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Long-term outlook for the German statutory pension system

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Non-technical summary

Research Question

Over the past few years, the financial situation of the statutory pension system in Germany has been relatively free of tension. This was due to past reforms, a pause in demographic change, and positive developments on the labour market. The contribution rate decreased and several benefits were expanded. Demographic developments will be putting pension funding under pressure in future, however. Life expectancy is likely to go on rising and the large baby boomer cohorts will be entering retirement from the mid-2020s onwards. The German federal government is aiming for a long-term pension reform. Long-term projections are important for this and it is necessary to understand what are the effects of changes in the current pension system.

Contribution

This paper presents long term projections of the German pension system that are based on a general equilibrium model with overlapping generations (OLG). This framework takes into account the two way feedback of both micro and macroeconomic relationships, meaning that households, for example, react to changes in the statutory pension system, such as the retirement age or the replacement rate. Changes in households' behaviour, in turn, impact on macroeconomic developments and public finances.

Results

One approach to parametrically reform the pension system would be linking (indexing) the retirement age systematically to increasing life expectancy. This reform would reduce the burden of the pension system and the model shows that an increase in employment would further bolster social security contributions and taxes. Moreover, with a rising retirement age and the associated longer periods of work, pension entitlements would increase.

Nichttechnische Zusammenfassung

Fragestellung

In den vergangenen Jahren war die Finanzlage der gesetzlichen Rentenversicherung relativ entspannt. Gründe waren vorangegangene Reformen, eine Pause im demografischen Wandel und die gute Entwicklung am Arbeitsmarkt. Der Beitragssatz sank, und etliche Leistungen wurden ausgeweitet. Künftig setzt aber die demografische Entwicklung die Rentenfinanzen unter Druck. Es wird erwartet, dass die Lebenserwartung weiter steigt, und es treten die großen Baby-Boom-Kohorten ab Mitte der 2020er Jahre in den Ruhestand. Die Bundesregierung strebt eine Rentenreform für die längere Frist an. Langfristige Vorausberechnungen sind dabei wichtig – trotz aller Unsicherheit. Sie verdeutlichen zentrale Entwicklungen und machen transparent, wie sich Reformen aus heutiger Perspektive auf Versicherte und Steuerpflichtige auswirken.

Beitrag

Solche Vorausberechnungen werden hier vorgestellt. Sie veranschaulichen, wie die wesentlichen Stellgrößen der Rentenversicherung zusammenhängen: das gesetzliche Rentenalter, das Versorgungsniveau, der Beitragssatz und die Bundesmittel. Bei den Simulationen wird deutlich, dass sich die demografischen Lasten kaum überzeugend über einzelne Stellgrößen auffangen lassen.

Ergebnisse

Ein Reformansatz wäre dessen systematische Verknüpfung (Indexierung) mit der zunehmenden Lebenserwartung. Beispielsweise ließe sich das Rentenalter nach 2030 so anheben, dass die Relation von Renten- zu Beitragsjahren in etwa stabil bleibt (statt, wie derzeit angelegt, immer weiter zu steigen). Die zunehmende Lebenszeit wäre dann mit einer längeren Erwerbsphase verbunden, aber auch die Rentenphase würde sich verlängern. Eine daraus resultierende umfangreichere Erwerbstätigkeit stützt gleichzeitig die Sozialbeiträge und Steuern. Mit einem steigenden Rentenalter und längeren Erwerbsphasen wachsen zudem die Rentenansprüche.

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Long-term Outlook for the German Statutory Pension System*

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Abstract

This paper presents long term projections of the German pension system that are based on a general equilibrium model with overlapping generations (OLG). This framework takes into account the two way feedback of both micro and macroeconomic relationships, meaning that households, for example, react to changes in the statutory pension system, such as the retirement age or the replacement rate. Changes in households' behaviour, in turn, impact on macroeconomic developments and public finances. One approach to parametrically reform the pension system would be linking (indexing) the retirement age systematically to increasing life expectancy. The model shows that the resulting increase in employment would also bolster social security contributions and taxes. Moreover, with a rising retirement age and the associated longer periods of work, pension entitlements would increase.

Keywords: Demographic Change, Pension System, OLG Models

JEL classification: E27, E62, H55, J11, J26

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This paper represents the views of the author and does not necessarily reflect the views of the Deutsche Bundesbank, or the Eurosystem. This is a companion paper to Bundesbank (October 2019): *Langfristige Perspektiven der gesetzlichen Rentenversicherung.* It provides additional technical details to the applied model omitted in the monthly report for sake of brevity. The model development was conducted with strong support from Alexander Ludwig.

1 Introduction

Over the past few years, the financial situation of the statutory pension system in Germany has been relatively free of tension. This was due to past reforms, a pause in demographic change, and positive developments on the labour market. The contribution rate decreased and several benefits were expanded. Demographic developments will be putting pension funding under pressure in future, however. Life expectancy is likely to go on rising and the large baby boomer cohorts will be entering retirement from the mid-2020s onwards. The German federal government is aiming for a long-term pension reform.

Long-term projections are important for this – despite all the uncertainty involved. They highlight key developments and illustrate how reforms, from a current vantage point, are going to affect persons covered by the statutory pension system and taxpayers. They demonstrate how the key variables of the statutory pension system are correlated: replacement rate, contribution rate statutory retirement age, and federal government funds.

This paper presents long term projections that are based on a general equilibrium model with overlapping generations (OLG). It contains rational utility-maximising households, profit-maximising firms and a government sector. A model framework of this nature captures both micro and macroeconomic relationships, meaning that households, for example, react to changes in the statutory pension system, such as the statutory retirement age or the replacement rate. Changes in households' behaviour, in turn, impact on macroeconomic developments and public finances. The statutory pension system is thus integrated into a macroeconomic model.

The simulation results clarify that it is not possible to capture the demographic burdens in a convincing manner using only a single parameter. The existing regulations distribute this pressure among the contribution rate and the replacement rate. One approach to reform would be to also consider the statutory retirement age and link (index) it systematically to increasing life expectancy.[1] More specifically, the statutory retirement age could be raised so that the ratio of years in retirement and years of contributions remains broadly stable. Increasing life expectancy would then be tied to a longer period of employment, although the period of pension payment would also become longer. Assuming a moderate variant of the life expectancy projections, the statutory retirement age would have to rise to 69 years and 4 months by 2070. Any resulting increase in employment would also bolster social security contributions and taxes.[2] Moreover, with a rising statutory retirement age and the associated longer periods of work, pension entitlements would increase.

Taking increasing life expectancy into account when setting the statutory retirement age would additionally make it possible, in particular, to cope with the financial pressure caused by the lower birth rates since the 1970s. However, even with an indexed statutory retirement age, the contribution rate and federal government funds would rise relatively sharply up to around 2040 and the replacement rate would fall. There would be much less

[1] An increase in the retirement age to address financial pressure of the demographic change on pension systems is also mentioned by several international organisations, e.g. OECD (2018), Internationl Monetary Fund (2019), European Commission (2019).

[2] The increased employment would also benefit other social insurance systems, e.g. health insurance and long term care insurance.

need for adjustment, however. After 2040, the replacement rate with a correspondingly greater number of contribution years would tend to remain flat. If consideration were given to a longer-term minimum threshold for the pension level, it is also an integral part of a reliable outlook that the resulting financial burdens appear sustainable. Even without an additional minimum threshold as reinforcement, such burdens are likely to increase considerably on those subject to compulsory contributions as well as on the federal government funds.

Our work is closely related to two strands of the economic literature. First, our work connects to papers that investigate the specific German pension system and its scope for reforms using microsimulation models, cf. Werding (2013), Börsch-Supan, Bucher-Koenen, and Rausch (2016), Börsch-Supan and Rausch (2018), Fenge and Peglow (2018). The focus of these papers lies on budgetary linkages between the pension system and the rest of the economy. In contrast to our approach, they do not account for the optimal household reactions or changes in factor prices triggered by a pension reform. Most of them also do not consider the interdependency between the pension system and the government budget.

This paper also includes an optimizing household sector in order to take into account household reactions. In this sense our paper relates to a vast number of papers that have analysed the economic consequences of demographic change and possible adjustment mechanisms. Important examples with a focus on social security adjustments include Imrohoroglu, Imrohoroglu, and Joines (1995), Huang, Imrohoroglu, and Sargent (1997), De Nardi, Imrohoroglu, and Sargent (1999), Fuster, Imrohoroglu, and Imrohoroglu (2007), Attanasio, Kitao, and Violante (2007), Attanasio, Bonfatti, Kitao, and Weber (2016), Kitao (2018).

These papers however are not tailored to the specific German situation. Our paper, tries to combine complex solution methods but also models the pension system in great detail. In this sense it is closely related to Ludwig, Krüger, and Börsch-Supan (2009), Börsch-Supan and Ludwig (2009), Ludwig and Reiter (2010) and Vogel, Ludwig, and Börsch-Supan (2017).

The remainder of our analysis is organized as follows. In section 2 we present the formal structure of the quantitative OLG model. Section 3 describes the calibration strategy. Results are presented in Section 4, for the current legal status in Section 4.1, for reform scenarios in Section 4.2. Finally, Section 5 concludes the paper. Detailed descriptions of the computational solution methods, parameter values, and additional results are relegated to separate appendices.

2 The Overlapping Generations Model

The following model is based on the work of Auerbach and Kotlikoff (1987) and its adoption by Ludwig (2005). It composes utility-maximising households, profit-maximising firms, a federal government and a pension system. The main purpose of the model is to evaluate different pension system reforms. Therefore, the pension system is modelled in greater detail than is the rest of the economy.

2.1 The Demographic Model

The demographic process is taken as exogenous and represents the main driving force of the model. Several cohorts that can be of varying size live in parallel in the model economy. A single cohort, c, per se is homogeneous and consists of identical households. At any point in time, t, the various cohorts are at different stages of life: households go through a life cycle in which they first work and then retire. At the end of each period, there is a given probability that households will die. The older the household, the greater is this probability. Households die with certainty at age J^T. Cohorts born later have a higher life expectancy. Note that point in time, t, and age of a household, j, uniquely determine its cohort, $c = t - j - 1$.

The size of the population of age j in period t is given recursively

$$N_{j,t} = N_{j-1,t-1}\pi_{j-1,t-1} + Z_{j,t}, \tag{1}$$

where $\pi_{j,t}$ denotes the age and time specific conditional survival rate and $Z_{j,t}$ is the net flow of people to Germany in a given period.[3]

Each year sees the entry of a new cohort. In each period newborns are determined by

$$N_{1,t} = \frac{1}{J^F}\sum_{j=1}^{J^F}\frac{N_{j,t-20}}{2} * f_{t-20} \tag{2}$$

where J^F is the maximum age a woman is assumed to bear children f_t is the fertility rate per woman over life. We concentrate on the economic life of agents and therefore let households enter the model at the biographical age of 20 which is in our model age of 1.

2.2 The Pension System

The German Pay As You Go (PAYG) pension system is characterized by a contribution rate, ϕ_t, and a replacement rate, γ_t. The budget of the PAYG pension system is balanced at any time t,

$$\phi_t w_t^g \sum_{j=1}^{J^T}\frac{\varepsilon_{j,t}}{\mathcal{E}_t} l_{j,t}\psi N_{j,t} + \phi_t w_t^g S^G + s^A Y_t = \Omega_t(1+\tfrac{1}{2}\varphi)\sum_{j=1}^{J^T} b_{j,t}^g p_{j,t}(1-l_{j,t})N_{j,t} \tag{3}$$

with

$$b_{j,t}^g = \begin{cases} 0 & \text{if } j < J_c^E \\ b_t^g = \gamma_t(1-\tfrac{1}{2}\psi\phi_t)w_t^g\frac{1}{D_t} & \text{if } j \geq J_c^E \end{cases}. \tag{4}$$

On the revenue side, w_t^g denotes gross wages of a fully working household. Labour supply resulting from optimal household decisions is denoted as $l_{j,t}$ and $\varepsilon_{j,t}$ is the age and time-specific individual labour productivity of a household. The parameter ψ accounts for the fact that not all employees are part of the pension system, e.g. self-employed, civil servants. They do not contribute to or benefit from the pension system.

[3]For computational reasons we assume that migrants enter Germany with the exact same amounts of assets and earnings points that households of the same age that already live in Germany possess.

In addition to contributions of households, the federal government provides funds to the pension system. These funds can be broadly classified into two types of grants. For the *General Federal Government Grant* the government acts like a fixed number of contributors, S^G. The size of this grant therefore varies with the contribution rate and the wage rate. The second type is the *Additional Federal Government Grant* that is a constant fraction of output, $s^A Y_t$. A main difference between these two grants is that the latter one is linked to the number of employees in the economy. In case of a declining workforce the *General Federal Government Grant* gains relative weight compared to the contributions and the *Additional Federal Government Grant*.

On the expenditure side of the pension budget equation are pension payments. Pensions are defined by an earnings point system. The paid out pension is calculated by multiplying the number of acquired earnings point, $p_{j,t}$, with the (gross) pension value, $b^g_{j,t}$. The pension value consists of the replacement rate, γ_t[4], times the wage (after pension contributions) at time t divided by D_t, the number of years in a standardized working life.

In each period of its working life a fully working household collects one earnings point when all of its members would be part of the pension system and its individual labour productivity is equal to the period's average labour productivity. If individual labour productivity is higher households earn more than one earnings point and vice versa. Average labour productivity is defined as

$$\mathcal{E}_t = \frac{\sum_{j=1}^{45} \varepsilon_{j,t} N_{j,t}}{\sum_{j=1}^{45} N_{j,t}}. \tag{5}$$

The German pension system allows for early retirement beginning at age, J^E_c.[5] For each year households retire before their cohort-specific statutory retirement age, J^R_c, the amount of earnings points is reduced by a pension penalty, Δ^-. It is also possible to work longer than the statutory retirement age. For each additional year the amount of earnings points is increased by a pension premium, Δ^+.[6]

$$p_{j+1,t+1} = \begin{cases} \frac{w^g_t \varepsilon_{j,t}}{w^g_t \mathcal{E}_t} l_{j,t}\psi + p_{j,t} & \text{if } j < J^E_c \\ \frac{w^g_t \varepsilon_{j,t}}{w^g_t \mathcal{E}_t} l_{j,t}\psi + p_{j,t}(1 - \Delta^-(1 - l_{j,t})\psi) & \text{if } J^E_c \leq j < J^R_c \\ \frac{w^g_t \varepsilon_{j,t}}{w^g_t \mathcal{E}_t} l_{j,t}\psi + p_{j,t}(1 + \Delta^+ \, l_{j,t}\psi) & \text{if } j \geq J^R_c \end{cases} \tag{6}$$

The pension system has to pay additionally the employer's share of other social in-

[4]In the German pension system the replacement rate is defined as the ratio of a standardized pension to the wage income of a fully working household before taxation but after deducting social insurance contribution: $\gamma_t = \frac{b^g_t D_t}{(1-\frac{1}{2}\psi\phi_t) w^g_t}$. As pensioners do not have to pay pension contributions the replacement rate is higher than the replacement rate before social insurances.

[5]In this model the retirement decision is a continuous choice. It is possible that a fraction, $1 - l_{j,t}$, of a household from age J^E_c already claims pension benefits while the rest of the household, $l_{j,t}$, still works in the labour market.

[6]This is a short cut of the actual German system as it includes compound interest. The exact modelling would change the results little but would require an additional state and is therefore computational burdensome.

surances like health insurance, with contribution rate of φ.[7] The parameter Ω_t stands for expenditures of the pension system that are not directly related to the actual earnings point system, e.g. disability insurance, survivor pensions or rehab.

The budget constraint of the pension system can be rewritten as

$$\phi_t w_t^g \left(\mathcal{L}_t + S^G\right) + s^A Y_t = \Omega_t (1 + \tfrac{1}{2}\varphi)\gamma_t (1 - \tfrac{1}{2}\psi\phi_t) w_t^g \mathcal{P}_t \tag{7}$$

with $\mathcal{L}_t = \sum_{j=1}^{J^T} \frac{\varepsilon_{j,t}}{\mathcal{E}_t} l_{j,t} \psi N_{j,t}$ defined as the number of *Equivalence Contributors* and $\mathcal{P}_t = \frac{1}{D_t} \sum_{j=J_c^E}^{J^T} p_{j,t}(1 - l_{j,t}) N_{j,t}$ defined as the number of *Equivalence Pensioners*. In general[8], the pension (value) annual adjustment is determined according to the following formula[9]

$$b_t^g = b_{t-1}^g \frac{w_t^g}{w_{t-1}^g} \frac{1 - \phi_{t-1}}{1 - \phi_{t-2}} \left[\left(1 - \frac{RQ_{t-1}}{RQ_{t-2}}\right) \times 0.25 + 1 \right] \frac{D_{t-1}}{D_t} \tag{8}$$

The pensioner ratio, RQ_t, is defined as the ratio of *Equivalence Pensioners* to *Equivalence Contributors*

$$RQ_t = \frac{\mathcal{P}_t}{\mathcal{L}_t}. \tag{9}$$

It can be seen as a summary statistic of the demographic and labour market developments. The pensioner ratio is closely related to the old age dependency ratio, $\text{OADR}_t = \frac{\sum_{j=J^R}^{J^T} N_{j,t}}{\sum_{j=1}^{J^R - 1} N_{j,t}}$ which relates the population share above and below the statutory retirement age. An increase of OADR_t due to demographic change also realizes in RQ_t. Additionally to changes in the population structure, the pensioner ratio reacts to changes in the employment rate. For example, if the employment rate suddenly increases the pensioner ratio would drop (while the OADR_t is unaffected). Therefore an increase in the employment rate instantaneously relieves some pressure caused by demographic change and c.p. raises the pension value.[10] However, in the long run a higher employment rate increases the number of earnings points (pension claims) in the economy, i.e. the number of *Equivalence Pensioners* increases. This offsets eventually the negative effect of the increased employment rate on the pensioner ratio and the positive effect on the pension value. Even though the employment rate has no long run effect on the pension value it should be noted that it leads to an increase in the pension system coverage. The percentage of pension recipients within a given cohort will rise over time.

[7]For simplicity we assume that employees and pensioners face the same contribution rate to other social insurances. This is a deviation from reality as pensioners do not contribute to unemployment insurance. They also have a different contribution rate to health insurance.

[8]In later simulations, this pension adjustment formula might be suspended, either temporarily or permanently.

[9]For simplicity we drop the factor for the supplementary private pension scheme, "Altersvorsorgeanteil (AVA)". We also do not distinguish between gross wages and salaries per employee and earnings subject to compulsory contributions per employee. For computational reasons the adjustment of the replacement rate to a change in wages is postponed by one period.

[10]The same logic applies if the share of socially insured employees increases, e.g. self-employed or civil servants would contribute to the pension system.

We obtain the adjustment formula for the replacement rate by inserting (4) into (8)

$$\gamma_t = \gamma_{t-1} \frac{1-\phi_{t-1}}{1-\phi_{t-2}} \left[\left(1 - \frac{RQ_{t-1}}{RQ_{t-2}} \right) \times 0.25 + 1 \right] \frac{1-\frac{1}{2}\psi\phi_{t-1}}{1-\frac{1}{2}\psi\phi_t}. \tag{10}$$

The contribution rate is determined endogenously so that the pension system's budget constraint is balanced in each period[11]

$$\phi_t = \frac{1 - \frac{s^A Y_t}{w_t^g \Omega_t \left(1+\frac{1}{2}\varphi\right)\gamma_t \mathcal{P}_t}}{\frac{1}{2}\psi + \frac{\mathcal{L}_t + S^G}{\Omega_t \left(1+\frac{1}{2}\varphi\right)\gamma_t \mathcal{P}_t}}. \tag{11}$$

2.3 The Firm Sector

Firms produce with a Cobb-Douglas production function employing capital and labour

$$Y_t = F(K_t, L_t) = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{12}$$

where K_t denotes the aggregate capital stock, L_t the aggregate labour input at time t. The output elasticity of capital is α. The total factor productivity (TFP) level is A_t and its growth rate is $\mu = \frac{A_{t+1}}{A_t} - 1$.

Aggregate labour input is a composite of four factors. It depends on, first, individual labour productivity, $\varepsilon_{j,t}$, second, the working hours per work contract, H_t, third, the labour supply decision of households, $l_{j,t}$, and fourth, the population structure at time t, $N_{j,t}$.

$$L_t = H_t \sum_{j=1}^{J^T} \varepsilon_{j,t} l_{j,t} N_{j,t} \tag{13}$$

A static firm maximises profits subject to capital accumulation condition

$$K_{t+1} = (1-\delta) K_t + I_t \tag{14}$$

where I_t is net investment, δ is the capital depreciation rate.[12]

The first order conditions from profit maximization give standard expressions for equilibrium factor prices. The gross return on capital is given by

$$r_t^g = r_t + \delta = \alpha A_t \left(\frac{K_t}{L_t} \right)^{\alpha-1} = \alpha \frac{Y_t}{K_t}. \tag{15}$$

On capital income households have to pay taxes with a tax rate of τ^k. Additionally, we assume that there are proportional administrative costs for investing in the capital market, ν. The net return on capital of households is then given by $r_t^n = (1-\tau^k)(1-\nu) r_t$.

[11] This is a deviation from the German pension system that has a fluctuation reserve. The actual adjustment rule is that the contribution rate must be raised if the fluctuation reserves would otherwise fall below their minimum permissible size. In the light of the demographic situation, the reserves are likely to dwindle from their currently high level to their minimum over the next few years.

[12] Capital adjustment costs in the firm sector are not considered.

Gross wages (for working the hours of a work contract, H_t) are given by

$$w_t^g \left(1 + \tfrac{1}{2}\psi\phi_t\right)\left(1 + \tfrac{1}{2}\varphi\right) = (1-\alpha)\, A_t \left(\frac{K_t}{L_t}\right)^{\alpha} = (1-\alpha)\,\frac{Y_t}{L_t}. \tag{16}$$

Half of the pension system contributions (plus contributions to other social insurances) are paid by the employer and the other half by the employee. The labour income tax rate is τ^y. Net wages are then given by $w_t^n = w_t^g\,(1-\tau^y)\left(1 - \tfrac{1}{2}\psi\phi_t\right)\left(1 - \tfrac{1}{2}\varphi\right)$.

2.4 The Household Sector

By choosing an optimal consumption and labour supply path, each cohort c maximizes at any age j and point in time $t = c + j - 1$ the sum of discounted future utility. The within period utility function exhibits constant relative risk aversion and preferences are additive and separable over time. Cohort c's maximization problem at $j = 1$ is given by

$$\underset{\{c_{j,t},l_{j,t}\}_{j=1}^{J^T}}{max} \sum_{j=1}^{J^T} \beta^j s_{j,t} U\left(c_{j,t}, \overline{H} - l_{j,t}H_t\right) + (1-s_{j,t})\Upsilon(a_{j+1,t+1}) \tag{17}$$

where β is the pure time discount factor. In addition to pure discounting, households discount future utility with their unconditional survival probability, $s_{j,t+j} = \prod_{m=1}^{j} \pi_{m-1,t}$. $c_{j,t}$ denotes consumption and leisure is $\overline{H} - l_{j,t}H_t$, where $\overline{H}$ is the maximum amount of time available to households. The labour supply of households may not exceed working hours of a work contract

$$0 \le l_{j,t} \le 1. \tag{18}$$

Households additionally derive utility from bequeathing assets, $\Upsilon(a_{j+1,t+1})$. All assets (including return on capital) of household that died at the end of one period are passed over to next periods younger households. So in each period households up to a specific inheritance age, J^Q, receive bequests

$$q_{j,t} = \begin{cases} \frac{(1+r_t^n)\sum_{i=1}^{J^T}(1-\pi_{i,t-1})a_{i,t-1}N_{i,t-1}}{\sum_{i=1}^{J^Q} N_{i,t}} & \text{if } j \le J^Q \\ 0 & \text{if } j > J^Q \end{cases}. \tag{19}$$

Denoting household assets by $a_{j,t}$, maximization of the household's inter-temporal utility is subject to a dynamic budget constraint given by

$$a_{j+1,t+1} = \left(1 + r_{t+1}^n\right)\left(a_{j,t} + q_{j,t} + (1-\tau^y)\, y_{j,t} - (1+\tau_t^c)\, c_{j,t}\right) \tag{20}$$

where τ_t^c is the time-varying consumption tax rate. Income, $y_{j,t}$, consists of labour income and pension income.

$$y_{j,t} = (1 - \tfrac{1}{2}\varphi)\left((1 - \tfrac{1}{2}\psi\phi_t) w_t^g \varepsilon_{j,t} l_{j,t} + b_{j,t} p_{j,t}(1 - l_{j,t})\right) \tag{21}$$

2.5 The Federal Government

The federal government levies taxes on consumption, capital income, labour income, and pension income[13]. The revenues from other social insurances are not counted as government revenue. With its revenues the government has to finance its consumption, G_t, and the federal government funds to the pension system. The government budget constraint reads as

$$\tau_t^c \sum_{j=1}^{J^T} c_{j,t} N_{j,t} + \tau^k \sum_{j=1}^{J^T} r_t a_{j,t} N_{j,t} + \tau^y \sum_{j=1}^{J^T} y_{j,t} N_{j,t} = G_t + \phi w_t^G S^G + s^A Y_t. \tag{22}$$

We assume that government consumption is a constant fraction of output, $G_t = \varrho Y_t$. The endogenously adjusted consumption tax rate balances the government budget

$$\tau_t^c = \frac{1}{C_t}(\varrho Y_t + \phi_t w_t^g S^G + s^A Y_t - \tau^k \sum_{j=1}^{J^T} r_t a_{j,t} N_{j,t} - \tau^y \sum_{j=1}^{J^T} y_{j,t} N_{j,t}) \tag{23}$$

where $C_t = \sum_{j=1}^{J^T} c_{j,t} N_{j,t}$ is aggregate private consumption.

2.6 Definition of Equilibrium

Given the exogenous population distribution and survival rates in all periods $\{N_{j,t}, \pi_{j,t}\}$, a competitive equilibrium of the economy is defined as a sequence of dis-aggregated variables, $\{c_{j,t}, l_{j,t}, a_{j,t}\}$, aggregate variables, C_t, L_t, K_t, a wage rate, w_t^g, a rate of return on capital, r_{t+1}^g, and pension policies, $\{\phi_t, \tau_t^c\}$ such that

1. Given initial conditions household maximize utility and $c_{j,t}$, $l_{j,t}$ are the resulting optimal policies.

2. Rates of return on capital and wages satisfy (15) and (16).

3. Government/pension policies satisfy equation (7) and (22) in every period.

4. Markets clear and allocations are feasible in all periods

$$L_t = H_t \sum_{j=1}^{J^T} \varepsilon_{j,t} l_{j,t} N_{j,t} \tag{24}$$

$$K_{t+1} = \sum_{j=1}^{J^T} a_{j+1,t+1} N_{j,t} \tag{25}$$

$$G_t + C_t + K_{t+1} - (\Omega_t - 1) b_t^g \mathcal{P}_t + \varphi \sum_{j=1}^{J^T} y_{j,t} N_{j,t} + \nu r_t K_t = Y_t + (1-\delta) K_t \tag{26}$$

[13]We assume that pension income is fully taxed (nachgelagerte Besteuerung). This is the legal situation in Germany from 2025 onwards.

3 Calibration

The aim of the calibration is to match the German economy and specifically its labour market in order to have a good baseline model for policy evaluation. Calibration of the model requires (i) data for the exogenous demographic processes and (ii) determination of values for several structural model parameters.[14]

The model period is one year. We assume that Germany was in a steady state in 1960. We then take 2018 to calibrate our model. The main focus of this model is then the forecast period of 2019 to 2070 where we assume that (almost) all structural parameters are the same as in the calibration period and only policy parameters change. We additionally assume that after 2100 all parameters (including policy parameters) remain unchanged and a new steady state is reached in the year 2500.

3.1 Demographics

For the demographic process the main data source is the German Federal Office of Statistics. We use actual German data for the age structure and the mortality rate for the period between 1960 and 2018. For the forecast period until 2060 we take projections of the recent *14th Coordinated Population Projection*.[15] After the end of this projection at 2060, we linearly extrapolate the mortality probabilities by age until 2100 and keep them constant afterwards. We assume that the maximum biological age is 109, in model terminology, $J^T = 90$.

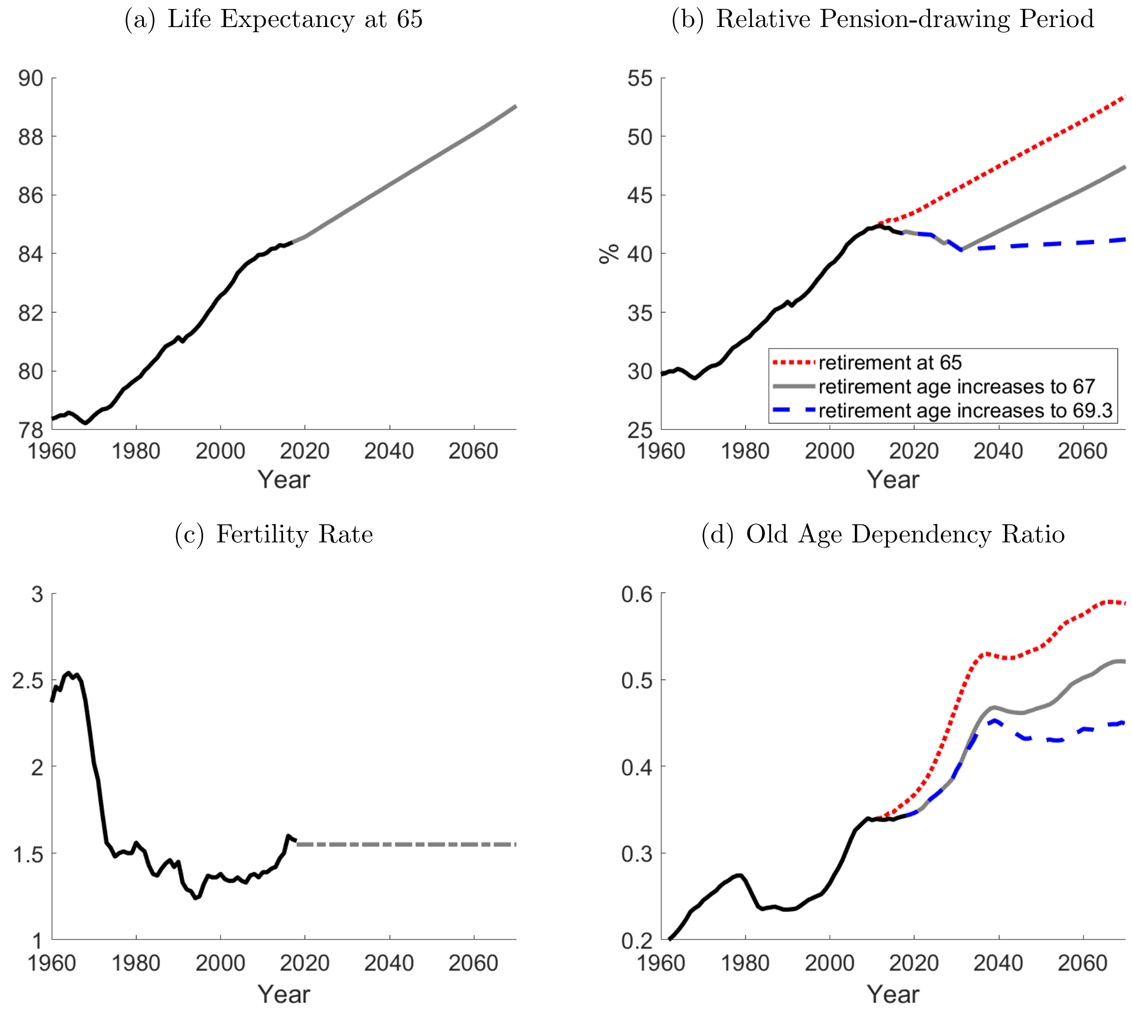
In 1960, remaining life expectancy at the age of 65 was 13.5 years (see Figure 1 (a)). Since then, it has increased to 19.5 years. We assume that it will have gone up by a further 4.5 years by 2070. With an unchanged statutory retirement age, there would be a steady increase in the pension-drawing period. The rise in the statutory retirement age to 67 will prevent increasing life expectancy from raising the relative pension-drawing period up until 2031. The relative pension-drawing period is defined as the ratio of years drawing a pension to years making pension contributions, assuming retirement at the statutory retirement age. Going forward (from the 2030s), a constant number of years of contributions will have to finance an increasing number of years in retirement again if the statutory retirement age remains unchanged from then on. This will increasingly weigh on the pension system. In the past, the relative pension-drawing period has risen sharply as a result of an increasing remaining life expectancy among the post-retirement cohort: it went up from 30.1% in 1960 to 42.2% in 2011. Without a further increase in the statutory retirement age, it would be 47.1% in 2070 (see Figure 1 (b)).

There has been a sharp fall in the birth rate since the mid-1960s (see Figure 1 (c)). It has fallen relatively swiftly from around 2.5 to somewhat below 1.5. Most recently, it was somewhat higher again at 1.57. In the baseline variant of its current population projection exercise, the Federal Statistical Office assumes a broadly unchanged birth rate of 1.55 which we also assume after 2060. The sharp decline about 50 years ago has led to a demographic hump. When the 1960s cohorts with relatively high birth rates (baby

[14] Tables with all parameters can be found in Appendix B.

[15] The 14th coordinated population projection includes various scenarios for the future trends of fertility, migration and mortality. The chosen assumptions are in the medium range of all scenarios (W2-L2-G2), cf. Statistisches Bundesamt (2019).

Figure 1: Demographics



Notes: Panel (a) shows average (men and women) life expectancies of 65 year old. Panel (b) shows ratios of pension-drawing periods (defined as remaining life expectancy as of statutory retirement age) to preceding contribution periods (defined as statutory retirement age minus 20 years) with statutory retirement age at 65 (red dotted line), current legal situation (grey solid line), statutory retirement age increases as in Scenario V – VII (blue dashed line). Panel (c) shows live births per female in the age range of 15 to 49 years calculated for the reporting year. Panel (d) shows old age dependency ratios defined as persons of statutory retirement age or older to persons aged between 20 and statutory retirement age. Statutory retirement age at 65 (red dotted line), current legal situation (grey solid line), statutory retirement age increases as in Scenario V – VII (blue dashed line).

boomers) enter retirement from the mid-2020s onwards, they will exert pressure on the pension system. This pressure caused by the extremely unequal cohort sizes will ease when the baby boom cohort dies out.

In recent years, there has been considerable net immigration. Over the past ten years, this has amounted to an annual average of around $400,000$ persons. What is crucial for the statutory pension system is the extent to which migration alters the number and structure of its contributor base and then, at a later date, the number and structure of pension recipients. Three things are of central importance: the age of those immigrating and emigrating, integration into the labour market, and the impact on future demographic developments. In the cited population projection, net migration falls to $206,000$ persons per year by 2026 (corresponds largely to the long-term median).[16] After this, the number of net migration remains constant. Migration is thus counteracting the effect of the low birth rate.

All three demographic factors affect the old-age dependency ratio. This is the ratio of older persons to people of working age. The working age is often defined as the age range from 20 to less than 65 years. As the statutory retirement age is being raised progressively, however, we define it as the range between 20 and the statutory retirement age. In 1990 the old-age dependency ratio defined in this way was 24.1% (see Figure 1 (d)). In other words, for every person of statutory retirement age and above, there were roughly four persons of working age. With the retirement of the baby boomer cohort, the old-age dependency ratio could rise to 44.9% by 2035. This ratio would then initially remain largely stable. Although life expectancy will continue to rise, the baby boomer cohorts will gradually die out. If the statutory retirement age were to remain unchanged at 67 years the expected rise in life expectancy will lead to a persistent increase in the old-age dependency ratio. In 2070, it would be around 53.0%. For every person of statutory retirement age and above, there would then be fewer than two persons of working age.

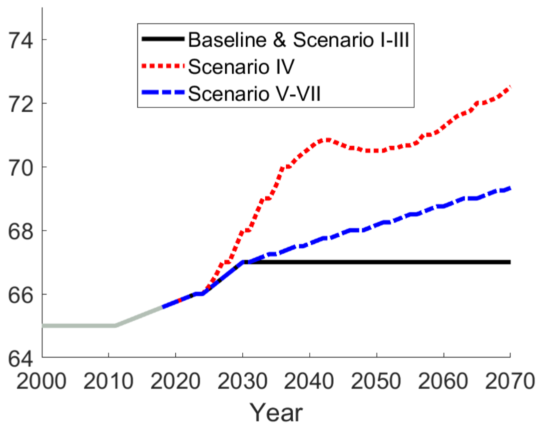
3.2 Pension Parameters

The statutory retirement age, J_c^R, of each household is determined by its cohort. Note that in the model the year a households enters its economic life (biological age of 20) defines its cohort. For example, someone born in 1940 belongs to model cohort 1960. For the current legal situation and all reform scenarios, the statutory retirement age for all households born before 1946 is 65. So the statutory retirement age of model cohorts 1960 to 1966 is $J_c^R = 46 \; \forall \; c \in \{1960, 1966\}$. For cohorts born between 1947 and 1958 statutory retirement age increases for the current legal situation and all reform by one month each year until it reaches 66, $J_{1978}^R = 47$. Then for the current legal situation and all reform scenarios except scenario IV, the statutory retirement age increases by two months each year until the cohort born in 1964 retires at 67, $J_{1984}^R = 48$. In the current legal situation and for reform scenarios I, II, and III the statutory retirement age is then kept constant at 67 for all future cohorts. For the reform scenarios V, VI, and VII the statutory retirement age increases further by $\frac{3}{4}$ of a month per year on average. To keep the step size of the increase at one month the statutory retirement age increases three years in a row and then one year is paused. By this pattern the statutory retirement age increases until it

[16] After 2060 we assume that all migration into Germany takes place at the age of 20.

reaches $J^R_{2044} = 72$.[17] For reform scenario IV the increase of the statutory retirement age is not linear. Here it has not only to absorb the increased life expectancy but also the effect of different cohort sizes on the old age dependency ratio. Eventually, the statutory retirement age in reform scenario IV reaches 74 for households born in 2026 (cohort 2046).

Figure 2: Statutory Retirement Age over Time



Notes: Statutory retirement age over time for current legal situation and reform scenarios I – III (solid black line), reform scenario IV (red dotted line), and reform scenarios V – VII (blue dashed line).

The statutory early retirement age is for all cohorts in the current legal situation and all reform scenarios two years before the statutory retirement age, $J^E_c = J^R_c - 2$. From this it directly follows that when the statutory retirement age increases also the early statutory retirement age increases by the same amount. The pension premium for late retirement of one year, $\Delta^+ = 6\%$, and the pension penalty for early retirement for one year, $\Delta^- = 3.6\%$, are taken from current pension legislation.

Not all employed people are insured in the pension system. Self-employed and civil servants do neither contribute to nor benefit from the pension system. The share of employees insured in the pension system is calibrated to match the number of *Equivalence Contributors* in 2018, $\psi = \frac{32 \text{ m}}{41 \text{ m}} = 78.0\%$.

The pension system expenditures included in the model comprises spending on ordinary old-age pensions as well as additional expenditure by the system: pensions for persons with reduced earnings capacity and for surviving dependents, contributions to the statutory health system and expenditure for rehabilitation and administration. These expenditures amounted in 2018 to a total of €98bn compared to €214bn in old age pension payments. These additional expenditures are captured in the pension budget constraint by the pension mark up $\Omega_{2018} = \frac{€214\text{bn}+€98\text{bn}}{€214\text{bn}} = 1.46$. With the exception of survivor pensions, it is assumed that these expenditures develop in line with spending on ordinary old-age pensions. In the past, survivor pensions have shown a clear downward trend. This is most likely due, not least, to the increase in labour market participation of women in particular (at the same time as tighter provisions for deductions). We assume that this

[17]Note that this cohort enters retirement in the year 2116.

downward trend continues up to 2070 and as a result the share of survivor pensions falls by roughly half (from the current level) by then. The expenditure mark-up falls until 2070, $\Omega_{2070} = 1.36$, and stays constant afterwards.

In 2018, the difference between the total revenue of the pension system and the compulsory contributions paid by employees (including the employer's share) amount to €100bn. In the model this difference is defined as the federal government funds provided to the pension system. Roughly 70% of the total federal government funds, or €70bn, we define as *General Federal Government Grant*.[18] For this grant the government acts like a defined number of contributors, $S^G = \frac{\text{€70bn}}{18.6\% \times \text{€36,085}} = 10.4\text{m}$. The rest of the federal government funds, €30bn, are classified as *Additional Federal Government Grant*.[19] This part of federal government funds is modelled as a constant fraction of the gross value added, $s^A = \frac{\text{€30bn}}{\text{€3,012bn}} = 1.0\%$. Hence, it is independent of the contribution rate.[20]

In later simulations the current legal situation requires that until 2025 the replacement rate may not fall below the minimum threshold of 48.0%. Additionally, the current legal situation provides a threshold for the contribution rate until 2025. The contribution rate is in this case capped at 20.0%. For this time period, if necessary, budget balance is achieved by a temporary additional grant from the government.

3.3 Government Sector

The government parameters that need to be chosen are $\left(\varrho, \tau^k, \tau^y, \varphi\right)$. Government consumption is calibrated to match the tax and contribution ratio in Germany in 2018 of 41.3%. The fraction of output that is needed for government consumption is then $\varrho = 22.6\%$. The capital income tax rate is calibrated to match the share of tax receipts on capital income[21] on total tax income in Germany in 2018, $\frac{\text{€221bn}}{\text{€774bn}} = 27.4\%$. The resulting capital income tax rate is $\tau^k = 30.7\%$. In the same way as the capital income tax, the labour and pension income tax rate is calibrated to match the share of labour and pension income tax receipts[22] on total tax income in 2018 of $\frac{\text{€220bn}}{\text{€774bn}} = 27.3\%$. The calibrated tax rate on labour and pension income is $\tau^y = 15.9\%$. The contribution rate to social insurances (except to the pension system), $\varphi = 21.5\%$, we set to match the social insurance contributions to output ratio of 17.1%.

3.4 Technology & Preferences

The TFP parameter A determines the level of output and is calibrated to match gross value added in 2018 of €3,012bn. The TFP growth rate is set to match the growth rate of

[18]The *General Federal Government Grant* comprises the *Allgemeiner Bundeszuschuss*, but also the *Zuschuss für Kindererziehungszeiten*.

[19]The *Additional Federal Government Grant* includes among other things the *Zusätzlicher Bundeszuschuss*, the *Erhöhungsbetrag*, *Beiträge der Krankenversicherung*, and *Beiträge der Agentur für Arbeit*.

[20]In Deutsche Bundesbank (2019), the *Additional Federal Government Grant* is modelled differently. There the grant depends additionally on the pension system contribution rate. Therefore the grant is slightly upward biased.

[21]We account as taxes on capital income: *Veranlagte Einkommensteuer*, *Körperschaftsteuer*, *Kapitalertragsteuer*, *Erbschaftsteuer*, *Gewerbesteuer*, *Grundsteuer*, *Grunderwerbsteuer*, and one third of *Solidaritätszuschlag*.

[22]Additionally to the income tax we add the remaining two thirds of *Solidaritätszuschlag*.

the GDP in the data. After 2018 we hold the TFP growth constant at a rate of $\mu = 0.6\%$.[23] The production elasticity of capital is calibrated such that we match the labour income share in Germany's national accounts in 2018, $1 - \alpha = \frac{wL}{Y} = \frac{€1{,}771\text{bn}}{€3{,}012\text{bn}} = 58.8\%$. We calibrate the model to target the capital stock in Germany 2018. To compute the German capital stock we subtract from total wealth in Germany the net foreign asset position and one half of private housing, $K_{2018} = €12,404\text{bn}$.[24] From this it follows that capital to output ratio is $\frac{K_{2018}}{Y_{2018}} = \frac{€12{,}400\text{bn}}{€3{,}012\text{bn}} = 4.1$. This capital to output ratio will be attained in the model by appropriate calibration of the preference parameters. The resulting time discount factor is $\beta = 0.9754$. Using data on output, capital, and national income, VE_{2018}, we derive the implied yearly depreciation rate of $\delta = \frac{Y_{2018} - \text{VE}_{2018}}{K_{2018}} = 4.1\%$.

Output elasticity of capital, depreciation rate and capital to output ratio determine the rate of return on capital, $r_{2018} = \alpha \frac{Y_{2018}}{K_{2018}} - \delta = 10.2\%$. To derive a more realistic private return on investment we model private administrative costs for saving. For a household net private return on savings of 3.4% we need additional administrative costs of $\nu = 17.2\%$.

We assume that the within period utility function is of the standard Cobb Douglas form given by

$$U\left(c_{j,t}, \overline{H} - l_{j,t}H_t\right) = \begin{cases} \frac{1}{1-\theta}\left(c_{j,t}^{\xi_t}\left(\overline{H} - l_{j,t}H_t\right)^{1-\xi_t}\right)^{1-\theta} & \text{if} \quad \theta \neq 1 \\ ln\left(c_{j,t}^{\xi_t}\left(\overline{H} - l_{j,t}H_t\right)^{1-\xi_t}\right) & \text{if} \quad \theta = 1 \end{cases} \tag{27}$$

θ is the inverse of the inter-temporal elasticity of substitution we set to 1.[25]

We set the maximum disposable hours of a household to 50 hours per week, $\overline{H} = 50 \times 52 = 2600$. The working hours of a contract we compute by dividing aggregate hours worked by the number of employees, $H_{2018} = \frac{61\text{bn}}{44.7\text{m}} = 1366$. For the projection period we keep the working hours per contract constant.

The consumption share parameter, ξ_t, determines the weight of consumption relative to leisure in household's utility. The utility weight of consumption is determined to match the model's total employment rate with the data. The model incorporates that the overall employment rate in Germany increased over the past 55 years, especially in the last 10 years.[26] We shift the consumption utility weight over time to match overall employment in the data in 1970, 1998, 2008, and 2018. For the projection period we assume a further but (more moderate) increase in the employment rate (see Figure 3 (a)). We set values for ξ_t to match our predictions of the employment rate for 2030, 2050, and 2070.[27]

The employment rates do not only differ over time but also has a pronounced hump

[23]In Appendix D we provide a sensitivity analysis for the TFP growth rate. It shows that the general developments of the pension system variables are robust to changes in the growth rate.

[24]We take data from the German Wealth Statistics Account, cf. Statistisches Bundesamt (2018).

[25]In Appendix D we provide a sensitivity analysis for the risk aversion parameter. It shows that the general developments of the pension system variables are robust to changes in the rate of time preference.

[26]The increase in labour supply of women reflects changes in household structures and higher old age working hours reflect better health in later life. These are all subsumed into changes in preferences over time.

[27]In assuming an increase in the employment rate in the future we are in line with many other studies even though we are in the lower range, cf. Börsch-Supan and Rausch (2018). It is important to note that the assumptions on the development of the employment rate are crucial for the replacement rate in the transition until a new steady state is reached. See discussion in Section 2.2.

shape profile over the life cycle. To reflect this pattern in the model we assume that labour productivity is age dependent, $\varepsilon_{j,t}$.[28] In younger ages there is a strong increase in individual labour productivity. This labour productivity stabilizes at a high level until it peaks around the biographical age of 50 when it is about 42% higher than at labour market entry. This is broadly in line with estimates by Hujer, Fitzenberger, Schnabel, and MaCurdy (2001). Labour productivity then slowly falls towards the end of working life and then faster after the statutory retirement age.

$$\varepsilon_{j+1,t+1} = \begin{cases} \varepsilon_{j,t} * \hat{\varepsilon}^1 & \text{if} \quad j < 10 \\ \varepsilon_{j,t} * \hat{\varepsilon}^2 & \text{if} \quad 10 \leq j < 30 \\ \varepsilon_{j,t} * \hat{\varepsilon}^3 & \text{if} \quad 30 \leq j < J_c^E \\ \varepsilon_{j,t} * \hat{\varepsilon}^4 & \text{if} \quad J_c^E \leq j < J_c^R \\ \varepsilon_{j,t} * \hat{\varepsilon}^5 & \text{if} \quad j \geq J_c^R \end{cases} \tag{28}$$

An increase in the statutory retirement age also affects the life cycle profile. By assuming that high labour productivity phase between 50 and J^E is extended we increase the employment rate in higher ages (see Figure 3 (b)).[29] The idea of an interrelation between labour productivity and pension reforms is not new. Starting with Ben-Porath (1967) and Becker (1962), human capital theory predicts that the value of human capital investment increases with the payout period. Many papers adopted this idea and investigated the effects of pension reforms on education and growth, cf. Buyse, Heylen, and de Kerckhove (2013), Buyse, Heylen, and Van De Kerckhove (2017), Gohl, Haan, Kurz, and Weinhardt (2020).

We normalize the average labour productivity in 2018, $\mathcal{E}_{2018} = 1$.

The functional form of bequests utility is

$$\Upsilon(a_{j,t}) = \begin{cases} \frac{1}{1-\theta}\left(a_{j,t}^{\upsilon_j}\right)^{1-\theta} & \text{if} \quad \theta \neq 1 \\ \upsilon_j ln\left(a_{j,t}\right) & \text{if} \quad \theta = 1 \end{cases} \tag{29}$$

The utility weight of the bequests, υ, is zero until retirement and then gradually increases. It is calibrated to match the aggregate amount of assets that are bequeathed to the next generation in Germany. We assume that of total annual bequests in Germany in 2018 €255bn[30], one third is bequeathed within the same cohort (to the spouse). The remaining two thirds, €171bn, is the calibration target of the bequest motive in our model. We assume that bequests are distributed to households within the first 10 periods of their life, $J^Q = 20$.

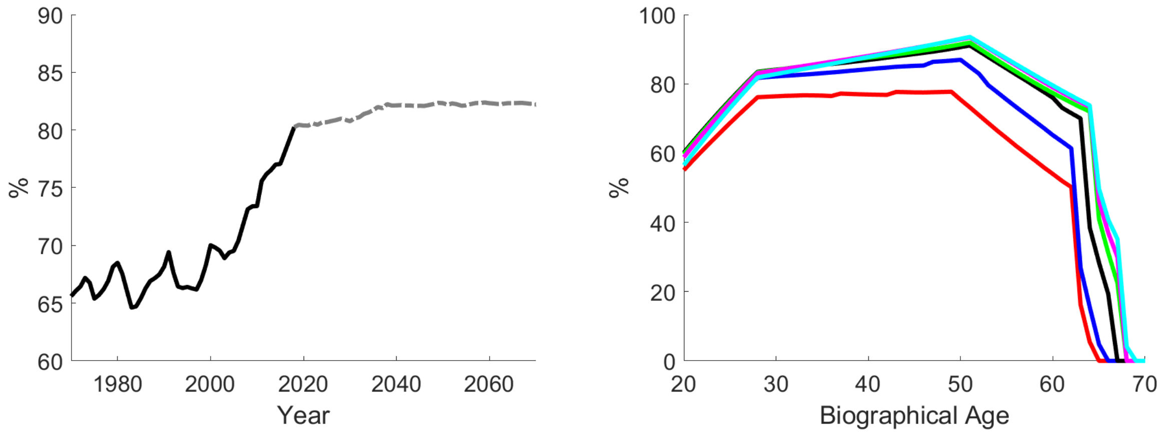
[28] The hump shape profile could also be achieved by making the consumption share parameter, ξ, age-dependent. In order to separate time effects and life cycle effects in the employment rate we use individual labour productivity for the hump shape life cycle profile and the consumption share parameter for the overall trend of the employment rate.

[29] This is in line with the past trends that show a strong increase in the employment rate of 55 – 69 year old, cf. Börsch-Supan, Rausch, and Goll (2019).

[30] A recent study from Tiefensee and Grabka (2017) estimates that total annual bequests in Germany are between €256bn and €385bn. We take the mean of both estimates and also deduct 20%. This accounts for the assumption that self-used real estate is not part of the capital stock.

Figure 3: Employment Rate...

(a) ...over Time Cross Section (b) ...over Live Cycle by Cohort



Notes: Panel (a) shows the ratio of employed persons to population between age 20 and statutory retirement age. Defining the upper age boundary to 65 would result in overall higher rates and a stronger increase in the future. The reason for this is that employment rate for older ages is lower than for younger ages. Panel (b) shows employment rates of cohorts over the life cycle. Cohort born 1945 (red line), cohort born 1960 (blue line), cohort born 1970 (black line), cohort born 1980 (green line).

4 Model Results

The following section will start by presenting the results based on the current legal situation.[31] After that, further reform scenarios will be used to illustrate the importance of key pension variables. The main focus of this paper is the impact of demographic change on the pension system. Hence, we concentrate on the pension variables. The applied model is, however, rich enough to also investigate other interesting effects of population ageing in Germany.[32]

4.1 Current Legal Situation

The entry of the baby boomer cohort in the mid 2020s will sharply increase the number of pensioners and reduce the number of employees. Therefore the pensioner ratio increases sharply in the next decade. Even though the pensioner ratio is directly linked to the replacement rate the minimum threshold of 48% will prevent the replacement rate from falling until 2025. After that the replacement rate drops by more than 5%-points. At the end of the 2030s, the pressure of the baby boomer cohorts will vanish. These strong cohorts relief the pension system when more and more of them reach the end of their life

[31]Results for pension system's replacement and contribution rates for the current legal situation are within the spectrum of findings of other studies, e.g. Werding (2013), Bundesministerium für Arbeit und Soziales (2018), Börsch-Supan and Rausch (2018). Deviations result, inter alia, from differences in the model class, the assumptions made, the starting year (and thus the data used for comparisons) as well as the underlying legal provisions.

[32]Further simulation results on macroeconomic variables can be found in the appendix.

cycle. However, this relief is counteracted by the entry of cohorts with histories of high employment over their live. The replacement rate will temporarily stabilise at around 43%.[33] However, the underlying downward pressure from the increasing life expectancy will lead again to a declining replacement rate which reaches 40% in 2070. This decline will permanently continue under the current legal situation unless the increase in life expectancy stops.

The current pension formula shares the burden of the demographic change between the replacement rate, the contribution rate and the federal government funds. So the contribution rate mirrors the picture of the replacement rate but in reverse. After the year 2025 the contribution rate will rise particularly sharply (to a magnitude of 24%) up until the end of the 2030s as the baby boomers enter retirement. The increase pauses around the year 2040 but gains momentum again. Up until 2070, growth will remain substantial, albeit slower to 26%. The contribution rate increase continues as long as mortality rates drops.

The funds that the German federal government provides for the pension system will rise sharply in the longer term. In large part, they will go up in line with per capita wages and the contribution rate. As a consequence, federal government funds overall are likely to outpace significantly the overall basis for receipts from contributions and taxes. In the following, output will be used as an aggregate indicator for the tax base. In the model, it captures macroeconomic developments. The increase in federal government funds relative to output is the result, first, of the sharply higher contribution rate. The second reason is the contracting employment headcount. As a result, the total wage bill and output are growing more slowly than per capita wages, to which the majority of federal government funds are linked. The fact that government funding is rising much faster than the tax base will put the federal budget under considerable and permanent pressure. This is captured by the rise in federal government funds relative to output. The results show that the need for federal government funds would expand substantially compared with 2018. By 2070, the requirements amounts to almost 5% of output a year.

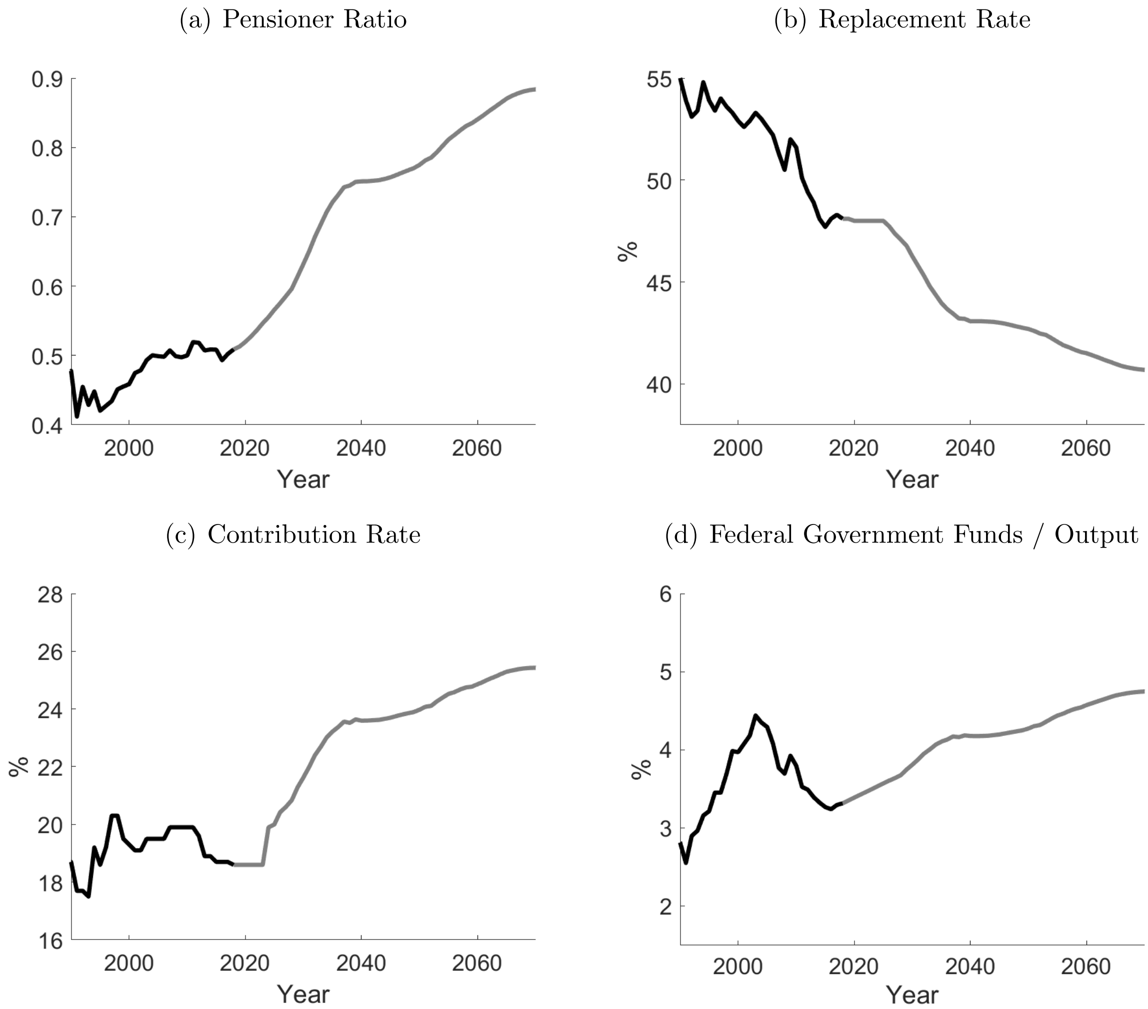
4.2 Reform Scenarios

The German government has announced a reform of the pension system for the period after 2025 – after the minimum and maximum thresholds for the replacement rate and the contribution rate expire. A key issue in all of this is the future distribution of the adjustment burdens. As compared with the current legal situation, the reform scenarios I – III distribute the adjustment burden relatively one-sidedly, with either the replacement rate or the contributions burden bearing the brunt. Reform scenario IV increases sharply the statutory retirement age. After that, examples of broader-based burden sharing are presented.

As the model focuses on Germany, the respective parameters are tailored to the situation there. The development of the return on capital in Germany is likely to hinge primarily on developments in the international capital market. However, this market has not been modelled in the present single country model. In the baseline, it is thus domestic

[33] It should be noted that although the replacement rate will drop for a constant 45 years of contributions, the number of years that contributions are paid will increase as the statutory retirement age rises to 67.

Figure 4: Current Legal Situation – Pension System



Notes: Panel (a) shows the pensioner ratio defined as *Equivalence pensioners* to *Equivalence contributors*. Panel (b) shows the wage replacement rates for 45 earnings points. Panel (d) shows the ratio of total federal government funds (*General Federal Government Grant* +*Additional Federal Government Grant*) to output.

households' propensity to save, which increases as the population ages, that drives the return on capital. This appears to be justified as international demographic developments are all in all comparable. Thus, if the model were to include an international capital market, developments would likely be similar. In this case, demographic change in itself would also lead to a lower return on capital. However, it seems plausible that German pension reforms have only a limited impact on the international return on capital. In this respect, the yield curve in the reform scenarios has been left unchanged.

4.2.1 Relatively One-sided Burden Sharing

Reform scenario I freezes the replacement rate at 48%: in other words, the current minimum threshold is extended beyond 2025. The statutory retirement age remains constant at 67 years from the 2030s onwards as in the current legal situation. As a result, the contribution rate rises very sharply, as pension expenditure now increases much more strongly than under the current legal situation. The contribution rate is significantly higher in 2070 (in the region of 31%) than in the current legal situation. In addition, federal government funds also grow more sharply as they are linked to the contribution rate. By 2070, the ratio of federal government funds relative to output would expand by almost 3 percentage points.

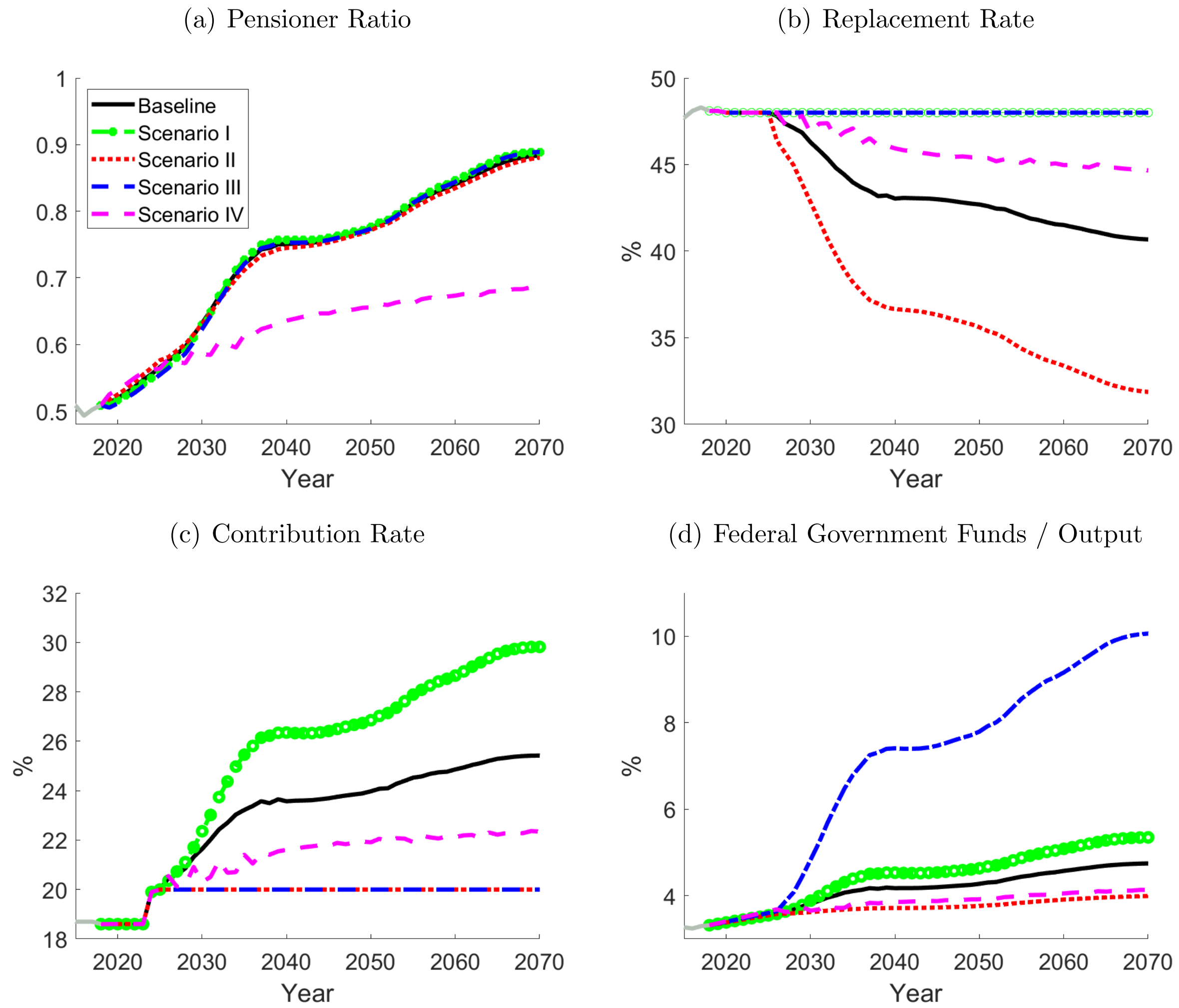
In reform scenario II, the contribution rate rather than the replacement rate is fixed at the level it reaches in 2025, namely 20%. This also considerably dampens the increase in most of the associated federal government funds. The replacement rate now bears the brunt of demographic change. Up until the end of the 2030s, it drops into the region of 35% and by 2070 to 30%. The coverage provided by the statutory pension system thus shrinks considerably.

In reform scenario III, the thresholds for both the replacement rate and the contribution rate remain in place after 2025. The full adjustment burden therefore lies on the federal budget, and the federal government funds employed shoot upwards. In 2070, they are, relative to output, almost 8 percentage points higher than in 2018. The percentage of the statutory pension system's receipts funded by the Federal Government rises to well over 50% (currently: 29%).

Besides replacement rate, contribution rate and federal government funds also the statutory retirement age could bear the brunt of the demographic change. Under the current legal situation, it will increase until the early 2030s, before remaining unchanged at 67 years thereafter. However, life expectancy is likely to continue to rise even after that. As the statutory retirement age rises, the actual age at which people enter retirement is also likely to shift back – as has been the case to date. This expands the work-force and is therefore also beneficial for overall economic growth and incomes. Receipts from pension contributions as well as from the other social security contributions and taxes then likewise develop more favourably. The number of pensions in payment grows more slowly if the statutory retirement age is raised, which, in turn, supports the replacement rate via the sustainability factor.

In principle, there are various conceivable approaches to increasing the statutory retirement age. A very sweeping approach would be for all demographic burdens, i.e. both rising life expectancy and lower birth rates (fluctuating cohort sizes), to be absorbed through increases in the statutory retirement age. However, the statutory retirement age

Figure 5: One-sided Burden Sharing – Pension System



Notes: Panel (a) shows pensioner ratio defined as equivalence pensioners to equivalence contributors. Panel (b) shows wage replacement rate for 45 earnings points. Panel (d) shows ratio of total federal government funds (*General Federal Government Grant* +*Additional Federal Government Grant*) to output. Results for the current legal situation (solid black line). Reform scenario I (circle green line). Reform scenario II (dotted red line). Reform scenario III (dotted dashed blue line). Reform scenario IV (dashed magenta line).

would have to rise very sharply in this case. It would also have to be raised significantly faster than currently envisaged, particularly when the baby boomer cohorts enter retirement between the mid-2020s and the mid-2030s. Reform scenario IV shows results for a simulation where an increase in the statutory retirement age stabilises the old age dependency ratio at the 2018 level for the future. The statutory retirement age therefore has to increase until 2070 to 74. Even though the direct demographic effect is offset the pensioner ratio increases (and therefore the replacement rate falls and the contribution rate rises). The reason for this is that the increased number of working years also increases the claims of households against the pension system. These additional claims drive a wedge between the OADR and the pensioner ratio. This effect is dealt with in the Section 4.2.2 by introducing an adjusted replacement rate.

In terms of private consumption per capita the reform scenarios also differ on impact and in the long run.[34] This is mainly due to the changed incentives for private old age provision. In scenario II, the pension level is strongly reduced. Households need to save in order to maintain their level of consumption after retirement. In the short run this increases the savings rate and reduces per capita consumption. In the long run the build-up of assets increases the income generated in the economy and strengthens per capita consumption. Another channel how a pension reform might affect consumption is via the expansion of the labour force. An increase in retirement age (scenario IV) reduced on impact the incentive to supply labour as households want to stabilize their life time labour supply. In the long run however the longer working life increases aggregate labour supply and therefore output and consumption per capita.

4.2.2 Broader-based Burden Sharing

The reform scenarios described above concentrate the burden of demographic adjustment on individual variables in a rather one-sided fashion. They thereby illustrate key correlations, and the results show why broader-based burden sharing makes sense. In the following reform scenarios, by contrast to the previous scenarios, the adjustment burdens are spread more broadly.

Indexed statutory retirement age The statutory retirement age rises as planned until the beginning of the 2030s, followed by additional rule-based increases. In concrete terms, the statutory retirement age is adjusted so that the ratio of years in retirement to years of contributions – i.e. the relative pension-drawing period – remains broadly stable as of the 2030s. Essentially, therefore, the current approach continues until the beginning of the 2030s, and even within this time-frame the increasing statutory retirement age largely stabilises the relative pension-drawing period (see Figure 1 (b)). The relative pension-drawing period therefore stands at around 40% on a lasting basis. In other words, given the life expectancy projections used here, the statutory retirement age would have to rise, on average, by three quarters of a month per year. For example, a person entering retirement at the age of 67 in 2031 has a life expectancy of 86 years. In 2070, the statutory retirement age would be 69 years and three months and life expectancy 89 years and six months (see Figure 2). The period of pension payment would then be just over 20 years

[34] See Table 8 and Table Table 9 in Appendix C.

and thus more than one year longer than in 2031.[35]

Dynamically adjusted replacement rate In addition to a longer retirement period, members of the statutory pension system gain more pension entitlements as they pay contributions for longer. A household that works full time between 20 and its cohort specific retirement age (with average labour productivity) collects J_c^R earnings points which are more than assumed in a standard pension of $D_t = 45$. The individual replacement rate consequently rises compared to the replacement rate definition used so far. As the statutory retirement age goes up, it would therefore make sense to stipulate a higher number of years of contributions in the definition of the standard pension and thus in the replacement rate. If, say, the statutory retirement age is set at 67, the standard pension and, consequently, the replacement rate, would have to be calculated for 47 instead of 45 years of contributions (dynamically adjusted replacement rate).

There could be various ways to achieve a dynamically adjusted replacement rate. One way would be to raise the definition of the standardized pension such that $D_t = J_t^r$. This would however reduce the value of earnings points also for people that are already retired and do not collect more points due to an increased statutory retirement age. To guarantee that the value of an earnings point does not decrease while in retirement we follow another approach. We connect the amount of earnings points a household can collect in one year to its cohort specific statutory retirement age by a cohort specific earnings point factor, $d_c = \frac{45}{J_c^R}$. The maximum amount of earnings points a fully working household could collect when it is working until its cohort specific statutory retirement age is still 45. In the following broader-based burden sharing reform scenarios the earnings point accumulation follows

$$p_{j+1,t+1} = \begin{cases} \frac{w_t^g \varepsilon_{j,t}}{w_t^g \mathcal{E}_t} l_{j,t} \psi d_c + p_{j,t} & \text{if } j < J_c^E \\ \frac{w_t^g \varepsilon_{j,t}}{w_t^g \mathcal{E}_t} l_{j,t} \psi d_c + p_{j,t}(1 - \Delta^-(1 - l_{j,t}\psi)) & \text{if } J_c^E \leq j < J_c^R \\ \frac{w_t^g \varepsilon_{j,t}}{w_t^g \mathcal{E}_t} l_{j,t} \psi d_c + p_{j,t}(1 + \Delta^+ l_{j,t}\psi) & \text{if } j \geq J_c^R \end{cases} \tag{30}$$

Compared to the replacement rate based on a static contribution period of just 45 years, the dynamically adjusted replacement rate is higher.

Current legal situation with an indexed statutory retirement age and a dynamically adjusted replacement rate (reform scenario V) Apart from the described indexing of the statutory retirement age, the current legal situation continues to apply, i.e. the burden is distributed relatively broadly across the other variables. They thus cushion the burden arising from the decline in the birth rate. As of 2026, pensions therefore once again need to be adjusted in accordance with the pension adjustment formula. The dynamically adjusted replacement rate falls chiefly due to the strain of baby boomers entering retirement (to around 44% by the end of the 2030s). It then stabilises. Although baby boomers pass away, cohorts with higher labour force participation rates retire. The

[35]In practice, an indexation would take into account the uncertainty connected with future life expectancy. If life expectancy projections were to change, there would be corresponding rule-based adjustments to the statutory retirement age as well. For example, the statutory retirement age would remain constant if life expectancy no longer increased.

cohort sizes then change only moderately and, due to the rising statutory retirement age, rising life expectancy no longer exerts any pressure. At the same time, the increasing number of contribution years supports the dynamically calculated replacement rate (see Figure 6).

In this reform scenario V, the contribution rate still increases significantly to around 24% in 2070. However, the increase is much smaller than would be the case without a further rise in the statutory retirement age. Contribution payers and the federal budget come under less strain. First, the pressure is eased by the smaller number of people drawing a pension, and second, the higher degree of employment leads to a marked increase in the tax base. As a percentage of output, federal government funds rise by more than 1 percentage points on their 2018 level. Ultimately, the additional burdens from the lower birth rates are thus distributed, on the one hand, among pension recipients (via the replacement rate), and, on the other hand, among contribution payers and taxpayers.

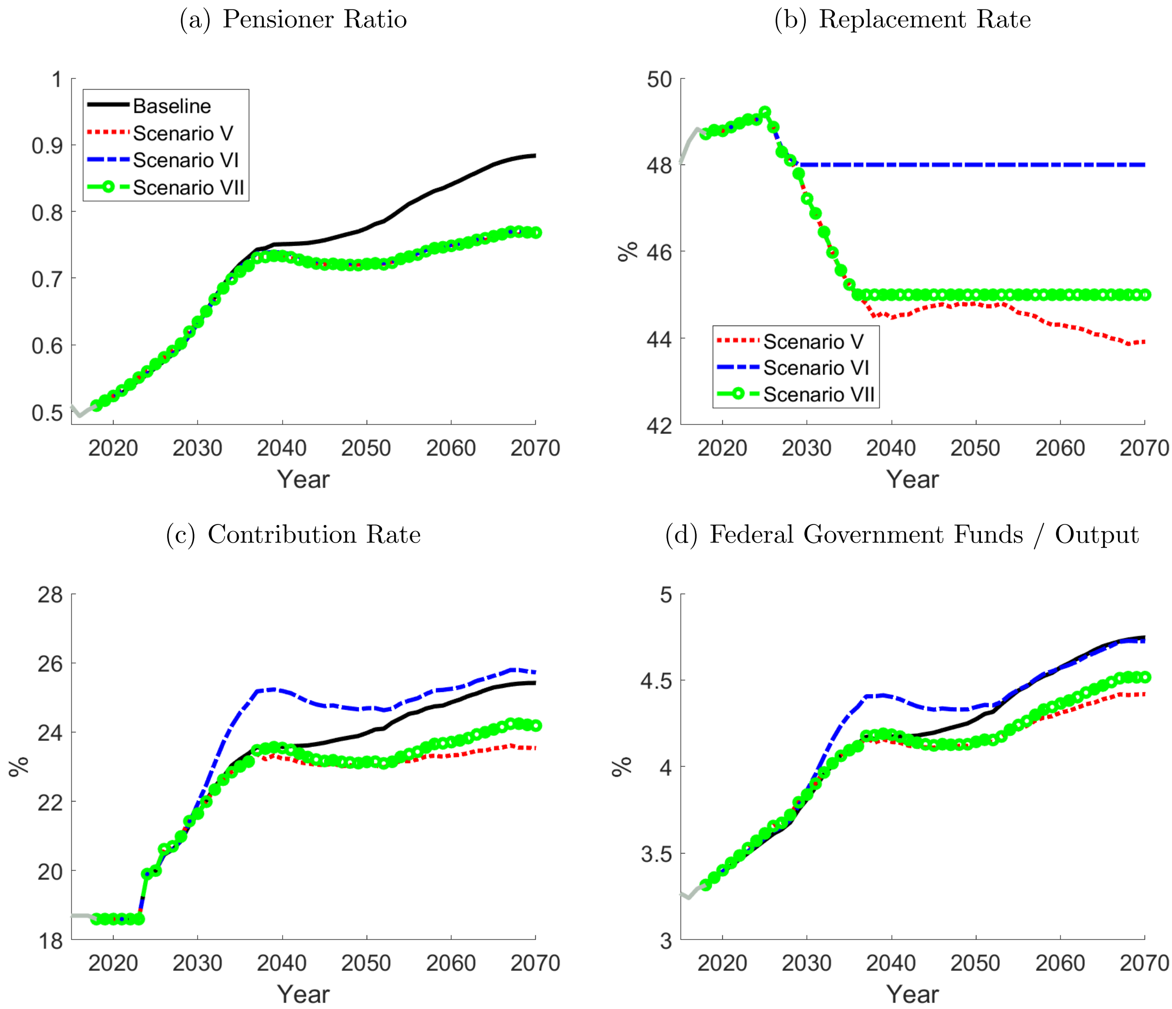
Reform scenario VI & VII with an indexed statutory retirement age and an additional threshold The replacement rate is a key topic in the pension debate. The preceding reform scenario V shows that it falls even when the statutory retirement age is indexed as described above. However, in the long term, it stabilises at 43% to 44%. Calls are frequently made for the replacement rate to be prevented from falling any further or for at least thresholds to be put in place.

For that reason, the following reform scenarios VI and VII include thresholds for the dynamically adjusted replacement rate described above. The adjustment burden thereby shifts further to the receipts side, i.e. to contribution rates and federal government funds. Risks of more unfavourable developments would therefore be borne by contribution payers and taxpayers.

The costs of thresholds increase significantly after 2025. Once the baby boomers reach statutory retirement age, costs continue to rise, though their trajectory flattens. The reform scenarios VI & VII show, as a rule of thumb, that a 1%-point higher threshold for the dynamically adjusted replacement rate requires the contribution rate in 2070 to be around 0.5%-point higher. At the same time, federal government funds as a percentage of output are 0.1%-point higher.

For example, a contribution rate of 27% is necessary in 2070 if the dynamically adjusted replacement rate is to remain at 48% after 2025. The federal government funds required then also rise more sharply by a total of 1.75% of output. However, the burden on contribution payers and taxpayers is significantly lower than if the statutory retirement age remains unchanged and a threshold is applied for the non-dynamically adjusted replacement rate based on a static contribution period of 45 years. If the threshold for the replacement rate is set at 45%, the contribution rate required is correspondingly lower at around 25%. The increase in federal government funds is also lower, at 1.5% of output.

Figure 6: Broader Based Burden Sharing – Pension System



Notes: Panel (a) shows pensioner ratio defined as equivalence pensioners to equivalence contributors. Panel (b) shows wage replacement rate for 45 earnings points. Panel (d) shows ratio of total federal government funds (*General Federal Government Grant* +*Additional Federal Government Grant*) to output. Results for the current legal situation (solid black line). Reform scenario V (dashed green line). Reform scenario VI (dotted red line). Reform scenario VII (dotted dashed blue line).

5 Conclusions

As a result of demographic trends, the pay-as-you-go statutory pension system will come under considerable pressure in the future, especially from the mid-2020s onwards. The German government has announced a pension reform which is intended to come into effect as of 2026 and put the pension system on a long-term stable footing. The key parameters are the statutory retirement age, the replacement rate and the contribution rate. They affect the future scope of the statutory pension system and the distribution of demographic burdens across cohorts. A role is also played by federal government funds, which are provided by all taxpayers.

The model results show that individual pension parameters would have to be adjusted very sharply if they alone had to absorb the demographic pressure. The statutory retirement age is an important factor in further reforms. It will increase to 67 years by 2031. As a result, the ratio of the pension-drawing period to the contribution period will not increase, despite the fact that life expectancy is rising. If the statutory retirement age subsequently remained constant, a static number of contribution years would once again be set against a continually growing period of pension payment, which would put pension funding under pressure.

With an indexed statutory retirement age, further targeted stabilisation of the relative pension-drawing period from the beginning of the 2030s onwards would be possible, for example. Persons covered by the statutory pension system in future would first have to contribute to the system for longer, but would subsequently also draw a pension for longer. They would therefore be no worse off in terms of the ratio of the period of pension payment to the contribution period. According to current life expectancy projections, under such an approach the statutory retirement age would rise by an average of three quarters of a month per year as of 2032. Those born in 2001 would enter regular retirement at the age of 69 and four months from 2070 onwards. If life expectancy were to develop differently, this would also have an impact on the statutory retirement age, provided it were indexed. In order to give those affected time to adjust, changes to the statutory retirement age could be smoothed and set out well in advance.

This adjustment to the statutory retirement age would not only ease the burden on the statutory pension system. Through increased employment, it would also strengthen macroeconomic potential and thus boost the assessment bases for taxes and social security contributions.

Longer periods of employment and more years of contributions also lead to greater pension entitlements. It would therefore be logical to take this into account in the projections of the replacement rate and the thresholds applying to it. For instance, the number of contribution years factored into the calculation of the replacement rate could rise in line with the statutory retirement age. For example, such a dynamically adjusted replacement rate would require 46 years of contributions on the basis of average earnings in 2024, and 47 years of contributions in 2031.

Indexing the statutory retirement age as described would absorb the pressure caused by longer life expectancy. However, other factors would still need to be addressed, including, in particular, the impact of lower birth rates since the 1970s. The vast majority of these adjustments would be concluded by the end of the 2030s. This means that, from this point onwards, almost no additional pressure on pension funding would arise. In the reform scenarios presented here, the dynamically adjusted replacement rate – with adjustment mechanisms otherwise remaining unchanged – falls from about 48% today to about 43% by 2070, before stabilising at roughly 44% thereafter. The contribution rate increases from almost 19% to about 24%. Over time, federal government funds also increase significantly relative to output. The respective developments are, however, much milder than is the case if the statutory retirement age is not indexed.

Nevertheless, if the threshold is extended, both the impact of lower birth rates as well as the remaining funding risks would be shifted almost entirely to contribution payers and taxpayers. The burden of taxes and contributions would potentially rise sharply. Furthermore, this problem cannot be solved by additionally capping the contribution rate, for although this would relieve adjustment pressure on the statutory pension system, it would place additional burdens on the federal budget and thus on taxpayers. The current legal situation will already lead to a sharp increase in financing needs, which is sometimes neglected in the debate on pension policy. It is essential that this aspect is taken into account in the specific design of the pension reform. At the very least, the financial impact of a reform should be disclosed on the basis of official projections over the very long term and as comprehensively as possible.

A Model Solution

A.1 General Equilibrium Solution Method

The time line of the model has four periods: a phase in period $t = 0, ..., T^C - 1$, a calibration period (2018), $t = T^C$, a projection period (2019-2070), $t = T^C + 1, ..., T^P$ and a phase out period, $t = T^P + 1, ..., T$ that lasts until 2500 when the model reaches its final steady state.

As for the solution of the model, we seek for a fixed point such that all markets clear. The general solution method does not differ much between steady state and transition calculations.

The fixed point iteration is written as an iteration searching for the equilibrium return on capital, contribution rate, consumption tax rate, and average labour productivity.

Steady State Solution

1. Start with an initial guess for the return on capital, r_0^g, contribution rate, ϕ_0, consumption tax rate ,τ_0^c, and average labour productivity, $\mathcal{E}_0$.

2. In each iteration m for $r_m^g, \phi_m, \tau_m^c, \mathcal{E}_m$

   (a) Calculate the gross wage $w_m^g = \frac{1-\alpha}{(1+\frac{1}{2}\psi\phi_m)(1+\frac{1}{2}\varphi)} \left(\frac{\alpha}{r_m^g}\right)^{\frac{\alpha}{1-\alpha}}$ from appropriately transforming the first order conditions of the firm sector (15) and (16).

   (b) Given net return on capital rate $r_m^n = \left(1 - \tau^k\right)(1 - \nu)(r_m^g - \delta)$, net wages $w_m^n = (1 - \frac{1}{2}\psi\phi_m)(1 - \frac{1}{2}\varphi)(1 - \tau^y) w_m^g$, average labour productivity $\mathcal{E}_m$, net pension value $b_m^n = (1 - \frac{1}{2}\psi\phi_m)(1 - \frac{1}{2}\varphi)(1 - \tau^y)\gamma w_m^g \mathcal{E}_m \frac{1}{D}$, and consumption tax rate, τ_m^c, solve the household model (see Section A.3).

   (c) Aggregate across all households living in the steady state to get the aggregate capital stock, $K = \sum_{j=1}^{JT} a_j N_j$.

   (d) Aggregate across all households living in the steady state to get aggregate labour supply $L = H \sum_{j=1}^{JT} \varepsilon_j l_j N_j$ and aggregate (claimed) earnings points $P = \sum_{j=J_c^E}^{JT} (1 - l_j) p_j N_j$.

   (e) Calculate $\tilde{r}_m^g = \alpha A \left(\frac{K}{L}\right)^{\alpha-1}$ as the corresponding capital return rate.

   (f) Calculate $\tilde{\mathcal{E}}_m = \frac{\sum_{j=1}^{45} \varepsilon_j N_j}{\sum_{j=1}^{45} N_j}$ as the average labour productivity.

   (g) Calculate $\tilde{\phi}_m$ using (11) as the contribution rate that equalize the budget of the pension system.

   (h) Calculate $\tilde{\tau}_m^c$ using (23) as the consumption tax rate that equalize the budget of the government.

   (i) If $\parallel x_m - \tilde{x}_m \parallel < \epsilon \; \forall \; x \in \{r^g, \phi, \tau^c, \mathcal{E}\}$, where ϵ is some pre-specified tolerance level STOP, ELSE form an update $x_{m+1} = \omega \cdot x_m + (1 - \omega) \cdot \tilde{x}_m \; \forall \; x \in \{r^g, \phi, \tau^c, \mathcal{E}\}$ where ω is some dampening factor. The lower the value of ω, the more conservative is the update of the guess for the equilibrium value of r^g, ϕ, τ^c and $\mathcal{E}$ to be used in the next iteration. Continue with step (a).

Solution for Transition Dynamics As for the transition period, the solution method described next requires that first the initial and the final steady state have been calculated. By linear interpolating between the values of the rate of return on capital, replacement rate, consumption tax rate, and average labour productivity in the initial period $r_0^g, \phi_0, \tau_0^c, \mathcal{E}_0$, and the final period T, $r_T^g, \phi_T, \tau_T^c, \mathcal{E}_T$ one gets initial guesses for the entire time path of $\overrightarrow{r}_t^g$, $\overrightarrow{\phi}_t, \overrightarrow{\tau}_t^c$, and $\overrightarrow{\mathcal{E}}_t$. Furthermore, the steady state solution of the model in period 0 gives an initial distribution of assets and earnings points for those households alive in period 0, $\{a_{j,0}\}_{j=1}^{J^T}, \{p_{j,0}\}_{j=1}^{J^T}$ and therefore also $\{a_{j,1}\}_{j=1}^{J^T}, \{p_{j,1}\}_{j=1}^{J^T}$ is known. The solution outside the steady state then is as follows

1. Solve the model for the initial steady state at $t = 0$.
2. Solve the model for a final steady state at $t = T$.
3. Form an initial guess for the capital return rates, $\{\overrightarrow{r}_{t,0}^g\}_{t=1}^{T-1}$, contribution rates $\{\overrightarrow{\phi}_{t,0}\}_{t=1}^{T-1}$, consumption tax rates $\{\overrightarrow{\tau}_{t,0}^c\}_{t=1}^{T-1}$ and average labour productivities $\{\overrightarrow{\mathcal{E}}_{t,0}\}_{t=1}^{T-1}$ obtained from linear interpolation between the two steady states. Form an additional initial guess for the entire time path of the replacement rate, $\overrightarrow{\gamma}_t$.
4. In each iteration m for $\{\overrightarrow{r}_{t,m}^g\}_{t=1}^{T-1}$, $\{\overrightarrow{\phi}_{t,m}\}_{t=1}^{T-1}$, $\{\overrightarrow{\tau}_{t,m}^c\}_{t=1}^{T-1}$, $\{\overrightarrow{\mathcal{E}}_{t,m}\}_{t=1}^{T-1}$ and $\{\overrightarrow{\gamma}_{t,m}\}_{t=1}^{T-1}$

   (a) Calculate $\overrightarrow{w}_{t,m}^g = \frac{1-\alpha}{(1+\frac{1}{2}\psi\overrightarrow{\phi}_{t,m})(1+\frac{1}{2}\varphi)}\left(\frac{\alpha}{\overrightarrow{r}_{t,m}^g}\right)^{\frac{\alpha}{1-\alpha}}$ from transforming the first order conditions of the firm (15 and 16.

   (b) Given net capital return rates $\overrightarrow{r}_{t,m}^n = \left(1-\tau^k\right)(1-\nu)\left(\overrightarrow{r}_{t,m}^g - \delta\right)$, net wages $\overrightarrow{w}_{t,m}^n = (1-\tau^y)(1-\frac{1}{2}\psi\overrightarrow{\phi}_{t,m})(1-\frac{1}{2}\varphi)\overrightarrow{w}_{t,m}^g$, consumption tax rate, $\overrightarrow{\tau}_{t,m}^c$, net pension value $\overrightarrow{b}_{t,m}^n = (1-\tau^y)(1-\frac{1}{2}\psi\overrightarrow{\phi}_{t,m})(1-\frac{1}{2}\varphi)\overrightarrow{\gamma}_{t,m}\overrightarrow{w}_{t,m}^g\overrightarrow{\mathcal{E}}_{t,m}\frac{1}{D_t}$ and average labour productivity $\overrightarrow{\mathcal{E}}_{t,m}$, solve the household model for all households born in $t = 1, ..., T$ (see Section A.3). Also solve the household model for those households already alive in period 1 using the initial conditions,$\{a_{j,1}\}_{j=1}^{J^T}$, $\{p_{j,1}\}_{j=1}^{J^T}$.

   (c) Aggregate across all households to get the aggregate capital stock, $K_t = \sum_{j=1}^{J^T} a_{j,t}N_{j,t}$ for all $t = 1, ..., T-1$.

   (d) Aggregate across all households to get the aggregate labour supply $L_t = \sum_{j=1}^{J^T} \varepsilon_{j,t}l_{j,t}N_{j,t}$ and the aggregate (claimed) earnings points $P_t = \sum_{j=J_c^E}^{J^T}(1-l_{j,t})p_{j,t}N_{j,t}$ for all $t = 1, ..., T-1$.

   (e) Calculate $\overrightarrow{r}_{t,m}^g = \alpha A_t\left(\frac{K_t}{L_t}\right)^{\alpha-1}$ for all $t = 1, ..., T$ as the corresponding capital return rate.

   (f) Calculate $\overrightarrow{\mathcal{E}}_{t,m} = \frac{\sum_{j=1}^{45}\varepsilon_{j,t}N_{j,t}}{\sum_{j=1}^{45}N_{j,t}}$ for all $t = 1, ..., T-1$ as the average labour productivity.

   (g) Given $\tilde{\gamma}_{0,m}$, calculate $\overrightarrow{\tilde{\gamma}}_{t,m}$ according to the pension formula (10) for all $t = 1, ...T-1$.

(h) Calculate $\overrightarrow{\phi}_{t,m}$ using (11) for all $t = 1, ...T - 1$ as the contribution rate that equalize the budget of the pension system.

(i) Calculate $\overrightarrow{\tau}^{c}_{t,m}$ using (23) for all $t = 1, ...T - 1$ as the consumption tax rate that equalize the government budget.

(j) If $\parallel \overrightarrow{x}_{t,m} - \overrightarrow{\tilde{x}}_{t,m} \parallel < \epsilon \ \forall \ x \in \{r^g, \phi, \tau^c, \gamma, \mathcal{E}\}$, where ϵ is some pre-specified tolerance level STOP, ELSE form an update $\overrightarrow{x}_{t,m+1} = \omega \cdot \overrightarrow{x}_{t,m} + (1-\omega) \cdot \overrightarrow{\tilde{x}}_{t,m} \ \forall \ x \in \{r^g, \phi, \tau^c, \gamma, \mathcal{E}\}$ where ω is some dampening factor. Continue with step (a).

A.2 Recursive Solution to Household Problem

Detrending the household problem

$$A_{j+1,t+1} = \left(1 + r^n_{t+1}\right)\left(A_{j,t} + Y^n_{j,t} - \left(1 + \tau^c_t\right) C_{j,t}\right) \tag{31}$$

and the detrended version is

$$a_{j+1,t+1} F_{t+1} = \left(1 + r^n_{t+1}\right)\left(a_{j,t} F_t + y^n_{j,t} F_t - \left(1 + \tau^c_t\right) c_{j,t} F_t\right) \tag{32}$$

$$a_{j+1,t+1} = \frac{F_t}{F_{t+1}}\left(1 + r^n_{t+1}\right)\left(a_{j,t} + y^n_{j,t} - \left(1 + \tau^c_t\right) c_{j,t}\right) \tag{33}$$

$$a_{j+1,t+1} = \frac{1}{1+\mu_t}\left(1 + r^n_{t+1}\right)\left(a_{j,t} + y^n_{j,t} - \left(1 + \tau^c_t\right) c_{j,t}\right) \tag{34}$$

The household problem[36] can be written in the following set-up

$$V_{j,t}\left(a_{j,t}, p_{j,t}\right) = \max_{c_{j,t}, l_{j,t}, a_{j+1,t+1}, p_{j+1,t+1}} \left\{U\left(c_{j,t}, \overline{H} - l_{j,t} H_t\right) + \tilde{\beta}_{j,t}\pi_{j,t} V_{j+1,t+1}\left(a_{j+1,t+1}, p_{j+1,t+1}\right) + (1-\pi_{j,t})\Upsilon(a_{j+1,t+1})\right\} \tag{35}$$

s.t.

$$0 \le l_{j,t} \le 1 \tag{36}$$

$$a_{j+1,t+1} = \left(1 + r^n_{t+1}\right)\left(a_{j,t} + y^n_{j,t} - \left(1 + \tau^c_t\right) c_{j,t}\right) \tag{37}$$

with

$$y^n_{j,t} = w^n_t \varepsilon_{j,t} l_{j,t} + b^n_{j,t} p_{j,t}(1 - l_{j,t}) \tag{38}$$

$$w^n_t = (1 - \tfrac{1}{2}\psi\phi_t)(1 - \tfrac{1}{2}\varphi)\left(1 - \tau^y\right) w^g_t \tag{39}$$

$$b^n_{j,t} = \left(1 - \tau^y\right)(1 - \tfrac{1}{2}\varphi) b^g_{j,t} = \begin{cases} 0 & \text{if } j < J^E_c \\ \left(1 - \tau^y\right)(1 - \tfrac{1}{2}\varphi)(1 - \tfrac{1}{2}\psi\phi_t)\gamma_t w^g_t \mathcal{E}_t \frac{1}{D_t} & \text{if } j \ge J^E_c \end{cases} \tag{40}$$

[36] Where $\tilde{\beta}_{j,t} = \beta\pi_{j,t}\left(1 + \mu\right)^{1-\theta}$ is the growth and survival rate adjusted time discount factor.

$$p_{j+1,t+1} = \begin{cases} \frac{\varepsilon_{j,t}}{\mathcal{E}_t} l_{j,t}\psi + p_{j,t} & \text{if } j < J_c^E \\ \frac{\varepsilon_{j,t}}{\mathcal{E}_t} l_{j,t}\psi + p_{j,t}(1 - \Delta^-(1 - l_{j,t}\psi)) & \text{if } J_c^E \leq j < J_c^R \\ \frac{\varepsilon_{j,t}}{\mathcal{E}_t} l_{j,t}\psi + p_{j,t}(1 + \Delta^+ l_{j,t}\psi) & \text{if } j \geq J_c^R \end{cases} \quad (41)$$

Solution using First order Conditions For readability we omit the arguments of the utility function, the value function and its derivatives. Also we omit the time subscript of derivatives of the utility functions. Additionally we denote $R_{t+1} = \frac{1+r_{t+1}^n}{1+\mu_t}$. From this we derive the first order conditions with respect to consumption, leisure and savings

$$\frac{\partial V_{j,t}}{\partial c_{j,t}} = U_c + \tilde{\beta}_{j,t}\left(\pi_{j,t}V_{j+1,t+1}^a + (1-\pi_{j,t})\Upsilon_{a'}\right) R_{t+1}(-1)(1+\tau_t^c) = 0 \quad \text{if } j < J^T \quad (42)$$

$$\frac{\partial V_{j,t}}{\partial l_{j,t}} = U_l + \tilde{\beta}_{j,t}\times$$
$$\begin{cases} ((\pi_{j,t}V_{j+1,t+1}^a + (1-\pi_{j,t})\Upsilon_{a'})R_{t+1}(w_t^n\varepsilon_{j,t}) & +\pi_{j,t}V_{j+1,t+1}^p\psi\frac{\varepsilon_{j,t}}{\mathcal{E}_t}) & = 0 & \text{if } j < J_c^E \\ ((\pi_{j,t}V_{j+1,t+1}^a + (1-\pi_{j,t})\Upsilon_{a'})R_{t+1}(w_t^n\varepsilon_{j,t} - b_t^n p_{j,t}) & +\pi_{j,t}V_{j+1,t+1}^p\psi(\frac{\varepsilon_{j,t}}{\mathcal{E}_t} + p_{j,t}\Delta^-)) & = 0 & \text{if } J_c^E \leq j < J_c^R \\ ((\pi_{j,t}V_{j+1,t+1}^a + (1-\pi_{j,t})\Upsilon_{a'})R_{t+1}(w_t^n\varepsilon_{j,t} - b_t^n p_{j,t}) & +\pi_{j,t}V_{j+1,t+1}^p\psi(\frac{\varepsilon_{j,t}}{\mathcal{E}_t} + p_{j,t}\Delta^+)) & = 0 & \text{if } J_c^R \leq j < J^T \\ (\Upsilon_{a'}R_{t+1}(w_t^n\varepsilon_{j,t} - b_t^n p_{j,t})) & & = 0 & \text{if } j = J^T \end{cases} \quad (43)$$

Updating the derivatives of the value function we use the following functions stemming from the envelope conditions[37]

$$\frac{\partial V_{j,t}}{\partial a_{j,t}} \triangleq V_{j,t}^a = \tilde{\beta}_{j,t}(\pi_{j,t}V_{j+1,t+1}^a + (1-\pi_{j,t})\Upsilon_{a'})R_{t+1} \quad (44)$$

Combined with (42) results

$$V_{j,t}^a = \frac{1}{1+\tau_t^c}U_c \quad (45)$$

that is used later on in the computational solution of the household problem

$$\frac{\partial V_{j,t}}{\partial p_{j,t}} \triangleq V_{j,t}^p = \tilde{\beta}_{j,t}\times$$
$$\begin{cases} (0 & +\pi_{j,t}V_{j+1,t+1}^p) & \text{if } j < J_c^E \\ ((\pi_{j,t}V_{j+1,t+1}^a + (1-\pi_{j,t})\Upsilon_{a'})R_{t+1}b_t^n & +\pi_{j,t}V_{j+1,t+1}^p(1-\Delta^-\psi(1-l_{j,t}))) & \text{if } J_c^E \leq j < J_c^R \\ ((\pi_{j,t}V_{j+1,t+1}^a + (1-\pi_{j,t})\Upsilon_{a'})R_{t+1}b_t^n & +\pi_{j,t}V_{j+1,t+1}^p(1+\Delta^+\psi l_{j,t})) & \text{if } J_c^R \leq j < J^T \end{cases} \quad (46)$$

Combined with (42) and (43) results in

$$V_{j,t}^p = \begin{cases} 0 & -\left[U_l + \frac{1}{1+\tau_t^c}U_c w_t^n\varepsilon_{j,t}\right]\frac{1}{\psi\frac{\varepsilon_{j,t}}{\mathcal{E}_t}} & \text{if } j < J_c^E \\ \frac{1}{1+\tau_t^c}U_{c_{j,t}}b_t^n & -\left[U_l + \frac{1}{1+\tau_t^c}U_c(w_t^n\varepsilon_{j,t} - b_t^n p_{j,t})\right]\frac{1-\Delta^-\psi(1-l_{j,t})}{\psi(\frac{\varepsilon_{j,t}}{\mathcal{E}_t}+p_{j,t}\Delta^-)} & \text{if } J_c^E \leq j < J_c^R \\ \frac{1}{1+\tau_t^c}U_c b_t^n & -\left[U_l + \frac{1}{1+\tau_t^c}U_c(w_t^n\varepsilon_{j,t} - b_t^n p_{j,t})\right]\frac{1-\Delta^+\psi l_{j,t}}{\psi(\frac{\varepsilon_{j,t}}{\mathcal{E}_t}+p_{j,t}\Delta^+)} & \text{if } J_c^R \leq j < J^T \end{cases} \quad (47)$$

that is used later on in the computational solution of the household problem

The solution to the inter-temporal optimization problem is characterized by two first order conditions. First, the intra-temporal Euler equation relates current period con-

[37] Using (43) and (42) that $\frac{\partial V}{\partial c}\cdot\frac{\partial c}{\partial a} = 0$ and $\frac{\partial V}{\partial l}\cdot\frac{\partial l}{\partial a} = 0$

sumption to current period leisure choice, if $l_{j,t}=0$ is not a binding constraint, by

$$-\frac{U_l}{U_c}=\begin{cases}\frac{1}{1+\tau_t^c}\left[w_t^n\varepsilon_{j,t}+\frac{\pi_{j,t}V^p_{j+1,t+1}}{R_{t+1}(\pi_{j,t}V^a_{j+1,t+1}+(1-\pi_{j,t})\Upsilon_{a'})}\psi\frac{\varepsilon_{j,t}}{\mathcal{E}_t}\right] & \text{if } j<J_c^E\\ \frac{1}{1+\tau_t^c}\left[w_t^n\varepsilon_{j,t}-b_t^np_{j,t}+\frac{\pi_{j,t}V^p_{j+1,t+1}}{R_{t+1}(\pi_{j,t}V^a_{j+1,t+1}+(1-\pi_{j,t})\Upsilon_{a'})}\psi\left(\frac{\varepsilon_{j,t}}{\mathcal{E}_t}+p_{j,t}\Delta^+\right)\right] & \text{if } J_c^E\le j<J_c^R\\ \frac{1}{1+\tau_t^c}\left[w_t^n\varepsilon_{j,t}-b_t^np_{j,t}+\frac{\pi_{j,t}V^p_{j+1,t+1}}{R_{t+1}(\pi_{j,t}V^a_{j+1,t+1}+(1-\pi_{j,t})\Upsilon_{a'})}\psi\left(\frac{\varepsilon_{j,t}}{\mathcal{E}_t}+p_{j,t}\Delta^-\right)\right] & \text{if } J_c^R\le j<J^T\end{cases} \quad (48)$$

or (using the functional form of the utility function) [38]

$$\frac{c_{j,t}}{\overline{H}-l_{j,t}H_t}=$$
$$\begin{cases}\frac{1}{1+\tau_t^c}\frac{\xi_t}{1-\xi_t}\frac{1}{H_t}\left[w_t^n\varepsilon_{j,t}+\frac{\pi_{j,t}V^p_{j+1,t+1}}{R_{t+1}(\pi_{j,t}V^a_{j+1,t+1}+(1-\pi_{j,t})\Upsilon_{a'})}\psi\frac{\varepsilon_{j,t}}{\mathcal{E}_t}\right] & \text{if } j<J_c^E\\ \frac{1}{1+\tau_t^c}\frac{\xi_t}{1-\xi_t}\frac{1}{H_t}\left[w_t^n\varepsilon_{j,t}-b_t^np_{j,t}+\frac{\pi_{j,t}V^p_{j+1,t+1}}{R_{t+1}(\pi_{j,t}V^a_{j+1,t+1}+(1-\pi_{j,t})\Upsilon_{a'})}\psi\left(\frac{\varepsilon_{j,t}}{\mathcal{E}_t}+p_{j,t}\Delta^+\right)\right] & \text{if } J_c^E\le j<J_c^R\\ \frac{1}{1+\tau_t^c}\frac{\xi_t}{1-\xi_t}\frac{1}{H_t}\left[w_t^n\varepsilon_{j,t}-b_t^np_{j,t}+\frac{\pi_{j,t}V^p_{j+1,t+1}}{R_{t+1}(\pi_{j,t}V^a_{j+1,t+1}+(1-\pi_{j,t})\Upsilon_{a'})}\psi\left(\frac{\varepsilon_{j,t}}{\mathcal{E}_t}+p_{j,t}\Delta^-\right)\right] & \text{if } J_c^R\le j<J^T\end{cases} \quad (49)$$

Second, the inter-temporal Euler equation describes the consumption growth rate of each household (unconditional whether $l_{j,t}=0$ is a binding constraint), given by

$$U_c=(1+\tau_t^c)\left(\pi_{j,t}V^a_{j+1,t+1}+(1-\pi_{j,t})\Upsilon_{a'}\right)\tilde{\beta}_{j,t}R_{t+1} \quad (50)$$

or (using the functional form of the utility function)

$$c_{j,t}=\left[(1+\tau_t^c)\,\tilde{\beta}_{j,t}R_{t+1}(\pi_{j,t}V^a_{j+1,t+1}+(1-\pi_{j,t})\Upsilon_{a'})\frac{1}{\xi_t}\left(\frac{c_{j,t}}{\overline{H}-l_{j,t}H_t}\right)^{(1-\xi_t)(1-\theta)}\right]^{-\frac{1}{\theta}} \quad (51)$$

for $t\in\left\{1,J^T-1\right\}$

For given factor prices (i.e. wages and capital return) and the parameters of the public pension system (i.e. contribution rate and replacement rate) the life time consumption paths of all cohorts can be computed using the Euler equations (51) and (49) as well as the budget constraints.

For the numerical solution method it is also necessary to invert the earnings point function, i.e. a function of earnings points today, $p_{j,t}$, as a function of labour today, $l_{j,t}$, and earnings points tomorrow, $p_{j,t}$.

$$p_{j,t}=\begin{cases}p_{j+1,t+1}-\psi\frac{\varepsilon_{j,t}}{\mathcal{E}_t}l_{j,t} & \text{if } j<J_c^E\\ \frac{p_{j+1,t+1}-\psi\frac{\varepsilon_{j,t}}{\mathcal{E}_t}l_{j,t}}{1-\Delta^-\psi(1-l_{j,t})} & \text{if } J_c^E\le j<J_c^R\\ \frac{p_{j+1,t+1}-\psi\frac{\varepsilon_{j,t}}{\mathcal{E}_t}l_{j,t}}{1+\Delta^+\psi\, l_{j,t}} & \text{if } j\ge J_c^R\end{cases} \quad (52)$$

In the period after its final age earnings points do not bear any value and assets only as bequests. We also assume that labour productivity is zero in the last period of life. Therefore $l_{J^T,t}=0$, $V^p_{J^T,t+1}=0$ and $V^a_{J^T,t+1}=0$. The optimal consumption $c_{J^T,t}$ and end

[38] $-\frac{U_l}{U_c}=\frac{c^{\xi}(1-\xi)(1-l)^{-\xi}(H)^{1-\xi}}{\xi c^{\xi-1}(H(1-l))^{1-\xi}}$

of life bequests are therefore given by using (42)

$$a_{J^T,t} = \left(\frac{1}{U_c} \tilde{\beta}_{J^T,t} R_{t+1} \left(1 + \tau_t^c\right) \upsilon_{J^T} \right)^{-1} \tag{53}$$

A.3 Solution Method to Household Problem

The fixed point iteration is written as an iteration searching for the utility maximizing final household consumption and earnings points of the household at the end of life that are in line with (51) and (49) and fulfil the household budget constraint (20) for given net capital return $\overrightarrow{r}^n$, net wages $\overrightarrow{w}^n$, net pension values $\overrightarrow{b^n}$ and average labour productivity $\overrightarrow{\mathcal{E}}$.

1. Start with an initial guess for the end of life consumption, $c_{J^T,0}$ and the end of life pension earnings points, $p_{J^T,0}$.
2. Assuming that earnings points have no value and assets only as bequests after the maximum age as well as assuming labour productivity is zero set $V^p_{J^T+1} = 0$, $V^a_{J^T+1} = 0$ and $l_{J^T} = 0$.
3. In each iteration k for $c_{J^T,k}, p_{J^T,k}$
   (a) Given $c_{J^T,k}$ calculate optimal bequest at the end of life $a_{J^T+1,k}$ by using (53).
   (b) Given $c_{J^T,k}$, $l_{J^T,k}$ and $a_{J^T+1,k}$ calculate assets at the end of life $a_{J^T,k}$ by using the budget constraint (20).
   (c) Compute the derivatives of the value function at the end of life $V^a_{J^T,k}$ and $V^p_{J^T,k}$ using (45) and (47) and the derivative of the bequest function with respect to asset holding $\Upsilon_{a_{J^T,k}} = \upsilon_{J^T} a^{-1}_{J^T,k}$.
   (d) For each age $j = J^T - 1, ...1$
      i. Compute initial guess for earnings points $p_{j,k}$ by using $l_{j,k} = l_{j+1,k}$ in (52).[39]
      ii. In each iteration i for $p_{j,k}$
         A. Derive the optimal consumption labour ratio $\frac{c_{j,k}}{\overline{H} - l_{j,k} H_t}$ given $V^p_{j+1,k}$, $V^a_{j+1,k}$, $\Upsilon_{a_{j+1,k}}$ and $p_{j,k}$ by using (49).
         B. Derive household consumption $c_{j,k}$ by using (51).
         C. Compute labour supply $l_{j,k} = \frac{1}{H_t} \left(\overline{H} - c_{j,k} \left[\frac{c_{j,k}}{\overline{H} - l_{j,k} H_t} \right]^{-1} \right)$
         D. Check IF $l_{j,k} < 0$ THEN $l_{j,k} = 0$ and compute $c_{j,k}$ by using (51).
         E. Update earnings points $\hat{p}_{j,k,1}$ by using $l_{j,k}$ by using (52).
         F. If $\parallel p_{j,k} - \hat{p}_{j,k} \parallel < \varepsilon$, where ε is some pre-specified tolerance level STOP, ELSE set $p_{j,k} = \tilde{p}_{j,k}$ and continue with step (a).
      iii. Calculate assets $a_{j,k}$ by using the budget constraint (20).
      iv. Update $V^a_{j,k}$ and $V^p_{j,k}$ using (45) and (47)and the derivative of the bequest function with respect to asset holding $\Upsilon_{a_{j,k}} = \upsilon_j a^{-1}_{j,k}$.

[39]Here we take $l_{j+1,k}$ as initial guess for the unknown $l_{j,k}$

(e) Compute consumption growth rate as $g^c_{j,k} = \frac{c_{j,k}}{c_{j-1,k}}$ for all $j = 1, ..., J^T$.

(f) Compute present value of consumption, C^{PV}, and present value of income, I^{PV}, given $c_{\bullet,k}$ and $l_{\bullet,k}$.

(g) Adjust consumption at age $j = 1$, $\tilde{c}_{1,k} = c_{1,k}\frac{I^{PV}}{C^{PV}+a_{J+1,k}}$.[40]

(h) For each age $j = 2, ..., J^T$

   i. Compute new consumption $\tilde{c}_{j,k} = \tilde{c}_{j-1,k} \times g^c_{j,k}$

   ii. Compute new pension earnings points $\tilde{p}_{j,k}$ with $l_{j,k}$, using (6) and starting with $\tilde{p}_{1,k} = 0$.

(i) If $\parallel x_k - \tilde{x}_k \parallel < \epsilon \; \forall \; x \in \{c_{J^T}, p_{J^T}\}$, where ϵ is some pre-specified tolerance level STOP, ELSE form an update of the capital return rate as $x_{k+1} = \omega \cdot x_k + (1 - \omega) \cdot \tilde{x}_k \; \forall \; x \in \{c_{J^T}, p_{J^T}\}$, where ω is some dampening factor independent of age. Continue with step (a).

[40] This ensures that the life time budget constraint holds.

B Parameter Values for Projection Phase

Table 1: Demographic Parameters

Parameter	Interpretation	Value
Set parameter		
f	Fertility rate per woman over life	1.55
J^F	Maximum child bearing age	20
Z_t	Net migration flow	400T $\searrow$ 206T

Table 2: Pension Parameters

Parameter	Interpretation	Value
Set parameter		
J_c^R	Statutory retirement age (current legal situation)	46 $\nearrow$ 48
J_c^E	Statutory early retirement age (current legal situation)	44 $\nearrow$ 46
D	Definition of standardized pension	45
Δ^+	Pension penalty per year for early retirement	6.0%
Δ^-	Pension premium per year for late retirement	3.6%
Parameter calibrated in equilibrium (targets in brackets)		
ψ	Share of employees in social insurance system	78.2%
	(Equivalence contributors)	(31m)
Ω_t	Mark-up pension expenditures	1.46 $\searrow$ 1.36
	(Non old-age pension expenditures of pension system)	(€98.0bn)
S^G	Artificial government contributors	10.4m
	(General federal government grant)	(€70bn)
s^a	Fraction of output for additional grant	1.0%
	(Additional federal government grant)	(€30bn)

Table 3: Government Parameters

Parameter	Interpretation	Value
Parameter calibrated in equilibrium (targets in brackets)		
ϱ	Government consumption share of output	26.7%
	(Taxes and social insurance contributions to output)	(41.3%)
τ^k	Capital income tax rate	30.7%
	(Share of capital income taxes in total taxes)	(27.4%)
τ^y	Labour income tax rate	15.9%
	(Share of labour income taxes in total taxes)	(27.3%)
φ	Contribution rate of other social insurances	21.5%
	(Social insurance contributions to output ratio)	(17.1%)

Table 4: Technology Parameters

Parameter	Interpretation	Value
Set parameter		
μ	Total factor productivity growth	0.6%
Parameter calibrated in equilibrium (targets in brackets)		
A	Total factor productivity	5.79
	(Gross value added)	(€$3,012$bn)
α	Output elasticity of capital	0.588
	(Compensation of employees)	(€$1,771$bn)
δ	Capital depreciation rate	4.1%
	(Aggregated depreciation)	(€509bn)
$\hat{\varepsilon}^1$	Individual labour productivity growth	2.8%
	(Employment rate diff. (biographical age 20 – 30))	(25.1%-point)
$\hat{\varepsilon}^2$	Individual labour productivity growth	0.0%
	(Employment rate diff. (biographical age 30 – 50))	(6.2%-point)
$\hat{\varepsilon}^3$	Individual labour productivity growth	−2.2%
	(Employment rate diff. (biographical age 50 – 63))	(−23.5%-point)
$\hat{\varepsilon}^4$	Individual labour productivity growth	−11.1%
	(Employment rate diff. (biographical age 50 – 63))	(−23.5%-point)
$\hat{\varepsilon}^5$	Individual labour productivity growth	−23.0%
	(Employment rate diff. (biographical age 65 – 66))	(−5.4%-point)
ν	Private capital cost	17.2%
	(Effective return on capital after tax)	(3.4%)

Table 5: Preference Parameters

Parameter	Interpretation	Value
Set parameter		
θ	Relative risk aversion	1
$\overline{H}$	Maximum hours of disposable time	2600
J^Q	Maximum age for receiving bequests	20
Parameter calibrated in equilibrium (targets in brackets)		
β	Time discount factor	0.9754
	(Aggregated assets)	(€12,404bn)
H	Working hours per work contract	1366
	(Aggregated hours worked)	(62bn)
ξ_{2018}	Consumption utility weight	0.4412
	(Employment rate 2018)	(80.3%)
ξ_{2030}	Consumption utility weight	0.4465
	(Employment rate 2030)	(80.9%)
ξ_{2040}	Consumption utility weight	0.4478
	(Employment rate 2040)	(82.1%)
ξ_{2050}	Consumption utility weight	0.4511
	(Employment rate 2050)	(82.3%)
ξ_{2070}	Consumption utility weight	0.4417
	(Employment rate 2070)	(82.3%)
υ_{JT}	Bequest motive utility weight	5.76
	(Aggregated bequests 2015)	(€170bn)

C Macro Variables for Reform Scenarios

Figure 7: Current Legal Situation – Macro Variables

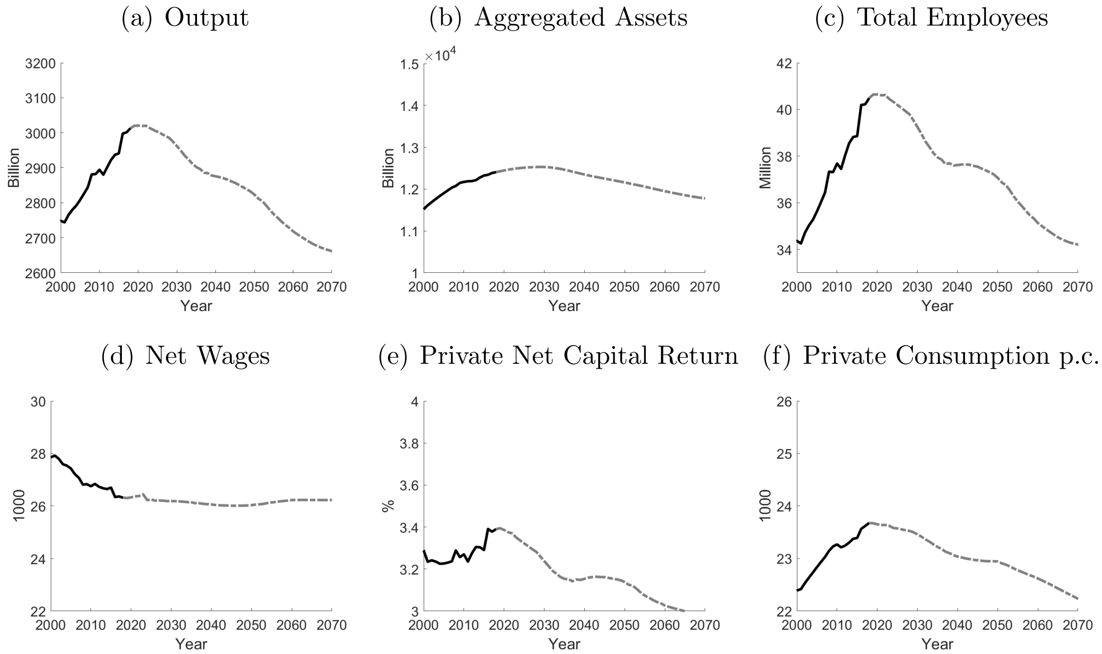
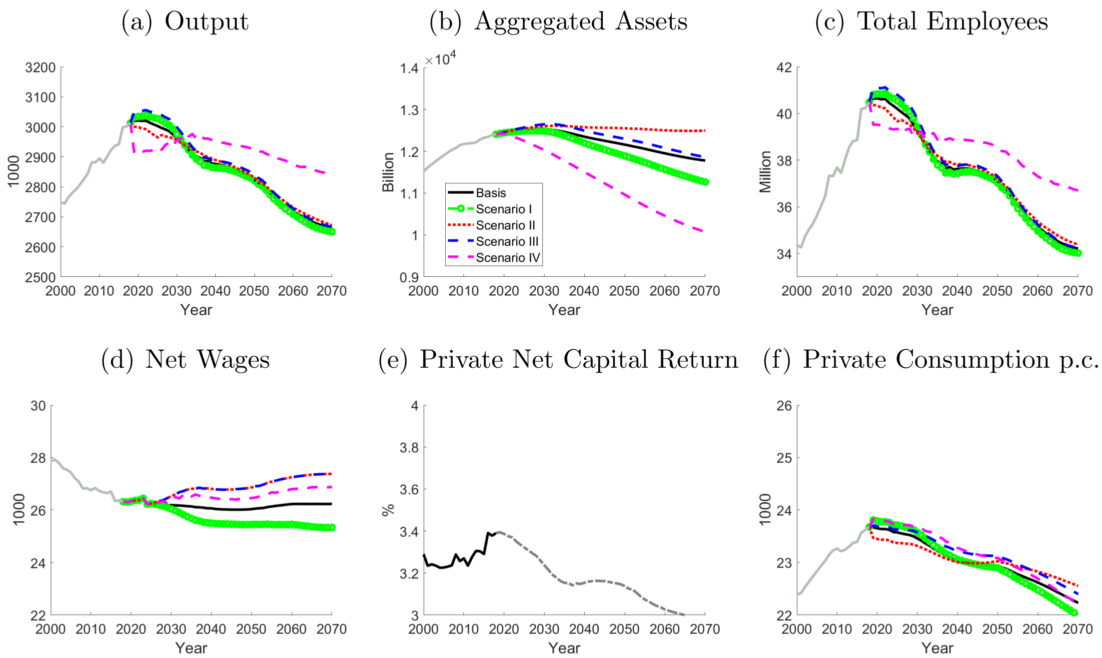
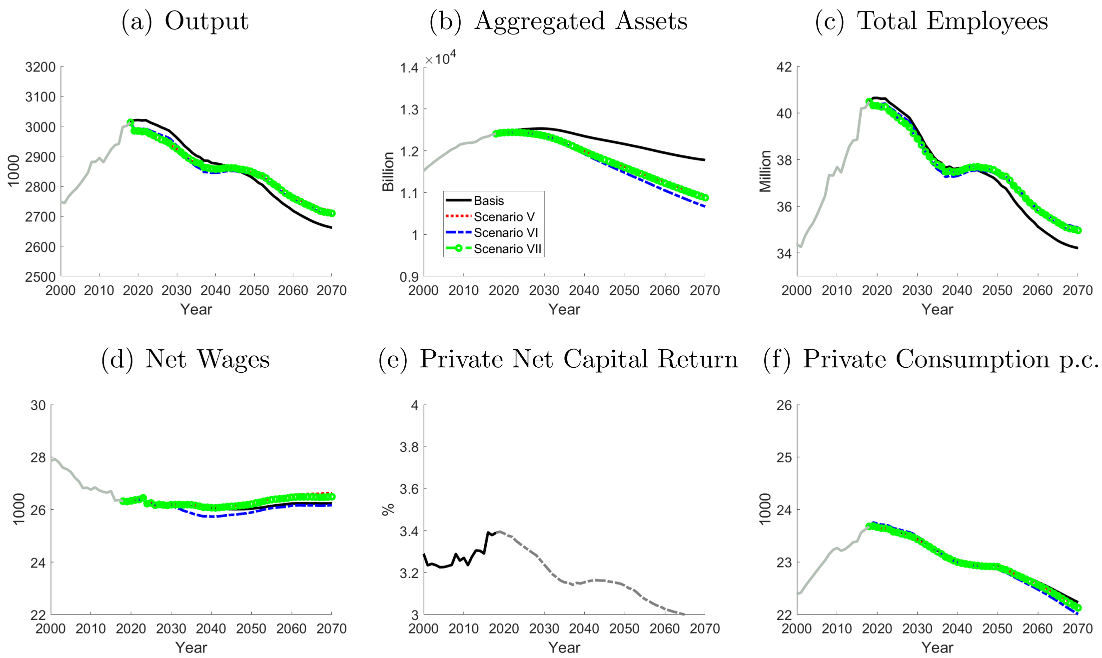


Figure 8: One-sided Burden Sharing – Macro Variables



Notes: Current legal situation (solid black line). Reform scenario I (circle green line). Reform scenario II (dotted red line). Reform scenario III (dotted dashed blue line). Reform scenario IV (dashed magenta line).

Figure 9: Broader-based Burden Sharing – Macro Variables

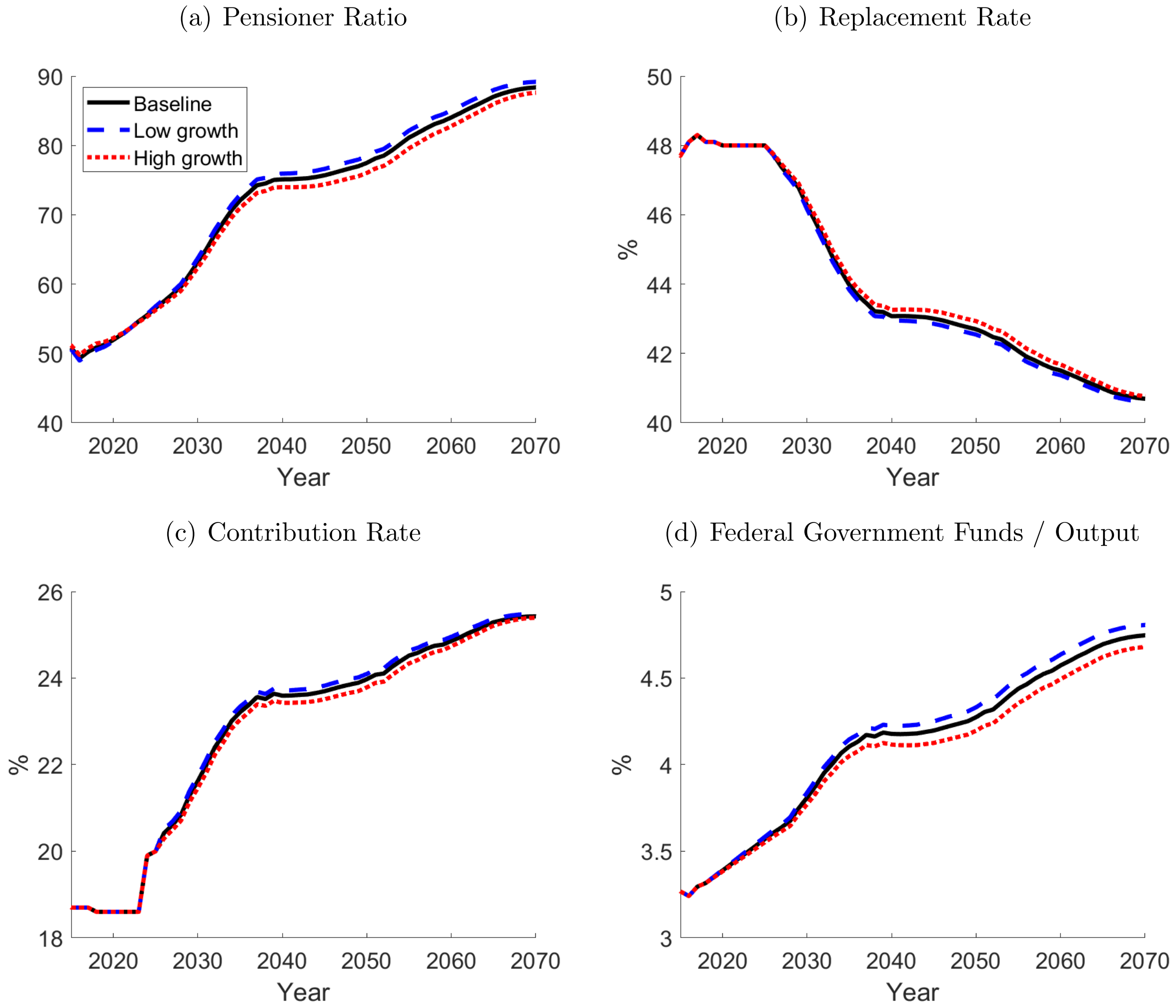


Notes: Current legal situation (solid black line). Reform scenario V (dashed green line). Reform scenario VI (dotted red line). Reform scenario VII (dotted dashed blue line).

D Sensitivity Analysis

In Figure 10 we show pension variables for the current legal status where we alter the TFP growth rate in the model. (Baseline: $\mu = 0.6\%$; sensitivity scenario low growth: $\mu = 0.0\%$; sensitivity scenario high growth: $\mu = 1.5\%$.

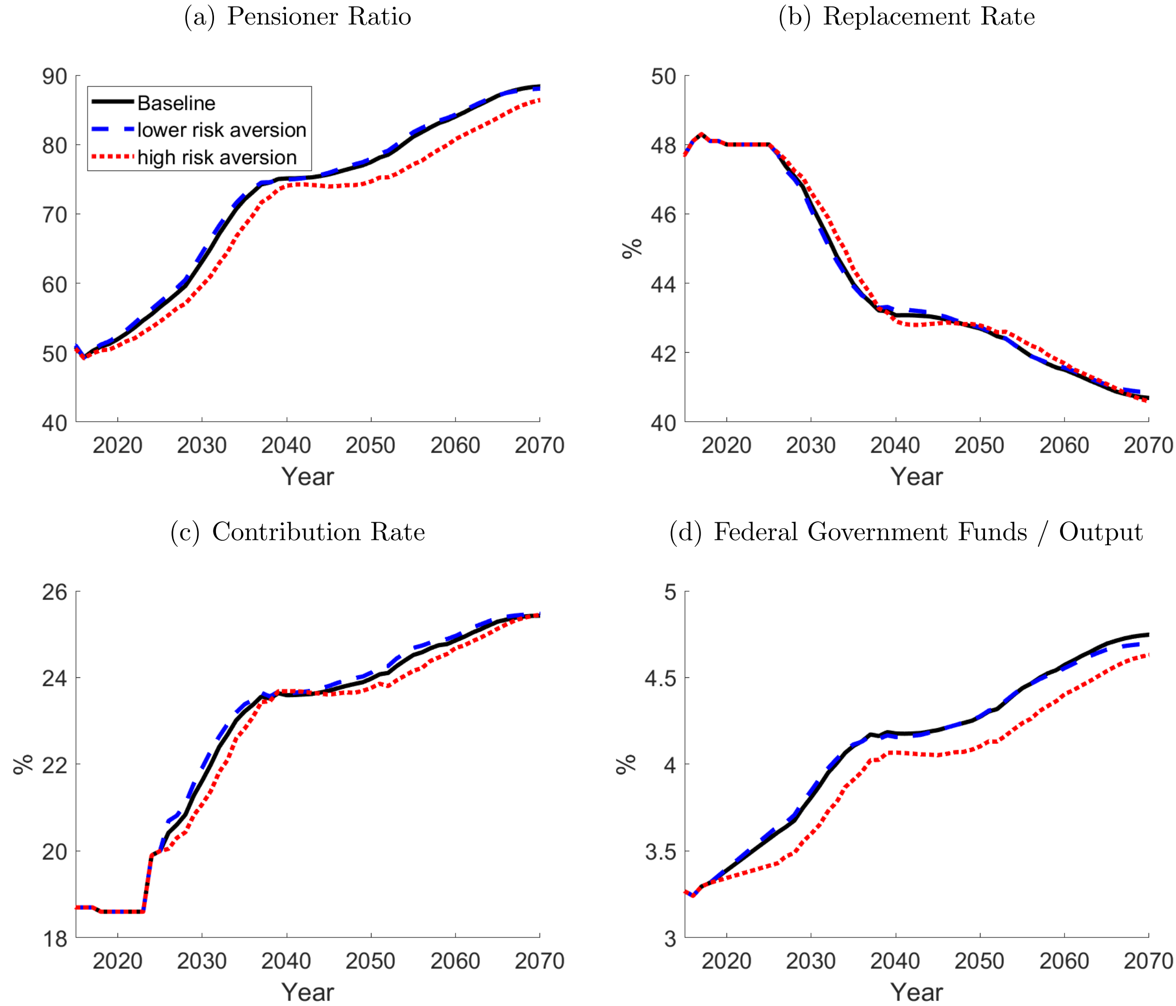
Figure 10: Sensitivity Total Factor Productivity Growth



Notes: Panel (a) shows the pensioner ratio defined as *Equivalence pensioners* to *Equivalence contributors*. Panel (b) shows the wage replacement rates for 45 earnings points. Panel (d) shows the ratio of total federal government funds (*General Federal Government Grant* +*Additional Federal Government Grant*) to output.

In Figure 11 we show pension variables for the current legal status where we alter the risk aversion parameter θ in the model. (Baseline: $\theta = 1.0$; sensitivity scenario lower risk aversion: $\theta = 0.8$; sensitivity scenario higher risk aversion: $\theta = 1.2$.

Figure 11: Sensitivity for Risk Aversion



Notes: Panel (a) shows the pensioner ratio defined as *Equivalence pensioners* to *Equivalence contributors*. Panel (b) shows the wage replacement rates for 45 earnings points. Panel (d) shows the ratio of total federal government funds (*General Federal Government Grant* +*Additional Federal Government Grant*) to output.

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