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Hitting the elusive inflation target

Francesco Bianchi
(Johns Hopkins, Duke, CEPR and NBER)

Leonardo Melosi
(FRB Chicago and CEPR)

Matthias Rottner
(Deutsche Bundesbank)

Deutsche Bundesbank, Wilhelm-Epstein-Straße 14, 60431 Frankfurt am Main,
Postfach 10 06 02, 60006 Frankfurt am Main

Tel +49 69 9566-0

Please address all orders in writing to: Deutsche Bundesbank,
Press and Public Relations Division, at the above address or via fax +49 69 9566-3077

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Non-technical summary

Research Question

Since the 2001 recession, core inflation has been on average below the Federal Reserve's implicit 2% target. This phenomenon has become even more severe in the aftermath of the 2008 recession. In other words, the "conquest of US inflation" that started with the Volcker disinflation seems to have gone too far. Inflation, instead of stabilizing around the desired 2% inflation target, has kept falling down. What factors can explain the disinflationary bias and how can monetary policy bring inflation back to its target?

Contribution

We show that the disinflationary bias is a consequence of a low nominal interest rate environment in which the central bank follows a symmetric strategy to stabilize inflation. An asymmetric rule according to which the central bank responds less aggressively to above-target inflation corrects the disinflationary bias, re-anchors long-term inflation expectations to the desired two-percent target and reduces the risk of encountering the zero lower bound in the future. Thus, an apparent paradox emerges: In order to interpret its inflation target as symmetric, the central bank should follow an asymmetric strategy. This paradox is only apparent, because the asymmetric strategy corrects for the constraint represented by the zero lower bound.

Results

The central bank can remove the disinflationary bias and can raise social welfare by committing to adjust the policy rate less aggressively when inflation is above target than when inflation is below target. We use our calibrated model to run a counterfactual analysis showing that if the asymmetric strategy had been adopted in 2000, the U.S. economy would have not experienced the growing disinflationary bias. Furthermore, the asymmetric rule does not entail any history dependence or commitment to overshoot the inflation target and can be implemented with an asymmetric target range.

Nichttechnische Zusammenfassung

Fragestellung

Seit der Rezession des Jahres 2001 lag die Kerninflationsrate im Durchschnitt unter dem von der US-Notenbank (Federal Reserve) implizit anvisierten Inflationsziel von 2%. Dieses Phänomen ist in der Folge der Rezession von 2008 noch stärker zu Tage getreten. Es scheint, als sei die „Überwindung der US-Inflation“, die mit der Volcker Disinflation begann, über das Ziel hinausgeschossen. Anstatt sich um den erwünschten Zielwert von 2% zu stabilisieren, fiel die Inflation immer weiter. Es stellt sich die Frage, mit welchen Faktoren sich diese persistente Disinflation erklären lässt und wie die Geldpolitik die Inflation wieder an das angestrebte Inflationsziel annähern kann?

Beitrag

Es wird gezeigt, dass diese Tendenz zur Disinflation die Folge eines Umfelds niedriger nominaler Zinssätze ist, in dem die Notenbank eine symmetrische Strategie zur Stabilisierung der Inflation verfolgt. Eine asymmetrische Vorgehensweise, bei der die Notenbank weniger aggressiv auf über dem Ziel liegende Inflationsraten reagiert, korrigiert den disinflationären Verlauf, verankert die langfristigen Inflationserwartungen wieder auf dem angestrebten Zielwert von 2% und verringert die Gefahr, künftig an die Nullzinsgrenze zu stoßen. Es ergibt sich somit ein scheinbares Paradoxon: Um ihr Inflationsziel als symmetrisch interpretieren zu können, sollte die Notenbank eine asymmetrische Strategie verfolgen. Dabei handelt es sich aber lediglich um ein scheinbares Paradoxon, da die asymmetrische Vorgehensweise eine Korrektur für die sich aus der Nullzinsgrenze ergebende Begrenzung darstellt.

Ergebnisse

Die Notenbank kann der persistenten Disinflation entgegenwirken und die gesellschaftliche Wohlfahrt steigern, indem sie sich verpflichtet, den Leitzins weniger stark anzupassen, wenn die Inflation das Inflationsziel überschreitet, als wenn sie es unterschreitet. Anhand eines kalibrierten Modells wird eine kontrafaktische Analyse durchgeführt, die zeigt, dass die zunehmende persistente Disinflation in der US-Wirtschaft ausgeblieben wäre, wenn im Jahr 2000 eine asymmetrische Strategie beschlossen worden wäre. Darüber hinaus ist die asymmetrische Vorgehensweise nicht vergangenheitsabhängig und zieht keine Verpflichtung nach sich, das Inflationsziel zu überschießen. Zudem lässt sie sich auch mit einem asymmetrischen Zielkorridor umsetzen.

DEUTSCHE BUNDESBANK DISCUSSION PAPER NO 40/2021

Hitting the Elusive Inflation Target*

Francesco Bianchi
Johns Hopkins, Duke
CEPR, NBER

Leonardo Melosi
FRB Chicago
CEPR

Matthias Rottner
Deutsche Bundesbank

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Abstract

Since the 2001 recession, average core inflation has been below the Federal Reserve's 2% target. This deflationary bias is a predictable consequence of a symmetric monetary policy strategy that fails to recognize the risk of encountering the zero-lower-bound. An asymmetric rule according to which the central bank responds less aggressively to above-target inflation corrects the bias, improves welfare, and reduces the risk of deflationary spirals – a pathological situation in which inflation keeps falling indefinitely. This approach does not entail any history dependence or commitment to overshoot the inflation target and can be implemented with an asymmetric target range. A counterfactual simulation shows that a modest level of asymmetry would have removed the deflationary bias observed in the United States.

JEL Codes: E31, E52.

Keywords: Asymmetric monetary policy, deflationary bias, deflationary spiral, target range, framework review

*Emails: francesco.bianchi@duke.edu, lmelosi@frbchi.org, matthias.rottner@bundesbank.de. We thank Marco Bassetto, Richard Clarida, Charlie Evans, Jonas Fisher, Spencer Krane, and Chris Sims for their very helpful suggestions. We thank the seminar participants at the NBER Summer Institute Monetary Economics Group, Bank of Finland and CEPR Conference 2020, 23rd Annual DNB Research Conference, Duke University, Chicago Fed, European Central Bank, European University Institute, Fordham University, and University of Warwick. The views in this paper are solely those of the authors and should not be interpreted as reflecting the views of the Deutsche Bundesbank, the Eurosystem, the Federal Reserve Bank of Chicago or any other person associated with the Federal Reserve System.

1 Introduction

Since the 2001 recession, core inflation has been on average below the Federal Reserve's implicit 2% target, as shown in Figure 1. This phenomenon has become even more severe in the aftermath of the 2008 recession. In other words, the "conquest of US inflation" that started with the Volcker disinflation seems to have gone too far. Inflation, instead of stabilizing around the desired 2% inflation target, has kept falling down. In a low nominal interest rate environment, this deflationary bias is a predictable consequence of a symmetric strategy to stabilize inflation, like the one followed by the Federal Reserve until the revision of its framework announced in August 2020. A clear indication of this symmetric approach to the conduct of monetary policy was in the former *Statement on Longer-Run Goals and Monetary Policy Strategy*, which read: "The Committee would be concerned if inflation were running persistently above or below this objective. Communicating this symmetric inflation goal clearly to the public helps keep longer-term inflation expectations firmly anchored [. . .]".

We argue that in the current low interest rate environment, it is advantageous for a central bank to be more concerned about inflation running below target than about inflation going above target. A low inflation target should be combined with an asymmetric monetary policy strategy calling for more aggressive actions when inflation is below target than when inflation is above target. Thus, an apparent paradox emerges: In order to interpret its inflation target as symmetric, the central bank should follow an asymmetric strategy. This paradox is only apparent, because the asymmetric strategy corrects for the constraint represented by the zero lower bound (ZLB). On August 27, 2020, the Federal Open Market Committee (FOMC) revised its *Statement on Longer-Run Goals and Monetary Policy Strategy*. In commenting on the revised statement, Vice Chairman Richard Clarida seems to echo the insights of our paper stating that "*[...] the aim to achieve symmetric outcomes for inflation (as would be the case under flexible inflation targeting in the absence of the ELB constraint) requires an asymmetric monetary policy reaction function in a low r* world with binding ELB constraints in economic downturns.*"Clarida (2020).

We obtain our results using a nonlinear quantitative New Keynesian model. Unlike the standard approach in the literature that studies linearized models with a kink in the monetary policy reaction function, we solve the fully non-linear specification of the model with global methods. This approach allows us (*i*) to take into account the highly nonlinear effects of macroeconomic volatility and of the low long-run real interest rate on the deflationary bias and (*ii*) to study the implications of asymmetric rules and asymmetric target ranges in general equilibrium models. When the long-run real interest rate is calibrated to the low values that seem plausible today (Laubach and Williams 2003), the model predicts that average inflation will remain below target even during expansions.

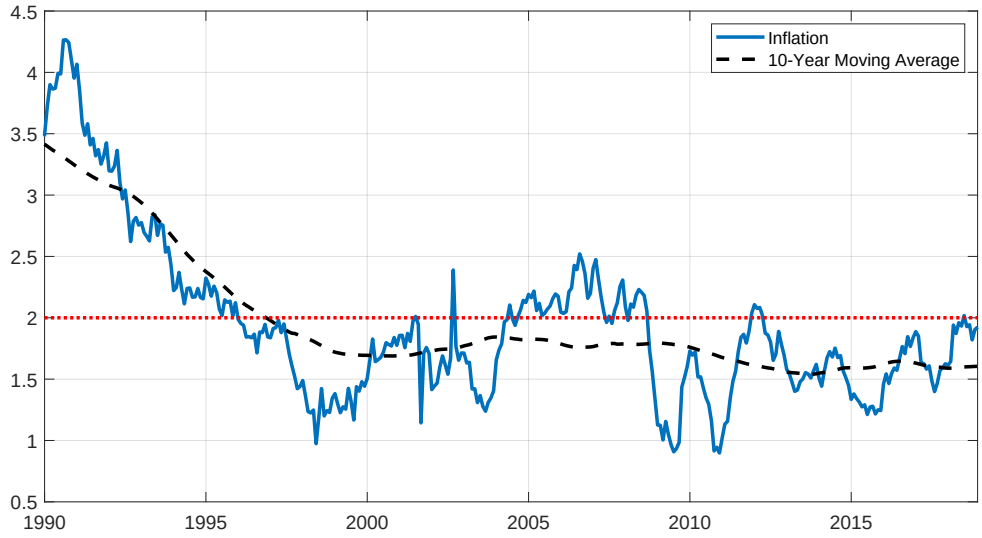

Figure 1: Year-to-year PCE core inflation and its ten-year moving average. Unit: Annualized percentage rates.

Forward-looking price setters anticipate that in the case of a large negative shock the central bank will be unable to fully stabilize inflation due to the ZLB constraint on nominal rates. These beliefs bring about deflationary pressures and depress inflation dynamics even when the economy is away from the ZLB.

A large and increasing deflationary bias is the harbinger of deflationary spirals. Deflationary spirals represent a pathological situation in which inflation keeps falling unboundedly. The deflationary bias arises when the probability of hitting the zero lower bound is nonzero. To counteract this deflationary pressure, the central bank keeps the interest rate low even when the economy is healthy and away from the zero lower bound. This deflationary pressure can become so large that the ZLB becomes binding also in good states. Lacking the offsetting effects of monetary policy, the real interest rate starts increasing and, in doing so, depresses aggregate demand, exacerbating the deflationary pressure. This vicious circle of low inflation, rising real interest rates, and even lower inflation sets the stage for deflationary spirals and implies that no stable rational expectations equilibrium exists. Given the persistent and increasing deflationary bias observed in the last twenty years, the US economy might currently be in the proximity of this scenario, implying that remedying the deflationary bias is an issue of first order importance.

The central bank can remove the deflationary bias and can raise social welfare by committing to adjust the policy rate less aggressively when inflation is above target than when inflation is below target. We use our calibrated model to run a counterfactual analysis showing that if the asymmetric strategy had been adopted in 2000, the U.S. economy would have not experienced the growing deflationary bias shown in Figure 1. By removing the deflationary bias, this asymmetric strategy re-anchor long-term inflation

expectations to the desired two-percent target, reduces the risk of encountering the ZLB in the future, and makes deflationary spirals less likely. The proposed strategy achieves all these goals because it raises the probability of inflation on the upside and, in doing so, offsets the downside risk due to the ZLB, reducing macroeconomic volatility.

In the minutes of the meeting of September 17-18 2019, the Federal Open Market Committee (FOMC) discussed whether the then long-run framework could be improved by adopting asymmetric strategies that require to "respond more aggressively to below-target inflation than to above-target inflation," in line with what advocated in this paper. Furthermore, according to the minutes, several participants suggested a target range as an effective way to communicate this asymmetric strategy. We show that the introduction of such a range can indeed close the deflationary bias provided that the range itself is asymmetric around the desired inflation objective. For instance, if the central bank is committed not to respond to inflation when inflation is within the target range, specifying a range between 1.5 percent and 3.1 percent will remove the deflationary bias. While the degree of asymmetry in the range necessary to remove the bias depends on the strength of the central bank's in-range response to inflation, the required degree of asymmetry is generally fairly modest.

Adam and Billi (2007) and Nakov (2008) were among the first to formally show that the deflationary bias and the corresponding output bias arise in New Keynesian models in which the nominal interest rate is occasionally constrained by the zero lower bound. With respect to these papers, we emphasize that the symmetry of standard monetary policy rules (e.g., the Taylor rule) plays an important role for these biases to arise and show that adopting an asymmetric strategy can remove these biases.

Basu and Bundick (2015) and Richter and Throckmorton (2015) also document that New Keynesian models with an occasionally binding ZLB constraint do not admit a solution when the volatility of the shocks is too large. Unlike those papers, we provide a graphical proof that no Rational Expectations equilibrium exists for a sufficiently large volatility of the shocks. We also show that the deflationary bias and the non-existence of Rational Expectations equilibrium, which we call deflationary spirals, are intertwined. Finally, unlike those papers, we show that the deflationary spirals can be avoided by adopting an asymmetric monetary policy strategy.

Kiley and Roberts (2017) and Bernanke et al. (2019) study a set of symmetric rules to mitigate the severity of recurrent ZLB episodes. Mertens and Williams (2019) evaluate a large variety of monetary policy rules and conclude that dynamic rules, which make up for forgone accommodation after the ZLB episode, can eliminate the deflationary biases and deliver better macroeconomic outcomes than static rules (such as the Taylor rule). Unlike dynamic rules studied in those papers, the asymmetric strategy we propose does not rely on history dependence to remove the deflationary bias. Consequently, the central bank is not committed to engineer deflation following a period of above-

target inflation. Similarly, the asymmetric strategy does not require the central bank to overshoot inflation in expansions in order to correct the deflationary bias. Werning (2011) studies optimal monetary and fiscal policies at the ZLB. Gust et al. (2017b) show that the deflationary bias can be mitigated if policymakers view output losses as asymmetric. Nakata and Schmidt (2019) show that the deflationary bias can be mitigated by appointing a conservative central banker a la Rogoff.

2 The Model

In this section, we introduce a quantitative model with the zero lower bound constraint based on Gust et al. (2017a), who expand the traditional linearized New Keynesian model (Clarida et al. 2000; Woodford 2003; Galí, 2008). The model is solved with global methods in its non-linear specification.

2.1 Model description

The economy consists of households, final goods producers, a continuum of monopolistic intermediate goods firms, a monetary authority, and a fiscal authority. Households buy and consume the final goods from producers, trade one-period government bonds, and supply labor to firms. The final goods producers buy intermediate goods and aggregate them into a homogenous final good using a CES technology. The intermediate goods firms set the price of their differentiated good subject to price adjustment costs a la Rotemberg. They demand labor to produce the amount of differentiated goods to be sold to households in a monopolistic competitive market. Labor is the only factor of production. The fiscal authority balances its budget in every period. The monetary authority sets the interest rate for the government bonds.

The economy features preference and monetary policy shocks as well as shocks to the technological trend of the economy. Preference shocks are included because they are often found to play a leading role in explaining business cycle fluctuations in estimated New-Keynesian DSGE models (Smets and Wouters 2007, Christiano et al. 2005, and Campbell et al. 2012). Furthermore, this is the shock typically used to model zero lower bound events (e.g. Eggertsson and Woodford 2003). We do not include price markup shocks for two main reasons. First, these shocks are well-known to give rise to a trade-off between output and inflation stabilization, which would make it harder to evaluate the role of the asymmetric rule in mitigating the deflationary bias –which is the main object of the paper. Second, these shocks are found to play a negligible role in explaining business cycles in estimated DSGE models and only account for high frequency movements in inflation that can be attributed to observation errors (Justiniano et al. 2013).

The Representative Household In every period, the representative household chooses consumption C_t, labor H_t, and government bonds B_t so as to maximize the expected discounted stream of utility

$$E_0 \sum\nolimits_{t=0}^{\infty} \beta^t \zeta_t^d \left[\frac{\left(\left(C_t - hC_{t-1}^A \right) / Z_t \right)^{1-\sigma}}{1-\sigma} - \chi \frac{H_t^{1+\eta}}{1+\eta} \right] \tag{1}$$

subject to the flow budget constraint

$$P_t C_t + B_t = P_t W_t H_t + R_{t-1} B_{t-1} + T_t + P_t Div_t \tag{2}$$

where C_t^A is aggregate consumption, P_t is the price level, W_t is the real wage, R_t is the gross interest rate, T_t are lump-sum taxes and Div_t are real profits from the intermediate good firms. The parameter h determines the degree of external consumption habits. B_t denotes the one-period government bonds in zero net supply. Z_t denotes the non-stationary aggregate level of technology and is introduced to allow us to conduct welfare analysis in a model in which consumption follows a balance growth path. The preference shock ζ_t^d follows an AR(1) process in logs $\ln(\zeta_t^d) = \rho_\zeta \ln(\zeta_{t-1}^d) + \sigma^{\zeta^d} \epsilon_t^{\zeta^d}$, where $\epsilon_t^{\zeta^d} \sim N(0,1)$.

Final Goods Producers Final goods producers transform intermediate goods into the homogeneous good through the following aggregation technology:

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} df \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{3}$$

where $Y_t(j)$ is the consumption of the good of the variety produced by firm j. The price index for the aggregate homogeneous good is:

$$P_t = \left[\int_0^1 P_t(j)^{1-\epsilon} df \right]^{\frac{1}{1-\epsilon}}, \tag{4}$$

and the demand for the differentiated good $j \in (0,1)$ is $Y_t(j) = \left(P_t(j)/P_t \right)^{-\epsilon} Y_t$.

Intermediate Goods Firms The firm j produces output with labor as the only input $Y_t(j) = Z_t \, H_t(j)$. The aggregate level of technology Z_t has a trend growth g_t: $Z_t = g_t Z_{t-1}$. The growth rate follows a stochastic trend g_t, with average $\bar{g}$ and subject to idiosyncratic shocks: $g_t = \bar{g} + \sigma^g \epsilon_t^g$, where $\epsilon_t^g \sim N(0,1)$. The firm j sets the price $P_t(j)$ of its differentiated goods j so as to maximize its profits:

$$Div_t(j) = P_t(j) \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} \frac{Y_t}{P_t} - \; MC_t \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} Y_t - \frac{\varphi}{2} \left(\frac{P_t(j)}{\Pi P_{t-1}(j)} - 1 \right)^2 Y_t, \tag{5}$$

subject to the downward sloping demand curve for intermediate goods. The parameter $\varphi > 0$ measures the cost of price adjustment in units of the final good.

Policy makers and resource constraint The monetary authority sets the interest rate R_t responding to inflation and output from their corresponding targets. The monetary authority faces a zero lower bound constraint. The policy rule reads as follows

$$R_t = \max\left[1, R_t^N\right]. \tag{6}$$

R_t^N denotes the notional rate that the monetary authority would set without the zero lower bound constraint

$$\frac{R_t^N}{R} = \left(\frac{R_{t-1}^N}{R}\right)^{\rho_R} \left[\left(\frac{\Pi_t}{\Pi}\right)^{\theta_\Pi} \left(\frac{Y_t}{Y_t^*}\right)^{\theta_Y}\right]^{1-\rho_R} \exp\left(\sigma^m \epsilon_t^m\right). \tag{7}$$

where Π denote the inflation target that pins down the inflation rate in the trend-stationary deterministic steady state and Y_t^* is the level of output in the flexible-price economy. Additionally, the monetary authority faces an iid monetary policy shock, where $\epsilon_t^m \sim N(0,1)$. The inertial component is introduced in the specification of the monetary rule to help the model explain critical moments in the data. The fiscal authority is assumed to follow a passive policy rule, moving a lump-sum tax to keep debt on a stable path.

The resource constraint is $C_t = Y_t\left[1 - .5\varphi\left(\Pi_t/\Pi - 1\right)^2\right]$.

2.2 Model Solution and Calibration of Parameters

We solve the model with time iterations and linear interpolation as in Richter et al. (2014). Expectations are evaluated with Gauss-Hermite Quadrature. A detailed description of the solution method and an assessment of the numerical accuracy is provided in Appendix A. The model parameters are calibrated using key moments of U.S. quarterly data computed from 2000:Q1 through 2019:Q4. This period has been characterized by record low interest rates and by a prolonged period of a binding zero lower bound constraint. Table 1 summarizes the calibration, sources and targeted moments.

The discount factor β is set to 0.9993 to obtain an annualized real interest rate of 1.5%, which is broadly in line with the estimates of Laubach and Williams (2003) for this period. The Rotemberg parameter φ is set to 1000 so that the slope of the New Keynesian Phillips curve is 0.01. The calibrated value for the demand elasticity ϵ implies a steady-state markup of 10 percent. The parameter governing the degree of external consumption habits is set to 0.5. The inverse Frisch elasticity is set in line with Chetty

a) Conventional Parameters		Value	Target/Source
β	Steady state discount rate	0.9993	Real interest rate= 1.5% p.a.
σ	Relative risk aversion	1	Log utility
η	Inverse Frisch elasticity	1.33	Chetty et al. (2011)
h	External consumption habit	0.5	Conventional
ϵ	Price elasticity of demand	11	Mark-up = 10%
χ	Disutility labor	1.82	Deterministic SS labor supply = 1
φ	Rotemberg pricing	1000	Slope of NKPC = 0.01
$4\log(\Pi)$	Annualized Inflation target	2%	Inflation target

b) Specific Parameters		Value	Moment		Data	Model
$\bar{g}$	Trend growth rate	1.0031	$\mu(\Delta Y)$	Mean GDP growth rate	0.31%	0.31%
θ_Π	MP inflation response	2.5	$\mu(\Pi)$	Mean inflation rate	1.72	1.72
θ_Y	MP output response	0.7	$\sigma(\Delta Y)$	Std. dev. GDP growth rate	0.6	0.6
$100\sigma_{\zeta^d}$	Std. dev. preference shock	2.16	$\sigma(\Pi)$	Std. dev. inflation	0.6	0.5
$100\sigma^m$	Std. dev. MP shock	0.42	$\rho(\Delta Y, \Pi)$	Correlation inflation, GDP growth	0.22	0.21
$100\sigma^g$	Std. dev. growth shock	0.56	$\sigma(\Delta Y \vert R=0)$	Std. dev. output growth at ZLB	0.5	0.5
ρ_{ζ^d}	Persistence preference shock	0.9	$\sigma(\Pi \vert R=0)$	Std. dev. inflation at ZLB	0.5	0.6
ρ_R	Persistence MP rule	0.7	$\rho(\Delta Y, \Pi \vert R=0)$	Corr. inflation, GDP growth ZLB	0.30	0.52

Table 1: Benchmark calibration: Parameter values and targeted moments

et al. (2011). The parameter controlling the disutility of labor χ is set to normalize the steady-state level of employment to unity. We set the inflation target to 2%.[1]

The remaining eight parameters are set to target selected moments of PCE core inflation and per capita real GDP growth. The steady-state TFP growth rate $\bar{g}$ is calibrated to match the average output growth rate. The inflation response of the monetary policy rule θ_Π is pinned down by the annualized average inflation rate of 1.72. The monetary policy response to output θ_Y is pinned down by the standard deviation of output growth. The standard deviations of the demand and monetary policy shocks are set to match the standard deviation of inflation and the correlation between GDP growth and inflation. In addition to these moments, we target selected moments conditional on a binding zero lower bound: i) the standard deviation of GDP growth, ii) the standard deviation of inflation, and iii) the correlation between inflation and GDP growth. To target these moments, we calibrate the standard deviation of the technological growth rate shock, the persistence of the preference shock, and the persistence of the monetary policy rule. As shown in Table 1, the calibrated model does a fairly good job at replicating the moments we target.

3 Deflationary Bias and Deflationary Spirals

To gain intuition about the causes of the deflationary bias and its relation with the deflationary spirals, we consider a simplified version of the model presented in the previous

[1]There is some disagreement about what the Federal Reserve's effective inflation objective was before 2012 (Shapiro and Wilson 2019). However, there is a strong consensus that the objective has been 2 percent since 2010.

section. The external consumption habit and the persistence in the monetary policy rule are shut down, that is $h = 0 = \rho_R = 0$. We assume that the central bank does not respond to the output gap ($\theta_Y = 0$) and that the economy is stationary ($\overline{g} = 1$) and is buffeted only by the preference shock ($\sigma^g = \sigma^m = 0$). Furthermore, the preference shock is assumed to take only two values low (bad state) and high (good state); i.e., $\zeta_t^d \in \left\{\zeta_L^d, \zeta_H^d\right\}$ with $\zeta_H^d > \zeta_L^d$. When the realizations of the preference shock are binary, equilibrium outcomes can be conditioned on the high or low value of the preference shock and hence can be characterized by solving a set of nonlinear equations as explained in greater detail in B. This simplified version of the model is useful for understanding the causes behind the deflationary bias, when deflationary spirals (i.e., non-existence of stable rational expectations equilibria) emerge, and why these two outcomes are intertwined. Once we have established these points, we will go back to the benchmark model and the calibration introduced in the previous section.

Given the structure of the simplified model, we can partition the model equilibrium conditions into two blocks of equations, one for the good state and one for the bad state. In what follows, we focus on the equilibrium in the good state because - as we will see - this is the state where the deflationary bias arises. The red dashed line in Figure 2 represents the interest rate R^H as function of inflation Π^H as implied by the Taylor rule in the good state, subject to the ZLB constraint. The blue line in the same figure conflates the restrictions imposed on the inflation rate and the nominal interest rate in the good state by all the other equations. Importantly, this curve also takes into account the equilibrium conditions for the bad state because agents in the model are forward looking. The intersections between the red dashed line and the blue solid line give us the (stable) Rational Expectations equilibria and their interest rate and inflation outcomes in the good state. Appendix B describes how these two lines are worked out.

The blue line is upward sloping because a fall in the equilibrium inflation rate in the good state, Π^H, lowers inflation expectations and hence the nominal interest rate in the good state, R^H.[2] The blue line also presents a kink and gets steeper for low values of inflation in the good state. When inflation in the good state declines, the partial equilibrium effect is such that expected inflation declines under both states, depressing inflation in the bad state. When the ZLB is not binding, the central bank responds by lowering the interest rate in the bad state. However, for sufficiently low levels of inflation *in the good state*, the central bank encounters the zero lower bound *in the bad state*. The existence of this threshold creates the kink in the blue line. When inflation is below this threshold, the ZLB constraint is binding in the bad state and any further decline in inflation in the good state implies an increase in the real interest rate in the

[2]Next period's inflation expectations are the weighted average of the equilibrium inflation expectations in the two states. In symbols, $E_t\Pi_{t+1} = p_{HH}\ \Pi^H + (1 - p_{HH})\ \Pi^L$, where p_{HH} is the probability that the economy will stay in the good state in the next period and Π^i, $i \in \{H, L\}$, denotes the equilibrium inflation in the state $\zeta_{t+1}^d = \zeta_i$.

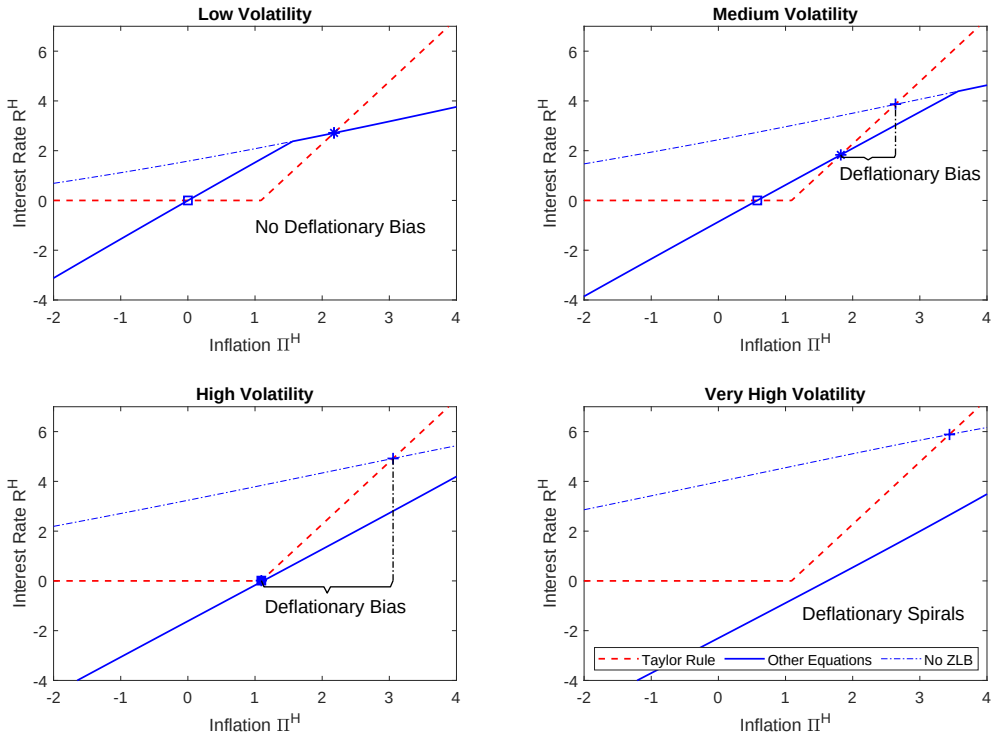

Figure 2: Equilibrium interest rate and inflation when the preference shock is high (good state) for various volatilities of shocks. The red dashed line in this figure represents the Taylor rule in the good state, subject to the ZLB constraint. The blue line in the same figure conflates the restrictions imposed on the inflation rate and the nominal interest rate in the good state by all the remaining equations –including the equations conditional on the bad state. The intersections between the red dashed line and the blue solid line are the (stable) Rational Expectations equilibria in the good state. The blue dashed-dotted line captures the counterfactual case in which we do not impose the ZLB constraint on the nominal interest rate in the bad state and hence the slope of the blue line does not change.

bad state, which exacerbates the recession and the drop in inflation in the bad state. In the good state, agents anticipate that the recession and deflation in the bad state will be more severe and these beliefs determine a steeper decline in inflation expectations and the nominal interest rate in the good state. For comparison, the blue dashed-dotted line captures the counterfactual case in which we do not impose the ZLB constraint on the nominal interest rate in the bad state and hence the slope of the blue dashed-dotted line does not change.

The four plots of Figure 2 show the equilibrium in the good state for various levels of volatility (low, medium, high, very high).[3] Across the four plots, we can see that as the volatility of the demand shock increases, the kink in the blue line occurs for larger values of Π^H, implying that the ZLB becomes a more relevant concern, even if the economy is

[3]The mean of the binary random variable ζ_t^d is unchanged when we raise its variance throughout this exercise. We consider scenarios of low volatility ($\zeta_L^d = 0.975$, $\zeta_H^d = 1.01$), medium volatility ($\zeta_L^d = 0.9062$, $\zeta_H^d = 1.0375$), high volatility ($\zeta_L^d = 0.8375$, $\zeta_H^d = 1.065$) and very high volatility ($\zeta_L^d = 0.7687$, $\zeta_H^d = 1.0925$) with a transition probability of staying in the good state $p = 0.9$ and that of staying in the bad state $q = 0.75$ fixed across these four scenarios.

currently in the good state.

In the upper left graph of Figure 2, we consider a low-volatility scenario. The volatility is relatively low and hence the severity of the negative preference shock is contained. In this case, there are two equilibria in the good state of the economy. One equilibrium implies that the nominal interest rate is not constrained (the star mark in the plot) and the other one is constrained by the ZLB (the square mark in the plot) in the good state.[4] In what follows, we disregard the equilibrium implying that the ZLB is binding in the good state and focus on the other equilibrium, corresponding to the star mark in the plot. In the upper-left plot, the economy is away from the ZLB. Furthermore, in this case the negative preference shock is too small to make the ZLB constraint binding in the bad state. This can be seen by observing that the equilibrium of interest, which is denoted by the star mark in the graph, lies on the flatter part of the blue line.

We now slightly increase the volatility of the preference shock, which implies that the negative preference shock is now larger than what it was in the previous case. Now the target equilibrium lies on the steeper part of the blue line, implying that the economy will go to the ZLB if a negative preference shock will hit tomorrow. These expectations have important effects on today's equilibrium outcomes. Now inflation is lower than what it would have been if the blue line were less steep as in the case in which we do not impose the ZLB constraint (the dashed-dotted blue line in the graph). We call the lower inflation rate in the good state due to the binding ZLB constraint in the bad state the *deflationary bias*. The magnitude of the deflationary bias is shown in the graph.

A further increase in the volatility of the binary preference shock causes the nominal rate and inflation to fall further, as illustrated in the lower left graph of Figure 2. Now the deflationary consequences of hitting the ZLB in the bad state are even more severe. As a result, the inflation rate in the good state falls further down and the deflationary bias widens. To respond to this large deflationary bias, the central bank has to drive the nominal interest rate to the ZLB even in the good state. This can be seen in the graph where the solid blue line intersects the kink of the red dashed line, implying that the two equilibria now coincide in the graph and the ZLB is binding in the good state under both equilibria. Furthermore, note that the deflationary bias is now larger than that in the previous case.

What happens if the volatility increases even further and the realization of the preference shock in the bad state becomes even worse? The central bank would like to lower the nominal interest rate further in the good state in order to mitigate the deflationary pressures owing to the severe deflation expected in the bad state. However, the binding ZLB constraint in the good state prevents the central bank from doing so. As a result,

[4]This result is reminiscent of the two steady-state equilibria characterized in a perfect-foresight environment in the influential paper by Benhabib et al. (2001). However, the equilibria in upper left plot are derived in a stochastic environment where agents take into account the probability that the economy may be hit by preference shocks in future periods.

the fall in inflation expectations combined with the forced inaction of the central bank leads to an increase in the real interest rate in the good state, which depresses inflation expectations even further. We call this vicious circle of lower and lower inflation *deflationary spirals*. In the lower right graph, the blue solid line and the dashed red line do not intersect, implying that no stable Rational Expectations equilibrium exists.

Three interesting lessons emerge from the analysis carried out in this section. First, the deflationary bias emerges when agents expect with some probability that the interest rate will become constrained by the ZLB in the future. Second, the deflationary bias and the deflationary spirals are intertwined: deflationary spirals occur when the deflationary bias is so large that the central bank cannot prevent inflation expectations from spiraling down. Third, when the deflationary bias widens over time a New Keynesian model solved globally in its nonlinear specification predicts that the economy will eventually slip into a deflationary spiral.

4 ZLB Risk and Macroeconomic Biases

The previous section illustrated the origins of the deflationary bias and the link between the deflationary bias and deflationary spirals. We can now return to our full-fledged quantitative model described in Section 2. In this section, we provide two formal definitions of deflationary bias and use the calibrated model to quantify the size of the bias for the U.S. economy.

The Deflationary Bias To define the deflationary bias, it is useful to define the stochastic steady-state equilibrium of the model.[5] We define the deflationary bias as the difference between the rate of inflation at the stochastic steady-state equilibrium and the central bank's inflation target, which coincides with the rate of inflation at the deterministic steady state. The deflationary bias arises when inflation at the stochastic steady state is lower than the central bank's target.

Both the deterministic and stochastic steady states define an economy that has not been hit by shocks for a sufficiently long number of periods so that their variables have stabilized around their steady-state values and do not vary anymore (unless a shock suddenly hits). However, in the deterministic steady state, agents fail to appreciate the macroeconomic risk due to future realizations of the shocks. Instead, in the stochastic steady state, agents appreciate the macroeconomic risks due to future realizations of the shocks and adjust their behavior accordingly. While in a linear model these two concepts of steady-state equilibria lead to the same macroeconomic outcome, in non-linear models whether agents act in response to future macroeconomic risks matters.

[5]Some scholars use the terms "risky steady state" to refer to what we call stochastic steady state. See, for instance, Coeurdacier et al. (2011).

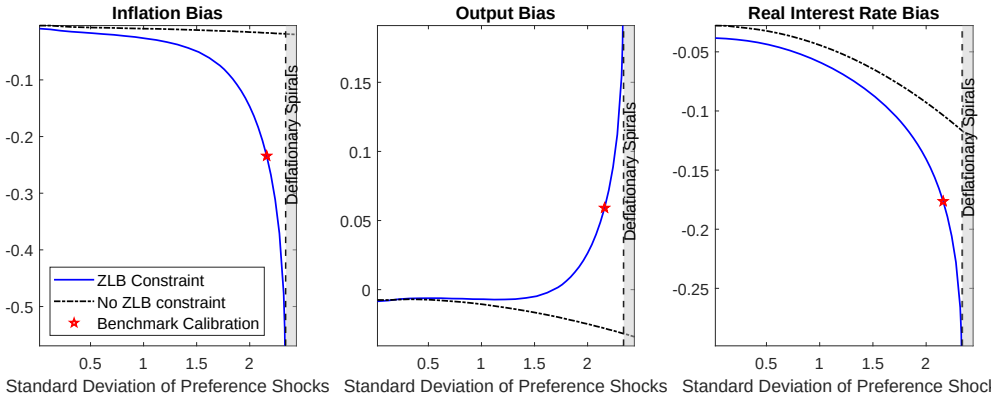

Figure 3: Macroeconomic distortions due the zero lower bound as the volatility of the preference shocks varies. Left graph: The inflationary bias due to model's non-linearities. The red star denotes the calibrated value of the standard deviation of this shock. The difference between the blue solid line and the black dot-dashed line captures the deflationary effects of a risk of a recession that pushes the nominal interest rate to its lower bound. Center graph: the same as the left graph but the bias is computed with respect to output (level). Right graph: the same as the left graph but the bias is computed with respect to the real interest rate. The gray area marks the region of the values for the standard deviation of the preference which trigger deflationary spirals. Units: Inflation and real interest bias is measured in percentage points of annualized rates while the output bias and the standard deviation of the preference shocks are in percent.

Unlike the stochastic steady state equilibrium, the deterministic steady-state equilibrium of our model can be characterized analytically.[6] The real interest rate in the deterministic steady state, r^*, coincides with $\overline{g}\beta^{-1}$ and captures the long-run level of the real interest rate in the absence of risk. The deterministic steady state of inflation is pinned down by the inflation target of the central bank, Π, and can be effectively dealt with as a parameter. The deterministic steady state is not affected by macroeconomic uncertainty, which influences the optimal behavior of rational agents in non-linear models. Such volatility drives a wedge between the outcomes of these two steady-state equilibria and hence fuels the deflationary bias.

The left graph of Figure 3 shows the difference between the inflation rate at the stochastic steady state and inflation at the deterministic steady state with (blue solid line) and without the zero lower bound constraint (black dash-dotted line). Comparing the blue solid line with the black dash-dotted line allows us to isolate the effects of the ZLB constraint on the inflation bias. From the figure, it is easy to conclude that when removing the ZLB constraint, the gap between the deterministic and stochastic steady state is quite low. Instead, the risk of hitting the zero lower bound can lead to large discrepancies between the desired and realized levels of inflation.

The red star denotes the deflationary bias that arises for the baseline calibration.

[6] As shown by Benhabib et al. (2001), there exist two deterministic steady-state equilibria once the zero lower bound on nominal interest rates is taken into account. The first steady state is characterized by positive inflation and a positive policy rate. The second steady state is characterized by a liquidity trap, that is, a situation in which the nominal interest rate is near zero and inflation is possibly negative. In line with most of the literature studying new-Keynesian models, we focus on the positive-inflation deterministic steady state.

Inflation undershoots the central bank's inflation target by 23 basis points because of the risk of hitting the ZLB in the future, which is broadly in line with findings in other empirical papers (e.g., Hills et al. 2019 and Amano et al. 2019). We then study the effects of an increase in the volatility of the preference shock, the shock that is more likely to trigger the ZLB. As the macroeconomic volatility increases, the bias widens up exponentially. A 10 basis-point increase in the standard deviation of preference shocks causes a 11-basis-points reduction in the model's long-run inflation rate. Furthermore, it would take just around an 18 basis-point increase in the standard deviation of preference shocks to make deflationary spirals possible. Since our calibration targets moments based on a period of low macroeconomic volatility, these results suggest a concrete risk of deflationary spirals.

The deflationary bias grows at a faster pace as the standard deviation of the shocks increases because so does the probability of hitting the ZLB. Appendix C shows how the probability of hitting the ZLB varies strongly nonlinearly in response to the volatility of preference shocks. This result can also be inferred by noting that the slope of the black dash-dotted line, which captures the counterfactual case where the ZLB constraint is not enforced, is tiny and close to constant.

The Output Bias The center graph of Figure 3 shows the effects of the risk of hitting the ZLB on the long-run level of output. As before, the long-term output bias due to the zero lower bound is given by the vertical difference between the blue solid line and the solid dashed-dot line, which corresponds to the bias when the ZLB constraint is not imposed. For sufficiently large values of volatility, the output bias is positive (output is higher than its level at the deterministic steady state equilibrium) because the central bank conducts an accommodative monetary policy to respond to the deflationary bias. Since the central bank applies the Taylor principle ($\theta_{\Pi} > 1$), this expansionary monetary policy leads to a negative bias in the real interest rate, as shown in the right graph of Figure 3.

It should be noted that absent the ZLB constraint or for sufficiently low volatility of shocks, there would be a small downward output bias due to to precautionary motives. However, the positive bias due to the lower bound constraint dominates these other effects for our benchmark calibration, which is marked by the red star in the plot.

Implications of a low interest rate environment The results that we have discussed so far rely on the assumption that the long-run real rate of interest is fixed and equal to 1.5 percent. Figure 4 shows the effects of changing both the standard deviation of the shocks and the long-term real rate of interest r^* on the inflationary, output, and real interest rate biases. The important takeaway from this graph is that for sufficiently large values of the long-term real interest rate r^*, the deflationary bias disappears. The

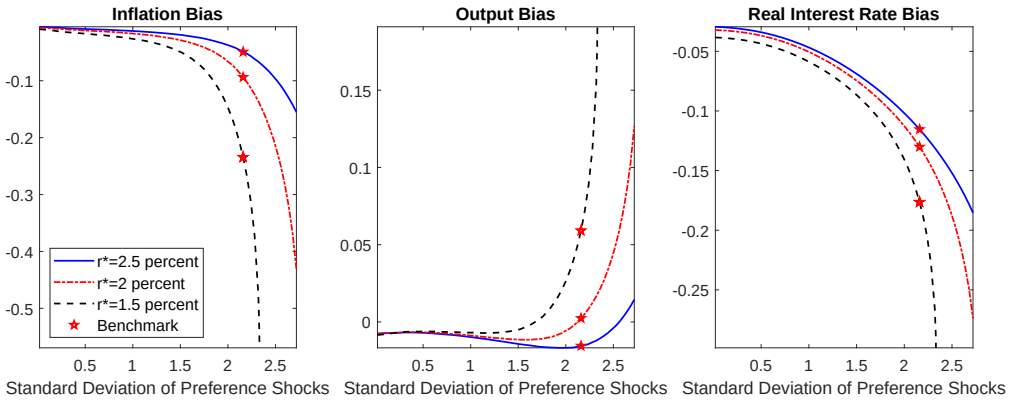

Figure 4: Macroeconomic distortions due the zero lower bound as the standard deviation of preference shocks varies (x-axis) and for alternative values of the steady-state real rate of interest. Left graph: The inflationary bias due to the zero lower bound constraint. The red star denotes the calibrated value of the standard deviation of this shock. Center graph: the same as the left graph but the bias is computed with respect to output (level). Right graph: the same as the left graph but the bias is computed with respect to the real interest rate. Units: Inflation and real interest bias is measured in percentage points of annualized rates while the output bias and the standard deviation of the preference shocks are in percent.

intuition is straightforward: when the long-term real interest rate is higher, it takes a bigger shock to make the ZLB constraint binding. Thus, the probability that the ZLB constraint will become binding falls, leading to a reduction in the deflationary bias (see Figure 11).

A higher real rate of interest r^* would make the function of the deflationary bias less steep and therefore would increase the threshold of the shock volatility that triggers the deflationary spirals. It is also interesting to notice that an increase in the long-term real rate of interest of one percentage point more than halves the deflationary bias in our benchmark calibration, denoted by the red star in the graph. The size of the bias due to non-linearities in the model other than the ZLB does not vary with the long-term real interest rate (not shown), suggesting that the long-term macroeconomic biases linked to a low-interest-rate environment is entirely due to one specific source of non-linearity in the New Keynesian model: the zero lower bound.

To sum up, the deflationary bias brought about by the risk of hitting the ZLB constraint in the future can generate first-order distortions for a central bank that tries to anchor long-term inflation expectations to its desired target Π. Furthermore, we noticed that the combination of a low long-term real interest rate, r^*, and moderate macroeconomic risk can trigger the long-run bias in inflation and output or, even worse, deflationary spirals.

An Alternative Definition of the Deflationary Bias: The Average Bias The notion of deflationary bias introduced in the previous section can be measured only within the context of a structural model. A concept of deflationary bias that can be observed

more directly in the data is the *average deflationary bias*, which we define as the difference between the model's unconditional mean of inflation and the central bank's inflation target Π. This alternative definition of deflationary bias does not only reflect the risk of hitting the ZLB, but it also reflects the inflation outcomes observed when ZLB episodes actually materialize.

To compute the unconditional inflation bias, we simulate the model for several periods and then compute the mean of the variables of interest. The behavior of the average inflationary bias mimics that of the deflationary bias based on the notion of stochastic steady state as shown in Figure 3. The average deflationary bias predicted by the calibrated model is 28 annualized basis points, which is consistent with the deflationary bias shown in Figure 1.

5 The Asymmetric Rule

We have shown that the deflationary bias induced by the ZLB increases when the real interest rate r^* declines or macroeconomic volatility rises. We now turn our attention to what the central bank can do to address the deflationary bias under the two definitions introduced in the previous section.

5.1 The Asymmetric Strategy

The policy strategy that we study in this paper implies a smaller response to inflation when inflation is above target.[7] Specifically, we consider the following modified policy rule:

$$\frac{R_t^N}{R} = \left(\frac{R_{t-1}^N}{R}\right)^{\rho_R} \left(\left[\mathbf{1}_{\Pi_t<\Pi} \left(\frac{\Pi_t}{\Pi}\right)^{\underline{\theta_\Pi}} + (1 - \mathbf{1}_{\Pi_t<\Pi}) \left(\frac{\Pi_t}{\Pi}\right)^{\overline{\theta_\Pi}} \right] \left(\frac{Y_t}{Y_t^*}\right)^{\theta_Y} \right)^{1-\rho_R} \exp\left(\sigma^m \epsilon_t^m\right), \tag{8}$$

where $\underline{\theta_\Pi}$ denotes the response to inflation when inflation is below target, $\overline{\theta_\Pi}$ stands for the response to inflation when inflation is above target, and $\mathbf{1}_{\Pi_t<\Pi}$ is an indicator function that is equal to one when inflation is below target ($\Pi_t < \Pi$). In what follows, we set $\underline{\theta_\Pi} = 2.5$ as in the benchmark calibration of Section 2.2 and study how the average and stochastic steady state biases vary in response to changes in $\overline{\theta_\Pi}$.

The asymmetric rule (8) allows for an autoregressive component. This is to make sure that the only difference with respect to the symmetric rule used to calibrate the model

[7] Alternatively, we can study an asymmetric strategy that implies a stronger response to inflation when inflation is below target. This strategy would also remove the bias as analyzed in Appendix G.

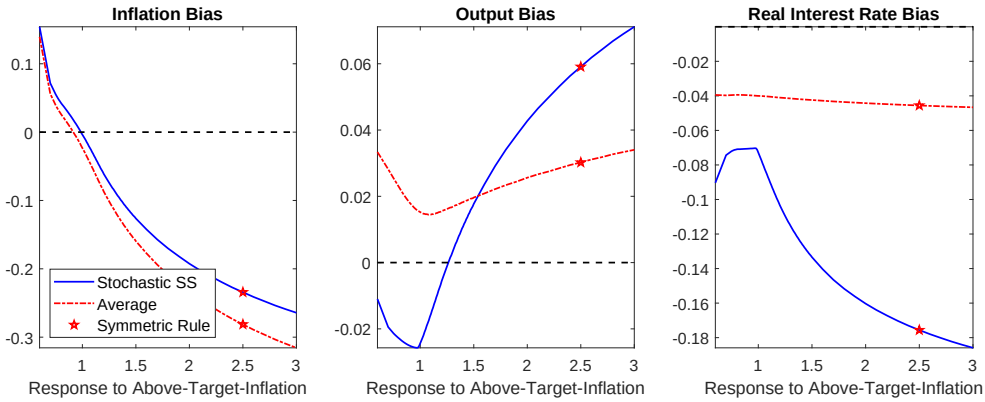

Figure 5: Macroeconomic biases due to the ZLB constraint as the central bank varies its response to positive deviations of inflation from target. The inflation bias (left plot), the output bias (center plot), and the real interest rate bias (the right plot) are computed by taking the difference between these variables at the stochastic steady state and their value at the deterministic steady state (blue solid line). These biases are also computed as the difference between the unconditional mean of these three variables and their value at the deterministic steady state (red dashed-dotted line). The response when inflation is below target is always equal to 2.5 as in the benchmark calibration. The red star marks the symmetric case in which the central bank responds with equal strength to inflation or deflation. Units: The inflation and the real interest rate biases are expressed in annualized percentage points and the output gap in percentage points.

– equation (7) – consists of an asymmetric response to inflation. In turn, we allowed for an autoregressive component in the symmetric rule to match the observed smoothness in interest rates. As shown in a working paper version of this paper (Bianchi et al. 2019), asymmetric rules can completely close the deflationary bias even if no interest rate smoothing is embedded in the rule.

The asymmetric rule in equation (8) can be interpreted as a strategy according to which the central bank is slower in raising rates when inflation goes above target. This strategy reduces the risk of encountering the zero lower bound and its undesirable effects. It is therefore particularly effective in a low interest rates environment, like the current one, in which the biases on key macroeconomic variables can be sizable.

Figure 5 shows how the macroeconomic distortions due to the zero lower bound vary as a function of the central bank's response to above-target inflation. We examine the behavior of the bias away from the zero lower bound (stochastic steady state, the blue solid line) and its unconditional mean (average bias, the red dashed-dotted line).[8] The red stars denote the distortion under a symmetric rule with a response to inflation equal to 2.5, as in the benchmark calibration.

We observe that being less aggressive when inflation is above target helps to mitigate all three macroeconomic biases shown in three plots of Figure 3. Specifically, for a response $\overline{\theta_\Pi}$ close to one, the ZLB-driven macroeconomic distortions become negligible. In a nutshell, to remove the macroeconomic distortions due to the ZLB constraint,

[8]The average bias is computed by taking the mean of inflation, output, and the real interest based on a simulation lasting 250,000 periods. We drop the first 50,000 observations to minimize the effects of initial conditions.

policymakers need to be willing to be less proactive in increasing the interest rate when inflation is running above target. This strategy makes deviations of inflation above the target more likely, offsetting the downside risk of inflation due to ZLB risk. As a result, the probability and the frequency of the ZLB constraint fall, mitigating or even eliminating the deflationary bias under either definition.

Importantly, Figure 5 shows that the two concepts of bias move closely together. This should not be surprising since the driver of the stochastic bias and the average bias is the same: the probability of hitting the ZLB. Indeed, by substantially reducing this probability, the asymmetric strategy closes both notions of deflationary bias. By reanchoring the long-run inflation expectations to the desired target Π, the asymmetric strategy also makes the deflationary spirals less likely. This is an important point to which we will return in Section 5.4.

It should be noted that the unconditional deflationary bias (the red dashed line) is always larger than the deflationary bias (the blue solid line). When computing the unconditional bias, the zero lower bound is not a mere possibility, but an event that occasionally occurs and, in fact, depresses the dynamics of inflation. Thus, average inflation bias is generally even further away from the desired inflation target because the economy experiences the deflationary pressures associated with the ZLB period.

The Asymmetric Strategy Is Not a Makeup Strategy The asymmetric strategy proposed in this paper removes the deflationary bias because it raises the probability of inflation on the upside and, in doing so, offsets the downside risk due to the ZLB. Hence, our strategy differs from the so-called makeup strategies (e.g., price-level targeting, and average inflation targeting) that correct the deflationary bias by committing the central bank to overheat the economy after a ZLB episode. Consequently, makeup strategies rely on history dependence which – it is often argued – makes these strategies hard to communicate to the public and possibly risky as policymakers should also commit to cause deflation if the price level or average inflation have been too high in the past.

While both approaches require the central bank to make some sort of commitment, the nature of the commitment is very different. The asymmetric strategy commits the central bank to respond asymmetrically to deviations of inflation from the central bank's target with no account for the past dynamics of inflation. The asymmetric strategy never requires the central bank to engineer an overshooting in inflation or a recession after a period of above-target inflation. In Appendix D, this important property of the asymmetric strategy is illustrated using a simulation exercise.

In the academic literature, there has been an ample discussion about the possibility of increasing the inflation target as a way to avoid the perils of the zero lower bound. An increase in the target would reduce the possibility of hitting the zero lower bound, as shown by Coibion et al. (2012). However, Nakamura et al. (2018) show that standard

models are unreliable when it comes to assess the welfare implications for the optimal inflation target. Moreover, policymakers have been quite reluctant to reconsider the target of inflation because they fear losses of reputation and argue that higher inflation is historically associated with more volatile inflation.

5.2 Counterfactual Analysis of the Asymmetric Rule

So far, the efficacy of the asymmetric rule was studied either in the absence of past shocks or by simulating the model with shocks drawn from their theoretical Gaussian distributions - the so-called average deflationary bias. Now we move a step forward and evaluate whether the asymmetric strategy would have been effective in removing the deflationary bias observed in the US over the past twenty years.

We use the calibrated model to compute core PCE inflation under the (counterfactual) assumption that the central bank had adopted the asymmetric rule in the first quarter 2000. First, we use the particle filter (see e.g. Fernández-Villaverde and Rubio-Ramírez, 2007) to estimate the structural shocks that explain the time series of real percapita GDP growth, core PCE inflation, and the federal funds rate using the model with the symmetric rule (benchmark calibration in Table 1).[9] Second we use these estimated shocks to simulate the model assuming that in the first quarter of 2000, the central bank (unexpectedly) switches to an asymmetric rule.[10] The Appendix H provides further details on the particle filter and the counterfactual analysis.

This exercise covers the first quarter of 1990 through the fourth quarter of 2019. The observables are real GDP per capita growth, PCE core inflation, and the effective federal funds rate. We calibrate the asymmetric rule so as to minimize the gap between the 10-year moving average of inflation in 2019:Q4 and the two-percent inflation target.

In the left plot of Figure 6, we compare core PCE inflation in the data to the counterfactual series of inflation, which our model with the switch to the asymmetric rule in 2000 predicts when simulated with the estimated shocks. The asymmetric strategy would have pushed inflation slightly upward throughout the entire sample. The right plot displays the 10-year moving average of core PCE inflation and the counterfactual inflation. We observe that counterfactual trend inflation fluctuates symmetrically around two percent. This finding suggests that the asymmetric strategy would have corrected the observed downward trend in the average core PCE inflation, shown in Figure 1.

[9]The policy rule is assumed to be symmetric in line with the previous framework of the Federal Reserve. The particle filter can estimate the sequence of shocks for non-linear models. We use an adapted particle filter following Herbst and Schorfheide (2015) and as applied in Aruoba et al. (2018) and Atkinson et al. (2020), among others.

[10]Rottner (2021) also uses this two-step procedure, which rests on using the particle filter to estimate the realizations of the shocks, to conduct counterfactuals.

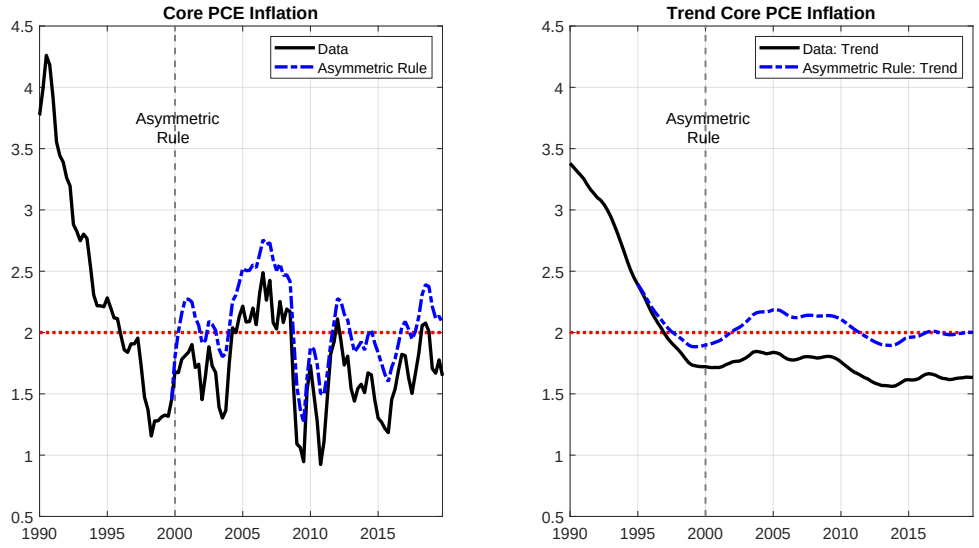

Figure 6: Counterfactual (trend) inflation dynamics with an asymmetric monetary policy rule rule. Year-to-year PCE core inflation and its ten-year moving average in the data relative to a scenario, in which the central bank adopted an asymmetric rule in 2000:Q1 onwards. The counterfactual scenario simulates the economy with an asymmetric rule using estimated structural shocks from a particle filter. Unit: Annualized percentage rates.

5.3 Welfare Analysis

In the previous section, we showed that the central bank would have hit the elusive inflation target if it had adopted an asymmetric inflation target. We now evaluate the appeal of the asymmetric strategy by measuring its impact on households' welfare W_0, defined in equation (1).

Figure 7 shows welfare W_t (left axis) and the inflation bias (right axis) as a function of the central bank response to above-target inflation in the asymmetric rule. As the central bank deviates from the symmetric strategy (the red star) by lowering the response to above-target inflation, welfare increases. The adoption of the asymmetric strategy allows the central bank to mitigate the deflationary bias, raising long-term inflation expectations and reducing the probability of falling into the ZLB in the future. The diminished risk of being constrained by the ZLB lowers macroeconomic volatility, improving welfare. When this response is close to 1.1, the welfare peaks – denoted by the blue star marker – and then it declines as the response to positive inflation deviations from target is further decreased.

It should be noticed that the asymmetric strategy that completely removes the deflationary bias, is suboptimal in that it allows too large and persistent positive deviations of inflation from the central bank's target. To see this, note that the optimal asymmetric rule solves the following trade-off. On the one hand, by tolerating some persistent positive deviations of inflation from its target the central bank manages to mitigate the deflationary bias. On the other hand, the central bank allows larger positive deviations

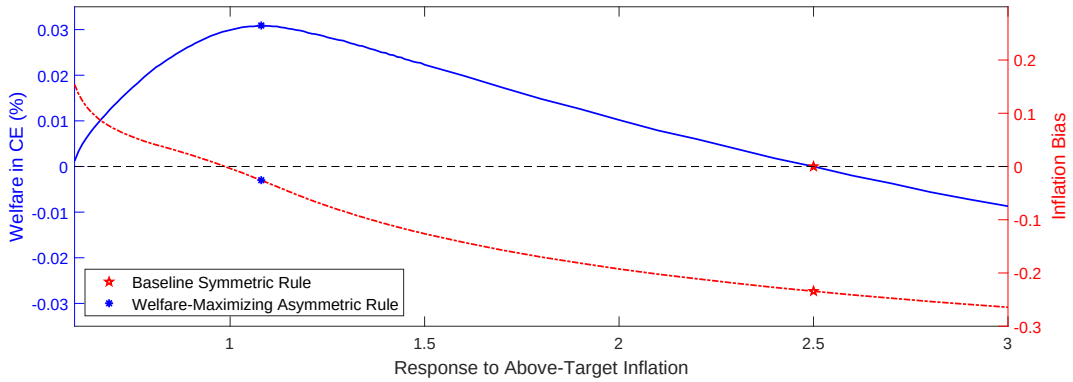

Figure 7: Welfare and inflation bias as the response to positive deviations of inflation from target varies in magnitude. Welfare bias on the left axis is shown by the blue solid line and is reported on the left axis. The inflation bias on the right axis is shown by the red dashed-dotted line and is defined as the difference between the annualized percentage rate of inflation at the stochastic steady state and the annualized percentage rate of inflation at the deterministic steady state.

of inflation from its target.

Opportunistic Reflation While we showed in Figure 7 that abandoning the symmetric rule to adopt an asymmetric strategy improves welfare, there may be cases in which it is arguably hard for the central bank to convince the public that it has adopted an asymmetric strategy. For instance, the central bank could be perceived to be myopic or unable to fully understand the functioning of the economy. In this case, the central bank needs an opportunity to show the public its commitment to the new asymmetric rule. The arrival of a shock that pushes inflation above target is such an opportunity. We call this scenario opportunistic reflation.

In this scenario, the optimal asymmetric rule widens the output and inflation gaps in the short run relative to the symmetric rule, whereas it mitigates the macroeconomic gaps in the long run. However, welfare raises both in the short run and in the longer run because the welfare gains associated with the mitigation of the macroeconomic biases outweigh the short-term losses due to the larger inflationary consequences of the shocks. In Appendix F, we show the effects of an opportunistic reflation with a simulation exercise and study the implications of a myopic central banker who does not internalize the long-term benefits of the opportunistic reflation.

5.4 Asymmetric Rules and Deflationary Spirals

As already discussed in Section 4, adopting an asymmetric strategy does not only remove the deflationary bias but it also lowers the risk for the economy of experiencing deflationary spirals. Since in our model parameters are fixed, welfare is not directly affected by this risk. Nevertheless, falling into a deflationary spiral may be very costly for the economy. The gray areas in Figure 8 denote the values of the standard deviation of preference

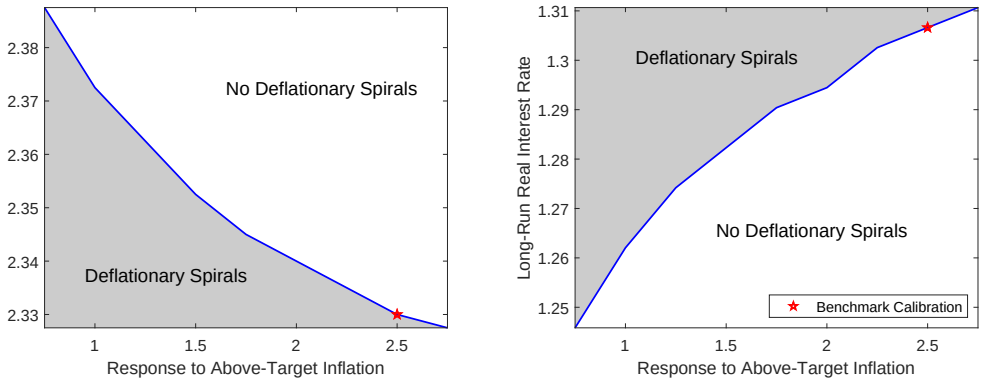

Figure 8: Asymmetric Rule and Deflationary Spirals. The left plot: the values of the standard deviation of preference shocks above which deflationary spirals arise as the above-target response to inflation varies and the below-target response is set to be equal to 2.5. The right plot: the values of the real long-term interest rate below which deflationary spirals arise as the above-target response to inflation varies and the below-target response is set to be equal to 2.5.

shocks and the values of the long-term real interest rate that trigger the deflationary spirals for any given above-target response to inflation. The bigger the asymmetry in the parameters of the rule, the larger the macroeconomic uncertainty (the smaller the real rate of interest) has to be to trigger deflationary spirals. This is because asymmetric rules lower the risk of encountering the ZLB.

Mertens and Williams (2019) study a rule according to which the Federal Reserve enforces an upper bound on the federal funds rate to resolve the deflationary bias. This rule, while correcting the bias, would imply an increase in the probability of inflationary spirals because effectively monetary policy becomes passive when inflation goes above a certain level. Therefore, such a rule reduces the risk of deflationary spirals at the cost of increasing the risk of triggering inflationary spirals. Instead, our asymmetric rule implies active responses to inflation deviations from the target and hence does not expose the economy to the risk of indeterminately large increases in inflation.

6 Target Ranges

In a recent meeting, the FOMC focused on two classes of alternative proposals to revisit the long-run monetary policy framework. The first class involves dynamic strategies that make up for periods of below-target inflation. The second class is in line with what advocated in this paper and it includes "those [strategies] that respond more aggressively to below-target inflation than to above-target inflation," (minutes of the FOMC meeting, September 17–18, 2019). According to the minutes, several FOMC members also proposed a specific way to implement the asymmetric strategy: "In this context, several participants suggested that the adoption of a target range for inflation could be helpful in achieving the Committee's objective of 2 percent inflation, on average, as it could

help communicate to the public that periods in which the Committee judged inflation to be moderately away from its 2 percent objective were appropriate." In what follows, we show that the asymmetric strategy proposed in this paper can in fact be implemented using target ranges as long as the target range is in itself asymmetric around the inflation objective.

To illustrate this point, we consider the following policy rule:

$$\frac{R_t^N}{R} = \left(\frac{R_{t-1}^N}{R}\right)^{\rho_R} \left(\left[\mathbf{1}_{\Pi_t \notin [\Pi_L, \Pi_H]} \left(\frac{\Pi_t}{\Pi}\right)^{\theta_\Pi^O} + \mathbf{1}_{\Pi_t \in [\Pi_L, \Pi_H]} \left(\frac{\Pi_t}{\Pi}\right)^{\theta_\Pi^I} \right] \left(\frac{Y_t}{Y_t^*}\right)^{\theta_Y} \right)^{1-\rho_R} \exp\left(\sigma^m \epsilon_t^m\right). \tag{9}$$

This policy rule prescribes a different response to deviations of inflation from the objective Π depending on how far inflation is from the desired level. Specifically, when inflation is inside the target range $[\Pi_L, \Pi_H]$, the central bank adjusts the interest rate less aggressively than what it does when inflation is outside the target range: $\theta_\Pi^I < \theta_\Pi^O$.[11] Such a rule is arguably easy to communicate. For example, if the in-range response θ_Π^I is set to zero, the central bank could simply announce that levels of inflation inside the target range are not reason of concern. However, an asymmetric target range is required to correct the deflationary bias. To assess the target range, we simplify the model and consider only preference shocks.[12]

In the left panel of Figure 9, we fix the in-range response to inflation to zero ($\theta_\Pi^I = 0$), while keeping the out-of-range response unchanged with respect to the benchmark case ($\theta_\Pi^O = 2.5$). We then report the target ranges that remove the deflationary bias (the solid blue line). Specifically, for each value of the lower bound of the target range, Π_L, we report on the y-axis the upper bound, Π_H, that corrects the deflationary bias. Thus, the U-shaped line reported in the panel represents all the pairs $[\Pi_L, \Pi_H]$ such that the deflationary bias is fully corrected.

We start with a lower-bound Π_L equal to 1.5%. In this case the upper bound needs to be only slightly larger than 3.0%, implying a modest level of asymmetry around the 2% objective. As the lower bound keeps increasing, the upper bound starts declining, but the asymmetry always remains. For instance, a target range $[1.75\%, 2.7\%]$ would also allow the central bank to remove the deflationary bias. To see this, note that the solid blue curve is always above the red-dashed line that implies a symmetric target range around the two-percent target. When the lower bound reaches the 2% objective, the upper bound is around 2.6%. Thus, a target region $[2\%, 2.6\%]$ is necessary to achieve the

[11] The target range rule could also be expressed in deviations from the boundaries of the target range. We prefer this formulation because it nests both a standard Taylor rule and the asymmetric rule presented above.

[12] The standard deviation of the preference shock is set to 2.50% so that the same asymmetric rule closes the bias in the simplified model.

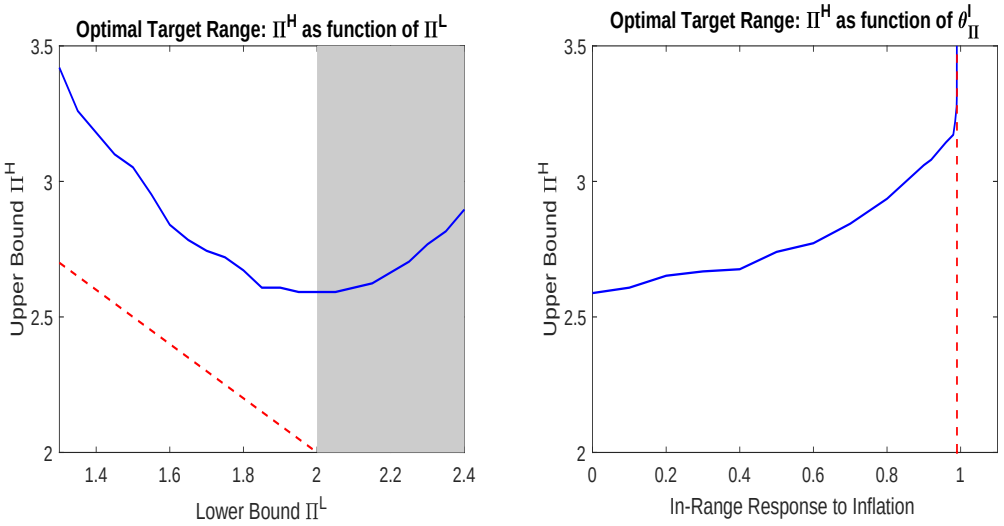

Figure 9: The target range required to close the deflationary bias. The left plot: the blue line shows the lower and upper bounds of the range that closes the deflationary bias when the central bank's in-range response to inflation is zero. The dashed red line marks the bounds implied by the symmetric target range. The right plot: the blue line shows the upper bound of the range as the central bank's in-range response to inflation varies on the horizontal axis. The lower bound of the range is fixed to 2 percent. The vertical red-dashed line is an asymptote that arises when the in-range response to inflation equals the above-target response to inflation in the asymmetric rule that removes the deflationary bias.

2% objective under the assumption of an in-range response to inflation equal to zero.

It should be noted that a target region with a lower bound equal to the 2% target is conceptually very similar to the asymmetric rule presented in Section 5. When inflation is below the objective, the response of the policy rate is strong. When inflation is above the target the response is weaker, but in a piecewise fashion. The advantage of the target range is arguably that it preserves the message that excessively high levels of inflation will not be tolerated.

The gray area of the graph denotes values of the lower bound Π_L that are *larger* than the objective 2%. While these target ranges also succeed in eliminating the deflationary bias, we believe that they are less interesting because they are not so easy to communicate: The target range now excludes the inflation objective ($\Pi_L > \Pi$). Nevertheless, we review this case for completeness. Once the lower bound become larger than the inflation objective, the upper bound of the target range starts increasing again. This is consistent with the results presented so far. Recall that in order to correct the deflationary bias, a rule needs to feature more tolerance to high inflation than to low inflation. When the target range is above the desired objective, higher and higher levels of inflation become progressively acceptable.

The right panel of Figure 9 shows that the amount of asymmetry required to correct the deflationary bias depends on the strength with which the central bank responds to inflation inside the target range. In this exercise, the lower bound of the target range is fixed to 2%. On the x-axis, we report different values of the in-range response to inflation

θ_{Π}^{I}. For each of them, the y-axis reports the upper-bound Π_H required to remove the deflationary bias. When the in-range response is equal to zero, the upper bound is around 2.6%, implying only a mild level of asymmetry around the 2% objective: $[2\%, 2.6\%]$. However, as the in-range response θ_{Π}^{I} increases, the required level of asymmetry of the target range increases. For example, with an in-range response θ_{Π}^{I} equal to 0.5, the required target range becomes: $[2\%, 2.74\%]$. This pattern accelerates as the inside-range response is raised until the blue line approaches a vertical asymptote. The level of asymmetry goes to infinity as the in-range response θ_{Π}^{I} approaches one and the target range rule collapses to the asymmetric rule of Section 5 that removes the deflationary bias. Indeed, the rule presented in Section 5 can be thought as a degenerate target range rule in which the upper bound of the target range goes to infinity.

Summarizing, a target range can be an effective way to implement an asymmetric policy strategy. However, the target range needs to be asymmetric around the desired objective for inflation. The extent of the asymmetry depends on the response to inflation inside the target range. In the benchmark case of a zero response inside the range, we show that the range needed to remove the deflationary bias is only modestly asymmetric. An asymmetric target range is arguably easy to communicate. For example, if the in-range response is set to zero, the central bank could simply announce that levels of inflation inside the target range are not reason of concern. At the same time, a target range allows the central bank to preserve the message that excessively high inflation will not be tolerated. As such, this asymmetric target range can be viewed as a good compromise between those policymakers who prefer a hawkish approach toward inflation stabilization and those who hold more dovish positions.

7 Conclusions

In an environment in which monetary policy faces the risk of encountering the zero lower bound, inflation tends to remain persistently below target, even if monetary policy is not constrained. We provide a proof of the non-existence of Rational Expectations equilibrium that arises when either long-run real interest rates or the volatility of shocks make the deflationary bias sufficiently large. An asymmetric strategy –according to which the central bank reacts less aggressively to positive deviations of inflation from its target than to negative deviations– can effectively remove this deflationary bias, improve social welfare, and reduce the risk for the economy to fall into highly costly deflationary spirals. We use a counterfactual simulation to show that this asymmetric rule would have removed the deflationary bias observed in the United States over the past twenty years.

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A Non-linear Solution Method

The model features a trend in the level of technology so that the model is detrended to induce stationarity. We outline the solution to the detrended model, where detrended variables are defined as follows $\tilde{X} = \frac{X_t}{Z_t}$.

Solving the representative household's problem yields the Euler equation

$$1 = \beta R_t \mathbb{E}_t \left[\frac{\zeta_{t+1}^d}{\zeta_t^d} \frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\Pi_{t+1} g_{t+1}} \right], \tag{10}$$

where $\lambda_t = \left(\tilde{C}_t - h\tilde{C}_{t-1}/g_t \right)^{-\sigma}$ is the adjusted multiplier on the budget constraint, $\Pi_t = P_t/P_{t-1}$ is gross inflation, and the labor supply

$$\tilde{W}_t = \chi H_t^{\eta} \lambda_t^{-1}. \tag{11}$$

The firm j produces output with labor as the only input

$$\tilde{Y}_t(j) = H_t(j) \tag{12}$$

The firm j sets the price $P_t(j)$ of its differentiated goods j so as to maximize its profits:

$$Div_t(j) = P_t(j) \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} \frac{Y_t}{P_t} - MC_t \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} Y_t - \frac{\varphi}{2} \left(\frac{P_t(j)}{\Pi P_{t-1}(j)} - 1 \right) Y_t, \tag{13}$$

subject to the downward sloping demand curve for intermediate goods. The parameter $\varphi > 0$ measures the cost of price adjustment in units of the final good.

The first order condition is

$$\begin{aligned}(\epsilon - 1) \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} \frac{Y_t}{P_t} = \epsilon \, MC_t \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon-1} \frac{Y_t}{P_t} - \varphi \left(\frac{P_t(j)}{\Pi P_{t-1}(j)} - 1 \right) \frac{Y_t}{\Pi P_{t-1}(j)} + \\ \varphi E_t \Lambda_{t,t+1} \left(\frac{P_{t+1}(j)}{\Pi P_t(j)} - 1 \right) \frac{P_{t+1}(j)}{\Pi P_t(j)} \frac{Y_{t+1}}{P_t(j)}\end{aligned} \tag{14}$$

where the stochastic discount factor $\Lambda_{t,t+1}$ is

$$\Lambda_{t,t+1} = \beta E_t \left[\left(\frac{\zeta_{t+1}^d}{\zeta_t^d} \right) \left(\frac{\lambda_{t+1}}{\lambda_t} \right) \frac{1}{g_{t+1}} \right] \tag{15}$$

In equilibrium all firms choose the same price. Thus, the New Keynesian Phillips curve is

$$\left[\varphi \left(\frac{\Pi_t}{\Pi} - 1 \right) \frac{\Pi_t}{\Pi} \right] = (1 - \epsilon) + \epsilon \, MC_t + \varphi \beta \mathbb{E}_t \left[\left(\frac{\zeta_{t+1}^d}{\zeta_t^d} \right) \left(\frac{\lambda_{t+1}}{\lambda_t} \right) \left(\frac{\Pi_{t+1}}{\Pi} - 1 \right) \frac{\Pi_{t+1}}{\Pi} \frac{\tilde{Y}_{t+1}}{\tilde{Y}_t} \right] \tag{16}$$

The monetary authority sets the interest rate R_t responding to inflation and output from their corresponding targets. The monetary authority faces a zero lower bound constraint. The policy rule reads as follows

$$R_t = \max\left[1, R_t^N\right], \tag{17}$$

$$R_t^N = \left(R_{t-1}^N\right)^{\rho_R}\left[R\left(\frac{\Pi_t}{\Pi}\right)^{\theta_\Pi}\left(\frac{\tilde{Y}_t}{\tilde{Y}}\right)^{\theta_Y}\right]^{1-\rho_R}\exp\left(\sigma^m\epsilon_t^m\right). \tag{18}$$

where R_t^N denotes the notional rate that the monetary authority would set without the zero lower bound constraint, Π and Y denote the inflation target which pins down the inflation rate in the deterministic steady state and the natural detrended output level, which is the level output that would arise if prices were flexible.

The resource constraint is

$$C_t = Y_t\left[1 - \frac{\varphi}{2}\left(\frac{\Pi_t}{\Pi} - 1\right)^2\right] \tag{19}$$

The model is solved with global methods. The agents take the presence of the zero lower bound into account and form their expectations accordingly. Therefore, the possibility of hitting the zero lower bound in the future affects potentially the equilibrium outcome in times of unconstrained monetary policy. We use time iteration with piecewise linear interpolation of policy functions as in Richter et al. (2014).[13] Expectations are calculated using numerical integration based on Gauss-Hermite quadrature.

The state variables $\mathbb{X}_t$ are $\tilde{C}_{t-1}^A$, R_{t-1}^N, ϵ_t^m, g_t and ζ_t^d while the policy variables are Π_t and labor H_t:

$$\Pi_t = g^1(\tilde{C}_{t-1}, R_{t-1}^N, \epsilon_t^m, g_t, \zeta_t^d) \tag{20}$$

$$H_t = g^2(\tilde{C}_{t-1}, R_{t-1}^N, \epsilon_t^m, g_t, \zeta_t^d) \tag{21}$$

where $g = (g^1, g^2)$ and $g^i : R^1 \to R^1$. To solve the model, we approximate the unknown policy functions with piecewise linear functions $\tilde{g}^i$ that can be written as:

$$\Pi_t = \tilde{g}^1(\tilde{C}_{t-1}, R_{t-1}^N, \epsilon_t^m, g_t, \zeta_t^d) \tag{22}$$

$$H_t = \tilde{g}^2(\tilde{C}_{t-1}, R_{t-1}^N, \epsilon_t^m, g_t, \zeta_t^d) \tag{23}$$

The time iteration algorithm to solve for the policy functions is summarized below:

1. Define a discretized grid for the states $\left\{\left[\underline{C}, \overline{C}\right], \left[\underline{R}^N, \overline{R}^N\right], \left[\underline{g}, \overline{g}\right], \left[\underline{\epsilon}^m, \overline{\epsilon}^m\right], \left[\underline{\zeta}^d, \overline{\zeta}^d\right]\right\}$ and the integration nodes $\epsilon = \left\{\left[\underline{\epsilon}^{g,I}, \overline{\epsilon}^{g,I}\right], \left[\underline{\epsilon}^{m,I}, \overline{\epsilon}^{m,I}\right], \left[\underline{\epsilon}^{\zeta^d,I}, \overline{\epsilon}^{\zeta^d,I}\right]\right\}$.

[13] This approach can handle the non-linearities associated with zero lower bound.

2. Guess the piece-wise linear policy functions $\tilde{g}(\tilde{C}_{t-1}, R^N_{t-1}, \epsilon^m_t, g_t, \zeta^d_t)$.

3. Solve for all time t variables for a given state vector ζ^d_t. The policy variables are:

$$\Pi_t = \tilde{g}^1(\tilde{C}_{t-1}, R^N_{t-1}, \epsilon^m_t, g_t, \zeta^d_t) \tag{24}$$
$$H_t = \tilde{g}^2(\tilde{C}_{t-1}, R^N_{t-1}, \epsilon^m_t, g_t, \zeta^d_t) \tag{25}$$

so that the remaining variables are given as:

$$\tilde{Y}_t = H_t \tag{26}$$
$$\tilde{C}_t = \tilde{Y}_t(1 - 0.5\varphi\left(\frac{\Pi_t}{\Pi} - 1\right)^2) \tag{27}$$
$$R^N_t = \left(R^N_{t-1}\right)^{\rho_R}\left[R\left(\frac{\Pi_t}{\Pi}\right)^{\theta_\Pi}\left(\frac{\tilde{Y}_t}{\tilde{Y}}\right)^{\theta_Y}\right]^{1-\rho_R}\exp\left(\sigma^m\epsilon^m_t\right) \tag{28}$$
$$R_t = \max\left[1, R^N_t\right] \tag{29}$$
$$\lambda_t = \left(\tilde{C}_t - h\tilde{C}_{t-1}/g_t\right)^{-\sigma} \tag{30}$$
$$W_t = \chi H^\eta_t \lambda^{-1}_t \tag{31}$$
$$MC_t = \tilde{W}_t \tag{32}$$

Calculate the state variable for period $t+1$ at each integration node i:

$$\zeta^{d,i}_{t+1} = \exp\left(\rho_\zeta \log(\zeta^d_t) + \epsilon^{\zeta^d,i}_{t+1}\right) \tag{33}$$
$$g^i_{t+1} = \bar{g} + \epsilon^{g,i}_{t+1} \tag{34}$$
$$\epsilon^{m,i}_{t+1} = \epsilon^{m,i}_{t+1} \tag{35}$$

For each integration node $g^i_{t+1}, \epsilon^{m,i}_{t+1}\zeta^{i,d}_{t+1}$, calculate the policy variables and solve for output and consumption:

$$\Pi^i_{t+1} = \tilde{g}^1(\tilde{C}_t, R^N_t, \epsilon^{m,i}_{t+1}, g^i_t, \zeta^{d,i}_t) \tag{36}$$
$$H^i_{t+1} = \tilde{g}^2(\tilde{C}_t, R^N_t, \epsilon^{m,i}_{t+1}, g^i_t, \zeta^{d,i}_t) \tag{37}$$
$$\tilde{Y}^i_{t+1} = H^i_{t+1} \tag{38}$$
$$\tilde{C}^i_{t+1} = \tilde{Y}^i_{t+1}(1 - 0.5\varphi\left(\frac{\Pi^i_{t+1}}{\Pi} - 1\right)^2) \tag{39}$$

Calculate the errors for the Euler Equation and the New Keynesian Phillips curve

$$err_1 = 1 - \beta R_t E_t\Big[\frac{\zeta^d_{t+1}}{\zeta^d_t}\frac{\lambda_{t+1}}{\lambda_t}\frac{1}{\Pi_{t+1}g_{t+1}}\Big] \tag{40}$$

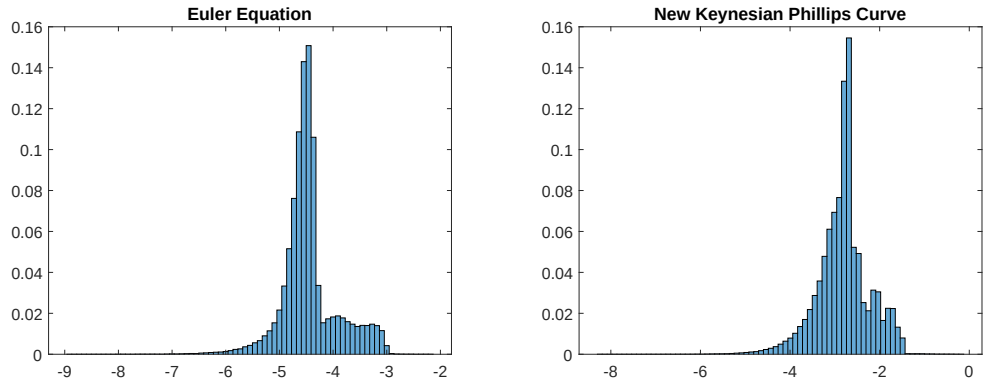

Figure 10: Histogram of the residual errors in the Euler equation and New Keynesian Phillips Curve based on a simulation of 200000 periods. The residual errors, which are displayed on the vertical axis, is transformed with the common logarithm.

$$err_2 = \varphi\left(\frac{\Pi_{t+1}}{\Pi} - 1\right)\frac{\Pi_t}{\Pi} - (1-\epsilon) - \epsilon MC_t - \beta E_t \varphi \left(\frac{\zeta^d_{t+1}}{\zeta^d_t}\right)\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left(\frac{\Pi_{t+1}}{\Pi} - 1\right)\left(\frac{\Pi_{t+1}}{\Pi}\right)\frac{\tilde{Y}_{t+1}}{\tilde{Y}_t} \tag{41}$$

where the expectations are numerically integrated across the integration nodes. The nodes and weights are based on Gaussian-Hermite quadrature.

4. Use a numerical root finder to minimize the errors for the equations.

5. Update the policy functions until the errors at each point of the discretized state are sufficiently small.

We discretize the two endogenous state variables R^N and $\tilde{C}$ in 11 evenly-spaced points with bounds at $\pm 2\%$ and $\pm 3.75\%$ around their respective deterministic steady state. The preference shock ζ^d_t is discretized in 15 evenly-spaced points with bounds chosen to be $\pm 6\sigma^{\zeta^d}$ around the deterministic steady state. The remaining two shocks g_t and ϵ^m_t are discretized in 7 evenly-spaced points, where the bounds are chosen to be $\pm 3\sigma^g$ and $\pm 3\sigma^m$, respectively, around the deterministic steady state. This results in a total of 88935 nodes. The Gauss-Hermiture quadradutre nodes provides the integration nodes $\left[\epsilon^{g,i}, \epsilon^{m,i}, \epsilon^{\zeta^d,i}\right]$ and the corresponding weights $\xi(i)$ for all integration nodes $i \in \{1, 2, \ldots, I\}$. We use 9 nodes for the preference shock, 5 nodes for the monetary policy shock and 5 nodes for the growth shocks so that we evluate the expectations using $I = 225$ weighted points.

An overview of the numerical accuracy is provided in Figure 10, where the distribution of the residual error of the Euler equation and the New Keynesian Phillips Curve for the baseline economy with the zero lower bound and symmetric monetary policy based on a simulation of 20000 periods is shown.

B A Model with Binary Realizations of the Shock

In this binary case, we treat the Taylor rule in the good state and all the other remaining equilibrium equations separately. Using different candidates of inflation for the good state (Π^H), we calculate two nominal interest rates for the good state $R^{H1}(\Pi^H)$ and $R^{H2}(\Pi^H)$. The first one stems from the Taylor rule, while the other one results from the other remaining equations.

The candidate for the nominal interest rate $R^{H1}(\Pi^H)$ resulting from of the Taylor rule in the good state reads as follows:

$$R^{H1} = \max\left[1, R\left(\frac{\Pi^H}{\Pi}\right)^{\theta_\Pi}\right]$$

This equation corresponds to the red line in Figure 2.

The other equilibrium equations in the good state give another solution for the nominal interest conditionally on Π^H. The remaining equations in the good state are given as:

$$1 = \beta R^{H2}\Big[(1-p)\frac{\zeta_L^d}{\zeta_H^d}\Big(\frac{C^H}{C^L}\Big)^\sigma \frac{1}{\Pi^L} + p\frac{1}{\Pi^H}\Big], \tag{42}$$

$$Y^H = H^H, \tag{43}$$

$$MC^H = \chi H_t^{H\eta} c^{H\sigma}, \tag{44}$$

$$C^H = Y^H\Big(1 - \varphi\Big(\frac{\Pi^H}{\Pi} - 1\Big)^2/2\Big) \tag{45}$$

$$\varphi\Big(\frac{\Pi^H}{\Pi} - 1\Big)\frac{\Pi^H}{\Pi} = (1-\epsilon) + \epsilon MC^H \tag{46}$$

$$+ \varphi\beta\left[(1-p)\frac{\zeta_L^d}{\zeta_H^d}\Big(\frac{C^H}{C^L}\Big)^\sigma\Big(\frac{\Pi^L}{\Pi} - 1\Big)\Big(\frac{\Pi^L}{\Pi}\Big)\frac{Y^L}{Y^H} + p\Big(\frac{\Pi^H}{\Pi} - 1\Big)\Big(\frac{\Pi^H}{\Pi}\Big)\right]$$

Since the good-state equilibrium outcomes depend on the bad state, we have to solve for the equilibrium in the bad state. An equilibrium in the bad state satisfies the following equations:

$$R^L = \max\left[1, R\Big(\frac{\Pi^L}{\Pi}\Big)^{\theta_\Pi}\right] \tag{47}$$

$$1 = \beta R^L\Big[(1-q)\frac{\zeta_H^d}{\zeta_L^d}\Big(\frac{C^L}{C^H}\Big)^\sigma \frac{1}{\Pi^H} + q\frac{1}{\Pi^L}\Big], \tag{48}$$

$$Y^L = H^L, \tag{49}$$

$$MC^L A = \chi H_t^{L\eta} c^{L\sigma}, \tag{50}$$

$$C^L = Y^L\Big(1 - \varphi\Big(\frac{\Pi^L}{\Pi} - 1\Big)^2/2\Big) \tag{51}$$

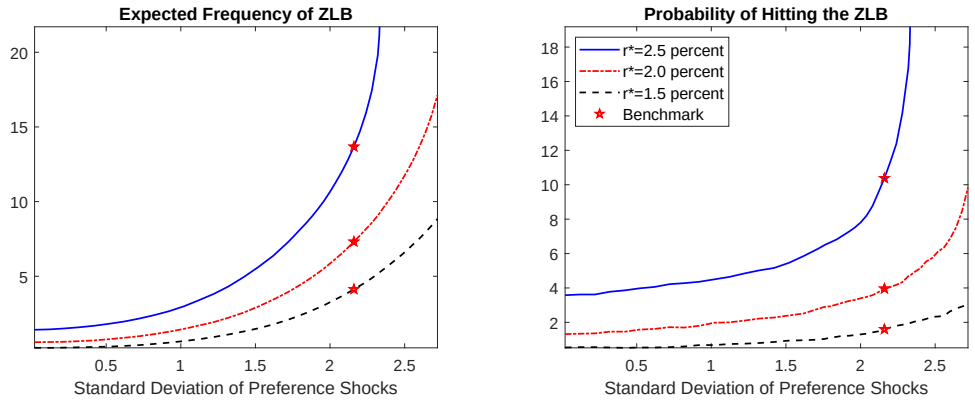

Figure 11: The risk of the zero lower bound. Left graph: Expected frequency of the zero lower bound as the variance of preference shocks varies and for different values of the long-run real rate. The frequency is in percentage points and it is computed as the ratio between the number of periods spent at the zero lower bound and the total sample size (200,000). Right graph: Probability of hitting the zero lower bound in the next year conditional on being at the stochastic steady state in the current period for different values of the variance of preference shocks and of the steady-state real rate. The probability is expressed in percentage points.

$$\varphi\left(\frac{\Pi^L}{\Pi}-1\right)\frac{\Pi^L}{\Pi}=(1-\epsilon)+\epsilon MC^L \tag{52}$$
$$+\varphi\beta\left[(1-q)\frac{\zeta_H^d}{\zeta_L^d}\left(\frac{C^L}{C^H}\right)^{\sigma}\left(\frac{\Pi^H}{\Pi}-1\right)\left(\frac{\Pi^H}{\Pi}\right)\frac{Y^H}{Y^L}+q\left(\frac{\Pi^L}{\Pi}-1\right)\left(\frac{\Pi^L}{\Pi}\right)\right]$$

Equations (42) to (47) give us a solution for the nominal interest rate $R^{H2}(\Pi^H)$. The nonlinear root solver is applied at this step as this system cannot be solved analytically.[14] The mapping of Π^H to R^{H2} corresponds to the blue solid line in Figure 2. To calculate a hypothetical economy without a zero lower bound in the bad state, we we assume that the ZLB constraint is not binding in that state. This gives us the dash-dotted blue line in Figure 2.

An equilibrium for the economy exists for a given inflation in the good state Π^H if $R^{H1}(\Pi^H)=R^{H2}(\Pi^H)$. This corresponds to an intersection of the red and the blue line in Figure 2. Looping over Π^H allows to check the existence of equilibria and find all possible solutions of the economy with binary realizations of the preference shock.

[14]To handle the kink in the Taylor rule in the low state, we use a guess and verify approach in practice. First, we solve the whole system assuming that the Taylor rule is not binding in the bad state. We keep the results if the result does not violate the zero lower bound in the bad state. Then, we guess that zero lower bound is binding in the bad state and keep the results if this is indeed the bad-equilibirum outcome.

C The Probability of Hitting the ZLB

The left plot of Figure 11 shows the percentage of periods spent at the ZLB when the model is simulated for a long period of time (200,000 periods).[15] In technical jargon, this is the ergodic probability of being constrained by the ZLB. As shown in the figure, this probability is affected by how volatile the preference shocks are (x-axis). The different lines are associated with different assumptions about the long-run annualized real rate of interest $r^* = \overline{g}\beta^{-1}$. Our benchmark calibration for this parameter is 1.5 percent. The red stars on the lines denote the calibrated standard deviation of the preference shock.

A lower long-term real interest rate raises the expected frequency of the ZLB as it shrinks the central bank's room of maneuver to counter the deflationary effects of recessionary shocks. We are closer to the bound on average so the central bank is expected to hit the lower bound more often. Note that the expected frequency of the ZLB as a function of macroeconomic volatility grows at an increasing speed as the long-term real interest rate r^* falls. Symmetrically, a given drop in the long term real interest rate r^* implies larger increases in the probability of encountering the ZLB if the volatility of the shock is higher. Thus, the more volatile shocks are and the lower r^* is, the higher the expected frequency of the ZLB, with the two effects reinforcing each other.

The graph on the right shows how likely it is for monetary policy to become constrained by the ZLB in the next year conditional on being currently at the (stochastic) steady state. As for the expected frequency of the ZLB, we study how this probability varies as we change the standard deviation of the preference shocks and the steady-state real rate of interest r^*. The larger the volatility of the shock, the more likely it is that the ZLB will be binding in the next year. It should be noted that the probability rises exponentially with the volatility of the shock. Lowering the long-term real rate of interest leads to similar results.

The worrying finding highlighted by both graphs is that in a low real-interest rate environment (low r^*, black dashed lines) the two functions are very steep. This means that even a small increase in the volatility of the shocks can lead to substantial increases in the probability of encountering the zero lower bound. Recall that our benchmark calibration for the .volatility of the preference shock is arguably very low for the U.S., given that it was chosen to match the level of volatility during the Great Moderation. The results above imply that even a small increase in macroeconomic volatility may lead agents to believe that the ZLB constraint has become a pervasive problem for monetary policy. These beliefs cause serious macroeconomic biases and distortions and can potentially lead to deflationary spirals

[15] Fernández-Villaverde et al. (2015) discuss the challenge to capture the length and duration of a zero lower bound spell.

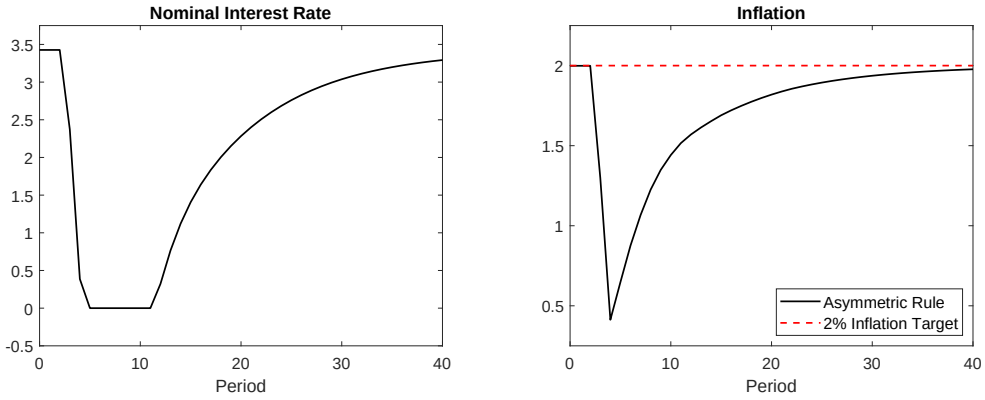

Figure 12: Simulations of inflation and nominal interest rate during an artificial recession. The economy is at its stochastic steady state in period 0, 1, and 2. From period 3 through period 4, the economy is hit by a three-standard-deviation negative preference shock in every period. Starting from period 5 no more shocks occur and the economy evolves back to its stochastic steady-state equilibrium. Units: percentage points of annualized rates.

D The Asymmetric Strategy is Not a Makeup Strategy

In this appendix we will show that the asymmetric strategy does not require the central bank to engineer an overshooting in inflation after a ZLB episode as makeup strategies (e.g., price-level targeting, average inflation targeting. etc.) do. To this end, we simulate the economy under a sequence of negative shocks large enough to bring the economy to the zero lower bound for a certain number of periods. We assume that the central bank is following the asymmetric rule that removes the deflationary bias. Figure 12 shows the path for the endogenous variables. We assume that the economy is initially at its stochastic steady states. In period 3 and 4, negative demand shocks hits the economy. The size of each shock is three standard deviations. Starting from period 9 no more shocks occur and the economy slowly goes back to the stochastic steady state.

In the left plot of Figure 12, the ZLB is binding after the negative preference shocks hit the economy. After the ZLB period, no more shocks hit the economy and the central bank lifts the nominal interest rate off the ZLB constraint. In the right plot of Figure 12, the dynamics of inflation in the simulation is reported. Inflation falls as the economy is hit by the negative preference shocks. As the effects of these shocks fade away, the inflation rate converges to the desired two-percent inflation target. Note that inflation converges to the desired target from below because the central bank does not try to overshoot its inflation target as it would have done if it had adopted a makeup strategy.

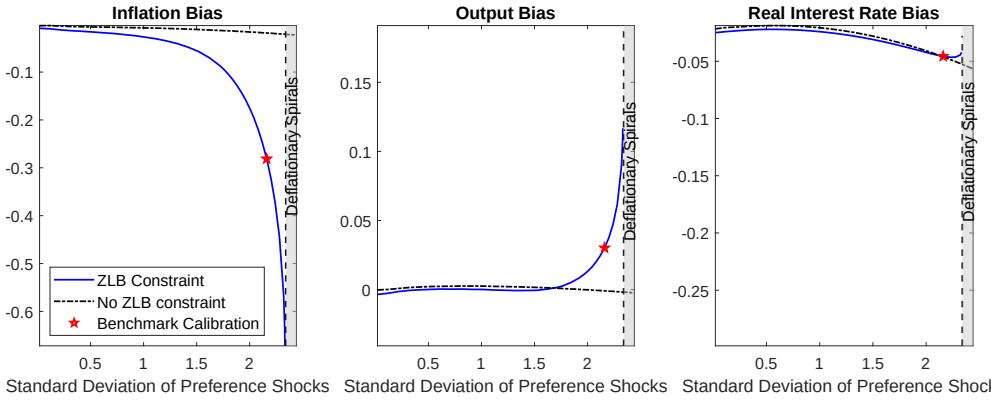

Figure 13: Average macroeconomic biases as the volatility of the preference shock varies. The bias is computed by taking the mean of inflation, output, and the real interest based on a simulation lasting 250,000 periods. We drop the first 50,000 observations to minimize the effects of initial conditions. The biases are reported on the same scale used in Figure 3.

E The Average Bias

Figure 13 reports the average bias as the volatility of the preference shock varies. The average bias is computed by taking the mean of inflation, output, and the real interest based on a simulation lasting 250,000 periods. We drop the first 50,000 observations to minimize the effects of initial conditions.

While the average deflationary bias is always larger, it turns out to be highly correlated with the other definition of deflationary bias based on the notion of stochastic steady-state equilibrium. When it comes to the behavior of output and the real interest rate, the bias is largely gone (unless the economy gets very close to deflationary spirals and then the output bias opens up). When looking at the average bias for the real interest rate, there is a countereffect that pushes the bias to be positive. This countereffect is brought about by the presence of the ZLB itself that truncates the left tail of the distribution of the nominal interest rate. Thus, the negative bias that arises away from the zero lower bound is compensated by the fact that at the zero lower bound the central bank cannot further lower the interest rate, making the effective real interest rate too high. Importantly, the two phenomena are just the two sides of the same coin: The negative bias away from the zero lower bound is generated by the deflationary pressure that arises exactly because at the zero lower bound the central bank is not able to lower the interest rate to mitigate the fall in inflation.

F Opportunistic Reflation

We investigate the implications for welfare and the macroeconomic outcomes of a central bank pursuing an opportunistic reflation with a simulation exercise. Let us assume that the economy is initially at the stochastic steady state associated with the symmetric rule

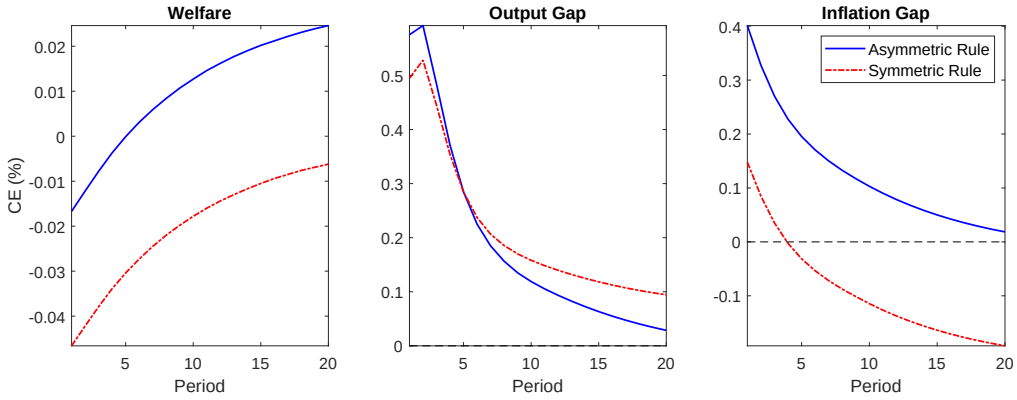

Figure 14: The dynamics of welfare, the output gap, and the inflation gap after a two-standard-deviation positive preference shock hits the economy in period 1. Two cases are reported: the case in which the central bank adopts the optimal asymmetric rule and conducts an opportunistic reflation of the economy (solid blue line) and the case in which the central bank does not take this opportunity and sticks to the symmetric rule (red dashed-dotted line). In both cases, the economy is initialized at its stochastic steady state. Units: Inflation gap is measured in percentage points of annualized rates while the output bias is expressed in percentage points.

when it gets hit by a positive preference shock that boosts consumption and aggregate demand. The central bank receives now the opportunity to show to the private sector that it is willing to commit to the optimal asymmetric rule by responding less aggressively to the inflation consequences of this shock. It is assumed that by observing the muted response to inflation, the private sector immediately believes that the central bank will follow the asymmetric rule forever.

In Figure 14, we show the impulse response function of welfare and the macroeconomic gaps (inflation and output) to a two standard deviation positive preference shock under the symmetric rule and under the optimal asymmetric rule. The output gap is measured in deviations from the flexible price economy whereas the inflation gap is expressed in deviations from the central bank's two-percent target. The optimal asymmetric rule raises the output and inflation gaps in the short run relative to the symmetric rule whereas it mitigates the macroeconomic gaps in the longer run. Welfare is reported in the left graph of Figure 14, which shows that the optimal asymmetric rule raises welfare both in the short run and in the longer run.

Why is welfare higher in every period when the central bank adopts the asymmetric rule even though this rule causes output and inflation gaps to widen more at the beginning? Welfare does not depend only on the current inflation and output gaps but it is also affected by the expected discounted stream of welfare gains that will be accrued over time. The short-term responses of social welfare to a two-standard-deviation positive preference shock implies that the long-term welfare gains associated with the mitigation of the macroeconomic biases outweigh the short-term welfare losses.[16]

[16]Under the asymmetric rule, the weaker systematic response to inflation raises agents' long-run uncertainty about inflation and hence, everything else being equal, lowers welfare in the long-run. However, in our model these losses are dominated by the gains from removing the deflationary bias.

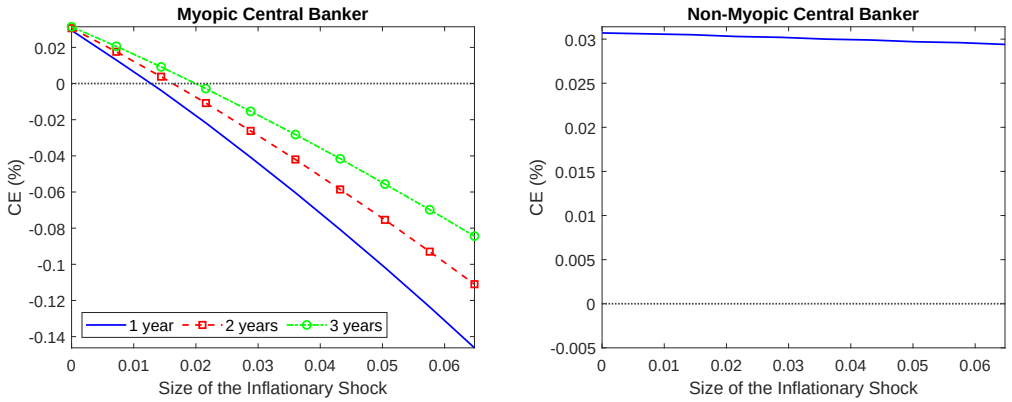

Figure 15: Welfare gains/losses from carrying out an opportunistic reflation as the size of the inflationary shock varies under different assumptions about how forward looking the central banker is. The left plot shows the myopic central banker's case and the different lines refer to different degrees of myopia; that is, the horizon k the central banker cares about when computing welfare gains/losses. The right plot shows the case of the benevolent central banker who maximizes the households' utility and thereby cares about the welfare gains at all horizons. Welfare gains/losses are computed as the difference between the welfare associated with adopting the optimal asymmetric rule and the welfare associated with sticking to the benchmark symmetric rule in the period when the inflationary shock hits the economy.

The opportunistic reflation involves a trade-off between short-term and long-term macroeconomic stabilization. Hence, a myopic central bank may refrain from seizing this opportunity as welfare costs are mostly front-loaded.[17] To further investigate this issue, we tweak the welfare function 1 to study the behaviors of a myopic central banker who only cares about the welfare gains accrued up to a finite time horizon k. The welfare of the myopic central banker is denoted by $\widetilde{W}_0^k$, which is defined as follows:

$$\widetilde{W}_0^k = E_0 \sum_{t=0}^{k} \beta^t \zeta_t^d \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{H_t^{1+\eta}}{1+\eta} \right] \tag{53}$$

The left plot of Figure 15 shows the myopic central bank's welfare gains from carrying out an opportunistic reflation following a positive preference shock as the size of the shock varies. The gains are computed by taking the difference of the welfare under the asymmetric rule and welfare under the benchmark symmetric rule at the time the inflationary shock hits the economy. The level of asymmetry is the one we find to be optimal for the non-myopic central banker. The different lines are associated with four degrees of the central banker's myopia, which is captured by the relevant horizons $k = 4$, 8, and 12 quarters. The shorter the horizon k, the more myopic the central banker. The gains are shown as a function of the size of the shock. The myopic central banker's gains decline as the size of the preference shocks increases and, hence, the short-run response of inflation to the shock is more pronounced. The speed of this decline increases as the myopia of the central banker becomes less severe.

[17] In what follows, a myopic central bank can also be interpreted as a conservative central bank that cares too much about the short-term inflation consequences of its actions.

If the relevant horizon is less or equal than four quarters ($k \leq 4$), gains are negative for all positive shock sizes. Such high levels of myopia dissuade the central bank from seizing the opportunity of reflating the economy as the policymaker is more allured by the short-run welfare gains, which stem from mitigating the immediate inflationary consequences of the shock. If the myopic central bank has a horizon of two years, it will opportunistically reflate the economy if the standard deviation of preference shocks is lower than two. Lower degrees of myopia (higher k) lead the central bank to carry out the opportunistic reflation even when the magnitude of the shock is large and the likely short-run inflationary consequences of the shock are considerable.

The right plot of Figure 15 shows the welfare gains from opportunistic reflation for the case of the non-myopic/benevolent central banker ($k \longrightarrow \infty$). In this case, the optimal asymmetric rule dominates the symmetric rule if the size of the shock is less than 6 times the calibrated standard deviations of the shocks (i.e., $100\sigma_{\zeta^d} = 1.175$). We consider this value as fairly high, which suggests that opportunistic reflation increases the economy's welfare by removing the deflationary bias, as long as the central bank internalizes the long term benefits of the policy.

Finally, if no opportunity to reinflate the economy occurs, the central bank can implement the asymmetric strategy by cutting the rate more aggressively when inflation is below target. This action shows to the public that the central bank has credibly adopted an asymmetric strategy. Appendix G shows that this alternative asymmetric strategy also removes the deflationary bias by lowering the probability of hitting the ZLB.

G Strategic Interest Rate Cuts

We showed that if the central bank seizes the opportunity of reflating the economy by adopting an asymmetric rule after an inflationary shock arises, social welfare generally increases. If no opportunity to reflating the economy arises, the central bank can still remove the deflationary bias and improves welfare by cutting more aggressively the interest rate if inflation is below target while clarifying that the response to inflation above target is unchanged.

This alternative asymmetric rule also eliminates the macroeconomic biases. The upper panels of Figure 16 report the behavior of the macroeconomic biases defined with respect to the stochastic steady state (blue solid lines) and the observable averages (red dashed lines) as the response to below-target inflation, θ_Π, varies. The response to positive deviations of inflation from the target is the same as in the symmetric rule ($\overline{\theta_\Pi} = 2.5$). The red star denotes the distortions under a symmetric rule ($\underline{\theta_\Pi} = \overline{\theta_\Pi} = 2.5$) as in the baseline calibration. The response to inflation below target that zeroes the biases is approximately 4.3.

The effects of adopting this asymmetric rule on the probability of hitting the ZLB

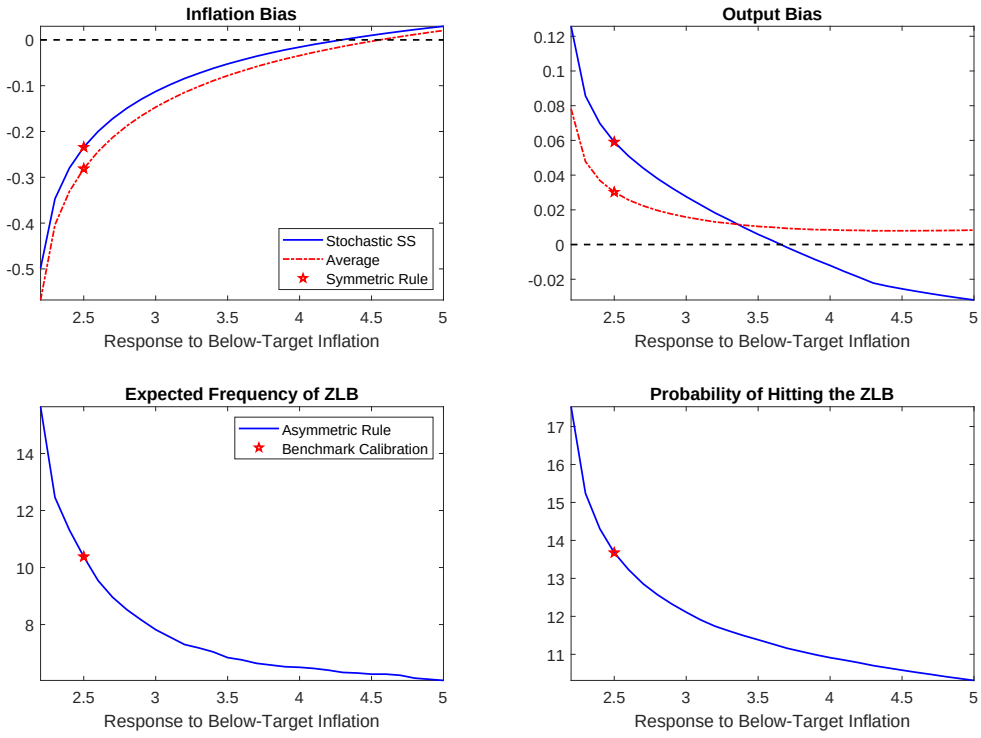

Figure 16: Macroeconomic biases due to risk of hitting ZLB under the asymmetric rule. The biases are computed relatively to the stochastic steady state (blue solid line) or the average inflation (red dashed-dotted line) and are shown in the upper panels. The output gap is expressed in percentage points and inflation gap is expressed in percentage points of annualized rates. The lower panels show the expected frequency of the ZLB (left) and the risk of hitting the ZLB in the next four quarters (right) as the response to inflation below target varies. The frequency is in percentage points and it is computed as the ratio between the number of periods spent at the zero lower bound and the total sample size (200,000). The probability of hitting the zero lower bound in the next period is conditional on being at the stochastic steady state in the current period and is expressed in percentage points.

and the frequency of ZLB episodes is ambiguous ex ante. On the one hand, lowering more vigorously the nominal interest rate to fight against deflationary pressures could increase the probability of hitting the zero lower bound. On the other hand, committing to respond more aggressively to negative deviations of inflation from target eliminates the deflationary bias and thereby raises the long-term nominal interest rate. Higher nominal rates cause the likelihood of hitting the ZLB to fall. As shown in the lower panels of Figure 16, the asymmetric rule that allows the central bank to remove the macroeconomic bias ($\theta_\Pi = 4.3$) lowers the probability of hitting the ZLB and the expected frequency of ZLB episodes.

H Particle Filter and Counterfactual Analysis

In this part, we provide further details on the algorithm for the particle filter, specifiy the measurement equation in detail and show some additional results.

We estimate the sequence of shocks with the the adapted particle filter outlined in Herbst and Schorfheide (2015) and Aruoba et al. (2018). We also illustrate how to use the estimated shocks to conduct then a counterfactual policy analysis.

Measurement Equation For the estimation of the shocks, we use a measurement equation that connects the observables to the non-linear model outcomes:

$$\mathbb{Y}_t = h(\mathbb{X}_t) + \nu_t, \tag{54}$$

where $\mathbb{Y}_t$ are the observables, $\mathbb{X}_t$ are the state variables and the measurement error ν_t follows a normal distribution $\nu_t \sim N(0, \Sigma_\nu)$.[18] The function h maps the state variables to the observables. In our context, the observables are the quarterly growth rate of GDP per capita, the annualized PCE core inflation rate and the annualized Federal Funds rate, so that the observation equation can be written as

$$\begin{bmatrix} \text{GDP Growth Per Capita} \\ \text{PCE Core Inflation Rate} \\ \text{Federal Funds Rate} \end{bmatrix} = \begin{bmatrix} 100 \frac{\tilde{Y}_t - \tilde{Y}_{t-1}/g_t}{\tilde{Y}_{t-1}/g_t} \\ 400\left(\Pi_t - 1\right) \\ 400\left(R_t - 1\right) \end{bmatrix} + \nu_t \tag{55}$$

where the variance Σ_ν of the measurement error is set to 5% of the sample variance of the data. Our sample covers 1990:Q1 to 2019:Q4 so that the number of periods T is 140.

Algorithm The description of the algorithm follows in large part (in particular step 1 and 2) Atkinson et al. (2020) and Rottner (2021) and is included for completeness. Before moving to the algorithm, it is helpful to define the structural shocks as $\epsilon_t \equiv \{\epsilon_t^{\zeta^d}, \epsilon_t^g, \epsilon_t^m\}$ and the state variables as $\mathbb{X}_t \equiv \{\tilde{C}_{t-1}, R_{t-1}^n, \zeta_t^d, g_t, \epsilon_t^m\}$. The number of particles Q is set to 100000.

1. **Initialization**: A sequence of random shocks for 25 periods for each particle is drawn: $\{\nu_{t,q}\}_{-24}^{0}$ $\forall q \in \{1, \ldots, Q\}$. Starting from the stochastic steady state, we use this sequence to simulate the economy forward. This provides then the starting point for the state variables.

2. **Recursion**: This step is repeated for periods $t = 1, \ldots, T$

   (a) The structural shocks are drawn from an adapted proposal distribution:

   $$\epsilon_{t,q} \sim N\left(\bar{\epsilon}_t, I\right), \tag{56}$$

   which is derived as follows:

[18] The measurement error is necessary to avoid a degeneracy of the likelihood.

i. The solution of the model for the average state vector $\overline{\mathbb{X}}_{t-1} = 1/Q \sum_{q=1}^{Q} \mathbb{X}_{t-1,q}$ and a guess of $\overline{\epsilon_t}$ is used to update $\mathbb{X}_t$ and calculate the observables of the model as defined in equation (55).

ii. The measurement error ν_t as defined in equation 55 is then calculated, which follows a multivariate normal distribution σ_ν. This gives us then the probability of observing the measurement error:

$$p(\nu_t|\mathbb{X}_t) = (.5\pi)^{-n/2}|\sigma_\nu|^{-0.5} \exp\left(-0.5\nu_t'\sigma_\nu^{-1}\nu_t\right), \tag{57}$$

where n is the number of observables. This would be 3 as we include output growth, inflation and the nominal interest rate.

iii. The probability of observing $\mathbb{X}_t$ conditional on the average state vector $\overline{\mathbb{X}}_{t-1}$:

$$p(\mathbb{X}_t|\overline{\mathbb{X}}_{t-1}) = (.5\pi)^{-n/2} \exp\left(-0.5\overline{\epsilon}_t'\overline{\epsilon}_t\right), \tag{58}$$

iv. The proposal distribution is determined by the $\overline{\epsilon}_t$ that maximizes

$$p(\nu_t|\mathbb{X}_t)p(\mathbb{X}_t|\overline{\mathbb{X}}_{t-1}) \propto \exp\left(-0.5\nu_t'\sigma_\nu^{-1}\nu_t\right) \exp\left(-0.5\overline{\epsilon}_t'\overline{\epsilon}_t\right), \tag{59}$$

We use a numerical root finder to determine $\overline{\epsilon}_t$.

(b) The drawn shocks $\epsilon_{t,q}$ are used to simulate the economy one period forward to obtain the new state variables $\mathbb{X}_{t,q}$ based on $\mathbb{X}_{t-1,q}$.

(c) The measurement error $\nu_{t,m}$ is calculated for all particles, which can be used to determine the incremental weights of each particle q:

$$w_{t,q} = \frac{p(\nu_{t,q}|\mathbb{X}_{t,q})p(\mathbb{X}_{t,q}|\mathbb{X}_{t-1,q})}{g(\mathbb{X}_{t,q}|\mathbb{X}_{t,q-1})} \propto \frac{\exp\left(-0.5\nu_{t,q}'\sigma_\nu^{-1}\nu_{t,q}\right) \exp\left(-0.5\epsilon_{t,q}'\epsilon_{t,q}\right)}{\exp\left(-0.5(\epsilon_{t,q} - \overline{\epsilon}_t)'(\epsilon_{t,q} - \overline{\epsilon}_t)\right)} \tag{60}$$

(d) The particles are resampled based on their normalized weights, which are given as

$$W_{t,q} = \frac{w_{t,q}}{\sum_{q=1}^{Q} w_{t,q}} \tag{61}$$

We resample the particles based on their weights and obtain the distribution of state variables $\mathbb{X}_t$.[19]

[19]The particle filter can approximate the log-likelihood function of the model, which is given as $\ln(L) = \sum_{t=1}^{T} \ln(l_t)$ with $\ln(l_t) = \ln\left(\frac{1}{Q}\sum_{q=1}^{Q} w_{t,q}\right)$.

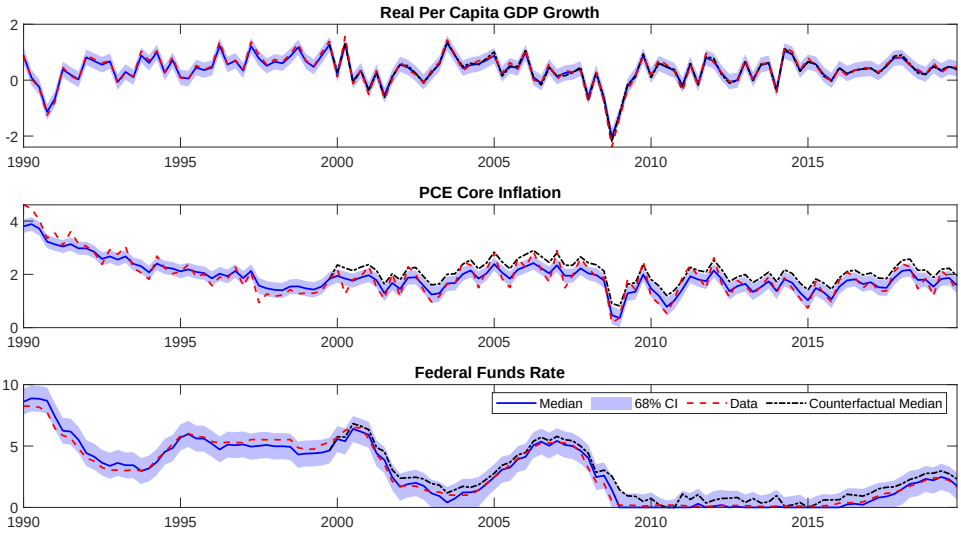

Figure 17: Comparison of the observables (quarter-to-quarter real per capita GDP growth rate, quarter-to-quarter PCE core inflation and the federal funds rate). The solid blue line is the median and the shaded area is the 68% CI. The dash-dotted black line is the median for counterfactual scenario with an asymmetric rule. Units: Annualized rate for inflation and the interest rate

3. **Counterfactual**: The particle filter estimates the sequence of shocks $\left\{\{\epsilon_{t,q}\}_{q=1}^{Q}\right\}_{t=1}^{T}$ with its normalized weights $\left\{\{W_{t,q}\}_{q=1}^{Q}\right\}_{t=1}^{T}$. We now return to the point of initialization and use the estimated shock series to propagate the economy forward (we also use the obtained weights from step 2 to resample the state vector $\mathbb{X}_{t,q}$.) However, we now use the asymmetric rule from 2000:Q1 forward to propagate the economy. This gives us now a counterfactual path for the observable variables of growth rate of real GDP per capita, PCE core inflation and the Federal Funds Rate.

Additional results Figure 17 shows the dynamics of the observables (quarter-to-quarter real per capita GDP growth rate, quarter-to-quarter PCE core inflation and the federal funds rate).[20] The solid blue line is the filtered median with its 68% confidence interval (blue shaded area) and the red line is the data. The model can captures the dynamics of the observables. The filtered median tracks well the period of a binding ZLB as the median suggests a binding zero lower bound most of the time between 2009:Q1 and 2015:Q4.[21] The black dash-dotted scenario shows the counterfactual with an asymmetric rule. This highlights how an asymmetric rule can push inflation upwards. We use this filtered results as input for the counterfactual analysis of trend inflation under an

[20]Instead of directly moving from the initialization to the recursion, we additionally estimate the sequence of shocks from 1985:Q1 1985:Q1 to 1989:Q4 to better initialize the particle filter.

[21]We leave the federal funds rate unchanged, which implies that the particle filter needs to use the measurement error to capture a zero lower bound episode. The results are robust to setting the federal funds rate to zero for the period from 2009:Q1 until 2015:Q4.

asymmetric rule as shown in Figure 6. For this picture, we map the quarter-to-quarter counterfactual to a year-to-year inflation measure.