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Demographic change and the rate of return in PAYG pension systems

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Non-technical summary

Research Question

How do a longer life expectancy and fluctuating cohort sizes affect pay-as-you-go (PAYG) pension systems? Who benefits and who loses due to these effects within a PAYG pension system? How does the type of the PAYG pension system alter the results?

Contribution

The paper decomposes the demographic change into a cohort effect and an ageing effect, taking Germany as an example. A simple theoretical model provides intuition for the underlying mechanisms. It illustrates how the two effects in general alter the payment structure within various kinds of PAYG pensions systems. The paper then applies a large quantitative overlapping generations (OLG) model calibrated to the German population. This model shows that these effects are economically sizeable by quantifying the rate of return that each cohort realizes within different PAYG pension systems.

Results

Contrary to the widespread view, decreasing mortality rates have an unambiguously positive impact on the rates of return generated within a PAYG pension system. Longer payout periods overcompensate decreasing pension levels or increasing contribution rates. Varying cohort sizes, by contrast, result in winners and losers in PAYG pension system. The key difference to the rise in life expectancy is the transitory nature of large/small cohorts within the population. All gains from expanding the pension system are offset in the long run by losses.

Overall, the demographic change in Germany has a positive impact on the rates of return participants realize in the German PAYG pension system. However, the benefits are not equally distributed. With an additional rate of return of 1.2 percentage points per year over the whole life cycle the birth cohort of 1939 benefits the most from the demographic change.

Nichttechnische Zusammenfassung

Fragestellung

Wie wirken sich die beiden Teilaspekte des demografischen Wandels, eine längere Lebenserwartung und eine variierende Kohortengröße, auf die implizite Rendite in einem umlagefinanzierten Rentensystem aus? Präziser gefragt, wer ist durch diese Effekte im Umlageverfahren begünstigt und wer wird benachteiligt? Wie unterscheiden sich diese Ergebnisse bei unterschiedlichen Ausgestaltungen eines Umlageverfahrens?

Beitrag

Die Studie zerlegt den demografischen Wandel am Beispiel Deutschlands zunächst in einen Alterungs- und in einen Kohorten-Effekt. Anhand eines theoretischen Modells werden die unterschiedlichen Wirkungsweisen der beiden Effekte auf ein umlagefinanziertes Rentensystem erläutert. Ein komplexes quantitatives Modell überlappender Generationen, kalibriert auf die deutsche Bevölkerungsentwicklung, quantifiziert die beiden Effekte und zeigt, dass sie ökonomisch bedeutsam sind.

Ergebnisse

Eine steigende Lebenserwartung beeinflusst die Rendite innerhalb eines umlagefinanzierten Rentensystems eindeutig positiv. Ein sinkendes Rentenniveau oder steigende Beiträge werden von einer längeren Auszahlungsphase überkompensiert. Eine schwankende Kohortengröße führt hingegen zu Gewinnern und Verlierern in einem Umlageverfahren. Der Hauptunterschied zur steigenden Lebenserwartung ist, dass es hierbei nicht zu einem dauerhaften Bevölkerungsanstieg kommt sondern die Wirkung nur vorübergehend ist. Allen Gewinnen stehen somit langfristig Verluste gegenüber, wobei diese von unterschiedlichen Kohorten getragen werden müssen.

Alles in allem beeinflusst der demografische Wandel in Deutschland die Renditen des Umlageverfahrens positiv. Die Verteilung der Gewinne unter den Kohorten ist jedoch ungleich. Mit einer nur aufgrund des demografischen Wandels um 1.2 Prozentpunkte höheren Rendite profitiert der Jahrgang 1939 am meisten.

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Demographic Change and the Rate of Return in PAYG pension systems*

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Abstract

The currently observed demographic change consists of two independent developments that differ in structure and persistence: (1) A slow, monotonic and (presumably) permanent ageing effect caused by an increasing life expectancy; (2) a more rapidly changing, non-monotonic and less permanent cohort effect caused by fluctuations in the size of cohorts. This paper shows the ageing effect has a positive impact on the rates of return households generate within pay-as-you-go (PAYG) pension system. The cohort effect, by contrast, results in winners and losers in PAYG systems. Taking Germany as an example and using a quantitative OLG model the paper shows that the two effects cause rate of return differentials within the pension system of almost 1.3 percentage points between generations.

Keywords: Demographic Change, Pension System, OLG Models

JEL classification: E27, E62, H55, J11, J26

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1 Introduction

In most advanced economies, pay-as-you-go (PAYG) pension systems are under enormous pressure as the proportion of older people in the overall population is rising. This increase, also known as demographic change, is often regarded as a one-dimensional phenomenon. In reality, though, demographic change is a combination of several developments occurring simultaneously. The two most important causes are that people are living longer (ageing effect) and that large "baby boom" cohorts are currently close to retirement (cohort effect). These two effects differ substantially. The ageing effect is slow and monotonic and will (probably) be sustained. By contrast, the number of births per year and net migration can move in either direction and changes more rapidly over time. The impact of past birth rates and migration on the demographic structure diminishes in the long term, however.

This paper illustrates how a longer life expectancy and fluctuating cohort sizes differently affect PAYG pension systems. More specifically, the paper asks who achieves higher returns due to these effects within a PAYG pension system. Furthermore, the paper answers the question how the type of the PAYG pension system alters the results. For developing sustainable solution for PAYG pension systems it is necessary to separate both effects.

To answer the research questions the paper decomposes the effect of demographic change into cohort and ageing effects, taking Germany as an example. A simple theoretical model provides intuition for the underlying mechanisms. It illustrates how the two effects in general alter the payment structure within various kinds of PAYG pensions systems. The paper then applies a large quantitative overlapping generation (OLG) calibrated to the German population. This model shows that differences between the rates of return of cohorts within different PAYG pension systems are economically sizeable. The model also disaggregates the overall rate of return differential into parts stemming from the ageing effect, the cohort effect and a third employment effect originating from the rising labour force participation.

Contrary to the widespread view, decreasing mortality rates have an unambiguously positive impact on the rates of return generated within a PAYG pension system. Holding the cohort size at birth constant, a longer life of individuals clearly results in a population increase. In turn, the population increase expands the budget of the pension system. In a PAYG pension system (permanent) expansions result in gains which are distributed depending on the specific pension system.

Varying cohort sizes, by contrast, result in winners and losers in PAYG systems. The key difference to the rise in life expectancy is that the impact of a large cohort is transitory. So each expansionary gain to the system is offset in the long run with a loss. However, cohorts benefiting from gains are not necessarily those bearing losses.

Overall, the demographic change in Germany has a positive impact on the rates of return participants realize in the German PAYG pension system. The distribution among the cohorts is unequal with a difference 1.3 percentage points per year over the whole life cycle. The cohort that benefited the most is the birth cohort of 1939 with an excess return of 1.2 percentage points compared to a world without demographic change. The birth cohort of 2026 will have a slightly negative excess return of -0.1 percentage points. Most of the excess return is driven by the ageing effect. The maximum excess return due to the ageing effect realized by birth cohort 1937 is 0.8 percentage points. The cohort effect is responsible for a maximum excess return of 0.3 percentage points for the cohort born in 1936. The 2005 cohort loses out on returns to the tune of 0.5 percentage points. So the maximum return differential between cohorts is 0.8 percentage points compared with a demography without cohort fluctuations.

This paper is related to papers that investigate the role of demographic change for pension systems and its scope for reforms using microsimulation models, cf. Werding (2013), Börsch-Supan, Bucher-Koenen, and Rausch (2016). Fenge and Peglow (2018) identify the isolated effects of mortality, fertility and migration developments on the dynamics of the German pension system. The focus of these papers lies on budgetary linkages between pension system and the rest of the economy. In contrast to the approach of this paper, they do not account for the optimal household reactions triggered by a pension reform.

In an empirical approach, Lüthen (2016) applies individual accounting of rates of return within the German pension system. He uses individual data to compute the returns of cohorts between 1935 and 1945 with respect to an early retirement pension reform. The approach of this paper is not to exactly keep track of individual contributions. The focus of the paper is on the effect of the demographic change on the general population.

The quantitative model that is used in this paper includes an optimizing household sector in order to take into account household reactions. In this sense this paper relates to a vast number of papers that have analysed the economic consequences of demographic change and possible adjustment mechanisms. Important examples with a focus on social security adjustments include Imrohoroglu, Imrohoroglu, and Joines (1995), De Nardi, Imrohoroglu, and Sargent (1999), Fuster, Imrohoroglu, and Imrohoroglu (2007), Attanasio, Kitao, and Violante (2007), Krueger and Ludwig (2007), Golosov, Shourideh, Troshkin, and Tsyvinski (2013), Attanasio, Bonfatti, Kitao, and Weber (2016), Kitao (2018).

This paper tries to combine complex solution methods but also model the pension system in great detail. In this sense it is closely related to Ludwig, Krüger, and Börsch-Supan (2009), Börsch-Supan and Ludwig (2009), Ludwig and Reiter (2010), Buyse, Heylen, and de Kerckhove (2013) and Vogel, Ludwig, and Börsch-Supan (2017). In contrast to these papers, this paper gives a description of the effects of different aspects of ageing on the rates of return within a PAYG pension system. It focuses on the interplay between demographic change, balanced intratemporal pension budgets and lifetime cash flows of individuals. It assumes that the individuals cannot optimize their participation in the state run PAYG pension system.[1] The purpose of this paper is not to do a welfare analysis of PAYG pension systems or proposing an optimal pension reform. This paper evaluates, given the current structure of such system, the redistributional effect of the demographic change within PAYG pension systems.

The remainder of this analysis is organized as follows. In Section 2 the paper presents the empirical details on the demographic change in Germany. It contains a decomposition of the general demographic change into its components. In Section 3 a simple analytical framework illustrates the effects these demographic developments pose for different types of PAYG pension systems. In Section 4 a quantitative OLG model which is presented that incorporate these effects in a more complex setting. The calibration of the quantitative model is shown in Section 5. Quantitative results are then presented in Section 6 for PAYG systems. Finally, Section 7 concludes the paper.

[1] This is the case in most countries, cf. OECD (2018).

2 Demographic Change in Germany

Demographic change is prevalent in all developed economies. For the purpose of decomposing the overall demographic developments into ageing and cohort effects the paper uses Germany as an example. It is especially suitable for the following analysis as Germany due to World War I and World War II exhibits a distinctly large baby boomer cohort that were born in the 1960s.

The main data source for the demographic process in Germany is the German Federal Office of Statistics (GFOS). For the period between 1960 and 2018 the actual German data for the age distribution and mortality rates are used. For the period between 2019 and 2060 the projections of the recent *14th Coordinated Population Projection* are used.[2] After the end of this projection at 2060, the mortality probabilities by age until 2100 are linearly extrapolated and kept constant afterwards. Migration and fertility rates are kept constant at the projected level of 2060.

The alteration of the age composition of the population (demographic change) currently being observed in Germany is essentially made up of two different, simultaneously occurring effects: an ageing effect and a cohort effect.

The ageing effect is reflected in an ever longer life expectancy. Life expectancy has risen in all developed countries with minor interruptions over the past 150 years. In 1960, life expectancy in Germany at the age of 20 was 73 years (Figure 1 (a)). Since then, it has increased by 9 years to nearly 82 years. The GFOS predicts that the average lifespan will continue to increase in future, too. Based on their projection, it will have gone up by a further 10 years by 2100. This further increase in life expectancy stems almost entirely from declining mortality of people older than 65 years. Consequently, longer life expectancy translates into a growing number of people who are older than 65 years. Already, the probability of dying at a younger age is practically zero. The number of persons under the age of 65 years remains virtually unchanged by the ageing effect.

The cohort effect is primarily driven by a varying birth rate. Figure 1 (b) shows the number of twenty-year-old in Germany over time. In the middle of the last century there was a strong fluctuation in the cohort strength in Germany. The sharp drop in the beginning of the 1960s originates partly from a lower birth rate during the World War II. This effect was enhanced by the fact that the cohorts in childbearing age at this time were themselves reduced by low birth rates during the World War I. After this sharp decline, the number of people born exploded in the mid-1960s (baby boomers). At the beginning of the 1970s the birth rate again plummeted sharply and stayed low for the next decades. In the baseline variant of its current population projection exercise, the GFOS assumes a broadly unchanged birth rate of 1.55 in the future. Both the sharp increase and the decline of cohort strength 50 years ago have led to a hump within the age distribution of the German population. At the beginning of their life cycle, relatively large cohorts make the overall population on average younger. This effect, however, reverses when the cohorts reach a more advanced age. Then the baby boomer cohort disproportionally increases the average age of the overall population. So this hump plays an important role in understanding the currently observed demographic pressure.

[2] The 14th coordinated population projection includes various scenarios for the future trends of fertility, migration and mortality. The chosen assumptions are in the medium range of all scenarios (W2-L2-G2), cf. Statistisches Bundesamt (2019).

Besides birth rates, cohort size is affected by migration. In recent years, there has been considerable net immigration in Germany. Over the past ten years, this has amounted to an annual average of around $400,000$ persons.[3] In the cited population projection, net migration falls to $206,000$ persons per year by 2026 (corresponds largely to the long-term median). Afterwards, the number of net migration remains constant. Migration is thus currently counteracting the effect of the low birth rate. With a net migration of 200,000 people per year and a birth rate of 1.55 children per woman the German population will converge to 80 million people.

All three demographic factors thus have an impact on the ratio of older to younger people in a population. A commonly used metric for this ratio is the old-age dependency ratio (OADR). This is defined as the ratio of the population group aged over 65 years to that aged under 65 years. Figure 1 (c) shows how ageing and cohort effect affect the OADR.

To separate the ageing effect, a population distribution is simulated where cohort sizes at age 20 years are identical over time. Therefore, the differences in the population age structure results solely from the decline in mortality rates. The ageing effect (blue dashed line) causes a monotonic increase in the OADR. This increase is also permanent.

The variations in cohort size have a different effect (black solid line).[4] To separate the cohort effect, the population is simulated by taking the cohort size over time from the data and keep the mortality rates constant (at the 1960 level).[5] The cohort effect on the old-age dependency ratio is, first, not monotonic and, second, only temporary. In the very long term, this effect vanishes entirely.[6]

Both effects impact the old-age dependency ratio of Germany (Figure 1 (d)). In 1960 the OADR was 25%. In other words, for every person of 65 years and above, there were roughly four persons of working age. This ratio remained virtually stable until 1990. The reason for that is that the ageing effect and the cohort effect offset each other. The ageing effect exerted upward pressure where the cohort effect for itself lowered the OADR. After 1990 the OADR started to sluggishly increase and reached 35% in 2018. With the retirement of the baby boomer cohort, the sign of both effects align and the old-age dependency ratio sharply rises to 52.9% by 2037. At this point in time, for every person of statutory retirement age and above, there are then fewer than two persons younger than 65 years. Although life expectancy continues to rise, the baby boomer cohorts gradually die out. The pressure from the cohort effect vanishes. This briefly stabilizes the OADR between 2040 and 2050. However, the OADR then again starts to increase to values above 65% and further as long as the life expectancy rises.

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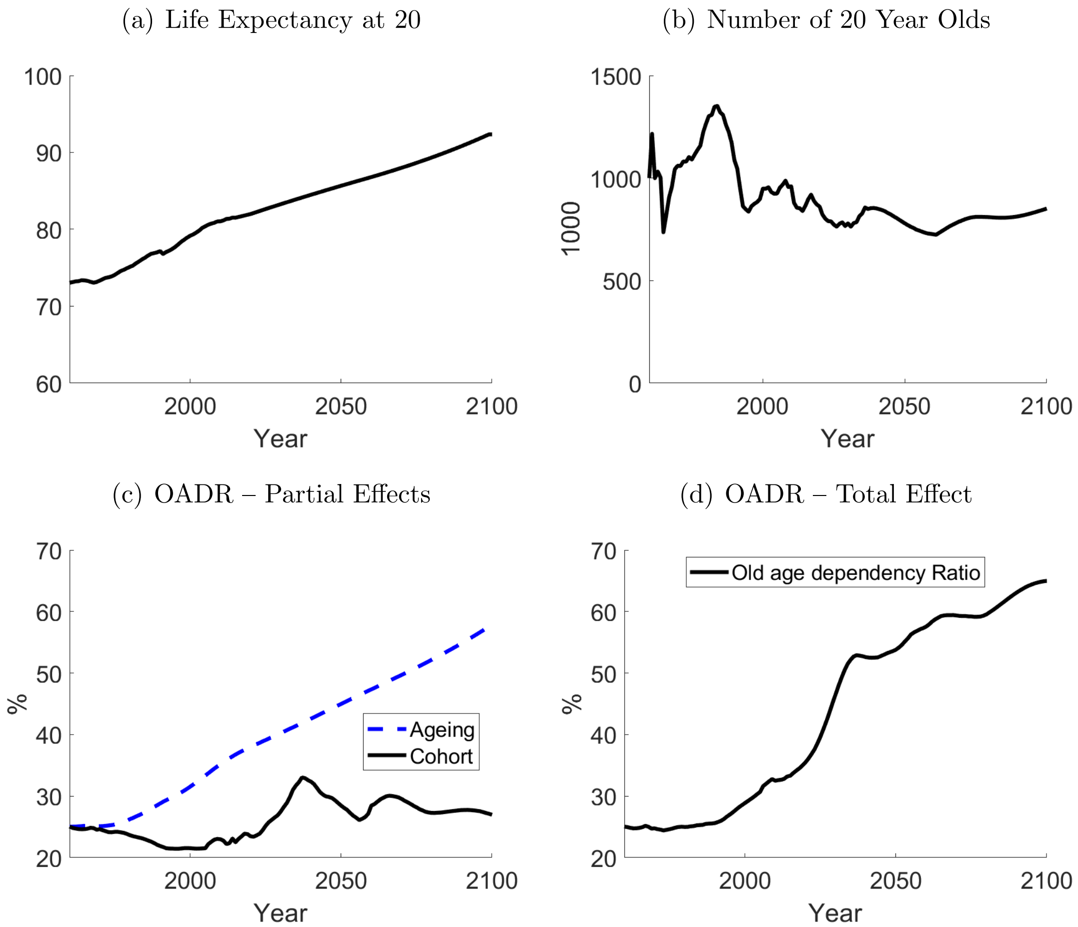
[3] What is crucial for pension systems is the extent to which migration alters the number and structure of its contributor base and at a later date, the number and structure of pension recipients. Three things are of central importance: the age of those immigrating and emigrating, integration into the labour market, and the impact on future demographic developments.

[4] In the remainder of this paper, the effects of birth rate and migration are consolidated as the cohort effect.

[5] This separation is not perfect as the magnitude of the cohort effect depends on the mortality rates. If the life expectancy is higher the size of a cohort matters over a longer time period for the OADR.

[6] The size of a cohort affects the demographic structure beyond its lifespan. A large cohort has many descendants in absolute as well as relative terms. Assuming that each cohort were to have children in just one year of its life, a high one-off birth rate would continue indefinitely. However, since reproduction occurs over multiple years, cohort sizes even out again over time.

Figure 1: Demographic Change in Germany



Notes: Panel (a) shows (averaged over men and women) life expectancies of 20 year olds. Panel (b) shows number 20 year olds. Panel (c) shows the old-age dependency ratio defined as persons older than 65 years to persons aged between 20 years and 65 years for ageing effect and cohort effect, separately. Panel (d) shows the overall old-age dependency ratio.

3 Demographic Challenges for PAYG Systems

Developments in the old-age dependency ratio are of central importance to the financial situation of a PAYG pension system. While the ageing and the cohort effect affect the old-age dependency ratio in different ways, both pose serious challenges for sustainability of PAYG pension systems. In the following section, a simple OLG model is used to illustrate how the discussed demographic effects vary (implicit) rates of return within a PAYG system. Additionally, it shows that these return differentials alter with the type of PAYG system.

3.1 A Simple Model

In a PAYG pension system, the implicit rate of return corresponds to the growth rate of the total wage bill. If the demographic structure remains constant, this return corresponds to productivity growth. Under the assumption of constant productivity growth, the returns within the PAYG pension system would be the same for all cohorts. Given uniform population growth, the return will rise/fall by the population growth rate. The simple model assumes that there is neither wage growth nor a (permanent) increase in newborns.[7]

Cohorts live J periods of equal length. At the beginning of their lives, cohorts work and contribute to the pension system. At age R, they retire and receive from then on a pension. The budget equation for the PAYG pension system is as follows:

$$\phi_t * w_t * L_t = \gamma_t * w_t * P_t \tag{1}$$

where ϕ_t denotes the contribution rate, w_t the wage rate, L_t the number of workers, γ_t the pension level, P_t the number of retirees. Equation (1) can be rearranged into

$$\frac{\phi_t}{\gamma_t} = \frac{P_t}{L_t} = \mathcal{Q}_t \tag{2}$$

where $\mathcal{Q}_t$ is the old-age dependency ratio. The simple analysis distinguishes between two types of PAYG systems, a fixed (or defined) contribution (FC) system and a fixed (or defined) benefit system (FB).[8]

In a fixed contribution PAYG system, the contribution rate is fixed, $\phi_t = \phi$. The (endogenous) pension level evens out the pension budget:

$$\gamma_t = \frac{\phi}{\mathcal{Q}_t}. \tag{3}$$

In a fixed benefit PAYG system, the pension level is fixed, $\gamma_t = \gamma$. The pension system budget now evens out the (endogenous) contribution rate.

$$\phi_t = \gamma * \mathcal{Q}_t \tag{4}$$

[7] It is also assumed that the number of workers has no impact on wages. This is the case, for example, in a small open economy.

[8] In OECD countries both types exist, cf. OECD (2018). Public PAYG schemes follow fixed benefit rules in 17 OECD countries, e.g. Japan, Spain and Switzerland. Fixed contribution schemes exist in five OECD countries, e.g. Italy, Poland and Sweden.

The rate of return a cohort born in period t realizes within the PAYG system is equal to the internal rate of return, i_t, of the inpayments/outpayments over its lifetime.

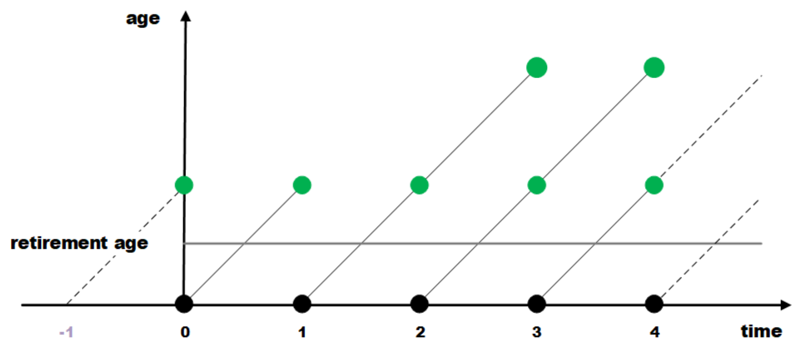
$$\sum_{j=1}^{J} \frac{\left(\gamma_{t+j-1}\mathbb{1}_{\{j\geq R\}} - \phi_{t+j-1}(1-\mathbb{1}_{\{j\geq R\}})\right) w_{t+j-1}}{(1+i_t)^{j-1}} = 0 \tag{5}$$

If the population remains constant and $R = \frac{1}{2}J$, there are exactly as many workers as there are retirees. In this case, the old-age dependency ratio is $\mathcal{Q} = 1$ and the contribution rate corresponds to the pension level, $\phi = \gamma$. The qualitative results of the simple model are independent of the pension level in the initial steady state. In this numerical example, the pension level and the contribution rate in the initial state ($t = 0$) are set to 30%. As there is no population or productivity growth the rate of return in the PAYG system is 0% in initial and final steady state.

3.2 Ageing Effect

An increasing life expectancy ceteris paribus increases the payout period of pensions. In a PAYG pension system this leads to a distortion of the rates of return within the system. The contribution rate during the working life of a cohort is linked via the intratemporal budget constraint of the pension system to a then existing shorter benefit period.

Figure 2: Longer Life Span and Retirement Age



	0	1	2	3	4
∑ population	2	2	2	3	3
∑ retirees	1	1	1	2	2
∑ workers	1	1	1	1	1
old age dependency ratio	1	1	1	2	2

Notes: Diagonal lines symbolize life cycles of agents through age and time. Longer lines mean longer lives. Black dots denote years in working life. Green dots denote years in retirement. The table shows summary statistics of the population.

The Lexis diagram in Figure 2 illustrates how increasing life spans affect size and structure of a population. In each period one cohort enters the population. Cohorts born in or before $t = 0$ live for two periods. All cohorts born in or after $t = 1$ have a longer life span of three periods. This increase in life span maps into an increased population. In $t = 3$ the size of the population increases from two to three cohorts that simultaneously live in the economy. This population increase will induce an expansionary effect in the pension system. Similar to an introduction of a PAYG pension system, any expansion of PAYG systems generates gains for the system. As the increase in life spans is (by assumption) permanent, these gains will never be offset by losses. As these gains can be distributed among the pension system participants, a longer life span increases the rates of return within PAYG systems for some cohorts without decreasing it for others.[9]

In the numerical example the number of retirees increases in $t = 3$ to three. As the number of workers remains unchanged, the old age dependency ratio increases to 2. Depending on the specific pension system either the pension level has to drop (FC) or the contribution rate has to increase (FB) (see Table 1). The pension level and the contribution rate deviate from their initial values for the entire future.

In terms of rate of return within the pension system it is for all cohorts either positive or zero. In a FC system, the cohort born in $t = 1$ profit from it. Their rates of return within the system are 36%. All cohorts born later have again a return of zero (equal to the wage growth of the model). In a FB system, the cohort born in $t = 2$ profits from the ageing effect. The rates of return are even higher than in a FC system. The reason for this is that in a FB system the size of the PAYG system increases even more than in a FC system. The gains are proportionate to the expansion and therefore higher in a FB system.

Table 1: Ageing Effect with Constant Retirement Age (Scenario 1)

Period		$t = 0$	$t = 1$	$t = 2$	$t = 3$	$t = 4$
OADR		1	1	1	2	2
Contribution rate	FC	30%	30%	30%	30%	30%
	FB	30%	30%	30%	60%	60%
Pension level	FC	30%	30%	30%	15%	15%
	FB	30%	30%	30%	30%	30%
Rate of return of	FC	0%	36%	0%	0%	0%
cohort born in t	FB	0%	0%	61%	0%	0%

3.3 Cohort Effect

In a PAYG system, cohort sizes influence the returns of those cohorts. A difference to the ageing effect is that the cohort effect has in PAYG systems not just beneficiaries but also losers. Here, it is especially important to distinguish between a fixed benefit system and a fixed contribution system.

[9]These expansionary gains do not vanish by adjusting the retirement age. When the retirement age is indexed to life expectancy, the ageing effect has no long run impact on the contribution rate or the pension level. The adjustment of the retirement age, however, does not prevent fluctuations of the pension variables in the short run. These fluctuations map into different rates of return of cohorts. This shows that an adjustment of the retirement age does have distributional effects. However, it does not offset the expansionary gains of the ageing effect.

In the following numerical example (Table 2), in $t = 1$ (only), twice as many people are born than in any other period. Life expectancy is $J = 2$ and retirement age is $R = 2$ for all cohorts. The rate of return simplifies to

$$i_t = \frac{\gamma_{t+1}}{\phi_t} - 1 \tag{6}$$

In a fixed contribution system, belonging to a large cohort (here born in $t = 1$) is a disadvantage. In the contribution period, large cohorts have the same per capita burden as all other cohorts. Because there is in $t = 1$ a large number of contribution-payers, the pension system has a higher income. The PAYG system distributes this income in the same period; the pension level doubles to 60%. This increase raises the return for the cohort born in period $t = 0$ to 100%. Cohort effects in PAYG pension systems are zero sum games. That means that if there are winners there must be losers. Because the cohort born in $t = 2$ is again of size one and the large cohort is now in retirement, the pension level must fall in period $t = 2$ to 15%. It is now even lower than its initial level. The return for cohort born in $t = 1$ declines as a result. For all subsequent cohorts the return is again at the initial value of zero.

A fixed benefit system is advantageous for relatively large cohorts. Based on the large number of contribution payers, the contribution rate, and hence per capita contributions, can be lower during the contribution period. Despite the large number of recipients, the pension level is guaranteed during the pension period. This ultimately leads to a high return for the cohort born in $t = 1$. Similar to the FC system, the gain of one cohort comes of a cost of another. In this system the cohort born in $t = 2$ has to bear the brunt. This cohort has to fund the greater pension expenditure through increased contributions. However, it cannot expect to receive a higher pension level itself.

Table 2: Cohort Effect in PAYG Systems

Period		$t = 0$	$t = 1$	$t = 2$	$t = 3$
Workers		1	2	1	1
Retirees		1	1	2	1
OADR		1	0,5	2	1
Contribution rate	FC	30%	30%	30%	30%
	FB	30%	15%	60%	30%
	PF	30%	30%	30%	30%
Pension level	FC	30%	60%	15%	30%
	FB	30%	30%	30%	30%
	PF	30%	30%	30%	30%
Rate of return of cohort born in t	FC	100%	-50%	0%	0%
	FB	0%	100%	-50%	0%
	PF	0%	0%	0%	0%
Pension fund	PF	0	15% w L	0	0

In contrast to the ageing effect the return differentials owing to the cohort effect can theoretically be evened out. However, the problem cannot be fully eliminated by adjusting the contribution rate, the pension level or the retirement age. The risk of belonging to a relatively large/small cohort can only be eliminated by structural reform of the PAYG system. One approach would be no longer requiring the pension system to run a balanced budget each period. The budget equation of the pension system would then be

$$\phi_t * w_t * L_t = \gamma_t * w_t * P_t + \Delta D_t \tag{7}$$

where ΔD is the change in a potential pension fund.

The basic idea is this: as long as a relatively large cohort is working, any surpluses accrued are not paid out to current retirees (of whom there are relatively few). Instead, these pension system surpluses are saved up. When the relatively large cohort enters the pension payment period, the surplus saved up previously will finance the higher level of expenditure. Once the relatively large cohort dies, the pension fund would be used up. The pension system is thus partially funded by capital. In a PAYG system with a pension fund (PF), the return is independent of the size of the cohort. All cohorts receive a return that is equal to wage growth. The size of the cohorts only determines whether the pension fund holds assets or liabilities. In the simple model presented here, the change in the pension fund corresponds to

$$\Delta D_t = (1 - \mathcal{Q}_t) * \phi_t * w_t * L_t \tag{8}$$

In the example figures selected, the pension fund would amount to 25% of the total wage bill at the end of the first period. At the end of the second period, the pension fund would be used up again.

4 A Quantitative OLG Model

The simplified problem presented in the previous section is now embedded into a quantitative model. The quantitative model contains a far more complex population structure. The rates of return determined in such quantitative model provide more realistic estimates than the simple calculations above.

Besides the population age structure, pension systems are also affected by labour force participation (LFP) rates.[10] A change in labour force participation resembles the cohort effect. Here it is important whether the observed increase in labour force participation is permanent or not.

Additionally, the quantitative model allows for more complex PAYG systems such as the German statutory pension system (Deutsche Rentenversicherung, or DRV) to be analysed. The DRV, which is also funded on a PAYG basis, is neither a pure FC nor a pure FB system. Its contribution rate and pension level are not fixed, but have a reciprocal effect on one another. However, the DRV, too, features the above-described redistributive effects owing to increasing in life expectancy and varying cohort sizes.

[10]Pension systems depend also on the share of employment subject to social security contributions. This share may equally hinge on demographics, but is not explored further in this paper. In the subsequent quantitative analysis, the share of employment subject to social security contributions is constant over time.

The following set-up is a small open economy overlapping generations model.[11] It is based on the work of Auerbach and Kotlikoff (1987) and its adoption by Schön (2020). It composes utility-maximising households, profit-maximising firms, and a PAYG pension system.

4.1 The Demographic Model

The demographic process is taken as exogenous and represents the main driving force of the model. Several cohorts that can be of varying size live simultaneously in the model economy. A single cohort, c, per se is homogeneous and consists of identical households. At any point in time, t, the various cohorts are at different stages of life: households go through a life cycle in which they first work and then retire. At the end of each period, there is a given probability that households will die.[12] The older the household, the greater is this probability. Households die with certainty at age J^T. Cohorts born later have a higher life expectancy. Note that point in time, t, and age of a household, j, uniquely determine its cohort, $k = t - j + 1$.

The size of the population of age j in period t is given recursively

$$N_{j,t} = N_{j-1,t-1}\pi_{j-1,t-1} + Z_{j,t}, \tag{9}$$

where $\pi_{j,t}$ denotes the age and time specific conditional survival rate and $Z_{j,t}$ is the net flow of people to Germany in a given period.[13]

Each year sees the entry of a new cohort. In each period newborns are determined by

$$N_{1,t} = \frac{1}{J^F} \sum_{j=1}^{J^F} \frac{N_{j,t-20}}{2} * f_{t-20} \tag{10}$$

where J^F is the maximum age a woman is assumed to bear children f_t is the fertility rate per woman over life.

[11] The small open economy setting neglects changes in factor prices. Changing factor prices in a closed (or larger open) economy due to demographic change (labour scarcity and abundance of capital) affect the rates of return of the PAYG pension system. Ageing economies exhibit increasing wages and lower capital returns. Higher wages also increase the benefits of most pension system and therefore alter their rate of return. Welfare effects stemming from this are analysed for example in Krueger and Ludwig (2007). This paper focuses on the effect of demographic change on the individual rates of return via the necessary balanced budget of PAYG systems. For this reason, the factor price channel is shut down by modelling a small open economy with a fixed world interest rate.

[12] The following analysis does not distinguish between men and women. The age specific mortality rates are the average of female and male mortality rates.

[13] For computational reasons migrants enter the economy with the exact same amounts of assets and earnings points that households of the same age that already live in Germany possess.

4.2 The Pension System

The PAYG pension system is characterized by a contribution rate, ϕ_t, and a pension level, γ_t. The budget of the PAYG pension system is balanced at any time t,

$$\phi_t w_t \sum_{j=1}^{J_k^R-1} l_{j,t} N_{j,t} = \sum_{j=J_k^R}^{J^T} b_{j,t} p_{j,t} N_{j,t} \tag{11}$$

with

$$b_{j,t} = \begin{cases} 0 & \text{if } j < J_k^R \\ b_t = \gamma_t(1-\phi_t) w_t \frac{1}{J_k^R-1} & \text{if } j \geq J_k^R \end{cases} \tag{12}$$

On the revenue side, w_t denotes the wage rate. Individual labour supply from households denotes as $l_{j,t}$. On the expenditure side of the pension budget equation are pension payments. Pensions are defined by an earnings point system. The paid out pension is calculated by multiplying the number of acquired earnings point, $p_{j,t}$, with the pension value, $b_{j,t}$. The pension value consists of the replacement rate, γ_t, times the wage (after pension contributions) at time t divided by $J_k^R - 1$, the number of years in a standardized working life. In each period of its working life a fully working household would collect one earnings point.

$$p_{j+1,t+1} = \begin{cases} l_{j,t} + p_{j,t} & \text{if } j < J_k^R \\ p_{j,t} & \text{if } j \geq J_k^R \end{cases} \tag{13}$$

Rewriting the budget constraint of the pension system gives

$$\phi_t w_t L_t = \gamma_t (1-\phi_t) w_t P_t \tag{14}$$

with $L_t = \sum_{j=1}^{J_k^R-1} l_{j,t} N_{j,t}$ defined as the number of contributors and $P_t = \frac{1}{J_k^R} \sum_{j=J_k^R}^{J^T} p_{j,t} N_{j,t}$ defined as the number of retirees.

In the following quantitative exercise three pension systems are analysed. They differ in how the pension level and the contribution rate are determined.

Fixed Benefit System In a fixed benefit system, the pension level is set to a constant level

$$\gamma_t = \overline{\gamma}. \tag{15}$$

The contribution rate is determined endogenously to balance the pension systems budget.

$$\phi_t = \left[1 + \frac{L_t}{\overline{\gamma} P_t}\right]^{-1}. \tag{16}$$

Fixed Contribution System In the fixed contribution system the contribution rate is fixed.

$$\phi_t = \overline{\phi}. \tag{17}$$

To balance the budget, the pension level has to adjust accordingly

$$\gamma_t = \frac{L_t}{P_t} \frac{\overline{\phi}}{1 - \overline{\phi}}. \tag{18}$$

German Pension System In the German pension system (DRV), the pension (value) annual adjustment is determined according to the following formula

$$b_t = b_{t-1} \frac{w_t}{w_{t-1}} \frac{1 - \phi_{t-1}}{1 - \phi_{t-2}} \left[\left(1 - \frac{Q_{t-1}}{Q_{t-2}} \right) \times 0.25 + 1 \right] \tag{19}$$

with retiree ratio, Q_t, defined as the ratio of retirees to contributors

$$Q_t = \frac{P_t}{L_t}. \tag{20}$$

It can be seen as a summary statistics of the demographic and labour market developments.[14] The adjustment formula for the replacement rate is obtained by inserting (12) into (19)

$$\gamma_t = \gamma_{t-1} \frac{1 - \phi_{t-1}}{1 - \phi_{t-2}} \left[\left(1 - \frac{Q_{t-1}}{Q_{t-2}} \right) \times 0.25 + 1 \right] \frac{1 - \phi_{t-1}}{1 - \phi_t}. \tag{21}$$

The contribution rate is determined endogenously so that the pension system's budget constraint is balanced in each period[15]

$$\phi_t = \left[1 + \frac{L_t}{\gamma_t P_t} \right]^{-1}. \tag{22}$$

4.3 The Firm Sector

Firms produce with a Cobb-Douglas production function employing capital and labour

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{23}$$

where K_t denotes the aggregate capital stock, L_t the aggregate labour input at time t. The output elasticity of capital is α. The total factor productivity (TFP) level is A_t and its growth rate is $\mu = \frac{A_{t+1}}{A_t} - 1$.

Aggregate labour input is the labour supply of households, $l_{j,t}$, times the population structure at time t, $N_{j,t}$.

$$L_t = \sum_{j=1}^{J^R - 1} l_{j,t} N_{j,t} \tag{24}$$

[14] The retiree ratio is closely related to the old-age dependency ratio, which relates the population share above and below the statutory retirement age. An increase of OADR due to the demographic change also realizes in Q.

[15] This is a deviation from the German pension system that has a fluctuation reserve. The actual adjustment rule is that the contribution rate must be raised if the fluctuation reserves would otherwise fall below their minimum permissible size. In the light of the demographic situation, the reserves are likely to dwindle from their currently high level to their minimum over the next few years.

A static firm maximises profits subject to capital accumulation condition

$$K_{t+1} = (1-\delta) K_t + I_t \tag{25}$$

where I_t is net investment, δ is the capital depreciation rate.[16]

The first order conditions from profit maximization give standard expressions for equilibrium factor prices. The return on capital is given by

$$r_t = r_t + \delta = \alpha A_t \left(\frac{K_t}{L_t}\right)^{\alpha-1} = \alpha \frac{Y_t}{K_t}. \tag{26}$$

Wages are given by

$$w_t = (1-\alpha) A_t \left(\frac{K_t}{L_t}\right)^{\alpha} = (1-\alpha) \frac{Y_t}{L_t}. \tag{27}$$

4.4 The Household Sector

By choosing an optimal consumption path, each cohort c maximizes at any age j and point in time $t = c + j$ the sum of discounted future utility. The within period utility function exhibits constant inter-temporal elasticity of substitution and preferences are additive and separable over time. Cohort c's maximization problem at $j = 1$ is given by

$$\underset{\{c_{j,t}\}_{j=1}^{J^T}}{max} \sum_{j=1}^{J^T} \beta^j s_{j,t} U(c_{j,t}) \tag{28}$$

where β is the pure time discount factor. In addition to pure discounting households discount future utility with their unconditional survival probability, $s_{j,t+j} = \prod_{m=1}^{j} \pi_{m-1,t}$. $c_{j,t}$ denotes consumption.

All assets (including return on capital) of household that died at the end of one period are passed over to next periods younger households. So in each period households up to a specific inheritance age, J^Q, receive bequests

$$q_{j,t} = \begin{cases} \frac{(1+r_t)\sum_{i=1}^{J^T}(1-\pi_{i,t-1})a_{i,t-1}N_{i,t-1}}{\sum_{i=1}^{J^Q} N_{i,t}} & \text{if } j \le J^Q \\ 0 & \text{if } j > J^Q \end{cases}. \tag{29}$$

Denoting household assets by $a_{j,t}$, maximization of the household's inter-temporal utility is subject to a dynamic budget constraint given by

$$a_{j+1,t+1} = (1 + r_{t+1}) (a_{j,t} + q_{j,t} + y_{j,t} - c_{j,t}) \tag{30}$$

Income, $y_{j,t}$, consists of labour income and pension income.

$$y_{j,t} = (1 - \phi_t) w_t l_{j,t} + b_{j,t} p_{j,t} \tag{31}$$

Households supply labour, $l_{j,t}$, exogenously. The labour supply varies over time and age.

[16] Capital adjustment costs in the firm sector are not considered.

4.5 Definition of Equilibrium

Given the exogenous population distribution and survival rates in all periods $\{N_{j,t}, \pi_{j,t}\}$ and a world interest rate, $\overline{r}_t$, a competitive equilibrium of the economy is defined as a sequence of dis-aggregated variables, $\{c_{j,t}, a_{j,t}\}$, aggregate variables, C_t, L_t, K_t, a wage rate, w_t, and government/pension policies, $\{\phi_t, \tau_t^y\}$ such that

1. Given initial conditions household maximize utility with $c_{j,t}$ as the resulting optimal policies.
2. Pension policies satisfy equation (14) in every period.
3. Wages satisfy (26) and (27).
4. Markets clear and allocations are feasible in all periods

$$L_t = \sum_{j=1}^{J_k^R - 1} l_{j,t} N_{j,t} \tag{32}$$

This small open economy setting requires that the rate of return on investment is equalized to the world interest rate,

$$r_t = \overline{r} \tag{33}$$

The net foreign assets, defined as the difference between assets and capital stock is,

$$F_{t+1} = \sum_{j=1}^{J^T} a_{j+1,t+1} N_{j,t} - K_{t+1} \tag{34}$$

The goods market clears

$$G_t + C_t + I_t + \delta K_{t+1} = Y_t + (r - \delta) F_t \tag{35}$$

5 Calibration

The aim of the calibration is to match the German economy and specifically its demographic structure. Calibration of the model requires (i) data for the exogenous demographic processes and (ii) determination of values for several structural model parameters. The model period is one year and it is concentrated on the economic life of agents. Therefore, households enter the model at the biographical age of 20 which is model age of 1. The maximum biological age is 109, in model terminology, $J^T = 90$. By assumption, Germany was in a steady state in 1960. The population projections and its parameters are already discussed in Section 2.

5.1 Labour Force Participation and Retirement Age

For the purpose of this analysis it is necessary to see how labour force participation (LFP) for each cohort developed over time. The historical data used for the labour force participation are taken from the historical data set of the GFOS and are based on the German micro-census. Figure 3 (a) shows the LFP for different cohorts over their life cycle. Solid lines represent actual data whereas dotted lines represent own projections (see below).

All LFP profiles show a pronounced hump shaped pattern. LFP increases between the beginning of working life between age 20 and age 30. The LFP profiles reach their respective maximum between the age of 40 years to the age of 50 years. After the age of 50 years the LFP falls until the end of working life.

Over time the hump shaped profiles rotate and have an overall higher level of labour force participation. Employment among those below the age of 30 is lower and decreases over time.[17] The increase in the LFP between age 30 and age 50 was sustained almost exclusively by a higher level of female employment. While the labour force participation of males was stagnating at a high level, the female labour force participation rate rose especially after the age of 30.[18]

At the end of working life, especially between 55 and 65, disability and early retirement (for other reasons) explain the low level of employment. The employment rate in between 55 and 65 reveals a marked upward trend over time. This could be a sign of improved health within the population. Another reason might be the reduced pressure to retire early owing to the considerably improved situation on the labour market. But it might also be the result of the more restrictive regulations on early retirement, cf. Bodnar and Nerlich (2020). While no more than roughly 40% of women aged between 55 and 60 were in employment in 1991, the figure was already just under 80% in 2015. The gradual convergence since 2000 of the statutory retirement age for women towards that for men is also likely to have played a part in this.[19]

The basis of the extrapolation assumed in the model is the observed labour force participation of the cohorts (Figure 3 (a) solid lines). There are three main assumptions on the future LFP development based on the observed past trends. First, the labour force participation rate decreases in younger years. Secondly, the labour force participation rate in the middle period of life in the past but now stabilises around 90%. And, thirdly, there is a strong and continuing increase in LFP between the age of 50 and 65.

On aggregate these assumptions lead to an increase of the overall LFP over time. The black solid line in Figure 3 (b) shows the sum of labour force participation over the life cycle (age 20-65) for different birth cohorts. Solid lines are calculated with actual data only and dotted line signals that actual data and projections are used. The labour force participation increases from 64% for 1940 cohort to almost 84% for the cohort born in 1990. The assumptions made result in more or less constant life cycle labour force participation for all cohorts after 1990.

The statutory retirement age, J_k^R, of each household is determined by its cohort. The

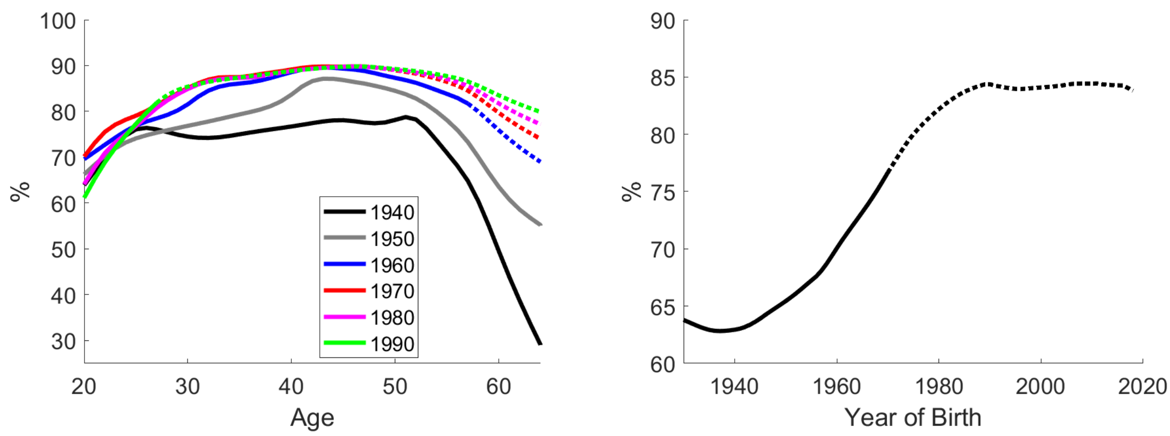
[17] One explanation for this might be that many persons at this age are still enrolled in an institute of higher education. This means that the trend might be a sign of an increasingly large segment of the population attaining a high level of formal education.

[18] One reason for the higher level of female employment along with improved childcare facilities may have been social reforms which made it less attractive to leave the labour force at a younger age.

[19] The fraction of people older than 65 years also rises over time. In 2010 only 3.4% of those over 65 were working. In 2016 this figure was as high as 6.4%. The fact that the statutory retirement age was raised from 65 years to 65 years and five months during this period is likely to have had a significant impact in this context.

Figure 3: Cohort Labour Force Participation

(a) ... over life cycle (b) ... aggregated over life



Notes: Panel (a) shows labour force participation for different cohorts over their life cycle. Solid lines represent actual data taken from GFOS; dotted lines represent projections. Panel (b) shows the sum of labour force participation over the life cycle. Solid lines are calculated actual data only; dotted line includes actual data and projections.

statutory retirement age for all cohorts is set to $J_k^R = 45$ (age 65 years).

5.2 Technology & Preferences

The TFP parameter A determines the level of output and is calibrated to match gross value added in 1960 of €185bn.[20] The TFP growth rate and the world interest rate are set to zero $r_t = \mu_t = 0$. The production elasticity of capital is calibrated such that the model matches the labour income share in Germany's national accounts in 1960, $1 - \alpha = 1 - 60\% = 0.4$.[21] The model assumes s capital to output ratio of 2.7. This capital to output ratio will be attained in the model by appropriate calibration of the preference parameters. The resulting time discount factor is $\beta = 0.9754$. Using data on output, capital, and national income, VE_t, the implied yearly depreciation rate is $\delta = \frac{Y_t - \text{VE}_t}{K_t} = 4.1\%$.

The within period utility function is given by

$$U(c_{j,t}) = ln(c_{j,t}) \tag{36}$$

Bequests are distributed to household within the first 10 periods of their life, $J^Q = 20$.

[20] The basis of this calibration is the GDP of West Germany. It is then extrapolated to also account for the GDP of East Germany.

[21] See Grömling (2006).

6 Rate of Return within PAYG Systems

The quantitative model allows us to calculate the return generated by cohort c within the PAYG system. It compares the contributions that a cohort makes over its lifetime with the benefits that a cohort receives. The return in a PAYG system is calculated as the internal rate of return produced by the following equation:

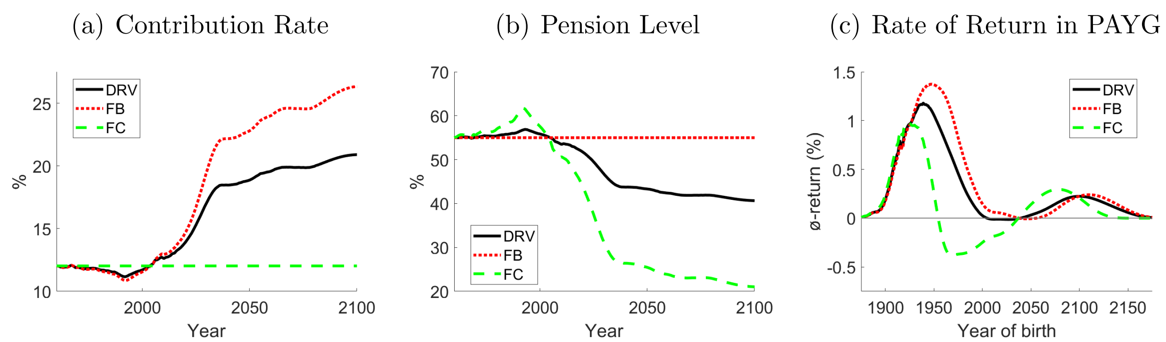
$$\sum_{j=1}^{90} \pi_{j,k} \frac{\mathcal{C}_{j,k+j} - \mathcal{B}_{j,k+j}}{(1+i_k)^{j-1}} = 0 \tag{37}$$

where $\pi_{j,k}$ stands for the unconditional survival probability of cohort c becoming j years old, $\mathcal{C}$ are contributions to the pension system, and $\mathcal{B}$ are benefits from the pension system.

6.1 Full Demographic Change

Figure 4 first maps the key variables of the pension system in the context of complete demographic change (ageing effect + cohort effect + changing labour force participation). The general pattern in all described PAYG systems is that between 1960 and 2004 pension systems benefit from the demographic change. Afterwards PAYG pension systems experience enormous financing pressure. This pressure is especially strong in the period between 2020 and 2035. In a FB system this pressure is borne by the contribution rate. Initially demographics and increased labour force participation result in a slightly lower contribution rate. In the long run however the contribution rate has to more than double to balance the pension budget. In a FC system the endogenous pension level initially increases. As time progresses the financing pressure due to the demographic change cuts the pension level in half in the long run. In the DRV, the pension level eventually decreases by a third and the contribution rate almost doubles.

Figure 4: Full Demographic Change



Notes: Panel (a) shows the contribution rate to the pension system. Panel (b) shows the pension level. Panel (c) shows the implicit rate of return in PAYG systems. Black solid line: DRV; red dotted line: FB system, green dashed line: FC system.

Figure 4 (c) shows the implicit rates of return cohorts realize within the PAYG pension systems. Sign and magnitude of the rate of return vary over time and depend on the design of the particular pension system. In the long term, in all PAYG systems rates of return converge towards wage growth (here 0). It is however noteworthy that for the majority of cohorts the demographic change has a positive overall impact on their returns. In a FB system all cohorts benefits from the ageing population and the increase in labour force participation. The cohort that profits most is born in 1948 with a return of 1.4%. The returns decrease for cohorts that are born later and will be almost neutral for the cohort born in 2035.

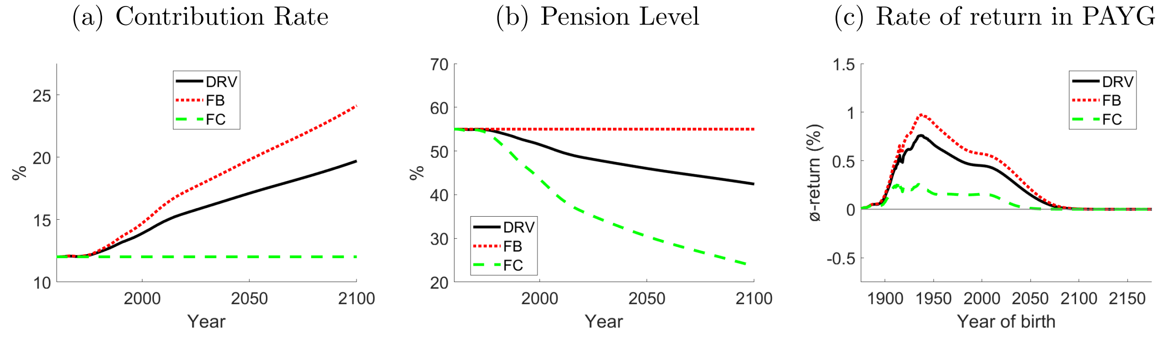
In a FC system it is especially profitable to be born early. The maximum return in a FC system is 1.0% for cohort 1924. Afterwards the rates of return decrease as in the FB system. Contrary to the FB system, in a FC system the returns of cohorts born between 1955 and 2037 are even negatively affected. The return of cohort of 1969 is −0.4%. The course of the rates of return of the German DRV system confirms its hybrid nature. The cohort born in 1939 benefits the most from demographic change. It is 1.2 percentage points higher than without the demographic change.

6.2 Ageing Effect

For the most part, the overall effect is determined by the ageing effect. To isolate the ageing effect, now all cohorts have the same size (the size of the 1960 cohort). The demographic structure only changes owing to the rising life expectancy as projected by GFOS. The old-age dependency ratio thus increases monotonically over time and, from 2100, remains at a consistently high level. Additionally, the life cycle profile of the labour force participation is kept constant over time (LFP of the 1960 cohort).

Figure 5 shows the pension variables as they would develop only on the basis of the ageing effect. The rising and then constantly high life expectancy results in a higher number of retirees. This leads to a constantly increasing contribution rate in a FB system and to a constantly declining pension level in a FC system. The German pension system occupies the middle ground here, with rising contribution rate and declining pension level. The shown trend would continue as long as the life expectancy increases.

Figure 5: Ageing Effect



Notes: Panel (a) shows the contribution rate. Panel (b) shows the pension level. Panel (c) shows the implicit rate of return in PAYG systems. Black solid line: DRV; red dotted line: FB system, green dashed line: FC system.

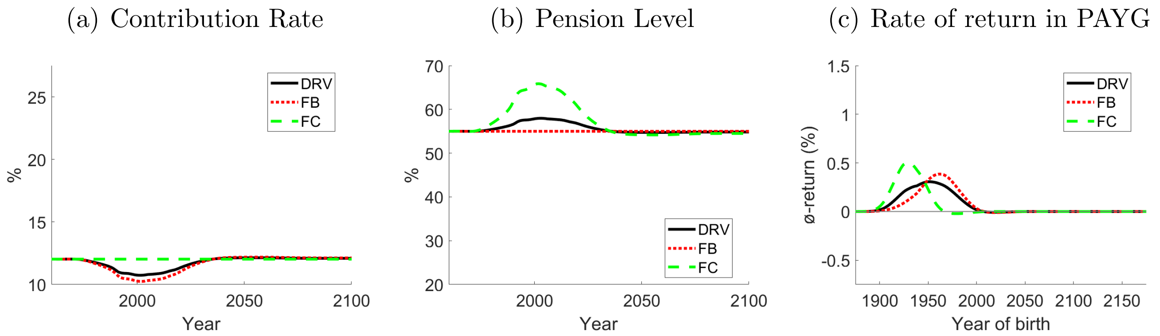
In all PAYG systems, the return is positive for all cohorts, with a maximum return of 1.0% for the 1939 cohort in the fixed benefit system. The reason for this positive return is that the longer life expectancy increases the amount paid out under the PAYG system. Similar to an introduction of a PAYG system, a permanent expansion creates gains. These gains are paid out to households over time, thus raising the implicit returns in the PAYG system. In a FC system the expansion of the system is much smaller and therefore the additional return is lower. The maximum additional return would be 0.3% for the 1934 cohort.

6.3 Labour Force Participation Effect

In this section the labour force participation effect is isolated. This is achieved by assuming that the mortality rates and therefore the life expectancies are identical for all households to that of cohort born in 1960. It is also assumed that all cohorts have the same size (the size of the 1960 cohort). Only the labour force participation varies over time.

An increase in LFP is for pension systems equivalent to an increase in the cohort size. The main difference to a fluctuation in cohort size is that the labour force participation is assumed to stay high and not bounce back.[22] In this sense the increase in LFP expands the PAYG system permanently and generates gains like the ageing effect. Contrary to the ageing effect, the pension level and the contribution rate are not disturbed in the long run. The reason for this is that higher LFP increases eventually the claims against the pension system.

Figure 6: Labour Force Participation Effect



Notes: Panel (a) shows the contribution rate. Panel (b) shows the pension level. Panel (c) shows the implicit rate of return in PAYG systems. Black solid line: DRV; red dotted line: FB system, green dashed line: FC system.

The LFP effect on the implicit rates of return is also similar to the ageing effect. No cohort loses due to the LFP effect and some cohort benefit from it. Depending on the pension system the magnitude of the benefits differs. In a FB system the benefits are distributed broader over cohorts. The maximum return is 0.4% for cohort born in

[22]A different result would occur if the observed increase in the employment is not permanent. In this case the LFP effect resembles in terms of rate of return the cohort effect.

1962. In a FC system cohorts born early benefit more than in the FB system. Here the maximum implicit rate of return is 0.5% for cohort born in 1929.

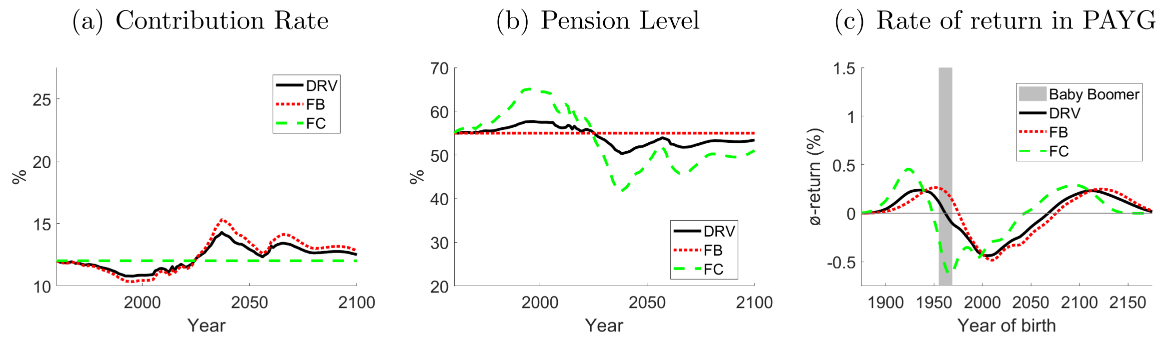
6.4 Cohort Effect

In the following section, the cohort effect will now also be analysed separately. It is assumed that the mortality rates and labour force participation of all households are identical to that of the cohort born in 1960. The size of the relevant cohorts is taken from the data and the projection of the GFOS.

The contribution rate and the pension level are not monotonic. For the fixed benefit system the endogenous contribution rate is initially decreasing. Beginning in the late 1990s it starts to increase until 2037. Then it converges back to its initial value. A mirrored but reversed pattern shows the endogenous pension level in a fixed contribution system. It initially increases and then drops to eventually return to its initial value. Again the German pension system holds the middle ground.

As demonstrated by the simple model, the cohort effect (Figure 7) has winners and losers. The quantitative model shows that the size of a cohort can only ever be seen in the context of the overall population. The baby boomers (cohorts from 1955-1969, grey shaded area) are at an advantage in FB systems. Here, there is a certain delay before the set-up of the system takes effect. The ordering of the pension systems in terms of rate of return changes several times. The variance of the rate of return in the FC system is higher with a maximum excess return of 0.5% for the cohort born in 1924. For some cohorts the FC system is most beneficial, for other cohorts it is the FB system. The greatest return differential between FB and FC system of 0.8% would occur for the 1967 cohort, for example. While a positive return of 0.2% would be generated in a FB system owing to the cohort effect, it would be −0.6% in a FC system. Here, too, it is clear that the German pension system plays a kind of intermediary role between a FB and a FC system. Even though the contribution rate and the pension level vary less than in the ageing effect case, the size of the variation in rates of return is comparable.

Figure 7: Cohort Effect



Notes: Panel (a) shows the contribution rate. Panel (b) shows the pension level. Panel (c) shows the implicit rate of return in PAYG systems. Black solid line: DRV; red dotted line: FB system, green dashed line: FC system.

The previous results illustrate that both a fixed benefit system as well as a fixed contribution system exhibits cohort-dependent rates of return. To provide equal rates of return for all cohorts one has to adjust the pension system by allowing for a pension fund. In this pension system the contribution and the replacement rate are fixed to the values in the initial steady state. In this example the contribution rate is set to $\phi_t = \overline{\phi} = 12\%$, and the replacement rate is set to $\gamma_t = \overline{\gamma} = 55\%$.

In contrast to the previous pension systems the budget of the pension system is not balanced in every period. The only condition is that the present value of contribution and expenditures must be equal.

$$\overline{\phi} \sum_{t=1}^{T} \frac{w_t L_t}{\Pi_{i=1}^{t}(1+r_i)} = \overline{\gamma} \sum_{t=1}^{T} \frac{w_t P_t}{\Pi_{i=1}^{t}(1+r_i)} \tag{38}$$

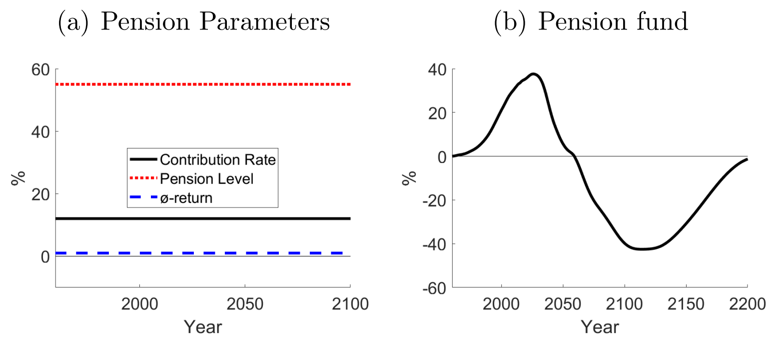
The pension fund, D_t, develops according to following equation

$$D_{t+1} = (1+r_t)D_t + \overline{\phi} w_t L_t - \overline{\gamma}(1-\overline{\phi}) w_t P_t \tag{39}$$

with $D_0 = 0$.

The pension budget is not balanced with a constant replacement rate and contribution rate due to the cohort effect. The differences in rates of return within the pension system due to the cohort effect cannot be sufficiently addressed by simple parameter adjustments. This makes it necessary to have a pension fund to buffer variation in cohort size. We assume that the pension system would have introduced the pension fund in 1960. The development of the pension fund is shown in Figure 8 (c). It starts at zero in 1960 then quickly accumulates assets until 2025. There the pension fund would amount to 41.2% of GDP. With the entry of the baby boomer cohort in the following years the pension fund would melt down. After 2060 the assets would have been exhausted completely and the pension fund has to issue debt to pay the pension. In 2110 the debt of the pension fund would reach its maximum and amounts to 46% of GDP. In the following the pension would pay back the debt and after 2200 the pension fund is again zero.

Figure 8: Reform in 1960



Notes: Panel (a) shows the contribution rate, the pension level, and the implicit rate of return in PAYG system. Panel (b) shows the pension fund in relation to GDP.

7 Conclusions

Demographic change consists of two different developments: first, increasing life expectancy, and second, fluctuations in the size of birth year cohorts. The increase in life expectancy is slow and monotonic and also (presumably) a sustained trend. The number of births per year can both decrease and increase, and changes more rapidly over time. However, fluctuating birth rates of the past have less and less of an impact on the demographic structure as time progresses. This paper shows that the ageing effect has a positive impact on the returns households generate within the PAYG pension system. The cohort effect, by contrast, results in winners and losers in a PAYG system in terms of the return on that system.

Overall, the demographic change in Germany has a positive impact on the rates of return participants realize in the German PAYG pension system. The distribution among the cohorts is unequal with a difference 1.3 percentage points per year over the whole life cycle. The cohort that benefited the most is the birth cohort of 1939 with an excess return of 1.2 percentage points compared to a world without demographic change. The birth cohort of 2026 will have a slightly negative excess return of -0.1 percentage points.

This paper illustrates the interconnectivity of demographic change, the requirement of PAYG pension systems to run a balanced budget and the contribution/benefits of individuals within the pension system. To focuses solely on this direct effect, it assumes many processes that are affected by the demographic change and the pension system as exogenous. An interesting deepening of this analysis would consider the resulting welfare effects of this redistribution. Such a welfare analysis could then be used to propose optimal pension reforms. To this end endogenous labour supply of household is an important channel. Higher rates of return within a Bismarckian type pension system increase the incentives to supply labour. Therefore, labour supply would be distorted by the ageing effect and the cohort effect. In turn, the rate of return would be additionally shifted. Another important channel would include endogenous migration. There is a huge literature discussing the endogeneity of the migration choice to existing welfare systems. With endogenous migration the described effects of changes in the demographic structure of the population will affect pre-tax and after tax income, thereby affecting the migration choice. Another potential question is whether the fertility rate could react to life expectancy and rate of returns of the pension system. Although including these feedback channels would be a promising and interesting approach for future research it is beyond the scope of this paper.

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