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Coin migration between Germany and other euro area countries

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Non-technical summary

Research Question

Euro coins can be linked to a country of origin based on their national sides. For example, as a consequence of the movements of travellers between euro area countries, euro coins migrate to other countries. The paper studies the mixing of national coin stocks with coins having different national sides, which allows us to analyse the question as to whether coin flows between Germany and other euro area countries are balanced. This question is relevant because euro area countries with net outflows of coins are able to collect larger coin revenues than net receiving countries.

Contribution

The paper develops a model to characterise the mixing of euro coins held in transaction balances throughout the euro area as well as the development of the coin demand components. The model sets itself apart from existing models of coin migration by differentiating between domestic transaction balances, domestic hoarding and foreign demand. The branches of the Deutsche Bundesbank regularly collect data on the composition of transaction balances of €2, €1, 50 cent and 20 cent coins of coins with different national sides. These data as well as comparable data for other euro area countries are used to calibrate the model parameters. The paper thus provides the first comprehensive estimates on the importance of coin outflows for the German issuance of euro coins.

Results

The model developed in this paper indicates that the ratio of domestic to foreign coins held in transaction balances in the euro area countries should stabilise over time. Further empirical implications can be studied by setting values for the model parameters. According to the estimates, slightly more €2, €1, 50 cent and 20 cent coins have migrated to Germany from the other euro area countries than the other way round. It is thus largely domestic factors which explain the relatively large quantities of coins issued by the Federal Republic of Germany.

Nichttechnische Zusammenfassung

Fragestellung

Euro-Münzen können anhand ihrer nationalen Seite einem Herkunftsland zugeordnet werden. Beispielsweise in Folge von Münzmitnahmen im Reiseverkehr müssen Euro-Münzen nicht mehr in ihrem Herkunftsland umlaufen. Die Arbeit untersucht die Durchmischung der nationalen Umläufe im Euroraum durch Münzen mit verschiedenen nationalen Seiten. Dadurch wird auch die Frage betrachtet, ob sich Münzflüsse zwischen Deutschland und dem Euroraum ohne Deutschland die Waage halten. Diese Frage ist relevant, da Euro-Mitgliedsländer mit Nettoabflüssen von Münzen höhere Münzeinnahmen erzielen können als solche mit Nettozuflüssen.

Beitrag

Die Arbeit stellt ein Modell zur Beschreibung der Durchmischung der inländischen Transaktionskassenbestände im Euroraum durch Euro-Münzen mit unterschiedlichen nationalen Seiten sowie der Entwicklung der Münznachfragekomponenten vor. Durch Unterscheidung der inländischen Transaktionskasse, der inländischen Hortung sowie der Auslandsnachfrage erweitert die Arbeit vorhandene Modelle zur Beschreibung der Münzmigration. Es werden Daten aus den Filialen der Deutschen Bundesbank zur Zusammensetzung der Transaktionskassenbestände von 2-Euro-, 1-Euro-, 50-Cent- und 20-Cent-Münzen aus Münzen mit verschiedenen nationalen Seiten sowie vergleichbare Daten für andere Euro-Mitgliedsländer zur Modellkalibration verwendet. Dadurch kann zum ersten Mal umfassend die Bedeutung von Münzabflüssen für die deutschen Münzemissionen untersucht werden.

Ergebnisse

Das entwickelte Modell legt nahe, dass sich das Verhältnis aus inländischen zu ausländischen Münzen in den Transaktionskassen der Euro-Mitgliedsländer über die Zeit stabilisiert. Weitere empirische Implikationen ergeben sich durch Annahmen zur Festlegung der Modellparameter. Den Schätzungen zufolge sind bei den betrachteten Münzstückelungen etwas mehr Münzen aus dem restlichen Euroraum nach Deutschland geflossen als umgekehrt. Die relativ hohen Münzemissionen der Bundesrepublik Deutschland erklären sich demnach vornehmlich durch nationale Bestimmungsfaktoren.

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Coin migration between Germany and other euro area countries[1]

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Abstract

Euro coins have a common European side and an individual national side. Thanks to coin migration, coins bearing a panoply of national sides are in circulation throughout the euro area. In this paper, we model the mixing of coins circulating in the euro area countries and in particular the extent of coin migration in the euro area. A model calibration suggests that, for the coin denominations €2, €1, 50 cent and 20 cent roughly the same quantity of euro coins migrate from Germany to the rest of the euro area as vice versa. Accordingly, the relatively large quantities of coins issued by the Federal Republic of Germany are not materially explained by exports of coins to other euro area countries.

Keywords: Euro coins, coin circulation, coin mixing
JEL classification: E41

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1 Introduction

Euro coins have a common European side and a national side. As a result of coins being taken along by travellers, amongst other reasons, national coin holdings in the euro area consist of coins with a mixture of national sides. One reason for studying coin migration is its potential implications for the distribution of coin revenues between the euro area countries. Whereas the revenue from issuing banknotes is pooled in the euro area, national coin revenues are directly linked to the size of the demand for coins among national coin-issuing authorities. If more coins migrate from one euro area country to the rest of the euro area than vice versa, this euro area country is, to a degree, also meeting coin demand in other euro area countries. A member state reporting net outflows of euro coins can thus obtain more coin revenues than other member states.

At end-2017, a total value of €28 billion worth of euro coins were in circulation, of which the Bundesbank brought into circulation €8.4 billion net. The Bundesbank thus brought a net amount of 29.9% of all circulating euro coins into circulation, whereas it accounted only for 25.6% of the European Central Bank's fully paid-up capital. Apparently, Germany has issued relatively large quantities of euro coins, raising the question as to whether the German coin issuance is primarily driven by domestic demand or net outflows to other euro area countries. In the latter case, Germany would be able to collect larger coin revenues by exporting coins to other euro area countries. If the German coin issuance, on the contrary, is primarily driven by domestic demand, this would provide a justification for the relatively large German coin revenue. The purpose of the present paper is to examine coin migration between Germany and the other euro area countries using a calibrated model of the mixture of national coin stocks.

We will begin by developing a model which describes the mixture of national coin stocks with coins bearing a variety of different national sides. In the model presented below, coins transition between regions as well as between transaction balances and coin hoards. For the purposes of this paper, a coin hoard shall denote coins which are saved or collected or which are permanently lost (Deutsche Bundesbank, 2015). The model parameters are then calibrated on the basis of the available data on the euro coin circulation and the mixture of transaction balances as well as of assumptions regarding the evolution of transaction balances. The estimates conducted in this way indicate that roughly the same amount of coins travel from Germany to the rest of the euro area as vice versa. Accordingly, it is not coin exports but instead primarily national determinants which shape the issuance of coins by the Federal Republic of Germany. As discussed below, a set of assumptions is necessary when choosing the model parameters, but the results are robust to deviations from the baseline assumptions.

Overall, the literature analysing coin demand is scarce. Seitz, Stoyan and Tödter (2012) also propose a model for coin migration in the euro area. A common idea in our work and in Seitz et al. (2012) is that coins transition between coin demand components at fixed transition rates. However, this paper expands on the approach applied by Seitz et al. (2012) by developing a substantially extended model of coin migration and by using a new dataset on the mixing of euro coins. One important contribution is that this paper allows coins to disappear from active circulation into coin hoards, which provides an empirically relevant extension compared to Seitz et al. (2012). At end-February 2002, i.e. two months after the introduction of euro coins and notes, 68% of DM coins in circulation at the end of 2001 were still outstanding. This observation suggests that a large fraction of coins in circulation might well indeed wind up in hoards. Another important contribution of this paper is related to the calibration of the model parameters. In order to calibrate their model, Seitz et al. (2012) set net coin migration at zero in a baseline period. For the first time, we are able to use a dataset describing the composition of national coin stocks in Germany and the euro area excluding Germany, thus enabling us to estimate coin in- and outflows from the data. In addition, we are able to cover the €2, €1, 50 cent and 20 cent coin denominations, while Seitz et al. (2012) cover only the case of the €1 coin. Taken together, this paper provides the first comprehensive estimates on the importance of coin outflows for the German coin issuance.

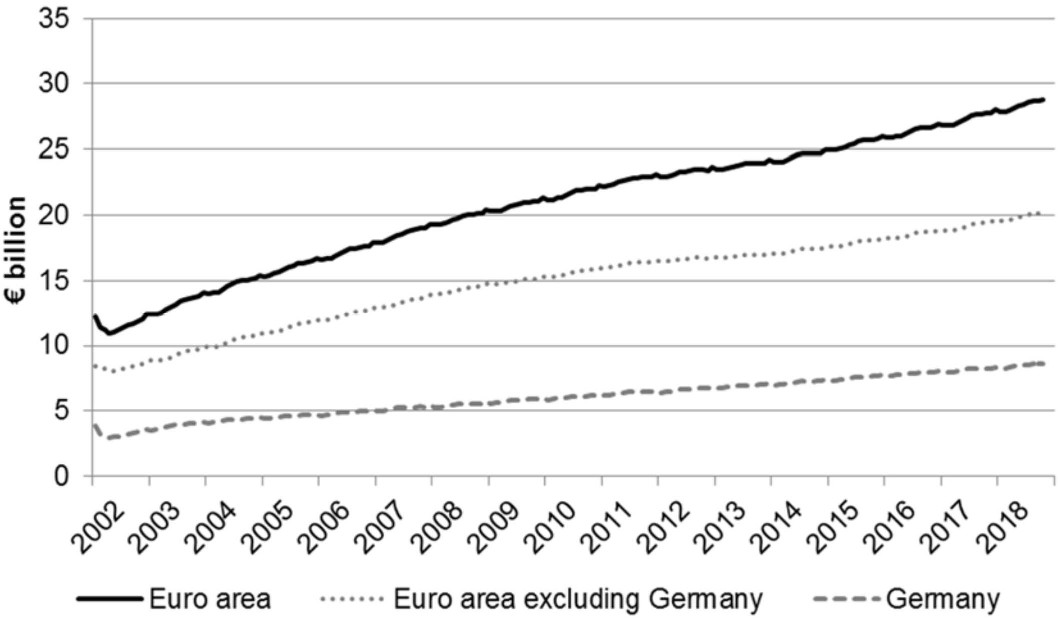
Altmann and Bartzsch (2014) and Deutsche Bundesbank (2015) study the holding of euro coins for transaction purposes in Germany using what is called the seasonal method. According to the results, transaction balances of euro coins in Germany amounted to €2.3 billion in 2011. That corresponded to 36% of all euro coins in circulation in Germany at that particular point in time, which also suggests that coin hoards could explain an empirically relevant share of the coin circulation. Goldin (1985) studies the lifetime and transaction balances of Israeli coins by looking at the dates stamped on each coin. However, knowledge of transaction balances does not translate directly to information on coin migration. The remaining coins could either be being hoarded or have migrated abroad.

The remainder of the paper is structured as follows. Section 2 describes the circulation of euro coins and sets out the dataset used for the subsequent analyses. Section 3 contains the model for coin migration. In Section 4, plausible parameters for this model are set, from which empirical implications are derived. Section 5 summarises and concludes.

2 Euro coin circulation

Figure 1 shows the development of the euro coin circulation over time. The cumulative net issuance of euro coins, defined as the cumulated difference between outpayments and inpayments, has risen evenly in both Germany and the euro area excluding Germany. According to the results presented in Altmann and Bartzsch (2014) and Deutsche Bundesbank (2015), changes in domestic transaction balances cannot explain the increase in German coin issuance. One purpose of this paper is to investigate the relative importance of coin hoarding and coin migration for the development of the national coin issuances using a calibrated coin mixture model.

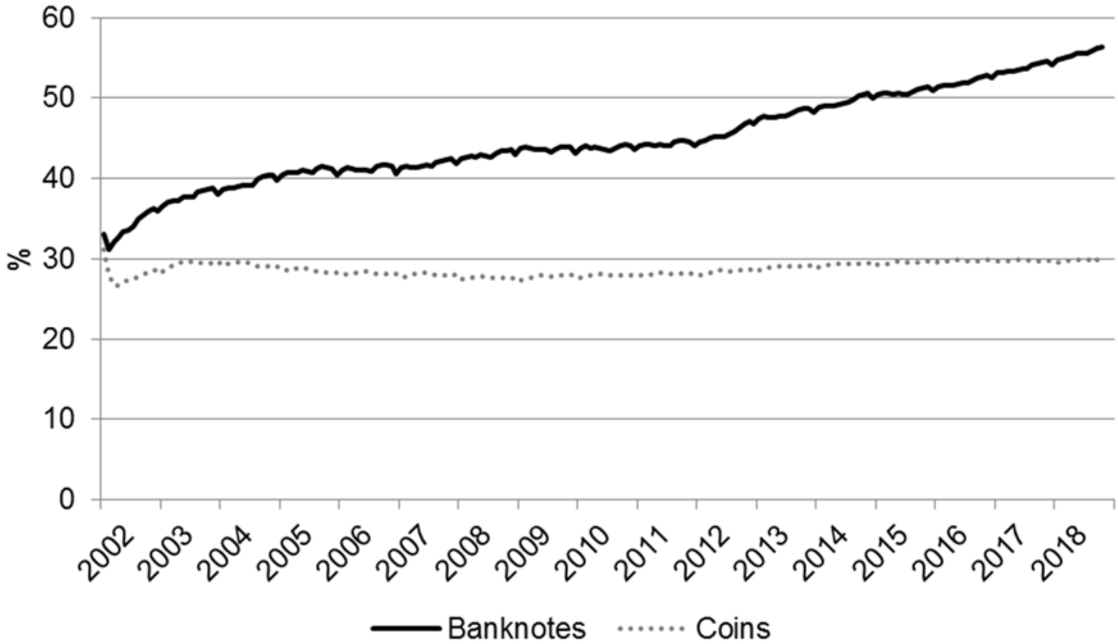
Figure 1: Circulation of euro coins in the euro area



Sources: Deutsche Bundesbank and European Central Bank.

It is well known that a considerable quantity of euro banknotes has migrated from Germany to other euro area countries and to non-euro area foreign countries (Bartzsch, Rösl, Seitz, 2011a; Bartzsch, Rösl, Seitz, 2011b; Bartzsch and Uhl, 2017; Deutsche Bundesbank, 2018a). There has been a clear growth in the Deutsche Bundesbank's share of cumulative net issuance of euro banknotes (Figure 2), rising from around 35% towards the end of 2002 to already as much as around 55% in December 2017. On the other hand, German euro coins' share of cumulative net coin issuance has remained fairly stable at around 30% for many years. Where domestic demand trends are identical in two countries in a monetary union, cash migration is reflected in a rising share for the country sending cash and a falling share for the country receiving cash. Whereas the Bundesbank's share in the case of banknotes actually is rising, Germany's share in the case of coins is relatively constant. This would be consistent with net outflows of coins from Germany to the rest of the euro area only if the domestic demand in the rest of the euro area were to rise more quickly than in Germany. There is no empirical evidence of this, however. Travellers using banknotes to cover their expected travel expenses are one important driver of the observed banknote movements across the euro area. Coins, however, are unsuitable for covering large expenses, indicating that travellers only carry the coins in their wallets. Therefore, there is no reason to expect coin stocks to be larger when travelling from Germany to another euro area country than when travelling in the other direction. This observation suggests that the value of coins leaving Germany should be broadly equal to the value of coins entering Germany.

Figure 2: Deutsche Bundesbank's share in cumulative net issuance



Sources: Deutsche Bundesbank and European Central Bank.

Based on the foregoing, we form the hypothesis that coin outflows from Germany to other euro area countries and coin inflows to Germany are balanced. The objective of this paper is to review this hypothesis using a coin mixture model. Empirically plausible parameters for the model are to be set using available data on coin mixture. For random samples of

€1 and €2 and 20 and 50 cent coins taken from cash lodgements in its branches, the Bundesbank establishes composition by national reverse sides on an annual basis. These cash lodgements result from the use of cash as a means of payment and thus originate from coin stocks held for transaction purposes. Consequently, this exercise tells us the country of manufacture for holdings of coins as transaction balances. Comparable surveys are also conducted in the other euro area countries. Table 1 provides an overview of the results. These show that the share of German euro coins for the above denominations ranged from 49% to 62% in Germany and between 13% and 20% in the rest of the euro area. For the first time, this dataset can be used to study coin migration in a scientific paper.

Table 1: Share of German euro coins in Germany and other parts of the euro area

	in Germany	in other parts of the euro area
€2 coin	61%	20%
€1 coin	49%	16%
50 cent coin	57%	15%
20 cent coin	62%	13%

Sources: Deutsche Bundesbank and Mint Directors Working Group.

Notes: Data from 2016. The share of German euro coins in the rest of the euro area is calculated as an unweighted average of the shares in 15 euro area countries.

3 Coin migration model

3.1 Coin shares in case of positive net issuances

We will develop a model for the mixing of coins held in transaction balances in two countries, called D for domestic and A for abroad, below. The model shares several ideas with the model developed in Seitz et al. (2012), but, as argued in Section 1 and in the appendix, expands it in several important dimensions. In the application presented in Section 4, the domestic country will be Germany and the other region will be the euro area excluding Germany. Let $T_{D,t}$ denote the number of coins held in transaction balances in country D at time t and $T_{A,t}$ the number of coins in active circulation in country A. Euro coins can be assigned by national sides to their countries of first issue.[2] However, thanks to coin migration, a given coin may not necessarily be in the country of first issue anymore. Let thus $T_{DA,t}$ denote the number of coins with the national side of country D located in transaction balances in country A at time t. $T_{DD,t}$, $T_{AA,t}$ and $T_{AD,t}$ are defined in similar fashion.

To begin with, we fix the time path of the transaction balances in country D, $T_{D,t}$, and in country A, $T_{A,t}$, by assuming fixed exogenous growth rates g_D and g_A. Thus, the following equations describe the size of the transaction balances in the two regions at time t.

(1) $T_{D,t}=(1+g_D)^t T_{D,0}$, t=0,1,…

(2) $T_{A,t}=(1+g_A)^t T_{A,0}$, t=0,1,…

We are not aware of any theoretical model for the determinants or size of the transaction balance of euro coins. Our paper does not attempt to close this gap, but focuses instead on describing coin mixture, assuming an exogenous path for the size of the transaction balances.

There are two conceivable channels of coin migration: coins being taken along by travellers and coin transports effected by professional cash handlers. In both cases, a coin is removed from active domestic circulation and transported abroad. As in Seitz et al. (2012) our model allows coins to migrate between the two regions at fixed rates. Let α_{DA} denote the share of coins which migrate from transaction balances in country D to transaction balances in country A in each period. Likewise, α_{AD} is the share of coins which migrate from active circulation in country A to active circulation in country D in each period.[3] Then, in period t, $\alpha_{DA}T_{D,t-1}$ coins migrate from country D to country A, of which $\alpha_{DA}T_{DD,t-1}$ bear the national side of country D. Likewise, $\alpha_{AD}T_{A,t-1}$ coins migrate from country A to country D, of which $\alpha_{AD}T_{AA,t-1}$ coins bear the national side of country A.

Overall, coin hoardings – coins which are saved or collected or which are permanently lost – are likely an important component of coin demand (Deutsche Bundesbank, 2015). One contribution of our paper is that coins are allowed to disappear from active

[2] We make the assumption that newly issued euro coins bear the national side of the issuer. However, where cross-border transports of euro coins between coin-issuing authorities occur, it would no longer be possible to accurately assign these coins to their country of origin. For the coin denominations analysed below, there have been no coin shipments between the Deutsche Bundesbank and coin-issuing authorities in other euro area countries. This means that the attribution in this paper of euro coins by national sides to Germany and to the euro area excluding Germany is reliable.

[3] The discussion paper by Seitz, Stoyan and Tödter (2009) interprets the coin mixing process as a Markov chain. In this view, a single coin has a probability α_{DA} to switch from region D to region A within a year and a probability α_{AD} to move from region A to region D. Due to the law of large numbers, this implies that a total of $\alpha_{DA}T_{D,t-1}$ coins migrate from region D to region A at time t and α_{AD} $T_{A,t-1}$ coins move from region A to region D at time t.

circulation into coin hoards.[4] Coins migrate from transaction balances to hoards, for instance, if they are lost or if consumers remove coins from their wallets and put them into some sort of collection receptacle. We assume that each period, a certain share of coins α_D is withdrawn from active circulation in country D and a share of coins α_A is withdrawn from active circulation in country A. Thus, $\alpha_D T_{D,t-1}$ coins disappear from active circulation in country D at time t, of which $\alpha_D T_{DD,t-1}$ bear the national side of country D. $\alpha_A T_{A,t-1}$ coins disappear from active circulation in country A at time t, of which $\alpha_A T_{AA,t-1}$ bear the national side of country A.

We also allow for the issuance of new coins. Let $\Delta N_{D,t}$ denote the net issuance of coins in country D at time t and $\Delta N_{A,t}$ the net issuance of country A. For the time being, we assume that both net issuances are positive, $\Delta N_{D,t}>0$ and $\Delta N_{A,t}>0$. By assumption, new issuances of coins bear the national side of the issuing country.[5] New coins are issued to satisfy the additional demand for transaction purposes, $g_D T_{D,t-1}$ and $g_A T_{A,t-1}$, and to replace coins that have disappeared into hoardings, $\alpha_D T_{D,t-1}$ and $\alpha_A T_{A,t-1}$. In addition, from the perspective of country D, $\alpha_{DA} T_{D,t-1}-\alpha_{AD} T_{A,t-1}$ coins have migrated to country A in net terms and have to be replaced by new coins. Thus, the following equations apply.

(3) $\Delta N_{D,t}=(g_D+\alpha_D+\alpha_{DA})T_{D,t-1}-\alpha_{AD}T_{A,t-1}$

(4) $\Delta N_{A,t}=(g_A+\alpha_A+\alpha_{AD})T_{A,t-1}-\alpha_{DA}T_{D,t-1}$

New issuance $\Delta N_{D,t}$ and $\Delta N_{A,t}$ offsets the difference between desired transaction balances $(1+g_D)T_{D,t-1}$ and $(1+g_A)T_{A,t-1}$ and coins actually existing in both regions following hoarding and migration, $(1-\alpha_D-\alpha_{DA})T_{D,t-1}+\alpha_{AD}T_{A,t-1}$ and $(1-\alpha_A-\alpha_{AD})T_{A,t-1}+\alpha_{DA}T_{D,t-1}$.

We are now in the position to characterise the development of the composition of transaction balances over time. Let the following apply to the share of coins bearing the national side of country D in transaction balances in country A:

$$\tau_{DA,t} = \frac{T_{DA,t}}{T_{A,t}};\ t = 0,1,\dots$$

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[4] Coins in circulation outside the euro area are not explicitly modelled. In the literature, the assumption that coin stocks outside the euro area are insignificant is regarded as uncontroversial (European Central Bank, 2017). Coins in circulation outside the euro area could be notionally assigned to domestic hoards and thus be captured indirectly. The analysis presented here hinges on the assumption that both hoarded coins and coins circulating outside the euro area have disappeared completely from active domestic circulation.

[5] This assumption is open to criticism if, previously, $\Delta N_{D,t^*}<0$ held for at least one $t^*<t$ as, in that case, the coin-issuing authority of country D had to collect coins in a previous period, and it would be plausible that the coin-issuing authority initially disburses vault holdings, which consist of coins with mixed national sides. In order not to make the description unnecessarily complex, we will abstract from this special feature here. However, this case is covered in the technical appendix.

$\tau_{DD,t}$, $\tau_{AA,t}$ and $\tau_{AD,t}$ are defined accordingly, implying that $1=\tau_{DD,t}+\tau_{AD,t}$ and $1=\tau_{DA,t}+\tau_{AA,t}$ hold. As argued above, the number of national coins held in national transaction balances is characterised by

(5) $$T_{DD,t}=(1-\alpha_D-\alpha_{DA})T_{DD,t-1}+\alpha_{AD}T_{DA,t-1}+\Delta N_{D,t}$$

(6) $$T_{AA,t}=(1-\alpha_A-\alpha_{AD})T_{AA,t-1}+\alpha_{DA}T_{AD,t-1}+\Delta N_{A,t}$$

To summarise the above discussion, $\alpha_D T_{DD,t-1}$ and $\alpha_A T_{AA,t-1}$ coins bearing the national side disappear from active circulation into coin hoards. $\alpha_{DA}T_{DD,t-1}$ coins bearing the national side of country D have migrated abroad, while $\alpha_{AD}T_{DA,t-1}$ coins bearing the national side of country D have returned from country A. Likewise, $\alpha_{AD}T_{AA,t-1}$ coins bearing the national side of country A have migrated to country D and $\alpha_{DA}T_{AD,t-1}$ coins have returned from country D to country A. $\Delta N_{D,t}$ and $\Delta N_{A,t}$ reflect the issuance of new coins. After scaling equation (5) by $T_{D,t}$ and equation (6) by $T_{A,t}$, we obtain the following difference equations for the evolution of the coin shares $\tau_{DD,t}$ and $\tau_{AA,t}$.

(7) $$(1+g_D)\tau_{DD,t}=g_D+\alpha_D+\alpha_{DA}+(1-\alpha_D-\alpha_{DA})\tau_{DD,t-1}-\alpha_{AD}\eta_{t-1}\tau_{AA,t-1}$$

(8) $$(1+g_A)\tau_{AA,t}=g_A+\alpha_A+\alpha_{AD}+(1-\alpha_A-\alpha_{AD})\tau_{AA,t-1}-\alpha_{DA}\eta_{t-1}^{-1}\tau_{DD,t-1}$$

$\eta_{t-1}=T_{A,t-1}/T_{D,t-1}$ denotes the ratio between the two sets of coins in domestic circulation at time t-1.

The development of the coin shares $\tau_t=(\tau_{DD,t}, \tau_{AA,t})'$ is thus described by a linear difference equation $\tau_t=b+A_{t-1}\tau_{t-1}$ with some starting value τ_0. In case the national coin issuances are both positive, $\Delta N_{D,t}>0$ and $\Delta N_{A,t}>0$, the elements of this equation are defined as follows.

$$b=\begin{pmatrix}1+g_D & 0\\ 0 & 1+g_A\end{pmatrix}^{-1}\begin{pmatrix}g_D+\alpha_D+\alpha_{DA}\\ g_A+\alpha_A+\alpha_{AD}\end{pmatrix}$$

$$A_{t-1}=\begin{pmatrix}1+g_D & 0\\ 0 & 1+g_A\end{pmatrix}^{-1}\begin{pmatrix}1-\alpha_D-\alpha_{DA} & -\alpha_{AD}\eta_{t-1}\\ -\alpha_{DA}\eta_{t-1}^{-1} & 1-\alpha_A-\alpha_{AD}\end{pmatrix}$$

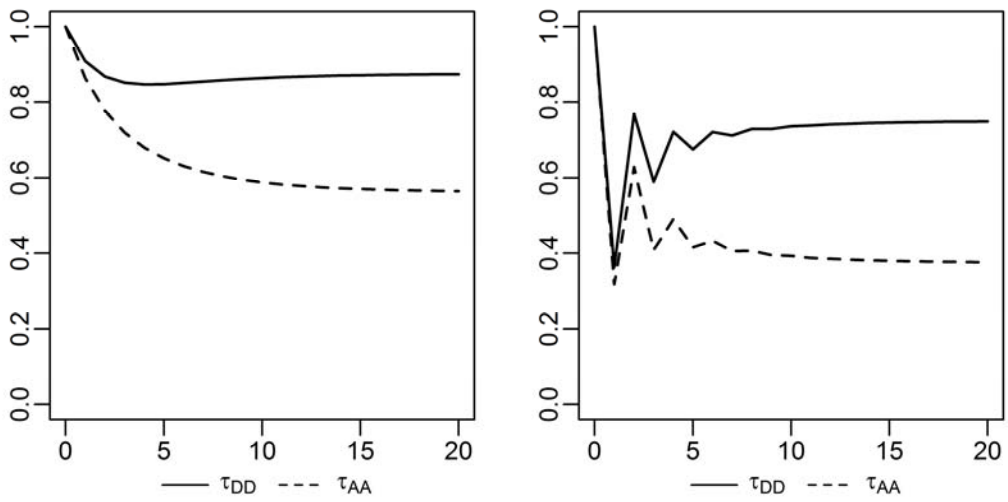
If $g_D=g_A$, then η is constant and the linear difference equation $\tau_t=b+A\tau_{t-1}$ continues to hold for all t.[6] As demonstrated in a technical appendix at the end of the paper, τ_t will then converge towards $(I-A)^{-1}b$, where I is the two-dimensional identity matrix.

The convergence of coin shares in the two countries under observation in the case of positive net issuance is shown in Figure 3 for hypothetical parameters. Due to a higher hoarding parameter α_D and a positive quantity of coins in circulation abroad – $\alpha_{DA}-\alpha_{AD}\eta>0$ – coin issuance by country D is higher than that of country A. This is reflected in

[6] If $g_D\neq g_A$, either the domestic net issuance $\Delta N_{D,t}$ or the foreign net issuance $\Delta N_{A,t}$ will eventually turn negative, as shown in the technical appendix at the end of the paper. The coin shares will still converge, however. The time indices have been dropped to reflect that the parameters are constant.

country D's higher share of coins in circulation. The right-hand panel of Figure 3 shows a special case which is probably empirically implausible. In this case, the migration parameters α_{DA} and α_{AD} take on such high values that more coins migrate abroad from transaction balances than remain in the country. As a consequence of the large migration parameters, the coin shares fluctuate sharply before then converging.

Figure 3: Convergence of coin shares given positive coin issuance



Notes: The left-hand panel shows a case with little migration; the parameters are $\tau_{DD,0}=\tau_{AA,0}=1$, $g_D=g_A=0.1$, $\alpha_D=0.2$, $\alpha_A=0.1$, $\alpha_{DA}=0.15$, $\alpha_{AD}=0.1$, $\eta_0=1$. The right-hand panel shows a case with a lot of migration; the parameters are $\tau_{DD,0}=\tau_{AA,0}=1$, $g_D=g_A=0.1$, $\alpha_D=0.2$, $\alpha_A=0.1$, $\alpha_{DA}=0.75$, $\alpha_{AD}=0.7$, $\eta_0=1$.

Additional material is contained in an appendix at the end of this paper. For $\Delta N_{D,t}>0$ and $\Delta N_{A,t}>0$, it presents an analysis of the effects of parameter changes on long-run coin shares as well as a detailed comparison of our model with the results presented in Seitz et al. (2012). The appendix also contains an extension of the model for the case of non-negative net issuances.

3.2 Coin demand components

The assumptions made in the previous section characterise the mixture of coins held in transaction balances, but also imply equations for the development of the overall cumulative net coin issuance and its components. The cumulative net coin issuance of a region D (A) in year t, $N_{D,t}$ ($N_{A,t}$), can be decomposed into the domestic transaction balance $T_{D,t}$ ($T_{A,t}$), domestic hoarding $H_{D,t}$ ($H_{A,t}$) and circulating coins abroad $A_{D,t}$ ($A_{A,t}$).

$$N_{D,t}=T_{D,t}+H_{D,t}+A_{D,t};\ t=0,1,\ldots$$

$$N_{A,t}=T_{A,t}+H_{A,t}+A_{A,t};\ t=0,1,\ldots$$

According to Equations (1) and (2), domestic transaction balances are given by $T_{D,t}=(1+g_D)^t T_{D,0}$ and $T_{A,t}=(1+g_A)^t T_{A,0}$. α_D and α_A denote the share of domestic transaction balances hoarded annually. Then, the development of domestic hoarding is described by the difference equations shown below.

$$H_{D,t}=H_{D,t-1}+\alpha_D T_{D,t-1};\ t=1,\ldots$$

$$H_{A,t}=H_{A,t-1}+\alpha_A T_{A,t-1};\ t=1,\ldots$$

Defining

$$c_{D,t}=\begin{cases} t, g_D=0 \\ \frac{(1+g_D)^t-1}{g_D}, g_D\neq 0 \end{cases}$$

and

$$c_{A,t}=\begin{cases} t, g_A=0 \\ \frac{(1+g_A)^t-1}{g_A}, g_A\neq 0 \end{cases}$$

and assuming $H_{D,0}=H_{A,0}=0$, the solutions to these difference equations are $H_{D,t}=c_{D,t}\alpha_D T_{D,0}$ and $H_{A,t}=c_{A,t}\alpha_A T_{A,0}$.

α_{DA} and α_{AD} denote the share of coins from the domestic transaction balances of country D and country A which migrate annually to, respectively, country A and country D. Thus, $\Delta A_{D,t}=\alpha_{DA}T_{D,t-1}-\alpha_{AD}T_{A,t-1}$ and $\Delta A_{A,t}=\alpha_{AD}T_{A,t-1}-\alpha_{DA}T_{D,t-1}$ represent coin net migration within period t. Taking the sums across periods, the difference equations below describe the development of foreign demand $A_{D,t}$ and $A_{A,t}$.

$$A_{D,t}=A_{D,t-1}+\alpha_{DA}T_{D,t-1}-\alpha_{AD}T_{A,t-1};\ t=1,\ldots$$

$$A_{A,t}=A_{A,t-1}+\alpha_{AD}T_{A,t-1}-\alpha_{DA}T_{D,t-1};\ t=1,\ldots$$

Assuming $A_{D,0}=A_{A,0}=0$, the solutions to these difference equations are $A_{D,t}=c_{D,t}\alpha_{DA}T_{D,0}-c_{A,t}\alpha_{AD}T_{A,0}$ and $A_{A,t}=-A_{D,t}$.

This distinction of domestic transaction balances, domestic hoarding and foreign demand is common in the literature analysing cash demand (see, e.g., Deutsche Bundesbank, 2018a). Domestic transaction balances and domestic hoarding reflect domestic demand and are thus positive numbers. For regions within a monetary union, foreign demand can turn negative, however. If $A_{D,t}>0$, more coins have migrated from region D to region A than vice versa and the cumulative net issuance, $N_{D,t}$, exceeds domestic demand for coins, $T_{D,t}+H_{D,t}$. In the opposite case, $A_{D,t}<0$, region D is a net receiver of coins and the cumulative net issuance, $N_{D,t}$, is smaller than domestic demand for coins, $T_{D,t}+H_{D,t}$. Thus, domestic demand for coins is partially satisfied by coins from region A. The main objective of the paper is to analyse $A_{D,t}$ and $A_{A,t}$, as these variables summarise coin net flows between the two regions of a monetary union.

4 Model implications with pre-set parameters

4.1 Choosing model parameters

The model presented in the last section describes the time path of coin demand components and the transitional dynamics of shares of each national side in transaction balances. We can study the empirical implications by setting the starting values as well as the model parameters, with the €2 and €1 and 50 and 20 cent denominations each being modelled separately. We will look at two geographical units below: Germany and the euro area excluding Germany. Figure 1 shows that the net issuance of Germany and of the euro area excluding Germany has typically been positive since the introduction of euro coins and notes. This means that the case of the model presented in Section 3 in which $\Delta N_{D,t}>0$ and $\Delta N_{A,t}>0$ hold is relevant.

The discussion from the previous section thus implies the following two equations for the development of the cumulative net issuance

$$N_{D,t}=N_{D,0}+(g_D+\alpha_D+\alpha_{DA})c_{D,t}T_{D,0}-\alpha_{AD}c_{A,t}T_{A,0}$$

$$N_{A,t}=N_{A,0}+(g_A+\alpha_A+\alpha_{AD})c_{A,t}T_{A,0}-\alpha_{DA}c_{D,t}T_{D,0}$$

and the following equation for the coin shares at time t

$$\tau_t=(I+A_{t-1}+\ldots+A_{t-1}\cdot\ldots\cdot A_1)b+A_{t-1}\cdot\ldots\cdot A_0\tau_0$$

with b and A_i, i=0,…,t-1 defined as in Section 3.1. While the model yields four equations, it has six unknown parameters (g_D, α_D, α_{DA}, g_A, α_A, α_{AD}) and unknown starting values ($T_{D,0}$, $T_{A,0}$). The general strategy applied in the following will be to fix the growth rates of domestic transaction balances g_D and g_A and the unknown starting values. A numerical procedure is then used to determine α_D, α_A, α_{DA} and α_{AD} such that the implied values for $N_{D,t}$, $N_{A,t}$, $\tau_{DD,t}$ and $\tau_{AA,t}$ correspond to their empirically observed values.[7] Table 2 shows an overview of the preferred parametrisation. All analyses conducted in this paper depend

[7] The system of equations is solved using the multivariate Newton procedure for solving non-linear equation systems (Judd, 1998). Owing to numerical studies, the existence of a unique solution is suspected, but no formal derivation of this statement exists. The robustness of the numerical results is corroborated by the fact that different starting values lead to identical conclusions. Comparable results are also produced if the parameter values are determined in a non-approximative fashion using the limits of $\tau_{DD,t}$ and $\tau_{AA,t}$.

on the assumptions made when setting up the model as well as the parameter choices and should thus be interpreted with care. To reflect the latter caveat, we implement a set of robustness exercises.

The model developed in the previous section describes the path of coin demand components as a function of the holding of transaction balances; assumptions regarding the time path of transaction balances therefore play a central role. Merely defining the growth rates g_D and g_A already yields plausible values for the further model parameters. In what follows, it is assumed that the holdings of euro coins for transaction purposes remain constant; therefore, $g_D=g_A=0$. While consumers acquire banknotes from automated teller machines or bank tellers and thus can shape their transaction balances of banknotes by deciding how much to withdraw, coin holdings in wallets are created by receiving change. Consumers therefore have comparatively less scope for manipulating their euro coin holdings for transaction purposes. Seen from this perspective, coin holdings for transaction purposes are a by-product of the use of cash as a medium of payment and are less the outcome of conscious decisions concerning the size of transaction balances held. This underlies the assumption that holdings of euro coins for transaction purposes are constant. Estimates regarding holdings of euro banknotes for transaction purposes in Germany are available as from the year 2008; over the observed estimation horizon, holdings of euro banknotes for transaction purposes are constant (Bartzsch and Uhl, 2017; Deutsche Bundesbank, 2018a). This is consistent with the assumption that transaction balances of euro coins are likewise constant. According to a Deutsche Bundesbank survey of the public, at the end of 2002 respondents in Germany held, on average, around 16 coins per person in their wallets with a total value of €5.62 (Deutsche Bundesbank, 2003). According to the results of the Deutsche Bundesbank's payment behaviour study, in 2008 respondents were carrying an average value of €6.70 worth of euro coins on their person; in 2011, the figure was €5.90; in 2014, €5.73; and in 2017, €6.29 (Deutsche Bundesbank, 2018b). This means that each German carries, on average, somewhere around €6 worth of coins in their wallets. These data likewise indicate constant holdings of euro coins for transaction purposes in Germany. The appendix at the end of this paper investigates possible consequences of deviating from the assumption $g_D=g_A=0$ for the growth rates of domestic transaction balances.

We will choose the end of 2002 as the starting point of our analysis. At this point in time, one-off effects on coins in circulation caused by the introduction of euro cash had already dissipated to a large extent (see Figure 1). For small values of the parameters α_D, α_{DA}, α_A and α_{AD}, migration and hoarding during the early stages should be negligible, which means it should be possible to set the domestic transaction balances T_D and T_A via coins in circulation $N_{D,0}$ and $N_{A,0}$ at the end of 2002.[8] For the starting values $\tau_{DD,0}$ and $\tau_{AA,0}$, 1 is chosen, which means that the initial supply was effected using domestic coins.[9]

Data on shares of German coins in transaction balances are available for Germany and 15 other euro area countries for the year 2016. $\tau_{DD,t}$ and $\tau_{AA,t}$ are determined for this period, with the share of foreign euro coins in foreign transaction balances, $\tau_{AA,t}$, being calculated as the weighted mean of data for individual euro area countries.[10] The selected weights derive from the respective size of transaction balances. Since the model uses an annual frequency, t takes on the value of 14 in 2016. Cumulative net issuance $N_{D,t}$ and $N_{A,t}$ at the end of 2016 is likewise shown in Table 2. Seitz et al. (2012) only use information on the share of German €1 coins in Germany for the model calibration. Compared to their work, we are able to cover a wider range of coin denominations and have information on the share of German coins in Germany and the euro area excluding Germany. The latter allows us to freely analyse coin flows between Germany and the other euro area countries, while Seitz et al. (2012) make the assumption that coin flows are balanced.

With the exception of the €1 coin, the hoarding parameters calculated for Germany, α_D, are much larger than the hoarding parameters calculated for the euro area excluding Germany, α_A. This suggests that coin demand in Germany is driven by hoarding to a larger extent than coin demand in other parts of the euro area. As can be inferred from Table 2, the migration parameters α_{DA} exceed the migration parameters α_{AD}. This is a plausible result as the euro area excluding Germany is larger than Germany, making it less probable that a coin from this region ends up in Germany. According to the information given in Table 2, Germany accounts for 28.2% of the euro area's transaction balances aggregated across the denominations included in this paper. The German share in coins actively in circulation throughout the euro area thus exceeds Germany's population share of 24.2%, indicating that Germans hold somewhat larger transaction balances of euro coins. Moderately larger transaction balances of coins in Germany are in line with previous research that finds that cash balances in wallets are typically larger

[8] The euro cash changeover could have been associated with special demand for coins for hoarding purposes. For example, consumers in the euro area might have shown an increased interest in collecting the new coins. These special effects can be incorporated into the analysis by adjusting the initial size of the domestic transaction balances. Varying the initial size of the transaction balances has little impact on the estimates for net coin migration.

[9] In an investigation of the robustness of the results, $T_{D,0}$, $T_{A,0}$, $\tau_{DD,0}$ and $\tau_{AA,0}$ are calculated under the assumption that the usual annual extent of hoarding and migration had already taken place by the end of 2002. The values shown in Table 2 for α_D, α_{DA}, α_A and α_{AD} are used for this adjustment. This does not make any meaningful changes to the results of this study.

[10] For the denominations under investigation, the 12 euro area countries which introduced euro cash in 2002 accounted for a share of between 97.1% and 98.3% of all euro coins in circulation as at the end of 2016. The subsequent expansion of the euro area is therefore not likely to have had any major impact on the results.

in Germany than in other countries (Esselink and Hernández, 2017; Bagnall, Bounie, Huynh, Kosse, Schmidt, Schuh and Stix, 2016). Coins held in wallets, however, are only a subset of the sum total of coins in active circulation, which comprise coins held for transaction purposes by households, retailers and credit institutions.

Table 2: Model parametrisation

	€2 coin	€1 coin	50 cent coin	20 cent coin
$N_{D,0}$	832	921	897	1352
$N_{A,0}$	1727	2762	2860	3797
$N_{D,14}$	2082	1590	1599	3213
$N_{A,14}$	3733	5388	4302	7565
$T_{D,0}$	832	921	897	1352
$T_{A,0}$	1727	2762	2860	3797
$\tau_{DD,0}$	1	1	1	1
$\tau_{AA,0}$	1	1	1	1
$\tau_{DD,14}$	0.61	0.49	0.57	0.62
$\tau_{AA,14}$	0.80	0.87	0.87	0.88
g_D	0	0	0	0
g_A	0	0	0	0
α_D	0.117	0.065	0.067	0.119
α_{DA}	0.094	0.080	0.060	0.060
α_A	0.078	0.063	0.032	0.064
α_{AD}	0.050	0.031	0.022	0.029

Sources: Deutsche Bundesbank, European Central Bank, Mint Directors Working Group and own calculations.

Note: Coin demand data in million pieces. t=0 corresponds to the year 2002, t=14 to the year 2016.

4.2 Model implications

Once the model parameters have been set, the model equations can be simulated up to the current end of the data, i.e. the year 2016. This results in values for the coin demand components $T_{D,t}$ and $T_{A,t}$, $H_{D,t}$ and $H_{A,t}$ as well as $A_{D,t}$ and $A_{A,t}$. The results are shown in Table 3. It is striking that, especially for €2, 50 cent and 20 cent coins, hoards in Germany account for a larger share of coins in circulation than do hoards in the euro area excluding Germany. Aggregated across the denominations included in Table 3, coin hoards account for 61.6% of the German coin circulation. This estimate is corroborated by the observed trend pattern of DM coins in circulation after the introduction of euro cash. After the euro was introduced in physical form in 2002, DM and euro cash could be used in parallel until the end of February 2002. By then, coins in active circulation in Germany had been replaced by euro coins, suggesting that the remaining DM coins in circulation at that point in time have been hoarded. By comparing the amount of DM coins in circulation at end-February 2002 with that as at end-2001, we estimate that roughly 68% of the DM coins in circulation were being hoarded as at end-2001, which exceeds the figure for the share of euro coins hoarded in Germany presented above. Taken together, the results presented in Table 3 indicate that one of the primary explanations for the relatively high issuance of coins in Germany is hoarding activity. It is possible that euro coins are being collected relatively frequently in Germany or are being held as a store of value.[11]

[11] The Federal Republic of Germany issues a relatively large number of €2 commemorative coins which are legal tender like other circulating coins; these are €2 coins with a special commemorative design on the national side (see https://www.ecb.europa.eu/euro/coins/comm/html/index.en.html, accessed on 26 February 2019). This is consistent with a relatively pronounced propensity to collect coins in Germany. On the other hand, these €2 commemorative coins are in normal circulation and the Federal Republic of Germany can issue more of these coins as the demand for German €2 coins is relatively high. A higher propensity to collect coins in Germany is also supported by the fact that the Federal Republic of Germany issues a particularly large number of collectors' coins.

Table 3: Implications of the model

	€2 coin	€1 coin	50 cent coin	20 cent coin
$T_{D,14}$	832	921	897	1352
$T_{A,14}$	1727	2762	2860	3797
$H_{D,14}$	1360	841	847	2251
$H_{A,14}$	1896	2454	1297	3378
$A_{D,14}$	-110	-172	-145	-390
$A_{A,14}$	110	172	145	390
τ_{DD}	0.62	0.44	0.54	0.61
τ_{AA}	0.78	0.88	0.81	0.86

Note: t=14 corresponds to the year 2016. Coin demand data in million pieces. τ_{DD} and τ_{AA} denote the long-run shares described in Section 3.1 and the technical appendix.

The results for foreign demand $A_{D,t}$ and $A_{A,t}$ are relevant for the main question addressed in this paper, i.e. whether more coins have migrated from Germany to the rest of the euro area than vice versa. According to the results, foreign demand for German euro coins is negative in all denominations under observation. Accordingly, more euro coins have migrated to Germany than vice versa, i.e. from Germany to the rest of the euro area. Compared with total coins in circulation, however, the value of coins in circulation abroad is small for all denominations. Aggregated across the coin denominations covered in our study, German euro coins in circulation abroad amount to -€0.5 billion, or -7.6% of the value of German euro coins in circulation in these denominations.[12] Overall, our results indicate that the relatively large German coin issuance is not explained by coin exports to other euro area countries. Neither Germany nor the euro area excluding Germany is able to generate significant coin revenues by exporting coins to other countries.

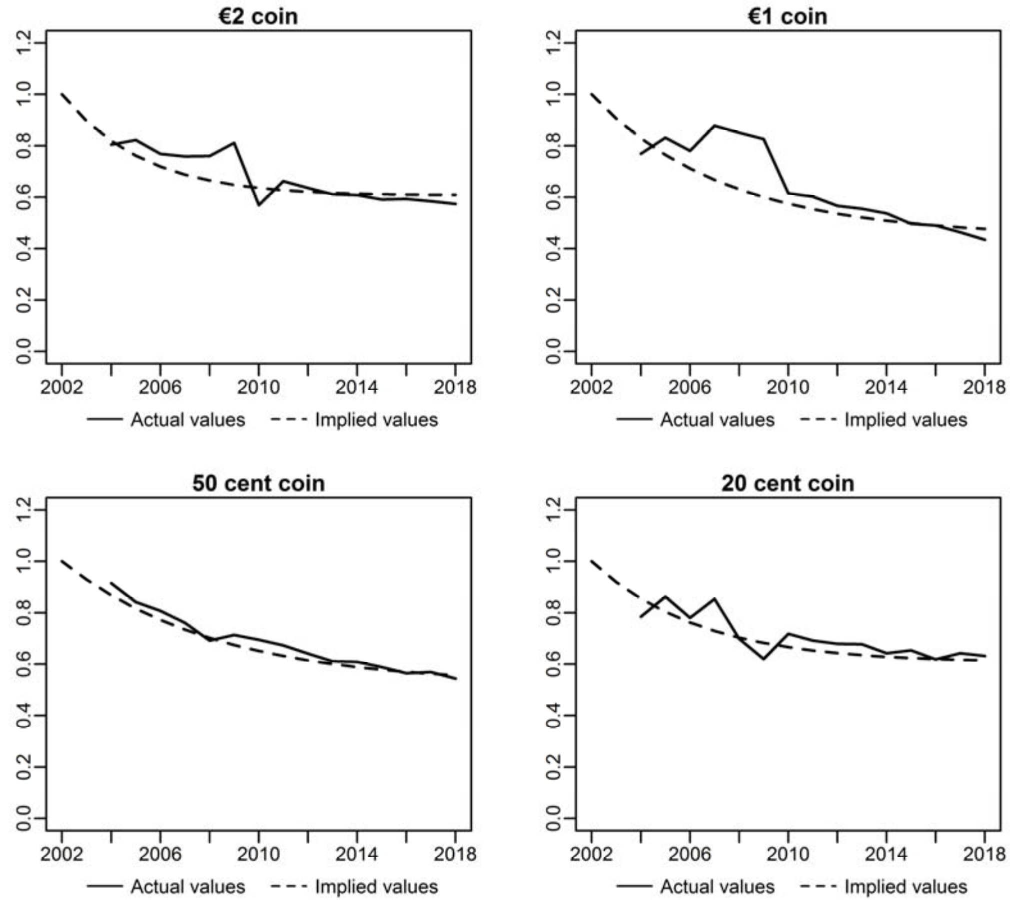
According to Table 3, in the long run 62% of the €2 coins in active circulation in Germany will be German, compared to 44% of the €1 coins, 54% of the 50 cent coins and 61% of the 20 cent coins. It is interesting to note that the current coin shares shown in Table 2 are

[12] Towards the end of 2016, the value of euro coins with a face value of 1 cent to 10 cent brought into circulation by the Bundesbank amounted to €0.9 billion, or 10.7% of all German coins in circulation. Even if all these euro coins were circulating abroad, the volume of all German coins circulating abroad would be, on the whole, moderate. We suspect, however, that it is precisely small coins which are being hoarded in Germany to a relatively large extent, as they are lost more frequently than higher-value coins (Deutsche Bundesbank, 2015). The key takeaways from the study, which are based on coins with a face value of more than 20 cent, can therefore be held to be true for the entire volume of German euro coins in circulation.

already quite close to these theoretically implied thresholds. Thus, we can conjecture that the distribution of national sides has already stabilised in the euro area. It is worth noting that German coins are less prevalent in transaction balances in the euro area excluding Germany than foreign coins in transaction balances in Germany. This is to be expected, given that Germany is smaller than the euro area excluding Germany.

For the shares of German euro coins in Germany, data are available as from 2004, though they are not based on a uniform data collection mode. In particular, data from 2011 onwards are based on a considerably larger sample and should thus be regarded as more reliable. Figure 4 shows the available actual values for the share of German euro coins compared with the values implied by the model. On the whole, the time series of the values implied by the model are close to the actual values. In the years up to and including 2010, the actual values fluctuate quite strongly; the variation of the sample is possibly higher in this period on account of the small sample size. In the case of the €2 and €1 coin, the values implied by the model are, in some cases, well below the actual values from 2005 to 2009. Over this period, the actual values for the share of German coins of these denominations exhibit behaviour which, at least in some cases, is quite surprising: coin shares rise in some years only to drop back down considerably between 2009 and 2010 and converge towards the values implied by the model. It is possible that the observed deviation between actual values and those implied by the model are, to a degree, attributable to an imprecise calculation of the actual coin shares.

Figure 4: Actual and implied shares of German coins in Germany



Sources: Deutsche Bundesbank and author's own calculations.

Notes: The data collection method used for coin shares is not fully comparable across the observation period. For instance, in 2011 the data collection method was thoroughly revised and the sample size enlarged considerably.

5 Conclusions

This paper looks at the migration of coins between Germany and the euro area excluding Germany, thereby addressing the question whether the Bundesbank is primarily satisfying domestic coin demand or whether Germany is able to achieve higher coin revenues by exporting coins to other euro area countries.

At end-2017, the total euro coin circulation stood at €28 billion, €8.4 billion of which was put into circulation by the Bundesbank. The share of German euro coins in the total euro coin circulation has remained fairly stable at around 30% since the introduction of euro cash. Given significant and persistent net outflows of coins from Germany to other euro area countries, however, one would expect a rising share of German euro coins. Based on the pattern of coin circulation, we formulate the hypothesis that coin flows between Germany and the euro area excluding Germany are balanced. The paper sets out to test this hypothesis using a calibrated coin mixture model. We start by characterising the movements of coins between regions and between transaction balances and coin hoards. This model of coin migration indicates a convergence of the ratio of German coins to foreign coins in transaction balances in Germany as well as in the euro area excluding Germany.

Empirical implications of the model can be derived by setting parameters. The results of the preferred parameter settings indicate that a total of 110 million more €2 coins, 172 million more €1 coins, 145 million more 50 cent coins and 390 million more 20 cent coins have migrated from other euro area countries to Germany than vice versa. These results indicate that net migration of coins between Germany and the rest of the euro area is small. To reflect the fact that the results are derived from assumptions, we implement a set of robustness exercises. The main conclusion of the paper – that net migration of coins between Germany and other parts of the euro area is small – is robust.

According to these results, the theory that the Federal Republic of Germany would obtain particularly large seigniorage income by meeting the demand for coins in other euro area countries does not hold water. Rather, it is national determinants which are primarily responsible for the trend path of national coin issuance in Germany.

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Appendix

Appendix 1: Further material for the case of positive net issuance

In the following, we analyse the development of the coin shares if $\Delta N_{D,t}>0$ and $\Delta N_{A,t}>0$ for all t in more detail. According to the discussion in Section 3, the development of the coin shares $\tau_t=(\tau_{DD,t}, \tau_{AA,t})'$ is described by a linear difference equation $\tau_t=b+A_{t-1}\tau_{t-1}$ with some starting value τ_0 and

$$b = \begin{pmatrix} 1+g_D & 0 \\ 0 & 1+g_A \end{pmatrix}^{-1} \begin{pmatrix} g_D + \alpha_D + \alpha_{DA} \\ g_A + \alpha_A + \alpha_{AD} \end{pmatrix}$$

$$A_{t-1} = \begin{pmatrix} 1+g_D & 0 \\ 0 & 1+g_A \end{pmatrix}^{-1} \begin{pmatrix} 1 - \alpha_D - \alpha_{DA} & -\alpha_{AD}\eta_{t-1} \\ -\alpha_{DA}\eta_{t-1}^{-1} & 1 - \alpha_A - \alpha_{AD} \end{pmatrix}$$

As argued below, the assumption $\Delta N_{D,t}>0$ and $\Delta N_{A,t}>0$ for all t requires that $g_D=g_A$, implying that the parameters of the linear difference equation are constant. Here and below, indexed elements such as $\tau_{1,t}$ denote the corresponding components of the vectors and matrices being looked at here. The calculations below are simplified if we decompose $b_t=D_t\beta_t$ and $A_t=D_t\alpha_t$. Here, D_t is the diagonal matrix which contains the terms $(1+g_D)^{-1}$ and $(1+g_A)^{-1}$, and β_t and α_t are, respectively, vectors and matrices which contain the remaining terms.

To ensure that the difference equation is well behaved, we will apply some parameter restrictions. First, we assume $\alpha_D\geq 0$, $\alpha_{DA}>0$, $\alpha_A\geq 0$ and $\alpha_{AD}>0$. The growth rate of domestic transaction balances shall be restricted to $-1<g_D, g_A<1$, which should cover empirically realistic growth rates. We also require that $1-\alpha_D-\alpha_{DA}>0$ and $1-\alpha_A-\alpha_{AD}>0$ to ensure that, over a given period, the number of coins that can migrate from transaction balances to hoards or abroad is smaller than the quantity of coins contained in transaction balances themselves. The assumptions $\Delta N_{D,t}>0$ and $\Delta N_{A,t}>0$ require that $(g_D+\alpha_D+\alpha_{DA})>\alpha_{AD}\eta_{t-1}$ and $(g_A+\alpha_A+\alpha_{AD})>\alpha_{DA}\eta_{t-1}^{-1}$, respectively. These conditions imply, among other things, that $0\leq\tau_{DD,t}\leq 1$ and $0\leq\tau_{AA,t}\leq 1$ as long as $0\leq\tau_{DD,t-1}\leq 1$ and $0\leq\tau_{AA,t-1}\leq 1$.

If the absolute values of the eigenvalues λ_1 and λ_2 of A are both less than 1, then τ_t converges towards the limit $\tau=(I-A)^{-1}b$ (Galor, 2007). The proof of $|\lambda_1|<1$ and $|\lambda_2|<1$, however, will require an extensive auxiliary calculation. The eigenvalues of A are

$$\lambda_{1,2} = \tfrac{1}{2}(a_{11}+a_{22}) \pm \tfrac{1}{2}\sqrt{(a_{11}+a_{22})^2 - 4(a_{11}a_{22} - a_{12}a_{21})}$$

Since $(a_{11}-a_{22})^2+4a_{12}a_{21}>0$, the eigenvalues are real and different from one another. The relation $|\lambda_2|\leq|\lambda_1|=\lambda_1$ applies. To prove that $\lambda_1<1$, $\lambda_1(g)$ is defined as follows.

$$\lambda_1(g) = \frac{1}{1+g}\lambda_1(0) \text{ where } \frac{\partial\lambda_1(g)}{\partial g} = -\frac{1}{(1+g)^2}\lambda_1(0) < 0$$

and $\lambda_1(0)$ is the largest eigenvalue of the matrix α. Let us initially assume that $\alpha_{11}-\alpha_{12}=\alpha_{22}-\alpha_{21}$ and we can directly show that $\lambda_1(g)<1$ for all permitted values of g. Then, without loss of generality, let us assume $\alpha_{11}-\alpha_{12}>\alpha_{22}-\alpha_{21}$. We can show that $\lambda_1(\alpha_{11}-\alpha_{12}-1)<1$. Then, $\lambda_1(g)<1$ for all permitted values of $g>\alpha_{11}-\alpha_{12}-1$. Therefore, $\lambda_1<1$. Accordingly, τ_t will converge towards $(I-A)^{-1}b$, where I is the two-dimensional identity matrix.

Table A1 shows the signs of the partial derivatives of the limits τ_{DD} and τ_{AA} of $\tau_{DD,t}$ and $\tau_{AA,t}$ with respect to the different model parameters. The impact of the common growth rate on the limiting values is undetermined and depends, inter alia, on the relative size of the net issuances in the two countries. An increase in the common growth rate of transaction balances g ceteris paribus increases the net issuance in both regions. Both countries are thus able to issue more coins with their national side, pushing up the spread of their coins. However, larger transaction balances in the other country also imply that more foreign coins enter the national circulation, depressing the share of national coins. Larger coin hoarding in country D, as represented by an increase in the parameter α_D, increases τ_{DD} and decreases τ_{AA}. Intuitively, hoarded coins have mixed national sides and are replaced by new coins with the national side of country D. These coins thus become more prevalent in both countries. Likewise, the partial derivative of τ_{DD} with respect to the migration parameter α_{DA} is positive, while the partial derivative of τ_{AA} with respect to α_{DA} is negative. Coins flowing out of country D have mixed national sides and are replaced by new coins showing the national side of country D. Over time, this implies that coins with the national side of country D become more common in both regions. An increase in the relative size of the transaction balance in country A compared to country D, η, decreases τ_{DD} and increases τ_{AA}. In this scenario, coin flows from country D to country A will be less important relative to the size of transaction balances in country A, which explains why coins from country D will become less common in country A.

Table A1: Comparative statics

	$\partial\tau_{DD}/\partial x$	$\partial\tau_{AA}/\partial x$
g	~	~
α_D	+	-
α_{DA}	+	-
α_A	-	+
α_{AD}	-	+
η	-	+

Note: τ_{DD} and τ_{AA} are the limit values of $\tau_{DD,t}$ and $\tau_{AA,t}$. Table shows the sign of the partial derivatives of the coin shares τ_{DD} and τ_{AA} to the model parameters. ~ indicates that the sign is undetermined.

In the coin migration model of Seitz et al. (2012), a given share of coins likewise migrates from national circulation outside of their respective countries. However, the model set up in this paper differs in several important ways. This paper adds to Seitz et al. (2012) by additionally factoring in hoarding. This extension should give a more complete picture of the determinants of national coin demand. One further subtle, yet important, difference between the two models of coin migration is that we scale the number of German coins in active circulation in Germany, $T_{DD,t}$, by the size of domestic transaction balances, $T_{D,t}$. In contrast, Seitz et al. (2012) scale the domestic circulation of German coins, say $N_{DD,t}$, by the German cumulative net issuance, $N_{D,t}$. Hence, in our approach, we analyse the composition of domestic transaction balances by coins with different national sides, while Seitz et al. (2012) analyse the spread of German coins across Germany and the euro area excluding Germany. By taking a sample of coins from the active circulation, it is possible to determine the share of the different national sides in transaction balances. Data on $\tau_{DD,t}$ and $\tau_{AA,t}$ collected in this way are shown in Table 2 and will be used for the model calibration in this paper. In contrast, data on $N_{DD,t}/N_{D,t}$ and $N_{AA,t}/N_{A,t}$ is not directly available, presenting a challenge in applying the model in Seitz et al. (2012).

Appendix 2: Extension to non-positive net issuance

According to Figure 1, the net issuance of Germany and the euro area excluding Germany has typically been positive. Hence, the model as presented in Section 3 is appropriate to characterise the development of the coin shares in Germany and the euro area excluding Germany. However, coin issuance could turn negative for some regions, denominations and time periods. Against this background, we present an empirically relevant extension of the model to the case of negative net issuance.

If $\Delta N_{D,t} \leq 0$, then the central bank is a net recipient of coins at time t. In particular, $(1+g_D)T_{D,t-1} \leq (1-\alpha_D-\alpha_{DA})T_{D,t-1}+\alpha_{AD}T_{A,t-1}$, which means that the desired transaction balances of coins, $(1+g_D)T_{D,t-1}$, are smaller than (or equal to) the existing stocks of coins after coin hoarding and migration $(1-\alpha_D-\alpha_{DA})T_{D,t-1}+\alpha_{AD}T_{A,t-1}$. The central bank will accordingly have to take in $\Delta N_{D,t}$ quantity of coins. In this situation, the percentage of coins received bearing the national side of country D should precisely correspond to the share of coins from country D which is located in country D after hoarding and migration.

$$T_{DD,t} = (1-\alpha_D-\alpha_{DA})T_{DD,t-1} + \alpha_{AD}T_{DA,t-1} + \frac{(1-\alpha_D-\alpha_{DA})T_{DD,t-1} + \alpha_{AD}T_{DA,t-1}}{(1-\alpha_D-\alpha_{DA})T_{D,t-1} + \alpha_{AD}T_{A,t-1}} \Delta N_{D,t}$$

This leads us to equations (A1):

(A1) $$(1-\alpha_D-\alpha_{DA}+\alpha_{AD}\eta_{t-1})\tau_{DD,t}=\alpha_{AD}\eta_{t-1}+(1-\alpha_D-\alpha_{DA})\tau_{DD,t-1}-\alpha_{AD}\eta_{t-1}\tau_{AA,t-1}$$

A commensurate equation for the development of $\tau_{AA,t}$ in the case where $\Delta N_{A,t} \leq 0$ follows.

(A2) $(1-\alpha_A-\alpha_{AD}+\alpha_{DA}\eta_{t-1}^{-1})\tau_{AA,t}=\alpha_{DA}\eta_{t-1}^{-1}+(1-\alpha_A-\alpha_{AD})\tau_{AA,t-1}-\alpha_{DA}\eta_{t-1}^{-1}\tau_{DD,t-1}$

Depending on the starting values $T_{D,0}$ and $T_{A,0}$ and the parameters, for any given point in time it should be possible to determine the sign of $\Delta N_{D,t}$ and $\Delta N_{A,t}$ according to equations (3) and (4). From this, it follows whether equation (7) or equation (A1) or corresponding equations for $\tau_{AA,t}$ describe the development of $\tau_{DD,t}$ and $\tau_{AA,t}$.

The variables $\tau_{DD,t}$ and $\tau_{AA,t}$ can be merged to form the column vector $\tau_t=(\tau_{DD,t}, \tau_{AA,t})'$. Given a starting value of τ_0, the development of τ_t can be described by a linear difference equation with time-varying parameters of the form

(A1) $\tau_{t+1}=b_t+A_t\tau_t$, t=0,1,…

where

$$(1+g_D)\tau_{1,t+1}=g_D+\alpha_D+\alpha_{DA}+(1-\alpha_D-\alpha_{DA})\tau_{1,t}-\alpha_{AD}\eta_t\tau_{2,t}$$

in the event that $\Delta N_{D,t+1}>0$ and

$$(1-\alpha_D-\alpha_{DA}+\alpha_{AD}\eta_t)\tau_{1,t+1}=\alpha_{AD}\eta_t+(1-\alpha_D-\alpha_{DA})\tau_{1,t}-\alpha_{AD}\eta_t\tau_{2,t}$$

in the event that $\Delta N_{D,t+1}\leq 0$. Similar equations hold for the time path of $\tau_{2,t}$. The calculations below are again simplified if we decompose $b_t=D_t\beta_t$ and $A_t=D_t\alpha_t$. Here, D_t is the diagonal matrix which contains the terms of the type $(1+g_D)^{-1}$ and $(1-\alpha_D-\alpha_{DA}+\alpha_{AD}\eta_t)^{-1}$, and β_t and α_t are, respectively, vectors and matrices which contain the remaining terms. We will show that τ_t converges.

The analysis is simplified because net issuance $\Delta N_{D,t}$ and $\Delta N_{A,t}$ change their signs no more than once. The conditions for the signs of $\Delta N_{D,t}$ and $\Delta N_{A,t}$ are: $\Delta N_{D,t}>0 \Leftrightarrow g_D+\alpha_D+\alpha_{DA}>\alpha_{AD}\eta_{t-1}$ and $\Delta N_{A,t}>0 \Leftrightarrow g_A+\alpha_A+\alpha_{AD}>\alpha_{DA}\eta_{t-1}^{-1}$. If $g_D=g_A$, then η_t is constant, and for all values of t these conditions are equally either met or not met. We will next look at $g_D>g_A$. Then, $\eta_t<\eta_{t-1}$ with $\eta_t\rightarrow 0$ and $\eta_t^{-1}>\eta_{t-1}^{-1}$ with $\eta_t^{-1}\rightarrow\infty$. If $g_D+\alpha_D+\alpha_{DA}\leq 0$, then $\Delta N_{D,t}\leq 0$ for all values of t. Otherwise, there exists a t*, which gives us $\Delta N_{D,t}>0$ for all values of $t\geq t^*$. There furthermore exists a $t^\#$, which gives us $\Delta N_{A,t}\leq 0$ for all values of $t\geq t^\#$. In the case of $g_D>g_A$, therefore, for large values of t either $\Delta N_{D,t}>0$ or $\Delta N_{D,t}\leq 0$ and $\Delta N_{A,t}\leq 0$. By analogy, in the case of $g_D<g_A$, for large values of t $\Delta N_{D,t}\leq 0$ and either $\Delta N_{A,t}>0$ or $\Delta N_{A,t}\leq 0$. These ideas show that, when looking at the convergence of τ_t, we can confine ourselves to four cases: $\Delta N_{D,t}>0$ and $\Delta N_{A,t}>0$, $\Delta N_{D,t}>0$ and $\Delta N_{A,t}\leq 0$, $\Delta N_{D,t}\leq 0$ and $\Delta N_{A,t}>0$ as well as $\Delta N_{D,t}\leq 0$ and $\Delta N_{A,t}\leq 0$ for all large values of t. Below, the time index has been chosen such that one of the four cases holds for all values of $t\geq 1$. The convergence of τ_t is shown for any given starting value τ_0, which renders the behaviour of the difference equation for $t\leq 0$ irrelevant to convergence.

The case in which $\Delta N_{D,t}>0$ and $\Delta N_{A,t}>0$ for all t has been studied in the previous section. The next case we will look at is $\Delta N_{D,t}>0$ as well as $\Delta N_{A,t}\leq 0$ (the case where $\Delta N_{D,t}\leq 0$ as well as $\Delta N_{A,t}>0$ can be solved by analogy). Let $\tau=(I-A_t)^{-1}b_t=(1, 0)^{\iota}$ for all values of t. Since, for all values of t, $\tau=b_t+A_t\tau$, instead of $\tau_{t+1}=b_t+A_t\tau_t$ we can look at the equivalent equation $\tau_{t+1}-\tau=A_t(\tau_t-\tau)$. From $A_tA_{t-1}\ldots A_0\rightarrow 0$ it follows that $\tau_{t+1}\rightarrow\tau$.

If A_t is constant, this follows directly from the fact that $|\lambda_1|<1$ and $|\lambda_2|<1$. The general proof is more difficult to achieve, however. The row sum norm of a 2x2 matrix A is defined as

$$\|A\| = max_{i=1,2}\sum_{j=1}^{2}|a_{ij}|$$

$||AB||\leq||A||||B||$ for 2x2 matrices A and B and $\|A_t\|=\max\{a_{11}-a_{12,t},-a_{21,t}+a_{22,t}\}=1$, since $a_{22,t}-a_{21,t}=1$ and $a_{11}-a_{12,t}<1$. Furthermore, the following holds:

$$\|A_tA_{t-1}\|=\max\{a_{11}(a_{11}-a_{12,t-1})-a_{12,t}(a_{22,t-1}-a_{21,t-1}),-a_{21,t}(a_{11}-a_{12,t-1})+a_{22,t}(a_{22,t-1}-a_{21,t-1})\}$$

$$\leq\max\{a_{11}(a_{11}-a_{12,0})-a_{12,0},1+a_{21,0}(1-(a_{11}-a_{12,0}))\}$$

We shall set $\gamma=\max\{a_{11}(a_{11}-a_{12,0})-a_{12,0},1+a_{21,0}(1-(a_{11}-a_{12,0}))\}$. $\gamma<1$, from which, for odd values of t, it follows that

$$\|A_tA_{t-1}\ldots A_0\|\leq\|A_tA_{t-1}\|\|A_{t-2}A_{t-3}\|\ldots\|A_1A_0\|\leq\gamma^{(t+1)/2}\rightarrow 0$$

For even values of t,

$$\|A_tA_{t-1}\ldots A_0\|\leq\|A_t\|\|A_{t-1}A_{t-2}\|\|A_{t-3}A_{t-4}\|\ldots\|A_1A_0\|\leq\gamma^{t/2}\rightarrow 0$$

Therefore, $A_tA_{t-1}A_{t-2}\ldots A_0\rightarrow 0$, from which it follows that $\tau_t\rightarrow\tau$.

If, initially, $\Delta N_{D,t}\leq 0$ and then $\Delta N_{D,t}>0$, the assumption that net issuance is always effected in coins of country D fails to convince. After all, it is conceivable that the central bank initially uses up the mixed holdings in its vaults before minting new coins. If there is a time t* at which vault holdings have been used up, this time can then be chosen as a starting point, and convergence takes place as described above. This scenario applies notably for $g_D\geq 0$. Otherwise, let γ be the ratio of coins of country D to coins of country A being held in the vaults of the central bank of country D. Equation (5) can then be written as $T_{DD,t}=(1-\alpha_D-\alpha_{DA})T_{DD,t-1}+\alpha_{AD}T_{DA,t-1}+\gamma\Delta N_{D,t}$; τ_t converges towards $\tau=(\gamma, 1-\gamma)$.

Finally, we will look at the case where $\Delta N_{D,t}\leq 0$ as well as $\Delta N_{A,t}\leq 0$. The convergence of τ_t is shown by the fact that $\tau_t\in\mathbb{R}^2$ is a Cauchy sequence and therefore converges. We will look at the maximum norm $||x||=\max_{i=1,\ldots,n}|x_i|$, where x_i are the elements of a vector $x\in\mathbb{R}^n$.

We will initially show that the column sum of τ_t converges towards 1. It holds namely that

$$|\tau_{1,t+1}+\tau_{2,t+1}-1|=(\alpha_{11}-\alpha_{12,t})^{-1}(\alpha_{22}-\alpha_{21,t})^{-1}|\alpha_{11}\alpha_{22}-\alpha_{12,t}\alpha_{21,t}||\tau_{1,t}+\tau_{2,t}-1|$$

Let us set $\gamma=(\alpha_{11}\alpha_{22}+\alpha_{12,t}\alpha_{21,t})^{-1}|\alpha_{11}\alpha_{22}-\alpha_{12,t}\alpha_{21,t}|$, which gives us $|\tau_{1,t+1}+\tau_{2,t+1}-1|\leq\gamma|\tau_{1,t}+\tau_{2,t}-1|$ where $\gamma<1$ and therefore $|\tau_{1,t+1}+\tau_{2,t+1}-1|\leq\gamma^{t+1}|\tau_{1,0}+\tau_{2,0}-1|\rightarrow 0$. By way of an auxiliary calculation, we can continue to show the following equation:

$$\|\tau_{t+k+1}-\tau_{t+k}\| \leq |\tau_{1,t+k}+\tau_{2,t+k}-1|$$

These results enable us to show now that τ_t is a Cauchy sequence and thus converges. We shall select t such that $|\tau_{1,t}+\tau_{2,t}-1|<(1-\gamma)\varepsilon$. Then, for all values of $m,n\geq t$

$$\|\tau_m-\tau_n\| \leq \sum_{k=0}^{\infty}\|\tau_{t+k+1}-\tau_{t+k}\| \leq \sum_{k=0}^{\infty}\gamma^k\,|\tau_{1,t}+\tau_{2,t}-1|$$

The right-hand expression is a geometric series with a known limit, which means that, all in all,

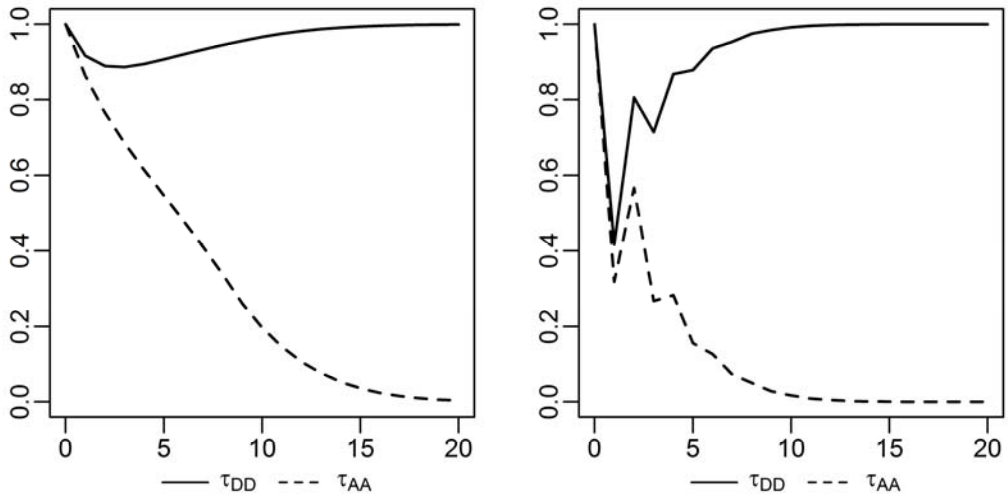
$$\|\tau_m-\tau_n\| \leq \frac{|\tau_{1,t}+\tau_{2,t}-1|}{1-\gamma} < \varepsilon$$

τ_t is thus actually a Cauchy sequence in $\mathbb{R}^2$.

The limit τ of τ_t generally depends on the parameters and the starting value and can be numerically approximated with an accuracy of ε. Let us simulate τ_t until $|\tau_{1,t}+\tau_{2,t}-1|<(1-\gamma)\varepsilon/2$. There exists an $m\geq t$ for which $\|\tau_m-\tau\|<\varepsilon/2$. Therefore, $\|\tau_t-\tau\|\leq\|\tau_t-\tau_m\|+\|\tau_m-\tau\|<\varepsilon$.

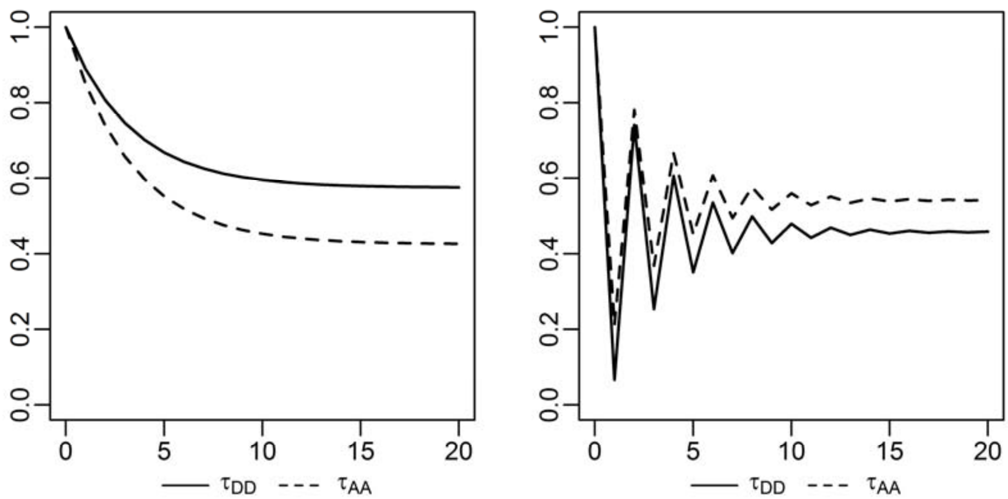
To illustrate the convergence of coin shares with negative net issuance, Figure A1 looks at the case where $\Delta N_{D,t}>0$ and $\Delta N_{A,t}\leq 0$ for large values of t. As expected, in the end only country D's coins are still in circulation. Figure A2 illustrates the case of $\Delta N_{D,t}\leq 0$ and $\Delta N_{A,t}\leq 0$, for which coin shares follow a pattern broadly similar to Figure 3.

Figure A1: Convergence in the case of coin issuance with mixed signs



Notes: The left-hand panel shows a case with little migration; the parameters are $\tau_{DD,0}$=$\tau_{AA,0}$=1, g_D=0.2, g_A=0.1, α_D=0.2, α_A=0.1, α_{DA}=0.15, α_{AD}=0.1, η_0=1. The right-hand panel shows a case with a lot of migration; the parameters are $\tau_{DD,0}$=$\tau_{AA,0}$=1, g_D=0.2, g_A=0.1, α_D=0.2, α_A=0.1, α_{DA}=0.75, α_{AD}=0.7, η_0=1. In both cases, the net issuance of country A is initially positive but then becomes negative.

Figure A2: Convergence of coin shares given negative coin issuance



Notes: The left-hand panel shows a case with little migration; the parameters are $\tau_{DD,0}$=$\tau_{AA,0}$=1, g_D=g_A=-0.2, α_D=α_A=0.05, α_{DA}=0.15, α_{AD}=0.1, η_0=1. The right-hand panel shows a case with a lot of migration; the parameters are $\tau_{DD,0}$=$\tau_{AA,0}$=1, g_D=g_A=-0.4, α_D=0.2, α_A=0.1, α_{DA}=0.75, α_{AD}=0.7, η_0=1.

One interesting special case results for g=α_D=α_A=0 and α_{DA}=$\alpha_{AD}\eta$. The first condition implies that transaction balances are constant and that coin hoarding equals zero. Under the second condition, coin net migration is zero. In this case, $N_D=T_D$ and $N_A=T_A$ as well as $\Delta N_D=0$ and $\Delta N_A=0$. It can be shown that the long-run coin shares in this case are:

$$\tau_{DD} = \frac{\eta^{-1}}{1+\eta^{-1}} = \frac{N_D}{N_D + N_A}$$

$$\tau_{AA} = \frac{1}{1+\eta^{-1}} = \frac{N_A}{N_A + N_D}$$

Hence, in the long run, the share of coins in country D with the national side of country D will correspond exactly to the share of coins with the national side of country D in the total amount of coins in circulation. In this special case, the model predicts complete mixing of national coin stocks.

Appendix 3: Additional model implications

One important assumption underlying the model parameter settings presented in Section 4 rests in the choice of $g_D=g_A=0$ for the growth rates of domestic transaction balances. To investigate the importance of the assumption on the development of the transaction balances, we calculate the volume of German euro coins circulating abroad for different assumptions of the growth rates g_D and g_A of domestic transaction balances. Since the introduction of euro coins and notes, German cumulative net issuance of coins has grown by an average of 5.8% per year, and that of the euro area excluding Germany by 5.5%. For this reason, and allowing for domestic hoarding to contribute to the overall development of the coin circulation, it is assumed that the -4% to 4% band is bound to contain all conceivable values for the growth rate of domestic transaction balances. For each combination of g_D and g_A, we solve the resulting equations for $N_{D,t}$, $N_{A,t}$, $\tau_{DD,t}$ and $\tau_{DD,t}$ for α_D, α_A, α_{DA} and α_{AD}. The other values will be selected as shown in Table 2. Given the ideas presented above on the size of the growth rate of national transaction balances, the scenario of $g_D=g_A=0$ is still assumed to be the preferred scenario. The only motivation for our further analysis of the importance of the growth rates of transaction balances is to secure the key finding of this paper – that the volume of German euro coins in circulation abroad is quantitatively negligible – against potential deviations from this assumption.

The results of this analysis are shown in Table A2 for the €2 coin, with the cumulated coin net flows between Germany and the euro area excluding Germany shown as a percentage of German coins in circulation for various combinations of the growth rates g_D and g_A. For the special case of $g_D=g_A=0$ we looked at earlier, we see cumulated coin net flows which amount to -5% of the €2 coins in circulation. Thus, slightly more €2 coins have migrated from the euro area excluding Germany to Germany than in the other direction. What we see is that the volume of German euro coins circulating abroad rises along with the difference between the foreign growth rate of transaction balances g_A and the corresponding domestic growth rate g_D. In the extreme case – in which the foreign transaction balances rise by 4% annually, while domestic transaction balances shrink by 4% annually – we still end up with a moderate volume of German euro coins in circulation abroad corresponding to 19% of German €2 coins in circulation. These results also hold for the other coin denominations. In the most extreme scenario, the circulation of €1 coins abroad amounts to 18%, the circulation of 50 cent coins to 12% and the circulation of 20 cent coins to 8% of the cumulative net issuance of these denominations. All in all, the size of the growth rates of domestic transaction balances may be perfectly relevant for the quantitative amount of German euro coins in circulation abroad; the presented results secure the conclusions of this paper, however, inasmuch as German euro coins in circulation abroad still end up being relatively moderate in amount even under extreme assumptions for growth rates g_D and g_A.

Table A2: €2 coins in circulation abroad, by growth rate of transaction balances

	g_A=-4%	g_A=-3%	g_A=-2%	g_A=-1%	g_A=0%	g_A=1%	g_A=2%	g_A=3%	g_A=4%
g_D=-4%	-7%	-2%	3%	6%	10%	12%	15%	17%	19%
g_D=-3%	-12%	-7%	-2%	2%	6%	9%	12%	14%	16%
g_D=-2%	-17%	-11%	-6%	-2%	2%	5%	8%	11%	13%
g_D=-1%	-22%	-16%	-10%	-6%	-2%	2%	5%	8%	11%
g_D=0%	-27%	-20%	-14%	-10%	-5%	-2%	2%	5%	8%
g_D=1%	-31%	-24%	-18%	-13%	-9%	-5%	-1%	2%	5%
g_D=2%	-34%	-27%	-21%	-16%	-12%	-8%	-4%	-1%	2%
g_D=3%	-36%	-30%	-24%	-19%	-15%	-11%	-7%	-4%	-1%
g_D=4%	-37%	-32%	-27%	-22%	-18%	-14%	-10%	-7%	-4%

Note: The table shows the cumulated net flows of €2 coins between Germany and the euro area excluding Germany as a percentage of the German cumulative net issuance of €2 coins for varying assumptions regarding the growth rates of domestic transaction balances g_D and g_A. Negative values imply that more coins have moved from the euro area excluding Germany to Germany than in the other direction.