A Factor Structure of Disagreement

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Deutsche Bundesbank, 10/17/2019

Motivation

- Forecasters disagree about everything
- What are the aspects of the economy that they disagree most about?
- How should we model multivariate disagreement parsimoniously?
- Dominant way of thinking:
    - ignore multivariate structure of disagreement
    - dispersion statistics summarize disagreement (e.g. s.d.)
- But the data can take us much further

Example

- Dispersion of forecasts about inflation π and output y both increase in recessions:
  $\text{Cov}_t\left(\text{Var}_i\left(\hat{\pi}_{it}\right), \text{Var}_i\hat{y}_{it}\right) > 0$
- Cross-sectional correlation can tell us about the source of disagreement
- Example: Lorenzoni (2009)-type heterogenous information model
  - disagreement about demand shocks:
    $\text{Cov}_i\left(\hat{\pi}_{it}, \hat{y}_{it}\right) > 0$
  - disagreement about supply shocks:
    $\text{Cov}_i\left(\hat{\pi}_{it}, \hat{y}_{it}\right) < 0$
- Challenges:
  - Need some structure
  - Many forecast variables
  - Lots of missing data

Paper summary

Method:

- Estimate a factor structure on individual SPF forecasts using full-information Bayesian methods
- Factors extract the most important comovement relationships across variables
- Interpret with semi-structural model of heterogenous expectations

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Main findings:

- First two factors capture supply and demand side disagreement
  - supply disagreement more prominent before Great Moderation
  - demand disagreement more prominent in Great Recession
- Monetary policy disagreement plays minor role

Related Literature

- Disagreement:
  Lahiri and Sheng (2008), Patton and Timmermann (2010), Andrade and Le Bihan (2013), Dovern (2014), Andrade et al. (2016), Rich and Tracy (2017), Bordalo et al. (2018), ...
- Structural models of heterogenous expectations:
  Brock and Hommes (1997), Lorenzoni (2009), Melosi (2014), ...
- Theory-consistency of forecasts:
  Carvalho and Nechio (2014), Draeger et al. (2016), ...

Dynamic factor model

- Predictions of forecaster $i = 1, \ldots, n$ for variable j and horizon h at time t:

$$\hat{y}_{jt+h|it} = \bar{y}_{jt+h|t} + \sum_{k=1}^{p} \lambda_{jhk} f_{kit} + \xi_{ijht} \tag{1}$$

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- Average ("consensus") forecast $\bar{y}_{jt+h|t} = 1/ \mid \mathcal{I}_{jht} \mid \sum_{i \in \mathcal{I}_{jht}} \hat{y}_{jt+h|it}$ where $\mathcal{I}_{jht} \subseteq \{1, \ldots, n\}$ are non-missing observations

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- Separate factors f_{kit} for each forecaster with identical loadings Λ and:

$$f_{kit} = \phi_k f_{kit-1} + u_{kit}, \;\; u_{kit} \sim \mathcal{N}(0, 1) \tag{2}$$

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- All disturbances are iid across forecasters and variables.

2-D representation

- Dataset has 3 dimensions: time t, forecaster i, variable (j, h)
- Can stack forecasters to obtain 2-D DFM with restrictions:

$$\overset{mn\times 1}{\begin{pmatrix} \hat{y}_{|1t} \\ \hat{y}_{|2t} \\ \vdots \\ \hat{y}_{|nt} \end{pmatrix}} = \begin{pmatrix} I_m \\ I_m \\ \vdots \\ I_m \end{pmatrix} \bar{y}_t + \overset{mn\times pn}{\begin{pmatrix} \Lambda & 0 & \cdots & 0 \\ 0 & \Lambda & & 0 \\ \vdots & \vdots & & \ddots \\ 0 & 0 & \cdots & \Lambda \end{pmatrix}} \overset{pn\times 1}{\begin{pmatrix} f_{1t} \\ f_{2t} \\ \vdots \\ f_{nt} \end{pmatrix}} + \overset{mn\times 1}{\begin{pmatrix} \xi_{1t} \\ \xi_{2t} \\ \vdots \\ \xi_{nt} \end{pmatrix}}$$

$$\begin{pmatrix} f_{1t} \\ f_{2t} \\ \vdots \\ f_{nt} \end{pmatrix} = \begin{pmatrix} \Phi & 0 & \cdots & 0 \\ 0 & \Phi & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \Phi \end{pmatrix} \begin{pmatrix} f_{1t-1} \\ f_{2t-1} \\ \vdots \\ f_{nt-1} \end{pmatrix} + \begin{pmatrix} u_{1t} \\ u_{2t} \\ \vdots \\ u_{nt} \end{pmatrix}, \; u_t \sim \mathcal{N}(0, I_n \otimes I_p)$$

$$\begin{pmatrix} \xi_{1t} \\ \xi_{2t} \\ \vdots \\ \xi_{nt} \end{pmatrix} = \begin{pmatrix} P & 0 & \cdots & 0 \\ 0 & P & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & P \end{pmatrix} \begin{pmatrix} \xi_{1t-1} \\ \xi_{2t-1} \\ \vdots \\ \xi_{nt-1} \end{pmatrix} + \begin{pmatrix} v_{1t} \\ v_{2t} \\ \vdots \\ v_{nt} \end{pmatrix}, \; v_t \sim \mathcal{N}(0, I_n \otimes \Sigma)$$

Heterogenous information model

Consider a generic model of heterogenous information.

- The state and observation equations of the economy are:

$$\tilde{y}_t = C\tilde{x}_t + \tilde{\eta}_t,\ \tilde{\eta}_t \sim \mathcal{N}\left(0, I_m\right)$$
$$\tilde{x}_t = A\tilde{x}_{t-1} + B\tilde{\varepsilon}_t,\ \tilde{\varepsilon}_t \sim \mathcal{N}\left(0, I_q\right).$$

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- At time t, agents observe x_{t-1}, and receive signals about ε_{kt}, and η_{jt+h}.
- Signal about ε_{kt} for agent i has the form:

$$s_{\varepsilon ikt} = \tilde{\varepsilon}_{kt} + \tilde{u}_{\varepsilon ikt} + \tilde{\omega}_{\varepsilon ikt}$$
$$\tilde{u}_{\varepsilon ikt} = \rho_{\varepsilon k}\tilde{u}_{\varepsilon ikt-1} + \tilde{v}_{\varepsilon ikt}.$$

- $s_{\varepsilon ip+1t}, s_{\varepsilon ip+2t}, \ldots$ perfectly correlated across agents $\Rightarrow$ no disagreement.
- Signals about η_{jt+h} have analogous forms.

Mapping the factor structure

Forecasts of rational Bayesian forecasters have a factor structure:

$$\hat{y}_{jt+h|it} = \delta_{jht} + \sum_{k=1}^{p} \lambda_{jhk}\varepsilon_{ikt} + \eta_{ijht} \tag{4}$$

- Factors and idiosyncratic processes follow ARMA(1,2) processes
- Factor loadings $\propto$ IRFs of the shocks forecasters disagree about:

$$\lambda_{jhk} = C_{j.}A^{h}B_{.k} \tag{5}$$

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- Can generalize this to current heterogenous-information DSGE models (e.g. Lorenzoni, 2009):
  - unobserved x_{t-1}
  - correlation of signals across shocks
  - signals about states

Example: New-Keynesian model

- Consider the standard New-Keynesian model:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa y_t + u_t$$
$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n\right)$$
$$i_t = \phi_\pi \pi_t + \phi_y y_t + e_t$$

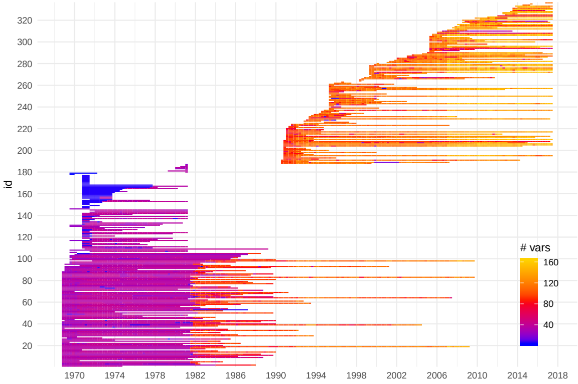
- Factor loadings λ:

Shock	y_{t+h}	π_{t+h}	i_{t+h}
supply u_t	(+)	(−)	(−)
demand r_t^n	(+)	(+)	(+)
monetary policy e_t	(+)	(+)	(−)

Data

Survey of Professional Forecasters (SPF):

- Quarterly survey 1968Q3-2018Q1
- about 30 forecasters per quarter
- many variables and forecast horizons
- missing data:
  - forecasters entry and exit
  - incomplete responses
  - variables and horizons added over the sample



Estimation

- Take all SPF variables at four-quarter and ten-year horizon
  - Data transformations follow Stock and Watson (2002)
- $p = 2$, no restrictions on loadings
  - factors identified by dynamic restrictions as long as $\phi_1 \neq \phi_2$
- Bayesian approach, group parameters of DFM into θ, then

$$p(\theta|Y) = \frac{p(Y|\theta)p(\theta)}{p(Y)}.$$

  - Elicit draws from posterior distribution via Gibbs sampling
  - Conjugate, uninformative priors

Posterior estimates (I)

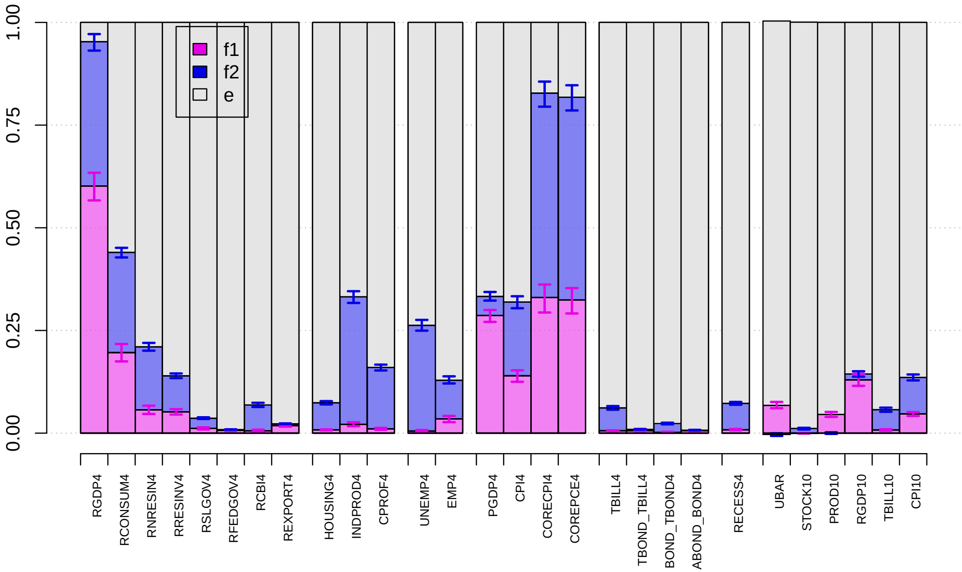
	mean $\Lambda_{\cdot 1}$	[5, 95]	mean $\Lambda_{\cdot 2}$	[5, 95]
RGDP4	0.79	[0.76, 0.81]	0.41	[0.37, 0.45]
RCONSUM4	0.56	[0.52, 0.59]	0.31	[0.28, 0.34]
RNRESIN4	0.92	[0.82, 1.03]	0.76	[0.68, 0.83]
RRESINV4	1.50	[1.32, 1.67]	1.12	[0.98, 1.26]
RSLGOV4	0.27	[0.22, 0.31]	0.15	[0.11, 0.18]
RFEDGOV4	0.26	[0.17, 0.35]	-0.06	[-0.12, 0.00]
RCBI4	0.03	[0.02, 0.04]	0.04	[0.03, 0.04]
REXPORT4	0.07	[0.05, 0.09]	-0.02	[-0.03, 0.00]
PGDP4	-0.39	[-0.41, -0.37]	0.12	[0.10, 0.15]
CPI4	-0.27	[-0.30, -0.24]	0.18	[0.16, 0.20]
CORECPI4	-0.25	[-0.29, -0.22]	0.18	[0.16, 0.20]
COREPCE4	-0.23	[-0.26, -0.20]	0.17	[0.15, 0.18]
UNEMP4	-0.03	[-0.04, -0.02]	-0.12	[-0.13, -0.12]
EMP4	0.15	[0.12, 0.18]	0.12	[0.10, 0.14]
TBILL4	-0.03	[-0.05, -0.00]	0.10	[0.08, 0.12]
TBONDTBILL4	0.04	[0.02, 0.06]	-0.01	[-0.03, 0.00]

Posterior estimates (II)

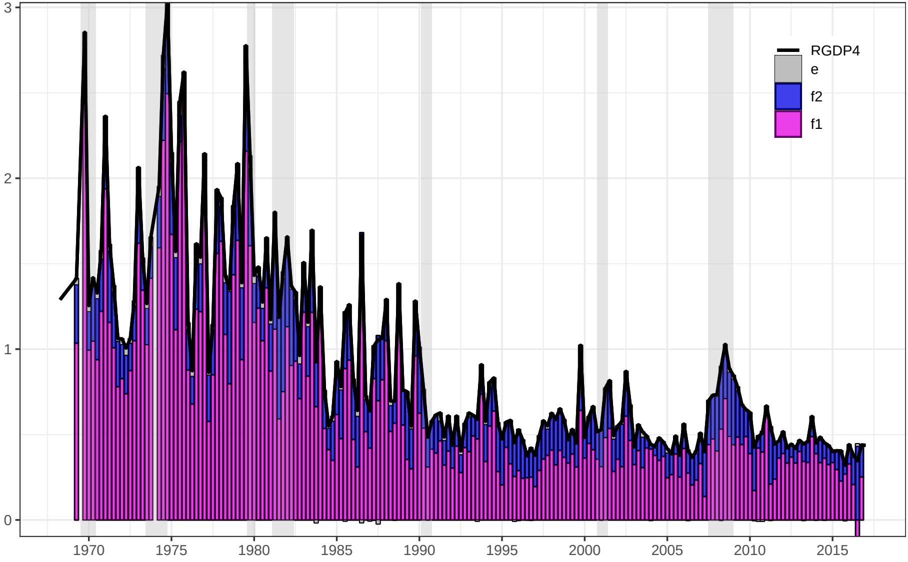
	mean $\Lambda_{\cdot 1}$	[5, 95]	mean $\Lambda_{\cdot 2}$	[5, 95]
BONDTBOND4	-0.04	[-0.05, -0.02]	-0.04	[-0.05, -0.03]
BAABONDBOND4	-0.03	[-0.05, 0.00]	-0.01	[-0.03, 0.01]
HOUSING4	0.50	[0.31, 0.70]	1.54	[1.35, 1.73]
INDPROD4	0.20	[0.15, 0.24]	0.61	[0.57, 0.65]
CPROF4	0.41	[0.26, 0.56]	1.37	[1.24, 1.50]
RECESS4	-0.98	[-1.24, -0.72]	-1.88	[-2.13, -1.63]
UBAR	-0.14	[-0.20, -0.08]	-0.03	[-0.07, 0.00]
STOCK10	0.04	[-0.16, 0.27]	0.12	[0.01, 0.24]
PROD10	0.13	[0.08, 0.17]	0.03	[0.00, 0.05]
RGDP10	0.18	[0.14, 0.21]	0.05	[0.03, 0.07]
TBILL10	-0.07	[-0.16, 0.03]	0.13	[0.08, 0.18]
CPI10	-0.09	[-0.11, -0.07]	0.08	[0.07, 0.10]

	mean	[5, 95]
ϕ_1	0.45	[0.42, 0.49]
ϕ_2	0.80	[0.78, 0.82]

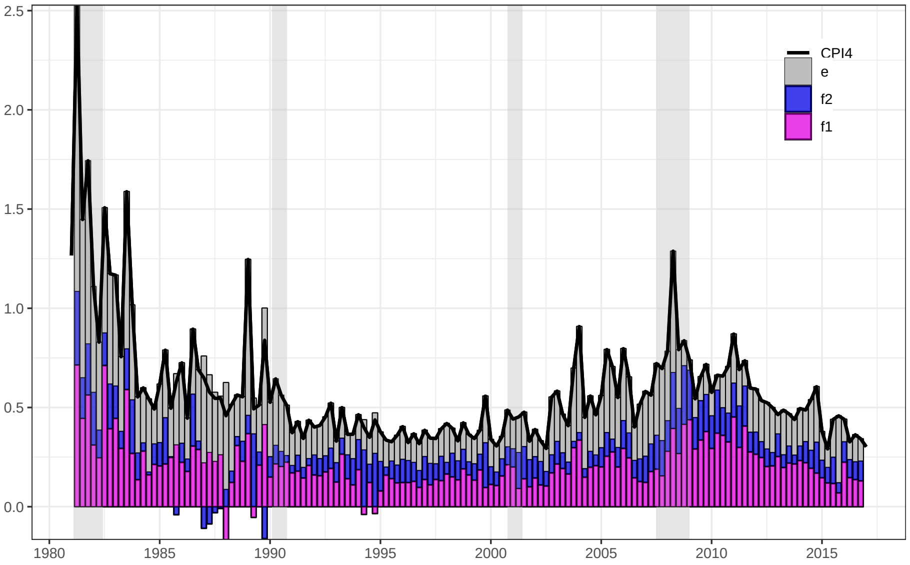
Variance decomposition



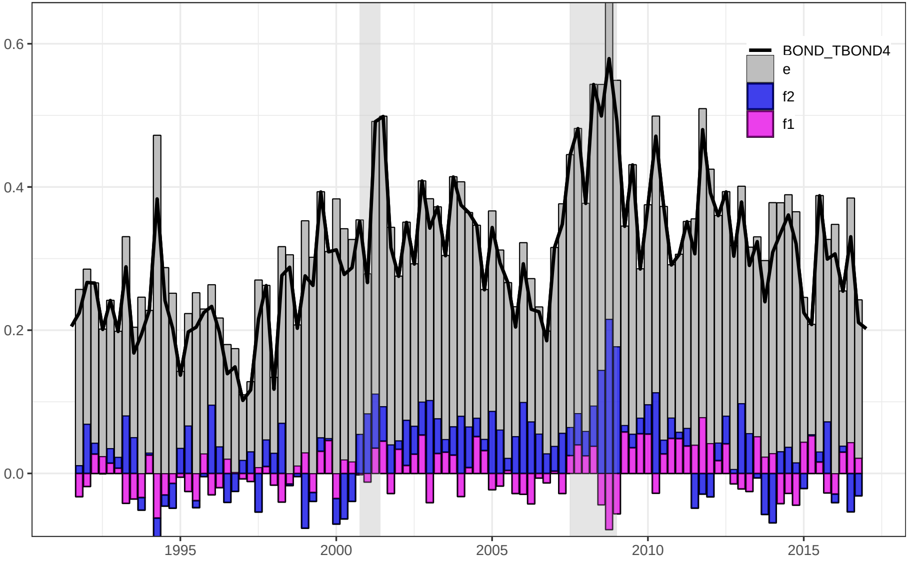
Decomposition of dispersion: Real GDP



Decomposition of dispersion: Inflation



Decomposition of dispersion: Corporate bond spread



Dispersion of factors

Subsamples

Subsample 1:
1968Q3–1984Q4

mean	$\Lambda_{\cdot 1}$	$\Lambda_{\cdot 2}$
RGDP4	0.73	0.77
CPROF4	-0.55	2.26
UNEMP4	0.01	-0.18
PGDP4	-0.81	-0.10
TBILL4	0.02	-0.09

- supply disagreement dominant
- interest rates little related to factors

Subsample 2:
1985Q1–2008Q2

mean	$\Lambda_{\cdot 1}$	$\Lambda_{\cdot 2}$
RGDP4	0.32	0.31
CPROF4	0.95	0.77
UNEMP4	-0.03	-0.10
PGDP4	-0.06	0.03
TBILL4	-0.23	0.32

- inflation disagreement less related to output
- interest rates respond more strongly

Subsample 3:
2008Q3–2016Q4

mean	$\Lambda_{\cdot 1}$	$\Lambda_{\cdot 2}$
RGDP4	0.22	0.27
CPROF4	0.80	0.77
UNEMP4	-0.01	-0.11
PGDP4	-0.16	0.16
TBILL4	-0.04	0.07

- demand disagreement more important
- interest rates respond less strongly again

Conclusion

- parsimonious factor structure of individual-level forecasts
- extracted factor loadings capture comovement of disagreement across variables
- interpretation with semi-structural model
- results:
  - supply disagreement dominates before Great Moderation, demand disagreement afterwards and during Great Recession
  - monetary policy disagreement not important
- next steps:
  - include more forecast horizons
  - optimal number of factors
  - apply to other datasets