Understanding HANK: Insights from a PRANK

Sushant Acharya Keshav Dogra

Federal Reserve Bank of New York

October 2019

Motivation

- Huge interest in how heterogeneity, incomplete markets affect aggregate outcomes

- Which features of market incompleteness can "solve RANK puzzles"?
  - Determinacy of equilibrium
  - Forward guidance too strong?
  - Fiscal spending multipliers too big at ZLB?

- Which features of HANKs $\Rightarrow$ difference from RANKs?
  - precautionary savings motive?
  - MPC heterogeneity?

Environment

- We use a tractable model to explain the distinct effects of
  - precautionary savings and the cyclicality of risk
  - MPC heterogeneity and the cyclicality of HTM income

  on determinacy, forward guidance puzzle, spending multipliers

- CARA utility + idiosyncratic income risk $\rightarrow$ linear aggregation (**P**seudo-**R**epresentative-**ANK**)
  - exact aggregate Euler equation
  - no need to keep track of wealth distribution

- Isolate the effect of cyclicality of risk, since MPC heterogeneity is wholly absent in our baseline (but we can put it back in)

Related literature

- quantitative models: Kaplan et al. (2018), McKay et al. (2016)
- stylized "zero-liquidity limit" models: Werning (2015), Ravn and Sterk (2018), McKay et al. (2017), Debortoli and Galí (2018), Bilbiie (2008, 2019a,b)
- MPC heterogeneity, sufficient statistics approach, determinacy of equilibrium - numerical: Auclert et al. (2018)

Household problem

Discrete time, no aggregate risk, measure 1 of households solve

$$\max_{\{c_t^i, A_{t+1}^i\}_{t=0}^{\infty}} \quad -\frac{1}{\gamma}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t e^{-\gamma c_t^i}$$

subject to

$$P_t c_t^i + \frac{1}{1+i_t} A_{t+1}^i = A_t^i + P_t \overbrace{\left[(1-\tau_t)\,\omega_t \ell_t^i + d_t + \frac{T_t}{P_t} \right]}^{y_t^i}$$

$$\ell_t^i \sim \text{i.i.d.} N\left(1, \sigma_\ell^2(y_t)\right)$$

Firms

- combine labor, Dixit-Stiglitz aggregate of intermediates inputs $M_t(j)$ to produce

$$x_t(j) = z m_t(j)^{\alpha} n_t(j)^{1-\alpha}$$

- Net output in symmetric eq'm is defined as: $Y_t = x_t - x_t^{\frac{1}{\alpha}}$

- face Rotemberg (1982) costs of price adjustment, max

$$\sum_{s=0}^{\infty} Q_{t|0} \left\{ \left(\frac{P_t(k)}{P_t} - mc_t \right) \left(\frac{P_t(k)}{P_t} \right)^{-\theta} - \frac{\Psi}{2} \left(\frac{P_t(k)}{P_{t-1}(k)} - 1 \right)^2 \right\} x_t$$

where $Q_{t|0} = \prod_{k=0}^{t-1} \frac{1}{1+r_k}$ and $mc_t = \frac{\omega_t^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}$

Policy

- Monetary policy:

$$1 + i_t = (1 + r)\Pi_t^{\phi_\pi}$$

given steady state real interest rate $1 + r$

- Fiscal policy

$$B_t + P_t g_t + T_t = P_t \tau_t \omega_t + \frac{1}{1 + i_t} B_{t+1}$$

- $\tau_t = \boldsymbol{\tau}(Y_t)$, lump-sum transfers T_t adjust as needed to ensure fiscal solvency: fiscal policy is 'passive' (Leeper, 1991)

Household decisions

$$c_t^i = \mathcal{C}_t + \mu_t \left(\frac{A_t^i}{P_t} + y_t^i \right)$$

Household decisions

$$c_t^i = \color{red}{\mathcal{C}_t} + \mu_t \left(\frac{A_t^i}{P_t} + y_t^i \right)$$

$$\mathcal{C}_t = \underbrace{\sum_{s=1}^{\infty} Q_{t+s|t} \frac{\mu_t}{\gamma \mu_{t+s}} \ln \left[\frac{1}{\beta \left(1 + r_{t+s-1}\right)} \right]}_{\text{impatience}} + \underbrace{\mu_t \sum_{s=1}^{\infty} Q_{t+s|t} \bar{y}_{t+s}}_{\text{PIH}} - \underbrace{\frac{\gamma \mu_t}{2} \sum_{s=1}^{\infty} Q_{t+s|t} \mu_{t+s} \sigma_{y,t+s}^2}_{\text{precautionary savings}}$$

Household decisions

$$c_t^i = \mathcal{C}_t + \mu_t \left(\frac{A_t^i}{P_t} + y_t^i \right)$$

$$\mathcal{C}_t = \sum_{s=1}^{\infty} Q_{t+s|t} \frac{\mu_t}{\gamma \mu_{t+s}} \ln \left[\frac{1}{\beta \left(1 + r_{t+s-1}\right)} \right] + \mu_t \sum_{s=1}^{\infty} Q_{t+s|t} \bar{y}_{t+s} - \frac{\gamma \mu_t}{2} \sum_{s=1}^{\infty} Q_{t+s|t} \mu_{t+s} \sigma_{y,t+s}^2$$

MPC: $$\mu_t = \frac{\mu_{t+1} \left(1 + r_t\right)}{1 + \mu_{t+1} \left(1 + r_t\right)}$$

- if $r_t = r$ for all t, $\mu_t = \frac{r}{1+r}$ precautionary savings

Aggregation

- Model linearly aggregates:

$$c_t = \int_0^1 c_t^i di = \mathcal{C}_t + \mu_t y_t$$

- Impose goods market clearing + use Govt. BC: "Aggregate Euler equation"

$$y_t = y_{t+1} - \frac{\ln \beta \left(1 + r_t\right)}{\gamma} - \frac{\gamma \mu_{t+1}^2}{2} \boldsymbol{\sigma}^2 \left(y_{t+1}\right) + g_t - g_{t+1}$$

The cyclicality of income risk

In equilibrium, y_t^i is i.i.d. with variance

$$\boldsymbol{\sigma}^{\mathbf{2}}(y_t) = \left[\Big(1 - \boldsymbol{\tau}(y_t)\Big)\boldsymbol{\omega}(y_t)^{1/\alpha}\right]^2 \boldsymbol{\sigma}_{\boldsymbol{\ell}}^{\mathbf{2}}(y_t)$$

so cyclicality of income risk $\dfrac{d\boldsymbol{\sigma}^{\mathbf{2}}(y)}{dy}$ equals

$$2\boldsymbol{\sigma}(y)\boldsymbol{\sigma}_{\boldsymbol{\ell}}(y)\left\{\underbrace{\left(1 - \boldsymbol{\tau}\left(Y\right)\right)\boldsymbol{\omega}'\left(y\right)}_{\substack{\text{cyclicality of}\\ \text{real wages}}} - \underbrace{\boldsymbol{\tau}'\left(y\right)\boldsymbol{\omega}\left(y\right)}_{\substack{\text{cyclicality of}\\ \text{taxes}}}\right\} + \underbrace{\frac{\boldsymbol{\sigma}^{\mathbf{2}}\left(y\right)}{\boldsymbol{\sigma}_{\boldsymbol{\ell}}^{\mathbf{2}}\left(y\right)}\frac{d\boldsymbol{\sigma}_{\boldsymbol{\ell}}^{\mathbf{2}}(y)}{dy}}_{\substack{\text{cyclicality of}\\ \text{employment risk}}}$$

endogenous - depends on tax-transfer system

Linearized demand block

$$\hat{y}_t = \left[1 - \frac{\gamma\mu^2}{2}\frac{d\boldsymbol{\sigma}^{\mathbf{2}}(\boldsymbol{y}^*)}{dY}\right]\hat{y}_{t+1} - \frac{1}{\gamma}(i_t - \pi_{t+1}) - \gamma\mu\boldsymbol{\sigma}(y^*)\hat{\mu}_{t+1}$$

$$\hat{\mu}_t = \tilde{\beta}\hat{\mu}_{t+1} + \tilde{\beta}(i_t - \pi_{t+1})$$

Linearized demand block

$$\begin{aligned}\hat{y}_t &= \Theta \hat{y}_{t+1} - \frac{1}{\gamma}(i_t - \pi_{t+1}) - \Lambda \hat{\mu}_{t+1} \\ \hat{\mu}_t &= \tilde{\beta}\hat{\mu}_{t+1} + \tilde{\beta}(i_t - \pi_{t+1})\end{aligned}$$

where

$$\Theta = 1 - \frac{\gamma \mu^2}{2} \frac{d\boldsymbol{\sigma^2(y^*)}}{dy} \qquad \text{and} \qquad \Lambda = \gamma\mu\boldsymbol{\sigma}(y^*)$$

Linearized demand block

$$\begin{aligned}\hat{y}_t &= \Theta \hat{y}_{t+1} - \frac{1}{\gamma}(i_t - \pi_{t+1}) - \Lambda \hat{\mu}_{t+1} \\ \hat{\mu}_t &= \tilde{\beta}\hat{\mu}_{t+1} + \tilde{\beta}(i_t - \pi_{t+1})\end{aligned}$$

where

$$\Theta = 1 - \frac{\gamma \mu^2}{2} \frac{d\boldsymbol{\sigma^2(y^*)}}{dy} \qquad \text{and} \qquad \Lambda = \gamma \mu \boldsymbol{\sigma}(y^*)$$

- RANK $(\sigma = 0)$: $\Theta = 1$, $\Lambda = 0$

Linearized demand block

$$\hat{y}_t = \Theta \hat{y}_{t+1} - \frac{1}{\gamma}(i_t - \pi_{t+1}) - \Lambda \hat{\mu}_{t+1}$$
$$\hat{\mu}_t = \tilde{\beta}\hat{\mu}_{t+1} + \tilde{\beta}(i_t - \pi_{t+1})$$

where

$$\Theta = 1 - \frac{\gamma \mu^2}{2} \frac{d\boldsymbol{\sigma}^2(\boldsymbol{y}^*)}{dy} \qquad \text{and} \qquad \Lambda = \gamma \mu \boldsymbol{\sigma}(y^*)$$

- RANK $(\sigma = 0)$: $\Theta = 1$, $\Lambda = 0$
- Procyclical risk $\left(\frac{d\boldsymbol{\sigma}^2}{dy} > 0\right)$: $\Theta < 1$, discounted Euler eq
- Acyclical risk $\left(\frac{d\boldsymbol{\sigma}^2}{dy} = 0\right)$: $\Theta = 1$, but still $\Lambda > 0$: precautionary savings channel
- Countercyclical risk $\left(\frac{d\boldsymbol{\sigma}^2}{dy} < 0\right)$: $\Theta > 1$, explosive Euler eq

Linearized supply block

Standard Phillips curve, Taylor rule:

$$\begin{aligned} \pi_t &= \kappa \hat{y}_t + \tilde{\beta}\pi_{t+1} \\ i_t &= \Phi_\pi \pi_t \end{aligned}$$

where $\tilde{\beta} = \frac{1}{1+r}$

Determinacy under a peg $(\Phi_\pi = 0)$ in the rigid price limit $\pi_t = 0$

$$\hat{y}_t = \Theta \hat{y}_{t+1} - \frac{1}{\gamma} i_t - \Lambda \hat{\mu}_{t+1}$$
$$\hat{\mu}_t = \tilde{\beta} \hat{\mu}_{t+1} + \tilde{\beta} i_t$$

Determinacy under a peg ($\Phi_\pi = 0$) in the rigid price limit $\pi_t = 0$

$$\hat{y}_t = \Theta \hat{y}_{t+1}$$

Determinacy under a peg ($\Phi_\pi = 0$) in the rigid price limit $\pi_t = 0$

$$\hat{y}_{t+1} = \Theta^{-1}\hat{y}_t$$

Does a unique bounded $\{\hat{y}_t\}$ solve this? YES (determinacy), NO (indeterminacy)

Determinacy under a peg $(\Phi_\pi = 0)$ in the rigid price limit $\pi_t = 0$

$$\hat{y}_{t+1} = \Theta^{-1}\hat{y}_t$$

Does a unique bounded $\{\hat{y}_t\}$ solve this? YES (determinacy), NO (indeterminacy)

- HANK - acyclical risk $(\Theta = 1)$/RANK: Indeterminacy
- HANK - procyclical risk $(\Theta < 1)$: Determinacy
- HANK - countercyclical risk $(\Theta > 1)$: Indeterminacy

An income risk-adjusted Taylor principle

With sticky prices and Taylor rule, equilibrium is locally determinate if

$$\Phi_\pi > 1 + \frac{\gamma}{\kappa}\left[\frac{\left(1-\tilde{\beta}\right)^2}{\left(1-\tilde{\beta}\right)+\gamma\tilde{\beta}\Lambda}\right](\Theta - 1)$$

- procyclical risk ($\Theta < 1$): determinacy more likely (Auclert et al., 2018)
- acyclical risk ($\Theta = 1$): determinacy requires $\Phi_\pi > 1$ as in RANK
- countercyclical risk ($\Theta > 1$): determinacy less likely (Ravn and Sterk, 2018)

Forward guidance

- Suppose Fed announces at t a rate cut at date $t+k$
- In RANK

$$\hat{y}_t = -\frac{1}{\gamma}\sum_{k=0}^{\infty}(i_{t+k} - \pi_{t+k+1})$$

- With fixed prices, date $t+k$ rate cut equally as effective as date t cut
- With sticky prices, date $t+k$ rate cut more effective than date t cut
  - 'forward guidance puzzle' (Del Negro et al., 2015)

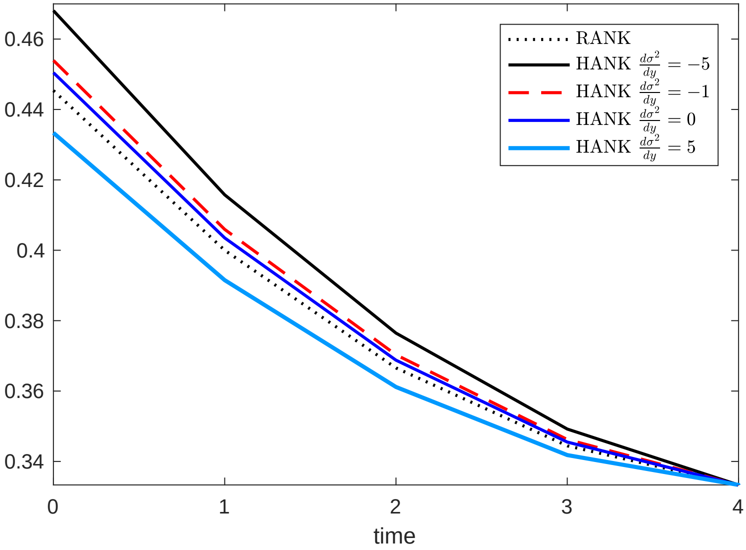
Forward guidance

- In HANK

$$\hat{y}_t = -\frac{1}{\gamma}\sum_{k=0}^{\infty}\Theta^k(i_{t+k} - \pi_{t+k+1}) - \Lambda\sum_{k=0}^{\infty}\Theta^k\sum_{s=1}^{\infty}\tilde{\beta}(i_{t+k+s} - \pi_{t+k+s+1})$$

- With fixed prices:
  - with sufficiently procyclical risk ($\Theta << 1$), date $t+k$ rate cut less effective than date t rate cut
  - with acyclical risk ($\Theta = 1$), date $t+k$ rate cut more effective (precautionary savings channel)
    - Lower future $r_t \Rightarrow \mu_t \downarrow$. Lower pass through of income risk into consumption risk, weakens precautionary savings motive.
  - with countercyclical risk ($\Theta > 1$), date $t+k$ rate cut more effective

Response of y_t to cut in i_t 5 periods in the future



Fiscal multipliers

- Consider liquidity trap lasting T periods, $\hat{g}_t = g > 0$ during trap, zero thereafter
- In RANK:
  - with fixed prices

$$\frac{\partial \hat{y}_t}{\partial g} = 1, 0 \leq t \leq T$$

    independent of duration of trap

  - With sticky prices, multiplier increasing in duration of trap ($\mathbb{E}\pi$ channel)

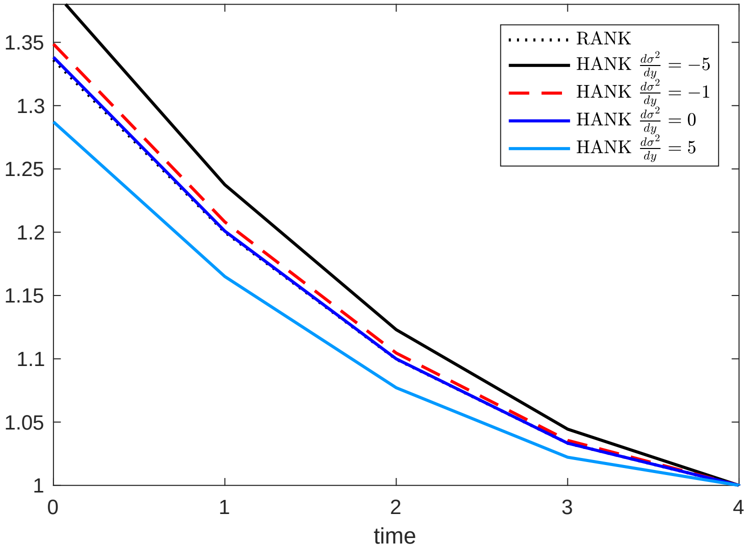
Fiscal multipliers

- In HANK with fixed prices:

$$\frac{\partial \hat{y}_t}{\partial g} = \Theta^{T-t-1}, 0 \leq t \leq T$$

  - with procyclical risk ($\Theta < 1$), decreasing in duration of trap
  - with acyclical risk ($\Theta = 1$), independent of duration of trap
  - with countercyclical risk ($\Theta > 1$), increasing in duration of trap
- With sticky prices...

$\frac{d\hat{y}_t}{dg}$ in a 10 period liquidity trap



Introducing MPC heterogeneity

- Suppose $\eta \in (0,1)$ households hand to mouth, income $y_t^i = \chi y_t$ (Bilbiie, 2008)
    - $\frac{dy_t^i}{dy_t} = \chi$: cyclical sensitivity of income of constrained $\chi \neq 1$, e.g., fiscal transfers
    - Avg. MPC $= (1-\eta) \times \mu_t + \eta \times 1 > \mu_t$
- Aggregate Euler eq becomes

$$y_t = y_{t+1} - \frac{\Xi}{\gamma} \ln(\beta(1+r_t)) - \Xi \frac{\gamma \mu_{t+1}^2 \boldsymbol{\sigma}^2(y_t)}{2}, \qquad \Xi = \frac{1-\eta}{1-\eta\chi}$$

- Resource constraint:

$$y_t = c_t = \eta\chi y_t + (1-\eta) c_t^u \qquad \Rightarrow \qquad y_t = \Xi c_t^u$$

Ξ is 'static' response of GDP to consumption of unconstrained

MPC heterogeneity

- direct effect of unit increase in c_t^u:

$$\Delta y_t^{\text{direct effect}} = \Delta c_t^u = 1 - \eta$$

- increases total income and consumption of constrained $\eta \times \chi(1-\eta)$
- and so on ...
- total effect:

$$\Delta y^{\text{total effect}} = 1 - \eta + \eta\chi(1-\eta) + \ldots = \frac{1-\eta}{1-\eta\chi} = \Xi$$

Affects contemporaneous response to r_t

$$y_t = y_{t+1} - \frac{\Xi}{\gamma}\ln(\beta(1+r_t)) - \Xi\frac{\gamma\mu_{t+1}^2\boldsymbol{\sigma}^2(y_t)}{2}, \qquad \Xi = \frac{1-\eta}{1-\eta\chi}$$

- HTM income less cyclically sensitive ($\Xi < 1$): dampens response to interest rates
- HTM income equally cyclically sensitive ($\Xi = 1$): no effect
- HTM income more cyclically sensitive ($\Xi > 1$): stronger response to interest rates

cyclicality of risk does not affect this (contra Werning (2015))

..but has less effect on determinacy and 'puzzles'

Linearizing:

$$\hat{y}_t = -\frac{\Xi}{\gamma}(i_t - \pi_{t+1}) + \underbrace{(\Xi\Theta + 1 - \Xi)}_{\tilde{\Theta}}\hat{y}_{t+1} - \Xi\Lambda\hat{\mu}_{t+1}$$

- MPC heterogeneity does not affect determinacy
- FGP: affects response to interest rates at all horizons, but not the slope
  - If $\Xi = 1$, then $\widetilde{\Theta} = \Theta$
  - If $\Xi < 1$, then $\widetilde{\Theta}$ is a linear combination of Θ and 1
  - If $\Xi > 1$, then $\widetilde{\Theta}$ closer to 1 than Θ

Fiscal policy

- Both cyclicality of risk Θ and cyclical sensitivity of HTM income Ξ depend crucially on fiscal policy
  - different tax-transfer scheme can change Θ, Ξ and thus change transmission mechanism
- This channel of fiscal policy is distinct from others:
  - active fiscal (FTPL)
  - passive fiscal but Δr_t requires changes in surpluses, and how surpluses are adjusted affects outcomes in non-Ricardian economies (Kaplan et al., 2018)

Conclusion

- Whether and how HANKs differ from RANK depends on both cyclicality of risk and MPC heterogeneity/cyclical sensitivity of HTM income

- They have different effects
  - procyclical risk makes determinacy more likely, moderates FGP, reduces multipliers; countercyclical risk does the opposite
  - MPC heterogeneity reduces contemporaneous response to r_t if HTM income less cyclical; increases it if HTM income more cyclical

- Both depend crucially on fiscal policy

- Very tractable framework. Easy extensions to persistent idiosyncratic income

- Acharya, Challe and Dogra (2019) study optimal monetary policy in similar environment + endogenous labor supply. cyclicality of risk: key determinant in how monetary policy should respond

END

References

Auclert, A., M. Rognlie, and L. Straub (2018): "The Intertemporal Keynesian Cross," Tech. rep.

Bilbiie, F. O. (2008): "Limited asset markets participation, monetary policy and (inverted) aggregate demand logic," Journal of Economic Theory, 140, 162–196.

——— (2019a): "Monetary Policy and Heterogeneity: An Analytical Framework," Tech. rep.

——— (2019b): "The New Keynesian cross," Journal of Monetary Economics.

Cagetti, M. (2003): "Wealth Accumulation over the Life Cycle and Precautionary Savings," Journal of Business & Economic Statistics, 21, 339–353.

Christelis, D., D. Georgarakos, T. Jappelli, and M. v. Rooij (forthcoming): "Consumption Uncertainty and Precautionary Saving," The Review of Economics and Statistics.

Debortoli, D. and J. Galí (2018): "Monetary Policy with Heterogeneous Agents: Insights from TANK models," Tech. rep.

Del Negro, M., M. Giannoni, and C. Patterson (2015): "The forward guidance puzzle," Staff Reports 574, Federal Reserve Bank of New York.

Fagereng, A., L. Guiso, and L. Pistaferri (2017): "Firm-Related Risk and Precautionary Saving Response," American Economic Review, 107, 393–97.

Guvenen, F., S. Ozkan, and J. Song (2014): "The Nature of Countercyclical Income Risk," Journal of Political Economy, 122, 621–660.

Kaplan, G., B. Moll, and G. L. Violante (2018): "Monetary Policy According to HANK," American Economic Review, 108, 697–743.

Leeper, E. M. (1991): "Equilibria under 'active' and 'passive' monetary and fiscal policies," Journal of Monetary Economics, 27, 129–147.

McKay, A., E. Nakamura, and J. Steinsson (2016): "The Power of Forward Guidance Revisited," American Economic Review, 106, 3133–58.

——— (2017): "The Discounted Euler Equation: A Note," Economica, 84, 820–831.

Ravn, M. O. and V. Sterk (2018): "Macroeconomic Fluctuations with HANK & SAM: An Analytical Approach," Tech. rep.

Rotemberg, J. J. (1982): "Monopolistic Price Adjustment and Aggregate Output," Review of Economic Studies, 49, 517–531.

Schorfheide, F. (2008): "DSGE model-based estimation of the New Keynesian Phillips curve," Economic Quarterly, 397–433.

Storesletten, K., C. Telmer, and A. Yaron (2004): "Cyclical Dynamics in Idiosyncratic Labor Market Risk," Journal of Political Economy, 112, 695–717.

Werning, I. (2015): "Incomplete Markets and Aggregate Demand," Working Paper 21448, National Bureau of Economic Research.

Strength of precautionary savings motive

Unlike zero-liquidity models: distinction between consumption and income risk.

- hh consumes μ_t of additional dollar at date t, saves $1-\mu_t$

$$dc_t^i = \mu_t \qquad \text{and} \qquad da_{t+1}^i = (1+r_t)(1-\mu_t) \qquad \text{and} \qquad dc_{t+1}^i = \mu_{t+1} da_{t+1}^i$$

- consumption smoothing $dc_t^i = dc_{t+1}^i \Rightarrow \mu_t = \frac{\mu_{t+1}(1+r_t)}{1+\mu_{t+1}(1+r_t)}$
- $\mu_t \uparrow$ when temp. higher path of interest rates in future $\mu_t = \left(\sum_{s=0}^{\infty} Q_{t+s|t}\right)^{-1}$
- when r_t high, curr. inc. larger fraction of lifetime inc. $\Rightarrow c_t^i$ responds more to y_t^i.

$$y_{i,t}^p = \frac{1}{\sum_{s=0}^{\infty} Q_{t+s|t}} y_t^i + \sum_{s=1}^{\infty} \left(\frac{Q_{t+s|t}}{\sum_{s=0}^{\infty} Q_{t+s|t}} \right) \mathbb{E}_t y_{t+s}^i$$

- mon. pol. affects pass-through of income risk to consumption risk back

Calibration

- Normalize $y^* = 1$ in steady state
- annual frequency, $\sigma_y = 0.5$ (Guvenen et al., 2014)
- $\kappa = 0.1$ (Schorfheide, 2008)
- coefficient of relative/absolute prudence $\gamma = 3$ (Cagetti, 2003; Fagereng et al., 2017; Christelis et al., forthcoming)
- $r = 4\%$
- range of values for $d\sigma^2/dy$, baseline -1 (Storesletten et al., 2004)

Phillips Curve

$$\Psi \Pi_t \left(\Pi_t - 1 \right) = 1 - \theta \left(1 - x_t^{\frac{1-\alpha}{\alpha}} \right) + \Psi \left(\Pi_{t+1} - 1 \right) \Pi_{t+1} \left[\frac{1}{1 + r_t} \frac{x_{t+1}}{x_t} \right]$$