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The incentive effects of monetary policy on fiscal policy behaviour

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Joost Röttger
Rafael Gerke

Deutsche Bundesbank, Wilhelm-Epstein-Straße 14, 60431 Frankfurt am Main,
Postfach 10 06 02, 60006 Frankfurt am Main

Tel +49 69 9566-0

Please address all orders in writing to: Deutsche Bundesbank,
Press and Public Relations Division, at the above address or via fax +49 69 9566-3077

Internet http://www.bundesbank.de

Non-technical summary

In January 2020, the Eurosystem launched its first monetary policy strategy review since 2003. The review process, which culminated in the publication of the monetary policy strategy statement in July 2021, involved various seminars and work streams dedicated to a wide range of topics deemed relevant from a monetary policy perspective (see Deutsche Bundesbank, 2021). The covered topics included the potential negative side effects of expansionary monetary policy measures. One example of such side effects is the potential deterioration of fiscal discipline due to persistently low interest rates. This topic is the subject of this paper, which was developed by staff of the Deutsche Bundesbank as a contribution to the strategy review work stream on monetary-fiscal policy interactions (see Work Stream on Monetary-Fiscal Policy Interactions, 2021).

To assess the consequences of sustained low-interest-rate episodes for the behaviour of fiscal policy, this paper uses a quantitative macroeconomic model in which the government of a country within a monetary union solves an intertemporal decision problem. By treating fiscal policies as the outcome of a political decision problem, the model can capture how policy incentives respond to changes in interest rates. The government in the model is impatient and unable to credibly commit to policies in the future. Both of these assumptions can be motivated by the political decision process in democratic societies. Among the decisions made by the government is how much debt to issue and whether to repay outstanding debt or not. The possibility of default is rationally taken into account by bond market investors. As a result, government bond prices command a default risk premium that reflects not only how the incentive to repay in the future depends on the public debt burden but also the interest rate environment.

The model, which can replicate various properties of public debt management and government bond yields observed for a number of selected euro area countries, is used to assess *(i)* how fiscal policy responds to a persistent interest rate reduction and *(ii)* the consequences of a subsequent interest rate normalisation. The quantitative analysis shows that low-interest-rate episodes tend to incentivise governments to increase deficit spending, resulting in the accumulation of more public debt. While the default risk premium is lower on average during a sustained low-interest-rate episode, the debt build-up increases the government's vulnerability to rollover crises after an interest rate reversal. The longer the low-interest-rate episode is expected to last, the greater the magnitude of the fiscal response will be.

Nichttechnische Zusammenfassung

Im Januar 2020 initiierte das Eurosystem die erste Überprüfung seiner geldpolitischen Strategie seit dem Jahr 2003. Der Überprüfungsprozess, der im Juli 2021 mit der Veröffentlichung der Erklärung zur geldpolitischen Strategie endete, umfasste diverse Seminare und Arbeitsgruppen zu einer breiten Anzahl geldpolitisch relevanter Themen (vgl. Deutsche Bundesbank, 2021). Unter den behandelten Themen waren auch potenziell negative Nebeneffekte expansionärer geldpolitischer Maßnahmen. Ein Beispiel für solche Nebeneffekte stellt die mögliche Verschlechterung fiskalischer Disziplin aufgrund anhaltend niedriger Zinsen dar. Diesem Thema widmet sich die vorliegende Arbeit, die von Mitarbeitern der Deutschen Bundesbank im Rahmen der Strategieüberprüfung als Beitrag für die Arbeitsgruppe zu geld- und fiskalpolitischen Interaktionen angefertigt wurde (vgl. Work Stream on Monetary-Fiscal Policy Interactions, 2021).

Um die Konsequenzen anhaltend niedriger Zinsen für das Verhalten der Fiskalpolitik abzuschätzen, verwendet die vorliegende Arbeit ein quantitatives makroökonomisches Modell, in dem die Regierung eines Mitgliedstaates einer Währungsunion ein intertemporales Entscheidungsproblem löst. Indem das Modell die Fiskalpolitik als Ergebnis eines politischen Entscheidungsprozesses auffasst, kann es abbilden wie fiskalische Anreize auf Zinsänderungen reagieren. Die Regierung wird als ungeduldig modelliert sowie als unfähig sich glaubhaft an eine zukünftigen Politik zu binden. Beide Annahmen können über den politischen Entscheidungsprozess in demokratischen Gesellschaften motiviert werden. Die Regierung entscheidet im Modell darüber, wie viele Schulden sie emittiert und ob sie ausstehende Schulden bedient. Ein möglicher Zahlungsausfall wird von den Investoren auf dem Anleihemarkt in rationaler Weise berücksichtigt. Der Anleihepreis enthält folglich eine Ausfallprämie, die reflektiert, wie der staatliche Anreiz zur Schuldenrückzahlung in der Zukunft von der Schuldenlast sowie der Zinsumgebung abhängen wird.

Das Modell kann eine Reihe beobachteter Eigenschaften des Staatsschuldenmanagements und der Staatsanleiherenditen ausgewählter Eurostaaten erfolgreich replizieren. Es wird in dieser Arbeit eingesetzt um auszuwerten *(i)* wie Fiskalpolitik auf eine persistente Zinssenkung reagiert, und *(ii)* welche Konsequenzen sich aus einer anschließenden Zinsnormalisierung ergeben. Die quantitative Analyse zeigt, dass Niedrigzinsphasen Regierungen tendenziell dazu verleiten, defizitfinanzierte Ausgaben zu erhöhen, was mit der Akkumulation zusätzlicher Staatsschulden einhergeht. Obgleich die Ausfallprämie während anhaltender Niedrigzinsphasen im Durchschnitt niedriger ausfällt, erhöht der zusätzliche Schuldenaufbau das Überwälzungsrisiko für Staaten im Fall einer Zinserhöhung. Je höher die erwartete Dauer der Niedrigzinsphase ist, desto stärker fällt die fiskalische Reaktion aus.

The Incentive Effects of Monetary Policy on Fiscal Policy Behaviour*

JOOST RÖTTGER† RAFAEL GERKE‡

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Abstract

How do prolonged low-interest-rate episodes affect fiscal discipline? This paper investigates this question by using a quantitative model with endogenous public debt management and sovereign default. Following a persistent interest rate reduction, sovereign risk and government bond yields decline. An impatient fiscal policy maker responds to improved financing conditions by relaxing its policy stance and accumulating more debt. Due to the increased debt burden, a subsequent interest rate reversal can put substantial pressure on the public budget, raising the likelihood of default. The longer the interest rate cut is expected to last, the more pronounced the fiscal response will be.

Keywords: Public Debt, Sovereign Risk, Low-Interest-Rate Policies

JEL Classification: E52, E62, H63

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†Deutsche Bundesbank, Directorate General Economics, Wilhelm-Epstein-Str. 14, 60431 Frankfurt am Main, Germany. Email: joost.roettger@bundesbank.de.

‡Deutsche Bundesbank, Directorate General Economics, Wilhelm-Epstein-Str. 14, 60431 Frankfurt am Main, Germany. Email: rafael.gerke@bundesbank.de.

1 Introduction

In recent years, the expansionary monetary policy measures of the Eurosystem, including its large-scale purchases of government securities, have contributed to extraordinarily low financing costs for the public sector. One concern associated with these developments is that they could reduce incentives for necessary fiscal consolidation or economic reforms and make deficit-financed policies more attractive. As a result, following a prolonged low-interest-rate episode, the sustainability of government finances might be at risk once interest rates rise due to a normalisation of monetary policy. The possibility of sovereign default could in turn threaten central bank independence by introducing a potentially challenging trade-off between price stability and debt stabilisation.

To understand the fiscal incentive effects of low-interest-rate policies and assess how vulnerable government solvency might be to interest rate reversals, a structural model framework is needed that takes into account how forward-looking fiscal policy makers optimally respond to changes in the macroeconomic environment, including the level of interest rates. A counterfactual analysis of the fiscal response to interest rate developments via – potentially estimated – fiscal policy rules, as typically used in dynamic stochastic general equilibrium (DSGE) models (see e.g. Forni et al., 2009; Leeper et al., 2010), would, by contrast, be subject to the Lucas critique (see Lucas, 1976) and therefore not be appropriate for such an assessment.

To take the endogenous response of governments to exogenous interest rate movements into account, this paper builds on recent literature that uses quantitative models of sovereign debt and default to study policy-related questions in the context of the European debt crisis (see Hatchondo et al., 2016; Bocola et al., 2019; Bocola and Dovis, 2019).[1] Compared to DSGE models used for monetary policy analysis (see e.g. Deutsche Bundesbank, 2008, 2015, 2016), these models are typically stylised along several dimensions. Unlike these models, they are, however, capable of replicating empirically observed dynamics of public debt and interest rate spreads as the equilibrium outcome of an intertemporal decision problem solved by a government that lacks commitment to future policies.

This paper focuses on the government of a country within a monetary union (MU). The model used in this paper takes a willingness-to-repay approach to sovereign default (see Eaton and Gersovitz, 1981). Whether the government repays outstanding debt thus reflects its political will to do so. By introducing a time-varying real risk-free rate into the model, this paper studies how changes in interest rates affect the dynamics and sustainability of public debt, taking into account how the incentives of a forward-looking but impatient fiscal policy maker respond to a changing interest rate environment.[2]

[1] Initially, these quantitative models were used to study business cycles in emerging economies (see Aguiar and Gopinath, 2006; Arellano, 2008). See Aguiar and Amador (2014) and Aguiar et al. (2016) for recent surveys of this literature.

[2] This paper does not study how higher public debt levels could pressure a central bank to neglect its price

The main finding of this paper is that if an interest rate reduction is expected to last for an extended period of time, the government tends to have an incentive to increase current spending by issuing more debt. By lowering the opportunity cost for financial market participants, an interest rate reduction directly lowers the cost of financing for the government. This in turn makes it cheaper to roll over maturing debt, raising the incentive to repay outstanding debt today as well as in future periods, where the interest rate is expected to remain low. The resulting decrease in default risk premia additionally improves financing conditions beyond the initial interest rate reduction, which amplifies the build-up of public debt. Once interest rates rise again, debt service becomes more expensive and the incentive to repay declines. As a result, the likelihood of a default increases given an interest rate reversal. Only after the government has reduced its debt position do default risk premia decline again. Lastly, the longer the low-interest-rate episode is expected to last, the stronger the responses of fiscal policy and bond yields will be.

2 Model

In the model, the government of a country within a monetary union finances public spending with stochastic tax revenues and by issuing of long-term government bonds that are traded with domestic and foreign financial investors. The government acts under discretion and is hence not able to credibly commit to policies that are not optimal ex post, i.e. it faces a time-inconsistency problem. Lack of commitment also concerns debt repayment, such that the government may decide to renege on its debt obligations in the future, which results in certain costs for the economy.[3] Investors are rational, such that the risk of a default is priced adequately by the market, reflecting how the incentive to default in the future depends on fundamentals, such as the level of debt and tax revenues. The risk of default also depends on the real risk-free rate, which – like tax revenues – is stochastic and fluctuates over time. The risk-free rate is the opportunity cost of investment for financial market participants. It therefore affects the government bond price via a no-arbitrage relationship. The time-varying risk-free rate allows us to study the response of fiscal policy to changes in monetary policy, whose measures ultimately affect the evolution of interest rates in the MU. While the real risk-free rate will, in reality, change over time due to factors other than monetary policy, this paper interprets changes in the real rate as a reflection of the (time-varying) monetary policy stance in the MU.[4]

stability mandate in order to maintain government solvency by keeping interest rates low. Since the focus of the paper is on the incentive effect of interest rate changes on fiscal policy, it will instead (implicitly) assume that monetary policy does not respond to developments in individual member states of the MU. This assumption is consistent with the notion of monetary dominance and reflected by a risk-free rate that is exogenous from the government's perspective.

[3] These costs involve the temporary exclusion from financial markets as well as an income loss during periods of default and financial autarky (see Arellano, 2008).

[4] Since monetary policy and hence the risk-free rate in a MU do not respond to country-specific developments, it is plausible to assume that the risk-free rate is exogenous from the perspective of a member state.

2.1 Model details

The government's objective is to maximise expected life time utility $\mathbb{E}_0\left[\sum_{t=0}^{\infty} u(G_t)\right]$, with discretionary spending G_t, discount factor β, strictly concave utility function $u(\cdot)$, and expectation operator $\mathbb{E}_t[\cdot]$. In each period t, the government receives stochastic tax revenues τY_t, which depend on the constant income tax rate τ and aggregate income Y_t, which follows a discrete first-order Markov process.[5] To smooth spending G_t across states and time, the government can trade bonds with risk-neutral investors.

To allow for a realistic debt maturity structure but keep the model tractable at the same time, the government is assumed to issue only real zero-coupon bonds that mature with an exogenous probability as in Chatterjee and Eyigungor (2012). Specifically, a bond issued in period t will mature in period $t+1$ with probability λ and pay out one unit of a tradable consumption good. With counterprobability $1-\lambda$, the bond does not mature in $t+1$ but may mature in $t+2$, again with probability λ. The memoryless nature of these random-maturity zero-coupon bonds implies that, at the beginning of any period, it is sufficient to know the amount of bonds that have not matured in the past. When exactly these bonds were issued is not relevant. The law of motion for aggregate public debt can then simply be written as $B_{t+1} = (1-\lambda)B_t + I_t$, where B_{t+1} denotes the face value of end-of-period public debt and I_t the amount of newly issued debt. The average maturity of the debt portfolio equals $1/\lambda$.

The government's period budget constraint for period t is

$$\tau Y_t + q_t\left(B_{t+1} - (1-\lambda)B_t\right) = G_t + \lambda B_t, \tag{1}$$

with bond price q_t.

In each period, the government has the option to refuse debt service ($d_t = 1$), which results in income loss $\phi(Y_t) \geq 0$ and temporary exclusion from financial markets. Conditional on being in financial autarky, the government re-enters financial markets with probability $0 < \theta \leq 1$, and remains in autarky with counterprobability $1-\theta$. Once the government leaves autarky after a default, a haircut h is applied (see Hatchondo et al., 2016) and the debt position is reduced by $1-h$. If the government repays its debt ($d_t = 0$), it can issue new debt, which it can again choose to (not) repay in the next period. Importantly, the government optimises sequentially and can neither commit to future repayment nor debt issuance.[6]

Additionally, the impact of inflation in a single member state on union-wide inflation is negligible, such that the nominal interest rate can also be viewed as exogenous from a single country's perspective. Finally, it is common to use changes in the real rate to study the effect of monetary policy on the economy (see e.g. Martinez-Miera and Repullo, 2021; McKay and Wieland, 2021; Whited et al., 2021).

[5] The assumption of a constant tax rate (i) simplifies numerical computation and (ii) reflects that it is usually easier to adjust public spending in the short run than to change the tax code.

[6] Lack of commitment to future debt issuance matters because the government does not internalise how its borrowing behaviour affects the price of debt issued in the past. This debt dilution problem will, however, be

Government bonds are bought by risk-neutral investors with deep pockets who can freely borrow and save on financial markets at the time-varying real risk-free rate r_t. The risk-free rate represents the opportunity cost for investors on financial markets.

The bond price then satisfies the risk-neutral pricing condition

$$q_t = e^{-r_t}\mathbb{E}_t\left[(1-d_{t+1})(\lambda+(1-\lambda)q_{t+1})+d_{t+1}p_{t+1}\right], \tag{2}$$

where d_{t+1} denotes the government's repayment decision in period $t+1$ and p_{t+1} the market price of a distressed government bond. A distressed government bond is priced by the market according to condition

$$p_t = \omega(1-h)q_t + (1-\omega)e^{-r_t}\mathbb{E}_t\left[p_{t+1}\right], \tag{3}$$

which reflects haircut h and the probability of re-entry ω, which governs the average duration of debt restructuring, $1/\omega$.

Since the government optimises sequentially, i.e. it chooses (B_{t+1}, d_t) on a period-by-period basis, the repayment decision in period $t+1$ is a function of the predetermined debt position B_{t+1} and the uncertain future states $S_{t+1} = (Y_{t+1}, r_{t+1})$. If the government issues more debt today, it reduces fiscal space tomorrow, increasing the incentive not to repay the debt to avoid spending cuts. As a result, an increase in B_{t+1} will tend to raise the probability of default, $\delta_t = \mathbb{E}_t[d_{t+1}]$, and reduce the bond price q_t accordingly.[7] This feedback from government actions to financing conditions captures the impact of market discipline on public debt issuance. As long as there remains a positive risk of default, debt sustainability is, however, not ensured and default events may happen in equilibrium. Ultimately, it is the government's decision whether it wants to accumulate risky debt positions or avoid the possibility of default altogether.

2.2 Government decision problem

The government decision problem is formulated recursively. In the remainder, time indices are hence dropped and values with a prime denote next period values.

If the government has access to financial markets, it first chooses whether to repay its debt or not,

$$\mathcal{V}^O(B,S) = \max_{d\in\{0,1\}}\left\{(1-d)\mathcal{V}^R(B,S) + d\mathcal{V}^D(B,S)\right\}, \tag{4}$$

where $\mathcal{V}^O(B,S)$ denotes the option value of default, $\mathcal{V}^R(B,S)$ the value of repayment and $\mathcal{V}^D(B,S)$ the value of default. The policy function for the optimal repayment decision is

anticipated by investors who demand a higher expected return to be compensated for the resulting price risk (see e.g. Hatchondo et al., 2016).

[7] For a given debt position, the incentive to default also increases with the real risk-free rate and decreases with tax revenues.

denoted as $\mathcal{D}(B,S)$.

Conditional on debt repayment, the government's optimal debt policy, $\mathcal{B}(B,S)$, solves the Bellman equation

$$\mathcal{V}^R(B,S) = \max_{B'} \left\{ u\left(\tau Y + q(B',S)(B' - (1-\lambda)B) - \lambda B\right) + \beta \mathbb{E}_{S'|S}\left[\mathcal{V}^O(B',S')\right] \right\}, \quad (5)$$

with bond price schedule

$$q(B',S) = e^{-r}\mathbb{E}_{S'|S}\left[(1-\mathcal{D}(B',S'))(\lambda + (1-\lambda)q(B'',S')) + \mathcal{D}(B',S')p(B',S')\right]. \quad (6)$$

The discounted expected value on the right-hand side of the bond price schedule reflects the different payment scenarios for investors. If the government repays its debt, i.e. $\mathcal{D}(B',S') = 0$, investors receive their payments with probability λ. If the government repays but the bond does not mature, which will be the case with probability $1-\lambda$, it will be valued at tomorrow's market price, $q(B'',S')$, reflecting expected payments in future periods. This market price will reflect the borrowing decision of the government in the next period, $B'' = \mathcal{B}(B',S')$, which in turn depends on the debt choice today, B'. If the government does not repay its debt, i.e. $\mathcal{D}(B',S') = 1$, debt service is suspended and investors do not receive a payment in that period. Since the government may, however, repay part of its debt at a later point, the distressed bond will be valued at the market price $p(B',S')$.[8] Written in recursive notation, this bond price satisfies the functional equation

$$p(B,S) = \omega(1-h)q(B(1-h),S) + (1-\omega)e^{-r}\mathbb{E}_{S'|S}\left[p(B,S')\right]. \quad (7)$$

The bond price reflects that, with probability ω, the government re-enters financial markets in the next period and the haircut h is applied. Given that the government may immediately decide to again default on its debt, the investors' payoff per bond is captured by $(1-h)q(B(1-h),S)$. With counterprobability $1-\omega$, debt restructuring will continue and the bond will be priced according to $p(B,S')$, taking into account changes in future states S'.

The government's Bellman equation for periods of default and financial autarky is

$$\mathcal{V}^D(B,S) = u(\tau[Y - \phi(Y)]) + \beta\mathbb{E}_{S'|S}\left[\omega\mathcal{V}^O(B(1-h),S') + (1-\omega)\mathcal{V}^D(B,S')\right]. \quad (8)$$

2.3 Recursive equilibrium

Definition 1 *A recursive equilibrium consists of value functions* $\{\mathcal{V}^O(\cdot),\mathcal{V}^R(\cdot),\mathcal{V}^D(\cdot)\}$ *that satisfy (4), (5) and (8), policy functions* $\{\mathcal{B}(\cdot),\mathcal{D}(\cdot)\}$ *that solve (4) and (5), and bond price functions* $\{q(\cdot),p(\cdot)\}$ *that satisfy (6) and (7).*

[8] Debt restructuring is modelled following Hatchondo et al. (2016).

3 Quantitative analysis

This section first discusses the model specification and calibration, followed by a brief discussion of the numerical solution method. It then presents results from a quantitative model experiment.

3.1 Model specification and calibration

To discipline the quantitative analysis, the model is calibrated to match selected statistics for Italy, Portugal and Spain between 2002Q1 and 2012Q3.[9]

Following Bocola and Dovis (2019), the utility function is specified as

$$u(G_t) = \frac{(G_t - \underline{G})^{(1-\sigma)}}{1-\sigma}.$$

As shown in detail by Bocola et al. (2019), the minimum (or subsistence) spending level $\underline{G} > 0$ is important to generate countercyclical debt in the presence of countercyclical default risk. With $\underline{G} = 0$, government debt as well as primary deficits are procyclical in quantitative models of sovereign debt and default. For a given level of debt, a negative income shock lowers bond prices by increasing the probability of default. The government can counteract this bond price decline by reducing its debt, and would indeed decide to do so for calibrations with $\underline{G} = 0$. Such behaviour would, however, be inconsistent with empirical evidence for developed economies as well as many emerging market economies, where public debt-to-GDP ratios and the primary deficit are both countercyclical, even in the presence of countercyclical default risk. A sufficiently large minimum spending level causes low-income states to be much more costly, such that the government seeks to avoid large spending cuts associated with debt deleveraging, making public debt go up in response to negative income shocks despite rising default risk premia.

The income loss function is specified as $\phi(Y_t) = \max\left\{0, Y_t - \overline{Y}\right\}$ (see Arellano, 2008). In the default case – as well as in periods of financial autarky – the tax base thus equals $\overline{Y}$ if $\overline{Y} \leq Y_t$ and Y_t if $\overline{Y} > Y_t$. This functional form implies that a default is less costly in low-income states, which – consistent with empirical evidence – results in countercyclical default risk. Income $Y_t = e^{y_t}$ follows a log-normal AR(1) process, $y_t = (1-\rho_y)\mu_y + \rho_y y_{t-1} + \sigma_y \varepsilon_{y,t}$, where $\mu_y = -0.5\sigma_y^2$ and $\varepsilon_{y,t}$ is an i.i.d. shock drawn from the standard normal distribution. The income process is discretised by using the method proposed by Tauchen (1986).

The real risk-free rate follows a discrete first-order Markov process. Specifically, r_t can take on two values, which correspond to a frequent normal-interest-rate regime (r^N)

[9] The time period chosen ends in 2012 to avoid an overlap with the Outright Monetary Transactions (OMT) programme as well as the public sector purchase programme (PSPP), which the model cannot adequately capture. Data are taken from Bocola et al. (2019) and Bocola and Dovis (2019).

and an infrequent low-interest-rate regime (r^L).[10] Whereas the normal-interest-rate regime captures normal times, the low-interest-rate regime is intended to capture times of extraordinarily low interest rates. By interpreting persistent changes in the real rate as being due to a time-varying monetary policy stance, the risk-free rate will provide the link between monetary and fiscal policy in the model. The value for the real risk-free rate in the normal-interest-rate regime (r^N) is set to 0.1%, which equals the average (quarterly) real return on German government bonds with a residual maturity of three months for the time period under observation. In the low-interest-rate regime, the real risk-free rate is (on a quarterly basis) 25 basis points below the value in the normal-interest-rate regime. The transition probabilities are chosen to study the impact of a persistent real rate decline on fiscal policy behaviour in a transparent way, with a focus on how the fiscal response depends on the expected duration of a low-interest-rate episode. Specifically, a switch to the low-interest-rate regime occurs with a very low probability of 0.01%. As a result, the government does not effectively anticipate a regime change during normal times.

Once a switch to the low-interest-rate regime occurs, the new regime is expected to remain in place for $1/\pi$ periods, with π denoting the probability of transitioning back to the normal-interest-rate regime. By varying π, one can assess how the expected duration of a low-interest-rate episode affects government behaviour during such times and what the consequences of a return back to the normal-interest-rate regime will be. The two regimes for the real rate are modelled in the spirit of Eggertsson and Woodford (2003), where a similar two-state Markov process is used to push the economy to the zero lower bound.[11] In the context of this paper, this modelling choice allows us to isolate the role of the expected duration of a low-interest-rate episode for the fiscal policy response because the behaviour of the government hardly changes with π during normal times.[12] As a result, the stochastic steady state will be almost invariant to different π values and hence provides a reasonable reference point.

The parameter values for the income process are averages of estimates for Italy, Portugal and Spain. The values are $(\rho_y, \sigma_y) = (0.97, 0.01)$. The tax rate τ is set to 0.41 following Bocola and Dovis (2019), the haircut h to 0.37 as in Hatchondo et al. (2016). The re-entry probability ω is set to 0.25, implying that the government spends on average one year in financial autarky after a default. Targeting an average debt maturity of around 6.5 years, the bond parameter λ is set to 0.0385. The inverse of the intertemporal elasticity of substitution σ is set to a standard value of 2.

[10] Almeida et al. (2019) also consider two persistent regimes for the risk-free rate in an application to Mexico's default experience in 1982. As in this paper, the authors interpret exogenous shifts in the risk-free rate as being due to monetary policy. In their model, a frequent normal-interest-rate regime alternates, however, with an infrequent high-interest-rate regime, which is intended to capture the "Volcker shock", rather than a low-interest-rate regime as in this paper.

[11] Eggertsson et al. (2021) provide a numerical toolkit to solve such types of models.

[12] Choosing a higher value for the transition probability from the normal- to the low-interest-rate regime does not, however, affect the main results of this paper.

The remaining three model parameters, $(\beta, \underline{G}, \overline{Y})$, are chosen to match an average debt-to-GDP ratio of 81.3%, an average annualised interest rate spread of 120 basis points, and a negative correlation between debt-to-GDP and income Y_t.[13] The quantitative experiment analysed in this paper will assess how the duration of low-interest-rate episodes affects debt accumulation and the likelihood of default once the economy moves back to the normal-interest-rate regime. To facilitate the comparison of different scenarios, the model is calibrated under the assumption that only the normal regime is operative. The values for the model parameters are $(\beta, \underline{G}, \overline{Y}) = (0.9745, 0.847, 0.860)$.

The calibration implies that the government is impatient relative to the market, i.e. its discount factor β is below the market discount factor e^{-r_t}. This feature is standard in the literature (see e.g. Arellano, 2008; Chatterjee and Eyigungor, 2012) and needed to match the average public debt-to-GDP ratio observed for the considered country sample.[14]

3.2 Numerical solution

The model is solved numerically via value function iteration.[15] As is well-known in the literature (see Chatterjee and Eyigungor, 2012), the numerical solution of sovereign default models with long-term bonds suffers from convergence problems when discrete state space methods are applied. To address this issue, this paper follows Gordon (2019) and solves a version of the model from Section 2 with preference shocks.

Suppose the government can only choose among a finite number of debt values contained in $\mathbb{B} = \{B_1, B_2, ..., B_{N_B}\}$. Let $\epsilon(B_i)$ denote an i.i.d. preference shock associated with debt value $B_i \in \mathbb{B}$ and define $\epsilon = (\epsilon(B_1), \epsilon(B_2), ..., \epsilon(B_{N_B}))$. Furthermore, assume that these preference shocks are drawn from a continuous distribution and uncorrelated with S.

Assuming preference shocks are realised after the repayment decision is made, the Bellman equation for the repayment case (5) is replaced by

$$\mathcal{V}^R(B,S) = \mathbb{E}_\epsilon \left[\max_{B' \in \mathbb{B}} \left\{ \tilde{\mathcal{V}}^R(B,S,B') + \epsilon(B') \right\} \right], \tag{9}$$

where

$$\tilde{\mathcal{V}}^R(B,S,B') = u\left(\tau Y + q(B',S)(B' - (1-\lambda)B) - \lambda B\right) + \beta \mathbb{E}_{S'|S}\left[\mathcal{V}^O(B',S')\right].$$

[13] All of these targets are again average values for Italy, Portugal and Spain for the time period under observation. The interest rate spread is the difference between the annualised yield on long-term bonds for the aforementioned countries and that on German government bonds with the same maturity.

[14] The assumption of an impatient government can be motivated by political economy considerations (see Alesina and Tabellini, 1990; Battaglini and Coate, 2008; Cuadra and Sapriza, 2008). In the academic literature, political frictions provide the most prominent explanation of the increase in public debt observed for advanced economies over the last few decades (see e.g. Yared, 2019). The necessary degree of impatience in this paper is, however, modest compared to applications to emerging market economies.

[15] As recommended by Hatchondo et al. (2010), we simultaneously iterate over value functions and bond price schedules instead of using an outer loop for bond price schedules and an inner loop for value functions as in Arellano (2008).

The government thus experiences the specific (additive) preference shock $\epsilon(B')$, depending on which debt value $B' \in \mathbb{B}$ it chooses. Given that the optimal debt choice depends on ϵ, the debt policy now becomes a function of the preference shocks, $\mathcal{B}(B,S,\epsilon)$.[16]

The bond price schedule (6) is replaced by

$$q(B',S) = e^{-r}\mathbb{E}_{S'|S}\Big[(1-\mathcal{D}(B',S'))(\lambda+(1-\lambda)\mathbb{E}_{\epsilon'}\left[q(\mathcal{B}(B',S',\epsilon'),S')\right]) \\ +\mathcal{D}(B',S')p(B',S')\Big]. \tag{10}$$

The purpose of the preference shocks is to randomise over debt choices, which smooths out non-convexities in the model that prohibit convergence when iterating on the value functions and bond price schedules.[17] In principle, one could also accomplish this by introducing an i.i.d. disturbance with continuous support that affects income (see Chatterjee and Eyigungor, 2012). The advantage of using preference shocks is, however, that expectations with respect to ϵ can be computed in closed form if preference shocks are drawn from the Gumbel distribution with scaling parameter σ_ϵ and location parameter μ_ϵ (see Rust, 1987; Gordon, 2019; Chatterjee et al., 2020).[18] To normalise the expected value of $\epsilon(B')$ to zero, we set the location parameter to $-\sigma_\epsilon\gamma$, where γ denotes the Euler-Mascheroni constant.[19]

The conditional choice probability for $B' \in \mathbb{B}$ is given by

$$\Pr\left(B'|B,S\right) = \frac{\exp\left(\tilde{\mathcal{V}}^R(B,S,B')/\sigma_\epsilon\right)}{\sum_{B_i\in\mathbb{B}}\exp\left(\tilde{\mathcal{V}}^R(B,S,B_i)/\sigma_\epsilon\right)},$$

[16]In models with defaultable long-term bonds, positive debt recovery can incentivise the government to issue as much debt as possible when a default is inevitable (see Chatterjee and Eyigungor, 2015; Hatchondo et al., 2016). This feature arises because the bond price remains positive even if the default probability equals one, such that spending can still be increased in this case by issuing more bonds. To address this issue, we follow Chatterjee and Eyigungor (2015) and assume for the numerical model solution that – due to underwriting standards – the government is not allowed to issue new debt ($B'-(1-\lambda)B>0$) if the default probability $\mathbb{E}_{S'|S}[\mathcal{D}(B',S')]$ exceeds a certain threshold $\iota\in[0,1]$. As in Chatterjee and Eyigungor (2015), we set $\iota=0.75$, such that the government is not permitted to issue new bonds if the probability of a default within the next year exceeds 99.96%. The findings of this paper are robust to this particular parameter value. To enforce the constraint on bond issuance, we add the penalty term $-\chi\times\mathbf{1}_{\Omega(B,S)}(B')$ next to the preference shock $\epsilon(B')$ in (9), where $\mathbf{1}_{\Omega(B,S)}(B')$ denotes the indicator function for the subset $\Omega(B,S)\subseteq\mathbb{B}$, which is given as $\Omega(B,S)=\{B'\in\mathbb{B}:\mathbb{E}_{S'|S}[\mathcal{D}(B',S')]>\iota\ \wedge\ B'-(1-\lambda)B>0\}$. A sufficiently high value is chosen for χ to prevent the government from choosing debt values contained in $\Omega(B,S)$.

[17]Without preference shocks, a small change in B' can have a discontinuous impact on bond revenues and hence utility due to jumps in the bond price schedule along the debt dimension. As a result, the algorithm can "get stuck" between iterations: The optimal debt choices alternate between certain on-grid debt values, causing the value functions and bond price schedules not to converge. The preference shocks smooth out the jumps in the bond price schedule and thereby ensure that the algorithm does not run into such cycles. See Chatterjee and Eyigungor (2012) for more details.

[18]The cumulative distribution function of the Gumbel distribution, also known as type I generalised extreme value distribution, is given as $F(x)=\exp(-\exp(-(x-\mu_\epsilon)/\sigma_\epsilon))$.

[19]The Euler-Mascheroni constant is defined as $\gamma=\lim_{n\to\infty}\left\{-\log n+\sum_{s=1}^{n}s^{-1}\right\}$. See Havil (2003) for details.

such that the expectations term with respect to ϵ' in (10) can simply be written as

$$\mathbb{E}_{\epsilon'}\left[q(\mathcal{B}(B',S',\epsilon'),S'))\right] = \sum_{B_i\in\mathbb{B}} \Pr\left(B_i|B',S'\right) q(B_i,S').$$

The expectations term in (9) cannot simply be calculated based on the choice probabilities because preference shocks directly affect utility, which needs to be taken into account when computing expectations. Fortunately however, a closed form expression also exists in this case:

$$\mathbb{E}_{\epsilon}\left[\mathcal{V}^R(B,S,\epsilon)\right] = \sigma_\epsilon \log\left\{\sum_{B_i\in\mathbb{B}} \exp\left(\tilde{\mathcal{V}}^R(B,S,B_i)/\sigma_\epsilon\right)\right\}.$$

Expectations with respect to S' are computed by using the conditional transition probabilities for Y' and r' (see Section 3.1).

Given initial guesses for the value functions and bond price schedules, the numerical solution algorithm *(i)* computes the conditional choice probabilities based on the formula above, and *(ii)* updates the value functions and bond price schedules accordingly. These two steps are repeated until the equilibrium objects of interest converge according to some criterion.[20]

Since the preference shocks introduce additional noise into the model, we check the extent to which their presence affects our model predictions. As in Dvorkin et al. (2021), we do so by also simulating the model when all preference shock realisations are set to zero. Compared to the full model simulation with randomly drawn preference shocks, we find that the predictions of this "ϵ-zero model" (Dvorkin et al., 2021) are qualitatively unchanged and quantitatively very close.

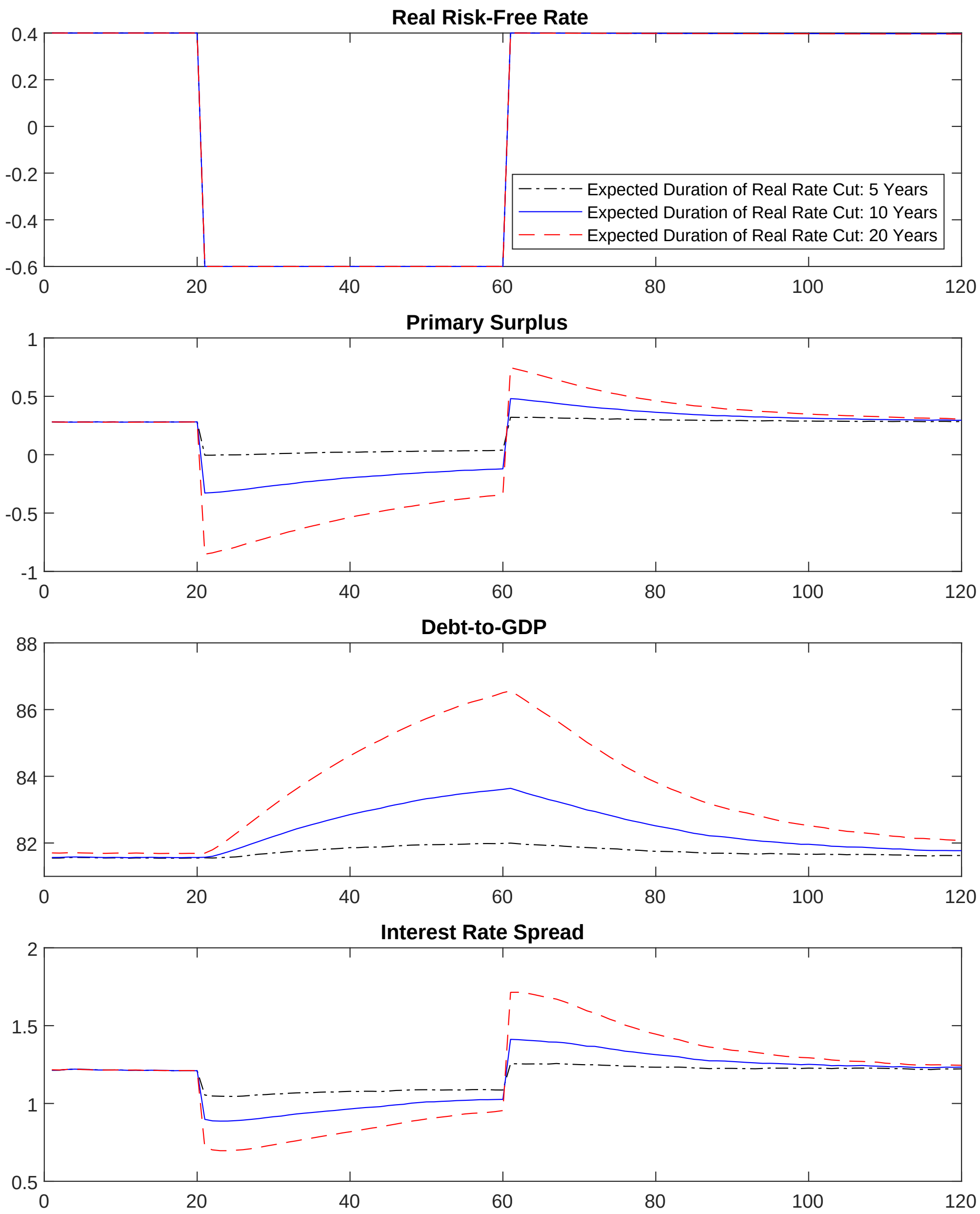
3.3 Results

The behaviour of the government during the normal-interest-rate regime provides the starting point for the quantitative analysis. The considered experiment is as follows. After simulating the economy in the normal-interest-rate regime for 400 periods, the economy suddenly experiences a switch to the low-interest-rate regime, which immediately lowers the real rate. The economy remains in this regime for 40 periods (10 years), after which it returns to the normal-interest-rate regime. For different expected durations of the low-interest-rate episode $1/\pi$, Figure 1 displays averaged time series for the respective model variables based on 250,000 model simulations, which feature the same time path for the real risk-free rate but different income shock realisations.

As one can see, after the switch to the low-interest-rate regime, the government immediately increases the primary deficit to take advantage of improved financing conditions and

[20] Specifically, we use the sup norm $||\cdot||_\infty$ to check for convergence after step *(ii)*. Let $X_{i+1}(\cdot)$ denote the update of equilibrium object X after step *(ii)* in iteration number i. The algorithm then stops if $||X_{i+1}(B,S)-X_i(B,S)||_\infty < 10^{-5}\times(1+||X_{i+1}(B,S)||_\infty)$, for $X\in\{\mathcal{V}^R,\mathcal{V}^D,q,p\}$.

Figure 1: Response to Persistent Interest Rate Cut and Reversal



Notes: All variables are expressed in percentage points. The real rate and interest rate spread are annualised. The primary surplus is in percent of quarterly GDP. Debt-to-GDP is defined as the face value of public debt over annual GDP. The horizontal axis depicts time in quarters.

raise public spending. The persistent reduction in the real rate directly lowers public financing costs by reducing the opportunity cost of investment for financial market participants. In addition, given that it will likely be cheaper for the government to roll over debt in the near future, the incentive to default declines, such that the probability of default in the next few periods declines on impact as well. This indirect effect is captured by the spread between the annualised yields of a risky government bond and a bond with the same maturity structure but no default risk.

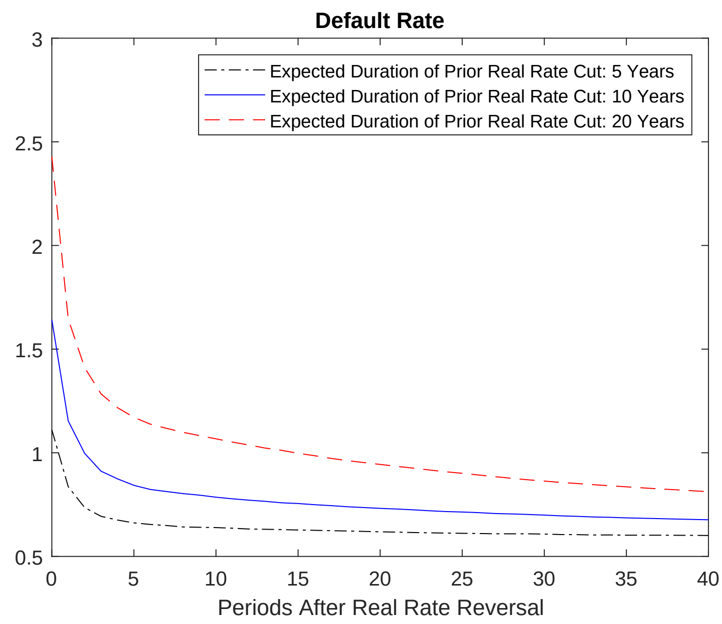
While the real rate turns negative after the regime change, this is not sufficient to compensate for the increase in the primary deficit. As a result, the public debt-to-GDP position goes up during the low-interest-rate regime. This highlights that a negative interest-rate-growth differential per se does not guarantee a stabilisation of public debt if the government adjusts to this scenario by raising the primary deficit (see Wyplosz, 2019).[21] Due to the government's impatience vis-à-vis financial investors, a widening of the gap between its rate of time preference and the real rate increases the incentive to frontload spending via deficit financing.

Note, however, that the marginal increase in debt declines over time in the low-interest-rate regime, which reflects the simultaneous widening of the interest rate spread. Since a higher public debt position raises the risk of default, investors (i.e. the market) demand compensation in return, which drives up the default risk premium and disciplines the government to some extent. Due to its impatience and the still low financing costs, the government is ultimately not discouraged from running a primary deficit. This behaviour changes once the shift back to the normal-interest-rate regime occurs. The immediate increase in the real rate, as well as the associated expectation that future interest rates will be higher, raises financing conditions on impact and forces the government to run a primary surplus. Repaying the debt is, however, only one option for the government in the model. The alternative response is to default. This cannot directly be seen in Figure 1 since it only displays averaged time series, where periods that featured a default will enter as a balanced budget due to financial autarky. Figure 2 therefore plots the fraction of sample paths in which a default occurred after the real rate switched back to the normal-interest-rate regime following the low-interest-rate episode displayed in Figure 1 (period 21 to period 60).

The response of the variables displayed in Figures 1 and 2 is qualitatively the same for all expected durations considered for the low-interest-rate episode in the simulation experiment. Quantitatively, however, the response differs and increases in magnitude with the expected duration of the low-interest-rate episode. The longer the government expects financing conditions to remain cheap, the more it takes advantage and frontloads spending by running a primary deficit. The size of the debt build-up therefore increases with the expected duration $1/\pi$, which in turn requires the government to make larger spending cuts

[21] Since the model does not feature a growth trend, the real risk-free rate is equal to the interest-rate-growth differential in the model.

Figure 2: Likelihood of Default *After* Real Rate Reversal



Notes: The default rate, expressed in percentage points, is defined as the fraction of simulated periods that featured a default *following* a persistent (40-quarter) real rate increase. It is calculated based on observations up to the respective time period (quarter) on the horizontal axis. The average values for the default rate in the period *prior* to the real rate reversal are 0.46% (5-year duration), 0.38% (10-year duration) and 0.36% (20-year duration).

once the low-interest-rate episode ends, amplifying the likelihood of default because it is costly to pursue fiscal consolidation by cutting spending.

Given that Figures 1 and 2 show averaged time paths for the variables, the government response can of course differ from the time paths shown, to the extent that tax revenues and/or initial debt differ from their average values. Specifically, if tax revenues are below (above) average, the fiscal response tends to be more (less) pronounced. Finally, note that the magnitude of the fiscal responses reflects not only the expected duration of low-interest-rate episodes but also the size of the interest rate reduction. An interest rate cut that is bigger (smaller) relative to the one considered in this paper would strengthen (weaken) the fiscal response.

4 Final remarks

The model used in this paper is highly stylised and abstracts from a number of potentially relevant features. Tax revenues, for instance, are exogenous (since output follows an exogenous process) and hence do not respond to changes in the real rate or the default risk premium. How important is this assumption for the results presented in this paper? Suppose that tax revenues are a decreasing function of the real rate and the default risk premium. On the one hand, a persistent real rate cut would then likely attenuate the debt build-up. On the other hand, however, a reversal of the real rate would likely lower tax revenues by curbing economic activity, which will likely increase the risk of a default.

Another feature that the model abstracts from is public investment because the government can only use its funds for public consumption by assumption. Public investment might raise the economic growth potential of the economy. Yet, given that the government is impatient, it is not obvious whether it would want to delay consumption today to increase consumption tomorrow via public investment.

In practice, interest rate changes induced by monetary policy decisions are likely to be more gradual compared to the experiment from the previous section. One way to incorporate such gradual changes into the model is to allow for additional, less persistent interest rate shocks within each of the two regimes. In this case, the two regimes would determine the expected real rate but not pin down the specific realisations. As a result, a regime change would not necessarily lead to an immediate change in the short-term real rate and only shift the likelihood of particular real rate realisations. Since the government issues long-term bonds, expected changes in future short-term rates would, however, immediately affect bond yields by changing the yield curve. Regime changes, which can capture a switch in the monetary policy stance, therefore have a very similar impact in the model regardless of whether the short-term real rate changes immediately or not.

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