

# Technical Paper

A comparison of monetary policy rules  
in a HANK model

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## Non-technical summary

In January 2020, the Eurosystem started a comprehensive review of its monetary policy strategy. The review process included a sequence of seminars dedicated to a range of topics supported by material prepared by various Eurosystem work streams. [Deutsche Bundesbank \(2021\)](#) summarises the main results of the strategy review, which ended in July 2021, and discusses the findings made in selected work streams, including the work stream on the price stability objective (WS PSO). Among the topics covered by this work stream was the potential of alternative monetary policy strategies to address limitations faced by conventional monetary policy due to an effective lower bound (ELB) on the short-term nominal interest rate.

Specifically, the WS PSO carried out simulations based on a large suite of macroeconomic models using a common simulation protocol to enhance comparability. The key take-away from these simulations is that monetary policy strategies with history-dependent (or “make-up”) elements have good macroeconomic stabilisation properties and can successfully attenuate negative biases in inflation and economic activity due to the ELB. While asymmetric monetary policy strategies can address these negative biases as well, they can result in a substantial overshooting of the inflation target if they also include make-up elements.

The model suite used by the WS PSO largely involved representative agent New Keynesian (RANK) models. Though frequently used for monetary policy analysis, RANK models have a number of shortcomings. These include a counterfactually strong interest rate sensitivity of household consumption. As a result, the predictions of the WS PSO about the effectiveness of certain monetary policy strategies could be biased.

The contribution of this paper is to assess the robustness of the findings of the WS PSO by using a heterogeneous agent New Keynesian (HANK) model with a more realistic depiction of household behaviour. In the model, households face idiosyncratic income risk and a borrowing constraint. Imperfect insurance against income risk leads to an economy with households that are heterogeneous with respect to their income and wealth. Moreover, households exhibit different marginal propensities to consume which matters for the transmission of monetary policy. The model economy features nominal price and wage rigidities, an occasionally binding ELB as well as aggregate cost-push and demand shocks. Closely following the simulation protocol of the WS PSO, monetary policy is modelled via simple feedback rules for the short-term nominal interest rate that are augmented with history-dependent and/or asymmetric elements.

Based on the HANK model simulations, this paper concludes that the simulation results of the WS PSO do not materially change if household heterogeneity is accounted for.

## Nichttechnische Zusammenfassung

Im Januar 2020 startete das Eurosystem die Überprüfung seiner geldpolitischen Strategie. Der Überprüfungsprozess beinhaltete eine Reihe von Seminaren zu diversen Themen und wurde unterstützt durch mehrere Arbeitsgruppen mit Mitgliedern aus dem gesamten Eurosystem. [Deutsche Bundesbank \(2021\)](#) stellt die zentralen Ergebnisse der im Juli 2021 beendeten Strategieüberprüfung vor und diskutiert die Resultate ausgewählter Arbeitsgruppen, inklusive der Arbeitsgruppe zum Preisstabilitätsziel (AG PSZ). Zu den Themen, die in dieser Arbeitsgruppe bearbeitet wurden, gehörte das Stabilisierungspotenzial alternativer geldpolitischer Strategien zur Bewältigung der Einschränkungen, denen sich die konventionelle Geldpolitik aufgrund einer effektiven Untergrenze für den kurzfristigen Nominalzins gegenüber sieht.

Zu diesem Zweck führte die AG PSZ Simulationen auf Basis einer Reihe verschiedener makroökonomischer Modelle durch. Zwecks Vergleichbarkeit wurde hierbei ein harmonisiertes Simulationsprotokoll verwendet. Das wesentliche Ergebnis der Modellsimulationen lautet, dass vergangenheitsabhängige geldpolitische Strategien gute Stabilitätseigenschaften besitzen und durch die Zinsuntergrenze bedingte negative Verzerrungen bei der Inflationsrate und der ökonomischen Aktivität erfolgreich abschwächen können. Obgleich asymmetrische geldpolitische Strategien diese negativen Verzerrungen ebenfalls reduzieren können, gehen sie mit einem beträchtlichen Überschießen des Inflationsziels einher, wenn sie zusätzlich über vergangenheitsabhängige Elemente verfügen.

Die in der Arbeitsgruppe verwendeten Modelle sind überwiegend neukeynesianische Modelle mit einem repräsentativen Agenten (auch RANK-Modelle genannt). Auch wenn dieser Modelltyp häufig für geldpolitische Analysen herangezogen wird, verfügt er über eine Reihe von Schwachstellen. Zu diesen gehört ein Konsumverhalten von Haushalten, das kontrafaktisch stark auf Zinsänderungen reagiert. Die Prognosen der Arbeitsgruppe hinsichtlich der Effektivität bestimmter Regeln könnten daher verzerrt sein.

Die vorliegende Arbeit überprüft die Robustheit der Ergebnisse der Arbeitsgruppe indem es ein neukeynesianisches Modell mit heterogenen Haushalten (auch HANK-Modell genannt) verwendet, das das Konsumverhalten der Haushalte realitätsnäher abbilden kann. Im Modell sind Haushalte einem individuellen Einkommensrisiko und einer Verschuldungsrestriktion ausgesetzt. Da die Haushalte sich nicht vollständig gegenüber individuellen Einkommensrisiken absichern können, unterscheiden sie sich im Modell hinsichtlich der Höhe ihres Einkommens und ihres Vermögens. Weiterhin unterscheiden sie sich in ihrer marginalen Konsumneigung, was von Bedeutung für die geldpolitische Transmission ist. Die Modellökonomie umfasst nominale Preis- und Lohnrigiditäten, eine gelegentlich bindende Zinsuntergrenze sowie sogenannte Cost-Push-Schocks und Schocks der Konsumnachfrage. Die Spezifikation und Simulation des Modells orientiert sich am harmonisierten Protokoll der Arbeitsgruppe, das die Geldpolitik durch eine einfache Feedback-Regel für den

kurzfristigen Nominalzins abbildet, die vergangenheitsabhängige und/oder asymmetrische Elemente aufweisen kann.

Auf Basis der Simulationsergebnisse kommt die vorliegende Arbeit zu dem Schluss, dass sich die Befunde der Arbeitsgruppe hinsichtlich der Effektivität alternativer geldpolitischer Strategien durch die Berücksichtigung von Ungleichheiten zwischen Haushalten nicht wesentlich ändern.

# A Comparison of Monetary Policy Rules in a HANK Model\*

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## Abstract

This paper provides a comparison of monetary policy rules with make-up and/or asymmetric elements for a heterogeneous agent New Keynesian (HANK) model. The model features incomplete financial markets, nominal price and wage rigidities, rational expectations, an occasionally binding effective lower bound (ELB) on the short-term nominal interest rate as well as aggregate demand and cost-push shocks. Simulations show that symmetric policy rules with make-up elements can substantially lower the downward inflation bias induced by the ELB and reduce macroeconomic volatility. Asymmetric policy rules can address the downward inflation bias as well but lead to a substantial overshooting of the inflation target if they also feature make-up elements. The predictions of the HANK model for the considered policy rules are close to those obtained for a corresponding (representative agent) model version with complete markets.

*Keywords:* Monetary Policy, HANK, Household Heterogeneity, Effective Lower Bound, Make-Up Strategies, Asymmetric Policy Rules, Inequality

*JEL Classification:* D31, E21, E31, E52, E58, J31

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# 1 Introduction

Since 2014, the Eurosystem has faced the challenge of providing economic stimulus with short-term nominal interest rates near or at the effective lower bound (ELB).<sup>1</sup> In response, the Eurosystem has expanded its policy toolkit and engaged in a number of unconventional monetary policies, such as asset purchases, forward guidance, negative interest rate policy and targeted longer-term refinancing operations. While these policies helped stimulate economic activity in the euro area they can have undesirable side effects (see e.g. [Altavilla et al., 2021](#); [Work Stream on Monetary-Fiscal Policy Interactions, 2021](#)). One other way of adjusting monetary policy to the “new normal” is to adopt a new policy strategy that is better suited to addressing the current challenges. The most prominent examples of such alternative strategies are (i) history-dependent (or make-up) strategies, and (ii) asymmetric strategies, which can – in theory – both help mitigate the problem posed by the ELB.<sup>2</sup>

As part of the Eurosystem’s 2020-21 strategy review, the work stream on the price stability objective (WS PSO) was tasked with assessing such alternative monetary policy strategies. To this end, the WS PSO conducted stochastic simulations using a large suite of macroeconomic models for the euro area with a common simulation protocol to enhance comparability. This protocol specifies alternative monetary policy strategies via simple feedback rules for the short-term nominal interest rate. Based on the model simulations, the WS PSO arrived at the conclusion that overall, monetary policy rules with a higher degree of history dependence tend to perform better in terms of macroeconomic stabilisation. They attenuate negative biases in inflation and economic activity resulting from an occasionally binding ELB and reduce macroeconomic volatility (see [Work Stream on the Price Stability Objective, 2021](#)). Asymmetric policy rules are found to be capable of addressing these negative biases as well, but may be associated with a substantial positive inflation bias, i.e. an average inflation rate above the target value, when they feature history-dependent elements.

The majority of the simulations used representative agent New Keynesian (RANK) models.<sup>3</sup> This class of models has certainly proven its worth in the conceptual and quantitative analysis of a variety of monetary policy questions. Yet, it exhibits well-known shortcomings that matter particularly when analysing monetary policy at the ELB. Most notable is the forward guidance puzzle, whereby a known future accommodative policy stance leads

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<sup>1</sup>Developments that led to this scenario include declining real interest rates (see e.g. [Holston et al., 2017](#); [Del Negro et al., 2019](#)) and disinflationary trends and shocks (see e.g. [Koester et al., 2021](#)).

<sup>2</sup>See e.g. [Mertens and Williams \(2019\)](#), [Arias et al. \(2020\)](#), [Bianchi et al. \(2021\)](#).

<sup>3</sup>There is one notable exception, which – based on [Fernández-Villaverde et al. \(2021\)](#) – involved some quantitative exercises for the WS PSO based on a New Keynesian model with heterogeneous households. However, a systematic comparison of different monetary policy rules based on the common simulation protocol was not carried out in this case. To the best of our knowledge, currently only [Feiveson et al. \(2020\)](#) have compared different monetary policy rules for a model with heterogeneous households but without clarity as to the exact model employed for this analysis.

to implausibly large contemporary effects of monetary policy.<sup>4</sup> The source of these puzzles lies in the counterfactual consumption behaviour of households. In RANK models, households are permanent-income consumers and therefore excessively sensitive to current and future interest rate changes. In addition, their consumption behaviour hardly responds to temporary income changes, i.e. they have very low marginal propensities to consume (MPCs). Empirically, however, a sizeable fraction of households barely react to interest rate changes but exhibit large MPCs out of temporary income changes.<sup>5</sup>

Taking these shortcomings into account, we build a heterogeneous agent New Keynesian (HANK) model to conduct a quantitative comparison of monetary policy rules with make-up and/or asymmetric elements.<sup>6</sup> Our model features households that are subject to idiosyncratic income risk and an ad-hoc borrowing constraint. Due to market incompleteness, income risk is partially uninsurable, implying that households are heterogeneous ex post. In addition to sticky goods market prices, our model features sticky nominal wages, which generates meaningful monetary policy trade-offs in the presence of aggregate supply shocks.

Relative to RANK models, our HANK model offers at least three advantages. First, under our calibration the model is not subject to the forward guidance puzzle.<sup>7</sup> Second, as in the data, households in our model have different MPCs and therefore react differently to income changes. Third, household heterogeneity in nominal assets leads to distributional effects of monetary policy between savers and borrowers.

To enhance the comparability of our results with those of the WS PSO, we closely follow its simulation protocol for the calibration and specification of our model (see [Work Stream on the Price Stability Objective, 2021](#)). Our key findings are as follows:

- Compared to inflation targeting (IT), history-dependent rules lead to overall improved stabilisation properties and mitigate (or even undo) the downward inflation bias induced by the ELB.
- A higher degree of history dependence implies stronger macroeconomic stabilisation and a lower downward inflation bias.
- History-dependent rules are better able to alleviate household borrowing constraints.
- Asymmetric inflation targeting leads to improvements over standard IT.

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<sup>4</sup>A related puzzle is known as the reversal puzzle (see [Carlstrom et al., 2015](#); [Gerke et al., 2020](#)), which involves a counterintuitive contraction of inflation in response to an interest rate peg.

<sup>5</sup>This finding has been established using both aggregate time series data (e.g. [Deaton, 1987](#); [Campbell and Mankiw, 1989](#)) as well as individual-level panel data (see e.g. [Vissing-Jørgensen, 2002](#); [Kaplan et al., 2014](#)).

<sup>6</sup>See [Kaplan and Violante \(2018\)](#) for a review of the HANK literature, which integrates household heterogeneity into New Keynesian models.

<sup>7</sup>By that, we mean that the output and inflation responses remain bounded if the distance between the announcement of a future (one-period) real rate cut and its implementation goes to infinity.



- Asymmetric average inflation targeting substantially reduces ELB occurrences and durations but comes with inflation overshooting and increased macroeconomic volatility.

Overall, we conclude that our results are in line with the central findings of the WS PSO.<sup>8</sup>

## 2 Model

The model economy is similar to [Hagedorn et al. \(2019\)](#) and [Ferrante and Paustian \(2019\)](#). Households are subject to uninsurable idiosyncratic income risk and an ad hoc borrowing constraint giving rise to a non-degenerate distribution of household wealth. At the aggregate level, the model economy features sticky goods prices as well as sticky nominal wages, incorporates an effective lower bound on the short-term monetary policy rate and – unlike [Hagedorn et al. \(2019\)](#) and [Ferrante and Paustian \(2019\)](#) – is subject to mark-up (“cost-push”) and discount factor (“demand”) shocks. The goods and labour markets are both modelled in a fairly standard fashion. Intermediate-good firms produce by using labour supplied by aggregate labour packers.<sup>9</sup> They face monopolistic competition and set the price of their product subject to quadratic adjustment costs (see [Rotemberg, 1982](#)). A competitive final-good sector aggregates intermediate inputs and bundles them into a final good available for households to consume. As in [Auclert et al. \(2021\)](#), households elastically provide labour services to a continuum of labour unions. Unions set wages for their members (the households) taking as given labour demand from labour packers, who aggregate labour services provided by unions, as well as the consumption-saving decision of households. When setting nominal wages, unions incur quadratic adjustment costs. Labour packers then sell labour services to intermediate-good producers.

### 2.1 Model details

This section provides a more detailed description of the model components.

**Households** There is a unit-mass continuum of households in the economy, indexed as  $i \in [0, 1]$ . Households choose consumption and bond holdings  $\{c_{i,t}, b_{i,t}\}_{t=0}^{\infty}$  to maximise

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<sup>8</sup>Note, however, that most of the WS simulations were – as in our model – conducted under the assumption of rational expectations. Our results as well as the results of the WS are therefore subject to this caveat. This needs to be kept in mind when deriving potential policy implications, since different (non-rational) expectations processes could very well yield different conclusions. See [Deutsche Bundesbank \(2021\)](#) for a discussion of potential difficulties related to make-up strategies both from a theoretical and a practical perspective.

<sup>9</sup>As is common in the New Keynesian literature (see [Galí, 2015](#)), we abstract from capital formation and consider labour as the only aggregate production factor.

their expected lifetime utility,

$$\mathbb{E}_t \left[ \sum_{s=0}^{\infty} \left( \prod_{k=0}^s \beta_{t+k-1} \right) \left\{ \frac{c_{i,t+s}^{1-\sigma} - 1}{1-\sigma} - \varphi_1 \frac{n_{i,t+s}^{1+\varphi_2}}{1+\varphi_2} \right\} \right],$$

subject to the period budget constraint  $c_{i,t} + b_{i,t} = b_{i,t-1} \frac{R_{t-1}}{\Pi_t} + w_t e_{i,t} n_{i,t} + d_t e_{i,t}$  and the debt limit  $b_{i,t} \geq \underline{b}$ . Bonds in the economy  $b_{i,t}$  are in zero net supply. A bond bought in period  $t-1$  yields a real return of  $r_t = R_{t-1}/\Pi_t$  in period  $t$ , with  $R_{t-1}$  denoting the nominal interest rate and  $\Pi_t$  the inflation rate. Households own the firms in the economy but cannot trade shares with each other, such that equity is a completely illiquid asset. Aggregate firm dividends  $d_t$  are allocated to households in proportion to their productivity state  $e_{i,t}$ , which follows a log-normal first-order autoregressive process with persistence parameter  $\rho_e$  and shock standard deviation  $\sigma_e$ . The support of individual household productivity  $e_{i,t}$  is normalised such that aggregate productivity equals one, i.e.  $\sum_i \Pr(e_i) e_i = 1$ , where  $\Pr(e_i)$  denotes the time-invariant share of households in productivity state  $e_i$ . Individual labour supply  $n_{i,t}$  is perfectly elastic and entirely determined by firm demand (see below). The real wage earned by households per efficiency unit of labour is  $w_t$ . The household discount factor  $\beta_t$  is subject to aggregate shocks and therefore carries a time index  $t$ . Specifically, it follows the process

$$\log(\beta_t) = (1 - \rho_\beta) \log(\beta) + \rho_\beta \log(\beta_{t-1}) + \sigma_\beta \varepsilon_{\beta,t}.$$

The optimal consumption-saving decision of a household  $i$  satisfies  $c_{i,t}^{-\sigma} \geq \beta_t \mathbb{E}_t \left[ c_{i,t+1}^{-\sigma} \frac{R_t}{\Pi_{t+1}} \right]$ , which holds with equality if  $b_{i,t} > \underline{b}$ .

**Goods market** There are two types of firms in the model: final-good producers and intermediate-good producers. Final-good firms produce  $Y_t$  by combining intermediate production inputs  $Y_{j,t}$  according to a constant elasticity of substitution (CES) technology

$$Y_t = \left( \int_j Y_{j,t}^{\frac{\varepsilon_t - 1}{\varepsilon_t}} dj \right)^{\frac{\varepsilon_t}{\varepsilon_t - 1}}.$$

Final-good producers' demand for intermediate goods is  $Y_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\varepsilon_t} Y_t$ , where  $P_{j,t}$  denotes the price set by intermediate-good producer  $j$  and  $P_t$  is the price of the final good. Intermediate goods are produced with linear technology  $Y_{j,t} = N_{j,t}$ , such that the real marginal production cost for firm  $j$  is  $mc_{j,t} = w_t$ . Price setting is subject to quadratic adjustment costs,  $\frac{\varepsilon_t}{2k} \log \left( \frac{P_{j,t}}{P_{j,t-1} \Pi} \right)^2 Y_t$  (see [Auclert et al., 2021](#)). As in [Hagedorn et al. \(2019\)](#), these costs are assumed to be virtual (or “as if”), i.e. they affect the firms' decision problem but do not take away actual resources in equilibrium. Intermediate-good producers also face the fixed production cost  $\Phi$ , which we introduce to calibrate steady-state profits (see [Hagedorn et al.](#),

2019).

Taking the demand for its goods as given, firm  $j$  chooses prices  $\{P_{j,t}\}_{t=0}^{\infty}$  to maximise expected profits

$$\mathbb{E}_t \left[ \sum_{s=0}^{\infty} \beta^s \left\{ \left( \frac{P_{j,t+s}}{P_{t+s}} - w_{t+s} \right) \left( \frac{P_{j,t+s}}{P_{t+s}} \right)^{-\varepsilon_t} Y_{t+s} - \frac{\varepsilon_t}{2\kappa} \log \left( \frac{P_{j,t+s}}{P_{j,t+s-1}\Pi} \right)^2 Y_{t+s} - \Phi \right\} \right],$$

where  $\beta$  is the long-run value of the household discount factor.

After imposing symmetry across firms, the first-order condition for the intermediate-good firm problem can be written as the New Keynesian Phillips Curve (NKPC),

$$\log \left( \frac{\Pi_t}{\Pi} \right) = \kappa(w_t - 1/\mu_t) + \beta \mathbb{E}_t \left[ \log \left( \frac{\Pi_{t+1}}{\Pi} \right) \frac{Y_{t+1}}{Y_t} \right],$$

with gross quarterly inflation rate  $\Pi_t = P_t/P_{t-1}$ , aggregate output  $Y_t = N_t$ , and price mark-up  $\mu_t = \varepsilon_t/(\varepsilon_t - 1)$ , which follows the first-order autoregressive process

$$\log(\mu_t) = (1 - \rho_\mu) \log(\mu) + \rho_\mu \log(\mu_{t-1}) + \sigma_\mu \varepsilon_{\mu,t}.$$

**Labour market** We set up the labour market following [Auclert et al. \(2021\)](#). Labour services offered to intermediate-good firms at price  $W_t$  are provided by competitive labour packers who combine specialised labour services  $N_{k,t}$  offered by a continuum of labour unions, indexed with  $k$ , according to a CES technology,

$$N_t = \left( \int_k N_{k,t}^{\frac{\varepsilon_W - 1}{\varepsilon_W}} dk \right)^{\frac{\varepsilon_W}{\varepsilon_W - 1}}.$$

Labour packers' demand for labour services is given as  $N_{k,t} = \left( \frac{W_{k,t}}{W_t} \right)^{-\varepsilon_W} N_t$ , where  $W_{k,t}$  denotes the nominal wage set by union  $k$ .

The role of union  $k$  is to aggregate efficiency units of labour supplied by households  $i$  into the union-specific task  $N_{k,t} = \int_i e_{i,t} n_{i,k,t} di$ . Unions are subject to quadratic wage adjustment costs,  $\frac{\varepsilon_W}{2\kappa_W} \log \left( \frac{W_{k,t}}{W_{k,t-1}\Pi_W} \right)^2$ , which are measured in terms of utility. Taking the demand for its labour services as well as household consumption-saving decisions as given, union  $k$  chooses nominal wages  $\{W_{k,t}\}_{t=0}^{\infty}$  to maximise expected average household utility,

$$\mathbb{E}_t \left[ \sum_{s=0}^{\infty} \beta^s \left\{ \int_i \left\{ \frac{c_{i,t+s}^{1-\sigma} - 1}{1-\sigma} - \varphi_1 \frac{n_{i,t+s}^{1+\varphi_2}}{1+\varphi_2} \right\} di - \frac{\varepsilon_W}{2\kappa_W} \log \left( \frac{W_{k,t+s}}{W_{k,t+s-1}\Pi_W} \right)^2 \right\} \right].$$

Each union  $k$  uses individual household labour services based on the uniform rule  $n_{i,k,t} = N_{k,t}$ .<sup>10</sup> Since each household  $i$  supplies  $n_{i,k,t}$  to union  $k$ , its total labour supply is  $n_{i,t} =$

<sup>10</sup>If individual household labour is demanded by union  $k$  in a uniform fashion, all households supply the

$\int_k n_{i,k,t} dk$ . In a symmetric equilibrium, it holds that  $W_{k,t} = W_t$  and  $N_{k,t} = N_t$ , such that  $n_{i,t} = N_t$ . The normalisation of aggregate productivity then implies that aggregate labour  $N_t$  and productivity-weighted aggregate labour  $\sum_i \Pr(e_i) e_i n_{i,t}$  coincide.

After applying some algebra, one arrives at the New Keynesian wage Phillips Curve (NKWPC),

$$\log\left(\frac{\Pi_{W,t}}{\Pi_W}\right) = \kappa_W \left( \varphi_1 N_t^{1+\varphi_2} - (1/\mu_W) w_t N_t \int_i e_{i,t} c_{i,t}^{-\sigma} di \right) + \beta \mathbb{E}_t \left[ \log\left(\frac{\Pi_{W,t+1}}{\Pi_W}\right) \right],$$

from the first-order condition associated with the union's problem, with nominal wage inflation  $\Pi_{W,t} = W_t/W_{t-1}$  and wage mark-up  $\mu_W = \varepsilon_W/(\varepsilon_W - 1)$ . The real wage rate evolves according to the law of motion  $w_t = w_{t-1}(\Pi_{W,t}/\Pi_t)$ .

**Market clearing** Finally, the market clearing conditions are  $\int_i b_{i,t} di = 0$ ,  $\int_i c_{i,t} di = C_t$ , and  $N_t = C_t + \Phi$ .<sup>11</sup>

**Monetary authority** We describe the conduct of monetary policy via simple feedback rules, with the short-term interest rate  $R_t$  as the only instrument set by the monetary authority. Specifically, we consider five potential monetary policy rules in total, closely following the simulation protocols used by [Work Stream on the Price Stability Objective \(2021\)](#). Let  $\tilde{R}_t^{(4)}$  denote the annualised nominal shadow interest rate,  $R^{(4)}$  the annualised long-run nominal interest rate,  $\bar{\Pi}_t^{(4)} = \prod_{k=1}^4 \Pi_{t-k+1}$  annual (year-on-year) price inflation,  $\bar{\Pi}_t^{(4T)} = (1/T) \prod_{k=1}^{4T} \Pi_{t-k+1}$  average annual price inflation over the past  $T$  years,  $\Pi^{(4)} = (\Pi)^{(4)}$  the annual inflation (point) target and  $Y_t$  quarterly real GDP with long-run value  $Y$ .

The monetary policy rules are as follows:

1. Inflation targeting (IT):

$$\frac{\tilde{R}_t^{(4)}}{R^{(4)}} = \left( \frac{\tilde{R}_{t-1}^{(4)}}{R^{(4)}} \right)^{0.85} \left( \frac{Y_t}{Y} \left( \frac{\bar{\Pi}_t^{(4)}}{\Pi^{(4)}} \right)^{1.5} \right)^{0.15}$$

2. Price level targeting (PLT):

$$\frac{\tilde{R}_t^{(4)}}{R^{(4)}} = \left( \frac{\tilde{R}_{t-1}^{(4)}}{R^{(4)}} \right)^{0.85} \left( \frac{Y_t \bar{\Pi}_t^{(4)}}{Y \Pi^{(4)}} \prod_{k=1}^{\infty} \frac{\Pi_{t-k+1}}{\Pi} \right)^{0.15}$$

same amount of labour to union  $k$ , such that  $N_{k,t} = \int_i e_{i,t} n_{i,k,t} di$  implies  $n_{i,k,t} = N_{k,t}$  due to  $\int_i e_{i,t} di = 1$ .

<sup>11</sup>For the numerical model solution we aggregate individual variables by using a finite-dimensional approximation of the infinite-dimensional distribution of wealth across households (see Section 3.2 for details). In practice, the integrals will therefore be replaced by sums.

3. Average inflation targeting (AIT):

$$\frac{\tilde{R}_t^{(4)}}{R^{(4)}} = \left( \frac{\tilde{R}_{t-1}^{(4)}}{R^{(4)}} \right)^{0.85} \left( \frac{Y_t \bar{\Pi}_t^{(4)}}{Y \bar{\Pi}^{(4)}} \left( \frac{\bar{\Pi}_t^{(4T)}}{\bar{\Pi}^{(4)}} \right)^T \right)^{0.15}$$

4. Asymmetric inflation targeting (AsymIT):

$$\frac{\tilde{R}_t^{(4)}}{R^{(4)}} = \left( \frac{\tilde{R}_{t-1}^{(4)}}{R^{(4)}} \right)^{0.85} \left( \frac{Y_t}{Y} \left( \frac{\bar{\Pi}_t^{(4)}}{\bar{\Pi}^{(4)}} \right)^{1.5} \left( \frac{\bar{\Pi}_t^{(4)}}{\bar{\Pi}^{(4)}} \right)^{\varphi \times \mathcal{I}_{\{\bar{\Pi}_t^{(4)} < \bar{\Pi}^{(4)}\}}} \right)^{0.15}$$

5. Asymmetric average inflation targeting (AsymAIT):

$$\frac{\tilde{R}_t^{(4)}}{R^{(4)}} = \left( \frac{\tilde{R}_{t-1}^{(4)}}{R^{(4)}} \right)^{0.85} \left( \frac{Y_t \bar{\Pi}_t^{(4)}}{Y \bar{\Pi}^{(4)}} \left( \frac{\bar{\Pi}_t^{(4)}}{\bar{\Pi}^{(4)}} \right)^{0.5 \times \mathcal{I}_{\{\bar{\Pi}_t^{(4T)} \geq \bar{\Pi}^{(4)}\}}} \right)^{0.15} \times \left( \frac{\bar{\Pi}_t^{(4T)}}{\bar{\Pi}^{(4)}} \right)^{\varphi \times \mathcal{I}_{\{\bar{\Pi}_t^{(4T)} < \bar{\Pi}^{(4)}\}}}$$

For AsymIT, we set  $\varphi = 0.5$ , whereas we use  $\varphi = T$  for AsymAIT. For a given monetary policy rule, the annualised nominal interest rate set by the monetary authority then is

$$R_t^{(4)} = \max \left\{ \tilde{R}_t^{(4)}, R_{ELB}^{(4)} \right\},$$

where  $R_{ELB}^{(4)}$  denotes the (annualised) effective lower bound. Since we formulate the model in quarterly terms, households can borrow or save at the nominal rate  $R_t = (R_t^{(4)})^{1/4}$ .

The IT rule is a version of the celebrated Taylor rule (see [Taylor, 1993](#)) with policy inertia. Note, however, that the specification of the rule differs from most of the macroeconomic literature in two ways. First, the year-on-year inflation rate, not the contemporaneous (quarterly) inflation rate, enters the rule as an argument. Second, policy inertia relates to the lagged value of the shadow rate, not the realised rate. Both features imply that there is already some history dependence built into the benchmark IT rule, which helps attenuate problems due to the ELB. Whereas the dependence on the lagged shadow rate introduces a lower-for-longer element that is similar to forward guidance (see e.g. [Coenen et al., 2020](#)), a year-on-year inflation target in a quarterly model implies that IT has some make-up properties since – ceteris paribus – a below-target inflation rate three quarters ago implies a more expansionary policy stance today.

This make-up element is the defining feature of the AIT rule, which differs from the IT rule because of the time horizon  $T \geq 1$  used to calculate the average inflation rate. The longer this time horizon  $T$  is, the more history-dependence the policy rule exhibits, such that monetary policy reacts to deviations in the inflation rate from the target further away in time.

If viewed as credible, the commitment to make up for below-target (above-target) inflation in future periods will stimulate (contract) economic activity not only in the future but in the present already since firms, unions and households are forward-looking decision makers. The ability of monetary policy to make use of this mechanism depends – among other things – on how willing or able households are to substitute consumption over time (“intertemporal substitution channel”). The more flexible households are with respect to their consumption-savings choice, the more they respond to (expected) changes in future income and inflation. The ability to stimulate the economy by managing expectations is particularly valuable if the monetary authority is constrained by the ELB, such that it is unable to stimulate the economy any further by cutting the nominal interest rate. The commitment to make up for below-target (average) inflation by overshooting the inflation target in the future can, however, provide additional stimulus by incentivising households to save less / borrow more today due to lower real interest rates and higher future income.

An alternative way of addressing the downward inflation bias under IT is to augment the policy rule by incorporating a state-dependent asymmetry (see rule 4). Specifically, acknowledging the asymmetry in the monetary policy response introduced by the ELB, the asymmetric version of IT dictates a more forceful response if the inflation rate is below target. This state-dependent asymmetry, which is captured by the indicator function  $\mathcal{I}_{\{\cdot\}}$  and the additional response coefficient  $\varphi$ , makes it less likely that the economy will reach the ELB by preemptively switching to a more expansionary monetary policy as soon as the probability of such a scenario increases due to below-target inflation. This asymmetry can also be combined with a make-up element (see rule 5), which might help address time-inconsistency problems associated with history-dependent strategies for periods with above-target inflation.

### 3 Quantitative analysis

This section presents the quantitative evaluation of the model described in the previous section for different monetary policy rules. Sections 3.1 and 3.2 discuss how the model is calibrated and solved numerically. Section 3.3 discusses the forward guidance puzzle in the context of our model and its implications for history-dependent monetary policy strategies. Section 3.4 presents the simulation results.

#### 3.1 Model calibration

The values for the model parameters are summarised by Table 1. The annual long-run (“natural”) real interest rate  $r^{(4)} = (r)^4$ , the inflation target  $\Pi^{(4)}$ , and the ELB  $R_{ELB}^{(4)}$  are all specified as suggested by [Work Stream on the Price Stability Objective \(2021\)](#), which

Table 1: Model parameters for HANK model

Parameter	Description	Value
$\beta$	Discount factor	0.9689*
$\kappa$	Slope NKPC	0.1
$\kappa_W$	Slope NKWPC	0.1
$\mu$	Price mark-up	1.1
$\mu_W$	Wage mark-up	1.1
$\Pi^{(4)}$	Long-run inflation target (annual)	1.02
$\Pi_W$	Long-run nominal wage inflation	1.005
$\rho_\beta$	Persist. discount factor	0.85
$\rho_\mu$	Persist. mark-up	0.85
$\rho_e$	Persist. idiosync. productivity	0.966
$\sigma$	Coefficient of relative risk aversion	2
$\sigma_\beta$	Std. dev. discount factor shock	0.0082
$\sigma_\mu$	Std. dev. mark-up shock	0.036
$\sigma_e$	Std. dev. idiosync. productivity shock	0.966
$\Phi$	Fixed cost of production	0.1
$\varphi_1$	Weight labour disutility	1*
$\varphi_2$	Inverse Frisch elasticity	2
$\underline{b}$	Household debt limit	-1.67
$R_{ELB}^{(4)}$	ELB (annual)	0.995

\* For the RANK model version,  $(\beta, \varphi_1) = (0.9988, 0.82)$  is used instead.

tries to capture the current conditions in the euro area. The respective (annual) net values are 0.5%, 2% and -0.5%. Whereas the first two parameters can be set directly for the model, the real rate is calibrated via the household discount factor  $\beta$ . In the RANK model, the real rate  $r$  is exogenous and equal to the inverse of the household discount factor. By contrast, the real rate is endogenous in the HANK model and below the inverse of the household discount factor due to the households' precautionary saving motive. For that case, we calibrate the discount factor to obtain our real rate target for the stationary equilibrium of the model. The coefficient of relative risk aversion  $\sigma$  and the inverse Frisch elasticity  $\varphi_2$  are both set to a standard value of 2. For the HANK model version, the labour disutility parameter  $\varphi_1$  is normalised to one, whereas it is set to 0.82 for the RANK version to obtain the same long-run output level  $Y$  as for the HANK version. The parameters for the goods and labour markets are set to rather standard values from the literature (see e.g. Auclert et al., 2018). Following Hagedorn et al. (2019), the fixed cost of production  $\Phi$ , which intermediate-good producers face, is set such that profits (and therefore dividend payments to households) are zero in the deterministic steady state of the model. The household debt limit  $\underline{b}$  equals -1.67, which is 5 times the monthly average household income (see Ferrante and Paustian, 2019). The persistence ( $\rho_e$ ) and volatility ( $\sigma_e$ ) of idiosyncratic productivity is specified as

in [McKay et al. \(2016\)](#). To calibrate the parameters for the aggregate shock processes, we proceed in two steps.<sup>12</sup> First, assuming that only demand shocks are operative, we choose a shock standard deviation such that the ELB binds 20% of the time in a RANK version of our model, leading to a value of  $\sigma_\beta = 0.0082$ .<sup>13</sup> Then, we set the standard deviation of the mark-up shock such that mark-up shocks roughly account for 30% of output fluctuations. The resulting value is  $\sigma_\mu = 0.036$ .

### 3.2 Model solution

The numerical solution of the model faces two non-trivial challenges. First, it has to deal with an infinite-dimensional and time-varying household wealth distribution, which enters the aggregate state space of the model. Second, it needs to capture nonlinearities at the aggregate level, which are introduced by the occasionally-binding ELB constraint and potential asymmetries in monetary policy rules. To address these challenges, we approximate the infinite-dimensional distribution of bonds across households with a finite-dimensional histogram (see [Young, 2010](#); [Reiter, 2009](#))<sup>14</sup> and rely on the extended path method (see [Fair and Taylor, 1983](#); [Gagnon, 1990](#)) to solve and simulate the nonlinear model at the aggregate level.<sup>15</sup> This entails that for each simulation period  $t$  we first draw values for the aggregate shocks in that period. We then use a nonlinear numerical solver to find the perfect-foresight transition path back to the deterministic steady state. This path in turn pins down the period- $t$  values of the endogenous model variables including state variables like the wealth distribution. Since computing such a transition path for each simulation period is quite costly we use a perturbation-based solution of the model (see [Reiter, 2009](#)) without an ELB to obtain a good initial guess for the solver and speed up our algorithm.<sup>16</sup>

While the extended path method is often used in macroeconomics to handle nonlinearities like the one induced by the ELB (see e.g. [Christiano et al., 2016](#); [Coenen et al., 2020](#)), to the best of our knowledge we are the first to apply the method in the context of a fully-fledged heterogeneous agent model with aggregate risk. Given that the extended

<sup>12</sup>This calibration procedure follows the simulation protocol used by the WGEM expert group on expectations formation and monetary policy.

<sup>13</sup>The persistence parameters for the processes are both set to 0.85. We calibrate the shock processes based on the RANK version of our model since it is substantially less time-consuming to solve and simulate it than the HANK version (see Section 3.2 for details).

<sup>14</sup>The discretised version of the distribution, denoted as  $\Psi_t(e, b)$ , will then be used to approximate the integrals contained in the NKWPC as well as the market clearing conditions. The productivity-weighted marginal utility of consumption for example, which appears in the NKWPC, is approximated as  $\int_i e_{i,t} c_{i,t}^{-\sigma} di \approx \sum_{i_e=1}^{n_e} \sum_{i_b=1}^{n_b} e_{i_e} c_t(e_{i_e}, b_{i_b})^{-\sigma} \Psi_t(e_{i_e}, b_{i_b})$ , where  $c_t(\cdot)$  denotes the individual consumption policy function and  $n_x$  the number of grid points for the individual state  $x \in \{b, e\}$ .

<sup>15</sup>At the individual level, nonlinearities due to the borrowing constraint are fully preserved by solving the household problem via the endogenous grid method (see [Carroll, 2006](#)).

<sup>16</sup>We also solve the RANK version of the model via the extended path method. Since there is no wealth distribution to track in this case, the associated computational cost is several orders of magnitude lower relative to the HANK case.



path method assumes certainty equivalence, an occasionally-binding ELB does not induce a precautionary saving motive at the aggregate level as in [Bianchi et al. \(2021\)](#) or [Fernández-Villaverde et al. \(2021\)](#). However, we show in [Section 3.4](#) that the ELB will nevertheless lead to a downward inflation bias under standard inflation targeting. This in turn will affect the households’ precautionary saving behaviour. Compared to a fully nonlinear approach à la [Krusell and Smith \(1998\)](#), the extended path method has the major advantage that it is not computationally costly to introduce additional aggregate state variables into the model.<sup>17</sup> This advantage is key for our model analysis as the aggregate state space is non-trivial, containing the discretised wealth distribution, two random variables and past values of the real wage rate, the shadow rate and the inflation rate.

### 3.3 Forward guidance puzzle in HANK and RANK

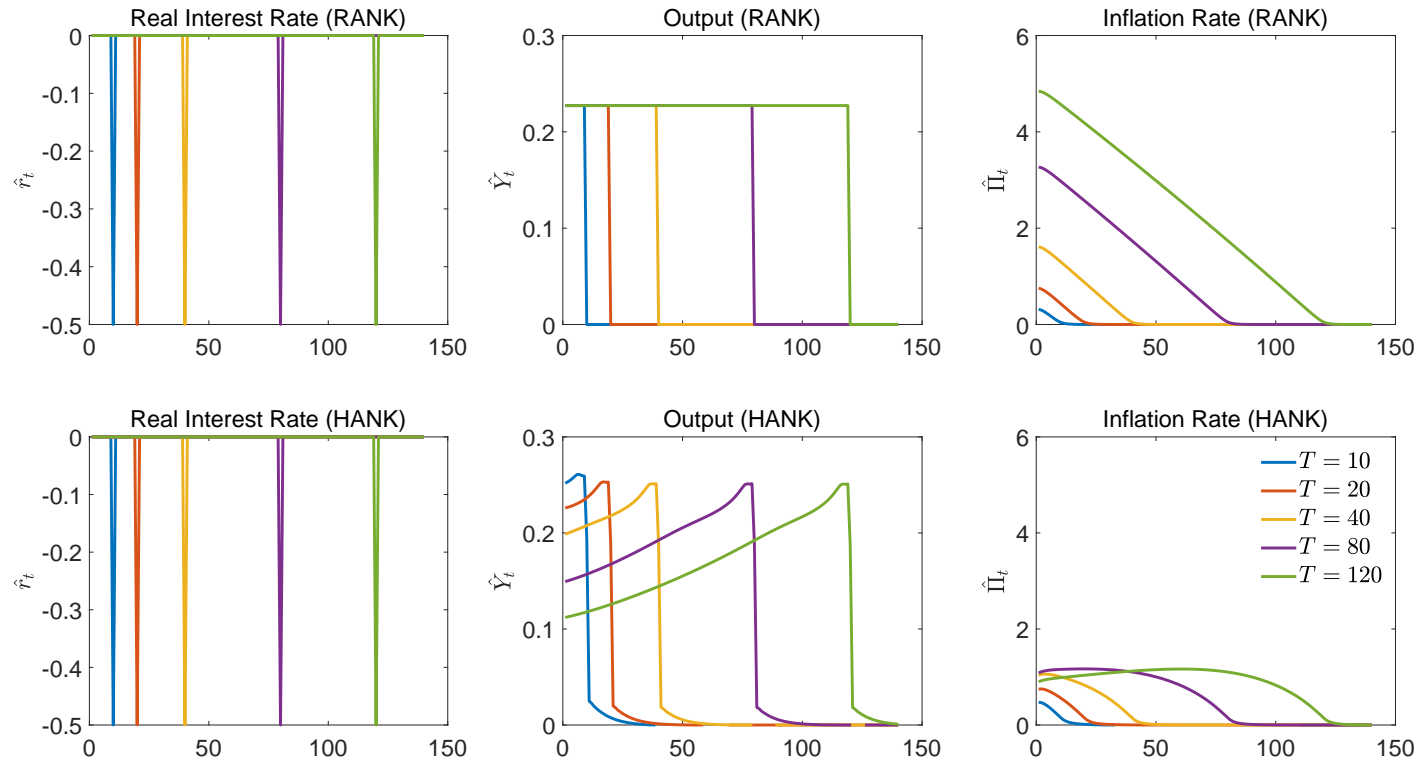
As argued in [Section 2.1](#), history-dependent monetary policy strategies depend on an intertemporal substitution channel to stabilise the economy. This also applies to forward guidance, i.e. an announcement about the future path of interest rates, which in principle allows monetary policy to stimulate the economy when policy rates are already at the ELB (see [Eggertsson and Woodford, 2003](#); [Jung et al., 2005](#)). In RANK models, the effects of forward guidance are known to usually be implausibly large. Specifically, the greater the distance between the date of the announcement of a future interest rate cut and the date of the cut itself, the stronger the impact on economic activity at the time of the announcement will be. This property, referred to as the forward guidance puzzle (FGP) in the literature (see [Del Negro et al., 2015](#)), has motivated subsequent research to find ways to resolve it. Since forward guidance and make-up strategies operate through similar channels, models that are subject to the FGP will likely be biased in favour of make-up strategies over standard IT. The FGP and its resolution (or at least attenuation) thus also matter when assessing the costs and benefits of make-up strategies.

Given the importance of the intertemporal substitution channel for the FGP, mechanisms that reduce agents’ willingness or ability to substitute resources over time will *ceteris paribus* attenuate or even eliminate the FGP. One way to accomplish this is by allowing for financial frictions that limit households’ ability to borrow (see [McKay et al., 2016](#)).<sup>18</sup> In principle, the presence of borrowing-constrained households can therefore allow HANK models to attenuate the FGP compared to corresponding RANK model versions. However, there are additional redistributive channels in HANK models relative to RANK models

<sup>17</sup>The reason for this is that, in contrast to a fully nonlinear solution, the extended path method does not require the approximation of aggregate state variables on discrete grids, such that the “curse of dimensionality” ([Bellman, 1957](#)) does not apply.

<sup>18</sup>Alternatively, the intertemporal substitution channel can also be weakened by the introduction of informational or behavioural frictions (see e.g. [Angeletos and Lian, 2018](#); [Woodford, 2018](#); [García-Schmidt and Woodford, 2019](#); [Farhi and Werning, 2019](#); [Gabaix, 2020](#)).

Figure 1: The impact of forward guidance: HANK vs. RANK



Notes: All variables are expressed in percent relative to steady state, i.e.  $\hat{x}_t = 100 \times \log(x_t/x)$  with  $x$  denoting the steady-state value of variable  $x_t$

that can potentially weaken or amplify the power of forward guidance (see [Werning, 2015](#); [Ferrante and Paustian, 2019](#); [Hagedorn et al., 2019](#)). To reduce the importance of redistributive effects that are not (directly) related to the monetary transmission mechanism, we (i) abstract from fiscal policy, and (ii) mute the role played by the distribution of firm profits across households.<sup>19</sup> Our model does, however, allow for direct redistribution between borrowers and savers via inflation since bonds are nominal and the household debt limit is non-zero. As discussed by [Ferrante and Paustian \(2019\)](#), this strengthens the power of forward guidance and therefore counteracts the attenuating effect that the presence of borrowing-constrained households has on the impact of lower-for-longer policies.

Due to these counteracting forces it is ultimately a quantitative question whether forward guidance is less effective under incomplete markets. Figure 1 illustrates the effect of an announced, one-period real rate cut of 0.5%  $T \in \{10, 20, 40, 80, 120\}$  periods ahead for our HANK model and the corresponding RANK model version.<sup>20</sup> First consider the RANK model version. In this case, the FGP is clearly noticeable: The stimulating effect of the announcement in period  $t = 1$  on inflation in the same period increases with the distance  $T$ , whereas the output response does not depend on  $T$ .<sup>21</sup> By contrast, the stimulating effect on output in the announcement period declines with  $T$  in the HANK model. While the inflation response in  $t = 1$  is positive for all  $T$ -values, the marginal impact declines with  $T$  and turns negative for large values of  $T$ . As a result, the inflation response in the announcement period is lower for  $T = 120$  relative to  $T \in \{40, 80\}$  (see Figure 1). Note that the magnitude of the impact that forward guidance has on inflation in the HANK model is about the same for  $T \in \{10, 20\}$  as in the RANK model, but substantially lower in the HANK model for  $T \in \{40, 80, 120\}$ .<sup>22</sup> Furthermore, note that although the output stimulus provided by forward guidance declines with  $T$  for the HANK model, it is larger for  $T = 10$  in the announcement period relative to the RANK version. While our HANK model does not exhibit explosive inflation dynamics as  $T$  approaches infinity and hence “solves the FGP”, it nevertheless predicts that forward guidance will have a sizeable impact on economic activity, particularly at shorter horizons. History-dependent monetary policy strategies like AIT or PLT will thus likely still provide some benefits for the HANK model studied in this paper. On the other hand, for an economy subject to cost-push shocks and wage rigidities as in this paper, make-up strategies can also be quite costly. Specifically, a positive cost-push shock would require strong reductions in economic activity to ensure

<sup>19</sup>As in [Hagedorn et al. \(2019\)](#), the latter is accomplished by calibrating steady-state firm profits (and hence dividend payments) to zero via the fixed production cost  $\Phi$ , adopting a progressive allocation rule for dividends, and by allowing for nominal wage rigidity, which dampens fluctuations in firm profits. The omission of fiscal policy implies that there is no redistribution between households via a tax system.

<sup>20</sup>[McKay et al. \(2016\)](#) also conduct this experiment to understand the power of forward guidance.

<sup>21</sup>The cumulative output response increases linearly with  $T$ .

<sup>22</sup>The cumulative inflation response is higher in the HANK model for  $T = 10$ . The inflation response in  $t = 1$  is slightly higher in the HANK model for  $T = 10$  and about the same for  $T = 20$ .

that the commitment to make up for past deviations from the inflation (or price level) target is met. The next section will discuss whether – and if so to what extent – history-dependent (as well as asymmetric) monetary policy rules improve macroeconomic stabilisation.

### 3.4 Simulation results

We split the discussion of our simulation results into two parts, focusing either on outcomes under symmetric policy rules (rule 1-3) or asymmetric ones (rules 4 and 5).

#### 3.4.1 Symmetric policy rules

##### **History-dependent rules can address the downward inflation bias in the HANK model.**

Table 2 shows the simulation results for IT, AIT with either 4- or 8-year averaging and PLT.<sup>23</sup> IT, defined as the baseline, leads to an average ELB frequency of 18.6% and an average duration of 7.5 quarters in the HANK model. There is a considerable downward inflation bias as evidenced by a mean annual inflation rate of only 1.72%. By contrast, all the history-dependent rules are able to reduce the downward inflation bias to a few basis points (AIT4) or even almost eliminate it altogether (AIT8, PLT). Furthermore, history-dependent rules show a lower ELB incidence and shorter durations. Compared to IT, the frequency of ELB episodes is at least 4 percentage points lower, whereas the duration decreases by at least a month and up to two quarters.

##### **Additionally, history dependent rules lead to overall improved stabilisation properties.**

Not only do they mitigate or even avoid the downward inflation bias; they also decrease inflation volatility – as measured by the standard deviation of year-on-year inflation – by almost 50%. Similarly, the output gap and short-term nominal rate are less volatile as well, reflecting overall reduced fluctuations in the economy.

##### **The higher the degree of history dependence, the stronger the stabilising properties.**

PLT, as the most history-dependent rule, outperforms AIT4 and AIT8 when it comes to reducing ELB incidences, the downward inflation bias and macroeconomic volatility in the HANK model. Nevertheless, AIT8 comes quite close to a PLT rule. The mean values for the inflation rate and output gap are only 0.01 percentage point lower, and the standard deviations for inflation, the output gap and nominal rates are close. The same holds true for the average ELB duration and particularly for the frequency of ELB episodes, which is only 2 percentage points higher under AIT8.

##### **History-dependent rules are better able to alleviate household borrowing constraints.**

Part of the reason why history-dependent rules perform better is their ability to move house-

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<sup>23</sup>The model statistics are calculated based on 5,000 simulations with 200 periods (100 burn-in) for each rule.

Table 2: Simulation results for HANK model with symmetric policy rules

Policy rule	ELB incidence		Inflation (annual, %)		Output gap (%)		Short-term nominal rate (annual, %)		Share of borr.- constrained HHs (%)	
	Freq. (%)	Avg. duration (quarters)	Mean	Std	Mean	Std	Mean	Std	Mean	Std
IT w/o ELB	0	0	2.00	3.01	0.00	2.29	2.50	2.75	21.58	6.17
IT	18.60	7.53	1.72	3.71	-0.19	2.79	2.69	2.44	22.73	7.95
AIT (4 years)	14.44	7.23	1.95	2.29	-0.04	2.31	2.60	2.17	22.04	5.43
AIT (8 years)	12.06	5.64	1.99	2.31	-0.02	2.26	2.59	2.05	21.78	4.95
PLT	10.18	5.34	2.00	1.99	-0.01	2.21	2.59	1.94	21.42	4.10

Table 3: Simulation results for RANK model with symmetric policy rules

Policy rule	ELB incidence		Inflation (annual, %)		Output gap (%)		Short-term nominal rate (annual, %)	
	Freq. (%)	Avg. duration (quarters)	Mean	Std	Mean	Std	Mean	Std
IT w/o ELB	0	0	2.00	3.35	0.02	2.20	2.53	3.27
IT	22.29	9.17	1.81	3.74	-0.16	2.46	2.89	2.77
AIT (4 years)	17.67	9.22	1.95	2.20	-0.06	2.25	2.72	2.35
AIT (8 years)	14.69	6.95	1.99	2.26	-0.02	2.20	2.66	2.19
PLT	12.64	6.58	2.00	2.06	-0.02	2.17	2.66	2.10

holds away from the borrowing constraint.<sup>24</sup> This provides additional economic stimulus as it directly improves economic conditions for households with high MPCs. The mean share of constrained households in the simulations is up to 1.31 percentage points lower relative to IT and the volatility of the share is up to almost 50% smaller.

**The ELB binds less frequently for HANK compared to RANK.** Table 3 shows the corresponding simulation results for a RANK model version. The model's structure, its parametrisation and the shock sizes are the same as in our HANK model, but there is no household heterogeneity and no debt limit.<sup>25</sup> In this setting, IT leads to more frequent ELB episodes in the RANK version (22.3% vs. 18.6%) with a longer average duration (9.2 vs. 7.5 quarters). At the same time, mean average inflation is higher (1.81% vs. 1.72%) with a similar volatility. The average size and volatility of the output gap are quite similar, while short-term rates are slightly higher on average and more volatile compared to the HANK model.

**The relative gains of history-dependent rules are similar for HANK and RANK.** For example, ELB episodes under PLT are 45% less likely for HANK versus 43% for RANK. Similarly, PLT reduces the average ELB duration by 29% for HANK versus 28% for RANK. Given that ELB episodes are a little more frequent in RANK, history-dependent rules are somewhat less able to stabilise economic fluctuations as measured by the volatility of inflation, the output gap and short-term rates.

**Taken together, the results imply that the main qualitative statements apply for HANK and RANK.** Furthermore, when it comes to macroeconomic stabilisation, the relative ranking of policy rules in our HANK model is consistent with the relative performance identified by the WS PSO. In this sense, the previous statements can be considered robust to model variations.

### 3.4.2 Asymmetric policy rules

**Asymmetric IT leads to improvements over standard IT.** Asymmetric IT reduces the frequency of ELB episodes by a mere 0.7 percentage point and the duration by roughly 0.3 quarter (see Table 4). Nevertheless, the average inflation rate is significantly higher (2.16% vs. 1.72%) but less volatile (3.24% vs. 3.71%). While AsymIT reduces the output gap by

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<sup>24</sup>History-dependent rules generally exhibit better stabilisation properties and thereby keep the economy closer to the steady state. This implies that household incomes are higher, meaning that fewer households reach the borrowing constraint in the first place. Furthermore, households on average can expect higher future inflation and incomes under history-dependent rules and are therefore less often expect to be borrowing-constrained, which weakens their precautionary saving motive and provides additional stimulus.

<sup>25</sup>The only exceptions are the discount factor  $\beta$  and the labour disutility parameter  $\varphi_1$ , which are chosen for the RANK model version to obtain the same long-run values for the real rate and aggregate labour supply as in the HANK version (see 3.1).

Table 4: Simulation results for HANK model with asymmetric policy rules

Policy rule	ELB incidence		Inflation (annual, %)		Output gap (%)		Short-term nominal rate (annual, %)		Share of borr.- constrained HHs (%)	
	Freq. (%)	Avg. duration (quarters)	Mean	Std	Mean	Std	Mean	Std	Mean	Std
IT w/o ELB	0	0	2.00	3.01	0.00	2.29	2.50	2.75	21.58	6.17
IT	18.60	7.52	1.72	3.71	-0.19	2.79	2.69	2.44	22.73	7.95
AsymAIT (4 years)	10.83	5.82	3.24	4.03	0.16	2.59	3.35	2.52	22.01	8.60
AsymAIT (8 years)	9.60	5.23	3.27	4.48	0.08	2.76	3.50	2.49	22.82	10.04
AsymIT	17.90	7.19	2.16	3.24	-0.04	2.55	2.80	2.49	21.94	6.86



Table 5: Simulation results for RANK model with asymmetric policy rules

Policy rule	ELB incidence		Inflation (annual, %)		Output gap (%)		Short-term nominal rate (annual, %)	
	Freq. (%)	Avg. duration (quarters)	Mean	Std	Mean	Std	Mean	Std
IT w/o ELB	0	0	2.00	3.35	0.02	2.20	2.53	3.27
IT	22.29	9.17	1.81	3.74	-0.16	2.46	2.89	2.77
AsymAIT (4 years)	13.81	7.56	3.05	3.44	0.16	2.38	3.32	2.57
AsymAIT (8 years)	10.98	5.94	3.08	3.68	0.11	2.39	3.46	2.48
AsymIT	22.80	9.14	2.14	3.48	-0.05	2.39	2.92	2.80

more than half (-0.04% vs. -0.19%), these are quite small improvements in absolute terms, and AsymIT still yields a negative output gap along with only a modest reduction in output gap volatility. The average size and volatility of the short-term nominal rate are similar across IT and AsymIT, as is the share of borrowing constrained households.

**Asymmetric AIT substantially reduces ELB occurrences and durations.** In fact, the performance of AsymAIT (8-year averaging) is better relative to PLT with respect to ELB occurrences and durations. On average, the ELB binds only 9.60% of the time under AsymAIT, compared to 10.17% under PLT and 18.6% under IT. While the average duration under PLT is 5.34 quarters, it is 5.23 quarters under AsymAIT8, and somewhat higher for AsymAIT4 at 5.82 quarters.

**However, asymmetric AIT leads to inflation overshooting and increases macroeconomic volatility.** Average inflation under AsymAIT is above 3% for both variants and therefore strongly overshoots the inflation target of 2%. This is mainly because, at least for the current parametrisation of the policy rule, upward deviations in the inflation rate from the target are much less stabilised than downward deviations due to the asymmetric nature of the rule. Inflation volatility under AsymAIT is furthermore higher than under a standard IT rule. The same holds true for the share of borrowing-constrained households, while the output gap and short-term nominal rates are about as volatile as under IT.

**Asymmetric rules again have very similar implications in the HANK and RANK cases.** Similar to the case of symmetric rules, adopting more sophisticated asymmetric rules yields the same qualitative predictions across the HANK and RANK model versions (see Table 5). The most notable difference concerns AsymIT, which leads to slightly more frequent ELB episodes in RANK but slightly less frequent ELB episodes in HANK compared to baseline IT. Besides this, the magnitude of the changes in all macroeconomic variables of interest are quite close across both model versions.

## 4 Conclusion

In this paper, we have studied a variety of monetary policy rules using a New Keynesian model with heterogeneous households, nominal price and wage rigidities, an effective lower bound on the short-term nominal interest rate as well as aggregate demand and cost-push shocks. Since the model allows for a more realistic consumption behaviour of households, it is less prone to well-known shortcomings of representative agent New Keynesian models. To deal with the computational complexity arising from the combination of the ELB and household heterogeneity, we use a version of the extended path method. Closely following the simulation protocol of the WS PSO to check the robustness of its main findings, we

mostly arrive at the same conclusions as the WS based on representative agent models. Our results therefore underscore the main messages of the WS PSO, and we conclude that the introduction of household heterogeneity and an attenuation of the forward guidance puzzle per se do not materially change the conclusions of the WS PSO. It is, however, important to keep in mind that – like most of the WS simulations – our results are derived under the assumption of rational expectations. Future work is needed to evaluate how bounded rationality might change the effectiveness of alternative monetary policy strategies in RANK and HANK models.

## References

- ALTAVILLA, C., W. LEMKE, T. LINZERT, J. TAPKING, AND J. VON LANDESBERGER (2021): “Assessing the Efficacy, Efficiency and Potential Side Effects of the ECB’s Monetary Policy Instruments since 2014,” Occasional Paper Series 278, European Central Bank.
- ANGELETOS, G.-M. AND C. LIAN (2018): “Forward Guidance without Common Knowledge,” *American Economic Review*, 108, 2477–2512.
- ARIAS, J., M. BODENSTEIN, H. CHUNG, T. DRAUTZBURG, AND A. RAFFO (2020): “Alternative Strategies: How Do They Work? How Might They Help?” Finance and Economics Discussion Series 2020-068.
- AUCLERT, A., B. BARDÓCZY, AND M. ROGNLIE (2021): “MPCs, MPEs and Multipliers: A Trilemma for New Keynesian Models,” *Review of Economics and Statistics*, forthcoming.
- AUCLERT, A., M. ROGNLIE, AND L. STRAUB (2018): “The Intertemporal Keynesian Cross,” Mimeo.
- BELLMAN, R. (1957): *Dynamic Programming*, Princeton, NJ: Princeton University Press.
- BIANCHI, F., L. MELOSI, AND M. ROTTNER (2021): “Hitting the Elusive Inflation Target,” Mimeo.
- CAMPBELL, J. AND N. MANKIW (1989): “Consumption, Income and Interest Rates: Reinterpreting the Time Series Evidence,” in *NBER Macroeconomics Annual*, ed. by O. Blanchard and S. Fischer, Cambridge, MA: MIT Press, vol. 4, 185–246.
- CARLSTROM, C., T. FUERST, AND M. PAUSTIAN (2015): “Inflation and Output in New Keynesian Models With a Transient Interest Rate Peg,” *Journal of Monetary Economics*, 76, 230–243.
- CARROLL, C. (2006): “The Method of Endogenous Gridpoints for Solving Dynamic Stochastic Optimization Problems,” *Economics Letters*, 91, 312–320.
- CHRISTIANO, L., M. EICHENBAUM, AND M. TRABANDT (2016): “Unemployment and Business Cycles,” *Econometrica*, 84, 1523–1569.
- COENEN, G., C. MONTES-GALDÓN, AND F. SMETS (2020): “Effects of State-Dependent Forward Guidance, Large-Scale Asset Purchases and Fiscal Stimulus in a Low-Interest-Rate Environment,” ECB Working Paper Series No. 2352.

- DEATON, A. (1987): “Life-Cycle Models of Consumption: Is the Evidence Consistent With the Theory?” in *Advances in Econometrics: Fifth World Congress*, ed. by T. Bewley, Cambridge, UK: Cambridge University Press, 121–148.
- DEL NEGRO, M., D. GIANNONE, M. GIANNONI, AND A. TAMBALOTTI (2019): “Global Trends in Interest Rates,” *Journal of International Economics*, 118, 248–262.
- DEL NEGRO, M., M. GIANNONI, AND C. PATTERSON (2015): “The Forward Guidance Puzzle,” Federal Reserve Bank of New York Staff Reports No. 574.
- DEUTSCHE BUNDESBANK (2021): “The Eurosystem’s Monetary Policy Strategy,” Monthly Report, September, forthcoming.
- EGGERTSSON, G. AND M. WOODFORD (2003): “The Zero Bound on Interest Rates and Optimal Monetary Policy,” *Brookings Papers on Economic Activity*, 34, 139–211.
- FAIR, R. AND J. TAYLOR (1983): “Solution and Maximum Likelihood Estimation of Dynamic Nonlinear Rational Expectations Models,” *Econometrica*, 51, 1169–1185.
- FARHI, E. AND I. WERNING (2019): “Monetary Policy, Bounded Rationality, and Incomplete Markets,” *American Economic Review*, 109, 3887–3928.
- FEIVESON, L., N. GOERNEMANN, J. HOTCHKISS, K. MERTENS, AND J. SIM (2020): “Distributional Considerations for Monetary Policy Strategy,” Finance and Economics Discussion Series 2020-073.
- FERNÁNDEZ-VILLAYERDE, J., J. MARBET, G. NUÑO, AND O. RACHEDI (2021): “Inequality and the Zero Lower Bound,” Mimeo.
- FERRANTE, F. AND M. PAUSTIAN (2019): “Household Debt and the Heterogeneous Effects of Forward Guidance,” International Finance Discussion Papers 1267.
- GABAIX, X. (2020): “A Behavioral New Keynesian Model,” *American Economic Review*, 110, 2271–2327.
- GAGNON, J. (1990): “Solving the Stochastic Growth Model by Deterministic Extended Path,” *Journal of Business & Economic Statistics*, 8, 35–36.
- GALÍ, J. (2015): *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*, Princeton, NJ: Princeton University Press.
- GARCÍA-SCHMIDT, M. AND M. WOODFORD (2019): “Are Low Interest Rates Deflationary? A Paradox of Perfect-Foresight Analysis,” *American Economic Review*, 109, 86–120.

- GERKE, R., S. GIESEN, AND D. KIENZLER (2020): “Interest Rate Pegs and the Reversal Puzzle: On the Role of Anticipation,” Deutsche Bundesbank Discussion Paper No. 50/2020.
- HAGEDORN, M., J. LUO, I. MANOVSKII, AND K. MITMAN (2019): “Forward Guidance,” *Journal of Monetary Economics*, 102, 1–23.
- HOLSTON, K., T. LAUBACH, AND J. WILLIAMS (2017): “Measuring the Natural Rate of Interest: International Trends and Determinants,” *Journal of International Economics*, 108, S59–S75.
- JUNG, T., Y. TERANISHI, AND T. WATANABE (2005): “Optimal Monetary Policy at the Zero-Interest-Rate Bound,” *Journal of Money, Credit and Banking*, 37, 813–835.
- KAPLAN, G. AND G. VIOLANTE (2018): “Microeconomic Heterogeneity and Macroeconomic Shocks,” *Journal of Economic Perspectives*, 32, 167–94.
- KAPLAN, G., G. VIOLANTE, AND J. WEIDNER (2014): “The Wealthy Hand-to-Mouth,” *Brookings Papers on Economic Activity*, 45, 77–138.
- KOESTER, G., E. LIS, C. NICKEL, C. OSBAT, AND F. SMETS (2021): “Understanding Low Inflation in the Euro Area from 2013 to 2019: Cyclical and Structural Drivers,” Occasional Paper Series 280, European Central Bank.
- KRUSELL, P. AND A. SMITH (1998): “Income and Wealth Heterogeneity in the Macroeconomy,” *Journal of Political Economy*, 106, 867–896.
- MCKAY, A., E. NAKAMURA, AND J. STEINSSON (2016): “The Power of Forward Guidance Revisited,” *American Economic Review*, 106, 3133–58.
- MERTENS, T. AND J. WILLIAMS (2019): “Tying Down the Anchor: Monetary Policy Rules and the Lower Bound on Interest Rates,” Federal Reserve Bank of San Francisco Working Paper 2019-14.
- REITER, M. (2009): “Solving Heterogeneous-Agent Models by Projection and Perturbation,” *Journal of Economic Dynamics and Control*, 33, 649–665.
- ROTEMBERG, J. (1982): “Sticky Prices in the United States,” *Journal of Political Economy*, 90, 1187–1211.
- TAYLOR, J. (1993): “Discretion Versus Policy Rules in Practice,” *Carnegie-Rochester Conference Series on Public Policy*, 39, 195–214.
- VISSING-JØRGENSEN, A. (2002): “Limited Asset Market Participation and the Elasticity of Intertemporal Substitution,” *Journal of Political Economy*, 110, 825–853.

- WERNING, I. (2015): “Incomplete Markets and Aggregate Demand,” Mimeo.
- WOODFORD, M. (2018): “Monetary Policy Analysis When Planning Horizons Are Finite,” in *NBER Macroeconomics Annual*, ed. by M. Eichenbaum and J. Parker, Cambridge, MA: MIT Press, vol. 33, 1–50.
- WORK STREAM ON MONETARY-FISCAL POLICY INTERACTIONS (2021): “Monetary-Fiscal Policy Interactions in the Euro Area,” Occasional Paper Series 273, European Central Bank.
- WORK STREAM ON THE PRICE STABILITY OBJECTIVE (2021): “The ECB’s Price Stability Framework: Past Experience, and Current and Future Challenges,” Occasional Paper Series 269, European Central Bank.
- YOUNG, E. (2010): “Solving the Incomplete Markets Model with Aggregate Uncertainty Using the Krusell-Smith Algorithm and Non-Stochastic Simulations,” *Journal of Economic Dynamics and Control*, 34, 36–41.