLB_{i,T+1}, 0\}, \quad (4)$$

including only those banks with insufficient liquidity in a crisis. In this way, banks with sufficiently large liquidity positions do not offset illiquid banks. While the SLB measures the amount of liquid assets available in the system after the funding shock, the shortfall SLS is informative about the level of liquidity that is needed in the banking system to ensure that all banks are able to withstand a liquidity shock. Notice that the SLS is directly derived from the SLB, so for brevity we will sometimes just refer to the SLB in the following sections.

We refer to the buffer as a systemic liquidity buffer for two reasons. First, we examine an extreme scenario in which all banks, or a substantial part of the banking system, faces liquidity outflows at the same time. Therefore, the buffer results from a system-wide liquidity shock. Second, a bank's asset sales can push market prices down, which affects the market value of securities held by other banks and is therefore likely to have an effect on the liquidity management of other banks in the system.

Our model makes three specific assumptions. First, we disregard the role of the central bank as a lender of last resort, as we are interested in the resilience of the banking system to liquidity risk without assistance from the central bank.[7] Consequently, we assume banks do not have access to central bank funding during the shock scenario. In particular, repo transactions vis-à-vis the central bank are excluded.[8] This assumption is guided by the macroprudential, i.e. preventive focus of our liquidity metrics. Second, we disregard

[7]We consider a model variant which permits central bank funding and compare the results with those of the basic model in Section 4.2.2. In reality, central banks have played an important role in liquidity crises. Empirical evidence shows that during the global financial crises in 2007/08 funding from private sources was replaced by central bank funding for German banks, e.g. German banks used securities as collateral to obtain funding from the central bank, see also Podlich, Schnabel, and Tischer (2017).

[8]Having said that, we do not completely disregard central banks. Banks' reserves with the central bank have become an important part of banks' liquidity buffers. As central banks all over the world have adopted extraordinary monetary policy measures in recent years, their decisions and actions have implications for the data that we use in the empirical section of the paper (see Section 4).

interbank credit as a potential funding source during episodes of stress. As we take a widespread funding shock in the banking system as given, we assume the functioning of interbank funding markets is disrupted. In particular, we assume banks cannot obtain liquidity through the interbank repo market. Hence, we only consider outright asset sales as a short-term funding source for banks.[9] Third, we assume banks restrict asset sales to securities designated as HQLA according to the CRR, which are eligible for the short-term LCR. We are thus following the underlying concept of the LCR where banks should build up a buffer of HQLA that can be used in times of stress. Adopting such a regulatory perspective allows us to compare readily the results produced by our model with established microprudential liquidity measures, such as the LCR. In this respect, our model tests the fungibility of securities designated as HQLA on private markets during a system-wide funding shock.

Rather, we study the connectedness within the banking system arising from changes in asset prices through distress sales.[10] To this end, the following section presents each individual bank's decision-making in this setup.

2.2 Banks' liquidity management under stress

We now describe the analytical framework for the systemic liquidity buffer. This framework underlies both the theoretical analysis in Section 3 and the empirical analysis in Section 4 and involves the bank's outflows, the evolution of market prices, and the bank's objective to stay liquid at all times.

[9]While in Germany, the majority of all repo transactions are traded on the interbank repo market (see ECB (2017) and ICMA (2021)) some large non-bank financial intermediaries, such as investment funds and insurances, also have access to the repo market and could provide short-term liquidity to the banking system through the repo channel. We leave it for future research to incorporate private repo markets in the analyses.

[10]It is worth mentioning that we do not consider the usage of derivatives for hedging purposes when modeling the distress sale, e.g. derivatives for hedging banks' bond portfolio.

Outflows

First, banks have issued liabilities that differ in terms of their volume, maturity or collateralisation, and hence give rise to different types of outflows under stress. For example, unsecured funding, such as unsecured deposits, are subject to larger outflows than secured or guaranteed funding (e.g. deposits with deposit insurance). In the empirical analysis (see Section 4), we determine the outflows for several types of liabilities using regulatory data. We take these total liquidity outflows, denoted by $l_{i,t}$, as given and model the bank's liquidity management in response to these outflows.

Price impact ratio

Second, suppose the bank's portfolio consists of K different types of assets. In each period t, bank i decides to sell a fraction $\omega_{i,k,t}$ of its assets of type k to the market. By definition we have $0 \leq \omega_{i,k,t} \leq 1$ for $k = 1, 2, \ldots, K$. Total sales in asset class k and period t are then given by

$$S_{k,t} = \sum_{i=1}^{N} \omega_{i,k,t} a_{i,k,t},$$

in which $a_{i,k,t}$ denotes the market value of asset k in bank i's portfolio at time t.

The total volume of sales by the entire banking system invokes a price reaction on asset markets. Let $p_{k,t}$ denote the market price of asset k at time t. The gross return in asset class k is denoted as $R^k_{t,t+1} := p_{k,t+1}/p_{k,t}$ and the net return as $r^k_{t,t+1} := R^k_{t,t+1} - 1$. A widely used empirical measure of market liquidity suggested by Amihud (2002) considers the ratio of the absolute value of the net return and the dollar trading volume. It shows the (absolute value of the) price change per dollar that is traded in an asset. If even small trading volumes are associated with large price changes, the Amihud measure is large and indicates illiquid markets.

We consider a modified version of the Amihud measure in our model. We restrict attention to falling prices in the stress episode, i.e. $R^k_{t,t+1} < 1$, and assume the constant relationship

$$\lambda_k = \frac{R^k_{t,t+1} - 1}{V^k_{t,t+1}} \tag{5}$$

between returns and the trading volume. Here, $V^k_{t,t+1}$ denotes the trading volume for asset class k measured at market prices at time $t+1$. The ratio λ_k indicates the relative price decline per nominal amount traded in the market. We refer to λ_k as the price impact ratio in asset class k.

Banks decide in period t on their portfolio sales and offer a total amount of $S_{k,t}$ to the market. However, during the stress episode banks can sell their securities only at a discounted price, which is captured by the gross return $R^k_{t,t+1} < 1$. The trading volume $V^k_{t,t+1}$ measured at the market price for the assets of type k is therefore obtained by

$$V^k_{t,t+1} = S_{k,t} R^k_{t,t+1}. \tag{6}$$

Combining (5) and (6), we obtain

$$R^k_{t,t+1}\left(S_{k,t}\right) = R^k_{t,t+1} = \frac{1}{1 - \lambda_k S_{k,t}}. \tag{7}$$

for the price change between the two periods t and $t+1$. Notice that λ_k (together with the simulated sales volume $S_{k,t}$) governs the price decline over the entire scenario horizon of T periods , and that $\lambda_k < 0$ in this model. We make a few remarks about the price impact ratio λ_k and the role of banks in this setup.

The price impact ratio λ_k does not depend on t. Conceptually, we think of λ_k as an average price decline in a market downturn, possibly reflecting periods of liquidity stress. The price impact ratio we suggest here is solely motivated by a variation of an empirical measure of market liquidity. Together with the simulated sales volume, we use λ_k to model the price adjustment. The sales volume is an endogenous quantity that results from having several banks of different sizes and with different asset/liability structures simultaneously sell their assets when they need cash to cover their immediate refinancing needs. These externalities give rise to systemic liquidity risk in our paper.[11] We focus on the fire-sales externality and use constant but rather conservative price impact ratios λ_k as an important input parameter in the empirical analysis.

Finally, we would like to point out that our model takes a narrow view of banks' transactions. Once the stress event begins, banks sell securities to maintain liquidity, but do not become buyers in these markets. Instead, during the stress event, other intermediaries in the financial system such as insurance companies or mutual funds buy securities from banks. We assume that actions of these other intermediaries have a price impact of zero on aggregate.

Optimisation

Third, we consider the decision-making of banks as an optimisation problem for bank i. The decisions to be made at time t are captured by the vector

$$\omega_{i,t} = (\omega_{i,1,t}, \ldots, \omega_{i,k,t}, \ldots, \omega_{i,K,t}) \, .$$

Recall that each component $\omega_{i,k,t}$ describes the share of assets of type k the banks intends to sell. For bank i the optimisation problem is:

$$\min_{(\omega_{i,t})_{t=1}^{T}} \sum_{t=1}^{T} \sum_{k=1}^{K} a_{i,t,k}(1 - R^k_{t,t+1}(S_{k,t})), \tag{8}$$

such that, for all $t = 1, 2, \ldots, T,$ and $k = 1, 2, \ldots, K,$

(L) $\quad c_{i,t+1} \geq 0,$

(C) $\quad c_{i,t+1} = c_{i,t} + \left(\sum_{k=1}^{K} \omega_{i,k,t} a_{i,k,t} R^k_{t,t+1}(S_{k,t}) \right) - l_{i,t},$

[11] If we additionally allowed the price impact ratio to be time-varying, a not so straight-forward analysis of which of the two factors is responsible for changes in the SLB/SLS and to what extent, would be necessary.

$$(B) \quad 0 \leq \omega_{i,k,t} \leq 1,$$

$$(V) \quad a_{i,k,t+1} = (1 - \omega_{i,k,t})\, a_{i,k,t} R^k_{t,t+1} (S_{k,t})\,.$$

Here, $a_{i,k,1}$, $c_{i,1} \geq 0$, and $l_{i,t}$, $t = 1, 2, \ldots, T$, are given, while $R^k_{t,t+1} (S_{k,t}) = 1/(1 - \lambda_k S_{k,t})$, in which λ_k is a fixed parameter, see (7).

Once the funding shock materialises, the bank's goal is to stay liquid at all points in time, such that its liquid funds are non-negative. This restriction corresponds to condition *(L)*.

The liquid funds evolve according to condition *(C)*. The available cash in period $t+1$ is given by previous cash holdings $c_{i,t}$ less net liquidity outflows. Moreover, the bank may decide to liquidate a share of its asset portfolio to restore or to increase liquidity. These shares are bounded between zero and one. This requirement corresponds to condition *(B)* and implies that short selling is ruled out in this setup.

Note that in condition *(C)*, the term $\omega_{i,k,t} a_{i,k,t} R^k_{t,t+1} (S_{k,t})$ is the liquidation value of asset k under market conditions determined by $R^k_{t,t+1}$.[12] The larger the drop in market prices, the less a sale of an asset will contribute to the cash holdings $c_{i,t+1}$. The exact nature of this price adjustment is given by (7).

Similar to condition *(C)*, the market value of an asset in period $t+1$ is the remaining stock of assets after liquidation in period t, evaluated at market prices implied by $R^k_{t,t+1}$, as specified in condition *(V)*.

Thus, price adjustments affect the bank both through the market prices at which it can sell assets (see condition *(C)*) and through the value adjustments of the remaining assets in the portfolio (see condition *(V)*).

The conditions *(L)*-*(V)* describe each bank's freedom of action when deciding on $\omega_{i,1}, \omega_{i,2}, \ldots, \omega_{i,T}$ to minimise losses in the market value of its portfolio over the entire sce-

[12]Comparing this condition *(C)* with (1), we see that $\omega_{i,k,t} a_{i,k,t} R^k_{t,t+1} (S_{k,t})$ corresponds to $v_{i,t}$ in (1). Strictly speaking, the definition of security sales in that equation should be denoted by $v_{i,t,t+1}$ to emphasise that the immediate price adjustment $R_{t,t+1}$ is taken into account. We chose the notation $v_{i,t}$ to simplify the exposition in Section 2.1.

nario horizon, as specified by the objective function $\sum_{t=1}^{T}\sum_{k=1}^{K} a_{i,k,t}\left(1-R_{t,t+1}^{k}\right)$.

The cash position $c_{i,T+1}$ and the market value of the entire portfolio $a_{i,T+1} = \sum_{k=1}^{K} a_{i,k,T+1}$ are then used to compute the Systemic Liquidity Buffer, see (2) and (3).

Behavioural assumption

Note that in the setup the decision variable at time t is a vector of length K, meaning that a separate decision is required for each asset class. More specifically, the bank has to decide on the fraction $\omega_{i,k,t}$ of the sub-portfolio k that it sells in each period. In order to simplify the problem this requirement will be relaxed. We will consider a pro-rata approach, i.e.

$$\omega_{i,t,k} = \omega_{i,t}\ , k = 1, 2, \ldots, K.$$

The bank spreads sales equally across asset classes. This pro-rata assumption is supported by the work from Van den End and Tabbae (2012). Their statistical tests show that in a crisis situation banks tend to liquidate assets in proportion to their balance sheet and do not follow a pecking order (e.g. by making larger adjustments to the most liquid balance sheet items compared to less liquid items).[13] In addition, as outlined in Section 4.2.1 it turns out that banks portfolio of securities designated as HQLA is highly concentrated on the most liquid securities (so-called level 1 assets). Government bonds, the most liquid type of security, accounts more than two-thirds of all securities designated as HQLA. The second most liquid type of security, covered bonds, accounts for a share of nearly 20%. That means when following a pro-rata approach banks effectively do sell predominantly the most liquid assets, i.e. government bonds. Hence, following a pecking order that sells

[13] Although we have not replicated the analysis by Van den End and Tabbae (2012), we have considered changes in the stock of liquid assets before, during and after the financial crisis of 2007-2009. It turns out that for the aggregate banking system, the relative decrease in debt securities, shares, covered bonds and shares in investment funds are of similar magnitude during the financial crisis. The results of this analysis are available on request.

the most liquid securities first would not materially change the results.

3 Optimal liquidity management for two banks in the system

In this section, we study the simplest version of the above model with two banks, two time periods and one asset in more detail. Hence, in (8), we let $N = 2$, $K = 1$ and $T = 2$. We reconsider the more general setup outlined in the previous section in the empirical application in Section 4.

The purpose of this section is twofold: first, we characterise optimal liquidity management if funding suddenly evaporates in the banking system. Here, banks choose a sequence of sales volumes to generate cash and to meet liquidity outflows. Second, we show that Nash equilibria exist in this setup so that these asset sales are mutually compatible and banks avoid becoming illiquid.

With one asset only, let $r_{t,t+1} := p_{t+1}/p_t - 1$ and let $v_{i,t}$ be the dollar volume of the asset sold by bank i for $i = 1, 2$. Following the discussion after equation (5) in the previous section, these amounts are measured at the price in the next period p_{t+1}. The modified Amihud ratio in this setup is then given by $\lambda = r_{t,t+1}/\left(v_{1,t} + v_{2,t}\right)$. Throughout this section we make the following assumption:

Assumption 1

a) In problem (8), let $N = 2$, $T = 2$, and $K = 1$,

b) $\lambda = \left(R_{t,t+1} - 1\right)/V_t$ is strictly negative, i.e. $\lambda < 0$,

c) $l_{i,1} - c_{i,1} > l_{i,2}$ for $i = 1, 2$.

Assumption 1 b) states that during the stress episode, the price of the asset declines.

According to Assumption 1 c) banks' liabilities net of liquid funds follow a monotonic decreasing pattern over time. The assumed liability profile should be largely consistent with the real conditions since banks' short-term liabilities usually exceed their long-term liabilities.[14] We make this assumption for technical reasons. It helps us to prove that in such a setting at least one equilibrium always exists, and we will refer to it later in more detail.

Note that Assumption 1 c) particularly implies $l_{i,1} > c_{i,1}$. Consequently, the financial obligations of bank 1 and bank 2 exceed its liquid funds at the beginning of the stress episode. Hence the risk of becoming illiquid already exists from the outset. Moreover, as shown in Appendix A, $l_{i,1} > c_{i,1}$ ensures $V_t > 0$ for $t = 1$ such that λ is well-defined.

We begin by reformulating the original optimisation problem (8). Using this alternative formulation, we can apply standard techniques from constrained optimisation theory. As shown in Appendix A (A.1), the optimisation problem (8) and its reformulation below are equivalent in the following sense: if there is an optimal solution to the reformulation, then we can uniquely identify a corresponding optimal solution to the original optimisation problem. Now, in terms of $v_{1,t}$, the optimisation problem for bank 1 is

$$\min_{\{v_{1,1},v_{1,2}\}} \left\{-\lambda a_{1,1}(v_{1,1}+v_{2,1}) - \lambda(a_{1,1}(1+\lambda(v_{1,1}+v_{2,1})) - v_{1,1})(v_{1,2}+v_{2,2})\right\}, \tag{9}$$

w.r.t. the transition equations

$$c_{1,t+1} = c_{1,t} + v_{1,t} - l_{1,t},$$

$$a_{1,t+1} = a_{1,t} \cdot (1 + \lambda \cdot (v_{1,t} + v_{2,t})) - v_{1,t},$$

and additional constraints

$$\begin{aligned} -c_{1,t} &\leq 0, \\ -v_{1,t} &\leq 0, \\ -v_{1,t} - c_{1,t} + l_{1,t} &\leq 0, \\ v_{1,t} &\leq \left(\frac{a_{1,t}}{1-\lambda a_{1,t}}\right) \cdot (1 + \lambda v_{2,t}), \quad \text{for} \quad t = 1, 2. \end{aligned}$$

[14] This can be seen for the German banking system in figures 1(a) and 1(b).

The transition equations show the evolution of cash holdings and the market value of the asset that the bank has. The first two inequalities are non-negativity constraints on the cash holdings and sales volumes. The third constraint ensures that the bank serves its liquidity providers. The fourth constraint is an upper bound to the sales volume which results from the availability of assets and the reduction in market prices.

The expression $1/(1-\lambda a_{1,1})$ which appears in the last constraint measures the potential impact on the market price that bank 1 has in the first period. In light of (7), it would be the resulting price discount if bank 1 sold its entire portfolio $a_{1,1}$ in the first period and bank 2 made no sale. Put differently,

$$\frac{1}{1-\lambda a_{1,1}} - 1 = \frac{\lambda a_{1,1}}{1-\lambda a_{1,1}} \tag{10}$$

measures the largest price impact that bank 1 can have in isolation in the first period. For instance, if $1/(1-\lambda a_{1,t}) = 0.9$, then the change in the market price induced by the sale of bank 1 would be -10%. As we will discuss in more detail below and in Section (A.2) in the Appendix A, the expressions in (10) are crucial for decision-making. First, the higher the resulting market value $a_{1,t}R_{t,t+1} = a_{1,t}/(1-\lambda a_{1,t})$, the higher the potential sales volume $v_{1,t}$. Second, sales by the other bank, denoted by $v_{2,t}$, push the price of the asset down, and therefore lower the value $v_{1,t}$ that bank 1 can possibly generate.

We then obtain the following result. Note here that bank 1 takes sales of bank 2 as given.[15]

Theorem 1 *Let*

$$d_{1,1}(v_{2,1}, v_{2,2}) := \frac{1}{2}(l_{1,1} + l_{1,2} - c_{1,1}) + \frac{1}{2}\left(v_{2,2} + \left(\frac{\lambda a_{1,1}}{1-\lambda a_{1,1}}\right) v_{2,1}\right) \quad \textit{and}$$
$$d_{1,2}(v_{2,1}, v_{2,2}) := \frac{1}{2}(l_{1,1} + l_{1,2} - c_{1,1}) - \frac{1}{2}\left(v_{2,2} + \left(\frac{\lambda a_{1,1}}{1-\lambda a_{1,1}}\right) v_{2,1}\right).$$

[15] A symmetric version of the theorem holds for bank 2 given that bank 1 has decided on its strategy. In the remainder of the text we will also use the relevant variables for bank 2. Especially we will make use of the decision variable $d_{2,1}$ and the variable $d_{2,2}$. Their definitions can be left out because they are symmetric and therefore straightforward.

Then, under assumption 1, the optimal solution to problem (9), denoted by $\left(v^*_{1,1}, v^*_{1,2}\right)$*, is given by*

1. *Just-in-time:* $v^*_{1,1} = l_{1,1} - c_{1,1}$*,* $v^*_{1,2} = l_{1,2}$ *if* $d_{1,1}(v_{2,1}, v_{2,2}) < l_{1,1} - c_{1,1}$*,*

2. *Smoothing:* $v^*_{1,1} = d_{11}(v_{2,1}, v_{2,2}), v^*_{1,2} = d_{1,2}(v_{2,1}, v_{2,2})$ *if* $l_{1,1} - c_{1,1} \leq d_{1,1}(v_{2,1}, v_{2,2}) < l_{1,1} + l_{1,2} - c_{1,1}$*,*

3. *Front-Servicing:* $v^*_{1,1} = l_{1,1} + l_{1,2} - c_{1,1}$*,* $v^*_{1,2} = 0$*, if* $l_{1,1} + l_{1,2} - c_{1,1} \leq d_{1,1}(v_{2,1}, v_{2,2})$ *and* $\frac{a_{1,1}}{1-\lambda a_{1,1}} \geq v_{2,2}$*,*

4. *Distress-Sale:* $v^*_{1,1} = \left(\frac{a_{1,1}}{1-\lambda a_{1,1}}\right)(1 + \lambda v_{2,1})$*,* $v^*_{1,2} = 0$*, if* $l_{1,1} + l_{1,2} - c_{1,1} \leq d_{1,1}(v_{2,1}, v_{2,2})$ *and* $\frac{a_{1,1}}{1-\lambda a_{1,1}} < v_{2,2}$*.*

A key variable in the theorem is $d_{1,1}$. It determines the decision of bank 1 which of the four possible strategies is the optimal one. Note that the optimal strategy for bank 1 in period 1 in the Smoothing strategy coincides with the variable $d_{1,1}$, which determines the decision on which of the four strategies is the optimal one. The four strategies ensure that the bank 1 can raise sufficient cash and simultaneously minimises the price drops caused by its selling behaviour. If it anticipates significant sales by bank 2 in period 2, it may prefer to sell some of its holdings immediately to trade at a relatively favorable price. Being aware of its own influence on the market price, however, the bank may restrain sales in period 1 to avoid driving down the price. After all, slumping prices not only affect securities that are traded, but also reduce the market value of the remaining portion of the assets.

We refer to Appendix A (A.2) for some more detailed explanations regarding the intuition of the expressions for each of the four cases and to Appendix A (A.3) for the proof.

Note that in the above optimisation problem, the price impact is affected not only by banks i's selling strategy, but also by the strategy of the other bank. The strategic interaction created through the bank's influence on the price impact of the asset means that the problem takes the form of a 2-period game in pure strategies under complete information, in which banks choose selling strategies to minimise their losses in the market value of its portfolio and to meet liquidity outflows. Next, we show that Nash equilibria in this setup always exist so that the individually optimal selling strategies are mutually compatible and banks avoid becoming illiquid.

Theorem 2 *Under assumption 1, a Nash equilibrium exists. In other words, for each initial parameter setting* $w = (a_{1,1}, a_{2,1}, c_{1,1}, c_{2,1}, l_{1,1}, l_{1,2}, l_{2,1}, l_{2,2}, \lambda)$ *a combination of strategies* $(v_{1,1}, v_{1,2})$ *for bank 1 and* $(v_{2,1}, v_{2,2})$ *for bank 2 exists such that simultaneously the following two statements hold true:*

1. *The strategy vector* $(v_{1,1}, v_{1,2})$ *of bank 1 is an optimal solution to the optimisation problem w.r.t. the strategy* $(v_{2,1}, v_{2,2})$ *of bank 2.*

2. *The strategy vector* $(v_{2,1}, v_{2,2})$ *of bank 2 is an optimal solution to the optimisation problem w.r.t. the strategy* $(v_{1,1}, v_{1,2})$ *of bank 1.*

While the proof of Theorem 2 can be found in Appendix A (A.4) we proceed with some explanatory notes about the intuition behind the possible Nash equilibria.

Table A1 displays all permissible combinations of strategies describing different possible formats for Nash equilibria, depending on which of the four possible strategies the two involved banks pursue. First, it is remarkable that equilibria in which both banks sell assets only in period 1 (i.e. Front-Servicing or Distress-Sale) are ruled out. This may appear counterintuitive at first glance. The reason is that in our setting banks act under perfect information. For example, if bank 1 expects bank 2 to sell all its assets in period 1, then bank 1 has no chance to sell its assets before bank 2, but could only sell at the same time at best. Put differently, in any case bank 1 has to take into account the price drop caused by bank 2 in period 1, no matter whether bank 1 sells simultaneously with bank 2 in period 1 or sells later in period 2. Banks' decision-making would be different if we incorporated uncertainty into the model, i.e. if in the above mentioned scenario the sales of bank 1 still took into account the possibility that they would precede the sales of bank 2 with some positive probability. Under such an extended framework a Nash equilibrium would still be possible given that all banks follow the objective to sell their assets as early as possible in order to anticipate a price slump caused by the sale of the other bank.

Second, two opposing incentives determine banks' optimal strategy in this model. On the one hand, banks will individually strive to sell their assets as quickly as possible in order to be ahead of competing banks and secure favourable prices in accordance with the discussion above. On the other hand, they will try to divide the sale up into small portions so as not to singlehandedly accelerate the price drop. Hence, large banks will therefore tend to act more cautiously than a bank that has little influence over the market price.

A banking system in which the portfolio of assets and the liabilities of one bank is much larger relative to those of the other bank tends to reach a Nash equilibrium where the large bank chooses Just-in-time or Smoothing and the small bank chooses Front-Servicing or Distress-Sale.

If the size of the portfolio of assets and liabilities becomes more similar between the two banks, the game transforms into a symmetric game with a symmetric Nash equilibrium. In such a scenario both banks can choose either Just-in-time or Smoothing (see Table A1). Which of the two Nash equilibria will be reached depends on the maturity structure of banks' liabilities. If banks' liabilities maturing in t=1 are much larger than those maturing in t=2, then banks' liquidity constraints (see constraint (C) in Section 2.2) will more likely become binding, resulting in a Nash equilibrium where both banks choose the Just-in-time strategy.

The optimisation problem (9) models the individual perspective of a bank under the assumption that the other bank's strategy is fixed. From Theorem 2 the existence of a Nash equilibrium as a combination of the strategies of bank 1 and bank 2 has been inferred. Now, we analyse the strategies that banks would pursue if they take a collective perspective. To this end, we assume their objective is to minimise the sum of banks' market losses. In such a setting where banks decision-making takes into account the liquidity situation of the banking system as a whole indirect contagion via distress-sale prices can be contained. Later, in Section 4.2.1, we quantify more precisely by how much the simulated market value loss in the system would be reduced if the banks were to forgo strategic thinking, but coordinate their actions. More explicitly, we are interested in the optimisation problem

$$\min_{\{v_{1,1},v_{1,2},v_{2,1},v_{2,2}\}} \Big\{ -\lambda a_{1,1}(v_{1,1}+v_{2,1}) - \lambda(a_{1,1}(1+\lambda(v_{1,1}+v_{2,1})) - v_{1,1})(v_{1,2}+v_{2,2}) \\ - \lambda a_{2,1}(v_{1,1}+v_{2,1}) - \lambda(a_{2,1}(1+\lambda(v_{1,1}+v_{2,1})) - v_{2,1})(v_{1,2}+v_{2,2}) \Big\}. \tag{11}$$

Note that the objective function adds the objective function of problem (9) to the analogous function for the same optimisation problem for bank 2. For simplicity we do not include the constraints here. The same transition equations and additional constraints as in problem (9) are applied to both bank 1 and bank 2. We then obtain:

Theorem 3 *If Assumption 1 holds then the optimal combination of strategies* $(v_{1,1}, v_{1,2})$ *and* $(v_{2,1}, v_{2,2})$ *w.r.t. optimisation problem (11) is Just-in-time for both banks.*

We refer to Appendix A (A.5) for the proof.

4 Empirical analysis

In this section we assess the systemic liquidity risk in the German banking system by using the *SLB*. First, we describe the regulatory and market data that we analyze and provide computational details in Section 4.1. Next, we motivate four policy issues and present the empirical results in Section 4.2. This section also addresses the development of the *SLB* and *SLS* after the outbreak of the COVID-19 pandemic in spring 2020. Our respective analyses use data available up to June 2020.

4.1 Data and Computation

Net outflows, liquid assets and cash

Data on net outflows, the initial stock of liquid assets are obtained from the Common Reporting framework (COREP). This standardised reporting framework includes two different data sets: regulatory reports on (1) Additional Monitoring Metrics for Liquidity (AMM) and (2) the LCR. The two types of reports differ in the granularity level of the maturity buckets of the net outflows. The regulatory reports on the AMM are the main source for our empirical analysis, as three applications (see Section 4.2.1, Section 4.2.4 and Section 4.2.5) rely on that data set. Data from regulatory reports on the LCR are used to compare the SLB with microprudential measures (see Section 4.2.2).

Both data sets are reported by German banks on a monthly basis and are both available at the solo and consolidated level.[16] Generally, we use consolidated data as we believe that looking at the entire banking group gives the necessary comprehensive perspective on the liquidity position.[17] This is especially true for larger banks as their intra-group liquidity management practices might involve several banks spread across different countries.[18] For banks that are not part of a larger group we use reporting information at the solo level.[19] Furthermore, we take the special properties of the banking associations'

[16]Regulatory data from the AMM are available on a monthly basis for large banks and on a quarterly basis for smaller banks.

[17]The implicit assumption is that in a stress period liquidity can quickly be transferred within the same banking group. While this might be true for the savings banks and cooperative banks, where the respective central institutions exercise intra-group cash management activities and the group units are domiciled in one jurisdiction, the assumption could turn out to be a strong one for large banks, which conduct world-wide business operations across several jurisdictions. See, for example, Financial Stability Board (2018) on the ongoing regulatory discussion regarding complexities associated with liquidity in resolution for global systemically important banks.

[18]During the initial phase-in transition of the reporting requirements not all banking groups reported their respective figures at the consolidated level. In those cases, we resort to data available at the solo level.

[19]Whenever banks which are part of a larger group, for which we have data at the consolidated level, additionally report figures for their respective units we omit those. Information on units that belong to larger financial groups is only available to us at the end of each calender year. This means that, whenever there is a change in the composition of such groups (i.e. certain parts are acquired by other banks/merged into other units) we implicitly assume that such a change takes place at the end of the year. While this way of handling the data issue might temporarily result in double-counting or omission of reports, we consider it to be a minor issue in practice.

liquidity management into account. As the savings banks and cooperative banks are integrated in a central cash management (e.g. cash pooling) controlled by their respective central institutions it is not reasonable to model them as independently acting players in a systemic liquidity crisis. Thus, we assume that savings bank coordinate within their respective regional associations and cooperative banks coordinate within their respective banking association so that each association would act like a single consolidated banking group when selling securities. Our sample includes 1,420 banks for June 2020 based on the regulatory reports for the AMM and 1,447 banks based on the regulatory reports for the LCR.

Both reports contain banks' liquid assets and their respective funding obligations. Banks' liquid assets are aggregated into different asset classes (e.g. central bank assets, government assets, different types of bonds, and shares). For each asset class, market values are reported which we use as initial values when modeling distress sale losses associated with the liquidation of banks' assets in times of funding squeezes.

We construct six asset classes: cash, government assets, uncovered bonds, covered bonds, shares and asset-backed securities. We assume that these classes properly reflect differences in liquidity in times of stress, and we assign different price impact ratios to each class. Further details regarding their computation can be found below.[20]

Panel A of Table 1 shows a decomposition of the balance sheet of the German banking system in June 2020. About half of total assets can be attributed to 12 systemically important German institutions. Liquid assets account for almost 19 % of the banking system's total assets. The stock of liquid assets mainly comprises central bank reserves and government bonds, while banks' holdings of corporate bonds, shares or asset-backed securities are relatively small. These observations hold for the system as a whole and

[20]Due to high computational effort and insufficient data granularity that prevents us to apply price impact ratios on an individual asset basis we have to aggregate assets and assign common price impact ratios to each asset class. The choice to aggregate assets implies that all assets within one class have a correlation equal to one. This assumption tends to overestimate the simulated distress sale losses. However, the imprecision should not be very large. First, banks' largest security class is 'government bonds' which primarily consists of only one asset, namely 'Bundesanleihen' (Bunds). Second, in times of severe market liquidity stress downward price pressure is often exerted simultaneously on different securities, i.e. correlations are usually high in a liquidity crisis.

the 12 systemically important institutions. Panel B of Table 1 presents an analogous decomposition specifically for US dollar positions (if available). Assets in US dollar are held mostly by the 12 systemically important banks. Again, about 20 % of US dollar assets qualify as liquid assets.

We determine the daily net outflows for each bank based on outflows from funding obligations net of inflows from non-fungible assets (e.g. inflows from interest income). The corresponding amounts are readily available from both data sets. For the regulatory reports on the AMM we obtain outflows according to their contractual obligations. For deposits we also take into account so-called behavioural outflows (see Section 4.2.1 for further details). The AMM data set provides a granular breakdown of banks' maturities of obligations. In particular, for the first 7 days, inflows and outflows are reported by day. To ensure that the stress scenario is sufficiently severe, we restrict daily net outflows to be floored by zero, i.e. inflows from non-fungible assets cannot overcompensate outflows from payment obligations.[21]

The regulatory reports for the LCR refer to a period of 30 calendar days for the net outflows. As no more detailed breakdown by maturity is provided, the daily net outflows for each bank are calculated based on the simplifying assumption of uniformly distributed outflows over 30 working days.[22]

Data on regulatory reports for the LCR are not available before September 2016. To provide a long-term comparison between the SLB and microprudential measures we obtain banks' net outflows and the initial stock of liquid assets from reports in accordance with the German Liquidity Directive. These reports cover all banks domiciled in Germany for the period from the end of 2000 to the end of 2017. The year-end figure for 2017 includes 1,635 banks.[23] The reports contain two tables: one reflects bank's asset liquidity

[21]In contrast, in the regulatory reports for the LCR net outflows are floored at 25% of inflows, i.e. inflows from non-fungible assets can compensate outflows from payment obligations with a maximum of 75%.

[22]We confirmed the robustness of our results by a distribution of net outflows over 30 days that is skewed to the right, where most outflows take place during the first few days of financial stress.

[23]Two larger banks have not been covered by the German Liquidity Directive since 2010 and 2014, respectively, because they have reported the liquidity position based on proprietary models.

positions and the other illustrates bank's funding obligations. The asset liquidity table contains data by product type for fungible assets (e.g. cash, bonds, and shares) and non-fungible claims (e.g. loans and receivables) as well as different residual maturity buckets ranging from on-demand to one year.

In general, fungible assets are reported with their market value. We choose six asset classes according to the prescribed instrument breakdown of the German Liquidity Directive, which is cash (equivalents), bonds, covered bonds, money market papers, equities (shares and investments) and collateral eligible for re-financing at (zero-weighted) central banks. For each product type we calculate the price impact ratios.

The table on banks' funding obligations shows the different product types (e.g. customer deposits, interbank liabilities) further broken down by residual maturity buckets which also range from on-demand to one year. The funding obligations are reported based on their nominal amounts with a specific haircut, indicating greater protection for insured liabilities, such as (retail) deposits which fall under a deposit guarantee scheme.

We determine the daily net outflows for each bank based on outflows from funding obligations net of (contractual) inflows from non-fungible assets. As the granularity of the maturity breakdown does not reflect a daily but monthly view, we take the first maturity bucket which ranges from 'on demand' to '1 month' and make the simplifying assumption that flows within one month from funding obligations and non-fungible claims are evenly distributed across 20 days. For prudential purposes, we assume again that daily net outflows are floored at zero, i.e. inflows from non-fungible assets cannot overcompensate outflows from payment obligations.[24]

Price impact ratio

A key parameter of the empirical model is the price impact ratio λ. We calculate the daily associations between the aggregated trading volume and the trading volume weighted average price decline across all securities that belong to a certain asset class k over a

[24] Alternatively, it is possible to impose a factor which further limits the weight of the non-fungible assets, say 75%, such as is done by the LCR and which we apply for the empirical analysis in Section 4.2.2.

specific observation period. Among the calculated daily associations we select the smallest value (which reflects the largest daily price impact) and assign it to the price impact ratio λ_k.[25]

In principle, the approach follows the basic concept of the Amihud-Ratio, which is defined as the average daily association between a unit of trading volume (measured in USD) and the relative price change for individual security over a certain period of time (e.g. one year) (see Amihud (2002)).

As banks' portfolios may consist of up to several thousand different securities it is not operationally feasible to simulate the price impact for each security based on security-specific price impact ratios. To keep the right balance between computational efforts and ensuring the necessary accuracy, we follow a practical approach. In order to form asset classes we assign different securities with similar characteristics (e.g. asset classes for government bonds, covered bonds, uncovered corporate bonds, equities) to supersets. The price impact ratios are then calculated for these supersets.[26]

Since our price impact ratio is a constant value in the empirical model, it is reasonable to determine λ for each asset in the most conservative way possible. Therefore, we either look at studies, which report extreme λ for financial stress periods or, as in the case of corporate bonds, covered bonds and government bonds, we calculate price impact ratios from the data of such periods. Proceeding in this way ensures that the price depreciations determined by our algorithm relative to the selling volume match those which have been observed during past liquidity stress periods.

The trading volume and price data for (corporate) bonds and covered bonds not traded on a centralised exchange are captured based on data from the TRACE reporting system with Bloomberg's TACT analysis and valuation function. The sample covers the period from June 2016 to August 2016. For government bonds (which make up the largest part of

[25] We use price impact ratios which could be observed during periods of financial turmoil and can be argued to be representative of such periods. See also the explanation referring to (5) in Section 2.2.

[26] Even if we only consider five different asset classes, this approach to model price impact ratios is more granular than the one taken by Greenwood et al. (2015).

the portfolio relevant for sale), we use MTS data[27] on daily bond prices and turnovers and calculate the average price impact ratio for Italian government bonds during the period from the beginning of May to the end of June 2012, which was just prior to the ECB president Mario Draghi's famous speech on July 26, 2012 at UKTI's Global Investment Conference over the 'irreversibility' of the euro and ECB's preparedness to do 'whatever it takes'. During this turbulent period of the European Sovereign debt crisis, spreads of 10-year Italian bonds over the corresponding German government bonds interest rate have been the largest in recent history.

Panel C of Table 1 lists the price impact ratios for each product type calculated. For (corporate) bonds the ratio amounts to -0.015 per billion USD, which lies in the lower range of other empirical estimates for this type of securities (i.e. the value used in our model is more conservative and result in a larger price drop).[28]

Empirical implementation and computation

Due to its complexity finding a closed form solution to the optimisation problem goes beyond the scope of this paper. To reach a satisfactory solution for the general optimisation problem, we develop a heuristic approach which successively determines each bank's optimal strategy conditional on the strategies of the other banks. Details are laid out in Appendix B.

4.2 Applications

We use the *SLB* to address four policy issues. First, we use the *SLB* to examine the impact of a severe funding shock on systemic liquidity over the course of 5 days. Second, we contrast the *SLB* with microprudential liquidity requirements, including the LCR. Third, we analyse systemic liquidity risk that may stem from the US dollar business of

[27]MTS is one Europe's leading electronic fixed income trading markets and a significant fraction of Italian government bonds is said to be traded via this market. Italy was one of the countries most affected by the European Sovereign debt crisis.

[28]For further details see Feldhütter (2012), Dick-Nielsen, Feldhütter, and Lando (2012) and Ellul, Jotikasthira, and Lundblad (2011).

German banks. Fourth, we study the impact of suddenly rising interest rates on systemic liquidity in the banking system.

4.2.1 How resilient is the banking system in the short run?

As Gorton and Metrick (2012) point out, the GFC was a system-wide run on the banking system. Funding risks materialised at various stages of the crisis and affected several segments of wholesale funding markets, including a dry-up of the (asset-backed) commercial paper market in August 2007 and in September 2008 (Kacperczyk and Schnabl, 2010; Brunnermeier, 2009) and a sharp increase in haircuts in the repo markets (Gorton and Metrick, 2012). Moreover, at the height of the crisis in September 2008, non-financial firms heavily relied on existing credit lines or loan commitments (Ivashina and Scharfstein, 2010; Cornett, McNutt, Strahan, and Tehranian, 2011). Some institutions even faced a decline in their retail deposits (Shin, 2009). Empirical evidence of distress sales in the market for residential mortgage-backed securities during the GFC is provided by Merrill, Nadauld, Stulz, and Sherlund (2014).

Against this background, we examine an extreme scenario, in which there is a widespread run on financial institutions. In such a scenario banks experience a combination of some of the funding difficulties illustrated above over the course of, say, five days. This time horizon mirrors some of the disruptive episodes mentioned above and allows us to keep the simulation of distress-sale spirals computationally tractable.

In this scenario, we assume that liabilities become due according to their contractual maturity. Therefore, existing wholesale debt instruments (such as repos) are not rolled over, unused credit lines are fully drawn, and some or even all wholesale and retail deposits vanish. Banks react by selling some of their assets to meet all obligations in due course. Using detailed information about banks' liquid assets as well as the outstanding amounts of banks' liabilities and their contractual maturities, we obtain the SLB to measure the resilience of the banking system to such a run, and the SLS as a measure of the aggregate liquidity need in such an extreme event. We also explore the cross-sectional distribution of liquidity shortages, to find out if systemically relevant institutions are vulnerable to liquidity risks.[29]

Finally, we investigate by how much strategic bank behaviour can further amplify downward price spirals. As outlined in the theoretical section, when banks expect the market price of their security holdings to fall, it may be rational for them to dispose them as early as possible. Banks selling securities on the basis of this strategic thinking can compound a fall in prices. This can result in much larger market value losses in the system. To quantify this effect, we simulate the market value loss in the system if the banks were to forgo this kind of strategic thinking and instead gear their sales towards covering payment outflows punctually, i.e. choose the Just-in-time strategy.

We assume that the run on the German banking system lasts five days and that banks face net liquidity outflows according to the contractual maturities of their liabilities during this time. We deviate from this approach with respect to deposits. Deposits include sight deposits held by retail investors, some of which are subject to deposit insurance and are therefore less run-prone. In their data reports, banks provide both contractual and so-called behavioural outflows for deposits. These behavioural outflows take the banks' estimates of actual business dynamics of deposit outflows into account and reflect the experience that they are a lot stickier than their contractual maturities suggest. Therefore, we consider this scenario with behavioural deposit outflows to be the baseline case, but we also examine results when the contractual deposit outflows are used instead. Panel (a) of Figure 1 shows that outflows account for more than 20% of liquid assets in this horizon of five days, and that outflows beyond five days up to 30 days would add little stress in this scenario. Note also the large difference between net outflows according to contractual maturities (blue) and net outflows adjusted to incorporate behavioural

[29] Note that the analysis has some limitations. For example, the run on the banking system does not take a reallocation of funds within the German banking system into account. For instance, in times of stress investors may shift funds that they have provided to some institutions to other institutions that are perceived as high-quality banks, see Pérignon, Thesmar, and Vuillemey (2018) for such effects in the market for certificates of deposits. Therefore, the above scenario assumes that funds leave the German banking system entirely and are shifted to other banking systems. In this view, the scenario may serve as an upper bound for funding risk materialising in the German banking system.

outflows from deposits (red). Taking these behavioural outflows into account, we obtain a scenario that may resemble a break-down of wholesale funding markets.

We analyze quarterly data on liquid assets and net outflows starting in the first quarter of 2018. At the beginning of each quarter, we take the stock of liquid assets and the net outflows for a five-day horizon as given, and compute the SLB as described in Section 4.1. Figure 2(a) shows the evolution of the SLB and the SLS until the second quarter of 2020. The SLB has fluctuated in a range of EUR 725 bn to EUR 1,200 bn, except in June 2018, in which it was negative. This negative value is likely due to data quality issues at the beginning of the sample period. In the course of 2020 the SLB rose strongly, indicating that the banking system has become more resilient to a system-wide funding shock with distress sales. Against the background of the outbreak of the COVID-19 pandemic, this result is surprising at first glance. In Section 4.2.3 we pinpoint the reason for the SLB's increase from January 2020 to June 2020.

While the SLB is an overall measure of resilience of the banking system to the funding shock, the SLS estimates the actual liquidity need that the banking system has. By design, it is non-positive, in which a SLS of zero would mean that all banks in the system can withstand the shock and thus have no liquidity need. From December 2019 to June 2020, the liquidity need almost halved to EUR 18 bn. In this view, the German banking system would need less liquidity of about EUR 18 bn as of June 2020, if the funding shock assumed here materialises for a five-day period. In addition, we report results for the SLB in Figure 2(b) when contractual deposit outflows are used. In this case, the scenario becomes even more severe and the SLB is negative for the entire sample period. This result shows that the treatment of deposit outflows can have a large impact in any liquidity analysis.

We now turn to the results for the latest quarter, June 2020, in more detail. In Panel A of Table 2, we present the SLB and the SLS for several types of institutions according to their business model. We learn that there is substantial heterogeneity in the cross section: while there is no shortfall among cooperative and savings banks, commercial banks have a shortfall of about EUR 18 bn. If we sort the banking system according to the systemic importance of institutions, we observe a shortfall of about EUR 6 bn for systemically important institutions in this scenario. Overall, the SLS is helpful in assessing the liquidity needs of the banking system and also helps finding potentially vulnerable institutions within the system.

In addition to the liquidity shortfall, we consider the loss in market value that banks experience during the funding shock. Here, we compare the stock of liquid assets valued at the end of the stress episode at possibly depressed market prices relative to the value of the liquid assets before the run on the banking system started. There is an overall loss in market value of EUR 52 bn in this period. While this loss is 3% of liquid assets, it is significant for most banks in terms of Tier 1 capital. For instance, the loss of EUR 32 bn for commercial banks corresponds to a loss of 13% in Tier 1 capital if it were written off immediately. The loss is also relevant for systemically important institutions, who suffer a loss equal to 15% of their Tier 1 capital. Hence, these results, show that our methodology also sheds some light on the consequences of distress sales for the level of capital. By taking both the liquidity shortfall and the loss of capital into account, we obtain a broader view on the impact of a funding shock on the banking system.

Furthermore, in Figure 3, we depict the evolution of the gross returns $R^k_{t,t+1}$ for $k = 1, 2, \ldots, 5$ asset classes to illustrate the dynamics of the simulated downward price spiral. Note that we adopt the pro-rata assumption in our model, so that assets sales are distributed across the whole portfolio of the banks. Therefore, we observe a price impact for all types of assets. Figure 3 shows a sharp fall in prices over the first two days, then a flattening. This is because most banks, in particularly smaller institutions, engage in distress sales, in which they sell their stock of liquid assets as early as possible. The cumulative price declines vary between 2% and 9%. Notably, government bonds suffer the largest decline, although they have the smallest (absolute) price impact ratio. Importantly, the price impact across assets classes depends on the commonality and level of banks' security holdings. As government bonds account for the bulk of banks' liquid securities (roughly two-thirds) the relatively large price drop experienced by sovereign bonds is driven by their large total selling volume. It is important to add, however, that in a crisis, government bonds may be subject to flight-to-liquidity effects as a result of increased demand by institutional investors, which could dampen the price declines shown here if these effects are not fully captured in the price impact parameter (Santis, 2014; Beber, Brandt, and Kavajecz, 2008).

Finally, we investigate by how much strategic bank behaviour can further amplify downward price spirals. Figure 4 compares the evolution of the gross return $R_{t,t+1}$ (i.e. for government bonds) when banks choose the individually optimal strategy (blue curve) with that when banks simply choose the Just-in-Time strategy (orange curve). Although both price curves have a similar pattern, it shows that when banks choose the individually optimal strategy the price falls much more steeply over the first two days and the cumulative price impact after five days is nearly two times higher. That means when banks choose the individually optimal strategy they sell securities far earlier and also in larger quantities than would be necessary for the purposes of honouring the payment outflows in a timely manner. If the banks were to forgo this kind of strategic thinking, and instead gear their sales towards covering payment outflows punctually (i.e. choose Just-in-time), the simulated price decline and market value loss in the system would be reduced by roughly one third.

4.2.2 How does the systemic liquidity buffer differ from the microprudential view on liquidity risk?

After the GFC liquidity requirements for banks were substantially revised and harmonised, resulting in the Basel III regulatory standard. In the EU, the LCR was introduced in 2015 and relates a liquidity buffer to a net liquidity outflow over a 30-calendar-day period.

The liquidity buffer of the LCR is a weighted sum of liquid assets. The weights capture the ease with which an asset can be expected to raise cash at short notice. For example, cash has a weight of 100%, while corporate bonds are assigned weights of 50% or 85%, depending on the rating of the bond. The net liquidity outflow is the difference between liquidity outflows and inflows a bank faces. Outflows are derived from the bank's liabilities. For instance, it is assumed that 5% of stable retail deposits, but 100% of deposits of other commercial banks are withdrawn in a stress period. An analogous approach applies to the inflows a bank expects, including, for example, repayments on interest and principal made by non-financial customers. Since 2018, the liquidity buffer must be at least as large as the net outflow, such that the bank has enough liquid assets to withstand the hypothetical liquidity shock.

Figure 6 shows the evolution of the aggregated LCR and its normalised components over time. Since its introduction the aggregated LCR has risen to almost 171 % in June 2021. This is reflected by an increase in the HQLA as well as the net outflows during 2020, where the former rose faster than the latter. Table 4 summarizes moments of the distribution of German banks' LCR and its components over time. One interesting observation is that for a given bank, the LCR and its component can be rather volatile and subject to non-negligible monthly changes.

Note that the weights assigned to liquid assets are fixed and therefore do not change over time. For example, most government bonds receive a weight of 100% and are thus measured at their current market price. Equivalently, they are subject to a haircut of 0% if their risk weight for credit risk is also 0% under the Basel Capital Adequacy Rules. Thus, in effect, each bank individually assumes that government bonds, say, can be sold at the current market price, but this assumption may neglect downward price pressure exerted by banks collectively. The SLB takes the effect of distress sales on market prices into account, resulting in time-varying liquidity weights over the course of the stress period. In this sense, the SLB offers a macroprudential extension to a microprudential liquidity measure.

We also investigate systemic liquidity risk over a longer period of time, including the period before the GFC in 2008. The goal of this exercise is to present a long-run view on systemic liquidity risk. Before the EU-wide harmonised framework for liquidity regulation was introduced, banks' liquidity was regulated nationally in the German Liquidity Regulation (LiqV). Conceptually similar to the LCR, an institution's liquidity was deemed sufficient if available liquid assets cover the expected outflows for the next month, which is a 20-business-day period in this case.

We begin with a long-run view on systemic liquidity from 2000 to 2018 using data from the German liquidity regulation. Every six months, we examine the impact of a liquidity stress event in which outflows materialise for each bank according to the information provided in the national regulation. Using the corresponding data on the amount of liquid assets, we then obtain the SLB in each period. In this way, we observe the evolution of systemic liquidity risk over time. In addition, we define the microprudential excess liquidity as the difference between liquid assets and net outflows according to the national regulation, and obtain the aggregate excess liquidity in the entire banking system as the sum of the individual excess liquidity. Notice that the SLB and the microprudential excess liquidity differ in terms of the valuation of liquid assets: in each period, the SLB simulates the evolution of distress sale prices over the next 20 business days, giving rise to time-varying market valuations of banks' assets. In the microprudential approach, a fixed weight is attached to each type of liquid asset which is simply 100% in most cases according to the German liquidity regulation.[30] The funding shock, represented by the net outflows for each bank, is the same in both measures.

In contrast to the short-run shock that we investigated in Section 4.2.1, we observe the net outflow over the entire 20-day period, but do not know the distribution of outflows within this period. To simplify the computation of the SLB, we assume that outflows are uniformly distributed among the days of the scenario horizon. As we have shown above, banks most likely face larger outflows at the beginning than at the end of the horizon, which may increase the need for some banks to engage in distress sales. Thus, the results shown below possibly overestimate the level of the SLB or underestimate the effect of distress sales on liquidity in the system.

[30] That means the actual market price of the liquid assets is considered as the basis of assessment.

Figure 5(a) presents the evolution of the SLB and the aggregate excess liquidity according to the national, microprudential regulation from 2000 to 2018. We make three observations.

First, the SLB is lower than the aggregate excess liquidity throughout. Hence, taking the impact of potential distress sales into account, systemic liquidity is lower than the aggregate of the individual liquidity measures. This is plausible given that the microprudential approach does not consider a reduction of the actual market price for most types of liquid assets.

Second, the SLB falls below zero in the second quarter of 2006 and reaches its low point in the second quarter of 2007, while the microprudential excess liquidity continues to rise. Therefore, ahead of the most intense period of the GFC in September 2008, the SLB indicates that the banking system as a whole is vulnerable to liquidity risk. The reason for the diverging patterns of the SLB and the microprudential excess liquidity is that the banking system increased its short-term funding on a large scale in the run-up to the crisis from June 2003 to June 2007, resulting in an increase of net outflows of nearly 65% to EUR 763 bn. Importantly, an increase in net outflows has a twofold effect on the SLB. Net outflows are directly deducted from the SLB (as for the microprudential excess liquidity) and, in addition, an increase in net outflows results in lower prices of liquid assets once banks start selling assets to raise cash, which depresses the SLB further. In this sense, the SLB has the potential to serve as an early warning indicator of systemic liquidity risk induced by excessive short-term refinancing in the banking system.

Third, in 2008, both measures of liquidity risk have decreased slightly since the GFC. German banks drastically reduced their interbank borrowing and hoarded liquidity. These effects result in substantially lower net outflows and a larger cash position, which increases both the SLB and aggregate excess liquidity.

We adopt an analogous approach to compare the SLB with the excess liquidity according to the LCR, which builds the foundation for microprudential liquidity requirements. Again, the two measures differ only in terms of the valuation of liquid assets, and we distribute LCR outflows uniformly among the 30-day horizon to obtain the SLB. We depict the evolution of the SLB and the aggregate excess liquidity since September 2016 in Figure 5(b). Both measures have been rising since the end of 2016, which is mostly due to an increase in cash or cash equivalents. As the excess liquidity and the SLB are larger than zero, both measures indicate that there is sufficient liquidity in the system to withstand the underlying funding shock. The SLB is substantially lower, however, than the excess liquidity at times.

There are two opposing features of banks' liquid assets that help understand differences and similarities between the two measures. First, government bonds are the main driver of the difference between the SLB and the aggregate excess liquidity. Government bonds account for roughly two-thirds of banks' liquid securities designated as HQLA and enter with a weight of 100% into the microprudential excess liquidity. In contrast the SLB allows for price declines in this asset class. Consequently, their weights will effectively fall below 100% in times of severe market turbulence. Second, most banks have built up large central bank funds (cash reserves) following the introduction of extraordinary monetary policy measures in the euro area. Accordingly, in June 2020, cash reserves accounted for 58% of aggregate HQLA. Cash reserves are taken into account equally in both the SLB and the LCR framework. As they form such a large part of banks' liquid assets, the need to liquidate other assets in times of stress is relatively low. Thus, a large share of cash reserves causes the level of both indicators to converge.

Like for the analysis in Section 4.2.1 we observe that the SLB rose strongly in the course of 2020. In addition, the gap between the SLB and the LCR decreased significantly from EUR 150 bn in December 2019 to EUR 85 bn in June 2020. In Section 4.2.3 we pinpoint the reason for these observations.

In Figure 5(b) we also show the graph of a model variant in which banks have access to central bank funding (green line). In this model variant we denote those HQLA-securities which are eligible for central bank borrowing as cash-equivalent liquidity for banks, though subject to a fix haircut of 5%.[31] Effectively, this leads to a decline in the securities portfolio relevant for sale and a corresponding increase in cash reserves. As the price impact of the distress sale is dampened due to central bank funding, the SLB for the model variant is higher on average as the SLB for the baseline model throughout the observation period. Notably, the SLB for the model variant is almost congruent with the microprudential excess liquidity based on data for the LCR. The technical reason is that government bonds, which is the main driver of the difference between the SLB and aggregate excess liquidity as mentioned above, enter with similar (low) fix liquidity weights into the model variant and the aggregate excess liquidity. The intuitive reason is that the central bank as a (credible) lender of last resort is able to provide abundance of liquidity, thereby avoiding strategic interactions via adverse price dynamics in distress asset sales. In other words, the endogenous response of banks and its amplifying effects on liquidity risk, as the main notion of systemic liquidity risk, is interrupted by the central bank as a lender of last resort. Under such circumstances liquidity risk follows rather the notion of exogenous liquidity risk, which is similar to the microprudential perspective.

4.2.3 What happened to banks' liquidity situation in 2020?

The time series analyses for the SLB described in Section 4.2.1 (referring to a 5-day shock scenario based on regulatory reports on AMM) and Section 4.2.2 (referring to a 30-day shock scenario based on regulatory reports on LCR) reveal that the SLB rose strongly in the course of 2020. The significant increase as illustrated by Figure 2(a) and Figure 5(b) respectively indicates that the banking system has become more resilient to a system-wide funding shock with distress sales. Against the background of the outbreak of the COVID-19 pandemic, this result is surprising at first glance. We look in more detail

[31] As our data set does not provide the information which of the securities designated as HQLA is accepted by the central bank as collateral we make the plausible assumption that within the portfolio of securities designated as HQLA all government bonds and all covered bonds are eligible for central bank borrowing. A fix haircut of 5% reflects a value in the upper range for haircuts applied to eligible marketable assets for category I, such as government bonds and covered bonds. For the remaining assets in the portfolio of securities designated as HQLA (which are uncovered bonds, shares and ABS) we make the conservative assumption that those are not accepted by the central bank as collateral.

at the development of the individual components of the SLB, i.e. net outflows and liquid assets, to pinpoint the reason for the SLB's increase from January 2020 to June 2020.

First, banks' cash levels increased markedly. This development was supported by banks' conservative liquidity management and, in particular, by an expansion of extra-ordinary monetary policy measures, such as the Targeted Longer Term Refinancing Operations (TLTRO-III) and the Pandemic Emergency Longer-Term Refinancing Operations (PELTROs). Table 3 shows banks' cash levels for December 2019 and June 2020. According to the table, the level of central bank funds have increased by almost 45% to EUR 923 bn. Its share of HQLA has increased as well. While before the outbreak of the pandemic central bank funds accounted for around 48% of HQLA, they make up more than 58% as of June 2020.

Second, net outflows increased together with the stock of HQLA at the beginning of the pandemic. However, while the latter continued to rise strongly, net outflows remained relatively stable in the course of 2020 as shown in Figure 6. Net outflows were affected by various (opposing) factors in 2020. For example, on the one hand non-retail deposits for several banks rose, which is plausible given companies typically hoard liquidity in times of uncertainty and stress. This had a positive effect on net outflows. On the other hand, many companies covered the increased liquidity requirements by drawing down existing credit lines with their house banks.[32] As some banks were reluctant to grant new credit lines this had a negative effect on net outflows.[33]

Figure 5(b) also shows that in the course of 2020 the gap between the microprudential excess liquidity (LCR) and the SLB decreased significantly. To explain this observation Table 3 bridges the LCR excess liquidity to the SLB for December 2019 and for June 2020. As mentioned above banks' cash levels increased significantly. With increased cash

[32]Due to the abrupt drop in sales following the imposed lockdown starting in March 2020 for Germany, many companies were no longer able to cover their running costs with their income. Therefore, they were forced to draw from their liquidity reserves.

[33]Especially some big banks were reluctant to grant new lines of credit. One reason could be that larger companies are more strongly represented in the loan portfolios of the large banks, which in many cases also cover their liquidity needs via the capital market. In March 2020, the number of new bonds issued rose sharply.

buffers some banks will rather first use existing cash reserves before selling securities to service the outflows. Consequently, the simulated market losses due to distress sales of securities held by banks decreased by roughly 40% between December 2019 and June 2020. As a result, the differences between the LCR excess liquidity and the SLB declined as well.

As shown in Table 3 the difference between the SLB and the LCR excess liquidity is mainly driven by the different valuation of government bonds, and covered bonds. While for government bonds and covered bonds the LCR haircut is zero or very small, their simulated distress sale loss is large. As government bonds and covered bonds account for the bulk of banks' liquid securities (roughly two-thirds and one-fourth respectively) the relatively large price drop experienced by sovereign bonds and covered bonds is driven by their large total selling volume. Notably, for uncovered bonds, shares and ABS the losses due to distress sales are lower than the LCR haircut. While the (absolut) price impact ratios for uncovered bonds, shares and ABS are relatively high, their stock of securities held by banks is relatively low. Consequently, the simulated selling volume is relatively low and thus the downward pressure on market prices. These observations regarding the haircuts reveal an interesting point regarding our model. The haircuts computed for the SLB are not necessarily more conservative than the LCR haircuts. Rather they are dynamic and reflect the characteristics of a certain crisis situation (e.g. large amounts of cash despite the COVID-19 crisis). The price drops measured in percent are higher, the bigger the absolute selling volume of assets. Even more specifically, the percentage drop in prices increases overproportionally with the amount of securities to be sold.[34]

[34] We are aware that our model does not capture the entire financial system. Rather it represents the sales of the banking system during a liquidity crisis. The price drops for covered and sovereign bonds should be quite accurate given the large holdings of the banking system in these segments. The share of government bonds held by German banks in the amount of all German government bonds outstanding has been around 15% for the past decade and recently declined to 12%. As this market share is not negligible, considerable price reactions are to be expected if many banks were to sell government bonds at the same time. The price movements determined for the other asset classes with smaller portfolios held by banks might be slightly upwardly biased. Nevertheless, since the volumes of securities that need to be sold properly reflect the funding needs of banks in a crisis scenario, our statements regarding the strategic interaction of banks and the numeric results should represent good proxies.

4.2.4 How does banks' US dollar business affect systemic liquidity risk?

Some of the largest financial institutions operate internationally, often in multiple jurisdictions and multiple currencies. Following McCauley, McGuire, and von Peter (2010), multinational banks establish a physical presence in a market abroad, and do business via branches or subsidiaries. The experience of the GFC of 2008 and the European sovereign debt crisis of 2011 has shown that funding shocks can propagate within internationally active banking groups. Cetorelli and Goldberg (2012) highlight that parent institutions of international banking groups used the internal funding market to cope with a sudden loss in wholesale funding after the Great Recession. Affiliated institutions, in turn, reduced lending to borrowers in foreign markets to local non-financial firms. Ivashina et al. (2015) and Correa et al. (2016) examine the consequences of the European sovereign debt crisis in 2011. Branches of European institutions experienced a run on their wholesale funding by money market funds in the United States in the autumn 2011. Typically these branches cannot rely on a retail funding base. Therefore this run was a severe shock to their funding model (see also Goulding and Nolle (2012)). When trying to rely on internal funding markets it turned out that losses in wholesale funding could not be fully off-set. Therefore, the institutions issued additional debt in euro, and used FX swaps to obtain US dollar funding. Increasing demand for this synthetic dollar funding led to market turbulence in the FX swap market as measured by deviations from the covered interest rate parity (Du, Tepper, and Verdelhan (2018)).

Thus, US dollar business plays an important role for internationally active banks, and funding of European institutions in this currency has tended to be vulnerable in times of market-wide distress.[35] Moreover, funding shocks can reduce lending to the real economy, which is also of concern for financial stability. Therefore, we specifically analyze systemic liquidity risk in US dollar. To this end, we re-do the analysis outlined in Section 4.2.1, but we restrict attention to the dollar assets and contractual liabilities.

[35] For an overview on the role of the US dollar and risks from US dollar funding in the international financial system after the financial crisis of 2007-2009, see Bank For International Settlements (2020).

We consider a funding shock analogous to the baseline case in Panel A of Table 2, but which is limited to liquid assets and net outflows in US dollar. Panel (b) of Figure 1 shows the net outflows in US dollars across maturity bands, as a percentage of liquid assets in US dollars. Note that in contrast to the case in which we consider all maturities, adjusted net outflows now exceed the net outflows with contractual outflows from deposits. There are two reasons for this result. First, banks do not entertain a sizeable retail business in US dollars, so that replacing contractual outflows from deposits with behavioural deposits has little effect. Second, the adjusted net outflow also incorporates contingent outflows from credit lines, which then pushes the adjusted net outflow over the net outflow without contingent outflows. This figure also highlights that banks may be vulnerable to liquidity shocks in certain currencies as in this case the system faces net outflows over a five-day period which exceed liquid assets in that currency.

The results of this exercise are presented in Panel B of Table 2. We see that the overall SLB is negative, indicating that the banking system is especially vulnerable to a funding shock with distress sales for exposures in US-dollars. The overall shortfall of USD 53 bn is mostly due to a shortfall for systemically important institutions. These institutions dominate the US dollar business conducted by German banks. Note also that the shortfall is large relative to the shortfall in the baseline case, which includes all currencies. As this shortfall is mostly concentrated on systemically important institutions, these findings may suggest that the German banking system is vulnerable to a funding shock in US dollars.

Note that a simplification had to be considered. The analysis assumes that banks only use liquid assets in US dollars to deal with the funding shock. In practice, banks can issue additional debt in euro, and then transform these funds into US dollar by using FX swaps. Similarly, they can use existing cash in Euro to buy US dollar on the spot market. Incorporating these features into the model requires additional assumptions on the nature of the EUR/USD swap market or the evolution of the EUR/USD spot rate. We leave these extensions for future work.

4.2.5 What is the impact of suddenly rising interest rates on liquidity in the banking system?

We examine the interaction between interest rate and liquidity risk. We combine the liquidity risk studied above with an exogenous interest rate shock, which shifts the yield curve upwards. In this way, we examine three channels that have an impact on the resilience of the banking system: On the liability side, banks *ceteris paribus* face a substantial increase in outflows. On the asset side, there are now two effects: First, suddenly rising interest rates lower the present value of banks' assets immediately. Most importantly from a liquidity management perspective, the market value of fixed income securities decreases. Second, as the run on the banks goes on, some banks may engage in distress sales to restore liquidity. These sales lower the market value of these assets, which in turn affects all banks in the system holding these assets. In this sense, the asset valuation channel opens up in two ways, an instantaneous repricing effect and a distress-sale effect that materialises over the course of the stress horizon.

The goal of the exercise is to investigate the impact of this combined shock on liquidity in the system as measured by the SLB. Notice that this combined shock is adopted in a pure ad-hoc fashion. We do not claim that rising interest rates may cause a run on the banking system or vice versa. Furthermore, we neither model the effect of a rise in interest rates on the value or composition of banks' liabilities, nor do we consider income-related effects on capital.

We start by describing the interest rate shock that affects the market value of the bond portfolio. The Bundesbank's Securities Holdings Statistics (SHS) lists securities at the level of individual ISIN for all German monetary financial institutions (MFI), excluding money market funds. Using these data, we obtained a sample of the stock of government bonds that German banks held in March 2018. We focus on bonds issues by the following countries: Austria, Germany, Spain, France, Greece, Great Britain, Italy, Portugal and the US. We accompany these securities with market data obtain from Thomson Reuters Datastream, including the modified duration.

In June 2020, systemically relevant institutions held government bonds with a market value of EUR 256 bn. Smaller institutions (known as less significant institutions) had a bond portfolio with a market value of EUR 197 bn. According to the modified duration, an overnight increase in interest rates of 100 basis points is associated with an average loss in market value of 8.5% and a median loss of 6%.

We view this measure of the sensitivity of banks' bond holdings to changes in interest rates as a guideline in this exercise: We examine a scenario in which a rise in interest rates results in an instantaneous loss in the market value of banks' bonds of 10%. Given the information from the sample described above, this loss is larger and more widespread, as we adopt this drop in the value of bonds to the entire portfolio, including government bonds and other types of bonds.

The initial shock of 10% corresponds to a market value loss of about EUR 70 bn. In Panel C of Table 2, we present the results for the combined interest rate and funding shocks. The total shortfall in the system increases only slightly, but the losses in market value on banks' securities increase substantially relative to the baseline funding shock, from EUR 52 bn to EUR 114 bn. Accordingly, the loss as a share in aggregate Tier 1 capital increases from 10% to 20%. Hence, in this exercise, the additional interest rate shock increases the level of market value losses, but does not change the dynamics of the funding shock significantly, as the shortfall in combined scenario (Panel C) does not change much relative to the baseline scenario (Panel A).

5 Conclusion

In this paper, we measure systemic liquidity risk by analysing banks' strategic interaction via adverse price dynamics. The model tests the resilience of the banking system to an exogenous funding shock. It gauges the joint impact of a funding liquidity shock and distress sales on financial institutions, illustrating how strategic bank behaviour can further amplify price declines. In addition, we propose two indicators termed SLB and

SLS. The first metric measures the resilience of the banking system to such a funding shock scenario with distress sales, and the latter metric measures the aggregate liquidity need in such an extreme event. Both measures are expressed in nominal terms and are therefore easy to interpret. We demonstrate the practicality of our framework with four examples: we compute the impact of a severe funding shock in the short run, compare the SLB with microprudential measures, determine the impact of a funding shock in US dollars, and finally, combine the funding shock with a jump in interest rates.

This framework is useful for policy makers in the context of macroprudential surveillance. Like the SLB, the microprudential LCR assumes a stress event where funding suddenly evaporates and banks face projected outflows over a specified period of time. However, the LCR assigns fix liquidity weights (haircuts) to securities designated as HQLA, whereas the SLB assigns distress prices that varies over time depending on system-level factors, in particular the aggregated short-term funding in the banking system. The higher the aggregated short-term funding, the lower the simulated distress prices for securities according to the model underlying the SLB. In this respect, the SLB is more sensitive than the LCR to changes in the aggregated short-term funding. It has the potential to provide early-warning of mounting vulnerabilities in the banking system caused by excessive short-term borrowing. The SLB signalled growing systemic liquidity risks ahead of the GFC 2007-08 by way of a decline in the corresponding liquidity buffers.

In addition, established microprudential indicators might be too optimistic in terms of systemic liquidity because they do not account for distress sales. For example, the LCR applies a haircut of zero to most government bonds eligible for re-financing at the central bank. While from a microprudential point of view a liquidity risk-weight of zero for these safe and liquid securities is meaningful, from a macroprudential view such an approach may underestimate systemic liquidity risk at times. In a financial crisis, the market liquidity of government bonds can deteriorate suddenly. Likewise, the eligibility of re-financing at the central bank may be restrained (as was the case for Greek government bonds during the European Sovereign debt crisis in 2012).

In this respect, our contribution is to provide an indicator that signals systemic liquidity stress in time. Finding suitable macroprudential instruments in order to deal with the identified systemic liquidity risks is beyond the scope of this paper and left for future research.[36]

When working with complex strategic interactions between multiple decision-makers, we have to make simplifying assumptions. For example, we take a narrow view of banks set of strategies. We assume that banks sell securities to maintain liquidity, but do not become buyers in these markets. As a consequence, we do not consider adverse effects stemming from predatory trading which may amplify price declines in a liquidity crisis as modelled by Brunnermeier and Pedersen (2005). We also assume that banks sell their securities in proportion to their actual holdings (pro-rata) but do not follow a pecking order. While there is empirical evidence[37] that banks tend to sell securities in such a way in a crisis event, this leaves certainly room for future research. Another area for improving the current framework is to integrate the interactions between banks and other sectors of the financial system or the real economy. Caccioli, Ferrara, and Ramadiah (2021) find that ignoring the common asset holdings between banks and the non-banks financial sector can lead to a significant underestimation of losses in distressed sales. In this vein, Deutsche Bundesbank (2020) shows that other financial intermediaries, such as insurance companies or mutual funds, often played a key role in the course of past liquidity crises. The same applies to the role of the central bank. For example, in a liquidity crisis central banks, as lenders of last resort, may provide emergency liquidity assistance. Integrating such crisis responses by the central bank in the model would allow an ex ante policy evaluation.

[36] From the policy perspective, the fundamental question that arises is how systemic liquidity risk should be adequately addressed. One dimension of it refers to the question of whether regulators should consider a (possibly time-varying) requirement to the current microprudential measures such as the LCR and/or the NSFR or whether other complementary instruments are needed (see e.g. European Systemic Risk Board (2014)). This issue is currently being discussed in different regulatory forums (see e.g. European Central Bank Task Force on Systemic Liquidity (2018)).

[37] Van den End and Tabbae (2012).

References

Adrian, T. and H. S. Shin (2010). Liquidity and leverage. *Journal of Financial Intermediation 19*(3), 418–437.

Allen, F. and D. Gale (2010). Financial fragility, liquidity, and asset prices. *Journal of the European Economic Association 2*(6), 1015–1048.

Amihud, Y. (2002). Illiquidity and stock returns: cross-section and time-series effects. *Journal of Financial Markets 5*(1), 31–56.

Bagehot, W. (1873). *Lombard Street, a Description of the Money Market.* London: Kegan Paul. Rpt., London: John Murray, 1920.

Bai, J., A. Krishnamurty, and C.-H. Weymuller (2018). Measuring liquidity mismatch in the banking sector. *Journal of Finance 73*(1), 51–93.

Bank For International Settlements (2020). US dollar funding: an international perspective. CGFS Papers, No 65, June.

Basel Committee on Banking Supervision (2010). Basel III: International framework for liquidity risk measurement, standards and monitoring. *December 2010.*

Basel Committee on Banking Supervision (2013). Basel III: The liquidity coverage ratio and liquidity risk monitoring tools. *January 2013.*

Basel Committee on Banking Supervision (2014). Basel III: The net stable funding ratio. *October 2014.*

Beber, A., M. W. Brandt, and K. A. Kavajecz (2008). Flight-to-quality or flight-to-liquidity? evidence from the euro-area bond market. *The Review of Financial Studies 22*(3), 925–957.

Brunnermeier, M., G. Gorton, and A. Krishnamurthy (2014). Liquidity mismatch measurement. In M. Brunnermeier and A. Krishnamurthy (Eds.), *Risk Topography: Systemic Risk and Macro Modeling*, Chapter 7, pp. 99–112. University of Chicago Press.

Brunnermeier, M. K. (2009). Deciphering the liquidity and credit crunch 2007-2008. *Journal of Economic Perspectives 23*(1), 77–100.

Brunnermeier, M. K., G. Gorton, and A. Krishnamurthy (2011). Risk topography. *NBER Macroeconomics Annual 26*(1), 149–176.

Brunnermeier, M. K. and L. H. Pedersen (2005, August). Predatory Trading. *Journal of Finance 60*(4), 1825–1863.

Brunnermeier, M. K. and L. H. Pedersen (2009). Market liquidity and funding liquidity. *Review of Financial Studies 22*(6), 2201–2238.

Caccioli, F., G. Ferrara, and A. Ramadiah (2021). Modelling fire sale contagion across banks and non-banks. *Bank of England Staff Working Paper*.

Cetorelli, N. and L. S. Goldberg (2012). Follow the money: Quantifying domestic effects of foreign bank shocks in the great recession. *American Economic Review: Papers & Proceedings 102*(3), 213–218.

Cifuentes, R., G. Ferrucci, and H. S. Shin (2005). Liquidity risk and contagion. *Journal of the European Economic Association 3*(2/3), 556–566.

Cornett, M. M., J. J. McNutt, P. E. Strahan, and H. Tehranian (2011). Liquidity risk management and credit supply in the financial crisis. *Journal of Financial Economics 101*, 297–312.

Correa, R., H. Sapriza, and A. Zlate (2016). Liquidity shocks, dollar funding costs, and the bank lending channel during the European sovereign debt crisis. Federal Reserve Bank of Boston, Working Paper, September.

Deutsche Bundesbank (2020). Financial Stability Review. October.

Dick-Nielsen, J., P. Feldhütter, and D. Lando (2012). Corporate bond liquidity before and after the onset of the subprime crisis. *The Journal of Financial Stability 103*, 471–492.

Du, W., A. Tepper, and A. Verdelhan (2018). Deviations from covered interest parity. *Journal of Finance 73*(3), 915–957.

ECB (November 2017). Financial stability review. *Financial Stability Review*.

Ellul, A., C. Jotikasthira, and C. Lundblad (2011). Regulatory pressure and fire sales in the corporate bond market. *Journal of Financial Economics 101*, 596–620.

English, W. B., S. J. van den Heuvel, and E. Zakrajsek (2018). Interest rate risk and bank equity valuations. *Journal of Monetary Economics 98*, 80–97.

European Central Bank Task Force on Systemic Liquidity (2018). Systemic liquidity concept, measurement and macroprudential instruments. *Occasional Paper Series 214/October 2018*.

European Systemic Risk Board (2014). The ESRB handbook on operationalising macroprudential policy in the banking sector.

Feldhütter, P. (2012). The same bond at different prices: identifying search frictions and selling pressures. *Review of Financial Studies 25*, 1155–1206.

Financial Stability Board (2018). Funding strategy elements of an implementable resolution plan. *FSB Policy Document*.

Gatev, E., T. Schuermann, and P. E. Strahan (2007). How do banks manage liquidity risk? evidenc from the equity and deposit markets in the fall of 1998. In M. Carey and R. M. Stulz (Eds.), *The risks of financial institutions*, Chapter 3, pp. 105–132. University of Chicago Press.

Geanakoplos, J. (2009). The leverage cycle. *NBER Macroeconomics Annual 24*, 1–66.

Gorton, G. and A. Metrick (2012). Securitized banking and the run on repo. *Journal of Financial Economics 104*, 425–451.

Goulding, W. and D. E. Nolle (2012). Foreign banks in the U.S.: A primer. Board of Governors of the Federal Reserve System, International Finance Discussion Papers, Number 1064r.

Greenwood, R., A. Landier, and D. Thesmar (2015). Vulnerable banks. *Journal of Financial Economics 115*, 471–485.

Grossmann, C. and J. Terno (1993). *Numerik der Optimierung*. Teubner Studienbücher Mathematik.

ICMA (March 2021). European repo market survey. *European Repo Market Survey Number 40.*

International Monetary Fund (2011). How to address the systemic part of liquidity risk. *Global Financial Stability Report, Chapter 2*.

Ivashina, V. and D. Scharfstein (2010). Bank lending during the financial crisis of 2008. *Journal of Financial Economics 97*, 319–338.

Ivashina, V., D. Scharfstein, and J. Stein (2015). Dollar funding and the lending behavior of global banks. *Quarterly Journal of Economics 130*(3), 1241–1281.

Kacperczyk, M. and P. Schnabl (2010). When safe proved risky: Commercial paper during the financial crisis of 2007-2009. *Journal of Economic Perspectives 24*(1), 29–50.

Krishnamurty, A. (2010). Amplification mechanisms in liquidity crises. *American Economic Journal: Macroeconomics 2*, 1–30.

McCauley, R. P., P. McGuire, and G. von Peter (2010). The architecture of global banking: From international to multinational? BIS Quarterly Review, May.

Merrill, C. B., T. Nadauld, R. M. Stulz, and S. M. Sherlund (2014). Were there fire sales in the RMBS market? Dice Center Working Paper 2014-09.

Morris, S. and H. S. Shin (2004). Liquidity black holes. *Review of Finance 8*, 1–18.

Pérignon, C., D. Thesmar, and G. Vuillemey (2018). Wholesale funding dry-ups. *Journal of Financial Economics 73*(2), 575–617.

Podlich, N., I. Schnabel, and J. Tischer (2017). Banks' trading after the Lehmann crisis - The role of unconventional monetary policy. *Deutsche Bundesbank Discussion Paper No 19/2017.*

Santis, R. A. D. (2014). The euro area sovereign debt crisis: Identifying flight-to-liquidity and the spillover mechanisms. *Journal of Empirical Finance 26*, 150–170.

Schmidt, K., F. Noth, and L. Tonzer (2021). A note of caution on quantifying banks' recapitalization effects. *Deutsche Bundesbank Discussion Paper No 02/2021.*

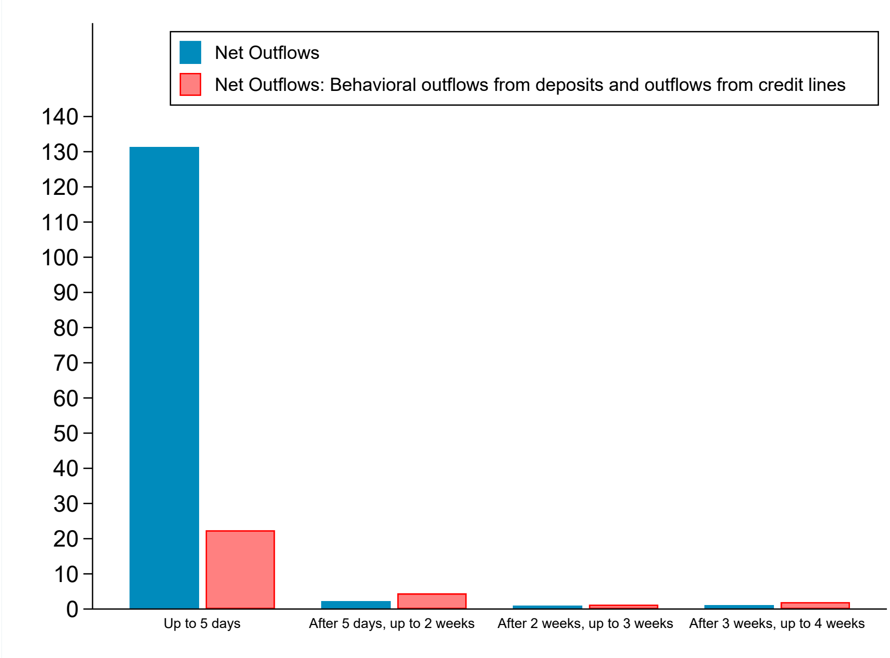
Shin, H. S. (2009). Reflections on Northern Rock: The bank run that heralded the global financial crisis. *Journal of Economic Perspectives 23*(1), 101–119.

Shin, H. S. (2010). *Risk and Liquidity.* Oxford University Press.

Thakor, A. V. (2015). The financial crisis of 2007–2009: Why did it happen and what did we learn? *The Review of Corporate Finance Studies 4*(2), 155–205.

Van den End, J. W. and M. Tabbae (2012). When liquidity risk becomes a systemic issue: Empirical evidence of bank behaviour. *The Journal of Financial Stability 8*, 107–120.

(a) All currencies



(b) US dollar

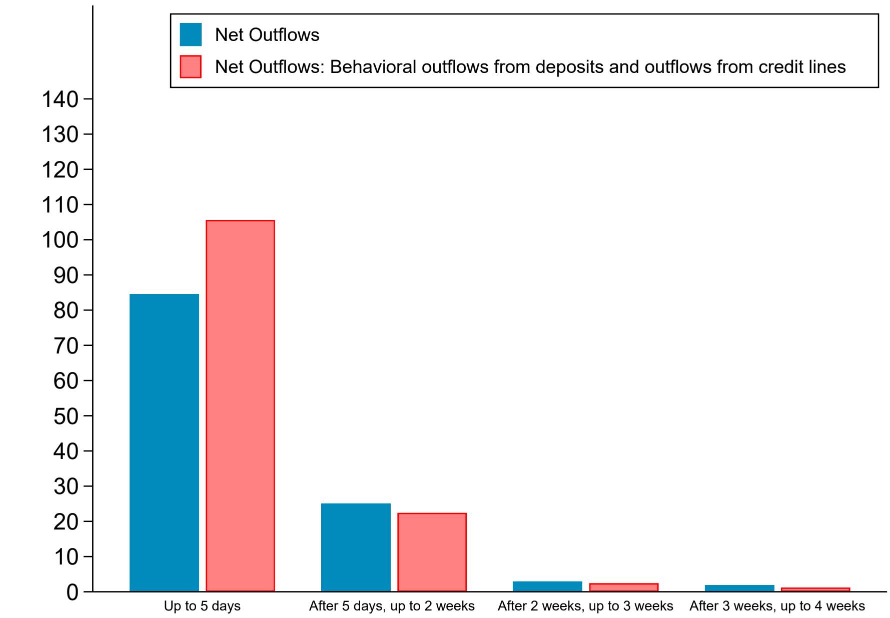
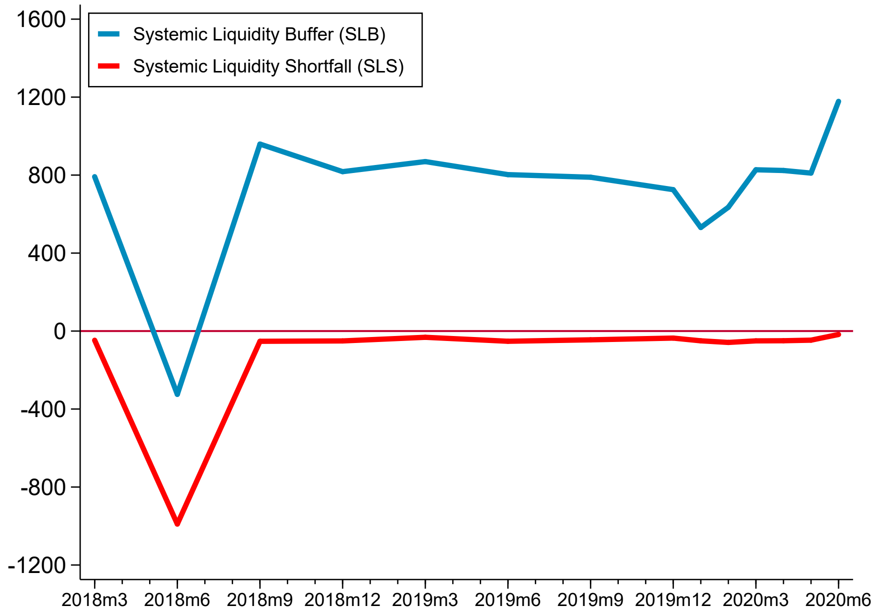


Figure 1:
Net liquidity outflows across maturities (as a percentage of liquid assets, June 2020)
This figure shows the aggregate net outflow (liquidity outflow - liquidity inflow) of the German banking system in June 2020, as a percentage of liquid assets (blue). The net outflow is sorted into four maturity buckets, spanning 30 calendar days in total. The net outflow is based on contractual maturities, and similarly to the liquidity coverage ratio (LCR), net outflows are restricted to be non-negative, so that a net inflow is not allowed. In addition, the figure displays the net outflow when contractual outflows from deposits are replaced by behavioural outflows from deposits (red). These behavioural outflows are reported by banks. Furthermore, this adjusted net outflow adds contingent outflows from committed credit and liquidity facilities, denoted as credit lines. Panel (a) shows overall positions (all currencies), while Panel (b) reports positions only in US dollars.

(a) The Systemic Liquidity Buffer and the Systemic Liquidity Shortfall



(b) The Systemic Liquidity Buffer: behavioural vs. contractual deposit outflows

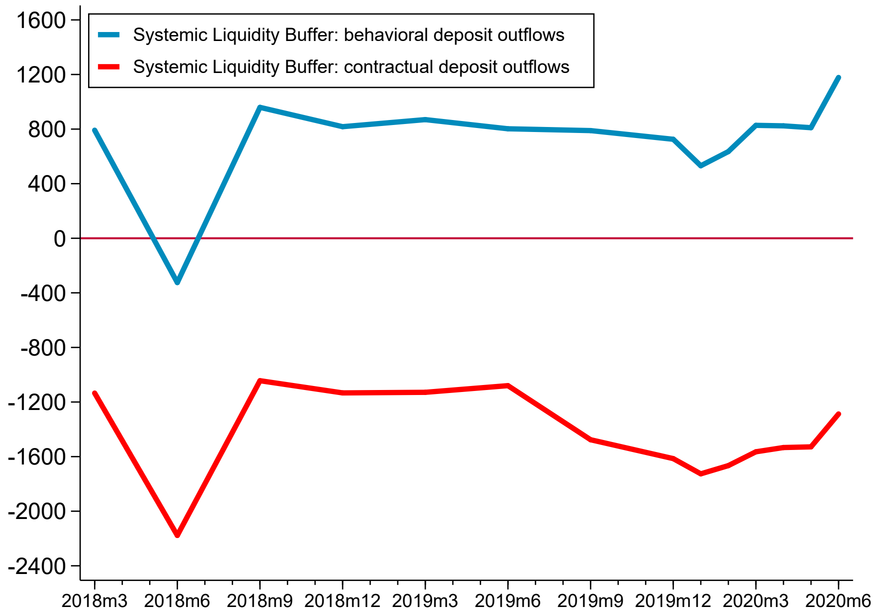


Figure 2:
The Systemic Liquidity Buffer and the Systemic Liquidity Shortfall for the German banking system (in EUR bn)
This figure shows the *SLB* and the *SLS* over time, assuming a five-day run on the German banking system at the beginning of each quarter (March 2018 - December 2019) or month (January 2020 - June 2020). In Figure 2(b), we depict the *SLB* for two cases: in the baseline case, we consider behavioural deposit outflows (blue), while in a more severe scenario, we adopt contractual deposit outflows. For all other types of liabilities besides deposits, contractual outflows are examined in both cases.

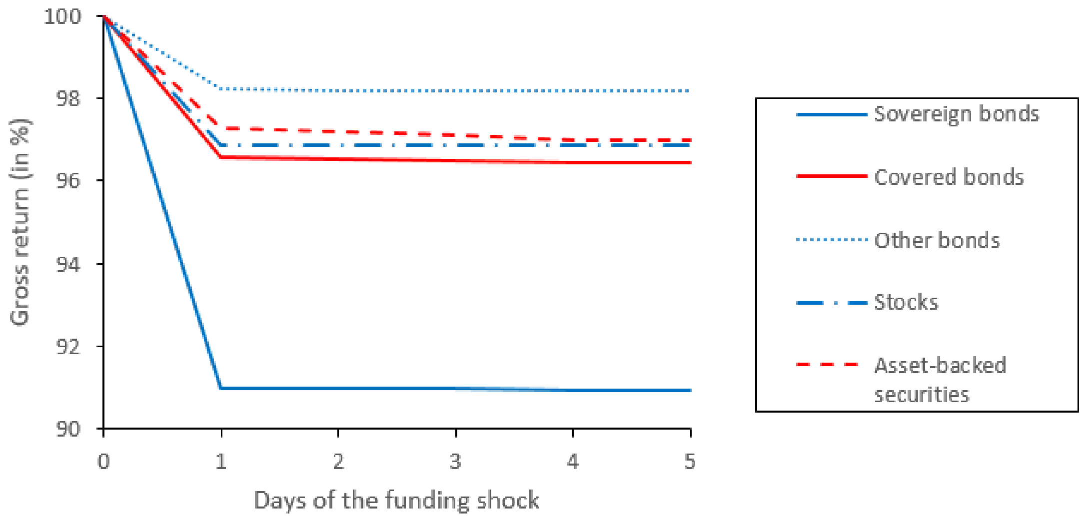


Figure 3:
The gross returns for liquid assets according to the Systemic Liquidity Buffer (SLB)
This figure shows the gross returns $R_{t,t+1}$ attached to several types of assets classes over the course of the scenario horizon for the funding shock in June 2020. See also Section 2.2 for a definition of the gross return.

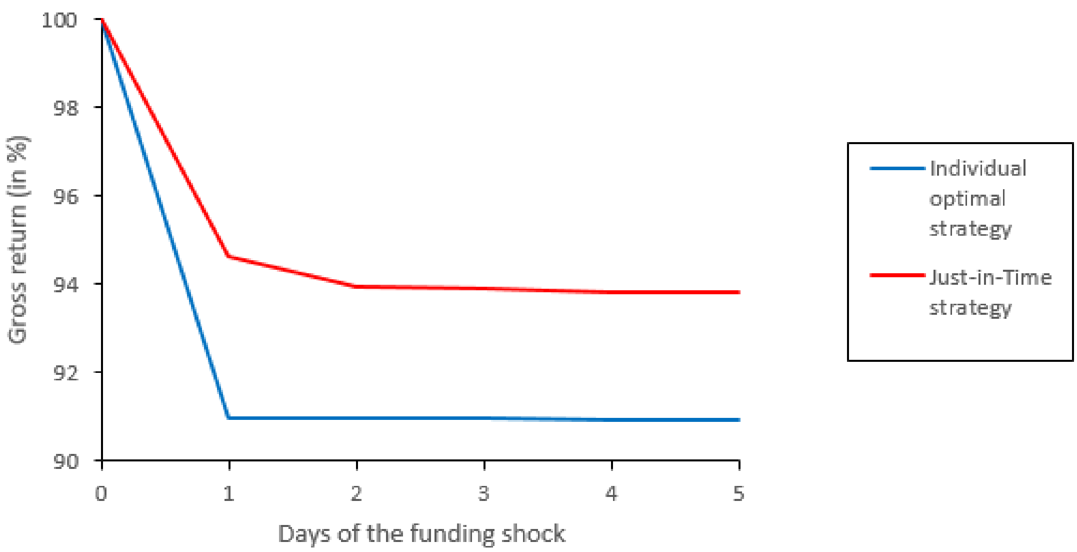
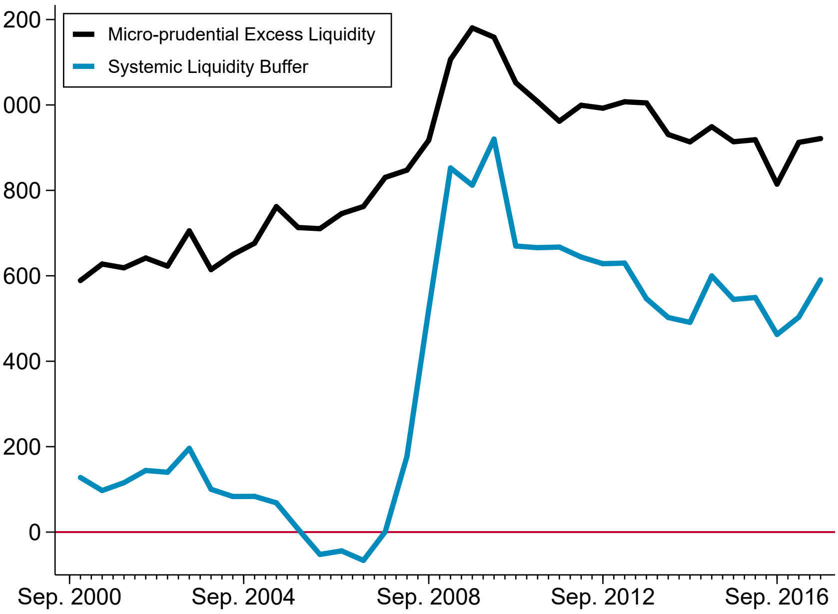


Figure 4:
The gross returns for government bonds
This figure shows the gross returns $R_{t,t+1}$ attached to government bonds for different selling strategies over the course of the scenario horizon for the funding shock in June 2020.

(a) The Systemic Liquidity Buffer and the microprudential excess liquidity according to a national liquidity measure



(b) The Systemic Liquidity Buffer and the microprudential excess liquidity according to the Liquidity Coverage Ratio (LCR)

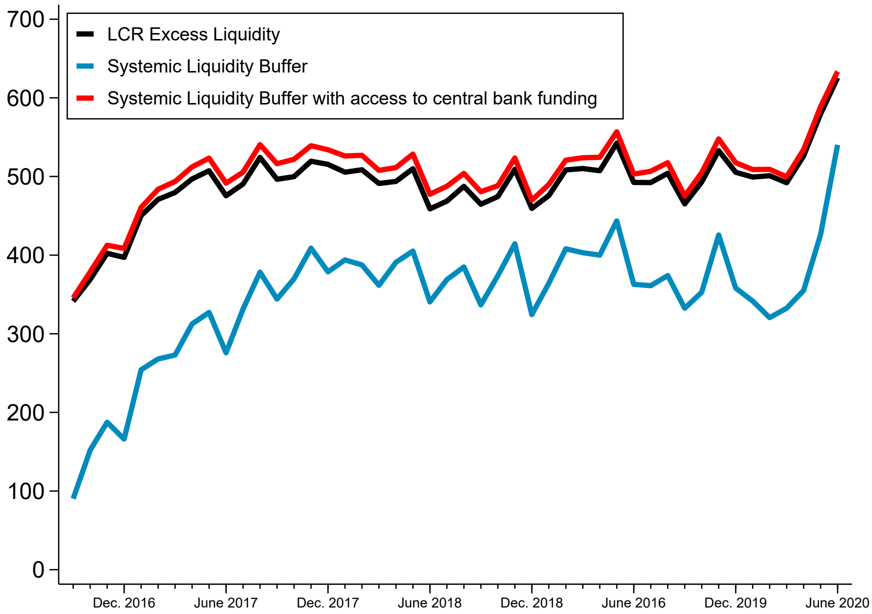


Figure 5:
The Systemic Liquidity Buffer (SLB) over time (in EUR bn)
This figure shows the SLB over time as discussed in section 4.2 from 2000 to 2019. In addition, we depict the aggregate excess liquidity (liquid assets - net outflows) derived from microprudential requirements. The excess liquidity is derived from a German liquidity measure from 2000 to 2018 in Figure 5(a), and from the Liquidity Coverage Ratio (LCR) in Figure 5(b). In either case, the net outflows underlying the SLB and the excess liquidity coincide, but the two approaches differ in the valuation of liquid assets, as the SLB takes distress sales into account.

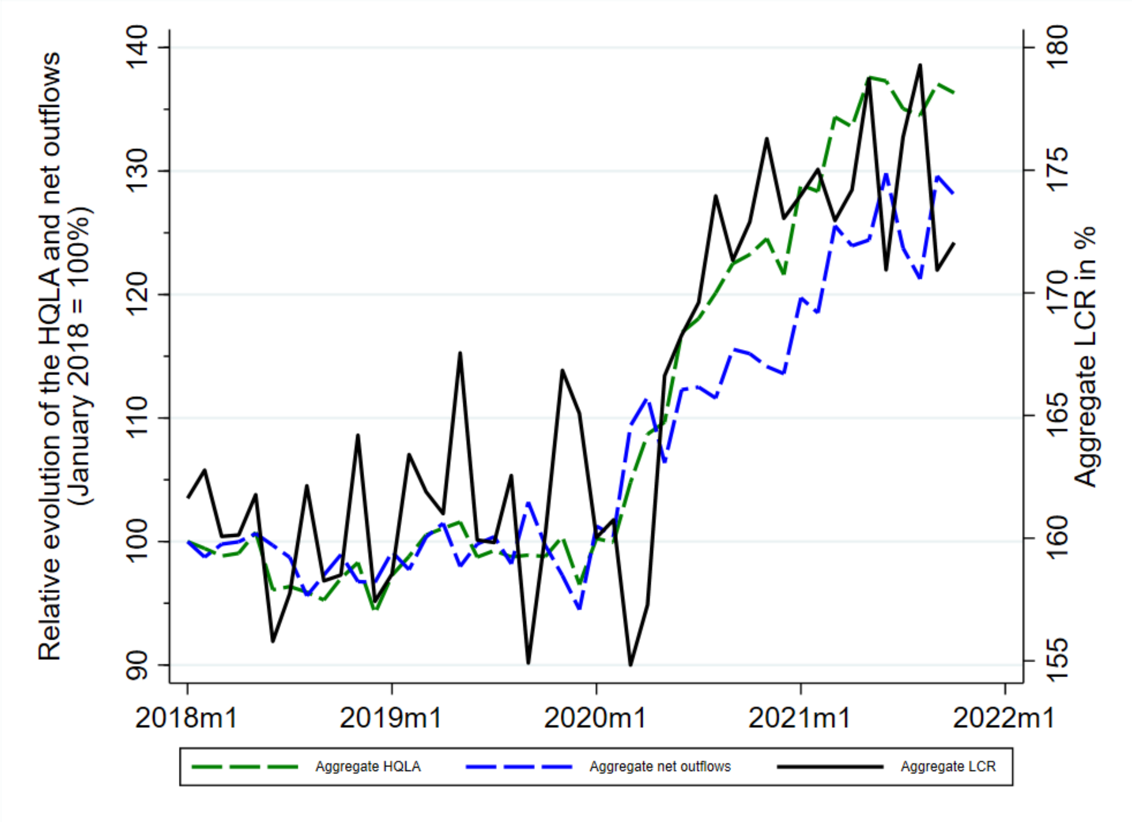


Figure 6:
Evolution of the LCR, the HQLA and the net outflows
This figure shows the aggregated LCR, and the evolution of the aggregated normalised HQLA and net outflows (January 2018 = 100%) for German banks at the consolidated level.

Table 1:
Liquid assets of financial institutions in Germany, June 2020
This table presents a break-down of the liquid assets of financial institutions in Germany. In Panel A, the overall balance sheet items are shown, which include all currencies. These overall positions are reported by financial institutions in EUR. In Panel B, an analogous break-down is shown specifically for items in US dollars. Risk-weighted assets and common equity tier 1 capital, however, are not separately available in US dollars. Data are obtained from the Common Reporting framework (COREP). This standardised reporting framework includes the Additional Monitoring Metrics for Liquidity (AMM), which is the main source of data in this analysis. In the table, liquid assets correspond to the so-called counterbalancing capacity in the AMM framework.

Panel A: All currencies	**All banks** (1,400 institutions)		**Systemically important banks** (12 institutions)	
	Total (EUR bn)	Share (in %)	Total (EUR bn)	Share (in %)
Total assets	8,121.1	100.0	4,239.0	100.0
Risk-weighted assets	3,056.2	37.6	1,163.1	27.4
Common equity tier 1 capital	493.4	6.1	176.1	4.2
Liquid assets	1529.2	18.8	841.9	19.9
of which:				
Cash and central bank reserves	808.6	10.0	433.5	10.2
Government and public sector bonds	501.4	6.2	310.2	7.3
Covered bonds	143.3	1.8	70.5	1.7
Corporate bonds	33.9	0.4	5.2	0.1
Shares	19.2	0.2	15.7	0.4
Asset-backed securities	22.9	0.3	6.8	0.2
of which:				
High-quality liquid assets (HQLA)	1482.0	18.2	818.8	19.3
Other assets	358.9	4.4	116.3	2.7
of which:				
Other tradable assets (non-HQLA)	304.7	3.8	89.7	2.1
Non-tradable assets eligible for central banks	34.5	0.4	21.0	0.5
Undrawn, irrevocable facilities	19.7	0.2	5.6	0.1

Continued on next page

Table 1 – *Continued from previous page*

Panel B: US dollar	**All banks** (61 institutions)		**Systemically important banks** (12 institutions)	
	Total (USD bn)	Share (in %)	Total (USD bn)	Share (in %)
Total assets	721.7	100.0	623.4	100.0
Risk-weighted assets	–	–	–	–
Common equity tier 1 capital	–	–	–	–
Liquid assets	148.3	20.5	132.5	21.3
of which:				
Cash and central bank reserves	65.4	9.1	65.3	10.5
Government and public sector bonds	71.3	9.9	56.7	9.1
Covered bonds	4.2	0.6	3.7	0.6
Corporate bonds	1.5	0.2	1.3	0.2
Shares	5.3	0.7	5.0	0.8
Asset-backed securities	0.6	0.1	0.5	0.1
of which:				
High-quality liquid assets (HQLA)	146.5	20.3	131.3	21.1
Other assets	43.2	6.0	34.2	5.5
of which:				
Other tradable assets (non-HQLA)	39.1	5.4	31.3	5.0
Non-tradeable assets eligible for central banks	3.0	0.4	2.9	0.5
Undrawn, irrevocable facilities	1.1	0.2	0.0	0.0

Panel C: Mapping price impacts to liquid assets	Price impact λ (in % per billion USD or EUR)
Government and Public Sector bonds	-0.1
Covered bonds	-0.3
Corporate bonds	-1.5
Asset-backed securities	-1.5
Shares	-1.7

Table 2:
Systemic liquidity risk in the cross section
This table presents the Systemic Liquidity Buffer (SLB) and the Shortfall (SLS) for several banking groups in column (1) and column (2), respectively. In Panel A, we present the results of the baseline funding shock for a 5-day period. Here, net outflows (outflows - inflows) materialise according to their contractual maturity, except for deposits, for which behavioural maturities are used. This baseline shock incorporates liquid assets and net outflows in all currencies. In Panel B, we consider a funding shock similar to the shock underlying the results shown in Panel A, but we restrict attention to liquid assets and net outflows denominated in US dollars. In all cases, the net outflows and liquid assets are based on supervisory data (Additional Monitoring Metrics for Liquidity) as of June 2020. In Panel C, we combine the funding shock in Panel A with an instantaneous repricing of banks' bond portfolio, in which all bonds lose 10% of their market value at the beginning of the stress horizon. For details on the computation of the SLB and the SLS, see Section 4.1. The loss in market value in column (3) is the decline in the value of the portfolio of assets when they are evaluated at the market prices at the end of the scenario horizon relative to the market value of the portfolio before the stress event. This loss is expressed as a percentage of aggregate liquid assets in column (4) and as a percentage of aggregate Equity Tier 1 capital assuming fair-value accounting in column (5).

	(1)	(2)	(3)	(4)	(5)
	Liquidity Risk		**Asset valuation**		
	SLB (EUR bn)	SLS (EUR bn)	Loss in market value (EUR bn)	in % of liquid assets	in % of Tier 1 capital
Panel A: Baseline funding shock					
Commercial banks	724	-18	32	3	13
Savings banks	264	0	13	3	8
Cooperative Banks	190	0	7	3	6
Total	1.178	-18	52	3	10
of which: Systemically important banks	790	-6	41	4	15
Panel B: US-Dollar funding shock					
Total	-13	-53	5	6	1
of which: Systemically important institutions	-19	-44	4	6	1
Panel C: Combined interest rate and funding shock					
Total	1.107	-22	114	7	20
of which: Systemically important institutions	742	-10	74	7	20

Table 3:
Bridge between microprudential excess liquidity (LCR) and SLB

This table bridges the microprudential excess liquidity and the SLB before and after the outbreak of the COVID-19 pandemic (i.e. December 2019 and June 2020) to illustrate the evolution of the differences between the two liquidity indicators. Data on liquid assets and net outflows are extracted from regulatory reports on the LCR.

			Jun 2020	Dec 2019
	Amounts in EUR bn		**Jun 2020**	**Dec 2019**
I	**HQLA at current market prices**		**1,582**	**1,322**
	thereof	Cash	923	639
		Government bonds	455	452
		Covered bonds	126	153
		Uncovered bonds	62	52
		Shares	11	17
		ABS	5	8
II	**LCR haircut deduction from HQLA**		**37**	**42**
	thereof	Cash	0	0
		Government bonds	0	0
		Covered bonds	11	13
		Uncovered bonds	19	18
		Shares	6	9
		ABS	1	2
III	**Net Outflows**		**920**	**774**
I-II-III = IV	**LCR excess liquidity**		**626**	**505**
V	**Delta between distress sale losses and LCR haircut**		**85**	**150**
	thereof	Cash	0	0
		Government bonds	94	136
		Covered bonds	4	26
		Uncovered bonds	- 7	-5
		Shares	- 5	-5
		ABS	- 1	-2
IV-V=VI	**SLB**		**540**	**355**

Table 4:
Liquidity Coverage Ratio in the cross section and over time
This table presents the distribution of the Liquidity Coverage Ratio (LCR), the HQLA and the net outflows for banks over a time span between January 2018 (full phase-in date of the LCR) and June 2021. Due to extreme outliers the sample has been winsorised at the 99%-percentile.

Liquidity Coverage Ratio	**obs**	**p10**	**p25**	**p50**	**p75**	**p90**	**mean**	**sd**
LCR (in %)	61,440	131	147	177	236	360	228	169
HQLA (in EUR mn)	61,440	9	23	89	265	822	974	7.734
of which:								
Cash and central bank reserves	61,440	2	5	20	107	362	572	5.435
Government and public sector bonds	61,440	1	6	22	78	256	289	2.064
Covered bonds	61,440	0	0	6	35	109	87	558
Corporate bonds	61,440	0	0	3	17	44	34	360
Shares	61,440	0	0	0	0	0	14	312
Asset-backed securities	61,440	0	0	0	0	0	4	99
Net outflows (in EUR mn)	61,440	4	12	45	139	431	592	5.271

A Proofs

A.1 Two versions of the problem of minimising distress sale losses and proof of their equivalence

Despite the fact that the optimisation problem (8) in Section 2 is presented using a standard format for nonlinear dynamic control problems it turned out to be difficult to tackle analytically by using standard tools from Lagrangean optimisation. The difficulties arise from the arbitrary length of the time horizon which comprises T periods and also from the undefined number n of banks. A further reason is that assets are considered before price adjustment in each period.

We therefore present the optimisation problem for the special case of two banks ($N = 2$) two periods ($T = 2$). As a further simplification we assume that the two banks only hold one asset class besides cash, i.e. we assume $K = 1$. By using (7) and by applying the transition equations for the assets $a_{i,t+1} = (1 - \omega_{i,t})\, a_{i,t} R_{t,t+1}(S_t)$, we obtain the objective function for bank 1

$$\sum_{t=1}^{2} a_{1,t} \left(1 - R_{t,t+1}(S_t)\right) = -\frac{\lambda a_{1,1} S_1}{1 - \lambda S_1} - \frac{(1 - \omega_{1,1})\, a_{1,1}}{1 - \lambda S_1} \cdot \frac{\lambda W(\omega)}{1 - \lambda S_1 - \lambda W(\omega)}$$

in which $S_1 = \omega_{1,1} a_{1,1} + \omega_{2,1} a_{2,1}$ and

$$W(\omega) := a_{1,1}(1 - \omega_{1,1})\omega_{1,2} + a_{2,1}(1 - \omega_{2,1})\omega_{2,2}.$$

While S_1 describes the total amount of assets which are sold by both bank 1 and bank 2 at time 1, the variable $W(\omega)$ describes the total sales in both periods 1 and 2. However, both amounts abstract from price adjustments, i.e. would describe the sales volumes under the assumption that price adjustments are 0%. Note, that the price adjustments captured in the objective function can be split into components. The first summand captures the price reduction in period 1. The subsequent price reduction in period 2 is captured by the product in the second summand. The volume of assets sold in period 2 is expressed as a share of the remaining assets after sales in period 1.

The optimisation problem for bank 1 is then

$$\min_{\{\omega_{1,1},\omega_{1,2}\}} -\left(\frac{\lambda a_{1,1}S_1}{1-\lambda S_1}+\frac{a_{1,1}\left(1-\omega_{1,1}\right)}{1-\lambda S_1}\frac{\lambda W\left(\omega\right)}{1-\lambda S_1-\lambda W\left(\omega\right)}\right) \tag{A1}$$

$$\begin{aligned}
&\text{subject to}\\
&-c_{1,1}-\frac{a_{1,1}\omega_{1,1}}{1-\lambda S_1}+l_{1,1}\leq 0,\\
&-c_{1,1}-\frac{a_{1,1}\omega_{1,1}}{1-\lambda S_1}-\frac{a_{1,1}(1-\omega_{1,1})\omega_{1,2}}{1-\lambda S_1-\lambda W\left(\omega\right)}+l_{1,1}+l_{1,2}\leq 0,\\
&-\omega_{1,1},-\omega_{1,2}\leq 0,\\
&\omega_{1,1}-1,\omega_{1,2}-1\leq 0.
\end{aligned}$$

Now let the sales volume of bank i be $v_{i,t}=\omega_{i,t}a_{i,t}R_{t,t+1}\left(S_t\right)$. We can optimise $v_{1,t}$ instead of the proportion of the portfolio that is sold. To this end, consider the optimisation problem in terms of sales volumes,

$$\begin{aligned}
\min_{\{v_{1,1},v_{1,2}\}} &\sum_{t=1}^{2}-\lambda a_{1,t}(v_{1,t}+v_{2,t})\\
&\text{such that, for } t=1,2,\\
&c_{1,t+1}=c_{1,t}+v_{1,t}-l_{1,t},\\
&a_{1,t+1}=a_{1,t}(1+\lambda(v_{1,t}+v_{2,t}))-v_{1,t},\\
&c_{1,t+1}\geq 0,\\
&v_{1,t}\geq 0,\\
&v_{1,t}\leq\left(\frac{a_{1,t}}{1-\lambda a_{1,t}}\right)\left(1+\lambda v_{2,t}\right),
\end{aligned}$$

in which we use $\lambda=\left(R_{t,t+1}\left(S_t\right)-1\right)/\left(v_{1,t}+v_{2,t}\right)$. Note that $\lambda<0$ by assumption 1 b), and therefore the denominators on the right-hand side in the last restrictions are well-defined.

By substituting $a_{1,2}$ and by rearranging the constraints, we can formulate the optimisation problem as

$$\min_{(v_{1,1},v_{1,2})} -\lambda a_{1,1}(v_{1,1}+v_{2,1}) - \lambda(a_{1,1}(1+\lambda(v_{1,1}+v_{2,1})) - v_{1,1})(v_{1,2}+v_{2,2}), \tag{A2}$$

$$\begin{aligned}
&\text{subject to}\\
&-v_{1,1}, -v_{1,2} \leq 0\\
&-v_{1,1} - c_{1,1} + l_{1,1} \leq 0\\
&-v_{1,1} - v_{1,2} - c_{1,1} + l_{1,1} + l_{1,2} \leq 0\\
&v_{1,1}\,(1-\lambda a_{1,1}) - a_{1,1}\,(1+\lambda v_{2,1}) \leq 0\\
&v_{1,2}\,(1-\lambda a_{1,2}) - a_{1,2}\,(1+\lambda v_{2,2}) \leq 0
\end{aligned}$$

The problems (A1) and (A2) are equivalent in the following sense.

Lemma 1 *Define the mapping*

$$\begin{aligned}
\phi: \quad [0,1]^4 \quad &\longrightarrow \quad \mathbb{R}^4_{\geq 0}\\
(\omega_{11}, \omega_{12}, \omega_{21}, \omega_{22}) &\longrightarrow (v_{11}, v_{12}, v_{21}, v_{22})
\end{aligned}$$

by

$$\begin{aligned}
v_{1,1} &= \frac{a_{1,1}\omega_{1,1}}{1-\lambda S_1}\\
v_{1,2} &= \frac{a_{1,1}(1-\omega_{1,1})\omega_{1,2}}{1-\lambda(S_1+W(\omega))}
\end{aligned}$$

in which S_1 and $W(\omega)$ are defined as introduced above and analogous assignments are made w.r.t. to $v_{2,1}$, $v_{2,2}$. Then ϕ is a bijective mapping of the set of feasible solutions for the optimisation problem (A1) and its alternative form (A2). Moreover, the mapping transforms the minimiser of (A1) into the minimiser of (A2).

Proof. The variables $v_{1,1}, v_{1,2}$ describe the amount of assets after price adjustments in the respective period. In contrast, the variables $\omega_{1,1}, \omega_{1,2}$ are shares of assets to be sold where assets are measured before price adjustments. This interpretation of the strategy vectors of the two problems gives the following system of four equations:

$$
\begin{aligned}
v_{1,1} &= a_{1,1}(\lambda(v_{1,1}+v_{2,1})+1)\omega_{1,1}\\
v_{2,1} &= a_{2,1}(\lambda(v_{1,1}+v_{2,1})+1)\omega_{2,1}\\
v_{1,2} &= a_{1,1}((\lambda(v_{1,1}+v_{2,1})+1)-v_{1,1})(\lambda(v_{1,2}+v_{2,2})+1)\omega_{1,2}\\
v_{2,2} &= a_{2,1}((\lambda(v_{1,1}+v_{2,1})+1)-v_{2,1})(\lambda(v_{1,2}+v_{2,2})+1)\omega_{2,2}
\end{aligned}
$$

The mapping ϕ is obtained by rearranging the update equation for $a_{i,t+1}$ and plugging the resulting expression for $v_{i,1}$ into the equation for $v_{i,2}$, $i = 1, 2$. On the other hand we can rearrange the upper equations to get the mapping $\psi : \mathbb{R}^4_{\geq 0} \to [0,1]^4,\ v \mapsto \omega$ by dividing by all factors of the right-hand side except $\omega_{i,j}$. Evaluating $\phi \circ \psi$ and $\psi \circ \phi$, we observe that ψ is in fact the inverse function of ϕ. Furthermore, ϕ directly transforms the optimisation problem (A1) and all constraints into the form (A2) and the corresponding constraints. Hence, ϕ is bijective on its domain and we can consider minima of the transformed optimisation problem (A2), i.e. in terms of sales volumes instead of shares of each banks assets. ■

A.2 Further explanations on the four selling strategies in Theorem 1

The goal of this subsection it to develop some intuition regarding the four possible strategies $v_{1,2}, v_{1,2}$ and their drivers.

Where the Just-in-time solution applies, banks 1 sells the precise amount of the asset that just satisfy the outflows in each period.

In the Smoothing solution, bank 1 chooses sales volumes such that the total liquidity need $l_{1,1} + l_{1,2} - c_{1,1}$ is distributed in a certain way across both periods depending on the strategy chosen by bank 2. To be more precisely, bank 1 makes two adjustments to this simple rule to balance two opposing motives.

First, the optimal sale of bank 1 in the first period increases in $v_{2,2}$: if there is a large drop in the price in period 2 due to a sale by bank 2, bank 1 sells a larger amount in period 1 to trade at the relatively high price at that time.

Second, the sale of bank 1 in the first period decreases in $v_{2,1}$. Building on (10), the expression $\lambda a_{1,1}/\left(1-\lambda a_{1,1}\right)$ measures bank 1's potential influence on the market price. As bank 2 increases its sale in the first period, bank 1 has a tendency to curb its sale so that it does not accelerate the price decline. The larger the price impact of bank 1 (in absolute value), the larger this tendency to restrain the sale of the asset is.

As the first adjustment is positive and the second adjustment is negative, it is not clear which effect prevails. In any case, the negative value of any adjustment to $v_{1,1}^{*}$ applies to the optimal amount $v_{1,2}^{*}$, which bank 1 sells in the second period such that the total liquidity need $l_{1,1}+l_{1,2}-c_{1,1}$ is served.

Before discussing the last two cases, let us take a closer look at $d_{1,1}(v_{2,1}, v_{2,2})$, which drives the decision made by bank 1. It has two components: the liquidity need of bank 1 and the impact of the actions of bank 2 on bank 1. Regarding the second component, $\lambda v_{2,1}$ describes the price drop stemming from a sale by bank 2 in the first period. Thus, $a_{1,1}\lambda v_{2,1}$ is the resulting loss for bank 1 due to the decline in the market value of the asset. As pointed out above, the level of $v_{2,2}$ determines the magnitude of the price reduction in the second period. Both components are combined in the expression

$$\frac{1}{2}\left(v_{2,2}+\left(\frac{\lambda a_{1,1}}{1-\lambda a_{1,1}}\right)v_{2,1}\right), \tag{A3}$$

which describes the impact of the actions of bank 2 that are relevant for bank 1 in absolute terms. Roughly speaking, if (A3) is small relative to the liquidity need of bank 1, the actions of bank 2 have only a limited impact on bank 1's decision-making. Then, if $l_{1,2}$ is also small, bank 1 focuses on meeting the dominant short-run liquidity need $l_{1,1}-c_{1,1}$ in the first period. As $l_{1,2}$ rises, bank 1 splits the liquidity need more equally among both periods. Therefore, bank 1 decides to increase its sales in the first period above the short-run need and generates more cash at the relatively high price at that time.

In contrast, as (A3) rises, d_1 eventually surpasses the bank's total liquidity need, and so sales by bank 2 have a significant bearing on the decision made by bank 1. In the Front-Servicing solution, banks 1 already sells an amount equal to the total liquidity need in the first period and does not make a sale thereafter. Finally, in the Distress-Sale solution, bank 1 sells the maximum amount available in the first period. So in contrast to the Just-In-Time or Smoothing solution, bank 1 restricts its sales entirely to the first period in both of the last two cases.

As a consequence, any change in the market price in the second period is driven entirely by bank 2 as $v_{1,2}^* = 0$ in these two cases. Note further that the additional condition $a_{1,1}/(1-\lambda a_{1,1}) \geq v_{2,2}$ is equivalent to $\lambda a_{1,1}/(1-\lambda a_{1,1}) \leq \lambda v_{2,2}$. As explained above, $\lambda a_{1,1}/(1-\lambda a_{1,1})$ is the drop in the market price in the first period induced solely by bank 1 in the extreme scenario that bank 1 sells all assets in period 1. Moreover, as $v_{1,2}^* = 0$, $\lambda v_{2,2} = r_{2,3}$ is the price drop in the second period.

Thus, bank 1 compares a price drop in the first period with the price drop in the second period: if $\lambda v_{2,2}$ was large (in absolute value) relative to $\lambda a_{1,1}/(1-\lambda a_{1,1})$, then the price would decline very sharply in the second period. Consequently, bank 1 would suffer a large loss in the market value of the asset at that time. To avoid such a loss, bank 1 prefers to liquidate as much as possible of the asset in the first period, leading to the Distress-Sale solution. Conversely, in the Front-Servicing solution, bank 1 just covers its total liquidity need in the first period, while the potential loss in market value of the assets held by bank 1 occuring in period 2 is comparatively low.

A.3 Proof of Theorem 1

We start by determining the set of all feasible strategies $(v_{1,1}, v_{1,2})$ of (A2). From the constraints of this problem we see that the set of feasible solutions is non-empty only if

$$l_{1,1} - c_{1,1} \leq v_{1,1} \leq \left(\frac{a_{1,1}}{1 - \lambda a_{1,1}}\right)(1 + \lambda v_{2,1}) \tag{A4}$$

holds. Note further that $l_{1,1} - c_{1,1} > 0$ by Assumption 1 c), so that any feasible solution has a strictly positive $v_{1,1}$. Similarly, any feasible $v_{1,2}$ must satisfy

$$\max\{0, l_{1,1} + l_{1,2} - c_{1,1} - v_{1,1}\} \leq v_{1,2} \leq \frac{a_{1,2}(1 + \lambda v_{2,2})}{1 - \lambda a_{1,2}}. \tag{A5}$$

Now we are going to tackle the FOCs of the optimisation problem (A2). Denoting the Lagrangian by ℓ and the Lagrange multipliers associated with the six constraints in (A2) by $\mu_1 - \mu_6$, the FOCs of the problem are

$$\begin{aligned}
\frac{\partial \ell(v_{1,1}, v_{1,2})}{\partial v_{1,1}} =& - \lambda a_{1,1} - \lambda\left(\lambda a_{1,1} - 1\right)\left(v_{1,2} + v_{2,2}\right) - \mu_1 - \mu_3 - \mu_4 + \left(1 - \lambda a_{1,1}\right)\mu_5 \\
&+ \left(1 - \lambda a_{1,1}\right)\left(1 + \lambda\left(v_{1,2} + v_{2,2}\right)\right)\mu_6 = 0, \\
\frac{\partial \ell(v_{1,1}, v_{1,2})}{\partial v_{1,2}} =& - \lambda a_{1,2} - \mu_2 - \mu_4 + (1 - \lambda a_{1,2})\mu_6 = 0.
\end{aligned}$$

Regarding the first condition, notice that $a_{1,2}$ in the last restriction in problem (A2) is a function of $v_{1,1}$ with partial derivative $\partial a_{1,2}/\partial v_{1,1} = \lambda a_{1,1} - 1$. The FOCs comprise the following conditions for the Lagrange multipliers:

$$\begin{aligned}
0 &= \mu_1 v_{1,1} \\
0 &= \mu_2 v_{1,2} \\
0 &= \mu_3(-v_{1,1} - c_{1,1} + l_{1,1}) \\
0 &= \mu_4(-v_{1,1} - v_{1,2} - c_{1,1} + l_{1,1} + l_{1,2})
\end{aligned}$$

$$0 = \mu_5((1-\lambda a_{1,1})\cdot v_{1,1} - a_{1,1} - \lambda a_{1,1}v_{2,1})$$

$$0 = \mu_6((1-\lambda a_{1,2}(v_{1,1}))v_{1,2} - a_{1,2}(v_{1,1}) - \lambda a_{1,2}(v_{1,1})v_{2,2}).$$

We call any vector $(v^*_{1,1}, v^*_{1,2})$ and any 6-tuple $(\mu^*_1, \ldots, \mu^*_6)$ which satisfies the FOCs a Karush-Kuhn-Tucker point.

We use a simple fact regarding the derivative of the objective function of problem (A2) with respect to $v_{1,2}$ as a starting point for the analysis of several cases:

$$\frac{\partial(-\lambda a_{1,1}(v_{1,1}+v_{2,1}) - \lambda a_{1,2}(v_{1,2}+v_{2,2}))}{\partial v_{1,2}} = -\lambda a_{1,2}.$$

Consequently, the objective function does not decrease in $v_{1,2}$. The subsequent discussion of cases is linked to the position of $v^*_{1,1}$ and $v^*_{1,2}$ in the intervals (A4) and (A5). As we will see below, different situations may occur depending on whether these variables equal either the left or right boundaries of these intervals or whether there are interior points.[38] Besides that the position of $v^*_{1,2}$ w.r.t. (A5) determines whether $a_{1,2} > 0$ or $a_{1,2} = 0$. In the first case $v^*_{1,2}$ coincides with the left side of the interval (A5), i.e. $v^*_{1,2} = \max\{0, -v_{1,1} - c_{1,1} + l_{1,1} + l_{1,2}\}$ since otherwise $(v^*_{1,1}, v^*_{2,1})$ would not be a minimum.

The determination of $v^*_{1,1}$ requires to consider three subcases depending on whether it coincides with the leftmost position with respect to (A4), with an inner point of the interval, or with the rightmost position.

Obviously, assets are exhausted after period 1 if and only if $v^*_{1,1} = \left(\frac{a_{1,1}}{1-\lambda a_{1,1}}\right)\left(1+\lambda v^*_{2,1}\right)$, i.e. if $v^*_{1,1}$ equals the rightmost position of (A4). Consequently, the condition for the third subcase is equal to $a_{1,2} = 0$.

Based on these three different possible scenarios for $v^*_{1,2}$ we determine the existence of

[38]Without loss of generality we can assume that both intervals (A4) and (A5) are true intervals having non-zero length. Consequently, we can assume for these intervals that the left boundary is smaller than the right one and that they have inner points. Theoretically one could construct parameter combinations in terms of initial assets, liabilities, cash and a strategy for bank 2 such that the intervals collapse into single points. However, in such a case a small perturbation of one of the parameters, e.g. initial assets, by an ϵ could be applied to establish a situation with “true” intervals. This assumption regarding the intervals is particulary relevant when Langrange multipliers are inferred to be zero and also when the criterion for sufficiency is dealt with at a later point of the exposition.

Karush-Kuhn-Tucker points for the problem (A2). After determining feasible strategies for each of the three cases we show that these feasible strategies satisfy the sufficient conditions for optimality w.r.t. the optimisation problem (A2). In order to do so we rely on the so-called Mangasarian-Fromowitz constraint qualification (Grossmann and Terno, 1993, p. 25).

This condition refers to the so-called active constraints of the optimisation problem (A2). In order to describe the condition it is necessary to think of the left sides of the six constraints of (A2) as functions $h_1(v_{1,1}, v_{1,2}), \ldots, h_6(v_{1,1}, v_{1,2})$. Using common terminology from optimisation theory a constraint h_i is called "active" for a strategy $(v^*_{1,1}, v^*_{1,2})$ if $h_i(v^*_{1,1}, v^*_{1,2}) = 0$. Then, the Mangasarian-Fromowitz constraint qualification states that the optimality of $(v^*_{1,1}, v^*_{1,2})$ is satisfied if the gradient vectors

$$\left(\frac{\partial h_i(v^*_{1,1}, v^*_{1,2})}{\partial v_{1,1}}, \frac{\partial h_i(v^*_{1,1}, v^*_{1,2})}{\partial v_{1,2}}\right)$$

establish a set of linearly independent vectors.

Case 1 (Just-in-time). $v^*_{1,1}$ is minimal w.r.t. (A4). Consequently, the Langrange multipliers $\mu^*_1, \mu^*_2, \mu^*_5$ and μ^*_6 are 0. The FOCs give

$$v^*_{1,1} = l_{1,1} - c_{1,1},\ \ v^*_{1,2} = l_{1,2}.$$

Taking into consideration that

$$\frac{\partial \ell}{\partial v_{1,1}} - \frac{\partial \ell}{\partial v_{1,2}} = 0$$

which results to

$$-\lambda a_{1,1} - \lambda(\lambda a_{1,1} - 1)(v_{1,2} + v_{2,2}) - \mu^*_3 + \lambda(a_{1,1}(1 + \lambda(v_{1,1} + v_{2,1})) - v_{1,1}) = 0$$

we obtain an explicit expression for μ^*_3. Taking the non-negativity of μ^*_3 into account and plugging in the expression for $v^*_{1,1}$ and $v^*_{1,2}$ as shown above we have

$$\frac{l_{1,1} + l_{1,2} - c_{1,1} + v_{2,2}}{2} - \frac{\lambda a_{1,1} v_{2,1}}{2} \leq v^*_{1,1} = l_{1,1} - c_{1,1},$$

which corresponds to the respective condition in Theorem 1.

The active constraints correspond to inequalities 3 and 4. Their gradients $(-1, 0)^T$ and $(-1, -1)^T$ are linearly independent.

Case 2. We assume that the selling amount $v^*_{1,1}$ of bank 1 at time $t = 1$ neither equals the maximum nor the minimum possible amounts w.r.t. (A4), in particular $v^*_{1,1} > l_{1,1} - c_{1,1}$. Under this assumption we can conclude μ_1, μ_3, μ_5 and μ_6 equal 0. Regarding the selling amount $v^*_{1,2}$ of bank 1 at time $t = 2$ two cases need to be distinguished.

Case 2.1 (Smoothing). $l_{1,1} + l_{1,2} - c_{1,1} - v^*_{1,1} > 0$, which implies

$$v^*_{1,2} = l_{1,1} + l_{1,2} - c_{1,1} - v^*_{1,1} > 0. \tag{A6}$$

This additionally implies $\mu^*_2 = 0$. The only remaining non-negative Lagrange multiplier is μ^*_4. Plugging this information into the FOCs gives

$$\begin{aligned} -\lambda a_{1,1} - \lambda(\lambda a_{1,1} - 1)(l_{1,1} + l_{1,2} - c_{1,1} - v^*_{1,1} + v_{2,2}) \\ + \lambda(a_{1,1}(1 + \lambda(v^*_{1,1} + v^*_{1,2})) - v^*_{1,1}) = 0. \end{aligned}$$

Some further arithmetic operations lead to

$$\begin{aligned} v^*_{1,1} &= \frac{1}{2}(l_{1,1} + l_{1,2} - c_{1,1} + v_{2,2}) + \frac{\lambda a_{1,1} v_{2,1}}{2(1 - \lambda a_{1,1})} = d_{1,1}(v_{2,1}, v_{2,2}) \\ v^*_{1,2} &= \frac{1}{2}(l_{1,1} + l_{1,2} - c_{1,1} - v_{2,2}) - \frac{\lambda a_{1,1} v_{2,1}}{2(1 - \lambda a_{1,1})}. \end{aligned}$$

From $\mu^*_3 > 0$ we obtain $v^*_{1,1} \leq l_{1,1} - c_{1,1}$ and the lower bound $d_{1,1}(v^*_{2,1}, v^*_{2,2}) \geq l_{1,1} - c_{1,1}$.

Finally, from (A6) we obtain the upper bound

$$l_{1,1} + l_{1,2} - c_{1,1} > d_{1,1}(v_{2,1}^*, v_{2,2}^*),$$

such that both conditions as stated in Theorem 1 are valid.

The only active constraint is the inequality 4. Consequently, its gradient $(-1,-1)^T$ forms a set of independent vectors.

Case 2.2 (Front-Servicing). The inequality $l_{1,1}+l_{1,2}-c_{1,1}-v_{1,1}^* \leq 0$ implies $v_{1,2}^* = 0$. The FOCs give $v_{1,1}^* = l_{1,1} + l_{1,2} - c_{1,1}$ and $\frac{\partial \ell}{\partial v_{1,2}} - \frac{\partial \ell}{\partial v_{1,1}} = 0$ which corresponds to

$$\lambda a_{1,1} + \lambda(\lambda a_{1,1} - 1)(v_{1,2} + v_{2,2}) + \mu_2^* - \lambda(a_{1,1}(1 + \lambda(v_{1,1} + v_{2,1})) - v_{1,1}) = 0.$$

Plugging in the expressions for the partial derivatives, substituting $v_{1,1}^*$ with $l_{1,1}+l_{1,2}-c_{1,1}$ and taking into account the non-negativity of μ_2 gives the desired results, i.e.

$$\begin{aligned} l_{1,1} + l_{1,2} - c_{1,1} &\leq \frac{l_{1,1} + l_{1,2} - c_{1,1} + v_{2,2}}{2} - \frac{\lambda a_{1,1} v_{2,1}}{2(\lambda a_{1,1} - 1)} \\ v_{2,2} &\leq \frac{a_{1,1}}{1 - \lambda a_{1,1}}. \end{aligned}$$

The inequalities 2 and 4 are the active constraints in this case. The gradients are $(0,-1)^T$ and $(-1,1)^T$. Obviously, they are linearly independent.

Case 3 (Distress-Sale). We assume that bank 1 sells the maximum possible amount of assets, ie $v_{1,1}^* = a_{1,1}\frac{1+\lambda \cdot v_{2,1}}{1-\lambda \cdot a_{1,1}}$. Consequently, $v_{1,2}^* = 0$. We leave out the trivial case that the interval described by (A5) collapses into a single point and may therefore assume that the three Lagrange multipliers μ_1^*, μ_3^* and μ_4^* are 0.

From the FOC's we obtain

$$\begin{aligned} -\lambda a_{1,1} - \lambda(\lambda a_{1,1} - 1)v_{2,2} + (1 - \lambda a_{1,1})\mu_5 + (1 - \lambda a_{1,1})(1 + \lambda v_{2,2})\mu_6 &= 0 \\ -\mu_2 + \mu_6 &= 0 \end{aligned}$$

for the remaining Lagrange-multipliers μ_2, μ_5 and μ_6. After re-arranging terms we obtain

$$\mu_5^* = -\lambda(v_{2,2} - \frac{a_{1,1}}{1-\lambda a_{1,1}}) - (1+\lambda v_{2,2})\mu_6^*.$$

The non-negativity of μ_5 gives

$$v_{2,2} \geq \frac{a_{1,1}}{1-\lambda a_{1,1}}.$$

Consequently we obtain

$$\frac{l_{1,1}+l_{1,1}-c_{1,1}+v_{2,2}}{2} - \frac{\lambda a_{1,1} v_{2,1}}{2(\lambda a_{1,1}-1)} = \frac{l_{1,1}+l_{1,2}-c_{1,1}}{2} + \frac{a_{1,1}(1+\lambda a_{1,1} v_{2,1})}{2(1-\lambda a_{1,1})},$$

which results to

$$\frac{l_{1,1}+l_{1,2}-c_{1,1}}{2} + \frac{a_{1,1}(1+\lambda a_{1,1}, v_{2,1})}{2(1-\lambda a_{1,1})} \geq l_{1,1}+l_{1,2}-c_{1,1}$$

because of $v_{1,1}^* \geq l_{1,1}+l_{1,2}-c_{1,1}$. Consequently, $d_{1,1}(v_{1,1}^*, v_{1,2}^*) \geq l_{1,1}+l_{1,2}-c_{1,1}$ follows as a necessary condition and both conditions as stated in Theorem 1 are met. The gradients of the active constraints 2 and 5 are $(0,-1)^T$ and $(1-\lambda a_{1,1}, 0)^T$, respectively. They are independent. Thus, the optimality of the strategy $(v_{1,1}^*, v_{1,2}^*)$ is guaranteed. ■

A.4 Proof of Theorem 2

As banks' objective functions are not quasi-concave we cannot refer to established existence theorems.[39] For the proof we rely on Table A1, which displays all permissible combinations of strategies, i.e. all combinations of strategies which reflect non-empty domains. From simple conditions it follows that eight non-empty domains exist (1 to 4.1). In other words, from the $4 \cdot 4 = 16$ possible combinations of strategies we can exclude eight combinations, i.e. four combinations where both banks choose an "early selling"

[39]The non-quasiconcavity of the objective function makes the proof of the existence of the Nash equilibrium more complex. However, the chosen objective function simplifies the empirical implementation since it builds on a modified version of a widely used empirical measure of market liquidity suggested by Amihud (2002) which can be easiliy calibrated based on market data.

strategy (i.e. Front Servicing or Distress-Sale)[40] and four combinations where one bank chooses an early selling strategy and the other bank chooses Smoothing[41].

The main idea of the proof is an investigation of the inequalities which determine the decisions of each of the two banks. The goal is to show that there exists no sample $w = (a_{1,1}, a_{2,1}, c_{1,1}, c_{2,1}, l_{1,1}, l_{1,2}, l_{2,1}, l_{2,2}, \lambda)$ of initial parameters, which cannot be assigned to at least one of the eight non-empty domains. An assignment would immediately imply that the strategies which are associated with this domain describe a Nash equilibrium because optimality of each bank's strategy conditional on the other bank's strategy immediately follows from Theorem 1. However, looking at Table A1 it becomes clear that the two arguments of $d_{1,1}$ and $d_{2,1}$ are not necessarily uniform across different domains, i.e. the variables $d_{1,1}$ and $d_{1,2}$ appear with different arguments. As a result, it is not obvious that every w in fact can be assigned.

Roughly speaking the proof works as follows:

- Firstly, we define transformed variables $\tilde{d}_{1,1}, \tilde{d}_{2,1}$. The difference between the variables that have a ˜ and the original variable without ˜ will be that instead of the original arguments which appear in Table A1 and differ across domains uniform arguments will be used instead.

- Secondly, we create nine transformed domains which compare the values of $\tilde{d}_{1,1}$ and $\tilde{d}_{2,1}$ with the thresholds in Theorem 1, i.e. $l_{1,1} - c_{1,1}$, $l_{1,1} + l_{1,2} - c_{1,1}$, $l_{2,1} - c_{2,1}$ and $l_{2,1} + l_{2,2} - c_{2,1}$

- Thirdly, as the nine transformed domains fully cover the space of $\mathbb{R}^2$ we know that any w can be assigned to one of the nine transformed domains.

[40]Following Theorem 1, the conditions for Front-Servicing and Distress-Sale cannot be met by, say bank 1, if bank 2 chooses Front-Servicing or Distress-Sale, each with $v_{2,2} = 0$. Consequently, bank 1's decision variable equals $d_{1,1} = \frac{l_{1,1}+l_{1,2}-c_{1,1}}{2} + \frac{\lambda a_{1,1}}{2(1-\lambda a_{1,1})} v^*_{2,1}$. Inserting this into the conditions for Front-Servicing or Distress-Sale gives $\frac{l_{1,1}+l_{1,2}-c_{1,1}}{2} \leq \frac{\lambda a_{1,1}}{2(1-\lambda a_{1,1})} v^*_{2,1}$. As the left side is positive and the right side is negative, the inequality is a contradiction.

[41]Again, these combinations violate Theorem 1. If, say, bank 1, chooses Smoothing and bank 2 chooses Front-Servicing or Distress-Sale, each with $v_{2,2} = 0$, bank 1's decision variable equals $d_{1,1} = \frac{l_{1,1}+l_{1,2}-c_{1,1}}{2} + \frac{\lambda a_{1,1}}{2(1-\lambda a_{1,1})} v^*_{2,1}$. Inserting this into the conditions for Smoothing gives us $\frac{l_{1,1}-l_{1,2}-c_{1,1}}{2} < \frac{\lambda a_{1,1}}{2(1-\lambda a_{1,1})} v^*_{2,1}$. As the left side is positive (see Assumption 1 c) and the right side is negative, the inequality has no solution.

- Fourthly, we exclude three transformed domains from the analysis because they are empty, i.e. not permissible under the set of assumptions of the model.
- Finally, we show that given w is assigned to one non-empty transformed domain, this implies the assignment to one of the eight domains shown in Table A1.

Let

$$\tilde{d}_{1,1} := \frac{l_{1,1} + l_{1,2} - c_{1,1} + l_{2,2}}{2} + \frac{\lambda a_{1,1}}{2(1 - \lambda a_{1,1})}(l_{2,1} - c_{2,1}) \text{ and analogously}$$
$$\tilde{d}_{2,1} := \frac{l_{2,1} + l_{2,2} - c_{2,1} + l_{1,2}}{2} + \frac{\lambda a_{2,1}}{2(1 - \lambda a_{2,1})}(l_{1,1} - c_{1,1}).$$

The variable $\tilde{d}_{1,1}$ equals the decision variable $d_{1,1}(l_{2,1} - c_{2,1}, l_{2,2})$, where the arguments reflect that the opposite bank 2 chooses Just-in-time. An analogous statement holds true for the variable $\tilde{d}_{2,1}$ and bank 1.

We link $\tilde{d}_{1,1}$ and $\tilde{d}_{2,1}$ to the thresholds $l_{1,1} - c_{1,1}$, $l_{1,1} + l_{1,2} - c_{1,1}$, $l_{2,1} - c_{2,1}$ and $l_{2,1} + l_{2,2} - c_{2,1}$, thereby creating a partition consisting of nine transformed domains (I to IX) as shown in Table A2. Looking at the conditions in Table A2, it becomes obvious that the nine transformed domains fully cover the space of $\mathbb{R}^2$, and hence any sample w can be assigned to one of the nine transformed domains.

First we investigate whether the domain VII is non-empty, i.e. whether

$$\begin{aligned} l_{1,1} - c_{1,1} \leq \tilde{d}_{1,1} < l_{1,1} + l_{1,2} - c_{1,1} \\ l_{2,1} + l_{2,2} - c_{2,1} \leq \tilde{d}_{2,1} \end{aligned} \tag{A7}$$

can hold simultaneously. It can be easily seen that the left inequality in (A7) is already violated by any arbitrary sample w of initial parameters because it is equivalent to

$$\frac{\lambda a_{1,1}}{2(1 - \lambda a_{1,1})}(l_{2,1} - c_{2,1}) \geq \frac{l_{1,1} - c_{1,1} - l_{1,2}}{2} + \frac{l_{2,2}}{2}.$$

This inequality includes a contradiction because the left side is negative while the right side is positive, which follows from Assumption 1 c).

The same reasoning applies to the conditions of domain VIII, where bank 1 and bank 2 simply need to be exchanged. Ultimately we can state that both domains VII and VIII are empty.

We investigate whether the conditions of domain IX, i.e. whether

$$l_{1,1} + l_{1,2} - c_{1,1} \leq \tilde{d}_{1,1}, \;\; l_{2,1} + l_{2,2} - c_{2,1} \leq \tilde{d}_{2,1}$$

can hold simultaneously. These two conditions are equivalent to

$$\frac{\lambda a_{1,1}}{2(1-\lambda a_{1,1})} \geq \frac{l_{1,1} + l_{1,2} - c_{1,1} - l_{2,2}}{2(l_{2,1} - c_{2,1})}, \tag{A8}$$

$$\frac{\lambda a_{2,1}}{2(1-\lambda a_{2,1})} \geq \frac{l_{2,1} + l_{2,2} - c_{2,1} - l_{1,2}}{2(l_{1,1} - c_{1,1})}. \tag{A9}$$

Since $\frac{\lambda a_{1,1}}{2(1-\lambda a_{1,1})}$ and $\frac{\lambda a_{2,1}}{2(1-\lambda a_{2,1})}$ are restricted to the interval $[-\frac{1}{2}, 0]$, each of the two conditions

$$l_{1,1} + l_{1,2} - c_{1,1} \geq l_{2,2}, \;\; l_{2,1} + l_{2,2} - c_{2,1} \geq l_{1,2}$$

would ensure that the two defining conditions of domain IX cannot hold simultaneously. It is straightforward to see that at least one of those conditions always holds. Consequently, domain IX described by the two conditions is empty.

Next, we show that given w is assigned to one of the six transformed domains I to VI, it implies the assignment to one of the eight domains shown in Table A1.

Domain I. Given that the conditions of domain I hold simultaneously, the conditions of domain 1 hold simultaneously as well since both sets of conditions are equivalent.

Domain II. If the conditions of domain II hold simultaneously, i.e.

$$l_{1,1} - c_{1,1} > \frac{l_{1,1} + l_{1,2} - c_{1,1} + l_{2,2}}{2} + \frac{\lambda a_{1,1}}{2(1-\lambda a_{1,1})}(l_{2,1} - c_{2,1}) \tag{A10}$$

$$l_{2,1} - c_{2,1} \leq \frac{l_{2,1} + l_{2,2} - c_{2,1} + l_{1,2}}{2} + \frac{\lambda a_{2,1}}{2(1-\lambda a_{2,1})}(l_{1,1} - c_{1,1}), \tag{A11}$$

then the conditions of domain 2.1 hold simultaneously as well. The conditions for domain 2.1 are

$$l_{1,1} - c_{1,1} > \frac{l_{1,1} + l_{1,2} - c_{1,1} + \tilde{d}_{2,2}}{2} + \frac{\lambda a_{1,1}}{2(1 - \lambda a_{1,1})} \tilde{d}_{2,1}$$

and (A11).

The second condition of domain II is obviously satisfied because $\tilde{d}_{2,2} = d_{2,2}$. The first condition is also satisfied. As $\tilde{d}_{2,1} > l_{2,1} - c_{2,1}$ and $\tilde{d}_{2,2} \leq l_{2,2}$[42] and as $d_{1,1}(\tilde{d}_{2,1}, \tilde{d}_{2,2})$ decreases in $\tilde{d}_{2,1}$ and increases in $\tilde{d}_{2,2}$, it follows $d_{1,1}(\tilde{d}_{2,1}, \tilde{d}_{2,2}) < \tilde{d}_{1,1}$.

The same reasoning applies for domain IV and domain 3.1, where bank 1 and bank 2 simply need to be exchanged.

Domain III. If the conditions of domain III hold simultaneously, i.e. (A10) and

$$l_{2,1} - c_{2,1} + l_{2,1} \leq \frac{l_{2,1} + l_{2,2} - c_{2,1} + l_{1,2}}{2} + \frac{\lambda a_{2,1}}{2(1 - \lambda a_{2,1})} (l_{1,1} - c_{1,1}),$$

then the conditions of domain 2.2 or 2.3 hold simultaneously as well. The first two conditions for domain 2.2 are

$$l_{1,1} - c_{1,1} > \frac{l_{1,1} + l_{1,2} - c_{1,1}}{2} + \frac{\lambda a_{1,1}}{2(1 - \lambda a_{1,1})} (l_{2,1} + l_{2,2} - c_{2,1})$$
$$l_{2,1} - c_{2,1} + l_{2,1} \leq \frac{l_{2,1} + l_{2,2} - c_{2,1} + l_{1,2}}{2} + \frac{\lambda a_{2,1}}{2(1 - \lambda a_{2,1})} (l_{1,1} - c_{1,1}).$$

The second condition of domain III is obviously satisfied because $\tilde{d}_{2,2} = d_{2,2}$. The first condition is also satisfied. As $l_{2,1} - c_{2,1} + l_{2,2} > l_{2,1} - c_{2,1}$ and $0 \leq l_{2,2}$ and due to monotonicity properties of $d_{1,1}(l_{2,1} - c_{2,1} + l_{2,2}, 0)$ it follows $d_{1,1}(l_{2,1} - c_{2,1} + l_{2,2}, 0) < d_{1,1}(l_{2,1} - c_{2,1}, l_{2,2})$. The same reasoning applies to domain 2.3, i.e., due to monotonicity

[42] This follows from $\tilde{d}_{2,1} + \tilde{d}_{2,2} = l_{2,1} + l_{2,2} - c_{2,1}$ and the conditions for domain 2.1.

properties the first two conditions of domain 2.3 are satisfied. Since the third conditions

$$l_{1,2} < \frac{a_{2,1}}{1 - \lambda a_{2,1}} \quad \textit{and } l_{1,2} \geq \frac{a_{2,1}}{1 - \lambda a_{2,1}}$$

of the domains 2.2 and 2.3 form the half-line $[0, \infty)$, it is obvious that exactly one of them is satisfied.

The same reasoning applies for domain V and domain 3.2 or 3.3, where bank 1 and bank 2 simply need to be exchanged.

Domain VI. We show that under the assumption that a sample w of initial parameters belongs to domain VI at least one set of conditions of the domain 2.1, 3.1 or 4.1, respectively, holds true. [43]

For this purpose we start by assuming that the conditions of domain 2.1 do not hold. We can then assume [44]

$$l_{1,1} - c_{1,1} \leq d_{1,1}(d_{2,1}(l_{1,1} - c_{1,1}, l_{1,2}), d_{2,2}(l_{1,1} - c_{1,1}, l_{1,2})). \tag{A12}$$

By means of equivalent transformation it can be shown that (A12) is equivalent to

$$l_{1,1} - c_{1,1} \leq d_{1,1}(d_{2,1}(d_{1,1}, d_{1,2}), d_{2,2}(d_{1,1}, d_{1,2})). \tag{A13}$$

The equivalence of (A12) and (A13) is not straightforward. Instead of presenting all details of the computations here, we only sketch the main lines of reasoning.

In a first step the term $l_{1,1} - c_{1,1}$ which appears on both sides of (A12) is isolated and

[43] The domains 1, 2.2, 2.3, 3.2 and 3.3 can be excluded from the analysis because their conditions obviously cannot be met simultaneously. These domains include at least one condition with $\tilde{d}_{1,1}$ or $\tilde{d}_{2,1}$. If this condition holds, it implies that the condition of domain VI cannot hold, e.g according to domain 2.2 the condition for bank 2 applies $l_{2,1} + l_{2,2} - c_{2,1} \leq \tilde{d}_{2,1}$ which is in direct conflict with the condition of domain VI for bank 2.

[44] The alternative assumption that bank 2's condition is violated would result in one of the cases 1, 2.2, 2.3, which have been already excluded from the analysis.

put on the left side of the inequality, which results in

$$l_{1,1} - c_{1,1} \leq \frac{(\lambda a_{2,1} - 1)(l_{2,1} - c_{2,1} + l_{1,2} + l_{2,2})}{2\lambda a_{1,1} + \lambda a_{2,1} - 2}. \tag{A14}$$

Through some further modifications it can be shown that the expression of the right side of (A14) is equal to

$$\frac{(\lambda a_{2,1} - 1)(l_{1,1} - c_{1,1} + l_{1,2} + l_{2,1} - c_{2,1} + l_{2,2})}{2\lambda a_{1,1} + 2\lambda a_{2,1} - 3}. \tag{A15}$$

The same expression is obtained if the right side of (A13) is broken down to the basic parameters, i.e. to an expression only using $a_{1,1}, a_{2,1}, c_{1,1}, c_{2,1}, l_{1,1}, l_{1,2}, l_{2,1}, l_{2,2}$. Doing so requires us to consider relationships between the quantities $d_{1,1}$, $d_{1,2}$, $d_{2,1}$ and $d_{2,1}$. More precisely, we consider a system of four equations

$$\begin{aligned} d_{1,1} &= A_{1,1}d_{2,1} + \frac{B_{1,2} + d_{2,2}}{2} \qquad & d_{1,2} &= -A_{1,1}d_{2,1} + \frac{B_{1,2} + d_{2,2}}{2} \\ d_{2,1} &= A_{2,1}d_{1,1} + \frac{B_{2,2} + d_{1,2}}{2} \qquad & d_{2,2} &= -A_{2,1}d_{1,1} + \frac{B_{2,2} + d_{1,2}}{2}, \end{aligned}$$

with the A's and B's defined by $A_{1,1} := \frac{\lambda a_{1,1}}{2(1-\lambda a_{1,1})}, A_{2,1} := \frac{a_{2,1}}{1-\lambda a_{2,1}}, B_{1,1} := l_{1,1} - c_{1,1}, B_{2,1} := l_{2,1} - c_{2,1}, B_{1,2} := l_{1,1} - c_{1,1} + l_{1,2}, B_{2,2} := l_{2,1} - c_{2,1} + l_{2,2}$. Solving this system of equations gives the desired result for the equivalence of the expression $d_{1,1}$ with (A15).

By analogy with the above considerations, we can show that if the conditions of domain 3.1 do not hold simultaneously, then

$$l_{2,1} - c_{2,1} \leq d_{2,1}(d_{1,1}(l_{2,1} - c_{2,1}, l_{2,2}), d_{1,2}(l_{2,1} - c_{2,1}, l_{2,2}))$$

can be assumed and

$$l_{1,1} - c_{1,1} \leq d_{2,1}(d_{1,1}(d_{2,1}, d_{2,2}), d_{1,2}(d_{2,1}, d_{2,2}))$$

follows.

Table A1: **Conditions for permissible combinations of strategies**

Domain	Conditions		Strategies	
	Bank1	Bank2	Bank1	Bank2
1	$l_{1,1} - c_{1,1} > d_{1,1}(l_{2,1} - c_{2,1}, l_{2,2})$	$l_{2,1} - c_{2,1} > d_{2,1}(l_{1,1} - c_{1,1}, l_{1,2})$	Just-In-Time	Just-In-Time
2.1	$l_{1,1} - c_{1,1} > d_{1,1}(d_{2,1}(l_{1,1} - c_{1,1}, l_{1,2}), d_{2,2}(l_{1,1} - c_{1,1}, l_{1,2}))$	$l_{2,1} - c_{2,1} \leq d_{2,1}(l_{1,1} - c_{1,1}, l_{1,2}) < l_{2,1} - c_{2,1} + l_{2,2}$	Just-In-Time	Smoothing
2.2	$l_{1,1} - c_{1,1} > d_{1,1}(l_{2,1} + l_{2,2} - c_{2,1}, 0)$	$l_{2,1} - c_{2,1} + l_{2,2} \leq d_{2,1}(l_{1,1} - c_{1,1}, l_{1,2}),\ l_{1,2} < \frac{a_{2,1}}{1-\lambda \cdot a_{2,1}}$	Just-In-Time	Front-Servicing
2.3	$l_{1,1} - c_{1,1} > d_{1,1}(\frac{a_{2,1} \cdot (1+\lambda \cdot (l_{1,1} - c_{1,1}))}{1-\lambda \cdot a_{2,1}}, 0)$	$l_{2,1} - c_{2,1} + l_{2,2} \leq d_{2,1}(l_{1,1} - c_{1,1}, l_{1,2}),\ l_{1,2} \geq \frac{a_{2,1}}{1-\lambda \cdot a_{2,1}}$	Just-In-Time	Distress-Sale
3.1	$l_{1,1} - c_{1,1} \leq d_{1,1}(l_{2,1} - c_{2,1}, l_{2,2}) < l_{1,1} - c_{1,1} + l_{1,2}$	$l_{2,1} - c_{2,1} > d_{2,1}(d_{1,1}(l_{2,1} - c_{2,1}, l_{2,2}), d_{1,2}(l_{2,1} - c_{2,1}, l_{2,2}))$	Smoothing	Just-In-Time
3.2	$l_{1,1} - c_{1,1} + l_{1,2} \leq d_{1,1}(l_{2,1} - c_{2,1}, l_{2,2}),\ l_{2,2} < \frac{a_{1,1}}{1-\lambda \cdot a_{1,1}}$	$l_{2,1} - c_{2,1} > d_{2,1}(l_{1,1} + l_{1,2} - c_{1,1}, 0)$	Front-Servicing	Just-In-Time
3.3	$l_{1,1} - c_{1,1} + l_{1,2} \leq d_{1,1}(l_{2,1} - c_{2,1}, l_{2,2}),\ l_{2,2} \geq \frac{a_{1,1}}{1-\lambda \cdot a_{1,1}}$	$l_{2,1} - c_{2,1} > d_{2,1}(\frac{a_{11} \cdot (1+\lambda \cdot (l_{2,1} - c_{2,1}))}{1-\lambda \cdot a_{1,1}}, 0)$	Distress-Sale	Just-In-Time
4.1	$l_{1,1} - c_{1,1} \leq d_{1,1}(d_{2,1}(d_{1,1}, d_{1,2}), d_{2,2}(d_{1,1}, d_{1,2})) < l_{1,1} - c_{1,1} + l_{1,2}$	$l_{2,1} - c_{2,1} \leq d_{2,1}(d_{1,1}(d_{2,1}, d_{2,2}), d_{1,2}(d_{2,1}, d_{2,2})) < l_{2,1} - c_{2,1} + l_{2,2}$	Smoothing*	Smoothing *

* Note that the variable $d_{1,1}$ appears both as argument as well as a function in the left column for bank 1. The analogous statement holds true for variable $d_{2,1}$ in the column for bank 2.
The decision variable $d_{1,1}$ refers to the optimal strategy of bank 1 given that bank 2 pursues the Smoothing strategy. Similarly, $d_{2,1}$ refers to the optimal strategy of bank 2 given that bank 1 pursues Smoothing.
It can be shown that explicit expressions exist for the four variables $d_{1,1}$, $d_{1,2}$, $d_{2,1}$ and $d_{2,2}$ such that they are compliant with the definitions in Theorem 1. More explicitly, by choosing $v_{1,1} = d_{1,1}$, $v_{1,2} = d_{1,2}$, $v_{2,1} = d_{2,1}$ and $v_{2,2} = d_{2,2}$ it can be made sure that both strategies $(v_{1,1}, v_{1,2})$ and $(v_{2,1}, v_{2,2})$ represent the optimal strategies in the sense of case 2 (Smoothing) of Theorem 1 w.r.t. the strategy chosen by the other bank.

Table A2: **Conditions of transformed domains**

Trans. Domain	Conditions		Permissible
	Bank1	Bank2	
I	$l_{1,1} - c_{1,1} > \tilde{d}_{1,1}$	$l_{2,1} - c_{2,1} > \tilde{d}_{2,1}$	yes
II	$l_{1,1} - c_{1,1} > \tilde{d}_{1,1}$	$l_{2,1} - c_{2,1} \leq \tilde{d}_{2,1} < l_{2,1} - c_{2,1} + l_{2,2}$	yes
III	$l_{1,1} - c_{1,1} > \tilde{d}_{1,1}$	$l_{2,1} - c_{2,1} + l_{2,2} \leq \tilde{d}_{2,1}$	yes
IV	$l_{1,1} - c_{1,1} \leq \tilde{d}_{1,1} < l_{1,1} - c_{1,1} + l_{1,2}$	$l_{2,1} - c_{2,1} > \tilde{d}_{2,1}$	yes
V	$l_{1,1} - c_{1,1} + l_{1,2} \leq \tilde{d}_{1,1}$	$l_{2,1} - c_{2,1} > \tilde{d}_{2,1}$	yes
VI	$l_{1,1} - c_{1,1} \leq \tilde{d}_{1,1} < l_{1,1} - c_{1,1} + l_{1,2}$	$l_{2,1} - c_{2,1} \leq \tilde{d}_{2,1} < l_{2,1} - c_{2,1} + l_{2,2}$	yes
VII	$l_{1,1} - c_{1,1} \leq \tilde{d}_{1,1} < l_{1,1} - c_{1,1} + l_{1,2}$	$l_{2,1} - c_{2,1} + l_{2,2} \leq \tilde{d}_{2,1}$	no
VIII	$l_{1,1} - c_{1,1} + l_{1,2} \leq \tilde{d}_{1,1}$	$l_{2,1} - c_{2,1} \leq \tilde{d}_{2,1} < l_{2,1} - c_{2,1} + l_{2,2}$	no
IX	$l_{1,1} - c_{1,1} + l_{1,2} \leq \tilde{d}_{1,1}$	$l_{2,1} - c_{2,1} + l_{2,2} \leq \tilde{d}_{2,1}$	no

Consequently, if the conditions of domain 2.1 do not hold simultaneously and the conditions of domain 3.1 do not hold simultaneously, then the conditions of domain 4.1 must hold simultaneously.[45] ■

A.5 Proof of Theorem 3

Since we consider one optimisation problem instead of two separate problems, namely one for each bank, we introduce $v_1 := v_{1,1} + v_{2,1}$, $v_2 := v_{1,2} + v_{2,2}$ (analogous definitions for the variables describing assets, liabilities and cash) as new variables. Then, the optimisation problem

$$\min_{\{v_1,v_2\}} \Big\{ -\lambda a_1 v_1 - \lambda(a_1(1 + \lambda(v_1)) - v_1)v_2 \Big\}. \tag{A16}$$

is equivalent to (11) in the sense that each pair of strategies $(v_{1,1}, v_{1,2})$ and $(v_{2,1}, v_{2,2})$ is optimal w.r.t. (A16) if and only if v_1, v_2 are optimal w.r.t. (11). The coordination

[45] As already noted above combinations of strategies where both banks choose an early selling strategy, or where one bank chooses an early selling strategy and the other bank chooses Smoothing reflect empty domains. Hence, if $l_{1,1} - c_{1,1} \leq d_{1,1}(d_{2,1}(d_{1,1}, d_{1,2}), d_{2,2}(d_{1,1}, d_{1,2}))$ and $l_{1,1} - c_{1,1} \leq d_{2,1}(d_{1,1}(d_{2,1}, d_{2,2}), d_{1,2}(d_{2,1}, d_{2,2}))$ hold simultaneously, then the conditions for domain 4.1 are met.

of strategies also implies that the Distress-Sale strategy as described by 4. in Theorem 1 does not make any sense anymore. To be more explicit we can state: While it can make sense for an individual bank to sell more than necessary to serve the liquidity outflows in a Nash equilibrium, the strategy would contradict the minimisation of distress sale losses, i.e. cannot be optimal w.r.t. problem (A16). Consequently, we can assume $v_1 + v_2 = l_1 + l_2 - c_1$. Plugging $v_2 = l_1 + l_2 - c_1 - v_1$ into (A16) simplifies the objective function to

$$F(v_1) = -\lambda a_1 v_1 - \lambda(a_1(1 + \lambda v_1) - v_1)(l_1 + l_2 - c_1 - v_1).$$

Its derivative is

$$F'(v_1) = \lambda((\lambda a_1 - 1)(2v_1 + c_1 - l_1 - l_2)).$$

A reconciliation with optimisation problem (A16) implies that the inequality

$$-v_1 - c_1 + l_1 \leq 0 \tag{A17}$$

remains as the only relevant restriction besides the non-negativity constraint for v_1.[46] An obvious consequence of constraint (A17) is that the derivative of the objective function is negative for all feasible v_1. The minimum is therefore attained at the maximum v_1 within the interval $[0, c_1 - l_1]$, i.e. $v_1^* = c_1 - l_1$ and $v_2^* = l_2$ is the optimal solution of (A16). The only allocation of v_1^* and v_2^* to feasible $v_{1,1}, v_{1,2}$ w.r.t. the initial problem (11) is represented by the Just-in-time strategy for both banks. ■

B Computational details: Distress sale algorithm

We apply an iterative procedure: Before the iteration process starts, banks are numbered based on a random ranking to determine which bank optimises first given the strategies of

[46] Adding up the last two restrictions of optimisation problem (A2) gives $v_1 \leq a_1(1 + \lambda v_1)$ and $v_2 \leq a_2(1 + \lambda v_2)$. It is straightforward to see, that for sufficiently large values for the assets a_1 and a_2, these two restrictions are always satisfied. In particular, they are satisfied if $a \leq 1/\lambda$. Any violation of this inequality could lead to negative selling volumes and would therefore contradict our modelling framework.

the other banks. The starting values of the iteration are the initial values of banks' strategies which are set to zero. In each iteration step, the algorithm calculates successively each banks' optimal selling strategy given the selling strategies of the other banks. The implementation of this optimisation step relies on numerical procedures from the Matlab software.

The algorithm stops after a finite number of M iterations, once for all banks the change in their strategies from iteration step m to iteration step $m+1$ is smaller than a small, positive value ϵ, which we set to 0.001. If the abort criterion is not fulfilled after $m = 50$ iterations, a second (less strict) abort criterion checks if the simulated SLB aggregated across all banks does not change by more than 1%. The second criterion ensures that at least the overall result remains stable and reliable conclusions regarding the overall liquidity situation of the banking system can be made.[47]

Another important aspect of the empirical model is the treatment of illiquid banks. If a bank has few liquid funds or security holdings it is possible that it becomes technically illiquid at a certain step of the iteration. Technically speaking, this means that the non-negative constraints (L) and (B) as introduced in Section 2 cannot be met by the bank and no feasible solution exists given the other banks' strategies as determined during the iteration. For such a case, we need to make specific assumptions about the selling strategy of such illiquid banks. First, we assume that once a bank becomes illiquid during the iteration those banks are immediately liquidated by a hypothetical resolution authority and banks' entire security holdings are sold on day one for the following iterations. The chosen behavioural assumption reflects a conservative approach and ensures that illiquid banks will tend to further decrease the SLB compared with the impact liquid banks have on the SLB.[48] Second, we assume that once a bank becomes illiquid during the iteration

[47]In our applications introduced below one abort criterion is always satisfied during the iteration process. Our simulations have demonstrated that the more complex the application becomes in terms of a longer shock period or a larger number of banks, the more likely it is that the first criterion is not fulfilled but the second criterion is.

[48]One might argue that the chosen assumption reflects a non-realistic extreme scenario. Another possible alternative could be to allow illiquid banks more time to liquidate their assets, e.g. for illiquid banks their optimisation problem should be applied without the non-negative constraints. It would ensure that these banks can still minimise losses during the distress sale spiral (and therefore would act according

it stays in that state until the final iteration. That means when the algorithm calculates a new iteration and banks optimise their strategies based on the updated strategies of the other banks, those banks found to be illiquid in the previous iteration will stay in that state. This approach ensures that banks do not keep on switching back and forth between the liquid and illiquid state from iteration to iteration, thereby supporting the convergence of the algorithm.

The computational details of the algorithm are laid out in Appendix B.

The box below includes the implementation of our heuristic approach to tackle the optimisation problem 8. We refer to this algorithm as Distress-sale algorithm.

Note that the first step in the iteration loop is necessary. Illiquidity can not only become obvious immediately when a bank is hit by a shock but can also occur because a bank cannot service its funding providers because of deteriorating market values of assets.

to the interests of the banks' investors). However, this alternative may eventually lead to stark perverse effects on the SLB. Specifically, illiquid banks would 'contribute' to a higher SLB relative to liquid banks. in other words, the space of feasible selling strategies for liquid banks is bound by constraints which induces those banks to sell securities earlier in the distress sale spiral than they otherwise would, and therefore exacerbate the decline in the market prices. Instead, the behavioural assumption we choose ensures that illiquid banks will tend to further decrease the SLB relative to liquid banks.

Distress-sale algorithm

In this box we use m to count the number of iterations. We use the symbol $||\cdot||$ to assign the maximum norm to a vector of N components, i.e $||\omega|| = \max_{i=1}^{N}\{\omega_i\}$. We rely on two different termination criteria which have to be tested before a new iteration step, denoted by m, is carried out.

Criterion 1 $||\omega_i|| < 0.0001$ for all banks $i = 1, \ldots, N$,

Criterion 2

$$\left| \frac{\sum_{i=1}^{N} SLB_{m+1} - \sum_{i}^{N} SLB_m}{\sum_{i=1}^{N} SLB_m} \right| < 1\%.$$

Note that criterion 1 is equivalent to the requirement that all components of the vector ω_m should be smaller than 0.001.

Initialise the strategy vectors for all banks, i.e. $\omega_{i,t} = 0$ for $i = 1, \ldots N$ and $t = 1, \ldots, T$. Set $SLB_0 = 0$. Set $m = 1$.

While both of the two termination criteria are not satisfied (to be tested for $m > 1$).

Begin iterate

For $i = 1$ **to** N

1. If the bank cannot service outflows, it is forced to sell all liquid assets in $t = 1$, i.e. $\omega_{i,1} = 1$ and $\omega_{i,t} = 0$ for $t = 2, \ldots, T$. This assignment is kept fixed throughout all remaining iterations.

2. Determine a strategy vector ω_i such that ω_i is optimal w.r.t. optimisation problem (8) in the main text under the additional assumption that all other strategy vectors ω_j for $j \neq i$ are kept fixed. For the ω_j's the strategy vectors from the earlier iterations steps are taken into consideration for all banks with index $j > i$ and from the current iteration step for banks with index $i < j$.

End

$SLB_m = \sum_{i=1}^{N} c_{i,T+1} + a_{i,T+1}$ in line with formula (2) in the main text.

$m = m + 1$

End iterate