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Interest rate pegs and the reversal puzzle: On the role of anticipation

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Non-technical Summary

Research question

The so-called *reversal puzzle* describes a phenomenon according to which the macroeconomic effect of forward guidance – technically implemented by a perfectly anticipated interest rate peg – can switch from expansionary to contractionary, depending on the duration of the interest rate peg. We ask whether the reversal puzzle still occurs if we leave the perfect foresight framework and allow for varying degrees of anticipation.

Contribution

We show that the degree of anticipation of the interest rate peg plays a key role for the appearance of the reversal puzzle. We consider three different assumptions about the degree of anticipation: perfect anticipation, no anticipation, and imperfect anticipation. We model the case of imperfect anticipation by means of a Markov-switching approach in which the regime of a pegged interest rate stochastically recurs.

Results

If agents perfectly anticipate an interest rate peg, the reversal puzzle is a robust phenomenon. If the agents do not anticipate the interest rate peg, the reversal puzzle is absent. If agents imperfectly anticipate an interest rate peg, the occurrence and the duration of each single peg episode are stochastic, but – as the agents are aware of the transition probabilities between the regimes – the frequency and the average duration of an interest rate peg are known. The degree of anticipation then depends on the average duration and the frequency of the peg. For the large range of average durations of the peg we consider, the reversal puzzle is absent, even if the frequency of the peg takes on a value that is twice as large as an empirically plausible value as measured by the post-WWII zero lower bound frequency in the US. Only for extreme and arguably implausible assumptions about the frequency of the peg, reversals occur.

Nichttechnische Zusammenfassung

Fragestellung

Das sogenannte *reversal puzzle* beschreibt ein Phänomen, demzufolge der Effekt von *forward guidance* – technisch implementiert als perfekt antizipierte Zinsbindung – von expansiv zu kontraktiv wechseln kann, je nachdem wie lange die Zinsbindung anhält. Wir untersuchen, ob das *reversal puzzle* noch immer auftritt, wenn wir die Annahme der perfekten Antizipation aufgeben und unterschiedliche Grade der Antizipation zulassen.

Beitrag

Wir zeigen, dass der Grad der Antizipation für das Auftreten des *reversal puzzles* eine Schlüsselrolle spielt. Dafür analysieren wir drei unterschiedliche Annahmen über den Grad der Antizipation: perfekte Antizipation, keine Antizipation und unvollkommene Antizipation. Wir implementieren den Fall der unvollkommenen Antizipation mittels eines Markov-switching-Ansatzes, in dem das Regime der Zinsbindung stochastisch wiederkehrt.

Ergebnisse

Antizipieren die Agenten die Zinsbindung perfekt, ist das *reversal puzzle* ein robustes Simulationsergebnis. Wenn die Agenten eine zukünftige Zinsbindung nicht antizipieren, tritt kein *reversal puzzle* auf. Wenn die Agenten eine Zinsbindung lediglich unvollständig antizipieren können, ist das Auftreten und die Dauer einer einzelnen Zinsbindungsepisode stochastisch, aber die Häufigkeit und die durchschnittliche Dauer des Regimes der Zinsbindung sind bekannt, da die Agenten die Übergangswahrscheinlichkeiten von einem Regime zum anderen kennen. Der Grad der Antizipation hängt dann von der durchschnittlichen Dauer und der Häufigkeit des Zinsbindungsregimes ab. Für eine große Spannbreite der durchschnittlichen Dauer des Zinsbindungsregimes tritt das *reversal puzzle* nicht auf, selbst wenn die Häufigkeit des Zinsbindungsregimes doppelt so hoch ist wie ein empirisch relevanter Wert (gemessen an der Häufigkeit einer bindenden Zinsuntergrenze in den USA nach dem Zweiten Weltkrieg). Das *reversal puzzle* tritt nur für extreme und eher unplausible Annahmen über die Häufigkeit des Zinsbindungsregimes auf.

Interest rate pegs and the reversal puzzle: On the role of anticipation*

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Abstract

We revisit the reversal puzzle: A counterintuitive contraction of inflation in response to an interest rate peg. We show that it is intimately related to the degree of agents' anticipation. If agents perfectly anticipate the peg, reversals occur depending on the duration of the peg. If they do not anticipate the peg, reversals are absent. In the case of imperfect anticipation, implemented by a Markov-switching framework, we measure the degree of anticipation by the frequency of the peg regime. Even if the frequency of the peg takes on a value twice as large as empirically observed, the reversal puzzle is absent.

Keywords: Interest rate peg, Reversal puzzle, Regime-switching model

JEL Classification: E32, E52

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1 Introduction

In the wake of the financial crisis, major central banks adopted a series of unconventional monetary policy measures both to restore the functioning of the monetary transmission mechanism and to provide further policy accommodation. In particular, central banks implemented sovereign bond purchase programmes, often referred to as quantitative easing (QE), and forward guidance as policy responses to anaemic growth and too low inflation rates. Such unconventional measures are widely conceived as alternative instruments when central banks have reached the effective lower bound on nominal short-term interest rates. While there is no consensus yet about the strength of the effects of QE and forward guidance, the literature generally supports the view that these policy measures are expansionary (see, for example, Carlstrom, Fuerst and Paustian, 2017; Gertler and Karadi, 2013; Chen, Cúrdia and Ferrero, 2012, and the references therein).

However, Carlstrom, Fuerst and Paustian (2015) describe the puzzling result that in standard New Keynesian models the effect of forward guidance – technically implemented by a perfectly anticipated interest rate peg – can switch from expansionary to contractionary for varying durations of the interest rate peg. Carlstrom et al. (2015) call these counter-intuitive sign reversals in the path of the endogenous variables for varying durations of the interest rate peg the reversal puzzle.[1]

In this paper, we show that agents' degree of anticipation of an interest rate peg is crucial for the appearance of the reversal puzzle. First, we reproduce Carlstrom et al. (2015)'s result that a perfectly anticipated interest rate peg leads to reversals, depending on the duration of the peg. An increasing duration of an interest rate peg first increases the response of inflation and then tends to make it explode as the duration of the peg approaches some critical value. If the duration of the peg exceeds this critical value, the model predicts a counterintuitive sign reversal (i.e., the reversal puzzle) or, put differently, a sizeable deflation instead of inflation. In our baseline scenario, this critical value is eight quarters.[2] If we continue to hold the interest rate fixed beyond the critical value, the duration of the peg approaches another critical value after which the sign of the inflation response changes once again. Thus, the qualitative response of output and inflation oscillates as the duration of the peg expands more into the future. We provide analytical intuition for reversals and show that their occurrence critically hinges on the assumption that the interest rate peg is perfectly anticipated.

We then show that the reversal puzzle does not occur if agents take the nominal interest rate peg into account only contemporaneously, but expect the peg to be absent in the future. In such

[1] Similarly, several studies, such as Lindé, Smets and Wouters (2016) and Binning and Maih (2017), document the occurrence of such sign reversals when modelling scenarios in which the interest rate is constrained due to the zero lower bound or an announcement to fix the interest rate for some periods (as is the case with forward guidance).

[2] In the appendix, we document that the puzzle is not due to a specific calibration of the model. Under the assumption of perfect foresight, different calibrations merely allow a longer duration of the interest rate peg until the reversal first appears.

a scenario there is effectively no anticipation of a future transient interest rate peg. Reversals are absent irrespective of the peg duration. We conclude from this experiment that a sufficient degree of anticipation is necessary for the occurrence of the reversal puzzle.

The polar cases perfect anticipation and no anticipation describe arguably unrealistic scenarios, and the case of no anticipation is obviously not a suitable solution for the reversal puzzle. We thus move beyond those polar cases and analyse a scenario where agents imperfectly anticipate a transient interest rate peg. We do so using a Markov-switching framework (Maih, 2015) in which agents attach non-zero transition probabilities to entering and exiting a prevailing regime of pegged interest rates. The occurrence and the duration of each single episode of pegged interest rates are then stochastic but – as the agents are aware of the transition probabilities – the frequency and the average duration of an interest rate peg are known. Since for a higher frequency agents consider an interest rate peg to be more likely, the frequency of the peg can be interpreted as the degree of anticipation in the stochastic scenario. Given a sufficient degree of anticipation, reversals can occur depending on the average duration of the peg. We find that the reversal puzzle is absent for empirically plausible calibrations of the peg frequency. In particular, even for frequencies that are much higher than 10 % — the post-WWII zero lower bound experience in the US[3] — reversals do not occur irrespective of the average duration of the peg. Reversals may still occur under imperfect anticipation but only for implausibly high frequencies of the peg. Thus, for realistic scenarios in which agents imperfectly anticipate an interest rate peg, the reversal puzzle is absent. Intuitively, as long as it is sufficiently unlikely to enter a peg episode, the degree of anticipation of a peg is small and no reversal occurs. As it becomes more likely to enter a peg episode, at some point the degree of anticipation is sufficiently strong for the possibility of reversals to occur. Whether they actually occur depends on the average peg duration, similar as in the perfect foresight scenario.

For an interest rate peg to potentially produce the reversal puzzle, an initial impulse must hit the economy. An interest rate peg has typically been associated with the zero lower bound on interest rates and forward guidance. In such an environment, many central banks have reverted to QE. Therefore, a natural choice of the initial impulse is the launch of a QE programme. As a laboratory for our experiments we implement the by now well-known model of Carlstrom et al. (2017) which features funding constraints and market segmentation, such that QE policies have an effect on real economic activity and inflation. In the absence of the peg, the model predicts the orthodox view, that is, an increase in inflation in response to the launch of a QE programme. In the presence of an interest rate peg, however, it is possible that the model implies a reversal puzzle.

Similarly to our work, other papers in the literature explicitly deal with the reversal puzzle. Carlstrom et al. (2015) are the first to analyse the reversal puzzle and point out that a necessary condition for the puzzle to occur in a perfect foresight setting is the existence of endogenous

[3]This value for the zero lower bound frequency is also used by Dordal-i-Carreras, Coibion, Gorodnichenko and Wieland (2016).

state variables which imply complex eigenvalues.[4] They show that if they change the model structure and switch from a sticky-price to a sticky-information framework, the reversal puzzle disappears.[5] Our paper is complementary to Carlstrom et al. (2015) in that we go beyond the perfect foresight setting and show that the degree of anticipation plays a key role for the occurrence of the reversal puzzle.

Several other papers that primarily analyse the effects of forward guidance also mention the occurrence of the reversal puzzle (see De Graeve, Ilbas and Wouters, 2014; Maliar and Taylor, 2019; Bundick and Smith, 2020). These authors argue that for an empirically realistic calibration of their specific models, forward guidance is less effective and, as a byproduct, sign switches in impulse responses are less likely to occur. Our approach differs in that we do not only focus on a realistic calibration of our model to solve the reversal puzzle but rather refer to a more realistic modelling of expectations. Similarly to our approach, de Groot and Mazelis (2020) argue that empirically realistic forward guidance scenarios may imply less powerful forward guidance, and counterintuitive reversals in response to forward guidance do not occur in their analysis. However, in contrast to the Markov-switching framework we use, their proposed method to implement forward guidance experiments does not imply that the interest rate peg is a stochastic event that may reoccur in the future. In their analysis, agents are fully aware of the interest rate peg and the authors propose to modify the solution of the linearised model so as to mimic deviations from the standard rational expectations behaviour regarding announcements of future monetary policy.

From a methodological viewpoint, a paper that is close to ours is the one by Chen (2017). She analyses the outcomes of implementing the zero lower bound under the perfect foresight approach and compares them to the outcomes of implementing the zero lower bound under a Markov-regime switching approach. Her focus is on the quantitative difference of the two approaches regarding the predicted path of macro variables and the government spending multiplier, as well as the qualitative difference when positive supply shocks hit the economy at the zero lower bound. Our paper is complementary to her analysis in that we compare the perfect foresight and the Markov-switching approach regarding their qualitative differences in producing the reversal puzzle. In line with Chen (2017), we find that the Markov-switching approach delivers more plausible model outcomes than the perfect foresight approach.

We organize our paper as follows. In the next section, we sketch the model and briefly describe the transmission channel through which a QE programme affects the economy. Section 3 then illustrates the effects of QE in combination with an interest rate peg of variable duration. Subsection 3.1 analyses the scenario of perfect anticipation, Subsection 3.2 the scenario of no

[4]Note that in models larger than the canonical 3eq. New Keynesian model, complex eigenvalues in the solution of the model are only a necessary but not a sufficient condition for the reversal puzzle to occur. Already the canonical model with price indexation, but with a price level targeting rule instead of a standard Taylor rule might come with complex eigenvalues in its solution, but the reversal puzzle is absent. For a more detailed exposition see also Gerke, Giesen, Kienzler and Tenhofen (2017), Section 6.2.

[5]Kiley (2016) also mentions that a sticky-information approach can mitigate the power of forward guidance.

anticipation, and Subsection 3.3 the scenario of imperfect anticipation. Section 4 concludes.

2 Model and transmission of QE shock without interest rate peg

In the standard New Keynesian model, asset purchases are neutral (the so-called Wallace neutrality holds), in that they do not have an effect on real economic activity and inflation (Eggertsson and Woodford, 2003). To assess the effects of QE, we therefore rely on a DSGE model by Carlstrom et al. (2017), which features funding constraints and market segmentation such that the Wallace neutrality breaks down. More precisely, in this model both households and financial intermediaries (henceforth FIs) face financial constraints. The bond market is segmented in that only FIs can purchase long-term debt instruments. These include public (i.e., government) and private (i.e., investment) bonds. From the perspective of the FIs, these bonds are perfect substitutes and, hence, yield the same returns. However, the ability of the FIs to adjust their liability position is limited by two constraints. First, they are leverage constrained because the amount of deposits they can attract is constrained by their net worth (due to a hold-up problem). Second, FIs face net worth adjustment costs. Households need to finance their investments by way of issuing (long-term) investment bonds and, thus, face a funding restriction with respect to their investments (a so-called loan-in-advance constraint). The purchase of government bonds increases the FIs' demand for investment bonds since the liability side of the FIs balance sheet cannot adjust easily due to the aforementioned constraints. This in turn alleviates the households' loan-in-advance constraint.

Otherwise, the model exhibits familiar New Keynesian features. It comprises households that consume with habits, save in (short-term) deposits and supply labour. There is monopolistic competition in intermediate goods production. Prices and wages are subject to rigidities as in Erceg, Henderson and Levin (2000) and are indexed as in Christiano, Eichenbaum and Evans (2005). Investment is subject to adjustment costs. If the interest rate is not pegged, monetary policy follows a standard Taylor rule with some degree of interest rate smoothing. The non-linear model equations are summarised in Table 1. A complete derivation of the model and the corresponding estimation results are delegated to the Appendices A & B.

Table 1: Nonlinear model equations

Model equations:	
HH cons. decision	$\Lambda_t = \frac{\mathrm{b}_t}{C_t - hC_{t-1}} - E_t \frac{\beta h \mathrm{b}_{t+1}}{C_{t+1} - hC_t}$
Euler equation	$\Lambda_t = E_t \beta \frac{\Lambda_{t+1}}{\Pi_{t+1}} R_t^d$
Wage curve (WC)	${w_t}^{1+\varepsilon_w \eta} = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{X_t^{wn}}{X_t^{wd}}$
WC nominator	$X_t^{wn} = \lambda_{w,t} b_t \chi w_t^{\varepsilon_w (1+\eta)} H_t^{1+\eta}$
	$+E_t \left\{ \theta_w \beta \Pi_{t+1}^{\varepsilon_w (1+\eta)} \Pi_t^{-\iota_w \varepsilon_w (1+\eta)} X_{t+1}^{wn} \right\}$
WC denominator	$X_t^{wd} = \Lambda_t w_t^{\varepsilon_w} H_t + \theta_w \beta \Pi_t^{-\iota_w (\varepsilon_w - 1)} \Pi_{t+1}^{(\varepsilon_w - 1)} E_t \left\{ X_{t+1}^{wd} \right\}$

Table 1: continued

Model equations:	
Wages law of motion	${w_t}^{1-\varepsilon_w} = (1-\theta_w)(w_t^*)^{1-\varepsilon_w} + \theta_w \left(\frac{\Pi_{t-1}^{\iota w} w_{t-1}}{\Pi_t}\right)^{1-\varepsilon_w}$
HH decision capital	$\Lambda_t M_t P_t^k = E_t \beta \Lambda_{t+1} \left[R_{t+1}^k + M_{t+1} P_{t+1}^k (1-\delta)\right]$
HH decision inv. bonds	$\Lambda_t M_t Q_t = E_t \frac{\beta \Lambda_{t+1}(1+\kappa Q_{t+1} \mathrm{M}_{t+1})}{\Pi_{t+1}}$
Welfare	$V_t^h = b_t \left\{\ln(C_t - hC_{t-1}) - D_t^w B \frac{H_t^{1+\eta}}{1+\eta}\right\} + \beta E_t V_{t+1}^h$
Price of capital	$R_t^k = mc_t \mathrm{MPK}_t$
Real wages	$w_t = mc_t \mathrm{MPL}_t$
Phillips curve (PC)	$\Pi_t^* = \frac{\varepsilon_p}{\varepsilon_p - 1} \frac{X_t^{pn}}{X_t^{pd}} \Pi_t$
PC nominator	$X_t^{pn} = Y_t \lambda_{p,t} mc_t(i) + E_t \left\{\theta_p \frac{\beta \Lambda_{t+1}}{\Lambda_t} \Pi_t^{-\iota_p \varepsilon_p} \Pi_{t+1}^{\varepsilon_p} X_{t+1}^{pn}\right\}$
PC denominator	$X_t^{pd} = Y_t + E_t \left\{\theta_p \frac{\beta \Lambda_{t+1}}{\Lambda_t} \Pi_t^{\iota_p(1-\varepsilon_p)} \Pi_{t+1}^{\varepsilon_p - 1} X_{t+1}^{pd}\right\}$
Infl. law of motion	$(\Pi_t)^{1-\varepsilon_p} = (1-\theta_p)(\Pi_t^*)^{1-\varepsilon_p} + \theta_p \left(\Pi_{t-1}^{\iota_p}\right)^{1-\varepsilon_p}$
Price dispersion	$D_t^p = \Pi_t^{\varepsilon_p} \left[(1-\theta_p)\Pi_t^{*-\varepsilon_p} + \theta_p \left(\Pi_{t-1}^{\iota_p}\right)^{-\varepsilon_p} D_{pt-1}\right]$
Wage dispersion	$D_t^w = \theta_w \left(\frac{\Pi_t}{\Pi_{t-1}^{\iota_w}}\right)^{\varepsilon_w} \left(\frac{w_t}{w_{t-1}}\right)^{\varepsilon_w} D_{wt-1} + (1-\theta_w)\left(\frac{w_t^*}{w_t}\right)^{-\varepsilon_w}$
Resource constraint	$Y_t = C_t + I_t$
Production function	$Y_t = A_t K_t^\alpha H_t^{1-\alpha} / D_t^p$
Firm's capital decision	$K_t = (1-\delta) K_{t-1} + \mu \left(1 - \psi_I \left(\frac{1}{2}\right)\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) I_t$
Investment decision	$P_t^k \mu_t \left\{1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right) \frac{I_t}{I_{t-1}}\right\} =$
	$1 - \beta P_{t+1}^k \frac{\Lambda_{t+1}}{\Lambda_t} \mu_{t+1} \left\{-S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right\}$
FI's balance sheet	$\bar{B}_t + \bar{F}_t = N_t + L_t$
Leverage ratio	$L_t = \frac{E_t \frac{\Lambda_{t+1}}{\Pi_{t+1}}}{\left[E_t \frac{\Lambda_{t+1}}{\Pi_{t+1}} + (\Phi_t - 1) E_t \frac{\Lambda_{t+1}}{\Pi_{t+1}} \frac{R_{t+1}^L}{R_t^d}\right]}$
Loan in advance constraint	$P_t^k I_t = \bar{F}_t - \kappa \frac{\bar{F}_t}{\Pi_t} \frac{Q_t}{Q_{t-1}}$
FI's net worth decision	$\Lambda_t \left[1 + f(N_t) + N_t f'(N_t)\right] =$
	$E_t \Lambda_{t+1} \beta \zeta \frac{P_t}{P_{t+1}} \left[\left(R_{t+1}^L - R_t^d\right) L_t + R_t^d\right]$
Long-term interest rate	$R_t^L = \frac{(1+\kappa Q_t)}{Q_{t-1}}$
Yield to maturity	$R_t^{10} = Q_t^{-1} + \kappa$
Marginal prod. of capital	$MPK_t = \alpha A_t K_{t-1}(i)^{\alpha-1} H_t(i)^{1-\alpha}$
Marginal prod. of labour	$MPL_t = (1-\alpha) A_t K_{t-1}(i)^\alpha H_t(i)^{-\alpha}$
Taylor rule	$R_t = (R_{t-1})^\rho \left(R_{ss} \Pi_t^{\tau_\Pi} \left(\frac{Y_t}{Y_{t-1}}\right)^{\tau_y}\right)^{1-\rho} \varepsilon_t^R$

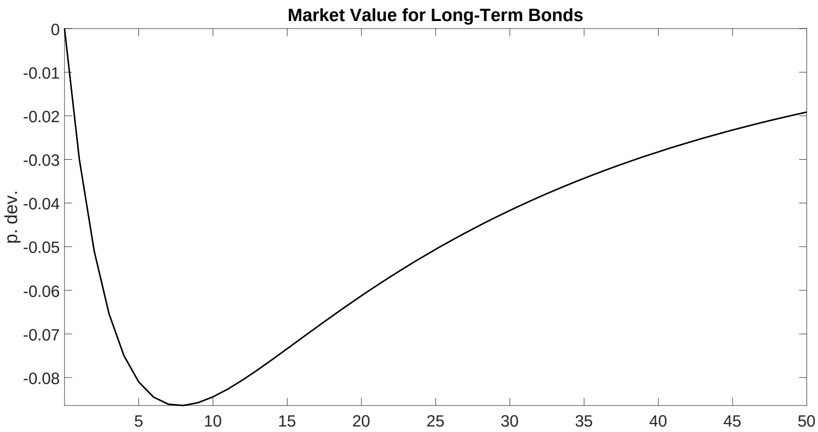
Notes: b_t = discount factor shock, C_t = consumption, Λ_t = Lagrange multiplier, Π_t = inflation, R_t = nominal interest rate, w_t = real wage, X_t^{wn} = & X_t^{wd} = auxiliary variables for wage curve, X_t^{pn} = & X_t^{pd} = auxiliary variables for Phillips curve, MPL_t = marginal product of labour, MPK_t = marginal product of capital, R_t^L = Long-term rate, R_t^{10} = yield to maturity, I_t = Investment, P_t^k = price of investment, $\bar{F}_t$ = investment bonds, $\bar{B}_t$ = government bonds, Q_t = price of bond, H_t = labour, A_t = technology shock, N_t = net worth, L_t = leverage, D_t^p = price dispersion, D_t^w = wage dispersion, K_t = capital, mc_t = marginal costs, μ_t = investment shock, Φ_t = financial shock, $\lambda_{w,t}$ = wage markup shock, $\lambda_{p,t}$ = price markup shock, Y_t = output.

We assume that the government controls the supply of long-term bonds independently of macroeconomic conditions. As in Carlstrom et al. (2017), a QE programme is implemented by a persistent AR(2) process for the real market value of long-term bonds available to the financial intermediaries:

$$\bar{B}_t = \bar{B}_{ss}^{(1-\bar{\rho}_1+\bar{\rho}_2)} \left(\bar{B}_{t-1}\right)^{\bar{\rho}_1} \left(\bar{B}_{t-2}\right)^{-\bar{\rho}_2} \varepsilon_t^{\bar{B}} \ . \tag{1}$$

This assumption is useful for two reasons: First, the AR(2) process is part of the model's equilibrium conditions and therefore taken into account by every agent. Thus, agents perfectly anticipate the path of the outstanding stock (value) of government bonds in the economy once a QE programme has been started. This will help us focusing on the degree of anticipation of the interest rate peg. Second, the (inverse) hump shape implied by an AR(2) process is well suited to representing a plausible QE programme: During the phase of purchases, the total value of outstanding bonds held by the public (i.e., excluding the central bank) declines, while it returns only gradually to the steady state after the purchases stop eventually – in our case after 6 quarters. Technically, the QE programme is triggered by a single shock, i.e., $\varepsilon_t^{\bar{B}}$, that occurs in the first period of the model simulation (see Figure 1).

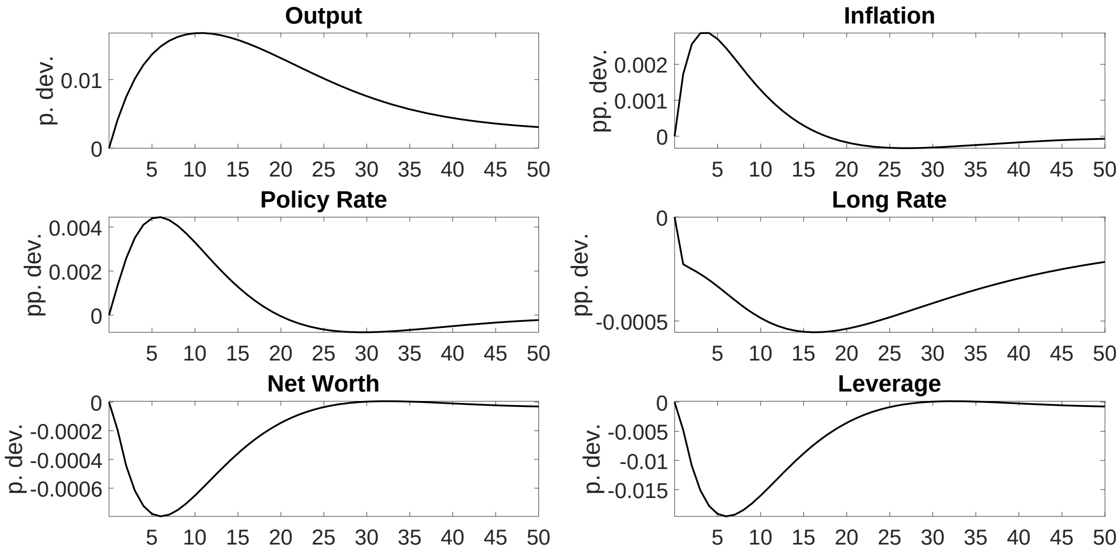
Figure 1: Total value of long-term bonds held by the public



Note: The solid line represents the evolution of $\bar{B}_t$ (i.e., the market value of long-term bonds) in percentage deviation from the steady state over 25 quarters.

We first consider a QE shock without considering an interest rate peg. This shows that the model is perfectly able to reproduce the conventional result that QE has inflationary effects.

Figure 2: Simulation of a QE shock without considering an interest rate peg



Note: The figure shows responses of (quarterly) output, net worth, and leverage in percent deviations from the steady state. Inflation, as well as the short- and long-term rates are measured in percentage point deviations from the steady state.

Figure 2 shows the transmission of a QE shock in this model. The decreasing supply of long-term government bonds available for FIs implies an upward pressure on its price and, correspondingly, lowers its yield to maturity – moderately but persistently. The term premium, too, decreases (which in the present model is essentially the distortion that is related to the loan-in-advance constraint).[6] The decrease in available bonds leads to a reduction in banks' net worth and leverage. Thus, the purchase of bonds shortens the FIs' balance sheet, but net worth mobility is limited due to portfolio adjustment costs. Correspondingly, FIs demand for investment bonds increases (portfolio adjustment). Since investment bonds and government bonds are perfect substitutes, the price of investment bonds also rises. Therefore, the households' loan-in-advance constraint is relaxed and, as a result, investment demand increases. Higher investment demand, in turn, increases aggregate output and so does the inflation rate. As a response, monetary policy increases its policy rate if it follows a Taylor rule.

3 The role of anticipation for the reversal puzzle

In this section, we analyse the effects of a QE programme when monetary policy pegs the interest rate. We do so under three different assumptions about the degree of the anticipation of the interest rate peg and show that this degree is crucial for the appearance of the reversal puzzle.

[6]Accordingly, the QE programme reduces the distortion that is due to market segmentation.

3.1 Perfect anticipation

We first conduct our analysis by solving the model under perfect foresight, such that the agents in the economy perfectly anticipate a temporary interest rate peg.[7] We implement the interest rate peg via a sequence of future shocks that consist of binary dummy variables, $\varepsilon_t^{TR} \in \{0,1\}$.[8] These are set to one for periods of pegged nominal rates and zero otherwise:

$$R_t = \varepsilon_t^{TR}\left(R_{ss}\right) + \left(1-\varepsilon_t^{TR}\right)\left(R_{t-1}\right)^{\rho}\left(R_{ss}\Pi_t^{\tau_\Pi}\left(\frac{Y_t}{Y_{t-1}}\right)^{\tau_y}\right)^{1-\rho}. \tag{2}$$

If the interest rate is pegged, the central bank does not follow the Taylor rule but instead keeps the short-term nominal interest rate unchanged for a pre-announced period of time, P (alongside its QE programme) such that ε_t^{TR} is equal to one for P successive quarters.

Figure 3 presents simulated time paths of inflation and the interest rate for different durations of the interest rate peg under the assumption of perfect foresight. Panel (a) shows outcomes for an interest rate peg of up to eight periods, i.e., $\{P \in \mathbb{N}_0 \mid 0 \leq P \leq 8\}$, panel (b) for $\{P \in \mathbb{N} \mid 9 \leq P \leq 14\}$, panel (c) for $\{P \in \mathbb{N} \mid 15 \leq P \leq 23\}$, and panel (d) for $\{P \in \mathbb{N} \mid 24 \leq P \leq 50\}$.

For up to seven periods of interest rate peg, the QE programme leads to a comparatively modest increase in inflation. For a duration of eight quarters, the responses of inflation increases dramatically. If we increase the duration of pegged rates further, i.e., $9 \leq P \leq 14$ (see panel (b)), inflation reverses its sign, that is, it decreases after the inception of QE. If we further increase the duration of the interest rate peg beyond 14 quarters (see panel (c)), the sign of the inflation response switches once again, predicting an expansionary effect until a duration of 23 periods. For durations of the peg from 24 to 50 periods, inflation responses turn negative again. Thus, the model responses oscillate with the duration of the interest rate peg.

To gain some intuition why the simulations oscillate with the duration of the peg, we look at the forward solution of the linearized model, for which we provide the derivation in Appendix C. The solution of the forward-looking (explosive) variables of the system of equilibrium conditions, $w_{2,t}$, is essentially a function of future disturbances ε_{t+n}:

$$w_{2,t} = -E_t\left\{\sum_{n=1}^{\infty} J^{n-1}\Omega_{22}^{-1}Q_2\Phi\varepsilon_{t+n}\right\}. \tag{3}$$

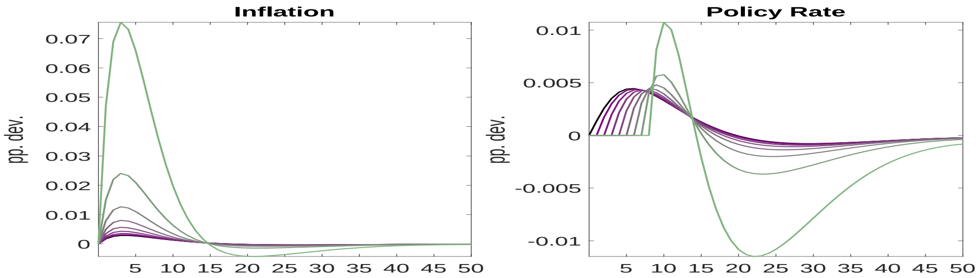
Under perfect foresight, the temporary interest rate peg is perfectly known to the agents in the economy, such that $E_t\left[\varepsilon_{t+n}\right]$ (recall that ε^{TR} is the only shock in our simulations) will be nonzero for the time period the central bank actually fixes the policy rate (i.e., $P > 0$) and

[7] A detailed description of the solution method we implement can be found in Adjemian and Juillard (2014).

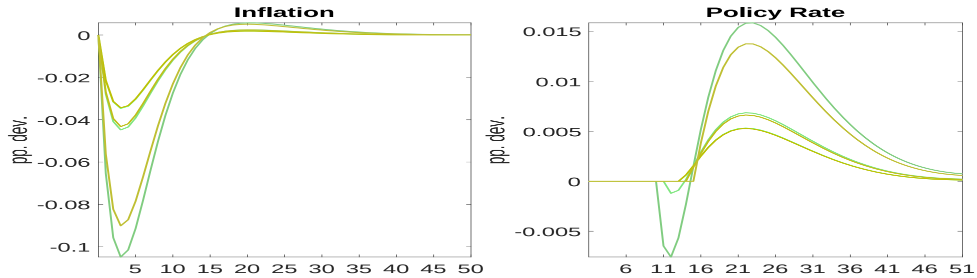
[8] One could implement the transient interest rate peg via a non-differentiable function (i.e., a min- or max-operator). However, this would render the peg endogenous with regard to its duration. Implementing the peg via the dummy approach allows us to set the duration of the peg in a completely exogenous way.

Figure 3: Simulation results of a QE shock in combination with an interest rate peg of variable duration under perfect foresight

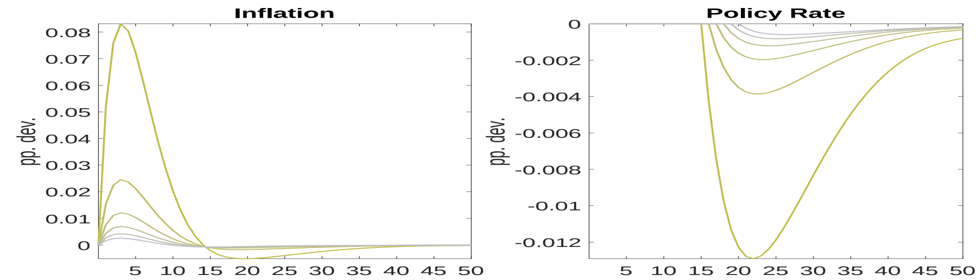
(a) Duration of interest rate peg: 0 to 8 periods



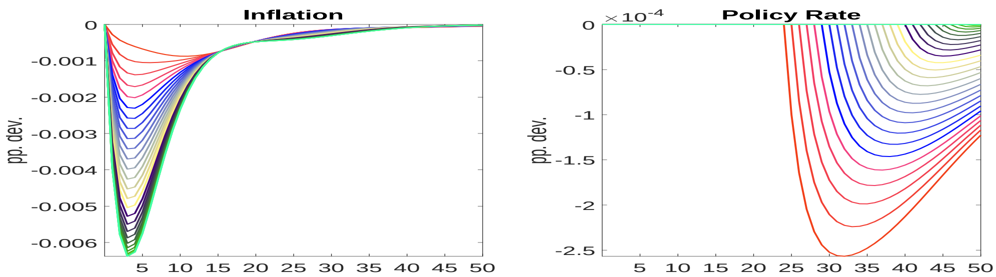
(b) Duration of interest rate peg: 9 to 14 periods



(c) Duration of interest rate peg: 15 to 23 periods



(d) Duration of interest rate peg: 24 to 50 periods



Note: The figure shows perfect foresight simulations for inflation and the short-term interest rate in response to a QE shock in combination with an interest rate peg of duration P. In period zero, the system is in the steady-state. Panel (a) shows results for $0 \leq P \leq 8$, panel (b) shows results for $9 \leq P \leq 14$, panel (c) shows results for $15 \leq P \leq 23$, and panel (d) shows results for $24 \leq P \leq 50$. The vertical axis shows percentage point deviations from steady state.

zero afterwards.[9] If some of the diagonal elements of J, which contains the unstable generalised eigenvalues of the system, turn out to be complex, they can be written in polar form. Let z_{jj} denote these complex diagonal elements, so that we can write $z_{jj} = a + bi$ or in polar form $z_{jj} = r(\cos\phi + i\sin\phi)$.[10] If now – because of known nonzero future ε_{t+n} – also powers of J enter the solution for $w_{2,t}$, we can write (by de Moivre's formula):

$$z_{jj}^k = r^k(\cos k\phi + i\sin k\phi), \quad \text{for } k = 0, ..., P-1. \tag{4}$$

The forward solution of the system, thus, involves trigonometric functions, which depend on the length P of a given interest rate peg. The longer the central bank keeps the policy rate fixed (i.e., the bigger P is), the farther we 'move' along the trigonometric functions contained on the diagonal elements of matrix J. As a consequence, with an increasing duration of pegged policy rates, the simulations presented in Figure 3 first approach an asymptote (i.e., the effect of an additional period of pegged policy rates grows exponentially) and afterwards the simulations switch their sign before they reach another asymptote and switch their sign again, and so on. The role of complex eigenvalues for the occurrence of the reversal puzzle has been made clear by Carlstrom et al. (2015).[11] Our exposition of the forward looking part of the system of equilibrium conditions' solution is meant to clarify the role of the degree of anticipation of the peg for the reversal puzzle.

The results presented thus far are robust to different parameterisations of the model. Appendix D provides an extensive grid search over the model's structural parameters and shows that the reversal puzzle is a very tenacious problem when the agents perfectly anticipate the peg.

3.2 No anticipation

Due to the presence of complex-valued eigenvalues, the model's dynamics switch sign, depending on the duration of a temporary interest rate peg. However, the complex eigenvalues on the main diagonal of the matrix J only imply a sign switch in the model simulations if the agents actually anticipate the interest rate peg (i.e., $E_t[\varepsilon_{t+n}] \neq 0$). Accordingly, the reversal puzzle should disappear when agents do not anticipate the interest rate peg at all. In such a scenario,

[9]The definitions of Ω_{22}^{-1} and Q_2 can be found in Appendix C.

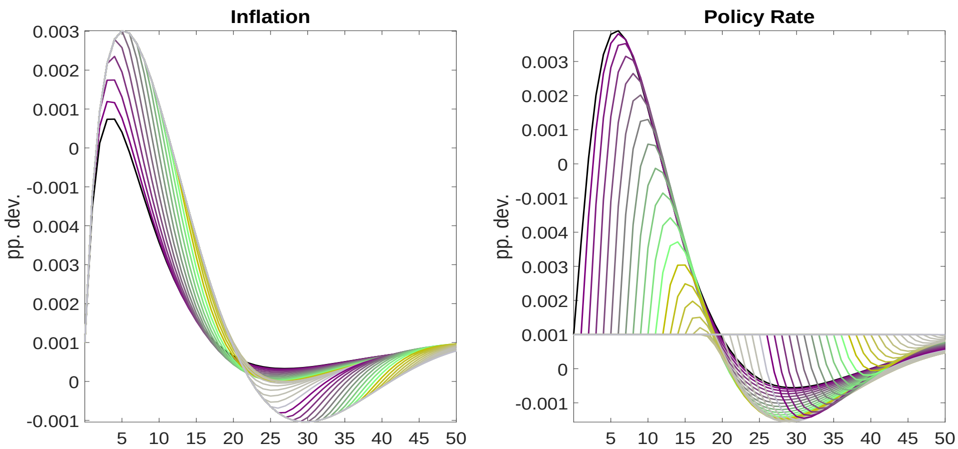
[10]a describes the real part of a complex eigenvalue and bi describes the imaginary part. While a and b are real numbers describing a pair of numerical Cartesian coordinates, r and ϕ denote the corresponding polar coordinates (i.e., distance and angle).

[11]While the reversal completely vanishes if one shuts down inflation indexation in the context of a small New Keynesian model, as shown by Carlstrom et al. (2015), this should not be expected for the medium-sized model presented here. It contains several other endogenous state variables (like capital, wages, net worth, etc.), that can give rise to complex-valued eigenvalues and, thus, sign switches in the model's simulations. However, it should be noted that, even under perfect foresight, the mere existence of complex eigenvalues is only a necessary but not a sufficient condition for the occurrence of the reversal puzzle. For example, the model by Erceg et al. (2000) augmented with price indexation and habit persistence in consumption shows no reversal puzzle even for very high degrees of indexation and consumption habit that imply complex eigenvalues in the solution of the model.

agents are surprised each period that the interest rate is still kept constant.

To implement this scenario, we solve and simulate the model with the extended path method, a procedure that has recently also been employed by Adjemian and Juillard (2013), Arias, Erceg and Trabandt (2016), and Christiano, Eichenbaum and Trabandt (2015). In contrast to perfect foresight, the paths for the endogenous variables are now computed by running a deterministic simulation for each period of the simulation horizon with the previous period as an initial condition for the next period and the steady state as terminal condition. In each period, agents now expect that the exogenous shocks will be zero for all future periods, i.e., they assume $E_t\left(\varepsilon_{t+n}\right) = 0$. Thus, agents do not anticipate at all that monetary policy pegs the interest rate for an extended period of time P. Figure 4 presents the time paths for inflation and the policy rate for $0 \leq P \leq 50$.

Figure 4: Simulation results of a QE shock in combination with an unanticipated interest rate peg of variable duration



Note: The figure shows extended path simulations for inflation and the short-term interest rate in response to a QE shock in combination with an interest rate peg of duration P, where $0 \leq P \leq 50$. In period zero, the system is in the steady-state. The vertical axis shows percentage deviations from steady state.

In the absence of anticipation, the reversal puzzle is absent. The initial response of inflation is always positive, irrespective of the duration of the peg. Consider once again equation (3), which we show here again for convenience:

$$w_{2,t} = -E_t \left\{ \sum_{n=1}^{\infty} J^{n-1} \Omega_{22}^{-1} Q_2 \Phi \varepsilon_{t+n} \right\}. \tag{5}$$

Recall that we solve the model in each period of the entire simulation horizon under the assumption that $\varepsilon_{t+n} = 0$ for all $n > 0$. Thus, now the solution for $w_{2,t}$ does not depend anymore on powers of the matrix J. As a consequence, the simulated time paths of the model will not 'move' along the trigonometric functions resulting from the complex elements on the main diagonal of matrix J. Thus, the explosive complex eigenvalues cannot induce explosive or cyclical effects in the solution of $w_{2,t}$. The model-implied dynamics following a QE shock together with the implementation of a temporary interest rate peg therefore deliver orthodox results. Thus, the mere occurrence of complex eigenvalues in the solution of the model do not necessarily imply appearance of reversals. Without a sufficient degree of anticipation, reversals cannot occur.

3.3 Imperfect anticipation

The two polar cases in which agents either perfectly or not anticipate an announced interest rate peg are arguably unrealistic. To analyse a more realistic scenario in which agents imperfectly anticipate an interest rate peg, we turn to a stochastic scenario. We implement imperfect anticipation by using a Markov-switching approach along the lines of Maih (2015) in which agents attach non-zero transition probabilities to entering and exiting a regime of pegged interest rates. The occurrence and the duration of each single episode of pegged interest rates are then stochastic but – as the agents are aware of the transition probabilities – the frequency and the average duration of an interest rate peg are known. As a higher frequency implies that the agents consider an interest rate peg more likely, variations in the peg frequency can be interpreted as variations in the degree of anticipation in the stochastic scenario. Given a sufficient degree of anticipation, reversals can occur depending on the average duration of the peg.

In our Markov-switching framework there are two different regimes regarding the policy rule. Regime 1 describes an economy in which the central bank follows the Taylor-type interest rate rule as specified in Section 2, i.e., a regime without an interest rate peg. Regime 2 is characterized by a central bank that does not respond to economic developments anymore, it thus pegs the interest rate (similar scenarios have been analysed by, e.g. Bianchi and Melosi, 2017; Chen, 2017). The regime switching policy rule is written in the following form:

$$R_t = (R_{t-1})^{\rho(S_t)} \left(R_{ss} \Pi_t^{\tau_\Pi(S_t)} \left(\frac{Y_t}{Y_{t-1}} \right)^{\tau_y(S_t)} \right)^{1-\rho(S_t)}, \tag{6}$$

where all parameters are functions of the prevailing regime S_t. $S_t = 1$ denotes the regime in which the central bank reacts to economic developments according to the Taylor-type rule and $S_t = 2$ denotes the regime in which the central bank pegs the interest rate. Correspondingly, in regime $S_t = 1$, the interest rate rule's coefficients are $\rho(S_t = 1) = 0.7409$, $\tau_\Pi(S_t = 1) = 1.5912$, and $\tau_y(S_t = 1) = 0.5725$ according to the (single regime) estimation described in Appendix B, whereas in regime $S_t = 2$ the coefficients take on the values $\rho(S_t = 2) = 0$, $\tau_\Pi(S_t = 2) = 0$, and $\tau_y(S_t = 2) = 0$.

We analyse the response of inflation to a QE shock for different combinations of the average duration and the frequency of the peg, which are determined by the transition probabilities of the model. The transition probability for going from regime 1, where the central bank follows the Taylor-type rule, to regime 2, where the central bank pegs the interest rate, is denoted

Table 2: Mapping from transition probabilities to average duration and frequency of regime 2 (interest rate peg)

	$\mathcal{F}_2 = 10\%$		$\mathcal{F}_2 = 15\%$		$\mathcal{F}_2 = 20\%$	
$\mathcal{AD}_2$	p^{12}	p^{21}	p^{12}	p^{21}	p^{12}	p^{21}
4 qrt.	2.78%	25.0%	4.41%	25.0%	6.25%	25.0%
11.5 qrt.	0.97%	8.70%	1.53%	8.70%	2.17%	8.70%
19 qrt.	0.58%	5.26%	0.93%	5.26%	1.32%	5.26%
37 qrt.	0.30%	2.70%	0.48%	2.70%	0.68%	2.70%
50 qrt.	0.22%	2.00%	0.35%	2.00%	0.50%	2.00%
	$\mathcal{F}_2 = 30\%$		$\mathcal{F}_2 = 40\%$		$\mathcal{F}_2 = 50\%$	
$\mathcal{AD}_2$	p^{12}	p^{21}	p^{12}	p^{21}	p^{12}	p^{21}
4 qrt.	10.71%	25.0%	16.67%	25.0%	25.0%	25.0%
11.5 qrt.	3.73%	8.70%	5.80%	8.70%	8.70%	8.70%
19 qrt.	2.26%	5.26%	3.51%	5.26%	5.26%	5.26%
37 qrt.	1.16%	2.70%	1.80%	2.70%	2.70%	2.70%
50 qrt.	0.86%	2.00%	1.33%	2.00%	2.00%	2.00%

Note: p^{12} denotes the transition probability for going from regime 1 to regime 2; p^{21} denotes the transition probability for going from regime 2 to regime 1; $\mathcal{AD}_2$ denotes the average duration of regime 2; $\mathcal{F}_2$ denotes the frequency of regime 2. Regime 2 is the regime where the central bank pegs the interest rate.

by p^{12}. The transition probability for going from regime 2 to regime 1 is denoted by p^{21}. The average duration of a peg episode, $\mathcal{AD}_2$, can be pinned down by a suitable choice of p^{21}: $\mathcal{AD}_2 = \frac{1}{p^{21}}$. Given p^{21}, the frequency of the peg regime, $\mathcal{F}_2$ can then be pinned down by p^{12}: $\mathcal{F}_2 = \frac{\mathcal{AD}_2}{\mathcal{AD}_1 + \mathcal{AD}_2}$, where $\mathcal{AD}_1 = \frac{1}{p^{12}}$ is the average duration of regime 1.

In the perfect foresight scenario of Section 3.1, the different durations of an interest rate peg could be divided into four different sub-ranges of durations according to whether a reversal occurs or not (see Figure 3): 0-8 periods (no reversal), 9-14 periods (reversal), 15-23 periods (no reversal), 24-50 periods (reversal). For expositional reasons, in the stochastic scenario we consider average durations of an interest rate peg that are in the middle of these sub-ranges, i.e., average durations of 4, 11.5, 19, and 37 quarters. To cover an even wider range of average durations in the stochastic scenario, we also consider 50 quarters. For each of these average durations, we analyse the inflation response for different frequencies. An empirically relevant frequency of an interest rate peg is based on the post-WWII zero lower bound frequency in the US of around 10% (7 years at the zero lower bound in 73 years). We additionally consider frequencies of 15%, 20%, 30% and 50% to cover a wide range of different frequencies and hence degrees of anticipation. The transition probabilities that imply the various combinations of average duration and frequency of the peg are shown in Table 2.

For each calibration of the transition probability matrix, we solve and simulate the regime-switching model and calculate generalized impulse response functions (GIRFs) of inflation for a given QE shock.[12] The results are presented in the left column of Figure 5. The different rows show the GIRFs for different frequencies of the interest rate peg. The top graph of the left column shows the response of inflation to the QE shock for the empirically relevant peg frequency of 10% for the different average durations of the peg. For all average durations of the peg, inflation increases after a QE shock, i.e., no reversals occur. The same is true for a frequency of 15% (second row, left column), and even for a frequency of the peg of 20% (third row, left column), reversals do not occur irrespective of the average peg durations. Thus, even for frequencies of the peg as large as double the empirically relevant value, reversals are absent for the large range of average peg durations we consider. Intuitively, if agents consider a peg episode to be very unlikely, the degree of anticipation of the peg will be low. The absence of a strong anticipation effect results in the absence of the reversal puzzle.

Only if the peg frequency is increased to large and arguably implausibly high values, we start to see reversals in the stochastic scenario, depending on the average peg duration. For a frequency of 30% (fourth row, left column in Figure 5), a reversal occurs for an average duration of four quarters. For a frequency of 40% (fifth row, left column), average durations of the peg of 4, 11.5, and 19 quarters imply a reversal, and all average durations of the peg imply a reversal for a frequency of 50% (sixth row, left column).

Compared to the case of perfect anticipation, the reversal pattern across the different average durations seems to be different in the case of imperfect anticipation.[13] Specifically, given a frequency of 30% and 40%, a reversal occurs for the lowest average peg duration.[14] Additionally, reversals seem to be "grouped" in the sense that they occur for adjacent average durations. Finally, for the frequency of 50%, all average durations display a reversal. Since the GIRFs incorporate simulations of the model over both the regime with and without interest rate peg, we look at the regime-specific IRFs to better understand these patterns.

The regime-specific IRFs of regime 2 (peg) are shown in the right column of Figure 5. Given the peg regime, low frequencies (10%, 15%, and 20%, top three rows of right column) imply a sufficient degree of anticipation for reversals to be possible. Specifically, given that the economy is in a peg, higher average durations lead to a reversal and lower average durations do not. If the frequency is increased to 30%, 40% or 50%, all average durations imply a reversal.

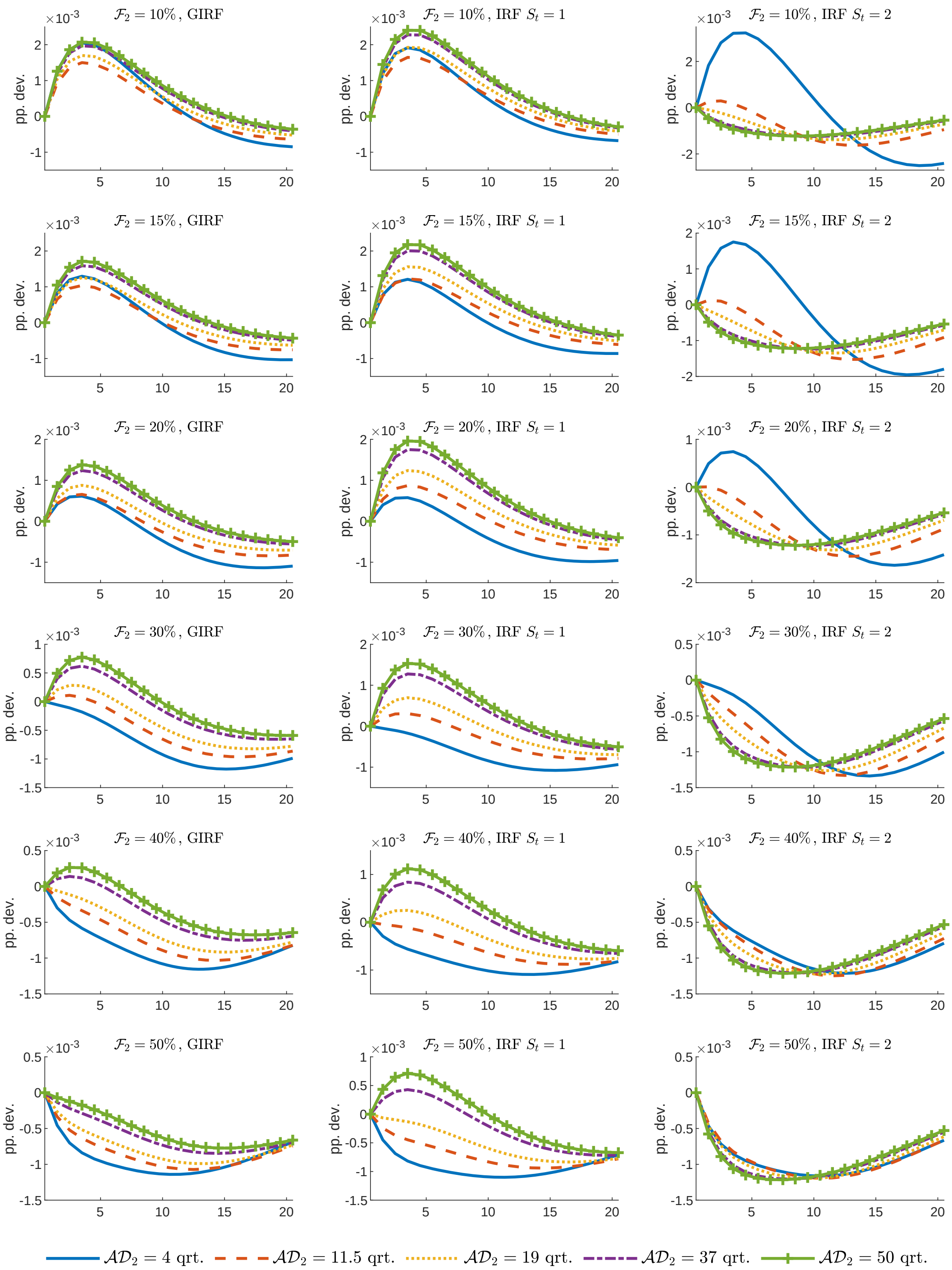
In contrast, as shown in the middle column of Figure 5, given regime 1 (no peg), low frequencies do not imply a sufficient degree of anticipation for reversals to be possible, and hence no

[12] We use the RISE toolbox, described in Maih (2015), to implement the different scenarios for the regime-switching model. The toolbox can be downloaded from `https://github.com/jmaih/RISE_toolbox`. For each of the calibrations outlined in Table 2, we first check the mean square stability condition. We compute the generalized impulse responses based on 50,000 draws.

[13] Recall that the average durations in the imperfect anticipation case were chosen so as to represent sub-ranges of durations (according to whether a reversal occurs or not) in the perfect anticipation case. The comparison is hence between the reversal pattern of the average durations in the imperfect anticipation case that represent the respective sub-ranges of durations in the perfect anticipation case, and the reversal pattern of the sub-ranges of durations in the perfect anticipation case.

[14] In the case of perfect anticipation, the lowest sub-range of durations did not show a reversal.

Figure 5: Impulse responses for the Markov-switching model



Note: The figure shows generalized and regime-specific impulse responses of inflation to a QE shock for five different scenarios. $\mathcal{F}_2$ denotes the frequency of regime 2 (peg regime); $\mathcal{AD}_2$ denotes the average duration of the interest rate peg. GIRF abbreviates generalized impulse response function; $IRF_{S_t=1}$ denotes regime-specific impulse response function for regime 1; $IRF_{S_t=2}$ denotes regime-specific impulse response function for regime 2. The vertical axis shows percentage point deviations from steady state.

average duration displays a reversal. Only frequencies of 30% or beyond are sufficient for the possibility of a reversal, but now, lower average durations lead to a reversal but not higher average durations.

Hence, as in the case of perfect anticipation, the qualitative response of inflation in the case of imperfect anticipation can alternate across the average durations we consider. However, the regime-switching setup implies a more complex pattern of qualitative responses: it depends on the regime, the peg frequency and the average peg duration. How this pattern is reflected in the GIRFs, the ultimately relevant statistics of interest, depends on the frequency of regimes: A relatively high frequency of regime 1 (regime 2) implies a high share of regime 1 (regime 2) in the GIRF simulations and hence a high resemblance of the GIRFs with the regime-specific IRFs for regime 1 (regime 2).

4 Conclusion

The reversal puzzle describes a counterintuitive contraction of inflation in response to a supposedly expansionary interest rate peg in New Keynesian models. In this study, we show that the reversal puzzle only occurs when agents exhibit a very high and arguably implausible degree of anticipation. If agents have perfect foresight and, thus, fully anticipate an interest rate peg, reversals are a robust phenomenon that occur for certain durations of the interest rate peg. If the agents do not anticipate the interest rate peg at all, reversals are absent. If agents imperfectly anticipate an interest rate peg in a Markov-switching framework, the occurrence and the duration of a single peg episode are stochastic, and the degree of anticipation depends on the frequency of the peg. For empirically relevant peg frequencies, reversals are absent for the large range of average durations of the peg we consider. Only for extreme and arguably implausible assumptions about the frequency and hence the degree of anticipation of the peg, reversals occur.

Our results bear important implications for the analysis of policy scenarios. Due to the occurrence of the zero lower bound and forward guidance in recent times, there is a need to account for those features in model simulations. Insofar as these features are addressed in the form of an interest rate peg, policy evaluations might face the problem of reversals in model outcomes. Our results show that a Markov-switching approach is a promising tool to circumvent this pathology and provide qualitatively plausible model outcomes.

References

Adjemian, S., Bastani, H., Karamé, F., Juillard, M., Maih, J., Mihoubi, F., Perendia, G., Pfeifer, J., Ratto, M. and Villemot, S. (2018). Dynare: Reference Manual, Version 4.5.4, *Dynare Working Papers 1*, CEPREMAP.

Adjemian, S. and Juillard, M. (2013). Stochastic Extended Path Approach. unpublished manuscript.

Adjemian, S. and Juillard, M. (2014). Assessing long run risk in a DSGE model under ZLB with the stochastic extended path approach. unpublished manuscript.

Arias, J. E., Erceg, C. and Trabandt, M. (2016). The Macroeconomic Risks of Undesirably Low Inflation, *European Economic Review* **88**: 88–107.

Bianchi, F. and Melosi, L. (2017). Escaping the Great Recession, *American Economic Review* **107**(4): 1030–1058.

Binning, A. and Maih, J. (2017). Modelling Occasionally Binding Constraints Using Regime-Switching, *Technical report*, Norges Bank.

Brooks, S. P. and Gelman, A. (1998). General Methods for Monitoring Convergence of Iterative Simulations, *Journal of Computational and Graphical Statistics* **7**(4): 434–455.

Bundick, B. and Smith, A. L. (2020). Should We Be Puzzled by Forward Guidance?, *Technical Report 20-01*, Federal Reserve Bank. http://doi.org/10.18651/RWP2020-01.

Carlstrom, C. T., Fuerst, T. S. and Paustian, M. (2015). Inflation and Output in New Keynesian Models with a Transient Interest Rate Peg, *Journal of Monetary Economics* **76**: 230–243.

Carlstrom, C. T., Fuerst, T. S. and Paustian, M. (2017). Targeting Long Rates in a Model with Segmented Markets, *American Economic Journal: Macroeconomics* **9**(1): 205–242.

Chen, H. (2017). The effects of the near-zero interest rate policy in a regime-switching dynamic stochastic general equilibrium model, *Journal of Monetary Economics* **90**: 176–192.

Chen, H., Cúrdia, V. and Ferrero, A. (2012). The Macroeconomic Effects of Large-Scale Asset Purchase Programmes, *Economic Journal* **122**: F289–F315.

Christiano, L., Eichenbaum, M. and Trabandt, M. (2015). Understanding the Great Recession, *American Economic Journal: Macroeconomics* **7**(1): 110–167.

Christiano, L. J., Eichenbaum, M. and Evans, C. L. (2005). Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy, *Journal of Political Economy* **113**(1): 1–45.

Christiano, L. J., Motto, R. and Rostagno, M. (2010). Financial Factors in Economic Fluctuations, *Technical Report 1192*, ECB Working Paper.

De Graeve, F., Ilbas, P. and Wouters, R. (2014). Forward Guidance and Long Term Interest Rates: Inspecting the Mechanism, *Technical Report 292*, Sveriges Riksbank Working Paper Series.

de Groot, O. and Mazelis, F. (2020). Mitigating the forward guidance puzzle: inattention, credibility, finite planning horizons and learning, *Working Paper 2426*, ECB.

Dordal-i-Carreras, M., Coibion, O., Gorodnichenko, Y. and Wieland, J. (2016). Infrequent but Long-Lived Zero Lower Bound Episodes and the Optimal Rate of Inflation, *Annual Review of Economics* .

Eggertsson, G. and Woodford, M. (2003). The Zero Bound on Interest Rates and Optimal Monetary Policy, *Brookings Papers on Economic Activity* **34**(1): 139–233.

Erceg, C. J., Henderson, D. W. and Levin, A. T. (2000). Optimal Monetary Policy with Staggered Wage and Price Contracts, *Journal of Monetary Economics* **46**(2): 281–313.

Gerke, R., Giesen, S., Kienzler, D. and Tenhofen, J. (2017). Interest-rate üegs, central bank asset purchases and the reversal puzzle, *Discussion Paper 21/2017*, Deutsche Bundesbank.

Gertler, M. and Karadi, P. (2013). QE1 vs. 2 vs. 3...: A Framework for Analyzing Large-Scale Asset Purchases as a Monetary Policy Tool, *International Journal of Central Banking* **9**(S1): 5–53.

Kiley, M. T. (2016). Policy Paradoxes in the New Keynesian Model, *Review of Economic Dynamics* **21**: 1–15.

Lindé, J., Smets, F. and Wouters, R. (2016). Challenges for Macro Models Used at Central Banks, *Research Paper Series 147*, Sveriges Riksbank.

Maih, J. (2015). Efficient perturbation methods for solving regime-switching DSGE models, *Technical report*, Norges Bank.

Maliar, L. and Taylor, J. B. (2019). Forward Guidance: Is It Useful Away from the Lower Bound?, *Technical Report 26053*, NBER Working Paper.

McKay, A., Nakamura, E. and Steinsson, J. (2016). The Discounted Euler Equation: A Note, *Working Paper 22129*, NBER.

Sims, C. A. (2001). Solving Linear Rational Expectations Models, *Computational Economics* **20**: 1–20.

A Model derivation

In our analysis we employ the model of Carlstrom et al. (2017).

A.1 Households and bond market structure

A.1.1 Households' intertemporal consumption decision

Households maximise their intertemporal utility:

$$E_t \sum_{s=0}^{\infty} \beta^s b_{t+s} \left\{ \ln \left(C_{t+s} - hC_{t+s-1} \right) - B \frac{H_{t+s}^{1+\eta}(j)}{1+\eta} \right\},$$

where C_t is consumption, h is habit formation, $H_t(j)$ is the individual labour input from household j, and b_t is a shock to the discount factor. Lifetime utility would evaluate to:

$$V_t^h = b_t \left\{ \ln \left(C_t - hC_{t-1} \right) - D_t^w B \frac{H_t^{1+\eta}}{1+\eta} \right\} + \beta E_t V_{t+1}^h$$

The law of motion for capital is:

$$K_t \leq \left(1 - \delta\right) K_{t-1} + I_t$$

Based on the households' nominal liability,

$$F_{t-1} = CI_{t-1} + \kappa CI_{t-2} + \kappa^2 CI_{t-3} + ...,$$

one can show that $CI_t = (F_t - \kappa F_{t-1})$, where CI_t is the number of bonds newly issued, and F_t is the households' nominal liability on new issues. New investments must be financed by issuing sufficient long term investment bonds which are purchased by the FI. Perpetual bonds are used with cash flows of $1, \kappa, \kappa^2$, etc.

The loan-in-advance constraint can be written as:

$$P_t^k I_t \leq \frac{Q_t \left(F_t - \kappa F_{t-1} \right)}{P_t} \left(= \frac{Q_t CI_t}{P_t} \right),$$

where Q_t is the time-t price of a new issue, P_t is the price level and P_t^k is the real price of capital. Moreover, the usual budget constraint is given by:

$$\overbrace{C_t + \underbrace{\frac{D_t}{P_t}}_{\substack{\text{HH real}\\ \text{deposits}}} + P_t^k I_t + \underbrace{\frac{F_{t-1}}{P_t}}_{\substack{\text{HH real liability}\\ \text{on past issues}}}}^{\text{Expenditure Side}} \leq W_t H_t + R_t^k K_t - T_t + \frac{D_{t-1}}{P_t} R_{t-1}^d + \underbrace{\frac{Q_t \left(F_t - \kappa F_{t-1}\right)}{P_t}}_{\substack{\text{HH newly issued real}\\ \text{investment bonds}}} + div_t$$

A.1.2 Households' Lagrangian

The corresponding Lagrangian maximising household utility is:

$$\mathcal{L} = E_t \sum_{s=0}^{\infty} \beta^s \left[\begin{array}{l} b_{t+s} \left\{ \ln\left(C_{t+s} - hC_{t+s-1}\right) - B \frac{H_{t+s}^{1+\eta}(j)}{1+\eta} \right\} \\ -\Lambda_{t+s} \left(\begin{array}{l} C_{t+s} + \frac{D_{t+s}}{P_{t+s}} + P_{t+s}^k I_{t+s} + \frac{F_{t+s-1}}{P_{t+s}} - W_{t+s} H_{t+s} - R_{t+s}^k K_{t+s} + T_{t+s} \\ -\frac{D_{t+s-1}}{P_{t+s}} R_{t+s-1}^d - \frac{Q_{t+s}(F_{t+s} - \kappa F_{t+s-1})}{P_{t+s}} - div_{t+s} \end{array} \right) \\ -\Lambda_{t+s}^K \left(K_{t+s} - (1-\delta) K_{t+s-1} - I_{t+s}\right) \\ \underbrace{-\vartheta_{t+s} \left(P_{t+s}^k I_{t+s} - \frac{Q_{t+s}\left(F_{t+s} - \kappa F_{t+s-1}\right)}{P_{t+s}} \right)}_{\text{Loan in advance constraint}} \end{array} \right]$$

The first-order conditions evaluate to:

$$\frac{\partial \mathcal{L}}{\partial C_t} : \Lambda_t = \frac{b_t}{C_t - hC_{t-1}} - E_t \frac{\beta h b_{t+1}}{C_{t+1} - hC_t}$$

$$\frac{\partial \mathcal{L}}{\partial D_t} : \Lambda_t = E_t \beta \frac{\Lambda_{t+1}}{\Pi_{t+1}} R_t^d \quad \text{with} \quad \Pi_{t+1} = \frac{P_{t+1}}{P_t}$$

$$\frac{\partial \mathcal{L}}{\partial I_t} : \Lambda_t^K = \vartheta_t P_t^k + \Lambda_t P_t^k = \left(\vartheta_t + \Lambda_t\right) P_t^k = M_t \Lambda_t P_t^k$$

$$\frac{\partial \mathcal{L}}{\partial F_t} : \Lambda_t M_t Q_t = E_t \frac{\beta \Lambda_{t+1} \left(1 + \kappa Q_{t+1} \mathrm{M}_{t+1}\right)}{\Pi_{t+1}},$$

with $\mathrm{M}_t = 1 + \frac{\vartheta_t}{\Lambda_t}$ or $\Lambda_t \mathrm{M}_t = \Lambda_t + \vartheta_t$.

$$\frac{\partial \mathcal{L}}{\partial K_t} : \Lambda_t M_t P_t^k = E_t \beta \Lambda_{t+1} \left[R_{t+1}^k + M_{t+1} P_{t+1}^k (1-\delta) \right],$$

A.1.3 Financial intermediaries

The FI choose dividends div_t and their net worth N_t to maximise the value function:

$$V_t = E_t \sum_{s=0}^{\infty} (\beta\zeta)^s \Lambda_{t+s} div_{t+s}$$

where ζ is a parameter for additional impatience using the basic household kernel for discounting.

This maximisation is subject to the budget constraint which represents the law of motion for net worth, with

$$R^L_{t+1} \equiv \left(\frac{\overbrace{1}^{Coupon} + \overbrace{\kappa Q_{t+1}}^{\text{t + 1 Principal/face value of issues from t}}}{\underbrace{Q_t}_{\text{Market Price}}} \right)$$

$$div_t + N_t \underbrace{[1 + f(N_t)]}_{\text{Diminishing net worth by adjustment costs}} \leq \underbrace{\frac{P_{t-1}}{P_t} \left[\underbrace{\left(R^L_t - R^d_{t-1}\right) L_{t-1}}_{\text{Earnings from leveraged net worth: lending - deposits}} + \underbrace{R^d_{t-1}}_{\text{For own net worth no interest on deposit has to be paid}} \right] N_{t-1}}_{\text{Profit FI(Change in net worth)}}$$

The net worth adjustment costs which limit the ability of the FI to adjust their portfolio deviating from its steady state are:

$$f(N_t) \equiv \frac{\psi_n}{2} \left(\frac{N_t - N_{ss}}{N_{ss}} \right)^2$$

The according Lagrangian becomes:

$$\mathcal{L} = E_t \sum_{s=0}^{\infty} (\beta\zeta)^s \left[\Lambda_{t+s} div_{t+s} - \Lambda^N_{t+s} \left\{ \begin{array}{l} div_{t+s} + N_{t+s} [1 + f(N_{t+s})] - \\ \frac{P_{t+s-1}}{P_{t+s}} \left[\left(R^L_{t+s} - R^d_{t+s-1}\right) L_{t+s-1} + R^d_{t+s-1} \right] N_{t+s-1} \end{array} \right\} \right]$$

This yields the following first-order conditions:

$$\frac{\partial \mathcal{L}}{\partial div_t} : \Lambda_{t+s} = \Lambda^N_{t+s}$$

$$\frac{\partial \mathcal{L}}{\partial N_t} : \Lambda_t \left[1 + f\left(N_t\right) + N_t f'\left(N_t\right)\right] = E_t \Lambda_{t+1} \beta \zeta \frac{P_t}{P_{t+1}} \left[\left(R^L_{t+1} - R^d_t\right) L_t + R^d_t\right]$$

The FIs are subject to a simple hold-up problem which limits their ability to attract deposits. When they choose to default they can seize a fraction μ_t from the household deposits. The incentive constraint for the FI not to default, because their income is greater than the assets they can keep in default, is:

$$\underbrace{E_t V_{t+1}}_{\substack{\text{Expected}\\ \text{future income}}} \geq \underbrace{\mu_t L_t N_t}_{\substack{\text{Fraction of their}\\ \text{balance sheet}}} \underbrace{E_t \Lambda_{t+1} \frac{P_t}{P_{t+1}} R^L_{t+1}}_{\substack{\text{Times next periods consumption}\\ \text{value plus earnings from lending}}}$$

The model can be calibrated for it to be binding. By choosing the fraction of assets the FI can keep in case of default to be

$$\mu_t = \Phi_t \left[1 + \frac{1}{N_t} E_t \left(\frac{g_{t+1}}{X_{t+1}}\right)\right],$$

with Φ_t an exogenous stochastic process that represents exogenous changes in the financial friction. It follows an AR(1) process:

$$\Phi_t = \left(1 - \rho_\Phi\right) \Phi_{ss} + \rho_\Phi \Phi_{t-1} + \varepsilon_{\Phi,t}.$$

Choosing this fraction ensures that leverage is a function independent of net worth. Hence, the FIs take leverage as given and we can aggregate the firms as they are just scaled equivalents. g_t is a function of current and forecasted market spreads z_t independent of N_{t-1}. Confirming the leverage equation, it follows:

$$E_t \frac{P_t}{P_{t+1}} \Lambda_{t+1} \left[\left(\frac{R^L_{t+1}}{R^d_t} - 1\right) L_t + 1\right] = \Phi_t L_t E_t \Lambda_{t+1} \frac{P_t}{P_{t+1}} \frac{R^L_{t+1}}{R^d_t}$$

$$\Leftrightarrow L_t = \frac{E_t \frac{\Lambda_{t+1}}{\Pi_{t+1}}}{\left[E_t \frac{\Lambda_{t+1}}{\Pi_{t+1}} + \left(\Phi_t - 1\right) E_t \frac{\Lambda_{t+1}}{\Pi_{t+1}} \frac{R^L_{t+1}}{R^d_t}\right]}$$

Using the derivation

$$\frac{\partial L_t}{\partial R^L_{t+1}} = \frac{-\frac{(\Phi_t - 1)}{R^d_t}}{\left[1 + (\Phi_t - 1)\frac{R^L_{t+1}}{R^d_t}\right]^2} \geq 0 \quad \text{for} \quad \Phi_t < 1,$$

this can be simplified to

$$L_t = \frac{1}{\left[1 + (\Phi_t - 1) E_t \frac{R^L_{t+1}}{R^d_t}\right]}.$$

Regarding the balance sheet of the FI and its composition, leveraged net worth is divided into holdings of long term government bonds and investment bonds:

$$N_t L_t = \overline{B}_t + \overline{F}_t,$$

with $\overline{B}_t \equiv Q_t \frac{B_t}{P_t}$ and $\overline{F}_t \equiv Q^I_t \frac{F_t}{P_t}$.

The time-t asset value of current and past issues of investment is:

$$Q_t F_t = Q_t CI_t + \kappa Q_t \left[CI_{t-1} + \kappa CI_{t-2} + \kappa^2 CI_{t-3}\right],$$

where the time-t price of the perpetuity issued in t-1 is κQ_t.

A.1.4 Term premium and price of capital mark-up

Rewriting the log-linearised version of the households' first-order condition with respect to K_t yields:

$$\lambda_t + p^k_t + m_t = E_t \left\{\lambda_{t+1} + [1 - \beta(1-\delta)] r^k_{t+1} + \beta(1-\delta)\left(p^k_{t+1} + m_{t+1}\right)\right\}$$

From the log-linearised version of the households first-order condition with respect to D_t, we know that $E_t\lambda_{t+1} - \lambda_t = E_t\pi_{t+1} - r_t$, and hence

$$p^k_t + m_t = E_t \left\{[1 - \beta(1-\delta)] r^k_{t+1} - (r_t - \pi_{t+1}) + \beta(1-\delta)\left(p^k_{t+1} + m_{t+1}\right)\right\}.$$

Iterative substitution then yields the mark-up character of m_t on the price of capital p^k_t:

$$p^k_t + m_t = E_t \sum_{j=0}^{\infty} [\beta(1-\delta)]^j \left\{[1 - \beta(1-\delta)] r^k_{t+j+1} - (r_{t+j} - \pi_{t+j+1})\right\}$$

Similarly, one can show that iterative substitution can also be applied to the log-linearised form of the households first-order condition with respect to F_t, which then can be written as:

$$m_t = E_t \sum_{j=0}^{\infty} [\beta\kappa]^j \{\beta\kappa q_{t+j+1} - q_{t+j} - r_{t+j}\}.$$

And since $r^L_{t+1} = \frac{\kappa q_{t+1}}{R^L} - q_t = \frac{\beta\zeta}{\Pi}\kappa q_{t+1} - q_t \approx \beta\kappa q_{t+1} - q_t$, this can be written as the discounted sum of future loan to deposit spreads:

$$m_t \approx E_t \sum_{j=0}^{\infty} [\beta\kappa]^j \left\{r^L_{t+j+1} - r_{t+j}\right\} = E_t \sum_{j=0}^{\infty} [\beta\kappa]^j \Xi_{t+j}$$

$$\Xi_{t+j} \equiv \beta\kappa q^i_{t+j+1} - q^i_{t+j} - r_{t+j} \approx r^L_{t+j+1} - r_{t+j}$$

A.2 Labour agencies

Perfectly competitive labour agencies combine differentiated labour inputs into a homogenous labour composite H_t according to the technology:

$$H_t = \left[\int_0^1 H_t(j)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dj\right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}}$$

where $\varepsilon_w \geq 1$ is the elasticity of substitution between different varieties of labour. The labour agencies purchase labour $H_t(j)$ at a nominal wage $W_t(j)$. Profit maximisation (i.e., cost minimisation) leads to the following problem:

$$\min_{H_t(j)} \int_0^1 W_t(j) H_t(j) dj$$

subject to (at least obtaining a bundle H_t):

$$\left[\int_0^1 H_t(j)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dj\right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}} \geq H_t$$

The corresponding Lagrangian is:

$$\mathcal{L}= \int_0^1 W_t(j)H_t(j)dj - \psi_t \left\{ \left[\int_0^1 H_t(j)^{\frac{\varepsilon_w-1}{\varepsilon_w}} dj \right]^{\frac{\varepsilon_w}{\varepsilon_w-1}} - H_t \right\}$$

$$\frac{\partial \mathcal{L}}{\partial H_t(j)} : W_t(j) = \psi_t \left[\int_0^1 H_t(j)^{\frac{\varepsilon_w-1}{\varepsilon_w}} dj \right]^{\frac{1}{\varepsilon_w-1}} H_t(j)^{-\frac{1}{\varepsilon_w}}$$

$$\Leftrightarrow H_t(j) = \left(\frac{W_t(j)}{\psi_t} \right)^{-\varepsilon_w} H_t$$

Using the definition of H_t leads to:

$$H_t = \left[\int_0^1 \left(\left(\frac{W_t(j)}{\psi_t} \right)^{-\varepsilon_w} H_t \right)^{\frac{\varepsilon_w-1}{\varepsilon_w}} dj \right]^{\frac{\varepsilon_w}{\varepsilon_w-1}}$$

$$\Leftrightarrow 1 = \left(\frac{1}{\psi_t} \right)^{-\varepsilon_w} \left[\int_0^1 W_t(j)^{1-\varepsilon_w} dj \right]^{\frac{\varepsilon_w}{\varepsilon_w-1}}$$

$$\Leftrightarrow \psi_t = \left[\int_0^1 W_t(j)^{1-\varepsilon_w} dj \right]^{\frac{1}{1-\varepsilon_w}} \equiv W_t$$

Plugging this into the demand function results in:

$$H_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\varepsilon_w} H_t$$

A.2.1 Optimal wage

Households are monopolistic suppliers of differentiated labour inputs $H_t(j)$ and set wages on a staggered basis (à la Calvo). In each period, the probability of resetting the wage is $(1 - \theta_w)$, while with the complementary probability (θ_w) the wage is automatically increased following the indexing rule:

$$W_t(j) = \Pi_{t-1}^{\iota_w} W_{t-1}(j)$$

The problem for a household j who can reset its wage at time t is:

$$\max_{W_t(j)} E_t \sum_{s=0}^{\infty} \theta_w^s \beta^s \left\{ \underbrace{-B \frac{H_{t+s}(j)^{1+\psi}}{1+\psi}}_{\text{Disutiliy of labour at t + s}} \underbrace{b_{t+s}}_{\substack{\text{discount}\\ \text{factor shock}}} \underbrace{\lambda_{w,t+s}}_{\substack{\text{markup}\\ \text{factor}}} \underbrace{+\Lambda_{t+s} \underbrace{\frac{W_t(j)}{P_{t+s}} H_{t+s}(j)}_{\text{real wage income at t + s}}}_{\text{Utility consequence of this income}} \right\}$$

The maximisation problem follows as:

$$\max_{W_t(j)} \Omega_t = E_t \sum_{s=0}^{\infty} \theta_w^s \beta^s \left\{ \begin{array}{l} -\lambda_{w,t+s} b_{t+s} \frac{B}{1+\psi} \left(\left(\frac{W_t(j)\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota w}\right)}{W_{t+s}} \right)^{-\varepsilon_w} H_{t+s} \right)^{1+\psi} \\ +\Lambda_{t+s} \frac{W_t(j)\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota w}\right)}{P_{t+s}} \left(\frac{W_t(j)\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota w}\right)}{W_{t+s}} \right)^{-\varepsilon_w} H_{t+s} \end{array} \right\}$$

This can be rewritten in the following way:

$$\left\{ \begin{array}{l} -\lambda_{w,t+s} b_{t+s} \frac{B}{1+\psi} \left(\left(\frac{W_t(j)\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota w}\right)}{W_{t+s}} \right)^{-\varepsilon_w} H_{t+s} \right)^{1+\psi} \\ +\Lambda_{t+s} \frac{W_t(j)\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota w}\right)}{P_{t+s}} \left(\frac{W_t(j)\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota w}\right)}{W_{t+s}} \right)^{-\varepsilon_w} H_{t+s} \end{array} \right\}$$

$$= E_t \sum_{s=0}^{\infty} \theta_w^s \beta^s \left\{ \begin{array}{l} -\lambda_{w,t+s} b_{t+s} \frac{B}{1+\psi} W_t(j)^{-\varepsilon_w(1+\psi)} \left(\left(\frac{\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota w}\right)}{W_{t+s}} \right)^{-\varepsilon_w} H_{t+s} \right)^{1+\psi} \\ +\Lambda_{t+s} W_t(j)^{1-\varepsilon_w} \frac{\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota w}\right)^{1-\varepsilon_w}}{P_{t+s}} W_{t+s}^{\varepsilon_w} H_{t+s} \end{array} \right\}$$

$$\frac{\partial \Omega_t}{\partial W_t(j)} : E_t \sum_{s=0}^{\infty} \theta_w^s \beta^s \left\{ \Lambda_{t+s} (1-\varepsilon_w) W_t(j)^{-\varepsilon_w} \frac{1}{P_{t+s}} \left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_w} \right)^{1-\varepsilon_w} W_{t+s}^{\varepsilon_w} H_{t+s} \right\}$$

$$= E_t \sum_{s=0}^{\infty} \theta_w^s \beta^s \left\{ \lambda_{w,t+s} b_{t+s} B (-\varepsilon_w) W_t(j)^{-\varepsilon_w(1+\psi)-1} \left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_w} \right)^{-\varepsilon_w(1+\psi)} W_{t+s}^{\varepsilon_w(1+\psi)} H_{t+s}^{1+\psi} \right\}$$

$$\Leftrightarrow W_t(j)^{1+\varepsilon_w\psi} E_t \sum_{s=0}^{\infty} \theta_w^s \beta^s \left\{ \Lambda_{t+s} \frac{1}{P_{t+s}} \left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_w} \right)^{1-\varepsilon_w} W_{t+s}^{\varepsilon_w} H_{t+s} \right\}$$

$$= \frac{\varepsilon_w}{\varepsilon_w - 1} E_t \sum_{s=0}^{\infty} \theta_w^s \beta^s \left\{ \lambda_{w,t+s} b_{t+s} B \left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_w} \right)^{-\varepsilon_w(1+\psi)} W_{t+s}^{\varepsilon_w(1+\psi)} H_{t+s}^{1+\psi} \right\}$$

$$\Leftrightarrow W_t(j)^{1+\varepsilon_w\psi} = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{E_t \sum_{s=0}^{\infty} \theta_w^s \beta^s \left\{ \lambda_{w,t+s} b_{t+s} B \left[\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_w} \right)^{-\varepsilon_w} W_{t+s}^{\varepsilon_w} H_{t+s} \right]^{1+\psi} \right\}}{E_t \sum_{s=0}^{\infty} \theta_w^s \beta^s \left\{ \Lambda_{t+s} \frac{1}{P_{t+s}} \left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_w} \right)^{1-\varepsilon_w} W_{t+s}^{\varepsilon_w} H_{t+s} \right\}}$$

Rewritten in terms of real wages $\left(w_t = \frac{W_t}{P_t} \right)$:

$$W_t(j)^{1+\varepsilon_w\psi} \frac{1}{P_t^{1+\varepsilon_w\psi}} = \frac{\varepsilon_w}{\varepsilon_w - 1} \underbrace{\frac{1}{P_t^{1+\varepsilon_w\psi}}}_{=\frac{1}{P_t^{1-\varepsilon_w+\varepsilon_w(1+\psi)}}} \frac{E_t \sum_{s=0}^{\infty} \theta_w^s \beta^s \left\{ \lambda_{w,t+s} b_{t+s} B \left[\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_w} \right)^{-\varepsilon_w} W_{t+s}^{\varepsilon_w} H_{t+s} \right]^{1+\psi} \right\}}{E_t \sum_{s=0}^{\infty} \theta_w^s \beta^s \left\{ \Lambda_{t+s} \frac{1}{P_{t+s}} \left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_w} \right)^{1-\varepsilon_w} W_{t+s}^{\varepsilon_w} H_{t+s} \right\}}$$

$$\Leftrightarrow w_t(j)^{1+\varepsilon_w\psi} = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{E_t \sum_{s=0}^{\infty} \theta_w^s \beta^s \left\{ \lambda_{w,t+s} b_{t+s} B \left[\left(\frac{\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_w}}{\prod_{k=1}^{s} \Pi_{t+k}} \right)^{-\varepsilon_w} w_{t+s}^{\varepsilon_w} H_{t+s} \right]^{1+\psi} \right\}}{E_t \sum_{s=0}^{\infty} \theta_w^s \beta^s \left\{ \Lambda_{t+s} \left(\frac{\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_w}}{\prod_{k=1}^{s} \Pi_{t+k}} \right)^{1-\varepsilon_w} w_{t+s}^{\varepsilon_w} H_{t+s} \right\}} = {w_t}^{1+\varepsilon_w\psi}$$

All agents choose the same $w_t(j)$ as derived in the labour agencies first-order condition with respect to $H_t(j)$. Letting the numerator be X_t^{wn} and the denominator X_t^{wd}, then this equation can be rewritten as:

$${w_t}^{1+\varepsilon_w\psi} = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{X_t^{wn}}{X_t^{wd}},$$

where the numerator is:

$$X_t^{wn} =$$

$$\lambda_{w,t} b_t B w_t^{\varepsilon_w(1+\psi)} H_t^{1+\psi} + E_t \left\{ \theta_w \beta \underbrace{E_{t+1} \left[\begin{array}{l} \sum_{s=1}^{\infty} \theta_w^{s-1} \beta^{s-1} \lambda_{w,t+s} b_{t+s} B \\ \left(\prod_{k=1}^{s} \Pi_{t+k} \right)^{\varepsilon_w(1+\psi)} \left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_w} \right)^{-\varepsilon_w(1+\psi)} w_{t+s}^{\varepsilon_w(1+\psi)} H_{t+s}^{1+\psi} \end{array} \right]}_{= X_{t+1}^{wn} \Pi_{t+1}^{\varepsilon_w(1+\psi)} \Pi_t^{-\iota_w \varepsilon_w(1+\psi)}} \right\}$$

and the denominator:

$$X_t^{wd} = \Lambda_t w_t^{\varepsilon_w} H_t + E_t \left\{ \theta_w \beta \underbrace{E_{t+1} \left[\sum_{s=1}^{\infty} \theta_w^{s-1} \beta^{s-1} \Lambda_{t+s} \left(\frac{\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_w}}{\prod_{k=1}^{s} \Pi_{t+k}} \right)^{1-\varepsilon_w} w_{t+s}^{\varepsilon_w} H_{t+s} \right]}_{= X_{t+1}^{wd} \Pi_t^{-\iota_w(\varepsilon_w-1)} \Pi_t^{(\varepsilon_w-1)}} \right\}$$

The equation for $w_t(i) = w_t^*$ can be written in the following way:

$$(w_t^*)^{1+\varepsilon_w \psi} = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{X_t^{wn}}{X_t^{wd}}$$

The law of motion for wages then is:

$$W_t^{1-\varepsilon_w} = (1-\theta_w)\,(W_t^*)^{1-\varepsilon_w} + \theta_w \left(\Pi_{t-1}^{\iota_w} W_{t-1} \right)^{1-\varepsilon_w}$$

$$\Leftrightarrow w_t^{1-\varepsilon_w} = (1-\theta_w)\,(w_t^*)^{1-\varepsilon_w} + \theta_w \left(\frac{\Pi_{t-1}^{\iota_w} w_{t-1}}{\Pi_t} \right)^{1-\varepsilon_w}$$

A.2.2 Wage dispersion

From the demand for differentiated labour, we have differentiated labour supply from household j:

$$H_t(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\varepsilon_w} H_t$$

Taking the integral over households on both sides, we have:

$$\underbrace{\textstyle\int_0^1 H_t(j)dj}_{H_{ht}} = H_t \underbrace{\textstyle\int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\varepsilon_w} dj}_{D_{wt}} = H_t D_{wt}$$

Now regarding the evolution of D_{wt}, the period-t wage dispersion is:

$$D_{wt} = W_t^{\varepsilon_w} \left[\theta_w \left(\Pi_{t-1}^{\iota_w}\right)^{-\varepsilon_w} \frac{D_{wt-1}}{W_{t-1}^{\varepsilon_w}} + (1-\theta_w)({W_t}^*)^{-\varepsilon_w}\right]$$

$$\Leftrightarrow D_{wt} = \theta_w \left(\Pi_{t-1}^{\iota_w}\right)^{-\varepsilon_w} \left(\frac{W_t}{W_{t-1}}\right)^{\varepsilon_w} D_{wt-1} + (1-\theta_w)\left(\frac{{W_t}^*}{W_t}\right)^{-\varepsilon_w}$$

$$\Leftrightarrow D_{wt} = \theta_w \left(\frac{\Pi_t}{\Pi_{t-1}^{\iota_w}}\right)^{\varepsilon_w} \left(\frac{w_t}{w_{t-1}}\right)^{\varepsilon_w} D_{wt-1} + (1-\theta_w)\left(\frac{{w_t}^*}{w_t}\right)^{-\varepsilon_w}$$

From the evolution of the aggregate wage index, we have:

$${W_t}^{1-\varepsilon_w} = (1-\theta_w)\left(W_t^*\right)^{1-\varepsilon_w} + \theta_w \left(\Pi_{t-1}^{\iota_w} W_{t-1}\right)^{1-\varepsilon_w} \Leftrightarrow \left(\frac{W_t^*}{W_t}\right)^{-\varepsilon_w} = \left[\frac{1-\theta_w\left(\Pi_{t-1}^{\iota_w}\frac{W_{t-1}}{W_t}\right)^{1-\varepsilon_w}}{1-\theta_w}\right]^{\frac{-\varepsilon_w}{1-\varepsilon_w}}$$

Substituting this into the evolution of wage dispersion yields:

$$D_{wt} = \theta_w \left(\Pi_{t-1}^{\iota_w}\right)^{-\varepsilon_w} \left(\frac{W_t}{W_{t-1}}\right)^{\varepsilon_w} D_{wt-1} + (1-\theta_w)^{\frac{1}{1-\varepsilon_w}} \left[1-\theta_w\left(\Pi_{t-1}^{\iota_w}\frac{W_{t-1}}{W_t}\right)^{1-\varepsilon_w}\right]^{\frac{\varepsilon_w}{\varepsilon_w-1}}$$

Finally, rewriting this in terms of real wages:

$$D_{wt} = \theta_w \left(\Pi_{t-1}^{\iota_w}\right)^{-\varepsilon_w} \left(\frac{w_t}{w_{t-1}}\Pi_t\right)^{\varepsilon_w} D_{wt-1} + (1-\theta_w)^{\frac{1}{1-\varepsilon_w}} \left[1 - \theta_w \left(\Pi_{t-1}^{\iota_w} \frac{w_{t-1}}{w_t \Pi_t}\right)^{1-\varepsilon_w}\right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}}$$

A.3 Goods market

A.3.1 Final goods producers

Perfectly competitive final goods producers combine differentiated intermediate goods $Y_t(i)$ into a homogeneous good Y_t according to the technology:

$$Y_t = \left[\textstyle\int_0^1 Y_t(i)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} di\right]^{\frac{\varepsilon_p}{\varepsilon_p - 1}}$$

The final goods producers buy the intermediate goods on the market, package Y_t, and resell it to consumers. These firms maximise profits in a perfectly competitive environment. Their optimisation problem (cost minimisation) is:

$$\min_{Y_t(i)} \textstyle\int_0^1 P_t(i) Y_t(i) di$$

subject to (at least obtaining a bundle Y_t):

$$\left[\textstyle\int_0^1 Y_t(i)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} di\right]^{\frac{\varepsilon_p}{\varepsilon_p - 1}} \geq Y_t$$

Thus, the Lagrangian is:

$$\mathcal{L} = \textstyle\int_0^1 P_t(i) Y_t(i) di - \Psi_t \left(\left[\int_0^1 Y_t(i)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} di\right]^{\frac{\varepsilon_p}{\varepsilon_p - 1}} - Y_t\right)$$

The first order condition w.r.t. $Y_t(i)$ is:

$$\frac{\partial \mathcal{L}}{\partial Y_t(i)} = P_t(i) - \Psi_t \left(\frac{\varepsilon_p}{\varepsilon_p - 1}\left[\textstyle\int_0^1 Y_t(i)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} di\right]^{\frac{\varepsilon_p}{\varepsilon_p - 1} - 1} \frac{\varepsilon_p - 1}{\varepsilon_p} Y_t(i)^{\frac{\varepsilon_p - 1}{\varepsilon_p} - 1}\right) = 0$$

$$\Leftrightarrow P_t(i) - \Psi_t \left(\underbrace{\left[\textstyle\int_0^1 Y_t(i)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} di\right]^{\frac{1}{\varepsilon_p - 1}}}_{Y_t^{\frac{1}{\varepsilon_p}}} Y_t(i)^{-\frac{1}{\varepsilon_p}}\right) = 0$$

$$\Leftrightarrow Y_t(i) = \left(\frac{P_t(i)}{\Psi_t}\right)^{-\varepsilon_p} Y_t,$$

which is the demand function.

Using the definition of Y_t leads to:

$$Y_t = \left[\int_0^1 \left(\left(\frac{P_t(i)}{\Psi_t} \right)^{-\varepsilon_p} Y_t \right)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} di \right]^{\frac{\varepsilon_p}{\varepsilon_p - 1}}$$

$$\Leftrightarrow \Psi_t = \left[\int_0^1 P_t(i)^{1-\varepsilon_p} di \right]^{\frac{1}{1-\varepsilon_p}} \equiv P_t$$

Plugging this into the demand function results in:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon_p} Y_t$$

A.3.2 Intermediate goods producers

A continuum of monopolistically competitive firms combines capital K_{t-1} and labour H_t to produce intermediate goods according to a standard Cobb-Douglas technology.

The production function is given by:

$$Y_t(i) = A_t K_{t-1}(i)^{\alpha} H_t(i)^{1-\alpha}$$

The firms minimise their cost

$$\min \left\{ \frac{W_t}{P_t} H_t(i) + R_t^k K_{t-1}(i) \right\}$$

subject to their production function, such that the corresponding Lagrangian reads:

$$\mathcal{L} \;=\; \frac{W_t}{P_t} H_t(i) + R_t^k K_{t-1}(i) + \nu_t(i) \left[Y_t(i) - A_t K_{t-1}(i)^{\alpha} H_t(i)^{1-\alpha} \right]$$

Thus, the firms choose labour and capital as follows:

$$\frac{\partial \mathcal{L}_t}{\partial H_t(i)} = \frac{W_t}{P_t} - \nu_t(i) \underbrace{(1-\alpha) A_t K_{t-1}(i)^{\alpha} H_t(i)^{-\alpha}}_{\text{MPL}(i)_t} = 0$$

$$\frac{\partial \mathcal{L}_t}{\partial K_{t-1}(i)} = R_t^k - \nu_t(i) \underbrace{\alpha A_t K_{t-1}(i)^{\alpha-1} H_t(i)^{1-\alpha}}_{MPK(i)_t} = 0$$

As intermediate result we get the marginal product of labour (MPL) and capital (MPK), respectively. Solving the derivative w.r.t. K_{t-1} for $\nu_t(i)$ and putting the corresponding equation into the derivative w.r.t. L_t yields:

$$\frac{K_{t-1}(i)}{H_t(i)} = \frac{\alpha}{(1-\alpha)} \frac{W_t}{P_t R_t^k}$$

Real marginal costs are derived as the shadow price of production $\nu_t(i)$. From the derivative w.r.t. H_t we have:

$$\nu_t(i) = \frac{1}{(1-\alpha) A_t} \left(\frac{K_{t-1}(i)}{H_t(i)} \right)^{-\alpha} \frac{W_t}{P_t}$$

Then plugging in the optimal capital-labour ratio from above, we get:

$$\nu_t(i) = \alpha^{-\alpha} (1-\alpha)^{-(1-\alpha)} \frac{\left(\frac{W_t}{P_t}\right)^{1-\alpha} \left(R_t^k\right)^{\alpha}}{A_t} = mc_t(i) = \frac{MC_t(i)}{P_t}$$

A.3.3 Optimal price setting

The intermediate goods producers set prices based on Calvo contracts. In each period firms adjust their prices with probability $(1-\theta_p)$ independently form previous adjustments. However, we depart from Calvo in the following way: For those firms that cannot adjust their prices in a given period, prices will be reset according to the following indexation rule:

$$P_t(i) = \Pi_{t-1}^{\iota_p} P_{t-1}(i),$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$ is gross inflation.

The firms that adjust their prices face the following problem:

$$\max_{P_t(i)} \Omega_t = E_t \sum_{s=0}^{\infty} \theta_p^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_t} \left[\frac{P_t(i) \left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_p} \right)}{P_{t+s}} Y_{t+s}(i) - \frac{W_{t+s}}{P_{t+s}} H_{t+s}(i) - R_{t+s}^k K_{t-1+s}(i) \right],$$

with demand given by:

$$Y_{t+s}(i)=\left(\frac{P_t(i)\left(\prod_{k=1}^{s}\Pi_{t+k-1}^{\iota_p}\right)}{P_{t+s}}\right)^{-\varepsilon_p}Y_{t+s}.$$

The optimisation problem is:

$$\max_{P_t(i)}\Omega_t=E_t\sum_{s=0}^{\infty}\theta_p^s\frac{\beta^s\Lambda_{t+s}}{\Lambda_t}\left[\frac{P_t(i)\left(\prod_{k=1}^{s}\Pi_{t+k-1}^{\iota_p}\right)}{P_{t+s}}-\lambda_{p,t+s}mc_{t+s}(i)\right]Y_{t+s}(i)$$

Plugged in aggregate demand:

$$E_t\sum_{s=0}^{\infty}\theta_p^s\frac{\beta^s\Lambda_{t+s}}{\Lambda_t}\left[\frac{P_t(i)\left(\prod_{k=1}^{s}\Pi_{t+k-1}^{\iota_p}\right)}{P_{t+s}}-\lambda_{p,t+s}mc_{t+s}(i)\right]\left(\frac{P_t(i)\left(\prod_{k=1}^{s}\Pi_{t+k-1}^{\iota_p}\right)}{P_{t+s}}\right)^{-\varepsilon_p}Y_{t+s}$$
$$=E_t\sum_{s=0}^{\infty}\theta_p^s\frac{\beta^s\Lambda_{t+s}}{\Lambda_t}\left[P_t(i)^{1-\varepsilon_p}\left(\frac{\left(\prod_{k=1}^{s}\Pi_{t+k-1}^{\iota_p}\right)}{P_{t+s}}\right)^{1-\varepsilon_p}-\lambda_{p,t+s}mc_{t+s}(i)P_t(i)^{-\varepsilon_p}\left(\frac{\left(\prod_{k=1}^{s}\Pi_{t+k-1}^{\iota_p}\right)}{P_{t+s}}\right)^{-\varepsilon_p}\right]Y_{t+s}$$

and taking the derivative w.r.t. $P_t(i)$ - this leads to:

$$E_t\sum_{s=0}^{\infty}\theta_p^s\frac{\beta^s\Lambda_{t+s}}{\Lambda_t}\left(\frac{\left(\prod_{k=1}^{s}\Pi_{t+k-1}^{\iota_p}\right)}{P_{t+s}}\right)^{-\varepsilon_p}Y_{t+s}\left[(1-\varepsilon_p)\left(\frac{\left(\prod_{k=1}^{s}\Pi_{t+k-1}^{\iota_p}\right)}{P_{t+s}}\right)\right]$$

$$=E_t\sum_{s=0}^{\infty}\theta_p^s\frac{\beta^s\Lambda_{t+s}}{\Lambda_t}\left(\frac{\left(\prod_{k=1}^{s}\Pi_{t+k-1}^{\iota_p}\right)}{P_{t+s}}\right)^{-\varepsilon_p}Y_{t+s}\left[\lambda_{p,t+s}mc_{t+s}(i)\left(-\varepsilon_p\right)P_t(i)^{-1}\right]$$

$$\Leftrightarrow P_t(i)=\left(\frac{\varepsilon_p}{\varepsilon_p-1}\right)\frac{E_t\sum_{s=0}^{\infty}\theta_p^s\frac{\beta^s\Lambda_{t+s}}{\Lambda_t}\left(\frac{\left(\prod_{k=1}^{s}\Pi_{t+k-1}^{\iota_p}\right)}{P_{t+s}}\right)^{-\varepsilon_p}Y_{t+s}\lambda_{p,t+s}mc_{t+s}(i)}{E_t\sum_{s=0}^{\infty}\theta_p^s\frac{\beta^s\Lambda_{t+s}}{\Lambda_t}\left(\frac{\left(\prod_{k=1}^{s}\Pi_{t+k-1}^{\iota_p}\right)}{P_{t+s}}\right)^{-\varepsilon_p}Y_{t+s}\frac{\left(\prod_{k=1}^{s}\Pi_{t+k-1}^{\iota_p}\right)}{P_{t+s}}}$$

And since $P_{t+s} = P_t \prod_{k=1}^{s} \Pi_{t+k}$:

$$P_t(i) = P_t \left(\frac{\varepsilon_p}{\varepsilon_p - 1}\right) \left(\frac{P_t}{P_t}\right)^{-\varepsilon_p} \frac{E_t \sum_{s=0}^{\infty} \theta_p^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_t} \left(\frac{\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota p}\right)}{\prod_{k=1}^{s} \Pi_{t+k}}\right)^{-\varepsilon_p} Y_{t+s} \lambda_{p,t+s} mc_{t+s}(i)}{E_t \sum_{s=0}^{\infty} \theta_p^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_t} \left(\frac{\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota p}\right)}{\prod_{k=1}^{s} \Pi_{t+k}}\right)^{-\varepsilon_p} Y_{t+s} \frac{\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota p}\right)}{\prod_{k=1}^{s} \Pi_{t+k}}}$$

$$\Leftrightarrow \underbrace{\frac{P_t(i)}{P_{t-1}}}_{=\Pi_t^*} \underbrace{\frac{P_{t-1}}{P_t}}_{=\Pi_t^{-1}} = \left(\frac{\varepsilon_p}{\varepsilon_p - 1}\right) \frac{E_t \sum_{s=0}^{\infty} \theta_p^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_t} \left(\frac{\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota p}\right)}{\prod_{k=1}^{s} \Pi_{t+k}}\right)^{-\varepsilon_p} Y_{t+s} \lambda_{p,t+s} mc_{t+s}(i)}{E_t \sum_{s=0}^{\infty} \theta_p^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_t} \left(\frac{\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota p}\right)}{\prod_{k=1}^{s} \Pi_{t+k}}\right)^{-\varepsilon_p} Y_{t+s} \frac{\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota p}\right)}{\prod_{k=1}^{s} \Pi_{t+k}}}$$

$$\Leftrightarrow \Pi_t^* = \left(\frac{\varepsilon_p}{\varepsilon_p - 1}\right) \frac{E_t \sum_{s=0}^{\infty} \theta_p^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_t} \left(\frac{\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota p}\right)}{\prod_{k=1}^{s} \Pi_{t+k}}\right)^{-\varepsilon_p} Y_{t+s} \lambda_{p,t+s} mc_{t+s}(i)}{E_t \sum_{s=0}^{\infty} \theta_p^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_t} \left(\frac{\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota p}\right)}{\prod_{k=1}^{s} \Pi_{t+k}}\right)^{1-\varepsilon_p} Y_{t+s}} \Pi_t$$

Each of the parts of this equation can be defined as follows:

$$X_t^{pd} = E_t \sum_{s=0}^{\infty} \theta_p^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_t} \left(\frac{\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota p}\right)}{\prod_{k=1}^{s} \Pi_{t+k}}\right)^{1-\varepsilon_p} Y_{t+s},$$

$$X_t^{pn} = E_t \sum_{s=0}^{\infty} \theta_p^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_t} \left(\frac{\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota p}\right)}{\prod_{k=1}^{s} \Pi_{t+k}}\right)^{-\varepsilon_p} Y_{t+s} \lambda_{p,t+s} mc_{t+s}(i),$$

where, regarding X_t^{pd}:

$$X_t^{pd} = E_t \sum_{s=0}^{\infty} \theta_p^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_t} \left(\frac{\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota p}\right)}{\prod_{k=1}^{s} \Pi_{t+k}}\right)^{1-\varepsilon_p} Y_{t+s}$$

$$\Leftrightarrow X_t^{pd} = Y_t + E_t \left\{ \theta_p \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left(\frac{\Pi_t^{\iota_p}}{\Pi_{t+1}} \right)^{1-\varepsilon_p} Y_{t+1} + \sum_{s=2}^{\infty} \theta_p^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_t} \left(\frac{\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_p} \right)}{\prod_{k=1}^{s} \Pi_{t+k}} \right)^{1-\varepsilon_p} Y_{t+s} \right\}$$

$$\Leftrightarrow X_t^{pd} = Y_t + E_t \left\{ \theta_p \frac{\beta \Lambda_{t+1}}{\Lambda_t} \underbrace{E_{t+1} \left[\sum_{s=1}^{\infty} \theta_p^{s-1} \frac{\beta^{s-1} \Lambda_{t+s}}{\Lambda_{t+1}} \left(\frac{\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_p} \right)}{\prod_{k=1}^{s} \Pi_{t+k}} \right)^{1-\varepsilon_p} Y_{t+s} \right]}_{=\Pi_t^{\iota_p(1-\varepsilon_p)} \Pi_{t+1}^{\varepsilon_p - 1} X_{t+1}^{pd}} \right\}$$

$$\Leftrightarrow X_t^{pd} = Y_t + E_t \left\{ \theta_p \frac{\beta \Lambda_{t+1}}{\Lambda_t} \Pi_t^{\iota_p(1-\varepsilon_p)} \Pi_{t+1}^{\varepsilon_p - 1} X_{t+1}^{pd} \right\}$$

and, considering X_t^{pn}:

$$X_t^{pn} = E_t \sum_{s=0}^{\infty} \theta_p^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_t} \left(\frac{\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_p} \right)}{\prod_{k=1}^{s} \Pi_{t+k}} \right)^{-\varepsilon_p} Y_{t+s} \lambda_{p,t+s} mc_{t+s}(i)$$

$$\Leftrightarrow X_t^{pn} = Y_t \lambda_{p,t} mc_t(i) + E_t \left\{ \begin{array}{l} \theta_p \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left(\frac{\Pi_t^{\iota_p}}{\Pi_{t+1}} \right)^{-\varepsilon_p} Y_{t+1} \lambda_{p,t+1} mc_{t+1}(i) \\ + \sum_{s=2}^{\infty} \theta_p^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_t} \left(\frac{\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_p} \right)}{\prod_{k=1}^{s} \Pi_{t+k}} \right)^{-\varepsilon_p} Y_{t+s} \lambda_{p,t+s} mc_{t+s}(i) \end{array} \right\}$$

$$\Leftrightarrow X_t^{pn} = Y_t \lambda_{p,t} mc_t(i) + E_t \left\{ \theta_p \frac{\beta \Lambda_{t+1}}{\Lambda_t} \Pi_t^{-\iota_p \varepsilon_p} \Pi_{t+1}^{\varepsilon_p} X_{t+1}^{pn} \right\}$$

Thus, we can write the equation for Π_t^* in the following way:

$$\Pi_t^* = \frac{\varepsilon_p}{\varepsilon_p - 1} \frac{X_t^{pn}}{X_t^{pd}} \Pi_t$$

The law of motion for prices then is:

$$P_t^{1-\varepsilon_p} = (1-\theta_p)\left(P_t^*\right)^{1-\varepsilon_p} + \theta_p \left(\Pi_{t-1}^{\iota_p} P_{t-1}\right)^{1-\varepsilon_p}$$

$$\Leftrightarrow \left(\Pi_t\right)^{1-\varepsilon_p} = (1-\theta_p)\left(\Pi_t^*\right)^{1-\varepsilon_p} + \theta_p \left(\Pi_{t-1}^{\iota_p}\right)^{1-\varepsilon_p}$$

A.3.4 Price dispersion

From the demand for differentiated goods, we have:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon_p} Y_t$$

Taking the integral on both sides, it follows:

$$\underbrace{\textstyle\int_0^1 Y_t(i)di}_{Y_{ht}} = Y_t \underbrace{\textstyle\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon_p} di}_{D_{pt}}$$

Regarding the evolution of D_{pt}, the period-t price dispersion is:

$$D_{pt} = P_t^{\varepsilon_p} \left[\theta_p \left(\Pi_{t-1}^{\iota_p}\right)^{-\varepsilon_p} \frac{D_{pt-1}}{P_{t-1}^{\varepsilon_p}} + (1-\theta_p)(P_t{}^*)^{-\varepsilon_p}\right]$$

$$\Leftrightarrow D_{pt} = \Pi_t^{\varepsilon_p} \left[(1-\theta_p)\Pi_t{}^{*-\varepsilon_p} + \theta_p \left(\Pi_{t-1}^{\iota_p}\right)^{-\varepsilon_p} D_{pt-1}\right)$$

From the evolution of the aggregate price index, we have:

$$P_t{}^{1-\varepsilon_p} = (1-\theta_p)\left(P_t^*\right)^{1-\varepsilon_p} + \theta_p \left(\Pi_{t-1}^{\iota_p} P_{t-1}\right)^{1-\varepsilon_p}$$

$$\Leftrightarrow \left(\frac{P_t^*}{P_t}\right)^{-\varepsilon_p} = \left[\frac{1-\theta_p\left(\frac{\Pi_{t-1}^{\iota_p}}{\Pi_t}\right)^{1-\varepsilon_p}}{1-\theta_p}\right]^{\frac{\varepsilon_p}{\varepsilon_p - 1}}$$

Substituting this into the evolution of price dispersion yields:

$$D_{pt} = \theta_p \left(\Pi_{t-1}^{\iota_p}\right)^{-\varepsilon_p} \Pi_t^{\varepsilon_p} D_{pt-1} + (1-\theta_p)^{\frac{1}{1-\varepsilon_p}} \left[1 - \theta_p \left(\frac{\Pi_{t-1}^{\iota_p}}{\Pi_t}\right)^{1-\varepsilon_p}\right]^{\frac{\varepsilon_p}{\varepsilon_p - 1}}$$

A.3.5 Capital producers

The profits of the capital producers can be defined as follows:

$$\underbrace{P_t^k \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t}_{\text{Income}} - \underbrace{I_t}_{\text{Costs}}$$

The profit maximisation of the capital producers without constraint is described by:

$$\mathcal{L} = E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t+s} \left[P_{t+s}^k \mu_{t+s} \left[1 - S\left(\frac{I_{t+s}}{I_{t+s-1}}\right)\right] I_{t+s}) - I_{t+s}\right]$$

$$\frac{\partial \mathcal{L}}{\partial I_t} : P_t^k \mu_t \left\{1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right) \frac{I_t}{I_{t-1}}\right\} = 1 - \beta P_{t+1}^k \frac{\Lambda_{t+1}}{\Lambda_t} \mu_{t+1} \left\{-S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right\}$$

A.4 Government policies

When the central bank does not peg the interest rate, it follows a standard Taylor rule:

$$\ln(R_t) = (1-\rho)\ln(R) + \rho \ln(R_{t-1}) + (1-\rho)\left(\tau_\pi\left(\pi_t - \pi\right) + \tau_y(y_t - y_{t-1})\right) + \varepsilon_t^r$$

QE policies are implemented via the AR(2) process:

$$\bar{B} = (\bar{B}_{ss})^{(1-\rho_{B_1}+\rho_{B_2})} * (\bar{B}_{t-1})^{(\rho_{B_1})} * (\bar{B}_{t-2})^{(-\rho_{B_2})} * \epsilon_B$$

A.5 Resource constraints and exogenous shock processes

The resource constraint evaluates to:

$$Y_t = C_t + I_t.$$

In addition to the equilibrium conditions, the model comprises seven exogenous processes.

1. Technology shock: $A_t = (1-\rho_a) * log(A_{ss}) + \rho_a * A_{t-1} + \epsilon_{A,t}$.
2. Financial shock: $\Phi_t = (1-\rho_\phi) * log(\Phi_{ss}) + \rho_{phi} * (\Phi_{t-1}) + \epsilon_{\Phi,t}$.

3. Investment shock: $\mu_t = (1 - \rho_\mu) * log(\mu_{ss}) + \rho_\mu * (\mu_{t-1}) + \epsilon_{\mu,t}$.
4. Wage markup shock: $\lambda_{w,t} = (1 - \rho_{\lambda^w}) * log(\lambda_{w,ss}) + \rho_{\lambda^w} * (\lambda_{w,t-1}) + \epsilon_{\lambda_{w,t}}$.
5. Price markup shock: $\lambda_{p,t} = (1 - \rho_{\lambda_p}) * log(\lambda_{p,ss}) + \rho_{\lambda_p} * (\lambda_{p,t-1}) + \epsilon_{\lambda_{p,t}}$.
6. Discount factor shock: $b_t = (1 - \rho_b) * log(b_{ss}) + \rho_b * (b_{t-1}) + \epsilon_{b,t}$.
7. Monetary policy residual: $R^\epsilon_t = (1 - \rho_m) * log(R^\epsilon_{ss}) + \rho_m * R^\epsilon_{t-1} + \epsilon_{R,t}$;

B Data, Estimation and Calibration

B.1 Data

Definition of observables

Real per capita output growth: $\frac{(YER/LFN)-(YER(-1)/LFN(-1))}{(YER(-1)/LFN(-1))}$

Real per capita investment growth: $\frac{(ITR/LFN)-(ITR(-1)/LFN(-1))}{(ITR(-1)/LFN(-1))}$

Gross inflation: $1+\frac{HICPSA-HICPSA(-1)}{HICPSA(-1)}$

Employment growth: $\frac{LNN-LNN(-1)}{LNN(-1)}$

Real wage growth: $\frac{(WRN/HICPSA)-(WRN(-1)/HICPSA(-1))}{(WRN(-1)/HICPSA(-1))}$

First difference of short-term interset rate: $STN - STN(-1)$

First difference of long-term interest rate: $LTN - LTN(-1)$

Real bank net worth growth: $\frac{(NWB/HICPSA)-(NWB(-1)/HICPSA(-1))}{(NWB(-1)/HICPSA(-1))}$

Data description

All seasonal data are seasonally adjusted.

YER: Real GDP. Millions of ECU/euro corrected with reference year 1995. Source: Area-wide Model (AWM) database.

LFN: Labor force (persons). Source: AWM database.

ITR: Gross investment. Source: AWM database.

HICPSA: Overall Harmonised Index of Consumer Prices. Base year 1996=100. Source: AWM database.

LNN: Total employment (persons). Source: AWM database.

WRN: Nominal wage rate per head. Source: AWM database.

STN: Nominal net short-term interest rate in percent. Source: AWM database.

LTN: Nominal net long-term interest rate in percent. Source: AWM database.

NWB: Nominal capital and reserves of euro area monetary financial Institutions (excluding eurosystem) in millions of euro. Source: European Central Bank, MFI Balance Sheet Items Statistics.

B.2 Estimation

As is common in the literature, we calibrate a subset of structural parameters to ensure identification. For the calibration, we rely on CFP. Table 3 gives the values for the calibrated parameters. β is set to 0.99, yielding a steady state annual real interest rate of 4%. The labour income share α is set to 0.33 and the capital depreciation rate per year to 10%, implying $\delta = 0.025$. A 20% mark-up in both prices and wages is assumed, leading to $\epsilon_p = \epsilon_w = 5$. A leverage ratio of 6 leads to $\zeta = 0.9854$. The other structural parameters are estimated using Bayesian methods. For the estimation, we linearise the model around the steady state. We use eight observables for the euro area: real per capita output growth, real per capita investment growth, gross inflation, employment growth, real wage growth, the first difference of the short- and long-term interest rate, and real bank net worth growth. Data on bank net worth are taken from the European Central Bank's MFI Balance Sheet Items Statistics. All the other variables are taken from the Area-wide Model database.[15] All variables are demeaned. Since we have only seven structural shocks in the model, we add a measurement error to the observations equation for bank net worth in order to avoid stochastic singularity. The sample period is from 1998Q1 to 2013Q4.

Table 3: Calibrated parameters

Parameters	Description	Value
β	Household discount factor	0.99
ψ_I	Investment adjustment cost	2
κ	Coupon payment	0.975
L_{ss}	Steady state leverage	6
ϵ_p	Elasticity of substitution (goods)	5
ϵ_w	Elasticity of substitution (labour)	5
α	Capital share	0.33
δ	Depreciation rate	0.025

Table 4: Prior and posterior distributions of structural parameters

		Prior distribution			Posterior distribution			
Param.	Description	Dist.	Mean	St. Dev.	Median	Mean	HPD inf	HPD sup
h	Habit formation	Beta	0.5000	0.2000	0.8642	0.8635	0.8193	0.9074
η	Labor disutility	Gamma	2.0000	0.5000	1.8101	1.8496	1.1055	2.5857
ι_p	Price indexation	Beta	0.6000	0.1000	0.5261	0.5263	0.3658	0.6890
ι_w	Wage indexation	Beta	0.6000	0.1000	0.3761	0.3786	0.2573	0.4991
θ_p	Price rigidity	Beta	0.7000	0.1000	0.8144	0.8139	0.7567	0.8676
θ_w	Wage rigidity	Beta	0.7000	0.1000	0.8211	0.8194	0.7641	0.8726
ρ	Interest rate smoothing	Beta	0.7500	0.1000	0.7409	0.7390	0.6850	0.7947
τ_{pi}	Inflation coeff. in TR	Normal	1.5000	0.1000	1.5912	1.5919	1.4333	1.7482
τ_y	Output growth coeff. in TR	Normal	0.5000	0.1000	0.5725	0.5723	0.4163	0.7270
ψ_N	Net worth adjustm. costs	Gamma	3.0000	1.0000	6.7634	6.8273	4.9522	8.7945

Notes: Results based on 4 chains with 500,000 draws each. HPD inf and HPD sup denote the lower and upper bound, respectively, of the 90% highest posterior density interval.

The choice of the prior distributions of the structural parameters to be estimated correspond largely to those in CFP and Christiano, Motto and Rostagno (2010). In general, we use the Beta distribution for parameters between zero and one. For the Taylor rule parameters we use

[15] We make use of the 14th update of the Area-wide Model (AWM) database from September 2014); see `http://www.eabcn.org/sites/default/files/fck_uploads/awm_database_update_14.pdf`.

Table 5: Prior and posterior distributions of parameters in shock processes

		Prior distribution			Posterior distribution			
Param.	Description	Dist.	Mean	St. Dev.	Median	Mean	HPD inf	HPD sup
ρ_A	AR(1), productivity	Beta	0.6000	0.2000	0.9719	0.9662	0.9342	0.9976
ρ_Φ	AR(1), financial	Beta	0.6000	0.2000	0.6628	0.6608	0.5719	0.7527
ρ_μ	AR(1), investment	Beta	0.6000	0.2000	0.8370	0.8347	0.7713	0.8980
ρ_{λ_W}	AR(1), wage mark-up	Beta	0.6000	0.2000	0.1741	0.1868	0.0420	0.3260
ρ_{λ_P}	AR(1), price mark-up	Beta	0.6000	0.2000	0.4542	0.4484	0.2351	0.6620
ρ_d	AR(1), discount factor	Beta	0.6000	0.2000	0.4945	0.4930	0.3297	0.6585
ρ_R	AR(1), monetary	Beta	0.6000	0.2000	0.5030	0.4993	0.3636	0.6366
ϵ_A	SE, productivity	Invgam	0.0100	1.0000	0.0056	0.0056	0.0048	0.0064
ϵ_Φ	SE, financial	Invgam	0.0500	1.0000	0.1882	0.1913	0.1419	0.2394
ϵ_μ	SE, investment	Invgam	0.5000	1.0000	0.0881	0.0887	0.0740	0.1028
ϵ_{λ_W}	SE, wage mark-up	Invgam	0.1000	1.0000	0.5742	0.6359	0.2132	1.0417
ϵ_{λ_P}	SE, price mark-up	Invgam	0.1000	1.0000	0.0528	0.0608	0.0240	0.0954
ϵ_d	SE, discount factor	Invgam	0.1000	1.0000	0.0300	0.0314	0.0206	0.0416
ϵ_R	SE, monetary	Invgam	0.0100	1.0000	0.0033	0.0033	0.0028	0.0038
ϵ_{NW}	SE, M.E. bank net worth	Invgam	0.0013	1.0000	0.0147	0.0148	0.0126	0.0171

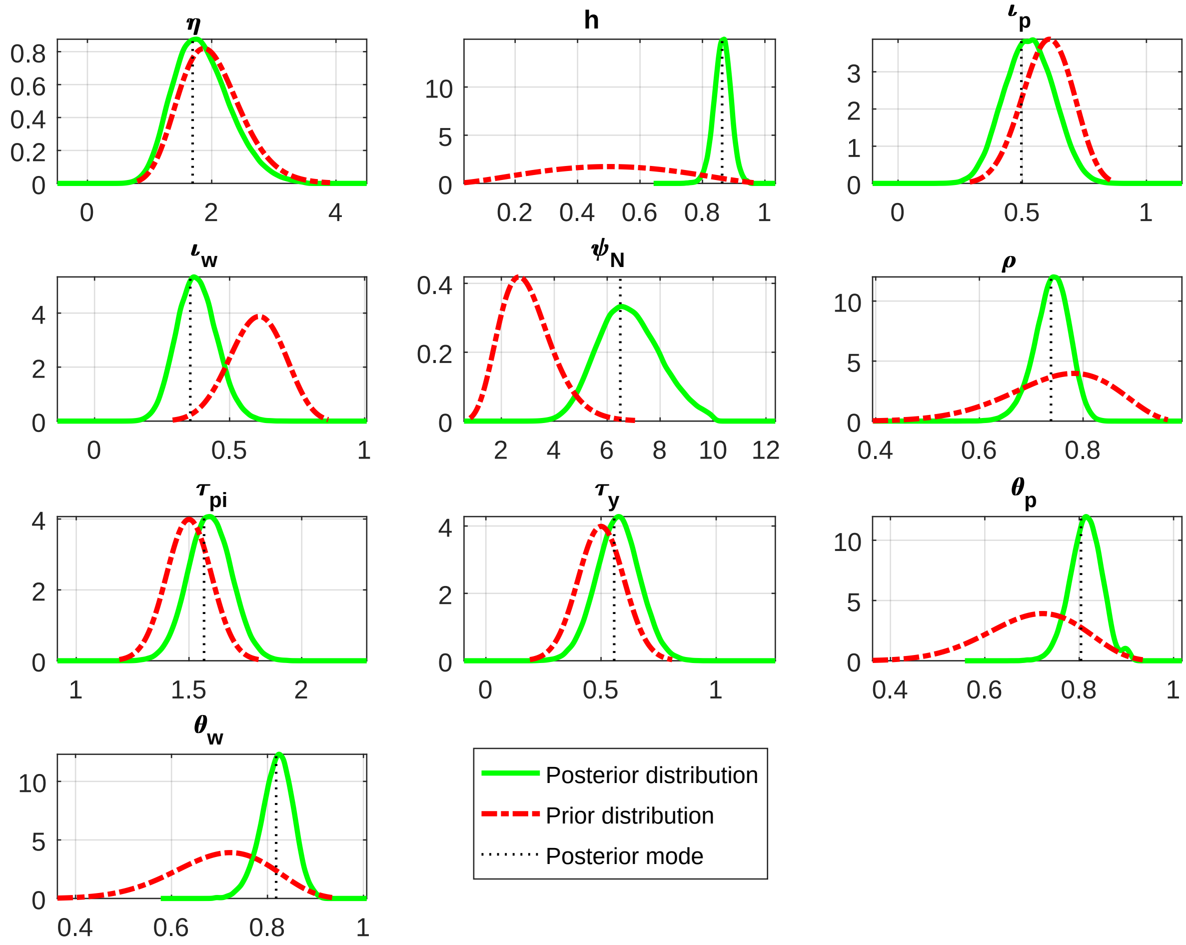
Notes: Results based on 4 chains with 500,000 draws each. HPD inf and HPD sup denote the lower and upper bound, respectively, of the 90% highest posterior density interval.

the normal distribution, which is typically used for unbounded parameters. For the financial sector parameter ψ_N, which governs the importance of net worth adjustment costs, we use a gamma distribution with mean 3 and standard deviation 1. The left parts of Tables 4 and 5 display the prior distributions of the estimated parameters.

Given the prior distributions of the parameters, we draw posterior distributions using the Metropolis-Hastings algorithm. We run four chains, each with 500,000 draws.[16] The right parts of Tables 4 and 5 report the posterior median, the posterior mean, and the lower and upper bounds of the 90% highest posterior density interval of the estimated parameters obtained by the Metropolis-Hastings algorithm. Further information, such as convergence statistics proposed by Brooks and Gelman (1998) as well as trace plots for the estimated structural parameters can be obtained from the authors. The posterior means of the habit formation parameter (0.86), the price rigidity parameter (0.81), and the price indexation parameter (0.53) are estimated to be somewhat higher than in CFP. The posterior means of the wage rigidity (0.82), wage indexation (0.38), and labour disutility (1.85) parameter are estimated to be somewhat lower than in CFP. The posterior means of the Taylor rule parameters are in line with commonly observed values in the literature. The most noticeable difference between our estimation result and CFP's is the posterior distribution for the net worth adjustment cost parameter ψ_N. Our posterior mean for this parameter (6.82) is vastly higher than the one in CFP (0.79). This could have several reasons: First, net worth elasticity could be different in Europe compared to the USA. Second, our sample ends in 2013Q4 and thus includes data of the financial crisis. Third, we use data on bank net worth to better identify the net worth adjustment cost parameter. On average, financial frictions could thus be more severe in our sample than in CFP's sample which ends in 2008Q4. Figure 6 shows the prior and posterior distributions of the structural parameters as well as their posterior modes.

[16] We use Dynare 4.5.4 for the estimation of the model, see Adjemian, Bastani, Karamé, Juillard, Maih, Mihoubi, Perendia, Pfeifer, Ratto and Villemot (2018).

Figure 6: Prior and posterior distribution of structural parameters



Note: Dashed-dotted lines are prior distributions, solid lines are posterior distributions, and the vertical dotted lines are the posterior modes.

C Forward solution of the linearized model

We consider the linearized version of the model, which we can write in the following general form:

$$\Gamma_0 Y_t = \Gamma_1 Y_{t-1} + \Phi\varepsilon_t + \Psi\eta_t. \tag{C.1}$$

Y_t denotes the endogenous variables, ε_t describes the fundamental shocks (for instance, the QE shock or the shock governing the interest rate peg), and η_t indicates the forecast errors. Following Sims (2001), we apply the QZ decomposition:

$$Q^{'}\Lambda Z^{'} = \Gamma_0 \tag{C.2}$$
$$Q^{'}\Omega Z^{'} = \Gamma_1. \tag{C.3}$$

As a result, we can rewrite equation (C.1) such that

$$Q^{'}\Lambda \underbrace{Z^{'}Y_t}_{\omega_t} = Q^{'}\Omega \underbrace{Z^{'}Y_{t-1}}_{\omega_{t-1}} + \Phi\varepsilon_t + \Psi\eta_t. \tag{C.4}$$

Premultiplying by Q and redefining $Z^{'}Y_t \equiv w_t$ implies

$$\Lambda w_t = \Omega w_{t-1} + Q\Phi\varepsilon_t + Q\Psi\eta_t. \tag{C.5}$$

Partitioning equation (C.5) into explosive and nonexplosive parts yields

$$\begin{bmatrix}\Lambda_{11} & \Lambda_{12}\\ 0 & \Lambda_{22}\end{bmatrix}\begin{bmatrix}w_{1,t}\\ w_{2,t}\end{bmatrix} = \begin{bmatrix}\Omega_{11} & \Omega_{12}\\ 0 & \Omega_{22}\end{bmatrix}\begin{bmatrix}w_{1,t-1}\\ w_{2,t-1}\end{bmatrix} + \begin{bmatrix}Q_1\\ Q_2\end{bmatrix}\left(\Phi\varepsilon_t + \Psi\eta_t\right), \tag{C.6}$$

where the second equation, i.e., the one containing the unstable eigenvalues, can separately be written as

$$\Lambda_{22}w_{2,t} = \Omega_{22}w_{2,t-1} + Q_2\left(\Phi\varepsilon_t + \Psi\eta_t\right). \tag{C.7}$$

Multiply equation (C.7) by Ω_{22}^{-1} to obtain

$$\Omega_{22}^{-1}\Lambda_{22}w_{2,t} = \Omega_{22}^{-1}\Omega_{22}w_{2,t-1} + \Omega_{22}^{-1}Q_2\left(\Phi\varepsilon_t + \Psi\eta_t\right). \tag{C.8}$$

Rewrite this expression as

$$Jw_{2,t} = w_{2,t-1} + \Omega_{22}^{-1}Q_2\left(\Phi\varepsilon_t + \Psi\eta_t\right). \tag{C.9}$$

In this expression, $J \equiv \Omega_{22}^{-1}\Lambda_{22}$ collects the ratios (i.e., the generalized eigenvalues) of the diagonal elements of Λ and Ω. Thus, J contains the generalized eigenvalues on its diagonal (i.e., when α_{jj} denotes the diagonal elements of matrix Λ and δ_{jj} denotes the diagonal elements of matrix Ω, then the generalized eigenvalues on the diagonal of matrix J are the ratios of these diagonal elements of Λ and Ω), such that

$$J = \begin{bmatrix}\frac{\alpha_{11}}{\delta_{11}} & & & *\\ & \frac{\alpha_{22}}{\delta_{22}} & & \\ & & \ddots & \\ * & & & \frac{\alpha_{jj}}{\delta_{jj}}\end{bmatrix}. \tag{C.10}$$

Shifting equation (C.9) one period forward and solving for $w_{2,t}$, we finally obtain

$$w_{2,t} = Jw_{2,t+1} - \Omega_{22}^{-1}Q_2\left(\Phi\varepsilon_{t+1} + \Psi\eta_{t+1}\right). \tag{C.11}$$

Iterating forward yields:

$$w_{2,t} = -\sum_{n=1}^{\infty} J^{n-1}\Omega_{22}^{-1}Q_2\left(\Phi\varepsilon_{t+n} + \Psi\eta_{t+n}\right). \tag{C.12}$$

Here, it is assumed that $\lim_{n\to\infty} J^n w_{2,t+n} = 0$. Since equation (C.12) contains future fundamental shocks and forecast errors, taking expectations leads to

$$w_{2,t} = -E_t\left\{\sum_{n=1}^{\infty} J^{n-1}\Omega_{22}^{-1}Q_2\Phi\varepsilon_{t+n}\right\}. \tag{C.13}$$

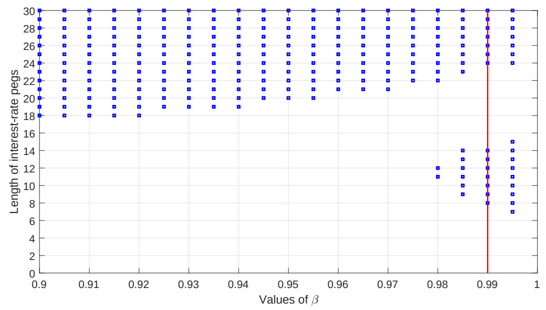
D Sensitivity analysis

One might conjecture that our results on the reversals depend very much on the specific parameterization of the model. In this section, we document that this is not the case. To this end, we conduct an extensive grid search over the model's structural parameters and illustrate for which duration of the anticipated interest rate peg reversals in the initial response of inflation occur. Specifically, we vary each parameter one-by-one, holding the other parameters constant at their benchmark values, to document that the reversal does not arise only for a very specific paramterisation of the model.

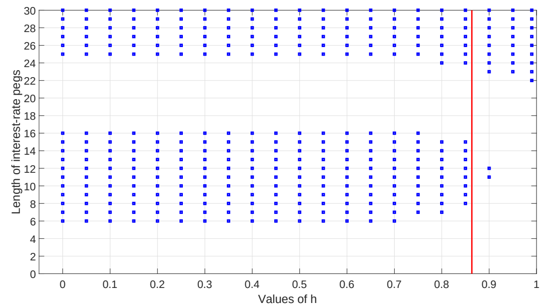
The household sector: Figure 7 shows that simply reducing the forward-lookingness of the agents in the economy by decreasing the discount parameter β (see subpanel (a) in the upper left), or increasing the backward-lookingness of the agents by increasing the consumption habit parameter, does not prevent reversals from occuring. We only observe that the duration of the peg that is required for the reversal to appear changes. For instance, if households discount future consumption more heavily (a smaller value for β), the peg has to be a few quarters longer in order for the reversal to appear (this is consistent with Carlstrom et al. (2015) who document that the implementation of a *discounted Euler equation*, along the lines of McKay, Nakamura and Steinsson (2016), does not resolve the reversal puzzle). In addition, variations in the parameters determining the investment decision of the households (i.e., the capital depreciation rate δ and the capital adjustment costs ψ_i shown in the subpanels (c) and (d)) do not prevent sign switches.

Figure 7: Duration of nominal interest rate peg for which the reversal puzzle occurs for different values of the household sector's structural parameters

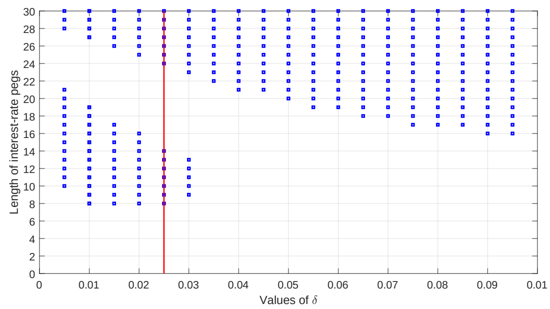
(a) Discount parameter β



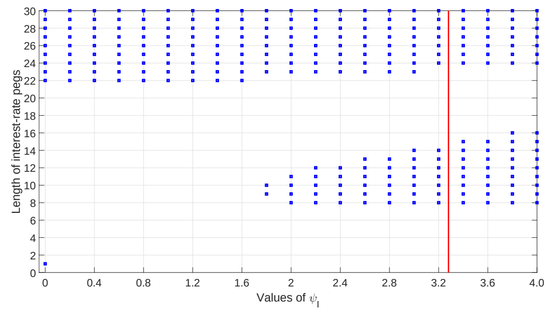
(b) Habit parameter h



(c) Capital depreciation δ



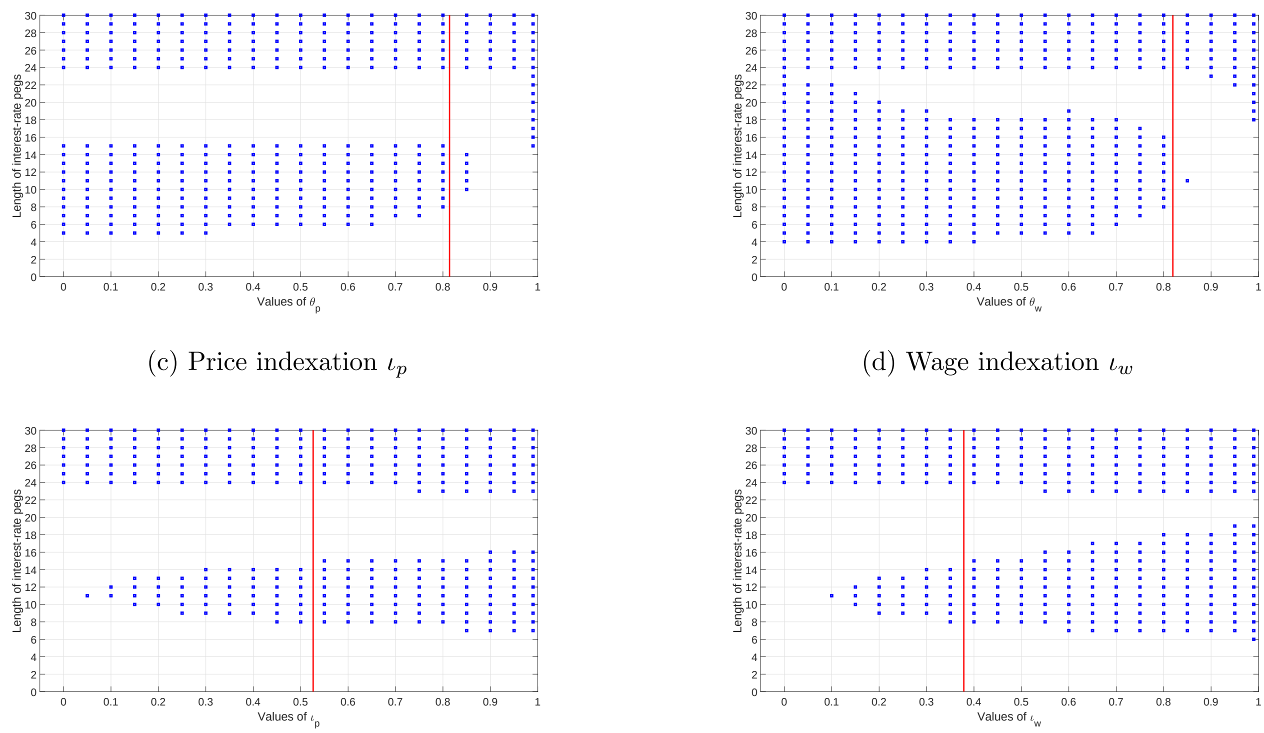
(d) Capital adjustment cost ψ_i



Note: The figure shows simulations for increasing durations of a pegged policy rate of up to thirty quarters for different parameter values. The blue points indicate a different sign compared to the scenario without interest rate peg. The red line marks the calibrated value for the respective parameters, which were used to carry out the analysis in Subsections 3.2 and 3.1.

The firm sector: Figure 8 presents results from our grid search over the parameter values which drive the behavior of the price and wage setters (i.e., the Calvo parameters for prices, θ_p, and wages, θ_w, as well as the parameters for price and wage indexation, ι_p and ι_w). Once again, we observe that the required duration of the interest rate peg in order for the reversal to appear, varies with different parameter values. However, we observe that if firms behave in a less forward-looking manner (i.e., for Calvo parameters for prices and wages > 0.9), the peg has to be a few years longer in order for the reversal to appear.

Figure 8: Duration of nominal interest rate peg for which the reversal puzzle occurs for different values of the firm sector's structural parameters



(a) Calvo parameter: prices θ_p

(b) Calvo parameter: wages θ_w

(c) Price indexation ι_p

(d) Wage indexation ι_w

Note: The figure shows simulations for increasing durations of a pegged policy rate of up to thirty quarters. Again, the blue points indicate a different sign compared to the scenario without interest rate peg. The red line marks the calibrated value for the respective parameters, which were used to carry out the analysis in Subsections 3.2 and 3.1.

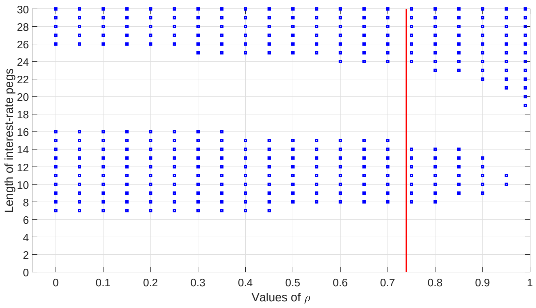
It should be noted that even if we shut down price and wage indexation jointly (i.e., $\iota_p = 0$ and $\iota_w = 0$) and re-run all the grids for all structural parameter values, sign reversals still occur. Thus, beyond indexation, there remain elements in the present model that produce sign switches.

Monetary policy: Figure 9, finally, presents the results from a grid search over the Taylor-rule coefficients, which become active, of course, only after the peg has ended.[17] As before, the occurrence of sign switches in the simulations does not depend on individual parameter values. Only the length of the peg, for which the reversal occurs, is affected by changes of the parameters. In particular, a more aggressive inflation stabilization (i.e., a higher coefficient τ_π) requires a longer duration of the interest rate peg in order for the reversal to occur. Introducing history dependence by means of interest rate smoothing does not prevent reversals from occurring, either; see upper left panel in Figure 9.

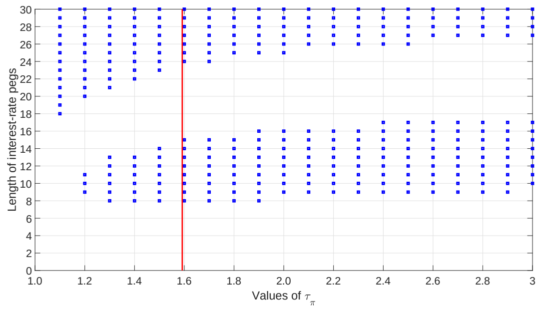
[17]Note that the agents perfectly anticipate the duration of forward guidance. Thus, they are perfectly aware of the point in time at which the Taylor rule is in place again.

Figure 9: Duration of nominal interest rate peg for which the reversal puzzle occurs for different values of the Taylor-rule parameters

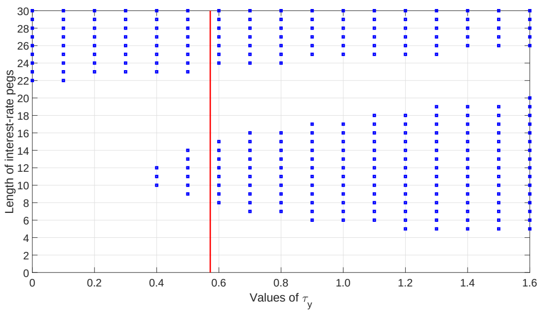
(a) Interest rate smoothing ρ



(b) MP reaction coefficient on inflation τ_π



(c) MP reaction coefficient on output τ_y



Note: The figure shows simulations for increasing durations of a pegged policy rate of up to thirty quarters. Once more, the blue points indicate a different sign compared to the scenario without interest rate peg. The red line marks the calibrated value for the respective parameters, which were used to carry out the analysis in Subsections 3.2 and 3.1.