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Unconventional monetary policies at the effective lower bound

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Non-technical summary

An article in a recent issue of the Monthly Report (Deutsche Bundesbank, 2021) surveys the main findings of the Eurosystem's recent review of its monetary policy strategy. This paper presents selected results of the model-based assessments on unconventional monetary policy documented in the aforementioned Monthly Report article and Work stream on the price stability objective (2021). In particular, it lays out the details of the model, the simulation design, and the implementation of various simulations. The analysis, however, does not incorporate or discuss any potential side effects of unconventional measures.

Based on an estimated medium-scale two-agent DSGE model for the euro area, various (stochastic) simulations are conducted. With respect to interest rate forward guidance, the simulations illustrate that a state-dependent element within the forward guidance has the potential to reduce macroeconomic volatility relative to purely calendar-based forward guidance. The reason is that a state-contingent promise, if credible and understood by the agents, "automatically" provides additional stimulus in bad states and reduces stimulus in good states of the world.

With respect to asset purchases, the simulations point to notable effects on economic output and inflation. The analysis is based on the assumption that net purchases are state-dependent. This comprises that net purchases take place only when interest rates are at the effective lower bound and that the size of net purchases depends on the inflation shortfall from its target. The implied state-dependency therefore mimics key features of the Asset Purchase Programme of the Eurosystem in the past. According to the simulations, unlimited asset purchases at the effective lower bound can provide significant inflation stabilisation. However, based on the estimated asset purchase rule, it cannot completely neutralise the negative inflation bias. This is even more true when upper purchase limits restrict the possible purchase volume.

An appropriate reinvestment strategy can increase the effectiveness of asset purchases in light of such upper purchase limits. Ultimately, a reinvestment strategy is a forward guidance on future asset purchases. This announcement, if credible and understood by the agents in the economy, already stimulates the economy already in the present. The reason is that higher future purchases lower the respective yields in the future, which – through anticipation and arbitrage conditions – already lower the long-term yields of today's assets.

Nichttechnische Zusammenfassung

Ein Artikel in einem kürzlich erschienenen Monatsbericht (Deutsche Bundesbank, 2021) gibt einen Überblick über die wichtigsten Ergebnisse der jüngsten Überprüfung der geldpolitischen Strategie des Eurosystems. Im vorliegenden Papier werden ausgewählte Ergebnisse vorgestellt, die im oben genannten Monatsbericht und im Workstream-Bericht zum Preisstabilitätsziel Teil der modellbasierten Analyse zur unkonventionellen Geldpolitik waren. Es legt insbesondere die Details des Modells, des Simulationsdesigns und der Durchführung verschiedener Simulationen dar. Die Analyse berücksichtigt oder diskutiert jedoch keine möglichen Nebenwirkungen der unkonventionellen Maßnahmen.

Basierend auf einem geschätzten mittelgroßen Zwei-Agenten-DSGE-Modell für den Euroraum werden verschiedene (stochastische) Simulationen durchgeführt. In Bezug auf die Zins-Forward-Guidance verdeutlichen die Simulationen, dass ein zustandsabhängiges Element das Potenzial hat, die makroökonomische Volatilität im Vergleich zu einer rein kalenderbasierten Forward Guidance zu reduzieren. Der Grund hierfür ist, dass ein zustandsabhängiges Versprechen, wenn es glaubwürdig ist und von den Agenten verstanden wird, in schlechten Zuständen "automatisch" zusätzlichen geldpolitischen Stimulus verspricht und in guten Zuständen den Stimulus reduziert.

In Bezug auf die Anleihekaufprogramme weisen die Simulationen auf nennenswerte Auswirkungen auf die Wirtschaftsleistung und die Inflation hin. Die Analyse basiert auf der Annahme zustandsabhängiger Nettokäufe. Dies beinhaltet, dass Nettokäufe nur dann getätigt werden, wenn sich die Zinssätze an der Zinsuntergrenze befinden, und dass die Höhe der Nettokäufe davon abhängt, wie stark die Inflationsrate ihren Zielwert unterschreitet. Die Zustandsabhängigkeit bildet daher wesentliche Merkmale des Anleihekaufprogramms des Eurosystems in der Vergangenheit ab. Den Simulationen zufolge können unbeschränkte Anleihekäufe an der effektiven Zinsuntergrenze eine signifikante Inflationsstabilisierung bewirken. Basierend auf der geschätzten Regel für Anleihekäufe kann die negative Inflationsverzerrung jedoch nicht vollständig neutralisiert werden. Dies gilt umso mehr, wenn Kaufobergrenzen das mögliche Kaufvolumen einschränken.

Eine geeignete Reinvestitionsstrategie kann angesichts von Kaufobergrenzen die Effektivität von Anleihekäufen erhöhen. Letztlich kann eine Reinvestitionsstrategie als eine Forward Guidance für zukünftige Anleihekäufe verstanden werden. Eine solche Ankündigung stimuliert die Wirtschaft bereits in der Gegenwart, wenn sie glaubwürdig ist und von den Wirtschaftsakteuren verstanden wird. Dies liegt darin begründet, dass höhere zukünftige Anleihekäufe die Renditen in der Zukunft senken, was – durch Antizipation und Arbitragebedingungen – die Renditen der heutigen Anleihen reduziert.

Unconventional monetary policies at the effective lower bound*

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Abstract

This paper presents selected results from a model-based assessment of the impact of unconventional monetary policies. The model is estimated on euro area data and features two types of agents, a financial sector, several real and nominal frictions as well as rational expectations. In our model simulations, we allow for multiple non-linearities: an occasionally binding effective lower bound (ELB) on the short-term policy rate; a state-dependent and calender-based interest rate forward guidance; state-dependent asset purchases conducted only at the ELB. There are four main results. First, state-dependent forward guidance can reduce the volatility of the economy in comparison to unconditional calendar-based forward guidance. Second, the ELB induces a large negative inflation bias if the central bank has only its short-term policy rate at its disposal. Third, asset purchases that are initiated only at the ELB can provide significant inflation stabilisation, but it remains uncertain whether they allow the distortions created by the ELB to be fully offset. This is the case, for instance, when an upper limit on purchases is imposed. Fourth, an appropriate reinvestment strategy might help to mitigate the limits of asset purchases.

Keywords: Effective lower bound, forward guidance, state-dependent asset purchases, reinvestment, make-up strategies, Bayesian estimation

JEL Classification: D78, E31, E43, E44, E52, E58

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1 Introduction

The effective lower bound (ELB) on short-term nominal interest rates is an important constraint for monetary policy. As is well known, the ELB induces a negative inflation bias when the central bank's only policy instrument is the short-term rate. The reason for this is that in the face of disinflationary or deflationary shocks, interest rate policy alone can only provide the needed inflationary stimulus until it reaches the ELB. Hence, on average, the inflation rate falls short of the monetary policy's inflation target (see, for instance, Bernanke, 2020; Kiley and Roberts, 2017; Coenen, Montes-Galdón and Smets, 2020; Work stream on the price stability objective, 2021).

Confronted with a binding ELB, central banks resorted to various unconventional measures to provide additional monetary stimulus, including asset purchase programmes or interest rate forward guidance. Those measures played a major role not only in the aftermath of the financial crisis of 2008/09 but also in the recent COVID-19 pandemic. Assessing the efficacy of such unconventional measures is important and was part of the Eurosystem's strategy review (Work stream on the price stability objective, 2021; Altavilla, Lemke, Linzert, Tapking and von Landesberger, 2021). As pointed out in Deutsche Bundesbank (2021) and shown in more detail in this paper, unconventional policies can significantly contribute to reducing the negative inflation bias and the heightened macroeconomic volatility generated by the lower bound.

In the following, we will present the details of the model, the simulation design and the implementation of various simulations referenced in Deutsche Bundesbank (2021). We proceed as follows. Section 2 provides the model details and its parametrisation via estimation on euro area data from 1999Q1 to 2014Q4. Section 3 presents the method that we use to implement the different non-linear simulation scenarios: a binding effective lower bound on nominal interest rates, a calendar- and threshold-based forward guidance policy, temporary and state-dependent asset purchases initiated at the ELB, asset purchases with binding upper limits and, lastly, a asset purchases with a reinvestment policy.

The subsequent Section 4, 5 and 6 present three different sets of policy experiments, all conducted within the same model framework, parametrisation and implementation method.[1]

Section 4 contrasts the implications of a threshold-based forward guidance (TBFG) with a pure calendar-based forward guidance (CBFG). We find that TBFG can reduce the volatility of the economy in comparison to unconditional CBFG. The state-contingent promise to hold rates at the ELB until inflation fulfils a certain threshold provides additional stimulus in bad states and reduces stimulus in good states of the world. Hence, with a TBFG the central bank can effectively reduce the negative bias associated with the ELB.

Section 5 illustrates how upper purchase limits can constrain the efficiency and efficacy of asset purchase programmes. Depending on the level of the upper limit (25% to 33% of all outstanding assets), the average inflation rate drops by around 10-15 basis points compared to a programme without an upper limit. The more assets the central bank can purchase at the ELB, when a monetary stimulus is needed the most, the better it can stabilise the economy. Specifically, a larger limit results in a lower ELB frequency and duration and less volatility of inflation and output. At the same time, it implies larger holdings of assets on the balance sheet.

Section 6 illustrates how an appropriate reinvestment strategy can increase the effectiveness of asset purchases in the presence of upper purchase limits. Ultimately, a reinvestment strategy can be conceived as a forward guidance on future asset purchases. This announcement, if credible and understood by the agents, stimulates the economy today as higher future purchases lower the respective yields in the future, which – through anticipation and no-arbitrage conditions – lower the long-term yields of today's assets. Hence, the central bank has two margins to increase the effects of its asset purchase programmes. It can increase the size or it can prolong the reinvestment horizon. The last section concludes.

[1] We assume that agents have rational expectations throughout all our simulations. Different (non-rational) expectations might therefore lead to different policy implications. See Deutsche Bundesbank (2021) or the WGEM Expert Group on "Expectations formation and monetary policy" for a discussion of potential difficulties related to alternative expectation formations.

2 Framework and estimation

This section lays out in detail the framework and the parametrisation/estimation of the model. We use the model of Gerke, Giesen and Scheer (2020a), which is a quantitative Two-Agent New Keynesian model that builds on the work of Carlstrom, Fuerst and Paustian (2017), Galí, López-Salido and Vallés (2007) and Bilbiie (2008). The economy consists of households, three types of firms (final good firm, intermediate good firm and capital good firm), a government, financial intermediaries and a segmented financial markets. The latter allow us to analyse the effects of asset purchases.

2.1 Households

The economy is populated by two types of households: A measure $1 - \lambda$ of households has complete access to financial markets and can smooth consumption through short-term deposits and the accumulation of real capital – we call them Ricardian or optimising households. The remaining fraction λ has no access to financial markets (it can neither borrow nor save) and in every period consumes its wage income and transfers entirely – we call them hand-to-mouth (rule-of-thumb or constrained) households.

Each optimising household (denoted by the superscript o) maximises expected lifetime utility

$$E_t \sum_{s=0}^{\infty} \beta^s d_{t+s} \left\{ \ln \left(C^o_{t+s} - hC^o_{t+s-1} \right) - B \frac{H^{1+\eta}_{t+s}}{1+\eta} \right\}, \tag{1}$$

where C^o_t denotes private consumption, h degree of habit, H_t the (individual) labour input (scaled by B to normalise labour input in the steady state) and d_t an exogenous process for a time-varying discount factor, given by:

$$\ln(d_t) = (1 - \rho_d) \ln(d) + \rho_d \ln(d_{t-1}) + \epsilon_{d,t}. \tag{2}$$

The budget constraint is given by

$$C_t^o + P_t^k I_t^o + \frac{D_t}{P_t} + (1 + \kappa Q_t)\frac{F_{t-1}}{P_t} = w_t H_t + R_t^k K_t + \frac{D_{t-1}}{P_t} R_{t-1}^d + div_t - T_t^o + \frac{Q_t F_t}{P_t} \tag{3}$$

Households invest in real capital I_t at the price P_t^k, save deposits D_t and repay their outstanding debt including a coupon payment of 1, $(1 + \kappa Q_t)\frac{F_{t-1}}{P_t}$ (see below for more details). They earn labour income $w_t H_t$ (to be specified below), a return on capital $R_t^k K_t$ and deposits $R_{t-1}^d \frac{D_{t-1}}{P_t}$ and dividends div_t net of taxes T_t^o (which consists of a lump-sum part and a re-distributive part; see subsection 2.4 for details). div_t comprises dividends from the financial intermediaries (div_t^{FI}), a capital goods producer (div_t^{CP}) and an intermediate goods producer (div_t^{IG}).

There is a need for intermediation through the financial system since all of the household's investment purchases must be financed beforehand by issuing new investment bonds (hence, there is a loan-in-advance constraint). The price of such bonds is denoted by Q_t and offers the payment stream $1, \kappa, \kappa^2, \ldots$, following Woodford (2001).[2] If CI_t denotes the number of new perpetuities issued in time t, the household's stock of nominal liabilities F_t is given by

$$F_t = \kappa F_{t-1} + CI_t \;\; \Leftrightarrow CI_t = F_t - \kappa F_{t-1}. \tag{4}$$

The loan-in-advance constraint is then given by:

$$P_t^k I_t \leq \frac{Q_t CI_t}{P_t} \tag{5}$$

The law of motion for capital follows:

$$K_t = (1 - \delta) K_{t-1} + I_t. \tag{6}$$

The Ricardian household therefore maximises utility (1) subject to the budget constraint (3), the loan-in-advance constraint (5) and the law of motion for capital (6).

[2]Due to the recursive structure, $\kappa^h Q_t$ is the time t price of such a bond that was issued in period $t - h$.

The first-order conditions are given by:

$$\Lambda_t = \frac{b_t}{C_t^o - hC_{t-1}^o} - E_t \frac{\beta h b_{t+1}}{C_{t+1}^o - hC_t^o} \tag{7}$$

$$\Lambda_t = E_t \beta \frac{\Lambda_{t+1}}{\Pi_{t+1}} R_t^d \quad \text{with} \quad \Pi_{t+1} = \frac{P_{t+1}}{P_t} \tag{8}$$

$$\Lambda_t M_t Q_t = E_t \frac{\beta \Lambda_{t+1} (1 + \kappa Q_{t+1} \mathrm{M}_{t+1})}{\Pi_{t+1}} \tag{9}$$

$$\Lambda_t M_t P_t^k = E_t \beta \Lambda_{t+1} \left[R_{t+1}^k + M_{t+1} P_{t+1}^k (1 - \delta) \right] \tag{10}$$

with $M_t = 1 + \frac{\vartheta_t}{\Lambda_t}$ or $\Lambda_t M_t = \Lambda_t + \vartheta_t$. The first two equations comprise the typical Euler equation for deposits, the third one for investment bonds. The fourth one, equation (10), describes the demand for capital. It is distorted by the time-varying wedge M_t which depends on the multiplier of the loan-in-advance constraint (5). As discussed in great detail in Carlstrom et al. (2017), this distortion acts as a mark-up on the price of new capital and is basically the term premium that exists due to the segmented markets and the leverage constraint of the banks that limit the arbitrage across the term structure (see next subsection).

The budget constraint of hand-to-mouth households is much simpler as they neither borrow nor save and only consume their labour income less taxes:[3]

$$C_t^h = w_t H_t - T_t^h, \tag{11}$$

where their consumption is C_t^h, labour income is $w_t H_t$ (see below) and T_t^h are taxes that hand-to-mouth households have to pay. Overall taxes are given by a time-invariant

[3]Such behaviour can be rationalised, for instance, by myopia, a lack of access to capital markets or ignorance of intertemporal trading opportunities. As pointed out by Galí et al. (2007), this is a rather extreme form of non-Ricardian behaviour, which nevertheless captures the observed heterogeneity in consumption responses and income as found in the data.

component T^h and a countercyclical transfer scheme:[4]

$$T_t^h = \frac{\tau}{\lambda}(Y_t - Y) + T^h. \tag{12}$$

$\tau \geq 0$ captures the degree of countercyclical transfers which rebates income whenever aggregate output is different from the steady state $(Y_t - Y)$.[5] Although this transfer scheme is stylised, it captures in a parsimonious way automatic stabilisers that are found in more complex settings (see for instance McKay and Reis, 2016; Leeper, Plante and Traum, 2010).[6] Additionally, it is the most direct way to introduce redistribution within the two types of households.

2.1.1 Labour agencies

Each household supplies a specialised type of labour H_t^j, independent of whether it is a Ricardian or a hand-to-mouth household (in the spirit of Erceg, Henderson and Levin, 2000). Since firms do not differentiate between the two household types when hiring labour for a specialised type j, the supply of hours and the wage rate are the same for both groups. The labour agencies bundle the specialised labour inputs into a homogeneous labour output that it sells to the intermediate good firm according to

$$H_t = \left[\int_0^1 \left(H_t^j\right)^{1/(1+\lambda_{w,t})} dj\right]^{1+\lambda_{w,t}} \tag{13}$$

where $\lambda_{w,t}$ is the wage mark-up, following (in linearised form)

$$\lambda_{w,t} = (1 - \rho_{\lambda^w})\ln(\lambda_w) + \rho_{\lambda^w}(\lambda_{w,t-1}) + \epsilon_{\lambda_{w,t}}. \tag{14}$$

[4]The time-invariant component ensures that, in the steady state, consumption is the same across households (i.e. $C^h = C^o$, see also Galí et al., 2007).

[5]See the discussion in Gerke, Giesen and Scheer (2020b) for rebating firm profits.

[6]The study of more complex transfer rules or distortionary taxes seems interesting, but is beyond the scope of the paper.

The demand for the different types of labour inputs is given by

$$H_t^j = \left(\frac{W_t^j}{W_t}\right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} H_t \tag{15}$$

In each period, the probability of resetting the wage is $(1-\theta_w)$, while with the complementary probability (θ_w) the wage is automatically increased following the indexation rule:

$$W_t^j = \Pi_{t-1}^{\iota_w} W_{t-1}^j$$

The maximisation problem of a given union for the specialised labour input j is given by:

$$\max_{\tilde{W}_t} E_t \sum_{s=0}^{\infty} (\beta\theta_w)^s \left\{(1-\lambda)\, u\left(C_{t+s}^o\right) + \lambda u\left(C_{t+s}^h\right) - d_{t+s}\Lambda_{t+s}^a B \frac{H_{t+s}^{1+\eta}}{1+\eta}\right\}$$

s.t. the budget constraints (3), (11) and labour demand (15) and with $\Lambda_{t+s}^a = (1-\lambda)\,\Lambda_{t+s}^o + \lambda\Lambda_{t+s}^h$ (similar to Colciago, 2011).[7]

2.2 Financial intermediaries

The financial intermediaries (FIs) in the model use accumulated net worth N_t and short-term deposits D_t to finance investment bonds F_t and long-term government bonds B_t. Their balance sheet is given by:

$$\underbrace{Q_t \frac{B_t}{P_t}}_{\overline{B}_t} + \underbrace{Q_t \frac{F_t}{P_t}}_{\overline{F}_t} = N_t + \frac{D_t}{P_t} = L_t N_t, \tag{16}$$

where L_t denotes leverage. Note that investment and government bonds are perfect substitutes since they offer the same payment streams and thus are valued at the same price Q_t. Define the return on those bonds as R_t^L:

$$R_t^L \equiv \frac{1+\kappa Q_t}{Q_{t-1}}. \tag{17}$$

[7]We define $\Lambda_{t+s}^h = d_{t+s}\frac{1}{c_{t+s}^h}$, i.e. without habit. The simulation results do not change qualitatively if we also introduce habit for the hand-to-mouth households.

Every period a FI receives the coupon payment of 1 from its old assets in $t-1$. Its income is thus $(1+\kappa Q_t)\left(\frac{B_{t-1}}{P_t}+\frac{F_{t-1}}{P_t}\right)$. It purchases new assets at price Q_t, such that the real value of these purchases is $Q_t\left(\frac{F_t}{P_t}+\frac{B_t}{P_t}\right)$. It further collects new deposits D_t and has to pay out interest rate expenses on the deposits of the previous period $R^d_{t-1}\frac{D_{t-1}}{P_t}$. Any deviation of net worth from the steady state will be costly: $f(N_t)N_t$, with $f(N_t)=\frac{\Psi_n}{2}\left(\frac{N_t-N}{N}\right)^2$.[8] Thus, the remaining dividend payments are given by interest income less the expenditures:

$$div_t^{FI}=(1+\kappa Q_t)\left(\frac{B_{t-1}}{P_t}+\frac{F_{t-1}}{P_t}\right)+\frac{D_t}{P_t}-Q_t\left(\frac{F_t}{P_t}+\frac{B_t}{P_t}\right)-R^d_{t-1}\frac{D_{t-1}}{P_t}-f(N_t)N_t$$

$$\Leftrightarrow div_t^{FI}+(1+N_t)f(N_t)=\underbrace{\frac{P_{t-1}}{P_t}\left(\left(R^L_t-R^d_{t-1}\right)L_{t-1}+R^d_{t-1}\right)N_{t-1}}_{\text{profits}}, \tag{18}$$

where the definition of the return R^L_t and the banks' balance sheet (16) were substituted. This equation shows that profits will be partly paid out as dividends div_t^{FI} to the (Ricardian) households while the rest is retained as net worth for subsequent activity. The FI discounts dividend flows using the (Ricardian) household's pricing kernel augmented with additional impatience $\zeta<1$, which allows for a positive excess return of long-term debt over deposits in the steady state.[9]

The FI then chooses dividends div_t^{FI} and net worth N_t to maximise expected dividend payments

$$V_t=E_t\sum_{s=0}^{\infty}(\beta\zeta)^s\Lambda_{t+s}div_{t+s}^{FI} \tag{19}$$

subject to (18). This yields the following first-order condition:

$$\Lambda_t\left[1+f(N_t)+N_tf'(N_t)\right]=E_t\Lambda_{t+1}\beta\zeta\frac{P_t}{P_{t+1}}\left[\left(R^L_{t+1}-R^d_t\right)L_t+R^d_t\right]. \tag{20}$$

[8] As will be shown below, a leverage constraint (due to a "hold-up" problem) limits the ability of the FI to attract deposits and thus eliminates the arbitrage opportunity between long and short rates. However, this limit to arbitrage could be undone by an increase in net worth (implicitly, that would be a lump-sum transfer (tax) on the (Ricardian) households). The net worth adjustment costs ensure that this does not happen.

[9] It can be shown that $R^L=R^d+\frac{1-\zeta}{\zeta L}R^d>R^d$ if $\zeta<1$. For that, evaluate equation (20) in the steady state.

The FIs are subject to a simple hold-up problem which limits their ability to attract deposits (similar in spirit to Gertler and Karadi, 2013). We follow the approach by Carlstrom et al. (2017) completely and arrive at the following expression for the leverage constraint L_t:

$$L_t = \frac{1}{\left[1 + (\Phi_t - 1)\, E_t \frac{R^L_{t+1}}{R^d_t}\right]}, \tag{21}$$

where Φ_t measures exogenous changes in the financial friction:

$$\Phi_t = (1 - \rho_\Phi)\,\Phi + \rho_\Phi \Phi_{t-1} + \varepsilon_{\Phi,t}. \tag{22}$$

2.3 Goods market

Perfectly competitive final goods producers combine differentiated intermediate goods $Y_t(i)$ into a homogeneous good Y_t according to the technology:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{1+\lambda_{p,t}}}\, di\right]^{1+\lambda_{p,t}}$$

where $\lambda_{p,t}$ is the time-varying price mark-up that evolves according to

$$\lambda_{p,t} = (1 - \rho_{\lambda_p})\ln(\lambda_p) + \rho_{\lambda_p}\lambda_{p,t-1} + \varepsilon_{\lambda_{p,t}}. \tag{23}$$

Profit maximisation leads to the following demand function:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} Y_t, \tag{24}$$

with

$$P_t = \left[\int_0^1 P_t(i)^{-\frac{1}{\lambda_{w,t}}}\, di\right]^{-\lambda_{w,t}}. \tag{25}$$

A continuum of monopolistic competitive firms combines capital K_{t-1} and labour H_t to produce intermediate goods according to a standard Cobb-Douglas technology. The production function is given by:

$$Y_t(i) = A_t K_{t-1}(i)^{\alpha} H_t(i)^{1-\alpha} \tag{26}$$

with

$$A_t = (1-\rho_a)\ln(A) + \rho_a A_{t-1} + \epsilon_{A,t}. \tag{27}$$

The intermediate goods producers set prices based on Calvo contracts. In each period firms adjust their prices with probability $(1-\theta_p)$ independently of previous adjustments. Those firms that cannot adjust their prices in a given period will re-set their prices according to the following indexation rule:

$$P_t(i) = \Pi_{t-1}^{\iota_p} P_{t-1}(i).$$

Firms that can adjust their prices face the following problem:

$$\max_{P_t^*} E_t \sum_{s=0}^{\infty} \theta_p^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_t} \left[\frac{P_t^* \left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_p} \right)}{P_{t+s}} Y_{t+s}(i) - \frac{W_{t+s}}{P_{t+s}} H_{t+s}(i) - R_{t+s}^k K_{t-1+s}(i) \right],$$

subject to labour demand (15) and $Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon_{p,t}} Y_t$. It holds that dividends are given by $div_t^{IG} = Y_t - w_t H_t - R_t^k K_{t-1}$.

The capital goods producers take final investment output I_t and sell it (with a mark-up) subject to adjustment costs to the households, therefore dividends $div_t^{CP} = P_t^k I_t^n - I_t = P_t^k \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t - I_t$, where the investment-specific technology shock follows an AR(1) process:

$$\mu_t = (1-\rho_\mu)\ln(\mu) + \rho_\mu \mu_{t-1} + \varepsilon_{\mu,t}. \tag{28}$$

The profit maximisation is then described by

$$\max_{I_t} E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t+s} \left[P_{t+s}^k \mu_{t+s} \left[1 - S\left(\frac{I_{t+s}}{I_{t+s-1}}\right)\right] I_{t+s}) - I_{t+s} \right]. \tag{29}$$

2.4 Government policies

The government consists of a two authorities. First, fiscal policy focuses on the redistribution between the two types of households. Second, a central bank sets the interest rate (and later also resorts to asset purchases).

2.4.1 Fiscal policy

The government collects taxes T_t in a lump-sum fashion and issues government bonds $\frac{Q_t B_t}{P_t}$ to finance its outstanding debt including coupon payments $(1+\kappa Q_t)\frac{B_{t-1}}{P_t}$. Its simple budget constraint is given by:

$$\frac{Q_t B_t}{P_t} + T_t = (1+\kappa Q_t)\frac{B_{t-1}}{P_t}. \tag{30}$$

Note that tax-income $T_t = \lambda T_t^h + (1-\lambda) T_t^o$ is net of the countercyclical transfers paid to hand-to-mouth households. Implicitly, there is redistribution of countercyclical transfers $\tau(Y_t - Y)$ from optimising to hand-to-mouth households (via the government). The respective tax rules for both agents are given by the following two equations:

$$T_t^o = \frac{1}{1-\lambda}\left(T^t + T^o - \tau\left(Y_t - Y\right)\right) \tag{31}$$

$$T_t^h = T^h + \frac{\tau}{\lambda}\left(Y_t - Y\right). \tag{32}$$

For simplicity, only the Ricardian households finance the government. Additionally, they are involved in the countercyclical transfer system in which the hand-to-mouth households participate as well. The degree of countercyclicality is given by τ. T^o and T^h are chosen such that consumption of hand-to-mouth and Ricardian households coincide in the steady state.[10]

[10] As the focus of this paper is on the effect of forward guidance when a fraction of households does not display forward-looking behaviour – and not so much about different consumption distributions – we view that assumption as being largely justifiable.

2.4.2 Central bank

The central bank follows a Taylor rule when setting its short-term policy rate R_t:[11]

$$\ln(R_t) = (1-\rho)\ln(R) + \rho\ln(R_{t-1}) + (1-\rho)\left(\tau_\pi(\pi_t - \pi) + \tau_y(y_t - y_{t-1})\right) + R_t^\epsilon$$

with

$$R_t^\epsilon = (1-\rho_m)\ln(R^\epsilon) + \rho_m R_{t-1}^\epsilon + \varepsilon_{R,t}. \tag{33}$$

This policy rate will be subject to the effective lower bound (ELB) during the simulations in sections 4 to 6. We will formally introduce the constraint there. For the estimation we abstain from an ELB.

Lastly, the central bank can resort to asset purchases. As we abstain from purchases during the estimation, we will specify them in sections 5 and 6, where we describe the quantitative implications of asset purchases with and without purchasing limits.

2.5 Aggregation

Taking the household and the government budget constraint, as well as all dividend payments, one arrives at the aggregate resource constraint

$$Y_t = C_t + I_t + f(N_t)N_t, \tag{34}$$

where aggregate consumption and investment are given by a weighted average of the respective variables for optimiser and hand-to-mouth households:

$$C_t = (1-\lambda)C_t^o + \lambda C_t^h \tag{35}$$

and

$$I_t = (1-\lambda)I_t^o. \tag{36}$$

[11] Since short-term government debt and bank deposits are perfect substitutes, it holds that $R_t^d = R_t$.

Similarly, the aggregate capital stock is given by

$$K_t = (1 - \lambda) K_t^o. \quad (37)$$

2.6 Estimation

After linearising the model around the deterministic steady state, we estimate it with Bayesian methods. We use eight quarterly euro area time series for the sample period 1999Q1 to 2014Q4.[12]

2.6.1 Data

We use a total of eight observables for the euro area: real GDP per capita, real investment, gross inflation, employment growth, real wage growth, shadow short-term interest rate, long-term interest rate and real bank net worth growth. The time series on bank net worth is taken from the European Central Bank's MFI Balance Sheet Items Statistics. All the other variables are taken from the Area-wide Model database of the ECB.[13] Since we have only seven structural shocks in the model, we add a measurement error to the observations equation for bank net worth in order to avoid stochastic singularity.

Per capita output and investment are obtained by dividing real GDP (YER) and investment (ITR) by the labour force (LFN). Growth rates are log-differences. Inflation is measured as the growth rate of the Harmonised Index of Consumer Prices (HICPSA). Employment growth is the log-difference of total employment (LNN). For the real wage series we first divide the nominal wage rate per head (WRN) by the HICPSA and then take the log-difference. Our short-term nominal interest rate is the 3-month Euribor rate (STN) and our long-term nominal interest rate the euro area 10-year government benchmark bond yield (LTN). Real bank net worth is obtained by dividing the nominal

[12]The estimation is closely related to previous work in Gerke, Giesen and Scheer (2020b). We use the Dynare software package for the estimation, see Adjemian, Bastani, Juillard, Karamé, Maih, Mihoubi, Mutschler, Perendia, Pfeifer, Ratto and Villemot (2011) for details.

[13]We use the 18th update of the Area-wide Model (AWM) database from August 2018.

capital and reserves of euro area monetary financial institutions (NWB) by HICPSA and taking the log-difference. All growth series are demeaned with their respective sample mean. The following table summarises the observation equations: [14]

$$\begin{bmatrix} \text{dlGDP}_t \\ \text{dlInvestment}_t \\ \text{dlHICPSA}_t \\ \text{ShortInterestRate}_t \\ \text{LongInterestRate}_t \\ \text{dlHours}_t \\ \text{dlWages}_t \\ \text{dlNetworth}_t \end{bmatrix} = 100 \cdot \begin{bmatrix} 0 \\ 0 \\ \log(\Pi) \\ \log(\Pi/\beta) \\ \log(\Pi/\beta) + 0.01/4 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \hat{y}_t - \hat{y}_{t-1} \\ \hat{x}_t - \hat{x}_{t-1} \\ \hat{\pi}_t \\ \hat{r}_t \\ \hat{r}_t^{L,10} \\ \hat{h}_t - \hat{h}_{t-1} \\ \hat{w}_t - \hat{w}_{t-1} \\ \hat{n}_t - \hat{n}_{t-1} + \varepsilon_{n,t} \end{bmatrix}.$$

We match the long-term interest rate time series to the yield-to-maturity of the 10-year government bond $\hat{r}_t^{L,10} = \log R_t^{L,10} - \log R^{L,10}$, with $R_t^{L,10} = \frac{1}{Q_t} + \kappa$ (see Carlstrom et al., 2017).

2.6.2 Calibration and prior distributions

As is common in the literature, we calibrate a subset of the structural parameters to ensure identification. We follow mostly the calibration of Carlstrom et al. (2017).[15] The time preference β is set to 0.99, yielding a steady-state annual real interest rate of roughly 4%. The labour income share α is set to 0.33 and the depreciation rate to $\delta = 0.025$, which implies a 10% annual depreciation of the capital stock. The steady-state mark-ups of prices and wages are set to 20%, i.e. $\lambda_w = \lambda_p = 0.2$. The leverage ratio is set to 6 which implies $\zeta = 0.9854$. We impose that in the steady state the annual long-term rate R^L is one percentage point above the short-term rate, i.e. $R^L = R^{L,10} = R + 0.01/4$ (in line with the data). In order to estimate the model with a 10-year government bond (similar to its empirical counterpart) we set $\kappa =$

[14] A ˆ denotes the log-deviation from the steady state, i.e. $\hat{y}_t = \ln(Y_t) - \ln(Y)$.

[15] We cross-check against values from Smets and Wouters (2003) who studied the euro area, but the results were largely unchanged.

Table 1: Prior and posterior distribution of estimated parameters

			Prior		Posterior			
		Dist	Mean	SE	Mode	Mean	5 percent	95 percent
Utility & technology								
h	Habit	B	0.5	0.2	0.7921	0.7897	0.7211	0.8576
η	Inverse Frisch	G	2	0.5	1.5801	1.7460	1.0043	2.4766
ψ_I	Investment adj.	G	10	1.0	14.1247	14.2112	12.3537	16.0382
ψ_N	Net worth adj.	G	3	1.0	6.2004	6.5485	4.6520	8.3655
Stickiness								
ι_p	Price indexation	B	0.6	0.1	0.4872	0.5187	0.3465	0.6859
ι_w	Wage indexation	B	0.6	0.1	0.3189	0.3443	0.2252	0.4591
θ_p	Price stickiness	B	0.7	0.1	0.8015	0.8127	0.7573	0.8692
θ_w	Wage stickiness	B	0.7	0.1	0.8646	0.8581	0.8131	0.9036
Government policy								
ρ_m	MP smoothing	B	0.75	0.1	0.7679	0.7655	0.7174	0.8148
τ_π	MP on inflation	N	1.5	0.1	1.5969	1.6280	1.4818	1.7696
τ_y	MP on output	N	0.5	0.1	0.6146	0.6217	0.4652	0.7758
τ	Redistribution	B	0.1	0.05	0.1666	0.1756	0.0642	0.2855
AR(1) shocks								
ρ_a	TFP	B	0.60	0.2	0.9846	0.9800	0.9637	0.9979
ρ_ϕ	Financial friction	B	0.60	0.2	0.7491	0.7366	0.6742	0.8010
ρ_μ	Investment	B	0.60	0.2	0.9316	0.9246	0.8849	0.9644
ρ_{λ_w}	Wage markup	B	0.60	0.2	0.2405	0.2760	0.0731	0.4565
ρ_{λ_p}	Price markup	B	0.60	0.2	0.5130	0.4569	0.2393	0.6647
ρ_d	Demand	B	0.60	0.2	0.5868	0.5874	0.4328	0.7493
ρ_{res}	Monetary policy	B	0.60	0.2	0.4843	0.4730	0.3328	0.6176
Std shocks								
ϵ_a	TFP	IG	0.01	1	0.0058	0.0059	0.0050	0.0067
ϵ_ϕ	Financial friction	IG	0.05	1	0.1623	0.1736	0.1290	0.2145
ϵ_μ	Investment	IG	0.5	1	0.0985	0.1007	0.0833	0.1179
ϵ_{λ_w}	Wage markup	IG	0.1	1	0.8377	0.9434	0.2504	1.6418
ϵ_{λ_p}	Price markup	IG	0.1	1	0.0438	0.0591	0.0242	0.0967
ϵ_d	Demand	IG	0.1	1	0.0346	0.0375	0.0258	0.0492
ϵ_r	Monetary policy	IG	0.01	1	0.0030	0.0031	0.0027	0.0035
ϵ_N^{ME}	ME on net worth	IG	0.001	1	0.0119	0.0122	0.0104	0.0139

Notes: B stands for the Beta, G for the Gamma, IG for the inverted Gamma and N for the Normal distribution.

0.975. It was not possible to identify the share of hand-to-mouth households λ and the redistribution coefficient τ simultaneously in the data. We calibrate the share of constrained households to 30% according to empirical evidence (e.g. Dolls, Fuest and Peichl, 2012; Bilbiie and Straub, 2013; Fève and Sahuc, 2017).[16]

The choices for the prior distributions are largely taken from Carlstrom et al. (2017) and are summarised in columns 2 to 4 of Table 1. The first block of parameters determine the shape of the utility and cost functions. For the level of consumption habit h, we use a beta distribution with a mean of 0.5 and a standard deviation of 0.2.

[16] As a cross-check we estimated the model with the calibrated redistribution τ (at the posterior mean of Table 1) and found a share of hand-to-mouth households of around 35%.

The inverse Frisch elasticity η has a relatively flat prior centred around 2. The prior mean and standard deviation for the investment adjustment costs Ψ_I are taken from the posterior mode of Coenen, Karadi, Schmidt and Warne (2018).

For the degree of indexation and stickiness, we use a beta distribution centred around 0.6 and 0.7, respectively, with a standard deviation of 0.1 for all four parameters, which is slightly below the values in Coenen et al. (2018)).

The prior for the persistence of monetary policy is a beta distribution with mean 0.75 and standard deviation of 0.1. The two Taylor coefficients on inflation and output growth both follow a normal distribution centred around 1.5 and 0.5 respectively. For the size of redistribution we take a relatively flat prior around 0.1, which is the posterior mean for a US estimate (Leeper et al., 2010).

We set the prior distribution for all autocorrelations of the exogenous shock processes as a beta distribution which is centred around 0.6 with a standard deviation of 0.2.[17] All priors for the standard deviations of shocks follow a relatively flat inverse gamma distribution with standard deviation of 1. The prior of the wage markup, price markup, demand and monetary policy are all centred around 0.1. For TFP (A_t), financial friction (Φ_t) and the investment specific technology (μ_t), we use slightly higher values of 0.5. The mean for the measurement error on net worth is set to 10% of the variance of the underlying data sample.

2.6.3 Posterior distribution

With the above specified prior distributions, we draw from the posterior distributions using the Metropolis-Hastings algorithm with two chains, each with 2,000,000 draws. In order to assess the convergence of the chains, we compute several measures following Brooks and Gelman (1998). The interval of the posterior distribution which is covered by the chains, as well as the second moment of the posterior distribution, seem to be stable for most parameters after approximately 1,000,000 draws. To ensure that results are reported based on parameter draws that have converged, we report results

[17]Note that for better identification of the autocorrelation of the monetary policy shock and the persistence in the Taylor rule we use a slightly tighter prior for the latter.

based on the last 100,000 draws of each chain.

The last columns of Table 1 report the posterior mode, the posterior mean, and the lower and upper bounds of the 90% posterior density interval of the estimated parameters obtained by the Metropolis-Hastings algorithm. Most of our estimates are in line with similar estimates for the euro area (e.g. Smets and Wouters, 2003; Coenen et al., 2018). In Appendix A, we plot the prior and posterior distribution of each parameter.

Compared to the above two studies, our data points to a slightly higher value of habit and wage stickiness as well as a much lower persistence of monetary policy (around 0.77 compared to above 0.9 in the other two studies). However, note that the monetary policy shock is also persistent. We estimate the degree of redistribution $\tau \sim 0.17$.[18] This is relatively close to Leeper et al. (2010), who find τ in the 0.05 to 0.25 range with a mean of 0.13 for a similar transfer rule in a representative agent model.

3 Overview of the solution and simulation method to capture multiple non-linearities

This section lays out the simulation method to implement various combinations of non-linearities at the same time. Specifically, we incorporate a binding ELB, a state-dependent forward guidance policy, a temporary and state-dependent asset purchase rule, an asset purchase rule with binding upper purchase limits and, lastly, a reinvestment policy.

We assume that the central bank only resorts to asset purchases when its short-term policy rate becomes constrained by the ELB. In such a case, it purchases long-term government bonds from the financial intermediaries. Due to limits to arbitrage (enforced by the hold-up constraint and net-worth adjustment costs), this raises the price of government bonds and thus decreases their yield. In turn, the financial intermediary

[18] In Gerke, Giesen and Scheer (2020a), the transfer coefficient $\bar{\tau}$ is around 0.5 with the transfer rule $T_t^h = \bar{\tau}(Y_t - Y) + T^h$. Here, we use the transfer rule $T_t^h = \frac{\tau}{\lambda}(Y_t - Y) + T^h$. Hence our value of τ is in principle a scaled version from Gerke, Giesen and Scheer (2020a), as the following holds: $\bar{\tau} = \frac{\tau}{\lambda}$.

increases its holding of relatively cheaper investment bonds (with which the households, by assumption, must pre-finance their investment), lowering their yields. Easier funding conditions for the households relax their loan-in-advance constraint which raises investment demand and eventually increases output and inflation. We capture the central bank's asset holdings by a feedback rule that makes the size of net purchases a function of the severity of the inflation shortfall from target. For details, see Section 5.

To solve the model, we rely on a piecewise-linear approach. Specifically, we employ the solution approach for structural changes developed by Kulish and Pagan (2017), which is similar to Guerrieri and Iacoviello (2015).[19] In particular, we assume that at any point in time, the economy is described by one of two regimes: an unconstrained regime (M1) and a constrained regime (M2).[20] In the former, the economy is described completely as in Section 2. For instance, in this case the central bank sets its policy rate according to the Taylor rule and does not resort to any asset purchases. In the constrained regime (M2), one or more of the non-linearities kick in. For instance, the ELB binds and the central bank conducts asset purchases. For each of the above-mentioned non-linearities, there is accordingly a new set of equations describing the economy. Once the dynamic equations of the respective regime are approximated up to first order, the regimes have the following representations.

Unconstrained regime:

$$Ax_t = C + Bx_{t-1} + DE_t x_{t+1} + F\varepsilon_t \tag{M1}$$

Constrained regime:

$$A_t^* x_t = C_t^* + B_t^* x_{t-1} + D_t^* E_t x_{t+1} + F_t^* \varepsilon_t \tag{M2}$$

[19] The difference between both papers is the context in which they developed their respective piecewise-linear approach. Kulish and Pagan (2017) simulate a model with a structural break that is determined by a policy maker. Guerrieri and Iacoviello (2015) model a structural change that was driven by an exogenous shock. If the (expected) duration of the regime is the same for both approaches, the reduced-form matrices associated with that duration are exactly the same in Kulish and Pagan (2017) and Guerrieri and Iacoviello (2015).

[20] If there are more non-linearities we have more than two regimes (see below).

In both regimes, E_t denotes the expectations operator, x_t the vector of endogenous and ε_t the vector of exogenous variables. The matrices A, B, C, D, F and $A_t^*, B_t^*, C_t^*, D_t^*, F_t^*$ are of conformable dimensions that capture the structural parameters of the economic system. Without loss of generality, assume the following timing:

- The constrained regime is in place in periods 1 until T^{constr}: M2 applies.
- The unconstrained regime is in place from $T^{constr} + 1$ onwards: M1 applies.

Note that the matrices in regime (M2) are time-varying. Hence, this notation allows for a wide range of application. The solution to such structural changes is then a time-varying (non-linear) policy function (Guerrieri and Iacoviello, 2015). It is obtained by an iterative procedure, where the underlying assumption is that after period $T^{constr}+1$, the unconstrained regime stays forever.[21] Therefore, the solution after period T^{constr} is obtained via standard solution techniques (e.g. Sims, 2002). It is given by

$$x_t = J + Qx_{t-1} + G\varepsilon_t, \quad \forall t > T^{constr} \tag{38}$$

Given this solution, one can substitute for the expectation in T^{constr}, i.e. regime (M1) and solve the model for T^{constr}. This continues until period 1. For a detailed description, see Kulish and Pagan (2017) or Guerrieri and Iacoviello (2015).

Overall, the policy function for (M1) is then given by

$$x_t = J_t + Q_t x_{t-1} + G_t \varepsilon_t \tag{39}$$

with

$J_t = J,\ Q_t = Q,\ G_t = G\ \ \forall t > T^{constr}$, i.e., the time-invariant policy function of (38)

and

[21]In other words, agents do not anticipate the initiation of the ELB/reinvestment policy, but once either regime applies, they fully incorporate the change in the structure of the economy. As a result, precautionary effects are not incorporated.

$$\Xi_t = \left(A_t^* - D_t^* Q_{t+1}\right)^{-1}, J_t = \Xi_t \left(C_t^* + D_t^* J_{t+1}\right), Q_t = \Xi_t B_t^*, G_t = \Xi_t F_t^* \text{ for } 1 \leq t \leq T^{constr}$$

Depending on the simulation scenarios, implementing the solution method requires two sorts of specifications. First, the exact form of the unconstrained regime (M2), namely which equations change in comparison to the unconstrained regime. For instance, if there exists a binding ELB, only the Taylor rule will change. When the central bank conducts net asset purchases only at the ELB, the Taylor rule and the asset purchase rule will change, etc. Second, the timing of the regimes might be more elaborate as sketched above. For example, it is conceivable that, within a given state of the economy, the constrained regime might already apply in expectations (expected ELB period), or that a mix of constrained and unconstrained regimes applies in the future (like an expected double-dip recession). Also, the timing of a lift-off might depend on a decisive policy choice, like in the case of a state-dependent forward guidance. We will be more specific about the exact timing and setup of the unconstrained regime in the following sections.

4 The merits of threshold-based forward guidance

When policy rates were stuck at the effective lower bound (ELB), central banks resorted to unconventional measures to expand the monetary policy stance. One of these measures was interest rate forward guidance. Through a credible announcement of an interest rate path, monetary policy can influence the level and the development of long-term interest rates. This can dampen the undesirable effects of the ELB, primarily, undesirably high real interest rates.

In mid 2013, the ECB announced for the first time its expectations on the future development of its policy rate: "The Governing Council expects the key ECB interest rates to remain at present or lower levels for an extended period of time".[22] In June 2018, the

[22]Strictly speaking, such a form of forward guidance is merely a projection or prediction based on the current state of information (sometimes referred to as "Delphic forward guidance"). It does not contain any explicit reference to a change in the future course of monetary policy.

ECB also introduced a specific calendar-based leg into its forward guidance and since then followed a so-called calendar-based forward guidance (CBFG): "...we expect [the key interest rates] to remain at their present levels at least through the summer of 2019", "...the end of 2019" (March 2019) or "...first half of 2020" (June 2019). In September 2019, the ECB reaffirmed and strengthened its state-based element in its forward guidance: "The Governing Council now expects the key ECB interest rates to remain at their present or lower levels until it has seen the inflation outlook robustly converge to a level sufficiently close to, but below, 2% ...".[23] As the lift-off is since then tied to a robust convergence, we classify it into the category of a threshold-based forward guidance (TBFG).[24]

In the following, we illustrate why a threshold-based forward guidance is associated with advantages in comparison to a purely calendar-based forward guidance. As shown below, TBFG reduces – in comparison to CBFG – the variance of the distribution of possible macroeconomic outcomes. This stabilises expectations and therefore the economy at large. Therefore, TBFG can be conceived as a hedge against the asymmetric effects generated by the ELB.

As an example, take a recession in which the policy rate is constrained by the ELB. The agents in the economy anticipate that the central bank will provide some stimulus through a forward guidance policy. A TBFG endogenously changes the lift-off date: If further negative shocks arise (which prolong a recession), the policy rate will be at the ELB for longer (as the threshold will be reached later). In contrast, if positive shocks arrive, causing the economy to recover more quickly than originally expected, the lift-off from the ELB will occur sooner (thus less monetary stimulus is needed).

[23] Earlier, the ECB additionally communicated a state-based leg next to the calendar-based leg. In particular, it refrained from a lift-off until the Governing Council sees "the continued sustained convergence of inflation to its aim over the medium term".

[24] An example of a more specific threshold-based forward guidance is the announcement of the Fed from December 2012: "...exceptionally low range for the federal funds rate will be appropriate at least as long as the unemployment rate remains above 6-1/2 percent, inflation between one and two years ahead is projected to be no more than a half percentage point above the Committee's 2 percent longer-run goal, and longer-term inflation expectations continue to be well anchored."

4.1 A simple illustration of TBFG's state-dependency

In order to illustrate the state-dependency of TBFG, it is useful to consider a baseline scenario. We thus first introduce a TBFG that captures some elements of the Governing Council's policy described above. Specifically, we use the following threshold:

The monetary authority keeps its policy rate at the effective lower bound until it expects the inflation rate to be above 1.7% for 4 quarters in a row.

In the baseline scenario, we simulate a negative demand shock (discount factor shock), which causes the ELB to bind for 4 quarters in the absence of TBFG. When we assume that, as a response to the shock, the central bank communicates a one-off TBFG policy as specified above, the ELB binds for 8 quarters. Hence, TBFG extends the ELB by 4 quarters. We model the exit as credible in the sense that once the inflation rate fulfils the threshold condition, there is no doubt regarding the lift-off.[25]

TBFG extends endogenously the period at the ELB (T^{constr}) until the threshold is reached, according to the following rule:[26]

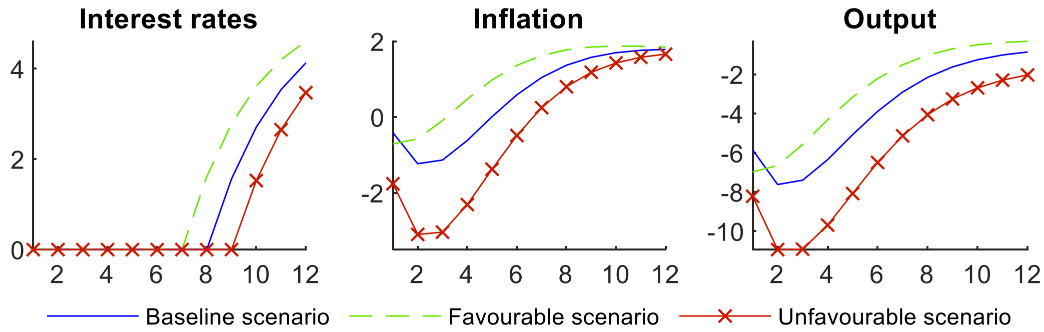
- If $E_t\bar{\pi}_{t+h} < 1.7\%$, $\forall h \in (1,2,3,4)$, set $T^{constr} = T^{constr} + 1$ (extend period at ELB).
- Else, if $E_t\bar{\pi}_{t+h} \geq 1.7\%$, $\forall h \in (1,2,3,4)$, lift interest rate according to interest rate rule.

Figure 1 illustrates the state-dependency of TBFG and the related built-in stabiliser by comparing the dynamic responses of interest rates, inflation and output in the context of three scenarios. In the baseline scenario, the central bank delays the lift-off until period nine. Hence, in quarters 9-12 (i.e. for 4 quarters), the inflation rate is above the threshold value of 1.7% (horizontal line in the middle panel). After the lift-off, monetary policy follows its prototypical interest rate rule.

[25]We assume the central bank is perfectly credible, i.e., agents fully take the central bank's announcements into account. We also assume just one threshold (for an example with a "dual threshold", see Boneva, Harrison and Waldron, 2018).

[26]In particular, it starts with $T^{constr} = 4$, which is the case without any TBFG element. However, the expected inflation rate within the next 4 quarters ($\pi_t, t \in (5,6,7,8)$) is below the threshold, so the period at the ELB is raised to $T^{constr} = 5$. Again, the expected inflation rate within the next 4 quarters ($\pi_t, t \in (6,7,8,9)$) is below the threshold, etc.

Figure 1: Simple illustration of TBFG's state-dependency



Note: Responses of interest rates, inflation and output in the context of three scenarios. In the TBFG scenario, the central bank delays the lift-off until it expects the inflation rate to be above 1.7% for 4 quarters in a row. In the favourable/unfavourable scenario, the underlying shock is less/more severe than in the TBFG scenario.

Now suppose the economy develops either more or less favourably compared to the baseline scenario. In particular, we assume that a further demand shock hits the economy. It is either a favourable one (in the sense that, overall, the negative demand shock is less severe than in the baseline scenario) or a less favourable one (overall, the negative demand shock is more severe than in the baseline scenario). Independently, the central bank still follows the same TBFG as described above.

In the favourable scenario (green dashed line), the recession is less pronounced (right panel), and accordingly the central bank starts the lift-off of its policy rate earlier (left panel) compared to the baseline. The reason is that the inflation rate crosses the threshold earlier (middle panel, the inflation rate is above 1.7% in period 8 to 11). This "deviation" from the baseline scenario is possible without changing the central bank's communication. In contrast, in the unfavourable scenario (red solid-x line), the time until the threshold is reached will be delayed (inflation response is more muted), and the central bank keeps the policy rate at the lower bound for longer. Again, there is no need to change the communication with the public.

4.2 Reduction of uncertainty

The automatic and state-dependent adjustment of the lift-off date allows a TBFG policy to reduce the volatility and therefore uncertainty of future macroeconomic developments in comparison to a pure, unconditional CBFG. This makes the conceptual advantage of TBFG most transparent.[27] Let us consider again a negative demand shock that brings the economy to the ELB for 4 quarters. With the above-defined TBFG, the lift-off starts in period 9. Alternatively, the central bank can implement this lower-for-longer policy also with CBFG by announcing that it will keep the interest rate at the ELB until period 8. This is an unconditional statement: Even if the economy develops more favourably than expected, the central bank will keep its policy rate at the ELB until period 8. If the economy develops less favourable, the interest rate remains at the ELB for longer (until the ELB ceases to bind).

Figure 2 compares the expected development of inflation and output for such a CBFG formulation with the above-specified TBFG. Here, we assume that a wide range of unexpected shocks hit the economy in the first period (only).[28] The contrast between CBFG (left column) and TBFG (right column) is stark. Due to the promise to keep the interest rate at the ELB until period 8, CBFG imparts a stimulus regardless of the state of the economy. This leads to very volatile inflation and output paths, since the promise to keep the interest rate at the ELB until period 8 provides too much stimulus in good states and insufficient stimulus in bad states. As a result, the variance of the distributions of inflation and the output gap is relatively wide under CBFG.

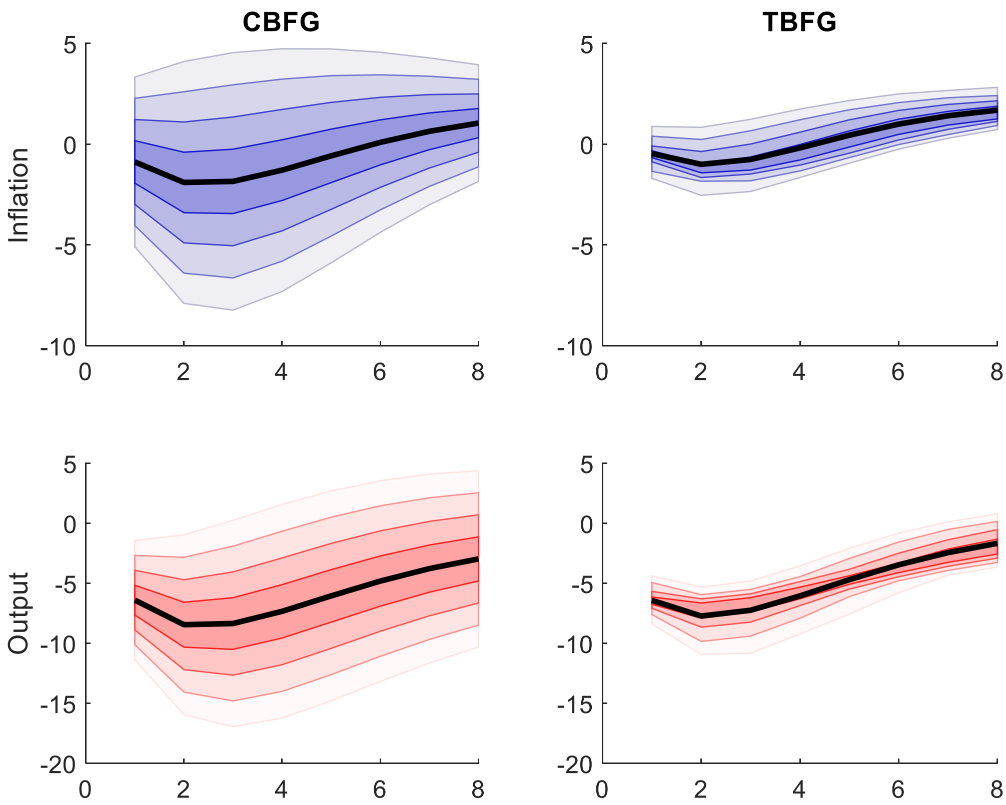
In contrast, the TBFG implemented above reduces the volatility of inflation and output substantially. The state-contingent promise to hold rates at the ELB until inflation fulfils the threshold provides additional stimulus in bad states of the world, hedging against some of the negative bias associated with the ELB. At the same time, an early lift-off in good states of the world dampens the positive bias compared to CBFG.[29]

[27] Recall that in practice the Eurosystem supplemented its calendar-based leg with its economic outlook. Hence, the following simulations do not compare the effects of the Eurosystem's forward guidance prior to September 2019 to that past September. Rather, they highlight the differences between an unconditional CBFG and TBFG. Nevertheless, and as we argue below, any change in communication is prone to possible misunderstandings or misinterpretations. Thus, TBFG (or the primacy of the state-leg) can be superior even to an "augmented" CBFG (if, of course, market participants understand the logic of TBFG properly).

[28] While the previous section considered two additional shocks (favourable/unfavourable), we simulate here from a complete distribution of further shocks (next to the shock in the baseline scenario).

[29] Nevertheless, the power of TBFG in reducing this form of uncertainty rests on the assumption that most economic agents understand the specific TBFG policy in place as well.

Figure 2: Comparison CBFG vs. TBFG



Note: Responses of inflation and output when the central bank resorts to an unconditional CBFG or a TBFG (it expects the inflation rate to be above 1.7% for 4 quarters in a row). In the first period, the economy is hit by a negative demand shock. The severity of this shock is varied to capture more and less severe recessions (captured by the areas in the figure).

A limitation of the analysis is that, in practice, a monetary authority could reduce the volatility by adjusting its calendar-based leg appropriately. However, this is problematic for two reasons. First, any deviation from past commitments might impair the central bank's credibility and render future commitments less powerful. Second, any change in communication is prone to misunderstandings or misinterpretations: Does the adaptation express a changed monetary policy orientation or merely reflect an adjustment of the economic assessment? Even if the central bank wants to give a decidedly more expansionary stimulus by extending the period at the ELB, this form of communication – contrary to its original intention – could have a contractionary influence (see, for instance, Woodford, 2012). At the same time, the effectiveness of TBFG in reducing the uncertainty is also not granted. It depends on how credible the

TBFG is and how well market participants understand the logic of TBFG.

5 Macroeconomic impact of upper limits for asset purchases

As is well known, the ELB induces a negative inflation bias when the central bank has only the short-term nominal interest rate at its disposal. To overcome such limitations, central banks have resorted to unconventional policies, notably asset purchase programmes. In principle, the larger the size of an asset purchase programme, the more expansionary it should be.

However, large asset purchases also increase the risk of undesirable side effects. For example, there is the possibility that extensive purchases of government bonds will make the central bank a dominant creditor of the governments. This could blur the boundaries between monetary and fiscal policy and thereby damage the independence of monetary policy.

In order to mitigate these and other undesirable side effects, asset purchase programmes are in practice subject to self-imposed and/or legal upper purchase limits, such as in the case of the Eurosystem's Public Sector Purchase Programme (PSPP)[30] or the Bank of England.[31] However, possible side effects – and thus the reasons for upper limits – are not part of the model analysis.

Whenever upper purchase limits indeed reduce the amount of possible asset purchases and thus the degree of monetary policy expansion, they reduce the effectiveness of asset purchase programmes. This section illustrates the relationship quantitatively in the following. Specifically, in order to illustrate the implications of asset purchases with and without limits, we first augment the model from Section 2 with the following

[30] See for instance Decision (EU) 2015/774 of the European Central Bank. The issue share limit refers to the maximum share of a single PSPP-eligible security that the Eurosystem is prepared to hold. The issuer limit refers to the maximum share of an issuer's outstanding securities that the ECB is prepared to buy.

[31] See the Consolidated Market Notice: Asset Purchase Facility: Gilt Purchases - Market Notice published on 11 June 2019: "The Bank does not currently intend to purchase gilts where the Bank holds more than 70% of the 'free float', i.e. the total amount in issue minus government holdings."

asset purchase rule:

$$\hat{b}_t = \rho_b \hat{b}_{t-1} + \mathbb{1}_{[R_t = ELB]} \phi_b \hat{\pi}_t, \tag{40}$$

where $\hat{b}_t$, denotes the volume of government debt on the central bank's balance sheet and $\hat{\pi}_t = \pi_t - \pi^*$ the deviation of net inflation from its target.[32] Thus, we model asset purchases as a state-dependent policy. The parameter ϕ_b captures the strength of this state-dependency. Higher values indicate higher purchases when the inflation rate falls below its target.

We parametrise the parameter $\phi_b = 0.37$. This implies that monthly net purchases increase by roughly 0.12% = 0.37/3 of GDP for every percentage point the inflation rate is below its target. This is in line with empirical estimates for the Eurosystem asset purchases (see Gerke, Kienzler and Scheer, 2021).[33] We also include a very persistent component by parametrising $\rho_b = 0.99$. This allows for a gradual build-up of overall central bank asset holdings, in line with actual asset purchases carried out by central banks, and a gradual unwinding of the balance sheet afterwards.

In addition, we limit asset purchases by enforcing an upper purchase limit. This restricts the volume that the central bank can hold relative to all outstanding debt. In the simulations, we either impose a limit of 33% or 25%, in line with the limits within the PSPP. Formally, we impose that $\hat{b}_t \leq 33\%$ or $\hat{b}_t \leq 25\%$.

The logic of the solution is similar to the two-regime example in Section 3, where the constrained regime is now captured by a binding ELB *and* state-dependent asset purchases. Once the dynamic equations of the respective regime are approximated up to first order, the regimes have the following representations.[34]

Unconstrained regime

$$Ax_t = C + Bx_{t-1} + DE_t x_{t+1} + F\varepsilon_t \tag{M1}$$

[32] Similar state-dependent purchase rules are analysed in Burlon, Notarpietro and Pisani (2019) and Coenen, Montes-Galdon and Schmidt (2021). While the former condition on the expected inflation rate, the latter condition on a latent (endogenous) shadow rate.

[33] They estimate the state-dependency via OLS. In particular, they regress the monthly net purchases on the inflation shortfall during 2015 (start of APP) and 2018 (PSPP first ended in December 2018).

[34] Note, again, that E_t denotes the expectations operator, x_t the vector of endogenous and ε_t the vector of exogenous variables. The other objects are matrices that capture the structural parameters of the economic system.

Constrained regime with ELB & asset purchases

$$A_t^* x_t = C_t^* + B_t^* x_{t-1} + D_t^* E_t x_{t+1} + F_t^* \varepsilon_t \tag{M2a}$$

Constrained regime with ELB & asset purchases at limit ($\hat{b}_t = 0.33$ or $\hat{b}_t = 0.25$)

$$A_t^+ x_t = C_t^+ + B_t^+ x_{t-1} + D_t^+ E_t x_{t+1} + F_t^+ \varepsilon_t \tag{M2b}$$

where regime (M2a) holds when purchases are unlimited or below the limit, and (M2b) when either limit binds.

5.1 Simulations

In order to assess the quantitative implications of purchase programmes with and without limits, we contrast three scenarios. In the first scenario, the central bank adjusts only its short-term policy rate to stimulate the economy. It does not resort to asset purchases. In the second scenario, the central bank can resort to (unlimited) asset purchases whenever the ELB is binding.[35] In the third scenario, the central bank can initiate asset purchases at the ELB, but it must obey an upper purchase limit of 25% or 33%.

For the simulations, we draw exogenous shocks from the estimated distributions except the investment specific shock.[36] Based on this, we generate 2000 simulations with a length of 200 periods each and discard the first 100 periods for initialisation. In order to roughly map the current configuration in the euro area, in the simulations we assume a symmetric inflation target of 2%, a long-term equilibrium real interest rate of 0.5% (Brand, Bielecki and Penalver, 2018) and an ELB of -0.5% (current value of the deposit

[35]Technically, there is a limit of 100% of all outstanding government debt.

[36]The investment specific shock $\varepsilon_{\mu,t}$ induced a lot of double-, triple- or quadruple-dip recessions. This reduced computing speed dramatically. Additionally, the investment specific shock elicited reversal puzzles (Carlstrom, Fuerst and Paustian, 2015; Gerke, Giesen and Kienzler, 2020) already for a relatively short ELB spell. However, without the investment-specific shock the ELB frequency in the first scenario dropped to around 10 to 15%. We thus scaled the other shocks in the simulations by a factor of 1.5 in order to generate a binding ELB frequency in the first scenario of around 30% - which is in line with the frequency when all shocks were included.

Figure 3: Average inflation rate for different policy scenarios

Note: Average annual inflation rates (in %) based on stochastic simulations that take the ELB into account. Annual inflation target is 2%, long-run level of the annual real rate is 0.5%, ELB is at -0.5%. Left marker: central bank has only interest rate at its disposal. Middle marker: central bank resorts to state-dependent asset purchases at the ELB without a limit. Right marker: central bank resorts to state-dependent asset purchases at the ELB with an upper purchase limit of 25% (lower end) and 33% (upper end).

facility rate). Note that possible side effects are not part of the model analysis. Taking these into account is important to better understand the central bank's options to fulfil its mandate. A complete analysis of those costs is beyond the scope of this technical paper (for an analysis of side effects, see, for instance, Altavilla et al., 2021).

The simulation results for the respective average inflation rates are depicted in Figure 3. Three main findings can be noted. First, the ELB causes a significant negative inflation bias if the central bank can only adjust the short-term nominal interest rate. Second, asset purchases that capture key features of the PSPP in the past can reduce the negative inflation bias, but do not completely eliminate it. Third, limits reduce the effectiveness of asset purchase programmes.

The left marker of Figure 3 illustrates the first main result. It shows an average inflation rate of around 1.5% in the baseline scenario, i.e. without an asset purchase programme. That is, the effective lower bound causes an average inflation rate that is around 50 basis points below the inflation target of 2%. Taken in isolation, this could jeopardise the central bank's credibility and make it difficult to anchor long-term inflation expectations. This would further limit the effectiveness of monetary policy.

If the central bank can resort to unconstrained asset purchases at the effective lower bound (scenario 2), the simulations show a noticeable increase in the average inflation rate towards the inflation target (middle marker). The simulations underline why such programmes have established themselves as part of unconventional monetary policy measures at the ELB. Nevertheless, the average inflation rate of just over 1.8% is still below the targeted rate of 2%.[37]

If monetary policy is faced with upper limits on its asset purchases, it will be more difficult to reach the inflation target compared to the unconstrained case. The right marker in Figure 3 shows to what extent an upper limit on the purchase volume of 25% or 33% reduces the effectiveness of asset purchase programmes. Depending on the level of the upper limit, the average inflation rate drops by around 10-15 basis points compared to a programme without an upper limit. Hence, in comparison to a case without asset purchases (scenario 1), the inflation rate is closer to its target. Nevertheless, with an average inflation rate of roughly 1.7% the central bank misses its target by more compared to the scenario without upper-bound bond purchases (scenario 2).

When central banks can resort to asset purchases with and without limits, this does not only affect the average inflation rate but the economy at large. This can be seen in Table 2, which provides selected summary statistics of the above simulations. Intuitively, if the central bank can purchase more assets at the ELB (25% limit to 33% limit to unlimited purchases), it stimulates the economy more. This results in a lower ELB frequency and duration, a reduced volatility of inflation and output and larger holdings of government debt on the balance sheet.

[37] One particular reason is the parametrisation of the state-dependent purchase rule. As described above, the strength of the state-dependency was estimated based on the ECB's PSPP. However, if we assume a stronger state-dependency, (unlimited) asset purchases can raise the average inflation rate to the inflation target.

Table 2: Summary statistics stochastic simulations

$\pi^* = 2\%$	No APP	APP (no limit)	APP (25% limit)	APP (33% limit)
Frequency ELB	28.01	21.74	22.58	22.28
Avg. duration ELB [q]	4.00	3.40	3.50	3.50
Mean inflation	1.52	1.82	1.67	1.73
Std inflation	7.52	4.56	6.43	5.77
Std output growth	13.58	5.08	8.93	7.28
Avg. CB bond holdings (% of total bonds)	-	23.68	16.06	18.99
Avg. CB bond holdings at ELB (% of total bonds)	-	28.26	18.87	22.48
Limit binding [% of time]	-	-	6.28	3.93

Note: Summary statistics based on stochastic simulations for the baseline scenario (no APP) and APP-scenarios 1 to 3, which are described in the main text of Section 5. Annual inflation target is 2%, long-run level of the annual real rate is 0.5%, ELB is at -0.5%. The inflation rate is annualised.

To sum up, the above simulations illustrate three main conclusions. First, the effective lower bound on interest rates constrains the central bank from reaching its inflation target. Second, even if central banks can resort to asset purchases without an upper limit, it is not necessarily sufficient to reach the inflation target. This is the case if the empirically estimated strength of the state dependency corresponds to that of the PSPP. Third, this is all the more true when upper limits restrict the possible purchase volume.

6 Mitigating limits of asset purchases: the case of reinvestment strategies

As illustrated in the previous section, the ELB induces a negative inflation bias when the central bank has only the short-term nominal interest rate at its disposal. To overcome such limitations central banks resort to unconventional policies, notably asset purchase programmes. However, in practice, self-imposed and legal limits can reduce the effectiveness of such programmes. Examples of such limits pertaining to the euro area are the above-mentioned issue share and issuer limits for the PSPP.

This section shows how an appropriate reinvestment strategy allows a central bank to increase its monetary stimulus, while still obeying the upper purchase limit. This question is not only pertinent given legal upper purchase limits. Additionally, large public debt holdings challenge monetary policy independence. In particular, mounting political pressure can lead to an increasingly blurred line between fiscal and monetary policy. It bears the risk of fiscal dominance, a scenario in which monetary policy can no longer guarantee price stability (for an overview of the interaction of monetary and fiscal policies, see Work stream on monetary-fiscal policy interactions, 2021).

Intuitively, through the reinvestment of maturing assets, the central bank commits to purchase more assets in the future relative to a case without such an announced reinvestment period. In essence, a reinvestment strategy is a forward guidance on future asset purchases. At the same time, the central bank does not conduct net purchases, which restricts the volume of the overall stock of bonds on its balance sheet. This allows it to obey the upper limits.

In our model, the announcement of future purchases stimulates the economy in the present. This is due to the fact that higher future purchases lower the respective yields in the future, which – through anticipation and arbitrage conditions – lower the long-term yields of today's assets. Hence, the macroeconomic effect of a bond purchase programme unfolds after its credible announcement. In other words, a stock effect is operative in our model.[38]

The stock view implicitly postulates that an intended macroeconomic impact can be achieved via two distinct programmes: one large in size but where the balance sheet is quickly reduced, and one small in size but where the reduction of the balance sheet takes longer. Hence, the central bank faces two margins of adjustments to alter the impact of asset purchase programmes. In order to raise the stimulus, it can either increase the size or it can shift purchases into the future, i.e., it can prolong or distinctly introduce the reinvestment horizon. We illustrate how much the central bank can reduce the

[38] In contrast to the stock view, the flow view emphasises the (macro-)effects of the individual purchases and their timing instead of the expected stock of assets over the entire programme. Empirically, the stock view seems to dominate (D'Amico and King, 2013; Sudo and Tanaka, 2021).

size of purchase programmes with an appropriate reinvestment strategy – for a given amount of aggregate stimulus.

With a reinvestment strategy the central bank announces not only the overall programme size but also a specific horizon after the end of net purchases during which all maturing assets will be reinvested. As an example, on the 4 June 2020, the Eurosystem not only extended the size of its Pandemic Emergency Purchase Programme (PEPP), but also introduced a specific reinvestment horizon after the end of net purchases: "The maturing principal payments from securities purchased under the PEPP will be reinvested until at least the end of 2022."(ECB, 2020). Such specific reference to reinvestments modifies previous monetary policy statements that largely emphasised the monthly or overall size of net purchases (Fed, 2012; ECB, 2018).

We implement the "PEPP" as of June 2020 with respect to the announced size, time and reinvestment.[39] In particular, we assume that the purchase programme reaches a size of €1350bn (∼ 11% of GDP) in 2021Q2, after which the reinvestment continues for another 6 quarters until 2022Q4. We contrast this scenario with counterfactual reinvestment strategies, specifically a case without reinvestment and a reinvestment period that is twice as long as the one originally announced.

6.1 Technical implementation

In order to match the size and timing of the "PEPP", we adapt the rule that governs asset purchases. We split this process into three distinct sub-parts: net purchases, reinvestment and reduction/unwinding of balance sheet. We consider each of them in turn.

1. We assume that the central bank implements an exogenous path of asset pur-

[39] We put the "PEPP" into quotation marks as we abstract from various issues that are important to assess its macroeconomic effects properly. For instance, we do not capture the origin of the pandemic (supply/demand shock) but assume a stylised demand-driven recession. We also cannot capture potential smaller effects of monetary policy during a pandemic, as economic agents are less forward-looking, reduce their marginal propensity to consume or because monetary policy cannot ensure an efficient allocation in an n-sector economy (Levin and Sinha, 2020; Lepetit and Fuentes-Albero, 2020; Woodford, 2020; Guerrieri, Lorenzoni, Straub and Werning, 2020).

chases with reference to the "PEPP". In particular, it evenly distributes the purchases of €1350bn over 5 quarters (2020Q2, to 2021Q2). The following (simple) functional form describes the stock of bonds on the central bank's balance sheet, $\hat{b}_t$, as follows:

$$\hat{b}_t = \bar{b}_t, \tag{41a}$$

where $\bar{b}_t$ denotes an exogenous value (which increases from 0 to €1350bn).

2. During the reinvestment period, the central bank reinvests all maturing assets for the above-mentioned 6 quarters.

$$\hat{b}_t = \hat{b}_{t-1} \tag{41b}$$

Such a process does not lead to explosive dynamics as the reinvestment horizon is finite.

3. After the reinvestment period, the central bank gradually unwinds its balance sheet. We use the following simple AR(1) process to allow for a gradual reduction:

$$\hat{b}_t = \rho_b \hat{b}_{t-1}. \tag{41c}$$

A compact representation of the asset purchase programme with reinvestment looks as follows:[40]

$$\hat{b}_t = \mathbb{1}_{[t \in \{2020Q2,\ldots,2021Q2\}]}\bar{b}_t + \mathbb{1}_{[t \in \{2021Q3,\ldots,2022Q4\}]}\hat{b}_{t-1} + \mathbb{1}_{[t > 2022Q4]}\rho_b \hat{b}_{t-1}. \tag{42}$$

We solve this system as sketched in Section 3 but with two constrained regimes. Constrained regime with exogenous purchases:

$$A_t^* x_t = C_t^* + B_t^* x_{t-1} + D_t^* E_t x_{t+1} + F_t^* \varepsilon_t. \tag{M2}$$

[40] For a more general version with state-dependent (instead of exogenous) net purchases and different reinvestment horizons, see Gerke et al. (2021).

Constrained regime with reinvestment:

$$A_t^+ x_t = C_t^+ + B_t^+ x_{t-1} + D_t^+ E_t x_{t+1} + F_t^+ \varepsilon_t. \tag{M3}$$

Given the above-mentioned timing, the time-varying policy function is then given by

$$x_t = J_t + Q_t x_{t-1} + G_t \varepsilon_t, \tag{43}$$

with

$$J_t = J,\ Q_t = Q,\ G_t = G \ \ \forall t > 2022Q4, \text{ the time-invariant policy function (38)}$$

$$\Xi_t = \left(A_t^+ - D_t^+ Q_{t+1}\right)^{-1}, J_t = \Xi_t\left(C_t^+ + D_t^+ J_{t+1}\right), Q_t = \Xi_t B_t^+, G_t = \Xi_t F_t^+ \text{ for } 2021Q2 < t \leq 2022Q4$$

$$\Xi_t = \left(A_t^* - D_t^* Q_{t+1}\right)^{-1}, J_t = \Xi_t\left(C_t^* + D_t^* J_{t+1}\right), Q_t = \Xi_t B_t^*, G_t = \Xi_t F_t^* \text{ for } 2020Q2 \leq t \leq 2021Q2$$

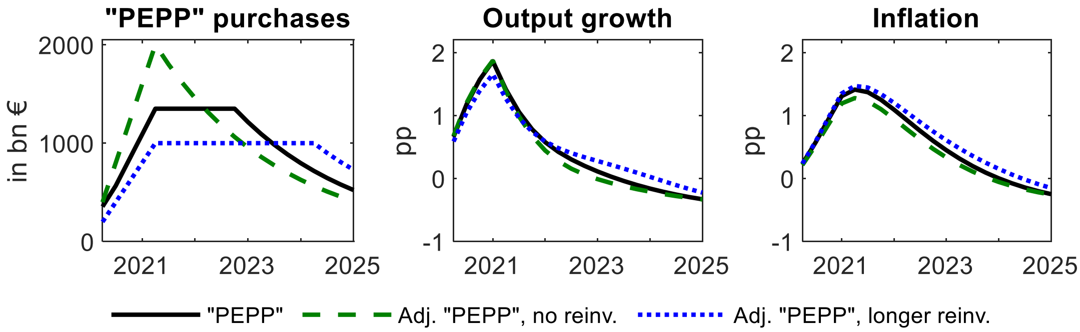
6.2 The impact of different reinvestment strategies

To illustrate the effect of different reinvestment strategies, we assume that the central bank launches a purchase programme "PEPP" in response to a severe recession. In particular, the economy is hit by a negative demand shock in 2020Q2. The shock is calibrated such that the short-term policy rate is constrained by the ELB for six quarters, i.e. until 2021Q3, when only the policy rate is at the central bank's disposal.

Figure 4 depicts the stock of purchases (left panel) and the macroeconomic response of output (middle panel) and inflation (right panel). The macroeconomic effects of the "PEPP" are depicted as a black solid line. It raises output and inflation relative to the baseline, in which the central bank adjusts only its policy rate.

We compare the "PEPP" with two alternative designs. First, we abstain from any reinvestment and ask how much higher the announced overall size should have been to induce the same stimulus as the "PEPP". Second, we prolong the reinvestment period

Figure 4: Similar macroeconomic effects for different purchase programmes



Note: Three different purchase programmes that differ in size and reinvestment horizon are depicted. However, the area below the curves are approximately the same. The results are depicted relative to a baseline scenario. In the baseline, the economy is hit by a negative demand shock and the central bank can only adjust its short-term policy rate to stabilise the economy. Output growth and inflation are annualised.

for a further six quarters (i.e. until mid-2024) and ask how much smaller the overall size could have been to achieve the same stimulus. For each programme, the overall size is chosen such that the area below the path of the central bank's balance sheet (left panel) is approximately the same.[41] According to the present simulations, the macroeconomic impact is relatively similar as can be seen from the middle and right panel (as the stock view predicts).

In particular, we find that without reinvestment (green dashed line), the Eurosystem would have needed to announce a €450bn (4% of GDP) larger volume of asset purchases, i.e. around €1800bn (or 15% of GDP) instead of €1350bn (11% of GDP). If the Eurosystem had instead extended the reinvestment horizon by a further 6 quarters, it could have reduced the volume of purchases by €350bn (3% of GDP). Hence, it could have announced a volume of "only" around €1000bn (around 8% of GDP, green dashed dotted line).

These simulations illustrate how a reinvestment strategy can, in principle, mitigate the impact of an upper limit of purchases. Reinvestment allows asset purchases to be shifted into the future – it is a forward guidance on asset purchases. The longer the future forward guidance on asset purchases (the longer the reinvestment horizon), the

[41]There are small deviations as the areas are not completely the same and there is some form of discounting if a large amount of purchases is delayed into the future.

longer the central bank leaves its public debt holdings on its balance sheet. This allows it to reduce the overall size of net purchases.

To sum up, reinvestment of maturing government bonds can soften the constraint imposed by an upper purchase limit. Hence, the central bank has two margins to increase the impact of its asset purchase programmes: it can increase the size or it can prolong the reinvestment horizon. Nevertheless, the stabilisation properties of reinvestment depend crucially on the private sector's understanding of such a forward guidance-type policy as well as its forward-lookingness – as we have already pointed to in the previous sections. We leave a quantitative exploration of the robustness with respect to more realistic expectation formation for future work.

7 Conclusion

Most central banks of advanced economies have resorted to unconventional policies as their short-term policy rate reached the effective lower bound. As part of its strategy review, the Eurosystem assessed the effectiveness of such measures, and a recent issue of the Bundesbank's Monthly Report (Deutsche Bundesbank, 2021) surveys the main results of the strategy review. One conclusion is that unconventional policies can significantly contribute to reducing the stabilisation bias and the heightened macroeconomic volatility generated by the effective lower bound (ELB) on interest rates.

In this paper we present selected results that have been produced for the model-based assessments on the impact of unconventional monetary policies discussed during the Eurosystem strategy review. In particular, we discuss in detail the methodology to produce these results and can draw four main conclusions with respect to various unconventional policies.

First, state-dependent forward guidance can reduce the volatility and hence uncertainty of future macroeconomic developments in comparison to unconditional calendar-based forward guidance. The state-contingent promise to hold rates at the ELB until inflation exceeds a certain threshold provides additional stimulus in bad states and reduces stimulus in good states of the world. This may be perceived as a hedge against some of the negative bias associated with the ELB.

Second, the effective lower bound induces a large negative inflation bias if the central bank has only its short-term policy rate at its disposal. The reason is that in the face of disinflationary or deflationary shocks, interest rate policy alone cannot provide inflationary stimulus beyond the ELB. Hence, the inflation rate falls short of monetary policy's target on average.

Third, asset purchases that are initiated when the policy rate is at the ELB can provide significant inflation stabilisation. However, it remains uncertain whether purchases fully offset the distortions created by the lower bound. This is all the more true whenever upper purchase limits restrict the possible purchase volume.

Fourth, an appropriate reinvestment strategy can help to mitigate the effects of upper purchase limits. In essence, a reinvestment strategy is a forward guidance on future asset purchases. This announcement stimulates the economy in the present as higher future purchases lower the respective yields in the future which – through anticipation and arbitrage conditions – lower the long-term yields of today's assets. Hence, the central bank has two margins to increase the effects of its asset purchase programmes: it can increase the size or it can extend the reinvestment horizon.

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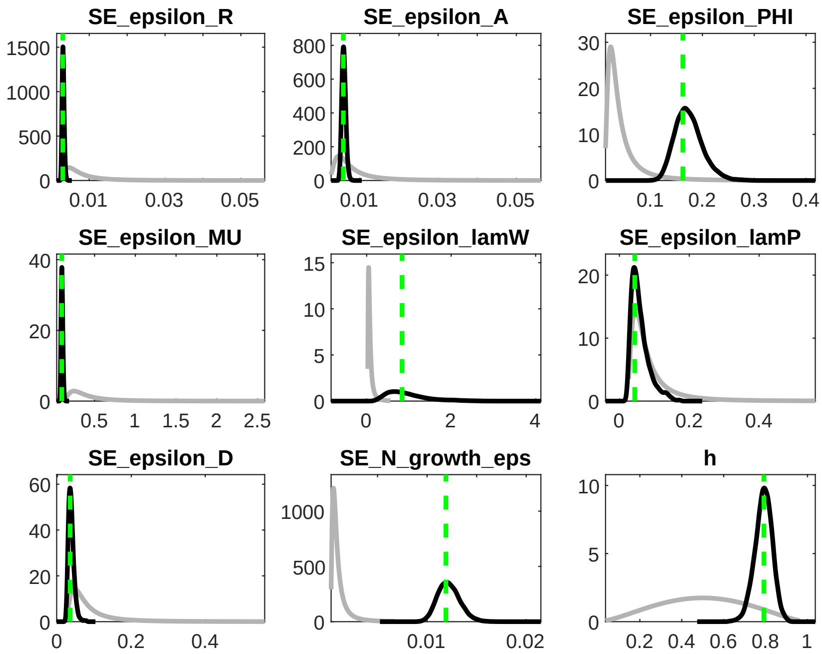
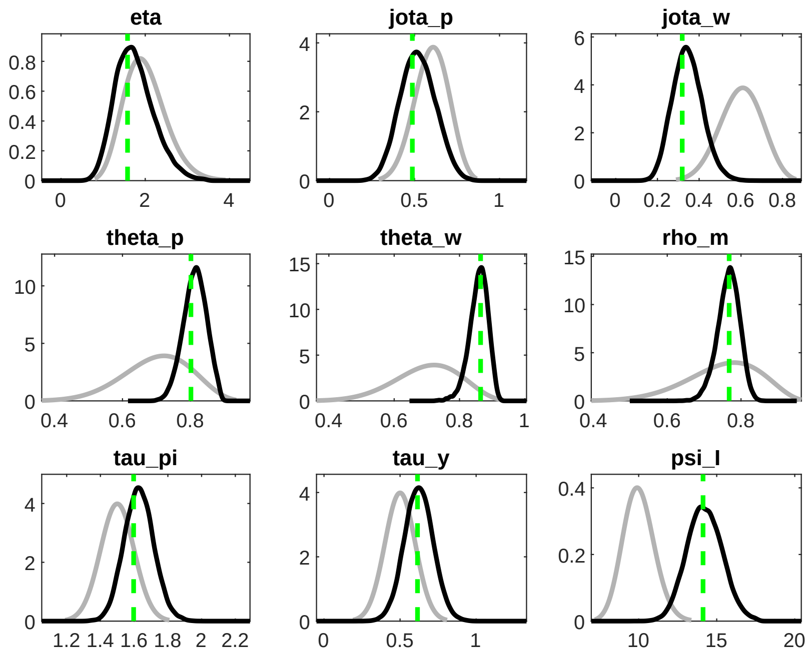
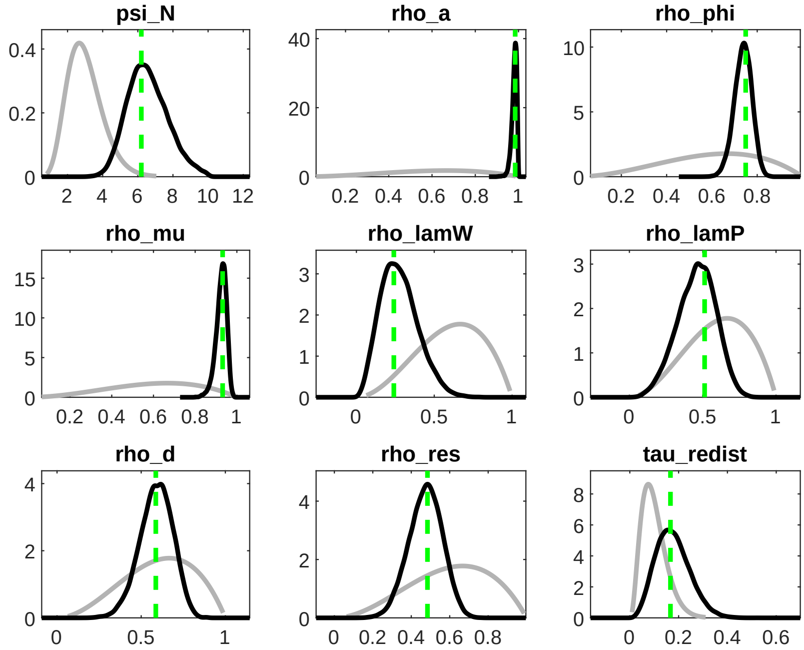
A Prior and posterior of estimated parameter







B Asset purchases and make-up strategies

As pointed out in Deutsche Bundesbank (2021), make-up approaches can have desirable stabilising properties if we assume rational expectations. However, the Eurosystem has decided against adopting a make-up strategy for good reasons (see Deutsche Bundesbank, 2021). For the sake of completeness, the following section shows that make-up strategies are able to eliminate the inflation bias in combination with asset purchases.

In particular, we contrast the effectiveness of an interest rate rule as in Section 2 with various combinations of limited asset purchases and make-up policies (each specified as an instrument rule). We consider a price-level targeting and two average-inflation targeting rules. For each combination of policies, we assume that the central bank conducts state-dependent net asset purchases as in the previous section. That is, the central bank conducts net purchases only if the ELB is binding and purchases a higher (lower) volume of bonds the lower (higher) the inflation shortfall from target. Purchases are again subject to a 25% or 33% limit. For convenience, we repeat the rule here:

$$\hat{b}_t = \rho_b \hat{b}_{t-1} + \mathbb{1}_{[R_t=ELB]} \phi_b \hat{\pi}_t, \tag{40}$$

When monetary policy adopts an average-inflation targeting (AIT) with a four- or eight-year-averaging window, it follows the rule

$$R_t = \rho_r R_{t-1} + (1-\rho_r)\left(\phi_\pi \hat{\pi}_t + T\hat{\pi}_t^{4T} + \phi_y [\hat{y}_t - \hat{y}_{t-1}]\right), \tag{44}$$

where T is the averaging window (4 or 8 years) and $\hat{\pi}_t^{4T}$ is the annualised average log-deviation of the inflation rate relative to the steady state over the past T years. This rule is similar to Coenen et al. (2021) and Arias, Bodenstein, Chung, Drautzburg and Raffo (2020). In case monetary policy adopts a price-level targeting (PLT), it follows the rule

$$R_t = \rho_r R_{t-1} + (1-\rho_r)\left(\phi_\pi \hat{\pi}_t + \hat{p}_t + \phi_y [\hat{y}_t - \hat{y}_{t-1}]\right), \tag{45}$$

with $\hat{p}_t = \hat{\pi}_t + \hat{p}_{t-1}$.

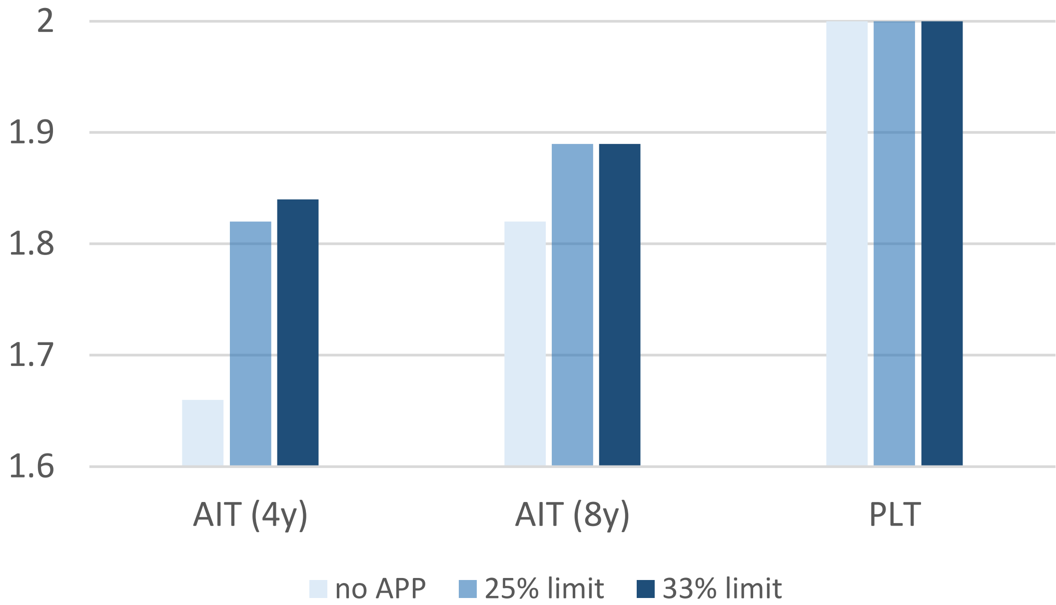
Figure 5 illustrates the stabilisation gains of the respective make-up element in addition to an asset purchase programme with a 25% and 33% upper purchase limit. Recall from Section 5 that IT in combination with an asset purchase programme with a 25% and 33% upper purchase limit implies an inflation bias of approximately 25 and 35 basis points. Compared to this, the negative inflation bias is reduced further to less than 20 to 10 basis points (first and second set of bars) for an AIT with a four- and eight-year averaging window, respectively.

In the case of a longer averaging window, the limits on asset purchases become less relevant as the make-up element stabilises the inflation rate quite well. As an upper purchase limit of 25% is hardly binding, the aggregate effects of a 33% limit are very similar (limits are binding less than 0.3% of the time: see Tables 3 and 4). The "irrelevance" of limits when asset purchase programmes are combined with make-up strategies is even more clearly visible when monetary policy adopts a PLT (third set of bars). In this case, the average inflation rate is at the inflation target independent of the use of asset purchases. Hence, in case of rational expectations, PLT can completely remove the inflation bias induced by the ELB.

The reason why make-up strategies in addition to limited asset purchases decrease the negative inflation bias is straightforward: Make-up strategies allow inflation expectations to act as automatic stabilisers. After a deviation from the inflation target, a make-up strategy entails a compensation by inducing an over- or undershooting of inflation in the future. To the extent that such a policy is credible, well understood and anticipated by the private sector (we assume rational expectations), it stabilises current inflation via its impact on the ex-ante real interest rate.

Moreover, Figure 6 highlights the fact that the adoption of make-up strategies potentially implies a marked decrease in the inflation and output volatility. Each combination of limited asset purchases and make-up strategy yields almost the same volatilities for the two different limits (25% and 33%), hence the reported reductions in the volatil-

Figure 5: Average inflation rate for AIT/PLT in combination with asset purchases



Note: Average inflation rates (in %) when the central bank adopts a make-up element. Results are based on stochastic simulations that take the ELB into account. Annual inflation target is 2%, long-run level of the annual real rate is 0.5%, ELB is -0.5%. In all scenarios, the central bank additionally adopts state-dependent net asset purchases at the ELB with limits (25% limit or 33% limit) or no asset purchases (no APP). Left set of bars: Central bank adopts average-inflation targeting (AIT) with 4 years of averaging. Middle set of bars: AIT with 8 years of averaging. Right set of bars: price-level targeting (PLT).

Table 3: Asset purchases with a 25% limit

	IT	AIT(4y)	AIT(8y)	PLT
Frequency ELB	22.58	19.56	18.70	24.04
Avg. duration ELB [q]	3.50	2.80	2.70	3.40
Mean inflation	1.67	1.82	1.89	2.00
Std inflation	6.43	3.21	3.22	2.34
Std output growth	8.93	4.71	4.78	4.74
Avg. CB bond holdings (% of total bonds)	16.06	9.62	9.53	4.96
Avg. CB bond holdings at ELB (% of total bonds)	18.87	11.46	10.59	5.90
Limit binding [% of time]	6.28	0.66	0.27	0.00

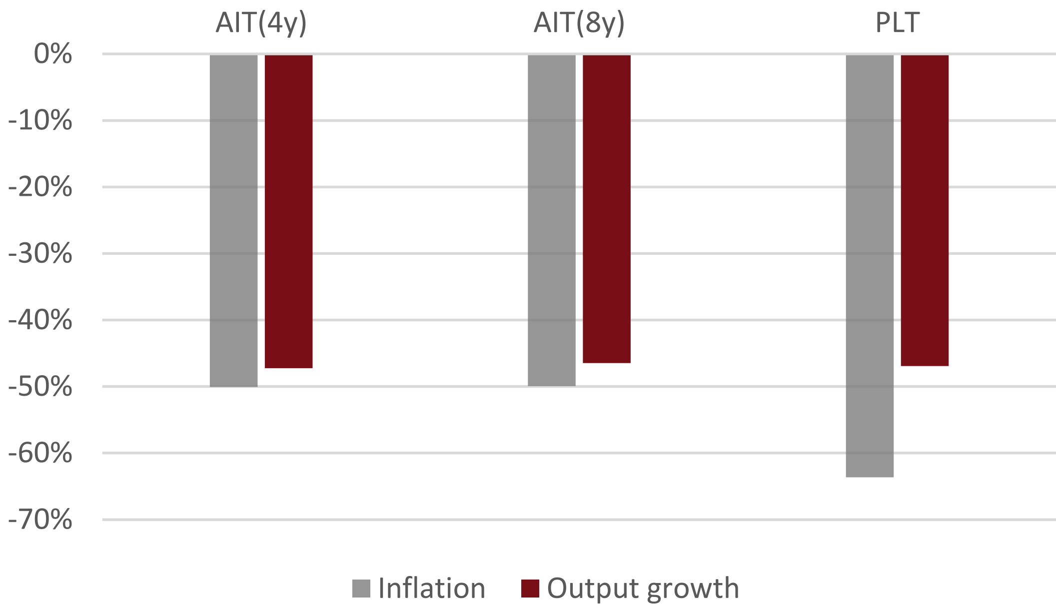
Note: Summary statistics based on stochastic simulations for different monetary policy rules (IT: inflation targeting (baseline); AIT: average inflation targeting with an averaging window of 4 or 8 years; PLT: price level targeting) combined with central bank (CB) asset purchases as specified in Section 5. Annual inflation target is 2%, long-run level of the annual real rate is 0.5%, ELB is at -0.5%. The inflation rate is annualised.

Table 4: Asset purchases with a 33% limit

	IT	AIT(4y)	AIT(8y)	PLT
Frequency ELB	22.28	19.50	18.69	24.04
Avg. duration ELB [q]	2.80	2.70	2.70	3.40
Mean inflation	1.73	1.84	1.89	2.00
Std inflation	5.77	3.16	3.21	2.34
Std output growth	7.28	4.69	4.78	4.74
Avg. CB bond holdings (% of total bonds)	18.99	9.82	9.63	4.96
Avg. CB bond holdings at ELB (% of total bonds)	22.48	11.71	11.08	5.90
Limit binding [% of time]	3.93	0.16	0.03	0.00

Note: Summary statistics based on stochastic simulations for different monetary policy rules (IT: inflation targeting (baseline); AIT: average inflation targeting with an averaging window of 4 or 8 years; PLT: price level targeting) combined with central bank (CB) asset purchases as specified in Section 5. Annual inflation target is 2%, long-run level of the annual real rate is 0.5%, ELB is at -0.5%. The inflation rate is annualised (a).

Figure 6: Decrease in volatility due to make-up element



Note: Change in the inflation volatility (grey bar) and output growth volatility (red bar) in % for three different make-up strategies. The results are relative to a scenario in which monetary policy follows its estimated policy rule and conducts state-dependent net asset purchases at the ELB that are subject to an upper limit of 25% (see Table 2). Results calculated from stochastic simulations taking into account the ELB. State-dependent net purchases conducted whenever the ELB is binding.

ity apply to both limits equally. Three aspects are noteworthy. First, all make-up elements cut the volatility of both output growth and inflation roughly in half. Second, the increased stabilisation properties for AIT are roughly similar for both lengths of the averaging windows (first two columns). Third, a price-level targeting element reduces the inflation volatility most whilst at the same time the decrease in output volatility is (slightly) less pronounced than for the different AIT-specifications.[42]

To summarise, the joint use of make-up strategies and asset purchases with limits could, given rational expectations, successfully reduce and even eliminate the bias, whilst at the same time reducing the volatility of the economy markedly. Of course, the stabilisation properties depend crucially on the private sector's understanding of the make-up strategies as well as its forward-lookingness. Model results show that make-up strategies typically preserve their favourable properties for (small) deviations from rational expectations (see, for instance, Deutsche Bundesbank, 2021).

[42]It is well known that make-up strategies can lead to a heightened volatility in real economic activity in the case of supply shocks. Nevertheless, monetary policy can mitigate this effect by a weaker response to deviations from the intended inflation rate, thus stretching out the time horizon over which monetary policy makes up for past inflation misses.