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Estimating regional wealth in Germany: How different are east and west really?

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Non-technical summary

Research Question

This paper provides estimates of households' average private net wealth at the regional level in Germany. Thereby, the analysis is focused on differences between regions located in eastern and western Germany 25 years after German reunification.

Contribution

Using the second wave of the Bundesbank's Panel on Household Finances (PHF) conducted in 2014, we contribute to the literature by estimating the regional distribution of private wealth in Germany. In contrast to official registry data, household surveys often face the problem of an insufficient number of observations from which to draw conclusions at a disaggregated level. We cope with this problem by enriching the survey data with regional indicators. In order to obtain estimates for average private net wealth for the 16 federal states as well as for the 96 planning regions, we apply a modified Fay-Herriot estimation accounting for characteristics of the survey data on private wealth.

Results

Even more than 25 years after German reunification, there is a clear dividing line at the former border with respect to household net wealth. Our estimates at federal state level show that in each eastern German federal state private wealth comes to less than half of the national mean. The consideration of the estimates at the level of planning regions provides evidence for heterogeneity within eastern and western Germany, respectively. The wealthiest planning regions in the east even exceed the level of several western German regions with the lowest estimates.

Nichttechnische Zusammenfassung

Fragestellung

Ziel dieser Arbeit ist es, Schätzungen für das durchschnittliche Nettovermögen privater Haushalte auf regionaler Ebene in Deutschland zu ermitteln. In diesem Zusammenhang wird untersucht, ob sich die Verteilung des Privatvermögens 25 Jahre nach der Wiedervereinigung zwischen den Alten und den Neuen Bundesländern unterscheidet.

Beitrag

Anhand der Befragungsdaten der zweiten Welle der Bundesbank-Studie "Private Haushalte und ihre Finanzen"(PHF) aus dem Jahr 2014 werden Indikatoren zur regionalen Verteilung des Privatvermögens in Deutschland ermittelt. Im Gegensatz zu offiziellen Registerdaten sind die Fallzahlen bei Haushaltsbefragungen oft zu gering, um Rückschlüsse auf die Verteilung ökonomischer Merkmale auf einer niedrigen regionalen Aggregationsebene ziehen zu können. Daher verknüpfen wir die Informationen aus den Befragungsdaten mit regionalen Indikatoren. Auf Basis einer modifizierten Version des Fay-Herriot-Modells werden Schätzer für das Durchschnittsnettovermögen privater Haushalte für die 16 Bundesländer sowie für die 96 Raumordnungsregionen in Deutschland ermittelt. Dabei werden Merkmale der Vermögensverteilung sowie die Eigenschaften der Haushaltsbefragung berücksichtigt.

Ergebnisse

In Bezug auf die Verteilung des Privatvermögens ist Deutschland selbst 25 Jahre nach der Wiedervereinigung weiterhin zweigeteilt. Unsere Schätzergebnisse zeigen, dass das durchschnittliche Nettovermögen privater Haushalte in allen fünf Neuen Bundesländern sowie in Berlin weniger als die Hälfte des gesamtdeutschen Durchschnitts beträgt. Die Analyse auf Raumordnungsebene zeigt jedoch, dass es sowohl innerhalb der Alten als auch innerhalb der Neuen Bundesländer deutliche Unterschiede bei der Verteilung des Privatvermögens gibt. In den vermögendsten Regionen in den Neuen Bundesländern liegt das private Durchschnittsnettovermögen sogar über den Werten der westdeutschen Raumordnungsregionen mit dem niedrigsten Schätzergebnissen.

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Estimating regional wealth in Germany: How different are east and west really? *

Ann-Kristin Kreutzmann† Philipp Marek‡ Nicola Salvati§ Timo Schmid¶

Abstract

More than 25 years after German reunification, key economic indicators for households living in eastern German regions are still below the western German levels. This particularly holds for private net wealth, which reaches only about 40% of the western German level. However, a more granular regional perspective may reveal a more diverse picture. Therefore, this study is designed to develop regional wealth indicators for the 16 federal states and for the 96 regional planning regions (*Raumordnungsregionen*) in Germany based on the second wave of the Panel on Household Finances (PHF) conducted by the Deutsche Bundesbank in 2014. These estimates are derived by means of a modified Fay-Herriot approach (Fay and Herriot, 1979) dealing with a) the skewness of the wealth distribution using a transformation, b) unit and item non-response, especially the multiple imputation used, and c) inconsistencies of the regional estimates with the national direct estimate. The results show that private wealth in all eastern German regions still remains far below the national average. However, the wealthiest planning regions in the east report higher private wealth figures than the western German regions with the lowest private wealth estimates. Although the paper is particularly focused on Germany, the approach proposed is applicable to surveys with a similar data structure.

Keywords: Small area estimation, non-response, multiple imputation, private wealth distribution

JEL classification: C13, C83, D13.

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1 Motivation

The financial crisis that began in 2007 uncovered the fragility of the global financial system. Private households' solvency is considered as one of the most important channels affecting financial stability. The tight linkage between private households and financial institutions is reflected by the large share of an economy's wealth, which is held by individuals (Ampudia, van Vlokhoven, and Żochowski, 2016). For instance, in 2017 total financial assets of private households in Germany accounted for almost EUR 6 trillion, which is nearly twice as large as the German annual GDP of EUR 3.3 trillion (Deutsche Bundesbank, 2018). A historical perspective undermines this important relationship as the majority of worldwide financial crises were preceded by an excessive credit supply in mortgage markets (Brunnermeier and Schnabel, 2016).

Several scholars have examined the important impact of the distribution of income and wealth on the stability of the financial system. For the United States (US), Kumhof, Rancière, and Winant (2015) assess the relationship between increasing income inequality and wealth concentration, and their impact on financial stability. The key mechanism lies in the negative marginal propensity to consume (see e.g. Carroll, 1998). Increasing income inequality results in a higher concentration of savings by top earners in the form of loans to bottom earners. This may lead to a rise in the debt-to-income ratio, which in turn may pose threats to the stability of the financial system.

Most of the literature on the economic importance of income and wealth distributions is focused on the national dimension. In this context, the spatial distribution of economic activity deserves additional attention, as economic activities are unevenly distributed across space (see e.g. Ottaviano and Puga, 1998). These agglomeration economies can be attributed to the importance of the local concentration of wealth, economic activity and innovative capacity (see e.g. Rodríguez-Pose and Crescenzi, 2008), but also to a process of rising inequality across regions.

The rising importance of agglomeration economies with their linkage to private wealth provides the motivation to assess the regional distribution of private financial resources (eurostat, 2017). In this context, the German reunification process provides a compelling example from an economic point of view. After a rapid catching-up process driven mostly by the construction sector, the convergence of eastern German regions came to an abrupt end in 1995. Since then the disposable income of households living in eastern Germany has been stagnating at about 80% of the western German level (Blum, Buscher, Gabrisch, Günther, Heimpold, Lang, Ludwig, Rosenfeld, and Schneider, 2010), while private net wealth in the east has caught up only to about 40% of the western German level (Deutsche Bundesbank, 2016).

Differences in the income and wealth gap between eastern and western Germany are rooted in the economic conditions after reunification. The ratio of disposable income of eastern German to western German households was 46% in 1991, whereas eastern German mean net wealth corresponded only to 30% of the average net wealth of households living in western Germany (Ammermüller, Weber, and Westerheide, 2005). Blum et al. (2010) provide several reasons behind the initial difference between income and wealth. First, the weak economy of East Germany at the start of the reunification process translated into differences in asset prices, such as house prices, between both parts of the country. Second, the institutional setting in the former German Democratic Republic (GDR), including low protection of property rights, provided weaker incentives for private capital accumulation. Third, these differences in incentives for wealth accumulation translated into lower home-ownership rates and lower saving rates in former East Germany. Fourth, financial assets were converted at the rate of 2:1 in contrast to the 1:1 conversion of wages. Furthermore, after reunification the savings ratio in eastern Germany remained below the western German contributing to the persistence of the wealth gap.

For the distribution of household income in German regions, several data sources are already available (Bundesinstitut für Bau-, Stadt-, und Raumforschung, 2017; RWI; microm, 2016). In particular, information on income obtained in the form of tax declarations or by notification of social security payments can help to develop indicators for the distribution of income at a regional level. In contrast, the measurement of the regional distribution of private wealth is not that trivial because the taxation of wealth is effective only in a few countries. Thus, surveys may be used to obtain figures of the distribution of private wealth in regions. In Germany, the Deutsche Bundesbank is responsible for the implemenation of the Panel on Household Finances (PHF) (Research Data and Service Centre (RDCS) of the Deutsche Bundesbank, 2014). This household survey is part of the Household Finance and Consumption Survey (HFCS) comprising harmonized surveys in 19 countries of the euro area under the umbrella of the European Central Bank (ECB).

With respect to the regional coverage, the Deutsche Bundesbank (2016) provides wealth figures for four larger regions, including a distinction between eastern and western Germany. The latter is divided into three subregions. Given the sample size of about 4,500 households, a more disaggregated analysis is subject to concerns about the precision of estimates. Therefore, this paper makes use of small area estimations (SAE) in order to obtain reliable estimates for private net wealth at a lower regional level, namely 16 federal states (*Bundesländer*) and 96 regional planning regions (*Raumordnungsregionen*). From an applied perspective, this paper contributes to the discussion of the wealth distribution at a regional level and on the convergence process after German reunification.

The estimates are obtained using a modified Fay-Herriot model (Fay and Herriot, 1979). In order to increase the accuracy of estimates at lower regional levels, direct estimates obtained from survey data are enriched with covariate information from other data sources such as registers. For the application of the Fay-Herriot model at a lower regional level, several aspects of the underlying data structure need to be taken into consideration. First, the skewness of the wealth distribution requires the usage of a log-transformation in order to fulfill the normality assumptions (Slud and Maiti, 2006; Neves, Silva, and Correa, 2013). Second, the present unit and item non-response need to be taken into account. The unit non-response is adjusted using weighting procedures. The Fay-Herriot model incorporates the produced survey weights by means of a weighted direct estimator in the model. The item non-response is handled with multiple imputation (Rubin, 1987). Therefore, our estimates are obtained using a combination of Rubin's rule and the Fay-Herriot approach. From a theoretical perspective, this leads to a modified (transformed) Fay-Herriot approach that accounts for the additional uncertainty due to the multiple imputation. Third, a benchmarking procedure ensures that the aggregated regional figures are consistent with the estimates at a higher level that are reported by the Deutsche Bundesbank (2016), namely for the whole country or the eastern and western regions, respectively.

The paper is structured as follows. Section 2 describes the data sources that are used in this work, particularly the PHF and the data sources for the covariate information. Section 3 describes the statistical method. In Section 4 the application of the Fay-Herriot model for the estimation of household net wealth is described including a discussion of the interpretation of the results. Section 5 discusses further potential research.

2 Data sources and initial analysis

In this section, the definition of household (HH) net wealth is introduced and the data sources used in the analysis are described. Wealth is composed of several assets and liabilities. It can be measured as gross wealth, i.e. the sum of assets, or as net wealth, i.e. the difference between assets and liabilities with the possibility of negative values. In this paper, net wealth serves as the variable of interest. A typical balance sheet of a HH is presented in the supplementary material of this paper.

2.1 The wealth survey: Panel on Household Finances

In 2008, the ECB's Governing Council decided to launch a survey on household finances in the euro area. This Household Finance and Consumption Survey (HFCS) is carried out by the national central banks under the umbrella of the ECB. The survey provides detailed data on various aspects of HH balance sheets and related economic and demographic variables, including income, private pensions, employment and measures of consumption (Eurosystem Household Finance and Consumption Network, 2013a,b). The HFCS is the first harmonized survey across the euro area addressing the distribution of private wealth and consumption and, thus, it is unique in enabling cross-country comparisons at a micro level. Therefore, many studies are already based on this data. For instance, some studies compare the wealth figures across countries obtained from this micro data source with macro-data sources such as national accounts (Kavonius and Honkkila, 2013; Andreasch and Lindner, 2016). The German part of the HFCS is named Panel on Household Finances (PHF). The PHF is a panel survey with the first wave conducted in 2010, a second wave in 2014, and the third in 2017. In comparison to other data sources covering wealth figures in Germany, such as the German Socio-Economic Panel (SOEP), the PHF is particularly designed to assess the wealth position of households. Hence, the questionnaire contains detailed components capturing wealth. The following analysis is based on data from the second wave.

For the purpose of this analysis, the PHF bears some methodological issues that need to be taken into consideration. First, the sampling design aims to overrepresent wealthy households (Knerr, Aust, Chudziak, Gilberg, and Kleudgen, 2015) as the distribution of assets and liabilities across households is quite concentrated. The sampling is conducted in three stages. In the first stage, German municipalities are divided into three strata depending on the size and proportion of wealthy households. In a second stage, the streets in cities with more than 100,000 citizens are categorized as wealthy or other streets. In the third stage, the public register is used to draw persons above the age of 18. This leads to a sample with 4,461 households. Second, the PHF has a high unit non-response rate. The response rate was 68% for households that had already participated in the first wave of the survey (panel households) and 18% for those contacted for the first time. Thus, the Bundesbank computes survey weights to adjust for the unequal probability survey design and for unit non-response. Knerr et al. (2015) provide a detailed description of the weighting procedure. Considering the sensitivity of the questionnaire, the low response rate is not surprising. Third, item non-response has to be taken into consideration. This is relatively low for many core variables even though very sensitive financial questions are asked (Eisele and Zhu, 2013). In order not to lose observations with missing values, the Deutsche Bundesbank implemented multiple imputations (MI) with $M = 5$ (Eisele and Zhu, 2013; Household Finance and Consumption Network, 2016a). For a proper imputation, Rubin (1996) suggests using as many variables as available. As described in Eisele and Zhu (2013), the imputation for the PHF is carried out using variables that are correlated with the imputed variable, characteristics explaining the non-response behavior and design weights. Disregarding the design could lead to a bias in the variance (Kott, 1995). Domain indicators are also considered (Household Finance and Consumption Network, 2016a). Eisele and Zhu (2013) also describe how they avoid overfitting using cross-validation methods for the variable selection. Since the item non-response is relatively low for many variables, especially for variables with a high impact on wealth, the difference between the five imputations of the variable net wealth is rather small leading to a high correlation between the imputations for net wealth (see Table 1).

Table 1: Pearson correlation coefficients of the variable net wealth in the five imputed data sets M1-M5 based on Research Data and Service Centre (RDCS) of the Deutsche Bundesbank (2014), Panel on Household Finances (PHF) 2014, authors' estimations.

	M1	M2	M3	M4	M5
M1	1.000	0.988	0.990	0.989	0.992
M2	0.988	1.000	0.995	0.993	0.994
M3	0.990	0.995	1.000	0.993	0.994
M4	0.989	0.993	0.993	1.000	0.993
M5	0.992	0.994	0.994	0.993	1.000

In order to obtain final estimates in the presence of multiple data sets, the estimates based on each data set need to be pooled. For the pooling, Rubin (1987) suggests a rule that is explained in more detail in Section 3. For the variance estimation of linear and non-linear indicators the PHF contains 1,000 replication weights. These replication weights are obtained from a Rao-Wu rescaled bootstrap (Rao and Wu, 1988; Household Finance and Consumption Network, 2016a).

Table 2 summarizes the average net wealth in thousand EUR (TEUR) for the regions reported by Deutsche Bundesbank (2016) illustrating the distinct differences between eastern and western Germany. Furthermore, it provides an indication of the heterogeneity of mean net wealth within the western part of Germany. Since our analysis is based on an updated version of the scientific use file for the second wave of the PHF issued in 2017, the reported values differ slightly from those reported by Deutsche Bundesbank (2016).

Table 2: Mean of HH net wealth in TEUR and sample sizes in the east, west and northern, southern and western federal states based on Research Data and Service Centre (RDCS) of the Deutsche Bundesbank (2014), Panel on Household Finances (PHF) 2014, authors' estimations.

Regional level			HH net wealth	Sample size
West			248.48	3610
	Northern states	Bremen, Hamburg, Lower Saxony, Schleswig-Holstein	256.66	752
	Southern states	Bavaria, Baden-Württemberg, Hesse	285.32	1714
	Western states	North Rhine-Westphalia, Rhineland-Palatinate, Saarland	196.83	1144
East		Berlin, Brandenburg, Mecklenburg -West Pomerania, Saxony, Saxony-Anhalt, Thuringia	90.23	851

This regional division (northern, western, southern and eastern states) is not based on administrative units, nor does the analysis highlight differences within the regions. Hence, it is the aim of this study to provide additional insights into the distribution of private wealth at a more granular regional level, namely for the 16 federal states and 96 regional planning regions located in Germany. The direct estimates for average household net wealth are shown in Figure 1.

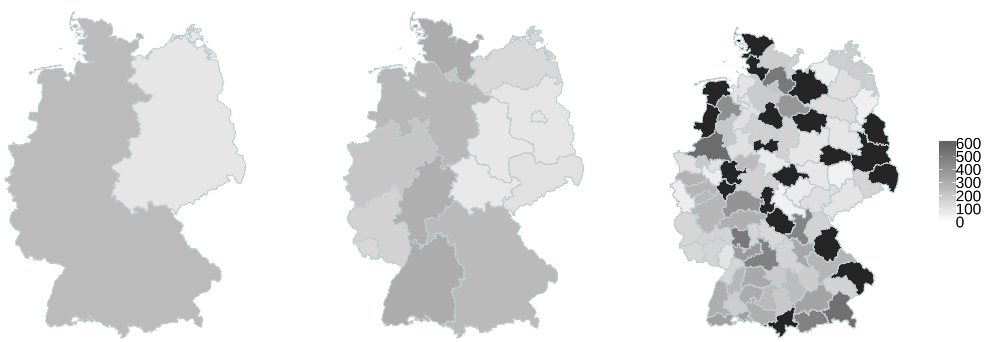

Figure 1: Map of the direct estimates of mean HH net wealth in TEUR for eastern and western Germany, the federal states and planning regions based on Research Data and Service Centre (RDCS) of the Deutsche Bundesbank (2014), Panel on Household Finances (PHF) 2014, authors' estimations. Regions where direct estimates are not available are colored in black.

Table 3: Summary of sample sizes in the federal states and planning regions based on Research Data and Service Centre (RDCS) of the Deutsche Bundesbank (2014), Panel on Household Finances (PHF) 2014, authors' estimations.

	Min.	P25	Median	P75	Max.
BL	32	98	189	358	925
ROR	9	28	40	65	340

While the map of estimates for the east and the west only shows the known wealth difference between them, the maps for the federal states and especially the estimates for the planning regions confirm the assumption of noticeable regional differences. However, Table 3 also shows a decrease in sample sizes compared to the sample sizes in Table 2. Furthermore, it was not possible to obtain direct estimates of HH net wealth for the black colored planning regions. Ten planning regions do not contain any households participating in the sample. For another nine regions the sample size is too small to obtain results not violating the Bundesbank's confidentiality issues. This means that the direct estimate would only be available for 77 out of 96 regions. Thus, the application of a direct estimator for the mean of household net wealth bears two issues. First, the direct estimates might be unreliable due to large variances in small areas. Second, direct estimates cannot be reported for regions with zero sample size or for direct estimates violating confidentiality issues. The combination of the survey data with available register data containing regional information may help to improve the accuracy of the estimates for both regional levels. Further, it provides predictions for the planning regions where direct estimation is not possible or not allowed to be published.

2.2 Register data of federal states and regional planning regions

The model used in this work requires additional information from administrative data sources in order to obtain estimates for regional HH net wealth. As already mentioned, net wealth is composed of several assets and liabilities. Thus, these components are natural predictors for the target variable. In the HFCS, home ownership and the value of the real estate have a very strong influence on household net wealth (Eurosystem Household Finance and Consumption Network, 2013b; Household Finance and Consumption Network, 2016b). But information on other assets, such as the number or value of vehicles, or information of liabilities may also be good predictors. While not a component of net wealth, higher income or income growth enables households to accumulate wealth according to income-to-wealth ratio proposed by Piketty and Zucman (2014). Since labor income represents the highest proportion of the average household income, employment figures such as the employment status should be taken into consideration. In particular, self-employed people are a group that tends to have higher wealth (Frick and Grabka, 2009). It can also be shown that net wealth initially increases with age and declines after retirement (Eurosystem Household Finance and Consumption Network, 2013b; Household Finance and Consumption Network, 2016b). Since we measure net wealth at the household level instead of personal net wealth, the household structure can also have an effect on the level of net wealth. Single HHs tend to have lower wealth, whereas wealth figures for HHs with two or more members do not increase proportionally with size (Eurosystem Household Finance and Consumption Network, 2013b). The variables that we use to proxy these effects are summarized in Table 4. Given that we estimate wealth figures at the level of regions, it should be noted that the register information is only available on an aggregated level and not for individual households.

For the federal states, aggregate statistics for most of these variables are obtained from the Federal Statistical Office and the Statistical Offices of the Länder (Statistische Ämter des Bundes und der Länder, 2018). For instance, the unemployment rate is obtained from the Regionaldatenbank Deutschland and the disposable income is obtained from the national accounts of the federal states. The covariate information for the planning regions is predominantly provided by the research data center FDZ Ruhr - RWI (Budde and Eilers, 2014; RWI; microm, 2016) and complemented by the German database Indikatoren und Karten zur Raum- und Stadtentwicklung (INKAR) (Bundesinstitut für Bau-, Stadt-, und Raumforschung, 2017). Information about the rental and purchase prices as well as the level of interior is delivered by empirica ag for both regional levels (Empirica ag, 2017). Most variables are obtained for 2014, the year of the survey, or for 2015. We assume that these variables are relatively time consistent at the aggregated level. For the same reason, we include the variables on employment and home ownership rates even though these are obtained from the German Census in 2011 (Statistische Ämter des Bundes und der Länder, 2011). Other factors that have an essential influence on wealth accumulation are inheritances and donations. However, data for these variables were not available for this work.

3 Statistical method

This section provides a description of how the survey data from the PHF were processed to obtain estimates for the mean of HH net wealth at a regional level in Germany. In order to derive estimates for the 16 federal states and 96 planning regions, the estimates should take the following aspects outlined in Section 2.1 into consideration:

- missing information for some regions due to zero sample size or confidentiality issues,
- the complex survey design including the weighting scheme,
- the skewness of the wealth distribution,
- the uncertainty of the data set based on multiple imputed values, and
- the inconsistency between the regional estimates and estimates at a more aggregated level (e.g. national).

Therefore, we propose a benchmarked Fay-Herriot (FH) estimator (Fay and Herriot, 1979) that additionally accounts for the variability due to multiple imputation (MI). The FH approach is one out of a wide range of small area estimation (SAE) methods that generally combine information from different data sources. For an overview of SAE methods, we refer to Pfeffermann (2013), Rao and Molina (2015) and Tzavidis, Zhang, Luna, Schmid, and Rojas-Perilla (2018). Compared to other SAE methods that require additional information at the unit level (see e.g., Molina and Rao, 2010; Molina, Nandram, and Rao,

Table 4: Identified variables that potentially help to predict HH net wealth. The numbers in parentheses state the number of variables for this group. References to the sources of the variables are given in the supplementary material.

Influence	**Variables**	**Year**	**Level**
Real estate	Ownership rate	2011	BL, ROR
	Rental and purchase prices per sqm, level of interior (3)	2014	BL, ROR
	Number of houses (2) and types of buildings (6)	2015	ROR
Vehicles	Density of cars and car segment (11)	2015	ROR
Savings	Savings ratio of HH	2014	BL
Liabilities	Default probability (8)	2015	ROR
	Private debtors per 100 inhabitants	2014	ROR
Income	Disposable income of private HHs per inhabitant	2014	BL
	Average HH income per inhabitant	2014	ROR
Employment status	Unemployment rate,	2014	BL, ROR
	percentage of employees, civil servants and self-employed	2011	BL
Age	Age groups (4)	2014	BL, ROR
	Youth dependency ratio, old-age dependency ratio	2014 2014	BL BL
Household structure	Single or couple	2015	ROR

2014), the main benefit of the FH model is that it only requires additional information at an aggregated level.

3.1 The Fay-Herriot model

This section provides an elaboration of the FH estimation designed to provide regional estimates for mean HH net wealth. Generally, a finite population of size N is partitioned into D domains of sizes $N_1, \ldots, N_D$, where $d = 1, \ldots, D$ refers to domain d. For the purpose of our analysis, each domain represents one region. Each observation or household, i, is assigned to one domain, $i = 1, \ldots, N_d$. As net wealth figures are not available for the entire population, a sample is drawn from this population using a complex sampling design with sample sizes $n_1, \ldots, n_D$ for the domains $d = 1, \ldots, D$.

The FH model is based on two model relations. The sampling model can be expressed as

$$\hat{\theta}_d^{\text{Dir}} = \theta_d + e_d, \quad d = 1, \ldots, D, \tag{1}$$

where $\hat{\theta}_d^{Dir}$ is a design-unbiased direct estimator of the population indicator θ_d, such as the mean. The direct estimator, which is only based on the survey data, is assumed to be equal to the population value, θ_d, plus a sampling error e_d. It allows the incorporation of survey weights, w. The indicator of interest in this analysis is the mean of HH net wealth, which is derived as the weighted mean for corresponding domain d:

$$\hat{\theta}_d^{\text{Dir}} = \frac{\sum_{i=1}^{n_d} w_{di} y_{di}}{\sum_{i=1}^{n_d} w_{di}}, \quad i = 1, \ldots, n_d, \quad d = 1, \ldots, D,$$

where y_{di} represents the net wealth figure of HH i in domain d, with the corresponding survey weight, w_{di}.

The second model links the population indicator θ_d with covariate information $\boldsymbol{x}$ for domain d in a linear relation:

$$\theta_d = \boldsymbol{x}_d^T \boldsymbol{\beta} + u_d, \quad d = 1, \ldots, D, \tag{2}$$

where $\boldsymbol{x}_d$ is a $p \times 1$ vector of domain-level covariate information and $\boldsymbol{\beta}$ is the vector of regression parameters with dimension $p \times 1$. The combination of the models (1) and (2) leads to a special linear mixed model that is defined as

$$\hat{\theta}_d^{\text{Dir}} = \boldsymbol{x}_d^T \boldsymbol{\beta} + u_d + e_d, \quad d = 1, \ldots, D,$$
$$u_d \overset{iid}{\sim} N(0, \sigma_u^2) \quad e_d \overset{ind}{\sim} N(0, \sigma_{e_d}^2),$$

with random effects u_d that are independent and identically normally distributed with variance σ_u^2 and sampling errors e_d that are independent normally distributed with variance $\sigma_{e_d}^2$. The two error terms are assumed to be independent. The estimates of the regression parameters $\hat{\boldsymbol{\beta}}$ are the empirical best linear unbiased estimators (EBLUE) of $\boldsymbol{\beta}$ (Rao and Molina, 2015). For the estimation of the variance of the random effect, σ_u^2, several approaches are available including, among others, the FH method-of-moments estimator, the maximum likelihood (ML) and the residual maximum likelihood (REML) method (Rao and Molina, 2015). A disadvantage of these approaches is the numerical possibility of a negative variance estimator, that is usually set to zero for negative values. This issue may especially arise in the case of a small number of domains. Therefore, adjusted estimation methods can be preferable when the number of domains is small since these always provide strictly positive variance estimates (Li and Lahiri, 2010; Yoshimori and Lahiri, 2014). Yoshimori and Lahiri (2014) propose an adjusted maximum residual likelihood approach (AMRL.YL).

The resulting FH estimator is an empirical best linear unbiased predictor (EBLUP) of θ_d^{FH}. It can be expressed as a weighted average of the direct estimator $\hat{\theta}_d^{\text{Dir}}$ and a synthetic part as follows:

$$\begin{aligned}\hat{\theta}_d^{\text{FH}} &= \boldsymbol{x}_d^T\hat{\boldsymbol{\beta}} + \hat{u}_d \\ &= \hat{\gamma}_d\hat{\theta}_d^{\text{Dir}} + (1-\hat{\gamma}_d)\boldsymbol{x}_d^T\hat{\boldsymbol{\beta}},\end{aligned}$$

where $\hat{\gamma}_d = \frac{\hat{\sigma}_u^2}{\hat{\sigma}_u^2+\sigma_{e_d}^2}$ is the ratio of the estimated variance of the random effects, $\hat{\sigma}_u^2$, and the total variance denoting the shrinkage factor for area d. For relatively small values of the sampling error variance $\sigma_{e_d}^2$, the weight on the direct estimate increases with higher values of $\hat{\gamma}_d$. For the domains with zero sample size or with estimates that are not allowed to be published (in the following called out-of-sample domains), the prediction shrinks to the synthetic part:

$$\hat{\theta}_{d,out}^{\text{FH}} = \boldsymbol{x}_d^T\hat{\boldsymbol{\beta}}.$$

In order to assess the accuracy of the FH estimates, the corresponding mean squared error (MSE) can be estimated. The composition of the MSE estimator depends on the selected estimation method for the variance of the random effect (Rao and Molina, 2015). Prasad and Rao (1990) as well as Datta and Lahiri (2000) describe the compositions when the REML and ML approaches are used, respectively. The MSE estimation of out-of-sample domains for both approaches follows Rao and Molina (2015). For the adjusted estimation methods, the MSE needs to be modified as described in Li and Lahiri (2010) and Yoshimori and Lahiri (2014).

3.2 The log-transformed Fay-Herriot model

If the relationship between the target variable and the covariate information is non-linear or the normality assumption of the error terms is not met, the log-transformed FH model can be used (Rao, 1999). According to Neves et al. (2013), the direct estimator and the sampling error variance can be transformed as follows:

$$\begin{aligned}\hat{\theta}_d^{\text{Dir}*} &= log(\hat{\theta}_d^{\text{Dir}}), \\ var(\hat{\theta}_d^{\text{Dir}*}) &= \left(\hat{\theta}_d^{\text{Dir}}\right)^{-2} var(\hat{\theta}_d^{\text{Dir}}),\end{aligned}$$

where the $*$ denotes the transformed scale.

The FH estimator on the transformed scale can be obtained by using the log-transformed direct estimator $\hat{\theta}_d^{\text{Dir}*}$ as dependent variable and the modified variance $var(\hat{\theta}_d^{\text{Dir}*})$ as estimate for the sampling error variance.

$$\hat{\theta}_d^{\text{FH}*} = \hat{\gamma}_d^*\hat{\theta}_d^{\text{Dir}*} + (1-\hat{\gamma}_d^*)\boldsymbol{x}_d^T\hat{\boldsymbol{\beta}},$$

where $\hat{\gamma}_d^* = \frac{\hat{\sigma}_u^2}{\hat{\sigma}_u^2 + \sigma_{e_{d*}}^2}$ with $\sigma_{e_{d*}}^2 = var(\hat{\theta}_d^{\text{Dir}*})$. For an appropriate interpretation of the results, the FH estimates need to be back-transformed to the original scale. A naive back-transformation using the exponential function may induce a bias to the estimates because of Jensen's inequality (Jensen, 1906). Therefore, several bias-corrected back-transformations are proposed in the literature. Neves et al. (2013) propose a back-transformation of $\hat{\theta}_d^{FH*}$ based on the properties of the log-normal distribution, which is also referred to as crude back-transformation in Rao and Molina (2015):

$$\hat{\theta}_d^{\text{FH, crude}} = exp\left\{\hat{\theta}_d^{\text{FH}*} + 0.5MSE(\hat{\theta}_d^{\text{FH}*})\right\}, \tag{3}$$
$$MSE(\hat{\theta}_d^{\text{FH, crude}}) = exp\left\{\hat{\theta}_d^{\text{FH}*}\right\}^2 MSE(\hat{\theta}_d^{\text{FH}*}),$$

where $\hat{\theta}_d^{\text{FH}*}$ is the FH estimate on the transformed scale and $MSE(\hat{\theta}_d^{\text{FH}*})$ the MSE estimate on the transformed scale, e.g. the Prasad-Rao MSE.

Slud and Maiti (2006) propose a bias-correction that considers the area-specific effects when the ML approach is used for the estimation of σ_u^2. Chandra, Aditya, and Kumar (2018) extend this work by taking the variability of the parameter estimation into account. The back-transformation for the point estimates differs slightly from the crude back-transformation:

$$\hat{\theta}_d^{\text{FH, Slud-Maiti}} = exp\left\{\hat{\theta}_d^{\text{FH}*} + 0.5\hat{\sigma}_u^2(1-\hat{\gamma}_d^*)\right\}, \tag{4}$$
$$\hat{\theta}_d^{\text{FH, Chandra et al.}} = c_d * exp\left\{\hat{\theta}_d^{\text{FH}*} + 0.5\hat{\sigma}_u^2(1-\hat{\gamma}_d^*)\right\},$$

where c_d is a bias term derived in Chandra et al. (2018).
Furthermore, a special MSE estimator for the log-transformed model is developed in Slud and Maiti (2006). The latter two bias-corrections face the disadvantage that they are only applicable for in-sample domains. Both approaches are based on the estimated $\hat{\gamma}_d^*$, which is only available for sampled domains.

3.3 Combination of multiple imputation and the Fay-Herriot approach

As already mentioned, key variables of the PHF survey are multiply imputed (see Eisele and Zhu, 2013). In the PHF, five values are imputed for each missing value, leading to five imputed data sets. The indicators of interest, namely the mean of HH net wealth, and its variance are estimated for each imputed data set. As outlined above, the variance estimation for each imputation is derived from the 1,000 replication weights provided in the PHF. In order to pool these estimates, Rubin's rule can be applied if the complete data estimates are approximately normal (Rubin, 1987). Consequently, we propose using the direct estimator and the corresponding variance after the application of Rubin's rule defined as $\hat{\theta}_d^{\text{RRDir}}$ and $\hat{\sigma}_{\epsilon_d}^2 = var(\hat{\theta}_d^{\text{RRDir}})$ in the Fay-Herriot approach. The variance $\hat{\sigma}_{\epsilon_d}^2$ covers the sampling variance and the variance due to missing values and imputation (Rubin, 1996; Kim, Brick, Fuller, and Kalton, 2006). The steps of the analysis are summarized as follows:

Step 1. Imputation: Impute the missing values. In the case of the PHF data set, the imputation is already conducted by the Deutsche Bundesbank (Eisele and Zhu, 2013).

Step 2. Analysis (Direct): Obtain $\hat{\theta}^{\text{Dir}}_{d,m}$ and $var(\hat{\theta}^{\text{Dir}}_{d,m})$ for $m = 1, \ldots, M$ where M is the number of imputed data sets. For the PHF data set, M equals 5.

Step 3. Pooling (Rubin's rule): Obtain $\hat{\theta}^{\text{RRDir}}_{d} = \frac{1}{M}\sum_{m=1}^{M}\hat{\theta}^{\text{Dir}}_{d,m}$ and

$$\hat{\sigma}^2_{\epsilon_d} = \frac{1}{M}\sum_{m=1}^{M} var(\hat{\theta}^{\text{Dir}}_{d,m}) + \left(1+\frac{1}{M}\right)\frac{1}{M-1}\sum_{m=1}^{M}(\hat{\theta}^{\text{Dir}}_{d,m} - \hat{\theta}^{\text{RRDir}}_{d})^2.$$

Step 4. Analysis (FH): Obtain the Fay-Herriot estimate for multiple imputed data sets (FH-MI) expressed by:

$$\hat{\theta}^{\text{FH-MI}}_{d} = \hat{\gamma}_d\hat{\theta}^{\text{RRDir}}_{d} + (1-\hat{\gamma}_d)\boldsymbol{x}_d^T\hat{\boldsymbol{\beta}},$$

where $\hat{\gamma}_d = \frac{\hat{\sigma}^2_u}{\hat{\sigma}^2_u + \hat{\sigma}^2_{\epsilon_d}}$.
Step 4*. Analysis (log-transformed FH): Log-transform the direct estimate and modify the variance estimate (here according to Neves et al. (2013)):

$$\hat{\theta}^{\text{RRDir*}}_{d} = log(\hat{\theta}^{\text{RRDir}}_{d}),$$
$$\hat{\sigma}^2_{\epsilon_{d,*}} = \left(\hat{\theta}^{\text{RRDir}}_{d}\right)^{-2}\hat{\sigma}^2_{\epsilon_d}.$$

Obtain the FH estimate for multiple imputed data sets (FH-MI) on the transformed scale expressed by:

$$\hat{\theta}^{\text{FH-MI*}}_{d} = \hat{\gamma}^*_d\hat{\theta}^{\text{RRDir*}}_{d} + (1-\hat{\gamma}^*_d)\boldsymbol{x}_d^T\hat{\boldsymbol{\beta}},$$

where $\hat{\gamma}^*_d = \frac{\hat{\sigma}^2_u}{\hat{\sigma}^2_u + \hat{\sigma}^2_{\epsilon_{d,*}}}$.
Back-transform the estimation results to the original scale.

Step 5. Computation of the MSE: Obtain the MSE estimate for $\hat{\theta}^{\text{FH-MI}}_{d}$. As discussed in Section 3.1, the choice of the MSE estimator depends on the chosen estimation method for $\hat{\sigma}^2_u$ in Step 4.
Step 5*. Computation of the back-transformed MSE: Obtain the MSE estimate for the back-transformed $\hat{\theta}^{\text{FH-MI*}}_{d}$. The MSE estimator depends on the chosen bias-correction in Step 4*: crude, Slud-Maiti, Chandra et al. (see also Section 3.2).

Please note that an alternative approach would be to fit the Fay-Herriot model on each combination of $\hat{\theta}^{\text{Dir}}_{d,m}$ and $var(\hat{\theta}^{\text{Dir}}_{d,m})$ for $m = 1, \ldots, M$. This approach leads to almost the same point estimates. However, the pooling of the MSE estimates is unclear and, thus, remains subject to further research.

3.4 Benchmarking for internal consistency

The aggregated regional FH-MI estimates may differ from the figures at the national level or at the regional level such as the distinction between east and west. For internal consistency of the estimates, Datta, Ghosh, Steorts, and Maples (2011) propose an approach assuming the following relationship between the benchmark value τ and the aggregated values of the single regions obtained from the FH-MI estimates:

$$\sum_{d=1}^{D} \xi_d \hat{\theta}_d^{\text{FH-MI,bench}} = \tau, \tag{5}$$

with $\xi_d = \frac{N_d}{N}$ representing the population share of region d in the total population. The benchmark value may be the national estimate or the estimate of larger regions such as the four regions reported in the PHF. The benchmarked FH-MI estimator can be expressed as

$$\hat{\theta}_d^{FH-MI,bench} = \hat{\theta}_d^{FH-MI} + \left(\sum_{d=1}^{D} \frac{\xi_d^2}{\phi_d} \right)^{-1} \left(\tau - \sum_{d=1}^{D} \xi_d \hat{\theta}_d^{FH-MI} \right) \frac{\xi_d}{\phi_d}. \tag{6}$$

Datta et al. (2011) present different ways to define ϕ_d, which can be regarded as an additional weighting factor for a multiple-objective decision process. Some depend on the value of the estimate or on its accuracy estimate. If $\phi_d = \xi_d$, all FH-MI estimates are adjusted equally. When dividing ξ_d by the corresponding point or MSE estimate the domains with a respectively larger value are adjusted more strongly. According to Steorts and Ghosh (2013), the benchmarking induces only a minor increase in the MSE of the FH estimates. The authors suggest deriving the corresponding MSE estimate by means of a parametric bootstrap.

4 Model specification and results

This section provides the estimates for mean HH net wealth obtained for the 16 federal states and the 96 planning regions in Germany. As outlined in Section 3, the estimation procedure is based on a modified version of the FH method (Fay and Herriot, 1979). In a first step, the model selection is presented. As described above, the estimates depend on covariate information at the domain level. In a second step, the accuracy gains from using the FH model are assessed by a comparison with the direct estimates. The third step contains the description of the benchmarking exercise. This section ends with an interpretation of the estimates obtained from the benchmarked FH model.

4.1 Model selection and diagnostic checking

Section 2 contains a list of predictors that can potentially explain household net wealth. As described, the regional dimension of the predictors needs to correspond to the domains chosen for the analysis. In total, the covariates add up to 18 for the federal states and to 46 for the planning regions, respectively. The number of possible predictors requires a variable reduction, which is conducted by means of an elastic net. Following Zou and Hastie (2005), an elastic net reduces the number of variables by eliminating trivial variables and including whole groups of closely related variables. The variable selection is based on the Kullback symmetric divergence criterion (KICb2) proposed by Marhuenda, Morales, and Pardo (2014), particularly designed for FH models. Finally, the model with the lowest value of KICb2 is chosen for our analysis (Marhuenda et al., 2014).

For the federal states, the final model includes two covariates, the savings ratio and the youth dependency ratio, capturing the relation between adolescents up to 19 years of age and individuals aged 20 to 64. Both factors are associated with a positive effect on the mean of HH net wealth. Since the number of domains is small, at 16 regions, the REML and the AMRL.YL method are considered. For both methods, the estimation of σ_u^2 is similar and far from 0. Thus, an adjustment to obtain a positive variance estimate is redundant. Hence, the REML approach and the Prasad-Rao MSE are used in the application for the federal states. The explanatory power of the selected model measured by the modified R^2 for FH models proposed by Lahiri and Suntornchost (2015) is 92%. The normality assumption of the two error terms in the FH model is assessed by the Shapiro-Wilk test and is not rejected for either error term. For the random effect (RE) the p-value equals 0.06 and for the standardized realized residuals (RRES) it is 0.85. Therefore, a transformation of estimates obtained for the FH model is not necessary.

For the planning regions, the normality assumption of the error terms does not hold for both terms in the original scale (RE: p = 0.001, std. RRES: p = 0.56). Thus, a log-transformation on the direct estimates of the mean of HH net wealth is applied. The transformed variable is used as dependent variable in the variable selection. The final model includes four covariates: the number of houses that are not for businesses, purchase price per sqm, the percentage of vans, and the percentage of HHs with a default probability below average. The effect of the variables is positive and the modified R^2 is 89%. The Shapiro-Wilk test supports the assumption of normally distributed error terms (RE: p = 0.45, std. RRES: p = 0.43) in the log-transformed FH model. The log-transformed FH-MI estimates for the planning regions are back-transformed to the original scale using the crude back-transformation. This choice is based on the fact that the crude back-transformation is applicable for in-sample and out-of-sample domains. Furthermore, the differences between the estimates for in-sample domains using the crude back-transformation and the back-transformations proposed by Slud and Maiti (2006) and Chandra et al. (2018) are quite small.

Table 5: Results for the goodness-of-fit test according to Brown et al. (2001).

Level	**W**	**df**	**p-value**
BL	4.23	16	0.99
ROR	22.84	77	1

One way of assessing the quality of the model-based estimates is a comparison with the direct estimates. Brown et al. (2001) propose a goodness-of-fit test for this assessment. The null hypothesis of the test assumes that the model-based estimates do not differ significantly from the direct estimates. The test statistic is defined as

$$W\left(\theta_d^{\text{FH-MI}}\right) = \sum_{d=1}^{D} \frac{\left(\theta_d^{\text{RRDir}} - \theta_d^{\text{FH-MI}}\right)^2}{var(\theta_d^{\text{RRDir}}) + MSE(\theta_d^{\text{FH-MI}})}, \tag{7}$$

where $\theta_d^{\text{FH-MI}}$ is the FH-MI estimate for the federal states and the back-transformed $\theta_d^{\text{FH-MI}*}$ estimate for the planning regions and $MSE(\theta_d^{\text{FH-MI}})$ are the corresponding MSE estimates. Note that only the estimates of in-sample domains are compared because direct estimates cannot be obtained for out-of-sample domains. The test statistic W is χ^2-distributed with D degrees of freedom under the null hypothesis. The results of the test show that the null hypothesis, that model-based estimates do not differ significantly from the direct estimates, cannot be rejected (see Table 5). According to Chandra, Salvati, and Chambers (2015), a useful diagnostic that measures the adequacy of the model is the correlation coefficient of the synthetic part of the FH-MI estimates and the direct estimates. For the federal states this correlation is 0.88, and for the planning regions it is 0.68. Both values are comparable to or higher than the value of 0.68 in Chandra et al. (2015).

4.2 Gain in accuracy

The accuracy of the FH-MI estimates is measured by the Prasad-Rao MSE for both the federal states and the planning regions since the estimation of σ_u^2 is based on the REML approach. The MSE for the planning regions is further back-transformed to the original scale using the crude bias-correction as described in Section 3.2. The variance of the direct estimates is obtained by using replication weights from the Rao-Wu rescaled bootstrap (see also Section 2.1).

Table 6: Distribution of the MSE of mean HH net wealth across federal states (BL) and planning regions (ROR) based on the Panel on Household Finances (PHF) 2014, authors' estimations.

Level	Estimate	Min.	P25	Median	Mean	P75	Max.
BL	Direct	140.47	430.41	821.16	1770.58	1906.36	8968.47
	FH-MI	143.54	371.31	552.89	592.14	844.02	1169.21
ROR	Direct	86.63	941.53	3701.64	19765.09	9610.40	561717.00
	FH-MI	81.40	766.19	1434.42	2432.02	3121.08	13846.04

Table 6 shows that the FH-MI estimates are more accurate than the direct estimates for the mean of HH net wealth for the federal states and planning regions. The gain in accuracy is especially large for the planning regions. From these results we can conclude that the FH approach helps to obtain more reliable results.

4.3 Benchmarking

For internal consistency, the benchmarking approach described in Section 3.4 is implemented. The Deutsche Bundesbank (2016) reports direct estimates with a regional distinction between east and west. These values are used as a benchmark. These estimates also almost add up to the national estimate with a negligible difference of 0.11% and 0.20% for the federal states and planning regions, respectively. Thus, benchmarking to the direct estimates of the regions ensures consistency with the national estimate.

Table 7: Mean difference of aggregated FH-MI estimates to the regional direct estimates for East and West in TEUR based on Research Data and Service Centre (RDCS) of the Deutsche Bundesbank (2014), Panel on Household Finances (PHF) 2014, authors' estimations.

Level	Benchmark	Regional direct estimate	Aggregated estimate
BL	East	90.23	89.77
	West	248.48	230.02
ROR	East	90.23	108.41
	West	248.48	238.40

Table 7 reports the direct estimates for east and west serving as the benchmark. For both regional levels, Table 7 also contains the aggregated FH-MI estimates prior to the application of the benchmarking specification. Thereby, the aggregation is based on the population figures. At the federal state level, the aggregated mean of HH net wealth of the FH-MI estimate is underestimated for regions in the western part and almost identical for ones in the east. For the planning regions, the aggregated mean value of the FH-MI overestimates the regional direct estimate for the east and underestimates the corresponding figures for the west. For the benchmarking specification, $\phi_d = \xi_d/\text{FH-MI}_d$ is chosen implying that regions with a larger estimate are adjusted more strongly. Thereby, the order of the regions with regard to the value of the mean of HH net wealth remains unchanged within the east and the west, respectively. After the application of the benchmarking approach, the aggregated benchmarked FH-MI estimates are equal to the direct estimates of the regions east and east and, thus, also equal to the national estimate.

Table 8: Distribution of mean HH net wealth across federal states (BL) and planning regions (ROR) in TEUR based on the Panel on Household Finances (PHF) 2014, authors' estimations.

Level	Estimate	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
BL	Direct	69.68	93.90	153.01	175.81	254.63	307.74
	FH-MI	74.83	87.23	155.27	159.79	223.51	283.91
	FH-MI, bench	75.87	87.67	167.73	170.11	241.44	306.69
ROR	Direct	51.45	112.92	178.40	220.68	294.17	621.41
	FH-MI	63.79	124.85	174.69	202.26	254.52	512.31
	FH-MI, bench	53.09	123.88	179.02	205.37	265.28	533.97

Table 8 shows the distribution of mean HH net wealth across the federal states and the planning regions for the direct, the FH-MI and the benchmarked FH-MI estimates. It can be seen that most of the benchmarked FH-MI results are larger than the FH-MI estimates. Given that 80% of the population lives in the western part of Germany, the adjustment for the underestimation of the aggregated estimate of the west has a larger effect on the benchmarked estimates than adjusting for the overestimation or for the slight underestimation of the aggregated estimates of the east (see Table 7). The following discussion of the results is based on the benchmarked FH-MI estimates, fulfilling the requirement to add up to the regional and national direct estimates.

4.4 Discussion of the estimation results

Figure 2 shows the regional distribution of benchmarked FH-MI estimates for the federal states and the planning regions. The map for the federal states shows the fairly known pattern of a clear dividing line at the former border between east and west. All federal states in the east report average private net wealth of EUR 90,000, which is more than 50% lower than the national mean, ranging from about EUR 75,000 in Saxony-Anhalt to EUR 110,000 in Brandenburg. As outlined by the Bundesbank (2016), the strong divergence with respect to private net wealth can be attributed to differences in financial wealth (EUR 30,000 in the east to EUR 60,000 in the west), home ownership (35% to 47%) as well as in the average value of owned dwellings (EUR 145,000 to EUR 250,000). This relationship is in line with the ratio of average prices per square metre for dwellings provided by bulwiengesa AG (2018). In 2014, the average prices per square metre for the purchase of real estate in the east was below EUR 1,200, whereas it was at EUR 1,900 in the west.

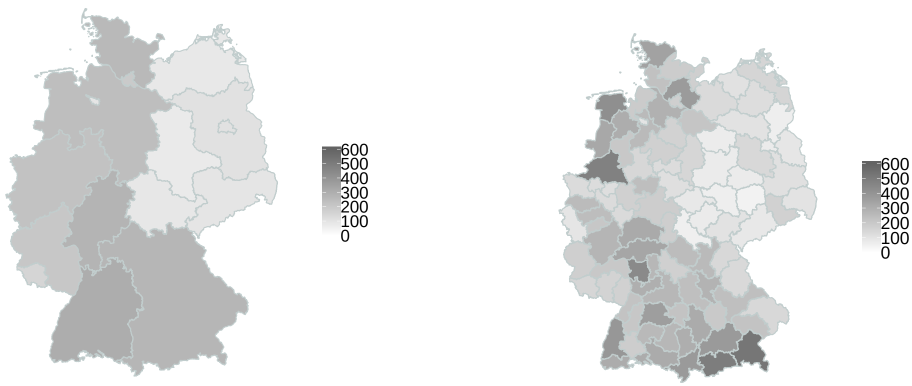

Figure 2: Map of the benchmarked FH-MI estimates for the federal states (left) and for the planning regions (right) of the mean of HH net wealth in TEUR based on Research Data and Service Centre (RDCS) of the Deutsche Bundesbank (2014), Panel on Household Finances (PHF) 2014, authors' estimations.

Furthermore, the estimates also provide evidence of the heterogeneity of private wealth across western German federal states. Our estimates confirm the findings by the Bundesbank (2016) reporting that the average net wealth of federal states in the south (Bavaria, Baden-Württemberg, and Hesse with an average figure of EUR 284,000) is about 50% higher than in federal states located in the West (North Rhine-Westphalia, Rhineland-Palatinate, and Saarland with an average net wealth of EUR 193,000). This difference is mostly driven by the conditional mean value of owner-occupied housing (EUR 275,000 in the south and EUR 197,000 in the west), financial assets (EUR 73,000 to EUR 53,000) and only slightly by home ownership rates (48% to 44%). Furthermore, the federal city state of Berlin reports an average net wealth below EUR 100,000. This observation is reasonable with regard to the home ownership rates, which are often lower in the urban regions. This especially holds for Berlin with a home ownership rate of 15.6% (Landesamt für Statistik Niedersachsen, 2014).

The analysis at the level of the planning regions enables further insights. Our results provide evidence for heterogeneity within western Germany. The regions around economically prosperous cities in the west - namely Munich, Frankfurt and Hamburg - report the highest private wealth levels in Germany. The top two regions (Südostoberbayern and Oberland) are located in the south of Munich, where average private net wealth is around EUR 520,000. The other end of the distribution in the west predominantly contains regions in the Ruhr area. This region was severely affected by the breakdown of the coal and steel industry, which is still reflected in the highest unemployment rates among western German planning regions. Note that the four planning regions of the Ruhr Area are listed among the five regions with the highest unemployment rates in western Germany in 2014: 1. Emscher-Lippe (11.7%), 2. Dortmund (11.5%), 3. *Bremen* (10.4%), 4. Duisburg/Essen (10.3%), and 5. Bochum/Hagen (9.1%) (Bundesinstitut für Bau-, Stadt-, und Raumforschung, 2017). The results show that average private net wealth figures in these four planning regions are quite similar, ranging from EUR 115,000 in Dortmund to EUR 125,000 in Duisburg/Essen.

The analysis of the 22 planning regions in eastern Germany including Berlin also captures regional differences with respect to the distribution of private wealth. The four regions with the lowest estimates for private wealth in eastern Germany are geographically dispersed across four different federal states: Westsachsen in Saxony (EUR 53,000), Südthüringen in Thuringia (EUR 62,000), Ueckermark-Barmin in Brandenburg (EUR 66,000) and Halle/Saale in Saxony-Anhalt (EUR 66,000). The regions with highest private wealth are located in the south-west of Berlin (Havelland-Fläming: EUR 130,000), at the Baltic Sea (Vorpommern: EUR 143,000) as well as in the region around the city of Dresden (Oberes Elbtal/Osterzgebirge: EUR 154,000).

The results for German planning regions show that wealth is geographically dispersed in both parts of the country. Furthermore, we can show that private wealth in all eastern German planning regions still remains far below the national average. However, the wealthiest planning regions in the East report higher private wealth figures than the western German regions with the lowest private wealth estimates.

5 Conclusion

The concentration of private income and wealth among countries and regions provides a motivation to assess the regional distribution of financial resources. While data sources for the estimation of regional income indicators are comprehensive, the current best source for the estimation of private wealth is, in most countries, survey data. In this context, the European Central Bank launched the HFCS in 2010, which is conducted in each country of the euro area. While the HFCS is, so far, used to report national estimates for private wealth, this work shows how to estimate average HH net wealth for low regional levels, namely the 16 federal states and 96 planning regions in Germany. We contribute to the literature by estimating the regional distribution of private wealth in Germany by means of a modified FH model, which

a) accounts for the skewness of the wealth distribution by means of a log-transformation in the estimation,

b) accounts for multiple imputation, and

c) ensures internal consistency of the estimates with a national benchmark.

The results of the estimation are very insightful and contribute to the discussion on the distribution of private wealth, which has strikingly gained attention in the scientific literature as well as in the public debate in recent years. Even 25 years after German reunification, there is clear dividing line at the former border with respect to private wealth. However, the wealthiest planning regions in the east report higher private wealth figures than the western German regions with the lowest private wealth estimates. This important finding can be relevant in the context of the discussion of the expiry of the *Solidarity Pact II* at the end of 2019. The *Solidarity Pact II* was the follow-up program of the *Solidarity Pact*, a package of measures allocating financial resources to east German federal states to foster the catching-up process of former East Germany. In this context, the German government concluded that new regional support schemes should be designed to foster all structurally lagging regions irrespective of their geographic location (Bundesministerium des Innern, für Bau und Heimat, 2019, see p.17).

Even though the application in this work concentrates on Germany, the theory is easily transferable to other survey data multiple imputation in order to account for item non-response, including the surveys that form part of the HFCS. Next to the estimation of regional estimates, the approach may help to obtain estimates for specific components of net wealth that do not contain sufficient observations or which would violate confidentiality standards. For the HFCS, estimates for various financial assets are either not reported for some countries because of a low precision of the estimates or because the sample size is below 25 (Household Finance and Consumption Network, 2016b).

For further research, it is of interest whether the proposed FH approach can also be used for other indicators. The application of the mean enables the usage of Rubin's rule. However, it is unclear whether the rule can also be applied for quantiles or non-linear poverty and inequality indicators like the headcount ratio (Foster, Greer, and Thorbecke, 1984) or the Gini coefficient (Gini, 1912). One way could be to use a transformation for indicators that do not fulfill the normality requirement before applying Rubin's rule as suggested in Marshall, Altman, Holder, and Royston (2009). A suitable transformation for the headcount ratio might be the arcsin transformation which is also used in the FH approach for binary dependent variables (Casas-Cordero, Encina, and Lahiri, 2016; Schmid, Bruckschen, Salvati, and Zbiranski, 2017). In this work, we propose an easy-to-apply approach by using the log-transformation to meet the model assumptions. Another approach to handle skewed data could be the assumption of a skewed normal distribution in the FH model (Moura, Neves, and do N. Silva, 2017). With regards to the back-transformation, the usage of the smearing estimator proposed by Duan (1983) could be transferred to the Fay-Herriot model as Li, Liu, and Zhang (2017) use it for the nested error linear regression model. Furthermore, future approaches could consider the panel structure of the survey.

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