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Do exchange rates absorb demand shocks at the ZLB?

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Non-technical summary

Research Question

According to the New Keynesian view, a flexible exchange rate regime cannot stabilize cyclical developments in response to adverse demand shocks when monetary policy is constrained at the zero lower bound on interest rates (ZLB). The reason is that negative demand conditions reduce inflation expectations and the real exchange rate appreciates as the policy rate cannot be lowered. However, the argument rests on the assumption that inflation expectations adjust implausibly strongly in response to economic conditions. Against this background, we address the question of whether the real exchange rate can absorb adverse demand shocks at the ZLB, when modeling inflation expectations in line with survey data.

Contribution

We embed imperfect information in the two-country New Keynesian framework to model inflation expectations in a survey-consistent way, so that they are heterogeneous across households and sluggish in response to shocks. Households only gradually learn about the true state of the economy, meaning that in the case of a severe adverse demand shock inflation expectations do not decline as strongly at the ZLB. The framework is sufficiently tractable to derive a set of analytical results. We also explore the dynamic effects in order to compare the theoretical implications to a set of empirical stylized facts.

Results

When modeling survey consistent inflation expectations using imperfect information, we find that exchange rates can absorb demand shocks at the ZLB. In contrast to the full information model: (i) A negative demand shock concentrated in the home country causes a real exchange rate depreciation that partially absorbs the demand shock. (ii) Empirical evidence is consistent with a real exchange rate depreciation at the ZLB. (iii) When the ZLB is binding in the home country, it is optimal for the foreign policymaker to reduce rather than increase foreign interest rates. (iv) When the central bank communicates the future path of monetary policy using "Forward Guidance" and reveals the true state of the economy, it exacerbates the negative output gap in the two countries.

Nichttechnische Zusammenfassung

Fragestellung

Aus Neu-Keynesianischer Sicht kann ein flexibles Wechselkursregime zyklische Schwankungen durch negative Nachfrageschocks an der Nullzinsgrenze nicht stabilisieren. Dies liegt daran, dass der Nachfragerückgang die Inflationserwartungen reduziert und damit der Wechselkurs aufwertet, da der Politikzins nicht weiter gesenkt werden kann. Allerdings basiert dieses Argument auf einer unplausibel starken Anpassung der Inflationserwartungen. Wir adressieren daher die Frage, ob der reale Wechselkurs negative Nachfrageschocks an der Nullzinsgrenze abfedern kann, wenn wir das Verhalten der Inflationserwartungen in einer Weise berücksichtigen, welches besser die Eigenschaften von tatsächlichen Umfragedaten widerspiegelt.

Beitrag

Wir erweitern ein zwei-Länder Neu-Keynesianisches Modell mit unvollständigen Informationen, um Inflationserwartungen konsistent mit Umfragedaten zu modellieren, so dass diese heterogen über die Haushalte sind als auch nur graduell auf Schocks reagieren. Haushalte lernen den tatsächlichen Zustand der Wirtschaft nur graduell, so dass bei einem gesamtwirtschaftlichen Nachfrageeinbruch an der Nullzinsgrenze die Inflationserwartungen deutlich weniger sinken. Der Modellrahmen erlaubt die Herleitung einiger analytischer Ergebnisse. Zudem untersuchen wir auch die dynamischen Effekte, um die theoretischen Ergebnissen mit empirischen Resultate zu vergleichen.

Ergebnisse

Auf Basis umfragekonsistenter Inflationserwartungen finden wir, dass der reale Wechselkurs nach einem negativen Nachfrageschock abwerten kann. Wir heben vier Resultate hervor: (i) Im Gegensatz zum Modell mit vollständigen Informationen, führt ein negativer Nachfrageschock an der Nullzinsgrenze zu einer realen Abwertung des Wechselkurses, welcher stabilisierend auf die Volkswirtschaft wirkt. (ii) Ein empirisch identifizierter Nachfrageschock ist konsistent mit einer realen Wechselkursabwertung an der Nullzinsgrenze. (iii) Wenn das Heimatland an der Nullzinsgrenze ist, dann ist es für das andere Land optimal die Zinsen zu reduzieren, anstatt sie wie unter vollständigen Informationen zu erhöhen. (iv) Wenn vorwärtsgerichtete Geldpolitik – "Forward Guidance" – Informationen über den tatsächlichen Zustand der Wirtschaft vermittelt, dann kann der negative Effekt auf die Produktionslücke der beiden Länder sogar verstärkt werden.

BUNDESBANK DISCUSSION PAPER NO 13/2021

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Mathias Hoffmann† Patrick Hürtgen‡

Abstract

According to the two-country full information New Keynesian model with flexible exchange rates, the real exchange rate appreciates in response to an asymmetric negative demand shock at the zero lower bound (ZLB) and exacerbates the adverse macroeconomic effects. This finding requires inflation expectations to adjust counterfactually large. When modeling inflation expectations consistent with survey expectations using imperfect information, we find that exchange rates can absorb demand shocks at the ZLB. In sharp contrast to the full information model: (i) A negative demand shock concentrated in the home country causes a real exchange rate depreciation that partially absorbs the demand shock. (ii) A VAR with an identified demand shock via sign restrictions is consistent with a real exchange rate depreciation at the ZLB. (iii) When the ZLB is binding in the home country, it is optimal for the foreign policymaker to reduce rather than increase foreign interest rates. (iv) Forward guidance that reveals the true state of the economy exacerbates the negative output gap in the two countries.

Keywords: Monetary policy, inflation expectations, imperfect information, real exchange rates.

JEL classification: F33, E31, E32.

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1 Introduction

There is a traditional view starting with Friedman (1953) that flexible exchange rate regimes allow the real exchange rate to depreciate in response to asymmetric negative demand disturbances.[1] This real depreciation then delivers efficient macroeconomic stabilization. However, when monetary policy is at the ZLB, the New Keynesian argument is that flexible exchange rate regimes cannot stabilize cyclical developments in response to adverse demand shocks (e.g. Cook and Devereux 2013, 2016). The reason is that negative demand conditions reduce inflation expectations tremendously and the real exchange rate appreciates as the policy rate cannot be lowered. Refining the modeling of inflation expectations is therefore crucial, because when nominal interest rates are zero, the driving forces of real exchange rate dynamics are inflation expectations.

We model inflation expectations in a survey-consistent way, so that they are heterogeneous across households and sluggish in response to shocks (e.g. Coibion and Gorodnichenko, 2012). We show analytically that in this case the exchange rate argument of a real appreciation at the ZLB is reversed: with realistic expectations formation the real exchange rate can depreciate and absorb asymmetric negative demand shocks at the ZLB. Our quantitative analysis confirms that the real exchange rate depreciates and that inflation expectations do not decline more strongly in response to an adverse demand shock at the ZLB, and also that macroeconomic volatility does not increase when the ZLB is binding.

At first glance, Figure 1 gives an emblematic but unconditional example for the euro area and the U.S. in this regard. While facing severe recessions and with monetary policy rates constrained by the ZLB, inflation expectations did not fall enormously and real exchange rate movements remained relatively stable during the ZLB periods (shaded gray) in Figure 1.

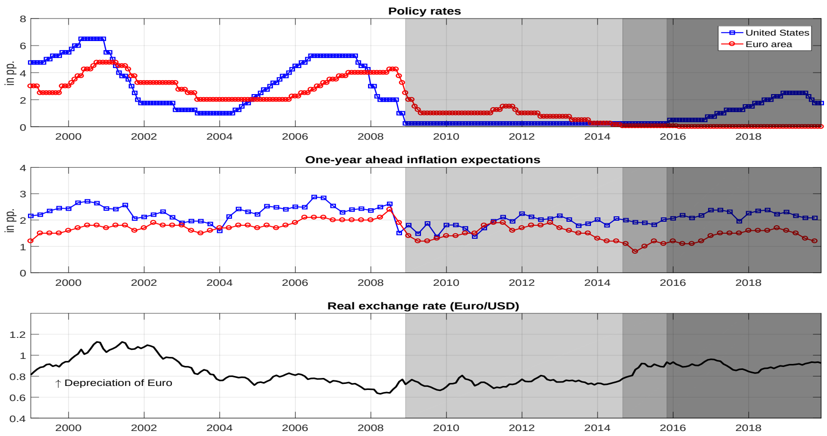

Figure 1: The euro area and the U.S. economy: 1999-2019.

Notes: The gray-shaded areas indicate ZLB periods. *Data sources:* OECD MEI, ECB SPF, and U.S. SPF.

[1]When the zero lower bound (ZLB) is not binding, the central bank reduces its nominal interest rate in response to a negative demand shock. This causes a real depreciation, implying an expenditure switching towards the cheaper goods.

Our work investigates this introductory example further and shows that the real exchange rate as well as inflation (expectations) and output in the euro area and the U.S. do not fluctuate more than before the ZLB episode. We also assess the conditional evidence by employing a sign-restricted vector autoregressive model using the sample from January 1999 to January 2020 to estimate the effect of a negative asymmetric demand shock on the real exchange rate and inflation expectations. The results indicate that the real exchange rate not only depreciates away from the ZLB, but also during the ZLB episode, whilst inflation expectations do not react more strongly when the ZLB is binding.[2] As mentioned before, the conventional two-country New Keynesian model is inconsistent with these stylized facts during the ZLB period, because in the model inflation expectations decline strongly and the real exchange rate appreciates.

We provide a novel theoretical explanation for these stylized facts. We allow for dispersed information amongst households about the underlying fundamentals as in Wiederholt (2019), so that households' inflation expectations are heterogeneous and sluggish. We assess this feature in a two-country New Keynesian model outlined by Clarida et al. (2002) and Engel (2010). The countries are highly integrated via financial markets, but less than perfectly integrated in goods markets, so that relative price adjustments across countries are required. Firms set their prices in their own currencies and the nominal exchange rate floats. The central banks follow a Taylor rule, unless the ZLB binds. Our framework allows us to provide analytic solutions to international negative demand shocks and we also illustrate their dynamic macroeconomic effects. We present our argument in two steps: following Figure 1, we examine the case of a severe global recession where either (i) both countries or (ii) one region is in a liquidity trap due to recessionary asymmetric demand shocks. We compare our results to the situation of full information, where the real exchange rate appreciates at the ZLB.

We highlight the following four theoretical results: First, in contrast to the full information New Keynesian model, an asymmetric negative demand shock, which constraints the home country by the ZLB, causes a real depreciation (rather than an appreciation). The reason is that households have imperfect information about the size of the demand shocks that hit the economies. Therefore, in the bad state of the world households perceive the asymmetric negative demand shock as being less contractionary. Consequently, inflation (expectations) increase in the two regions, compared to the full information scenario. Then the foreign central bank, which is unconstrained by the ZLB, would respond to higher inflation by increasing the policy rate by more, so that the real foreign interest rate would rise. Since households in the home country also perceive the demand shock to be less severe, the fall in inflation expectations at home is mitigated in comparison to the full information scenario. This and the induced rise in the real interest rate abroad reverses the real interest rate differential and causes a real depreciation. The slump in the home country is mitigated because the real depreciation enhances the country's competitiveness and insulates from deflationary pressure, while the volatility of output and inflation is mitigated. In comparison to the impact response under full information, output and inflation are cushioned by about 2/5 and 1/3, in a calibrated version with imperfect information. When the ZLB is binding in both countries,

[2] Corsetti et al. (2014) assess positive demand shocks for the U.S. prior to the ZLB period, finding a real appreciation.

the real exchange rate channel is also present, but less pronounced.

Second, trade openness mitigates macroeconomic volatility under imperfect informtion conditions in comparison to full information. Since the insulating effect of the real depreciation on output rises with openness, the efficient macroeconomic stabilization under imperfect information increases when countries are more open.

Third, when the home country's monetary policymaker cannot counter the negative demand shock at the ZLB but inflation expectations adjust sluggishly, it is optimal for the foreign policymaker to reduce foreign interest rates relative to the full information model. In the latter, the foreign country chooses to have a positive policy rate, though the world's natural real interest rate is below zero, to generate a real depreciation.

Fourth, for floating exchange rates to act as a shock absorber at the ZLB, i.e. to generate a real depreciation, the model with full information requires that the inflation rate be kept higher for an extended period via forward guidance. We show that when using imperfect information, this is not necessarily the case. By revealing the true bad state of the world, monetary policy might even increase the negative output gap in the two countries, causing a more severe recession in these circumstances.

Related literature: Within the extensive open macroeconomy literature, this work is closely related to a recent strand which studies the international dimensions of monetary policy at the ZLB (see Bodenstein et al., 2009; Corsetti et al., 2017; Jeanne, 2009; Werning, 2012; and Fujiwara et al., 2013). The paper most closely related to our work is Cook and Devereux (2013).[3] The authors characterize the optimal monetary and fiscal policy within a (global) liquidity trap under a flexible exchange rate regime. In their framework, an adverse macroeconomic shock at the ZLB induces a pervasive appreciation of the real exchange rate. We contribute to this literature by assessing the implications of survey-consistent inflation expectations for the international transmission of demand shocks at the ZLB as well as their implications for optimal policy and forward guidance. Unlike Cook and Devereux (2013) we show that the puzzlingly strong exchange rate dynamics at the ZLB disappear and the shock-absorbing properties of the real exchange rate remain at work when inflation expectations adjust only sluggishly to shocks.

In terms of modeling the sluggish adjustment of inflation expectations, our work is in line with contributions by Angeletos and Lian (2018) as well as Wiederholt (2019), who assess the effects of inflation expectations on monetary policy within a closed economy setting. To obtain closed-form solutions and to highlight the essence of the sluggish adjustment in inflation expectations for the international monetary policy stance, we utilize the approach of Wiederholt (2019). We show that his result on resolving the deflationary spiral not only carries over to an open economy environment, but also that the counter-intuitively large aggregate macroeconomic effects are mitigated further through the real exchange rate response. This feature stabilizes the international macroeconomic environment and moves the world economy closer to the natural equilibrium.

Gali (2021) demonstrates the existence of a forward guidance exchange rate puzzle under the as-

[3]Work by Müller et al. (2019) and Stavrakeva and Tang (2020) also assesses the role of expectation formation for the exchange rate. However, the authors focus on the role of monetary policy shocks or broker-dealer relationships in the exchange rate market.

sumption of full information in a small open economy model. We show that the same puzzle occurs in a two-country model. When modeling inflation expectations more consistently with survey data, our framework provides a potential explanation to cushion the forward guidance exchange rate puzzle.

Finally, Debortoli, Gali and Gambetti (2019) provide empirical evidence in support of the irrelevance hypothesis at the ZLB. According to this hypothesis output, inflation and long-term rates are not affected by the ZLB episode in the U.S. This hypothesis is at odds with the standard New Keynesian model. We show that this also holds true for the euro area and the EUR/USD real exchange rate. Under perfect information the exchange rate is much more volatile at the ZLB compared to a setting away from the ZLB. Imperfect information is one feature that mitigates these strong counterfactual predictions in our theoretical model.

The paper proceeds as follows: in the next section we lay out empirical evidence on the effects of a negative demand shock on the real exchange rate and macroeconomic volatility at the ZLB. Based on these stylized facts, in Section 3 we develop a two-country open economy model with sticky prices and imperfect information. In Section 4 we then compare the real exchange rate dynamics in response to an asymmetric negative demand shock away from and at the ZLB, when information is either complete or incomplete. Section 5 outlines the optimal monetary policy implications of imperfect information at the ZLB. Section 6 highlights the implications for forward guidance. Section 7 concludes.

2 An empirical assessment

In this section we provide several empirical facts that we later match in a theoretical dispersed information model. We investigate the changes in the volatility of the real exchange rate as well as inflation (expectations) and output during the period in which the ZLB constraint was binding in the U.S. and the euro area. In a second step we employ a sign-restricted vector autoregressive (VAR) model before and during the ZLB episode to estimate the effect of a negative demand shock on the real exchange rate.

	United States		Euro area	
Real exchange rate (EUR/USD)	0.58	0.68	0.29	0.30
Consumer price inflation	0.82	0.99	1.47	1.48
Inflation expectations	0.51	0.34	0.65	0.70
Industrial production	0.89	0.92	0.92	1.11
Great Recession?	yes	no	yes	no

Notes: Standard deviations are computed relative to the pre-ZLB period. Data on industrial production, CPI and RER is monthly and span the period from 01/1999 to 01/2020. Data on inflation expectations cover Q4 2002 to Q3 2018 (euro area) and Q1 1995 to Q3 2018 (U.S.). The left column for each country corresponds to the whole time span including the Great Recession (Q1 2008 to Q2 2009). The right column excludes this period both from ZLB and pre-ZLB periods. Industrial production and CPI are in log differences, the RER is in logs.

Table 1: Ratio of standard deviations at the ZLB relative to pre-ZLB

Table 1 shows that the volatility of the real exchange rate has not increased in the ZLB regime. This observation is in line with that for other macroeconomics variables as documented by Debortoli, Gali and Gambetti (2019). The business cycle statistics in Table 1 are based on the bilateral euro/dollar real exchange rate (RER), the consumer price index (CPI) and industrial production. The unconditional statistics in Table 1 provide little evidence that the real exchange rate as well as inflation (expectations) and output fluctuate more when the ZLB is binding, as measured by the relative standard deviation.[4]

To refine the analysis we run volatility regressions. The dependent variable is the absolute deviation of a variable from its period-specific mean. The real exchange rate volatility is significantly lower at the ZLB in the euro area. Note that the point estimate is also negative for the U.S. economy. When controlling for the Great Recession we still find an (insignificant) reduction in real exchange rate volatility. Overall, the results indicate no evidence of an increase in macroeconomic volatility in the ZLB period compared to before.

	Dependent variable:	
	RER (1)	RER (2)
ZLB	-0.085^{*}	-0.081
	(0.050)	(0.055)
Great Recession		0.033
		(0.073)
Constant	0.117^{**}	0.114^{**}
	(0.047)	(0.052)
Observations	253	253

Notes: *p<0.1; **p<0.05; ***p<0.01. Variables are in absolute deviations from the mean.

Table 2: Volatility regressions: euro area

In a second step we assess the response of the euro area macroeconomic variables to an asymmetric negative euro area demand shock. The VAR model contains four variables: output, CPI inflation, the euro/dollar real exchange rate as well as one-year-ahead inflation expectations. We estimate a structural VAR before the ZLB, at the ZLB, and the whole period. The model is expressed by the following equation:

$$Y_t = A_1 + A_2\tau + A_3\tau^2 + \sum_{p=1}^{P} B_p Y_{t-p} + \underbrace{C\epsilon_t}_{u_t}, \tag{1}$$

where Y_t is a vector of endogenous variables in logs, τ is a time trend and the lag length P equals six. The structural shocks are given by $\epsilon_t \sim N(0, I)$, while u_t are reduced form shocks. The data used in the VAR estimation are monthly from January 1999 to January 2020 and taken from the OECD MEI. Output is measured by total industrial production excluding construction (s.a.). CPI inflation is derived from the euro area's total consumer price index. The RER is the bilateral real exchange rate between the euro and the U.S. dollar, and is calculated using the nominal exchange rate and CPI indexes of the euro area and U.S. The one-year-ahead inflation expectations are taken from the ECB's SPF. The ZLB period for the euro area lasts from October 2014 until the end of the sample. To identify demand shocks, we impose

[4]The same holds true when using the BIS traded-weighted real exchange rate.

the following sign restrictions: a negative asymmetric euro area demand shock should decrease output and CPI inflation in the euro area, but we leave the response of the real exchange rate and inflation expectations unrestricted:

$$\begin{bmatrix} u_t^{Output} \\ u_t^{\pi^{CPI}} \\ u_t^{RER} \\ u_t^{\pi^e} \end{bmatrix} = \begin{bmatrix} * & * & * & + \\ * & * & * & + \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} \epsilon_t^1 \\ \epsilon_t^2 \\ \epsilon_t^3 \\ \epsilon_t^{demand} \end{bmatrix}$$

We compare results when imposing sign restrictions on output and inflation for one, three and twelve periods.

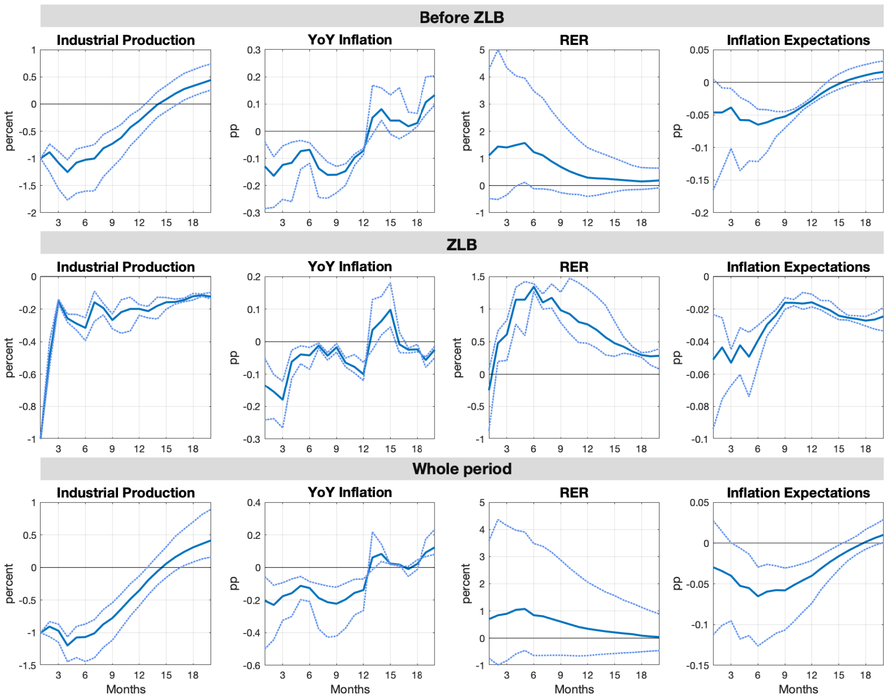

Figure 2: Impulse response to a 1% negative euro area demand shock

Notes: The solid line corresponds to the median impulse response for sign restrictions imposed for 12 months. The dotted lines reflects the 16th and 84th percentile confidence bands.

Figure 2 shows the results of our baseline regression, where we restrict industrial production and inflation for 12 months. We highlight two main findings: first, the real exchange rate depreciates significantly in both samples, i.e. before and at the ZLB. The size of the real depreciation is very similar and amounts to 1.5 percent in both samples (with tigher confidence bands in the ZLB period.). Since we are focusing on the role of inflation expectations, the second result we emphasize is that the decline in inflation expectations at the ZLB is not stronger than before the ZLB. The decline in both sub-samples is around 0.05 percentage point.

Figure 3 shows that our results are also robust when using either one or three months of restrictions on output and inflation.[5] When using fewer restrictions we also find a real exchange rate depreciation before and at the ZLB. The bottom row of Figure 3 confirms that the decline in inflation expectations is not more pronounced at the ZLB compared with the time period before the ZLB.

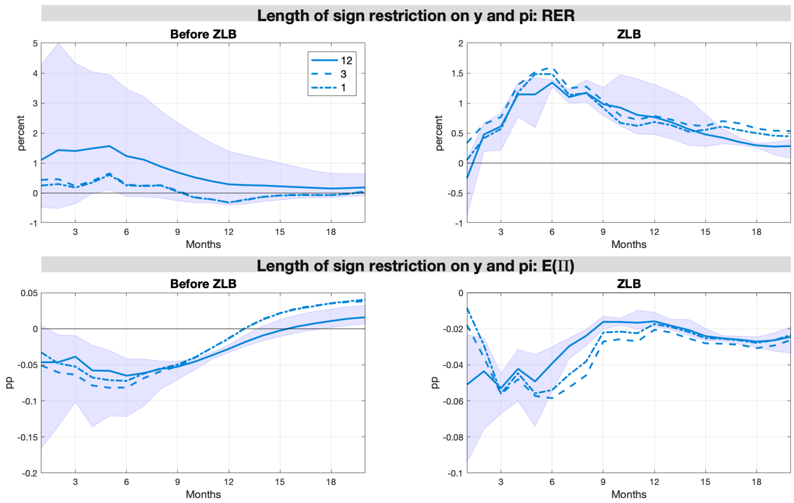

Figure 3: Sensitivity of impulse responses to a 1% negative euro area demand shock

Notes: Solid, dashed and dashed-dotted lines correspond to median impulse responses for sign restrictions for one, three, and twelve months. The shaded area reflects the 16th and 84th percentile confidence bands for the twelve-month restriction.

We show next that these stylized empirical facts are hard to reconcile with the full information two-country New Keynesian model. We embed imperfect information in the two-country New Keynesian model to account for the sluggish adjustment in inflation expectations. In this way we reconcile the above empirical facts, specifically the finding of no increase in the real exchange rate volatility at the ZLB and a real exchange rate depreciation at the ZLB in response to a negative demand shock.

3 The model

The model setup is similar to the two-country New Keynesian model as in Clarida et al. (2002). Following Engel (2010), we allow for less than perfectly integrated goods markets. There are two equally-sized countries, home (H) and foreign (F). In each of the economies there is a continuum of i households indexed by $i \,\epsilon\, [0,1]$, and a continuum of monopolistically competitive firms j indexed by $j \,\epsilon\, [0,1]$. The model is closed by a government and a central bank in each country.

Households In the home country, household i's utility is given by

$$E_0^i \left[\sum_{t=0}^{t=\infty} \exp(\xi_{i,t}) \beta^t \left(\frac{C_{i,t}^{1-\sigma} - 1}{1-\sigma} - N_{i,t} \right) \right], \tag{2}$$

[5] The results with respect to the real exchange rate are also robust when using the trade-weighted BIS real exchange rate.

and similarly for the foreign country's household i, which we denote henceforth with an asterisk: $*$. The utility of household i decreases with work effort, N_i, whereas the agent's utility depends positively on consumption, C_i. The intertemporal elasticity of substitution equals $1/\sigma$. The parameter $\beta \in (0, 1)$ is the discount factor.

Each household i faces a time preference shock ξ_i, which we also refer to as a demand shock. Our analysis focuses on a negative shock to ξ_i, which implies that households are willing to consume more in the future rather than today, and therefore, they increase savings and reduce their demand. The time preference shock $\xi_{i,t}$ is unanticipated and follows a stochastic decay. In every period $t \geq 1$ it holds that $\xi_{i,t} = \xi_{i,t-1}$ with a probability of $\mu < 1$.[6] With a probability of $(1 - \mu)$ the time preference shock $\xi_{i,t}$ returns to its normal value of zero. This occurs at the same period T for all households in the two countries.

Household i's expectation operator is given by E^i. To be consistent with survey expectations we introduce heterogeneity in the expectation formation through the time preference shock, following Wiederholt (2019). The time preference shock ξ_i can either take a low (l) or a high (h) value, so that $\xi_{i,0} \in \{\xi_l, \xi_h\}$, with $\xi_l < \xi_h$. The mass of households with $\xi_{i,0} = \xi_h$ is λ, while the remaining mass of households with $\xi_{i,0} = \xi_l$ is $(1 - \lambda)$. There are two possible aggregate states in period zero that differ in the mass of households λ which experience the high realization of the time preference shock: $\lambda \in \{\lambda_{bad}, \lambda_{good}\}$, with $0 < \lambda_{bad} < \lambda_{good} < 1$. This assumption is introduced so that the states $\mathsf{s} \in \{bad, good\}$ are not fully revealed in the case of imperfect information. Each realization is possible and the cross-sectional mean of the preference shock in state s in the home country equals $\xi_{\mathsf{s}} = \lambda_{\mathsf{s}}\xi_h + (1 - \lambda_{\mathsf{s}})\,\xi_l$, and similarly for the foreign country.

Households form expectations as follows: in the respective countries they have correct prior beliefs about the probability of the good state and of the bad state, the distribution of the states and their dynamics in the two states. The prior probability of households in the good state is given by $\theta \in (0, 1)$ and the prior probability of the bad state equals $(1-\theta)$. In period zero, households observe the realization of their own preference shock and update their beliefs about the evolution of aggregate shocks and future variables using Bayes' rule. They assign a conditional probability p_i^{good} (p_i^{bad}) of being in the good (bad) state of the economy.

In line with survey-based evidence we introduce a slow adjustment of expectations. Under incomplete information it is costly for households to update their beliefs so that only a fraction ϖ updates their beliefs about the true state of the economy and future macroeconomic variables, as in Mankiw and Reis (2002, 2006). In every period $0 \leq t \leq T-1$ a constant fraction $\varpi \in [0, 1]$ of randomly selected households learn the aggregate shock that has hit in period zero and moves back to the full information equilibrium. To derive analytic solutions we set $\varpi = 0$ and then show numerical results for alternative parameter values where the same economic intuition that is presented in the simple case prevails. We contrast results for the conventional case where households instantaneously know the full information equilibrium,

[6] For $0 < \mu < 1$, the ZLB will expire in expectations; see Eggertson and Woodford (2003). This ensures that inflation today is pinned down by expectations that inflation will be determined by the stable manifold in the future.

i.e. $\varpi = 1$.

In period T all households are the same, since they all receive the post-transfer of wealth in state-contingent assets traded in period -1. They demand state-contingent assets S in (international) financial markets in the period before the shocks $\xi_{i,0}$ and $\xi^*_{i,0}$ hit and all agents are still identical. Claims are contingent on λ, $\xi_{i,0}$ and $\xi^*_{i,0}$ as well as the payment period T. This all implies that in equilibrium all households will have the same post-transfer of wealth when the state-contingent claims are settled in period $T \geq 1$.[7] Households also demand domestic bonds B, pay domestic taxes and receive lump-sum transfers from the domestic government. The budget constraint equals

$$B_{i,t} = R_{t-1}B_{i,t-1} + W_{i,t}N_{i,t} + D_{i,t} + S_{i,t} - P_t C_{i,t}, \tag{3}$$

with $S_{i,t}$ referring to the flow budget constraint of internationally traded state-contingent claims. The difference between dividends and taxes is given by $D_{i,t}$, and $B_{i,t}$ reflects the bond holding between periods t and $t+1$. The nominal wage rate is $W_{i,t}$, while P_t represents the consumer price index (CPI).

Households consume goods produced by home and foreign firms. The preference for home and foreign goods subsumed in the consumption bundle C_i is represented by the parameter $0 < v \leq 2$. For $v > 1$ there is a home bias in consumption. Consumption of tradable goods in period t equals

$$C_{i,t} = C_{Hi,t}^{\frac{v}{2}} C_{Fi,t}^{1-\frac{v}{2}} \left(\frac{v}{2}\right)^{-\frac{v}{2}} \left(1 - \frac{v}{2}\right)^{-\left(1-\frac{v}{2}\right)} \text{ with } P_t = P_{Ht}^{\frac{v}{2}} P_{Ft}^{1-\frac{v}{2}}. \tag{4}$$

The relative preferences between the home-produced good, C_{Hi}, and foreign-produced traded good, C_{Fi}, are reflected by the value $\frac{v}{2}$ and $(1-\frac{v}{2})$, respectively. The CES aggregate of the composite goods produced by firm j in each country is given by

$$C_{Hi,t}^{\frac{\psi-1}{\psi}} = \int_0^1 C_{Hi,j,t}^{\frac{\psi-1}{\psi}} dj \text{ and } C_{Fi,t}^{\frac{\psi-1}{\psi}} = \int_0^1 C_{Fi,jt}^{\frac{\psi-1}{\psi}} dj, \text{ with } P_{Ht}^{1-\psi} = \int_0^1 P_{Hj,t}^{1-\psi} dj \text{ and } P_{Ft}^{1-\psi} = \int_0^1 P_{Fj,t}^{1-\psi} dj, \tag{5}$$

The elasticity of substitution between any two heterogeneous goods is $\psi > 1$. For $\psi \to \infty$ the $C_{Hi,j}$ and $C_{Fi,j}$ become perfect substitutes. For the foreign country a similar consumption and price index holds. Finally, households supply labour, $N_{i,t}$, to imperfectly competitive firms.

Firms' production technology and resource constraints Firms sell their differentiated goods to both domestic and foreign households. A Calvo (1983) lottery establishes which firms optimise their producer price in any period t. To produce the goods demanded by households for the home and foreign market, firms hire the labour at the given production prices derived below. The fixed unit mass of firms produce by

$$Y_{j,t} = N_{j,t}^{\varrho} = C_{Hj,t} + C^*_{Hj,t}, \text{ with } N_{j,t} = (N_{i,j,t}^{\frac{\eta-1}{\eta}} di)^{\frac{\eta}{\eta-1}}. \tag{6}$$

The amount of labour used by firm j is $N_{j,t}$ with $\varrho \leq 1$ being the elasticity of output with respect to labour $N_{j,t}$ and $\eta > 1$ the elasticity of output with respect to the composite $N_{i,j,t}$. Output $Y_{j,t}$ is used to accommodate the goods' demand $C_{Hj,t}$ and $C^*_{Hj,t}$. Similar conditions hold abroad.

[7] Similar assumptions have been made by Lucas (1990), Lorenzoni (2010) and Curdia and Woodford (2011).

Monetary and fiscal authority The monetary authorities decide on the underlying nominal interest rates while the country-specific fiscal authorities collect either lump-sum taxes or pay transfers to their residents. In particular, the monetary authority adopts the following policy rule:

$$R_t = \max\{1, R\Pi_{Ht}^{\phi}\}, \tag{7}$$

where $R = 1/\beta$ and $\Pi_{Ht} = P_{Ht}/P_{Ht-1}$ denotes the gross home producer price inflation rate. The Taylor principle holds and the reaction on inflation is given by $\phi > 1$. A similar condition holds for the foreign country. At the ZLB the monetary authorities can lower the nominal rate up to $\ln(R_t) - \ln(R) = r_t = -\ln(1/\beta)$. The fiscal authority collects either lump-sum taxes or pays transfers to its residents. The flow budget constraint of the government in country H equals $0 = \frac{\tau_t}{P_t} + \frac{B_t - (1+i_{t-1})B_{t-1}}{P_t} - \frac{P_{Ht}}{P_t}\frac{G_t}{P_{Ht}}$ and similarly for the foreign country F. The fiscal authority has to finance maturing government bonds and government purchases of domestic goods. It can collect lump-sum taxes τ or issue new government bonds. We assume that $G_t = G_t^* = 0$.

Equilibrium relationships From this section onwards we log-linearize the model around its steady-state values.[8] Lowercase letters reflect log deviations from the variable's X_t steady state X: $x_t = \log(X_t) - \log(X)$. With this in mind, we start by outlining the evolution of consumption and output. We go on to discuss the determination of inflation as well as the real exchange rate. Then, the Euler equation of household i is

$$c_{i,t} = E_t^i[c_{i,t+1} - \frac{1}{\sigma}(\xi_{i,t+1} - \xi_{i,t} + r_t - \pi_{t+1})], \tag{8}$$

with $E_t^i[r_t - \pi_{t+1}]$ reflecting household i's perceived consumption-based real interest rate, which depends on consumer price inflation (CPI), π, as well as the policy rate, r. The Markov property of the time preference (i.e. demand) shocks ξ_i and ξ_i^* implies that under independent monetary policy and flexible exchange rates, there are no predetermined state variables in the model. Hence, all endogenous variables in the world economy will inherit the same persistence as the shock itself, in expectation. The Euler equation for the highly-informed household (hi) in state s can then be written as

$$c_t^{hi} = \frac{1}{\sigma}(\xi_{h,t} - \mu\xi_{h,t+1}) - \frac{1}{\sigma}r_t + \frac{\mu}{\sigma}\pi_{t+1} + \mu c_{t+1}^{hi}, \text{ given } E_t^{hi}\left[r_t - \mu\pi_t\right] = (r_t - \mu\pi_{t+1}). \tag{9}$$

For the high-uninformed (hu) the expectations about the real interest rate are equal to

$$E_t^{hu}\left[r_t - \mu\pi_{t+1}\right] = \mathsf{p}_h^{good}\left(r_{good,t} - \mu\pi_{good,t}\right) + \mathsf{p}_h^{bad}\left(r_{bad,t} - \mu\pi_{bad,t}\right). \tag{10}$$

Thus, uninformed households attach a conditional probability p_h^{good} (p_h^{bad}) of being in the good (bad) state of the economy. The consumption Euler equation for the high-uninformed type of household equals

$$\begin{aligned} c_t^{hu} \quad = \quad & \frac{1}{\sigma}(\xi_{h,t} - \mu\xi_{h,t+1}) - \frac{\mathsf{p}_h^{good}}{\sigma}(r_{good,t} - \mu\pi_{good,t+1}) - \frac{\mathsf{p}_h^{bad}}{\sigma}(r_{bad,t} - \mu\pi_{bad,t+1}) \\ & + (1-\varpi)\,\mu c_{t+1}^{hu} + \varpi(\mathsf{p}_h^{good}\mu c_{good,t+1}^{hi} + \mathsf{p}_h^{bad}\mu c_{bad,t+1}^{hi}), \end{aligned} \tag{11}$$

[8] Appendix A.1 displays all relevant optimality conditions of households and firms.

given the fraction ϖ of becoming informed in the next period or remaining uninformed by $(1-\varpi)$. Similar equations hold for the low-informed (li) and low-uninformed (lu), respectively. Aggregated consumption then equals in each of state s:

$$c_t = \left(1-(1-\varpi)^{t+1}\right)\left(\lambda c_t^{hi} + (1-\lambda)\, c_t^{li}\right) + (1-\varpi)^{t+1}\left(\lambda c_t^{hu} + (1-\lambda)\, c_t^{lu}\right). \tag{12}$$

Similar conditions to (8)-(12) also hold for the foreign economy. To keep the exposition concise in the remaining part of this section, we summarize the expectations of households by average expectations defined as $\overline{E}_t\,[.] = \int_0^1 E_t^i\,[.]\,di$. Real wages are obtained from the consumption-leisure trade-off and by aggregating over the i types of households, $w_t - p_t = \sigma c_t$, with a similar condition holding in the foreign economy. To express the international linkages in a compact way, we use the following definitions: the variable $x_t^R = (x_t - x_t^*)/2$ denotes the world relative variables, while $x_t^W = (x_t + x_t^*)/2$ shows the world's average. From the firms' resource constraint and households demand conditions aggregate output becomes

$$y_t = c_t + (1 - \frac{(v-1)}{\delta})y_t^R - \frac{v(2-v)}{\delta}\xi_t^R. \tag{13}$$

The relative weight on home and foreign goods is $\delta = 1 + (\sigma-1)v\,(2-v) \geq 1$ for $v \geq 1$. If $v = 0$ (full foreign bias) or $v = 2$ (full home bias) it follows that $\delta = 1$. The intensity of the home bias is then expressed by $0 \leq (v-1)/\delta \geq 1$. If there is no home bias, $v = 1$ and $\delta = \sigma$. Given that goods markets are only imperfectly integrated, we set $v > 1$. From (8)-(13) a relation between interest rates, CPI and output is obtained:

$$r_t^R = \overline{E}_t[\pi_{t+1}^R + \sigma\frac{(v-1)}{\delta}\Delta y_{t+1}^R - \frac{(v-1)^2}{\delta}\Delta\xi_{t+1}^R], \text{ with } \pi^R = \pi_H^R - (2-v)\left(\pi_H^R - \Delta e/2\right), \tag{14}$$

denoting relative CPI. The log-change in the nominal exchange rate in terms of domestic currency units equals Δe. Expressed in terms of world averages, we have

$$r_t^W = \overline{E}_t[\pi_{t+1}^W + \sigma\Delta y_{t+1}^W - \Delta\xi_{t+1}^W], \text{ with } \pi^W = (\pi_H + \pi_F^*)/2. \tag{15}$$

Equations (14) and (15) describe the evolution of relative and world output in response to a demand shock. Later, we express home and foreign variables as functions of the world relative and world average variable: $x = x^W + x^R$ and $x^* = x^W - x^R$. Domestic price inflation π_H equals:[9]

$$\pi_{H,t} = \frac{\kappa}{2}y_t - \kappa_{(y-y^*)}y_t^R + \kappa_{(c-c^*)}\xi_t^R + \beta E_t\left[\pi_{Ht+1}\right], \text{ with} \tag{16}$$

$$\kappa \equiv 2(\sigma + \frac{1-\varrho}{\varrho})\frac{\frac{(1-\alpha)(1-\alpha\beta)}{\alpha}}{1+\psi\frac{1-\varrho}{\varrho}} > \kappa_{(y-y^*)} \equiv (\sigma - \frac{\sigma}{\delta})\frac{\frac{(1-\alpha)(1-\alpha\beta)}{\alpha}}{1+\psi\frac{1-\varrho}{\varrho}} \geq \kappa_{(c-c^*)} \equiv (1 - \frac{(v-1)}{\delta})\frac{\frac{(1-\alpha)(1-\alpha\beta)}{\alpha}}{1+\psi\frac{1-\varrho}{\varrho}} \geq 0,$$

for $v \geq 1$. The probability of a firm not adjusting its price in a given period is reflected by the parameter α, for $p_{H,t} = \int_0^1 p_{Hj,t}dj = \alpha\left(p_{H,t-1} + \log\overline{\pi}_H\right) + (1-\alpha)\int_0^1 \widetilde{p}_{Hj,t}dj$, which follows Calvo (1983). The coefficient κ defines the responsiveness of domestic inflation to domestic output. The responsiveness of domestic inflation to relative output is given by $\kappa_{(y-y^*)}$ and captures how strongly inflation adjusts to changes in relative output. Since $v > 1$, inflation responds more strongly to relative output changes by

[9] This follows from firms' optimal price setting. See also equation (62) in appendix A.1.

$\sigma - \sigma/\delta \geq 1$. The inflation response to relative time preference conditions is determined by $\kappa_{(c-c^*)}$. In the foreign country a similar condition to (16) holds, with the second and third term of the right-hand side in (16) taking the opposite sign. From (14) we obtain a relationship between relative interest rates, CPI and the real exchange rate q,

$$r_t^R = \overline{E}_t[\pi_{t+1}^R + \frac{\Delta q_{t+1}}{2}]. \tag{17}$$

Equation (17) mirrors the real UIP condition. The nominal exchange rate e is determined by the nominal counterpart of (17). Then, from (14) and (15) expected CPI inflation evolves by

$$\overline{E}_t\left[\pi_{t+1}\right] = \overline{E}_t[\pi_{Ht+1}] + (2-v)\,\overline{E}_t[r_t^R - \pi_{Ht+1}^R]. \tag{18}$$

From (17) it also becomes clear that when the Taylor rule (7) determines the nominal rate and the Taylor principle is satisfied, monetary policy crucially determines the real exchange rate. Under the assumption that $\lim_{T\to\infty} \overline{E}_t\left[q_T\right]$ is well defined and bounded, (17) can be solved forward. After taking $\lim_{T\to\infty}$ the equation can be written for (18) and the foreign counterpart as

$$\frac{q_t}{2} = -\overline{E}_t \sum_{l=0}^{\infty} (v-1)\left[r_{t+l}^R - \pi_{H,t+1+l}^R\right] + \lim_{T\to\infty} \frac{\overline{E}_t[q_T]}{2}, \tag{19}$$

implying that the real exchange rate can also be determined by the real interest differentials between the home and foreign country, $r_t^R - \overline{E}[\pi_{Ht+1}^R]$. Inflation expectations play an important part in defining the real rates. Hence, modeling inflation expectations is important when assessing the real exchange rate, particularly at the ZLB. We focus below on (i) real exchange rate dynamics at the ZLB and (ii) assess their efficacy as a shock absorber when inflation expectations are modeled in a survey-consistent way.

4 International effects of negative demand shocks

In this section we derive analytical solutions under full and dispersed information. To set the stage, Section 4.1 examines the outcome of international demand shocks in the natural economy with full information. We then compare these results in Section 4.2 to a situation where information is complete, prices are sticky and the economies are away from or at the ZLB. We also assess the case when the ZLB is binding in one country only. In Section 4.3 we then contrast these findings to the case of imperfect information.

We consider equilibria where the variables are constant from period zero until the preference shock reverts back to zero and the economy is in its non stochastic steady state thereafter.[10] Note from the above that the time preference shocks ξ_i and ξ_i^* are unanticipated and follow an stochastic decay. This means that consumption, output, inflation and the real exchange rate inherit the Markov property of the demand shock. They will take on the same values as long as the shock lasts and they will revert to zero once the shock ends.

[10]This builds on the work by Eggertson and Woodford (2003), Christiano, Eichenbaum and Rebelo (2011) and many others.

4.1 Full information and the natural economy

We first derive as a benchmark the flexible price equilibrium of the home and foreign economy, which would hold if prices in the world economy are purely flexible and information is complete, so that $\varpi = 1$. Households learn immediately about the exact size of the aggregate shocks and the individual's expectation is equal to the average expectation, $E_t^i = \overline{E}_t$. In the flexible price equilibrium the variable x is denoted by $\overline{x}$. In every period the Euler equation is satisfied and the flexible price value of inflation is zero. Given the Markov property of the demand shock, output in every period $0 \leq t \leq T-1$ is constant but depends on the shocks. Therefore, the time subscript t is replaced by the state subscript s $\epsilon \{bad, good\}$. Relative world output then equals

$$\overline{y}_{\mathsf{s}}^R = -\frac{\delta - (v-1)}{\frac{(1-\varrho)}{\varrho}\delta + \sigma}\xi_{\mathsf{s}}^R. \tag{20}$$

A negative relative demand shock, $\xi_{\mathsf{s}}^R < 0$, raises output at home relative to abroad, so that $\overline{y}_{\mathsf{s}}^R > 0$.[11] Expenditure switching towards home production is obtained by relative price movements, which are mirrored by the real exchange rate, given the flexible price counterpart of (17):

$$\frac{\overline{q}_{\mathsf{s}}}{2} = -\frac{\overline{r}_{\mathsf{s}}^R}{1-\mu}, \tag{21}$$

The real exchange rate depreciates, i.e. $\overline{q}_{\mathsf{s}} > 0$, when $\mu < 1$ and the relative natural rate $\overline{r}_{\mathsf{s}}^R < 0$. The fall in the relative natural rate of interest causes a real depreciation $\overline{q}_{\mathsf{s}} > 0$ and gears relative demand into the desired direction by expenditure switching towards the relatively cheaper goods, produced in the home country. The relative natural rate $\overline{r}_{\mathsf{s}}^R$ declines for $v > 1$ and $\xi_{\mathsf{s}}^R < 0$:

$$\overline{r}_{\mathsf{s}}^R = (1-\mu)\,(1 + v(v-2)\frac{\sigma}{\delta} + (v-1)\,\frac{\sigma}{\delta}\frac{\delta-(v-1)}{\frac{(1-\varrho)}{\varrho}\delta + \sigma})\xi_{\mathsf{s}}^R < 0. \tag{22}$$

If there is no home bias, i.e. $v = 1$, the relative natural rate is zero, $\overline{r}_{\mathsf{s}}^R = 0$, due to the fact that with no home bias financial market integration induces an equalization of the natural rates of interest at home and abroad and, consequently, the real exchange rate is zero. Given the world natural rate

$$\overline{r}_{\mathsf{s}}^W = (1-\mu)\,\xi_{\mathsf{s}}^W, \tag{23}$$

the home and foreign natural rates can be derived as $\overline{r}_{\mathsf{s}} = \overline{r}_{\mathsf{s}}^W + \overline{r}_{\mathsf{s}}^R$ and $\overline{r}_{\mathsf{s}}^* = \overline{r}_{\mathsf{s}}^W - \overline{r}_{\mathsf{s}}^R$, respectively. For $v > 1$, a negative relative demand shock causes the natural home rate $\overline{r}_{\mathsf{s}}$ to decline. Whether the foreign rate $\overline{r}_{\mathsf{s}}^*$ declines depends also on the relative shock size of ξ_{s}^* and ξ_{s}^R. For $v = 1$ the natural rate is the same for both countries: $\overline{r}_{\mathsf{s}} = \overline{r}_{\mathsf{s}}^* = \overline{r}_{\mathsf{s}}^W$. For a full home bias in goods, $v = 2$, which reflects closed economies, the natural rate of interest is equal to $\overline{r}_{\mathsf{s}} = (1-\mu)\,\xi_{\mathsf{s}}$ and $\overline{r}_{\mathsf{s}}^* = (1-\mu)\,\xi_{\mathsf{s}}^*$, respectively. Their decline in response to negative demand shocks then ensures that home and foreign flexible price output would be equal to zero.

[11] We outline the aggregate variables here, since in equilibrium all households have the same post-transfer of wealth in period T due to the trade in state-contingent claims in period -1.

We can now compare the natural rates to the nominal interest rates set by the monetary authorities in the sticky price, (im)perfect information environment, which are given by equation (7):

$$r_{\mathsf{s}} = \max\{-\ln(1/\beta), \phi\pi_{H\mathsf{s}}\} \text{ and } r^*_{\mathsf{s}} = \max\{-\ln(1/\beta), \phi\pi^*_{F\mathsf{s}}\}. \tag{24}$$

As long as the natural rates and policy rates under sticky prices are greater than $-\ln(1/\beta)$, monetary policy can efficiently stabilize the economies in response to negative demand shocks. When the ZLB is binding, monetary policy cannot stabilize the economies efficiently, since the policy rates r_{s} and r^*_{s} cannot fall below $-\ln(1/\beta)$, which $\overline{r}_{\mathsf{s}}$ and $\overline{r}^*_{\mathsf{s}}$, in contrast, can do. At the ZLB there is an inability to match the drop in the natural rates with a commensurate reduction in the policy rates below $-\ln(1/\beta)$. The ZLB on the nominal interest rate is binding when the cross-sectional means ξ_{s} and ξ^*_{s} in state s are sufficiently negative. Appendix A.2 shows the solution to the critical values ξ^{crit} and ξ^{*crit}. When $\xi_{\mathsf{s}} < \xi^{crit}_{\mathsf{s}}$ ($\xi^*_{\mathsf{s}} < \xi^{crit}_{\mathsf{s}}$) it follows that $\overline{r}_{\mathsf{s}} < -\ln(1/\beta)$ ($\overline{r}^*_{\mathsf{s}} < -\ln(1/\beta)$) and the ZLB is binding in the home (foreign) country under sticky prices.

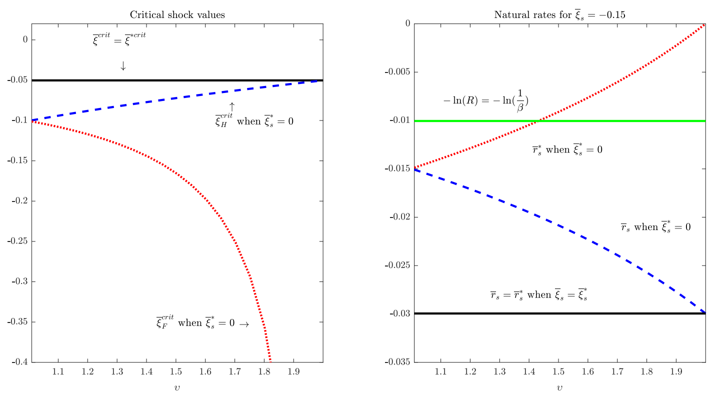

Figure 4: Critical shock values (left) and natural rates (right) as functions of openness (υ)

Notes: Parameter settings: $\beta = 0.99$. Intertemporal elasticity of substitution $1/\sigma = 0.5$. Elasticity of output w.r.t. labour $\varrho = 2/3$. Persistence of demand shock $\mu = 0.8$.

The left panel of Figure 4 plots the critical values ξ^{crit} as well as ξ^{crit}_H and ξ^{crit}_F as functions of $1 < \upsilon \leq 2$, the preference bias towards domestically produced goods.[12] In particular, the black solid line shows the evolution of $\xi^{crit} = \xi^{*crit}$.[13] Then, the preference bias towards domestically produced goods has no effect on the critical shock values. The blue dashed and red doted line mirror the evolution of the critical values ξ^{crit}_H and ξ^{crit}_F, so that the ZLB is binding for $\xi^*_{\mathsf{s}} = 0$. If υ is close to unity, ξ^{crit}_H and ξ^{crit}_F are very similar because the amount of domestic and foreign goods in the consumption basket is aligned. As υ increases, the preference bias towards domestically produced goods rises, too. The amount of home-produced goods consumed in the foreign country is then less important. Consequently, ξ^{crit}_F has to be very large to push the foreign country towards the ZLB, as shown by the red dashed line. The opposite is true for ξ^{crit}_H in the home country.

[12] ξ^{crit}_H and ξ^{crit}_F prevail when the foreign demand shock ξ^{*crit} would be zero.
[13] See equation (66) in appendix A.2, for ξ_{s} and ξ^*_{s} being non-zero and (67) for $\xi^*_{\mathsf{s}} = 0$.

The right panel of Figure 4 plots the natural rates based on equations (64) and (65) for $\xi_{\mathsf{s}} = -0.15$. When $\xi_{\mathsf{s}} = \xi_{\mathsf{s}}^*$ the natural rates are aligned as shown by the black solid line and $\overline{r}_{\mathsf{s}} = \overline{r}_{\mathsf{s}}^* < -\ln(1/\beta)$ holds. When $\xi_{\mathsf{s}}^* = 0$, then $\overline{r}_{\mathsf{s}}$ and $\overline{r}_{\mathsf{s}}^*$ almost coincide for v close to unity. As the countries move from more open to more closed, i.e. v increases, the impact of ξ_{s} on $\overline{r}_{\mathsf{s}}$ rises (the blue dashed line) while it falls for $\overline{r}_{\mathsf{s}}^*$ (red dotted line), so that the natural foreign rate equals zero for $v = 2$. In the example, at around $v > 1.4$, i.e. an import share of 30% or less, the ZLB is binding only in the home country because $\xi_{\mathsf{s}}^* > \xi_F^{crit}$ and $\overline{r}_{\mathsf{s}}^* > -\ln(1/\beta)$.

In the following sections we contrast the results of the flexible price equilibrium to a situation where prices are sticky and information is either full or incomplete. Thereafter, we express consumption and output in deviations from the flexible price analogously, $\overline{x}$, so that $\widetilde{x}_{\mathsf{s}} = x_{\mathsf{s}} - \overline{x}_{\mathsf{s}}$.

4.2 Full information and sticky prices

We first consider the case of sticky prices and full information ($\varpi = 1$). Then, from the Euler equation (8) and (12) and the natural equilibrium's counterpart, aggregate consumption evolves by

$$\widetilde{c}_t = \overline{E}_t[-\frac{1}{\sigma}(r_t - \pi_{t+1} - \overline{r}_t) + \widetilde{c}_{t+1}]. \tag{25}$$

In every period the Euler equation is satisfied, given the monetary policy rules (7). Since the variables are constant, but depend on the size of the time preference shock, we replace t by the state subscript s:

$$\widetilde{c}_{\mathsf{s}} = -\frac{\frac{1}{\sigma}}{1-\mu}[r_{\mathsf{s}} - \mu\pi_{\mathsf{s}} - \overline{r}_{\mathsf{s}}] \text{ with } [\mu\pi_{\mathsf{s}}] = [\mu\pi_{H,\mathsf{s}}] + (2-\nu)(r_{\mathsf{s}}^R - [\mu\pi_{H,\mathsf{s}}^R]). \tag{26}$$

Consumption is affected by the country-specific policy rate and expected CPI inflation in relation to the natural rate. Expected CPI inflation rises with an expected increase in domestic inflation, $[\mu\pi_{H,\mathsf{s}}]$, and the real interest rate differential between the home and foreign country, $r_{\mathsf{s}}^R - [\mu\pi_{H,\mathsf{s}}^R]$. The latter defines the real exchange rate, q. From the real UIP condition (17) the real exchange rate q follows by

$$\frac{q_{\mathsf{s}}}{2} = -(v-1)\frac{r_{\mathsf{s}}^R - [\mu\pi_{H,\mathsf{s}}^R]}{1-\mu}. \tag{27}$$

A fall in the relative real interest rate differential causes a real depreciation ($q_{\mathsf{s}} > 0$). The Phillips curve (16) defines domestic inflation:

$$\pi_{H,\mathsf{s}} = \pi_{H,\mathsf{s}}^W + \pi_{H\mathsf{s}}^R = \kappa_{\widetilde{y}}\widetilde{y}_{\mathsf{s}} - \left(\kappa_{\widetilde{y}} - \kappa_{\widetilde{y}^R}\right)\widetilde{y}_{\mathsf{s}}^R, \tag{28}$$

with $\kappa_{\widetilde{y}} \equiv \kappa/(2(1-\beta\mu)) > 0 \kappa_{\widetilde{y}^R} \equiv (\kappa - 2\kappa_{(y-y^*)})/(2(1-\beta\mu)) > 0$. The coefficient $\kappa_{\widetilde{y}}$ reflects the responsiveness of inflation to a change in $\widetilde{y}$. A rise in the home output gap $\widetilde{y}_{\mathsf{s}}$ leads to a rise in home inflation $\pi_{H\mathsf{s}}$ for a given relative output gap $\widetilde{y}_{\mathsf{s}}^R$, which is reflected by equation (14) in deviation from the natural equilibrium:

$$\widetilde{y}_{\mathsf{s}}^R = -\frac{\delta}{\sigma}\frac{r_{\mathsf{s}}^R - [\mu\pi_{H\mathsf{s}}^R] - \frac{\overline{r}_s^R}{(v-1)}}{(1-\mu)} \text{ with } \pi_{H\mathsf{s}}^R = \kappa_{\widetilde{y}^R}\widetilde{y}_{\mathsf{s}}^R. \tag{29}$$

In the foreign economy inflation evolves similarly to (28), with the second term taking the opposite sign. Then, equation (15) implies in terms of world averages, that world output and inflation equate to

$$\widetilde{y}_{\mathsf{s}}^W = -\frac{1}{\sigma}\frac{r_{\mathsf{s}}^W - [\mu\pi_{H\mathsf{s}}^W] - \overline{r}_{\mathsf{s}}^W}{1-\mu} \text{ with } \pi_{H,\mathsf{s}}^W = \kappa_{\widetilde{y}}\widetilde{y}_{\mathsf{s}}^W. \tag{30}$$

Summing over equations (29) and (30), home output evolves by

$$\widetilde{y}_{\mathsf{s}} = -\frac{\frac{1}{\sigma}[r_{\mathsf{s}} - \mu\pi_{H\mathsf{s}}]}{1-\mu} + \frac{\frac{1}{\sigma}\overline{r}^W_{\mathsf{s}} + \frac{1}{\sigma}\frac{\delta}{v-1}\overline{r}^R_{\mathsf{s}}}{1-\mu} + \frac{1}{\sigma}\frac{\delta-1}{v-1}\frac{q_{\mathsf{s}}}{2}. \tag{31}$$

Output is affected by the real interest rate $[r_{\mathsf{s}} - \mu\pi_{H\mathsf{s}}]$ in relation to the natural rates as well as the real exchange rate, q_{s}. A fall in the real interest rate and a real depreciation, i.e. a rise in the real exchange rate, affect output positively. Foreign output follows a similar expression as in (31), except that the terms on $\overline{r}^R_{\mathsf{s}}$ and q_{s} take the opposite sign.

The next section contrasts the case of being outside the ZLB to two situations where either the ZLB is binding in all states s of the world in both countries or only in the home country. Thus in the first case both countries are pushed into the ZLB, but the demand shocks are asymmetric, with the shock being more severe in the home country. In the second situation we assume that only the domestic economy is at the ZLB, while the foreign economy is able to utilize the nominal interest rate as its policy tool.[14]

4.2.1 ZLB is not binding in both countries

When the ZLB is not binding it holds that: $-\ln(1/\beta) < \overline{r}^R_{\mathsf{s}} < \overline{r}^W_{\mathsf{s}} < 0$.[15] The monetary policy conditions are then equal to $r_{\mathsf{s}} = \phi\pi_{H\mathsf{s}}$ as well as $r^*_{\mathsf{s}} = \phi\pi^*_{F\mathsf{s}}$. It follows that a fall in the relative natural rate of interest causes a fall in relative home inflation expectations $\left[\mu\pi^R_{H\mathsf{s}}\right]$, which from equation (27) results in a real depreciation:

$$\frac{q_{\mathsf{s}}}{2} = -(v-1)\frac{(\phi-\mu)}{(1-\mu)}\pi^R_{H\mathsf{s}} > 0, \text{ for } [\mu\pi^R_{H\mathsf{s}}] = \frac{\kappa_{\widetilde{y}^R}}{\gamma_{\widetilde{y}^R}}\frac{\mu\delta}{v-1}\overline{r}^R_{\mathsf{s}} < 0 \tag{32}$$

and $\gamma_{\widetilde{y}^R} \equiv (1-\mu)\sigma + (\phi-\mu)\delta\kappa_{\widetilde{y}^R} > 0$. The reason is that the Taylor principle holds, $\phi - \mu > 0$: a fall in relative home inflation leads to a fall in relative consumption-based real interest rates by $(v-1)(\phi-\mu)\pi^R_{H\mathsf{s}}$.[16] The Taylor principle also ensures that the domestic real interest rate, $(\phi-\mu)\pi_{H,\mathsf{s}}$, falls for

$$\pi_{H\mathsf{s}} = \frac{\kappa_{\widetilde{y}}}{\gamma_{\widetilde{y}}}\overline{r}^W_{\mathsf{s}} + \frac{\kappa_{\widetilde{y}^R}}{\gamma_{\widetilde{y}^R}}\frac{\delta}{v-1}\overline{r}^R_{\mathsf{s}} < 0, \tag{33}$$

and $\gamma_{\widetilde{y}} \equiv (1-\mu)\sigma + (\phi-\mu)\kappa_{\widetilde{y}} > 0$. This, together with the real exchange rate depreciation, mitigates the negative demand shock on output. To see this we decompose domestic output (31) into natural rates, inflation, and the real exchange rate:

$$\widetilde{y}_{\mathsf{s}} = \frac{\frac{1}{\sigma}\overline{r}^W_{\mathsf{s}} + \frac{1}{\sigma}\frac{\delta}{v-1}\overline{r}^R_{\mathsf{s}}}{1-\mu} - \frac{1}{\sigma}\frac{(\phi-\mu)}{1-\mu}\pi_{H\mathsf{s}} + \frac{1}{\sigma}\frac{\delta-1}{v-1}\frac{q_{\mathsf{s}}}{2} = \frac{\overline{r}^W_{\mathsf{s}}}{\gamma_{\widetilde{y}}} + \frac{\frac{\delta}{v-1}}{\gamma_{\widetilde{y}^R}}\overline{r}^R_{\mathsf{s}} < 0. \tag{34}$$

The first term reflects the negative impact of the natural rates. The second term shows that monetary policy accompanies the negative demand shock by reducing the domestic real interest rate, $(\phi-\mu)\pi_H <$

[14] Without loss of generality we assume that the demand shock is always more negative in the home than in the foreign country. Then, the relative demand shock is negative, $\xi^R_{\mathsf{s}} < 0$, and we consider only cases where also $\xi^W_{\mathsf{s}} < 0$ will hold. From (22) and (23) it therefore follows that the relative natural rate of interest falls, $\overline{r}^R_{\mathsf{s}} < 0$ and so does the world natural rate, $\overline{r}^W_{\mathsf{s}} < 0$.

[15] The shock sizes are then given by: $\xi^{crit} < \xi_{\mathsf{s}} < 0$ and $\xi^{*crit} < \xi^*_{\mathsf{s}}$ with $\xi_{\mathsf{s}} < \xi^*_{\mathsf{s}}$.

[16] The real depreciation is accompanied by a nominal depreciation. The nominal exchange rate can also be derived from equation (32) and the fact that $e_{\mathsf{s}} - e_{-1} = q_{\mathsf{s}} + \pi^R_{\mathsf{s}} - q_{-1}$. Then the initial nominal depreciation equals $-(\phi-1)/(1-\mu)\pi^R_{H\mathsf{s}}$.

0, so that the effect of falling inflation on output is positive. The last term of (34) reflects the impact of the real exchange rate. The real depreciation allows for the negative effects on output to be mitigated by the coefficient: $(\delta-1)/(2\sigma(v-1))>0$. Hence, the effect of the adverse demand shock on output is smaller in the open economy compared to the closed economy.[17] The last equality shows the closed-form solution of home output, which is negative, since $\overline{r}_{\mathsf{s}}^{W}<0$ and $\overline{r}_{\mathsf{s}}^{R}<0$.

If the authorities were to put a high weight on stabilizing inflation (ϕ large), the divine coincidence occurs and inflation as well as output in the two countries would be fully stabilized. This follows immediately from equations (33) and (34): a large ϕ causes $\gamma_{\widetilde{y}}$ and $\gamma_{\widetilde{y}^R}$ to rise. Consequently, when ϕ is sufficiently high, inflation and output are stabilized around their flexible price values under flexible exchange rates.

4.2.2 ZLB is binding in both countries

When the monetary authorities in both countries are constrained by the ZLB, $\overline{r}_{\mathsf{s}}^{W}<\overline{r}_{\mathsf{s}}^{R}<-\ln(1/\beta)<0$ holds.[18] It follows then from (7) that $r_{\mathsf{s}}=r_{\mathsf{s}}^{*}=-\ln(1/\beta)$. We show that in this case the effects of inflation and the real exchange rate on output are the opposite at the ZLB compared to the case away from the ZLB.

Equation (35) shows that the fall in the relative natural rate of interest and relative home inflation expectations $\left[\mu\pi_{H\mathsf{s}}^{R}\right]$ now causes a real appreciation:

$$\frac{q_{\mathsf{s}}}{2}=\frac{(v-1)}{1-\mu}[\mu\pi_{H\mathsf{s}}^{R}]<0,\text{ for }[\mu\pi_{H\mathsf{s}}^{R}]=\frac{\kappa_{\widetilde{y}^R}}{\gamma_{\widetilde{y}^R}^{zlb}}\frac{\mu\delta}{v-1}\overline{r}_{\mathsf{s}}^{R}<0 \tag{35}$$

and $\gamma_{\widetilde{y}^R}^{zlb}\equiv(1-\mu)\sigma-\mu\delta\kappa_{\widetilde{y}^R}>0$. At the ZLB the constrained monetary policy cannot accommodate a relative fall in (expected) home inflation. As a consequence, the fall in relative (expected) inflation leads to a rise in today's consumption-based relative real interest rates by $-(v-1)[\mu\pi_{H\mathsf{s}}^{R}]>0$ and, hence, a real appreciation.[19] The real exchange rate not only appreciates, but its magnitude is amplified in comparison to the case of no ZLB for a given relative natural interest rate $\overline{r}_{\mathsf{s}}^{R}$, since $\mu\gamma_{\widetilde{y}^R}>(\phi-\mu)\gamma_{\widetilde{y}^R}^{zlb}$. Under perfect information the model predicts for a plausible range of parameter values that the exchange rate is multiple times more volatile at the ZLB. Therefore, this finding extends the irrelevance hypothesis of Derbatoli, Gali and Gambetti (2019) to the exchange rate. To see how the real exchange rate and expected inflation, $[\mu\pi_{H,\mathsf{s}}]$, affect home output at the ZLB we decompose output from equation (31), so that

$$\widetilde{y}_{\mathsf{s}}=\frac{\frac{1}{\sigma}\overline{r}_{\mathsf{s}}^{W}+\frac{1}{\sigma}\frac{\delta}{v-1}\overline{r}_{\mathsf{s}}^{R}}{1-\mu}+\frac{\frac{1}{\sigma}\ln(1/\beta)}{1-\mu}+\frac{\frac{1}{\sigma}}{1-\mu}[\mu\pi_{H,\mathsf{s}}]+\frac{1}{\sigma}\frac{\delta-1}{v-1}\frac{q_{\mathsf{s}}}{2}=\frac{\overline{r}_{\mathsf{s}}^{W}+\ln(1/\beta)}{\gamma_{\widetilde{y}}^{zlb}}+\frac{\frac{\delta}{v-1}}{\gamma_{\widetilde{y}^R}^{zlb}}\overline{r}_{\mathsf{s}}^{R}<0, \tag{36}$$

with $\gamma_{\widetilde{y}}^{zlb}\equiv(1-\mu)\sigma-\mu\kappa_{\widetilde{y}}>0$.[20] The first term shows the impact of the natural rates, which are negative. The second term is the effect of the nominal interest rate. The monetary authorities are able

[17] This holds as long as $\overline{r}_{\mathsf{s}|v=2}/\gamma_{\widetilde{y}}<\overline{r}_{\mathsf{s}}^{W}/\gamma_{\widetilde{y}}+\delta\overline{r}_{\mathsf{s}}^{R}/((v-1)\gamma_{\widetilde{y}^R})$ for $\overline{r}_{\mathsf{s}|v=2}=(1-\mu)\xi_{\mathsf{s}}$.

[18] These conditions are satisfied under: $\xi_{\mathsf{s}}<\xi^{crit}$ and $\xi_{\mathsf{s}}^{*}<\xi^{*crit}$, but $\xi_{\mathsf{s}}<\xi_{\mathsf{s}}^{*}$.

[19] From (35) and $e_{\mathsf{s}}-e_{-1}=q_{\mathsf{s}}+\pi_{\mathsf{s}}^{R}-q_{-1}$ the initial appreciation of the nominal exchange rate equals $1/(1-\mu)\pi_{H\mathsf{s}}^{R}$. The anticipated movements in the subsequent periods are zero, so that uncovered interest rate parity is still satisfied.

[20] We are focusing on a parameter space where $\gamma_{\widetilde{y}}^{zlb}>0$ and $\gamma_{\widetilde{y}^R}^{zlb}>0$ is ensured.

to reduce the nominal interest rate to $\ln(1/\beta)$, which has a positive effect on output. The last two terms of the first equality reflect the impact of inflation expectations and the real exchange rate. In contrast to the case when the ZLB is not binding (see equation (34)), inflation expectations affect home output negatively by

$$[\mu\pi_{H,\mathsf{s}}] = \frac{\kappa_{\widetilde{y}}}{\gamma^{zlb}_{\widetilde{y}}}\mu\left(\overline{r}^{W}_{\mathsf{s}} + \ln(1/\beta)\right) + \frac{\kappa_{\widetilde{y}^R}}{\gamma^{zlb}_{\widetilde{y}^R}}\frac{\mu\delta}{\upsilon - 1}\overline{r}^{R}_{\mathsf{s}} < 0. \tag{37}$$

The reason is that at the ZLB there is an inability to match the drop in the natural rates with a commensurate reduction in the policy rates below $-\ln(1/\beta)$, so that the fall in expected inflation leads to a rise in today's domestic real interest rate. Furthermore, at the ZLB inflation expectations decline to a larger degree compared to the situation when the ZLB is not binding. The reason is that $\gamma^{zlb}_{\widetilde{y}}$ and $\gamma^{zlb}_{\widetilde{y}^R}$ can be small and the decline in inflation expectations is larger compared to equation (33). From equation (35) it follows that the real exchange rate appreciates strongly, which drives down demand for domestically produced goods. Given the strong appreciation in the real exchange rate and declining inflation expectations, the drop in home output is larger, so that $\widetilde{y}^{ZLB}_{\mathsf{s}} < \widetilde{y}_{\mathsf{s}} < 0$. The full solution to home output is shown by the second equality in equation (36). Thus the amplified decline in output is due to both the strong decline in inflation (expectations) and the real exchange rate appreciation, which amplifies the negative output effect compared to the closed economy.

4.2.3 ZLB is binding only in the home country: A Keynesian cross analysis

This section assesses the situation when the home country is at the ZLB, while the foreign economy can use monetary policy to counteract the consequences of asymmetric negative demand shocks. When the ZLB only binds in the home country, we maintain the assumption that the shocks are such that $\overline{r}^{W}_{\mathsf{s}} < \overline{r}^{R}_{\mathsf{s}} < -\ln(1/\beta) < 0$.[21] With the home country at the ZLB, monetary policy equals $r_{\mathsf{s}} = -\ln(1/\beta)$ and $r^*_{\mathsf{s}} = \phi\pi^*_{F\mathsf{s}}$. We show that even in this environment perverse responses of the real exchange rate can occur. Accounting for the monetary policy stance, the real exchange rate (27) and, hence the real interest rate differential evolves by

$$\frac{q_{\mathsf{s}}}{2} = \frac{(\upsilon - 1)}{(1-\mu)}\left(\frac{\ln(1/\beta) + \phi\pi^*_{F\mathsf{s}}}{2} + [\mu\pi^R_{H\mathsf{s}}]\right). \tag{38}$$

Equation (38) implies that not only relative inflation expectations, i.e. $[\mu\pi^R_{H\mathsf{s}}]$, matter for the real interest rate differential and, hence the real exchange rate, but also the foreign monetary policy and its ability to affect foreign inflation, $\phi\pi^*_{F\mathsf{s}}$. The expected inflation differential as well as foreign inflation are given by (29) and the foreign counterpart of (28): $[\mu\pi^R_{H\mathsf{s}}] = \mu\kappa_{\widetilde{y}^R}\widetilde{y}^R_{\mathsf{s}}$ and $\pi^*_{F\mathsf{s}} = \kappa_{\widetilde{y}}\widetilde{y}^W_{\mathsf{s}} - \kappa_{\widetilde{y}^R}\widetilde{y}^R_{\mathsf{s}}$. For example, a rise in foreign inflation $\pi^*_{F\mathsf{s}}$ would cause a rise in the real exchange rate compared to (35), as this is accompanied with a rise in the foreign real interest rate, $(\phi - \mu)\pi^*_F > 0$.[22] But when the ZLB in the home country is severe, relative inflation (expectations) deteriorate, so that $[\mu\pi^R_{H\mathsf{s}}] < (\ln(1/\beta) + \phi\pi^*_{F\mathsf{s}})$.

[21] In this case it holds that $\xi_{\mathsf{s}} < \xi^{crit} < 0$, but $\xi^{*crit} < \xi^*_{\mathsf{s}}$.

[22] A rise in foreign inflation occurs when the foreign demand shock is sufficiently positive, i.e. $\xi^*_{\mathsf{s}} > 0$. We will maintain this assumption. If the foreign demand shock were negative, i.e. $\xi^{*crit} < \xi^*_{\mathsf{s}} < 0$, foreign inflation would also fall. From (38) a real appreciation occurs unambiguously in the home country. For a full solution of $\widetilde{y}^R < \widetilde{y}^w < 0$, see (68) and (69) in appendix A.3.3.

Hence, the real interest rate differential rises and the real exchange rate appreciates. This situation occurs at the ZLB when the difference between the home and foreign demand shock is sufficiently large, so that $\overline{r}_{\mathsf{s}}^{W} < \overline{r}_{\mathsf{s}}^{R} < -\ln\left(1/\beta\right) < 0$. To keep the exposition concise, we will focus on an analysis based on the Keynesian cross. A complete derivation of all equilibrium conditions of this case are to be found in appendix A.3.3. Figure 5 shows the Keynesian cross when the ZLB is either binding or not binding in the home country.

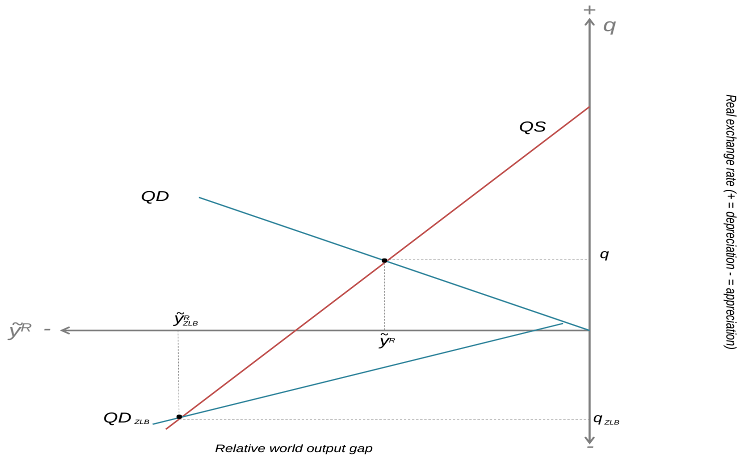

Figure 5: The real exchange rate equilibrium: no ZLB vs. a binding ZLB in the home country

Notes: The elasticity of substitution between differentiated goods, $\psi = 10$. The probability that a firm cannot adjust its price in a given quarter $\alpha = 0.66$. The values of $\upsilon = 1.5$ implies a steady-state import content of 25%. The average import content across the euro area countries varies between 15% and 32% according to EuroStat (2019). We therefore chose the values of $\upsilon = 1.5$ such that the steady-state import content equates to 25%, covering the intermediate region of trade openness. The Taylor rule coefficient is $\phi = 1.6$. All other parameters are as in Figure 4.

In particular, Figure 5 shows the relative output and real exchange rate response in the (y_{s}^{R}-q_{s}) space: The QS curve (red line) reflects the international goods market equilibrium. Subsuming firms' and households' aggregate supply and demand conditions of equations (27) and (29), we can write

$$\text{QS}: q_{\mathsf{s}} = 2\left(\upsilon - 1\right)\frac{\sigma}{\delta}\widetilde{y}_{\mathsf{s}}^{R} - \frac{2\overline{r}_{\mathsf{s}}^{R}}{1-\mu} \tag{39}$$

The QS curve is upward-sloping by $2\sigma\left(\upsilon - 1\right)/\delta$, because a relatively lower home supply of goods generates upward pressure on home goods prices so that the real exchange rate appreciates. The intercept of the QS curve is given by $-2\overline{r}_{\mathsf{s}}^{R}/(1-\mu)$. Reflecting the goods market, the QS curve is independent of monetary policy.

The QD curve mirrors the international money market equilibrium. It combines the real UIP condition with the conditional monetary policies and the firms' price setting, utilizing (24) with (27) and (29). First we describe the case when the ZLB is not binding. Given the monetary policies at home and abroad, we have

$$\text{QD}: q_{\mathsf{s}} = -\left(\upsilon - 1\right)\kappa_{\widetilde{y}^{R}}\frac{\left(\phi - \mu\right)}{\left(1-\mu\right)}2\widetilde{y}_{\mathsf{s}}^{R}. \tag{40}$$

In this case the QD curve (blue line) is downward-sloping by $-2(v-1)\kappa_{\widetilde{y}^R}(\phi-\mu)/(1-\mu)<0$, because a fall in relative home output is accompanied by a decline in the relative real interest rate differential $(\phi-\mu)(v-1)\pi^R_{H\mathsf{s}}<0$ and a real depreciation. The intersection point of the upward-sloping QS curve and the downward-sloping QD curve in Figure 5 is the equilibrium in the two economies when the ZLB is not binding. The asymmetric negative demand shock induces relative output and, hence inflation expectations to fall. The decline in relative inflation leads to a decline in relative real interest rates for $\phi-\mu>0$. This condition is satisfied when the Taylor principle holds ($\phi>1$) and the shock dynamics are stationary ($\mu<1$), resulting in a real depreciation.

We compare this scenario to the situation where the ZLB is binding in the home country. Such a situation occurs, for example, when the home country is constrained by the ZLB but the foreign country experiences a positive demand shock. This might reflect the situation between the euro area and the U.S. before the COVID-19 crisis, where the latter has left the ZLB and has adjusted its policy rate in response to rising inflation and output. For a better comparison, we impose identical rates on the two scenarios without a loss of generality: $\overline{r}^R_{\mathsf{s}}=\overline{r}^{R\ ZLB}_{\mathsf{s}}$. Since the QS curve is independent of monetary policy, it remains unchanged. The QD$_{ZLB}$ curve is now upward-sloping

$$\mathrm{QD}_{ZLB}: q_{\mathsf{s}}=(v-1)\,\kappa_{\widetilde{y}^R}\frac{\mu}{(1-\mu)}2\widetilde{y}^R_{\mathsf{s}}+\frac{(v-1)}{(1-\mu)}(\ln(1/\beta)+\phi\pi^*_{F\mathsf{s}}),$$

for $2(v-1)\kappa_{\widetilde{y}^R}\mu/(1-\mu)>0$. The intercept of the QD$_{ZLB}$ curve equals $(v-1)(\ln(1/\beta)+\phi\pi^*_{F,bad})/(1-\mu)>0$.[23] The reason for the upward-sloping QD$_{ZLB}$ curve is that the negative asymmetric demand shock pushes home inflation (expectations) downwards and the home country towards the ZLB. At the ZLB home monetary policy cannot accommodate this fall. The home real interest rate rises relative to abroad, $-(\ln(1/\beta)+[\mu\pi^R_H])>(\phi-\mu)\pi^*_F$, and the relative real interest rate differential increases. Hence, the real exchange rate appreciates when the ZLB is only binding in the home country, shown by the intersection of the QS and QD$_{ZLB}$ curve.[24]

4.2.4 Summary of the results under perfect information

In summary, when the ZLB is binding in at least one country, no expenditure switching is possible via the real exchange rate, since a real appreciation occurs for the country in which the ZLB is binding. This effect is caused by a strong decline in (relative) inflation expectations in the country hit most severely by the asymmetric negative demand shock. The model also predicts that output, inflation (expectations) and the real exchange rate are substantially more volatile at than outside of the ZLB. Those findings are add odds with the empirical evidence, established in Section 2. Next, we outline how these results compare to the case when inflation expectations are modelled in a more data-consistent way and adjust sluggishly.

[23] Appendix A.3.3 expresses $\pi^*_{F\mathsf{s}}$ as function of the natural real interest rates and, hence, demand shocks of the model. Therefore, the intercept accounts for foreign inflation.

[24] Since $\sigma(v-1)/\delta>(v-1)\kappa_{\widetilde{y}^R}\mu/(1-\mu)$, the intersection of the QS curve and the QD$_{ZLB}$ curve is the equilibrium outcome.

4.3 Imperfect information and sticky prices

With imperfect information at their disposal, households form beliefs based only on their local conditions, and a fraction $\varpi \,\epsilon\, [0,1]$ of randomly selected households move towards perfect information. As in equation (9) consumption of the informed households (with subscript hi) in period t equals

$$\widetilde{c}^{hi}_{\mathsf{s},t} = \frac{1}{\sigma}\overline{r}^{h}_{t} - \frac{1}{\sigma} r_{\mathsf{s},t} + \frac{\mu}{\sigma}\pi_{\mathsf{s},t+1} + \mu \widetilde{c}^{hi}_{\mathsf{s},t+1}, \tag{41}$$

and similarly for the low-informed type households (with subscript li). For the high-uninformed households (with subscript hu) equation (11) becomes

$$\begin{aligned} \widetilde{c}^{hu}_{t} = & \frac{1}{\sigma}\overline{r}^{h}_{t} - \frac{\mathsf{p}^{good}_{h}}{\sigma}\left(r_{good,t} - \mu\pi_{good,t+1}\right) - \frac{\mathsf{p}^{bad}_{h}}{\sigma}\left(r_{bad,t} - \mu\pi_{bad,t+1}\right) + \\ & \left(1-\varpi\right)\mu\widetilde{c}^{hu}_{t+1} + \varpi(\mathsf{p}^{good}_{h}\mu\widetilde{c}^{hi}_{good,t+1} + \mathsf{p}^{bad}_{h}\mu\widetilde{c}^{hi}_{bad,t+1}), \end{aligned} \tag{42}$$

with p^{good}_{h} (p^{good}_{l}) denoting the conditional probability that a high (low) type assigns to the good state:

$$\mathsf{p}^{good}_{h} = \frac{\lambda_{good}\theta}{\lambda_{good}\theta + \lambda_{bad}\left(1-\theta\right)} = 1 - \mathsf{p}^{bad}_{h} \text{ and } \mathsf{p}^{good}_{l} = \frac{\left(1-\lambda_{good}\right)\theta}{\left(1-\lambda_{good}\right)\theta + \left(1-\lambda_{bad}\right)\left(1-\theta\right)} = 1 - \mathsf{p}^{bad}_{l}\ .$$

The probabilities reflect two possible aggregate states in period zero that differ in terms of mass of households experiencing the high and low realization of the time preference shock, with $\lambda_{bad} < \lambda_{good} < 1$. A similar equation holds for low-uninformed households. Here, aggregate consumption (12) evolves by

$$\widetilde{c}_{good,t} = (1-(1-\varpi)^{t+1})(\lambda^{hi}_{good}\widetilde{c}^{hi}_{good,t} + \left(1-\lambda_{good}\right)\widetilde{c}^{li}_{good,t}) + \left(1-\varpi\right)^{t+1}\left(\lambda_{good}\widetilde{c}^{hu}_{t} + \left(1-\lambda_{good}\right)\widetilde{c}^{lu}_{t}\right), \text{ and}$$

$$\widetilde{c}_{bad,t} = (1-(1-\varpi)^{t+1})(\lambda^{hi}_{bad}\widetilde{c}^{hi}_{bad,t} + \left(1-\lambda_{bad}\right)\widetilde{c}^{li}_{bad,t}) + \left(1-\varpi\right)^{t+1}\left(\lambda_{bad}\widetilde{c}^{hu}_{t} + \left(1-\lambda_{bad}\right)\widetilde{c}^{lu}_{t}\right), \text{ respectively.}$$

Output in state $\mathsf{s}\,\epsilon\{good, bad\}$ is obtained from equation (13) whilst expected CPI inflation follows from (18). Similar equations apply to the foreign country.

To see the implications of imperfect information we set $\varpi = 0$: households form beliefs based exclusively on their local conditions. This assumption allows us to derive an analytic solution. We will show below that the analytical results will also hold true when $\varpi > 0$. For $\varpi = 0$ households remain uninformed over the next period. The average expectation about the real interest rate, $\int_0^1 E^i\left[r_{\mathsf{s}} - \mu\pi_{\mathsf{s}}\right]di = \overline{E}_{\mathsf{s}}\left[r_{\mathsf{s}} - \mu\pi_{\mathsf{s}}\right]$, then equals

$$\begin{aligned} \overline{E}_{\mathsf{s}}\left[r_{\mathsf{s}} - \mu\pi_{\mathsf{s}}\right] = & \overline{\mathsf{p}}^{good}_{\mathsf{s}}(r_{good} - \mu\pi_{H,good} + (2-v)(r^{R}_{good} - \mu\pi^{R}_{H,good})) \\ & +(1-\overline{\mathsf{p}}^{good}_{\mathsf{s}})(r_{bad} - \mu\pi_{H,bad} + (2-v)(r^{R}_{bad} - \mu\pi^{R}_{H,bad})). \end{aligned} \tag{43}$$

The difference between this case and a full information scenario is that consumption depends on the average expectation about the real interest rate in both states, $\mathsf{s}\,\epsilon\{good, bad\}$. The average probability assigned to the good (bad) state when the economy is in the bad (good) state equals

$$\overline{\mathsf{p}}^{good}_{bad} = \lambda_{bad}\mathsf{p}^{good}_{h} + \left(1-\lambda_{bad}\right)\mathsf{p}^{good}_{l} = 1 - \overline{\mathsf{p}}^{bad}_{bad} \text{ and } \overline{\mathsf{p}}^{bad}_{good} = \lambda_{good}\mathsf{p}^{bad}_{h} + \left(1-\lambda_{good}\right)\mathsf{p}^{bad}_{l} = 1 - \overline{\mathsf{p}}^{good}_{good}. \tag{44}$$

A similar condition holds for the foreign economy. This translates also to the real exchange rate, which mirrors the real interest rate differential between the two countries. From the real UIP condition (17) as well as equation (43) and its foreign counterpart, the real exchange rate under imperfect information evolves by

$$\frac{q_{\mathsf{s}}}{2} = -\left(v-1\right)\frac{\overline{E}_{\mathsf{s}}[r^R_{\mathsf{s}} - \mu\pi^R_{H,\mathsf{s}}]}{1-\mu} = -\left(v-1\right)\frac{\overline{\mathsf{p}}^{good}_{\mathsf{s}}\left(r^R_{good} - \mu\pi^R_{H,good}\right) + (1-\overline{\mathsf{p}}^{good}_{\mathsf{s}})\left(r^R_{bad} - \mu\pi^R_{H,bad}\right)}{1-\mu}. \tag{45}$$

The state dependence of equations (43) and (45) is an important difference to the full information case. As inflation expectations matter most when the ZLB is binding, we continue to compare the results to those of a full information scenario when the ZLB is either binding in both countries or only in the home country. Intuitively, a household in a bad state will assign a positive probability of being in the good state and vice versa. A useful property of the imperfect information solution is that we can separate the solution into a full information part and a new term that captures the impact of imperfect information. This is shown next.

4.3.1 ZLB is binding in both countries

We first compare the outcome of imperfect information conditions to that of a full information scenario when the ZLB is binding in all states in the home and foreign country, so that $r_{\mathsf{s}} = r^*_{\mathsf{s}} = -\ln(1/\beta)$ holds. We assume throughout that the same shock conditions apply, as outlined in Section 4.2. Then it follows from equation (45) that the real exchange rate, as a counterpart to (35) under full information, is now equal to

$$\frac{q_{bad}}{2} = \left(v-1\right)\frac{[\mu\pi^R_{H,bad}] + \overline{\mathsf{p}}^{good}_{bad}[\mu\pi^R_{H,good} - \mu\pi^R_{H,bad}]}{1-\mu}.$$

When information is incomplete the state of the economy is not fully revealed. In the bad state, households attach a positive probability $\overline{\mathsf{p}}^{good}_{bad} > 0$ to the good state in which the relative demand shock is less severe, so that expected relative world inflation is higher in comparison to the bad state:

$$[\mu\pi^R_{Hgood} - \mu\pi^R_{Hbad}] = \frac{\mu\kappa_{\widetilde{y}^R}\frac{\delta}{v-1}(\overline{r}^R_{good} - \overline{r}^R_{bad})}{\left(v-1\right)\left(1-\mu\right)\sigma - (1-\overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad})\delta\mu(\kappa_{\widetilde{y}^R} - \kappa_{\widetilde{e}})} > 0, \tag{46}$$

as shown in appendix A.4.3. Going on to fully solve the equation for the real exchange rate in the bad state, we have that

$$\frac{q_{bad}}{2} = \frac{(v-1)}{1-\mu}\left(\frac{\kappa_{\widetilde{y}^R}}{\gamma^{zlb}_{\widetilde{y}^R}}\frac{\mu\delta}{v-1}\overline{r}^R_{bad} + \overline{\mathsf{p}}^{good}_{bad}\delta(\kappa_{\widetilde{y}^R} - \kappa_{\widetilde{e}})\frac{[\mu\pi^R_{Hgood} - \mu\pi^R_{Hbad}]}{\gamma^{zlb}_{\widetilde{y}^R}}\right), \tag{47}$$

with $\kappa_{\widetilde{y}^R} \geq \kappa_{\widetilde{e}} \equiv (2-\nu)\,\sigma(1-\mu)/(\mu\delta)$. The first term of the right-hand side of equation (47) is equal to the full information solution in equation (35). Under imperfect information a second term enters that is strictly positive, because households attach a positive probability to the good state in which the relative demand shock is less severe and inflation expectations are perceived to fall by less. Therefore, the real exchange rate depreciates relative to full information by the amount

$$q_{bad}|_{\varpi=0} - q_{bad}|_{\varpi=1} = \overline{\mathsf{p}}^{good}_{bad}\delta(\kappa_{\widetilde{y}^R} - \kappa_{\widetilde{e}})\frac{[\mu\pi^R_{Hgood} - \mu\pi^R_{Hbad}]}{\gamma^{zlb}_{\widetilde{y}^R}} > 0. \tag{48}$$

Whether the absolute response of the real exchange rate becomes positive depends on the perceived inflation differential to be large enough. For example, if in the bad state households attach a large probability of being in the good state, the real exchange rate response is at least substantially muted compared to full information. In this way the real exchange rate absorbs relatively more of the adverse shock. This affects home output positively in comparison to the situation under full information. Using equations (**??**) and (48), appendix A.4.3 shows that the difference between output under full and imperfect information can be written as

$$\widetilde{y}_{bad}|_{\varpi=0} - \widetilde{y}_{bad}|_{\varpi=1} = \overline{\mathsf{p}}_{bad}^{good} \left(\frac{[\mu\pi_{Hgood}^{W} - \mu\pi_{Hbad}^{W}]}{\gamma_{\widetilde{y}}^{zlb}} + \frac{q_{good}|_{\varpi=0} - q_{good}|_{\varpi=1}}{\kappa_{\widetilde{y}^R}} \right) > 0. \tag{49}$$

The first term reflects the difference in the expected world inflation rates between the good and bad state, which is positive by $[\mu\pi_{Hgood}^{W} - \mu\pi_{Hbad}^{W}] = \mu\kappa_{\widetilde{y}}(\overline{r}_{good}^{W} - \overline{r}_{bad}^{W})/((1-\mu)\sigma - (1 - \overline{\mathsf{p}}_{good}^{bad} - \overline{\mathsf{p}}_{bad}^{good})\mu\kappa_{\widetilde{y}}) > 0$, as shown in appendix A.4.3. Since households perceive the good state to be less severe, not only relative expected inflation (see equation (46)) but also world inflation will be higher in the good state of the world. The second term reflects the real depreciation, discussed in equation (48). Because both terms are positive, output under imperfect information will be higher than under perfect information.

A similar economic intuition with the opposite effect applies when considering the good state: In the good state households assign a positive probability $\overline{\mathsf{p}}_{good}^{bad} > 0$ to the bad state, which is the state where households believe that inflation (expectations) decline by more than they would do under full information. This effect then causes an increase in (average expectations about) the real interest rate, so that the real exchange rate appreciates by more than under full information:

$$q_{good}|_{\varpi=0} - q_{good}|_{\varpi=1} = -\overline{\mathsf{p}}_{good}^{bad}\delta(\kappa_{\widetilde{y}^R} - \kappa_{\widetilde{e}})\frac{[\mu\pi_{Hgood}^{R} - \mu\pi_{Hbad}^{R}]}{\gamma_{\widetilde{y}^R}^{zlb}} < 0. \tag{50}$$

Equation (50) shows that the relative difference in the real exchange rate under imperfect information compared to full information is negative. Consequently, the relative appreciation is accompanied by a relative drop in home output under imperfect in comparison to full information by

$$\widetilde{y}_{good}|_{\varpi=0} - \widetilde{y}_{good}|_{\varpi=1} = -\overline{\mathsf{p}}_{good}^{bad} \left(\frac{[\mu\pi_{Hgood}^{W} - \mu\pi_{Hbad}^{W}]}{\gamma_{\widetilde{y}}^{zlb}} + \frac{q_{good}|_{\varpi=0} - q_{good}|_{\varpi=1}}{\kappa_{\widetilde{y}^R}} \right) < 0. \tag{51}$$

Note that when households expect that a severe recession in the bad state is relatively rare and, hence, the prior probability of such an event $1-\theta$ is very small. Consequently, the probability $\overline{\mathsf{p}}_{good}^{bad}$ is also small, reflecting that the differences between imperfect and full information within and across countries are not that distinct in the good state of the world.

4.3.2 ZLB is binding only in the home country: A Keynesian cross analysis

In this section we return to the situation where only the home country is constrained by the ZLB in all states, while the foreign economy can still use monetary policy to counteract the consequences of asymmetric demand shocks. Then $r_{\mathsf{s}} = -\ln(1/\beta)$ and $r_{\mathsf{s}}^{*} = \phi\pi_{F\mathsf{s}}^{*}$ holds, which is the mirror image of Section 4.2.3. To keep the exposition concise, we will focus on the key equations and an analysis based on the Keynesian cross. A complete derivation of all equilibrium conditions are in appendix A.4.4.

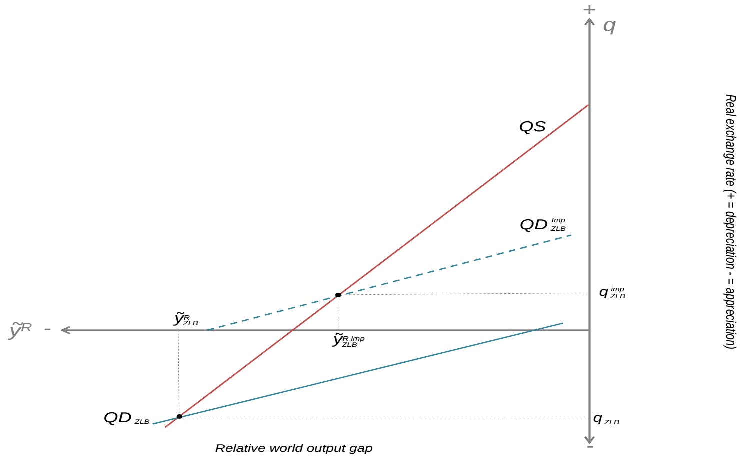

Figure 6: The real exchange rate equilibrium: full vs. imperfect information when the ZLB is binding in the home country

Notes: The prior probability of households in the good state is given by $\theta = 0.9$. This implies that the prior probability of being in the bad state is relatively rare and equates to 10%. The fraction of households being of high type in the good (bad) state equals $\lambda_{good} = 0.75$ ($\lambda_{bad} = 0.25$). All other parameters are as in Figure 5.

Figure 6 shows the bad state equilibrium of the real exchange rate under full and imperfect information at the ZLB. The QS curve reflects the goods market equilibrium at each point in time and is independent of monetary policy and inflation expectations. Therefore we continue to utilize equation (39) in the bad state. Since the QD curve subsumes the real UIP condition with the conditional monetary policies, inflation expectations directly impact on the international money market equilibrium. Following equations (38) and (45), the real interest rate differential in the bad state of the world under imperfect information evolves by

$$\frac{q_{bad}}{2} = \frac{v-1}{1-\mu}\left(\left(\frac{\ln(1/\beta)+\phi\pi^*_{F,bad}}{2}+[\mu\pi^R_{H,bad}]\right)+\overline{\mathsf{p}}^{good}_{bad}\frac{[\mu\pi_{H,good}-\mu\pi_{H,bad}]+(\phi-\mu)\left(\pi^*_{F,good}-\pi^*_{F,bad}\right)}{2}\right).$$

The first term of the right-hand side of this equation is the equivalent to equation (38) under full information. Since we continue to have that $[\mu\pi^R_{H,bad}] < (\ln(1/\beta)+\phi\pi^*_{F,bad})$, the first term causes the real exchange rate to appreciate. However, the second term reflects the perceived real interest rate differential under imperfect information in the bad state of the world. Households assign a positive probability of being in the good state where inflation is higher, so that the real interest rate differential would fall. Consequently, the second term is positive and will cause a real depreciation. In Figure 6 the equilibrium forces are assessed by combining the QS and QD curve. The latter is obtained by utilizing the above expression together with equation (29):

$$\begin{aligned}\mathrm{QD}^{Imp}_{ZLB} \quad : \quad q_{bad} &= (v-1)\,\kappa_{\widetilde{y}^R}\frac{\mu}{(1-\mu)}2\widetilde{y}^R_{bad}+\frac{(v-1)}{(1-\mu)}\left(\ln\left(1/\beta\right)+\phi\pi^*_{Fbad}\right) \qquad (52)\\ &+\frac{(v-1)}{(1-\mu)}\overline{\mathsf{p}}^{good}_{bad}\left([\mu\pi_{H,good}-\mu\pi_{H,bad}]+(\phi-\mu)\left(\pi^*_{F,good}-\pi^*_{F,bad}\right)\right).\end{aligned}$$

As in the case under full information, the imperfect information QD_{ZLB}^{Imp} curve in Figure 6 is upward-sloping by $2(v-1)\kappa_{\widetilde{y}^R}\mu/(1-\mu) > 0$. The last two terms on the right-hand side of equation (52) reflect the impact of imperfect information on the QD_{ZLB}^{Imp} curve's intercept. Households perceive the bad state to be less severe, so that inflation expectations do not deteriorate that strongly. This weighs on overall world inflation expectations, $[\mu\pi_{H,good}^W - \mu\pi_{H,bad}^W] > 0$ and the last two terms in equation (52) shift the intercept of the QD_{ZLB}^{Imp} upwards in comparison to the QD_{ZLB} curve under full information by the amount $(v-1)\bar{\mathsf{p}}_{bad}^{good}([\mu\pi_{H,good} - \mu\pi_{H,bad}] + (\phi-\mu)(\pi_{F,good}^* - \pi_{F,bad}^*))/(1-\mu) > 0$.[25] This is shown by the dashed line in Figure 6. Thus, the real exchange rate depreciates rather than appreciates, as it was the case under full information. This is shown graphically by the intersection of the QD_{ZLB}^{Imp} and QS curve in Figure 6.

When information is incomplete, households abroad attach a positive probability to the good state in which inflation would rise by a greater amount, $[\mu\pi_{Fgood}^* - \mu\pi_{Fbad}^*] > 0$, as shown in appendix A.4.4. Consequently, also actual inflation will rise abroad and the foreign central bank responds by increasing the policy rate, so that the real foreign interest rate rises, $(\phi-\mu)(\pi_{Fgood}^* - \mu\pi_{Fbad}^*) > 0$. Households in the home country perceive the demand shock to be less severe in the bad state of the world. Hence, the fall in output and (expected) inflation at home is mitigated in comparison to full information. This and the induced rise in the real interest rate abroad reverses the real interest rate differential and causes a depreciation of the real exchange rate under incomplete information. In contrast to full information, the disinflationary effects on the relative output gap are cushioned by a real depreciation. Since the additional imperfect information terms in the QD_{ZLB}^{Imp}-curve are positive, the real exchange rate generally depreciates compared to full information. The effect can be substantial and can compensate for the strong disinflationary effects under full information.

4.3.3 The role of openness and information frictions

In this section we illustrate the role of trade openness when the ZLB is only binding in the home country. We measure trade openness by the countries' import share, which we have set to 25 percent in our model so far. We examine two important dimensions: (i) trade openness and (ii) information frictions. Figure 7 shows that the difference between full and imperfect information is amplified the more open the countries are. For a realistic range of trade openness (an import share of 15-25 percent), the real exchange rate depreciates (rather than appreciates) under imperfect information. In addition, the top middle panel shows that in response to a negative relative demand shock at home the effect of the real exchange rate on output becomes more important when trade openness increases.[26] The real depreciation under imperfect information mitigates the negative impact response of output and inflation. This mitigation effect is stronger the more open the countries are to trade, which can be seen from the top middle and right panel of Figure 7.

[25] Appendix A.4.4 expresses $\left[\mu\pi_{H,good} - \mu\pi_{H,bad}\right] > 0$ and $(\phi-\mu)(\pi_{F,good}^* - \pi_{F,bad}^*) > 0$ as functions of the natural real interest rates and, hence, demand shocks of the model. Therefore, the intercept incorporates those terms in Figure 6.

[26] This takes into account that the real exchange rate response is transmitted to home output by $(\delta-1)/(\sigma(v-1))$.

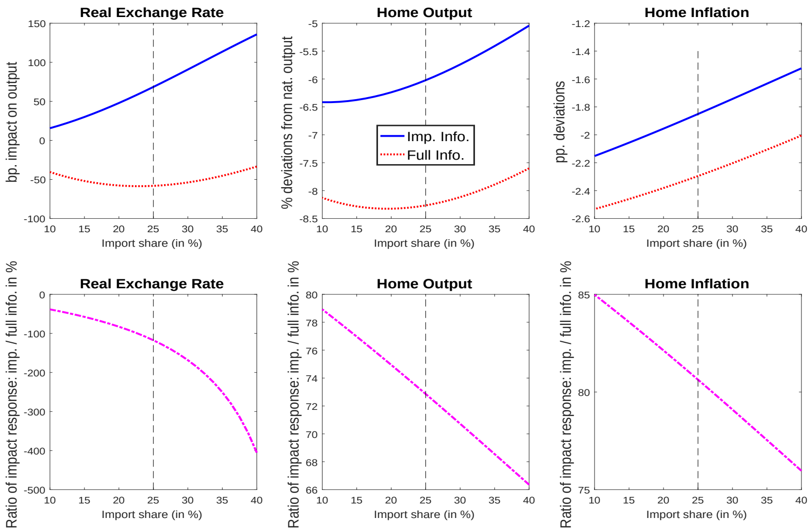

Figure 7: Impact multipliers and the role of openness

Notes: All parameters are as in Figure 6, except for openness, which is measured by the import share in % : $((2-v)/2)*100$. The vertical black dashed line reflects an import share of 25%.

The extent to which the impact responses under imperfect and full information differ is shown in the second row of Figure 7. The relative real exchange rate appreciation under full information amplifies the negative output and inflation response. For an import share of 25 percent, the decline in output (inflation) is increased by around 1/4 (1/5) under full information compared to imperfect information. The reason is that under full information the real appreciation pushes the economy further away from the natural equilibrium. Consequently, output and inflation adjust less efficiently under full information compared to imperfect information. This effect is more distinct the higher the degree of trade openness is.

4.3.4 Dynamic responses

Lastly, we generalize the above findings to a dynamic setting. As before, we consider the case when the ZLB is binding only in the home country. We extend the above analysis in two steps. First, when information is incomplete, we set the fraction of households updating their beliefs about the true state of the economy to $\omega = 0.125$, as estimated by Coibion and Gorodnichenko (2012). Thus, with imperfect information households fully learn about the true aggregate state within four years after the shock. In the case of full information we assume that $\omega = 1$. Second, we calibrate the duration of the ZLB to last for nine years.[27] Therefore, we follow Wiederholt (2019) and assume that the demand shock follows a deterministic decay. Under this assumption, the demand shock is linked between periods $t+1$ and t by $\xi_{t+1} = \rho\xi_t$, where $\rho > 0$ reflects the persistence of the shock. Then, for t periods ahead, the demand shock

[27] This is in light of the current crises within the euro area. Since September 2014, the euro area's policy rate is at the ZLB and given the outbreak of the coronavirus, we assume that it will remain there for another three years.

evolves by $(1-\mu\rho)\,\rho^t\xi_t$. To have the ZLB binding for nine years in the home country, we set $\mu = 0.9$ and $\rho = 0.99$.[28]

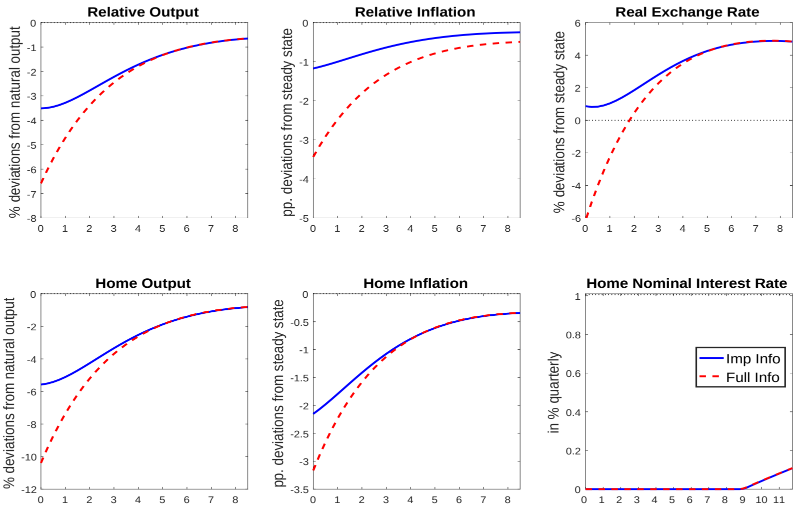

Figure 8: Dynamic responses when the ZLB is binding on home country

Notes: All parameters are as in Figure 6, while in the case of incomplete information we have that $(1-\omega) = 0.875$. For the duration of the ZLB we set $\mu = 0.9$ and $\rho = 0.99$, respectively.

Figure 8 confirms that our results also hold within the dynamic setting. The blue (red) solid (dashed) lines show the responses under in(complete) information. Under incomplete information the real exchange rate depreciates. This causes a mitigated response in home output and inflation. On impact, the negative output (inflation) response under imperfect information is smaller by 2/5 (1/3) in comparison to full information. Furthermore, Figure 8 shows that during the first periods (shown here in years) home output, inflation as well as the real exchange rate are less volatile at the ZLB when information is incomplete.

4.3.5 Summary of the results under imperfect information

Section 4.3 has documented a (relative) real depreciation when the ZLB is binding. This together with a less pronounced decline in inflation expectations strongly mitigates the decline in output under imperfect information. Therefore, with more realistic expectation formation in a ZLB environment, the shock absorbing capacity of the real exchange rate could be much more efficient than previously thought (under full information, e.g. Cook and Devereux (2013)) and home and foreign output move closer to their natural counterparts. Our results also suggest that macroeconomic volatility is not as large as under perfect information at the ZLB (see also Debortoli, Gali, and Gambetti, 2020) and confirms our empirical findings from Section 2.

[28] Up to the fourth year after the shock the ZLB is binding in all states, and from the fifth year onwards only in the bad state.

5 Optimal monetary policy

In Section 4 we applied a Taylor rule to the foreign country when the ZLB was not binding. We now assess how the foreign interest rate response changes under full and imperfect information when the foreign economy sets its policy rate optimally. In all states the home country is at the ZLB, but now the central banks are cooperating to maximize welfare.[29] The optimal monetary policy is set under discretion.[30]

To derive a second-order approximation to global welfare we continue to assume that ϖ is either zero (incomplete information) or unity (full information). We make use of the fact that $\widetilde{x}_{\mathsf{s}} = \widetilde{x}_{\mathsf{s}}^W + \widetilde{x}_{\mathsf{s}}^R$ and $\widetilde{x}_{\mathsf{s}}^* = \widetilde{x}_{\mathsf{s}}^W - \widetilde{x}_{\mathsf{s}}^R$ to express global welfare in a compact way in terms of relative and average world variables:

$$\widetilde{\mathcal{W}}_t \equiv -\textstyle\sum_{\mathsf{s}}(\lambda_\pi \frac{1}{2}(\pi_{H\mathsf{s},t}^W + \pi_{H\mathsf{s},t}^R)^2 + \lambda_\pi \frac{1}{2}\left(\pi_{H\mathsf{s},t}^W - \pi_{H\mathsf{s},t}^R\right)^2 + \lambda_{\widetilde{y}^W}(\widetilde{y}_{\mathsf{s},t}^W)^2 + \lambda_{\widetilde{y}^R}(\widetilde{y}_{\mathsf{s},t}^R)^2), \tag{53}$$

with $\lambda_\pi > \lambda_{\widetilde{y}^W} \geq \lambda_{\widetilde{y}^R}$, shown in appendix A.5. Under discretion the optimal monetary policy requires maximizing the global welfare function (53) subject to world relative and aggregate demand and supply conditions, equations (14) and (15), in deviations from the natural equilibrium in the good and bad state, taking as given the future values of those variables. The optimal cooperative policy problem is given by maximizing the Lagrangian $\mathcal{L}_t$ (see appendix A.5) with respect to $\widetilde{y}_{\mathsf{s},t}^R, \widetilde{y}_{\mathsf{s},t}^W, \pi_{H\mathsf{s},t}^R, \pi_{H\mathsf{s},t}^W, r_{\mathsf{s},t}^R$ and $r_{\mathsf{s},t}^W$ in the good and bad state. Equations (73)-(80) in appendix A.5 show the optimal policy solutions for these variables, which allow us to derive the foreign interest rate solution in the good and bad state:

$$r_{good}^* = \overline{r}_{good}^* - \gamma_{\overline{r}}\overline{r}_{good} + \overline{\mathsf{p}}_{good}^{bad}\gamma_{\overline{r}_{g-b}}\left(\overline{r}_{good} - \overline{r}_{bad}\right) \text{ and } r_{bad}^* = \overline{r}_{bad}^* - \gamma_{\overline{r}}\overline{r}_{bad} - \overline{\mathsf{p}}_{bad}^{good}\gamma_{\overline{r}_{g-b}}\left(\overline{r}_{good} - \overline{r}_{bad}\right).$$

The full solution to $\gamma_{\overline{r}}$ and $\gamma_{\overline{r}_{g-b}}$ as functions of the parameters of the model is provided in appendix A.5.

Let us consider first the case when information is complete ($\varpi = 1$) and the world economy is in the bad state. Households then immediately learn about the size of the shock in each state of the world so that $\overline{\mathsf{p}}_{bad}^{good} = 0$. Then, the above optimal interest rate response becomes

$$r_{bad}^*|_{\varpi=1} = \overline{r}_{bad}^* - \gamma_{\overline{r}}\overline{r}_{bad} = 2\overline{r}_{bad}^W - (1 + \gamma_{\overline{r}})\,\overline{r}_{bad}. \tag{54}$$

Focusing on the first equality of equation (54), when the economies would not trade, i.e. $\upsilon = 2$, it follows that $\gamma_{\overline{r}} = 0$ and the foreign policy rate would be set to equate to the foreign natural rate of interest, $\overline{r}_{\mathsf{s}}^*$. However, when the economies are open to trade and for an home bias in consumption, i.e. $1 < \upsilon < 2$, the assumption we have made throughout the paper, the coefficient on the home natural interest rate becomes $\gamma_{\overline{r}} > 0$ and the foreign interest rate increases when $\overline{r}_{\mathsf{s}} < 0$. For the calibration used before, the increase in the foreign policy rate would then ensure a depreciation of the real exchange rate in comparison to the outcome under full information in Section 4.2.3. In terms of welfare, this effect then mitigates the

[29]Clarida et al. (2002) as well as Beningo and Beningno (2006) focus on a non-cooperative monetary policy equilibrium. However, at the ZLB a Nash equilibrium becomes very complex due to the restricted strategy space of one of the two countries and would preclude us from deriving simple analytical solutions to the model.

[30]We focus on policy commitment in terms of forward guidance in the next section.

relative output gap, $\widetilde{y}_{\mathsf{s}}^{R}$. The rise in r_{s}^{*} will lead to a smaller foreign output gap, $\widetilde{y}_{\mathsf{s}}^{*}$. However, the overall reduction in $\widetilde{y}_{\mathsf{s}}^{W}$ might remain small. Turning to the second equality of equation (54), the foreign country chooses to set the optimal policy rate above zero, even if the world natural rate is negative (see also Cook and Devereux, 2013).

In the following we show that the introduction of imperfect information can reverse the results of optimal monetary policy under perfect information. In the case of imperfect information ($\varpi = 0$) inflation expectations are sluggish and their fall is mitigated, compared to full information. Consequently, the real interest rate differential rises by less. To generate a real depreciation, it is then optimal for the foreign policymaker to reduce the foreign interest rate in comparison to full information by

$$r_{bad}^{*}|_{\varpi=0} - r_{bad}^{*}|_{\varpi=1} = \overline{\mathsf{p}}_{bad}^{good} \gamma_{\overline{r}_{g-b}} \left(\overline{r}_{good} - \overline{r}_{bad}\right) < 0, \tag{55}$$

with $\overline{\mathsf{p}}_{bad}^{good} > 0$ and $\gamma_{\overline{r}_{g-b}} > 0$ for $1 < \upsilon < 2$. This response ensures that the foreign policy rate moves closer to the world's natural rate and, from the perspective of the global economy, reduces the deviations from the world's natural output for a given real depreciation. The real depreciation then also alleviates the negative effects the ZLB has on the home economy.

6 Forward guidance

In the previous section we have concentrated on the economic outcome when monetary policy acts optimally but does not commit to future actions. We now focus on the attempt by the central bank to manage expectations. The literature on the ZLB (e.g. Eggertsson and Woodford, 2003) highlights the benefits of forward guidance. We focus on the effects of forward guidance in an environment where only the home country is constrained by the ZLB. The foreign country pursues a positive interest rate policy, following the Taylor rule. Below we outline two possibilities for the home country to commit to monetary policy.

6.1 Central bank communication about the current state

The home country's central bank will communicate the *current state* of the economy. Since the current state is not perfectly revealed to all households, the central bank's communication matters. Let us assume that the home country's central bank statement reaches a fraction $\mathcal{C} \epsilon [0, 1]$ of randomly selected households. Note that the assumption about financial markets ensures that in equilibrium all households have the same post-transfer wealth at time T. Since a fraction of $\mathcal{C}$ households is reached by the central bank and correct their beliefs about the state of the economy appropriately, only a fraction of $(1 - \mathcal{C})$ remains uninformed. Then, the probabilities in equation (44) have to be multiplied by $(1 - \mathcal{C})$:

$$\begin{aligned} (\overline{\mathsf{p}}_{bad}^{good})^{com} &= (1 - \mathcal{C})\, \overline{\mathsf{p}}_{bad}^{good} = (1 - \mathcal{C}) \left(\lambda_{bad} \mathsf{p}_h^{good} + (1 - \lambda_{bad})\, \mathsf{p}_h^{good}\right) \\ (\overline{\mathsf{p}}_{good}^{bad})^{com} &= (1 - \mathcal{C})\, \overline{\mathsf{p}}_{good}^{bad} = (1 - \mathcal{C}) \left(\lambda_{good} \mathsf{p}_h^{bad} + (1 - \lambda_{good})\, \mathsf{p}_h^{bad}\right) \end{aligned}$$

It then follows from our discussion in Sections 4.3.1 and 4.3.2 that by fully revealing the actual *bad* state of the economy to all households, i.e. $\mathcal{C}$ =1, the home country would end up in the full information environment, with stronger decline in inflation and output compared to the outcome under imperfect information. Central bank communication can then even exacerbate a macroeconomic downturn (see also Wiederholt, 2019).

6.2 Central bank communication about the future state

We now assess the effects when the home country's central bank announces in the *bad* state a path of the *future state*. In particular, it will set the nominal interest rate in periods $t \geq T$ so as to achieve an inflation target of $\overline{\pi}_H > 0$, to raise inflation expectations in periods $t < T$. We assess the following scenario: At time $t = 1$ there is a relative negative demand shock to the home country which drives the world's natural real interest rate below zero, as described in Sections 4.2.3 and 4.3.2. For illustrative purposes, we focus on a special case where the central bank has perfect foresight and knows that the preference shock will last for T periods. The home country's central bank reaches a fraction $\mathcal{C} >0 \epsilon [0, 1]$ of randomly selected households in every period $0 \leq t$. We assume that the shock persists for $T = 4$ periods, while the home country's central bank will set the $\overline{\pi}_H > 0$, by 40 basis points annually for $t = 8$ periods.

Figure 9 shows that when the home country's policymaker commits to the future path of interest rates, it follows that under full information a real exchange rate depreciation (dashed line) can be generated by announcing higher inflation today. This also impacts positively on output and inflation in comparison to the situation described in Section 4.2.3.

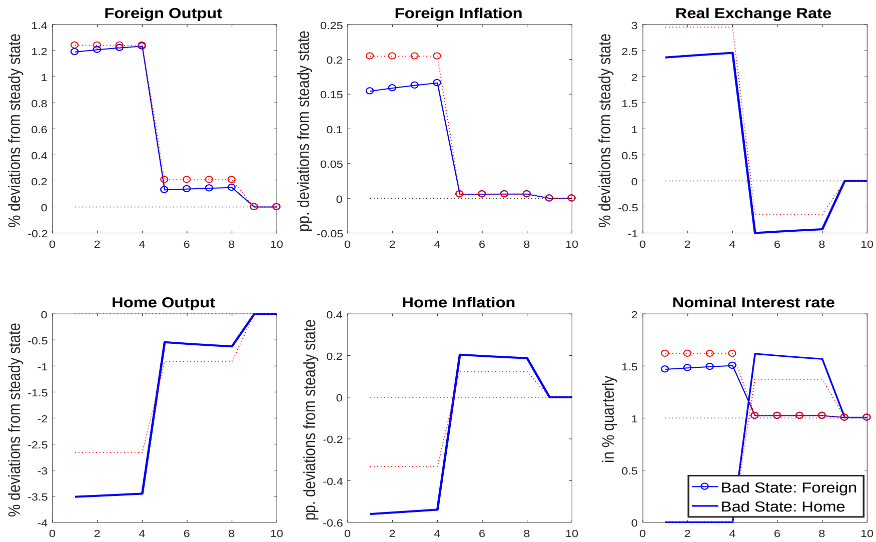

Figure 9: Forward guidance by the home country with a binding ZLB of four periods

Notes: All parameters are as in Figure 6.

When information is imperfect, solid lines in Figure 9, the picture looks different. Since not all households are reached by the central bank, the fraction of uninformed households still perceives itself as being in a state, where the central bank is not committing to a future path of its policy rate. Consequently, in comparison to the full information equilibrium the home country is now worse off, with output and inflation being lower and with a relative real exchange rate appreciation.

The central bank has committed itself to continue keeping inflation higher even after the shock reverts back to zero. However, not all households have been reached by the central bank under imperfect information. Consequently, they perceive higher inflation to be a positive relative demand shock, even though the (relative) demand shock has reverted back to zero in period $T = 4$. As a consequence of their perception, households under imperfect information will adjust home output and inflation upward more strongly in comparison to full information. This is shown by comparing the solid and dashed lines in Figure 9 at period $T = 4$. Thus, with imperfect information output gains are generated in the medium term, when the home central bank communicates policy about the future state of the economy.

6.3 Revisiting the forward guidance exchange rate puzzle

In this exercise we relate our model framework to the findings in Gali (2021) on the effectiveness of forward guidance on the real exchange rate. His work shows that within a small open economy model a forward guidance exchange rate puzzle occurs. This puzzle refers to the fact that the effect of a given anticipated change in the policy rate on the current exchange rate is larger the longer the implementation horizon is.[31]

We revisit this puzzle in a two-country setting and examine whether the puzzle is mitigated with survey-consistent inflation expectations. Two key properties of the two-country model are: no discounting in the real UIP condition and no discounting in the Euler equation. As a result, the same type of forward guidance exchange rate puzzle that is present in the small open economy model carries over to the two-country model.

To illustrate the puzzle in the most transparent way possible, we conduct the same policy experiment in the two-country model that is studied in Gali (2021). The experiment is a one-period increase in the nominal interest rate in the bad state of the home country. In period 1 the policymaker announces the policy which is implemented in periods $T > 1$. We assume that the monetary authority commits to keeping the interest rate at its steady state level from period 1 to T independently of inflation dynamics.

We first illustrate the exchange rate response at the time of the announcement for different implementation horizons under full information. The left panel in Figure 10 shows that the exchange rate appreciates more strongly the longer the implementation horizon T is. Therefore, the two-country model also inherits the same property that prevails in the small open economy.

Next we introduce imperfect information on the household side in a dynamic version of the model that features more realistic formation of inflation expectations. We assess how imperfect information mitigates the current exchange rate response in the right panel of Figure 10.[32] As before, under full information ($1 - \omega = 0$) the impact response of the exchange rate is around -8.5 percent. Under complete

[31] A second feature of the puzzle is that the effect on output of a given anticipated change in the real interest rate is invariant to the horizon of implementation of that change.

[32] In keeping with Wiederholt (2019) we set the autocorrelation of the demand shock $\rho = 0.99$ and $\mu = 0.95$.

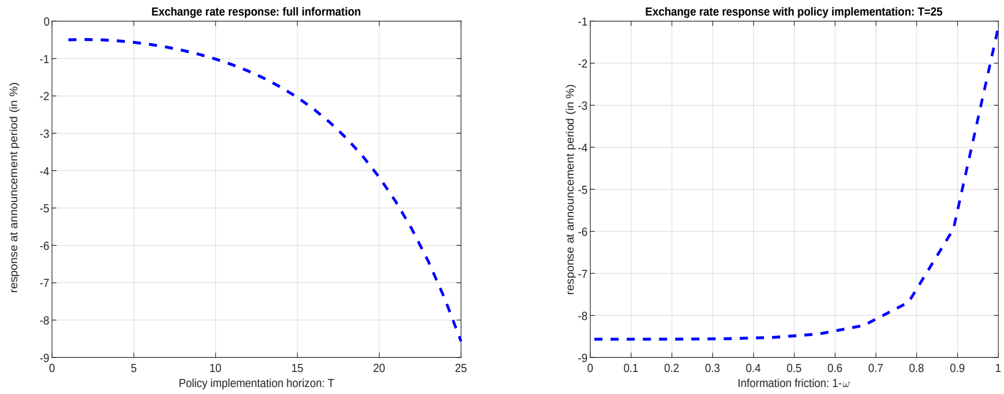

Figure 10: Forward guidance exchange rate puzzle

Notes: All parameters are as in Figure 6, unless otherwise stated in the text of Section 6.3.

imperfect information the effect is only -1 percent. For empirically plausible values of the information friction, i.e. $1 - \omega = 0.875$, the effect is diminished by about 30 percent.

Quantitatively the effect is comparable to alternative ways to mitigate the puzzle. Interestingly, Gali (2021) introduces a behavioral model based on Gabaix (2019) that deviates from rational expectations. He shows that the effect is also diminished by around 30 to 60 percent. For a plausible policy horizon the quantitative results are confirmed by imperfect information.

7 Conclusion

This paper embeds survey-consistent belief formation of inflation expectations through imperfect information into a two-country New Keynesian model. We show that the international propagation of asymmetric negative demand shocks at the ZLB can be considerably different compared to the full information benchmark. When inflation expectations adjust only sluggishly to shocks, the exchange rate's shock-absorbing mechanism can in fact remain at work under flexible exchange rates. Furthermore, the puzzlingly strong exchange rate dynamics at the ZLB under full information disappear. Hence, modeling expectation formation more closely in line with survey data is important if we are to better understand the international macroeconomic effects of asymmetric demand shocks at the ZLB. Accounting for realistic expectations formation is also crucial when assessing the effects of international central bank cooperation and communication within a liquidity trap.

With flexible exchange rates we find that a negative asymmetric demand shock concentrated in the home country causes a real exchange rate depreciation rather than an appreciation. This real depreciation then supports the macroeconomic stabilization at the ZLB. When the home country is constrained by the ZLB, it is optimal for the foreign policymaker to reduce rather than increase foreign interest rates in response to asymmetric negative demand shocks under imperfect information.

For floating exchange rates to work efficiently at the ZLB, i.e. to generate a real exchange rate depreciation in the full information New Keynesian model, forward guidance is required. However, when information is incomplete, by revealing the true current state the policymaker might initially even exacerbate the negative world output gap. However, with imperfect information gains are generated in the medium term, when the central bank communicates its policy about the future state of the economy.

Extending the analysis to scenarios where the central bank or firms also have imperfect information could be fruitful. We have also left out other potentially relevant factors such as sovereign debt constraints, quantitative easing or bubbles in financial markets. Finally, the framework could be extended to endogenize the information friction and assess feedback effects between central bank communication strategies and the private sector. We leave these extensions for future research.

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A Appendix

A.1 Optimality conditions of households and firms

Households' inter- and intratemporal choices From the objective function, (2), and the budget constraint the following first-order conditions can be derived for agent i: given the portfolio choices and noting in equilibrium that agents must be indifferent between the payoff of state-contingent claims and the return on the nominal government risk-free bond, consumption evolves according to

$$\exp(\xi_{i,t})C_{i,t}^{-\sigma} = \beta R_t E_t^i \left[\exp(\xi_{i,t+1})C_{i,t+1}^{-\sigma}\Pi_{t+1}^{-1}\right], \tag{56}$$

where $\Pi = \Pi_H^{\frac{v}{2}}\Pi_F^{1-\frac{v}{2}}$ denotes the gross home consumer price inflation rate, given (4). The gross producer price inflation rate of the foreign goods sold in the home country equals $\Pi_F = \Delta\mathcal{E}\Pi_F^*$. Thus, the law of one price (LOOP) holds and $\mathcal{E}$ is the nominal exchange rate defined as the domestic currency price of foreign exchange. Then, a depreciation is given by a rise in the exchange rate. The consumption-leisure trade-off equals

$$\frac{W_{i,t}}{P_t} = \frac{\eta}{\eta-1}C_{i,t}^{\sigma}. \tag{57}$$

Similar conditions hold in the foreign economy. The trade of state-contingent claims in international financial markets implies for initially symmetric conditions that

$$\frac{C_{i,t}}{C_{i,t}^*} = Q_t^{\frac{1}{\sigma}}\left(\frac{\exp(\xi_{i,t})}{\exp(\xi_{i,t}^*)}\right)^{\frac{1}{\sigma}}. \tag{58}$$

The real exchange rate Q is defined by $Q = \mathcal{E}P^*/P$. Maximizing the objective function (2) subject to equation (3) and the trade balance condition yields the total demand functions:

$$C_{Hi,t} = \frac{v}{2}\left(\frac{P_{Ht}}{P_t}\right)^{-1}C_{i,t} \text{ and } C_{Fi,t} = \left(1-\frac{v}{2}\right)\left(\frac{P_{Ft}}{P_t}\right)^{-1}C_{i,t}. \tag{59}$$

LOOP also holds for good j, so that $P_{Fj,t} = \mathcal{E}_t P_{Fj,t}^*$ and $P_{Hj,t}^* = P_{Hj,t}/\mathcal{E}_t$. The conditional demands are

$$C_{Hi,jt} = \left(\frac{P_{Hj,t}}{P_{Ht}}\right)^{-\psi}C_{Hi,t} \text{ and } C_{Fi,j,t} = \left(\frac{P_{Fj,t}}{P_{Ft}}\right)^{-\psi}C_{Fi,t}. \tag{60}$$

Firms' cost minimization and prices Firm j's cost minimization implies type i labour demand

$$N_{i,j,t} = (W_{i,t}/W_t)^{-\eta}N_{j,t}, \text{ with } W_t = (\textstyle\int_0^1 W_{i,t}^{1-\eta}di)^{\frac{1}{1-\eta}}. \tag{61}$$

The wage bill of firm j equals $W_tN_{j,t}$. Every period t some firms are allowed to re-optimize their price $\widehat{P}_{Hj,t}$. Firm j sets its price in its own producer currency. When re-optimizing prices, the firm solves

$$\max E_t[\textstyle\sum_{s=0}^{\infty}(\alpha\beta)^s\lambda_{t,t+s}^i(\Pi_H\widehat{P}_{Hj,t} - MC_{j,t+s}^n)Y_{t,t+s}(j)], \tag{62}$$

subject to the firm's resource constraint

$$Y_{j,t} = \left(\frac{P_{Hj,t}}{P_{H,t}}\right)^{-\psi}\left(C_{H,t} + C_{H,t}^*\right) = \left(\frac{P_{Hj,t}}{P_{H,t}}\right)^{-\psi}Y_t, \tag{63}$$

where the stochastic discount factor is $\lambda_{t,t+s}^i = \exp(\xi_{i,s})C_{i,s}^{-\sigma}P_t/(\exp(\xi_{i,t})C_{i,t}^{-\sigma}P_s)$ and firm j's nominal marginal cost equals $MC^n = WY^{\frac{1-\varrho}{\varrho}}/\varrho$. Similar conditions hold in the foreign country as well.

A.2 Natural equilibrium: critical shock values

The home and foreign natural interest rates can be expressed as follows:

$$\overline{r}_{\mathsf{s}} = (1-\mu)\,\xi_{\mathsf{s}} + (1-\mu)\,v(v-2)\frac{\sigma}{\delta}\xi_{\mathsf{s}}^{R} + (1-\mu)\,(v-1)\,\frac{\sigma}{\delta}\frac{\delta-(v-1)}{\frac{(1-\varrho)}{\varrho}\delta+\sigma}\xi_{\mathsf{s}}^{R} < 0. \tag{64}$$

For $v > 1$, a negative relative demand shock causes the natural home rate to decline. Whether the foreign rate declines depends also on the relative shock size of ξ_{s}^{*} and ξ_{s}^{R}:

$$\overline{r}_{\mathsf{s}}^{*} = (1-\mu)\,\xi_{\mathsf{s}}^{*} - (1-\mu)\,v(v-2)\frac{\sigma}{\delta}\xi_{\mathsf{s}}^{R} - (1-\mu)\,(v-1)\,\frac{\sigma}{\delta}\frac{\delta-(v-1)}{\frac{(1-\varrho)}{\varrho}\delta+\sigma}\xi_{\mathsf{s}}^{R}. \tag{65}$$

To obtain the critical values of ξ^{crit} and ξ^{*crit}, we equate $\overline{r}_{\mathsf{s}}$ and $\overline{r}_{\mathsf{s}}^{*}$ in (64) and (65) to $-\ln(1/\beta)$ and solve this for ξ_{s} and ξ_{s}^{*}:

$$\xi^{crit} = \xi^{*crit} = -\frac{\ln(1/\beta)\,\frac{1}{1-\mu}(1+\frac{\sigma}{\delta}(\frac{(v-1)(\delta-(v-1))}{2(\frac{1-\varrho}{\varrho}\delta+\sigma)} - \frac{v(2-v)}{2}))}{(1+\frac{\sigma}{\delta}(\frac{(v-1)(\delta-(v-1))}{2(\frac{1-\varrho}{\varrho}\delta+\sigma)} - \frac{v(2-v)}{2}))^2 - (\frac{\sigma}{\delta}(\frac{(v-1)(\delta-(v-1))}{2(\frac{1-\varrho}{\varrho}\delta+\sigma)} - \frac{v(2-v)}{2}))^2} < 0. \tag{66}$$

When $\xi_{\mathsf{s}} < \xi_{\mathsf{s}}^{*} < \xi_{\mathsf{s}}^{crit}$, it follows that $\overline{r}_{\mathsf{s}} < \overline{r}_{\mathsf{s}}^{*} < -\ln(1/\beta)$ and the ZLB is binding when prices are sticky. Since we assume that $\xi_{\mathsf{s}}^{R} < 0$, an alternative assumption is that the negative demand shock only occurs in the home country, so that $\xi_{\mathsf{s}}^{*} = 0$ equals

$$\xi_{H}^{crit} = -\frac{\ln(1/\beta)\,\frac{1}{1-\mu}}{1+\frac{\sigma}{\delta}\frac{(v-1)(\delta-(v-1))}{2(\frac{1-\varrho}{\varrho}\delta+\sigma)} - \frac{\sigma}{\delta}\frac{v(2-v)}{2}} < 0 \text{ and } \xi_{F}^{crit} = -\frac{\ln(1/\beta)\,\frac{1}{1-\mu}}{\frac{\sigma}{\delta}\frac{v(2-v)}{2} - \frac{\sigma}{\delta}\frac{(v-1)(\delta-(v-1))}{2(\frac{1-\varrho}{\varrho}\delta+\sigma)}} < 0. \tag{67}$$

For values smaller than ξ_{H}^{crit} (ξ_{F}^{crit}) the ZLB is binding in the sticky price economy, since $\overline{r}_{\mathsf{s}} < -\ln(1/\beta)$ ($\overline{r}_{\mathsf{s}}^{*} < -\ln(1/\beta)$) holds.

A.3 Full information and sticky prices: equilibrium

To obtain the variables outlined below, consider equation (13):

$$y_{\mathsf{s}} = c_{\mathsf{s}} + (1-\frac{(v-1)}{\delta})y_{\mathsf{s}}^{R} - \frac{v(2-v)}{\delta}\xi_{\mathsf{s}}^{R}.$$

and the Euler equation (26):

$$\widetilde{c}_{\mathsf{s}} = -\frac{\frac{1}{\sigma}}{1-\mu}\left[r_{\mathsf{s}} - \mu\pi_{\mathsf{s}} - \overline{r}_{\mathsf{s}}\right] \text{ with } [\mu\pi_{\mathsf{s}}] = [\mu\pi_{H,\mathsf{s}}] + (2-\nu)\,(r_{\mathsf{s}}^{R} - [\mu\pi_{H,\mathsf{s}}^{R}]).$$

Consequently, output can be written as

$$\widetilde{y}_{\mathsf{s}} = -\frac{\frac{1}{\sigma}}{1-\mu}\left[r_{\mathsf{s}} - \mu\pi_{H,\mathsf{s}} + (2-\nu)\left(\mu\pi_{H,\mathsf{s}}^{R} - r_{\mathsf{s}}^{R}\right) - \overline{r}_{\mathsf{s}}\right] + (1-\frac{(v-1)}{\delta})\widetilde{y}_{\mathsf{s}}^{R}$$

and similarly for the foreign country

$$\widetilde{y}_{\mathsf{s}}^{*} = -\frac{\frac{1}{\sigma}}{1-\mu}\left[r_{\mathsf{s}}^{*} - \mu\pi_{F,\mathsf{s}}^{*} - (2-\nu)\left(\mu\pi_{H,\mathsf{s}}^{R} - r_{\mathsf{s}}^{R}\right) - \overline{r}_{\mathsf{s}}^{*}\right] - (1-\frac{(v-1)}{\delta})\widetilde{y}_{\mathsf{s}}^{R}.$$

Note from (14) that

$$r_{t}^{R} = \overline{E}_{t}[\pi_{t+1}^{R} + \sigma\frac{(v-1)}{\delta}\Delta y_{t+1}^{R} - \frac{(v-1)^2}{\delta}\Delta\xi_{t+1}^{R}],$$

equals in deviations from the natural rate

$$r^{R} - \left[\mu\pi^{R}\right] = -(1-\mu)\,\sigma\frac{(v-1)}{\delta}\widetilde{y}^{R} + \overline{r}^{R}$$

and for

$$r^R - \left[\mu\pi^R\right] = (\nu - 1)\left[r_{\mathsf{s}}^R - \mu\pi_{H,\mathsf{s}}^R\right]$$

we can write

$$r_{\mathsf{s}}^R - \mu\pi_{H,\mathsf{s}}^R = -(1-\mu)\,\sigma\frac{1}{\delta}\widetilde{y}^R + \frac{\overline{r}^R}{(\nu-1)}.$$

Consequently, home and foreign output can be written in terms of their domestic inflation rates:

$$\begin{aligned}
\widetilde{y_{\mathsf{s}}} &= -\frac{\frac{1}{\sigma}}{1-\mu}\left[r_{\mathsf{s}} - \mu\pi_{H\mathsf{s}} - \overline{r}_{\mathsf{s}} - \frac{2-\nu}{\nu-1}\overline{r}_{\mathsf{s}}^R\right] + (1-\frac{1}{\delta})\widetilde{y}_{\mathsf{s}}^R \\
\widetilde{y_{\mathsf{s}}^*} &= -\frac{\frac{1}{\sigma}}{1-\mu}\left[r_{\mathsf{s}}^* - \mu\pi_{F,\mathsf{s}}^* - \overline{r}_{\mathsf{s}}^* + \frac{2-\nu}{\nu-1}\overline{r}_{\mathsf{s}}^R\right] - (1-\frac{1}{\delta})\widetilde{y}_{\mathsf{s}}^R.
\end{aligned}$$

Then the difference in home and foreign output is given by (29)

$$\widetilde{y}_{\mathsf{s}}^R = -\frac{\delta}{\sigma}\frac{r_{\mathsf{s}}^R - [\mu\pi_{H\mathsf{s}}^R] - \frac{\overline{r}_{\mathsf{s}}^R}{(v-1)}}{(1-\mu)} \text{ with } \pi_{H\mathsf{s}}^R = \kappa_{\widetilde{y}^R}\widetilde{y}_{\mathsf{s}}^R$$

whilst aggregate world output then follows as in (30)

$$\widetilde{y}_{\mathsf{s}}^W = -\frac{1}{\sigma}\frac{r_{\mathsf{s}}^W - [\mu\pi_{H\mathsf{s}}^W] - \overline{r}_{\mathsf{s}}^W}{1-\mu} \text{ with } \pi_{H,\mathsf{s}}^W = \kappa_{\widetilde{y}}\widetilde{y}_{\mathsf{s}}^W.$$

Based on these equations together with the monetary policy conditions outlined in equation (24)

$$r_{\mathsf{s}} = \max\{-\ln(1/\beta), \phi\pi_{H\mathsf{s}}\} \text{ and } r_{\mathsf{s}}^* = \max\{-\ln(1/\beta), \phi\pi_{F\mathsf{s}}^*\},$$

we derive the following relationships outside and at the ZLB.

A.3.1 ZLB is not binding in both countries

$$\begin{aligned}
\widetilde{y}_{\mathsf{s}}^R &= \frac{\frac{\frac{1}{\sigma}}{1-\mu}\frac{1}{v-1}}{\frac{1}{\delta} + \frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2} - \kappa_{(y-y^*)}}{(1-\beta\mu)}}\overline{r}_{\mathsf{s}}^R \\
\pi_{H\mathsf{s}}^R &= \frac{\frac{\kappa}{2} - \kappa_{(y-y^*)}}{(1-\beta\mu)}\widetilde{y}_{\mathsf{s}}^R \\
q_{\mathsf{s}} &= -(v-1)\frac{(\phi-\mu)}{1-\mu}\pi_{H,\mathsf{s}}^R \\
\widetilde{y_{\mathsf{s}}} &= \frac{(1-\frac{1}{\delta}) + \frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\kappa_{(y-y^*)}}{1-\beta\mu}}{1 + \frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\widetilde{y}_{\mathsf{s}}^R + \frac{\frac{\frac{1}{\sigma}}{1-\mu}\left(\overline{r}_{\mathsf{s}} + \frac{2-v}{v-1}\overline{r}_{\mathsf{s}}^R\right)}{1 + \frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}} \\
\pi_{H\mathsf{s}} &= \frac{\frac{\frac{\kappa}{2}}{1-\beta\mu}(1-\frac{1}{\delta}) - \frac{\kappa_{(y-y^*)}}{1-\beta\mu}}{1 + \frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\widetilde{y}_{\mathsf{s}}^R + \frac{\frac{\frac{\kappa}{2}}{1-\beta\mu}\frac{\frac{1}{\sigma}}{1-\mu}\left(\overline{r}_{\mathsf{s}} + \frac{2-v}{v-1}\overline{r}_{\mathsf{s}}^R\right)}{1 + \frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}} \\
\pi_{\mathsf{s}} &= \pi_{H,\mathsf{s}} - \frac{\kappa_e}{1-\mu\beta}\widetilde{y}_{\mathsf{s}}^R + \frac{1}{\mu}\frac{2-v}{v-1}\overline{r}_{\mathsf{s}}^R
\end{aligned}$$

In terms of relative and world averages:

$$\begin{aligned}
\widetilde{y}_{\mathsf{s}}^R &= \frac{\frac{\frac{1}{\sigma}}{1-\mu}\frac{1}{v-1}}{\frac{1}{\delta} + \frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2} - \kappa_{(y-y^*)}}{(1-\beta\mu)}}\overline{r}_{\mathsf{s}}^R \\
\pi_{H\mathsf{s}}^R &= \frac{\frac{\frac{\kappa}{2} - \kappa_{(y-y^*)}}{(1-\beta\mu)}\frac{\frac{1}{\sigma}}{1-\mu}\frac{1}{v-1}}{\frac{1}{\delta} + \frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2} - \kappa_{(y-y^*)}}{(1-\beta\mu)}}\overline{r}_{\mathsf{s}}^R \\
\widetilde{y}_{\mathsf{s}}^W &= \frac{\frac{\frac{1}{\sigma}}{1-\mu}}{1 + \frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\overline{r}_{\mathsf{s}}^W \\
\pi_{H\mathsf{s}}^W &= \frac{\frac{\frac{\kappa}{2}}{1-\beta\mu}\frac{\frac{1}{\sigma}}{1-\mu}}{1 + \frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\overline{r}_{\mathsf{s}}^W
\end{aligned}$$

Then we can write the home and foreign variables in terms of world and relative variables, by noting that

$$
\begin{aligned}
x_{\mathsf{s}} &= x_{\mathsf{s}}^W + x_{\mathsf{s}}^R \\
x_{\mathsf{s}}^* &= x_{\mathsf{s}}^W - x_{\mathsf{s}}^R
\end{aligned}
$$

so that

$$
\widetilde{y}_{\mathsf{s}} = \widetilde{y}_{\mathsf{s}}^W + \widetilde{y}_{\mathsf{s}}^R = \frac{\frac{\frac{1}{\sigma}}{1-\mu}}{1+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\overline{r}_{\mathsf{s}}^W + \frac{\frac{\frac{1}{\sigma}}{1-\mu}\frac{1}{v-1}}{\frac{1}{\delta}+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}-\kappa_{(y-y^*)}}{(1-\beta\mu)}}\overline{r}_{\mathsf{s}}^R
$$

Defining:

$$
\kappa_{\widetilde{y}} = \frac{\kappa}{2}/(1-\beta\mu) > 0 \qquad \kappa_{\widetilde{y}^R} = (\frac{\kappa}{2} - \kappa_{(y-y^*)})/(1-\beta\mu) > 0
$$

so that

$$
\begin{aligned}
\widetilde{y}_{\mathsf{s}}^R &= \frac{\frac{\frac{1}{\sigma}}{1-\mu}\frac{1}{v-1}}{\frac{1}{\delta}+\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\widetilde{y}^R}}{\sigma}}\overline{r}_{\mathsf{s}}^R = \frac{\frac{\delta}{v-1}\overline{r}_{\mathsf{s}}^R}{(1-\mu)\,\sigma + (\phi-\mu)\,\delta\kappa_{\widetilde{y}^R}} \\
\pi_{H\mathsf{s}}^R &= \frac{\kappa_{\widetilde{y}^R}\frac{1}{\sigma}\frac{1}{1-\mu}\frac{1}{v-1}}{\frac{1}{\delta}+\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\widetilde{y}^R}}{\sigma}}\overline{r}_{\mathsf{s}}^R = \frac{\kappa_{\widetilde{y}^R}\frac{\delta}{v-1}\overline{r}_{\mathsf{s}}^R}{(1-\mu)\,\sigma + (\phi-\mu)\,\delta\kappa_{\widetilde{y}^R}} \\
\widetilde{y}_{\mathsf{s}}^W &= \frac{\frac{\frac{1}{\sigma}}{1-\mu}}{1+\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\widetilde{y}}}{\sigma}}\overline{r}_{\mathsf{s}}^W = \frac{\overline{r}_{\mathsf{s}}^W}{(1-\mu)\,\sigma + (\phi-\mu)\,\kappa_{\widetilde{y}}} \\
\pi_{H\mathsf{s}}^W &= \frac{\frac{\kappa_{\widetilde{y}}}{\sigma}\frac{1}{1-\mu}}{1+\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\widetilde{y}}}{\sigma}}\overline{r}_{\mathsf{s}}^W = \frac{\kappa_{\widetilde{y}}\overline{r}_{\mathsf{s}}^W}{(1-\mu)\,\sigma + (\phi-\mu)\,\kappa_{\widetilde{y}}}
\end{aligned}
$$

and we can write output

$$
\widetilde{y}_{\mathsf{s}} = \widetilde{y}_{\mathsf{s}}^W + \widetilde{y}_{\mathsf{s}}^R = \frac{\frac{\frac{1}{\sigma}}{1-\mu}\overline{r}_{\mathsf{s}}^W}{1+\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\widetilde{y}}}{\sigma}} + \frac{\frac{\frac{1}{\sigma}}{1-\mu}\frac{\overline{r}_{\mathsf{s}}^R}{v-1}}{\frac{1}{\delta}+\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\widetilde{y}^R}}{\sigma}}
$$

and inflation

$$
\pi_{H\mathsf{s}} = \pi_{H\mathsf{s}}^W + \pi_{H\mathsf{s}}^R = \frac{\frac{\kappa_{\widetilde{y}}}{\sigma}\frac{\overline{r}_{\mathsf{s}}^W}{1-\mu}}{1+\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\widetilde{y}}}{\sigma}} + \frac{\frac{\kappa_{\widetilde{y}^R}}{\sigma}\frac{1}{v-1}\frac{\overline{r}_{\mathsf{s}}^R}{1-\mu}}{\frac{1}{\delta}+\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\widetilde{y}^R}}{\sigma}}
$$

Or rewriting the denominator:

$$
\begin{aligned}
\widetilde{y}_{\mathsf{s}} &= \frac{\frac{\frac{1}{\sigma}}{1-\mu}\overline{r}_{\mathsf{s}}^W}{1+\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\widetilde{y}}}{\sigma}} + \frac{\frac{\frac{1}{\sigma}}{1-\mu}\frac{\overline{r}_{\mathsf{s}}^R}{v-1}}{\frac{1}{\delta}+\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\widetilde{y}^R}}{\sigma}} = \frac{\overline{r}_{\mathsf{s}}^W}{(1-\mu)\,\sigma + (\phi-\mu)\,\kappa_{\widetilde{y}}} + \frac{\frac{\delta}{v-1}\overline{r}_{\mathsf{s}}^R}{(1-\mu)\,\sigma + (\phi-\mu)\,\delta\kappa_{\widetilde{y}^R}} \\
\pi_{H\mathsf{s}} &= \pi_{H\mathsf{s}}^W + \pi_{H\mathsf{s}}^R = \frac{\frac{\kappa_{\widetilde{y}}}{\sigma}\frac{\overline{r}_{\mathsf{s}}^W}{1-\mu}}{1+\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\widetilde{y}}}{\sigma}} + \frac{\frac{\kappa_{\widetilde{y}^R}}{\sigma}\frac{1}{v-1}\frac{\overline{r}_{\mathsf{s}}^R}{1-\mu}}{\frac{1}{\delta}+\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\widetilde{y}^R}}{\sigma}} = \frac{\kappa_{\widetilde{y}}\overline{r}_{\mathsf{s}}^W}{(1-\mu)\,\sigma + (\phi-\mu)\,\kappa_{\widetilde{y}}} + \frac{\kappa_{\widetilde{y}^R}\frac{\delta}{v-1}\overline{r}_{\mathsf{s}}^R}{(1-\mu)\,\sigma + (\phi-\mu)\,\delta\kappa_{\widetilde{y}^R}}
\end{aligned}
$$

Defining

$$
\begin{aligned}
\gamma_{\widetilde{y}} &= (1-\mu)\,\sigma + (\phi-\mu)\,\kappa_{\widetilde{y}} \\
\gamma_{\widetilde{y}^R} &= (1-\mu)\,\sigma + (\phi-\mu)\,\delta\kappa_{\widetilde{y}^R}
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{y}_{\mathsf{s}} &= \frac{1}{\gamma_{\widetilde{y}}}\overline{r}_{\mathsf{s}}^W + \frac{\frac{\delta}{v-1}}{\gamma_{\widetilde{y}^R}}\overline{r}_{\mathsf{s}}^R \\
\pi_{H\mathsf{s}} &= \frac{\kappa_{\widetilde{y}}}{\gamma_{\widetilde{y}}}\overline{r}_{\mathsf{s}}^W + \frac{\kappa_{\widetilde{y}^R}\frac{\delta}{v-1}}{\gamma_{\widetilde{y}^R}}\overline{r}_{\mathsf{s}}^R
\end{aligned}
$$

A.3.2 ZLB is binding in both countries

$$
\begin{aligned}
\widetilde{y}_{\mathsf{s}}^{R} &= \frac{\frac{\frac{1}{\sigma}}{1-\mu}\frac{1}{v-1}}{\frac{1}{\delta}-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}-\kappa_{(y-y^*)}}{(1-\beta\mu)}}\overline{r}_{\mathsf{s}}^{R} \\
\pi_{H\mathsf{s}}^{R} &= \frac{\frac{\kappa}{2}-\kappa_{(y-y^*)}}{(1-\beta\mu)}\widetilde{y}_{\mathsf{s}}^{R} \\
q_{\mathsf{s}} &= (v-1)\frac{\mu}{1-\mu}\pi_{H,\mathsf{s}}^{R} \\
\widetilde{y}_{\mathsf{s}} &= \frac{(1-\frac{1}{\delta})-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_{(y-y^*)}}{1-\beta\mu}}{1-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\widetilde{y}_{\mathsf{s}}^{R}+\frac{\frac{\frac{1}{\sigma}}{1-\mu}\left(\overline{r}_{\mathsf{s}}+\frac{2-v}{v-1}\overline{r}_{\mathsf{s}}^{R}+\ln(1/\beta)\right)}{1-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}} \\
\pi_{H\mathsf{s}} &= \frac{\frac{\frac{\kappa}{2}}{1-\beta\mu}(1-\frac{1}{\delta})-\frac{\kappa_{(y-y^*)}}{1-\beta\mu}}{1-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\widetilde{y}_{\mathsf{s}}^{R}+\frac{\frac{\frac{\kappa}{2}}{1-\beta\mu}\frac{\frac{1}{\sigma}}{1-\mu}\left(\overline{r}_{\mathsf{s}}+\frac{2-v}{v-1}\overline{r}_{\mathsf{s}}^{R}+\ln(1/\beta)\right)}{1-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}} \\
\pi_{\mathsf{s}} &= \pi_{H,\mathsf{s}}-\frac{\kappa_e}{1-\mu\beta}\widetilde{y}_{\mathsf{s}}^{R}+\frac{1}{\mu}\frac{2-v}{v-1}\overline{r}_{\mathsf{s}}^{R}
\end{aligned}
$$

In terms of relative and world averages:

Defining:

$$
\kappa_{\widetilde{y}}=\frac{\kappa}{2}/(1-\beta\mu)>0 \qquad \kappa_{\widetilde{y}^R}=(\frac{\kappa}{2}-\kappa_{(y-y^*)})/(1-\beta\mu)>0
$$

so that

$$
\begin{aligned}
\widetilde{y}_{\mathsf{s}}^{R} &= \frac{\frac{\frac{1}{\sigma}}{1-\mu}\frac{1}{v-1}}{\frac{1}{\delta}-\frac{\mu}{1-\mu}\frac{\kappa_{\widetilde{y}^R}}{\sigma}}\overline{r}_{\mathsf{s}}^{R} \\
\pi_{H\mathsf{s}}^{R} &= \frac{\frac{\kappa_{\widetilde{y}^R}}{\sigma}\frac{1}{1-\mu}\frac{1}{v-1}}{\frac{1}{\delta}-\frac{\mu}{1-\mu}\frac{\kappa_{\widetilde{y}^R}}{\sigma}}\overline{r}_{\mathsf{s}}^{R} \\
\widetilde{y}_{\mathsf{s}}^{W} &= \frac{\frac{\frac{1}{\sigma}}{1-\mu}}{1-\frac{\mu}{1-\mu}\frac{\kappa_{\widetilde{y}}}{\sigma}}\overline{r}_{\mathsf{s}}^{W} \\
\pi_{H\mathsf{s}}^{W} &= \frac{\frac{\kappa_{\widetilde{y}}}{\sigma}\frac{1}{1-\mu}}{1-\frac{\mu}{1-\mu}\frac{\kappa_{\widetilde{y}}}{\sigma}}\overline{r}_{\mathsf{s}}^{W}
\end{aligned}
$$

becomes

$$
\begin{aligned}
\widetilde{y}_{\mathsf{s}}^{R} &= \frac{\frac{\frac{1}{\sigma}}{1-\mu}\frac{1}{v-1}}{\frac{1}{\delta}-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}-\kappa_{(y-y^*)}}{(1-\beta\mu)}}\overline{r}_{\mathsf{s}}^{R} = \frac{\frac{\delta}{v-1}\overline{r}_{\mathsf{s}}^{R}}{(1-\mu)\,\sigma-\mu\delta\kappa_{\widetilde{y}^R}} \\
\pi_{H\mathsf{s}}^{R} &= \frac{\frac{\frac{\kappa}{2}-\kappa_{(y-y^*)}}{(1-\beta\mu)}\frac{\frac{1}{\sigma}}{1-\mu}\frac{1}{v-1}}{\frac{1}{\delta}-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}-\kappa_{(y-y^*)}}{(1-\beta\mu)}}\overline{r}_{\mathsf{s}}^{R} = \frac{\kappa_{\widetilde{y}^R}\frac{\delta}{v-1}\overline{r}_{\mathsf{s}}^{R}}{(1-\mu)\,\sigma-\mu\delta\kappa_{\widetilde{y}^R}} \\
\widetilde{y}_{\mathsf{s}}^{W} &= \frac{\frac{\frac{1}{\sigma}}{1-\mu}}{1-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\overline{r}_{\mathsf{s}}^{W} = \frac{\overline{r}_{\mathsf{s}}^{W}}{(1-\mu)\,\sigma-\mu\kappa_{\widetilde{y}}} \\
\pi_{H\mathsf{s}}^{W} &= \frac{\frac{\frac{\kappa}{2}}{1-\beta\mu}\frac{\frac{1}{\sigma}}{1-\mu}}{1-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\overline{r}_{\mathsf{s}}^{W} = \frac{\kappa_{\widetilde{y}^R}\overline{r}_{\mathsf{s}}^{W}}{(1-\mu)\,\sigma-\mu\kappa_{\widetilde{y}}}
\end{aligned}
$$

We can write output

$$
\widetilde{y}_{\mathsf{s}}=\widetilde{y}_{\mathsf{s}}^{W}+\widetilde{y}_{\mathsf{s}}^{R}=\frac{\frac{\frac{1}{\sigma}}{1-\mu}\overline{r}_{\mathsf{s}}^{W}}{1-\frac{\mu}{1-\mu}\frac{\kappa_{\widetilde{y}}}{\sigma}}+\frac{\frac{\frac{1}{\sigma}}{1-\mu}\frac{\overline{r}_{\mathsf{s}}^{R}}{v-1}}{\frac{1}{\delta}-\frac{\mu}{1-\mu}\frac{\kappa_{\widetilde{y}^R}}{\sigma}}
$$

and inflation

$$
\pi_{H\mathsf{s}}=\pi_{H\mathsf{s}}^{W}+\pi_{H\mathsf{s}}^{R}=\frac{\frac{\kappa_{\widetilde{y}}}{\sigma}\frac{\overline{r}_{\mathsf{s}}^{W}}{1-\mu}}{1-\frac{\mu}{1-\mu}\frac{\kappa_{\widetilde{y}}}{\sigma}}+\frac{\frac{\kappa_{\widetilde{y}^R}}{\sigma}\frac{1}{v-1}\frac{\overline{r}_{\mathsf{s}}^{R}}{1-\mu}}{\frac{1}{\delta}-\frac{\mu}{1-\mu}\frac{\kappa_{\widetilde{y}^R}}{\sigma}}
$$

Or rewriting the denominator:

$$\widetilde{y}_{\mathsf{s}} = \frac{\frac{\frac{1}{\sigma}}{1-\mu}\overline{r}_{\mathsf{s}}^{W}}{1-\frac{\mu}{1-\mu}\frac{\kappa_{\widetilde{y}}}{\sigma}} + \frac{\frac{\frac{1}{\sigma}}{1-\mu}\frac{\overline{r}_{\mathsf{s}}^{R}}{v-1}}{\frac{1}{\delta}-\frac{\mu}{1-\mu}\frac{\kappa_{\widetilde{y}^R}}{\sigma}} = \frac{\overline{r}_{\mathsf{s}}^{W}}{(1-\mu)\,\sigma-\mu\kappa_{\widetilde{y}}} + \frac{\frac{\delta}{v-1}\overline{r}_{\mathsf{s}}^{R}}{(1-\mu)\,\sigma-\mu\delta\kappa_{\widetilde{y}^R}}$$

$$\pi_{H\mathsf{s}} = \pi_{H\mathsf{s}}^{W}+\pi_{H\mathsf{s}}^{R} = \frac{\frac{\kappa_{\widetilde{y}}}{\sigma}\frac{\overline{r}_{\mathsf{s}}^{W}}{1-\mu}}{1-\frac{\mu}{1-\mu}\frac{\kappa_{\widetilde{y}}}{\sigma}} + \frac{\frac{\kappa_{\widetilde{y}^R}}{\sigma}\frac{1}{v-1}\frac{\overline{r}_{\mathsf{s}}^{R}}{1-\mu}}{\frac{1}{\delta}-\frac{\mu}{1-\mu}\frac{\kappa_{\widetilde{y}^R}}{\sigma}} = \frac{\kappa_{\widetilde{y}}\overline{r}_{\mathsf{s}}^{W}}{(1-\mu)\,\sigma-\mu\kappa_{\widetilde{y}}} + \frac{\kappa_{\widetilde{y}^R}\frac{\delta}{v-1}\overline{r}_{\mathsf{s}}^{R}}{(1-\mu)\,\sigma-\mu\delta\kappa_{\widetilde{y}^R}}$$

Defining

$$\gamma_{\widetilde{y}}^{zlb} = (1-\mu)\,\sigma-\mu\kappa_{\widetilde{y}}$$
$$\gamma_{\widetilde{y}^R}^{zlb} = (1-\mu)\,\sigma-\mu\delta\kappa_{\widetilde{y}^R}$$

$$\widetilde{y}_{\mathsf{s}} = \frac{1}{\gamma_{\widetilde{y}}^{zlb}}\overline{r}_{\mathsf{s}}^{W} + \frac{\frac{\delta}{v-1}}{\gamma_{\widetilde{y}^R}^{zlb}}\overline{r}_{\mathsf{s}}^{R}$$
$$\pi_{H\mathsf{s}} = \frac{\kappa_{\widetilde{y}}}{\gamma_{\widetilde{y}}^{zlb}}\overline{r}_{\mathsf{s}}^{W} + \frac{\frac{\delta}{v-1}\kappa_{\widetilde{y}^R}}{\gamma_{\widetilde{y}^R}^{zlb}}\overline{r}_{\mathsf{s}}^{R}$$

A.3.3 ZLB is only binding in the home country

$$\widetilde{y}_{\mathsf{s}} = \frac{(1-\frac{1}{\delta})-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa(y-y^*)}{1-\beta\mu}}{1-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\widetilde{y}_{\mathsf{s}}^{R} + \frac{\frac{\frac{1}{\sigma}}{1-\mu}\left(\overline{r}_{\mathsf{s}}+\frac{2-v}{v-1}\overline{r}_{\mathsf{s}}^{R}+\ln(1/\beta)\right)}{1-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}$$

$$\widetilde{y}_{\mathsf{s}} = \frac{(1-\frac{1}{\delta})-\frac{\frac{\mu}{\sigma}}{1-\mu}\left(\kappa_{\widetilde{y}}-\kappa_{\widetilde{y}^R}\right)}{1-\frac{\frac{\mu}{\sigma}}{1-\mu}\kappa_{\widetilde{y}}}\widetilde{y}_{\mathsf{s}}^{R} + \frac{\frac{\frac{1}{\sigma}}{1-\mu}\left(\overline{r}_{\mathsf{s}}+\frac{2-v}{v-1}\overline{r}_{\mathsf{s}}^{R}+\ln(1/\beta)\right)}{1-\frac{\frac{\mu}{\sigma}}{1-\mu}\kappa_{\widetilde{y}}}$$

$$\widetilde{y}_{\mathsf{s}} = \frac{\frac{\delta-1}{\delta}(1-\mu)\,\sigma-\mu\left(\kappa_{\widetilde{y}}-\kappa_{\widetilde{y}^R}\right)}{\left((1-\mu)\,\sigma-\mu\kappa_{\widetilde{y}}\right)}\widetilde{y}_{\mathsf{s}}^{R} + \frac{\left(\overline{r}_{\mathsf{s}}+\frac{2-v}{v-1}\overline{r}_{\mathsf{s}}^{R}+\ln(1/\beta)\right)}{\left((1-\mu)\,\sigma-\mu\kappa_{\widetilde{y}}\right)}$$

$$\widetilde{y}_{\mathsf{s}}^{*} = -\frac{(1-\frac{1}{\delta})+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\kappa(y-y^*)}{1-\beta\mu}}{1+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\widetilde{y}_{\mathsf{s}}^{R} + \frac{\frac{\frac{1}{\sigma}}{1-\mu}\left(\overline{r}_{\mathsf{s}}^{*}-\frac{2-v}{v-1}\overline{r}_{\mathsf{s}}^{R}\right)}{1+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}$$

$$\widetilde{y}_{\mathsf{s}}^{*} = -\frac{\frac{\delta-1}{\delta}+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\left(\kappa_{\widetilde{y}}-\kappa_{\widetilde{y}^R}\right)}{1+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\kappa_{\widetilde{y}}}\widetilde{y}_{\mathsf{s}}^{R} + \frac{\frac{\frac{1}{\sigma}}{1-\mu}\left(\overline{r}_{\mathsf{s}}^{*}-\frac{2-v}{v-1}\overline{r}_{\mathsf{s}}^{R}\right)}{1+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\kappa_{\widetilde{y}}}$$

$$\widetilde{y}_{\mathsf{s}}^{*} = -\frac{\frac{\delta-1}{\delta}(1-\mu)\,\sigma+(\phi-\mu)\left(\kappa_{\widetilde{y}}-\kappa_{\widetilde{y}^R}\right)}{\left((1-\mu)\,\sigma+(\phi-\mu)\,\kappa_{\widetilde{y}}\right)}\widetilde{y}_{\mathsf{s}}^{R} + \frac{\overline{r}_{\mathsf{s}}^{*}-\frac{2-v}{v-1}\overline{r}_{\mathsf{s}}^{R}}{\left((1-\mu)\,\sigma+(\phi-\mu)\,\kappa_{\widetilde{y}}\right)}$$

so that

$$\begin{aligned}\widetilde{y}_{\mathsf{s}}-\widetilde{y}_{\mathsf{s}}^{*} &= \frac{\frac{\delta-1}{\delta}(1-\mu)\,\sigma-\mu\left(\kappa_{\widetilde{y}}-\kappa_{\widetilde{y}^R}\right)}{\left((1-\mu)\,\sigma-\mu\kappa_{\widetilde{y}}\right)}\widetilde{y}_{\mathsf{s}}^{R} + \frac{\frac{\delta-1}{\delta}(1-\mu)\,\sigma+(\phi-\mu)\left(\kappa_{\widetilde{y}}-\kappa_{\widetilde{y}^R}\right)}{\left((1-\mu)\,\sigma+(\phi-\mu)\,\kappa_{\widetilde{y}}\right)}\widetilde{y}_{\mathsf{s}}^{R} \\ &\quad + \frac{\left(\overline{r}_{\mathsf{s}}+\frac{2-v}{v-1}\overline{r}_{\mathsf{s}}^{R}+\ln(1/\beta)\right)}{\left((1-\mu)\,\sigma-\mu\kappa_{\widetilde{y}}\right)} - \frac{\overline{r}_{\mathsf{s}}^{*}-\frac{2-v}{v-1}\overline{r}_{\mathsf{s}}^{R}}{\left((1-\mu)\,\sigma+(\phi-\mu)\,\kappa_{\widetilde{y}}\right)}\end{aligned}$$

$$\widetilde{y}_{\mathsf{s}}-\widetilde{y}_{\mathsf{s}}^{*} = \frac{\left((1-\mu)\,\sigma+(\phi-\mu)\,\kappa_{\widetilde{y}}\right)\left(\frac{\delta-1}{\delta}(1-\mu)\,\sigma-\mu\left(\kappa_{\widetilde{y}}-\kappa_{\widetilde{y}^R}\right)\right)}{\left((1-\mu)\,\sigma-\mu\kappa_{\widetilde{y}}\right)\left((1-\mu)\,\sigma+(\phi-\mu)\,\kappa_{\widetilde{y}}\right)}\widetilde{y}_{\mathsf{s}}^{R}$$

$$
\begin{aligned}
&+\frac{\left((1-\mu)\sigma-\mu\kappa_{\widetilde{y}}\right)\left(\frac{\delta-1}{\delta}(1-\mu)\sigma+(\phi-\mu)\left(\kappa_{\widetilde{y}}-\kappa_{\widetilde{y}^R}\right)\right)}{\left((1-\mu)\sigma-\mu\kappa_{\widetilde{y}}\right)\left((1-\mu)\sigma+(\phi-\mu)\kappa_{\widetilde{y}}\right)}\widetilde{y}^R_{\mathsf{s}} \\
&+\frac{\left((1-\mu)\sigma+(\phi-\mu)\kappa_{\widetilde{y}}\right)\left(\overline{r}_{\mathsf{s}}+\frac{2-v}{v-1}\overline{r}^R_{\mathsf{s}}+\ln(1/\beta)\right)}{\left((1-\mu)\sigma-\mu\kappa_{\widetilde{y}}\right)\left((1-\mu)\sigma+(\phi-\mu)\kappa_{\widetilde{y}}\right)} \\
&-\frac{\left((1-\mu)\sigma-\mu\kappa_{\widetilde{y}}\right)\left(\overline{r}^*_{\mathsf{s}}-\frac{2-v}{v-1}\overline{r}^R_{\mathsf{s}}\right)}{\left((1-\mu)\sigma-\mu\kappa_{\widetilde{y}}\right)\left((1-\mu)\sigma+(\phi-\mu)\kappa_{\widetilde{y}}\right)}
\end{aligned}
$$

$$
\begin{aligned}
&\widetilde{y}^R_{\mathsf{s}}\left(\frac{\left(\begin{array}{c}
2\left((1-\mu)\sigma-\mu\kappa_{\widetilde{y}}\right)\left((1-\mu)\sigma+(\phi-\mu)\kappa_{\widetilde{y}}\right) \\
-\left((1-\mu)\sigma+(\phi-\mu)\kappa_{\widetilde{y}}\right)\left(\frac{\delta-1}{\delta}(1-\mu)\sigma-\mu\left(\kappa_{\widetilde{y}}-\kappa_{\widetilde{y}^R}\right)\right) \\
-\left((1-\mu)\sigma-\mu\kappa_{\widetilde{y}}\right)\left(\frac{\delta-1}{\delta}(1-\mu)\sigma+(\phi-\mu)\left(\kappa_{\widetilde{y}}-\kappa_{\widetilde{y}^R}\right)\right)
\end{array}\right)}{\left((1-\mu)\sigma-\mu\kappa_{\widetilde{y}}\right)\left((1-\mu)\sigma+(\phi-\mu)\kappa_{\widetilde{y}}\right)}\right) \\
=\ &\frac{\left((1-\mu)\sigma+(\phi-\mu)\kappa_{\widetilde{y}}\right)\left(\overline{r}_{\mathsf{s}}+\frac{2-v}{v-1}\overline{r}^R_{\mathsf{s}}+\ln(1/\beta)\right)}{\left((1-\mu)\sigma-\mu\kappa_{\widetilde{y}}\right)\left((1-\mu)\sigma+(\phi-\mu)\kappa_{\widetilde{y}}\right)}-\frac{\left((1-\mu)\sigma-\mu\kappa_{\widetilde{y}}\right)\left(\overline{r}^*_{\mathsf{s}}-\frac{2-v}{v-1}\overline{r}^R_{\mathsf{s}}\right)}{\left((1-\mu)\sigma-\mu\kappa_{\widetilde{y}}\right)\left((1-\mu)\sigma+(\phi-\mu)\kappa_{\widetilde{y}}\right)}
\end{aligned}
$$

$$
\widetilde{y}^R_{\mathsf{s}}=\frac{\left((1-\mu)\sigma+(\phi-\mu)\kappa_{\widetilde{y}}\right)\left(\overline{r}_{\mathsf{s}}+\frac{2-v}{v-1}\overline{r}^R_{\mathsf{s}}+\ln(1/\beta)\right)-\left((1-\mu)\sigma-\mu\kappa_{\widetilde{y}}\right)\left(\overline{r}^*_{\mathsf{s}}-\frac{2-v}{v-1}\overline{r}^R_{\mathsf{s}}\right)}{\left(\begin{array}{c}
2\left((1-\mu)\sigma-\mu\kappa_{\widetilde{y}}\right)\left((1-\mu)\sigma+(\phi-\mu)\kappa_{\widetilde{y}}\right) \\
-\left((1-\mu)\sigma+(\phi-\mu)\kappa_{\widetilde{y}}\right)\left(\frac{\delta-1}{\delta}(1-\mu)\sigma-\mu\left(\kappa_{\widetilde{y}}-\kappa_{\widetilde{y}^R}\right)\right) \\
-\left((1-\mu)\sigma-\mu\kappa_{\widetilde{y}}\right)\left(\frac{\delta-1}{\delta}(1-\mu)\sigma+(\phi-\mu)\left(\kappa_{\widetilde{y}}-\kappa_{\widetilde{y}^R}\right)\right)
\end{array}\right)}
$$

Defining

$$
\begin{aligned}
\gamma^{zlb}_{\widetilde{y}} &= (1-\mu)\sigma-\mu\kappa_{\widetilde{y}} \\
\gamma^{zlb}_{\widetilde{y}^R} &= (1-\mu)\sigma-\mu\delta\kappa_{\widetilde{y}^R}
\end{aligned}
$$

and

$$
\begin{aligned}
\gamma_{\widetilde{y}} &= (1-\mu)\sigma+(\phi-\mu)\kappa_{\widetilde{y}} \\
\gamma_{\widetilde{y}^R} &= (1-\mu)\sigma+(\phi-\mu)\delta\kappa_{\widetilde{y}^R}
\end{aligned}
$$

and for

$$
\begin{aligned}
\mu\kappa_{\widetilde{y}^R} &= -\frac{\gamma^{zlb}_{\widetilde{y}^R}-(1-\mu)\sigma}{\delta} \\
\mu\kappa_{\widetilde{y}} &= -\gamma^{zlb}_{\widetilde{y}}+(1-\mu)\sigma
\end{aligned}
$$

$$
\begin{aligned}
-\mu\left(\kappa_{\widetilde{y}}-\kappa_{\widetilde{y}^R}\right) &= \mu\kappa_{\widetilde{y}^R}-\mu\kappa_{\widetilde{y}}=-\frac{\gamma^{zlb}_{\widetilde{y}^R}-(1-\mu)\sigma}{\delta}+\gamma^{zlb}_{\widetilde{y}}-(1-\mu)\sigma \\
-\mu\left(\kappa_{\widetilde{y}}-\kappa_{\widetilde{y}^R}\right) &= \frac{\delta\gamma^{zlb}_{\widetilde{y}}-\gamma^{zlb}_{\widetilde{y}^R}}{\delta}-\frac{\delta-1}{\delta}(1-\mu)\sigma
\end{aligned}
$$

and

$$
\begin{aligned}
(\phi-\mu)\left(\kappa_{\widetilde{y}}-\kappa_{\widetilde{y}^R}\right) &= \gamma_{\widetilde{y}}-(1-\mu)\,\sigma-\frac{\gamma_{\widetilde{y}^R}}{\delta}+\frac{(1-\mu)\,\sigma}{\delta} \\
&= \frac{\delta\gamma_{\widetilde{y}}-\gamma_{\widetilde{y}^R}}{\delta}-\frac{\delta-1}{\delta}(1-\mu)\,\sigma
\end{aligned}
$$

so that

$$
\begin{aligned}
\widetilde{y}_{\mathsf{s}}^R &= \frac{\left((1-\mu)\,\sigma+(\phi-\mu)\,\kappa_{\widetilde{y}}\right)\left(\overline{r}_{\mathsf{s}}+\frac{2-v}{v-1}\overline{r}_{\mathsf{s}}^R+\ln(1/\beta)\right)-\left((1-\mu)\,\sigma-\mu\kappa_{\widetilde{y}}\right)\left(\overline{r}_{\mathsf{s}}^*-\frac{2-v}{v-1}\overline{r}_{\mathsf{s}}^R\right)}{\left(2\gamma_{\widetilde{y}}^{zlb}\gamma_{\widetilde{y}}-\gamma_{\widetilde{y}}\frac{\delta\gamma_{\widetilde{y}}^{zlb}-\gamma_{\widetilde{y}^R}^{zlb}}{\delta}-\gamma_{\widetilde{y}}^{zlb}\frac{\delta\gamma_{\widetilde{y}}-\gamma_{\widetilde{y}^R}}{\delta}\right)} \\
\widetilde{y}_{\mathsf{s}}^R &= \frac{\delta\gamma_{\widetilde{y}}\left(\overline{r}_{\mathsf{s}}+\frac{2-v}{v-1}\overline{r}_{\mathsf{s}}^R+\ln(1/\beta)\right)-\delta\gamma_{\widetilde{y}}^{zlb}\left(\overline{r}_{\mathsf{s}}^*-\frac{2-v}{v-1}\overline{r}_{\mathsf{s}}^R\right)}{\delta\left(2\gamma_{\widetilde{y}}^{zlb}\gamma_{\widetilde{y}}-\gamma_{\widetilde{y}}\frac{\delta\gamma_{\widetilde{y}}^{zlb}-\gamma_{\widetilde{y}^R}^{zlb}}{\delta}-\gamma_{\widetilde{y}}^{zlb}\frac{\delta\gamma_{\widetilde{y}}-\gamma_{\widetilde{y}^R}}{\delta}\right)} \\
\widetilde{y}_{\mathsf{s}}^R &= \frac{\delta\gamma_{\widetilde{y}}\left(\overline{r}_{\mathsf{s}}+\frac{2-v}{v-1}\overline{r}_{\mathsf{s}}^R+\ln(1/\beta)\right)-\delta\gamma_{\widetilde{y}}^{zlb}\left(\overline{r}_{\mathsf{s}}^*-\frac{2-v}{v-1}\overline{r}_{\mathsf{s}}^R\right)}{\left(2\delta\gamma_{\widetilde{y}}^{zlb}\gamma_{\widetilde{y}}-\gamma_{\widetilde{y}}\left(\delta\gamma_{\widetilde{y}}^{zlb}-\gamma_{\widetilde{y}^R}^{zlb}\right)-\gamma_{\widetilde{y}}^{zlb}\left(\delta\gamma_{\widetilde{y}}-\gamma_{\widetilde{y}^R}\right)\right)} \\
\widetilde{y}_{\mathsf{s}}^R &= \frac{\delta\gamma_{\widetilde{y}}\left(\overline{r}_{\mathsf{s}}+\frac{2-v}{v-1}\overline{r}_{\mathsf{s}}^R+\ln(1/\beta)\right)-\delta\gamma_{\widetilde{y}}^{zlb}\left(\overline{r}_{\mathsf{s}}^*-\frac{2-v}{v-1}\overline{r}_{\mathsf{s}}^R\right)}{\left(2\delta\gamma_{\widetilde{y}}^{zlb}\gamma_{\widetilde{y}}-\delta\gamma_{\widetilde{y}}^{zlb}\gamma_{\widetilde{y}}+\gamma_{\widetilde{y}}\gamma_{\widetilde{y}^R}^{zlb}-\delta\gamma_{\widetilde{y}}^{zlb}\gamma_{\widetilde{y}}+\gamma_{\widetilde{y}}^{zlb}\gamma_{\widetilde{y}^R}\right)} \\
\widetilde{y}_{\mathsf{s}}^R &= \frac{\frac{\delta}{v-1}(2-v)\left(\gamma_{\widetilde{y}}+\gamma_{\widetilde{y}}^{zlb}\right)\overline{r}_{\mathsf{s}}^R+\delta\gamma_{\widetilde{y}}\left(\overline{r}_{\mathsf{s}}+\ln(1/\beta)\right)-\delta\gamma_{\widetilde{y}}^{zlb}\overline{r}_{\mathsf{s}}^*}{\left(\gamma_{\widetilde{y}^R}^{zlb}\gamma_{\widetilde{y}}+\gamma_{\widetilde{y}}^{zlb}\gamma_{\widetilde{y}^R}\right)}
\end{aligned}
$$

World output equals

$$
\begin{aligned}
\widetilde{y}_{\mathsf{s}}+\widetilde{y}_{\mathsf{s}}^* &= \frac{\frac{\delta-1}{\delta}(1-\mu)\,\sigma-\mu\left(\kappa_{\widetilde{y}}-\kappa_{\widetilde{y}^R}\right)}{\left((1-\mu)\,\sigma-\mu\kappa_{\widetilde{y}}\right)}\widetilde{y}_{\mathsf{s}}^R+\frac{\frac{\delta-1}{\delta}(1-\mu)\,\sigma+(\phi-\mu)\left(\kappa_{\widetilde{y}}-\kappa_{\widetilde{y}^R}\right)}{\left((1-\mu)\,\sigma+(\phi-\mu)\,\kappa_{\widetilde{y}}\right)}\widetilde{y}_{\mathsf{s}}^R \\
&\quad+\frac{\overline{r}_{\mathsf{s}}+\frac{2-v}{v-1}\overline{r}_{\mathsf{s}}^R+\ln(1/\beta)}{\left((1-\mu)\,\sigma-\mu\kappa_{\widetilde{y}}\right)}+\frac{\overline{r}_{\mathsf{s}}^*-\frac{2-v}{v-1}\overline{r}_{\mathsf{s}}^R}{\left((1-\mu)\,\sigma+(\phi-\mu)\,\kappa_{\widetilde{y}}\right)}
\end{aligned}
$$

This can be written as

$$
\begin{aligned}
\widetilde{y}_{\mathsf{s}}+\widetilde{y}_{\mathsf{s}}^* &= \frac{\delta\gamma_{\widetilde{y}}^{zlb}\gamma_{\widetilde{y}}-\gamma_{\widetilde{y}^R}^{zlb}\gamma_{\widetilde{y}}}{\delta\gamma_{\widetilde{y}}^{zlb}\gamma_{\widetilde{y}}}\widetilde{y}_{\mathsf{s}}^R+\frac{\delta\gamma_{\widetilde{y}}^{zlb}\gamma_{\widetilde{y}}-\gamma_{\widetilde{y}}^{zlb}\gamma_{\widetilde{y}^R}}{\delta\gamma_{\widetilde{y}}^{zlb}\gamma_{\widetilde{y}}}\widetilde{y}_{\mathsf{s}}^R \\
&\quad+\frac{\overline{r}_{\mathsf{s}}+\frac{2-v}{v-1}\overline{r}_{\mathsf{s}}^R+\ln(1/\beta)}{\left((1-\mu)\,\sigma-\mu\kappa_{\widetilde{y}}\right)}+\frac{\overline{r}_{\mathsf{s}}^*-\frac{2-v}{v-1}\overline{r}_{\mathsf{s}}^R}{\left((1-\mu)\,\sigma+(\phi-\mu)\,\kappa_{\widetilde{y}}\right)} \\
\widetilde{y}_{\mathsf{s}}+\widetilde{y}_{\mathsf{s}}^* &= \frac{2\delta\gamma_{\widetilde{y}}^{zlb}\gamma_{\widetilde{y}}-\gamma_{\widetilde{y}^R}^{zlb}\gamma_{\widetilde{y}}-\gamma_{\widetilde{y}}^{zlb}\gamma_{\widetilde{y}^R}}{\delta\gamma_{\widetilde{y}}^{zlb}\gamma_{\widetilde{y}}}\widetilde{y}_{\mathsf{s}}^R+\frac{\frac{\delta}{v-1}(2-v)\left(\gamma_{\widetilde{y}}-\gamma_{\widetilde{y}}^{zlb}\right)}{\delta\gamma_{\widetilde{y}}^{zlb}\gamma_{\widetilde{y}}}\overline{r}_{\mathsf{s}}^R \\
&\quad+\frac{\delta\gamma_{\widetilde{y}}\left(\overline{r}_{\mathsf{s}}+\ln(1/\beta)\right)}{\delta\gamma_{\widetilde{y}}^{zlb}\gamma_{\widetilde{y}}}+\frac{\delta\gamma_{\widetilde{y}}^{zlb}}{\delta\gamma_{\widetilde{y}}^{zlb}\gamma_{\widetilde{y}}}\overline{r}_{\mathsf{s}}^*
\end{aligned}
$$

Then from the above equations, average and relative world output can be written as

$$
\widetilde{y}_{\mathsf{s}}^W=\frac{\gamma_{\widetilde{y}^R}\gamma_{\widetilde{y}}^{zlb}-\gamma_{\widetilde{y}}\gamma_{\widetilde{y}^R}^{zlb}}{2\delta\gamma_{\widetilde{y}}\gamma_{\widetilde{y}}^{zlb}}\widetilde{y}_{\mathsf{s}}^R+\frac{\delta\left(\gamma_{\widetilde{y}}+\gamma_{\widetilde{y}}^{zlb}\right)\overline{r}_{\mathsf{s}}^W+\frac{\delta}{v-1}\left(\gamma_{\widetilde{y}}-\gamma_{\widetilde{y}}^{zlb}\right)\overline{r}_{\mathsf{s}}^R+\delta\gamma_{\widetilde{y}}\ln(1/\beta)}{2\delta\gamma_{\widetilde{y}}\gamma_{\widetilde{y}}^{zlb}} \tag{68}
$$

$$
\widetilde{y}_{\mathsf{s}}^R=\frac{\delta\left(\gamma_{\widetilde{y}}-\gamma_{\widetilde{y}}^{zlb}\right)\overline{r}_{\mathsf{s}}^W+\frac{\delta}{v-1}\left(\gamma_{\widetilde{y}}+\gamma_{\widetilde{y}}^{zlb}\right)\overline{r}_{\mathsf{s}}^R+\delta\gamma_{\widetilde{y}}\ln(1/\beta)}{\gamma_{\widetilde{y}}\gamma_{\widetilde{y}^R}^{zlb}+\gamma_{\widetilde{y}}^{zlb}\gamma_{\widetilde{y}^R}}, \tag{69}
$$

with $\widetilde{y}_{\mathsf{s}}^{R} < \widetilde{y}_{\mathsf{s}}^{W} < 0$. The expressions are negative for a negative relative home demand shock and so is home output, $\widetilde{y}_{\mathsf{s}} = \widetilde{y}_{\mathsf{s}}^{W} + \widetilde{y}_{\mathsf{s}}^{R} < 0$.

A.4 Imperfect information and sticky prices: equilibrium

A.4.1 Household's i Euler equation and aggregate consumption

Household i's Euler equation equals for $\varpi = 0$:

$$
\begin{aligned}
c_{i,t} &= \mu c_{i,t+1} - \frac{1}{\sigma}(\mu\xi_{i,t+1} - \xi_{i,t} + E_t^i[r_t - \pi_{t+1}]), \text{ or} \\
c_{i,t} &= \mu c_{i,t+1} - \frac{1}{\sigma}(\mu\xi_{i,t+1} - \xi_{i,t}) \\
&\quad -\frac{1}{\sigma}\mathsf{p}_i^{good}(r_{good,t} - \mu\pi_{good,t+1}) - \frac{1}{\sigma}\mathsf{p}_i^{bad}(r_{bad,t} - \mu\pi_{bad,t+1})
\end{aligned}
$$

Then integrating across households of high- and low-type in state s gives

$$
\begin{aligned}
\lambda_{\mathsf{s}} c_t^{hu} + (1-\lambda_{\mathsf{s}})\, c_t^{lu} &= \lambda_{\mathsf{s}}\mu c_{t+1}^{h} + (1-\lambda_{\mathsf{s}})\,\mu c_{t+1}^{l} - \frac{(\mu-1)}{\sigma}\lambda_{\mathsf{s}}\xi_h - \frac{(\mu-1)}{\sigma}(1-\lambda_{\mathsf{s}})\,\xi_l \\
&\quad -\lambda_{\mathsf{s}}\frac{1}{\sigma}\mathsf{p}_h^{good}(r_{good,t} - \mu\pi_{good,t+1}) - (1-\lambda_{\mathsf{s}})\,\frac{1}{\sigma}\mathsf{p}_l^{good}(r_{good,t} - \mu\pi_{good,t+1}) \\
&\quad -\lambda_{\mathsf{s}}\frac{1}{\sigma}\mathsf{p}_h^{bad}(r_{bad,t} - \mu\pi_{bad,t+1}) - (1-\lambda_{\lambda_{\mathsf{s}}})\,\frac{1}{\sigma}\mathsf{p}_h^{bad}(r_{bad,t} - \mu\pi_{bad,t+1})
\end{aligned}
$$

Then, from the cross-sectional mean we can write

$$
\begin{aligned}
c_{\mathsf{s},t} &= \mu c_{\mathsf{s},t+1} - \frac{(\mu-1)}{\sigma}\xi_{\mathsf{s}} - \frac{1}{\sigma}\overline{\mathsf{p}}_{\mathsf{s}}^{good}(r_{good,t} - \mu\pi_{good,t+1}) - \frac{1}{\sigma}\overline{\mathsf{p}}_{\mathsf{s}}^{bad}(r_{bad,t} - \mu\pi_{bad,t+1}) \\
c_{\mathsf{s},t} &= \mu c_{\mathsf{s},t+1} - \frac{(\mu-1)}{\sigma}\xi_{\mathsf{s}} - \frac{1}{\sigma}\overline{E}[r_t - \pi_{t+1}],
\end{aligned}
$$

with

$$
\begin{aligned}
\overline{E}[r_t - \pi_{t+1}] &= \overline{\mathsf{p}}_{\mathsf{s}}^{good}(r_{t,good} - \mu\pi_{Ht+1,good} + (2-v)(r_{t,good}^{R} - \mu\pi_{Ht+1,good}^{R})) \\
&\quad +(1-\overline{\mathsf{p}}_{\mathsf{s}}^{good})(r_{t,bad} - \mu\pi_{Ht+1,bad} + (2-v)(r_{t,bad}^{R} - \mu\pi_{Ht+1,bad}^{R})).
\end{aligned}
$$

Given the zero inflation condition in the natural economy we have consumption in deviation from the natural equilibrium

$$
\widetilde{c}_{\mathsf{s},t} = \mu\widetilde{c}_{\mathsf{s},t+1} + \frac{1}{\sigma}\overline{r}_{\mathsf{s},t} - \frac{1}{\sigma}\overline{\mathsf{p}}_{\mathsf{s}}^{good}(r_{good,t} - \mu\pi_{good,t+1}) - \frac{1}{\sigma}\overline{\mathsf{p}}_{\mathsf{s}}^{bad}(r_{bad,t} - \mu\pi_{bad,t+1}),
$$

as used in Section 3 and the following sections. The foreign country will have a similar Euler equation. Based on this, we conduct similar steps as outlined above to arrive at the following set of equilibrium equations under imperfect information.

A.4.2 ZLB is not binding in both countries

$$
\begin{aligned}
\widetilde{y}_{good}^{R} - \widetilde{y}_{bad}^{R} &= \frac{\frac{\frac{1}{\sigma}}{1-\mu}\frac{1}{v-1}}{\frac{(v-1)}{\delta} + \left(1 - \overline{\mathsf{p}}_{good}^{bad} - \overline{\mathsf{p}}_{bad}^{good}\right)\left(\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}-\kappa_{(y-y^*)}}{(1-\beta\mu)} + \frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_e}{(1-\beta\mu)}\right)}\left(\overline{r}_{good}^{R} - \overline{r}_{bad}^{R}\right) \\
\pi_{Hgood}^{R} - \pi_{Hbad}^{R} &= \frac{\frac{\kappa}{2} - \kappa_{(y-y^*)}}{(1-\beta\mu)}\frac{\frac{\frac{1}{\sigma}}{1-\mu}\frac{1}{v-1}}{\frac{(v-1)}{\delta} + \left(1 - \overline{\mathsf{p}}_{good}^{bad} - \overline{\mathsf{p}}_{bad}^{good}\right)\left(\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}-\kappa_{(y-y^*)}}{(1-\beta\mu)} + \frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_e}{(1-\beta\mu)}\right)}\left(\overline{r}_{good}^{R} - \overline{r}_{bad}^{R}\right) \\
\pi_{good}^{R} - \pi_{bad}^{R} &= \left(\pi_{Hgood}^{R} - \pi_{Hbad}^{R}\right) - \frac{\kappa_e}{(1-\beta\mu)}\left(\widetilde{y}_{good}^{R} - \widetilde{y}_{bad}^{R}\right) + \frac{1}{\mu}\frac{2-v}{v-1}\left(\overline{r}_{good}^{R} - \overline{r}_{bad}^{R}\right)
\end{aligned}
$$

and for the good state

$$
\begin{aligned}
\widetilde{y}^R_{good} &= \frac{\frac{\frac{1}{\sigma}}{1-\mu}\frac{1}{v-1}}{\frac{1}{\delta}+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}-\kappa_{(y-y^*)}}{(1-\beta\mu)}}\overline{r}^R_{good}+\overline{\mathsf{p}}^{bad}_{good}\frac{\left(\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}+\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_e}{\frac{\kappa}{2}-\kappa_{(y-y^*)}}\right)}{\frac{1}{\delta}+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}-\kappa_{(y-y^*)}}{(1-\beta\mu)}}\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right)\\
\pi^R_{Hgood} &= \frac{\frac{\kappa}{2}-\kappa_{(y-y^*)}}{(1-\beta\mu)}\widetilde{y}^R_{good}\\
\pi^R_{good} &= \pi^R_{Hgood}-\frac{\kappa_e}{(1-\beta\mu)}\widetilde{y}^R_{good}+\frac{1}{\mu}\frac{2-v}{v-1}\overline{r}^R_{good}\\
q_{good} &= -(v-1)\frac{(\phi-\mu)}{1-\mu}\pi^R_{Hgood}
\end{aligned}
$$

and in the bad state

$$
\begin{aligned}
\widetilde{y}^R_{bad} &= \frac{\frac{\frac{1}{\sigma}}{1-\mu}\frac{1}{v-1}}{\frac{1}{\delta}+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}-\kappa_{(y-y^*)}}{(1-\beta\mu)}}\overline{r}^R_{bad}-\overline{\mathsf{p}}^{good}_{bad}\frac{\left(\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}+\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_e}{\frac{\kappa}{2}-\kappa_{(y-y^*)}}\right)}{\frac{1}{\delta}+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}-\kappa_{(y-y^*)}}{(1-\beta\mu)}}\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right)\\
\pi^R_{Hbad} &= \frac{\frac{\kappa}{2}-\kappa_{(y-y^*)}}{(1-\beta\mu)}\widetilde{y}^R_{bad}\\
\pi^R_{bad} &= \pi^R_{Hgood}-\frac{\kappa_e}{(1-\beta\mu)}\widetilde{y}^R_{bad}+\frac{1}{\mu}\frac{2-v}{v-1}\overline{r}^R_{bad}\\
q_{bad} &= -(v-1)\frac{(\phi-\mu)}{1-\mu}\pi^R_{Hbad}
\end{aligned}
$$

and for home country equations

$$
\begin{aligned}
\widetilde{y}_{good}-\widetilde{y}_{bad} &= \frac{1-\frac{(v-1)}{\delta}+\left(1-\overline{\mathsf{p}}^{bad}_{good}-\overline{\mathsf{p}}^{good}_{bad}\right)\left(\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\kappa_{(y-y^*)}}{1-\beta\mu}-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_e}{(1-\beta\mu)}\right)}{1+\left(1-\overline{\mathsf{p}}^{bad}_{good}-\overline{\mathsf{p}}^{good}_{bad}\right)\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\left(\widetilde{y}^R_{good}-\widetilde{y}^R_{bad}\right)\\
&\quad+\frac{\frac{\frac{1}{\sigma}}{1-\mu}\left(\left(\overline{r}_{good}-\overline{r}_{bad}\right)+\frac{2-v}{v-1}\left(\overline{r}^R_{good}-\overline{r}^R_{bad}\right)\right)}{1+\left(1-\overline{\mathsf{p}}^{bad}_{good}-\overline{\mathsf{p}}^{good}_{bad}\right)\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\\
\pi_{Hgood}-\pi_{Hbad} &= \frac{\frac{\kappa}{2}}{1-\beta\mu}\left(\widetilde{y}_{good}-\widetilde{y}_{bad}\right)-\frac{\kappa_{(y-y^*)}}{1-\beta\mu}\left(\widetilde{y}^R_{good}-\widetilde{y}^R_{bad}\right)\\
\pi_{good}-\pi_{bad} &= \pi_{Hgood}-\pi_{Hbad}-\frac{\kappa_e}{1-\mu\beta}\left(\widetilde{y}^R_{good}-\widetilde{y}^R_{bad}\right)+\frac{1}{\mu}\frac{2-v}{v-1}\left(\overline{r}^R_{good}-\overline{r}^R_{bad}\right)
\end{aligned}
$$

Then in the good state

$$
\begin{aligned}
\widetilde{y}_{good} &= \frac{\left(1-\frac{1}{\delta}\right)+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\kappa_{(y-y^*)}}{1-\beta\mu}}{1+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\widetilde{y}^R_{good}+\frac{\frac{\frac{1}{\sigma}}{1-\mu}\left(\overline{r}_{good}+\frac{2-v}{v-1}\overline{r}^R_{good}\right)}{1+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\\
&\quad+\overline{\mathsf{p}}^{bad}_{good}\left(\frac{\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}}{1+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\left(\pi_{Hgood}-\pi_{Hbad}\right)+\frac{\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_e}{\frac{\kappa}{2}-\kappa_{(y-y^*)}}}{1+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right)\right)\\
\pi_{Hgood} &= \frac{\frac{\kappa}{2}}{1-\beta\mu}\widetilde{y}_{good}-\frac{\kappa_{(y-y^*)}}{1-\beta\mu}\widetilde{y}^R_{good}\\
\pi_{good} &= \pi_{Hgood}-\frac{\kappa_e}{1-\mu\beta}\widetilde{y}^R_{good}+\frac{1}{\mu}\frac{2-v}{v-1}\overline{r}^R_{good}
\end{aligned}
$$

Now we express everything in relative in world averages:

Noting that

$$
\begin{aligned}
\kappa_{\widetilde{y}} &= \frac{\kappa}{2}/(1-\beta\mu)>0 \qquad \kappa_{\widetilde{y}^R}=\left(\frac{\kappa}{2}-\kappa_{(y-y^*)}\right)/(1-\beta\mu)>0\\
\kappa_e &\equiv (1-\beta\mu)(2-\nu)\,\sigma(1-\mu)/(\mu\delta)\\
\kappa_{\widetilde{e}} &= \kappa_e/(1-\beta\mu)
\end{aligned}
$$

$$\widetilde{y}^R_{good} = \frac{\frac{\frac{1}{\sigma}}{1-\mu}\frac{1}{v-1}}{\frac{1}{\delta}+\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\widetilde{yR}}}{\sigma}}\overline{r}^R_{good} + \overline{\mathsf{p}}^{bad}_{good}\frac{\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}+\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_{\widetilde{e}}}{\kappa_{\widetilde{yR}}}}{\frac{1}{\delta}+\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\widetilde{yR}}}{\sigma}}\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right)$$

$$\widetilde{y}^R_{good} = \frac{\frac{\delta}{v-1}\overline{r}^R_{good}}{(1-\mu)\sigma+(\phi-\mu)\delta\kappa_{\widetilde{y}^R}} + \overline{\mathsf{p}}^{bad}_{good}\frac{\delta\left((\phi-\mu)+\mu\frac{\kappa_{\widetilde{e}}}{\kappa_{\widetilde{y}R}}\right)}{(1-\mu)\sigma+(\phi-\mu)\delta\kappa_{\widetilde{y}^R}}\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right)$$

$$\pi^R_{Hgood} = \kappa_{\widetilde{y}^R}\widetilde{y}^R_{good}$$

$$\pi^R_{Hgood} = \frac{\kappa_{\widetilde{y}^R}\frac{\delta}{v-1}\overline{r}^R_{good}}{(1-\mu)\sigma+(\phi-\mu)\delta\kappa_{\widetilde{y}^R}} + \overline{\mathsf{p}}^{bad}_{good}\frac{\delta\left(\kappa_{\widetilde{y}^R}(\phi-\mu)+\mu\kappa_{\widetilde{e}}\right)}{(1-\mu)\sigma+(\phi-\mu)\delta\kappa_{\widetilde{y}^R}}\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right)$$

$$\widetilde{y}^W_{good} = \frac{\frac{\frac{1}{\sigma}}{1-\mu}\overline{r}^W_{good}}{1+\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\widetilde{y}}}{\sigma}} + \overline{\mathsf{p}}^{bad}_{good}\frac{\frac{\phi-\mu}{1-\mu}\frac{1}{\sigma}}{1+\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\widetilde{y}}}{\sigma}}\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right)$$

$$\widetilde{y}^W_{good} = \frac{\overline{r}^W_{good}}{(1-\mu)\sigma+(\phi-\mu)\kappa_{\widetilde{y}}} + \overline{\mathsf{p}}^{bad}_{good}\frac{(\phi-\mu)}{(1-\mu)\sigma+(\phi-\mu)\kappa_{\widetilde{y}}}\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right)$$

$$\pi^W_{Hgood} = \kappa_{\widetilde{y}}\widetilde{y}^W_{good}$$

$$\pi^W_{Hgood} = \frac{\kappa_{\widetilde{y}}\overline{r}^W_{good}}{(1-\mu)\sigma+(\phi-\mu)\kappa_{\widetilde{y}}} + \overline{\mathsf{p}}^{bad}_{good}\frac{\kappa_{\widetilde{y}}(\phi-\mu)}{(1-\mu)\sigma+(\phi-\mu)\kappa_{\widetilde{y}}}\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right)$$

The last terms reflect the perceived real interest rate in the sticky price environment, whereby

$$\widetilde{y}^R_{good}-\widetilde{y}^R_{bad} = \frac{\frac{1}{\sigma}\frac{1}{1-\mu}\frac{\overline{r}^R_{good}-\overline{r}^R_{bad}}{v-1}}{\frac{(v-1)}{\delta}+\left(1-\overline{\mathsf{p}}^{bad}_{good}-\overline{\mathsf{p}}^{good}_{bad}\right)\left(\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\widetilde{yR}}}{\sigma}+\frac{\mu}{1-\mu}\frac{\kappa_{\widetilde{e}}}{\sigma}\right)}$$

$$\widetilde{y}^R_{good}-\widetilde{y}^R_{bad} = \frac{\frac{\delta}{v-1}(\overline{r}^R_{good}-\overline{r}^R_{bad})}{(v-1)(1-\mu)\sigma+(1-\overline{\mathsf{p}}^{bad}_{good}-\overline{\mathsf{p}}^{good}_{bad})\delta((\phi-\mu)\kappa_{\widetilde{y}^R}+\mu\kappa_{\widetilde{e}})}$$

$$\pi^R_{Hgood}-\pi^R_{Hbad} = \frac{\frac{\kappa_{\widetilde{yR}}}{\sigma}\frac{1}{1-\mu}\frac{\overline{r}^R_{good}-\overline{r}^R_{bad}}{v-1}}{\frac{(v-1)}{\delta}+\left(1-\overline{\mathsf{p}}^{bad}_{good}-\overline{\mathsf{p}}^{good}_{bad}\right)\left(\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\widetilde{yR}}}{\sigma}+\frac{\mu}{1-\mu}\frac{\kappa_{\widetilde{e}}}{\sigma}\right)}$$

$$\pi^R_{Hgood}-\pi^R_{Hbad} = \frac{\kappa_{\widetilde{y}^R}\frac{\delta}{v-1}(\overline{r}^R_{good}-\overline{r}^R_{bad})}{(v-1)(1-\mu)\sigma+(1-\overline{\mathsf{p}}^{bad}_{good}-\overline{\mathsf{p}}^{good}_{bad})\delta((\phi-\mu)\kappa_{\widetilde{y}^R}+\mu\kappa_{\widetilde{e}})}$$

$$\widetilde{y}^W_{good}-\widetilde{y}^W_{bad} = \frac{\frac{1}{\sigma}\frac{1}{1-\mu}\left(\overline{r}^W_{good}-\overline{r}^W_{bad}\right)}{1+\left(1-\overline{\mathsf{p}}^{bad}_{good}-\overline{\mathsf{p}}^{good}_{bad}\right)\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\widetilde{y}}}{\sigma}}$$

$$\widetilde{y}^W_{good}-\widetilde{y}^W_{bad} = \frac{\overline{r}^W_{good}-\overline{r}^W_{bad}}{(1-\mu)\sigma+(1-\overline{\mathsf{p}}^{bad}_{good}-\overline{\mathsf{p}}^{good}_{bad})(\phi-\mu)\kappa_{\widetilde{y}}}$$

$$\pi^W_{Hgood}-\pi^W_{Hbad} = \frac{\frac{\kappa_{\widetilde{y}}}{\sigma}\frac{1}{1-\mu}\left(\overline{r}^W_{good}-\overline{r}^W_{bad}\right)}{1+\left(1-\overline{\mathsf{p}}^{bad}_{good}-\overline{\mathsf{p}}^{good}_{bad}\right)\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\widetilde{y}}}{\sigma}}$$

$$\pi^W_{Hgood}-\pi^W_{Hbad} = \frac{\kappa_{\widetilde{y}}(\overline{r}^W_{good}-\overline{r}^W_{bad})}{(1-\mu)\sigma+(1-\overline{\mathsf{p}}^{bad}_{good}-\overline{\mathsf{p}}^{good}_{bad})(\phi-\mu)\kappa_{\widetilde{y}}}$$

Then

$$\widetilde{y}_{good} = \widetilde{y}^W_{good}+\widetilde{y}^R_{good}$$

$$\widetilde{y}_{good} = \frac{\frac{\frac{1}{\sigma}}{1-\mu}\overline{r}^W_{good}}{1+\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\widetilde{y}}}{\sigma}} + \frac{\frac{\frac{1}{\sigma}}{1-\mu}\frac{\overline{r}^R_{good}}{v-1}}{\frac{1}{\delta}+\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\widetilde{yR}}}{\sigma}} + \overline{\mathsf{p}}^{bad}_{good}\frac{\frac{\phi-\mu}{1-\mu}\frac{1}{\sigma}}{1+\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\widetilde{y}}}{\sigma}}\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right) + \overline{\mathsf{p}}^{bad}_{good}\frac{\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}+\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_{\widetilde{e}}}{\kappa_{\widetilde{yR}}}}{\frac{1}{\delta}+\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\widetilde{yR}}}{\sigma}}\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right)$$

$$\pi_{Hgood} = \pi^W_{Hgood}+\pi^R_{Hgood}$$

$$\pi_{Hgood} = \frac{\frac{\kappa_{\tilde{y}}}{\sigma}\frac{\bar{r}^W_{good}}{1-\mu}}{1+\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\tilde{y}}}{\sigma}} + \frac{\frac{\kappa_{\tilde{y}^R}}{\sigma}\frac{1}{v-1}\frac{\bar{r}^R_{good}}{1-\mu}}{\frac{1}{\delta}+\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\tilde{y}^R}}{\sigma}} + \bar{\mathsf{p}}^{bad}_{good}\frac{\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\tilde{y}}}{\sigma}}{1+\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\tilde{y}}}{\sigma}}\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right) + \bar{\mathsf{p}}^{bad}_{good}\frac{\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\kappa_{\tilde{y}^R}+\frac{\frac{\mu}{\sigma}}{1-\mu}\kappa_{\tilde{e}}}{\frac{1}{\delta}+\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\tilde{y}^R}}{\sigma}}\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right)$$

Or rewriting the denominator:

$$\tilde{y}_{good} = \frac{\bar{r}^W_{good}+\bar{\mathsf{p}}^{bad}_{good}(\phi-\mu)\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right)}{(1-\mu)\sigma+(\phi-\mu)\kappa_{\tilde{y}}} + \frac{\frac{\delta}{v-1}\bar{r}^R_{good}+\bar{\mathsf{p}}^{bad}_{good}\delta\left((\phi-\mu)+\mu\frac{\kappa_{\tilde{e}}}{\kappa_{\tilde{y}^R}}\right)\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right)}{(1-\mu)\sigma+(\phi-\mu)\delta\kappa_{\tilde{y}^R}}$$

$$\pi_{Hgood} = \frac{\kappa_{\tilde{y}}\bar{r}^W_{good}+\bar{\mathsf{p}}^{bad}_{good}\kappa_{\tilde{y}}(\phi-\mu)\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right)}{(1-\mu)\sigma+(\phi-\mu)\kappa_{\tilde{y}}} + \frac{\kappa_{\tilde{y}^R}\frac{\delta}{v-1}\bar{r}^R_{good}+\bar{\mathsf{p}}^{bad}_{good}\delta\left(\kappa_{\tilde{y}^R}(\phi-\mu)+\mu\kappa_{\tilde{e}}\right)\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right)}{(1-\mu)\sigma+(\phi-\mu)\delta\kappa_{\tilde{y}^R}}$$

Using the definitions

$$\begin{aligned}
\gamma_{\tilde{y}} &= (1-\mu)\sigma+(\phi-\mu)\kappa_{\tilde{y}} \\
\gamma_{\tilde{y}^R} &= (1-\mu)\sigma+(\phi-\mu)\delta\kappa_{\tilde{y}^R}
\end{aligned}$$

we can write

$$\begin{aligned}
\tilde{y}^R_{good} &= \frac{\frac{\delta}{v-1}\bar{r}^R_{good}}{\gamma_{\tilde{y}^R}} + \bar{\mathsf{p}}^{bad}_{good}\frac{\delta\left(1+\frac{\mu\kappa_{\tilde{e}}}{(\phi-\mu)\kappa_{\tilde{y}^R}}\right)}{\gamma_{\tilde{y}^R}}(\phi-\mu)\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right) \\
\pi^R_{Hgood} &= \frac{\kappa_{\tilde{y}^R}\frac{\delta}{v-1}\bar{r}^R_{good}}{\gamma_{\tilde{y}^R}} + \bar{\mathsf{p}}^{bad}_{good}\frac{\delta\left(\kappa_{\tilde{y}^R}+\frac{\mu\kappa_{\tilde{e}}}{(\phi-\mu)}\right)}{\gamma_{\tilde{y}^R}}(\phi-\mu)\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right) \\
\tilde{y}^W_{good} &= \frac{\bar{r}^W_{good}}{\gamma_{\tilde{y}}} + \bar{\mathsf{p}}^{bad}_{good}\frac{1}{\gamma_{\tilde{y}}}(\phi-\mu)\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right) \\
\pi^W_{Hgood} &= \frac{\kappa_{\tilde{y}}\bar{r}^W_{good}}{\gamma_{\tilde{y}}} + \bar{\mathsf{p}}^{bad}_{good}\frac{\kappa_{\tilde{y}}}{\gamma_{\tilde{y}}}(\phi-\mu)\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right),
\end{aligned}$$

with $(\phi-\mu)\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right)$ and $(\phi-\mu)\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right)$ defining the perceived interest rates. Then

$$\begin{aligned}
\tilde{y}_{good} &= \frac{\bar{r}^W_{good}+\bar{\mathsf{p}}^{bad}_{good}(\phi-\mu)\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right)}{\gamma_{\tilde{y}}} + \frac{\frac{\delta}{v-1}\bar{r}^R_{good}+\bar{\mathsf{p}}^{bad}_{good}\delta\left(1+\frac{\mu\kappa_{\tilde{e}}}{(\phi-\mu)\kappa_{\tilde{y}^R}}\right)(\phi-\mu)\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right)}{\gamma_{\tilde{y}^R}} \\
\tilde{y}_{good} &= \frac{\bar{r}^W_{good}}{\gamma_{\tilde{y}}} + \frac{\frac{\delta}{v-1}\bar{r}^R_{good}}{\gamma_{\tilde{y}^R}} + \bar{\mathsf{p}}^{bad}_{good}\frac{1}{\gamma_{\tilde{y}}}(\phi-\mu)\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right) + \bar{\mathsf{p}}^{bad}_{good}\frac{\delta\left(1+\frac{\mu\kappa_{\tilde{e}}}{(\phi-\mu)\kappa_{\tilde{y}^R}}\right)}{\gamma_{\tilde{y}^R}}(\phi-\mu)\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right) \\
\pi_{Hgood} &= \frac{\kappa_{\tilde{y}}\bar{r}^W_{good}+\bar{\mathsf{p}}^{bad}_{good}\kappa_{\tilde{y}}(\phi-\mu)\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right)}{\gamma_{\tilde{y}}} + \frac{\kappa_{\tilde{y}^R}\frac{\delta}{v-1}\bar{r}^R_{good}+\bar{\mathsf{p}}^{bad}_{good}\delta\left(\kappa_{\tilde{y}^R}+\frac{\mu\kappa_{\tilde{e}}}{(\phi-\mu)}\right)(\phi-\mu)\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right)}{\gamma_{\tilde{y}^R}} \\
\pi_{Hgood} &= \frac{\kappa_{\tilde{y}}}{\gamma_{\tilde{y}}}\bar{r}^W_{good} + \frac{\kappa_{\tilde{y}^R}\frac{\delta}{v-1}}{\gamma_{\tilde{y}^R}}\bar{r}^R_{good} + \bar{\mathsf{p}}^{bad}_{good}\frac{\kappa_{\tilde{y}}}{\gamma_{\tilde{y}}}(\phi-\mu)\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right) + \bar{\mathsf{p}}^{bad}_{good}\frac{\delta\left(\kappa_{\tilde{y}^R}+\frac{\mu\kappa_{\tilde{e}}}{(\phi-\mu)}\right)}{\gamma_{\tilde{y}^R}}(\phi-\mu)\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right)
\end{aligned}$$

In the bad state we have

$$\begin{aligned}
\tilde{y}_{bad} = {} & \frac{\left(1-\frac{1}{\delta}\right)+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\kappa_{(y-y^*)}}{1-\beta\mu}}{1+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\tilde{y}^R_{bad} + \frac{\frac{\frac{1}{\sigma}}{1-\mu}}{1+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\bar{r}_{bad} \\
& -\bar{\mathsf{p}}^{good}_{bad}\left(\frac{\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}}{1+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\left(\pi_{Hgood}-\pi_{Hbad}\right) + \frac{\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_e}{\frac{\kappa}{2}-\kappa_{(y-y^*)}}}{1+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right)\right)
\end{aligned}$$

$$
\begin{aligned}
\pi_{Hbad} &= \frac{\frac{\kappa}{2}}{1-\beta\mu}\widetilde{y}_{bad} - \frac{\kappa_{(y-y^*)}}{1-\beta\mu}\widetilde{y}^R_{bad} \\
\pi_{bad} &= \pi_{Hbad} - \frac{\kappa_e}{1-\mu\beta}\widetilde{y}^R_{bad} + \frac{1}{\mu}\frac{2-v}{v-1}\overline{r}^R_{bad}
\end{aligned}
$$

and in terms of world relatives and averages

$$
\begin{aligned}
\widetilde{y}^R_{bad} &= \frac{\frac{\frac{1}{\sigma}}{1-\mu}\frac{1}{v-1}}{\frac{1}{\delta}+\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\widetilde{y}^R}}{\sigma}}\overline{r}^R_{bad} - \overline{\mathsf{p}}^{good}_{bad}\frac{\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}+\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_{\widetilde{e}}}{\kappa_{\widetilde{y}^R}}}{\frac{1}{\delta}+\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\widetilde{y}^R}}{\sigma}}\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right) \\
\widetilde{y}^R_{bad} &= \frac{\frac{\delta}{v-1}\overline{r}^R_{bad}}{(1-\mu)\sigma+(\phi-\mu)\delta\kappa_{\widetilde{y}^R}} - \overline{\mathsf{p}}^{good}_{bad}\frac{\delta\left((\phi-\mu)+\mu\frac{\kappa_{\widetilde{e}}}{\kappa_{\widetilde{y}^R}}\right)}{(1-\mu)\sigma+(\phi-\mu)\delta\kappa_{\widetilde{y}^R}}\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right) \\
\pi^R_{Hbad} &= \kappa_{\widetilde{y}^R}\widetilde{y}^R_{bad} \\
\pi^R_{Hbad} &= \frac{\kappa_{\widetilde{y}^R}\frac{\delta}{v-1}\overline{r}^R_{bad}}{(1-\mu)\sigma+(\phi-\mu)\delta\kappa_{\widetilde{y}^R}} - \overline{\mathsf{p}}^{good}_{bad}\frac{\delta\left(\kappa_{\widetilde{y}^R}(\phi-\mu)+\mu\kappa_{\widetilde{e}}\right)}{(1-\mu)\sigma+(\phi-\mu)\delta\kappa_{\widetilde{y}^R}}\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right) \\
\widetilde{y}^W_{bad} &= \frac{\frac{\frac{1}{\sigma}}{1-\mu}\overline{r}^W_{bad}}{1+\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\widetilde{y}}}{\sigma}} - \overline{\mathsf{p}}^{good}_{bad}\frac{\frac{\phi-\mu}{1-\mu}\frac{1}{\sigma}}{1+\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\widetilde{y}}}{\sigma}}\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right) \\
\widetilde{y}^W_{good} &= \frac{\overline{r}^W_{bad}}{(1-\mu)\sigma+(\phi-\mu)\kappa_{\widetilde{y}}} - \overline{\mathsf{p}}^{good}_{bad}\frac{(\phi-\mu)}{(1-\mu)\sigma+(\phi-\mu)\kappa_{\widetilde{y}}}\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right) \\
\pi^W_{Hbad} &= \kappa_{\widetilde{y}}\widetilde{y}^W_{bad} \\
\pi^W_{Hbad} &= \frac{\kappa_{\widetilde{y}}\overline{r}^W_{bad}}{(1-\mu)\sigma+(\phi-\mu)\kappa_{\widetilde{y}}} - \overline{\mathsf{p}}^{good}_{bad}\frac{\kappa_{\widetilde{y}}(\phi-\mu)}{(1-\mu)\sigma+(\phi-\mu)\kappa_{\widetilde{y}}}\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right)
\end{aligned}
$$

so that

$$
\begin{aligned}
\widetilde{y}_{bad} &= \widetilde{y}^W_{bad}+\widetilde{y}^R_{bad} \\
\widetilde{y}_{bad} &= \frac{\frac{\frac{1}{\sigma}}{1-\mu}\overline{r}^W_{bad}}{1+\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\widetilde{y}}}{\sigma}} + \frac{\frac{\frac{1}{\sigma}}{1-\mu}\frac{\overline{r}^R_{bad}}{v-1}}{\frac{1}{\delta}+\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\widetilde{y}^R}}{\sigma}} - \overline{\mathsf{p}}^{good}_{bad}\frac{\frac{\phi-\mu}{1-\mu}\frac{1}{\sigma}}{1+\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\widetilde{y}}}{\sigma}}\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right) - \overline{\mathsf{p}}^{good}_{bad}\frac{\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}+\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_{\widetilde{e}}}{\kappa_{\widetilde{y}^R}}}{\frac{1}{\delta}+\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\widetilde{y}^R}}{\sigma}}\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right)
\end{aligned}
$$

$$
\begin{aligned}
\pi_{Hbad} &= \pi^W_{Hbad}+\pi^R_{Hbad} \\
\pi_{Hbad} &= \frac{\frac{\kappa_{\widetilde{y}}}{\sigma}\frac{\overline{r}^W_{\mathsf{s}}}{1-\mu}}{1+\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\widetilde{y}}}{\sigma}} + \frac{\frac{\kappa_{\widetilde{y}^R}}{\sigma}\frac{1}{v-1}\frac{\overline{r}^R_{\mathsf{s}}}{1-\mu}}{\frac{1}{\delta}+\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\widetilde{y}^R}}{\sigma}} - \overline{\mathsf{p}}^{good}_{bad}\frac{\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\widetilde{y}}}{\sigma}}{1+\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\widetilde{y}}}{\sigma}}\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right) - \overline{\mathsf{p}}^{good}_{bad}\frac{\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\kappa_{\widetilde{y}^R}+\frac{\frac{\mu}{\sigma}}{1-\mu}\kappa_{\widetilde{e}}}{\frac{1}{\delta}+\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\widetilde{y}^R}}{\sigma}}\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right)
\end{aligned}
$$

Or rewriting the denominator:

$$
\begin{aligned}
\widetilde{y}_{bad} &= \frac{\overline{r}^W_{bad}-\overline{\mathsf{p}}^{good}_{bad}(\phi-\mu)\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right)}{(1-\mu)\sigma+(\phi-\mu)\kappa_{\widetilde{y}}} + \frac{\frac{\delta}{v-1}\overline{r}^R_{bad}-\overline{\mathsf{p}}^{good}_{bad}\delta\left((\phi-\mu)+\mu\frac{\kappa_{\widetilde{e}}}{\kappa_{\widetilde{y}^R}}\right)\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right)}{(1-\mu)\sigma+(\phi-\mu)\delta\kappa_{\widetilde{y}^R}} \\
\pi_{Hbad} &= \frac{\kappa_{\widetilde{y}}\overline{r}^W_{bad}-\overline{\mathsf{p}}^{good}_{bad}\kappa_{\widetilde{y}}(\phi-\mu)\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right)}{(1-\mu)\sigma+(\phi-\mu)\kappa_{\widetilde{y}}} + \frac{\kappa_{\widetilde{y}^R}\frac{\delta}{v-1}\overline{r}^R_{bad}-\overline{\mathsf{p}}^{good}_{bad}\delta\left(\kappa_{\widetilde{y}^R}(\phi-\mu)+\mu\kappa_{\widetilde{e}}\right)\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right)}{(1-\mu)\sigma+(\phi-\mu)\delta\kappa_{\widetilde{y}^R}}
\end{aligned}
$$

And using the definitions from above

$$
\begin{aligned}
\widetilde{y}^R_{bad} &= \frac{\frac{\delta}{v-1}\overline{r}^R_{bad}}{\gamma_{\widetilde{y}^R}} - \overline{\mathsf{p}}^{good}_{bad}\frac{\delta\left(1+\frac{\mu\kappa_{\widetilde{e}}}{(\phi-\mu)\kappa_{\widetilde{y}^R}}\right)}{\gamma_{\widetilde{y}^R}}(\phi-\mu)\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right) \\
\pi^R_{Hbad} &= \frac{\kappa_{\widetilde{y}^R}\frac{\delta}{v-1}\overline{r}^R_{bad}}{\gamma_{\widetilde{y}^R}} - \overline{\mathsf{p}}^{good}_{bad}\frac{\delta\left(\kappa_{\widetilde{y}^R}+\frac{\mu\kappa_{\widetilde{e}}}{(\phi-\mu)}\right)}{\gamma_{\widetilde{y}^R}}(\phi-\mu)\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right)
\end{aligned}
$$

$$\widetilde{y}^W_{good} = \frac{\overline{r}^W_{bad}}{\gamma_{\widetilde{y}}} - \overline{\mathsf{p}}^{good}_{bad}\frac{1}{\gamma_{\widetilde{y}}}(\phi-\mu)\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right)$$

$$\pi^W_{Hbad} = \frac{\kappa_{\widetilde{y}}\overline{r}^W_{bad}}{\gamma_{\widetilde{y}}} - \overline{\mathsf{p}}^{good}_{bad}\frac{\kappa_{\widetilde{y}}}{\gamma_{\widetilde{y}}}(\phi-\mu)\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right)$$

$$\widetilde{y}_{bad} = \frac{\overline{r}^W_{bad} - \overline{\mathsf{p}}^{good}_{bad}(\phi-\mu)\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right)}{\gamma_{\widetilde{y}}} + \frac{\frac{\delta}{v-1}\overline{r}^R_{bad} - \overline{\mathsf{p}}^{good}_{bad}\delta\left(1+\frac{\mu\kappa_{\widetilde{e}}}{(\phi-\mu)\kappa_{\widetilde{y}^R}}\right)(\phi-\mu)\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right)}{\gamma_{\widetilde{y}^R}}$$

$$\widetilde{y}_{bad} = \frac{\overline{r}^W_{bad}}{\gamma_{\widetilde{y}}} + \frac{\frac{\delta}{v-1}\overline{r}^R_{bad}}{\gamma_{\widetilde{y}^R}} - \frac{\overline{\mathsf{p}}^{good}_{bad}}{\gamma_{\widetilde{y}}}(\phi-\mu)\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right) - \overline{\mathsf{p}}^{good}_{bad}\frac{\delta\left(1+\frac{\mu\kappa_{\widetilde{e}}}{(\phi-\mu)\kappa_{\widetilde{y}^R}}\right)}{\gamma_{\widetilde{y}^R}}(\phi-\mu)\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right)$$

$$\pi_{Hbad} = \frac{\kappa_{\widetilde{y}}\overline{r}^W_{bad} - \overline{\mathsf{p}}^{good}_{bad}\kappa_{\widetilde{y}}(\phi-\mu)\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right)}{\gamma_{\widetilde{y}}} + \frac{\kappa_{\widetilde{y}^R}\frac{\delta}{v-1}\overline{r}^R_{bad} - \overline{\mathsf{p}}^{good}_{bad}\delta\left(\kappa_{\widetilde{y}^R}+\frac{\mu\kappa_{\widetilde{e}}}{(\phi-\mu)}\right)(\phi-\mu)\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right)}{\gamma_{\widetilde{y}^R}}$$

$$\pi_{Hbad} = \frac{\kappa_{\widetilde{y}}}{\gamma_{\widetilde{y}}}\overline{r}^W_{bad} + \frac{\kappa_{\widetilde{y}^R}\frac{\delta}{v-1}}{\gamma_{\widetilde{y}^R}}\overline{r}^R_{bad} - \overline{\mathsf{p}}^{good}_{bad}\frac{\kappa_{\widetilde{y}}}{\gamma_{\widetilde{y}}}(\phi-\mu)\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right) - \overline{\mathsf{p}}^{good}_{bad}\frac{\delta\left(\kappa_{\widetilde{y}^R}+\frac{\mu\kappa_{\widetilde{e}}}{(\phi-\mu)}\right)}{\gamma_{\widetilde{y}^R}}(\phi-\mu)\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right)$$

A.4.3 ZLB is binding in both countries

$$\widetilde{y}^R_{good}-\widetilde{y}^R_{bad} = \frac{\frac{\frac{1}{\sigma}}{1-\mu}\frac{1}{v-1}}{\frac{(v-1)}{\delta} - \left(1-\overline{\mathsf{p}}^{bad}_{good}-\overline{\mathsf{p}}^{good}_{bad}\right)\left(\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}-\kappa_{(y-y^*)}}{(1-\beta\mu)} - \frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_e}{(1-\beta\mu)}\right)}\left(\overline{r}^R_{good}-\overline{r}^R_{bad}\right)$$

$$\pi^R_{Hgood}-\pi^R_{Hbad} = \frac{\frac{\kappa}{2}-\kappa_{(y-y^*)}}{(1-\beta\mu)}\frac{\frac{\frac{1}{\sigma}}{1-\mu}\frac{1}{v-1}}{\frac{(v-1)}{\delta} - \left(1-\overline{\mathsf{p}}^{bad}_{good}-\overline{\mathsf{p}}^{good}_{bad}\right)\left(\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}-\kappa_{(y-y^*)}}{(1-\beta\mu)} - \frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_e}{(1-\beta\mu)}\right)}\left(\overline{r}^R_{good}-\overline{r}^R_{bad}\right)$$

$$\pi^R_{good}-\pi^R_{bad} = \left(\pi^R_{Hgood}-\pi^R_{Hbad}\right) - \frac{\kappa_e}{(1-\beta\mu)}\left(\widetilde{y}^R_{good}-\widetilde{y}^R_{bad}\right) + \frac{1}{\mu}\frac{2-v}{v-1}\left(\overline{r}^R_{good}-\overline{r}^R_{bad}\right)$$

and for the good state

$$\widetilde{y}^R_{good} = \frac{\frac{\frac{1}{\sigma}}{1-\mu}\frac{1}{v-1}}{\frac{1}{\delta} - \frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}-\kappa_{(y-y^*)}}{(1-\beta\mu)}}\overline{r}^R_{good} - \overline{\mathsf{p}}^{bad}_{good}\frac{\left(\frac{\frac{\mu}{\sigma}}{1-\mu} - \frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_e}{\frac{\kappa}{2}-\kappa_{(y-y^*)}}\right)}{\frac{1}{\delta} - \frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}-\kappa_{(y-y^*)}}{(1-\beta\mu)}}\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right)$$

$$\pi^R_{Hgood} = \frac{\frac{\kappa}{2}-\kappa_{(y-y^*)}}{(1-\beta\mu)}\widetilde{y}^R_{good}$$

$$q_{good} = -(v-1)\frac{(\phi-\mu)}{1-\mu}\pi^R_{Hgood}$$

$$\pi^R_{good} = \pi^R_{Hgood} - \frac{\kappa_e}{(1-\beta\mu)}\widetilde{y}^R_{good} + \frac{1}{\mu}\frac{2-v}{v-1}\overline{r}^R_{good}$$

and in the bad state

$$\widetilde{y}^R_{bad} = \frac{\frac{\frac{1}{\sigma}}{1-\mu}}{\frac{1}{\delta} - \frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}-\kappa_{(y-y^*)}}{(1-\beta\mu)}}\overline{r}^R_{good} + \overline{\mathsf{p}}^{good}_{bad}\frac{\left(\frac{\frac{\mu}{\sigma}}{1-\mu} - \frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_e}{\frac{\kappa}{2}-\kappa_{(y-y^*)}}\right)}{\frac{1}{\delta} - \frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}-\kappa_{(y-y^*)}}{(1-\beta\mu)}}\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right)$$

$$\pi^R_{Hbad} = \frac{\frac{\kappa}{2}-\kappa_{(y-y^*)}}{(1-\beta\mu)}\widetilde{y}^R_{bad}$$

$$q_{bad} = (v-1)\frac{\mu}{1-\mu}\pi^R_{Hbad}$$

$$\pi^R_{bad} = \pi^R_{Hgood} - \frac{\kappa_e}{(1-\beta\mu)}\widetilde{y}^R_{bad} + \frac{1}{\mu}\frac{2-v}{v-1}\overline{r}^R_{bad}$$

and for home equations

$$\widetilde{y}_{good}-\widetilde{y}_{bad} = \frac{1-\frac{(v-1)}{\delta} - \left(1-\overline{\mathsf{p}}^{bad}_{good}-\overline{\mathsf{p}}^{good}_{bad}\right)\left(\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_{(y-y^*)}}{1-\beta\mu} + \frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_e}{(1-\beta\mu)}\right)}{1-\left(1-\overline{\mathsf{p}}^{bad}_{good}-\overline{\mathsf{p}}^{good}_{bad}\right)\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\left(\widetilde{y}^R_{good}-\widetilde{y}^R_{bad}\right)$$

$$
+\frac{\frac{\frac{1}{\sigma}}{1-\mu}\left(\left(\overline{r}_{good}-\overline{r}_{bad}\right)+\frac{2-v}{v-1}(\overline{r}^R_{good}-\overline{r}^R_{bad})\right)}{1-\left(1-\overline{\mathsf{p}}^{bad}_{good}-\overline{\mathsf{p}}^{good}_{bad}\right)\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}
$$

$$
\begin{aligned}
\pi_{Hgood}-\pi_{Hbad} &= \frac{\frac{\kappa}{2}}{1-\beta\mu}\left(\widetilde{y}_{good}-\widetilde{y}_{bad}\right)-\frac{\kappa_{(y-y^*)}}{1-\beta\mu}\left(\widetilde{y}^R_{good}-\widetilde{y}^R_{bad}\right) \\
\pi_{good}-\pi_{bad} &= \pi_{Hgood}-\pi_{Hbad}-\frac{\kappa_e}{1-\mu\beta}\left(\widetilde{y}^R_{good}-\widetilde{y}^R_{bad}\right)+\frac{1}{\mu}\frac{2-v}{v-1}\left(\overline{r}^R_{good}-\overline{r}^R_{bad}\right)
\end{aligned}
$$

Then in the good state

$$
\begin{aligned}
\widetilde{y}_{good} &= \frac{\left(1-\frac{1}{\delta}\right)-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_{(y-y^*)}}{1-\beta\mu}}{1-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\widetilde{y}^R_{good}+\frac{\frac{\frac{1}{\sigma}}{1-\mu}\left(\overline{r}_{good}+\frac{2-v}{v-1}\overline{r}^R_{good}+\ln(1/\beta)\right)}{1-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}} \\
&\quad -\overline{\mathsf{p}}^{bad}_{good}\left(\frac{\frac{\frac{\mu}{\sigma}}{1-\mu}}{1-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\left(\pi_{Hgood}-\pi_{Hbad}\right)-\frac{\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_e}{\frac{\kappa}{2}-\kappa_{(y-y^*)}}}{1-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right)\right) \\
\pi_{Hgood} &= \frac{\frac{\kappa}{2}}{1-\beta\mu}\widetilde{y}_{good}-\frac{\kappa_{(y-y^*)}}{1-\beta\mu}\widetilde{y}^R_{good} \\
\pi_{good} &= \pi_{Hgood}-\frac{\kappa_e}{1-\mu\beta}\widetilde{y}^R_{good}+\frac{1}{\mu}\frac{2-v}{v-1}\overline{r}^R_{good}
\end{aligned}
$$

or we can express everything in relative world averages, noting that

$$
\begin{aligned}
\kappa_{\widetilde{y}} &= \frac{\kappa}{2}/(1-\beta\mu)>0 \qquad \kappa_{\widetilde{y}^R}=(\frac{\kappa}{2}-\kappa_{(y-y^*)})/(1-\beta\mu)>0 \\
\kappa_e &\equiv (1-\beta\mu)\,(2-\nu)\,\sigma(1-\mu)/(\mu\delta) \\
\kappa_{\widetilde{e}} &= \kappa_e/\,(1-\beta\mu)
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{y}^R_{good} &= \frac{\frac{\frac{1}{\sigma}}{1-\mu}\frac{1}{v-1}}{\frac{1}{\delta}-\frac{\mu}{1-\mu}\frac{\kappa_{\widetilde{y}^R}}{\sigma}}\overline{r}^R_{good}-\overline{\mathsf{p}}^{bad}_{good}\frac{\frac{\frac{\mu}{\sigma}}{1-\mu}-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_{\widetilde{e}}}{\kappa_{\widetilde{y}^R}}}{\frac{1}{\delta}+\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\widetilde{y}^R}}{\sigma}}\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right) \\
\widetilde{y}^R_{good} &= \frac{\frac{\delta}{v-1}\overline{r}^R_{good}}{(1-\mu)\,\sigma-\mu\delta\kappa_{\widetilde{y}^R}}-\overline{\mathsf{p}}^{bad}_{good}\frac{\delta\left(\mu-\mu\frac{\kappa_{\widetilde{e}}}{\kappa_{\widetilde{y}^R}}\right)}{(1-\mu)\,\sigma-\mu\delta\kappa_{\widetilde{y}^R}}\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right) \\
\pi^R_{Hgood} &= \kappa_{\widetilde{y}^R}\widetilde{y}^R_{good} \\
\pi^R_{Hgood} &= \frac{\kappa_{\widetilde{y}^R}\frac{\delta}{v-1}\overline{r}^R_{good}}{(1-\mu)\,\sigma-\mu\delta\kappa_{\widetilde{y}^R}}-\overline{\mathsf{p}}^{bad}_{good}\frac{\delta\left(\mu\kappa_{\widetilde{y}^R}-\mu\kappa_{\widetilde{e}}\right)}{(1-\mu)\,\sigma-\mu\delta\kappa_{\widetilde{y}^R}}\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right) \\
\widetilde{y}^W_{good} &= \frac{\frac{\frac{1}{\sigma}}{1-\mu}\overline{r}^W_{good}}{1-\frac{\mu}{1-\mu}\frac{\kappa_{\widetilde{y}}}{\sigma}}-\overline{\mathsf{p}}^{bad}_{good}\frac{\frac{\mu}{1-\mu}\frac{1}{\sigma}}{1-\frac{\mu}{1-\mu}\frac{\kappa_{\widetilde{y}}}{\sigma}}\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right) \\
\widetilde{y}^W_{good} &= \frac{\overline{r}^W_{good}}{(1-\mu)\,\sigma-\mu\kappa_{\widetilde{y}}}-\overline{\mathsf{p}}^{bad}_{good}\frac{\mu}{(1-\mu)\,\sigma-\mu\kappa_{\widetilde{y}}}\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right) \\
\pi^W_{Hgood} &= \kappa_{\widetilde{y}}\widetilde{y}^W_{good} \\
\pi^W_{Hgood} &= \frac{\kappa_{\widetilde{y}}\overline{r}^W_{good}}{(1-\mu)\,\sigma-\mu\kappa_{\widetilde{y}}}-\overline{\mathsf{p}}^{bad}_{good}\frac{\mu\kappa_{\widetilde{y}}}{(1-\mu)\,\sigma-\mu\kappa_{\widetilde{y}}}\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right)
\end{aligned}
$$

The last term again shows the perceived real interest rate under sticky prices.

Note that

$$
\widetilde{y}^R_{good}-\widetilde{y}^R_{bad} = \frac{\frac{1}{\sigma}\frac{1}{1-\mu}\frac{\overline{r}^R_{good}-\overline{r}^R_{bad}}{v-1}}{\frac{(v-1)}{\delta}-\left(1-\overline{\mathsf{p}}^{bad}_{good}-\overline{\mathsf{p}}^{good}_{bad}\right)\left(\frac{\mu}{1-\mu}\frac{\kappa_{\widetilde{y}^R}}{\sigma}-\frac{\mu}{1-\mu}\frac{\kappa_{\widetilde{e}}}{\sigma}\right)}
$$

$$
\begin{aligned}
\widetilde{y}^R_{good} - \widetilde{y}^R_{bad} &= \frac{\frac{\delta}{v-1}(\overline{r}^R_{good} - \overline{r}^R_{bad})}{(v-1)(1-\mu)\sigma - (1 - \overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad})\delta\mu(\kappa_{\widetilde{y}^R} - \kappa_{\widetilde{e}})} \\
\pi^R_{Hgood} - \pi^R_{Hbad} &= \frac{\frac{\kappa_{\widetilde{y}^R}}{\sigma}\frac{1}{1-\mu}\frac{\overline{r}^R_{good}-\overline{r}^R_{bad}}{v-1}}{\frac{(v-1)}{\delta} - \left(1 - \overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}\right)\left(\frac{\mu}{1-\mu}\frac{\kappa_{\widetilde{y}^R}}{\sigma} - \frac{\mu}{1-\mu}\frac{\kappa_{\widetilde{e}}}{\sigma}\right)} \\
\pi^R_{Hgood} - \pi^R_{Hbad} &= \frac{\kappa_{\widetilde{y}^R}\frac{\delta}{v-1}(\overline{r}^R_{good} - \overline{r}^R_{bad})}{(v-1)(1-\mu)\sigma - (1 - \overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad})\delta\mu(\kappa_{\widetilde{y}^R} - \kappa_{\widetilde{e}})} \\
\widetilde{y}^W_{good} - \widetilde{y}^W_{bad} &= \frac{\frac{1}{\sigma}\frac{1}{1-\mu}\left(\overline{r}^W_{good} - \overline{r}^W_{bad}\right)}{1 - \left(1 - \overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}\right)\frac{\mu}{1-\mu}\frac{\kappa_{\widetilde{y}}}{\sigma}} \\
\widetilde{y}^W_{good} - \widetilde{y}^W_{bad} &= \frac{\overline{r}^W_{good} - \overline{r}^W_{bad}}{(1-\mu)\sigma - (1 - \overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad})\mu\kappa_{\widetilde{y}}} \\
\pi^W_{Hgood} - \pi^W_{Hbad} &= \frac{\frac{\kappa_{\widetilde{y}}}{\sigma}\frac{1}{1-\mu}\left(\overline{r}^W_{good} - \overline{r}^W_{bad}\right)}{1 - \left(1 - \overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}\right)\frac{\mu}{1-\mu}\frac{\kappa_{\widetilde{y}}}{\sigma}} \\
\pi^W_{Hgood} - \pi^W_{Hbad} &= \frac{\kappa_{\widetilde{y}}(\overline{r}^W_{good} - \overline{r}^W_{bad})}{(1-\mu)\sigma - (1 - \overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad})\mu\kappa_{\widetilde{y}}}
\end{aligned}
$$

Then

$$
\begin{aligned}
\widetilde{y}_{good} &= \widetilde{y}^W_{good} + \widetilde{y}^R_{good} \\
\widetilde{y}_{good} &= \frac{\frac{\frac{1}{\sigma}}{1-\mu}\overline{r}^W_{good}}{1 - \frac{\mu}{1-\mu}\frac{\kappa_{\widetilde{y}}}{\sigma}} + \frac{\frac{\frac{1}{\sigma}}{1-\mu}\frac{\overline{r}^R_{good}}{v-1}}{\frac{1}{\delta} - \frac{\mu}{1-\mu}\frac{\kappa_{\widetilde{y}^R}}{\sigma}} - \overline{\mathsf{p}}^{bad}_{good}\frac{\frac{\mu}{1-\mu}\frac{1}{\sigma}}{1 - \frac{\mu}{1-\mu}\frac{\kappa_{\widetilde{y}}}{\sigma}}\left(\pi^W_{Hgood} - \pi^W_{Hbad}\right) - \overline{\mathsf{p}}^{bad}_{good}\frac{\frac{\frac{\mu}{\sigma}}{1-\mu} - \frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_{\widetilde{e}}}{\kappa_{\widetilde{y}^R}}}{\frac{1}{\delta} - \frac{\mu}{1-\mu}\frac{\kappa_{\widetilde{y}^R}}{\sigma}}\left(\pi^R_{Hgood} - \pi^R_{Hbad}\right)
\end{aligned}
$$

$$
\begin{aligned}
\pi_{Hgood} &= \pi^W_{Hgood} + \pi^R_{Hgood} \\
\pi_{Hgood} &= \frac{\frac{\kappa_{\widetilde{y}}}{\sigma}\frac{\overline{r}^W_{good}}{1-\mu}}{1 - \frac{\mu}{1-\mu}\frac{\kappa_{\widetilde{y}}}{\sigma}} + \frac{\frac{\kappa_{\widetilde{y}^R}}{\sigma}\frac{1}{v-1}\frac{\overline{r}^R_{good}}{1-\mu}}{\frac{1}{\delta} - \frac{\mu}{1-\mu}\frac{\kappa_{\widetilde{y}^R}}{\sigma}} - \overline{\mathsf{p}}^{bad}_{good}\frac{\frac{\mu}{1-\mu}\frac{\kappa_{\widetilde{y}}}{\sigma}}{1 - \frac{\mu}{1-\mu}\frac{\kappa_{\widetilde{y}}}{\sigma}}\left(\pi^W_{Hgood} - \pi^W_{Hbad}\right) - \overline{\mathsf{p}}^{bad}_{good}\frac{\frac{\frac{\mu}{\sigma}}{1-\mu}\kappa_{\widetilde{y}^R} - \frac{\frac{\mu}{\sigma}}{1-\mu}\kappa_{\widetilde{e}}}{\frac{1}{\delta} - \frac{\mu}{1-\mu}\frac{\kappa_{\widetilde{y}^R}}{\sigma}}\left(\pi^R_{Hgood} - \pi^R_{Hbad}\right)
\end{aligned}
$$

Or rewriting the denominator:

$$
\begin{aligned}
\widetilde{y}_{good} &= \frac{\overline{r}^W_{good} - \overline{\mathsf{p}}^{bad}_{good}\mu\left(\pi^W_{Hgood} - \pi^W_{Hbad}\right)}{(1-\mu)\sigma - \mu\kappa_{\widetilde{y}}} + \frac{\frac{\delta}{v-1}\overline{r}^R_{good} - \overline{\mathsf{p}}^{bad}_{good}\delta\left(1 - \frac{\kappa_{\widetilde{e}}}{\kappa_{\widetilde{y}^R}}\right)\mu\left(\pi^R_{Hgood} - \pi^R_{Hbad}\right)}{(1-\mu)\sigma - \mu\delta\kappa_{\widetilde{y}^R}} \\
\pi_{Hgood} &= \frac{\kappa_{\widetilde{y}}\overline{r}^W_{good} - \overline{\mathsf{p}}^{bad}_{good}\kappa_{\widetilde{y}}\mu\left(\pi^W_{Hgood} - \pi^W_{Hbad}\right)}{(1-\mu)\sigma - \mu\kappa_{\widetilde{y}}} + \frac{\kappa_{\widetilde{y}^R}\frac{\delta}{v-1}\overline{r}^R_{good} - \overline{\mathsf{p}}^{bad}_{good}\delta\left(\kappa_{\widetilde{y}^R} - \kappa_{\widetilde{e}}\right)\mu\left(\pi^R_{Hgood} - \pi^R_{Hbad}\right)}{(1-\mu)\sigma - \mu\delta\kappa_{\widetilde{y}^R}}
\end{aligned}
$$

Using the definitions

$$
\begin{aligned}
\gamma^{zlb}_{\widetilde{y}} &= (1-\mu)\sigma - \mu\kappa_{\widetilde{y}} \\
\gamma^{zlb}_{\widetilde{y}^R} &= (1-\mu)\sigma - \mu\delta\kappa_{\widetilde{y}^R}
\end{aligned}
$$

we have

$$
\begin{aligned}
\widetilde{y}^R_{good} &= \frac{\frac{\delta}{v-1}\overline{r}^R_{good}}{\gamma^{zlb}_{\widetilde{y}^R}} + \overline{\mathsf{p}}^{good}_{bad}\frac{\delta\left(1 - \frac{\kappa_{\widetilde{e}}}{\kappa_{\widetilde{y}^R}}\right)}{\gamma^{zlb}_{\widetilde{y}^R}}\mu\left(\pi^R_{Hgood} - \pi^R_{Hbad}\right) \\
\pi^R_{Hgood} &= \frac{\kappa_{\widetilde{y}^R}\frac{\delta}{v-1}\overline{r}^R_{good}}{\gamma^{zlb}_{\widetilde{y}^R}} + \overline{\mathsf{p}}^{good}_{bad}\frac{\delta\left(\kappa_{\widetilde{y}^R} - \kappa_{\widetilde{e}}\right)}{\gamma^{zlb}_{\widetilde{y}^R}}\mu\left(\pi^R_{Hgood} - \pi^R_{Hbad}\right)
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{y}^{W}_{good} &= \frac{\overline{r}^{W}_{good}}{\gamma^{zlb}_{\widetilde{y}}} + \overline{\mathsf{p}}^{good}_{bad} \frac{1}{\gamma^{zlb}_{\widetilde{y}}} \mu \left(\pi^{W}_{Hgood} - \pi^{W}_{Hbad}\right) \\
\pi^{W}_{Hgood} &= \frac{\kappa_{\widetilde{y}} \overline{r}^{W}_{good}}{\gamma^{zlb}_{\widetilde{y}}} + \overline{\mathsf{p}}^{good}_{bad} \frac{\kappa_{\widetilde{y}}}{\gamma^{zlb}_{\widetilde{y}}} \mu \left(\pi^{W}_{Hgood} - \pi^{W}_{Hbad}\right)
\end{aligned}
$$

with $-\mu\left(\pi^{R}_{Hgood} - \pi^{R}_{Hbad}\right)$ and $-\mu\left(\pi^{W}_{Hgood} - \pi^{W}_{Hbad}\right)$ reflecting the perceived real interest rate. Then

$$
\begin{aligned}
\widetilde{y}_{good} &= \frac{\overline{r}^{W}_{good} - \overline{\mathsf{p}}^{bad}_{good} \mu \left(\pi^{W}_{Hgood} - \pi^{W}_{Hbad}\right)}{\gamma^{zlb}_{\widetilde{y}}} + \frac{\frac{\delta}{v-1} \overline{r}^{R}_{good} - \overline{\mathsf{p}}^{bad}_{good} \delta \left(1 - \frac{\kappa_{\widetilde{e}}}{\kappa_{\widetilde{y}^R}}\right) \mu \left(\pi^{R}_{Hgood} - \pi^{R}_{Hbad}\right)}{\gamma^{zlb}_{\widetilde{y}^R}} \\
\widetilde{y}_{good} &= \frac{\overline{r}^{W}_{good}}{\gamma^{zlb}_{\widetilde{y}}} + \frac{\frac{\delta}{v-1} \overline{r}^{R}_{good}}{\gamma^{zlb}_{\widetilde{y}^R}} - \overline{\mathsf{p}}^{bad}_{good} \frac{1}{\gamma^{zlb}_{\widetilde{y}}} \mu \left(\pi^{W}_{Hgood} - \pi^{W}_{Hbad}\right) - \overline{\mathsf{p}}^{bad}_{good} \frac{\delta \left(1 - \frac{\kappa_{\widetilde{e}}}{\kappa_{\widetilde{y}^R}}\right)}{\gamma^{zlb}_{\widetilde{y}^R}} \mu \left(\pi^{R}_{Hgood} - \pi^{R}_{Hbad}\right) \\
\pi_{Hgood} &= \frac{\kappa_{\widetilde{y}} \overline{r}^{W}_{good} - \overline{\mathsf{p}}^{bad}_{good} \kappa_{\widetilde{y}} \mu \left(\pi^{W}_{Hgood} - \pi^{W}_{Hbad}\right)}{\gamma^{zlb}_{\widetilde{y}}} + \frac{\kappa_{\widetilde{y}^R} \frac{\delta}{v-1} \overline{r}^{R}_{good} - \overline{\mathsf{p}}^{bad}_{good} \delta \left(\kappa_{\widetilde{y}^R} - \kappa_{\widetilde{e}}\right) \mu \left(\pi^{R}_{Hgood} - \pi^{R}_{Hbad}\right)}{\gamma^{zlb}_{\widetilde{y}^R}} \\
\pi_{Hgood} &= \frac{\kappa_{\widetilde{y}}}{\gamma^{zlb}_{\widetilde{y}}} \overline{r}^{W}_{good} + \frac{\kappa_{\widetilde{y}^R} \frac{\delta}{v-1}}{\gamma^{zlb}_{\widetilde{y}^R}} \overline{r}^{R}_{good} - \overline{\mathsf{p}}^{bad}_{good} \frac{\kappa_{\widetilde{y}}}{\gamma^{zlb}_{\widetilde{y}}} \mu \left(\pi^{W}_{Hgood} - \pi^{W}_{Hbad}\right) - \overline{\mathsf{p}}^{bad}_{good} \frac{\delta \left(\kappa_{\widetilde{y}^R} - \kappa_{\widetilde{e}}\right)}{\gamma^{zlb}_{\widetilde{y}^R}} \mu \left(\pi^{R}_{Hgood} - \pi^{R}_{Hbad}\right)
\end{aligned}
$$

and in the bad state

$$
\begin{aligned}
\widetilde{y}_{bad} &= \frac{\left(1 - \frac{1}{\delta}\right) - \frac{\frac{\mu}{\sigma}}{1-\mu} \frac{\kappa_{(y-y^*)}}{1-\beta\mu}}{1 - \frac{\frac{\mu}{\sigma}}{1-\mu} \frac{\frac{\kappa}{2}}{1-\beta\mu}} \widetilde{y}^{R}_{bad} + \frac{\frac{\frac{1}{\sigma}}{1-\mu} \left(\overline{r}_{bad} + \frac{2-v}{v-1} \overline{r}^{R}_{bad} + \ln(1/\beta)\right)}{1 - \frac{\frac{\mu}{\sigma}}{1-\mu} \frac{\frac{\kappa}{2}}{1-\beta\mu}} \\
&\quad + \overline{\mathsf{p}}^{good}_{bad} \left(\frac{\frac{\frac{\mu}{\sigma}}{1-\mu}}{1 - \frac{\frac{\mu}{\sigma}}{1-\mu} \frac{\frac{\kappa}{2}}{1-\beta\mu}} \left(\pi_{Hgood} - \pi_{Hbad}\right) - \frac{\frac{\frac{\mu}{\sigma}}{1-\mu} \frac{\kappa_e}{\frac{\kappa}{2} - \kappa_{(y-y^*)}}}{1 - \frac{\frac{\mu}{\sigma}}{1-\mu} \frac{\frac{\kappa}{2}}{1-\beta\mu}} \left(\pi^{R}_{Hgood} - \pi^{R}_{Hbad}\right) \right) \\
\pi_{Hbad} &= \frac{\frac{\kappa}{2}}{1-\beta\mu} \widetilde{y}_{bad} - \frac{\kappa_{(y-y^*)}}{1-\beta\mu} \widetilde{y}^{R}_{bad} \\
\pi_{bad} &= \pi_{Hbad} - \frac{\kappa_e}{1-\mu\beta} \widetilde{y}^{R}_{bad} + \frac{1}{\mu} \frac{2-v}{v-1} \overline{r}^{R}_{bad}
\end{aligned}
$$

Or in terms of world relatives and averages:

$$
\begin{aligned}
\widetilde{y}^{R}_{bad} &= \frac{\frac{\frac{1}{\sigma}}{1-\mu} \frac{1}{v-1}}{\frac{1}{\delta} - \frac{\mu}{1-\mu} \frac{\kappa_{\widetilde{y}^R}}{\sigma}} \overline{r}^{R}_{bad} + \overline{\mathsf{p}}^{good}_{bad} \frac{\frac{\frac{\mu}{\sigma}}{1-\mu} - \frac{\frac{\mu}{\sigma}}{1-\mu} \frac{\kappa_{\widetilde{e}}}{\kappa_{\widetilde{y}^R}}}{\frac{1}{\delta} - \frac{\mu}{1-\mu} \frac{\kappa_{\widetilde{y}^R}}{\sigma}} \left(\pi^{R}_{Hgood} - \pi^{R}_{Hbad}\right) \\
\widetilde{y}^{R}_{bad} &= \frac{\frac{\delta}{v-1} \overline{r}^{R}_{bad}}{(1-\mu)\sigma - \mu\delta\kappa_{\widetilde{y}^R}} + \overline{\mathsf{p}}^{good}_{bad} \frac{\delta \left(1 - \frac{\kappa_{\widetilde{e}}}{\kappa_{\widetilde{y}^R}}\right)}{(1-\mu)\sigma - \mu\delta\kappa_{\widetilde{y}^R}} \mu \left(\pi^{R}_{Hgood} - \pi^{R}_{Hbad}\right) \\
\pi^{R}_{Hbad} &= \kappa_{\widetilde{y}^R} \widetilde{y}^{R}_{bad} \\
\pi^{R}_{Hbad} &= \frac{\kappa_{\widetilde{y}^R} \frac{\delta}{v-1} \overline{r}^{R}_{bad}}{(1-\mu)\sigma - \mu\delta\kappa_{\widetilde{y}^R}} + \overline{\mathsf{p}}^{good}_{bad} \frac{\delta \left(\kappa_{\widetilde{y}^R} - \kappa_{\widetilde{e}}\right)}{(1-\mu)\sigma - \mu\delta\kappa_{\widetilde{y}^R}} \mu \left(\pi^{R}_{Hgood} - \pi^{R}_{Hbad}\right) \\
\widetilde{y}^{W}_{bad} &= \frac{\frac{\frac{1}{\sigma}}{1-\mu} \overline{r}^{W}_{bad}}{1 - \frac{\mu}{1-\mu} \frac{\kappa_{\widetilde{y}}}{\sigma}} + \overline{\mathsf{p}}^{good}_{bad} \frac{\frac{\mu}{1-\mu} \frac{1}{\sigma}}{1 - \frac{\mu}{1-\mu} \frac{\kappa_{\widetilde{y}}}{\sigma}} \left(\pi^{W}_{Hgood} - \pi^{W}_{Hbad}\right) \\
\widetilde{y}^{W}_{good} &= \frac{\overline{r}^{W}_{bad}}{(1-\mu)\sigma - \mu\kappa_{\widetilde{y}}} + \overline{\mathsf{p}}^{good}_{bad} \frac{1}{(1-\mu)\sigma - \mu\kappa_{\widetilde{y}}} \mu \left(\pi^{W}_{Hgood} - \pi^{W}_{Hbad}\right) \\
\pi^{W}_{Hbad} &= \kappa_{\widetilde{y}} \widetilde{y}^{W}_{bad} \\
\pi^{W}_{Hbad} &= \frac{\kappa_{\widetilde{y}} \overline{r}^{W}_{bad}}{(1-\mu)\sigma - \mu\kappa_{\widetilde{y}}} + \overline{\mathsf{p}}^{good}_{bad} \frac{\kappa_{\widetilde{y}}}{(1-\mu)\sigma - \mu\kappa_{\widetilde{y}}} \mu \left(\pi^{W}_{Hgood} - \pi^{W}_{Hbad}\right)
\end{aligned}
$$

so that

$$
\begin{aligned}
\tilde{y}_{bad} &= \tilde{y}^W_{bad} + \tilde{y}^R_{bad} \\
\tilde{y}_{bad} &= \frac{\frac{\frac{1}{\sigma}}{1-\mu}\overline{r}^W_{bad}}{1-\frac{\mu}{1-\mu}\frac{\kappa_{\tilde{y}}}{\sigma}} + \frac{\frac{\frac{1}{\sigma}}{1-\mu}\frac{\overline{r}^R_{bad}}{v-1}}{\frac{1}{\delta}-\frac{\mu}{1-\mu}\frac{\kappa_{\tilde{y}^R}}{\sigma}} + \overline{\mathsf{p}}^{good}_{bad}\frac{\frac{\mu}{1-\mu}\frac{1}{\sigma}}{1-\frac{\mu}{1-\mu}\frac{\kappa_{\tilde{y}}}{\sigma}}\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right) + \overline{\mathsf{p}}^{good}_{bad}\frac{\frac{\frac{\mu}{\sigma}}{1-\mu}-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_{\tilde{e}}}{\kappa_{\tilde{y}^R}}}{\frac{1}{\delta}+\frac{\phi-\mu}{1-\mu}\frac{\kappa_{\tilde{y}^R}}{\sigma}}\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right)
\end{aligned}
$$

$$
\begin{aligned}
\pi_{Hbad} &= \pi^W_{Hbad} + \pi^R_{Hbad} \\
\pi_{Hbad} &= \frac{\frac{\kappa_{\tilde{y}}}{\sigma}\frac{\overline{r}^W_{s}}{1-\mu}}{1-\frac{\mu}{1-\mu}\frac{\kappa_{\tilde{y}}}{\sigma}} + \frac{\frac{\kappa_{\tilde{y}^R}}{\sigma}\frac{1}{v-1}\frac{\overline{r}^R_{s}}{1-\mu}}{\frac{1}{\delta}-\frac{\mu}{1-\mu}\frac{\kappa_{\tilde{y}^R}}{\sigma}} + \overline{\mathsf{p}}^{good}_{bad}\frac{\frac{\mu}{1-\mu}\frac{\kappa_{\tilde{y}}}{\sigma}}{1-\frac{\mu}{1-\mu}\frac{\kappa_{\tilde{y}}}{\sigma}}\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right) + \overline{\mathsf{p}}^{good}_{bad}\frac{\frac{\frac{\mu}{\sigma}}{1-\mu}\kappa_{\tilde{y}^R}-\frac{\frac{\mu}{\sigma}}{1-\mu}\kappa_{\tilde{e}}}{\frac{1}{\delta}-\frac{\mu}{1-\mu}\frac{\kappa_{\tilde{y}^R}}{\sigma}}\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right)
\end{aligned}
$$

Or rewriting the denominator:

$$
\begin{aligned}
\tilde{y}_{bad} &= \frac{\overline{r}^W_{bad}+\overline{\mathsf{p}}^{good}_{bad}\mu\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right)}{(1-\mu)\,\sigma-\mu\kappa_{\tilde{y}}} + \frac{\frac{\delta}{v-1}\overline{r}^R_{bad}+\overline{\mathsf{p}}^{good}_{bad}\delta\left(1-\frac{\kappa_{\tilde{e}}}{\kappa_{\tilde{y}^R}}\right)\mu\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right)}{(1-\mu)\,\sigma-\mu\delta\kappa_{\tilde{y}^R}} \\
\pi_{Hbad} &= \frac{\kappa_{\tilde{y}}\overline{r}^W_{bad}+\overline{\mathsf{p}}^{good}_{bad}\kappa_{\tilde{y}}\mu\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right)}{(1-\mu)\,\sigma-\mu\kappa_{\tilde{y}}} + \frac{\kappa_{\tilde{y}^R}\frac{\delta}{v-1}\overline{r}^R_{bad}+\overline{\mathsf{p}}^{good}_{bad}\delta\left(\kappa_{\tilde{y}^R}-\kappa_{\tilde{e}}\right)\mu\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right)}{(1-\mu)\,\sigma-\mu\delta\kappa_{\tilde{y}^R}}
\end{aligned}
$$

Using the definitions

$$
\begin{aligned}
\gamma^{zlb}_{\tilde{y}} &= (1-\mu)\,\sigma-\mu\kappa_{\tilde{y}} \\
\gamma^{zlb}_{\tilde{y}^R} &= (1-\mu)\,\sigma-\mu\delta\kappa_{\tilde{y}^R}
\end{aligned}
$$

we have

$$
\begin{aligned}
\tilde{y}^R_{bad} &= \frac{\frac{\delta}{v-1}\overline{r}^R_{bad}}{\gamma^{zlb}_{\tilde{y}^R}} + \overline{\mathsf{p}}^{good}_{bad}\frac{\delta\left(1-\frac{\kappa_{\tilde{e}}}{\kappa_{\tilde{y}^R}}\right)}{\gamma^{zlb}_{\tilde{y}^R}}\mu\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right) \\
\pi^R_{Hbad} &= \frac{\kappa_{\tilde{y}^R}\frac{\delta}{v-1}\overline{r}^R_{bad}}{\gamma^{zlb}_{\tilde{y}^R}} + \overline{\mathsf{p}}^{good}_{bad}\frac{\delta\left(\kappa_{\tilde{y}^R}-\kappa_{\tilde{e}}\right)}{\gamma^{zlb}_{\tilde{y}^R}}\mu\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right) \\
\tilde{y}^W_{good} &= \frac{\overline{r}^W_{bad}}{\gamma^{zlb}_{\tilde{y}}} + \overline{\mathsf{p}}^{good}_{bad}\frac{1}{\gamma^{zlb}_{\tilde{y}}}\mu\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right) \\
\pi^W_{Hbad} &= \frac{\kappa_{\tilde{y}}\overline{r}^W_{bad}}{\gamma^{zlb}_{\tilde{y}}} + \overline{\mathsf{p}}^{good}_{bad}\frac{\kappa_{\tilde{y}}}{\gamma^{zlb}_{\tilde{y}}}\mu\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right)
\end{aligned}
$$

with $-\mu\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right)$ and $-\mu\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right)$ reflecting the perceived real interest rate. Then

$$
\begin{aligned}
\tilde{y}_{bad} &= \frac{\overline{r}^W_{bad}+\overline{\mathsf{p}}^{good}_{bad}\mu\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right)}{\gamma^{zlb}_{\tilde{y}}} + \frac{\frac{\delta}{v-1}\overline{r}^R_{bad}+\overline{\mathsf{p}}^{good}_{bad}\delta\left(1-\frac{\kappa_{\tilde{e}}}{\kappa_{\tilde{y}^R}}\right)\mu\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right)}{\gamma^{zlb}_{\tilde{y}^R}} \\
\tilde{y}_{bad} &= \frac{\overline{r}^W_{bad}}{\gamma^{zlb}_{\tilde{y}}} + \frac{\frac{\delta}{v-1}\overline{r}^R_{bad}}{\gamma^{zlb}_{\tilde{y}^R}} + \overline{\mathsf{p}}^{good}_{bad}\frac{1}{\gamma^{zlb}_{\tilde{y}}}\mu\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right) + \overline{\mathsf{p}}^{good}_{bad}\frac{\delta\left(1-\frac{\kappa_{\tilde{e}}}{\kappa_{\tilde{y}^R}}\right)}{\gamma^{zlb}_{\tilde{y}^R}}\mu\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right) \\
\pi_{Hbad} &= \frac{\kappa_{\tilde{y}}\overline{r}^W_{bad}+\overline{\mathsf{p}}^{good}_{bad}\kappa_{\tilde{y}}\mu\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right)}{\gamma^{zlb}_{\tilde{y}}} + \frac{\kappa_{\tilde{y}^R}\frac{\delta}{v-1}\overline{r}^R_{bad}+\overline{\mathsf{p}}^{good}_{bad}\delta\left(\kappa_{\tilde{y}^R}-\kappa_{\tilde{e}}\right)\mu\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right)}{\gamma^{zlb}_{\tilde{y}^R}} \\
\pi_{Hbad} &= \frac{\kappa_{\tilde{y}}}{\gamma^{zlb}_{\tilde{y}}}\overline{r}^W_{bad} + \frac{\kappa_{\tilde{y}^R}\frac{\delta}{v-1}}{\gamma^{zlb}_{\tilde{y}^R}}\overline{r}^R_{bad} + \overline{\mathsf{p}}^{good}_{bad}\frac{\kappa_{\tilde{y}}}{\gamma^{zlb}_{\tilde{y}}}\mu\left(\pi^W_{Hgood}-\pi^W_{Hbad}\right) + \overline{\mathsf{p}}^{good}_{bad}\frac{\delta\left(\kappa_{\tilde{y}^R}-\kappa_{\tilde{e}}\right)}{\gamma^{zlb}_{\tilde{y}^R}}\mu\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right)
\end{aligned}
$$

and

$$[\mu\pi^W_{Hgood} - \mu\pi^W_{Hbad}] = \frac{\mu\kappa_{\widetilde{y}}(\overline{r}^W_{good} - \overline{r}^W_{bad})}{(1-\mu)\,\sigma - (1 - \overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad})\mu\kappa_{\widetilde{y}}} > 0.$$

A.4.4 ZLB is only binding in the home country

$$\begin{aligned}
\widetilde{y}^R_{good} - \widetilde{y}^R_{bad} &= \frac{\frac{1}{\sigma}}{1-\mu}\frac{\begin{pmatrix} \left(1 + \left(1 - \overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}\right)\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right)\left((\overline{r}_{good} - \overline{r}_{bad}) + \frac{2-v}{v-1}\left(\overline{r}^R_{good} - \overline{r}^R_{bad}\right)\right) \\ -\left(1 - \left(1 - \overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}\right)\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right)\left(\left(\overline{r}^*_{good} - \overline{r}^*_{bad}\right) - \frac{2-v}{v-1}\left(\overline{r}^R_{good} - \overline{r}^R_{bad}\right)\right)\end{pmatrix}}{\begin{pmatrix} 2\left(1 - \left(1 - \overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}\right)\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right)\left(1 + \left(1 - \overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}\right)\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right) \\ -\left(1 + \left(1 - \overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}\right)\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right) \\ \left(1 - \frac{(v-1)}{\delta} - \left(1 - \overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}\right)\left(\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_{(y-y^*)}}{1-\beta\mu} + \frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_e}{1-\beta\mu}\right)\right) \\ -\left(1 - \left(1 - \overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}\right)\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right) \\ \left(1 - \frac{(v-1)}{\delta} + \left(1 - \overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}\right)\left(\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\kappa_{(y-y^*)}}{1-\beta\mu} - \frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_e}{(1-\beta\mu)}\right)\right)\end{pmatrix}} \\
\pi^R_{Hgood} - \pi^R_{Hbad} &= \frac{\frac{\kappa}{2} - \kappa_{(y-y^*)}}{(1-\beta\mu)}\left(\widetilde{y}^R_{good} - \widetilde{y}^R_{bad}\right) \\
\pi^R_{good} - \pi^R_{bad} &= \left(\pi^R_{Hgood} - \pi^R_{Hbad}\right) - \frac{\kappa_e}{(1-\beta\mu)}\left(\widetilde{y}^R_{good} - \widetilde{y}^R_{bad}\right) + \frac{1}{\mu}\frac{2-v}{v-1}\left(\overline{r}^R_{good} - \overline{r}^R_{bad}\right)
\end{aligned}$$

Let us rewrite this as

$$\begin{aligned}
\widetilde{y}^R_{good} - \widetilde{y}^R_{bad} &= \frac{\begin{pmatrix} \left(\gamma_{\widetilde{y}} - \left(\overline{\mathsf{p}}^{bad}_{good} + \overline{\mathsf{p}}^{good}_{bad}\right)(\phi-\mu)\,\kappa_{\widetilde{y}}\right)\left((\overline{r}_{good} - \overline{r}_{bad}) + \frac{2-v}{v-1}\left(\overline{r}^R_{good} - \overline{r}^R_{bad}\right)\right) \\ -\left(\gamma^{zlb}_{\widetilde{y}} + \left(\overline{\mathsf{p}}^{bad}_{good} + \overline{\mathsf{p}}^{good}_{bad}\right)\mu\kappa_{\widetilde{y}}\right)\left(\left(\overline{r}^*_{good} - \overline{r}^*_{bad}\right) - \frac{2-v}{v-1}\left(\overline{r}^R_{good} - \overline{r}^R_{bad}\right)\right)\end{pmatrix}}{\begin{pmatrix} 2\left(\gamma^{zlb}_{\widetilde{y}} + \left(\overline{\mathsf{p}}^{bad}_{good} + \overline{\mathsf{p}}^{good}_{bad}\right)\mu\kappa_{\widetilde{y}}\right)\left(\gamma_{\widetilde{y}} - \left(\overline{\mathsf{p}}^{bad}_{good} + \overline{\mathsf{p}}^{good}_{bad}\right)(\phi-\mu)\,\kappa_{\widetilde{y}}\right) \\ -\left(\gamma_{\widetilde{y}} - \left(\overline{\mathsf{p}}^{bad}_{good} + \overline{\mathsf{p}}^{good}_{bad}\right)(\phi-\mu)\,\kappa_{\widetilde{y}}\right) \\ \left((1-\mu)\,\sigma\frac{\delta-(v-1)}{\delta} - \left(1 - \overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}\right)\mu\left(\left(\kappa_{\widetilde{y}} - \kappa_{\widetilde{y}^R}\right) + \kappa_{\widetilde{e}}\right)\right) \\ -\left(\gamma^{zlb}_{\widetilde{y}} + \left(\overline{\mathsf{p}}^{bad}_{good} + \overline{\mathsf{p}}^{good}_{bad}\right)\mu\kappa_{\widetilde{y}}\right) \\ \left((1-\mu)\,\sigma\frac{\delta-(v-1)}{\delta} + \left(1 - \overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}\right)\left((\phi-\mu)\left(\kappa_{\widetilde{y}} - \kappa_{\widetilde{y}^R}\right) - \mu\kappa_{\widetilde{e}}\right)\right)\end{pmatrix}} \\
\pi^R_{Hgood} - \pi^R_{Hbad} &= \frac{\frac{\kappa}{2} - \kappa_{(y-y^*)}}{(1-\beta\mu)}\left(\widetilde{y}^R_{good} - \widetilde{y}^R_{bad}\right) \\
\pi^R_{good} - \pi^R_{bad} &= \left(\pi^R_{Hgood} - \pi^R_{Hbad}\right) - \frac{\kappa_e}{(1-\beta\mu)}\left(\widetilde{y}^R_{good} - \widetilde{y}^R_{bad}\right) + \frac{1}{\mu}\frac{2-v}{v-1}\left(\overline{r}^R_{good} - \overline{r}^R_{bad}\right)
\end{aligned}$$

$$\begin{aligned}
\widetilde{y}_{good} - \widetilde{y}_{bad} &= \frac{1 - \frac{(v-1)}{\delta} - \left(1 - \overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}\right)\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_{(y-y^*)}+\kappa_e}{1-\beta\mu}}{1 - \left(1 - \overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}\right)\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\left(\widetilde{y}^R_{good} - \widetilde{y}^R_{bad}\right) \\
&\quad + \frac{\frac{\frac{1}{\sigma}}{1-\mu}}{1 - \left(1 - \overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}\right)\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\left(\overline{r}_{good} - \overline{r}_{bad} + \frac{2-v}{v-1}\left(\overline{r}^R_{good} - \overline{r}^R_{badd}\right)\right) \\
\pi_{Hgood} - \pi_{Hbad} &= \frac{\frac{\kappa}{2}}{1-\beta\mu}\left(\widetilde{y}_{good} - \widetilde{y}_{bad}\right) - \frac{\kappa_{(y-y^*)}}{1-\beta\mu}\left(\widetilde{y}^R_{good} - \widetilde{y}^R_{bad}\right) \\
\pi_{good} - \pi_{bad} &= \pi_{Hgood} - \pi_{Hbad} - \frac{\kappa_e}{1-\mu\beta}\left(\widetilde{y}^R_{good} - \widetilde{y}^R_{bad}\right)\frac{1}{\mu}\frac{2-v}{v-1}\left(\overline{r}^R_{good} - \overline{r}^R_{bad}\right)
\end{aligned}$$

and similarly for the foreign country:

$$\widetilde{y}^*_{good} - \widetilde{y}^*_{bad} = -\frac{1 - \frac{(v-1)}{\delta} + \left(1 - \overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}\right)\left(\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\kappa_{(y-y^*)}}{1-\beta\mu} - \frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_e}{(1-\beta\mu)}\right)}{1 + \left(1 - \overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}\right)\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\left(\widetilde{y}^R_{good} - \widetilde{y}^R_{bad}\right)$$

$$
\begin{aligned}
& +\frac{\frac{\frac{1}{\sigma}}{1-\mu}}{1+\left(1-\bar{\mathsf{p}}^{bad}_{good}-\bar{\mathsf{p}}^{good}_{bad}\right)\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\left(\bar{r}^*_{good}-\bar{r}^*_{bad}-\frac{2-v}{v-1}\left(\bar{r}^R_{good}-\bar{r}^R_{badd}\right)\right)\\
\pi^*_{Fgood}-\pi^*_{Fbad} &= \frac{\frac{\kappa}{2}}{1-\beta\mu}\left(\widetilde{y}^*_{good}-\widetilde{y}^*_{bad}\right)+\frac{\kappa_{(y-y^*)}}{1-\beta\mu}\left(\widetilde{y}^R_{good}-\widetilde{y}^R_{bad}\right)\\
\pi_{good}-\pi_{bad} &= \pi_{Hgood}-\pi_{Hbad}-\frac{\kappa_e}{1-\mu\beta}\left(\widetilde{y}^R_{good}-\widetilde{y}^R_{bad}\right)-\frac{1}{\mu}\frac{2-v}{v-1}\left(\bar{r}^R_{good}-\bar{r}^R_{bad}\right)
\end{aligned}
$$

For the good state we have

$$
\begin{aligned}
\widetilde{y}^R_{good} &= \frac{\left(1+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right)\left(\bar{r}_{good}+\frac{2-v}{v-1}\bar{r}^R_{good}+\ln(1/\beta)\right)-\left(1-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right)\left(\bar{r}^*_{good}-\frac{2-v}{v-1}\bar{r}^R_{good}\right)}{\left(\begin{array}{c} 2\left(1-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right)\left(1+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right)\\ -\left(1+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right)\left(\left(1-\frac{1}{\delta}\right)-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_{(y-y^*)}}{1-\beta\mu}\right)-\left(1-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right)\left(\left(1-\frac{1}{\delta}\right)+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\kappa_{(y-y^*)}}{1-\beta\mu}\right)\end{array}\right)}\\
&\quad -\bar{\mathsf{p}}^{bad}_{good}\frac{\frac{\frac{\mu}{\sigma}}{1-\mu}\left(1+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right)\left(\pi_{Hgood}-\pi_{Hbad}\right)+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\left(1-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right)\left(\pi^*_{Fgood}-\pi^*_{Fbad}\right)}{\left(\begin{array}{c} 2\left(1-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right)\left(1+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right)\\ -\left(1+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right)\left(\left(1-\frac{1}{\delta}\right)-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_{(y-y^*)}}{1-\beta\mu}\right)-\left(1-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right)\left(\left(1-\frac{1}{\delta}\right)+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\kappa_{(y-y^*)}}{1-\beta\mu}\right)\end{array}\right)}\\
&\quad +\bar{\mathsf{p}}^{bad}_{good}\frac{\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_e}{\frac{\kappa}{2}-\kappa_{(y-y^*)}}\left(\left(1+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right)+\left(1-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right)\right)\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right)}{\left(\begin{array}{c} 2\left(1-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right)\left(1+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right)\\ -\left(1+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right)\left(\left(1-\frac{1}{\delta}\right)-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_{(y-y^*)}}{1-\beta\mu}\right)-\left(1-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right)\left(\left(1-\frac{1}{\delta}\right)+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\kappa_{(y-y^*)}}{1-\beta\mu}\right)\end{array}\right)}\\
\pi^R_{Hgood} &= \frac{\frac{\kappa}{2}-\kappa_{(y-y^*)}}{(1-\beta\mu)}\widetilde{y}^R_{good}\\
\pi^R_{good} &= \pi^R_{Hgood}-\frac{\kappa_e}{(1-\beta\mu)}\widetilde{y}^R_{good}+\frac{1}{\mu}\frac{2-v}{v-1}\bar{r}^R_{bad}
\end{aligned}
$$

which can be written as

$$
\begin{aligned}
\widetilde{y}^R_{good} &= \frac{\frac{\delta}{v-1}(2-v)\left(\gamma_{\widetilde{y}}+\gamma^{zlb}_{\widetilde{y}}\right)\bar{r}^R_{good}+\delta\gamma_{\widetilde{y}}\left(\bar{r}_{good}+\ln(1/\beta)\right)-\delta\gamma^{zlb}_{\widetilde{y}}\bar{r}^*_{\mathsf{s}}}{\left(\gamma^{zlb}_{\widetilde{y}^R}\gamma_{\widetilde{y}}+\gamma^{zlb}_{\widetilde{y}}\gamma_{\widetilde{y}^R}\right)}\\
&\quad -\bar{\mathsf{p}}^{bad}_{good}\frac{\frac{\gamma_{\widetilde{y}}\frac{\delta}{\sigma}}{1-\mu}\mu\left(\pi_{Hgood}-\pi_{Hbad}\right)+\frac{\gamma^{zlb}_{\widetilde{y}}\frac{\delta}{\sigma}}{1-\mu}(\phi-\mu)\left(\pi^*_{Fgood}-\pi^*_{Fbad}\right)}{\left(\gamma^{zlb}_{\widetilde{y}^R}\gamma_{\widetilde{y}}+\gamma^{zlb}_{\widetilde{y}}\gamma_{\widetilde{y}^R}\right)}\\
&\quad +\bar{\mathsf{p}}^{bad}_{good}\frac{\frac{\frac{\delta}{\sigma}}{1-\mu}\frac{\kappa_{\widetilde{e}}}{\kappa_{\widetilde{y}^R}}\left(\gamma^{zlb}_{\widetilde{y}}+\gamma_{\widetilde{y}}\right)\mu\left(\pi^R_{Hgood}-\pi^R_{Hbad}\right)}{\left(\gamma^{zlb}_{\widetilde{y}^R}\gamma_{\widetilde{y}}+\gamma^{zlb}_{\widetilde{y}}\gamma_{\widetilde{y}^R}\right)}\\
\pi^R_{Hgood} &= \frac{\frac{\kappa}{2}-\kappa_{(y-y^*)}}{(1-\beta\mu)}\widetilde{y}^R_{good}\\
\pi^R_{good} &= \pi^R_{Hgood}-\frac{\kappa_e}{(1-\beta\mu)}\widetilde{y}^R_{good}+\frac{1}{\mu}\frac{2-v}{v-1}\bar{r}^R_{bad}
\end{aligned}
$$

and in the bad state

$$
\begin{aligned}
\widetilde{y}^R_{bad} &= \frac{\left(1+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right)\frac{\frac{1}{\sigma}}{1-\mu}\left(\bar{r}_{bad}+\frac{2-v}{v-1}\bar{r}^R_{bad}+\ln(1/\beta)\right)-\left(1-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right)\frac{\frac{1}{\sigma}}{1-\mu}\left(\bar{r}^*_{bad}-\frac{2-v}{v-1}\bar{r}^R_{bad}\right)}{\left(\begin{array}{c} 2\left(1-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right)\left(1+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right)\\ -\left(1+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right)\left(\left(1-\frac{1}{\delta}\right)-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_{(y-y^*)}}{1-\beta\mu}\right)-\left(1-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right)\left(\left(1-\frac{1}{\delta}\right)+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\kappa_{(y-y^*)}}{1-\beta\mu}\right)\end{array}\right)}\\
&\quad +\bar{\mathsf{p}}^{good}_{bad}\frac{\frac{\frac{\mu}{\sigma}}{1-\mu}\left(1+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right)\left(\pi_{Hgood}-\pi_{Hbad}\right)+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\left(1-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right)\left(\pi^*_{Fgood}-\pi^*_{Fbad}\right)}{\left(\begin{array}{c} 2\left(1-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right)\left(1+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right)\\ -\left(1+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right)\left(\left(1-\frac{1}{\delta}\right)-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_{(y-y^*)}}{1-\beta\mu}\right)-\left(1-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right)\left(\left(1-\frac{1}{\delta}\right)+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\kappa_{(y-y^*)}}{1-\beta\mu}\right)\end{array}\right)}
\end{aligned}
$$

$$-\overline{\mathsf{p}}_{bad}^{good}\frac{\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_e}{\frac{\kappa}{2}-\kappa_{(y-y^*)}}\left(\left(1+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right)+\left(1-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right)\right)\left(\pi_{Hgood}^R-\pi_{Hbad}^R\right)}{\left(\begin{array}{c} 2\left(1-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right)\left(1+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right) \\ -\left(1+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right)\left(\left(1-\frac{1}{\delta}\right)-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_{(y-y^*)}}{1-\beta\mu}\right)-\left(1-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}\right)\left(\left(1-\frac{1}{\delta}\right)+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\kappa_{(y-y^*)}}{1-\beta\mu}\right)\end{array}\right)}$$

$$\begin{aligned}
\pi_{Hbad}^R &= \frac{\frac{\kappa}{2}-\kappa_{(y-y^*)}}{(1-\beta\mu)}\tilde{y}_{bad}^R \\
\pi_{bad}^R &= \pi_{Hgood}^R-\frac{\kappa_e}{(1-\beta\mu)}\tilde{y}_{bad}^R+\frac{1}{\mu}\frac{2-v}{v-1}\overline{r}_{bad}^R
\end{aligned}$$

which can be written as

$$\begin{aligned}
\tilde{y}_{bad}^R &= \frac{\frac{\delta}{v-1}(2-v)\left(\gamma_{\tilde{y}}+\gamma_{\tilde{y}}^{zlb}\right)\overline{r}_{bad}^R+\delta\gamma_{\tilde{y}}\left(\overline{r}_{bad}+\ln(1/\beta)\right)-\delta\gamma_{\tilde{y}}^{zlb}\overline{r}_{\mathsf{s}}^*}{\left(\gamma_{\tilde{y}^R}^{zlb}\gamma_{\tilde{y}}+\gamma_{\tilde{y}}^{zlb}\gamma_{\tilde{y}^R}\right)} \\
&\quad+\overline{\mathsf{p}}_{bad}^{good}\frac{\frac{\gamma_{\tilde{y}}\frac{\delta}{\sigma}}{1-\mu}\mu\left(\pi_{Hgood}-\pi_{Hbad}\right)+\frac{\gamma_{\tilde{y}}^{zlb}\frac{\delta}{\sigma}}{1-\mu}(\phi-\mu)\left(\pi_{Fgood}^*-\pi_{Fbad}^*\right)}{\left(\gamma_{\tilde{y}^R}^{zlb}\gamma_{\tilde{y}}+\gamma_{\tilde{y}}^{zlb}\gamma_{\tilde{y}^R}\right)} \\
&\quad-\overline{\mathsf{p}}_{bad}^{good}\frac{\frac{\frac{\delta}{\sigma}}{1-\mu}\frac{\kappa_{\tilde{e}}}{\kappa_{\tilde{y}^R}}\left(\gamma_{\tilde{y}}^{zlb}+\gamma_{\tilde{y}}\right)\mu\left(\pi_{Hgood}^R-\pi_{Hbad}^R\right)}{\left(\gamma_{\tilde{y}^R}^{zlb}\gamma_{\tilde{y}}+\gamma_{\tilde{y}}^{zlb}\gamma_{\tilde{y}^R}\right)} \\
\pi_{Hbad}^R &= \frac{\frac{\kappa}{2}-\kappa_{(y-y^*)}}{(1-\beta\mu)}\tilde{y}_{bad}^R \\
\pi_{bad}^R &= \pi_{Hgood}^R-\frac{\kappa_e}{(1-\beta\mu)}\tilde{y}_{bad}^R+\frac{1}{\mu}\frac{2-v}{v-1}\overline{r}_{bad}^R
\end{aligned}$$

Then in the good state

$$\begin{aligned}
\tilde{y}_{good} &= \frac{\left(1-\frac{1}{\delta}\right)-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_{(y-y^*)}}{1-\beta\mu}}{1-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\tilde{y}_{good}^R+\frac{\frac{\frac{1}{\sigma}}{1-\mu}}{1-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\left(\overline{r}_{good}+\frac{2-v}{v-1}\overline{r}_{good}^R+\ln(1/\beta\right)) \\
&\quad-\overline{\mathsf{p}}_{good}^{bad}\left(\frac{\frac{\frac{\mu}{\sigma}}{1-\mu}}{1-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\left(\pi_{Hgood}-\pi_{Hbad}\right)-\frac{\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_e}{\frac{\kappa}{2}-\kappa_{(y-y^*)}}}{1-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\left(\pi_{Hgood}^R-\pi_{Hbad}^R\right)\right) \\
\pi_{Hgood} &= \frac{\frac{\kappa}{2}}{1-\beta\mu}\tilde{y}_{good}-\frac{\kappa_{(y-y^*)}}{1-\beta\mu}\tilde{y}_{good}^R \\
\pi_{good} &= \pi_{Hgood}-\frac{\kappa_e}{1-\mu\beta}\tilde{y}_{good}^R+\frac{1}{\mu}\frac{2-v}{v-1}\overline{r}_{good}^R
\end{aligned}$$

which can be written as

$$\begin{aligned}
\gamma_{\tilde{y}} &= (1-\mu)\sigma+(\phi-\mu)\kappa_{\tilde{y}} \\
\gamma_{\tilde{y}^R} &= (1-\mu)\sigma+(\phi-\mu)\delta\kappa_{\tilde{y}^R}
\end{aligned}$$

$$\begin{aligned}
\gamma_{\tilde{y}} &= (1-\mu)\sigma-\mu\kappa_{\tilde{y}} \\
\gamma_{\tilde{y}^R} &= (1-\mu)\sigma-\mu\delta\kappa_{\tilde{y}^R}
\end{aligned}$$

$$\begin{aligned}
-\mu\left(\kappa_{\tilde{y}}-\kappa_{\tilde{y}^R}\right) &= \mu\kappa_{\tilde{y}^R}-\mu\kappa_{\tilde{y}}=-\frac{\gamma_{\tilde{y}^R}^{zlb}-(1-\mu)\sigma}{\delta}+\gamma_{\tilde{y}}^{zlb}-(1-\mu)\sigma \\
-\mu\left(\kappa_{\tilde{y}}-\kappa_{\tilde{y}^R}\right) &= \frac{\delta\gamma_{\tilde{y}}^{zlb}-\gamma_{\tilde{y}^R}^{zlb}}{\delta}-\frac{\delta-1}{\delta}(1-\mu)\sigma
\end{aligned}$$

and

$$
\begin{aligned}
(\phi-\mu)\left(\kappa_{\tilde{y}}-\kappa_{\tilde{y}^R}\right) &= \gamma_{\tilde{y}}-(1-\mu)\,\sigma-\frac{\gamma_{\tilde{y}^R}}{\delta}+\frac{(1-\mu)\,\sigma}{\delta} \\
&= \frac{\delta\gamma_{\tilde{y}}-\gamma_{\tilde{y}^R}}{\delta}-\frac{\delta-1}{\delta}(1-\mu)\,\sigma
\end{aligned}
$$

as

$$
\begin{aligned}
\widetilde{y}_{good} &= \frac{\delta\gamma_{\tilde{y}}^{zlb}-\gamma_{\tilde{y}^R}^{zlb}}{\delta\gamma_{\tilde{y}}^{zlb}}\widetilde{y}_{good}^{R}+\delta\frac{\overline{r}_{good}+\frac{2-v}{v-1}\overline{r}_{good}^{R}+\ln(1/\beta)}{\delta\gamma_{\tilde{y}}^{zlb}} \\
&\quad -\overline{\mathsf{p}}_{good}^{bad}\left(\frac{\delta}{\gamma_{\tilde{y}}^{zlb}}\frac{\mu}{\delta}\left(\pi_{Hgood}-\pi_{Hbad}\right)-\frac{\delta\frac{\kappa_{\tilde{e}}}{\kappa_{\tilde{y}^R}}}{\gamma_{\tilde{y}}^{zlb}}\frac{\mu}{\delta}\left(\pi_{Hgood}^{R}-\pi_{Hbad}^{R}\right)\right)
\end{aligned}
$$

and in the bad state

$$
\begin{aligned}
\widetilde{y}_{bad} &= \frac{\left(1-\frac{1}{\delta}\right)-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_{(y-y^*)}}{1-\beta\mu}}{1-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\widetilde{y}_{bad}^{R}+\frac{\frac{\frac{1}{\sigma}}{1-\mu}}{1-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\left(\overline{r}_{bad}+\frac{2-v}{v-1}\overline{r}_{bad}^{R}+\ln(1/\beta)\right)) \\
&\quad +\overline{\mathsf{p}}_{bad}^{good}\left(\frac{\frac{\frac{\mu}{\sigma}}{1-\mu}}{1-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\left(\pi_{Hgood}-\pi_{Hbad}\right)-\frac{\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_e}{\frac{\kappa}{2}-\kappa_{(y-y^*)}}}{1-\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\left(\pi_{Hgood}^{R}-\pi_{Hbad}^{R}\right)\right) \\
\pi_{Hbad} &= \frac{\frac{\kappa}{2}}{1-\beta\mu}\widetilde{y}_{bad}-\frac{\kappa_{(y-y^*)}}{1-\beta\mu}\widetilde{y}_{bad}^{R} \\
\pi_{bad} &= \pi_{Hbad}-\frac{\kappa_e}{1-\mu\beta}\widetilde{y}_{bad}^{R}-\frac{1}{\mu}\frac{2-v}{v-1}\overline{r}_{bad}^{R}
\end{aligned}
$$

which can be written as

$$
\begin{aligned}
\widetilde{y}_{bad} &= \frac{\delta\gamma_{\tilde{y}}^{zlb}-\gamma_{\tilde{y}^R}^{zlb}}{\delta\gamma_{\tilde{y}}^{zlb}}\widetilde{y}_{bad}^{R}+\delta\frac{\overline{r}_{bad}+\frac{2-v}{v-1}\overline{r}_{bad}^{R}+\ln(1/\beta)}{\delta\gamma_{\tilde{y}}^{zlb}} \\
&\quad +\overline{\mathsf{p}}_{bad}^{good}\left(\frac{\delta}{\gamma_{\tilde{y}}^{zlb}}\frac{\mu}{\delta}\left(\pi_{Hgood}-\pi_{Hbad}\right)-\frac{\delta\frac{\kappa_{\tilde{e}}}{\kappa_{\tilde{y}^R}}}{\gamma_{\tilde{y}}^{zlb}}\frac{\mu}{\delta}\left(\pi_{Hgood}^{R}-\pi_{Hbad}^{R}\right)\right)
\end{aligned}
$$

In the foreign country we have in the good state

$$
\begin{aligned}
\widetilde{y}_{good}^{*} &= -\frac{\left(1-\frac{1}{\delta}\right)+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\kappa_{(y-y^*)}}{1-\beta\mu}}{1+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\widetilde{y}_{good}^{R}+\frac{\frac{\frac{1}{\sigma}}{1-\mu}}{1+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\left(\overline{r}_{good}^{*}-\frac{2-v}{v-1}\overline{r}_{good}^{R}\right) \\
&\quad +\overline{\mathsf{p}}_{good}^{bad}\left(\frac{\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}}{1+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\left(\pi_{Fgood}^{*}-\pi_{Fbad}^{*}\right)-\frac{\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_e}{\frac{\kappa}{2}-\kappa_{(y-y^*)}}}{1+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\left(\pi_{Hgood}^{R}-\pi_{Hbad}^{R}\right)\right) \\
\pi_{Fgood}^{*} &= \frac{\frac{\kappa}{2}}{1-\beta\mu}\widetilde{y}_{good}^{*}+\frac{\kappa_{(y-y^*)}}{1-\beta\mu}\widetilde{y}_{good}^{R}
\end{aligned}
$$

and in the bad state

$$
\begin{aligned}
\widetilde{y}_{bad}^{*} &= -\frac{\left(1-\frac{1}{\delta}\right)+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\kappa_{(y-y^*)}}{1-\beta\mu}}{1+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\widetilde{y}_{bad}^{R}+\frac{\frac{\frac{1}{\sigma}}{1-\mu}}{1+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\left(\overline{r}_{bad}^{*}-\frac{2-v}{v-1}\overline{r}_{bad}^{R}\right) \\
&\quad -\overline{\mathsf{p}}_{bad}^{good}\left(\frac{\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}}{1+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\left(\pi_{Fgood}^{*}-\pi_{Fbad}^{*}\right)-\frac{\frac{\frac{\mu}{\sigma}}{1-\mu}\frac{\kappa_e}{\frac{\kappa}{2}-\kappa_{(y-y^*)}}}{1+\frac{\frac{\phi-\mu}{\sigma}}{1-\mu}\frac{\frac{\kappa}{2}}{1-\beta\mu}}\left(\pi_{Hgood}^{R}-\pi_{Hbad}^{R}\right)\right) \\
\pi_{Fbad}^{*} &= \frac{\frac{\kappa}{2}}{1-\beta\mu}\widetilde{y}_{bad}^{*}+\frac{\kappa_{(y-y^*)}}{1-\beta\mu}\widetilde{y}_{bad}^{R}
\end{aligned}
$$

which can be written as

$$\widetilde{y}^{*}_{good} = -\frac{\delta\gamma^{zlb}_{\widetilde{y}} - \gamma^{zlb}_{\widetilde{y^R}}}{\delta\gamma_{\widetilde{y}}}\widetilde{y}^{R}_{good} + \delta\frac{\overline{r}^{*}_{good} - \frac{2-v}{v-1}\overline{r}^{R}_{good}}{\delta\gamma_{\widetilde{y}}}$$
$$+\overline{\mathsf{p}}^{bad}_{good}\left(\frac{\delta}{\gamma_{\widetilde{y}}}\frac{(\phi-\mu)}{\delta}\left(\pi^{*}_{Fgood} - \pi^{*}_{Fbad}\right) - \frac{\delta\frac{\kappa_{\widetilde{e}}}{\kappa_{\widetilde{y^R}}}}{\gamma_{\widetilde{y}}}\frac{(\phi-\mu)}{\delta}\left(\pi^{R}_{Hgood} - \pi^{R}_{Hbad}\right)\right)$$

$$\widetilde{y}^{*}_{bad} = -\frac{\delta\gamma^{zlb}_{\widetilde{y}} - \gamma^{zlb}_{\widetilde{y^R}}}{\delta\gamma_{\widetilde{y}}}\widetilde{y}^{R}_{bad} + \delta\frac{\overline{r}^{*}_{bad} - \frac{2-v}{v-1}\overline{r}^{R}_{bad}}{\delta\gamma_{\widetilde{y}}}$$
$$-\overline{\mathsf{p}}^{good}_{bad}\left(\frac{\delta}{\gamma_{\widetilde{y}}}\frac{(\phi-\mu)}{\delta}\left(\pi^{*}_{Fgood} - \pi^{*}_{Fbad}\right) - \frac{\delta\frac{\kappa_{\widetilde{e}}}{\kappa_{\widetilde{y^R}}}}{\gamma_{\widetilde{y}}}\frac{(\phi-\mu)}{\delta}\left(\pi^{R}_{Hgood} - \pi^{R}_{Hbad}\right)\right)$$

Then,

$$\widetilde{y}^{*}_{bad}|_{\varpi=0} - \widetilde{y}^{*}_{bad}|_{\varpi=1} = -\overline{\mathsf{p}}^{good}_{bad}\frac{(\phi-\mu)}{\mu}\left(\frac{[\mu\pi^{*}_{Fgood} - \mu\pi^{*}_{Fbad}]}{\gamma_{\widetilde{y}}} - \frac{\kappa_{\widetilde{e}}}{\kappa_{\widetilde{y^R}}}\frac{[\mu\pi^{R}_{Hgood} - \mu\pi^{R}_{Hbad}]}{\gamma_{\widetilde{y^R}}}\right) < 0, \tag{70}$$

with $[\mu\pi^{*}_{Fgood} - \mu\pi^{*}_{Fbad}] > [\mu\pi^{R}_{Hgood} - \mu\pi^{R}_{Hbad}] > 0$. Finally, the real exchange rate in the good and bad state equals

$$\frac{q_{\mathsf{s}}}{2} = (\nu-1)\left(\frac{\mu}{1-\mu}\pi^{R}_{H\mathsf{s}} + \frac{\phi}{1-\mu}\frac{\pi^{*}_{F\mathsf{s}}}{2} + \frac{\ln(1/\beta)}{2}\right)$$

A.5 Optimal monetary policy

The Lagrangian is:

$$\begin{aligned}
\mathcal{L}_t = {} & \widetilde{\mathcal{W}}_t + k_{1,good}\Big(-(r^{R}_{t,good} - \overline{r}^{R}_{t,good}) + \mu\pi^{R}_{Ht+1,good} + \frac{\sigma}{\delta}(\mu\widetilde{y}^{R}_{good,t+1} - \widetilde{y}^{R}_{good,t}) \\
& +\overline{\mathsf{p}}^{bad}_{good}((r^{R}_{good,t} - r^{R}_{bad,t}) - (\overline{r}^{R}_{good,t} - \overline{r}^{R}_{bad,t}) - (\mu\pi^{R}_{H,good,t+1} - \mu\pi^{R}_{H,bad,t+1}))\Big) \\
& +k_{2,good}\Big(-(r^{W}_{t,good} - \overline{r}^{W}_{t,good}) + \mu\pi^{W}_{H,t+1,good} + \sigma(\mu\widetilde{y}^{W}_{good,t+1} - \widetilde{y}^{W}_{good,t}) \\
& +\overline{\mathsf{p}}^{bad}_{good}((r^{W}_{good,t} - r^{W}_{bad,t}) - (\overline{r}^{W}_{good,t} - \overline{r}^{W}_{bad,t}) - (\mu\pi^{W}_{H,good,t+1} - \mu\pi^{W}_{H,bad,t+1}))\Big) \\
& +k_{3,good}(\pi^{R}_{Hgood,t} - (\frac{\kappa}{2} - \kappa_{(y-y^*)})\widetilde{y}^{R}_{good,t} - \beta\pi^{R}_{Hgood,t+1}) + k_{4,good}(\pi^{W}_{Hgood,t} - \frac{\kappa}{2}\widetilde{y}^{W}_{good,t} - \beta\pi^{W}_{Hgood,t+1}) \\
& +k_{5,good}(r^{W}_{good,t} + r^{R}_{good,t}) + k_{6,good}(r^{W}_{good,t} - r^{R}_{good,t}) \\
& +k_{1,bad}\Big(-(r^{R}_{t,bad} - \overline{r}^{R}_{t,bad}) + \mu\pi^{R}_{Ht+1,bad} + \frac{\sigma}{\delta}(\mu\widetilde{y}^{R}_{bad,t+1} - \widetilde{y}^{R}_{bad,t}) \\
& -\overline{\mathsf{p}}^{good}_{bad}((r^{R}_{good,t} - r^{R}_{bad,t}) - (\overline{r}^{R}_{good,t} - \overline{r}^{R}_{bad,t}) - (\mu\pi^{R}_{H,good,t+1} - \mu\pi^{R}_{H,bad,t+1}))\Big) \\
& +k_{2,bad}\Big(-(r^{W}_{t,bad} - \overline{r}^{W}_{t,bad}) + \mu\pi^{W}_{H,t+1,bad} + \sigma(\mu\widetilde{y}^{W}_{bad,t+1} - \widetilde{y}^{W}_{bad,t}) \\
& -\overline{\mathsf{p}}^{good}_{bad}((r^{W}_{good,t} - r^{W}_{bad,t}) - (\overline{r}^{W}_{good,t} - \overline{r}^{W}_{bad,t}) - (\mu\pi^{W}_{H,good,t+1} - \mu\pi^{W}_{H,bad,t+1}))\Big) \\
& +k_{3,bad}(\pi^{R}_{Hbad,t} - (\frac{\kappa}{2} - \kappa_{(y-y^*)})\widetilde{y}^{R}_{bad,t} - \beta\pi^{R}_{Hbad,t+1}) + k_{4,bad}(\pi^{W}_{Hbad,t} - \frac{\kappa}{2}\widetilde{y}^{W}_{bad,t} - \beta\pi^{W}_{Hbad,t+1}) \\
& +k_{5,bad}(r^{W}_{bad,t} + r^{R}_{bad,t}) + k_{6,bad}(r^{W}_{bad,t} - r^{R}_{bad,t}),
\end{aligned}$$

Then the constraints in the Lagrangian $\mathcal{L}_t$ for the aggregate demand equations in relative and average terms are given by $k_{1,good}$, $k_{2,good}$ as well as $k_{1,bad}$, $k_{2,bad}$, respectively. The constraints $k_{3,good}$, $k_{4,good}$ as well as $k_{3,bad}$, $k_{4,bad}$ reflect those on relative and average world inflation. The last constraints $k_{5,good}$, $k_{6,good}$ as well as $k_{5,bad}$, $k_{6,bad}$ in the Lagrangian $\mathcal{L}_t$ are the non-negative constraints on the home and foreign interest rate. The implied comparative slackness conditions equals

$$k_{5,good}(r^W_{good,t} + r^R_{good,t}) = 0 \text{ and } k_{5,bad}(r^W_{bad,t} + r^R_{bad,t}) = 0 \tag{71}$$

$$k_{6,good}(r^W_{good,t} - r^R_{good,t}) = 0 \text{ and } k_{6,bad}(r^W_{bad,t} - r^R_{bad,t}) = 0. \tag{72}$$

Form this the first-order conditions for the variables $\widetilde{y}^R_{\mathsf{s},t}, \widetilde{y}^W_{\mathsf{s},t}, \pi^R_{H\mathsf{s},t}$ and $\pi^W_{H\mathsf{s},t}$ for $\mathsf{s}\ \epsilon\{bad, good\}$ follow:

$$k_{1,good} = -\gamma_{k_1}\widetilde{y}^R_{t,good} \text{ and } k_{1,bad} = -\gamma_{k_1}\widetilde{y}^R_{t,bad} \tag{73}$$

$$k_{2,good} = -\gamma_{k_2}\widetilde{y}^W_{t,good} \text{ and } k_{2,bad} = -\gamma_{k_2}\widetilde{y}^W_{t,bad} \tag{74}$$

$$k_{3,good} = \gamma_{k_3}\widetilde{y}^R_{t,good} \text{ and } k_{3,bad} = \gamma_{k_3}\widetilde{y}^R_{t,bad} \tag{75}$$

$$k_{4,good} = \gamma_{k_4}\widetilde{y}^W_{t,good} \text{ and } k_{4,bad} = \gamma_{k_4}\widetilde{y}^W_{t,bad} \tag{76}$$

The coefficients in front of the variables are given by

$$\gamma_{k_1} \equiv \frac{2\delta}{\sigma}(\lambda_{\widetilde{y}^R} + \frac{\lambda_\pi \kappa^2_{\widetilde{y}^R}}{(1-\mu\beta)^{-1}}) > 0,\ \gamma_{k_2} \equiv \frac{2}{\sigma}(\lambda_{\widetilde{y}^W} + \frac{\lambda_\pi \kappa^2_{\widetilde{y}}}{(1-\mu\beta)^{-1}}) > 0,\ \gamma_{k_3} \equiv 2\lambda_\pi \kappa^2_{\widetilde{y}^R} > 0,\ \gamma_{k_4} \equiv 2\lambda_\pi \kappa_{\widetilde{y}} > 0.$$

As we are assessing a situation where the ZLB in the home country is binding in every state, it follows that home output in the good and in the bad state will be negative. Consequently, the sum $k_{1,good}+k_{2,good}$ and $k_{1,bad}+k_{2,bad}$ will be positive. Then, given the comparative slackness condition (71) $k_{5,good} > 0$ and $k_{5,bad} > 0$, the home interest rate is at the ZLB. The foreign economy is not bounded by the ZLB. Consequently, $k_{2,good} - k_{1,good}$ and $k_{2,bad} - k_{1,bad}$ will be zero. The constraints on (71) and (72) are obtained from the first-order conditions for the variables $r^R_{\mathsf{s},t}$ and $r^W_{\mathsf{s},t}$, which result in

$$k_{5,good} = (1-\overline{\mathsf{p}}^{bad}_{good})\frac{(k_{1,good}+k_{2,good})}{2} + \overline{\mathsf{p}}^{good}_{bad}\frac{(k_{1,bad}+k_{2,bad})}{2} \tag{77}$$

$$k_{6,good} = (1-\overline{\mathsf{p}}^{bad}_{good})\frac{(k_{2,good}-k_{1,good})}{2} + \overline{\mathsf{p}}^{good}_{bad}\frac{(k_{2,bad}-k_{1,bad})}{2} \tag{78}$$

$$k_{5,bad} = (1-\overline{\mathsf{p}}^{good}_{bad})\frac{(k_{1,bad}+k_{2,bad})}{2} + \overline{\mathsf{p}}^{bad}_{good}\frac{(k_{1,good}+k_{2,good})}{2} \tag{79}$$

$$k_{6,bad} = (1-\overline{\mathsf{p}}^{good}_{bad})\frac{(k_{2,bad}-k_{1,bad})}{2} + \overline{\mathsf{p}}^{good}_{bad}\frac{(k_{2,good}-k_{1,good})}{2}. \tag{80}$$

Here we outline the complete problem for $k = \lambda$: for the two states in every period t we have

$$\begin{aligned}
\widetilde{\mathcal{W}} \equiv & -\Big[\lambda_\pi \frac{1}{2}\left(\pi^W_{Hgood,t}+\pi^R_{Hgood,t}\right)^2 + \lambda_\pi \frac{1}{2}\left(\pi^W_{Hgood,t}-\pi^R_{Hgood,t}\right)^2 + \left(\widetilde{y}^W_{good,t}\right)^2\lambda_{\widetilde{y}^W} + \left(\widetilde{y}^R_{good,t}\right)^2\lambda_{\widetilde{y}^R} \\
& +\lambda_\pi \frac{1}{2}\left(\pi^W_{Hbad,t}+\pi^R_{Hbad,t}\right)^2 + \lambda_\pi\left(\pi^W_{Hbad,t}-\pi^R_{Hbad,t}\right)^2 + \left(\widetilde{y}^W_{bad,t}\right)^2\lambda_{\widetilde{y}^W} + \left(\widetilde{y}^R_{bad,t}\right)^2\lambda_{\widetilde{y}^R}\Big] \\
& \max \widetilde{\mathcal{W}} \\
& +\gamma_{1,good}\left[\begin{array}{c} -\left(r^R_{t,good}-\overline{r}^R_{t,good}\right) + \mu\pi^R_{Ht+1,good} + \frac{\sigma}{\delta}\left(\mu\widetilde{y}^R_{good,t+1}-\widetilde{y}^R_{good,t}\right) \\ +\overline{\mathsf{p}}^{bad}_{good}\left(\left(r^R_{good,t}-r^R_{bad,t}\right)-\left(\overline{r}^R_{good,t}-\overline{r}^R_{bad,t}\right)-\left(\mu\pi^R_{H,good,t+1}-\mu\pi^R_{H,bad,t+1}\right)\right)\end{array}\right] \\
& +\gamma_{2,good}\left[\begin{array}{c} -\left(r^W_{t,good}-\overline{r}^W_{t,good}\right) + \mu\pi^W_{H,t+1,good} + \sigma\left(\mu\widetilde{y}^W_{good,t+1}-\widetilde{y}^W_{good,t}\right) \\ +\overline{\mathsf{p}}^{bad}_{good}\left(\left(r^W_{good,t}-r^W_{bad,t}\right)-\left(\overline{r}^W_{good,t}-\overline{r}^W_{bad,t}\right)-\left(\mu\pi^W_{H,good,t+1}-\mu\pi^W_{H,bad,t+1}\right)\right)\end{array}\right] \\
& +\gamma_{3,good}\left[\pi^R_{Hgood,t}-\left(\frac{\kappa}{2}-\kappa_{(y-y^*)}\right)\widetilde{y}^R_{good,t}-\beta\pi^R_{Hgood,t+1}\right] \\
& +\gamma_{4,good}\left[\pi^W_{Hgoodt}-\frac{\kappa}{2}\widetilde{y}^W_{good,t}-\beta\pi^W_{Hgood,t+1}\right] \\
& +\gamma_{5,good}\left[r^W_{good,t}+r^R_{good,t}\right] \\
& +\gamma_{6,good}\left[r^W_{good,t}-r^R_{good,t}\right] \\
& +\gamma_{1,bad}\left[\begin{array}{c} -\left(r^R_{t,bad}-\overline{r}^R_{t,bad}\right) + \mu\pi^R_{Ht+1,bad} + \frac{\sigma}{\delta}\left(\mu\widetilde{y}^R_{bad,t+1}-\widetilde{y}^R_{bad,t}\right) \\ -\overline{\mathsf{p}}^{good}_{bad}\left(\left(r^R_{good,t}-r^R_{bad,t}\right)-\left(\overline{r}^R_{good,t}-\overline{r}^R_{bad,t}\right)-\left(\mu\pi^R_{H,good,t+1}-\mu\pi^R_{H,bad,t+1}\right)\right)\end{array}\right]
\end{aligned}$$

$$
\begin{aligned}
&+\gamma_{2,bad}\left[\begin{array}{c} -\left(r^{W}_{t,bad}-\overline{r}^{W}_{t,bad}\right)+\mu\pi^{W}_{H,t+1,bad}+\sigma\left(\mu\widetilde{y}^{W}_{bad,t+1}-\widetilde{y}^{W}_{bad,t}\right) \\ -\overline{\mathsf{p}}^{good}_{bad}\left(\left(r^{W}_{good,t}-r^{W}_{bad,t}\right)-\left(\overline{r}^{W}_{good,t}-\overline{r}^{W}_{bad,t}\right)-\left(\mu\pi^{W}_{H,good,t+1}-\mu\pi^{W}_{H,bad,t+1}\right)\right)\end{array}\right] \\
&+\gamma_{3,bad}\left[\pi^{R}_{Hbad,t}-\left(\frac{\kappa}{2}-\kappa_{(y-y^{*})}\right)\widetilde{y}^{R}_{bad,t}-\beta\pi^{R}_{bad,Ht+1}\right] \\
&+\gamma_{4,bad}\left[\pi^{W}_{Hbad,t}-\left(\frac{\kappa}{2}\right)\widetilde{y}^{W}_{bad,t}-\beta\pi^{W}_{bad,Ht+1}\right] \\
&+\gamma_{5,bad}\left[r^{W}_{bad,t}+r^{R}_{bad,t}\right] \\
&+\gamma_{6,bad}\left[r^{W}_{bad,t}-r^{R}_{bad,t}\right]
\end{aligned}
$$

Then the first-order conditions are:

$$
\begin{aligned}
0 &= -\lambda_{\pi}2\pi^{W}_{Hgood,t}+\gamma_{4,good} \\
0 &= -\lambda_{\pi}2\pi^{R}_{Hgood,t}+\gamma_{3,good} \\
0 &= -\lambda_{\widetilde{y}^{W}}2\widetilde{y}^{W}_{good,t}-\sigma\gamma_{2,good}-\frac{\kappa}{2}\gamma_{4,good} \\
0 &= -\lambda_{\widetilde{y}^{R}}2\widetilde{y}^{R}_{good,t}-\frac{\sigma}{\delta}\gamma_{1,good}-\left(\frac{\kappa}{2}-\kappa_{(y-y^{*})}\right)\gamma_{3,good} \\
0 &= -\gamma_{2,good}\left(1-\overline{\mathsf{p}}^{bad}_{good}\right)-\gamma_{2,bad}\overline{\mathsf{p}}^{good}_{bad}+\gamma_{5,good}+\gamma_{6,good} \\
0 &= -\gamma_{1,good}\left(1-\overline{\mathsf{p}}^{bad}_{good}\right)-\gamma_{1,bad}\overline{\mathsf{p}}^{good}_{bad}+\gamma_{5,good}-\gamma_{6,good}
\end{aligned}
$$

$$
\begin{aligned}
0 &= -\lambda_{\pi}2\pi^{W}_{Hbad,t}+\gamma_{4,bad} \\
0 &= -\lambda_{\pi}2\pi^{R}_{Hbad,t}+\gamma_{3,bad} \\
0 &= -\lambda_{\widetilde{y}^{W}}2\widetilde{y}^{W}_{bad,t}-\sigma\gamma_{2,bad}-\frac{\kappa}{2}\gamma_{4,bad} \\
0 &= -\lambda_{\widetilde{y}^{R}}2\widetilde{y}^{R}_{bad,t}-\frac{\sigma}{\delta}\gamma_{1,bad}-\left(\frac{\kappa}{2}-\kappa_{(y-y^{*})}\right)\gamma_{3,bad} \\
0 &= -\gamma_{2,bad}\left(1-\overline{\mathsf{p}}^{good}_{bad}\right)-\gamma_{2,good}\overline{\mathsf{p}}^{bad}_{good}+\gamma_{5,bad}+\gamma_{6,bad} \\
0 &= -\gamma_{1,bad}\left(1-\overline{\mathsf{p}}^{good}_{bad}\right)-\gamma_{1,good}\overline{\mathsf{p}}^{bad}_{good}+\gamma_{5,bad}-\gamma_{6,bad}
\end{aligned}
$$

Simplifying yields in the good state

$$
\begin{aligned}
\gamma_{4,good} &= 2\lambda_{\pi}\pi^{W}_{Hgood,t} \\
\gamma_{3,good} &= 2\lambda_{\pi}\pi^{R}_{Hgood,t} \\
\gamma_{2,good} &= -2\frac{1}{\sigma}\lambda_{\widetilde{y}^{W}}\widetilde{y}^{W}_{good,t}-\frac{1}{\sigma}\frac{\kappa}{2}\gamma_{4,good} \\
\gamma_{1,good} &= -2\lambda_{\widetilde{y}^{R}}\frac{\delta}{\sigma}\widetilde{y}^{R}_{good,t}-\frac{\delta}{\sigma}\left(\frac{\kappa}{2}-\kappa_{(y-y^{*})}\right)\gamma_{3,good} \\
\gamma_{2,good} &= \gamma_{5,good}+\gamma_{6,good}+\gamma_{2,good}\overline{\mathsf{p}}^{bad}_{good}-\gamma_{2,bad}\overline{\mathsf{p}}^{good}_{bad} \\
\gamma_{1,good} &= \gamma_{5,good}-\gamma_{6,good}+\gamma_{1,good}\overline{\mathsf{p}}^{bad}_{good}-\gamma_{1,bad}\overline{\mathsf{p}}^{good}_{bad}
\end{aligned}
$$

and in the bad state

$$
\begin{aligned}
\gamma_{4,bad} &= 2\lambda_{\pi}\pi^{W}_{Hbad,t} \\
\gamma_{3,gbad} &= 2\lambda_{\pi}\pi^{R}_{Hbad,t} \\
\gamma_{2,bad} &= -2\frac{1}{\sigma}\lambda_{\widetilde{y}^{W}}\widetilde{y}^{W}_{bad,t}-\frac{1}{\sigma}\frac{\kappa}{2}\gamma_{4,bad} \\
\gamma_{1,bad} &= -2\lambda_{\widetilde{y}^{R}}\frac{\delta}{\sigma}\widetilde{y}^{R}_{bad,t}-\frac{\delta}{\sigma}\left(\frac{\kappa}{2}-\kappa_{(y-y^{*})}\right)\gamma_{3,bad} \\
\gamma_{2,bad} &= \gamma_{5,bad}+\gamma_{6,bad}+\gamma_{2,bad}\overline{\mathsf{p}}^{good}_{bad}-\gamma_{2,good}\overline{\mathsf{p}}^{bad}_{good} \\
\gamma_{1,bad} &= \gamma_{5,bad}-\gamma_{6,bad}+\gamma_{1,bad}\overline{\mathsf{p}}^{good}_{bad}-\gamma_{1,good}\overline{\mathsf{p}}^{bad}_{good}
\end{aligned}
$$

To summarize we can write:

$$\begin{aligned}
\gamma_{4,good} &= 2\lambda_\pi \pi^W_{Hgood,t} \\
\gamma_{3,good} &= 2\lambda_\pi \pi^R_{Hgood,t} \\
\gamma_{2,good} &= -2\frac{1}{\sigma}\lambda_{\widetilde{y}^W}\widetilde{y}^W_{good,t} - \frac{1}{\sigma}\frac{\kappa}{2}\gamma_{4,good} \\
\gamma_{1,good} &= -2\lambda_{\widetilde{y}^R}\frac{\delta}{\sigma}\widetilde{y}^R_{good,t} - \frac{\delta}{\sigma}\left(\frac{\kappa}{2} - \kappa_{(y-y^*)}\right)\gamma_{3,good} \\
2\gamma_{5,good} &= \left(1-\overline{\mathsf{p}}^{bad}_{good}\right)(\gamma_{2,good}+\gamma_{1,good}) + \overline{\mathsf{p}}^{good}_{bad}(\gamma_{2,bad}+\gamma_{1,bad}) \\
2\gamma_{6,good} &= \left(1-\overline{\mathsf{p}}^{bad}_{good}\right)(\gamma_{2,good}-\gamma_{1,good}) + \overline{\mathsf{p}}^{good}_{bad}(\gamma_{2,bad}-\gamma_{1,bad})
\end{aligned}$$

and

$$\begin{aligned}
\gamma_{4,bad} &= 2\lambda_\pi \pi^W_{Hbad,t} \\
\gamma_{3,bad} &= 2\lambda_\pi \pi^R_{Hbad,t} \\
\gamma_{2,bad} &= -2\frac{1}{\sigma}\lambda_{\widetilde{y}^W}\widetilde{y}^W_{bad,t} - \frac{1}{\sigma}\frac{\kappa}{2}\gamma_{4,bad} \\
\gamma_{1,bad} &= -2\lambda_{\widetilde{y}^R}\frac{\delta}{\sigma}\widetilde{y}^R_{bad,t} - \frac{\delta}{\sigma}\left(\frac{\kappa}{2} - \kappa_{(y-y^*)}\right)\gamma_{3,bad} \\
2\gamma_{5,bad} &= \left(1-\overline{\mathsf{p}}^{good}_{bad}\right)(\gamma_{2,bad}+\gamma_{1,bad}) + \overline{\mathsf{p}}^{bad}_{good}(\gamma_{2,good}+\gamma_{1,good}) \\
2\gamma_{6,bad} &= \left(1-\overline{\mathsf{p}}^{good}_{bad}\right)(\gamma_{2,bad}-\gamma_{1,bad}) + \overline{\mathsf{p}}^{bad}_{good}(\gamma_{2,good}-\gamma_{1,good})
\end{aligned}$$

Then for:

$$\begin{aligned}
\pi^R_{Hgood,t} &= \frac{\left(\frac{\kappa}{2} - \kappa_{(y-y^*)}\right)}{1-\mu\beta}\widetilde{y}^R_{good,t} \\
\pi^R_{Hbad,t} &= \frac{\left(\frac{\kappa}{2} - \kappa_{(y-y^*)}\right)}{1-\mu\beta}\widetilde{y}^R_{bad,t}
\end{aligned}$$

$$\begin{aligned}
\pi^W_{Hgood,t} &= \frac{\frac{\kappa}{2}}{1-\mu\beta}\widetilde{y}^W_{good,t} \\
\pi^W_{Hbad,t} &= \frac{\frac{\kappa}{2}}{1-\mu\beta}\widetilde{y}^W_{bad,t}
\end{aligned}$$

We have in the good state

$$\begin{aligned}
\gamma_{1,good} &= -2\frac{\delta}{\sigma}\left(\lambda_{\widetilde{y}^R} + \lambda_\pi\frac{\left(\frac{\kappa}{2} - \kappa_{(y-y^*)}\right)^2}{1-\mu\beta}\right)\widetilde{y}^R_{t,good} \\
\gamma_{2,good} &= -\frac{2}{\sigma}\left(\lambda_{\widetilde{y}^W} + \lambda_\pi\frac{\left(\frac{\kappa}{2}\right)^2}{1-\mu\beta}\right)\widetilde{y}^W_{t,good} \\
\gamma_{3,good} &= 2\lambda_\pi\frac{\frac{\kappa}{2} - \kappa_{(y-y^*)}}{1-\mu\beta}\widetilde{y}^R_{t,good} \\
\gamma_{4,good} &= 2\lambda_\pi\frac{\frac{\kappa}{2}}{1-\mu\beta}\widetilde{y}^W_{t,good}
\end{aligned}$$

and in the bad state

$$\begin{aligned}
\gamma_{1,bad} &= -2\frac{\delta}{\sigma}\left(\lambda_{\widetilde{y}^R} + \lambda_\pi\frac{\left(\frac{\kappa}{2} - \kappa_{(y-y^*)}\right)^2}{1-\mu\beta}\right)\widetilde{y}^R_{t,bad} \\
\gamma_{2,bad} &= -\frac{2}{\sigma}\left(\lambda_{\widetilde{y}^W} + \lambda_\pi\frac{\left(\frac{\kappa}{2}\right)^2}{1-\mu\beta}\right)\widetilde{y}^W_{t,bad} \\
\gamma_{3,bad} &= 2\lambda_\pi\frac{\frac{\kappa}{2} - \kappa_{(y-y^*)}}{1-\mu\beta}\widetilde{y}^R_{t,bad} \\
\gamma_{4,bad} &= 2\lambda_\pi\frac{\frac{\kappa}{2}}{1-\mu\beta}\widetilde{y}^W_{t,bad}
\end{aligned}$$

Now continue with the solution:

Using first

$$\pi^R_{Hgood,t} = \frac{\left(\frac{\kappa}{2} - \kappa_{(y-y^*)}\right)}{1-\mu\beta}\widetilde{y}^R_{good,t}$$
$$\pi^R_{Hbad,t} = \frac{\left(\frac{\kappa}{2} - \kappa_{(y-y^*)}\right)}{1-\mu\beta}\widetilde{y}^R_{bad,t}$$

as well as

$$\pi^R_{Hgood,t} - \pi^R_{Hbad,t} = \frac{\left(\frac{\kappa}{2} - \kappa_{(y-y^*)}\right)}{1-\mu\beta}\left(\widetilde{y}^R_{good,t} - \widetilde{y}^R_{bad,t}\right) \tag{81}$$

and

$$r^R_{t,good} - \overline{r}^R_{t,good} = \mu\pi^R_{Ht,good} - \frac{\sigma}{\delta}(1-\mu)\widetilde{y}^R_{good,t} + \overline{\mathsf{p}}^{bad}_{good}\left(\left(r^R_{good,t} - r^R_{bad,t}\right) - \left(\overline{r}^R_{good,t} - \overline{r}^R_{bad,t}\right) - \left(\mu\pi^R_{H,good,t} - \mu\pi^R_{H,bad,t}\right)\right)$$
$$r^R_{t,bad} - \overline{r}^R_{t,bad} = \mu\pi^R_{Ht,bad} - \frac{\sigma}{\delta}(1-\mu)\widetilde{y}^R_{bad,t} - \overline{\mathsf{p}}^{good}_{bad}\left(\left(r^R_{good,t} - r^R_{bad,t}\right) - \left(\overline{r}^R_{good,t} - \overline{r}^R_{bad,t}\right) - \left(\mu\pi^R_{H,good,t} - \mu\pi^R_{H,bad,t}\right)\right)$$

Subtracting bad from good yields:

$$\left(r^R_{good,t} - r^R_{bad,t}\right) - \left(\overline{r}^R_{good,t} - \overline{r}^R_{bad,t}\right) - \left(\mu\pi^R_{H,good,t} - \mu\pi^R_{H,bad,t}\right) = -\frac{\frac{\sigma}{\delta}(1-\mu)}{1-\overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}}\left(\widetilde{y}^R_{good,t} - \widetilde{y}^R_{bad,t}\right) \tag{82}$$

Note that (81) and (82) can be used to get a solution to output and, hence, inflation as a function of the interest rate differential.

For the moment continue with

$$r^R_{t,good} - \overline{r}^R_{t,good} = \mu\pi^R_{Ht,good} - \frac{\sigma}{\delta}(1-\mu)\widetilde{y}^R_{good,t} - \frac{\sigma}{\delta}(1-\mu)\frac{\overline{\mathsf{p}}^{bad}_{good}}{1-\overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}}\left(\widetilde{y}^R_{good,t} - \widetilde{y}^R_{bad,t}\right)$$
$$r^R_{t,bad} - \overline{r}^R_{t,bad} = \mu\pi^R_{Ht,bad} - \frac{\sigma}{\delta}(1-\mu)\widetilde{y}^R_{bad,t} + \frac{\sigma}{\delta}(1-\mu)\frac{\overline{\mathsf{p}}^{good}_{bad}}{1-\overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}}\left(\widetilde{y}^R_{good,t} - \widetilde{y}^R_{bad,t}\right)$$

Now plug the inflation equation in

$$r^R_{t,good} - \overline{r}^R_{t,good} = \mu\frac{\left(\frac{\kappa}{2} - \kappa_{(y-y^*)}\right)}{1-\mu\beta}\widetilde{y}^R_{good,t} - \frac{\sigma}{\delta}(1-\mu)\widetilde{y}^R_{good,t} - \frac{\sigma}{\delta}(1-\mu)\frac{\overline{\mathsf{p}}^{bad}_{good}}{1-\overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}}\left(\widetilde{y}^R_{good,t} - \widetilde{y}^R_{bad,t}\right)$$
$$r^R_{t,bad} - \overline{r}^R_{t,bad} = \mu\frac{\left(\frac{\kappa}{2} - \kappa_{(y-y^*)}\right)}{1-\mu\beta}\widetilde{y}^R_{bad,t} - \frac{\sigma}{\delta}(1-\mu)\widetilde{y}^R_{bad,t} + \frac{\sigma}{\delta}(1-\mu)\frac{\overline{\mathsf{p}}^{good}_{bad}}{1-\overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}}\left(\widetilde{y}^R_{good,t} - \widetilde{y}^R_{bad,t}\right)$$

$$\begin{aligned}(1-\mu\beta)\left(r^R_{t,good} - \overline{r}^R_{t,good}\right) &= -\left((1-\mu\beta)\frac{\sigma}{\delta}(1-\mu) - \mu\left(\frac{\kappa}{2} - \kappa_{(y-y^*)}\right)\right)\widetilde{y}^R_{good,t} \\ &\quad -\frac{\sigma}{\delta}(1-\mu)\frac{(1-\mu\beta)\overline{\mathsf{p}}^{bad}_{good}}{1-\overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}}\left(\widetilde{y}^R_{good,t} - \widetilde{y}^R_{bad,t}\right) \\ (1-\mu\beta)\left(r^R_{t,bad} - \overline{r}^R_{t,bad}\right) &= -\left((1-\mu\beta)\frac{\sigma}{\delta}(1-\mu) - \mu\left(\frac{\kappa}{2} - \kappa_{(y-y^*)}\right)\right)\widetilde{y}^R_{bad,t} \\ &\quad +\frac{\sigma}{\delta}(1-\mu)\frac{(1-\mu\beta)\overline{\mathsf{p}}^{good}_{bad}}{1-\overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}}\left(\widetilde{y}^R_{good,t} - \widetilde{y}^R_{bad,t}\right)\end{aligned}$$

with

$$\left((1-\mu\beta)\frac{\sigma}{\delta}(1-\mu) - \mu\left(\frac{\kappa}{2} - \kappa_{(y-y^*)}\right)\right) = \Delta^D_2$$

$$(1-\mu\beta)\left(r^R_{t,good} - \overline{r}^R_{t,good}\right) = -\Delta^D_2\widetilde{y}^R_{good,t} - \frac{\sigma}{\delta}(1-\mu)(1-\mu\beta)\frac{\overline{\mathsf{p}}^{bad}_{good}}{1-\overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}}\left(\widetilde{y}^R_{good,t} - \widetilde{y}^R_{bad,t}\right) \tag{83}$$
$$(1-\mu\beta)\left(r^R_{t,bad} - \overline{r}^R_{t,bad}\right) = -\Delta^D_2\widetilde{y}^R_{bad,t} + \frac{\sigma}{\delta}(1-\mu)(1-\mu\beta)\frac{\overline{\mathsf{p}}^{good}_{bad}}{1-\overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}}\left(\widetilde{y}^R_{good,t} - \widetilde{y}^R_{bad,t}\right) \tag{84}$$

Now do the same for world inflation and output:

$$\begin{aligned}\pi^W_{Hgood,t} &= \frac{\frac{\kappa}{2}}{1-\mu\beta}\widetilde{y}^W_{good,t}\\ \pi^W_{Hbad,t} &= \frac{\frac{\kappa}{2}}{1-\mu\beta}\widetilde{y}^W_{bad,t}\end{aligned}$$

so that

$$\pi^W_{Hgood,t}-\pi^W_{Hbad,t}=\frac{\frac{\kappa}{2}}{1-\mu\beta}\left(\widetilde{y}^W_{good,t}-\widetilde{y}^W_{bad,t}\right) \tag{85}$$

$$r^W_{t,good}-\overline{r}^W_{t,good}=\mu\pi^W_{H,t,good}-\sigma(1-\mu)\widetilde{y}^W_{good,t}+\overline{\mathsf{p}}^{bad}_{good}\left(\left(r^W_{good,t}-r^W_{bad,t}\right)-\left(\overline{r}^W_{good,t}-\overline{r}^W_{bad,t}\right)-\left(\mu\pi^W_{H,good,t}-\mu\pi^W_{H,bad,t}\right)\right)$$

$$r^W_{t,bad}-\overline{r}^W_{t,bad}=\mu\pi^W_{H,t,bad}-\sigma(1-\mu)\widetilde{y}^W_{bad,t}-\overline{\mathsf{p}}^{good}_{bad}\left(\left(r^W_{good,t}-r^W_{bad,t}\right)-\left(\overline{r}^W_{good,t}-\overline{r}^W_{bad,t}\right)-\left(\mu\pi^W_{H,good,t}-\mu\pi^W_{H,bad,t}\right)\right)$$

Substracting bad from good yields:

$$\left(r^W_{good,t}-r^W_{bad,t}\right)-\left(\overline{r}^W_{good,t}-\overline{r}^W_{bad,t}\right)-\left(\mu\pi^W_{H,good,t}-\mu\pi^W_{H,bad,t}\right)=-\frac{\sigma(1-\mu)}{1-\overline{\mathsf{p}}^{bad}_{good}-\overline{\mathsf{p}}^{good}_{bad}}\left(\widetilde{y}^W_{good,t}-\widetilde{y}^W_{bad,t}\right) \tag{86}$$

Note that (85) and (86) can be used to get a solution to output and, hence inflation as a function of the interest rate differential. Now plug the inflation equation in

$$\begin{aligned}(1-\mu\beta)\left(r^W_{t,good}-\overline{r}^W_{t,good}\right) &= -\left((1-\mu\beta)\sigma(1-\mu)-\mu\frac{\kappa}{2}\right)\widetilde{y}^W_{good,t}-\sigma(1-\mu)(1-\mu\beta)\frac{\overline{\mathsf{p}}^{bad}_{good}}{1-\overline{\mathsf{p}}^{bad}_{good}-\overline{\mathsf{p}}^{good}_{bad}}\left(\widetilde{y}^W_{good,t}-\widetilde{y}^W_{bad,t}\right)\\ (1-\mu\beta)\left(r^W_{t,bad}-\overline{r}^W_{t,bad}\right) &= -\left((1-\mu\beta)\sigma(1-\mu)-\mu\frac{\kappa}{2}\right)\widetilde{y}^W_{bad,t}+\sigma(1-\mu)(1-\mu\beta)\frac{\overline{\mathsf{p}}^{good}_{bad}}{1-\overline{\mathsf{p}}^{bad}_{good}-\overline{\mathsf{p}}^{good}_{bad}}\left(\widetilde{y}^W_{good,t}-\widetilde{y}^W_{bad,t}\right)\end{aligned}$$

with

$$\left((1-\mu\beta)\sigma(1-\mu)-\mu\frac{\kappa}{2}\right)=\Delta_2$$

$$(1-\mu\beta)\left(r^W_{t,good}-\overline{r}^W_{t,good}\right) = -\Delta_2\widetilde{y}^W_{good,t}-\sigma(1-\mu)(1-\mu\beta)\frac{\overline{\mathsf{p}}^{bad}_{good}}{1-\overline{\mathsf{p}}^{bad}_{good}-\overline{\mathsf{p}}^{good}_{bad}}\left(\widetilde{y}^W_{good,t}-\widetilde{y}^W_{bad,t}\right) \tag{87}$$

$$(1-\mu\beta)\left(r^W_{t,bad}-\overline{r}^W_{t,bad}\right) = -\Delta_2\widetilde{y}^W_{bad,t}+\sigma(1-\mu)(1-\mu\beta)\frac{\overline{\mathsf{p}}^{good}_{bad}}{1-\overline{\mathsf{p}}^{bad}_{good}-\overline{\mathsf{p}}^{good}_{bad}}\left(\widetilde{y}^W_{good,t}-\widetilde{y}^W_{bad,t}\right) \tag{88}$$

Using (83) yields

$$\begin{aligned}(1-\mu\beta)\left(r^R_{t,good}-\overline{r}^R_{t,good}\right) &= \frac{\Delta^D_2}{\Psi_D}(1-\mu\beta)\frac{1}{2}\frac{\sigma}{\delta}\gamma_{1,good}-\frac{\sigma}{\delta}(1-\mu)(1-\mu\beta)\frac{\overline{\mathsf{p}}^{bad}_{good}}{1-\overline{\mathsf{p}}^{bad}_{good}-\overline{\mathsf{p}}^{good}_{bad}}\left(\widetilde{y}^R_{good,t}-\widetilde{y}^R_{bad,t}\right)\\ \left(r^R_{t,good}-\overline{r}^R_{t,good}\right) &= \frac{\Delta^D_2}{\Psi_D}\frac{1}{2}\frac{\sigma}{\delta}\gamma_{1,good}-\frac{\sigma}{\delta}(1-\mu)\frac{\overline{\mathsf{p}}^{bad}_{good}}{1-\overline{\mathsf{p}}^{bad}_{good}-\overline{\mathsf{p}}^{good}_{bad}}\left(\widetilde{y}^R_{good,t}-\widetilde{y}^R_{bad,t}\right)\\ \gamma_{1,good} &= \frac{\Psi_D}{\Delta^D_2\frac{\sigma}{\delta}\frac{1}{2}}\left(r^R_{t,good}-\overline{r}^R_{t,good}\right)+\frac{\Psi_D}{\Delta^D_2\frac{\sigma}{\delta}\frac{1}{2}}\frac{\sigma}{\delta}(1-\mu)\frac{\overline{\mathsf{p}}^{bad}_{good}}{1-\overline{\mathsf{p}}^{bad}_{good}-\overline{\mathsf{p}}^{good}_{bad}}\left(\widetilde{y}^R_{good,t}-\widetilde{y}^R_{bad,t}\right)\\ \gamma_{1,good} &= \left(\frac{\Psi_D}{\Delta^D_2 S_D}\right)\left(r^R_{t,good}-\overline{r}^R_{t,good}\right)-\frac{\sigma}{\delta}\frac{(1-\mu)(1-\mu\beta)}{\Delta^D_2}\frac{\overline{\mathsf{p}}^{bad}_{good}}{1-\overline{\mathsf{p}}^{bad}_{good}-\overline{\mathsf{p}}^{good}_{bad}}\left(\gamma_{1,good}-\gamma_{1,bad}\right)\\ \gamma_{1,good} &= \Omega_D\left(r^R_{t,good}-\overline{r}^R_{t,good}\right)-\Omega^{g-b}_D\frac{\overline{\mathsf{p}}^{bad}_{good}}{1-\overline{\mathsf{p}}^{bad}_{good}-\overline{\mathsf{p}}^{good}_{bad}}\left(\gamma_{1,good}-\gamma_{1,bad}\right) \qquad (89)\end{aligned}$$

with

$$\begin{aligned}\frac{\sigma}{\delta}\frac{1}{2} &= S_D\\ \left(\frac{\Psi_D}{\Delta^D_2 S_D}\right) &= \Omega_D\\ \frac{\sigma}{\delta}\frac{(1-\mu)(1-\mu\beta)}{\Delta^D_2} &= \Omega^{g-b}_D\end{aligned}$$

and

$$\gamma_{1,bad} = \Omega_D \left(r^R_{t,bad} - \overline{r}^R_{t,bad} \right) + \Omega_D^{g-b} \frac{\overline{\mathsf{p}}^{good}_{bad}}{1 - \overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}} \left(\gamma_{1,good} - \gamma_{1,bad} \right) \tag{90}$$

Similarly, rewrite

$$\begin{aligned}
\gamma_{2,good} &= -\frac{2}{\sigma} \left(\lambda_{\widetilde{y}^W} + \lambda_\pi \frac{\left(\frac{\kappa}{2}\right)^2}{1 - \mu\beta} \right) \widetilde{y}^W_{t,good} \\
(1 - \mu\beta) \frac{\sigma}{2} \gamma_{2,good} &= \left((1 - \mu\beta) \lambda_{\widetilde{y}^W} + \lambda_\pi \left(\frac{\kappa}{2} \right)^2 \right) \widetilde{y}^W_{t,good} \\
(1 - \mu\beta) \frac{\sigma}{2} \gamma_{2,good} &= -\Psi \widetilde{y}^W_{t,good} \qquad (91) \\
(1 - \mu\beta) \frac{\sigma}{2} \gamma_{2,bad} &= -\Psi \widetilde{y}^W_{t,bad} \qquad (92)
\end{aligned}$$

with

$$\left((1 - \mu\beta) \lambda_{\widetilde{y}^W} + \lambda_\pi \left(\frac{\kappa}{2} \right)^2 \right) = \Psi$$

and use (87)

$$\begin{aligned}
(1 - \mu\beta) \left(r^W_{t,good} - \overline{r}^W_{t,good} \right) &= -\Delta_2 \widetilde{y}^W_{good,t} - \sigma (1 - \mu)(1 - \mu\beta) \frac{\overline{\mathsf{p}}^{bad}_{good}}{1 - \overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}} \left(\widetilde{y}^W_{good,t} - \widetilde{y}^W_{bad,t} \right) \\
\left(r^W_{t,good} - \overline{r}^W_{t,good} \right) &= \frac{\Delta_2}{\Psi} \frac{\sigma}{2} \gamma_{2,good} - \sigma (1 - \mu) \frac{\overline{\mathsf{p}}^{bad}_{good}}{1 - \overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}} \left(\widetilde{y}^W_{good,t} - \widetilde{y}^W_{bad,t} \right) \\
\gamma_{2,good} &= \frac{\Psi}{\Delta_2 S} \left(r^W_{t,good} - \overline{r}^W_{t,good} \right) + \frac{\Psi}{\Delta_2 S} \sigma (1 - \mu) \frac{\overline{\mathsf{p}}^{bad}_{good}}{1 - \overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}} \left(\widetilde{y}^W_{good,t} - \widetilde{y}^W_{bad,t} \right) \\
\gamma_{2,good} &= \left(\frac{\Psi}{\Delta_2 S} \right) \left(r^W_{t,good} - \overline{r}^W_{t,good} \right) + \frac{\Psi}{\Delta_2 S} \sigma (1 - \mu) \frac{\overline{\mathsf{p}}^{bad}_{good}}{1 - \overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}} \left(\widetilde{y}^W_{good,t} - \widetilde{y}^W_{bad,t} \right) \\
\gamma_{2,good} &= \Omega \left(r^W_{t,good} - \overline{r}^W_{t,good} \right) - \sigma \frac{(1 - \mu)(1 - \mu\beta)}{\Delta_2} \frac{\overline{\mathsf{p}}^{bad}_{good}}{1 - \overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}} \left(\gamma_{2,good} - \gamma_{2,bad} \right) \qquad (93)
\end{aligned}$$

with

$$\begin{aligned}
\frac{\sigma}{2} &= S \\
\left(\frac{\Psi}{\Delta_2 S} \right) &= \Omega \\
\sigma \frac{(1 - \mu)(1 - \mu\beta)}{\Delta_2} &= \Omega^{g-b}
\end{aligned}$$

and

$$\gamma_{2,bad} = \Omega \left(r^W_{t,bad} - \overline{r}^W_{t,bad} \right) + \Omega^{g-b} \frac{\overline{\mathsf{p}}^{good}_{bad}}{1 - \overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}} \left(\gamma_{2,good} - \gamma_{2,bad} \right) \tag{94}$$

In summary we have:

$$\begin{aligned}
\gamma_{1,good} &= \Omega_D \left(r^R_{t,good} - \overline{r}^R_{t,good} \right) - \Omega_D^{g-b} \frac{\overline{\mathsf{p}}^{bad}_{good}}{1 - \overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}} \left(\gamma_{1,good} - \gamma_{1,bad} \right) \\
\gamma_{1,bad} &= \Omega_D \left(r^R_{t,bad} - \overline{r}^R_{t,bad} \right) + \Omega_D^{g-b} \frac{\overline{\mathsf{p}}^{good}_{bad}}{1 - \overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}} \left(\gamma_{1,good} - \gamma_{1,bad} \right)
\end{aligned}$$

$$\begin{aligned}
\gamma_{2,good} &= \Omega \left(r^W_{t,good} - \overline{r}^W_{t,good} \right) - \Omega^{g-b} \frac{\overline{\mathsf{p}}^{bad}_{good}}{1 - \overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}} \left(\gamma_{2,good} - \gamma_{2,bad} \right) \\
\gamma_{2,bad} &= \Omega \left(r^W_{t,bad} - \overline{r}^W_{t,bad} \right) + \Omega^{g-b} \frac{\overline{\mathsf{p}}^{good}_{bad}}{1 - \overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}} \left(\gamma_{2,good} - \gamma_{2,bad} \right)
\end{aligned}$$

with

$$
\begin{aligned}
\Omega &= \frac{\Psi}{\Delta_2 S} \\
\Psi &= (1-\mu\beta)\,\lambda_{\widetilde{y}^W} + \lambda_\pi \left(\frac{\kappa}{2}\right)^2 \\
\Delta_2 &= (1-\mu\beta)\,\sigma\,(1-\mu) - \mu\frac{\kappa}{2} \\
S &= \frac{\sigma}{2} \\
\Omega_D &= \frac{\Psi_D}{\Delta_2^D S_D} \\
\Psi_D &= (1-\mu\beta)\,\lambda_{\widetilde{y}^R} + \lambda_\pi \left(\frac{\kappa}{2} - \kappa_{(y-y^*)}\right)^2 \\
\Delta_2^D &= (1-\mu\beta)\,\frac{\sigma}{\delta}\,(1-\mu) - \mu\left(\frac{\kappa}{2} - \kappa_{(y-y^*)}\right) \\
S_D &= \frac{\sigma}{\delta}\frac{1}{2} \\
\Omega^{g-b} &= \sigma\frac{(1-\mu)(1-\mu\beta)}{\Delta_2} \\
\Omega_D^{g-b} &= \frac{\sigma}{\delta}\frac{(1-\mu)(1-\mu\beta)}{\Delta_2^D}
\end{aligned}
$$

Let's take differences:

$$
\gamma_{1,good} - \gamma_{1,bad} = \Omega_D\left[\left(r^R_{t,good} - \overline{r}^R_{t,good}\right) - \left(r^R_{t,bad} - \overline{r}^R_{t,bad}\right)\right] - \frac{\Omega_D^{g-b}\left(\overline{\mathsf{p}}^{bad}_{good} + \overline{\mathsf{p}}^{good}_{bad}\right)}{1 - \overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}}\left(\gamma_{1,good} - \gamma_{1,bad}\right)
$$

$$
\left(\gamma_{1,good} - \gamma_{1,bad}\right)\frac{\left(1 - \left(\overline{\mathsf{p}}^{bad}_{good} + \overline{\mathsf{p}}^{good}_{bad}\right) + \Omega_D^{g-b}\left(\overline{\mathsf{p}}^{bad}_{good} + \overline{\mathsf{p}}^{good}_{bad}\right)\right)}{1 - \overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}} = \Omega_D\left[\left(r^R_{t,good} - \overline{r}^R_{t,good}\right) - \left(r^R_{t,bad} - \overline{r}^R_{t,bad}\right)\right]
$$

$$
\left(\gamma_{1,good} - \gamma_{1,bad}\right) = \Omega_D\frac{\left(1 - \overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}\right)}{1 - \left(1 - \Omega_D^{g-b}\right)\left(\overline{\mathsf{p}}^{bad}_{good} + \overline{\mathsf{p}}^{good}_{bad}\right)}\left[\left(r^R_{t,good} - \overline{r}^R_{t,good}\right) - \left(r^R_{t,bad} - \overline{r}^R_{t,bad}\right)\right]
$$

Similarly,

$$
\left(\gamma_{2,good} - \gamma_{2,bad}\right) = \Omega\frac{\left(1 - \overline{\mathsf{p}}^{bad}_{good} - \overline{\mathsf{p}}^{good}_{bad}\right)}{1 - \left(1 - \Omega^{g-b}\right)\left(\overline{\mathsf{p}}^{bad}_{good} + \overline{\mathsf{p}}^{good}_{bad}\right)}\left[\left(r^W_{t,good} - \overline{r}^W_{t,good}\right) - \left(r^W_{t,bad} - \overline{r}^W_{t,bad}\right)\right]
$$

$$
\begin{aligned}
\gamma_{1,good} &= \Omega_D\left(r^R_{t,good} - \overline{r}^R_{t,good}\right) - \overline{\mathsf{p}}^{bad}_{good}\frac{\Omega_D^{g-b}\Omega_D}{1 - \left(1 - \Omega_D^{g-b}\right)\left(\overline{\mathsf{p}}^{bad}_{good} + \overline{\mathsf{p}}^{good}_{bad}\right)}\left[\left(r^R_{t,good} - \overline{r}^R_{t,good}\right) - \left(r^R_{t,bad} - \overline{r}^R_{t,bad}\right)\right] \\
\gamma_{1,bad} &= \Omega_D\left(r^R_{t,bad} - \overline{r}^R_{t,bad}\right) + \overline{\mathsf{p}}^{good}_{bad}\frac{\Omega_D^{g-b}\Omega_D}{1 - \left(1 - \Omega_D^{g-b}\right)\left(\overline{\mathsf{p}}^{bad}_{good} + \overline{\mathsf{p}}^{good}_{bad}\right)}\left[\left(r^R_{t,good} - \overline{r}^R_{t,good}\right) - \left(r^R_{t,bad} - \overline{r}^R_{t,bad}\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
\gamma_{2,good} &= \Omega\left(r^W_{t,good} - \overline{r}^W_{t,good}\right) - \overline{\mathsf{p}}^{bad}_{good}\frac{\Omega^{g-b}\Omega}{1 - \left(1 - \Omega^{g-b}\right)\left(\overline{\mathsf{p}}^{bad}_{good} + \overline{\mathsf{p}}^{good}_{bad}\right)}\left[\left(r^W_{t,good} - \overline{r}^W_{t,good}\right) - \left(r^W_{t,bad} - \overline{r}^W_{t,bad}\right)\right] \\
\gamma_{2,bad} &= \Omega\left(r^W_{t,bad} - \overline{r}^W_{t,bad}\right) + \overline{\mathsf{p}}^{good}_{bad}\frac{\Omega^{g-b}\Omega}{1 - \left(1 - \Omega^{g-b}\right)\left(\overline{\mathsf{p}}^{bad}_{good} + \overline{\mathsf{p}}^{good}_{bad}\right)}\left[\left(r^W_{t,good} - \overline{r}^W_{t,good}\right) - \left(r^W_{t,bad} - \overline{r}^W_{t,bad}\right)\right]
\end{aligned}
$$

Then

$$
\begin{aligned}
\gamma_{1,good} + \gamma_{2,good} = {} & \Omega_D\left(r^R_{t,good} - \overline{r}^R_{t,good}\right) \\
& -\overline{\mathsf{p}}^{bad}_{good}\frac{\Omega_D^{g-b}\Omega_D}{1 - \left(1 - \Omega_D^{g-b}\right)\left(\overline{\mathsf{p}}^{bad}_{good} + \overline{\mathsf{p}}^{good}_{bad}\right)}\left[\left(r^R_{t,good} - \overline{r}^R_{t,good}\right) - \left(r^R_{t,bad} - \overline{r}^R_{t,bad}\right)\right] + \\
& \Omega\left(r^W_{t,good} - \overline{r}^W_{t,good}\right) \\
& -\overline{\mathsf{p}}^{bad}_{good}\frac{\Omega^{g-b}\Omega}{1 - \left(1 - \Omega^{g-b}\right)\left(\overline{\mathsf{p}}^{bad}_{good} + \overline{\mathsf{p}}^{good}_{bad}\right)}\left[\left(r^W_{t,good} - \overline{r}^W_{t,good}\right) - \left(r^W_{t,bad} - \overline{r}^W_{t,bad}\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
\gamma_{1,good}+\gamma_{2,good} &= (\Omega_D+\Omega)\frac{r_{t,good}-\overline{r}_{t,good}}{2}-(\Omega_D-\Omega)\frac{r^*_{t,good}-\overline{r}^*_{t,good}}{2} \\
&-\overline{\mathsf{p}}^{bad}_{good}\left(\frac{\Omega_D^{g-b}\Omega_D}{1-\left(1-\Omega_D^{g-b}\right)\left(\overline{\mathsf{p}}^{bad}_{good}+\overline{\mathsf{p}}^{good}_{bad}\right)}+\frac{\Omega^{g-b}\Omega}{1-\left(1-\Omega^{g-b}\right)\left(\overline{\mathsf{p}}^{bad}_{good}+\overline{\mathsf{p}}^{good}_{bad}\right)}\right)\frac{r_{t,good}-\overline{r}_{t,good}}{2} \\
&+\overline{\mathsf{p}}^{bad}_{good}\left(\frac{\Omega_D^{g-b}\Omega_D}{1-\left(1-\Omega_D^{g-b}\right)\left(\overline{\mathsf{p}}^{bad}_{good}+\overline{\mathsf{p}}^{good}_{bad}\right)}-\frac{\Omega^{g-b}\Omega}{1-\left(1-\Omega^{g-b}\right)\left(\overline{\mathsf{p}}^{bad}_{good}+\overline{\mathsf{p}}^{good}_{bad}\right)}\right)\frac{r^*_{t,good}-\overline{r}^*_{t,good}}{2} \\
&+\overline{\mathsf{p}}^{bad}_{good}\left(\frac{\Omega_D^{g-b}\Omega_D}{1-\left(1-\Omega_D^{g-b}\right)\left(\overline{\mathsf{p}}^{bad}_{good}+\overline{\mathsf{p}}^{good}_{bad}\right)}+\frac{\Omega^{g-b}\Omega}{1-\left(1-\Omega^{g-b}\right)\left(\overline{\mathsf{p}}^{bad}_{good}+\overline{\mathsf{p}}^{good}_{bad}\right)}\right)\frac{r_{t,bad}-\overline{r}_{t,bad}}{2} \\
&-\overline{\mathsf{p}}^{bad}_{good}\left(\frac{\Omega_D^{g-b}\Omega_D}{1-\left(1-\Omega_D^{g-b}\right)\left(\overline{\mathsf{p}}^{bad}_{good}+\overline{\mathsf{p}}^{good}_{bad}\right)}-\frac{\Omega^{g-b}\Omega}{1-\left(1-\Omega^{g-b}\right)\left(\overline{\mathsf{p}}^{bad}_{good}+\overline{\mathsf{p}}^{good}_{bad}\right)}\right)\frac{r^*_{t,bad}-\overline{r}^*_{t,bad}}{2}
\end{aligned}
$$

Let's define

$$
\begin{aligned}
\frac{\Omega_D^{g-b}\Omega_D}{1-\left(1-\Omega_D^{g-b}\right)\left(\overline{\mathsf{p}}^{bad}_{good}+\overline{\mathsf{p}}^{good}_{bad}\right)}+\frac{\Omega^{g-b}\Omega}{1-\left(1-\Omega^{g-b}\right)\left(\overline{\mathsf{p}}^{bad}_{good}+\overline{\mathsf{p}}^{good}_{bad}\right)} &= \Omega_{\gamma_1+\gamma_2} \\
\frac{\Omega_D^{g-b}\Omega_D}{1-\left(1-\Omega_D^{g-b}\right)\left(\overline{\mathsf{p}}^{bad}_{good}+\overline{\mathsf{p}}^{good}_{bad}\right)}-\frac{\Omega^{g-b}\Omega}{1-\left(1-\Omega^{g-b}\right)\left(\overline{\mathsf{p}}^{bad}_{good}+\overline{\mathsf{p}}^{good}_{bad}\right)} &= \Omega_{D\gamma_1+\gamma_2}
\end{aligned}
$$

so that

$$
\begin{aligned}
\gamma_{1,good}+\gamma_{2,good} &= \left(\begin{array}{c}(\Omega_D+\Omega) \\ -\overline{\mathsf{p}}^{bad}_{good}\left(\frac{\Omega_D^{g-b}\Omega_D}{1-\left(1-\Omega_D^{g-b}\right)\left(\overline{\mathsf{p}}^{bad}_{good}+\overline{\mathsf{p}}^{good}_{bad}\right)}+\frac{\Omega^{g-b}\Omega}{1-\left(1-\Omega^{g-b}\right)\left(\overline{\mathsf{p}}^{bad}_{good}+\overline{\mathsf{p}}^{good}_{bad}\right)}\right)\end{array}\right)\frac{r_{t,good}-\overline{r}_{t,good}}{2} \\
&-\left(\begin{array}{c}(\Omega_D-\Omega) \\ -\overline{\mathsf{p}}^{bad}_{good}\left(\frac{\Omega_D^{g-b}\Omega_D}{1-\left(1-\Omega_D^{g-b}\right)\left(\overline{\mathsf{p}}^{bad}_{good}+\overline{\mathsf{p}}^{good}_{bad}\right)}-\frac{\Omega^{g-b}\Omega}{1-\left(1-\Omega^{g-b}\right)\left(\overline{\mathsf{p}}^{bad}_{good}+\overline{\mathsf{p}}^{good}_{bad}\right)}\right)\end{array}\right)\frac{r^*_{t,good}-\overline{r}^*_{t,good}}{2} \\
&+\overline{\mathsf{p}}^{bad}_{good}\left(\frac{\Omega_D^{g-b}\Omega_D}{1-\left(1-\Omega_D^{g-b}\right)\left(\overline{\mathsf{p}}^{bad}_{good}+\overline{\mathsf{p}}^{good}_{bad}\right)}+\frac{\Omega^{g-b}\Omega}{1-\left(1-\Omega^{g-b}\right)\left(\overline{\mathsf{p}}^{bad}_{good}+\overline{\mathsf{p}}^{good}_{bad}\right)}\right)\frac{r_{t,bad}-\overline{r}_{t,bad}}{2} \\
&-\overline{\mathsf{p}}^{bad}_{good}\left(\frac{\Omega_D^{g-b}\Omega_D}{1-\left(1-\Omega_D^{g-b}\right)\left(\overline{\mathsf{p}}^{bad}_{good}+\overline{\mathsf{p}}^{good}_{bad}\right)}-\frac{\Omega^{g-b}\Omega}{1-\left(1-\Omega^{g-b}\right)\left(\overline{\mathsf{p}}^{bad}_{good}+\overline{\mathsf{p}}^{good}_{bad}\right)}\right)\frac{r^*_{t,bad}-\overline{r}^*_{t,bad}}{2}
\end{aligned}
$$

becomes

$$
\begin{aligned}
\gamma_{1,good}+\gamma_{2,good} &= \left((\Omega_D+\Omega)-\overline{\mathsf{p}}^{bad}_{good}\Omega_{\gamma_1+\gamma_2}\right)\frac{r_{t,good}-\overline{r}_{t,good}}{2}-\left((\Omega_D-\Omega)-\overline{\mathsf{p}}^{bad}_{good}\Omega_{D\gamma_1+\gamma_2}\right)\frac{r^*_{t,good}-\overline{r}^*_{t,good}}{2} \qquad (95)\\
&+\overline{\mathsf{p}}^{bad}_{good}\Omega_{\gamma_1+\gamma_2}\frac{r_{t,bad}-\overline{r}_{t,bad}}{2}-\overline{\mathsf{p}}^{bad}_{good}\Omega_{D\gamma_1+\gamma_2}\frac{r^*_{t,bad}-\overline{r}^*_{t,bad}}{2}
\end{aligned}
$$

In the bad state we have

$$
\begin{aligned}
\gamma_{1,bad}+\gamma_{2,bad} &= \Omega_D\left(r^R_{t,bad}-\overline{r}^R_{t,bad}\right)+\overline{\mathsf{p}}^{good}_{bad}\frac{\Omega_D^{g-b}\Omega_D}{1-\left(1-\Omega_D^{g-b}\right)\left(\overline{\mathsf{p}}^{bad}_{good}+\overline{\mathsf{p}}^{good}_{bad}\right)}\left[\left(r^R_{t,good}-\overline{r}^R_{t,good}\right)-\left(r^R_{t,bad}-\overline{r}^R_{t,bad}\right)\right]+ \\
&\Omega\left(r^W_{t,bad}-\overline{r}^W_{t,bad}\right)+\overline{\mathsf{p}}^{good}_{bad}\frac{\Omega^{g-b}\Omega}{1-\left(1-\Omega^{g-b}\right)\left(\overline{\mathsf{p}}^{bad}_{good}+\overline{\mathsf{p}}^{good}_{bad}\right)}\left[\left(r^W_{t,good}-\overline{r}^W_{t,good}\right)-\left(r^W_{t,bad}-\overline{r}^W_{t,bad}\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
\gamma_{1,bad}+\gamma_{2,bad} &= (\Omega_D+\Omega)\frac{r_{t,bad}-\overline{r}_{t,bad}}{2}-(\Omega_D-\Omega)\frac{r^*_{t,bad}-\overline{r}^*_{t,bad}}{2} \\
&+\overline{\mathsf{p}}^{good}_{bad}\left(\frac{\Omega_D^{g-b}\Omega_D}{1-\left(1-\Omega_D^{g-b}\right)\left(\overline{\mathsf{p}}^{bad}_{good}+\overline{\mathsf{p}}^{good}_{bad}\right)}+\frac{\Omega^{g-b}\Omega}{1-\left(1-\Omega^{g-b}\right)\left(\overline{\mathsf{p}}^{bad}_{good}+\overline{\mathsf{p}}^{good}_{bad}\right)}\right)\frac{r_{t,good}-\overline{r}_{t,good}}{2} \\
&-\overline{\mathsf{p}}^{good}_{bad}\left(\frac{\Omega_D^{g-b}\Omega_D}{1-\left(1-\Omega_D^{g-b}\right)\left(\overline{\mathsf{p}}^{bad}_{good}+\overline{\mathsf{p}}^{good}_{bad}\right)}-\frac{\Omega^{g-b}\Omega}{1-\left(1-\Omega^{g-b}\right)\left(\overline{\mathsf{p}}^{bad}_{good}+\overline{\mathsf{p}}^{good}_{bad}\right)}\right)\frac{r^*_{t,good}-\overline{r}^*_{t,good}}{2}
\end{aligned}
$$

$$
\begin{aligned}
&-\overline{\mathsf{p}}_{bad}^{good}\left(\frac{\Omega_D^{g-b}\Omega_D}{1-\left(1-\Omega_D^{g-b}\right)\left(\overline{\mathsf{p}}_{good}^{bad}+\overline{\mathsf{p}}_{bad}^{good}\right)}+\frac{\Omega^{g-b}\Omega}{1-\left(1-\Omega^{g-b}\right)\left(\overline{\mathsf{p}}_{good}^{bad}+\overline{\mathsf{p}}_{bad}^{good}\right)}\right)\frac{r_{t,bad}-\overline{r}_{t,bad}}{2}\\
&+\overline{\mathsf{p}}_{bad}^{good}\left(\frac{\Omega_D^{g-b}\Omega_D}{1-\left(1-\Omega_D^{g-b}\right)\left(\overline{\mathsf{p}}_{good}^{bad}+\overline{\mathsf{p}}_{bad}^{good}\right)}-\frac{\Omega^{g-b}\Omega}{1-\left(1-\Omega^{g-b}\right)\left(\overline{\mathsf{p}}_{good}^{bad}+\overline{\mathsf{p}}_{bad}^{good}\right)}\right)\frac{r_{t,bad}^{*}-\overline{r}_{t,bad}^{*}}{2}
\end{aligned}
$$

Let's define

$$
\begin{aligned}
\frac{\Omega_D^{g-b}\Omega_D}{1-\left(1-\Omega_D^{g-b}\right)\left(\overline{\mathsf{p}}_{good}^{bad}+\overline{\mathsf{p}}_{bad}^{good}\right)}+\frac{\Omega^{g-b}\Omega}{1-\left(1-\Omega^{g-b}\right)\left(\overline{\mathsf{p}}_{good}^{bad}+\overline{\mathsf{p}}_{bad}^{good}\right)} &= \Omega_{\gamma_1+\gamma_2}\\
\frac{\Omega_D^{g-b}\Omega_D}{1-\left(1-\Omega_D^{g-b}\right)\left(\overline{\mathsf{p}}_{good}^{bad}+\overline{\mathsf{p}}_{bad}^{good}\right)}-\frac{\Omega^{g-b}\Omega}{1-\left(1-\Omega^{g-b}\right)\left(\overline{\mathsf{p}}_{good}^{bad}+\overline{\mathsf{p}}_{bad}^{good}\right)} &= \Omega_{D\gamma_1+\gamma_2}
\end{aligned}
$$

so that

$$
\begin{aligned}
\gamma_{1,bad}+\gamma_{2,bad} &= (\Omega_D+\Omega)\frac{r_{t,bad}-\overline{r}_{t,bad}}{2}-(\Omega_D-\Omega)\frac{r_{t,bad}^{*}-\overline{r}_{t,bad}^{*}}{2}\\
&+\overline{\mathsf{p}}_{bad}^{good}\Omega_{\gamma_1+\gamma_2}\frac{r_{t,good}-\overline{r}_{t,good}}{2}\\
&-\overline{\mathsf{p}}_{bad}^{good}\Omega_{D\gamma_1+\gamma_2}\frac{r_{t,good}^{*}-\overline{r}_{t,good}^{*}}{2}\\
&-\overline{\mathsf{p}}_{bad}^{good}\Omega_{\gamma_1+\gamma_2}\frac{r_{t,bad}-\overline{r}_{t,bad}}{2}\\
&+\overline{\mathsf{p}}_{bad}^{good}\Omega_{D\gamma_1+\gamma_2}\frac{r_{t,bad}^{*}-\overline{r}_{t,bad}^{*}}{2}
\end{aligned}
$$

becomes

$$
\begin{aligned}
\gamma_{1,bad}+\gamma_{2,bad} &= \left((\Omega_D+\Omega)-\overline{\mathsf{p}}_{bad}^{good}\Omega_{\gamma_1+\gamma_2}\right)\frac{r_{t,bad}-\overline{r}_{t,bad}}{2}-\left((\Omega_D-\Omega)-\overline{\mathsf{p}}_{bad}^{good}\Omega_{D\gamma_1+\gamma_2}\right)\frac{r_{t,bad}^{*}-\overline{r}_{t,bad}^{*}}{2}\\
&+\overline{\mathsf{p}}_{bad}^{good}\Omega_{\gamma_1+\gamma_2}\frac{r_{t,good}-\overline{r}_{t,good}}{2}-\overline{\mathsf{p}}_{bad}^{good}\Omega_{D\gamma_1+\gamma_2}\frac{r_{t,good}^{*}-\overline{r}_{t,good}^{*}}{2}
\end{aligned}
\tag{96}
$$

Now we calculate the difference of the two multiplier:

In the bad state

$$
\begin{aligned}
\gamma_{2,bad}-\gamma_{1,bad} &= \Omega\left(r_{t,bad}^{W}-\overline{r}_{t,bad}^{W}\right)-\Omega_D\left(r_{t,bad}^{R}-\overline{r}_{t,bad}^{R}\right)\\
&+\overline{\mathsf{p}}_{bad}^{good}\frac{\Omega^{g-b}\Omega}{1-\left(1-\Omega^{g-b}\right)\left(\overline{\mathsf{p}}_{good}^{bad}+\overline{\mathsf{p}}_{bad}^{good}\right)}\left[\left(r_{t,good}^{W}-\overline{r}_{t,good}^{W}\right)-\left(r_{t,bad}^{W}-\overline{r}_{t,bad}^{W}\right)\right]\\
&-\overline{\mathsf{p}}_{bad}^{good}\frac{\Omega_D^{g-b}\Omega_D}{1-\left(1-\Omega_D^{g-b}\right)\left(\overline{\mathsf{p}}_{good}^{bad}+\overline{\mathsf{p}}_{bad}^{good}\right)}\left[\left(r_{t,good}^{R}-\overline{r}_{t,good}^{R}\right)-\left(r_{t,bad}^{R}-\overline{r}_{t,bad}^{R}\right)\right]
\end{aligned}
$$

In the good state

$$
\begin{aligned}
\gamma_{2,good}-\gamma_{1,good} &= \Omega\left(r_{t,good}^{W}-\overline{r}_{t,good}^{W}\right)-\Omega_D\left(r_{t,good}^{R}-\overline{r}_{t,good}^{R}\right)\\
&-\overline{\mathsf{p}}_{good}^{bad}\frac{\Omega^{g-b}\Omega}{1-\left(1-\Omega^{g-b}\right)\left(\overline{\mathsf{p}}_{good}^{bad}+\overline{\mathsf{p}}_{bad}^{good}\right)}\left[\left(r_{t,good}^{W}-\overline{r}_{t,good}^{W}\right)-\left(r_{t,bad}^{W}-\overline{r}_{t,bad}^{W}\right)\right]\\
&+\overline{\mathsf{p}}_{good}^{bad}\frac{\Omega_D^{g-b}\Omega_D}{1-\left(1-\Omega_D^{g-b}\right)\left(\overline{\mathsf{p}}_{good}^{bad}+\overline{\mathsf{p}}_{bad}^{good}\right)}\left[\left(r_{t,good}^{R}-\overline{r}_{t,good}^{R}\right)-\left(r_{t,bad}^{R}-\overline{r}_{t,bad}^{R}\right)\right]
\end{aligned}
$$

and for

$$
\begin{aligned}
\frac{\Omega_D^{g-b}\Omega_D}{1-\left(1-\Omega_D^{g-b}\right)\left(\overline{\mathsf{p}}_{good}^{bad}+\overline{\mathsf{p}}_{bad}^{good}\right)}+\frac{\Omega^{g-b}\Omega}{1-\left(1-\Omega^{g-b}\right)\left(\overline{\mathsf{p}}_{good}^{bad}+\overline{\mathsf{p}}_{bad}^{good}\right)} &= \Omega_{\gamma_1+\gamma_2}\\
\frac{\Omega_D^{g-b}\Omega_D}{1-\left(1-\Omega_D^{g-b}\right)\left(\overline{\mathsf{p}}_{good}^{bad}+\overline{\mathsf{p}}_{bad}^{good}\right)}-\frac{\Omega^{g-b}\Omega}{1-\left(1-\Omega^{g-b}\right)\left(\overline{\mathsf{p}}_{good}^{bad}+\overline{\mathsf{p}}_{bad}^{good}\right)} &= \Omega_{D\gamma_1+\gamma_2}
\end{aligned}
$$

$$\gamma_{2,good}-\gamma_{1,good} = \left((\Omega_D+\Omega)-\overline{\mathsf{p}}_{good}^{bad}\Omega_{\gamma_1+\gamma_2}\right)\frac{r^*_{t,good}-\overline{r}^*_{t,good}}{2}-\left((\Omega_D-\Omega)-\overline{\mathsf{p}}_{good}^{bad}\Omega_{D\gamma_1+\gamma_2}\right)\frac{r_{t,good}-\overline{r}_{t,good}}{2} \quad (97)$$
$$+\overline{\mathsf{p}}_{good}^{bad}\Omega_{\gamma_1+\gamma_2}\frac{r^*_{t,bad}-\overline{r}^*_{t,bad}}{2}-\overline{\mathsf{p}}_{good}^{bad}\Omega_{D\gamma_1+\gamma_2}\frac{r_{t,bad}-\overline{r}_{t,bad}}{2}$$

Doing similar steps for the bad state:

$$\gamma_{2,bad}-\gamma_{1,bad} = \left((\Omega_D+\Omega)-\overline{\mathsf{p}}_{bad}^{good}\Omega_{\gamma_1+\gamma_2}\right)\frac{r^*_{t,bad}-\overline{r}^*_{t,bad}}{2}-\left((\Omega_D-\Omega)-\overline{\mathsf{p}}_{bad}^{good}\Omega_{D\gamma_1+\gamma_2}\right)\frac{r_{t,bad}-\overline{r}_{t,bad}}{2} \quad (98)$$
$$+\overline{\mathsf{p}}_{bad}^{good}\Omega_{\gamma_1+\gamma_2}\frac{r^*_{t,good}-\overline{r}^*_{t,good}}{2}-\overline{\mathsf{p}}_{bad}^{good}\Omega_{D\gamma_1+\gamma_2}\frac{r_{t,good}-\overline{r}_{t,good}}{2}$$

If the foreign country is not at the ZLB we have

$$(\gamma_{2,\mathsf{s}}-\gamma_{1,\mathsf{s}})=0$$

Then

$$\left((\Omega_D+\Omega)-\overline{\mathsf{p}}_{good}^{bad}\Omega_{\gamma_1+\gamma_2}\right)\frac{r^*_{t,good}-\overline{r}^*_{t,good}}{2} = \left((\Omega_D-\Omega)-\overline{\mathsf{p}}_{good}^{bad}\Omega_{D\gamma_1+\gamma_2}\right)\frac{r_{t,good}-\overline{r}_{t,good}}{2}$$
$$-\overline{\mathsf{p}}_{good}^{bad}\Omega_{\gamma_1+\gamma_2}\frac{r^*_{t,bad}-\overline{r}^*_{t,bad}}{2}+\overline{\mathsf{p}}_{good}^{bad}\Omega_{D\gamma_1+\gamma_2}\frac{r_{t,bad}-\overline{r}_{t,bad}}{2}$$

$$\left((\Omega_D+\Omega)-\overline{\mathsf{p}}_{bad}^{good}\Omega_{\gamma_1+\gamma_2}\right)\frac{r^*_{t,bad}-\overline{r}^*_{t,bad}}{2} = \left((\Omega_D-\Omega)-\overline{\mathsf{p}}_{bad}^{good}\Omega_{D\gamma_1+\gamma_2}\right)\frac{r_{t,bad}-\overline{r}_{t,bad}}{2}$$
$$-\overline{\mathsf{p}}_{bad}^{good}\Omega_{\gamma_1+\gamma_2}\frac{r^*_{t,good}-\overline{r}^*_{t,good}}{2}+\overline{\mathsf{p}}_{bad}^{good}\Omega_{D\gamma_1+\gamma_2}\frac{r_{t,good}-\overline{r}_{t,good}}{2}$$

When the home country is at the ZLB:

$$\left((\Omega_D+\Omega)-\overline{\mathsf{p}}_{good}^{bad}\Omega_{\gamma_1+\gamma_2}\right)\frac{r^*_{t,good}-\overline{r}^*_{t,good}}{2} = -\left((\Omega_D-\Omega)-\overline{\mathsf{p}}_{good}^{bad}\Omega_{D\gamma_1+\gamma_2}\right)\frac{\overline{r}_{t,good}}{2}$$
$$-\overline{\mathsf{p}}_{good}^{bad}\Omega_{\gamma_1+\gamma_2}\frac{r^*_{t,bad}-\overline{r}^*_{t,bad}}{2}-\overline{\mathsf{p}}_{good}^{bad}\Omega_{D\gamma_1+\gamma_2}\frac{\overline{r}_{t,bad}}{2}$$

$$\left((\Omega_D+\Omega)-\overline{\mathsf{p}}_{bad}^{good}\Omega_{\gamma_1+\gamma_2}\right)\frac{r^*_{t,bad}-\overline{r}^*_{t,bad}}{2} = -\left((\Omega_D-\Omega)-\overline{\mathsf{p}}_{bad}^{good}\Omega_{D\gamma_1+\gamma_2}\right)\frac{\overline{r}_{t,bad}}{2}$$
$$-\overline{\mathsf{p}}_{bad}^{good}\Omega_{\gamma_1+\gamma_2}\frac{r^*_{t,good}-\overline{r}^*_{t,good}}{2}-\overline{\mathsf{p}}_{bad}^{good}\Omega_{D\gamma_1+\gamma_2}\frac{\overline{r}_{t,good}}{2}$$

Then substitute for $\frac{r^*_{t,bad}-\overline{r}^*_{t,bad}}{2}$ in the good state: First

$$\frac{r^*_{t,bad}-\overline{r}^*_{t,bad}}{2} = -\frac{\left((\Omega_D-\Omega)-\overline{\mathsf{p}}_{bad}^{good}\Omega_{D\gamma_1+\gamma_2}\right)}{\left((\Omega_D+\Omega)-\overline{\mathsf{p}}_{bad}^{good}\Omega_{\gamma_1+\gamma_2}\right)}\frac{\overline{r}_{t,bad}}{2}-\overline{\mathsf{p}}_{bad}^{good}\frac{\Omega_{\gamma_1+\gamma_2}}{\left((\Omega_D+\Omega)-\overline{\mathsf{p}}_{bad}^{good}\Omega_{\gamma_1+\gamma_2}\right)}\frac{r^*_{t,good}-\overline{r}^*_{t,good}}{2}$$
$$-\overline{\mathsf{p}}_{bad}^{good}\frac{\Omega_{D\gamma_1+\gamma_2}}{\left((\Omega_D+\Omega)-\overline{\mathsf{p}}_{bad}^{good}\Omega_{\gamma_1+\gamma_2}\right)}\frac{\overline{r}_{t,good}}{2}$$

so that

$$\left((\Omega_D+\Omega)-\overline{\mathsf{p}}_{good}^{bad}\Omega_{\gamma_1+\gamma_2}\right)\frac{r^*_{t,good}-\overline{r}^*_{t,good}}{2} = -\left((\Omega_D-\Omega)-\overline{\mathsf{p}}_{good}^{bad}\Omega_{D\gamma_1+\gamma_2}\right)\frac{\overline{r}_{t,good}}{2}$$
$$-\overline{\mathsf{p}}_{good}^{bad}\Omega_{\gamma_1+\gamma_2}\frac{r^*_{t,bad}-\overline{r}^*_{t,bad}}{2}-\overline{\mathsf{p}}_{good}^{bad}\Omega_{D\gamma_1+\gamma_2}\frac{\overline{r}_{t,bad}}{2}$$

becomes

$$\frac{r^*_{t,good}-\overline{r}^*_{t,good}}{2}$$
$$= -\frac{(\Omega_D-\Omega)\left((\Omega_D+\Omega)-\overline{\mathsf{p}}_{bad}^{good}\Omega_{\gamma_1+\gamma_2}\right)-\overline{\mathsf{p}}_{good}^{bad}\Omega_{D\gamma_1+\gamma_2}(\Omega_D+\Omega)}{(\Omega_D+\Omega)\left((\Omega_D+\Omega)-\Omega_{\gamma_1+\gamma_2}\left(\overline{\mathsf{p}}_{bad}^{good}+\overline{\mathsf{p}}_{good}^{bad}\right)\right)}\frac{\overline{r}_{t,good}}{2}$$
$$-\overline{\mathsf{p}}_{good}^{bad}\frac{(\Omega_D+\Omega)\,\Omega_{D\gamma_1+\gamma_2}-(\Omega_D-\Omega)\,\Omega_{\gamma_1+\gamma_2}}{(\Omega_D+\Omega)\left((\Omega_D+\Omega)-\Omega_{\gamma_1+\gamma_2}\left(\overline{\mathsf{p}}_{bad}^{good}+\overline{\mathsf{p}}_{good}^{bad}\right)\right)}\frac{\overline{r}_{t,bad}}{2}$$

Note that under full information we have

$$\frac{r^*_{t,good}-\overline{r}^*_{t,good}}{2}=-\frac{(\Omega_D-\Omega)}{(\Omega_D+\Omega)}\frac{\overline{r}_{t,good}}{2}$$

accounting for

$$\begin{aligned}
&\frac{r^*_{t,good}-\overline{r}^*_{t,good}}{2}\\
&= -\frac{(\Omega_D-\Omega)}{(\Omega_D+\Omega)}\frac{\overline{r}_{t,good}}{2}+\frac{(\Omega_D-\Omega)\left((\Omega_D+\Omega)-\Omega_{\gamma_1+\gamma_2}\left(\overline{\mathsf{p}}^{good}_{bad}+\overline{\mathsf{p}}^{bad}_{good}\right)\right)}{(\Omega_D+\Omega)\left((\Omega_D+\Omega)-\Omega_{\gamma_1+\gamma_2}\left(\overline{\mathsf{p}}^{good}_{bad}+\overline{\mathsf{p}}^{bad}_{good}\right)\right)}\frac{\overline{r}_{t,good}}{2}\\
&\quad-\frac{(\Omega_D-\Omega)\left((\Omega_D+\Omega)-\overline{\mathsf{p}}^{good}_{bad}\Omega_{\gamma_1+\gamma_2}\right)-\overline{\mathsf{p}}^{bad}_{good}\Omega_{D\gamma_1+\gamma_2}(\Omega_D+\Omega)}{(\Omega_D+\Omega)\left((\Omega_D+\Omega)-\Omega_{\gamma_1+\gamma_2}\left(\overline{\mathsf{p}}^{good}_{bad}+\overline{\mathsf{p}}^{bad}_{good}\right)\right)}\frac{\overline{r}_{t,good}}{2}\\
&\quad-\overline{\mathsf{p}}^{bad}_{good}\frac{(\Omega_D+\Omega)\Omega_{D\gamma_1+\gamma_2}-(\Omega_D-\Omega)\Omega_{\gamma_1+\gamma_2}}{(\Omega_D+\Omega)\left((\Omega_D+\Omega)-\Omega_{\gamma_1+\gamma_2}\left(\overline{\mathsf{p}}^{good}_{bad}+\overline{\mathsf{p}}^{bad}_{good}\right)\right)}\frac{\overline{r}_{t,bad}}{2}
\end{aligned}$$

$$\begin{aligned}
&\frac{r^*_{t,good}-\overline{r}^*_{t,good}}{2}\\
&= -\frac{(\Omega_D-\Omega)}{(\Omega_D+\Omega)}\frac{\overline{r}_{t,good}}{2}-\overline{\mathsf{p}}^{bad}_{good}\frac{(\Omega_D-\Omega)\Omega_{\gamma_1+\gamma_2}-(\Omega_D+\Omega)\Omega_{D\gamma_1+\gamma_2}}{(\Omega_D+\Omega)\left((\Omega_D+\Omega)-\Omega_{\gamma_1+\gamma_2}\left(\overline{\mathsf{p}}^{good}_{bad}+\overline{\mathsf{p}}^{bad}_{good}\right)\right)}\left(\frac{\overline{r}_{t,good}}{2}-\frac{\overline{r}_{t,bad}}{2}\right)
\end{aligned}$$

and finally

$$\begin{aligned}
r^*_{t,good} &= \overline{r}^*_{t,good}-\frac{(\Omega_D-\Omega)}{(\Omega_D+\Omega)}\overline{r}_{t,good}\\
&\quad-\overline{\mathsf{p}}^{bad}_{good}\frac{(\Omega_D-\Omega)\Omega_{\gamma_1+\gamma_2}-(\Omega_D+\Omega)\Omega_{D\gamma_1+\gamma_2}}{(\Omega_D+\Omega)\left((\Omega_D+\Omega)-\Omega_{\gamma_1+\gamma_2}\left(\overline{\mathsf{p}}^{good}_{bad}+\overline{\mathsf{p}}^{bad}_{good}\right)\right)}\left(\overline{r}_{t,good}-\overline{r}_{t,bad}\right)
\end{aligned}\tag{99}$$

Now we derive the situation in the bad state:

Take

$$\begin{aligned}
\left((\Omega_D+\Omega)-\overline{\mathsf{p}}^{good}_{bad}\Omega_{\gamma_1+\gamma_2}\right)\frac{r^*_{t,bad}-\overline{r}^*_{t,bad}}{2} &= -\left((\Omega_D-\Omega)-\overline{\mathsf{p}}^{good}_{bad}\Omega_{D\gamma_1+\gamma_2}\right)\frac{\overline{r}_{t,bad}}{2}\\
&\quad-\overline{\mathsf{p}}^{good}_{bad}\Omega_{\gamma_1+\gamma_2}\frac{r^*_{t,good}-\overline{r}^*_{t,good}}{2}-\overline{\mathsf{p}}^{good}_{bad}\Omega_{D\gamma_1+\gamma_2}\frac{\overline{r}_{t,good}}{2}
\end{aligned}$$

Then substitute for $\frac{r^*_{t,good}-\overline{r}^*_{t,good}}{2}$ in the good state: First

$$\begin{aligned}
\left((\Omega_D+\Omega)-\overline{\mathsf{p}}^{bad}_{good}\Omega_{\gamma_1+\gamma_2}\right)\frac{r^*_{t,good}-\overline{r}^*_{t,good}}{2} &= \left((\Omega_D-\Omega)-\overline{\mathsf{p}}^{bad}_{good}\Omega_{D\gamma_1+\gamma_2}\right)\frac{r_{t,good}-\overline{r}_{t,good}}{2}\\
&\quad-\overline{\mathsf{p}}^{bad}_{good}\Omega_{\gamma_1+\gamma_2}\frac{r^*_{t,bad}-\overline{r}^*_{t,bad}}{2}+\overline{\mathsf{p}}^{bad}_{good}\Omega_{D\gamma_1+\gamma_2}\frac{r_{t,bad}-\overline{r}_{t,bad}}{2}
\end{aligned}$$

$$\begin{aligned}
\frac{r^*_{t,good}-\overline{r}^*_{t,good}}{2} &= -\frac{\left((\Omega_D-\Omega)-\overline{\mathsf{p}}^{bad}_{good}\Omega_{D\gamma_1+\gamma_2}\right)}{\left((\Omega_D+\Omega)-\overline{\mathsf{p}}^{bad}_{good}\Omega_{\gamma_1+\gamma_2}\right)}\frac{\overline{r}_{t,good}}{2}-\overline{\mathsf{p}}^{bad}_{good}\frac{\Omega_{\gamma_1+\gamma_2}}{\left((\Omega_D+\Omega)-\overline{\mathsf{p}}^{bad}_{good}\Omega_{\gamma_1+\gamma_2}\right)}\frac{r^*_{t,bad}-\overline{r}^*_{t,bad}}{2}\\
&\quad-\overline{\mathsf{p}}^{bad}_{good}\frac{\Omega_{D\gamma_1+\gamma_2}}{\left((\Omega_D+\Omega)-\overline{\mathsf{p}}^{bad}_{good}\Omega_{\gamma_1+\gamma_2}\right)}\frac{\overline{r}_{t,bad}}{2}
\end{aligned}$$

so that

$$\begin{aligned}
\left((\Omega_D+\Omega)-\overline{\mathsf{p}}^{good}_{bad}\Omega_{\gamma_1+\gamma_2}\right)\frac{r^*_{t,bad}-\overline{r}^*_{t,bad}}{2} &= -\left((\Omega_D-\Omega)-\overline{\mathsf{p}}^{good}_{bad}\Omega_{D\gamma_1+\gamma_2}\right)\frac{\overline{r}_{t,bad}}{2}\\
&\quad-\overline{\mathsf{p}}^{good}_{bad}\Omega_{\gamma_1+\gamma_2}\frac{r^*_{t,good}-\overline{r}^*_{t,good}}{2}-\overline{\mathsf{p}}^{good}_{bad}\Omega_{D\gamma_1+\gamma_2}\frac{\overline{r}_{t,good}}{2}
\end{aligned}$$

becomes

$$\begin{aligned}
&\frac{\left((\Omega_D+\Omega)-\overline{\mathsf{p}}^{good}_{bad}\Omega_{\gamma_1+\gamma_2}\right)\left((\Omega_D+\Omega)-\overline{\mathsf{p}}^{bad}_{good}\Omega_{\gamma_1+\gamma_2}\right)-\overline{\mathsf{p}}^{good}_{bad}\Omega_{\gamma_1+\gamma_2}\Omega_{\gamma_1+\gamma_2}\overline{\mathsf{p}}^{bad}_{good}}{\left((\Omega_D+\Omega)-\overline{\mathsf{p}}^{bad}_{good}\Omega_{\gamma_1+\gamma_2}\right)}\frac{r^*_{t,bad}-\overline{r}^*_{t,bad}}{2}\\
&= -\frac{\left((\Omega_D-\Omega)-\overline{\mathsf{p}}^{good}_{bad}\Omega_{D\gamma_1+\gamma_2}\right)\left((\Omega_D+\Omega)-\overline{\mathsf{p}}^{bad}_{good}\Omega_{\gamma_1+\gamma_2}\right)-\overline{\mathsf{p}}^{good}_{bad}\Omega_{D\gamma_1+\gamma_2}\Omega_{\gamma_1+\gamma_2}\overline{\mathsf{p}}^{bad}_{good}}{\left((\Omega_D+\Omega)-\overline{\mathsf{p}}^{bad}_{good}\Omega_{\gamma_1+\gamma_2}\right)}\frac{\overline{r}_{t,bad}}{2}\\
&\quad+\overline{\mathsf{p}}^{good}_{bad}\frac{\Omega_{\gamma_1+\gamma_2}\left((\Omega_D-\Omega)-\overline{\mathsf{p}}^{bad}_{good}\Omega_{D\gamma_1+\gamma_2}\right)-\Omega_{D\gamma_1+\gamma_2}\left((\Omega_D+\Omega)-\overline{\mathsf{p}}^{bad}_{good}\Omega_{\gamma_1+\gamma_2}\right)}{\left((\Omega_D+\Omega)-\overline{\mathsf{p}}^{bad}_{good}\Omega_{\gamma_1+\gamma_2}\right)}\frac{\overline{r}_{t,good}}{2}
\end{aligned}$$

and then

$$\begin{aligned}&\frac{r^*_{t,bad}-\overline{r}^*_{t,bad}}{2}\\ =\ & -\frac{(\Omega_D-\Omega)\left((\Omega_D+\Omega)-\overline{\mathsf{p}}^{bad}_{good}\Omega_{\gamma_1+\gamma_2}\right)-\overline{\mathsf{p}}^{good}_{bad}\Omega_{D\gamma_1+\gamma_2}(\Omega_D+\Omega)}{(\Omega_D+\Omega)\left((\Omega_D+\Omega)-\left(\overline{\mathsf{p}}^{good}_{bad}+\overline{\mathsf{p}}^{bad}_{good}\right)\Omega_{\gamma_1+\gamma_2}\right)}\frac{\overline{r}_{t,bad}}{2}\\ &-\overline{\mathsf{p}}^{good}_{bad}\frac{(\Omega_D+\Omega)\Omega_{D\gamma_1+\gamma_2}-(\Omega_D-\Omega)\Omega_{\gamma_1+\gamma_2}}{(\Omega_D+\Omega)\left((\Omega_D+\Omega)-\left(\overline{\mathsf{p}}^{good}_{bad}+\overline{\mathsf{p}}^{bad}_{good}\right)\Omega_{\gamma_1+\gamma_2}\right)}\frac{\overline{r}_{t,good}}{2}\end{aligned}$$

Now note again that

$$\frac{r^*_{t,bad}-\overline{r}^*_{t,bad}}{2}=-\frac{(\Omega_D-\Omega)}{(\Omega_D+\Omega)}\frac{\overline{r}_{t,bad}}{2}$$

so that

$$\begin{aligned}&\frac{r^*_{t,bad}-\overline{r}^*_{t,bad}}{2}\\ =\ & -\frac{(\Omega_D-\Omega)}{(\Omega_D+\Omega)}\frac{\overline{r}_{t,bad}}{2}+\frac{(\Omega_D-\Omega)\left((\Omega_D+\Omega)-\overline{\mathsf{p}}^{good}_{bad}\Omega_{\gamma_1+\gamma_2}-\overline{\mathsf{p}}^{bad}_{good}\Omega_{\gamma_1+\gamma_2}\right)}{(\Omega_D+\Omega)\left((\Omega_D+\Omega)-\left(\overline{\mathsf{p}}^{good}_{bad}+\overline{\mathsf{p}}^{bad}_{good}\right)\Omega_{\gamma_1+\gamma_2}\right)}\frac{\overline{r}_{t,bad}}{2}\\ &-\frac{(\Omega_D-\Omega)\left((\Omega_D+\Omega)-\overline{\mathsf{p}}^{bad}_{good}\Omega_{\gamma_1+\gamma_2}\right)-\overline{\mathsf{p}}^{good}_{bad}\Omega_{D\gamma_1+\gamma_2}(\Omega_D+\Omega)}{(\Omega_D+\Omega)\left((\Omega_D+\Omega)-\left(\overline{\mathsf{p}}^{good}_{bad}+\overline{\mathsf{p}}^{bad}_{good}\right)\Omega_{\gamma_1+\gamma_2}\right)}\frac{\overline{r}_{t,bad}}{2}\\ &-\overline{\mathsf{p}}^{good}_{bad}\frac{(\Omega_D+\Omega)\Omega_{D\gamma_1+\gamma_2}-(\Omega_D-\Omega)\Omega_{\gamma_1+\gamma_2}}{(\Omega_D+\Omega)\left((\Omega_D+\Omega)-\left(\overline{\mathsf{p}}^{good}_{bad}+\overline{\mathsf{p}}^{bad}_{good}\right)\Omega_{\gamma_1+\gamma_2}\right)}\frac{\overline{r}_{t,good}}{2}\end{aligned}$$

Then we have that

$$\begin{aligned}&\frac{r^*_{t,bad}-\overline{r}^*_{t,bad}}{2}\\ =\ & -\frac{(\Omega_D-\Omega)}{(\Omega_D+\Omega)}\frac{\overline{r}_{t,bad}}{2}+\frac{(\Omega_D-\Omega)\left((\Omega_D+\Omega)-\overline{\mathsf{p}}^{good}_{bad}\Omega_{\gamma_1+\gamma_2}-\overline{\mathsf{p}}^{bad}_{good}\Omega_{\gamma_1+\gamma_2}\right)}{(\Omega_D+\Omega)\left((\Omega_D+\Omega)-\left(\overline{\mathsf{p}}^{good}_{bad}+\overline{\mathsf{p}}^{bad}_{good}\right)\Omega_{\gamma_1+\gamma_2}\right)}\frac{\overline{r}_{t,bad}}{2}\\ &-\frac{(\Omega_D-\Omega)\left((\Omega_D+\Omega)-\overline{\mathsf{p}}^{bad}_{good}\Omega_{\gamma_1+\gamma_2}\right)-\overline{\mathsf{p}}^{good}_{bad}\Omega_{D\gamma_1+\gamma_2}(\Omega_D+\Omega)}{(\Omega_D+\Omega)\left((\Omega_D+\Omega)-\left(\overline{\mathsf{p}}^{good}_{bad}+\overline{\mathsf{p}}^{bad}_{good}\right)\Omega_{\gamma_1+\gamma_2}\right)}\frac{\overline{r}_{t,bad}}{2}\\ &-\overline{\mathsf{p}}^{good}_{bad}\frac{(\Omega_D+\Omega)\Omega_{D\gamma_1+\gamma_2}-(\Omega_D-\Omega)\Omega_{\gamma_1+\gamma_2}}{(\Omega_D+\Omega)\left((\Omega_D+\Omega)-\left(\overline{\mathsf{p}}^{good}_{bad}+\overline{\mathsf{p}}^{bad}_{good}\right)\Omega_{\gamma_1+\gamma_2}\right)}\frac{\overline{r}_{t,good}}{2}\end{aligned}$$

becomes

$$\begin{aligned}\frac{r^*_{t,bad}-\overline{r}^*_{t,bad}}{2} =\ & -\frac{(\Omega_D-\Omega)}{(\Omega_D+\Omega)}\frac{\overline{r}_{t,bad}}{2}\\ &+\overline{\mathsf{p}}^{good}_{bad}\frac{(\Omega_D-\Omega)\Omega_{\gamma_1+\gamma_2}-(\Omega_D+\Omega)\Omega_{D\gamma_1+\gamma_2}}{(\Omega_D+\Omega)\left((\Omega_D+\Omega)-\left(\overline{\mathsf{p}}^{good}_{bad}+\overline{\mathsf{p}}^{bad}_{good}\right)\Omega_{\gamma_1+\gamma_2}\right)}\left(\frac{\overline{r}_{t,good}}{2}-\frac{\overline{r}_{t,bad}}{2}\right)\end{aligned}$$

and finally

$$\begin{aligned}r^*_{t,bad}-\overline{r}^*_{t,bad} =\ & -\frac{(\Omega_D-\Omega)}{(\Omega_D+\Omega)}\overline{r}_{t,bad}\\ &+\overline{\mathsf{p}}^{good}_{bad}\frac{(\Omega_D-\Omega)\Omega_{\gamma_1+\gamma_2}-(\Omega_D+\Omega)\Omega_{D\gamma_1+\gamma_2}}{(\Omega_D+\Omega)\left((\Omega_D+\Omega)-\left(\overline{\mathsf{p}}^{good}_{bad}+\overline{\mathsf{p}}^{bad}_{good}\right)\Omega_{\gamma_1+\gamma_2}\right)}\left(\overline{r}_{t,good}-\overline{r}_{t,bad}\right)\end{aligned} \tag{100}$$

In summary, if the ZLB is binding in the home country, it holds that

$$\begin{aligned}r^*_{t,good} =\ & \overline{r}^*_{t,good}-\frac{(\Omega_D-\Omega)}{(\Omega_D+\Omega)}\overline{r}_{t,good}\\ &+\overline{\mathsf{p}}^{bad}_{good}\frac{(\Omega_D+\Omega)\Omega_{D\gamma_1+\gamma_2}-(\Omega_D-\Omega)\Omega_{\gamma_1+\gamma_2}}{(\Omega_D+\Omega)\left((\Omega_D+\Omega)-\left(\overline{\mathsf{p}}^{good}_{bad}+\overline{\mathsf{p}}^{bad}_{good}\right)\Omega_{\gamma_1+\gamma_2}\right)}\left(\overline{r}_{t,good}-\overline{r}_{t,bad}\right)\end{aligned} \tag{101}$$

and

$$\begin{aligned}r^*_{t,bad} =\ & \overline{r}^*_{t,bad}-\frac{(\Omega_D-\Omega)}{(\Omega_D+\Omega)}\overline{r}_{t,bad}\\ &-\overline{\mathsf{p}}^{good}_{bad}\frac{(\Omega_D+\Omega)\Omega_{D\gamma_1+\gamma_2}-(\Omega_D-\Omega)\Omega_{\gamma_1+\gamma_2}}{(\Omega_D+\Omega)\left((\Omega_D+\Omega)-\left(\overline{\mathsf{p}}^{good}_{bad}+\overline{\mathsf{p}}^{bad}_{good}\right)\Omega_{\gamma_1+\gamma_2}\right)}\left(\overline{r}_{t,good}-\overline{r}_{t,bad}\right)\end{aligned} \tag{102}$$

Or as shown in Section 5:

$$
\begin{aligned}
r^*_{good} &= \overline{r}^*_{good} - \gamma_{\overline{r}}\overline{r}_{good} + \overline{\mathsf{p}}^{bad}_{good}\gamma_{\overline{r}_{g-b}}\left(\overline{r}_{good} - \overline{r}_{bad}\right) \\
r^*_{bad} &= \overline{r}^*_{bad} - \gamma_{\overline{r}}\overline{r}_{bad} - \overline{\mathsf{p}}^{good}_{bad}\gamma_{\overline{r}_{g-b}}\left(\overline{r}_{good} - \overline{r}_{bad}\right).
\end{aligned}
$$

with

$$
\begin{aligned}
\gamma_{\overline{r}} &= \frac{(\Omega_D - \Omega)}{(\Omega_D + \Omega)} \\
\gamma_{\overline{r}_{g-b}} &= \frac{(\Omega_D + \Omega)\,\Omega_{D\gamma_1+\gamma_2} - (\Omega_D - \Omega)\,\Omega_{\gamma_1+\gamma_2}}{(\Omega_D + \Omega)\left((\Omega_D + \Omega) - \left(\overline{\mathsf{p}}^{good}_{bad} + \overline{\mathsf{p}}^{bad}_{good}\right)\Omega_{\gamma_1+\gamma_2}\right)}.
\end{aligned}
$$