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On household labour supply in sticky-wage HANK models

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Non-technical summary

Research question

Heterogeneous-agent New Keynesian models with sticky nominal wages usually assume that wage-setting unions demand the same amount of hours from all households. Under this assumption, unions do not take account of the fact that *(i)* households are heterogeneous in their willingness to work, and that *(ii)* some households might have to work *more* hours than they would like to. In this paper, we study the macroeconomic implications of *(i)* and *(ii)*.

Contribution

To assess the relevance of *(i)* and *(ii)*, we consider two departures from the standard modelling approach adopted in the literature. In the first case, we look at a specification in which wage-setting unions can demand different hours from different households. As a result, unions take account of the fact that households are heterogeneous with respect to their productivity and wealth, addressing concern *(i)*. In the second case, we maintain the uniform labour assumption but require unions to take account of the fact that, to satisfy a given labour demand by firms, the wage rate has to incentivise all households to work the required amount. This modification is tailored to address concern *(ii)*.

Results

When allowing for heterogeneous household labour supply, we show that unions find it optimal to ration hours worked for all households, i.e. nobody has to work more than desired. Therefore, the specification with heterogeneous labour supply can directly address both concerns voiced above. By taking the individual financial situation of the households into account, adjustments in hours worked serve as a stabiliser against idiosyncratic shocks at the household level. This insurance role of labour supply dampens the macroeconomic response to aggregate shocks relative to the case with uniform household labour supply by attenuating the overall labour response. When unions demand all households to supply the same amount of hours but set wages to ensure everyone is incentivised to work the demanded hours, the macroeconomic response to aggregate shocks is notably different relative to the standard uniform labour case. Specifically, wages become much more flexible to ensure that no household has to work more hours than desired. This causes inflation to respond more strongly in this case as well, passing on changes in firm marginal costs to consumers.

Nichttechnische Zusammenfassung

Fragestellung

Neukeynesianische Modelle mit heterogenen Haushalten und trägen Löhnen nehmen gemeinhin an, dass lohnsetzende Gewerkschaften von allen Haushalten die gleiche Menge an Arbeit nachfragen. Unter dieser Annahme berücksichtigen Gewerkschaften nicht, dass *(i)* Haushalte sich hinsichtlich ihrer Bereitschaft zu arbeiten unterscheiden, und *(ii)* manche Haushalte unter Umständen mehr arbeiten müssen als sie eigentlich möchten. In dieser Arbeit widmen wir uns den makroökonomischen Konsequenzen von *(i)* und *(ii)*.

Beitrag

Wir betrachten zwei Variationen des weithin verwendeten Modellierungsansatzes, um die Bedeutung von *(i)* und *(ii)* herauszuarbeiten. Im ersten Fall betrachten wir eine Modellvariante, in der Gewerkschaften von unterschiedlichen Haushalten unterschiedlich viel Arbeit nachfragen können. Auf diese Weise können Gewerkschaften der Heterogenität der Haushalte hinsichtlich ihrer Produktivität und ihres Vermögens prinzipiell Rechnung tragen, d.h. *(i)* wird berücksichtigt. Im zweiten Fall erhalten wir die Annahme aufrecht, dass alle Haushalte gleich viel arbeiten. Um *(ii)* zu berücksichtigen, wird von den Gewerkschaften jedoch verlangt, die Lohnsetzung so vorzunehmen, dass kein Haushalt mehr arbeitet als er möchte.

Ergebnisse

Für den Fall unterschiedlich viel arbeitender Haushalte zeigen wir, dass Gewerkschaften es für optimal befinden, die Menge an Arbeitsstunden für sämtliche Haushalte zu beschränken. Dies bedeutet, dass niemand mehr arbeiten muss als er möchte. Diese Modellvariante trägt entsprechend beiden oben genannten Aspekten simultan Rechnung. Indem Gewerkschaften die finanzielle Lage der Haushalte bei der Arbeitsallokation berücksichtigen, kann die Wirkung idiosynkratischer Schocks auf das Budget einzelner Haushalte durch individuelle Anpassungen bei den Arbeitsstunden gesenkt werden. Auf gesamtwirtschaftlicher Ebene senkt diese Absicherungsfunktion individuell angepasster Arbeitsstunden den Effekt makroökonomischer Schocks im Vergleich zum Fall einheitlicher Arbeitsstunden. Hintergrund dieser Unterschiede ist die abgeschwächte Reaktion der gesamten Arbeitsstunden in der Ökonomie. Wenn Gewerkschaften von allen Haushalten die gleiche Menge an Arbeit nachfragen, jedoch über die Lohnsetzung hinreichend Anreize setzen müssen, damit niemand mehr arbeitet als er möchte, kommt es zu nennenswerten Unterschieden im Vergleich zum Referenzfall mit einheitlichem Arbeitsangebot. Insbesondere verhalten sich die Löhne deutlich flexibler, damit sichergestellt ist, dass kein Haushalt mehr arbeitet als er möchte. Infolgedessen reagiert auch die Inflation deutlich sensitiver, weil Firmen die stärkeren Änderungen bei ihren Kosten an die Verbraucher entsprechend weitergeben.

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On household labour supply in sticky-wage HANK models*

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November 24, 2023

Abstract

Heterogeneous-agent New Keynesian models with sticky nominal wages usually assume that wage-setting unions demand the same amount of hours from all households. As a result, unions do not take account of the fact that *(i)* households are heterogeneous in their willingness to work, and that *(ii)* some households might have to work *more* hours than they would like to. In this paper, we consider two departures from the standard modelling approach. First, we consider a model version in which unions can demand different hours from different households, directly taking household heterogeneity into account. In this case, we show that unions find it optimal to ration hours worked for all households, such that nobody works more than desired. Compared to the standard case in which all households work the same amount by assumption, the response of output, wages and inflation to monetary policy shocks becomes notably less pronounced. This attenuation reflects that hours worked respond differently across the income distribution. The second model version we consider maintains the assumption that all households work the same amount but prohibits unions from requiring any household to work more than it would like to. This modification substantially lowers the effective stickiness of nominal wages, resulting in markedly different wage and inflation dynamics.

Keywords: Heterogeneous households, HANK, labour supply, nominal wage rigidity, monetary policy

JEL Classification: D31, E21, E24, E31, E52, E58, J22

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1 Introduction

In the heterogeneous-agent New Keynesian (HANK) literature, nominal wage rigidities are now widely recognised as an essential model ingredient (see e.g. Hagedorn et al., 2019; Broer et al., 2020; Auclert et al., 2023a; Auclert et al., 2023b). While the introduction of sticky nominal wages into HANK models remedies various shortcomings of sticky-price flexible-wage models, such as countercyclical profits or implausibly large fiscal multipliers, it also raises the question of how to properly model how much individual households work if labour is demand-determined. Usually, following the representative-agent New Keynesian (RANK) literature (Erceg et al., 2000; Galí, 2015), labour unions set a wage rate and are assumed to demand the same amount of hours from all households at that rate (see e.g. Hagedorn et al., 2019; de Ferra et al., 2020; Auclert et al., 2023a).[1] This approach has two immediate implications. First, it does not take account of the fact that individuals differ in their willingness to work. As a result, individual labour adjustments are not available as an insurance device against idiosyncratic shocks. Second, households do not have a say in how much labour they supply, working as much as demanded by labour unions. Therefore, it may well be the case that labour demand is *higher* than what individual households would be willing to supply (if they had such a choice). Consequently, some households may be rationed and would like to work more hours, whereas others would prefer to work less. As argued by Huo and Ríos-Rull (2020) in a RANK context, the latter case is a violation of the principle of voluntary exchange, as households effectively work against their will. Whereas this only arises in a RANK model outside the steady state, in a HANK model, it already arises in the steady state due to household heterogeneity in productivity and wealth, giving rise to heterogeneity in the willingness to work.

In this paper, we set up a HANK model with nominal price and wage rigidities based on Rotemberg adjustment costs, and analyse how the determination of individual household labour supply affects the quantitative model predictions. Specifically, we focus on the transmission of monetary policy shocks (see e.g. McKay et al., 2016; Kaplan et al., 2018) and compare outcomes under three different labour supply specifications.[2] For the first specification, we assume that all households work the same amount of hours by assumption, as e.g. in Auclert et al. (2023a). We call this specification the *homogeneous labour supply case* and treat it as the baseline since it is the approach typically adopted in the recent HANK literature. Second, we consider a specification in which the wage-setting union can demand different hours from different households, thus explicitly taking the heterogeneity of households in terms of their productivity and wealth into account. In this setting, each household works a different number of hours, such that we refer to it as the *heterogeneous labour supply case*. We show that labour demanded by unions optimally

[1]Some papers use a different approach where there are two types of wages, a flexible wage faced by households that matters for individual labour supply, and a sticky wage faced by firms (see e.g. Bayer et al., 2022; Sims et al., 2022). Sometimes, a reduced-form assumption is also made that rations household labour supply exogenously to match empirical estimates (see Auclert and Rognlie, 2020), such as those in Guvenen et al. (2017). In RANK models, households are sometimes assumed to set wages directly in a monopolistic fashion. This assumption is difficult to implement in HANK models, as wage setting would directly interact with market incompleteness.

[2]In the paper, we also consider other types of shocks. Our main results do not depend on the types of aggregate shocks the economy faces.

rations labour supply of all individuals in this case, such that nobody works more than desired. This specification therefore directly addresses both concerns voiced above for the baseline case. Our third specification maintains the assumption of homogeneous labour supply, but forces the wage-setting unions to respect labour supply constraints (see Huo and Ríos-Rull, 2020). We call this the *homogeneous labour supply with labour supply constraints case*. Under this labour supply specification, unions set nominal wages by taking account of the fact that, to satisfy a given labour demand by firms, the wage rate has to incentivise the (marginal) household with the lowest willingness to work to supply the amount of hours demanded by the union.[3] Although all households work the same number of hours in this case, and some (most) of them would like to work more, the labour supply constraints ensure that no one works more than what is individually optimal at the wage set by the union.

In our quantitative analysis, we find that the determination of individual household labour supply has various implications for the transmission of a monetary policy shock. Qualitatively, we find that all three model specifications make predictions that are in line with what is typically found in the literature: Output, real wages and inflation all increase in response to a monetary expansion. While the baseline case with homogeneous labour supply and the heterogeneous labour supply case also make similar quantitative predictions, there are some notable differences. Specifically, we observe a weaker response of all variables in the case of heterogeneous labour supply, reflecting the insurance effect of individual labour supply adjustments. In line with the empirical findings in Cantore et al. (2022), households in the left tail of the income distribution work less following a monetary expansion, reflecting a substantially improved financial situation. This behaviour dampens the response of total labour, which increases overall but less so compared to the case with homogeneous household labour supply. The model version with homogeneous labour supply and labour supply constraints is markedly different in terms of the magnitude of initial real wage and inflation responses. Here, real wages increase sharply on impact, as a strong wage increase is needed to induce the household least willing to supply the amount of hours necessary to satisfy labour demand. Once monetary policy returns to normal, the labour supply constraints are eased sharply, which induces the wages to strongly revert back. The inflation response mirrors the behaviour of real wages, as they are driving firms' marginal costs.

Therefore, we find that the presence of labour supply constraints lowers the effective stickiness of wages in a HANK model. While Huo and Ríos-Rull (2020) make the same observation in a RANK context, there is an important difference. Whereas labour supply constraints in a RANK model bind only occasionally outside the steady state, they always affect the economy in a HANK model. The reason for this is that, due to household heterogeneity, there is always a household that needs to be incentivised appropriately by union

[3] Broer et al. (2023) consider sticky wages based on an ex-ante contract jointly set by worker-firm pairs. While hours worked are demand-determined in the model, workers do not provide extra hours without additional adequate compensation, such that workers do not work against their will. In this paper, we seek to only slightly modify the commonly used approach to nominal wage rigidity (see Erceg et al., 2000) and study the model implications. Making more elaborate adjustments in the context of a full-fledged HANK model, e.g. by introducing a wage contract as in Broer et al. (2023), would be a non-trivial task and is beyond the scope of this paper.

wage setting to work the demanded hours. Labour supply constraints are therefore already relevant in the steady state and they always matter for dynamics as well.[4] The permanent relevance of labour supply constraints in the HANK model in turn lowers the effective stickiness of wages much more relative to a RANK model.

By increasing the flexibility of wages, the introduction of labour supply constraints into a model with heterogeneous households but homogeneous labour supply also implies strongly countercyclical firm profits. This property is at odds with empirical evidence, which in turn has partly motivated the adoption of nominal wage rigidity in the literature. By contrast, the model version with heterogeneous household labour supply can address two concerns associated with the uniform labour assumption at the same time without making such counterfactual predictions. Furthermore, looking at a monetary policy shock, the model-implied response of household labour supply across the income distribution is broadly in line with empirical evidence. Giving up the assumption of homogeneous labour supply across households therefore appears to to have no obvious downside while having various upsides. The HANK literature might hence want to depart from this assumption in the future, especially since it could be highly relevant for welfare and optimal policy analysis in sticky-wage HANK models.[5]

Related literature Our paper is related to two strands of the literature. First, it relates to a recent literature that studies wage rigidity and labour supply in HANK models. Whenever wages are rigid, a choice has to be made about how labour is determined. One simple way to determine individual labour supply is to rely on estimated reduced-form incidence functions (see Alves et al., 2020).[6] More frequently, however, the issue of labour supply heterogeneity is abstracted from by assuming that all households supply the same amount of labour (Hagedorn et al., 2019; de Ferra et al., 2020; Auclert and Rognlie, 2020; Auclert et al., 2020; Auclert et al., 2021b; Auclert et al., 2023a; Auclert et al., 2023b).[7] This latter approach does not allow for any heterogeneity in individual labour supply. By contrast, we allow labour to differ across households.

Second, our paper relates to the rapidly growing body of work that investigates the transmission of conventional and unconventional monetary policy in HANK models. These papers include those that focus on standard interest rate policy (see e.g. Kaplan et al., 2018), as well as those that focus on forward guidance (see e.g. McKay et al., 2016). The transmission mechanisms, particularly with respect to forward guidance, have been discussed extensively in the HANK literature, starting with the pioneering work of Werning (2015) and McKay et al. (2016). These papers have examined the implications of the distribution

[4]This is similar to Schmitt-Grohé and Uribe (2022), where downward nominal wage rigidity always matters in the cross section and not as an occasionally-binding constraint at the aggregate level.

[5]See McKay and Wolf (2023) for a recent paper on optimal monetary policy in a sticky-wage HANK model, adopting the standard assumption that all households work the same amount. Recent papers that use the uniform labour assumption, such as Auclert and Rognlie (2020) and Auclert et al. (2023a), usually argue in favour of that assumption based on evidence that suggests households have low marginal propensities to earn. We discuss this argument at the end of the literature review below.

[6]Ma (2023) considers indivisible labour supply in a HANK model without nominal wage rigidities.

[7]There is also a search-and-matching literature (McKay and Reis, 2016; Gornemann et al., 2021; Herman and Lozej, 2023), although that strand is not directly related due to the explicit consideration of (involuntary) unemployment.

of income and the cyclicality of income risk for the effectiveness of forward guidance. A key finding of this literature is that countercyclical income risk raises the power of forward guidance compared to representative-agent New Keynesian models (see also Acharya and Dogra, 2020; Bilbiie, 2021). By contrast, we investigate how different modelling assumptions about individual household labour supply affect the transmission and effectiveness of monetary policy.

Recent empirical evidence shows that heterogeneity in the labour market matters for monetary policy transmission on the one hand, and that monetary policy affects labour market participants in an unequal way (Amberg et al., 2022, Broer et al., 2022, Cantore et al., 2022). Broer et al. (2022) document an unequal incidence of monetary policy on earnings and employment, finding particularly large effects on low-income workers. Cantore et al. (2022) report evidence of a strong income effect on individual labour supply for the left tail of the income distribution. Specifically, they observe an increase in real labour income for this group in response to an interest rate hike, which is entirely driven by an increase in individual hours worked. Aggregate hours and labour earnings, on the other hand, decline in response.

Regarding the importance of individual labour supply adjustments, there is some disagreement in the HANK literature. Auclert and Rognlie (2020) and Auclert et al. (2023a) have argued that flexible-wage HANK models are inconsistent with empirical evidence, as it would indicate low marginal propensities to earn (MPEs) at the micro-level. Households would therefore hardly adjust their labour supply in response to income changes. However, this argument relies on empirical evidence on MPEs for Europe, particularly Sweden, as reported in e.g. Cesarini et al. (2017). By contrast, empirical evidence for other European countries (see e.g. Cantore et al., 2022) suggests a more important role for income effects on individual labour supply. Moreover, empirical evidence for the United States indicates stronger individual labour supply effects compared to Europe (see Golosov et al., 2023).

Layout The remainder of this paper is structured as follows. Section 2 introduces our model and presents three cases for the determination of individual household labour supply. Section 3 covers our model calibration and solution. Section 4 shows results for the transmission of a monetary shock. Section 5 discusses the robustness of these results along various dimensions. Section 6 discusses other aggregate shocks. Section 7 concludes.

2 Model

This section provides a description of the model economy, in which households, firms, unions and a government interact on goods, asset and labour markets.

2.1 Households

There is a unit-one continuum of households, indexed with $i \in [0, 1]$, who can save/borrow via a non-contingent bond that yields a gross real return of r_{t+1} in the next period. The household decision problem involves choosing a sequence for consumption and savings,

$\left\{c_{i,t}, b_{i,t+1}\right\}_{t=0}^{\infty}$, that maximises expected lifetime utility,

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left\{ u(c_{i,t+s}) - \int_k v(n_{i,k,t+s}) dk \right\},$$

with felicity functions $u(c) = (c^{1-\sigma} - 1)/(1-\sigma)$ and $v(n) = \varphi_1 n^{1+\varphi_2}/(1+\varphi_2)$, subject to the period budget constraint

$$c_{i,t} + b_{i,t+1} = b_{i,t} r_t + e_{i,t} \int_k w_{k,t} n_{i,k,t} dk + (d_t - T_t)\, e_{i,t},$$

and ad-hoc debt limit $b_{i,t} \geq \underline{b}$.

Households own firms in the economy and receive associated profits. Total (real) firm profits d_t as well as real taxes T_t are allocated to households relative to their productivity $e_{i,t}$. Individual labour, $n_{i,k,t}$, is supplied elastically by the households to a continuum of labour unions $k \in [0, 1]$. The real wage earned by households per efficiency unit of labour supplied to union k, $e_{i,t} n_{i,k,t}$, is denoted as $w_{k,t}$. We normalise the support of individual household productivity $e_{i,t}$, such that aggregate productivity is equal to one, i.e. $\sum_i \Pr(e_i)\, e_i = 1$, with $\Pr(e_i)$ denoting the time-invariant share of households with productivity e_i. The optimal consumption-savings choice of household i in period t satisfies the condition

$$c_{i,t}^{-\sigma} \geq \beta_t \mathbb{E}_t c_{i,t+1}^{-\sigma} r_{t+1},$$

which holds with equality for $b_{i,t} > \underline{b}$.

2.2 Goods market

Final-good producing firms manufacture Y_t by combining intermediate production inputs $Y_{j,t}$ based on a CES technology,

$$Y_t = \left(\int_j Y_{j,t}^{(\epsilon_p - 1)/\epsilon_p} dj \right)^{\epsilon_p/(\epsilon_p - 1)}.$$

The demand for intermediate goods is hence given by $Y_{j,t} = \left(P_{j,t}/P_t\right)^{-\epsilon_p} Y_t$, with $P_{j,t}$ denoting the price set by intermediate good producer $j \in [0, 1]$ and P_t the price of the final good. Intermediate goods are produced with labour according to $Y_{j,t} = N_{j,t}$. Price setting by intermediate good producers is subject to Rotemberg-type price adjustment costs as in Auclert et al. (2023a),

$$\frac{1}{2\kappa_p \epsilon_p} \log\left(\frac{P_{j,t}}{P_{j,t-1}\Pi}\right)^2 Y_t,$$

where Π denotes the (credible) long-run inflation target. In addition, intermediate good producers face a fixed operating cost $\Phi \geq 0$.

Taking the demand for its goods as given, firm j chooses prices $\left\{P_{j,t}\right\}_{t=0}^{\infty}$ to maximise the stream of current and discounted future profits

$$\sum_{s=0}^{\infty} \beta^s \left\{ \left(\frac{P_{j,t+s}}{P_{t+s}} - w_{t+s} \right) \left(\frac{P_{j,t+s}}{P_{t+s}} \right)^{-\epsilon_p} Y_{t+s} - \frac{1}{2\kappa_p \epsilon_p} \log\left(\frac{P_{j,t+s}}{P_{j,t+s-1}\Pi} \right)^2 Y_{t+s} - \Phi \right\}.$$

It is straightforward to show that – after imposing symmetry across firms – the associated first-order condition gives rise to the New Keynesian Phillips curve (NKPC)

$$\log\left(\frac{\Pi_t}{\Pi}\right) = \kappa_p\left(w_t - 1/\mu_p\right) + \beta\log\left(\frac{\Pi_{t+1}}{\Pi}\right)\frac{Y_{t+1}}{Y_t}, \tag{1}$$

with inflation rate $\Pi_t = P_t/P_{t-1}$ and price mark-up $\mu_p \equiv \epsilon_p/(\epsilon_p - 1)$.

2.3 Labour market

The basic set-up of the labour market follows Auclert et al. (2023a). However, importantly, we do not restrict attention to a uniform labour demand rule. This rule states that all households are assumed to supply the same amount of labour, such that households are likely off their individually optimal labour supply curves. Before presenting two modifications of the labour market relative to Auclert et al. (2023a), we first present their version with homogeneous labour supply as a benchmark and discuss its implications.

Homogeneous labour supply Labour services offered to intermediate good firms at nominal wage W_t are provided by competitive labour packers who combine specialised labour services $N_{k,t}$ offered by a continuum of labour unions, indexed with k, according to a CES technology,

$$N_t = \left(\int_k N_{k,t}^{(\epsilon_w-1)/\epsilon_w} dk\right)^{\epsilon_w/(\epsilon_w-1)}.$$

The demand for labour services of type k thus is $N_{k,t} = \left(W_{k,t}/W_t\right)^{-\epsilon_w} N_t$, where $W_{k,t}$ denotes the nominal wage set by union k. Union k combines efficiency units of labour supplied by households into a union-specific labour services, $N_{k,t} = \int_i e_{i,t} n_{i,k,t} di$, and sets nominal wages $\left\{W_{k,t}\right\}_{t=0}^{\infty}$ subject to quadratic wage adjustment costs that are measured in utils,

$$\frac{\epsilon_w}{2\kappa_w}\log\left(\frac{W_{k,t}}{W_{k,t-1}\Pi_W}\right)^2.$$

Given the demand for its labour services and taking the households' consumption-savings decisions as given, union k chooses nominal wage $W_{k,t}$ and labour services $N_{k,t}$ to maximise average utility of its household members,

$$\sum_{s=0}^{\infty}\beta^s\left\{\int_i\left\{\frac{c_{i,t+s}^{1-\sigma}-1}{1-\sigma} - \int_k \varphi_1\frac{n_{i,k,t+s}^{1+\varphi_2}}{1+\varphi_2}dk\right\}di - \frac{\epsilon_w}{2\kappa_w}\log\left(\frac{W_{k,t+s}}{W_{k,t+s-1}\Pi_W}\right)^2\right\},$$

subject to the demand condition $N_{k,t} = \left(W_{k,t}/W_t\right)^{-\epsilon_w} N_t$.

A crucial assumption made by Auclert et al. (2023a), which simplifies the union problem considerably, is that each union k uses individual household labour services based on the uniform rule, $n_{i,k,t} = N_{k,t}$.[8]

After imposing symmetry, $W_{k,t} = W_t$ and $N_{k,t} = N_t$, and defining real wage $w_t = W_t/P_t$,

[8]Labour supplied by each household to union k, $n_{i,k,t}$, equals total demand for labour by union k, $N_{k,t}$, due to *(i)* the normalisation of household mass to one and *(ii)* the support of individual productivity being normalised such that aggregate labour productivity equals one as well, i.e. $\sum_i \Pr(e_i)\, e_i = 1$.

the union's first-order conditions can be written as

$$0 = -\frac{\varepsilon_w}{\kappa_w} \frac{\log\left(\frac{\Pi_{W,t}}{\Pi_W}\right)}{w_t} + \beta \frac{\varepsilon_w}{\kappa_w} \frac{\log\left(\frac{\Pi_{W,t+1}}{\Pi_W}\right)}{w_t} + \int_i e_{i,t} N_t c_{i,t}^{-\sigma} di + \lambda_t \left[-\varepsilon_w w_t^{-1} N_t\right], \tag{2}$$

and

$$0 = \int_i e_{i,t} w_t c_{i,t}^{-\sigma} di - \varphi_1 N_t^{\varphi_2} - \lambda_t, \tag{3}$$

where $\Pi_{W,t} = W_t / W_{t-1}$ denotes nominal wage inflation and $\lambda_{k,t} = \lambda_t$, after imposing symmetry, is the multiplier associated with the labour demand condition faced by each union.[9]

Combining the two conditions yields the New Keynesian wage Phillips curve (NKWPC) used by Auclert et al. (2023a),

$$\log\left(\frac{\Pi_{W,t}}{\Pi_W}\right) = \kappa_w \left(\varphi_1 N_t^{\varphi_2} - \frac{\epsilon_w - 1}{\epsilon_w} w_t \int_i e_{i,t} c_{i,t}^{-\sigma} di\right) N_t + \beta \log\left(\frac{\Pi_{W,t+1}}{\Pi_W}\right), \tag{4}$$

where $\mu_w \equiv \epsilon_w / (\epsilon_w - 1)$ denotes the wage mark-up. This NKWPC links current (nominal) wage inflation to discounted future wage inflation as well as a measure related to the average marginal rate of substitution between labour and consumption on the one hand, and the (marked-down) real wage on the other hand. The real wage rate satisfies $w_t = w_{t-1}\left(\Pi_{W,t}/\Pi_t\right)$.

Discussion To understand the implications of the uniform rule assumption for household labour supply, it is useful to take a look at the union first-order condition (3). For $\lambda_t > 0$, there is a wedge between the average marginal gain of working more, $\int_i e_{i,t} w_t c_{i,t}^{-\sigma} di$, and the associated average marginal cost, $\varphi_1 N_t^{\varphi_2}$, which coincides with individual disutility of labour under the uniform rule, $n_{i,k,t} = N_{k,t} = N_t$. The wedge reflects that, from the utilitarian perspective of the union, it is optimal not to equalise these marginal benefits and costs. Instead, as a price-setter, the union understands that it can exploit its market power and make households better off on average by receiving a higher wage and letting everyone work less.

By contrast, from the perspective of an individual price-taking household, labour supply implied by this condition is sub-optimal for almost all households. Given a real wage w_t, households would decide to equalise the marginal benefits and costs of working, $e_{i,t} w_t c_{i,t}^{-\sigma} = \varphi_1 \left(n_{i,t}^*\right)^{\varphi_2}$, where $n_{i,t}^*$ denotes the counterfactual household labour choice in this case.[10] In contrast to condition (3), this condition is *(i)* in terms of individual household quantities rather than cross-sectional household averages, and *(ii)* does not take market power into account. Since households are heterogeneous with respect to their wealth and labour productivity, the uniform labour demand rule implies that most households are off their (individually) optimal labour supply curves. Specifically, some households are rationed and would like to work more than N_t hours at the real wage w_t, whereas others would prefer

[9] See Appendix A for details on the derivation.

[10] As in Huo and Ríos-Rull (2020), we use actual consumption $c_{i,t}$, rather than counterfactual consumption $c_{i,t}^*$, which would be consistent with the counterfactual budget constraint and savings choice, to derive $n_{i,t}^*$. Using $c_{i,t}^*$ instead does not materially change our results. However, it increases the computational burden since $c_{i,t}^*$ and $n_{i,t}^*$ have to be solved for jointly and numerically in this case.

to work less and are therefore effectively working against their will.

As criticised by Huo and Ríos-Rull (2020) in a RANK context, having households effectively work against their will is a violation of the principle of voluntary exchange and hence problematic. Importantly, in RANK models, which do not feature idiosyncratic income risk and incomplete markets, a positive mark-up $\mu_W > 1$ ensures that households do not work more than desired at least in the steady state (see Huo and Ríos-Rull, 2020). Outside the steady state, aggregate shocks may, however, lead to a situation where households work more hours than what they would find optimal from an individual perspective. By contrast, in a standard HANK model, the uniform labour assumption implies that some households work more hours than what is individually optimal even in the steady state.

In this paper, we therefore consider two modifications of the benchmark case above to investigate the importance of the uniform labour assumption and address the problem of letting households work more than they want to. The first modification assumes that each union k can freely choose the amount of labour services to demand from each household i. The second modification maintains the assumption of uniform labour demand, but requires unions to respect *labour supply constraints* for all households, as in Huo and Ríos-Rull (2020), ensuring that no household works more hours than what is optimal from an individual perspective.

Heterogeneous labour supply Suppose that union k can now freely demand different amounts of hours from each household, i.e. choose household-specific sequences for labour $\left\{n_{i,k,t+s}\right\}_{i\in[0,1]}$, subject to the feasibility constraint $\int_i e_{i,t} n_{i,k,t} di = N_{k,t}$. After imposing symmetry, such that $W_{k,t} = W_t$, $N_{k,t} = N_t$, $\lambda_{k,t} = \lambda_t$ and $n_{i,k,t} = n_{i,t}$ for all i, the union's first-order conditions for the nominal wage and labour for household i can be written as

$$0 = -\frac{\varepsilon_w}{\kappa_w} \frac{\log\left(\frac{\Pi_{W,t}}{\Pi_W}\right)}{w_t} + \beta \frac{\varepsilon_w}{\kappa_w} \frac{\log\left(\frac{\Pi_{W,t+1}}{\Pi_W}\right)}{w_t} + \int_i e_{i,t} n_{i,t} {c_{i,t}}^{-\sigma} di + \lambda_t \left[-\varepsilon_w w_t^{-1} N_t\right], \tag{5}$$

and

$$0 = e_{i,t} w_t {c_{i,t}}^{-\sigma} - \varphi_1 {n_{i,t}}^{\varphi_2} - e_{i,t}\lambda_t, \tag{6}$$

in this case, respectively. As shown in Appendix A, these conditions can be combined to obtain the NKWPC under heterogeneous labour supply,

$$\begin{aligned} \log\left(\frac{\Pi_{W,t}}{\Pi_W}\right) =& \kappa_w \left(\int_i \varphi_1 n_{i,t}^{\varphi_2} di - w_t \int_i \frac{\epsilon_w - n_{i,t}/N_t}{\epsilon_w} e_{i,t} c_{i,t}^{-\sigma} di\right) N_t \\ &+ \beta \log\left(\frac{\Pi_{W,t+1}}{\Pi_W}\right). \end{aligned} \tag{7}$$

It is easy to see that this expression collapses to the NKWPC (4) under the uniform rule, i.e. when imposing $n_{i,t} = N_t$.

The union's first-order condition (6) now pins down labour supplied by household i, $n_{i,t}$, instead of the uniform rule $n_{i,t} = N_t$, with λ_t denoting the Lagrange multiplier on the union's constraint $\int_i e_{i,t} n_{i,k,t} di = \left(W_{k,t}/W_t\right)^{-\epsilon_w} N_t$ in a symmetric equilibrium.[11] Since

[11]In this case, the constraint simplifies to $\int_i e_{i,t} n_{i,t} di = N_t$.

$\lambda_t > 0$, $e_{i,t} w_t c_{i,t}^{-\sigma} > \varphi_1 n_{i,t}^{\varphi_2}$ holds. Individual labour supply is hence rationed for all households in the sense that every household would be willing to work more hours at the wage rate w_t. Huo and Ríos-Rull (2020)'s critique therefore does not apply in a sticky-wage HANK setting if hours worked are heterogeneous. As in the homogeneous labour supply case, the reason why labour supply is rationed by the union is that it jointly optimises over individual labour $\left\{n_{i,k,t+s}\right\}_{i\in[0,1]}$ and nominal wage $W_{k,t}$, given firm labour demand. In contrast to the optimal labour condition under homogeneous labour supply, the respective conditions in the heterogeneous labour supply case take into account how marginally allocating more hours to one particular household affects that household's utility as well as labour demand by firms. As a result, labour at the household level reflects the individual wealth and income situation, such that individual labour adjustments provide insurance against adverse idiosyncratic shocks, which is absent under the uniform labour demand rule. Heterogeneous labour supply also affects unions' wage setting by changing the extent to which wages need to be adjusted in response to a shock to satisfy labour demand.

Homogeneous labour supply with labour supply constraints Now suppose that unions apply the uniform labour demand rule but take account of the fact that no household can be required to work more than it is willing to. Specifically, union k maximises average household utility subject to the labour demand condition but – as in Huo and Ríos-Rull (2020) – now additionally faces the labour supply constraints $n_{i,k,t} \leq n_{i,k,t}^*$, where $n_{i,k,t}^*$ satisfies household i's (individually optimal) counterfactual labour supply condition $e_{i,t} w_{k,t} c_{i,t}^{-\sigma} = \varphi_1 \left(n_{i,k,t}^*\right)^{\varphi_2}$, given real wage rate $w_{k,t}$.

In a symmetric equilibrium with $W_{k,t} = W_t$ and $N_{k,t} = N_t$, the union first-order conditions can be written as

$$\begin{aligned} 0 = &- \frac{\varepsilon_w}{\kappa_w} \frac{\log\left(\frac{\Pi_{W,t}}{\Pi_W}\right)}{w_t} + \beta \frac{\varepsilon_w}{\kappa_w} \frac{\log\left(\frac{\Pi_{W,t+1}}{\Pi_W}\right)}{w_t} + \int_i e_{i,t} N_t c_{i,t}^{-\sigma} di + \lambda_t \left[-\varepsilon_w w_t^{-1} N_t\right] \\ &+ \int_i \zeta_{i,t} \frac{\partial n_{i,t}^*}{\partial w_t} di, \end{aligned} \tag{8}$$

and

$$0 = \int_i e_{i,t} w_t c_{i,t}^{-\sigma} di - \varphi_1 N_t^{\varphi_2} - \lambda_t - \int_i \zeta_{i,t} di, \tag{9}$$

with $\zeta_{i,t}$ denoting the multiplier on the labour supply constraint for household i.

Combining the two first-order conditions yields the NKWPC under labour supply constraints,

$$\begin{aligned} \log\left(\frac{\Pi_{W,t}}{\Pi_W}\right) =& \kappa_w \left(\varphi_1 N_t^{\varphi_2} - \frac{\epsilon_w - 1}{\epsilon_w} w_t \int_i e_{i,t} c_{i,t}^{-\sigma} di\right) N_t + \beta \log\left(\frac{\Pi_{W,t+1}}{\Pi_W}\right) \\ &+ \kappa_w \int_i \zeta_{i,t} \left\{N_t + \epsilon_w^{-1} w_t \frac{\partial n_{i,t}^*}{\partial w_t}\right\} di. \end{aligned} \tag{10}$$

This NKWPC is the same as presented in Auclert et al. (2021a), extended by an additional term on the RHS that captures how union wage setting is affected by labour supply constraints.[12] If these constraints do not matter, i.e. $\zeta_{i,t} = 0$ holds for all i, the NKWPC reduces to the standard NKWPC displayed in equation (4).

The introduction of labour supply constraints has a more noticeable impact on the model economy than the introduction of heterogeneous hours worked – both within and outside of the steady state. Given that in our HANK model, the households' willingness to work is heterogeneous, some households may end up working more hours than they would like to do under the uniform rule in the steady state. The labour supply constraints ensure that such a situation does not arise. In a steady state, where aggregate quantities and prices are constant, equation (10) determines aggregate labour supply – and hence output. In contrast to the standard case with homogeneous labour supply, the multiplier $\zeta_{i,t}$ ensures that aggregate labour adjusts such that all households work no more than their individually optimal amount of hours at the steady-state real wage $1/\mu_p$ (see NKPC (1)), i.e. $N_t \leq n^*_{i,t}$ for all i. As a result, aggregate labour, which coincides with individual labour in this case, will be determined by a marginal household $m \in [0, 1]$, who is the household least willing to work at the union-set wage rate.[13] Since output equals aggregate labour in the model, ceteris paribus, the introduction of supply constraints lowers average output in the economy. Of course, the constraints also matter for the unions' behaviour in response to an aggregate shock. Specifically, in response to an expansionary shock that raises aggregate labour demand, unions have to adjust wages further to meet this demand, since all households must be willing to work as much as demanded. This requirement to incentivise households to work more is captured by the derivative $\partial n^*_{i,t}/\partial w_t$ in equation (10). As in Huo and Ríos-Rull (2020), labour supply constraints affect the effective degree of nominal wage stickiness in the economy. However, in their RANK model, labour supply constraints bind for wage-setting unions only occasionally outside the steady state. By contrast, the labour supply constraint of the marginal household always matters, such that the presence of labour supply constraints affects the effective degree of wage stickiness much more in a HANK model than a RANK model.

2.4 Government

The government raises taxes to pay off maturing debt, i.e. $T_t = B(r_t - 1)$, where B denotes the time-invariant public debt position. As in McKay et al. (2016), monetary policy sets the nominal interest rate according to the rule

$$R_t = r \prod_{k=0}^{\infty} \exp(\epsilon^r_{t-s,t}) \Pi_{t+1}, \tag{11}$$

where r is the steady-state real rate and $\epsilon^r_{t-s,t}$ are i.i.d. policy (news) shocks, which are announced in period $t - s$ to take place in period t.[14] These policy shocks will be used to study the effects of contemporaneous and anticipated future monetary policy shocks.

[12] The detailed derivation of the NKWPC under labour supply constraints is delegated to Appendix A, which also lists the complementary slackness conditions associated with the labour supply constraints.

[13] In Section 3, we show how we identify this marginal household for our quantitative model analysis.

[14] Auclert et al. (2023b) also use this rule but abstract from news shocks. Allowing the shocks to be autocorrelated does not change our results.

Table 1: Shared model parameters

Parameter	Description	Value
Φ	Fixed cost of production	0.167
φ_2	Inverse Frisch elasticity	2
Π	Long-run inflation target	1.005
Π_W	Long-run nominal wage inflation	1.005
ϵ_p	Price elast. of subst. int. goods	6
ϵ_w	Wage elast. of subst. labour services	6
ρ_e	Persist. idiosync. productivity	0.966
σ	Coeff. of relative risk aversion	1
σ_e	Std. dev. idiosync. prod. shock	0.13
κ_p	Slope NKPC	0.11
κ_w	Slope NKWPC	0.03

In the absence of policy shocks, the interest rate rule implements a constant path for the (gross) real interest rate, $r_{t+1} \equiv R_t/\Pi_{t+1}$. A positive monetary policy shock $\epsilon^r_{t-s,t}$ then raises the real rate r_{t-k+1}, which enters the household optimality condition for $b_{i,t-k}$.[15]

2.5 Market clearing

The market clearing conditions for the economy are $\int_i b_{i,t} di = B$, $\int_i c_{i,t} di = C_t$, $\int_i e_{i,t} n_{i,t} di = N_t$, and $N_t = C_t + \Phi$.[16]

3 Model calibration and solution

In this section, we briefly present our model calibration and numerical solution method.

3.1 Calibration

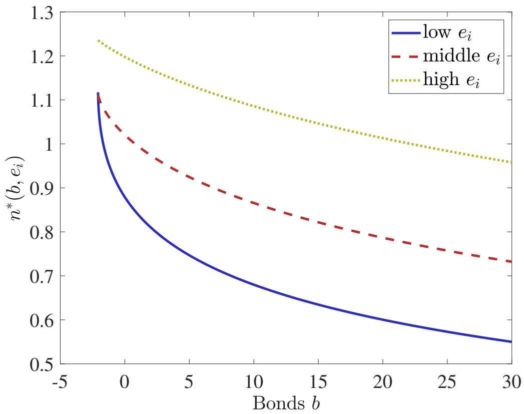
We calibrate the model at a quarterly frequency. To enhance comparability across the three modelling approaches to household labour supply considered in this paper, we recalibrate three model parameters that are linked to key model statistics for each of these cases. First, we calibrate the household discount factor β to match an annualised steady-state real rate of 2%. Second, we choose $\underline{b}$ to obtain a steady-state share 20% for borrowing-constrained households. Third, we set φ_1 to normalise steady-state aggregate hours – and hence output – to one. Differences across the three model cases regarding the transmission of monetary policy are therefore not driven by differences with respect to these three model variables in the steady state.[17] The remaining (shared) model parameters are in line with the literature

[15] In Section 6, we consider a more conventional Taylor-type interest rate rule and look at various other types of aggregate shocks.

[16] As in Hagedorn et al. (2019), we assume that price adjustment costs are "as if", i.e. they affect firm price-setting but do not materialise as resource costs and hence do not show up in the aggregate resource constraint.

[17] This calibration strategy implies different average marginal propensities to consume (MPC) for households in model versions with homogeneous and heterogeneous labour supply. We also perform our analyses for a different calibration that chooses $\underline{b}$ by targeting a value for the average MPC instead of the share of borrowing-constrained households. Our results do not depend on which of the two calibration strategies we adopt.

Figure 1: Steady-state policy functions for counterfactual labour supply in baseline model



(see Table 1).[18] We set the coefficient of relative risk aversion to $\sigma = 1$ and the inverse Frisch elasticity to $\varphi_2 = 2$. For the elasticity of substitution between intermediate goods ϵ_p and the Rotemberg cost parameter κ_p, we consider values of 6 and 0.11, respectively. As in Hagedorn et al. (2019), we set $\epsilon_w = \epsilon_p$. For κ_w, we consider a value of 0.03 (see Auclert et al., 2023b), capturing the fact that wages are usually more rigid than prices. Targeting zero steady-state firm profits, we set the fixed cost to $\Phi = 0.167$. Idiosyncratic labour productivity $e_{i,t}$ follows a log-normal AR(1) process with autocorrelation coefficient $\rho_e = 0.966$ and shock standard deviation $\sigma_e = 0.13$ (see McKay et al., 2016). The annual long-run inflation rate is set to 2%. In the long run, average nominal wage inflation is equal to long-run price inflation, i.e. $\Pi_W = \Pi$. We abstract from outside liquidity and set the supply of government bonds to zero, $B = 0$. Fiscal policy is therefore absent in our baseline calibration.[19]

3.2 Numerical solution

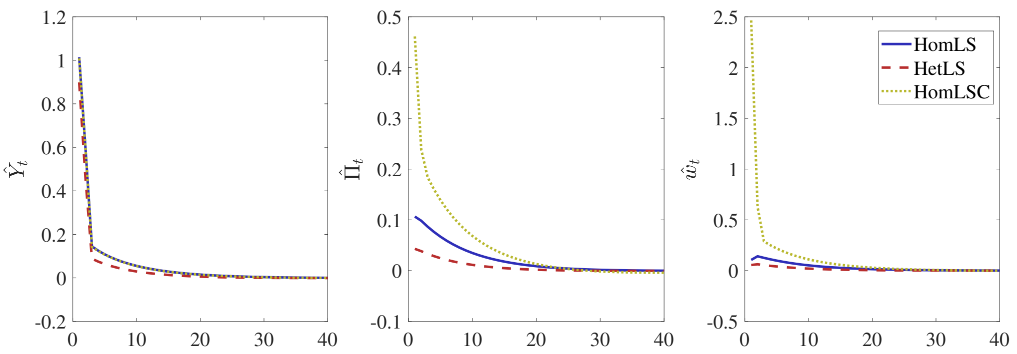
For the quantitative model evaluation, we simulate how the model responds if the economy is hit by an aggregate (MIT) shock in the deterministic steady state. To obtain this response, we use a nonlinear numerical solver to find the perfect foresight path that returns the economy back to the initial steady state (see e.g. McKay et al., 2016). Importantly, the solution method preserves nonlinearities due to occasionally-binding borrowing constraints or labour supply constraints at the individual level. Solving for the model's steady state is already a potentially non-trivial task when allowing for endogenously binding labour supply constraints. In principle, one has to solve for a continuum of Kuhn-Tucker multipliers that are associated with the labour supply constraints.[20] Fortunately, these multipliers are zero except for a marginal household $m \in [0, 1]$, who is the household least

[18] Section 5 provides a sensitivity analysis for key model parameters.

[19] Section 5.3 shows that our main results do not depend on this assumption.

[20] In practice, the continuous distribution of households across productivity and wealth is approximated with a discrete histogram (see Young, 2010), making the number of household types and thus the number of labour supply constraints finite. However, a large number of grid points is needed for a proper approximation, such that it remains a difficult task to solve for the multipliers associated with the labour supply constraints. Note that the model equilibrium conditions now also include the complementary slackness conditions for the labour supply constraints, which adds to the computational complexity.

Figure 2: Output, inflation and real wage after an expansionary monetary policy shock



Notes: IRFs for interest rate shock $\varepsilon^r_{t,t} = -0.01$ are expressed in percentage deviations from steady state. HomLS = homogeneous labour supply, HetLS = heterogeneous labour supply, HomLSC = homogeneous labour supply with labour supply constraints.

willing to work the demanded amount of hours at the wage rate set by the union. For the numerical solution, we exploit the fact that it is possible to identify this marginal household. Specifically, for our calibration, the marginal household is given by the wealthiest household among the least productive ones. This household can be identified by looking at the steady-state policy functions for counterfactual labour supply $n^*_{i,t}$ in the baseline model version with homogeneous labour supply (see Figure 1).[21] Only the multiplier for this particular household type is then added to the set of model equations, together with the associated (binding) labour supply constraint.

4 Results

We first consider a contemporaneous (one-off) expansionary monetary policy shock, where, at time t, the central bank temporarily lowers the real interest rate r_{t+1} by one (quarterly) percentage point, and returns it to its initial (steady-state) level the period thereafter. We then discuss a forward guidance experiment along the lines of McKay et al. (2016).

4.1 Contemporaneous monetary policy shock

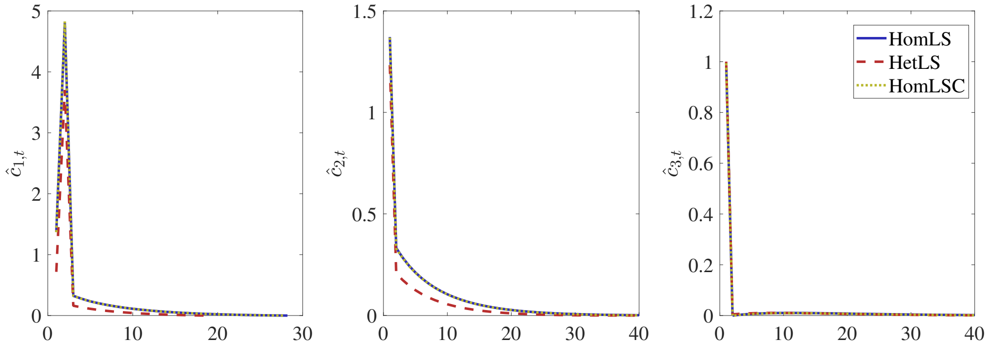
Figure 2 shows the paths for output, inflation, and real wages for a contemporaneous monetary policy shock ($k = 0$). Responses for all three model versions are displayed. The homogeneous labour supply case (HomLS) is shown with solid blue, the heterogeneous labour supply case (HetLS) with dashed red, and the homogeneous labour supply with labour supply constraints case (HomLSC) with dotted yellow lines. All variables are expressed in percentage deviations from their initial steady-state values.

In all three cases, the temporary reduction in real interest rates causes a temporary increase in consumption and therefore output.[22] This increase is driven in part by high-

[21]For our calibration, we find that 12% of households are working more relative to their desired hours if all households are assumed to work the same amount.

[22]Recall that in our setting, consumption and output differ only because intermediate good producers face a fixed production cost.

Figure 3: Consumption response of different income groups to an expansionary monetary policy shock



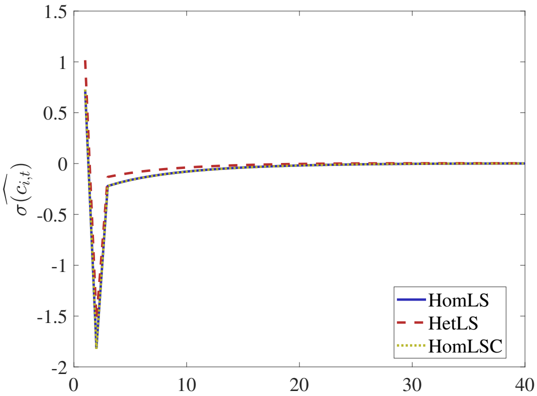
Notes: IRFs for interest rate shock $\varepsilon^r_{t,t} = -0.01$ are expressed in percentage deviations from steady state. The Figure displays consumption values aggregated for households with low ($\hat{c}_{1,t}$), medium ($\hat{c}_{2,t}$) and high productivity ($\hat{c}_{3,t}$). HomLS = homogeneous labour supply, HetLS = heterogeneous labour supply, HomLSC = homogeneous labour supply with labour supply constraints.

income households, who can increase consumption by borrowing against future income or dissaving. Kaplan et al. (2018) call this the direct effect of monetary policy. Another part of the increase in consumption is driven by low-income households with high marginal propensities to consume (MPC), who receive a boost in labour income due to the expansion in labour demand and the associated wage increase. In addition, these high-MPC households also benefit from the reduction in their interest payments.[23] Kaplan et al. (2018) call these (general equilibrium) responses indirect effects of monetary policy. Since output is demand-determined, the increase in consumption requires a corresponding increase in production, which induces an increase in labour demand. This labour demand has to be met by labour supply, which is induced by an increase in real wages. An increase in wages causes an increase in firms' marginal costs, which in turn implies an increase in goods prices and hence inflation. This general transmission mechanism is present under all three specifications of labour supply. However, there are notable differences in the details of the underlying transmission mechanism.

A comparison with the HomLS case shows that the increases in output, real wages and inflation are weaker in the HetLS case. In contrast to the HomLS case, unions jointly choose wages and hours demanded from each individual household to maximise average household utility. Based on this objective, unions trade off the marginal costs and benefits of letting a specific household work more hours, as illustrated by equation (6). Additional labour is therefore supplied by all households combined, but not necessarily to the same extent. Indeed, the sign of the labour adjustment does not even have to be the same across households. Also, recall that the labour supply of all households is rationed (see discussion in Section 2.3). While all households work an equal number of hours in the HomLS case, individual hours differ in the HetLS case. The latter case then necessitates (only) a lower

[23]Indebted households find it cheaper to roll over (or pay off) their debt, which boosts their income and hence consumption. Because their marginal propensities to consume out of temporary income are higher than those of wealthy households (who are the recipients of most interest payments), the redistribution from savers to borrowers stimulates aggregate demand.

Figure 4: Cross-sectional consumption dispersion after an expansionary monetary policy shock



Notes: IRFs for interest rate shock $\varepsilon^r_{t,t} = -0.01$ are expressed in percentage deviations from steady state. HomLS = homogeneous labour supply, HetLS = heterogeneous labour supply, HomLSC = homogeneous labour supply with labour supply constraints.

increase in the real wage to induce the amount of labour required to support the given output increase. Because wages increase by less, so do firms' real marginal costs and hence the response of inflation is less pronounced relative to HomLS. Overall, the same shock leads to an output responses that is roughly 10% lower in the HetLS relative to the HomLS case, whereas inflation and real wage responses are roughly 50% lower with heterogeneous labour supply.

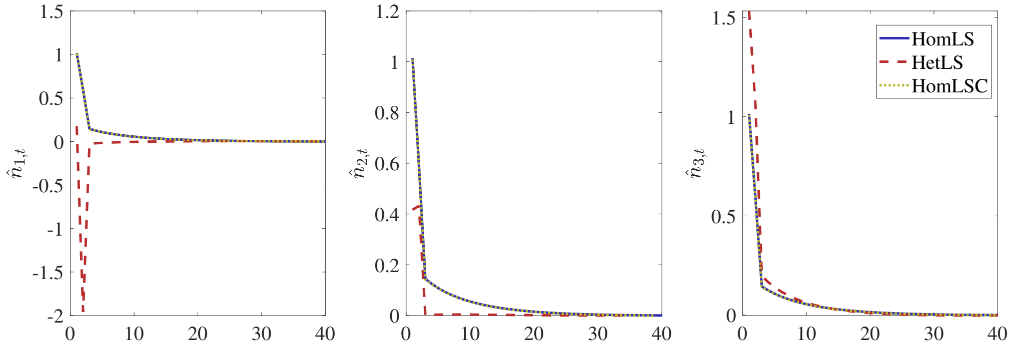
To better understand the differences in the response patterns with homogeneous and heterogeneous labour supply, we disaggregate consumption and labour responses by household income. Specifically, we look at three income groups: low-income households (group 1), middle-income households (group 2), and high-income households (group 3).[24]

Figure 3 shows the consumption paths for each of these income groups, again in percentage deviations from steady state. Note that in all cases, high-income households, whose steady-state consumption share is 41%, increase their consumption immediately as the real interest rate declines, and decrease their consumption immediately after the real interest rate returns to its initial steady-state level.[25] This reflects the short-lived nature of the policy shock, which does not exhibit any persistence. These households respond in this way because they are wealthy and far away from the borrowing constraint, making them sensitive to real interest rate fluctuations but not to income fluctuations. Middle-income households, whose consumption accounts for 47% of total consumption in steady state, respond similarly, but are more exposed to income fluctuations, as they are less wealthy and closer to the borrowing constraint. More specifically, their consumption response does not immediately fall to the previous level when the real interest rate returns to its initial level, but lingers on at a higher level for about 20 quarters after the shock. This mirrors the persistent wage increase. Low-income households, who have a steady-state consump-

[24]Group 1 and 3 account for 25% of the population each, whereas the population share of group 2 is 50%.

[25]Recall that the policy rule (11) implies that a shock $\epsilon^r_{t,t}$ lowers the real rate r_{t+1}, which is paid out in period $t+1$.

Figure 5: Labour supply response of different income groups to an expansionary monetary policy shock



Notes: IRFs for interest rate shock $\varepsilon^r_{t,t} = -0.01$ are expressed in percentage deviations from steady state. The Figure displays labour supply values aggregated for households with low ($\hat{n}_{1,t}$), medium ($\hat{n}_{2,t}$) and high productivity ($\hat{n}_{3,t}$). HomLS = homogeneous labour supply, HetLS = heterogeneous labour supply, HomLSC = homogeneous labour supply with labour supply constraints.

tion share of 12%, increase their consumption on impact of the monetary policy shock, driven mainly by the increase in wages. With the decrease in the interest rate, there is also a redistributive effect at work, which benefits the (indebted) low-income households and causes a sharp increase in their consumption when the decline in interest payments occurs.[26] Note that, relative to the steady state, the consumption response of low-income households is almost five times that of high-income households, reflecting a much higher MPC relative to other income groups.

Reflecting consumption movements at the individual level that are qualitatively similar across model versions, the cross-sectional dispersion of household consumption responds similarly for all three model versions as well (see Figure 4). Specifically, consumption inequality slightly increases when the shock is announced, which is caused by the strong increase in consumption for mostly high- and middle-income groups, whose level of consumption is higher than the level of consumption of the low-income households. This means that even though consumption of the low-income household group increases by more (percentage-wise) than that of the high-income group, the level effect dominates and dispersion increases on impact, before decreasing due to the faster decline in the high- and middle-income groups' consumption. Whether these findings are in line with empirical evidence is unclear because there is no consensus in the literature about the impact of monetary policy on consumption inequality. For instance, for the United States, Chang and Schorfheide (2022) find that negative interest rate shocks raise consumption inequality, whereas Coibion et al. (2017) find the opposite.

With respect to household labour supply, there are no cross-sectional differences under the uniform labour rule (HomLS and HomLSC) by assumption, as all households work the same amount of hours (see Figure 5). Specifically, individual and aggregate labour supply is pinned down by condition (3) in the HomLS case, which we reproduce in slightly

[26]See also Ferrante and Paustian (2019). This intuition is similar to the interest rate exposure channel discussed in Auclert et al. (2023a).

rearranged form here,

$$\varphi_1 N_t^{\varphi_2} = \int_i e_{i,t} \left(w_t c_{i,t}^{-\sigma} - \lambda_t \right) di.$$

Labour responses across groups then mirror the behaviour of output displayed in Figure 2 due to $N_t = Y_t$.

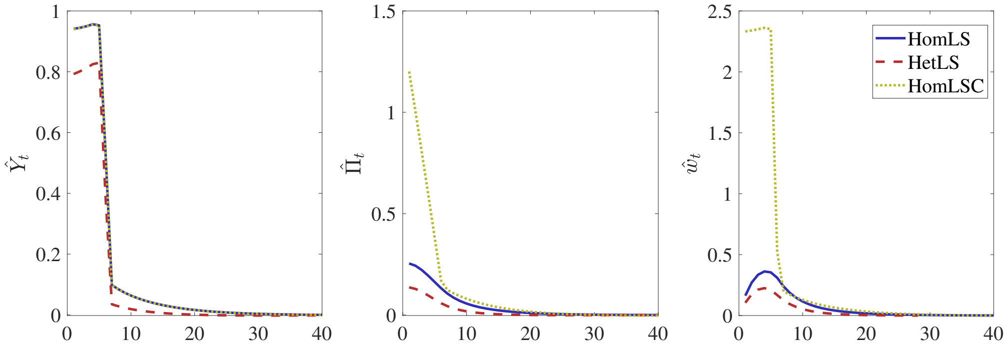
The HetLS case is more involved. Recall the first-order condition for labour demanded from household i, given by equation (6), which we reproduce slightly rearranged here,

$$\varphi_1 n_{i,t}^{\varphi_2} = e_{i,t} \left(w_t c_{i,t}^{-\sigma} - \lambda_t \right).$$

The LHS represents the marginal disutility of labour supplied by individual household i. The sign of the labour response depends on the RHS, which is given as the productivity-weighted difference between the aggregate wage times the marginal utility of consumption for household i and the Lagrange multiplier λ_t. If a household's consumption increases more compared to the change in the Lagrange multiplier, the corresponding fall in the marginal utility of consumption causes the RHS of the equation to fall and therefore labour supply to decrease accordingly. This is what we observe in the HetLS case, where the labour supply of low-income households barely moves on impact and declines in the period thereafter (when consumption increases much more), before returning (close) to the initial level (see Figure 5). The reason for this is that it is optimal for the unions (who act on behalf of households) to ration low-income households more in the second period, when the labour supply of middle- and high-income households (almost) returns to the steady-state level, in order to sustain the wage increase. They do so because it harms low-income households the least to reduce their labour supply, as they increased their consumption by more than other household groups, in percentage terms.

The results for the HetLS case are consistent with what Cantore et al. (2022) call the "dampening channel" in the context of a contractionary monetary policy shock. For the United States and the United Kingdom, the authors document that, although aggregate hours worked and labour earnings decline following a positive interest rate shock, labour supply increases for households in the left tail of the income distribution. The authors propose a two-agent New Keynesian model to rationalise this finding. Their key model ingredient is that income effects on labour supply are assumed to be larger for low-income households, which are modelled as hand-to-mouth agents. Alternatively, the empirical findings can also be rationalised in a two-agent model version with two types of households that are either permanently borrowers or savers due to differences in impatience. In our model, the key mechanism that delivers qualitatively different labour responses for the left tail of the income distribution is similar to that in the latter model version. However, in our case, whether a household is a borrower or a saver is not exogenously and permanently imposed but endogenous and the result of different histories of idiosyncratic shocks. The intuition for the mechanism is, however, similar to that in Cantore et al. (2022), as the low-income group contains a much higher share of households at or close to the borrowing constraint. If interest rates go up, these households are strongly affected by the increased debt burden and/or cost of borrowing. In the HetLS case, unions take the

Figure 6: Output, inflation and real wage responses to an anticipated expansionary monetary policy shock in 5 quarters



Notes: IRFs for interest rate shock $\varepsilon^r_{t,t+5} = -0.01$ are expressed in percentage deviations from steady state. HomLS = homogeneous labour supply, HetLS = heterogeneous labour supply, HomLSC = homogeneous labour supply with labour supply constraints.

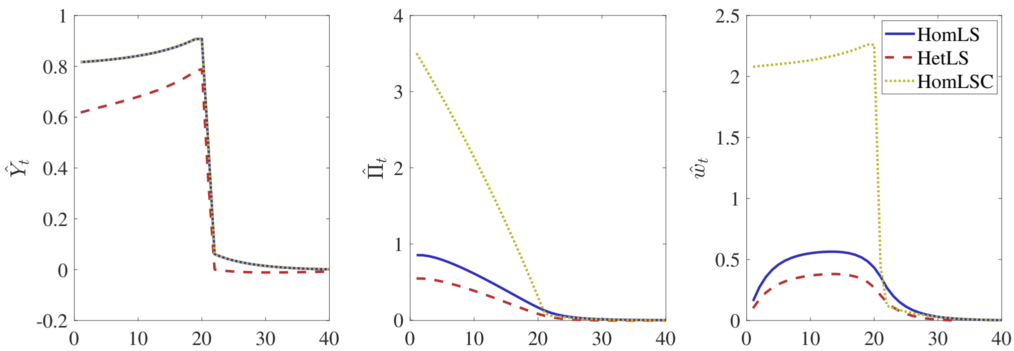
financial situation of individual households into account and demand more labour from low-income, i.e. low-productivity, households as they benefit disproportionally more from working additional hours due to their distressed financial situation (see equation (6)). This labour response partially offsets the response of the other groups of households, who work less on average, leading to an attenuated response of the economy at the aggregate level to the interest rate hike.[27] Importantly, in our paper, the relationship between the income type and the (effective) labour supply elasticity arises endogenously and is not imposed exogenously.

In the case of homogeneous labour supply and labour supply constraints (HomLSC), the output response coincides with the one in the HomLS case due to our modelling and calibration strategy. Specifically, we calibrate output (and hence aggregate labour supply), the real rate and the share of borrowing-constrained households to be the same in the steady state for all three cases. This, together with the model assumption that dividends – like labour income $w_t N_t$ – enter the household budget constraints in proportion to individual productivity, implies that total household (dividend plus labour) income, $e_{i,t}\,(Y_t - \Phi)$, moves identically in both model versions. This model property does not, however, imply that the composition of income coincides in both model versions. This can be seen in the centre and right panels of Figure 2, where wages for HomLSC exhibit a very different pattern than for HomLS (and HetLS). Due to individual labour supply constraints, real wages in the HomLSC case have to increase sharply on impact, because they have to incentivise the individual household with the lowest willingness to work to supply the demanded hours. Recall that it is this marginal household whose willingness to work determines hours worked for all households.[28] Accordingly, the resulting wage increase turns out to be much stronger on impact than in HomLS (and HetLS). This higher wage increase

[27]While the labour responses across the income distribution are qualitatively in line with Cantore et al. (2022), the relative magnitude is not. Although labour supply responds more for low-income households, in line with the data, the magnitude is not that much different relative to the other income groups.

[28]Therefore, the response of actual hours worked by each household corresponds to the response of aggregate hours worked, which in turn corresponds to the response of output in Figure 2.

Figure 7: Output, inflation and real wage responses to an anticipated expansionary monetary policy shock in 20 quarters



Notes: IRFs for interest rate shock $\varepsilon^r_{t,t+20} = -0.01$ are expressed in percentage deviations from steady state. HomLS = homogeneous labour supply, HetLS = heterogeneous labour supply, HomLSC = homogeneous labour supply with labour supply constraints. HomLS = homogeneous labour supply, HetLS = heterogeneous labour supply, HomLSC = homogeneous labour supply with labour supply constraints.

is needed to induce the marginal household with a binding labour supply constraint to work more. This can be seen from the following equation, which is a rewritten version of equation (6),

$$\varphi_1 N_t^{\varphi_2} = \int_i e_{i,t} \left(w_t c_{i,t}^{-\sigma} - \lambda_t \right) di - \zeta_{m,t}.$$

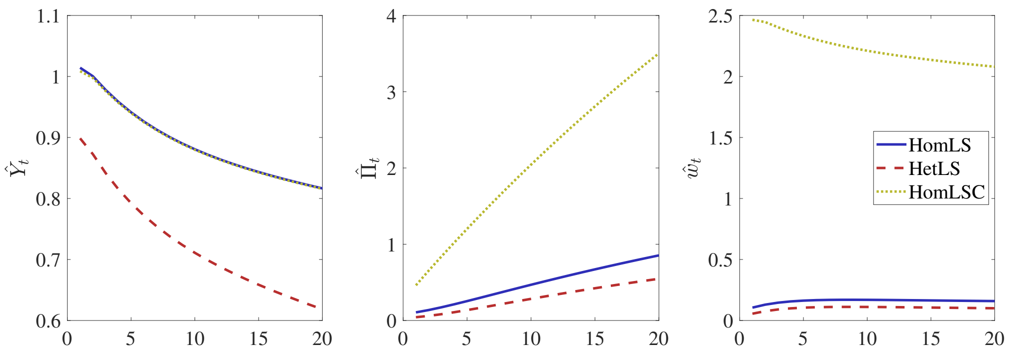
The multiplier $\zeta_{m,t}$ denotes the multiplier associated with the marginal household's labour supply constraint (and λ_t is defined as above). It introduces an additional wedge in the labour supply condition (and note that the multiplier $\zeta_{i,t}$ is zero for all $i \neq m$).[29]

As soon as real interest rates return to the initial steady-state level and aggregate demand falls, the initial response is undone. The marginal household is willing to work more for lower real wages and its labour supply constraint eases with the drop in consumption. Since real wages represent firms' marginal costs, inflation dynamics reflect the wage movement. However, price-setting is subject to adjustment costs, such that forward-looking firms do not change prices as abruptly as unions change wages (centre panel of Figure 2) – on impact and in the subsequent periods.[30]

The wage response in the HomLSC case shows that the union's need to respect the labour supply preferences of the marginal household effectively lowers wage stickiness. Indeed, the labour supply constraint of the marginal household becomes the dominant force behind the union's wage-setting decisions. Effectively, wage setting behaves as it would in a flex-wage setting where only the labour supply condition of the marginal household matters. Modifying the HomLSC NKWPC by raising the wage rigidity parameter κ_w (lower wage rigidity) or even discarding the forward-looking expectations term does not change the dynamics of output, wages and inflation. Only the magnitude of multiplier $\zeta_{m,t}$ changes in these instances. This distinguishes the relevance of labour supply constraints for model dynamics in HANK from that in RANK (see Huo and Ríos-Rull, 2020). In a RANK model, labour supply constraints bind only occasionally and at the aggregate

[29]Recall that we normalise aggregate labour productivity to one.
[30]This would still be the case if we did not assume $\kappa_p > \kappa_w$ but $\kappa_p = \kappa_w$ instead.

Figure 8: Strength of forward guidance effects in announcement period for different time horizons



Notes: Period-*t* effects of shock $\varepsilon^r_{t,t+k} = -0.01$ are expressed in percentage deviations from steady state. HomLS = homogeneous labour supply, HetLS = heterogeneous labour supply, HomLSC = homogeneous labour supply with labour supply constraints.

level. By contrast, the labour supply constraint of the marginal household is always (just) binding in our HANK model, where these constraints matter in the cross-section. As a result, labour supply constraints affect model dynamics much more profoundly relative to a RANK model with occasionally-binding labour supply constraints that are imposed at the aggregate level.[31]

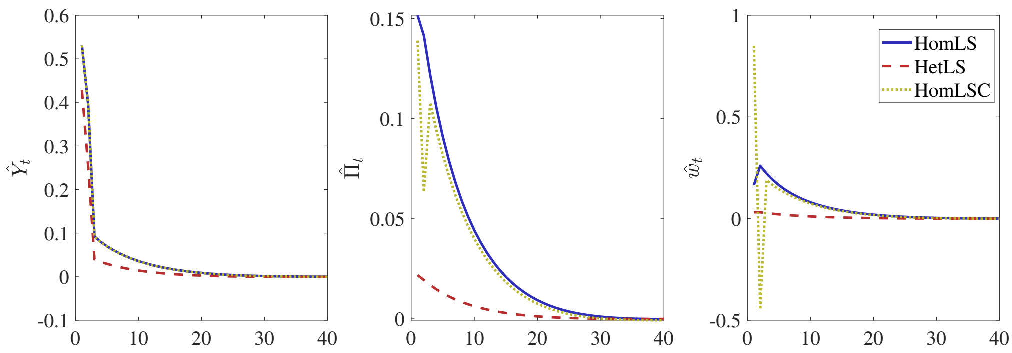
4.2 Forward guidance

We now turn to anticipated future monetary policy shocks (interest rate forward guidance). Figures 6 and 7 depict the impulse responses to forward guidance for all three versions of the model. Figure 6 depicts the effects of short-term (five-quarter) forward guidance ($k = 5$), while Figure 7 shows the effects of forward guidance at a twenty-quarter horizon ($k = 20$).

Qualitatively, the responses follow the patterns observed for the contemporaneous monetary policy shock. Again, the increase in real wages and inflation for HetLS lies below HomLS, for the same reason as for the contemporaneous monetary policy shock. Similarly, HomLSC again implies a more pronounced real wage response, which in turn leads to stronger inflation effects. With respect to the impact of interest rate forward guidance on outcomes in the announcement period, all three versions of the model again make the same qualitative predictions. This can be seen in Figure 8, which depicts the impact of the news shock at different horizons on the response of each variable upon announcement. In all three cases, the output effects decline with the forward guidance horizon. Quantitatively, this property is most pronounced in the HetLS case. The reason for this is that with HetLS, low-income households' consumption does not increase as much until the interest rate decrease materialises. Middle-income and high-income households, by contrast, increase their consumption the most on impact. As discussed in detail in the previous section, the real wage increase induced by monetary policy implies that middle-income and high-

[31] In Huo and Ríos-Rull (2020), the probability of labour supply constraints binding in the future can also affect model dynamics. However, the importance of this (additional) effect is likely limited.

Figure 9: Model responses after an expansionary monetary policy shock with lower intertemporal elasticity of substitution ($\sigma = 2$)



Notes: IRFs for interest rate shock $\varepsilon^r_{t,t} = -0.01$ are expressed in percentage deviations from steady state. HomLS = homogeneous labour supply, HetLS = heterogeneous labour supply, HomLSC = homogeneous labour supply with labour supply constraints.

income households work more, while low-income households work less. Unions can better accommodate the high-income households' desire to work more in the shock realisation period, the further away this period is. To some extent, this pattern is also present for middle-income households. While the effect on real wages and hence inflation does not decline with the forward guidance horizon, it does not explode either. Our model variants therefore attenuate the "forward guidance puzzle" (see Del Negro et al., 2023). However, in contrast to McKay et al. (2016), this attenuation is not due to the cyclical behaviour of dividends (or taxes).[32] In our setting, the cyclicality of dividends plays – by construction – only a negligible role. Specifically, our steady-state dividends are calibrated to zero and – within and outside the steady state – they are distributed proportionally to individual labour productivity, resulting in acyclical income risk.

5 Sensitivity

In this section, we discuss the robustness of our results with respect to assumptions about various model features and parameters made so far. Specifically, we assess the impact of the intertemporal elasticity of substitution, the cyclicality of income risk, the (non-)availability of outside liquidity and the distribution of dividends. Our main findings are robust to changes in the model calibration and specification.

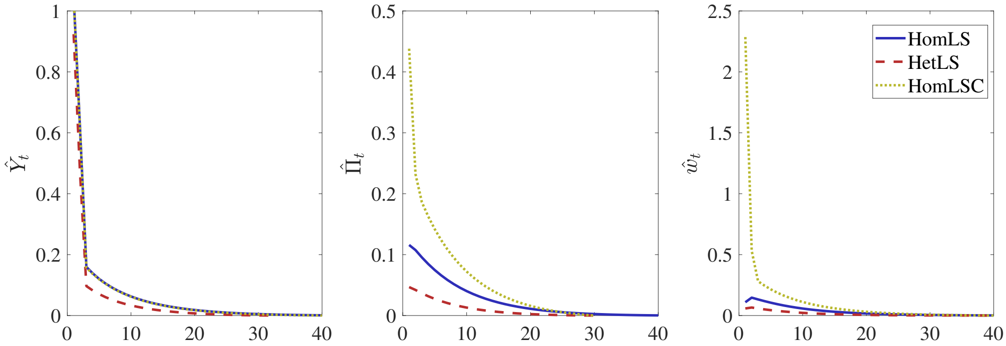
5.1 Intertemporal elasticity of substitution

In the first experiment, we lower households' intertemporal elasticity of substitution (IES) from $\sigma^{-1} = 1$ to $\sigma^{-1} = 0.5$. This change increases the desire to smooth consumption.[33] Figure 9 shows the results. Compared to the benchmark case with log utility shown in Figure 2, we now observe an attenuation of the output (and therefore consumption and labour)

[32]In McKay et al. (2016), dividends are countercyclical, implying procyclical income risk, which in turn reduces the power of forward guidance (see Werning, 2015).

[33]The model is again recalibrated to match the same long-run targets as for $\sigma = 1$.

Figure 10: Model responses after an expansionary monetary policy shock with countercyclical iodiosyncratic risk ($\xi < 0$)



Notes: IRFs for interest rate shock $\varepsilon^r_{t,t} = -0.01$ are expressed in percentage deviations from steady state. HomLS = homogeneous labour supply, HetLS = heterogeneous labour supply, HomLSC = homogeneous labour supply with labour supply constraints.

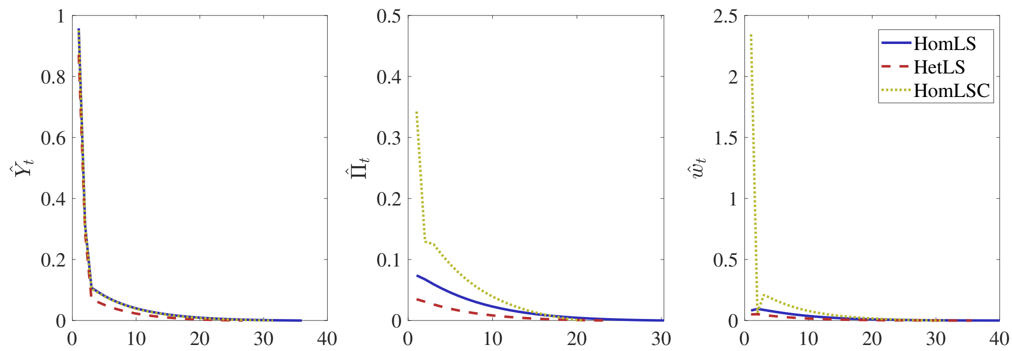
responses in all three cases. The response of real wages is also attenuated in the HetLS case and, on impact period, even more so in the HomLSC case. In the HomLS case, real wages increase more strongly than in the benchmark calibration. The response of inflation reflects the pattern of wage responses. In particular, the initial effect is now dampened in the HomLSC case, because the initial wage increase is weaker and and because wages decrease in period two, when the labour supply constraint binds less strongly and the willingness of the marginal household to supply labour increases sharply.

In all cases, a lower IES implies a lower consumption increase on impact and therefore a lower increase in the demand for labour. However, a lower IES (higher constant relative risk aversion) also means stronger wealth effects on labour supply, such that households' desire to work declines more strongly as they get wealthier. These two effects counter each other regarding their impact on real wages, and it is not obvious a priori which of the effects prevails. For the HetLS case, the lower demand for labour prevails over the wealth effect, which causes an attenuation of the real wage response. But in the HomLS case, the labour demand effect prevails over the (average) wealth effect (as viewed through the labour union's objective).[34] Note that this is the only case where households' individual labour supply preferences are ignored by the unions, who only take average household utility into account.

Compared to the log-utility case, we observe a strong attenuation of the real wage increase on impact for HomLSC in Figure 9 and a strong reversion in the subsequent period, with real wages suddenly, but only briefly, declining below their steady-state level. The inflation response mirrors these wage movements. However, due to the only short-lived real wage decline in the second period, we do not see inflation going below steady state, owing to the forward-looking nature of price-setting firms. The peculiar wage response in the HomLSC case reflects the fact that the incentive-provision component of the NKWPC

[34]The results go into the opposite direction when allowing for a higher IES, e.g. $\sigma^{-1} = 2$.

Figure 11: Model responses after an expansionary monetary policy shock with positive outside liquidity ($B > 0$)



Notes: IRFs for interest rate shock $\varepsilon^r_{t,t} = -0.01$ are expressed in percentage deviations from steady state. HomLS = homogeneous labour supply, HetLS = heterogeneous labour supply, HomLSC = homogeneous labour supply with labour supply constraints.

(10), as captured by the last term on the RHS of the equation with partial derivative

$$\frac{\partial n^*_{i,t}}{\partial w_t} = \varphi_2^{-1}\left(\varphi_1^{-1} e_{i,t} w_t c_{i,t}^{-\sigma}\right)^{\varphi_2^{-1}-1} \times \varphi_1^{-1} e_{i,t}\left[c_{i,t}^{-\sigma} + w_t\left(-\sigma\right) c_{i,t}^{-\sigma-1} e_{i,t} N_t\right],$$

depends on σ in non-trivial ways.

5.2 Cyclicality of income risk

The literature has highlighted the cyclicality of individual income risk as an important determinant of monetary policy transmission in HANK models (see e.g. Werning, 2015; Acharya and Dogra, 2020). We therefore check the sensitivity of our results along this dimension. To do so, we follow Auclert and Rognlie (2020) and assume that individual household productivity is given as $e_i \Gamma(e_i, Y_t)$, with

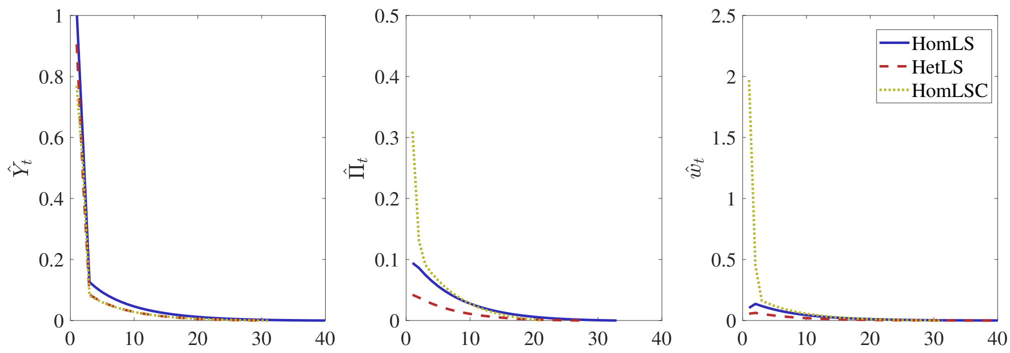
$$\Gamma(e_i, Y_t) = \frac{e_i^{\xi \log(Y_t/Y)}}{\sum_j \Pr\left(e_j\right) e_j^{1+\xi \log(Y_t/Y)}}.$$

Under this specification, the cyclicality of the cross-sectional variance of individual log labour productivity is directly governed by ξ. In the benchmark case considered so far, we implicitly assume $\xi = 0$, implying acyclical income risk. The variance becomes procyclical for $\xi > 0$ and countercyclical for $\xi < 0$. The latter case seems to be the empirically relevant one (see e.g. Ravn and Sterk, 2021).[35] In line with Werning (2015), the response of the economy to a monetary policy shock becomes stronger under countercyclical income risk (see Figure 10).[36] This property holds for all three of our model versions. However, the differences compared to the benchmark simulation are rather small.

[35] We consider the value $\xi = -0.25$.

[36] For procyclical income risk ($\zeta > 0$), the findings go in the opposite direction.

Figure 12: Model responses after an expansionary monetary policy shock under uniform dividend distribution



Notes: IRFs for interest rate shock $\varepsilon^r_{t,t} = -0.01$ are expressed in percentage deviations from steady state. HomLS = homogeneous labour supply, HetLS = heterogeneous labour supply, HomLSC = homogeneous labour supply with labour supply constraints.

5.3 Positive outside liquidity

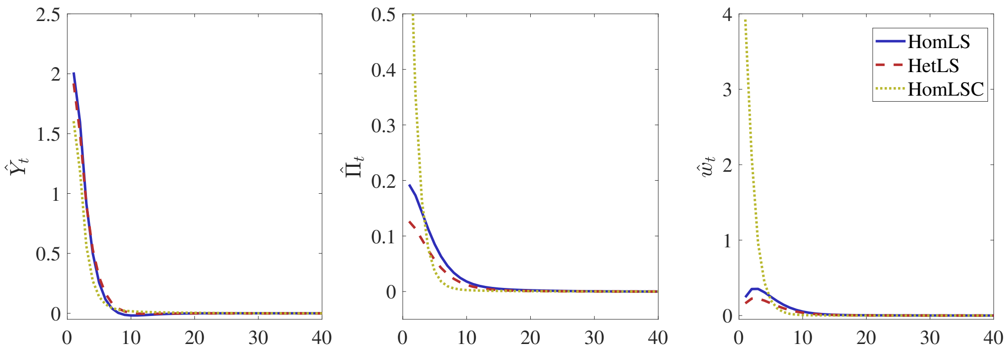
So far, we did not allow for outside liquidity in the model. Households' aggregate credit demand therefore had to be met by households' aggregate savings. As a result, taxes, which are levied to pay maturing government debt, were zero. Once the model is recalibrated to match the same targets for steady-state output, the real rate and the share of borrowing-constrained households, the results for a calibration with positive government debt are very close to those of our baseline model version (see Figure 11).[37]

5.4 Distribution of dividends

In the HANK literature, it is well known that the distribution of dividends among households can have potentially strong effects on the transmission of monetary policy (see Werning, 2015). So far, we neutralised the impact of the dividend distribution on outcomes by *(i)* targeting zero steady-state profits and hence dividends, and *(ii)* assuming dividends are distributed in proportion to individual productivity. The first assumption ensures that dividends are not an important source of income for households on average, but also in response to aggregate shocks. The second assumption implies that the composition of total income does not matter for the households' decisions under homogeneous labour supply. It also implies acyclical income risk. If we assume instead that all households receive the same amount of dividends, income risk becomes procyclical (even for $\xi = 0$ with the specification from Section 5.2), which dampens the power of forward guidance about future interest rates (see McKay et al., 2016). In addition, the real economy does not behave in the same way under homogeneous labour supply, regardless of whether labour supply constraints are imposed or not, because real wage dynamics now have distributional consequences. However, while the output response for HomLSC is now weaker compared to the HomLS case (see Figure 12), our main results do not change.

[37]Public debt supply is constant over time and equal to 52% of annual GDP. Taxes are paid by all households in proportion to their individual productivity.

Figure 13: Output, inflation and real wage after an expansionary nominal rate shock



Notes: IRFs for interest rate shock $\epsilon_t^R = -0.01$ are expressed in percentage deviations from steady state. HomLS = homogeneous labour supply, HetLS = heterogeneous labour supply, HomLSC = homogeneous labour supply with labour supply constraints.

6 Other types of shocks

Until now, we considered shocks to the real rate as the only aggregate disturbance. In this section, we compare the response of selected variables across our three model versions for other shocks as well. Specifically, we look at a standard (nominal) interest rate shock as well as aggregate supply and demand shocks.

6.1 Nominal rate shock

To evaluate the implications of a standard monetary policy shock, we consider the Taylor-type interest rate rule

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left(\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\Pi} \left(\frac{Y_t}{Y}\right)^{\phi_Y}\right)^{1-\rho_R} \exp(\epsilon_t^R), \tag{12}$$

instead of the real rate rule assumed previously.[38]

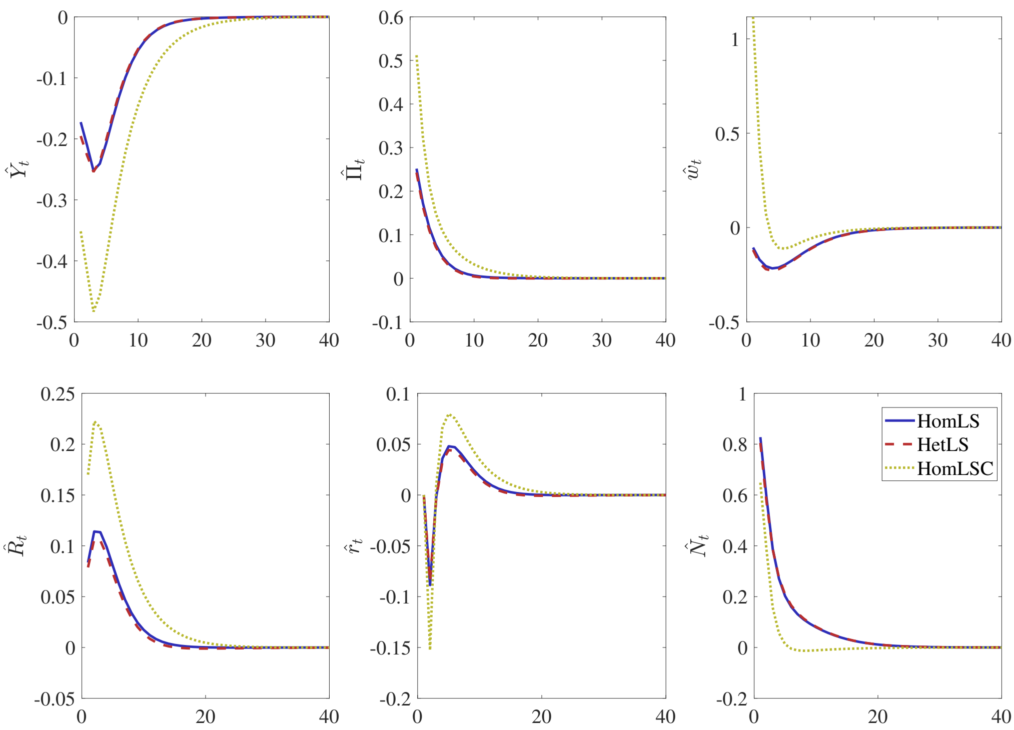
Figure 13 shows IRFs for an expansionary standard nominal interest rate shock $\epsilon_t^R = -0.01$. Broadly, the results are in line with those obtained for the real rate rule. Importantly, however, the evolution of the real rate does not coincide in the HomLS and HomLSC cases anymore. Since the consumption demand by households differs as a result, the associated output paths are not identical either, with the response being dampened in the HomLS case. Overall, because inflation increases on impact and persistently as well as due to interest rate smoothing implied by $\rho_R > 0$, the real interest rate decreases persistently as well, which is why we observe stronger and more persistent responses under the Taylor-type rule compared to the real rate rule considered previously.

6.2 Supply shock

We now turn to an aggregate shock that affects the supply side of the economy. Let the production function of intermediate good producers now be given as $A_t N_t$, with $A_t =$

[38] We assume the parameter values $\rho_R = 0.75$, $\phi_\Pi = 1.5$ and $\phi_Y = 0.25$ for the interest rate rule.

Figure 14: Selected variables after a contractionary supply shock



Notes: IRFs for productivity shock $\varepsilon_t^A = -0.01$ are expressed in percentage deviations from steady state. HomLS = homogeneous labour supply, HetLS = heterogeneous labour supply, HomLSC = homogeneous labour supply with labour supply constraints.

$A_{t-1}^{\rho_A} \exp(\epsilon_t^A)$ denoting exogenous aggregate productivity.[39] For the nominal interest rate, we again assume that equation (12) applies. Assuming $\rho_A = 0.8$, Figure 14 plots the responses of selected variables to a contractionary supply shock $\epsilon_t^A = -0.01$.[40] In all three model versions, output declines in response to the shock despite aggregate labour N_t going up, reflecting a negative income effect. Because output is demand-determined in the short run, and households wish to smooth their consumption, consumption demand and hence output fall by less than productivity. In order to supply the goods demanded, firms have to make up for this gap in production by increasing their labour demand. In the HomLSC case, this puts upward pressure on wages, causing inflation to increase more sharply relative to the other two cases. This stronger inflation effect in turn results in a more aggressive response of the central bank, leading to a notably deeper recession compared to the model versions without labour supply constraints.

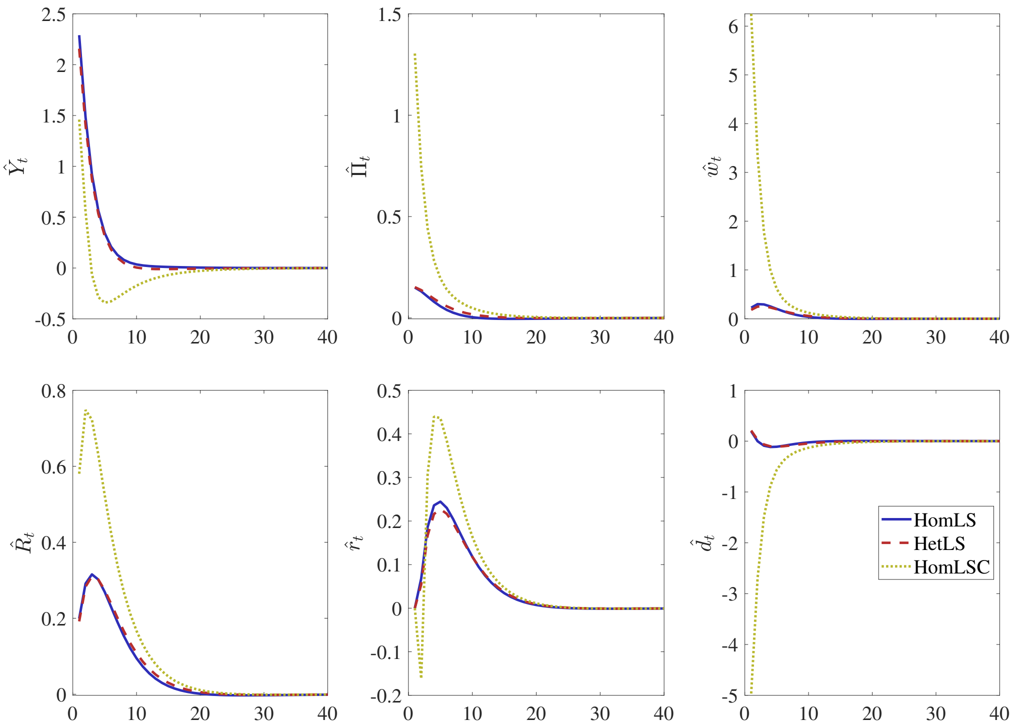
6.3 Demand shock

Finally, we consider an aggregate demand shock by assuming a time-varying household discount factor that follows the autoregressive process $\beta_t = \beta^{1-\rho_\beta} \beta_{t-1}^{\rho_\beta} \exp(\epsilon_t^\beta)$. Again, we assume the process to have an autocorrelation of 0.8 and consider the interest rate rule

[39] Since firms' marginal costs are now given by w_t/A_t, the NKPC will look slightly different in this case.

[40] In reduced form, such a shock could perhaps capture the macroeconomic conditions experienced by many economies in the aftermath of the COVID-19 pandemic.

Figure 15: Selected variables after a contractionary demand shock



Notes: IRFs for discount factor shock $\epsilon_t^{\beta} = -0.01$ are expressed in percentage deviations from steady state, except d_t which is in absolute deviations from steady state. HomLS = homogeneous labour supply, HetLS = heterogeneous labour supply, HomLSC = homogeneous labour supply with labour supply constraints.

(12). Figure 15 plots the results for an expansionary demand shock $\epsilon_t^{\beta} = -0.01$. In line with our previous results, the response of real wages is much more pronounced in the HomLSC case. The same holds true for output. In contrast to the supply shock, inflation now also responds much more strongly in the HomLSC case compared to the other two cases. The aggregate demand shock implies that all households suddenly want to front-load consumption, including the marginal household, who now requires a much higher wage rate to supply additional hours. To contain the inflationary response due to increased wage pressure, the central bank has to raise the nominal rate much further in the HomLSC case than in the other two cases, muting the output response to the shock as a result.

One of the reasons the HANK literature now commonly assumes sticky nominal wages is that models with flexible wages predict output and firm profits to move in opposite directions following a demand shock. This prediction is not a desirable one because the data suggests a positive comovement between those variables. However, if wages respond sluggishly following a demand shock, profits become procyclical – in line with the empirical evidence. As can be seen in Figure 15, the HomLS and HetLS cases do indeed exhibit this feature. By contrast, profits are countercyclical in the HomLSC case since this case involves more flexible wages. This model version thus eliminates a key feature that often motivates the introduction of sticky wages.

7 Conclusion

In this paper, we assess the importance of assumptions made in sticky-wage HANK models about hours worked at the household level. Following most of the recent HANK literature, our baseline model requires wage-setting unions to demand the same amount of labour from all households. As a result, unions do not take account of the fact that *(i)* households are heterogeneous in their willingness to work, and that *(ii)* some households might have work more hours than they would like to. We consider two modifications of this modelling approach in this paper. First, we allow unions to demand different hours from different households. We find that optimal labour demanded by unions leads to a rationing of labour supply for all households in this case. Not a single individual therefore works against his will. Compared to our baseline model with homogeneous labour supply, output, wages and inflation respond less strongly to monetary policy shocks under heterogeneous labour supply. In line with empirical evidence presented in Cantore et al. (2022), this model version predicts hours worked to behave qualitatively differently across the income distribution following a monetary policy shock. Our second modification of the standard approach maintains the homogeneous labour supply assumption but does not permit unions to require households to work more than they would like to. Although we show that this modification does not matter (much) for the impact of monetary policy on the real economy, it endogenously raises the flexibility of nominal wages and results in notably different responses of goods prices and wages to monetary policy shocks. An immediate consequence of increased wage flexibility is a strong countercyclicality of profits. This property is at odds with the data and absent in the two other model versions considered in this paper, likely disqualifying further application of this model version. By contrast, letting unions optimally allocate labour across households does not appear to have any serious drawback. Instead, this modelling assumption can successfully address concerns related to the commonly used uniform labour assumption. Addressing these concerns might be particularly relevant for welfare evaluations or optimal policy analysis in sticky-wage HANK models.

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A Appendix: Derivation of New Keynesian wage Phillips curves

This section derives the New Keynesian wage Phillips curves for the three cases covered in this paper (see Section 2.3). First, we derive the NKWPC under heterogeneous labour supply. Second, we consider the union problem under homogeneous labour supply and labour supply constraints. Third, we derive the NKWPC for the homogeneous labour supply case without labour supply constraints.

Heterogeneous labour supply (HetLS) Without loss of generality, we let unions directly set the real wage, given current and future inflation. In the HetLS case, the decision problem of a union k is then given by

$$\max_{\left\{w_{k,t+s},\{n_{i,k,t+s}\}_{i\in[0,1]}\right\}_{s=0}^{\infty}} \sum_{s=0}^{\infty} \beta^s \left\{ \int_i \left\{ \frac{c_{i,t+s}^{1-\sigma}-1}{1-\sigma} - \int_k \varphi_1 \frac{n_{i,k,t+s}^{1+\varphi_2}}{1+\varphi_2} dk \right\} di \right.$$
$$\left. - \frac{\varepsilon_w}{2\kappa_w}\left[\log\left(\frac{w_{k,t+s}}{w_{k,t+s-1}\Pi_W}\Pi_{t+s}\right)\right]^2 \right\}$$

subject to the constraint

$$\left(\frac{w_{k,t+s}}{w_{t+s}}\right)^{-\varepsilon_w} N_{t+s} = \int_i e_{i,t+s} n_{i,k,t+s} di,$$

which is derived from the two conditions

$$N_{k,t+s} = \left(\frac{w_{k,t+s}}{w_{t+s}}\right)^{-\varepsilon_w} N_{t+s},$$
$$N_{k,t+s} = \int_i e_{i,t+s} n_{i,k,t+s} di.$$

Union k takes as given the individual saving decisions, i.e. $b_{i,t+s}$.[41] The Lagrangian for its decision problem is

$$\min_{\{\lambda_{k,t+s}\}_{s=0}^{\infty}} \max_{\left\{w_{k,t+s},\{n_{i,k,t+s}\}_{i\in[0,1]}\right\}_{s=0}^{\infty}} \sum_{s=0}^{\infty} \beta^s \left\{ \int_i \left\{ \frac{c_{i,t+s}^{1-\sigma}-1}{1-\sigma} - \int_k \varphi_1 \frac{n_{i,k,t+s}^{1+\varphi_2}}{1+\varphi_2} dk \right\} di \right.$$
$$- \frac{\varepsilon_w}{2\kappa_w}\left[\log\left(\frac{w_{k,t+s}}{w_{k,t+s-1}\Pi_W}\Pi_{t+s}\right)\right]^2$$
$$\left. + \lambda_{k,t+s}\left[\left(\frac{w_{k,t+s}}{w_{t+s}}\right)^{-\varepsilon_w} N_{t+s} - \int_i e_{i,t+s} n_{i,k,t+s} di\right]\right\}.$$

[41]Taking the derivative of both sides of the household budget constraint, $c_{i,t+s} = e_{i,t+s}\int_k w_{k,t+s} n_{i,k,t+s} dk + e_{i,t+s}(d_{t+s} - T_{t+s}) + b_{i,t+s-1} r_{t+s} - b_{i,t+s}$ with respect to $w_{k,t+s}$ yields $\partial c_{i,t+s}/\partial w_{k,t+s} = e_{i,t+s} n_{i,k,t+s}$, whereas the derivative with respect to $n_{i,k,t+s}$ is $\partial c_{i,t+s}/\partial n_{i,k,t+s} = e_{i,t+s} w_{k,t+s}$.

The union's first-order conditions for $w_{k,t}$ and $n_{i,k,t}$ are

$$0 = -\frac{\varepsilon_w}{\kappa_w}\frac{\log\left(\frac{\Pi_{W,k,t}}{\Pi_W}\right)}{w_{k,t}} + \beta\frac{\varepsilon_w}{\kappa_w}\frac{\log\left(\frac{\Pi_{W,k,t+1}}{\Pi_W}\right)}{w_{k,t}} + \int_i e_{i,t}n_{i,k,t}c_{i,t}{}^{-\sigma}di$$
$$+ \lambda_{k,t}\left[-\varepsilon_w w_t^{-1}\left(\frac{w_{k,t}}{w_t}\right)^{-\varepsilon_w-1}N_t\right],$$

and

$$0 = e_{i,t}w_{k,t}c_{i,t}^{-\sigma} - \varphi_1 n_{i,k,t}^{\varphi_2} - \lambda_{k,t}e_{i,t},$$

respectively.

In a symmetric equilibrium, we have $w_{k,t} = w_t$, $\Pi_{W,k,t} = \Pi_{W,t}$, $N_{k,t} = N_t$, and $\lambda_{k,t} = \lambda_t$.[42] We can thus reduce the above conditions to

$$\log\left(\frac{\Pi_{W,t}}{\Pi_W}\right) = \beta\log\left(\frac{\Pi_{W,t+1}}{\Pi_W}\right) + \frac{\kappa_w}{\varepsilon_w}\int_i e_{i,t}w_t n_{i,t}c_{i,t}^{-\sigma}di + \lambda_t\left[-\kappa_w N_t\right],$$

and

$$0 = e_{i,t}w_t c_{i,t}^{-\sigma} - \varphi_1 n_{i,t}^{\varphi_2} - \lambda_t e_{i,t}.$$

We can now use that

$$\lambda_t = \frac{e_{i,t}w_t c_{i,t}^{-\sigma} - \varphi_1 n_{i,t}^{\varphi_2}}{e_{i,t}},$$

holds for all households i, such that

$$\lambda_t = \frac{\int_i\left\{e_{i,t}w_t c_{i,t}^{-\sigma} - \varphi_1 n_{i,t}^{\varphi_2}\right\}di}{\int_i e_{i,t}di},$$

holds as well.[43]

Since we normalised the support of $e_{i,t}$ such that $\int_i e_{i,t}di = 1$, we can write

$$\lambda_t = \int_i\left\{e_{i,t}w_t c_{i,t}^{-\sigma} - \varphi_1 n_{i,t}^{\varphi_2}\right\}di.$$

After using this condition to substitute out λ_t in the first-order condition for wages and collecting terms, we obtain

$$\log\left(\frac{\Pi_{W,t}}{\Pi_W}\right) = \kappa_w\left[N_t\int_i \varphi_1 n_{i,t}^{\varphi_2}di - w_t\int_i (N_t - \epsilon_w{}^{-1}n_{i,t})e_{i,t}c_{i,t}^{-\sigma}di\right] + \beta\log\left(\frac{\Pi_{W,t+1}}{\Pi_W}\right).$$

Homogeneous labour supply with labour supply constraints (HomLSC) Under the assumption that unions uniformly demand labour services from households, i.e. all households supply the same amount of hours, but face labour supply constraints, the de-

[42] In a symmetric equilibrium, $n_{i,k,t} = n_{i,t}$ holds as well, which should not be confused with the assumption of a uniform labour demand as in Auclert et al. (2023a).

[43] Due to the unit-mass assumption for the households, it holds that $\int_i \lambda_t di = \lambda_t \int_i di = \lambda_t$.

cision problem of a union k is given by

$$\max_{\{w_{k,t+s},N_{k,t+s}\}_{s=0}^{\infty}} \sum_{s=0}^{\infty} \beta^s \left\{ \int_i \left\{ \frac{c_{i,t+s}^{1-\sigma}}{1-\sigma} - \int_k \varphi_1 \frac{N_{k,t+s}^{1+\varphi_2}}{1+\varphi_2} \right\} di - \frac{\varepsilon_w}{2\kappa_w} \left[\log \left(\frac{w_{k,t+s}}{w_{k,t+s-1}\Pi_W} \Pi_{t+s} \right) \right]^2 \right\}$$

subject to

$$N_{k,t+s} = \left(\frac{w_{k,t+s}}{w_{t+s}} \right)^{-\varepsilon_w} N_{t+s},$$

and

$$N_{k,t+s} \leq n_{i,k,t+s}^*,$$

where $n_{i,k,t+s}^*$ is the maximum amount of labour that household i is willing to supply to union k at wage $w_{k,t+s}$, if it could freely choose. For the functional forms assumed in this paper, the counterfactual optimality condition $\varphi_1 \left(n_{i,k,t+s}^* \right)^{\varphi_2} = e_{i,t+s} w_{k,t+s} c_{i,t+s}^{-\sigma}$ characterises $n_{i,k,t+s}^*$. We can then simply express counterfactual labour supply as

$$n_{i,k,t+s}^* = \left(\varphi_1^{-1} e_{i,t+s} w_{k,t+s} c_{i,t+s}^{-\sigma} \right)^{\varphi_2^{-1}}.$$

The Lagrangian for union k's problem is

$$\begin{aligned}
\min_{\left\{\lambda_{k,t+s}, \{\zeta_{i,k,t+s}\}_{i\in[0,1]}\right\}_{s=0}^{\infty}} \max_{\{w_{k,t+s},N_{k,t+s}\}_{s=0}^{\infty}} \sum_{s=0}^{\infty} \beta^s \Bigg\{ & \int_i \left\{ \frac{c_{i,t+s}^{1-\sigma} - 1}{1-\sigma} - \int_k \varphi_1 \frac{N_{k,t+s}^{1+\varphi_2}}{1+\varphi_2} dk \right\} di \\
& - \frac{\varepsilon_w}{2\kappa_w} \left[\log \left(\frac{w_{k,t+s}}{w_{k,t+s-1}\Pi_W} \Pi_{t+s} \right) \right]^2 \\
& + \lambda_{k,t+s} \left[\left(\frac{w_{k,t+s}}{w_{t+s}} \right)^{-\varepsilon_w} N_{t+s} - N_{k,t+s} \right] \\
& + \int_i \zeta_{i,k,t+s} \left[n_{i,k,t+s}^* - N_{k,t+s} \right] di \Bigg\}.
\end{aligned}$$

The first-order conditions for $W_{k,t}$ and $N_{k,t}$ are

$$\begin{aligned}
0 = & - \frac{\varepsilon_w}{\kappa_w} \frac{\log\left(\frac{\Pi_{W,k,t}}{\Pi_W}\right)}{w_{k,t}} + \beta \frac{\varepsilon_w}{\kappa_w} \frac{\log\left(\frac{\Pi_{W,k,t+1}}{\Pi_W}\right)}{w_{k,t}} + \int_i e_{i,t} N_{k,t} c_{i,t}^{-\sigma} di \\
& + \lambda_{k,t} \left[-\varepsilon_w w_t^{-1} \left(\frac{w_{k,t}}{w_t} \right)^{-\varepsilon_w - 1} N_t \right] + \int_i \zeta_{i,k,t} \frac{\partial n_{i,k,t}^*}{\partial w_{k,t}} di,
\end{aligned}$$

and

$$0 = \int_i e_{i,t} w_{k,t} c_{i,t}^{-\sigma} di - \varphi_1 N_{k,t}^{\varphi_2} - \lambda_{k,t} - \int_i \zeta_{i,t} di,$$

with

$$\frac{\partial n_{i,k,t}^*}{\partial w_{k,t}} = \varphi_2^{-1} \left(\varphi_1^{-1} e_{i,t} w_{k,t} c_{i,t}^{-\sigma} \right)^{\varphi_2^{-1}-1} \times \varphi_1^{-1} e_{i,t} \left[c_{i,t}^{-\sigma} + w_{k,t} \left(-\sigma\right) c_{i,t}^{-\sigma-1} e_{i,t} N_{k,t} \right].$$

Now use the first-order condition for $N_{k,t}$ to replace the multiplier $\lambda_{i,t}$ in the first-order condition for $w_{k,t}$ and impose symmetry. We then arrive at the expression

$$\log\left(\frac{\Pi_{W,t}}{\Pi_W}\right) = \kappa_w \left[\varphi_1 N_t^{1+\varphi_2} - \frac{\varepsilon_w - 1}{\varepsilon_w}\int_i e_{i,t} w_t N_t c_{i,t}^{-\sigma} di\right] + \beta \log\left(\frac{\Pi_{W,t+1}}{\Pi_W}\right)$$
$$+ w_t \frac{\kappa_w}{\varepsilon_w}\int_i \zeta_{i,+t}\left\{\varepsilon_w w_t^{-1} N_t + \frac{\partial n_{i,t}^*}{\partial w_t}\right\} di.$$

This equation is the New Keynesian wage Phillips curve in Auclert et al. (2023a), extended by an additional term that reflects potentially binding labour supply constraints and their impact on wage-setting. In addition to this condition, the complementary slackness conditions

$$n_{i,t}^* - N_t \geq 0, \quad \zeta_{i,t} \geq 0, \quad \zeta_{i,t} \times \left(n_{i,t}^* - N_t\right) = 0,$$

need to be satisfied for all $i \in [0, 1]$.

Homogeneous labour supply (HomLS) Without labour supply constraints, union k's maximisation problem under the uniform labour assumption is

$$\max_{\{w_{k,t+s}, N_{k,t+s}\}_{s=0}^{\infty}} \sum_{s=0}^{\infty} \beta^s \left\{ \int_i \left\{ \frac{c_{i,t+s}^{1-\sigma}}{1-\sigma} - \int_k \varphi_1 \frac{N_{k,t+s}^{1+\varphi_2}}{1+\varphi_2} \right\} di - \frac{\varepsilon_w}{2\kappa_w}\left[\log\left(\frac{w_{k,t+s}}{w_{k,t+s-1}\Pi_W}\Pi_{t+s}\right)\right]^2 \right\}$$

subject to

$$N_{k,t+s} = \left(\frac{w_{k,t+s}}{w_{t+s}}\right)^{-\varepsilon_w} N_{t+s}.$$

The union problem is thus the same as in the previous case, except that the labour supply constraints are absent. Consequently, one arrives at the NKWPC for this case by following the same steps as above, assuming that the labour supply constraints are always slack, i.e. $\zeta_{i,t} = 0$ for all households i and periods t,

$$\log\left(\frac{\Pi_{W,t}}{\Pi_W}\right) = \kappa_w \left(\varphi_1 N_t^{1+\varphi_2} - \frac{\epsilon_w - 1}{\epsilon_w} N_t w_t \int_i e_{i,t} c_{i,t}^{-\sigma} di\right) + \beta \log\left(\frac{\Pi_{W,t+1}}{\Pi_W}\right).$$

Note that this NKWPC is also nested by the NKWPC for heterogeneous labour supply if we impose the uniform labour assumption, i.e. set $n_{i,t} = N_t$ for all $i \in [0, 1]$.