Explanatory notes

Yields

Unlike the nominal interest rate, the bond yield indicates the interest actually received per annum. Its calculation takes account of all the factors influencing the earnings from a debt security. These comprise not only the nominal interest rate but also the frequency of interest payments, the purchase and redemption price, maturity and the mode of redemption (falling due en bloc, falling due in instalments). Yields calculated in this way permit a comparison of the interest actually received on different bonds or a comparison between bonds and other forms of investment (eg savings deposits, the yield on which is wholly dependent on the interest rate).

The yield statistics cover debt securities issued by residents with a maximum maturity according to the terms of issue of over four years. Structured Products and the like, debt securities with unscheduled redemptions, zero coupon bonds, floating rate notes and bonds not denominated in Euro are not included. The yields of redeemable issues are based on the computed residual maturity. The calculation of total yield and of yield by type of security covers only securities with a residual maturity of more than three years. Group yields are weighted by the amounts outstanding at market prices (up to December 2001, by nominal amounts outstanding) or (in the case of issue yields) the amounts sold of the debt securities included in the calculation. Owing to the monthly changes in the composition of the debt securities included in the calculation, the movement of the changes in the interest rate level but - particularly in the computed yield is to be attributed not only to movement case of the issue yields - also to structural influences (e.g. changes in the maturity pattern). The monthly figures on yields outstanding are calculated on the basis of the yields based on the XETRA prices on all the trading days in a month; up to 23 May 2011 on the basis of spot rates. The annual figures are the unweighted means of the monthly figures; 12 monthly figures were not always available for the computation, especially in the case of issue yields.

Term structure of interest rates

The term structure of interest rates in the bond market shows the relation between the interest rates and maturities of zero coupon bonds. The data on the term structure of interest rates published here are estimates derived from the observed yields to maturity of coupon bonds outstanding. In contrast to the implied assumption calculating yields to maturity that all payment flows of a coupon bond carry the same rate of return (namely the yield to maturity), the estimation of the term structure of interest rates assumes a different rate of return for each payment flow of a coupon bond at the interest rate corresponding to the current market conditions on the respective payment date. The individual payment flows of a coupon bond are regarded as redemptions of zero coupon bonds with different maturities and interest rates. The prices and interest rates of these notional individual zero coupon bonds are unknown, however, since they are only traded as a bundle in the form of the coupon bond. The price of the coupon which is interpreted as the aggregate price of the component zero coupon bonds, reflects the interest rate expectations of market participants which are to be captured in the envisaged term structure of interest rates. If the individual payment flows of a coupon bond were discounted at the respective interest rates of this (unknown) term structure of interest rates, the sum of the present values should theoretically equal the market price of the coupon bond and hence also its market yield.

This being so, the term structure of interest rates can be calculated with the help of a non-linear optimisation procedure. In this process, the individual payment flows of the coupon bonds are first discounted at the interest rates of a tentatively specified term structure and the notional coupon bond yields to maturity which are derived from the sum of the present values of the payment flows are compared with the actual yields to maturity observed in the market. The specified term structure is varied until the deviations between the notional yields to maturity and the market yields of the coupon bonds included in the estimate are minimised. The term structure of interest rates thus derived then approximately matches the term structure in the bond market which determines the market prices of coupon bonds.

$$z(T, \beta, \tau) = \beta_0 + \beta_1 \left(\frac{1 - \exp(-T/\tau_1)}{(T/\tau_1)} \right)$$

$$+ \beta_2 \left(\frac{1 - \exp(-T/\tau_1)}{(T/\tau_1)} - \exp(-T/\tau_1) \right)$$

$$+ \beta_3 \left(\frac{1 - \exp(-T/\tau_2)}{(T/\tau_2)} - \exp(-T/\tau_2) \right)$$

The estimation of the term structure of interest rates is based on listed Federal bonds, five-year Federal notes and Federal Treasury notes as well as on listed mortgage and public Pfandbriefe. These securities are largely homogeneous. To avoid distortions at the short end of the term structure, securities with a residual maturity (time to maturity) of less than three months are not included. However, the inclusion of bonds with a residual maturity of between three months and one year ensures that the one-year interest rate can be reliably estimated.

For the purpose of the estimation, an assumption is made about the functional relation between interest rates and residual maturities. In the estimation approach used here, the interest rate is defined as the sum of a constant and various exponential terms (where the residual maturity has a negative sign in the exponent) and as a function of a total of six parameters: where $z\,(T,\,\beta,\,\nu)$ denotes the interest rate for maturity T in years as a function of the parameter vectors $(\beta_0,\,\beta_1,\,\beta_2,\,\beta_3)$ and $(\tau_1,\,\tau_2)$ denotes the parameters to be estimated.

This parametric approach is sufficiently flexible to reflect the data constellations observed in the market. These include monotonically rising, declining, U-shaped, inverted U-shaped and S-shaped curves. Unlike non-parametric approaches, this estimation procedure smooths out individual kinks in the curve so that the estimation results are relatively little influenced by individual observations. This makes them less suited to identify, for example, abnormalities in individual maturity segments or in individual securities. However, they provide curves which are relatively free of outliers and thus are easier to interpret for monetary policy analysis.

For further details of the process of estimating term structure data see Deutsche Bundesbank, Estimating the term structure of interest rates, Monthly Report, October 1997, pages 61-66.