DEUTSCHE
BUNDESBANK
EUROSYSTEM

Discussion Paper

Deutsche Bundesbank
No 15/2020

Demographics and the decline in firm entry: Lessons from a life-cycle model

Oke Röhe
Nikolai Stähler

Deutsche Bundesbank, Wilhelm-Epstein-Straße 14, 60431 Frankfurt am Main,
Postfach 10 06 02, 60006 Frankfurt am Main

Tel +49 69 9566-0

Please address all orders in writing to: Deutsche Bundesbank,
Press and Public Relations Division, at the above address or via fax +49 69 9566-3077

Internet http://www.bundesbank.de

ISBN 978–3–95729–688–7 (Printversion)
ISBN 978–3–95729–689–4 (Internetversion)

Non-technical summary

Research Question

Since the mid-1970s, firm entry rates in the United States have declined significantly. This also holds for other OECD countries over the past years. At the same time, these economies experienced a gradual process of population aging. Against this background, we address the following questions: Can lower firm entry rates (at least partly) be explained by the demographic transition? And if so, what is the underlying mechanism?

Contribution

The recent literature suggests a relation between population aging and business dynamism. For instance, it has been argued that entrepreneurship requires energy and creativity as well as business acumen, and that at least some (of these) factors decrease with age. In this paper, we use a general equilibrium life-cycle model with endogenous firm dynamics to analyze the effects of aging on US business dynamism. Thereby, our approach does not rely on the assumption that entrepreneur skills are age-dependent in order to link a decrease in the firm entry rate to population aging. Rather, our model explains the decline in business formation purely through aging-related general equilibrium effects. In this respect, our analysis should be seen as a supplement to the recent literature emphasizing the role of population demographics for business dynamism, which – by pointing out potential implications of increasing longevity – adds another layer to the understanding of the effects of demographic change on business churn.

Results

We derive three main results: First, the entry rate of firms falls because of the demographic transition. Specifically, in our simulations population aging explains up to 30% of the decline that we observe in the data. Second, the decline in the working-age population turns out to be an important factor explaining these developments, which confirms recent findings from dynamic firm models. Finally, by additionally allowing for endogenous firm exit, we find that an increase in longevity may also play a prominent role in explaining the observed decline in firm entry and exit rates.

Nichttechnische Zusammenfassung

Fragestellung

Seit Mitte der 1970er Jahre hat die Markteintrittsrate von Unternehmen in den USA deutlich abgenommen. Eine vergleichbare Entwicklung ist dabei auch in anderen OECD-Staaten zu beobachten. Gleichzeitig ist in diesen Volkswirtschaften eine zunehmende Alterung der Gesellschaft zu verzeichnen. Vor diesem Hintergrund stellen sich folgende Fragen: Lassen sich rückläufige Markteintrittsraten von Unternehmen (zumindest teilweise) durch eine voranschreitende Bevölkerungsalterung erklären? Und falls dies so ist: Wie sieht die Wechselbeziehung im Detail aus?

Beitrag

Jüngere Untersuchungen legen einen Zusammenhang zwischen Alterung und Unternehmensdynamik nahe. Unter anderem wird argumentiert, dass Unternehmensgründungen Fähigkeiten wie Innovationskraft, Kreativität und unternehmerisches Gespür benötigen, die (zumindest zum Teil) mit dem Alter abnehmen. Wir nutzen ein makroökonomisches „Lebenszyklusmodell“mit endogenem Marktein- und -austritt von Unternehmen, um den Einfluss von Bevölkerungsalterung auf die Unternehmensdynamik in den USA zu untersuchen. Unser Ansatz unterstellt dabei nicht, dass sich unternehmerische Fähigkeiten mit dem Alter verändern. Stattdessen wird der Zusammenhang zwischen alternder Bevölkerung und abnehmenden Markteintritten ausschließlich durch allgemeine Gleichgewichtseffekte erklärt, die sich im verwendeten Modell bei zunehmender Alterung ergeben. Unsere Analyse stellt damit eine Ergänzung zu bisherigen Untersuchungen zur Wechselwirkung von Bevölkerungsalterung und Unternehmesndynamik dar, indem sie der Debatte um die Rolle demographischer Veränderungen für den Marktein- und -austritt von Unternehmen durch die spezifische Betrachtung des Faktors „Langlebigkeit“eine neue Facette hinzufügt.

Ergebnisse

Unsere Analyse liefert drei zentrale Ergebnisse: Zum einen führt Bevölkerungsalterung zu einer Abnahme der Markteintrittsrate. Unsere Modellsimulationen können dabei bis zu 30% des in den Daten zu beobachtenden Rückgangs erklären. Die Analyse zeigt darüber hinaus, dass die rückläufige Erwerbsbevölkerung ein entscheidender Faktor für diese Entwicklung darstellt. Zudem deuten Simulationen unter Berücksichtigung eines endogenen Marktaustritts von Unternehmen an, dass eine höhere Lebensdauer ebenfalls ein wichtiger Faktor für den Rückgang von Marktein- und -austrittsraten sein kann.

Deutsche Bundesbank Discussion Paper No 15/2020

Demographics and the Decline in Firm Entry: Lessons from a Life-Cycle Model*

Oke Röhe
Deutsche Bundesbank

Nikolai Stähler
Deutsche Bundesbank

Abstract

Since the mid-1970s, firm entry rates in the United States have declined significantly. This also holds for other OECD countries over the past years. At the same time, these economies experienced a gradual process of population aging. Applying a tractable life-cycle model with endogenous firm dynamics, we show that falling US firm entry rates can be explained by demographic transition. Specifically, our model simulations suggest that aging can account for up to one third of the observed decrease in US firm entry rates. In addition to the negative effects of a slowdown in working-age population growth on firm entry, our analysis points out that an increase in longevity may also be an important factor contributing to the decline in business dynamism, weighing on both firm entry and exit rates.

Keywords: Life-Cycle model, Population aging, Business dynamism, Firm entry

JEL classification: H25, L52, E20, E62, L10, O30.

---

[*] Contact address: Deutsche Bundesbank, Wilhelm-Epstein-Strasse 14, 60431 Frankfurt, Germany. E-mail: oke.roehe@bundesbank.de, nikolai.staehler@bundesbank.de. The paper represents the authors' personal opinions and does not necessarily reflect the views of the Deutsche Bundesbank, of the Eurosystem or of its staff. We are grateful for comments from Johannes Hoffmann, Vivien Lewis, Julian Neira, Karsten Wendorff, and participants of the 2020 Public Economics Meeting in Barcelona.

1 Introduction

Since the mid-1970s, firm entry rates in the United States have declined significantly, which has been pointed out, amongst others, by Hathaway and Litan (2014), Decker, Haltiwanger, Jarmin, and Miranda (2014), and Alon, Berger, Dent, and Pugsley (2018). Similar patterns can be observed in Canada (Cao, Salameh, Seki, and St-Amant, 2017) and, at least since the beginning of this millennium, in other OECD economies such as Australia, Germany, or the United Kingdom (Emes, Jackson, and Globerman, 2018). In this respect several studies have raised concerns about the macroeconomic implications of a decline in business formation. Decker et al. (2014), for example, point out the adverse effects for job creation, while Clementi and Palazzo (2016), Gourio, Messer, and Siemer (2016) and Decker, Haltiwanger, Jarmin, and Miranda (2017, 2018) associate a decreasing startup rate with a reduction in productivity growth.

So far, several potential explanations for the decline in firm entry have been put forward, including a slowdown in knowledge diffusion from the frontier firms to the follower ones discouraging business formation (Akcigit and Ates, 2019a,b), entry barriers stemming from lobbying and regulations (Kozeniauskas, 2019; Gutiérrez, Jones, and Philippon, 2019), as well as changes in the demographic structure of the population (see, e.g., Hopenhayn, Neira, and Singhania, 2018; Pugsley and Sahin, 2018; Karahan, Pugsley, and Sahin, 2019).

This last explanation is remarkable insofar as all of the aforementioned economies did indeed experience a fall in population growth as well as longer life expectancy and thus an increase in the old age dependency ratio (OADR, henceforth), as documented by OECD (2017). Moreover, the increase in the OADR is projected to remain on an upward trend until at least 2080. In view of this, we address the following questions:

i) Can lower firm entry rates be explained by the demographic transition?

ii) And if so, what is the underlying mechanism?

Studies investigating the relationship between demographic developments and firm entry are scarce. In a recent contribution, Hopenhayn et al. (2018) propose a declining labor force growth as one explanation for the decrease in firm entry rates, emphasizing the interplay of population and firm demographics. Liang, von Hui, and Lazear (2018) show that countries with an older workforce have lower rates of entrepreneurship. They stress that entrepreneurship requires energy, creativity, as well as business acumen and that at least some of these factors decrease with age. A similar argumentation can be deduced from Ouimet and Zarutskie (2014), who find that young employees disproportionately join young firms with higher innovation potential. They point out that unique skills and greater risk tolerance of young workers contribute to their disproportionate share of employment in young firms and that there exists a causal relationship between the supply of young workers and the rate of new firm creation.

In this paper, we add to this literature by providing a tractable general equilibrium life-cycle model of the economy to investigate the relationship between population demographics and business dynamism. We find that both a reduction of the working-age population and an increase in longevity can have noticeable effects on firm entry. Thereby, our approach abstracts from age-dependent entrepreneur skills to link a decrease in firm entry rates and population aging. Rather, we show a mechanism that explains declining startup rates through aging-related general equilibrium effects on firm dynamics.

We start our analysis with a basic life-cycle model, extended to allow for endogenous firm entry. In this model, individuals are born as workers and supply one unit of work inelastically during their working life. After retirement, households consume out of their assets, which include physical capital, government bonds, and firms. Even though the model is highly stylized, we can derive two important insights: First, the entry rate of firms falls as a result of the demographic transition. Second, in this core model, where firm exit rates are assumed to be constant, the decreasing firm entry rate is completely attributed to the decline in population growth. Specifically, the rise in longevity (i.e. higher survival rates) alone would actually increase firm entry rates, at least along the transition.

What is the intuition behind these initial results? One the one hand, the economy ultimately grows with labor supply which, given the inelastic individual supply of labor, is exogenous. A slowdown in labor supply growth implies a decrease in the growth rates of both output and profits per firm. This implies that opportunities for potential entrants decline and the firm entry rate falls over time. An increase in longevity, on the other hand, induces households to save more, implying a rise in firm investment and therefore a higher firm entry rate, at least along the transition. All in all, however, the former effect dominates, implying a declining firm entry rate.

The framework sketched above, however, includes several simplifications. As a robustness exercise, we increase the richness of the model by adding a range of features neglected so far: Household labor supply is endogenous and may rise given that aging puts upward pressure on wages. On the production side, the assumption of a constant firm exit rate is not in line with recent empirical evidence, and is therefore replaced by endogenous producer exit. A similar argument applies to the assumption of constant price markups, which are therefore also endogenized in the extended setup. Although adding these extensions to the core model does not challenge the finding that aging can explain the decline in firm entry rates, it has some noticeable quantitative implications. In particular, in the extended model version, the aging-induced decline in the firm entry rate becomes more pronounced. While in the core model the entry rate falls by about 1.1 percentage points between 1970 and 2060, it declines by more than 1.4 percentage points when taking into account the model extensions.

In this respect, our results point to the critical role of endogenous market exit as an amplifying factor. Specifically, when the firm exit rate is endogenously determined by firm profitability, an increase in longevity does not only contribute negatively to the entry rate, it also becomes a substantial driver of the decline in the firm entry rate. The reason why longevity plays such a remarkable role is related to its strong impact on savings, leading to lower capital costs, an increase in output, and thus higher profits per firm. With endogenous market exit, higher firm profitability implies that incumbent firms are less likely to exit the market, thereby lowering the opportunities for potential entrants. As a consequence, entry and exit rate decline, both of which can be observed in the data.

The rest of the paper is organized as follows. In Section 2, we discuss the related literature. Section 3 presents the setup of the core model and describes its calibration, while the simulation design is laid out in Section 4. Section 5 discusses the benchmark results obtained from the core model. In Section 6, we introduce several model extensions and discuss the impact of aging on entry in this extended framework. Section 7 concludes. In an appendix, we show how adding each model extension separately affects the results in more detail.

2 Related literature

Our paper is linked to a number of different literatures. First, our analysis is related to a range of studies on US business dynamism pointing out a significant decline in firm entry rates over the past three decades (see, e.g., Decker et al., 2014; Decker, Haltiwanger, Jarmin, and Miranda, 2016; Decker et al., 2017; Pugsley and Sahin, 2018). Most of that literature is empirical, however, and refrains from offering an in-depth theoretical evaluation.

Second, we contribute to a recent strand of literature investigating the relationship between demographic developments and business dynamism. Using a (standard) general equilibrium firm dynamics model, Hopenhayn et al. (2018) show that the interplay of population and firm demographics can explain a substantial part of the observed downward trend in firm entry. Specifically, a decline in labor force growth is assumed to lead to changes in firm demographics, and therefore to changes in average firm size and aggregate exit rates. This allows labor force growth to account for a sizable fraction of the observed decline in US firm entry rates. Choosing a similar theoretical approach, Karahan et al. (2019) also point to the decline in the growth rate of the working-age population as an important driver of the decline in US firm entry. Using a fully-fledged life-cycle model, we confirm population growth to be a decisive factor driving the decline in firm entry. Our framework, however, enables us also to take a closer look at the implications of longevity for firm dynamism.

Rather than focussing on the supply of the workforce, Liang et al. (2018) employ a human capital framework coupled with an economic focus on fertility patterns to highlight that the age structure of the workforce can have a significant impact on economic performance through entrepreneurship. Testing the model using a detailed data set on cross-country entrepreneurship, the theoretical framework successfully predicts that countries with an older workforce have lower rates of entrepreneurship, while younger societies, which provide more opportunities for workers to acquire business skills early in their careers, create a more viable environment for entrepreneurship.[1] In a similar vein, Aksoy, Basso, Smith, and Grasl (2019) – although not considering the effects on firm entry – provide evidence on the link between the demographic structure and innovation. Engbom (2019), embedding endogenous growth through creative destruction in an equilibrium job ladder model, stresses the relation between the extent of mismatch in the labor market and incentives to innovate. In our analysis, we abstract from age-specific entrepreneur skills and instead focus on the general equilibrium effects stemming from an increasing OADR.

Finally, our analysis is related to a vast literature investigating the macroeconomic impact of demographic developments through the lens of life-cycle dynamic general equilibrium models, thereby allowing for a comprehensive view on the effects of demographic

---

[1] Also Kopecky (2017) refers to an age-dependent willingness to undertake a risky entrepreneurial investment, implying a hump-shaped age-entrepreneur relationship.

change on the various agents' savings behavior and interest rate effects (see, e.g., Gertler, 1999; Ferrero, 2010; Carvalho, Ferrero, and Nechio, 2016; Kara and von Thadden, 2016).

3 Core model

Our theoretical framework consists of a flexible-price model of the business cycle featuring a life-cycle structure, as well as endogenous entry of firms and time-varying markups. In line with Gertler (1999), Ferrero (2010), and Carvalho et al. (2016), the economy features three types of agents: households, firms, and the government. Workers consume final goods, work and save, while retirees exclusively consume out of their asset wealth. Agents save via physical capital, government bonds, and investments in new (intermediate goods producing) firms. Each of these firms produces a unique intermediate good using aggregate technology, capital, and labor services. The government sets its spending exogenously and finances its expenditures through a mix of lump-sum taxes and one-period debt.

As is standard in the life-cycle literature using perpetual youth models, we consider the effects of unexpected one-time changes in the demographic structure and abstract from aggregate uncertainty in an otherwise perfect-foresight environment. Retirement and death risks, however, affect agents' behavior, as these probabilities are a source of idiosyncratic uncertainty. To keep the model tractable, we assume that the probabilities of retirement and death are independent of age (see Blanchard, 1985; Weil, 1989). This simplifies aggregation without sacrificing the life-cycle dimension. In what follows, we will describe the model in more formal detail.

3.1 Life-cycle structure

At any point in time, individuals belong to one of two groups: workers (w) or retirees (r). New workers are born at rate $(1-\omega_t+n_t^w)$. Conditional on being a worker in the current period, an individual faces a probability ω_t of remaining a worker in the next period. Hence, the working-age population grows at rate n_t^w, and $(1-\omega_t+n_t^w)$ can be interpreted as the "fertility rate". Retirees face a survival probability γ_t and die with probability $(1-\gamma_t)$. Hence, the laws of motion for workers and retirees are

$$N_t^w = (1-\omega_t+n_t^w)\,N_{t-1}^w + \omega_t N_{t-1}^w = (1+n_t^w)\,N_{t-1}^w, \tag{1}$$

$$N_t^r = (1-\omega_t)\,N_{t-1}^w + \gamma_t N_{t-1}^r. \tag{2}$$

Defining the old age dependency ratio as $\Psi_t = N_t^r/N_t^w$, its law of motion can be written as

$$\Psi_t = \frac{1-\omega_t}{1+n_t^w} + \frac{\gamma_t}{1+n_t^w}\Psi_{t-1}. \tag{3}$$

3.2 Decision problem of retirees and workers

Workers inelastically supply one unit of labor each period, while retirees do not work. Preferences for an individual of group $z=\{w,r\}$ are a restricted version of the recursive non-expected utility family that assumes risk neutrality (see Epstein and Zin, 1989):

$$V_t^z = \left\{ (C_t^z)^\rho + \beta_{t+1}^z \left[E_t \left(V_{t+1} | z \right) \right]^\rho \right\}^{\frac{1}{\rho}}, \tag{4}$$

where C_t^z denotes consumption and V_t^z the value of utility in period t. To account for the probability of death, workers and retirees have different discount factors. Specifically, it holds that $\beta_{t+1}^r = \beta\gamma_{t+1}$ and $\beta^w = \beta$. Moreover, the expected continuation value, $E_t \left(V_{t+1} | z \right)$, differs between workers and retirees due to the transition probabilities between groups. In particular, $E_t \left(V_{t+1} | r \right) = V_{t+1}^r$ while $E_t \left(V_{t+1} | w \right) = \omega_{t+1} V_{t+1}^w + (1 - \omega_{t+1}) V_{t+1}^r$. As extensively discussed in the literature (see, e.g., Gertler, 1999; Ferrero, 2010; Carvalho et al., 2016), this life-cycle model is analytically tractable because the transition probabilities to retirement and death are independent of age. However, standard risk-averse preferences would imply disproportionately strong precautionary savings motives (see, e.g., Farmer, 1990). By separating the elasticity of intertemporal substitution, $\sigma \equiv (1-\rho)^{-1}$, from risk aversion, this preference specification allows for a reasonable response of consumption and savings to changes in interest rates.

Households decide on their purchases of the final good for consumption, C_t, and their savings, A_t, which they deposit with a financial intermediary (investment fund) that pays a pre-determined real gross return on the deposits.

Retirees: An individual born in period j and retired in period τ chooses consumption $C_t^r(j,\tau)$ and assets $A_t^r(j,\tau)$ for $t \geq \tau$ to solve equation (4) for $z = r$ subject to

$$C_t^r(j,\tau) + A_t^r(j,\tau) = \frac{1}{\gamma_t} R_{t-1} A_{t-1}^r(j,\tau).$$

A financial intermediary undertakes the final investment decision (which we will describe in more detail below) and pays a pre-determined real gross interest R_t to the household in the next period. In the case of retirees, we assume that a perfectly competitive mutual fund industry invests the proceeds and pays a premium over the market return to compensate for the probability of death (see Yaari, 1965; Blanchard, 1985). Therefore, the real return on asset investments for a retiree who has survived from period $t-1$ to t is R_t/γ_t.

Additionally, the optimization problem is subject to the consistency requirement that the retiree's initial asset holdings upon retirement correspond to the assets held in the last period as a worker, i.e. $A_{\tau-1}^r(j,\tau) = A_{\tau-1}^w(j)$. In the absence of aggregate uncertainty, the Euler equations of the maximization problem imply

$$C_{t+1}^r(j,\tau) = \left(\beta R_t \right)^\sigma \, C_t^r(j,\tau). \tag{5}$$

It can be shown that consumption of each retiree is a fraction of total wealth:[2]

$$C_t^r(j,\tau) = \xi_t^r \left(\frac{R_{t-1} A_{t-1}^r(j,\tau)}{\gamma_t} \right), \tag{6}$$

where the marginal propensity to consume out of wealth satisfies the following first-order

---

[2] For a detailed formal derivation see, among others, Carvalho et al. (2016).

non-linear difference equation

$$\xi_t^r = 1 - \gamma_{t+1}\beta^\sigma R_t^{\sigma-1}\frac{\xi_t^r}{\xi_{t+1}^r}. \tag{7}$$

From equations (5) and (6), it follows that asset holdings of retiree j evolve according to $A_t^r(j,\tau) = (1-\xi_t^r)R_{t-1}A_{t-1}^r(j,\tau)\gamma_t$. The value function for a retiree is linear in consumption: $V_t^r(j,\tau) = (\xi_t^r)^{\sigma/(1-\sigma)}C_t^r(j,\tau)$.

Workers: Workers start their lives with zero assets. They also invest their assets $A_t^w(j)$ via a financial intermediary and receive a real gross return of R_t. A worker born in j chooses consumption $C_t^w(j)$ and assets $A_t^w(j)$ for $t \geq j$ to maximize equation (4) for $z = w$ subject to

$$C_t^w(j) + A_t^w(j) = R_{t-1}A_{t-1}^w(j) + w_t - T_t^w \tag{8}$$

and $A_j^w(j) = 0$. The worker's budget constraint differs from that of a retiree in two aspects. First, in addition to the interest received from asset accumulation, the worker earns real wages, w_t, and has to pay lump-sum taxes T_t^w. Second, workers do not turn to the mutual fund industry and, hence, do not receive the additional return that compensates for the probability of death.[3] Furthermore, as discussed above, the expected continuation value of workers differs from that of retirees. Solving the worker's optimization problem shows that workers' consumption is a fraction of total wealth, defined as the sum of financial and non-financial (human) wealth

$$C_t^w(j) = \xi_t^w\left[R_{t-1}A_{t-1}^w(j) + H_t^w\right], \tag{9}$$

where the latter is independent of individual characteristics and captures the discounted value of current and future wage income net of taxation:

$$H_t^w = w_t - T_t^w + \frac{\omega_{t+1}H_{t+1}^w}{\Omega_{t+1}R_t}. \tag{10}$$

As for retirees, workers' marginal propensity to consume out of wealth evolves according to

$$\xi_t^w = 1 - \beta^\sigma\left(\Omega_{t+1}R_t\right)^{\sigma-1}\frac{\xi_t^w}{\xi_{t+1}^w}. \tag{11}$$

The adjustment term $\Omega_t \equiv \omega_t + (1-\omega_t)\left(\xi_t^r/\xi_t^w\right)^{1/(1-\sigma)}$ depends on the ratio of the marginal propensities to consume between retirees and workers and it can be shown that $\xi_t^r/\xi_t^w > 1$, $\forall t$. This indicates that retirees discount future income streams at an effectively higher rate than workers, reflecting the expected finiteness of their life. It makes the future less valuable than it is in conventional real business cycle models.

The dynamics of workers' asset holdings can be obtained from their budget constraints and the consumption function (9): $A_t^w(j) + \omega_{t+1}H_{t+1}^w/\left(\Omega_{t+1}R_t\right) = (1-\xi_t^w)\left[R_{t-1}A_{t-1}^w(j) + H_t^w\right]$. The workers' value function is also linear in their consumption: $V_t^w(j) = (\xi_t^w)^{\sigma/(1-\sigma)}c_t^w(j)$.

---

[3] Allowing them to do so would provide complete insurance against the probability of retirement and thus shut down most of the life-cycle dimension of the model.

3.3 Aggregation of households' decisions

Any aggregate variable S^z_t for group $z = \{w, r\}$ takes the form $S^z_t \equiv \int_0^{N^z_t} S^z_z(i)di$. Given the linearity of the consumption functions discussed above, consumption of workers and retirees is given by

$$C^w_t = \xi^w_t \left(R_{t-1}A^w_{t-1} + H_t\right), \tag{12}$$

$$C^r_t = \xi^r_t R_{t-1}A^r_{t-1}, \tag{13}$$

while aggregate economy-wide consumption is defined as

$$C_t = C^w_t + C^r_t. \tag{14}$$

Let A^z_{t-1} denote the total financial wealth that members of group $z = \{w, r\}$ carry from period $t-1$ to t. It must hold that $A_t = A^w_t + A^r_t$. The aggregate value for human wealth H_t evolves according to

$$H_t = w_t N^w_t - T_t + \frac{\omega_{t+1}H_{t+1}}{(1 + n^w_{t+1})\Omega_{t+1}R_t}, \tag{15}$$

where $T_t = N^w_t T^w_t$.

In contrast to the individual consumption decisions, the mutual fund no longer plays a role for the consumption of retirees as a group. This is because the assets left by those who pass away are transferred to the other retirees and remain in the same group. Analogously, we have to take into account working-age population growth for the aggregate value of human wealth.

If we let $\lambda_t \equiv A^r_t/A_t$ denote the share of total financial assets held by retirees and take into account that the aggregate consumption function (14) can be expressed as $C_t = \xi^w_t \left(R_{t-1}A^w_{t-1} + H_t\right) + \xi^r_t R_{t-1}A^r_{t-1}$, we can use equations (12) and (13) to derive the law of motion for the distribution of financial wealth across groups:

$$\lambda_t A_t = (1 - \omega_{t+1})A_t + \omega_{t+1}(1 - \xi^r_t)\lambda_{t-1}R_{t-1}A_{t-1}. \tag{16}$$

Relative to a standard neoclassical growth model, the distribution of assets across cohorts is an additional state variable.[4] It keeps track of the heterogeneity in wealth accumulation due to the life-cycle structure.

3.4 Investment fund and financial market clearing

Following Fujiwara and Teranishi (2008) and Schön and Stähler (2019), a financial intermediary collects deposits from households, A_t, and allocates household financial wealth among investments in physical capital, K_t, government bonds, B_t, and firms, N^f_t. Hence, non-labor income of households encompasses three different sources. First, the intermediary rents the capital stock to intermediate goods producing firms at the real rate r^k_t

---

[4] Aggregate assets for retirees depend on the total savings of those who are already retired plus the savings of those who retire now. Aggregate savings of workers depend only on the savings of those who remain in the labor force.

while bearing the cost of depreciation $\delta^k \in (0,1)$. In addition, government bonds pay a gross return R_t^G. Finally, investing in new intermediate goods producing firms guarantees a share of economy-wide firm profits, $N_t^f \Pi_t$, where Π_t denotes (average) profits per firm. Intermediate goods producer are initially assumed to exit the market at an exogenous rate $\delta^f \in (0,1)$.

The law of motion for the capital stock is given by $K_{t+1} = (1-\delta^k)K_t + I_t$, where I_t denotes physical capital investment. Following Lewis and Poilly (2012), we assume that firm creation is associated with an inefficient scramble of startups. Hence, the law of motion of the number of firms held by the financial intermediary is $N_{t+1}^f = (1-\delta^f)N_t^f + [1 - F_{N,t}(\cdot)]\, N_t^{f,e}$, where $N_t^{f,e}$ denotes the newly created firms. We assume that $F_{N,t}(\cdot)$ is an increasing function of the change in entry.[5] This implies that only a fraction of newly created firms, namely $1 - F_{N,t}(\cdot)$, becomes operational in the next period. It captures the idea that some newly created firms fail, and that this failure rate is related to the relative changes in firm creation from one period to the next (see Beaudry, Collard, and Portier, 2011; Mata and Portugal, 1994). Letting υ_t denote firm creation costs, the investment fund thus aims to maximize

$$F_t^{fund} = r_{t+1}^k K_t + R_t^G B_t + N_t^f \Pi_t - I_t - B_{t+1} - \upsilon_t N_t^{f,e} + A_{t+1} - R_t A_t,$$

subject to the laws of motion for capital and firms. Assuming that the fund operates under a zero-profit condition (in a perfectly competitive market), the Euler equations of the maximization problem imply

$$R_t = R_t^G = r_{t+1}^k + (1-\delta^k) = \frac{(1-\delta^f)p_{t+1}^f}{p_t^f - \Pi_t}, \tag{17}$$

where the (expected) value of setting up a new firm is equal to firm entry costs:

$$p_t^f \left[1 - F_{N,t}(\cdot) - F'_{N,t}(\cdot)N_t^{f,e}\right] + \left(p_{t+1}^f / R_{t+1}\right) F'_{N,t+1}(\cdot) N_{t+1}^{f,e} = \upsilon_t, \tag{18}$$

letting p_t^f denote the value of an incumbent firm. Hence, the financial market clearing condition is given by:

$$A_t = K_t + B_t + p_t^f \cdot N_t^f. \tag{19}$$

3.5 Firms, production and market entry/exit

Following Jaimovich (2007) and Jaimovich and Floetotto (2008), we assume a two-layer production structure. The final good Y_t is produced by a perfectly competitive representative firm. It aggregates a measure one continuum of industry goods $Q_t(j)$ according to the constant elasticity of substitution (CES) technology $Y_t = \left[\int_0^1 Q_t(j)^\mu dj\right]^{\frac{1}{\mu}}$, where $0 < \mu < 1$ determines the elasticity of substitution between industry goods $Q_t(j)$. Letting

---

[5] Specifically, we assume convex costs of the form $F_{N,t}(\cdot) = \frac{\kappa^f}{2}\left(\frac{N_t^{f,e}}{N_{t-1}^{f,e}} - 1\right)^2$. The assumption of inefficient firm creation is not critical for our results, but allows for a smoother transition when the economic agents learn about the changing parameters of the demographic developments.

$P_t(j)$ denote the price index of industry j in period t, the following demand function for industry goods is obtained from the profit maximization problem of a representative final goods producing firm:

$$Q_t(j) = \left[\frac{P_t(j)}{P_t}\right]^{\frac{1}{\mu-1}} Y_t, \tag{20}$$

where the price of the final output is given by $P_t = \left[\int_0^1 P_t(j)^{\frac{\mu}{\mu-1}} dj\right]^{\frac{\mu-1}{\mu}}$.

Within each industry j, there is a continuum $N_t^f(j)$ of firms, each producing one differentiated intermediate good. The intermediate goods are bundled into an industry good $Q_t(j)$ according to the CES aggregating function:

$$Q_t(j) = N_t^f(j)^{1-\frac{1}{\kappa}} \left[\int_{i=0}^{N_t^f(j)} x_t(j,i)^\kappa di\right]^{\frac{1}{\kappa}}, \tag{21}$$

where $x_t(j,i)$ denotes the output of intermediate goods producing firm i in industry j in period t, and $0 < \kappa < 1$ determines the elasticity of substitution between the intermediate goods. Given the demand function for industry goods (20), static profit maximization yields the following demand for intermediate good $x_t(j,i)$:

$$x_t(j,i) = \left[\frac{p_t(j,i)}{P_t(j)}\right]^{\frac{1}{\kappa-1}} \frac{Q_t(j)}{N_t^f(j)}, \tag{22}$$

where $p_t(j,i)$ denotes the period t output price of firm i in industry j.

Using labor $\tilde{N}_t^w(j,i)$ and capital $\tilde{K}_{t-1}(j,i)$, each intermediate good is produced by a single monopolistically competitive firm with access to the constant-returns-to-scale technology

$$x_t(j,i) \leq \tilde{K}_{t-1}(j,i)^\alpha \left[\tilde{N}_t^w(j,i)\right]^{1-\alpha}, \tag{23}$$

where $\alpha \in (0,1)$ represents the elasticity of output with respect to capital.

In every period, an intermediate goods producer maximizes its profits, $\Pi_t(j,i) = p_t(j,i)x_t(j,i) - W_t\tilde{N}_t^w(j,i) - R_t^k\tilde{K}_{t-1}(j,i)$, subject to the production technology (23) and the demand for industry goods (20) and intermediate goods (22). While each firm exerts some market power, it acts as a price taker in the factor markets with nominal factor prices for labor services and capital given by W_t and R_t^k. Cost minimization leads to the standard first order conditions for labor services and capital:

$$W_t = \varphi_t(j,i)\left\{(1-\alpha)\tilde{K}_{t-1}(j,i)^\alpha \left[\tilde{N}_t^w(j,i)\right]^{-\alpha}\right\} \tag{24}$$

and

$$R_t^k = \varphi_t(j,i)\left\{\alpha\tilde{K}_{t-1}(j,i)^{\alpha-1} \left[\tilde{N}_t^w(j,i)\right]^{1-\alpha}\right\}. \tag{25}$$

By letting $\varphi_t(j,i)$ denote the firm's nominal marginal costs and taking into account that its pricing decision affects the industry price level $P_t(j)$, but not the aggregate price level

P_t, the optimal price $p_t(j,i)$ each firm sets is a markup ϕ_t over marginal costs:

$$p_t(j,i) = \phi_t \varphi_t(j,i). \tag{26}$$

Restricting attention to a symmetric equilibrium, the markup can be expressed as

$$\phi_t = \phi = \frac{1}{\kappa}. \tag{27}$$

Using (27), total profits distributed to the household are given by

$$N_t^f \Pi_t = \left(\frac{\phi - 1}{\phi}\right) Y_t, \tag{28}$$

with $Y_t = N_t^f x_t$, while total output can be rewritten as

$$Y_t = \frac{K_{t-1}^{\alpha} \left(N_t^w\right)^{1-\alpha}}{\phi} + N_t^f \Pi_t, \tag{29}$$

with $K_{t-1} = N_t^f \tilde{K}_{t-1}$ and $N_t^w = N_t^f \tilde{N}_t^w$.

Regarding market entry, we assume that there exist N_t^f intermediate goods producing firms and an unbounded set of potential entrants. Entry decisions are made by a large group of potential entrepreneurs. To found a new firm, an entrepreneur faces an entry cost η, denominated in labor units (see, e.g., Ghironi and Melitz, 2005; Bilbiie, Ghironi, and Melitz, 2012, 2019). The entrepreneurs subsequently sell the firms to the financial intermediary, pricing them at their (expected) value. Entry occurs until the expected firm value and entry costs are equalized. Hence, the free entry condition can be expressed as

$$p_t^f \left[1 - F_{N,t}(\cdot) - F'_{N,t}(\cdot) N_t^{f,e}\right] + \left(p_{t+1}^f / R_{t+1}\right) F'_{N,t+1}(\cdot) N_{t+1}^{f,e} = \upsilon_t = \eta \cdot w_t, \tag{30}$$

where $w_t = W_t / P_t$.

3.6 Fiscal policy

The government issues one-period debt B_t and levies lump-sum taxes to finance a given stream of consumption G_t. The flow government budget constraint is

$$B_t = R_{t-1} B_{t-1} + G_t - T_t. \tag{31}$$

For simplicity, we follow Carvalho et al. (2016) and assume that the ratio between government spending and GDP is constant, $G_t = gY_t$. We also impose public debt to be a fixed share of GDP, i.e.: $B_t = bY_t$.

3.7 Equilibrium

Given the dynamics for the demographic processes n_t^w, ω_t, and γ_t, a competitive equilibrium for this economy is a sequence of quantities $\{C_t^r, C_t^w, A_t, \lambda_t, H_t, Y_t, K_t, I_t, N_t^f, N_t^{f,e}, V_t^r, V_t^w, B_t, T_t\}$, marginal propensities to consume $\{\xi_t^r, \xi_t^w, \Omega_t\}$, prices $\{R_t, R_t^G, r_t^k, \upsilon_t, w_t\}$, and the dependency ratio Ψ_t such that:

1. Retirees and workers maximize utility subject to their budget constraints, taking market prices as given, as described in Sections 3.2 and 3.3.
2. Firms maximize profits, set markups and enter the market subject to their technology and entry costs, as outlined in Section 3.5.
3. The fiscal authority chooses a mix of debt and lump-sum taxes to satisfy its budget constraint of Section 3.6.
4. Prices are such that the markets for labor, capital, goods and firm entry clear. In particular, the economy-wide resource constraint $Y_t = K_{t-1}^{\alpha} \left(N_t^w\right)^{1-\alpha} = C_t + I_t + G_t + v_t N_t^{f,e}$ must hold.

3.8 Calibration

We calibrate our model to an annual frequency. Individuals are born at the age of 15 and stay in the labor force for an average of $1/(1-\omega)$ years. They live, on average, $1/(1-\gamma)$ years after retirement. We choose $\omega = 0.98$ such that, in steady state, individuals retire at the age of 65 on average. This value is consistent with the current retirement age in the majority of OECD countries. The initial growth rate of the working-age population in the economy, n^w, is set to 1.56%, which is the average value of the United States from 1960 to 1977 according to U.S. Census Bureau data. We calibrate the survival probability $\gamma = 0.8965$ to match an OADR of 16% (conditional on the values of n^w and ω). The latter is the value reported by U.S. Census Bureau for 1970, which is the starting year of our simulations. This gives a life expectancy of about 74.6 years, which also corresponds to the value reported by OECD (2017).[6] Table 1 summarizes our calibration choices.

The other parameters are fairly standard in the literature. We choose a labor share in production of 2/3, assume that capital depreciates at an annual rate of 10% and set the elasticity of intertemporal substitution to $\sigma = 0.5$.[7] These values are consistent with estimates by Hall (1988) and Yogo (2004). Government spending represents 20% of GDP, while government debt corresponds to 60% of GDP. The individual discount factor β is set to 0.979 so that the real interest rate in the initial steady state equals 4%.

Turning to the firm entry decisions, we set the parameters governing the elasticities of substitution within and across sectors to $\kappa = 0.7692$, implying a steady-state markup of 30% percent. This is broadly in line with recent estimates reported in the literature (see, e.g., Edmond, Midrigan, and Xu, 2018). The steady-state entry rate is set to 14% in line with US data for 1970s. These assumptions are used to pin down the entry cost parameter $\eta = 2.1$ and allows us to derive the firm exit rate $\delta^f = 0.1266$, which is in line with standard values chosen in the literature (see, e.g., Jaimovich and Floetotto, 2008; Bilbiie et al., 2012, 2019). The adjustment cost parameter for new firms is set to $\kappa^f = 8$ to smooth out firm dynamics in line with Lewis and Poilly (2012).[8]

---

[6] These values are also in line with those chosen by Carvalho et al. (2016).

[7] As discussed in Ferrero (2010), the relatively low value for σ has become standard in this class of models since the seminal work of Auerbach and Kotlikoff (1987).

[8] Offick and Winkler (2019) estimate the adjustment cost parameter to be much lower (around 1.5).

Table 1: Baseline calibration

Variable/Parameter	Symbol	Value
Working-age population growth	n^w	0.96%
Old age dependency ratio^T	Ψ	0.160
Retirement probabilities	$1-\omega$	0.02
Survival probabilities^e	γ	0.894
Capital share in production	α	0.333
Capital depreciation rate	δ^k	0.100
Firm entry rate^T	$N^{f,e}/N^f$	0.140
Firm exit rate^e	δ^f	0.127
Failure rate parameter of new entrants	κ^f	8.000
Steady-state markup^T	ϕ	0.769
Elasticity of substitution btw. intermediate goods^e	κ	0.769
Entry cost parameter^e	η	2.134
Elasticity of intertemporal substitution	σ	0.500
Steady-state interest rate^T	R	4.0%
Discount factor^e	β	0.979
Government spending-to-GDP ratio^T	G/Y	0.200
Government debt-to-GDP ratio^T	B/Y	0.600
Lump-sum tax-to-GDP ratio^e	T/Y	0.218

Source: U.S. Census Bureau and OECD (2017) for demographic variables. Remaining variables/parameters as described in the main text. The superscript T marks targets, e endogenously derived values to meet these targets. Parameters without a mark are set exogenously as described in the main text.

4 Simulation design

According to the U.S. Census Bureau, the old age dependency ratio in the US increased from 16% in 1970 to 24.5% in 2018. It is projected to further increase to 38.9% in 2060. The growth rate of the working-age population, on average amounting to 1.56% during the period 1960 to 1970, is projected to continuously fall to 0.13% by 2060. Life expectancy is anticipated to increase from around 74 years in 1970 to about 86 years in 2060. We use our model to approximately replicate these demographic trends.

Specifically, we assume that, in the final steady state, the growth rate of the working-age population is reduced to 0.13% as projected by the OECD. Adjustment from the initial

---

Performing a robustness check on this parameter shows, however, that quantitative results are hardly altered by changing the calibration. Qualitatively, our findings are unaffected by a specific parameter choice.

to the final steady-state value follows the AR(1) process: $n_t^w = (1-\rho^{n^w})n^w + \rho^{n^w} n_{t-1}^w$. Starting at $n_{t=0}^w = 0.0156$ and setting $\rho^{n^w} = 0.97$ implies that the new steady-state value is roughly reached in 2060. We assume an analogous process for the increase in longevity, $\gamma_t = (1-\rho^\gamma)\gamma + \rho^\gamma \gamma_{t-1}$, where the final steady-state value $\gamma = 0.9525$ is set to reach the projected old age dependency ratio of 38.9% in the final steady state. We set $\rho^\gamma = 0.9$.[9]

As is standard in the literature (see, e.g., Ferrero, 2010), we assume that demographic variables change unexpectedly in 1970, the initial steady state. This implies that, after the initial period, agents perfectly anticipate the evolution of the exogenous variables, which become constant again in 2060 at their new steady-state values. The model and the simulations are solved in a fully non-linear fashion under perfect foresight.

5 Benchmark results

Figure 1 shows the main simulation results for the core model. In response to the demographic transition, i.e. an increase in the OADR (red crossed line, right axis), the entry rate of firms falls from its peak of almost 14.2% in 1977 to around 13.3% in 2016 (blue solid line, left axis).[10] According to the data, US startup rates have dropped by about 3.4 percentage points between 1970 and 2016.[11] Our core model therefore attributes roughly a fifth of that decline to aging. Furthermore, the entry rate is projected to fall almost another 0.4 percentage points before stabilizing at its new steady-state in 2060.

The figure also sketches the evolution of the entry rate in two counterfactual scenarios. The first (dashed green line) shows the evolution of the firm entry rate implied by the increase in life expectancy only, holding constant the population growth rate at its initial steady-state value. Conversely, the second (black circled line) shows the firm entry rate implied by a fall in the population growth rate only, holding constant the death rate at its initial steady-state value. This decomposition reveals that in the core model the decline in the entry rate is solely driven by the decline in population growth, which is in line with the findings of Hopenhayn et al. (2018) and Karahan et al. (2019). An increase in longevity ceteris paribus does not reduce the firm entry rate, but actually increases it, at least along the transition. As outlined in Section 6, this last finding changes when relaxing some of the simplifications of the core model (i.e., when allowing for endogenous firm exit, in particular).

What is the mechanism behind these results? Societal aging increases savings and reduces the real interest rate. This happens for two reasons. First, the increase in longevity (higher survival probability) induces retirees to save more in order to finance consumption during a longer retirement period (captured by an increase in discount factor $\beta\gamma_{t+1}$). Since workers become retirees with a certain probability at each point in time, it also

---

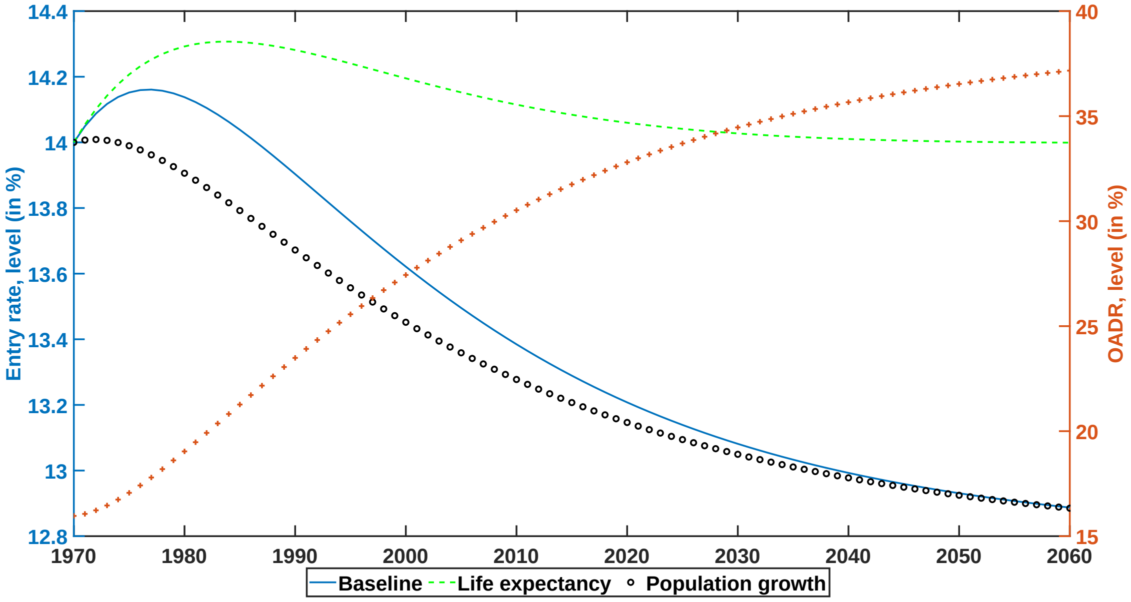
[9] In the appendix, we perform an analogous analysis feeding in US census data until 2060. We find the results to be qualitatively analogous.

[10] It increases slightly from 1970 to 1980 and steadily falls thereafter. As we explain in detail below, this is due to the fact that, to prepare for longevity, households save more and also do so by investing in (new) firms.

[11] US startup rates (between 1970 and 2016) are derived from U.S. Department of Commerce's Survey of Current Business and from U.S. Census Bureau's Business Dynamics Statistics (BDS). Since there are no official data on entry rates from 1963 to 1977, we follow the procedure in Hopenhayn et al. (2018) and linearly interpolate for the period 1963 to 1977.

increases their savings. Specifically, their continuation value is affected by the increase in the survival probability during their retirement period, which they will enter eventually. Figure 2 shows that an increase in the survival probability reduces the propensities to consume for workers and retirees, increases savings in the economy, and hence dampens the returns on bonds, capital, and firm holdings.

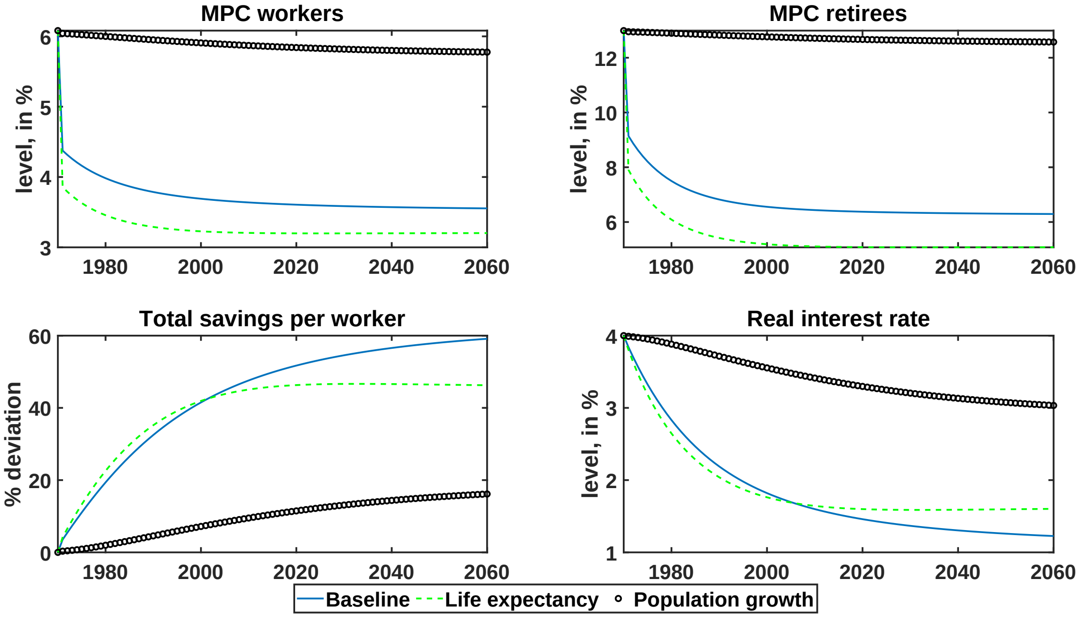
Figure 1: Effects of aging on entry rate



Notes: Figure plots the simulation-based evolution of the firm entry rate (left axis, solid blue line) against the OADR (right axis, crossed red line). The green dashed and circled black lines (left axis) plot the simulation-based evolution of the firm entry rate when only changing the survival probability and working-age population growth, respectively.

Second, the fall in the population growth rate also increases savings. However, reduced population growth brings about two opposing effects. On the one hand, the capital-labor ratio rises when the population shrinks (Figure 3). This reduces the rental rate of capital and, under the no-arbitrage assumption, also depresses the real interest rate and the return on firm holdings. On the other hand, the OADR increases. Because older individuals have a higher marginal propensity to consume, $\xi_t^r/\xi_t^w > 1 \forall t$, as discussed in Section 3.1. This mitigates the former effect. Overall, the capital-to-labor ratio effect dominates, implying that a reduction in population growth is accompanied by an increase in savings and reduction of the real interest rate (Figure 2). The effects of a reduced population growth rate on savings and interest rates are noticeably smaller than the effects of increased longevity, however. Taking both effects together, economy-wide savings increase and the interest rate falls, which is a familiar result in the literature (see, e.g., Carvalho et al., 2016).

Figure 2: Consumption and savings behavior



Notes: Figure plots the simulated marginal propensities to consume (MPC) for workers and retirees, the percentage deviations of savings and the evolution of the real interest rate resulting from population aging. It differentiates between the baseline simulation (blue solid lines) and simulations when only the survival probability (green dashed lines) or the population growth rate changes (black circled lines).

To understand the impact of aging on firm dynamics, it is again helpful to consider the role of a declining population growth and an increase in longevity separately. Starting with the latter, a rise in life expectancy induces both workers and retirees to save. To do so, they can invest in government bonds, physical capital, or (new) firms. As public debt is restricted to be a given share of GDP, it is "fixed" (and only changes when GDP does; see Figure 4). If the implied increase in public debt falls short of the rise in savings, which it does (see Figures 2 and 4), households have to decide whether to save by further investing into physical capital or in firms. As indicated by the permanent rise in the capital-to-labor ratio and the number of firms (see Figure 3), there is a noticeable, although in part transitory, increase in both types of investment. To get an intuition for this result, it is useful to abstract from entry adjustment costs for simplicity (by assuming $\kappa^{nf} = 0$, such that $p_t^f = v_t$, which does not affect the generality of the argument) and use equations (28), (29), (30), the production function, and the fact that the aggregate wage is given by $w_t = (1-\alpha)Y_t/N_t^w$ to re-state the no-arbitrage condition (17) as

$$R_t^w = \frac{(1-\alpha)\eta(1-\delta^f)\left(\frac{K_t^{int}}{K_{t-1}^{int}}\right)^{\alpha}}{(1-\alpha)\eta - \frac{\phi-1}{\phi}\frac{1}{\tilde{N}_t^f}}, \tag{32}$$

where $K_t^{int} = K_{t-1}/N_t^w$ is the capital-to-labor ratio (or "capital intensity") while $\tilde{N}_t^f = N_t^f/N_t^w$ describes the number of firms per active worker. As societal aging increases the capital-to-labor ratio K_t^{int}, implying a decrease of the return on capital and hence the real interest rate R_t, the no-arbitrage argument requires the number of firms per active worker, $\tilde{N}_t^f$, to increase as well. This is also reflected in the temporary rise of the firm entry rate. Since the startup rate is tied to the firm exit rate and population growth, both of which are unaltered by an increase in longevity, the entry rate returns to its initial steady-state, however. To see this, note that the (aggregate) law of motion for firms can be expressed as

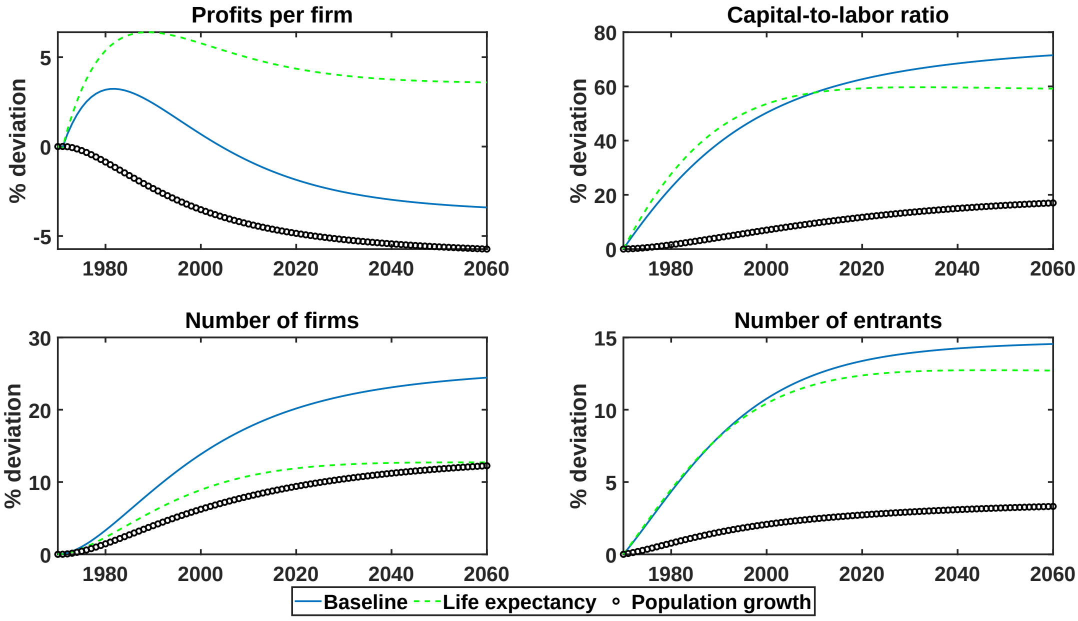
$$\tilde{N}_t^f = \frac{(1-\delta^f)}{(1+n_t^w)}\tilde{N}_{t-1}^f + \tilde{N}_t^{f,e},$$

implying that the entry rate evolves according to

$$\frac{\tilde{N}_t^{f,e}}{\tilde{N}_t^f} = 1 - \frac{(1-\delta^f)}{(1+n_t^w)}\frac{\tilde{N}_{t-1}^f}{\tilde{N}_t^f},$$

where $\tilde{N}_t^{f,e} = N_t^{f,e}/N_t^w$. Hence, along a balanced growth path, the entry rate is determined by $(n^w+\delta^f)/(1+n^w)$. Despite the unaltered long-run steady-state of the entry rate, Figure 3 shows a permanent increase in the number of entrants and firms per worker. This masks, however, that in per capita terms (i.e. relative to $N_t^w+N_t^r$) both the number of firms as wells the number of entrants will fall in the long-run, since an increase in life expectancy raises the OADR, thereby weighing on per capita output and firm profitability.

Figure 3: Firm sector developments

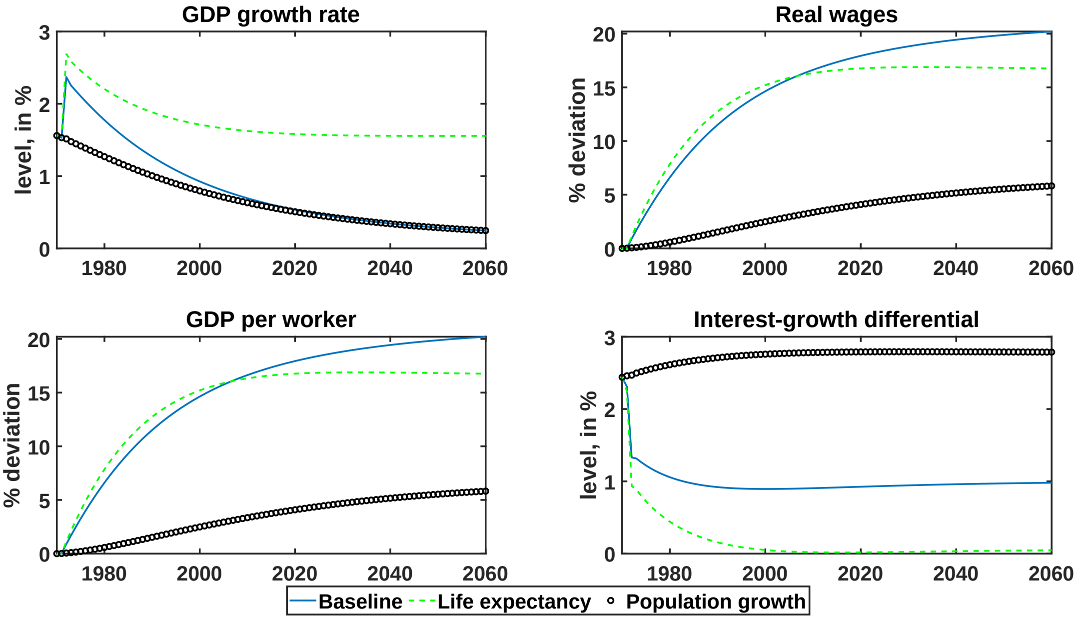


Notes: Figure plots the simulated evolution of firm profits, capital investment, the number of firms and new entrants (per worker) resulting from population aging. It differentiates between the baseline simulation (blue solid lines) and simulations when only the survival probability (green dashed lines) or the population growth rate changes (black circled lines).

Also a decline in population growth initially stimulates the firm entry rate. However, in line with the modest response of the capital-to-labor ratio and the real interest rate, the impact on firm entry is considerably weaker compared to a rise in longevity. Moreover, the rise is only very short-lived and the firm entry rate falls along the transition, resulting in a noticeable lower long-run steady-state. This drop is attributable to the gradual decline in labor growth, implying a decrease in the growth rate of both output and profits per firm. Since the entry decision of a firm critically depends on incumbents' profitability, the startup rate will fall. In addition, the increasing real wage also acts as a dampening factor, implying a rise in firm entry costs (see Figure 4).

The full demographic transition is a result of these two mechanisms. Thereby, we find the longevity effect to dominate for about 10 years, implying a noticeable increase in the firm entry rate. Subsequently, however, the impact of a declining population growth rate becomes evident resulting in a 1.1 percentage point decrease of the steady-state entry rate.

Figure 4: Macroeconomic developments



Notes: Figure plots the simulated evolution of GDP growth, GDP per worker, real wages and the interest-growth differential resulting from population aging. It differentiates between the baseline simulation (blue solid lines) and simulations when only the survival probability (green dashed lines) or the population growth rate changes (black circled lines).

6 Extended model

To inspect more closely the role of aging for business dynamism, we relax some of simplifications made so far. Specifically, we augment the core model by introducing endogenous individual labor supply, endogenous firm exit, and endogenous markups. First, we provide a formal description of the model extensions and then show how adding these features will affect the results obtained from the core model. In the appendix, we discuss how adding each extension separately changes the core model results.

Endogenous labor supply: If households are allowed to decide on the amount of labor they want to supply, we have to modify their utility function (4). Following Gertler (1999) by defining $(1-l_t^z)$ as leisure and υ^l as the marginal rate of transformation between consumption and leisure, and by assuming that workers and retirees can supply labor, it is $V_t^z = \left\{ \left[(C_t^z)^{\upsilon^l} (1-l_t^z)^{1-\upsilon^l} \right]^\rho + \beta_{t+1}^z \left[E_t \left(V_{t+1} | z \right) \right]^\rho \right\}^{\frac{1}{\rho}}$ for $z = \{w, r\}$. Retirees are assumed to be less productive, governed by a parameter $\varsigma \in (0,1)$ that determines the productivity of a retiree relative to a worker. Aggregate labor supply of workers is now given by $L_t^w = N_t^w - (1-\upsilon^l)/\upsilon^l C_t^w / W_t$, while for retirees it holds that $L_t^r = N_t^r - (1-\upsilon^l)/\upsilon^l c_t^r / (\varsigma \cdot W_t)$. Economy-wide labor supply is thus $L_t = L_t^w + \varsigma \cdot L^r$, which needs to be introduced in the production function (23). Output will now be produced by $Y_t = K_{t-1}^\alpha L_t^{1-\alpha}$. This implies that the wage equation and the return on capital, equations (24) and (25), have to be adjusted accordingly, too. We follow the literature and assume that, in the initial steady state, $L^w = 0.9$ and $L^r = 0.01$, the latter reflecting that only a small fraction of retirees actually supply labor (see, e.g., Kara and von Thadden, 2016). Given the calibration strategy described in Section 3.8, we additionally choose $\upsilon^l = 0.8781$ and $\varsigma = 0.0250$ to meet these targets.

Endogenous markups: In order to allow for time-varying markups, we follow the approach of Jaimovich (2007) and Jaimovich and Floetotto (2008). Instead of assuming that each intermediate goods producer is of measure zero within its sector, the output sectoral good j is now given by

$$Q_t(j) = N_t^f(j)^{1-\frac{1}{\kappa}} \left[\sum_{i=1}^{N_t^f(j)} x_t(j,i)^\kappa \right]^{\frac{1}{\kappa}}. \tag{33}$$

Hence, the price markup can be expressed as:

$$\phi_t = \left[\frac{(1-\mu)N_t^f - (\kappa - \mu)}{\kappa(1-\mu)N_t^f - (\kappa - \mu)} \right], \tag{34}$$

implying an inverse relationship between the market power and the number of incumbent firms, i.e. the degree of market competition. We target the same initial steady-state markup as described in Section 3.8. Therefore, we assume that the elasticity of substitution between any two goods within an industry ($\kappa = 0.949$) is higher than the elasticity of substitution across industries ($\mu = 0.001$), which is in line with empirical evidence presented in Broda and Weinstein (2006).

Endogenous firm exit: For the sake of simplicity, firm exit was assumed to occur at an exogenous rate δ^f in the core model. This stands, however, in contrast to recent empirical evidence for the US, pointing to a time-varying behavior of firm exit (see, e.g., Tian, 2018).[12] To make it endogenous, we draw on the industrial organization literature by letting exit decisions be based on a stochastic exit value (see, e.g., Doraszelski and Pakes, 2007; Pakes, Ostrovsky, and Berry, 2007; Weintraub, Benkard, and Van Roy,

---

[12] In this respect, it should also be noted that, according to U.S. Census Bureau's Business Dynamics Statistics (BDS), firm exit rates declined markedly over the 1977-2016 period.

2008; Moscoso Boedo and Mukoyama, 2012; Dunne, Klimek, Roberts, and Xu, 2013). Specifically, we assume that an intermediate goods producing firm observes its random exit value S_j at the beginning of each period (before production takes place). If the observed scrap value is higher than expected next-period (stationary) real profits $\tilde{\Pi}_{t+1} = \Pi_{t+1}/(1+n_t^w)$, the firm will leave the market. We thereby refrain from employing a frequently used modeling approach adopted from the trade literature which links firm survival to physical productivity (see, e.g., Melitz, 2003; Ghironi and Melitz, 2005). Instead, we account for empirical evidence, highlighting the role of profitability in determining firms' exit decision (see, e.g., Foster, Haltiwanger, and Syverson, 2008). In this respect, our exit mechanism is in line with recent micro-evidence of significant effects of short-run profitability on firms' decision to exit (see Golombek and Raknerud, 2018). Following Cavallari (2015), we assume that the scrap value is Pareto distributed across firms. Hence, the exit rate in period t is given by $\delta_{n,t} \equiv Pr(S_j > \tilde{\Pi}_{t+1}) = 1 - F(\tilde{\Pi}_{t+1})$, where

$$F(S_j) = \begin{cases} 1 - \left(\frac{S_j}{S_{min}}\right)^{-\kappa^{ex}} & S_j \geq S_{min} \\ 0 & S_j \leq S_{min} \end{cases}$$

is the cumulative distribution function of S_j and where κ^{ex} and S_{min} represent the respective shape and scale parameters of the distribution. We set $\kappa^{ex} = 2.015$ which, together with initial steady-state exit rate, pins down S_{min}, thereby allowing us to closely match the standard deviation of the firm exit rate observed in the data.[13]

The calibration of the model extensions is done in line with what we have described in Section 3.8, except for the additional parameters that we have to set according to the description of the model extensions. We also adjust the individual discount factor so that the real interest rate in the initial steady state remains at 4%, as is done in Carvalho et al. (2016) when introducing model extensions.

6.1 Simulation results in the extended model

In what follows, we repeat the simulations described in Section 4 with the extended model containing all the elements detailed in the previous subsection. Figure 5 shows the main results of this exercise: First, the overall finding that aging reduces the firm entry rate is robust to the model extensions. However, with the extended model the decline of entry rate becomes more pronounced. Specifically, up to 30% of the decline in the US firm entry rate observed between 1970 and 2016 can be attributed to aging, while we find the entry rate in 2060 to be almost 1.5 percentage points lower compared to its initial steady-state. Second, in contrast to the core model, it is the increase in longevity that now substantially contributes to this decline.

From Figure 6, we see that none of the extensions actually seem to modify the savings behavior of households. While the increase in savings and the reduction in the real interest rate are somewhat larger in the extended model, households in the extended model behave analogously to those in the core framework.[14]

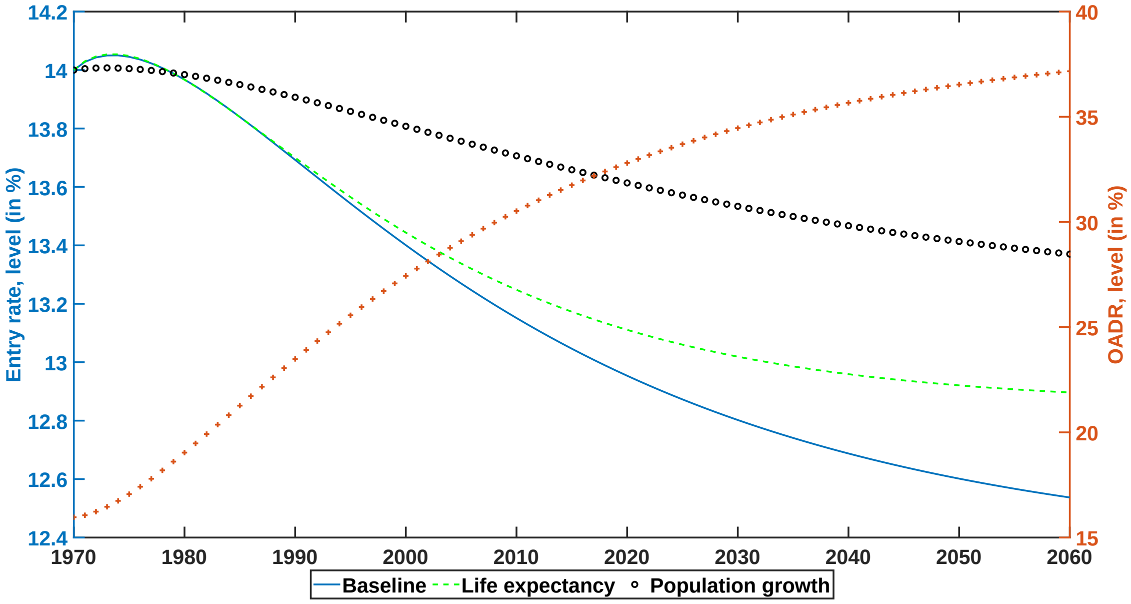
---

[13] Our results are unaffected by alternative calibrations of the scale parameter $\kappa^{ex} > 2$.

[14] As we discuss in more detail in the appendix, the larger increase in savings as well as the larger reduction in the real interest rate are primarily the result of increasing individual labor supply (due to

Although some quantitative differences also become evident when comparing the simulation paths of key macroeconomic variables in the core with those in the extended model (see Figures 4 and 7), the simulation results again appear to be hardly affected by the model extension from a qualitative perspective.[15]

Figure 5: Effects of aging on entry rate in the extended model



Notes: Figure plots the simulation-based evolution of the firm entry rate (left axis, solid blue line) against the OADR (right axis, dashed red line). The green dashed and circled black lines (left axis) plot the simulation-based evolution of the firm entry rate when only changing the survival probability and working-age population growth, respectively.

However, when turning to the firm sector, noticeable differences become evident (see Figure 8). First of all, the increase in the number of firms is significantly lower in the extended setup than in the core model. This is mainly a result of the endogenous markups. As can be seen from equation (34), the markup is an inverse function of the number of firms. Hence, an increase in the number of firms (competitors) reduces the markup firms can charge. This, in turn, has a double impact on the no-arbitrage condition (32), where ϕ is now replaced by ϕ_t indicating the time-varying nature of the markup. While the mechanisms in the core model still apply, the same increase in the number of firms (per worker), $\tilde{N}_t^f$, now has a much stronger effect on the right-hand side of equation (32). Hence, for a given reduction in the real interest rate, the number of firms has to increase much less in the extended model in order to comply with the no-arbitrage condition. Intuitively, the profitability of firms falls with the number of competitors. Hence, the incentive to invest in more firms is lower now. While the number of firms per worker in

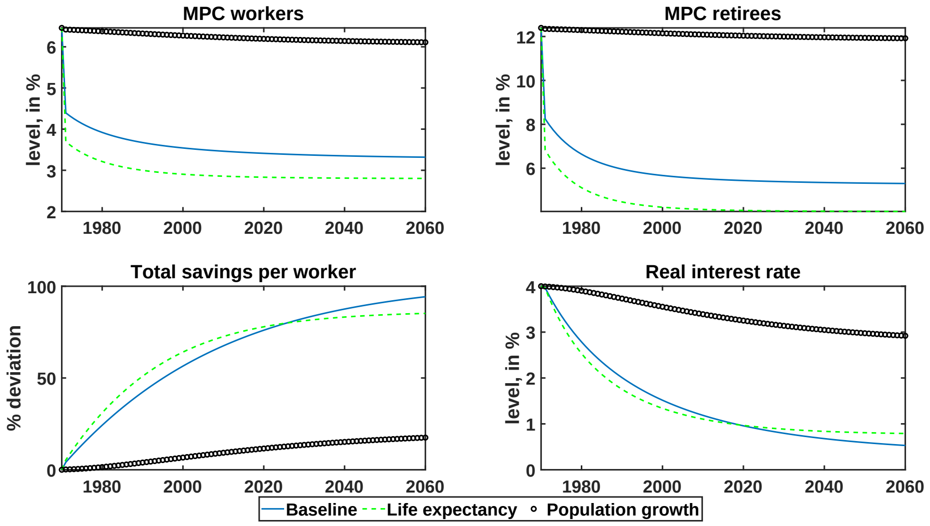
---

higher wages), which ultimately augments the incentive to save.

[15] As outlined in the appendix, the quantitative changes are mainly driven by endogenous individual labor supply.

the core model increases by nearly 25%, it only rises by slightly more than 6% in the extended setup.

Figure 6: Consumption and savings behavior in the extended model



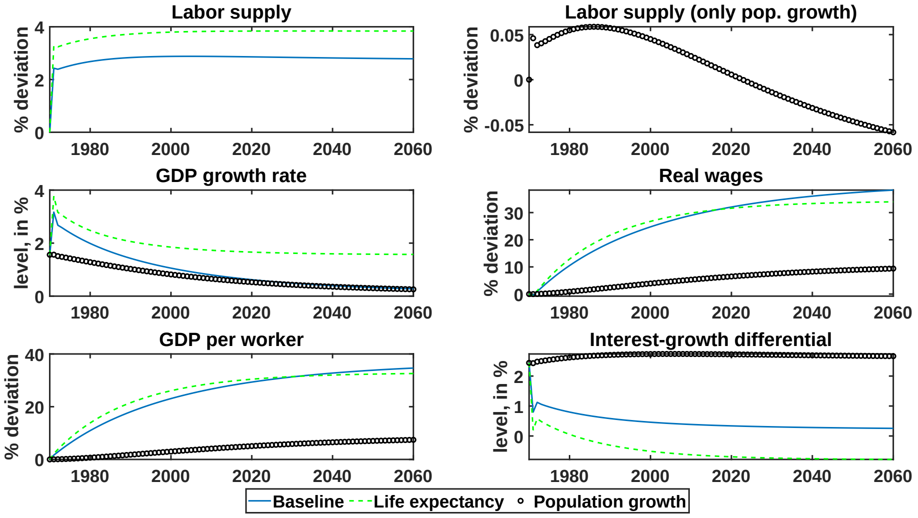
Notes: Figure plots the simulated marginal propensities to consume (MPC) for workers and retirees, the percentage deviations of savings and the evolution of the real interest rate resulting from population aging. It differentiates between the baseline simulation (blue solid lines) and simulations when only the survival probability (green dashed lines) or the population growth rate changes (black circled lines).

A second noticeable difference is the evolution of the number of entrants per worker. While it increases in the core model, it falls in the extended setup (after a short rise on impact driven by increased longevity). This is mainly due to two features: Endogenous markups and endogenous firm exit. As laid out, endogenous markups reduce the need to adjust the number of firms in order to comply with the no-arbitrage condition. Although lower markups initially depress firm profitability and hence entry, firm profits per worker rise along the transition. To get an intuition for this, one should note that while output and production costs evolve in a similar way in the core and in the extended model, the number of firms increases much less in the extended setup. Hence, by construction, profits per firm (per worker) increase noticeably more (or fall less, when only taking into account changes in population growth; see Figure 8). However, with a rise in profits, the endogenous exit mechanism implies that fewer firms will leave the market, implying a declining exit rate (see Figure 8). From a qualitative perspective, the latter also appears to be in line with the observed pattern in the data.[16] To see how this will affect the startup rate along a balanced growth path, it is useful to reconsider that under endogenous exit the steady-state entry rate is now determined by $(n^w + \delta^f(\pi))/(1 + n^w)$ with $\delta^f(\pi)' < 0$. In the event that aging induces an increase in firm profitability incumbents are less likely to exit the market implying a lower exit rate and therefore also a decline in the startup rate. Specifically, it is this mechanism which allows an increase in longevity to have a lasting impact on the firm entry rate.

---

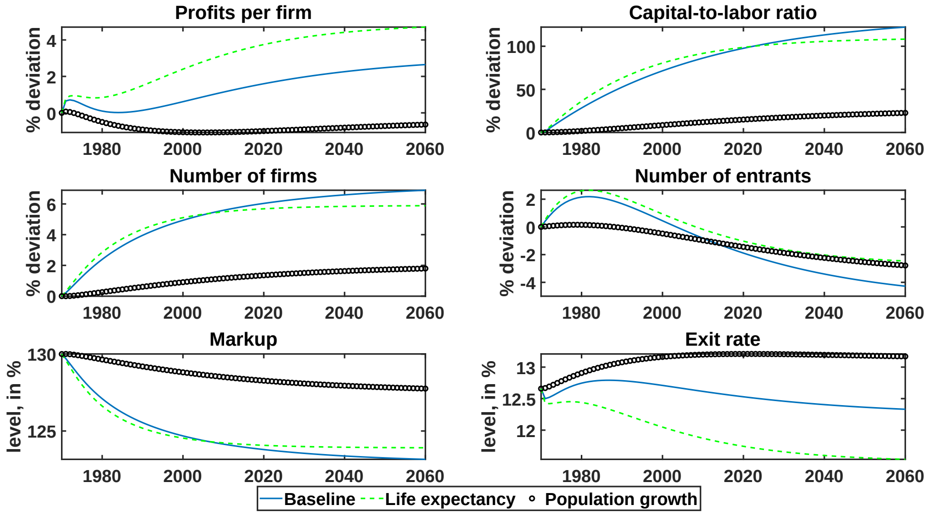
[16] According to BDS data, firm exit rates declined by almost 3 percentage points between 1978-2016.

Figure 7: Macroeconomic developments in the extended model



Notes: Figure plots the simulated evolution of GDP growth, GDP per worker, real wages and the interest-growth differential resulting from population aging. It differentiates between the baseline simulation (blue solid lines) and simulations when only the survival probability (green dashed lines) or the population growth rate changes (black circled lines).

Figure 8: Firm sector developments in the extended model



Notes: Figure plots the simulated evolution of firm profits, capital investment, the number of firms and new entrants (per worker) resulting from population aging. It differentiates between the baseline simulation (blue solid lines) and simulations when only the survival probability (green dashed lines) or the population growth rate changes (black circled lines).

7 Conclusion

We use a general equilibrium life-cycle model with endogenous firm entry to derive three main results. First, the entry rate of firms falls as a result of the demographic transition. According to our model simulations, population aging can account for up to 30% of the observed drop in US firm entry rates. Second, the decline in the population growth is an important factor explaining these developments, which confirms recent findings from dynamic firm models. Finally, by additionally allowing for endogenous firm exit, we find that an increase in longevity may also play a prominent role in explaining the observed decline in business dynamism, dampening both firm entry and exit rates. Our approach does not assume that entrepreneur skills are age-dependent in order to link a decrease in the firm entry rate to population aging. Rather, our model explains the decline in new business formation purely through aging-related general equilibrium effects. Our analysis might be seen as a complement to the recent literature emphasizing the role of demographics for business dynamism, while – by pointing out potential implications of increasing longevity – adding another layer to the understanding of the effects of demographic change on business churn.

References

Akcigit, U. and S. T. Ates (2019a). Ten facts on declining business dynamism and lessons from endogenous growth theory. NBER Working Paper 25755.

Akcigit, U. and S. T. Ates (2019b). What happened to U.S. business dynamism? NBER Working Paper 25756.

Aksoy, Y., H. S. Basso, R. P. Smith, and T. Grasl (2019). Demographic structure and macroeconomic trends. *American Economic Journal: Macroeconomics 11*(1), 193–222.

Alon, T., D. Berger, R. Dent, and B. Pugsley (2018). Older and slower: The startup deficit's lasting effects on aggregate productivity growth. *Journal of Monetary Economics 93*(C), 68–85.

Auerbach, A. J. and L. J. Kotlikoff (1987, May). Evaluating fiscal policy with a dynamic simulation model. *American Economic Review 77*(2), 49–55.

Beaudry, P., F. Collard, and F. Portier (2011). Gold rush fever in business cycles. *Journal of Monetary Economics 58*(2), 84–97.

Bilbiie, F. O., F. Ghironi, and M. J. Melitz (2012). Endogenous entry, product variety, and business cycles. *Journal of Political Economy 120*(2), 304–345.

Bilbiie, F. O., F. Ghironi, and M. J. Melitz (2019). Monopoly power and endogenous product variety: Distortions and remedies. *American Economic Journal: Macroeconomics 11*(4), 140–74.

Blanchard, O. J. (1985). Debt, deficits, and finite horizons. *Journal of Political Economy 93*(2), 223–247.

Broda, C. and D. E. Weinstein (2006). Globalization and the gains from variety. *The Quarterly Journal of Economics 121*(2), 541–585.

Cao, S., M. Salameh, M. Seki, and P. St-Amant (2017). Trends in firm entry and new entrepreneurship in canada. *Canadian Public Policy 43*(3), 202–220.

Carvalho, C., A. Ferrero, and F. Nechio (2016). Demographics and real interest rates: Inspecting the mechanism. *European Economic Review 88*(C), 208–226.

Cavallari, L. (2015). Entry costs and the dynamics of business formation. *Journal of Macroeconomics 44*, 312–326.

Clementi, G. L. and B. Palazzo (2016). Entry, Exit, Firm Dynamics, and Aggregate Fluctuations. *American Economic Journal: Macroeconomics 8*(3), 1–41.

Decker, R., J. Haltiwanger, R. Jarmin, and J. Miranda (2014). The role of entrepreneurship in us job creation and economic dynamism. *Journal of Economic Perspectives 28*(3), 3–24.

Decker, R. A., J. Haltiwanger, R. S. Jarmin, and J. Miranda (2016). Where has all the skewness gone? the decline in high-growth (young) firms in the U.S. *European Economic Review 86*, 4 – 23.

Decker, R. A., J. Haltiwanger, R. S. Jarmin, and J. Miranda (2017). Declining dynamism, allocative efficiency, and the productivity slowdown. *American Economic Review 107*(5), 322–26.

Decker, R. A., J. Haltiwanger, R. S. Jarmin, and J. Miranda (2018). Changing business dynamism and productivity: Shocks vs. responsiveness. NBER Working Paper 24236.

Doraszelski, U. and A. Pakes (2007). A framework for applied dynamic analysis in io. In M. Armstrong and R. Porter (Eds.), *Handbook of Industrial Organization*, Volume 3, Chapter 30, pp. 1887–1966.

Dunne, T., S. D. Klimek, M. J. Roberts, and D. Y. Xu (2013). Entry, exit, and the determinants of market structure. *The RAND Journal of Economics 44*(3), 462–487.

Edmond, C., V. Midrigan, and D. Y. Xu (2018). How costly are markups? NBER Working Paper 24800.

Emes, J., T. Jackson, and S. Globerman (2018). Small business entry rates, demography, and productivity performance in selected developed countries. In S. Globerman and J. Clemens (Eds.), *Demographics and Entrepreneurship: Mitigating the Effects of an Aging Population*, Chapter 3, pp. 77–109.

Engbom, N. (2019). Firm and worker dynamics in an aging labor market. Federal Reserve Bank of Minneapolis Working Paper 756.

Epstein, L. G. and S. E. Zin (1989). Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework. *Econometrica 57*(4), 937–969.

Farmer, R. E. A. (1990). RINCE preferences. *The Quarterly Journal of Economics 105*(1), 43–60.

Ferrero, A. (2010). A structural decomposition of the u.s. trade balance: Productivity, demographics and fiscal policy. *Journal of Monetary Economics 57*(4), 478–490.

Foster, L., J. Haltiwanger, and C. Syverson (2008). Reallocation, firm turnover, and efficiency: Selection on productivity or profitability? *The American Economic Review 98*(1), 394–425.

Fujiwara, I. and Y. Teranishi (2008). A dynamic New Keynesian life-cycle model: Societal aging, demographics, and monetary policy. *Journal of Economic Dynamics and Control 32*(8), 2398–2427.

Gertler, M. (1999). Government debt and social security in a life-cycle economy. *Carnegie-Rochester Conference Series on Public Policy 50*(1), 61–110.

Ghironi, F. and M. J. Melitz (2005). International trade and macroeconomic dynamics with heterogeneous firms. *The Quarterly Journal of Economics 120*(3), 865–915.

Golombek, R. and A. Raknerud (2018). Exit dynamics of start-up firms: Structural estimation using indirect inference. *Journal of Econometrics 205*(1), 204–225.

Gourio, F., T. Messer, and M. Siemer (2016). Firm entry and macroeconomic dynamics: A state-level analysis. *American Economic Review 106*(5), 214–218.

Gutiérrez, G., C. Jones, and T. Philippon (2019). Entry costs and the macroeconomy. NBER Working Paper 25609.

Hall, R. E. (1988). Intertemporal substitution in consumption. *Journal of Political Economy 96*(2), 339–357.

Hathaway, I. and R. E. Litan (2014). What's driving the decline in the firm formation rate? A partial explanation. *Brookings Institution.*

Hopenhayn, H., J. Neira, and R. Singhania (2018). From population growth to firm demographics: Implications for concentration, entrepreneurship and the labor share. NBER Working Paper 25382.

Jaimovich, N. (2007). Firm dynamics and markup variations: Implications for sunspot equilibria and endogenous economic fluctuations. *Journal of Economic Theory 137*(1), 300–325.

Jaimovich, N. and M. Floetotto (2008). Firm dynamics, markup variations, and the business cycle. *Journal of Monetary Economics 55*(7), 1238–1252.

Kara, E. and L. von Thadden (2016). Interest rate effects of demographic changes in a new keynesian life-cycle framework. *Macroeconomic Dynamics 20*(1), 120–164.

Karahan, F., B. Pugsley, and A. Sahin (2019). Demographic origins of the startup deficit. NBER Working Paper 25874.

Kopecky, J. (2017). An aging dynamo: demographic change and the decline of entrepreneurial activity in the united states. mimeo.

Kozeniauskas, N. (2019). What's driving the decline in entrepreneurship? mimeo.

Lewis, V. and C. Poilly (2012). Firm entry, markups and the monetary transmission mechanism. *Journal of Monetary Economics 59*(7), 670–685.

Liang, J., W. von Hui, and E. P. Lazear (2018). Demographics and entrepreneurship. *Journal of Political Economy 126*(S1), S140–S196.

Mata, J. and P. Portugal (1994). Life duration of new firms. *Journal of Industrial Economics 42*(3), 227–245.

Melitz, M. J. (2003). The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica 71*(6), 1695–1725.

Moscoso Boedo, H. J. and T. Mukoyama (2012). Evaluating the effects of entry regulations and firing costs on international income differences. *Journal of Economic Growth 17*(2), 143–170.

OECD (2017). *Pensions at a Glance 2017: OECD and G20 Indicators.*

Offick, S. and R. C. Winkler (2019). Endogenous firm entry in an estimated model of the u.s. business cycle. *Macroeconomic Dynamics 23*(1), 284–321.

Ouimet, P. and R. Zarutskie (2014). Who works for startups? the relation between firm age, employee age, and growth. *Journal of Financial Economics 112*(3), 386–407.

Pakes, A., M. Ostrovsky, and S. Berry (2007). Simple estimators for the parameters of discrete dynamic games (with entry/exit examples). *The RAND Journal of Economics 38*(2), 373–399.

Pugsley, B. and A. Sahin (2018). Grown-up business cycles. *The Review of Financial Studies 32*(3), 1102–1147.

Schön, M. and N. Stähler (2019). When old meets young? Germany's population ageing and the current account. *Economic Modelling forthcoming.*

Tian, C. (2018). Firm-level entry and exit dynamics over the business cycles. *European Economic Review 102*, 298 – 326.

Weil, P. (1989). Overlapping families of infinitely-lived agents. *Journal of Public Economics 38*(2), 183–198.

Weintraub, G. Y., C. L. Benkard, and B. Van Roy (2008). Markov perfect industry dynamics with many firms. *Econometrica 76*(6), 1375–1411.

Yaari, M. E. (1965). Uncertain lifetime, life insurance, and the theory of the consumer. *Review of Economic Studies 32*(2), 137–150.

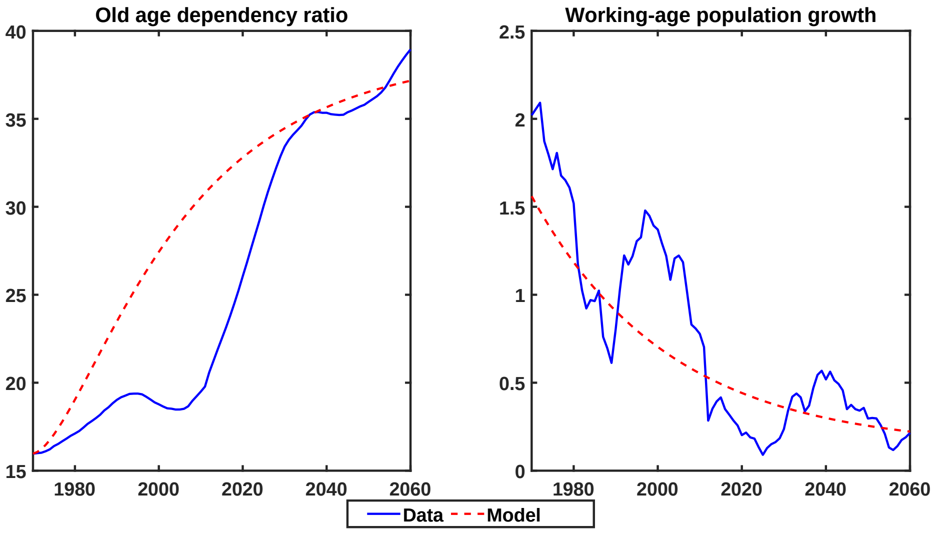
Yogo, M. (2004). Estimating the Elasticity of Intertemporal Substitution When Instruments Are Weak. *The Review of Economics and Statistics 86*(3), 797–810.

Appendix

A.1 Simulations based on official US population estimates and projections

In this appendix, we show core model simulations, feeding in official population estimates and projections sourced from the U.S. Census Bureau. Figure A.1 compares the evolution of the exogenous demographic variables (old age dependency ratio and population growth) that we used in the model simulations (see Section 4) with their data counterparts. The left panel shows that, in our baseline model simulation, we overestimate the increase in the OADR along the transition. In the right panel, we see that, on average, we map the decline in population growth quite well. However, we also overestimate the decline in population growth at the beginning, and underestimate it after 2020. Furthermore, the official (projected) population growth is more volatile than the approximation used in the previous simulations.

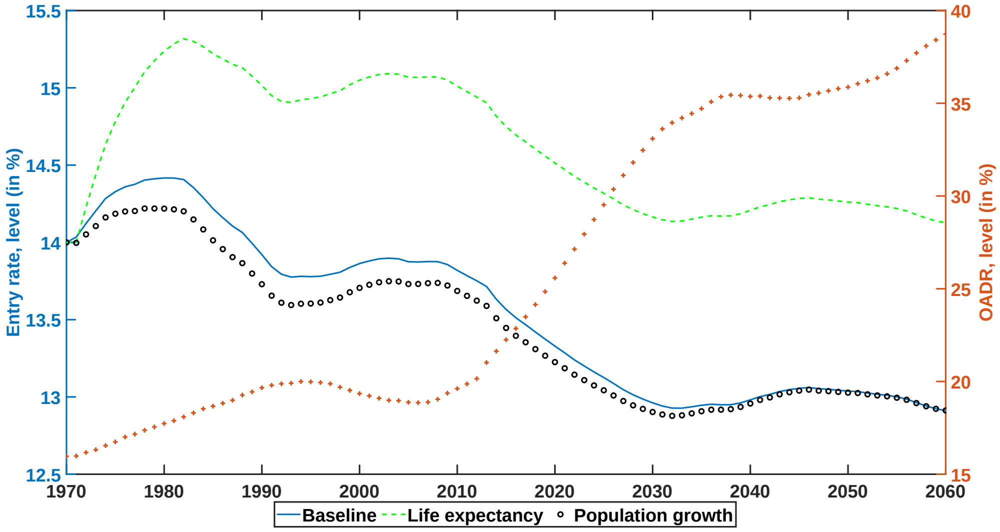
Figure A.1: Demographic trends



Notes: Figure plots the old age dependency ratio in the model (dashed red lines) and in the data (blue solid lines) as well as the working-age population growth rates in the model (dashed red lines) and in the data (blue solid lines).

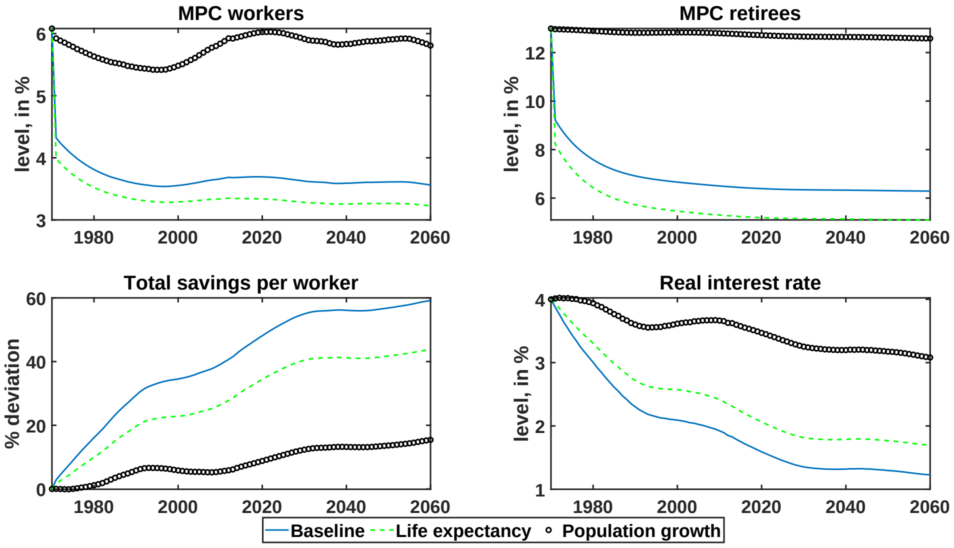
To describe the main mechanisms at work, the stylized demographic processes used in the main text are sufficient. However, we show here that a simulation which meets the demographic variables in the data produces analogous results. Not surprisingly, the transition paths contain more wiggles and, because of above-steady state population growth initially, the decline in the entry rate sets in later. Results are summarized in Figures A.2 to A.5. Interpretation is analogous to the main text.

Figure A.2: Effects of aging on entry rate feeding in official population estimates and projections



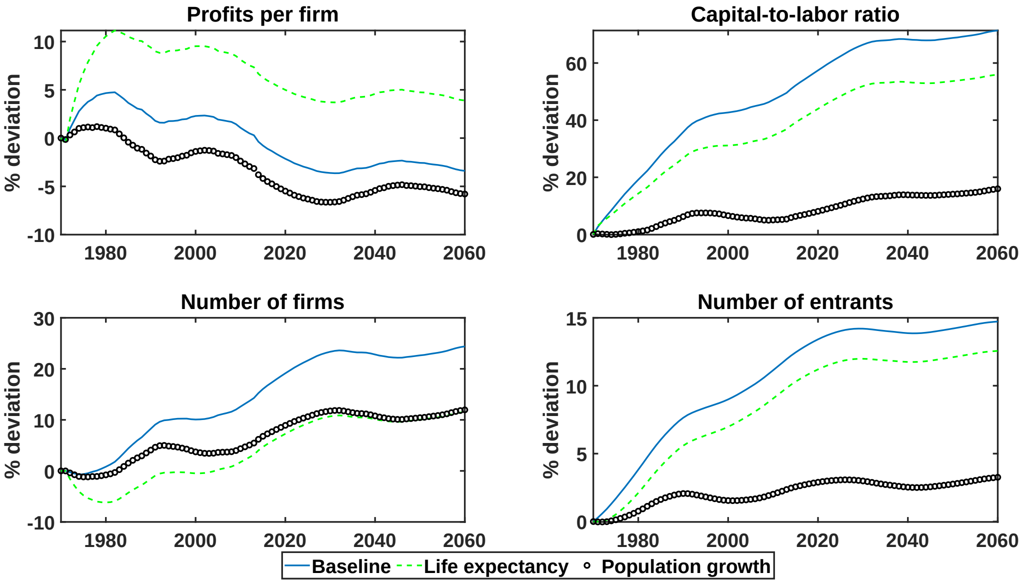
Notes: Figure is the analogue to Figure 1 with official population data.

Figure A.3: Consumption and savings behavior feeding in official population estimates and projections



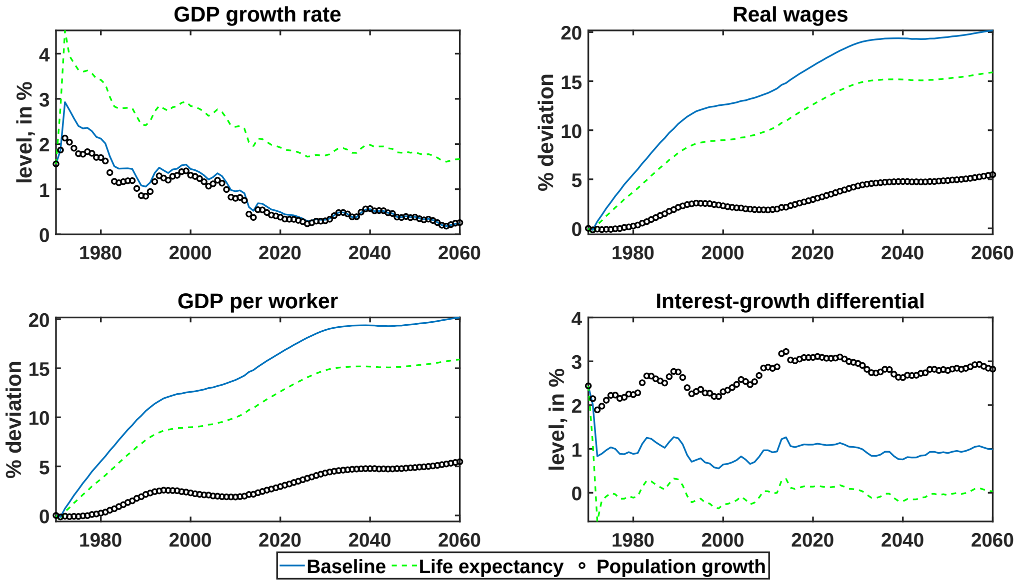
Notes: Figure is the analogue to Figure 2 with official population data.

Figure A.4: Firm sector developments feeding in official population estimates and projections



Notes: Figure is the analogue to Figure 3 with official population data.

Figure A.5: Macroeconomic developments feeding in official population estimates and projections



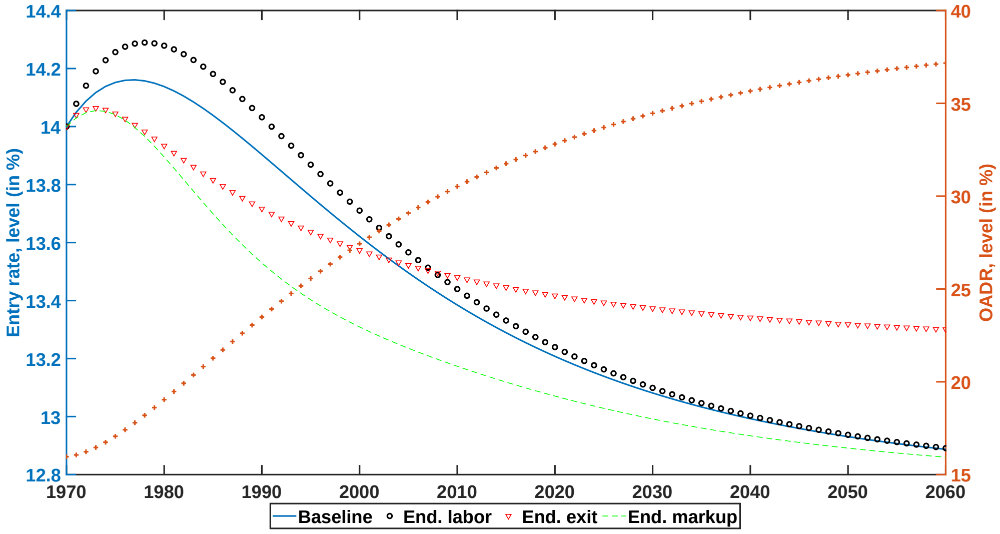
Notes: Figure is the analogue to Figure 4 with official population data.

B.1 Including each model extension separately

In this appendix, we dissect the effects of the various model extensions described in Section 6 by comparing the developments of consumption, savings, firm dynamics, output growth, and the interest rate that would result if each model extensions is integrated separately into the core model. Figures B.1 to B.4 are the analogue to Figures 1 to 4.

Endogenous labor supply: With endogenous labor supply, the rise in wages fosters the incentive of providing labor for individuals (see Figure B.4). For given wages, an increase in labor also reduces consumption as consumption and leisure are normal goods. Hence, relative to our core model with constant individual labor supply, the marginal propensity to consume falls relatively more (especially for workers; see Figure B.2). The wage increase mitigates this effect on consumption, although it is still present. Its magnitude is determined by the marginal transformation between labor and leisure, υ^l, and the elasticity of output with respect to labor, $(1-\alpha)$. The relatively larger decrease in consumption of workers positively affects savings which ultimately affects the real interest rate more negatively (Figure B.2). In the end, the increase in the capital-to-labor share is relatively larger, too, positively affecting firm profits and hence the incentive to invest in new firms (Figure B.3). Therefore, the positive impulse on the entry rate on impact is larger when labor supply is endogenous. As population shrinks, and the increase in individual labor supply no longer compensates for the reduction in working-age population, the positive impact dies out and the entry rate starts falling more quickly.

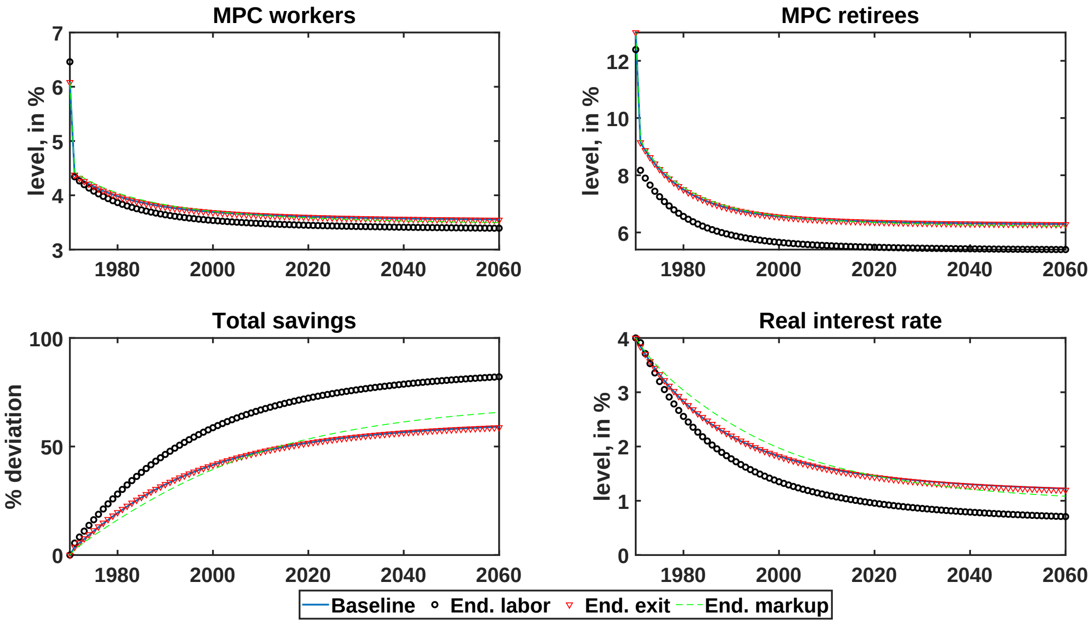
Figure B.1: Effects of aging on entry rate under alternative model specifications



Notes: Figure plots the simulation-based evolution of the firm entry rate (left axis) against the OADR (right axis, crossed light red line). It differentiates between the baseline simulation (blue solid lines) and simulations with the following model extensions: endogenous labor supply (black circled line), endogenous firm exit (red triangled line) and endogenous markups (green dashed line).

Endogenous exit rate: Because profits per firm increase when the population ages, as described above, the endogenous firm exit rate, depending on the expected profits next period, falls. A falling exit rate implies that, in order to increase the number of firms in the economy, the entry rate has to increase less. This reduces the positive impulse on the entry rate on impact and hence implies that the decline of the entry rate starts earlier. Once the profits per firm fall below their initial steady-state value, the exit rate starts to increase (Figure B.3). This dampens the fall in the entry rate, implying that it does not fall as much as it would without endogenous firm exit. To see this, note that the steady-state entry rate is given by $(n^w + \delta^f)/(1 + n^w)$.

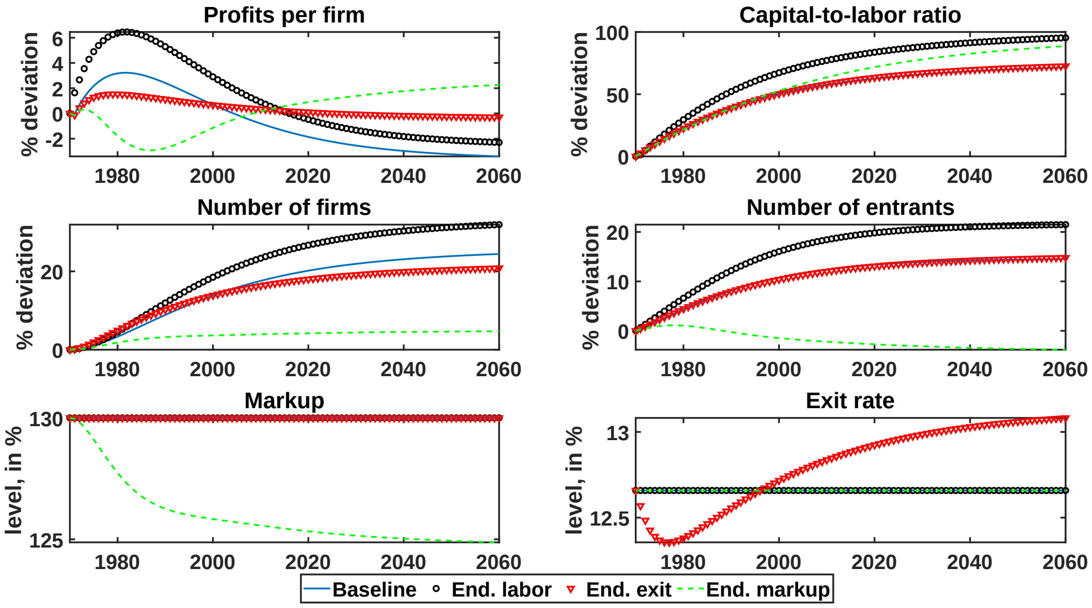
Figure B.2: Consumption and savings behavior under alternative model specifications



Notes: Figure plots the simulated marginal propensities to consume for workers and retirees, the percentage deviations of savings and the evolution of the real interest rate resulting from population aging. It differentiates between the baseline simulation (blue solid lines) and simulations with the following model extensions: endogenous labor supply (black circled line), endogenous firm exit (red triangled line) and endogenous markups (green dashed line).

Endogenous markups: When the markup firms demand for their profits is endogenous, an increase in the number of firms (per worker) has a twofold impact on the (modified) no-arbitrage condition (32). While all the channels described in the core scenario are still present, the increase in the number of firms now additionally reduces the markup firms (can) demand for their products as can be seen from equation (34). A reduction in the markup lowers the profitability of firms (in relative terms) and thus allows to meet the no-arbitrage condition with a smaller adjustment of firms (per worker). Hence, the entry rate must increase by less compared to a setup with constant markups, and the negative effects on the entry rate due to lower population growth become effective relatively faster and are therefore slightly stronger. It is interesting to note here that, although markups are reduced, profits per firm (per worker) tend to be the highest in the long-run compared to all other scenarios (see Figure B.3). This can be explained by the fact that, while the effects on the interest rate, the capital-to-labor ratio, output per worker and wages are more or less the same as in the core model without endogenous markups, the number of firms has to increase much less due to the twofold impact on the no-arbitrage condition. Given that, in this situation, the same output is produced at the same cost, but with fewer firms, profits per firm must be higher. It is also interesting to note that the number of entrants declines (Figure B.3). While in all other situations, the number of firms (per worker), $\tilde{N}_t^f$, has increased sufficiently to overcompensate the decrease in the entry rate, $(n^w + \delta^f)/(1 + n^w)$, this is no longer the case here.[17]

Figure B.3: Firm sector developments under alternative model specifications

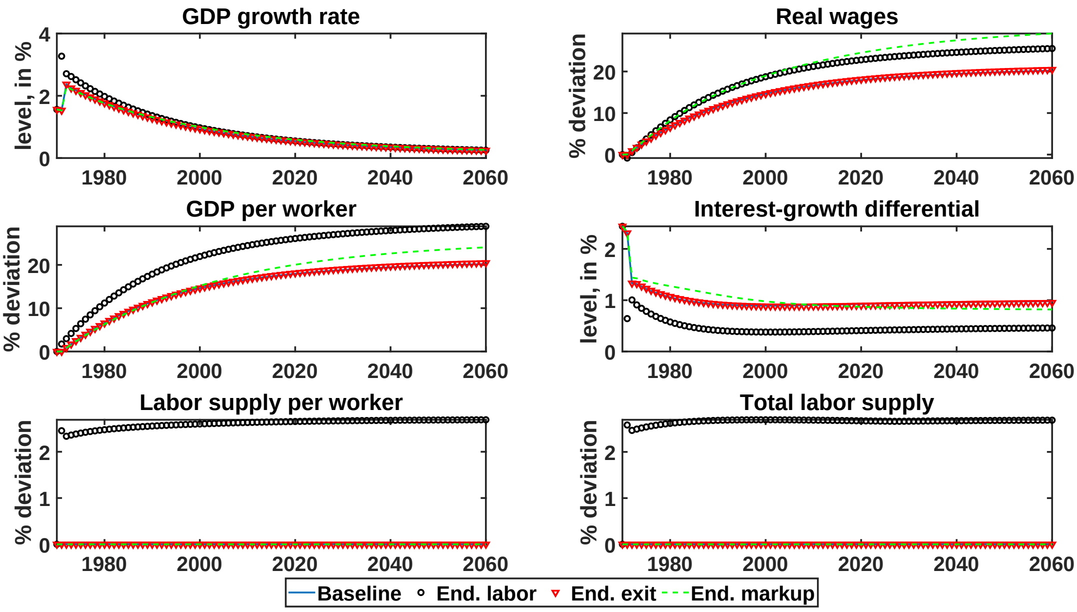


Notes: Figure plots the simulated evolution of firm profits, capital investment, the number of firms and new entrants (per worker) resulting from population aging. It differentiates between the baseline simulation (blue solid lines) and simulations with the following model extensions: endogenous labor supply (black circled line), endogenous firm exit (red triangled line) and endogenous markups (green dashed line).

---

[17] Remember that $\tilde{N}_t^{f,e} = \tilde{N}_t^f - \frac{(1-\delta^f)}{(1+n_t^w)}\tilde{N}_{t-1}^f$ (see Section 6.1). In steady state, it holds that $\tilde{N}^{f,e} = \frac{(n^w+\delta^f)}{(1+n^w)}\tilde{N}^f$. As the entry rate falls, $\tilde{N}^{f,e}$ can only increase if the number of firms increases sufficiently to overcompensate this fall. This is not the case when markups are endogenous.

Figure B.4: Macroeconomic developments under alternative model specifications

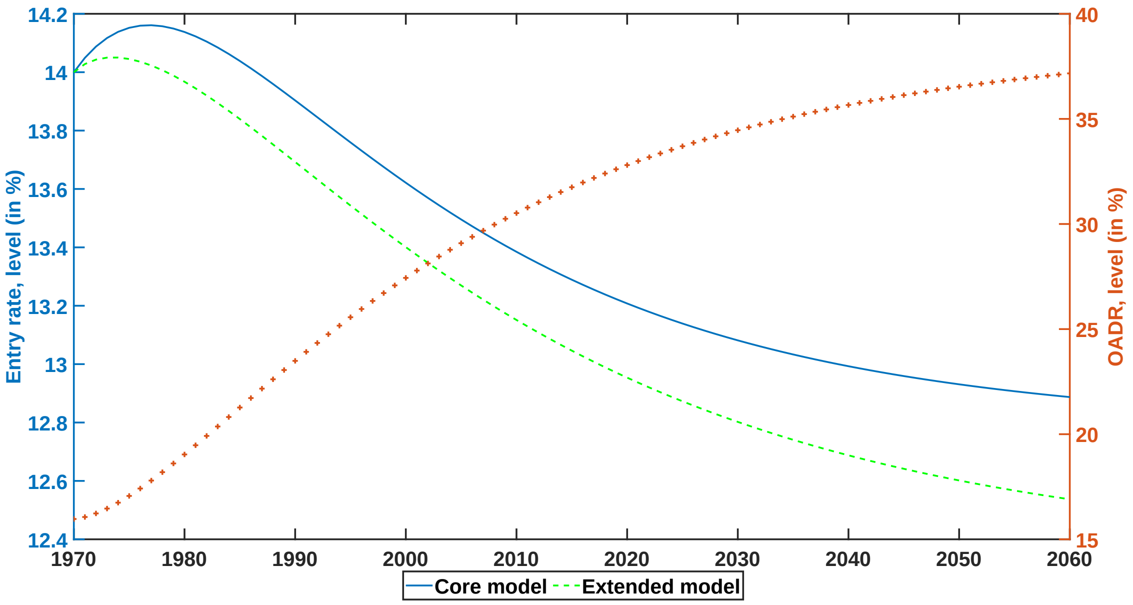


Notes: Figure plots the simulated evolution of GDP growth, GDP per worker, real wages and the interest-growth differential resulting from population aging. It differentiates between the baseline simulation (blue solid lines) and simulations with the following model extensions: endogenous labor supply (black circled line), endogenous firm exit (red triangled line) and endogenous markups (green dashed line).

C.1 Comparison of the core and the extended model

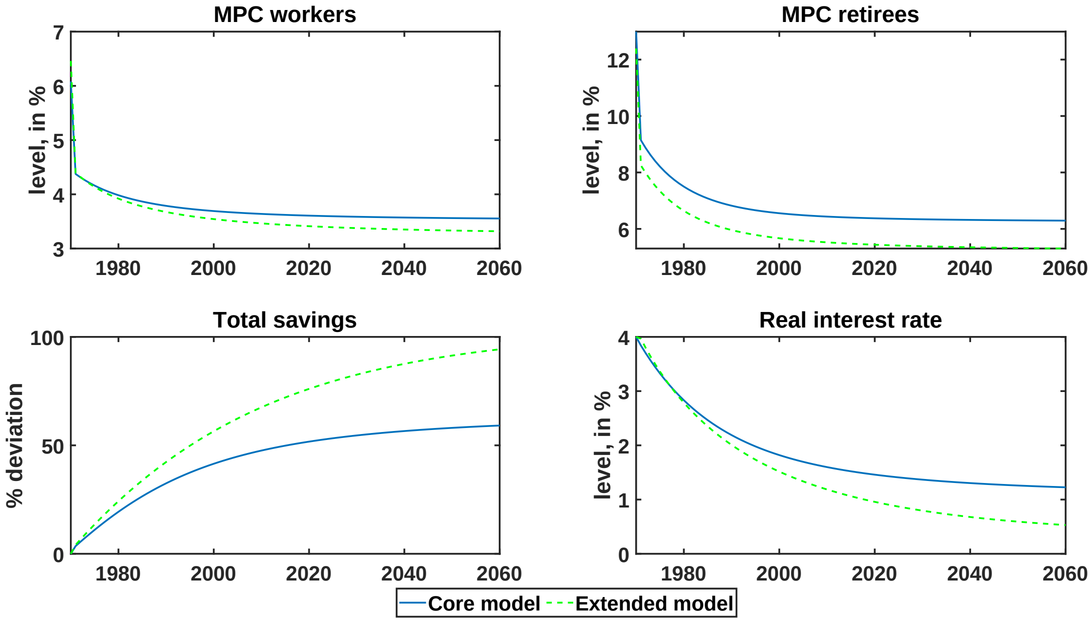
In this appendix, we compare the evolution of the entry rate in the core and the extended model. We only consider the full demographic transition (i.e. the simultaneous increase in longevity and the decline in working-age population growth). The underlying mechanisms are discussed in the main text.

Figure C.1: Effects of aging on entry rate in core and extended model



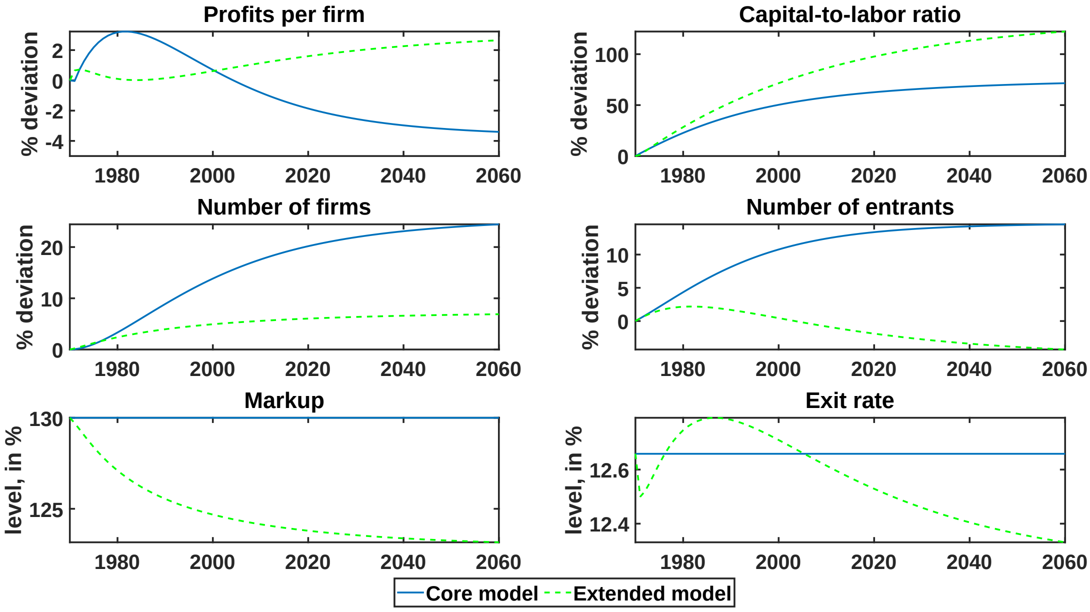
Notes: Figure is analogue to Figure 1 comparing the core and the extended model.

Figure C.2: Consumption and savings behavior under alternative model specifications



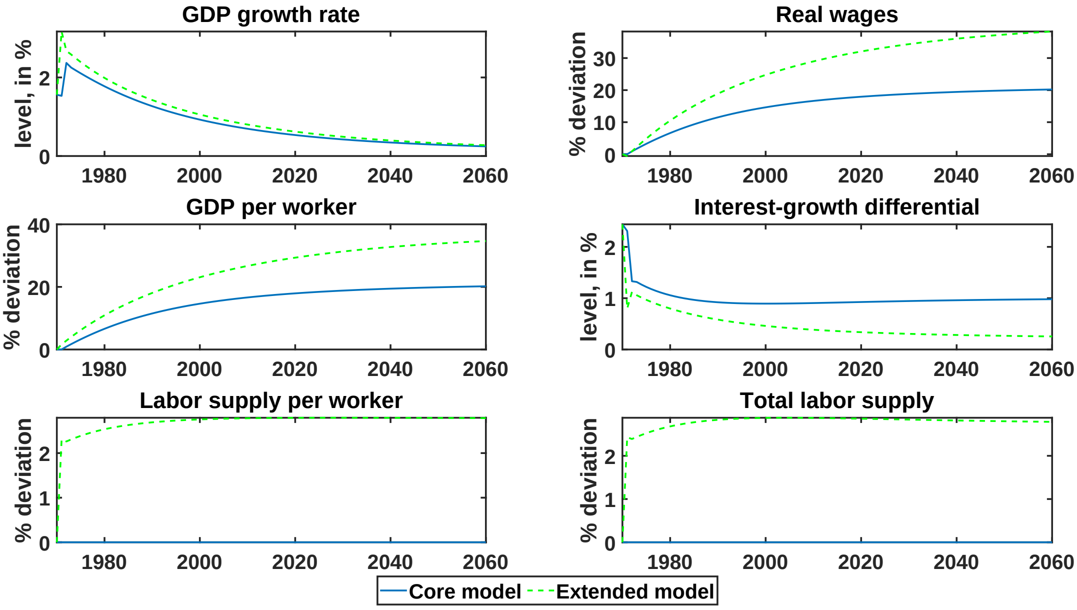
Notes: Figure is analogue to Figure 2 comparing the core and the extended model.

Figure C.3: Firm sector developments under alternative model specifications



Notes: Figure is analogue to Figure 3 comparing the core and the extended model.

Figure C.4: Macroeconomic developments under alternative model specifications



Notes: Figure is analogue to Figure 4 comparing the core and the extended model.