The Long-Run Effects of Corporate Tax Reforms

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Stabilization policies: Lessons from the COVID-19 crisis

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Corporate taxes

- Key **source of revenue** in OECD countries
  - Corporate income tax (CIT) is 10% total revenue →
- General **decreasing trend** over last 40 years
  - CIT decreased from 42% to 21% →
- Reforms are **very persistent** at the country-level
  - In U.S., three CIT reforms over last 40 years →

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"...raising the rate from 21 to 28%, to help fund critical investments in infrastructure, clean energy, R&D, and more to maintain the competitiveness of the US and grow the economy".

President Biden, American Jobs Plan, 3/21

What do we do

- How do corporate tax reforms affect ...

  aggregate productivity?
  firms' market value?
  business cycles?

  } Key: Private investment

What do we do

- How do corporate tax reforms affect ...

  aggregate productivity?
  firms' market value?
  business cycles?
  } Key: Private investment

- Develop a **micro-founded** investment model
  - Heterogeneity: firm-level productivity shocks
  - Investment frictions: fixed cost + irreversibility
  - Taxes: corporate/personal income, capital gains, deductions

- Study effect of corporate taxation on three **macro outcomes**
  1. Capital allocation: $\mathbb{V}[\log mpk]$
  2. Capital valuation: Aggregate marginal q
  3. Capital fluctuations: IRF to aggregate productivity shocks

Three new insights

- Result 1: Economy <u>with</u> taxes and frictions

$$\Longleftrightarrow$$

Economy <u>without</u> taxes and <u>re-scaled</u> frictions

- Result 2: Capital allocation drives valuation and fluctuations

- Result 3: Measure macro outcomes with a few micro moments

- Application: A reduction in corporate income tax...
  - ✓ improves capital allocation
  - ✓ decreases capital valuation
  - ✓ accelerates capital fluctuations

Step 1:

A parsimonious investment model

Technology, shocks, and frictions

- **Production technology**
  - Firms produce output using capital

$$y_s = u_s^{1-\alpha} k_s^{\alpha}, \quad \alpha < 1$$

  - u, idiosyncratic productivity: $\mathrm{d}\log(u_s) = \mu \, \mathrm{d}s + \sigma \, \mathrm{d}W_s$
  - k, uncontrolled capital: $\mathrm{d}\log(k_s) = -\xi^k \, \mathrm{d}s$

- **Fixed adjustment cost** →

$$\theta_s = \theta u_s$$

  - Paid for each non-zero investment $\Delta k_s \neq 0$

- **Price wedge**

$$p(\Delta k_s) = p^{buy} \mathbb{I}(\Delta k_s > 0) + p^{sell} \mathbb{I}(\Delta k_s < 0), \qquad p^{buy} - p^{sell} > 0$$

  - Used capital (adverse selection, specificity, search, VAT...)

Firm investment problem

- **When** and **how much** to invest $\{T_h,\ \Delta k_{T_h}\}_{h=1}^{\infty}$?

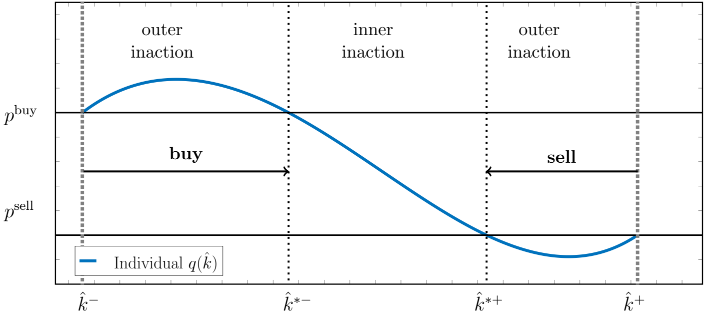
$$V(k_0, u_0) = \max_{\{T_h, \Delta k_{T_h}\}_{h=1}^{\infty}} \mathbb{E}_0 \left[\int_0^{\infty} Q_s \pi_s \, \mathrm{d}s - \sum_{h=1}^{\infty} Q_{T_h} \Big(\underbrace{\theta_{T_h}}_{\text{fixed cost}} + \underbrace{p(\Delta k_{T_h})\ \Delta k_{T_h}}_{\text{investment}} \Big) \right]$$

$$\begin{aligned}
&\text{(profits)} & \pi_s &= A y_s \\
&\text{(discount)} & Q_s &= e^{-\rho s} \\
&\text{(prices)} & p(\Delta k_s) &= p^{buy} \mathbb{I}(\Delta k_s > 0) + p^{sell} \mathbb{I}(\Delta k_s < 0)
\end{aligned}$$

- **Redefine state:** capital-productivity ratio $\hat{k} \equiv \log(k/u)$
  - Frictionless: $\hat{k}_s$ is constant $\forall s$
  - Uncontrolled: $\mathrm{d}\hat{k}_s = -\nu \, \mathrm{d}s + \sigma \, \mathrm{d}W_s, \quad \nu \equiv \xi^k + \mu$

Optimal investment policy

- **Policy:** $\mathcal{K} \equiv \{\hat{k}^- < \hat{k}^{*-} < \hat{k}^{*+} < \hat{k}^+\}$ HJB
- **Individual Tobin's q:** $q(\hat{k}) \equiv \frac{\partial V(k,u)}{\partial k} = v'(\hat{k})e^{-\hat{k}}$



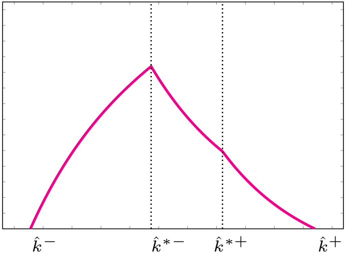
⋆ Individual $q(\hat{k})$ is not a sufficient statistic for investment

⋆ Correlated adjustment sign

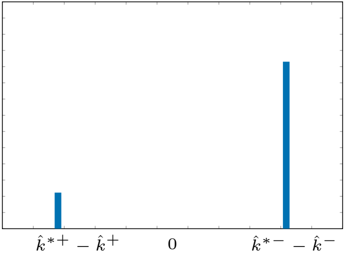
Aggregation

- Continuum of firms, all risk is idiosyncratic
- Steady-state density $g(\hat{k})$, key moments: $\mathbb{E}[\hat{k}]$, $\mathbb{V}[\hat{k}]$ KFE
- Observable statistics in microdata:
  - Investment: $\Delta \hat{k}_h = \hat{k}^{*-} - \hat{k}^{-} > 0 \quad (\hat{k}^{*+} - \hat{k}^{+} < 0)$
  - Duration of inaction: $\tau_h = T_h - T_{h-1}$
  - Age of capital: $a_s = s - \max\{T_h : T_h < s\}$

Distribution of $\hat{k}$



Distribution of $\Delta\hat{k}$



Step 2:

Three macroeconomic outcomes

- **Outcomes**: capital allocation, valuation, and fluctuations
- **Strategy**:

$$\underbrace{\text{Optimality}}_{\text{HJB}} + \underbrace{\text{Distribution}}_{\text{KFE}} = \underbrace{\text{Macro outcomes}}_{\mathbb{E}[\text{HJB} \times \text{KFE}]}$$

(1) Capital Allocation

(Mis)allocation $\equiv$ Dispersion of log marginal products

$$\mathbb{V}[\log mpk] \;=\; (1-\alpha)^2\mathbb{V}[\hat{k}]$$

- Investment frictions as a source of misallocation
  - **Frictionless**: $\mathbb{V}[\hat{k}] \;=\; 0$
  - **With frictions**: $\mathbb{V}[\hat{k}] \;>\; 0$

- Recovering $\mathbb{V}[\hat{k}]$ with microdata

$$(\hat{k}^{*\pm} = \mathbb{E}[\hat{k}], \nu > 0) \qquad \mathbb{V}[\hat{k}] \;=\; \overline{\mathbb{E}}[\Delta\hat{k}^2\phi(\Delta\hat{k})] \quad \text{with} \quad \phi(\Delta\hat{k}) \;=\; \frac{\Delta\hat{k}}{\overline{\mathbb{E}}[\Delta\hat{k}]}$$

  - Similar mappings for $\mathbb{E}[\hat{k}]$, $\hat{k}^{*-}, \hat{k}^{*+}$, σ, ν →
  - Variance decomposition for frictions (in paper)

(2) Capital Valuation

Aggregate q $\equiv$ weighted average of individual q

$$q \;=\; \frac{1}{p}\int q(\hat{k})\omega(\hat{k})g(\hat{k})d\hat{k} \;=\; \frac{\mathbb{E}[v'(\hat{k})]}{p\mathbb{E}[e^{\hat{k}}]}$$

where $\quad q(\hat{k}) = v'(\hat{k})e^{-\hat{k}}, \quad \omega(\hat{k}) = \frac{e^{\hat{k}}}{\mathbb{E}[e^{\hat{k}}]}, \quad p = \overline{\mathbb{E}}[p(\Delta\hat{k})]$

- Investment frictions affect marginal valuations
  - **Frictionless**: $q \;=\; 1$
  - **With frictions**: $q \;\neq\; 1$
- Define $\mathcal{P}$: capital gains/loses accrued after reseting

$$\mathcal{P}(\hat{k}) \;\equiv\; \begin{cases} p^{buy}/p - 1 & \text{for all} \quad \hat{k} \leq \hat{k}^{*-} \\ p^{sell}/p - 1 & \text{for all} \quad \hat{k} \geq \hat{k}^{*+} \end{cases}, \qquad \mathcal{P}(\hat{k}) \in \mathbb{C}^2$$

(2) Capital Valuation

Proposition: Aggregate q and steady-state moments

$$q = \frac{1}{r}\left(\underbrace{\frac{\alpha A\hat{Y}}{p\hat{K}} + \frac{\sigma^2}{2} - \nu}_{\text{Productivity}} - \underbrace{\frac{\overline{\mathbb{C}ov}\left[\Delta\hat{k}, \mathcal{P}(\Delta\hat{k})\right]}{\overline{\mathbb{E}}[\tau]}}_{\text{Irreversibility}}\right)$$

$$\text{where} \quad \frac{\hat{Y}}{\hat{K}} \equiv \frac{\mathbb{E}[e^{\alpha\hat{k}}]}{\mathbb{E}[e^{\hat{k}}]} \approx \exp\left\{-(1-\alpha)\left(\mathbb{E}[\hat{k}] + \frac{\alpha}{2}\mathbb{V}[\hat{k}]\right)\right\}$$

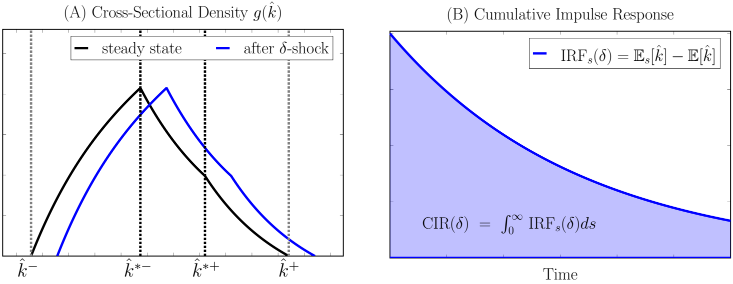
- **Productivity**
  - Scarcity ($\mathbb{E}[\hat{k}] \Uparrow$, $q \Downarrow$) + Misallocation ($\mathbb{V}[\hat{k}] \Downarrow$, $q \Uparrow$)
- **Irreversibility**
  - Firms buy expensive and sell cheap ($q \Downarrow$)

⋆ q **is monotonic!** Sufficient statistic for aggregate investment

(3) Capital Fluctuations

- MIT shock δ reduces all firms' productivity: $\hat{k}_0 = \hat{k}_{-1} + \delta$
- Aggregate capital's transitional dynamics

$$\text{CIR}(\delta) \equiv \int_0^\infty \left(\mathbb{E}_s[\hat{k}] - \mathbb{E}[\hat{k}]\right) \, \mathrm{d}s.$$



- Investment frictions as a source of persistence
  - **Frictionless:** CIR $= 0$
  - **With frictions:** CIR > 0

(3) Capital Fluctuations (cont...) →

Proposition: CIR and steady-state moments

$$\frac{\text{CIR}(\delta)}{\delta} = \underbrace{\frac{\mathbb{V}[\hat{k}]}{\sigma^2}}_{\text{Misallocation}} + \underbrace{\frac{\nu \mathbb{C}ov[\hat{k}, a]}{\sigma^2}}_{\text{Asymmetry}} - \underbrace{\frac{\overline{\mathbb{C}ov}[\Delta\hat{k}, \mathcal{M}(\Delta\hat{k})]}{\overline{\mathbb{E}}[\tau]}}_{\text{Irreversibility}} + o(\delta)$$

- **Misallocation**
  - Insensitivity to shocks (CIR ⇑)
- **Asymmetry**
  - Downsizing (CIR ⇑) vs. upsizing (CIR ⇓)
- **Irreversibility**
  - Persistent deviations above steady-state (CIR ⇑)

$$\mathcal{M}(\hat{k}) = \begin{cases} \frac{\mathbb{E}[\mathbb{P}^{+}(\hat{k})]}{\mathbb{P}^{-+}}(\mathbb{E}^{-}[\hat{k}] - \mathbb{E}[\hat{k}])\overline{\mathbb{E}}^{-}[\tau] & \text{for all} \quad \hat{k} \leq \hat{k}^{*-} \\ \frac{\mathbb{E}[\mathbb{P}^{-}(\hat{k})]}{\mathbb{P}^{+-}}(\mathbb{E}^{+}[\hat{k}] - \mathbb{E}[\hat{k}])\overline{\mathbb{E}}^{+}[\tau] & \text{for all} \quad \hat{k} \geq \hat{k}^{*+} \end{cases} \qquad \mathcal{M}(\hat{k}) \in \mathbb{C}^2$$

Step 3:

Introduce corporate taxes

Corporate tax schedule

- **4 instruments:**

t^c	Corporate income tax	t^p	Personal income tax
ξ^d	Depreciation allowance	t^g	Capital gains tax

- Pay t^c on cash flow net of deductions →

$$\textbf{Profitability:} \qquad A \to (1 - t^c)A$$

- No arbitrage between bonds and stocks →

$$\textbf{Discount:} \qquad \rho \to \left(\frac{1 - t^p}{1 - t^g}\right)\rho$$

- PDV of deductions

$$z \equiv \frac{\xi^d}{r\frac{1-t^p}{1-t^g} + \xi^d} < 1 \implies \begin{cases} \textbf{Prices:} & p(\Delta k) \to (1 - t^c z)p(\Delta k) \\ \textbf{Fixed costs:} & \theta \to (1 - t^c z)\theta \end{cases}$$

- Decompose value function

$$V(k, u, d) = \frac{1 - t^p}{1 - t^g}\left[uv(\hat{k}) + t^c z d\right]$$

After-tax investment frictions

$$\mathcal{K} \;=\; \underbrace{\hat{k}^{ss}}_{\text{frictionless}} \;+\; \underbrace{\mathcal{X}(\tilde{\theta}, \tilde{p}^{buy}, \tilde{p}^{sell})}_{\text{dynamic}}$$

a **Frictionless policy** $\hat{k}^{ss}$ reflects **user cost**:

$$\hat{k}^{ss} \;=\; \frac{1}{1-\alpha}\log\left(\left(\frac{1-t^c}{1-t^c z}\right)\frac{\alpha A}{p\tilde{\mathcal{U}}}\right), \qquad \tilde{\mathcal{U}} \;\equiv\; \frac{1-t^p}{1-t^g}\rho + \xi^k - \sigma^2$$

b **Dynamic policy** $\mathcal{X}$ depends exclusively on **after-tax frictions**

$$\tilde{\theta} \;\equiv\; \left(\frac{1-t^c z}{1-t^c}\right)\frac{\theta}{Ae^{\alpha\hat{k}^{ss}}}, \qquad \tilde{p}^{buy} - \tilde{p}^{sell} \;\equiv\; \left(\frac{1-t^c z}{1-t^c}\right)\frac{p^{buy} - p^{sell}}{Ae^{(\alpha-1)\hat{k}^{ss}}}$$

and solves standard "menu cost" model

$$\mathcal{V}(x) \;=\; \max_{\tau,\Delta x}\; \mathbb{E}\left[\int_0^\tau e^{-\tilde{r}\tau}\left(e^{\alpha x_s} - \alpha e^{x_s}\right)\mathrm{d}s \right.$$
$$\left. + \; e^{\tilde{r}\tau}\left(-\tilde{\theta} + \tilde{p}(\Delta x)(e^{x_\tau + \Delta x} - e^{x_\tau}) + \mathcal{V}(x_\tau + \Delta x)\right)\right]$$

Discussion of after-tax frictions

- Assume **reduction** in corporate income tax $t^c \downarrow$
- **<u>After-tax</u> fixed cost falls**:

$$\tilde{\theta} \propto \left(\frac{1 - t^c z}{1 - t^c} \right)^{\frac{1}{1-\alpha}} \theta$$

  - ⋆ Derivative of $\tilde{\theta}$ with t^c is positive
  - ⋆ Sectors with higher fixed costs more sensitive to tax cut

- **<u>After-tax</u> price wedge does not change**:

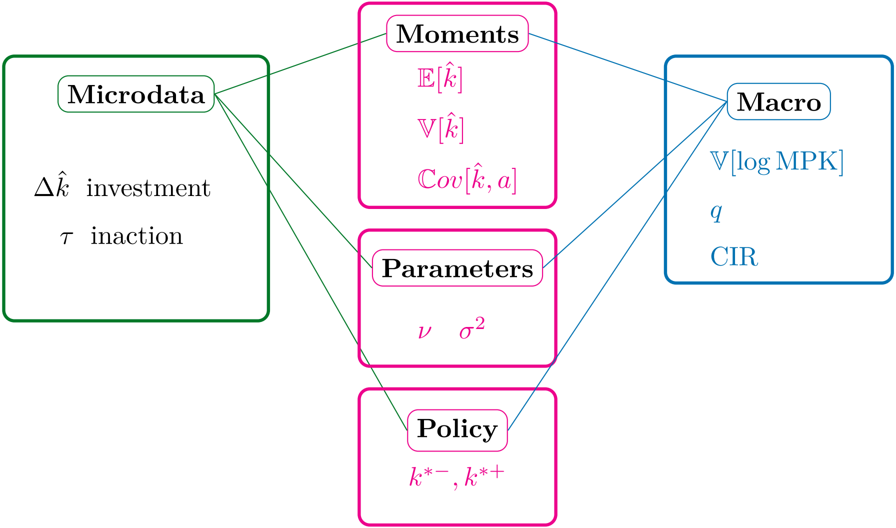
$$\tilde{p}^{buy} - \tilde{p}^{sell} = \frac{\alpha}{\tilde{\mathcal{U}}} \frac{p^{\text{buy}} - p^{\text{sell}}}{p}$$

  - ⋆ Derivative of $\tilde{p}^{buy} - \tilde{p}^{sell}$ with t^c is zero
  - ⋆ NO effects on price wedge!

Step 4:

Empirical application

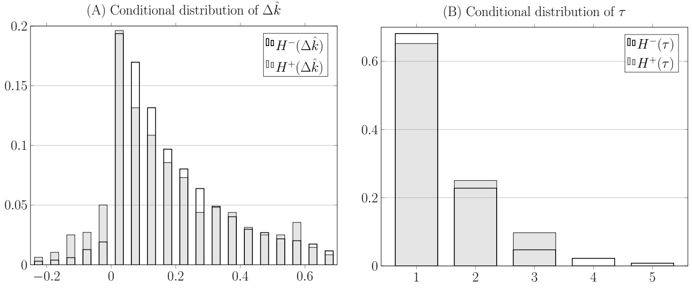
Measure macro outcomes with microdata



Microdata

- Establishment-level annual data from Chile, 1980-2011
- Capital stock k: Perpetual Inventory Method
- Changes in $\Delta\hat{k}$:

$$\Delta\hat{k}_s = \begin{cases} \log\left(1 + \Delta k_s / k_{s-1}\right) & \text{if} \quad |\Delta k_s| > 1\% \\ 0 & \text{if} \quad |\Delta k_s| < 1\% \end{cases}$$



- Recover cross-sectional moments $\mathbb{E}[\hat{k}], \mathbb{V}[\hat{k}]$

Calibration and Estimation

- Externally-set parameters: match Chilean averages 1980–2011
  - **Taxes:**

τ^p	τ^g	τ^c	ξ^d	z
0.471	0.471	0.260	0.070	0.547

  - **Technology:**

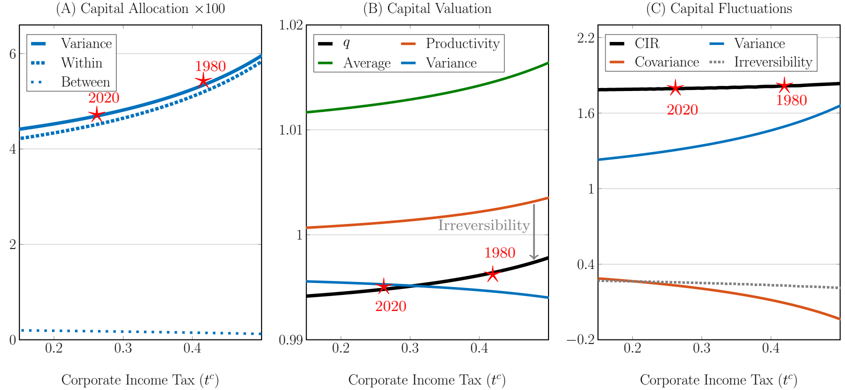
ρ	μ	α	p^{buy}	p^{sell}
0.066	0.033	0.720	2.000	1.900

    - $(p^{\text{buy}}, p^{\text{sell}}) : Y/K = 0.36$
    - $p^{\text{buy}} - p^{\text{sell}}$ to match price wedge of 5%

- Estimated parameters with microdata →
  - $(\nu, \sigma^2) = (0.118, 0.054)$
  - $\theta = 0.2$ to match $\mathbb{V}[\hat{k}]$ and $\mathbb{C}ov[\hat{k}, a]$ (SMM)

A reduction in corporate income tax t^c...

1. Decreases $\mathbb{V}[\log mpk]$ (lowers fixed cost, improves allocation)
2. Decreases q (abundant capital)
3. Decreases CIR (accelerates propagation)



Recap: Three new insights

- Result 1: Economy with taxes and frictions

$$\Longleftrightarrow$$

Economy without taxes and re-scaled frictions

- Result 2: Capital allocation drives valuation and fluctuations

- Result 3: Measure macro outcomes with a few micro moments

- Application: A reduction in corporate income tax...
  - ✓ improves capital allocation
  - ✓ decreases capital valuation
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Today's lesson

Corporate tax reforms **effectively change** investment frictions

... affecting the dynamic component of investment, and

... structurally changing how the macroeconomy works.

Backup

A1. Contributions

A2. Importance of Corporate Taxes

A3. General Hazard Model

A4. Firm Policy and HJB

A5. Distributions and KFE

A6. Measuring misallocation

A7. CIR and cumulative deviations

A8. Taxes in the model

A9. Two benchmark cases

A10. Observability

A1. Contributions

⋆ Contributions

- **Long-run effects of corporate tax reforms**
  - Summers (81), Poterba and Summers (83), King and Fullerton (84), Auerbach (86), Auerbach and Hines (86), Hassett and Hubbard (02), Barro and Furman (18)
  - Miao (19), Gourio and Miao (10), Miao and Wang (14)

We reduce complex interactions to rescaling of frictions, exploit microdata

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- **Short-run stimulus effects**
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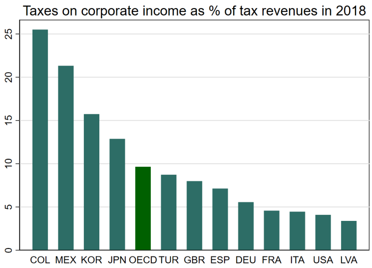
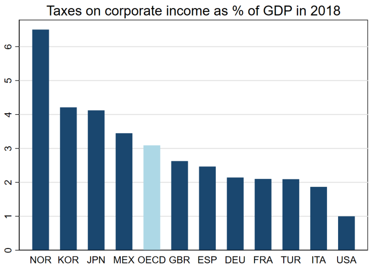
- **Role of micro-level frictions for macro**
  - Abel and Eberly (94, 96), Caballero and Engel (99, 07), Alvarez and Lippi (14), Alvarez, Le Bihan and Lippi (16), Baley and Blanco (21)

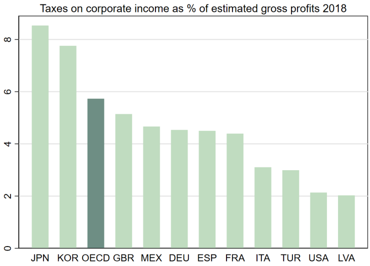
  We examine interaction of frictions and corporate taxes

A2. Importance of Corporate Taxes

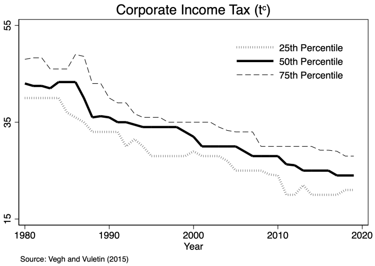
⋆ Importance of Corporate Taxes

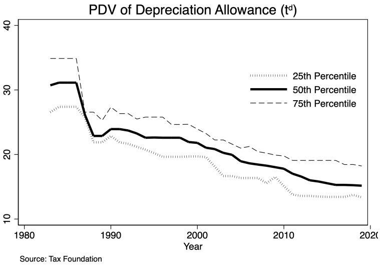


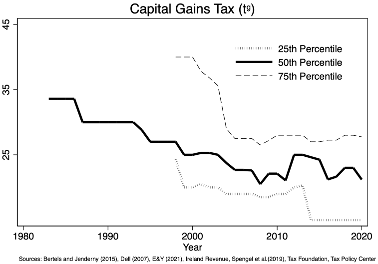


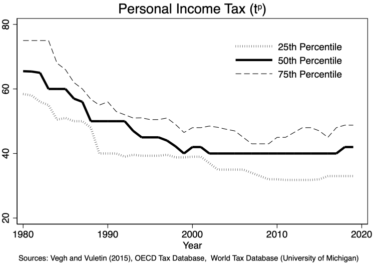


⋆ General decreasing trends

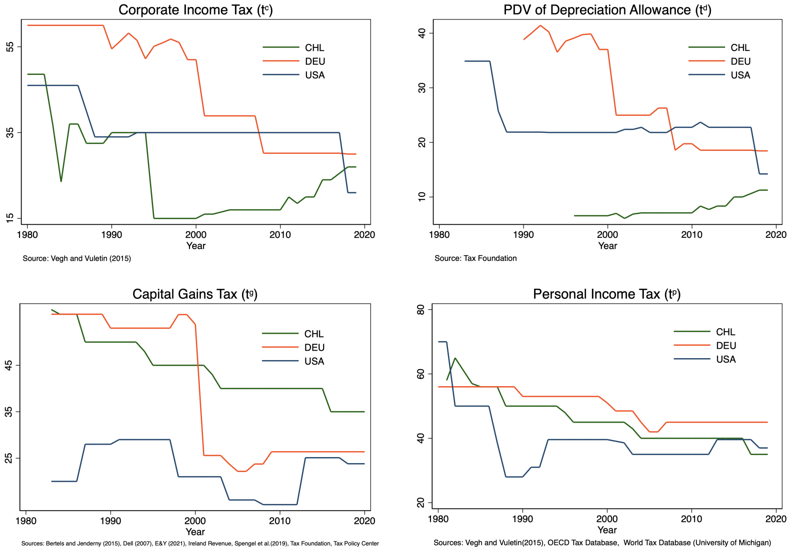








⋆ Very persistent reforms at country-level



A3. General Hazard Model

Asymmetric Generalized Hazard Model

- **Adjustment technology**:

$$\theta_s = \Theta(i_s, \mathrm{d}N_s^-, \mathrm{d}N_s^+, \vartheta_s^-, \vartheta_s^+)u_s$$

$$\Theta(i, \mathrm{d}N^+, \mathrm{d}N^-, \vartheta^-, \vartheta^+) = \begin{cases} 0 & \text{if} \quad i = 0 \\ \bar{\theta}^+(1 - \mathrm{d}N) \ + \ \mathrm{d}N\vartheta^+ & \text{if} \quad i < 0 \\ \bar{\theta}^-(1 - \mathrm{d}N) \ + \ \mathrm{d}N\vartheta^- & \text{if} \quad i > 0. \end{cases}$$

  - $N_t^\pm \sim Poisson(\lambda^\pm)$, $\vartheta^\pm \sim_{i.i.d.} J^\pm(\varphi)$, $\mathrm{Supp}(\vartheta^\pm) = [0, \bar{\theta}^\pm]$
  - Models of adjustment:
    - **Standard Ss model**: $\lambda = 0$ and $\bar{\theta}^+ = \bar{\theta}^-$
      Sheshinski and Weiss (77)
    - **Bernoulli fixed costs:** Free adj. opportunity $\vartheta^+ = \vartheta^- = 0$
      Baley and Blanco (21)
    - **Generalized hazard**: $\vartheta^+ = \vartheta^-$
      Caballero and Engel (93)

←

Asymmetric Generalized Hazard Model

- **Firm Problem**:

$$V(k_0, u_0) = \max_{\{T_h, i_{T_h}\}_{h=1}^{\infty}} \mathbb{E}\left[\int_0^{\infty} e^{-\rho s}\pi_s \, \mathrm{d}s - \sum_{h=1}^{\infty} e^{-\rho T_h}\left(\theta_{T_h} + p\left(i_{T_h}\right) i_{T_h}\right)\right]$$

- **Hazard rate of adjustment** $\Lambda(\hat{k})$: Adjustment prob. $\Lambda(\hat{k})\,\mathrm{d}t$

1. $\Lambda(\hat{k}) = 0$ for all $\hat{k} \in (\hat{k}^{*-}, \hat{k}^{*+})$
2. $\Lambda(\hat{k})$ weakly increasing in $|\hat{k} - \frac{\hat{k}^{*-}+\hat{k}^{*+}}{2}|$
3. If $J^{-}(0) > 0$, then $\Lambda(\hat{k}) \geq \lambda^{-} J^{-}(0)$ in $(\hat{k}^{-}, \hat{k}^{*-})$
4. If $J^{+}(0) > 0$, then $\Lambda(\hat{k}) \geq \lambda^{+} J^{+}(0)$ in $(\hat{k}^{*+}, \hat{k}^{+})$

←

A4. Firm policy and HJB

Sufficient optimality conditions

←

- Let $r \equiv \rho - \mu - \sigma^2/2$ and $\nu \equiv \mu + \xi^d$
- $v(\hat{k})$ and the optimal policy $\{\hat{k}^-, \hat{k}^{*-}, \hat{k}^{*+}, \hat{k}^+\}$ satisfy:

1. **HJB**:

$$rv(\hat{k}) \;=\; Ae^{\alpha\hat{k}} \;-\; \nu v'(\hat{k}) \;+\; \frac{\sigma^2}{2}v''(\hat{k})$$

2. **Value-matching:**

$$v(\hat{k}^-) \;=\; v(\hat{k}^{*-}) \;-\; \theta \;+\; p^{buy}(e^{\hat{k}^-} - e^{\hat{k}^{*-}})$$
$$v(\hat{k}^+) \;=\; v(\hat{k}^{*+}) \;-\; \theta \;+\; p^{sell}(e^{\hat{k}^+} - e^{\hat{k}^{*+}})$$

3. **Optimality and smooth-pasting:**

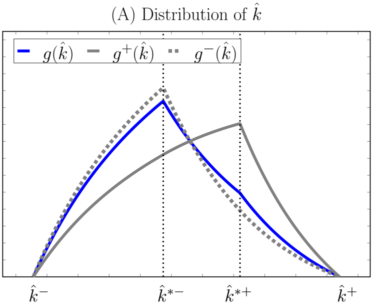
$$v'(\hat{k}) \;=\; p^{buy}e^{\hat{k}}, \quad \hat{k} \in \left\{\hat{k}^-, \hat{k}^{*-}\right\}$$
$$v'(\hat{k}) \;=\; p^{sell}e^{\hat{k}}, \quad \hat{k} \in \left\{\hat{k}^{*+}, \hat{k}^+\right\}$$

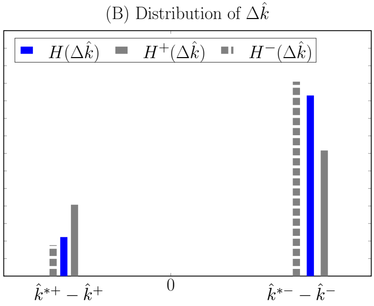
A5. Distributions and KFE

Cross-sectional distributions

←

- **Distribution of capital-productivity ratio** $g(\hat{k})$
  - Conditional on last reset point: $g^{\pm}(\hat{k})(\hat{k})$
  - Expectations in cross-section: $\mathbb{E}, \mathbb{E}^{\pm}$
- **Distribution of investment** $H(\Delta\hat{k}, \tau)$
  - Conditional on last reset point: $H^{\pm}(\Delta\hat{k})$
  - Expectations of adjusters: $\overline{\mathbb{E}}, \overline{\mathbb{E}}^{\pm}$





Characterizing cross-sectional distribution

←

- Characterizing $g(\hat{k}) \in \mathbb{C}$
  - KFE: $0 = \nu \frac{\mathrm{d}g(\hat{k})}{\mathrm{d}\hat{k}} + \frac{\sigma^2}{2}\frac{\mathrm{d}^2 g(\hat{k})}{\mathrm{d}\hat{k}^2}, \qquad \forall \hat{k} \in \left(\hat{k}^-, \hat{k}^+\right)/\{\hat{k}^{*-}, \hat{k}^{*+}\}$
  - Border conditions: $0 = g(\hat{k}^-) = g(\hat{k}^+) \quad ; \quad \int_{\hat{k}^-}^{\hat{k}^+} g(\hat{k})\,\mathrm{d}\hat{k} = 1$
  - Irreversibility

$$\underbrace{\frac{\sigma^2}{2}\lim_{\hat{k}\downarrow\hat{k}^-} g'(\hat{k})}_{\text{freq. with } \Delta\hat{k}>0} = \underbrace{\frac{\sigma^2}{2}\left[\lim_{\hat{k}\uparrow\hat{k}^{*-}} g'(\hat{k}) - \lim_{\hat{k}\downarrow\hat{k}^{*-}} g'(\hat{k})\right]}_{\text{discontinuity due to entry}}$$

- Characterizing $g^{\pm}(\hat{k}) \in \mathbb{C}$
  - KFE: $0 = \nu \frac{\mathrm{d}g^{\pm}(\hat{k})}{\mathrm{d}\hat{k}} + \frac{\sigma^2}{2}\frac{\mathrm{d}^2 g^{\pm}(\hat{k})}{\mathrm{d}\hat{k}^2}, \qquad \forall \hat{k} \in \left(\hat{k}^-, \hat{k}^+\right)/\{\hat{k}^{*^{\pm}}\}$
  - Border conditions: $0 = g(\hat{k}^-) = g(\hat{k}^+) \quad ; \quad \int_{\hat{k}^-}^{\hat{k}^+} g(\hat{k})\,\mathrm{d}\hat{k} = 1$

Steady-state distribution of firms

←

- $g(\hat{k})$: firms' distribution
- $H^{\pm}(\Delta\hat{k}, \tau)$: firms' distribution conditional on last reset $\hat{k}^{\pm}$
- $g^{\pm}(\hat{k})$: firms' distribution conditional on last reset $\hat{k}^{\pm}$
- $\mathcal{N}^{+}$ & $\mathcal{N}^{-}$: frequency of of $\Delta\hat{k} < 0$ and $\Delta\hat{k} > 0$
- Bayes' law

$$H(\Delta\hat{k}, \tau) = \frac{\mathcal{N}^{-}}{\mathcal{N}} H^{-}(\Delta\hat{k}, \tau) + \frac{\mathcal{N}^{+}}{\mathcal{N}} H^{+}(\Delta\hat{k}, \tau)$$

$$g(\hat{k}) = \frac{\mathcal{N}^{-}}{\mathcal{N}} \frac{\overline{\mathbb{E}}^{-}[\tau]}{\overline{\mathbb{E}}[\tau]} g^{-}(\hat{k}) + \frac{\mathcal{N}^{+}}{\mathcal{N}} \frac{\overline{\mathbb{E}}^{+}[\tau]}{\overline{\mathbb{E}}[\tau]} g^{+}(\hat{k})$$

A6. Measuring Misallocation

Measuring misallocation with microdata

- **Challenge:** $g(\hat{k})$ is not observed
- Let $\hat{k}^*(\Delta\hat{k})$ and $\hat{k}_\tau(\Delta\hat{k})$ be given by

$$\begin{aligned}\hat{k}^*(\Delta\hat{k}) &= \begin{cases}\hat{k}^{*-} & \text{if} \quad \Delta\hat{k} > 0\\ \hat{k}^{*+} & \text{if} \quad \Delta\hat{k} < 0,\end{cases}\\ \hat{k}_\tau(\Delta\hat{k}) &= \hat{k}^*(\Delta\hat{k}) - \Delta\hat{k}.\end{aligned}$$

- Two steps:
  1. Obtain ν, σ, $\hat{k}^{*-}$, $\hat{k}^{*+}$
  2. Obtain $\mathbb{E}[\hat{k}]$ and $\mathbb{V}[\hat{k}]$

←

Step 1

Let $\Phi(\nu,\sigma^2) \equiv \log\left(\alpha A/(r+\alpha\nu-\alpha^2\sigma^2/2)\right)$. Then

$$\nu = \frac{\overline{\mathbb{E}}[\Delta\hat{k}]}{\overline{\mathbb{E}}[\tau]}, \; \sigma^2 = \frac{\overline{\mathbb{E}}[(\hat{k}_\tau+\nu\tau)^2]-\overline{\mathbb{E}}[(\hat{k}^*)^2]}{\overline{\mathbb{E}}[\tau]}$$

$$\hat{k}^{*-} = \frac{1}{1-\alpha}\left[\Phi(\nu,\sigma^2)-\log(p^{buy})+\log\left(\frac{1-\overline{\mathbb{E}}^{-}\left[e^{-\hat{r}\tau+\alpha\left(\hat{k}_\tau-\hat{k}^{*+}\right)}\right]}{1-\overline{\mathbb{E}}^{-}\left[\frac{p(\Delta\hat{k})}{p^{\text{buy}}}e^{-\hat{r}\tau+\hat{k}_\tau-\hat{k}^{*+}}\right]}\right)\right]$$

$$\hat{k}^{*+} = \frac{1}{1-\alpha}\left[\Phi(\nu,\sigma^2)-\log(p^{sell})+\log\left(\frac{1-\overline{\mathbb{E}}^{+}\left[e^{-\hat{r}\tau+\alpha\left(\hat{k}_\tau-\hat{k}^{*-}\right)}\right]}{1-\overline{\mathbb{E}}^{+}\left[\frac{p(\Delta\hat{k})}{p^{\text{sell}}}e^{-\hat{r}\tau+\hat{k}_\tau-\hat{k}^{*-}}\right]}\right)\right]$$

- Drift =adjustment size × frequency of adjustment
- Volatility =quadratic size without trend × frequency of adjustment
- $\Phi(\cdot)$ = profitability to user cost $/p^{sell}, p^{buy}$ = cost of investment
- Last term= PDV marginal profits over expected resale value

←

Step 2

$$\mathbb{E}[\hat{k}] = \overline{\mathbb{E}}\left[\overline{\mathbb{E}}\left[\underbrace{\left(\frac{\hat{k}^* + \hat{k}_\tau}{2}\right)}_{\text{midpoint start-finish}} \overbrace{\left(\frac{\hat{k}^* - \hat{k}_\tau}{\overline{\mathbb{E}}[\Delta\hat{k}]}\right)}^{\text{renewal weight}} \middle| \Delta\hat{k}\right]\right] + \underbrace{\frac{\sigma^2}{2\nu}}_{\text{accum. drift correction}},$$

$$\mathbb{V}[\hat{k}] = \overline{\mathbb{E}}\left[\overline{\mathbb{E}}\left[\underbrace{\left((\hat{k}^* - \mathbb{E}[\hat{k}])(\hat{k}_\tau - \mathbb{E}[\hat{k}]) + \frac{(\hat{k}^* - \hat{k}_\tau)^2}{3}\right)}_{\text{distance start-finish}} \underbrace{\left(\frac{\hat{k}^* - \hat{k}_\tau}{\overline{\mathbb{E}}[\Delta\hat{k}]}\right)}_{\text{renewal weight}} \middle| \Delta\hat{k}\right]\right]$$

←

A7. CIR and cumulative deviations

CIR

- Let $\mathcal{M}(\hat{k})$ be equal to

$$\mathcal{M}(\hat{k}) = \begin{cases} \mathcal{M}^{buy} & \text{if } \hat{k} \in [\hat{k}^{-}, \hat{k}^{*-}] \\ \mathcal{M}^{sell} & \text{if } \hat{k} \in [\hat{k}^{*+}, \hat{k}^{+}], \end{cases}$$

$$\mathcal{M}^{buy} \equiv (\mathbb{E}^{-}[\hat{k}] - \mathbb{E}[\hat{k}])\overline{\mathbb{E}}^{-}[\tau]\,\frac{\mathbb{E}[\mathbb{P}^{+}]}{\mathbb{P}^{-+}} < 0,$$

$$\mathcal{M}^{sell} \equiv (\mathbb{E}^{+}[\hat{k}] - \mathbb{E}[\hat{k}])\overline{\mathbb{E}}^{+}[\tau]\,\frac{\mathbb{E}[\mathbb{P}^{-}]}{\mathbb{P}^{+-}} > 0.$$

  - $\mathbb{E}[\mathbb{P}^{+}] \equiv \Pr[\Delta\hat{k}' < 0]$ and $\mathbb{P}^{-+} \equiv \Pr[\Delta\hat{k}' < 0|\Delta\hat{k} > 0]$

- $\mathbb{C}ov[\hat{k}, a]$ can be obtained as

$$\mathbb{C}ov[\hat{k}, a] = \frac{1}{2\nu}\left(\mathbb{V}[\hat{k}] - \frac{\overline{\mathbb{E}}[(\hat{k}_{\tau} - \mathbb{E}[\hat{k}])^{2}\tau]}{\overline{\mathbb{E}}[\tau]} + \frac{\sigma^{2}}{2}\frac{\overline{\mathbb{E}}[\tau]}{2}(1 + \overline{\mathbb{CV}}^{2}[\tau])\right),$$

←

A7. Taxes

Corporate income tax t^c and deductions ξ^d

- Pay t^c on cash flow π_s, net of deductions $\xi^d k_s$
- Since $\xi^d \neq \xi^k$, we distinguish deductions d_s from capital k_s

$$\pi_s \;=\; Ay_s - t^c(Ay_s - \xi^d d_s) \;=\; \underbrace{(1-t^c)\;Au_s^{1-\alpha}k_s^\alpha}_{\text{after-tax profit rate}} + \underbrace{t^c\xi^d d_s}_{\text{deductions}}$$

where deductions evolve as:

$$\log d_s \;=\; \log d_0 \;-\; \xi^d s \;+\; \Sigma_{h:T_h\leq s}\;\left(1+\frac{\theta_{T_h}+p\left(\Delta k_{T_h}\right)\Delta k_{T_h}}{d_{T_h^-}}\right)$$

←

Personal income tax t^p and capital gains tax t^p

- Equity held by a stockholder, with access to risk-less bond return ρ

$$\textbf{No-arbitrage:} \quad \underbrace{(1-t^p)\rho\,\mathrm{d}s}_{\text{bond return}} = \underbrace{(1-t^g)\frac{\mathbb{E}[\mathrm{d}P_s]}{P_s}}_{\text{capital gains}} + \underbrace{(1-t^p)\frac{D_s}{P_s}\,\mathrm{d}s}_{\text{dividends}}$$

  - P_s price per share, 1 share (normalization)
  - D_s dividend per share

- Let V_0 be the firm's market value:

$$V_0 = P_0 = \frac{1-t^p}{1-t^g}\,\mathbb{E}_0\left[\int_0^\infty e^{-\frac{1-t^p}{1-t^g}\rho s} D_s\,\mathrm{d}s\right]$$

  - Firm maximizes cum-dividends market value of equity P_0
  - Uses stockholder's discount $(1-t^p)/(1-t^g)\rho$

- **Dividend policy**: tax capitalization view

$$D_s\,\mathrm{d}s = \pi_s\,\mathrm{d}s - [\theta_s + p(\Delta k_s)\Delta k_s]\,\mathbb{D}(\Delta k_s \neq 0), \quad \mathbb{D} \sim \mathit{Dirac}$$

←

A7. Two cases: Additional Material

Two benchmark cases

- Study macro outcomes under two polar cases
  1. Symmetry: $\nu \to 0$ and $\tilde{p}^{buy} = -\tilde{p}^{sell} = \tilde{p}$
  2. Small idiosyncratic shocks: $\sigma \to 0$
- Why?
  - Isolate the role of each friction
  - Characterize analytically macro elasticities to taxes

CASE 1: $\nu \to 0$ and $\tilde{p}^{buy} = -\tilde{p}^{sell} = \tilde{p}$

- **Only fixed costs**: $x^{*+} = x^{*-} = 0$ and $\bar{x} = \left(\frac{6\tilde{\theta}\sigma^2}{\alpha(1-\alpha)}\right)^{1/4}$

$$\mathbb{V}[\hat{k}] = \bar{x}^2/6; \qquad q = 1 - \frac{\tilde{\mathcal{U}}}{\tilde{r}}\frac{\alpha(1-\alpha)}{2}\mathbb{V}[\hat{k}]; \qquad \text{CIR} = \frac{1}{\sigma^2}\mathbb{V}[\hat{k}]$$

  - Lower t^c, decreases $\tilde{\theta}$
  - $\mathbb{V}[\hat{k}]$ and CIR fall, q increases if $\rho > \sigma^2$

- **Both frictions:** marginal increase of smaller friction has no effect

$$\left.\frac{\mathrm{d}M}{\mathrm{d}\tilde{\theta}}\right|_{\tilde{\theta}=0,\,\tilde{p}>0} = 0, \text{ for } M \in \{\mathbb{V}[\hat{k}], q, \text{CIR}\}.$$

CASE 2: $\sigma \to 0$

- Partial irreversibility has no effect
- Indifference curve for relevant steady-state moment

$$\mathbb{E}[x]\sqrt{\mathbb{V}[x]} \;=\; -\frac{\tilde{r}\tilde{\theta}}{\sqrt{12}\alpha(1-\alpha)}; \qquad \frac{\mathbb{E}[x]}{\mathbb{V}[x]+\mathbb{E}[x]^2} \;=\; -\left(\frac{\tilde{r}}{\nu}+\frac{\alpha+1}{2}\right),$$

- Macro outcomes

$$q \;=\; 1 \;-\; \frac{\tilde{U}}{\tilde{r}}(1-\alpha)\left(\mathbb{E}[x]+\frac{\alpha}{2}\mathbb{V}[x]\right); \qquad \text{CIR} \;=\; 0.$$

  - Lower t^c, decreases $\tilde{\theta}$
  - $\mathbb{V}[\hat{k}]$ and $|\mathbb{E}[x]|$ fall, ambiguous effect on q

A10. Observability

Parameters and policy from microdata ←

- Use $\overline{\mathbb{E}}[\cdot]$ to denote expectations conditional on adjustment
- Assume for simplicity $\hat{k}^{\pm} = \mathbb{E}[\hat{k}]$
- We recover **stochastic process** (ν, σ^2) as:

$$\nu = \frac{\overline{\mathbb{E}}[\Delta \hat{k}]}{\overline{\mathbb{E}}[\tau]} \quad ; \quad \sigma^2 = \frac{\overline{\mathbb{E}}[(\nu\tau - \Delta\hat{k})^2]}{\overline{\mathbb{E}}[\tau]}$$

  - Drift = frequency × average of investment
  - Volatility = frequency × dispersion of investment
- We recover the **reset capital** $\hat{k}^*$ as:

$$\hat{k}^* = \frac{1}{1-\alpha}\left[\Phi + \log\left(\frac{1-\overline{\mathbb{E}}\left[e^{-\hat{r}\tau-\alpha\Delta\hat{k}}\right]}{1-\overline{\mathbb{E}}\left[e^{-\hat{r}\tau-\Delta\hat{k}}\right]}\right)\right]$$

where $\Phi \equiv \log\left(\frac{\alpha(1-t^c)}{(1-t^d)p(\hat{r}+\alpha\nu-\alpha^2\sigma^2/2)}\right)$

Cross-sectional moments from microdata ←

- We recover **cross-sectional moments** as:

$$\mathbb{E}[\hat{k}] = \hat{k}^* + \frac{1}{2\nu}\left(\sigma^2 - \frac{\overline{\mathbb{E}}[\Delta\hat{k}^2]}{\overline{\mathbb{E}}[\tau]}\right)$$

$$\mathbb{V}[\hat{k}] = \frac{(\hat{k}^* - \mathbb{E}[\hat{k}])^3 - \overline{\mathbb{E}}\left[\left(\hat{k}_\tau - \mathbb{E}[\hat{k}]\right)^3\right]}{3\overline{\mathbb{E}}[\Delta\hat{k}]}$$

$$\mathbb{C}ov[\hat{k}, a] = \frac{1}{2\nu}\left[\mathbb{V}[\hat{k}] - \frac{\overline{\mathbb{E}}[\tau\hat{k}_\tau^2]}{\overline{\mathbb{E}}[\tau]} + \frac{\sigma^2}{2}\overline{\mathbb{E}}[\tau]\left(1 + \overline{\mathbb{CV}}^2[\tau]\right)\right]$$

where $\hat{k}_\tau = \hat{k}^* + \Delta\hat{k}$

- Intuition for $\mathbb{V}[\hat{k}]$:
  - If $\hat{k}^* = \mathbb{E}[\hat{k}]$: $\mathbb{V}[\hat{k}] = (1/3)\underbrace{\overline{\mathbb{E}}[\Delta k]^2}_{\text{size}} \underbrace{\overline{\mathbb{E}}\left[\left(\Delta k/\overline{\mathbb{E}}[\Delta k]\right)^3\right]}_{\text{dispersion}}$
  - Large investments $\Longrightarrow$ Signals large $\hat{k}$
  - Dispersed investments $\Longrightarrow$ Large $\hat{k}$ more representative

Calibration and Estimation

- Externally-set parameters: match Chilean averages 1980–2011
  - **Taxes:**

τ^p	τ^g	τ^c	ξ^d	z
0.471	0.471	0.260	0.070	0.547

  - **Technology:**

ρ	μ	α	p^{buy}	p^{sell}
0.066	0.033	0.720	2.000	1.900

    - $(p^{\text{buy}}, p^{\text{sell}}) : Y/K = 0.36$
    - $p^{\text{buy}} - p^{\text{sell}}$ to match price wedge of 5%

- Estimated parameters with microdata →
  - $(\nu, \sigma^2) = (0.118, 0.054)$
  - $\theta = 0.2$ to match $\mathbb{V}[\hat{k}]$ and $\mathbb{C}ov[\hat{k}, a]$ (SMM)

Recovering macro outcomes from microdata

Average macro outcomes in Chile 1980–2011

Investment Policy		**Capital Allocation**	
Difference in reset capitals ($\hat{k}^{*+} - \hat{k}^{*-}$)	0.372	Variance	0.099
Exogenous price wedge	0.183	Both frictions	0.067
PDV of capital-productivity ratio	0.189	Irreversibility	0.032

Capital Valuation		**Capital Fluctuations**	
Aggregate q	1.041	CIR	2.509
Productivity	1.050	Variance	1.821
Irreversibility	-0.009	Covariance	0.604
		Irreversibility	0.083

- Direct and indirect effects of after-tax frictions