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Going the extra mile: Effort by workers and job-seekers

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Non-technical summary

Research Question

We investigate how two types of effort, namely labor effort by the employed and search effort by the unemployed, affect the existence and uniqueness of the adjustment dynamics – 'determinacy' – in a labor search-and-matching model. Time use evidence reveals that effort at work varies procyclically and this can help explain the procyclicality of labor productivity observed across many countries and time periods. Variable search effort has empirical support; it amplifies employment fluctuations in the labor search model, thereby bringing the model closer to the data.

Contribution

In the search-and-matching model, hiring decisions are forward-looking: a firm compares the cost of hiring an additional worker with the stream of profits that this worker helps to generate in the future. Suppose that expectations of higher future profits make firms overly optimistic and they post too many vacancies. If such beliefs become self-fulfilling, this can lead to inefficient macroeconomic fluctuations and there is room for welfare-enhancing stabilization policies. As shown in the literature, self-fulfilling beliefs can arise in the labor search model if new job matches are very responsive to vacancies. In this case, firms are actually induced to post too many vacancies. They do not take into account the congestion effect by which an additional vacancy makes the labor market tighter, which increases the effective hiring cost of all firms in the market and thereby makes existing workers more valuable. We contribute to this literature by evaluating the impact of two empirically plausible model extensions, worker effort and search effort by job-seekers.

Results

We find that variable labor effort makes indeterminacy more likely relative to the standard search model. Variable labor effort generates increasing returns to hours in production. This implies that output increases more than labor input, such that firms' optimistic beliefs about future profits become self-validating. Variable search effort has the opposite effect; the indeterminacy region shrinks. This happens because of a strategic complementarity that reduces congestion. Vacancy posting by firms raises labor market tightness and the job finding rate, which leads to greater search effort by the unemployed, which in turn dampens the tightening of the labor market.

Nichttechnische Zusammenfassung

Fragestellung

In der vorliegenden Studie wird untersucht, wie zwei Arten von zeitvariierenden Anstrengungen am Arbeitsmarkt – der Arbeitseinsatz von Beschäftigten und die Anstrengungen der Stellensuche von Arbeitslosen – die Existenz und Eindeutigkeit des dynamischen Gleichgewichts in einem Such- und Matching-Modell beeinflussen. Gemäß Auswertungen des American Time Use Survey schwankt der Arbeitsaufwand der Beschäftigten prozyklisch im Konjunkturverlauf. Die explizite Berücksichtigung dieser Erkenntnis in einem Suchmodell kann dazu beitragen, die oftmals beobachtete Prozyklizität der Arbeitsproduktivität zu erklären. Eine prozyklische Arbeitsproduktivität könnte eine Ursache für Indeterminiertheit und erwartungsgetriebene Schwankungen darstellen, da erhöhter Arbeitseinsatz in diesem Fall zu steigenden Erträgen aus den Produktionsstunden führt. Auch für einen variablen Suchaufwand der Arbeitslosen finden sich empirische Belege in Befragungsdaten. Aufgrund einer strategischen Komplementarität zwischen den Suchanstrengungen der Beschäftigten einerseits und der Unternehmen andererseits verstärkt dieser Mechanismus – konsistent mit dem empirischen Befund – das Ausmaß der Beschäftigungsschwankungen im Suchmodell und verändert somit dessen dynamische Eigenschaften.

Beitrag

Die Einstellungsentscheidungen im Suchmodell sind zukunftsorientiert: Ein Unternehmen stellt die Kosten für einen zusätzlichen Beschäftigten dem zusätzlichen künftigen Gewinnstrom gegenüber. Die Erwartung höherer zukünftiger Gewinne kann Unternehmen zu übertriebenem Optimismus veranlassen. In diesem Fall werden die Unternehmen dazu veranlasst, übermäßig viele freie Stellen auszuschreiben. Sie berücksichtigen dabei nicht, dass eine zusätzliche freie Stelle die Anspannung am Arbeitsmarkt verstärkt. Dies wiederum führt bei allen Unternehmen am Markt zu höheren effektiven Kosten für Neueinstellungen und erhöht so den Wert bereits vorhandener Arbeitskräfte. Wenn solche Erwartungen selbsterfüllend werden, kann dies ineffiziente makroökonomische Schwankungen zur Folge haben und Spielraum für wohlfahrtssteigernde Stabilisierungsmaßnahmen schaffen. Unser Beitrag besteht darin, zwei empirisch plausible Wirkungskanäle – die Anstrengungen von Arbeitnehmern und die Anstrengungen von Arbeitsuchenden – in das Modell einzugliedern und quantitativ zu evaluieren.

Ergebnisse

Wir stellen fest, dass die Wahrscheinlichkeit der Indeterminiertheit steigt, falls der Aufwand der Beschäftigten im Suchmodell variabel ausgestaltet ist. Infolge steigender Erträge aus den geleisteten Arbeitsstunden nimmt die Arbeitsproduktivität stärker zu als der Arbeitseinsatz, so dass sich die optimistischen Gewinnerwartungen der Unternehmen selbst bestätigen. Ein variabler Suchaufwand bewirkt das Gegenteil: Der Bereich der Indeterminiertheit schrumpft. Ursache hierfür ist die folgende strategische Komplementarität: Erhöhen die Unternehmen die Zahl der ausgeschriebenen Stellen, erhöht sich dadurch die Anspannung am Arbeitsmarkt und somit die Übergangsrate in die Beschäftigung. Somit haben Arbeitslose größere Anreize nach einer Stelle zu suchen, wodurch die angespannte Lage am Arbeitsmarkt wiederum gedämpft wird.

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Abstract

We introduce two types of effort into an otherwise standard labor search model to examine indeterminacy and sunspot equilibria. Variable *labor* effort gives rise to increasing returns to hours in production. This makes workers more valuable and contributes to self-fulfilling profit expectations, raising the likelihood of indeterminacy. Variable *search* effort makes workers search more intensively in a tighter labor market, which alleviates congestion and reduces the likelihood of indeterminacy. Indeterminacy disappears completely when vacancy posting costs are replaced with hiring costs.

Keywords: determinacy, effort, hours, labor market frictions, search intensity.

JEL classification: E23, E24, E32, E64, E71.

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1 Introduction

In this paper, we investigate how two types of effort, namely labor effort by the employed and search effort by the unemployed, affect the existence and uniqueness of the adjustment dynamics in an otherwise standard labor search-and-matching model à la Diamond-Mortensen-Pissarides (Pissarides, 2000). Variable search effort is empirically relevant and has significant effects on labor market outcomes (Leduc and Liu, 2019). Variable labor effort – closely associated with the phenomenon of labor hoarding – also has empirical support (Burda, Genadek, and Hamermesh, 2017) and can explain the procyclicality of labor productivity observed across many countries and time periods.

As shown by Krause and Lubik (2010), sunspot equilibria can occur in the search-and-matching model of the labor market. Driven by expectations of higher future profits, firms post more vacancies. For a given number of job-seekers, this lowers the chance of filling these vacancies and thereby increases firms' effective recruitment costs. Firms do not take this negative congestion externality into account in their hiring decisions. When the number of matches is very elastic to vacancies, posting vacancies leads to more successful matches and the congestion effect is large. This raises the asset value of a worker, validating firms' higher profit expectations in equilibrium. The present paper examines the role of effort by workers and job seekers in this framework. Our main insight is that these two types of effort have opposing effects on equilibrium uniqueness.

Variable *labor* effort validates sunspot beliefs by firms by increasing the marginal product of labor. Equilibrium effort in our model is an increasing function of hours. A rise in hours worked implies a simultaneous rise in effort, generating increasing returns to hours in production. This implies that the marginal worker will be more valuable than in the constant effort model: the additional hours he puts in raise output, and thus future profits, more than proportionally. This helps to make sunspot beliefs of higher expected profits self-fulfilling.

Variable *search* effort alleviates the congestion externality that arises when firms post vacancies. As the labor market gets tighter, the job finding rate rises and this induces the unemployed to search more intensively, thereby limiting the rise in tightness and the associated rise in the workers' asset value. Consequently, indeterminacy becomes less likely when search intensity is endogenous.

While the search-and-matching model of the labor market is undoubtedly the most popular model of labor market frictions, it does suffer from certain deficiencies (see Shimer, 2005). In a competing approach vacancy posting costs are replaced with hiring costs (see, for example, Gertler and Trigari, 2009; Pissarides, 2009; Hertweck, 2013; Galí and van Rens, 2014; Christiano, Eichenbaum, and Trabandt, 2016). This leads to more plausible vacancy dynamics in response to technology shocks. We show that indeterminacy completely disappears in this alternative model, even if variable labor effort is allowed for. This can be explained by the absence of congestion effects that are inherent to the search model.

Our exercise is useful for the development and calibration of empirically plausible business cycle models with labor search, and for understanding the role of beliefs in business cycles. Business cycles driven by fundamental shocks, i.e. to technology or preferences, are not generally inefficient and as such do not warrant any policy response. Instead, under equilibrium indeterminacy and multiple equilibria, self-fulfilling beliefs can lead to inefficient fluctuations and macroeconomic volatility. In that case, there is room for policy to stabilize the economy and raise economic welfare (Farmer and Guo, 1994).

Related Literature. The paper speaks to two strands of the literature, labor search-and-matching models, and models with equilibrium indeterminacy. Regarding the former, it is now well established that labor markets are not perfectly flexible, but are instead characterized by considerable frictions as workers do not seamlessly move from one job to another. The search-and-matching framework presented by Diamond, Mortensen and Pissarides (Pissarides, 2000) has emerged as a consensus model to characterize the labor market, with Merz (1995) and Andolfatto (1996) integrating it into real business cycle theory.

The second strand of the literature is on indeterminacy in macroeconomics.[1] Farmer and Guo (1994) argue that sunspot shocks should be taken seriously as a potential source of business cycle fluctuations – rather than being a mere intellectual curiosity. As shown by Benhabib and Farmer (1994), increasing returns in the production function can be a source of indeterminacy, leading to multiple equilibria. Wen (1998) demonstrates that, in a real business cycle model with capacity utilization, indeterminacy can arise under an empirically plausible calibration in the case of (mildly) increasing returns. Our work is related in the sense that variable labor effort generates increasing returns to hours in production, providing a potential source of indeterminacy.

We build on Krause and Lubik (2010) and Lazaryan and Lubik (2018), who bring the two strands of the literature together.[2] Indeed, Farmer (2019) argues that a meaningful theory of involuntary unemployment and large welfare losses must allow for indeterminacy.

The remainder of the paper is structured as follows. Section 2 presents our model with two types of effort. In the linearized model written as a two-equation system, the transition matrix is derived and its roots determine whether a stable model solution exists and is unique. Section 3 discusses the calibration and derives the steady state. Then Section 4 first provides the economic intuition on how the congestion externality affects equilibrium existence and uniqueness. Second, it derives the condition for indeterminacy in the standard labor search model; this is done analytically in the case of risk neutrality. For the more general case with risk aversion, it conducts a numerical exercise showing how determinacy depends on a set of parameter values. Third, it shows how variable labor effort affects the determinacy results. Finally, it presents a similar analysis for variable search effort. Section 5 derives an alternative model with hiring costs, and discusses its determinacy properties. Finally, Section 6 concludes.

2 Model

In the following, we outline our search-and-matching model featuring two additional labor margins, hours and effort, as well as variable search intensity by job-seekers. In essence, we extend the analysis in Krause and Lubik (2010).[3]

Abstracting from a participation margin, we normalize the labor force to unity, such that

$$n_t + u_t = 1, \tag{1}$$

where n_t denotes employment and u_t is the unemployment rate. The law of motion for

[1] For surveys of this literature, see Benhabib and Farmer (1999) and Farmer (2019).

[2] Zanetti (2006) studies a New Keynesian model with search-and-matching frictions where the monetary policy rule generates indeterminacy.

[3] Unlike Hashimzade and Ortigueira (2005), we abstract from physical capital.

employment is

$$n_{t+1} = (1-\rho)(n_t + m_t), \tag{2}$$

with initial employment n_0 given. The parameter $\rho \in (0,1)$ captures the job separation rate and m_t is the number of new job matches. A constant separation rate is justified by the observation that – in comparison with the job creation margin – the empirical counterpart of ρ is fairly stable over the business cycle (for evidence on this, see Hall, 2005b; Fujita and Ramey, 2009; Shimer, 2012). The matching technology is a function of unemployed workers, their search intensity s_t, and vacancies v_t,

$$m_t = \chi(s_t u_t)^{\xi} v_t^{1-\xi}, \tag{3}$$

where $\xi \in (0,1)$ is the match elasticity to 'total search effort' (Merz, 1995) and $\chi > 0$ captures the efficiency of the matching process. Petrongolo and Pissarides (2001) argue that the Cobb-Douglas form for the matching function is a stylized fact compatible with a large number of empirical studies. By spending more time and resources searching for jobs, unemployed workers can raise the probability of match success. Search intensity entering the matching function multiplicatively with unemployment can be thought of as 'input-augmenting' (Pissarides, 2000), similar to technological progress in the production function for goods.

The representative household is composed of n_t workers whose wage income is $w_t h_t$ each, and u_t unemployed members who receive unemployment benefits b and spend resources $\mathcal{G}(s_t)$ on searching for a job. Households choose a path for consumption $\{C_t\}_{t=0}^{\infty}$ to maximize expected lifetime utility,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma} - 1}{1-\sigma} - n_t g(h_t, e_t) \right], \tag{4}$$

subject to the budget constraint $C_t + T_t = n_t w_t h_t + (1 - n_t)(b - \mathcal{G}(s_t))$, where T_t are lump-sum taxes, $\beta \in (0,1)$ is the subjective discount rate, $\sigma \geq 0$ is the constant parameter of relative risk aversion and $g(h_t, e_t)$ measures individual disutility of providing hours of work h_t and effort per hour e_t. Unemployment benefits are financed through lump-sum taxes. For simplicity, we abstract from public debt and stipulate that the government budget constraint is balanced in each period, i.e. $T_t = u_t b$ for all t.

In period t, an employed worker receives the wage income $w_t h_t$. In the next period, he is either still employed with probability $(1-\rho)$, in which case he has an expected value of $E_t\{\beta_{t,t+1} W_{t+1}\}$, or the employment relation is dissolved with probability ρ, then his expected value is $E_t\{\beta_{t,t+1} U_{t+1}\}$. The worker's asset value therefore is

$$W_t = w_t h_t + E_t\{\beta_{t,t+1}[(1-\rho) W_{t+1} + \rho U_{t+1}]\}, \tag{5}$$

where $\beta_{t-1,t} = \beta \frac{\lambda_t}{\Lambda_{t-1}}$ is the household's stochastic discount factor and $\lambda_t = C_t^{-\sigma}$ is the marginal utility of consumption. The value of being unemployed U_t is in turn given by

$$U_t = b - \mathcal{G}(s_t) + E_t\{\beta_{t,t+1}[p_t(1-\rho)W_{t+1} + (1 - p_t(1-\rho))U_{t+1}]\}. \tag{6}$$

The term $b - \mathcal{G}(s_t)$ can be thought of as the (net) value of leisure. In the next period, the unemployed person faces a probability p_t of finding a new job, which has an expected value of $E_t\{\beta_{t,t+1} W_{t+1}\}$, and a probability $1 - p_t$ of remaining unemployed, which has an expected value of $E_t\{\beta_{t,t+1} U_{t+1}\}$. The job finding rate is defined as the number of matches over unemployment, $p_t = m_t/u_t$. Defining the worker's surplus as $\mathcal{W}_t = W_t - U_t$, we can subtract (6) from (5) to write the match surplus going to the household as

$$\mathcal{W}_t = w_t h_t - g(h_t, e_t)/\lambda_t - (b - \mathcal{G}(s_t)) + (1-\rho)E_t\left\{\beta_{t,t+1}(1-p_t)\mathcal{W}_{t+1}\right\}. \tag{7}$$

One-worker firms produce consumption goods y_t. Let $\mathcal{J}_t$ denote the firm's match surplus, i.e. the value to the firm of hiring a worker. It is the sum of current profits, i.e. output minus the wage bill $w_t h_t$, and the firm's continuation value. The latter is the expected future match surplus in case the employment relationship continues, which happens with probability $(1-\rho)$. The firm's value is zero in case the worker and the firm separate, which happens with probability ρ. Thus,

$$\mathcal{J}_t = y_t - w_t h_t + (1-\rho)E_t\{\beta_{t,t+1}\mathcal{J}_{t+1}\}. \tag{8}$$

The value of posting a vacancy is given by the negative of the vacancy posting cost c, plus the expected future value of the vacancy. The latter is the weighted average of the value of filling the vacancy, i.e. the firm's match value in the next period, which has probability $q_t(1-\rho)$, and the future value of the unfilled vacancy, V_{t+1}, which has probability $(1 - q_t(1-\rho))$, where $q_t = m_t/v_t$ is the vacancy filling rate. Therefore,

$$V_t = -c + E_t\{\beta_{t,t+1}[q_t(1-\rho)\mathcal{J}_{t+1} + (1-q_t(1-\rho))V_{t+1}]\}. \tag{9}$$

Free entry drives the value of a vacancy to zero at each point in time, such that $V_t = 0$ for all t and thus (9) becomes

$$c/q_t = (1-\rho)E_t\{\beta_{t,t+1}\mathcal{J}_{t+1}\}. \tag{10}$$

Combining the firm's asset value (8) and the free entry condition (10), we get the following expression for the firm's match surplus: $\mathcal{J}_t = y_t - w_t h_t + (1-\rho)c/q_t$. Finally, using this to substitute out $\mathcal{J}_{t+1}$ in the free entry condition (10), we obtain the vacancy posting condition,

$$c/q_t = (1-\rho)E_t\left\{\beta_{t,t+1}\left(y_{t+1} - w_{t+1}h_{t+1} + c/q_{t+1}\right)\right\}. \tag{11}$$

Hiring is an investment decision where the intertemporal dimension, more specifically the expected value of a marginal worker, is key. Equation (11) states that the current cost of posting a vacancy, c/q_t, should equate the expected benefit of posting a vacancy which consists of three terms: (1) the output produced y_t, (2) wage payments $w_t h_t$, and (3) the savings on future vacancy posting costs due to a successful match. The transversality condition is

$$\lim_{T\to\infty} E_t\{\beta_{t,T}\mathcal{J}_T n_T\} = 0, \tag{12}$$

see also Mortensen (2009). Under Nash bargaining, the real wage maximizes the joint match surplus $\mathcal{W}_t^{\eta}\mathcal{J}_t^{1-\eta}$, where $\eta \in (0,1)$ is the worker's bargaining share. The surplus sharing rule is $(1-\eta)\mathcal{W}_t = \eta\mathcal{J}_t$, and the bargaining wage satisfies

$$w_t h_t = \eta(y_t + c\theta_t) + (1-\eta)[g(h_t, e_t)/\lambda_t + (b - \mathcal{G}(s_t))], \tag{13}$$

where $\theta_t \equiv v_t/u_t$ is labor market tightness. Finally, goods market clearing requires that consumption equals net aggregate output, $C_t = Y_t$. In a symmetric equilibrium, the latter is total output produced by all firms, less the resources used up in vacancy posting and search activities, $Y_t = y_t n_t - cv_t - \mathcal{G}(s_t)u_t$.

Labor effort. The firm's production function is $y_t = e_t h_t$. Worker effort is modelled as in Bils and Cho (1994), who assume that labor disutility is given by

$$g(h_t, e_t) = \frac{\lambda_h h_t^{1+\sigma_h}}{1+\sigma_h} + h_t \frac{\lambda_e e_t^{1+\sigma_e}}{1+\sigma_e}. \tag{14}$$

The parameters $\sigma_h > 0$ and $\sigma_e > 0$ measure, respectively, the curvature of the labor disutility function in hours and effort, while $\lambda_h > 0$ and $\lambda_e > 0$ are the weights on hours and effort in labor disutility.

Every period, firms and workers jointly choose hours and effort in order to minimize (14), subject to the production technology. Equilibrium effort is an increasing and convex function of hours per worker, $e_t = e_0 h_t^{\sigma_h/(1+\sigma_e)}$, where $e_0 = (\frac{1+\sigma_e}{\sigma_e}\frac{\lambda_h}{\lambda_e})^{1/(1+\sigma_e)}$. Using the optimal effort choice, we can rewrite labor disutility as a function of hours only, $g(h_t) = \lambda_h \frac{1+\sigma_h+\sigma_e}{(1+\sigma_h)\sigma_e} h_t^{1+\sigma_h}$, and the production function becomes

$$y_t = e_0 h_t^{\phi}, \tag{15}$$

with $\phi = 1 + \frac{\sigma_h}{1+\sigma_e}$ measuring the returns to hours in production. For a given elasticity of labor disutility to hours σ_h, a finite value for σ_e implies that there are increasing returns to hours in production ($\phi > 1$), i.e. one additional hour of work produces more than one additional unit of output. The no-effort model is recovered as the limiting case where $\sigma_e \to \infty$; any incremental rise in effort would lead to an overwhelmingly large utility loss, such that in equilibrium effort does not change.

Hours worked are determined jointly by the firm and the worker to maximize the sum of the firm's and worker's surpluses. Hours per worker thus satisfy

$$\phi e_0 h_t^{\phi-1} = g'(h_t)/\lambda_t, \tag{16}$$

where $g'(h_t)$ denotes the worker's disutility from working an additional hour. By (16), the marginal product of hours must equal the marginal rate of substitution between hours and consumption.

Search effort. The household chooses the optimal amount of search intensity up to the point where the marginal search costs and the benefits from searching just balance out. As explained in chapter 5 of Pissarides (2000), worker i chooses s_i, taking the aggregate job finding rate p_t and labor market tightness θ_t as given. His personal job finding rate does, however, depend upon his search intensity, $p_{it} = p_t(s_{it}; s_t, \theta_t)$. For each efficiency unit supplied in the search process, workers transition from unemployment to employment at rate $\frac{m_t}{s_t u_t}$. Therefore, the transition probability of worker i per period is given by $p_{it} = \frac{m_t}{s_t u_t} s_{it}$, and the derivative is $\frac{\partial p_{it}}{\partial s_{it}} = \frac{p_t}{s_t}$. In equilibrium, search intensity is positively related to labor market tightness,

$$\mathcal{G}'(s_t) = \frac{\eta}{1-\eta}\frac{c\theta_t}{s_t}. \tag{17}$$

The left hand side of (17) is the marginal cost of exerting search effort. The right hand side is the contribution of one efficiency unit of search to expected value of employment, $\frac{\partial p_{it}}{\partial s_{it}}(1-\rho)E_t\{\beta_{t,t+1}\mathcal{W}_{t+1}\}$, which we can combine with the surplus sharing rule and the free entry condition for vacancies (10), to obtain (17). Table 1 summarizes the equilibrium conditions.

Definition 1. *A decentralized equilibrium in the labor search model with two types of effort is a set of infinite sequences for quantities* $\{u_t, \theta_t, n_{t+1}, m_t, h_t, y_t, Y_t, v_t, s_t\}_{t=0}^{\infty}$, *matching rates* $\{q_t, p_t\}_{t=0}^{\infty}$ *and wages* $\{w_t\}_{t=0}^{\infty}$, *satisfying the transversality condition (12), such that:*

1. *given matching rates and wages, the quantities solve the household's problem,*
2. *given matching rates and wages, the quantities solve the firm's problem,*
3. *employment is determined by the law of motion (2),*
4. *matching rates are determined by the matching function (3),*
5. *wages solve the Nash bargaining problem,*
6. *goods markets clear.*

Table 1: Model equilibrium conditions

Unemployment	$u_t = 1 - n_t$
Tightness	$\theta_t = v_t/u_t$
Job finding rate	$p_t = m_t/u_t$
Vacancy filling rate	$q_t = m_t/v_t$
Employment	$n_{t+1} = (1-\rho)(n_t + m_t)$
Matches	$m_t = \chi(s_t u_t)^{\xi} v_t^{1-\xi}$
Hours	$\phi e_0 h_t^{\phi-1} = g'(h_t)/\lambda_t$
Production	$y_t = e_0 h_t^{\phi}$
Output	$Y_t = y_t n_t - c v_t - \mathcal{G}(s_t) u_t$
Vacancies	$c/q_t = (1-\rho)\beta E_t \left\{ (Y_t/Y_{t+1})^{\sigma} \left(y_{t+1} - w_{t+1}h_{t+1} + c/q_{t+1} \right) \right\}$
Wages	$w_t h_t = \eta(y_t + c\theta_t) + (1-\eta)[g(h_t)/\lambda_t + (b - \mathcal{G}(s_t))]$
Search intensity	$\mathcal{G}'(s_t)s_t = \frac{\eta}{1-\eta} c\theta_t$

We linearize the equilibrium conditions around their non-stochastic steady state. Letting a hat above a variable denote that variable's linear approximation, the system can be condensed into two equilibrium conditions determining one control variable, $\hat{\theta}_t$, and one state variable, $\hat{n}_{t+1}$,

$$\alpha_1 E_t\{\hat{\theta}_{t+1}\} = \left[\xi + \sigma \frac{1}{\delta_1} \left(\frac{cv}{Y} + \frac{\mathcal{G}(s)u}{Y} \right) + \sigma\alpha_2 \left(1 - \xi + \frac{\xi}{\iota} \right) \rho \right] \hat{\theta}_t - \sigma\alpha_2 \left(\frac{\rho}{u} - \frac{\delta_2 - 1}{\delta_2} \right) \hat{n}_t, \tag{18}$$

$$\hat{n}_{t+1} = \left(1 - \xi + \frac{\xi}{\iota} \right) \rho\hat{\theta}_t + \frac{u-\rho}{u} \hat{n}_t. \tag{19}$$

In (18), ι is the elasticity of the search cost function, $\frac{\mathcal{G}'(s_t)s_t}{\mathcal{G}(s_t)}$, while α_1 and α_2 are defined as follows,

$$\alpha_1 = \beta(1-\rho)\frac{\iota - 1}{\iota}\xi \left(1 - \frac{\eta}{\xi}p \right) + \sigma\frac{\delta_2}{\delta_1} \left(\frac{cv}{Y} + \frac{\mathcal{G}(s)u}{Y} \right),$$

$$\alpha_2 = \sigma\frac{\delta_2}{\delta_1} \left(1 + \frac{c\theta}{Y} + \frac{\mathcal{G}(s)}{Y} \right),$$

and we introduce the composite parameters $\delta_1 \geq 1$ and $\delta_2 \geq 1$,

$$\delta_1 = 1 + \sigma \left(1 + \frac{c\theta}{Y} + \frac{\mathcal{G}(s)}{Y}\right) \frac{\phi}{(1+\sigma_h) - \phi},$$

$$\delta_2 = 1 + \beta(1-\rho)(1-\eta)\frac{\phi}{1+\sigma_h}\frac{e_0 h^\phi}{c/q}.$$

We can write the two-equation system (18) and (19) in a more compact way:

$$\begin{bmatrix} E_t\{\hat{\theta}_{t+1}\} \\ \hat{n}_{t+1} \end{bmatrix} = \begin{bmatrix} \left\{\frac{\xi}{\alpha_1} + \frac{\sigma}{\alpha_1\delta_2}\left(\frac{cv}{Y} + \frac{\mathcal{G}(s)u}{Y}\right) + \sigma\frac{\alpha_2}{\alpha_1}\left(1-\xi+\frac{\xi}{\iota}\right)\rho\right\} & -\sigma\frac{\alpha_2}{\alpha_1}\left(\frac{\rho}{u} - \frac{\delta_2 - 1}{\delta_2}\right) \\ \left(1-\xi+\frac{\xi}{\iota}\right)\rho & 1 - \frac{\rho}{u} \end{bmatrix} \begin{bmatrix} \hat{\theta}_t \\ \hat{n}_t \end{bmatrix}. \quad (20)$$

The model has a (locally) unique stable solution if and only if the transition matrix in (20) has one stable root (i.e. smaller than 1) and one unstable root (i.e. greater than 1). Then θ_t can be solved forward in terms of the state variable n_t and the model is characterized by saddle-path stability. If, instead, both roots are unstable, the model solution is non-existent. Finally, if both roots are stable, the solution is indeterminate and multiple equilibria exist. This means that any initial value of θ is consistent with the model's equilibrium condition in Table 1.[4]

3 Parameterization

A few introductory words on our calibration strategy are in order. It is common practice to calibrate parameters (for example the leisure value b) and let steady state variables such as the unemployment rate be determined endogenously. However, we have a good reason to deviate from this common practice because we do not calibrate one model and examine its properties; rather, we calibrate and examine a two-dimensional continuum of models on the (ξ,η)-plane. To maintain comparability, steady state unemployment should remain constant across all calibrated models. Hence, we always adjust b simultaneously whenever a change in, say, the bargaining parameter η is examined. While this approach is not standard, it is in our view the most appropriate to address our research question.

We calibrate the model to a monthly frequency and set the discount factor to $\beta = 0.99^{1/3}$. A risk aversion parameter of $\sigma = 1$ yields logarithmic consumption utility and implies balanced growth. The steady state unemployment rate is calibrated to 6% in line with US post-war data. The cost of posting a vacancy is set to $c = 0.1$ as in Krause and Lubik (2010). This value is consistent with Hagedorn and Manovskii (2008), who propose a non-capital cost of posting a vacancy equal to 11% of labor productivity. We also note that our choice for c implies a share of vacancy posting costs over GDP close to 1%, which is in line with Andolfatto (1996), Blanchard and Galí (2010) among others.

Further, we calibrate the match elasticity to total search effort $\xi = 0.5$, which is at the lower end of the range proposed by Petrongolo and Pissarides (2001). Hall (2005a) estimates an even lower match elasticity using the US Job Openings and Labor Turnover Survey (JOLTS). He first computes tightness θ by dividing the number of vacancies by the unemployed; second, he computes the job finding rate p as the ratio of new hires ('matches')

[4] A more general condition for a determinate solution is that there are as many non-predetermined variables as non-explosive roots of Φ in the system $z_t = \Phi z_{t+1} + \Gamma e_{t+1}$, where z are the endogenous variables and e are the exogenous shocks (see Benhabib and Farmer, 1999).

to the unemployed; finally, he calculates the ratio of $\ln\Delta p$ to $\ln\Delta\theta$, where Δ measures the change over the period December 2000 to December 2002. This yields 0.765, such that the match elasticity ξ is 0.235. For different groups of unemployed job-seekers, Hall and Schulhofer-Wohl (2018) report a range of estimates of the match elasticity from 0.45 to 0.83.

Our calibration for the separation rate is based on Shimer (2012)'s estimate using US labor market micro data. His value $\rho = 0.034$ for the average monthly separation rate implies that jobs last for around two and a half years. Hobijn and Sahin (2009) present estimates of monthly separation rates, defined as the fraction of workers who leave their jobs, in different OECD countries ranging from 0.7% to 2%. Our calibration for the steady state vacancy filling rate, $q = 0.33$, follows den Haan, Ramey, and Watson (2000). Christoffel, Kuester, and Linzert (2009) also propose this value based on European data.

The worker's bargaining weight is set to $\eta = 0.5$, such that the Hosios condition is satisfied. Empirical evidence on the size of the bargaining weight is scant. In Hall and Milgrom (2008), the implied worker's share equals 0.54 as in Mortensen and Pissarides (1994), even though wage setting is rather different, resting on a bargaining model with different threat points than the ones assumed under Nash bargaining. In both models, this value is obtained by solving the zero profit condition to match the unemployment rate in the data.

The parameter governing the disutility of hours, σ_h, is calibrated to 2, which is in the middle of the range proposed by Keane and Rogerson (2012). The disutility of effort parameter, σ_e, is calibrated to target $\phi = 1.5$. This calibration of increasing returns in aggregate production follows Barnichon (2010) and is consistent with Bils and Cho (1994)'s work. In addition, Lewis, Villa, and Wolters (2019) estimate the parameter ϕ in a New Keynesian model with variable capital and labor utilization and find a value greater than 1.5.

From this calibration, we derive several other steady state variables and parameters recursively. At the steady state, employment is $n = 1 - u$. The number of matches is derived from the law of motion for employment, $m = \frac{\rho}{1-\rho}n$. Given that we pin down the vacancy filling rate q, vacancies are given by $v = m/q$. Labor market tightness is $\theta = v/u$. Without loss of generality, we normalize search intensity to unity, $s = 1$. Matching efficiency is computed as $\chi = q(\theta/s)^{\xi}$. We set hours h to unity and find the value of λ_h which achieves this normalization. Similarly, we calibrate λ_e to obtain $e_0 = 1$, which yields $\lambda_e = \frac{1+\sigma_e}{\sigma_e}\lambda_h$. Firm output y is equal to $e_0 h^{\phi}$, see the production function. GDP is aggregate production minus vacancy posting costs and job search costs, $Y = yn - cv - \mathcal{G}(s)u$. Finally, we solve the steady state job creation condition for the value of leisure b.

4 Determinacy analysis

Having derived the model and its linearized representation, we now analyze its determinacy properties. We proceed as follows. First, we discuss the intuition for congestion effects as the main source of indeterminacy in the labor search model. Second, we characterize the conditions for local equilibrium existence and uniqueness for three model variants: 1) the standard search model with constant labor and search effort, 2) the model with variable labor effort (and constant search effort), 3) the model with variable search effort (and constant labor effort). We do so analytically for the case of risk neutral households. To analyze the more general case with risk aversion, we resort to numerical methods.

4.1 Congestion externalities and indeterminacy

The search and matching model has inherent externalities. The probability of a vacancy being filled $q(\theta)$ and the job finding rate $p(\theta)$ both depend on labor market tightness. The vacancy filling rate decreases with the number of vacancies. As a firm posts more vacancies, it becomes more difficult for other firms to fill their open positions. This constitutes an externality, because a firm does not take into account that an additional vacancy increases the hiring costs to other firms. Similarly, the job finding rate decreases with the size of the unemployment pool. When an additional unemployed worker searches for a job, or when an unemployed worker exerts additional search effort, this reduces the chances of other job-hunters getting hired. This phenomenon of more agents searching on the same side of the market thus gives rise to a negative *congestion externality.* The probability of a vacancy being filled, instead, increases with the number of unemployed workers and the job finding rate increases with the number of vacancies.[5]

Vacancy posting is a forward-looking decision process driven by profit expectations. The firm posts a vacancy if it expects higher future profits from hiring an additional worker. However, by lowering the vacancy filling rate, more vacancies increase vacancy duration and thus effective hiring costs, reducing profits for all firms. The job creation condition (11) balances out these two effects on profits under a standard calibration, leading to a unique equilibrium. Under certain conditions, however, sunspot equilibria can arise. This means that an expectation or belief becomes self-fulfilling, even in the absence of a fundamental disturbance.[6] For instance, suppose that firms expect the value of a worker to rise in the future. They will be inclined to hire and hence post many vacancies today, taking advantage of the relatively low cost of doing so. This then leads to a fall in the vacancy filling rate and an actual rise in future effective hiring costs, thereby validating the firms' initial belief.

As explained by Petrongolo and Pissarides (2001), the elasticity of matches to the number of unemployed workers, parameter ξ in the matching function, governs the size of congestion externalities and thereby affects the occurrence of sunspots. Given the Cobb-Douglas form of the matching function, a lower elasticity, i.e. a higher value of $1-\xi$, means that a given expansion in the number of vacancies produces a larger number of matches, *ceteris paribus.* A larger number of matches today leads to higher future employment and thus greater savings on future hiring. In turn, greater savings on future hiring imply higher future profits, which induce firms to post more vacancies today, which raises future recruitment costs. The larger is this effect, the greater is the congestion externality that firms impose on one another. If the prospect of lower future hiring costs dominates the reduction in profits due to current effective hiring costs, then sunspot beliefs are validated (Lubik and Schorfheide, 2003).

4.2 Standard labor search model

To start with, we characterize determinacy in the standard labor search model with constant (labor and search) effort. These results can be found – presented in a slightly different way – in Krause and Lubik (2010).

[5]More agents searching on the other side of the market causes a positive *trading externality* (Yashiv, 2007) or *thick-market effect* (Petrongolo and Pissarides, 2001).

[6]For an extensive discussion of sunspots in macroeconomic models see Benhabib and Farmer (1999).

Risk neutrality. It is instructive to study the case of risk neutrality where $\sigma = 0$. In that case, $\alpha_2 = \delta_2 = 0$, $\delta_1 = 1$, and $\alpha_1 = \beta(1-\rho)\frac{\iota-1}{\iota}\xi(1-\frac{\eta}{\xi}p)$. The dynamic system simplifies to

$$\begin{bmatrix} E_t\{\hat{\theta}_{t+1}\} \\ \hat{n}_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{\beta(1-\rho)(1-\frac{\eta}{\xi}p)} & 0 \\ (1-\xi+\frac{\xi}{\zeta})\rho & 1-\frac{\rho}{u} \end{bmatrix} \begin{bmatrix} \hat{\theta}_t \\ \hat{n}_t \end{bmatrix}. \tag{21}$$

Since the transition matrix in (21) is lower triangular, its roots are given by its diagonal elements. Thus,

1. the model solution is **unique** if and only
   (a) either $|\frac{1}{\beta(1-\rho)(1-\frac{\eta}{\xi}p)}| < 1$ and $|1-\frac{\rho}{u} > 1|$, that is if $|\beta(1-\rho)(1-\frac{\eta}{\xi}p)| > 1$ and $2u < \rho$.
   (b) or $|\frac{1}{\beta(1-\rho)(1-\frac{\eta}{\xi}p)}| > 1$ and $|1-\frac{\rho}{u}| < 1$, that is if $|\beta(1-\rho)(1-\frac{\eta}{\xi}p)| < 1$ and $\rho < 2u$.
2. the model solution is **indeterminate** if $|\beta(1-\rho)(1-\frac{\eta}{\xi}p)| > 1$ and $\rho < 2u$.
3. the model solution is **non-existent** if $|\beta(1-\rho)(1-\frac{\eta}{\xi}p)| < 1$ and $2u < \rho$.

Regardless of parameter choices, the model itself provides further restrictions that influence equilibrium existence and uniqueness.

Proposition 1. *Under risk neutrality, the labor search model has at least one stable solution for all admissable values of the steady state unemployment rate and the job separation rate, i.e. those that lie on the unit interval.*

Proof. Combining the law of motion for employment (2) and the constant labor force assumption (1) at the steady state, the job finding rate can be expressed as $p = \frac{\rho}{1-\rho}\frac{1-u}{u}$. For the job finding rate to be strictly lower than unity at the steady state, the separation rate must not exceed the steady state unemployment rate, $\rho < u$. Therefore, under our calibration strategy that fixes u, and given the above cross-parameter restriction, the second root of the transition matrix is stable. This rules out equilibrium non-existence, Case 3, as well as Case 1(a). □

Proposition 2. *Under risk neutrality, the labor search model is characterized by equilibrium indeterminacy if the worker's bargaining weight exceeds the match elasticity to unemployment, thereby violating the Hosios (1990) condition, to a sufficiently large degree.*[7]

Proof. Consider the root $1/[\beta(1-\rho)(1-\frac{\eta}{\xi}p)]$. Notice that, since β, $(1-\rho)$ and p all lie between 0 and 1, it is clear that under the so-called 'Hosios condition', $\eta = \xi$, we have that $|\beta(1-\rho)(1-p)| < 1$ and thus the first root of the transition matrix is unstable. Therefore, the Hosios condition ensures equilibrium uniqueness.[8] Indeterminacy arises if the first root is stable, which occurs if the Hosios condition is violated to a sufficiently large degree. More specifically, we need that $|\beta(1-\rho)(1-\frac{\eta}{\xi}p)| > 1$, which requires that the workers' bargaining power, which measures the share of the match surplus going to workers, exceeds by a sufficiently large amount the workers' contribution to match success, which is captured

[7]The indeterminacy frontier derived below can also be found in Lazaryan and Lubik (2018) for the global solution to the simple search model.

[8]This result has been noted in Bhattacharya and Bunzel (2003), Krause and Lubik (2010), Lazaryan and Lubik (2018).

by the match elasticity to unemployment, i.e. when $\eta >> \xi$. Rearranging the indeterminacy condition, we find that indeterminacy arises if and only if

$$\frac{\eta}{\xi} > \frac{1+\beta(1-\rho)}{\beta(1-\rho)p}. \tag{22}$$

Condition (22) shows that, on the (ξ, η)-plane, the indeterminacy frontier is a straight line with slope greater than 1, where we again assume that $\rho < u$. All (ξ, η)-pairs above this line are associated with an indeterminate model solution. □

The intuition behind this result is that, on the one hand, a high bargaining share of workers means that the wage responds more to labor market tightness, equation (13). On the other, a low match elasticity to unemployment means that the number of matches responds more to labor market tightness, equation (3). Consequently, the vacancy filling rate falls by more when the labor market gets tighter, and this implies a greater rise in (future) hiring costs through the congestion externality.

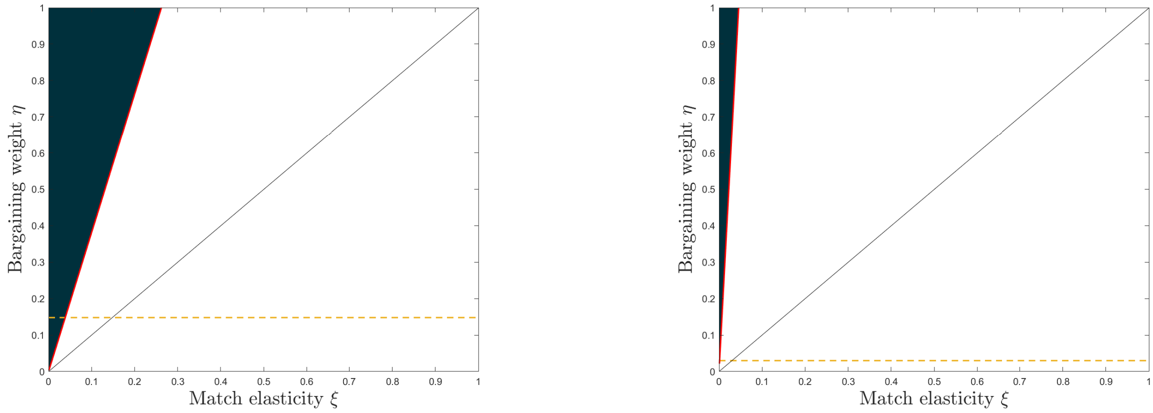
The slope of the indeterminacy frontier depends positively on the separation rate and negatively on the steady state job finding rate. This means that the indeterminacy region is larger, the lower is the separation rate and the higher is the steady state job finding rate. On the one hand, then, in labor markets characterized by frequent *outflows* from unemployment, indeterminacy is more of a concern than otherwise. The intuition for this result is that a high steady state job finding rate p implies more successful matches for a given number of job searchers, and thus more employment relationships in the next period. This helps firms to recoup current vacancy posting costs and raises the probability of an indeterminate equilibrium. On the other hand, labor markets characterized by infrequent *inflows* into unemployment are more prone to exhibit indeterminacy. Recall that in this framework, a worker is an 'asset' with a stream of future benefits in the form of profits and vacancy posting costs saved. Discounting takes into account both impatience, captured by β, and separations, captured by ρ. Lower discounting – either due to a higher β or a lower ρ raises the asset value, thereby making hiring today more attractive.

To illustrate the importance of the fluidity of the labor market on determinacy, we present in Figure 1 two calibrations: one for the US, the other for the Euro Area. For the US, we set $u = 0.06$ and $\rho = 0.033$; for the Euro Area, we set $u = 0.1$ and $\rho = 0.0101$ in line with the data. The parameter on the horizontal axis is the elasticity of the matching function to unemployment, ξ, the parameter on the vertical axis is the workers' share of the surplus, which in the standard labor search model is equivalent to the Nash bargaining weight, η. The indeterminacy region, the combination of ξ and η that satifies (22), shows up as a black triangle in the upper left corner of the figure, where workers appropriate a share of the wage bargain far above their contribution to the realization of the match surplus.
The figure shows that the difference in the size of the indeterminacy region is rather large between the US and the Euro Area. Labor market rigidities, which are greater in the Euro Area, appear to reduce the risk of indeterminacy visibly. Furthermore, the implied leisure value is negative only for extremely low values of the worker's bargaining weight.

Proposition 3. *Under risk neutrality, neither variable labor effort nor variable search effort has any bearing on equilibrium determinacy.*

Proof. The proof is straightforward from the fact that the two roots of the transition matrix are independent of ζ, as well as σ_h and σ_e. □

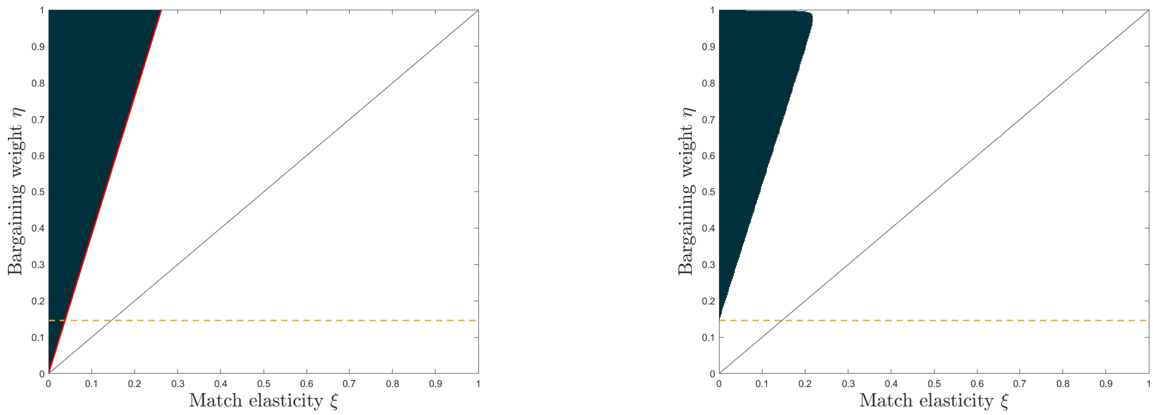
Figure 1: Standard search model: US vs. Euro Area calibration



Note: Indeterminacy regions are shaded black, uniqueness regions are white. Below the dashed line, the implied leisure value b is negative.

Risk aversion. Under the assumption of risk averse households ($\sigma > 0$), we can no longer characterize the determinacy properties analytically and need to use numerical techniques instead. In our model, we set $e_t = s_t = 1$ and we assume constant hours as well, $h_t = 1$. Constant hours and effort can be achieved with a calibration that sets the elasticity of disutility of hours and effort, as well as the elasticity of search costs, to a very high number.

Figure 2: Standard search model: risk neutrality ($\sigma = 0$) vs. risk aversion ($\sigma = 1$)



Note: Indeterminacy regions are shaded black, uniqueness regions are white. Below the dotted line, the implied leisure value b is negative.

First, Figure 2 again confirms that the Hosios condition (see Hosios, 1990) along the 45-degree line guarantees determinacy. In contrast, a bargaining weight which exceeds the match elasticity by a large amount gives rise to indeterminacy. Second, comparing the two panels in Figure 2, we see that indeterminacy is somewhat less likely under risk aversion. Why is this? Risk aversion implies greater intertemporal substitution, which leads to greater discounting, i.e. it lowers the effective discount factor. There is a negative relationship between the effective discount factor and the tolerance level. Therefore, under risk aversion, the determinacy frontier lies further away from the Hosios condition.

An additional model-implied restriction, which we have ignored so far, is that the leisure value b needs to be positive. Realistically, the unemployed receive welfare benefits rather than being taxed. We investigated under which parameter combinations b turns out to be negative. This happens if the worker's bargaining weight is rather low. The match elasticity has no effect on the implied leisure value. In Figure 2, the parameter combinations beneath the dashed line lead to a negative leisure value and are therefore not admissible.

4.3 Labor effort and determinacy

Employment flows are not the only form of labor adjustment. In many countries, hours worked per employee are an important margin along which labor varies (see the evidence in Ohanian and Raffo, 2012; Dossche, Lewis, and Poilly, 2019). Moreover, variable labor utilization, or effort, has been proposed as a third labor margin to help explain the observed procyclicality of labor productivity.[9] Burda et al. (2017) use the American Time Use Survey 2003-12 to show that 'non-work at work', which we might interpret as low effort per hour, is substantial and varies countercyclically. More specifically, they find that time spent in non-work conditional on any positive amount rises, while the fraction of workers reporting positive values declines with unemployment. Since the former effect dominates, there is a positive relationship between non-work and the unemployment rate. This evidence suggests that variable effort is a relevant labor adjustment margin in the US. In a business cycle model estimated for the Euro Area, Lewis et al. (2019) show that a model with labor effort outperforms one with variable capital utilization.

Benhabib and Farmer (1994) show that increasing returns can be a source of indeterminacy. In our model, hours and variable labor effort allow for increasing returns to hours in production. Through the effort channel, an additional worker produces an amount of output that exceeds his input in terms of hours worked. As a result, profits and hence the worker's asset value in the vacancy posting condition (11) becomes more elastic in response to shocks. This indicates that indeterminacy is more likely in the model with hours and effort than in the standard search model.

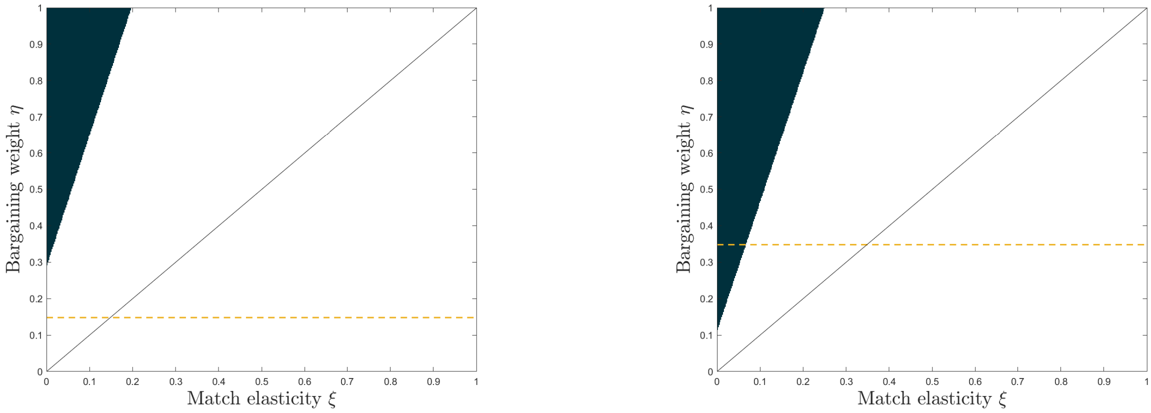
Figure 3 shows the determinacy regions in our model, setting the constant of relative risk aversion to $\sigma = 2$. The chart on the left is the standard labor search model without hours or effort; the one on the right is the model featuring both hours and effort. The figure shows that introducing hours and effort into the model expands the indeterminacy region somewhat, although for plausible parameter values the model still has a unique stable equilibrium.

The region where b is negative is larger in the model with two additional labor margins than they are in the standard labor search model. This is intuitive, since introducing hours and effort reduces the model-implied leisure value. We can write $b = b^s - \phi/(1 + \sigma_h)$, where b^s is the leisure value in the standard labor search model without hours and effort. This shows that introducing hours and effort into the model reduces the implied value of leisure.

We conclude from this exercise that increasing returns due to variable labor utilization have a rather small effect on indeterminacy when compared with congestion externalities of the search model. This contrasts with Wen (1998), who argues that variable capital utilization can generate indeterminacy under empirically relevant parameter choices. Variable labor effort has a larger impact on shrinking the admissible region for the worker's bargaining weight η that is consistent with a non-negative leisure value.

[9] A non-exhaustive list includes Oi (1962), Bils and Cho (1994), Rotemberg and Summers (1990), Barnichon (2010), and Galí and van Rens (2014).

Figure 3: Effect of labor effort on determinacy: constant vs. variable labor effort



Note: Indeterminacy regions are shaded black, uniqueness regions are white. Below the dashed line, the implied leisure value b is negative.

4.4 Search effort and determinacy

In the standard model we have found that a bargaining share of workers sufficiently above the value required for efficiency according to the Hosios condition results in an indeterminate equilibrium. Does this result depend on the (common) assumption of one-sided search – on the part of firms – which we have maintained thus far? In the standard labor search model, only the firms actively search by posting vacancies. Quite plausibly, though, unemployed workers could drive up their search intensity whenever it is advantageous to do so, i.e. whenever the expected return to searching more intensively exceeds the associated marginal cost. As search effort rises, firms find it easier to fill their vacancies; effective hiring costs fall as the vacancy filling rate rises. In other words, the congestion externality and associated sunspot mechanism is weakened under variable search intensity. Models with two-sided search, on the part of both firms and unemployed workers, have been developed in Merz (1995), Hashimzade and Ortigueira (2005) and Berentsen, Rocheteau, and Shi (2007), among others.[10]

Search intensity is an empirically relevant model ingredient. A large body of evidence, discussed in more detail below, suggests that search intensity by job-seekers varies over the business cycle. The standard search model suffers from the deficiency that it generates too little labor market volatility (Shimer, 2005). Gomme and Lkhagvasuren (2015) show that unemployment volatility is higher in a model with variable search intensity, thus bringing the search model closer to the data in this dimension.

The mechanism is the following. With constant search intensity, firms that expect a boost to profits (e.g. thanks to an expected technological improvement) post more vacancies, raising the job finding rate and thus the workers' outside option. The resulting rise in wages eats up much of the firm's expected rise in profits. Instead, with variable search intensity, the value of being unemployed rises by less – since exerting more search effort is costly –, and therefore the wage also rises by less. This leaves a larger surplus for the firm, which in turn amplifies the rise in vacancies. The mechanism is similar to the search complementarities in Fernández-Villaverde, Mandelman, Yu, and Zanetti (2019)'s model with inter-firm matching: as one party increases its search activities, it becomes advantageous for the other party to

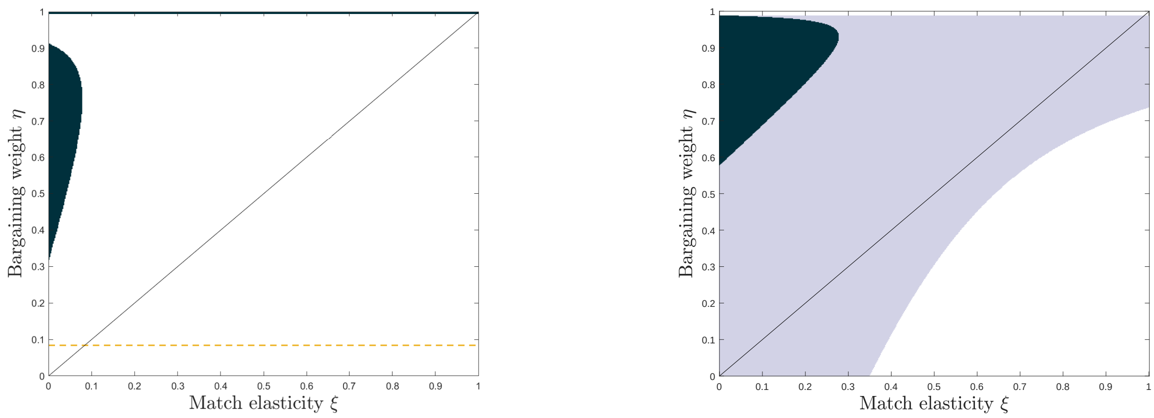
[10]For simplicity, we abstract from on-the-job search in the present analysis. Only unemployed workers engage in search.

do the same.

Figure 4 shows the determinacy regions in the model with variable search effort, where the coefficient of risk aversion is set to $\sigma = 1$. We can verify that a highly convex cost function, setting ζ to a value above 10, brings the two-sided search model close to our baseline model with constant search intensity shown on the right hand side of Figure 2. The indeterminacy region shrinks as we make the search cost function flatter, lowering ζ. In the panel on the left, search costs are quadratic following the evidence in Yashiv (2000). If we instead assume that the $\mathcal{G}(s_t)$-function is convex but fairly flat, following the argument in Shimer (2004), and set $\zeta = 1.1$, indeterminacy all but disappears (not shown).

Endogenous search intensity has the effect of dampening the congestion externality discussed above. As firms post more vacancies and the labor market tightens, unemployed workers see their job finding prospects improve and so they intensify their search. If the search cost function is fairly flat, the penalty for doing so will be low and search effort will rise a lot. This, in turn, limits the rise in labor market tightness and the resulting fall in the vacancy filling rate. As a consequence, effective hiring costs rise by less and the incidence of indeterminacy is reduced.

Figure 4: Variable search effort: quadratic vs. (mildly) concave search cost function



Note: Indeterminacy regions are shaded black, uniqueness regions are white, non-existence regions are gray. Below the dashed line, the implied leisure value b is negative.

On the right hand panel of Figure 4, we set $\zeta = 0.9$, i.e. the cost of searching is concave in search intensity. The figure reveals that along the 45-line where the Hosios condition is satisfied, the model solution is non-existent. We recall the result derived in chapter 8 of Pissarides (2000) that the Hosios rule remains the efficiency condition in the labor search model, even if the model is extended to allow for variable search intensity. It is therefore interesting to observe that the Hosios condition fails to ensure equilibrium existence if the search cost function is close to linear but (mildly) concave. Even lower values for ζ shrink the non-existence region (not shown).

The intuition for this non-existence or instability region is a positive feedback loop between vacancy posting by firms and greater search effort by the unemployed. More vacancy posting increases labor market tightness and the job finding rate, which in turn induces job-seekers to raise their search effort. An increase in search intensity is accompanied by sufficiently small resource cost, which encourages the use of the search intensity margin.

To summarize, a convex but fairly flat search cost function reduces or even eliminates indeterminacy. An equilibrium always exists under convex search costs, but not necessarily under concave search costs. Below, we discuss the literature on search intensity and the empirical evidence on the size of ζ.

A look at the literature. A large empirical literature documents that search intensity by the unemployed varies across time, lending support to the idea of endogenizing s_t. However, there is no agreement on its cyclical properties, which is in part related to the fact that search intensity is not directly observable. A number of proxies have been proposed.

Shimer (2004) uses the number of search methods from the Current Population Survey (CPS) and points out that this measure is countercyclical. Pan (2019) constructs search activity indices for different sectors in the US, based on Internet search volumes, which are also countercyclical. However, these approaches do not differentiate between on-the-job search and search activity by the unemployed, which according to Faberman, Mueller, Sahin, and Topa (2017) might differ to a large extent.

Krueger and Mueller (2010) consider the time the unemployed spend on search activities. Using the American Time Use Survey (ATUS), they show that job search time increases with the expected wage. Gomme and Lkhagvasuren (2015) argue that this is indirect evidence of procyclical search intensity, since expansions are times when expected wages are high, and individual wages are also highly procyclical as shown by Solon, Barsky, and Parker (1994).[11] Moreover, Gomme and Lkhagvasuren (2015) find that labor market tightness is highly correlated with search intensity by the short-term unemployed. The two-sided search model here and in Pissarides (2000) is consistent with search effort being procyclical.

The other important issue for our analysis is the shape of the search cost function. How large is ζ? While Stiglitz (1987) considers both convex and concave search costs, many studies impose convexity on the search cost function: Merz (1995), Kaas (2010), Gomme and Lkhagvasuren (2015) all do this. First, to the extent that search activity is time-intensive, the natural constraint imposed by the time endowment makes every additional unit of search more and more costly. This reasoning for convexity in search costs is arguably more applicable to on-the-job search, where a searching worker is already close to his time constraint. Second, as explained above, Gomme and Lkhagvasuren (2015) argue that convex search costs help to generate employment volatility and this goes some way in solving the so-called 'Shimer puzzle'. Empirical evidence in Christensen, Lentz, Mortensen, Neumann, and Werwatz (2005) supports a specification of search costs that is quadratic, although we note that, here also, the authors analyze on-the-job search. Instead, in Yashiv (2000), only unemployed workers exert effort; that paper also presents evidence of quadratic search costs.

In contrast, Fernández-Villaverde et al. (2019) employ a search cost function with a non-convexity. This feature is critical to the existence of multiple equilibria in their model. Since some agents will choose not to search at all, there exists an equilibrium with low output, low search and high unemployment in addition to an equilibrium with high output, high search activity and low unemployment. Cheron and Decreuse (2016) present evidence of postings by job-seekers (or recruiting firms) that testify from past search activity and remain online even after a match has taken place. Removing these postings is costly and they therefore live on as 'phantoms'. This evidence is in contradiction with the notion of convex search costs and suggests that searching entails some fixed costs.

[11] The argument here is that the observed acyclicality of average wages is driven in part by a compositional bias.

5 An alternative model with hiring costs

The search-and-matching model fails to replicate certain salient features of the labor market (Shimer, 2005). In particular, it predicts a strong immediate response of vacancies to productivity shocks, whereas in the data, we instead observe hump-shaped dynamics.[12] As explained in Hertweck (2013), the reason for this counterfactual prediction lies in the assumption of linear vacancy posting costs, c, which a firm incurs each period, irrespective of whether or not the matching process is successful. Effective hiring costs are in this case given by c/q_t, i.e. vacancy posting costs multiplied by the expected duration of a vacancy. After a positive productivity shock, vacancy duration $1/q_t$ and hence effective hiring costs increase sharply and persistently; this is due to congestion externalities as explained above. The persistence in elevated hiring costs induces firms to post many vacancies immediately, giving rise to a convex-shaped impulse response in the number of vacancies.

Replacing linear vacancy posting costs with (quadratic) hiring costs, as in Gertler and Trigari (2009) and Hertweck (2013), brings the model closer to the data in this dimension. Conceptually, an important difference between the two models is that in the hiring cost model, a firm that wishes to hire a worker always finds one. In other words, there is no unsatisfied demand for new workers because of unsuccessful matches, which nevertheless entail costs to firms, as in the search model. The hiring cost approach has become more popular recently; applications include Galí and van Rens (2014).

We would like to stress that our modelling approach differs from Mortensen and Nagypál (2007) and Ljungqvist and Sargent (2017), who maintain the search framework setup with the vacancy posting costs and merely add post-bargaining hiring costs, which make the firm's surplus more elastic to productivity changes.[13]

In the following, we investigate the determinacy properties of the hiring cost model.

5.1 Model setup

Let us introduce the hiring rate as the number of new matches over employment, $x_t = m_t/n_t$. Hiring costs to an individual firm depend on the aggregate hiring rate and are given by cx per new worker, such that the aggregate accounting equation changes to $Y_t = y_t n_t - cx_t^2 n_t$.[14] The vacancy posting condition of the labor search model (11) is replaced with

$$cx_t = (1-\rho)E_t\{\beta_{t,t+1}(y_{t+1} - w_{t+1}h_{t+1} + cx_{t+1})\}. \tag{23}$$

Notice from (23) that the firm's surplus from hiring is different from the standard labor search model. As a consequence, the bargaining wage is also different,

$$w_t h_t = \eta(y_t + p_t c x_t) + (1-\eta)\left[g(h_t)/\lambda_t + b\right]. \tag{24}$$

Equilibrium in the hiring cost model is defined as follows.

Definition 2. *A decentralized equilibrium in the hiring cost model is a set of infinite sequences for quantities* $\{u_t, m_t, n_{t+1}, Y_t, h_t\}_{t=0}^{\infty}$*, matching rates* $\{p_t, x_t\}_{t=0}^{\infty}$ *and wages* $\{w_t\}_{t=0}^{\infty}$*, satisfying the transversality condition, such that:*

1. *given aggregate matching rates and wages, the quantities solve the household's problem,*

[12]See Pissarides (2000).
[13]Pre-bargaining hiring costs as in Pissarides (2009) in addition have an effect on the equilibrium wage.
[14]We abstract from endogenous search effort.

2. *given aggregate matching rates and wages, the quantities solve the firm's problem,*
3. *employment is determined by the law of motion (2),*
4. *wages solve the Nash bargaining problem,*
5. *goods markets clear.*

Table 2: Equilibrium conditions: hiring cost model

Unemployment	$u_t = 1 - n_t$
Hiring rate	$x_t = m_t/n_t$
Job finding rate	$p_t = m_t/u_t$
Employment	$n_{t+1} = (1-\rho)(n_t + m_t)$
Hours	$\phi e_0 h_t^{\phi-1} = g'(h_t)/\lambda_t$
Production	$y_t = e_0 h_t^{\phi}$
Output	$Y_t = y_t n_t - c x_t^2 n_t$
Hiring	$c x_t = (1-\rho)\beta E_t \{(Y_t/Y_{t+1})^{\sigma}(y_{t+1} - w_{t+1}h_{t+1} + c x_{t+1})\}$
Wages	$w_t h_t = \eta(e_0 h_t^{\phi} + p_t c x_t) + (1-\eta)\left[g(h_t)/\lambda_t + b\right]$

The equilibrium conditions of the hiring cost model are presented in Table 2. In linearized form, the system can be written compactly in two equations describing one control variable, the hiring rate x_t, and one state variable, employment n_{t+1},

$$\alpha_1 E_t\left\{\hat{x}_{t+1}\right\} = \left[1 + \frac{\sigma}{\delta_1}\left(2\frac{cxm}{Y} + \delta_2\rho\right) + \beta(1-\rho)\frac{\rho}{u}\eta p\right]\hat{x}_t + \beta(1-\rho)\frac{\eta p}{u}\hat{n}_t, \tag{25}$$

$$\hat{n}_{t+1} = \rho\hat{x}_t + \hat{n}_t, \tag{26}$$

where we define α_1, $\delta_1 \geq 1$ and $\delta_2 \geq 1$ as follows,

$$\alpha_1 = \beta(1-\rho)(1-2\eta p) + \sigma\frac{\delta_2}{\delta_1}2\frac{cxm}{Y},$$

$$\delta_1 = 1 + \sigma\left(1 + \frac{c\theta}{Y}\right)\frac{\phi}{(1+\sigma_h) - \phi},$$

$$\delta_2 = 1 + \beta(1-\rho)(1-\eta)\frac{\phi}{1+\sigma_h}\frac{e_0 h^{\phi}}{cx}.$$

The two-equation system of the hiring cost model, (26) and (25), can be written in matrix notation,

$$\begin{bmatrix} E_t\{\hat{x}_{t+1}\} \\ \hat{n}_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{\alpha_1}\left[1 + \frac{\sigma}{\delta_1}\left(2\frac{cxm}{Y} + \delta_2\rho\right) + \beta(1-\rho)\frac{\rho}{u}\eta p\right] & \frac{1}{\alpha_1}\beta(1-\rho)\frac{\eta p}{u} \\ \rho & 1 \end{bmatrix}\begin{bmatrix} \hat{x}_t \\ \hat{n}_t \end{bmatrix}. \tag{27}$$

Notice that in this model, the hiring rate, rather than labor market tightness, is the relevant control variable.

5.2 Determinacy analysis

Let us again consider risk neutrality as a special case, where $\alpha_1 = \beta(1-\rho)(1-2\eta p)$ and $\delta_1 = 1$. The dynamic system simplifies to

$$\begin{bmatrix} E_t\{\hat{x}_{t+1}\} \\ \hat{n}_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{\beta(1-\rho)(1-2\eta p)} + \frac{\frac{\rho}{u}\eta p}{1-2\eta p} & \frac{\frac{1}{u}\eta p}{1-2\eta p} \\ \rho & 1 \end{bmatrix} \begin{bmatrix} \hat{x}_t \\ \hat{n}_t \end{bmatrix}. \tag{28}$$

Proposition 4. *Under risk neutrality, the model with hiring costs is determinate, i.e. it has a unique stable equilibrium.*

Proof. The trace and the determinant of the transition matrix are

$$\text{tr} = 1 + \frac{1}{\beta(1-\rho)(1-2\eta p)} + \frac{\frac{\rho}{u}\eta p}{1-2\eta p},$$

$$\text{D} = \frac{1}{\beta(1-\rho)(1-2\eta p)}.$$

The two roots of the system can be written in terms of the trace and the determinant,

$$\lambda_i = \frac{1}{2}(\text{tr} \pm \sqrt{\text{tr}^2 - 4\text{D}}) \text{ with } i = 1, 2.$$

Using a proof by contradiction, we can show that the first root of the transition matrix is smaller than unity for $2\eta p < 1$, which rules out non-existence. By a similar argument, the second root is necessarily unstable. Therefore, the system has a unique solution.[15] □

Turning to the general model with risk averse household, we analyse determinacy numerically. To this end, we use the same calibration as in the standard search model. The steady state hiring rate x, employment n, number of vacancies v, and labor market tightness θ are identical in both models. However, the implied leisure value b becomes,

$$b = \left(1 - \frac{\phi}{1+\sigma_h}\right) y - \frac{1-\beta(1-\rho)(1-\eta p)}{\beta(1-\rho)(1-\eta)} cx.$$

We find that the hiring cost model has a determinate solution, even when we allow for variable labor effort. We found no parameter combinations that gave rise to indeterminacy. What explains this result?

Let us go back to the equilibrium of the standard search-and-matching model with linear vacancy posting costs, which is saddle-path stable. The instability of the saddle point arises from the forward-looking nature of the representative firm's intertemporal optimization problem (see e.g. Atolia, Chatterjee, and Turnovsky, 2010, and the references cited therein). In particular, the representative firm needs to open a costly vacancy today in order to potentially employ one worker in the future. For this reason, firms treat vacancies as an asset, which entails that their supply is inherently unstable (Pissarides, 2000). Moreover, the standard model assumes that the effective cost of hiring an additional worker is proportional to the time the vacancy remains unfilled. Due to the presence of congestion externalities, the expected search time rises whenever firms intensify hiring activities (as the aggregate vacancy filling rate falls). When business expectations improve, firms with

[15] For details, see the online appendix.

unfilled positions plan to hire new workers before the vacancy filling rate falls. In a symmetric rational expectations equilibrium, this implies that all firms start posting vacancies immediately. Hence, vacancies first overshoot their long-run equilibrium level (Pissarides, 1985), and then fall back to normal levels.

As shown by Krause and Lubik (2010), the overshooting behavior of vacancies may lead to indeterminacy issues whenever the right hand side of the vacancy posting condition (the discounted net benefit of hiring a worker) responds more elastically to vacancy creation than the left hand side (the effective cost of hiring a worker). The right hand side might respond more strongly when employment reacts elastically to vacancy posting (which requires that the matching elasticity of vacancies is high and the separation rate is low) and the negative feedback mechanism of tightness through wages is weak (which requires that the worker's bargaining power is low).

The alternative specification with hiring costs changes the representative firm's optimization problem in that *filling* a vacancy entails a cost, rather than *posting* a vacancy. This means that the effective cost of hiring a worker is independent of the search time and, thus, independent of the aggregate vacancy filling rate. In the absence of congestion externalities, firms have much weaker incentives to prepone hiring activities. Instead, the effective hiring cost now is proportional to the aggregate gross hiring rate. When business expectations improve, the aggregate hiring rate increases somewhat, but much less than the expected search time to find a suitable worker. In addition, the rise in the aggregate hiring rate is much less persistent. This gives firms strong incentives to smooth hiring activities over several periods. As a result, both the left and the right hand side of the job creation condition respond much less elastically, which basically eliminates the possibility of indeterminacy at conventional parameter constellations.

6 Conclusion

We introduce two types of effort, worker effort and search effort by job-seekers, into a simple search-and-matching model. We analyze how these extensions affect the model's determinacy properties. A sunspot shock can lead to self-fulfilling dynamics when agents expect the asset value of a worker – the expected benefit of posting a vacancy – to increase and start hiring at a faster rate. If the asset value increases as the workforce expands, a belief-driven higher expected return on investing in new workers can be self-validating. As shown in the literature, self-fulfilling sunspot beliefs can arise in the canonical labor search model when the match elasticity to unemployment is low. In this case, firms are induced to post too many vacancies, not taking into account the negative congestion externality by which any additional vacancy increases the effective hiring cost of all firms, making existing workers more valuable.

The presence of variable labor effort expands the regions of indeterminacy compared to a model featuring employment only. This result is driven by the increasing returns to hours in production in the model with hours and effort as additional labor margins. However, this effect is rather limited and determinacy is preserved for empirically plausible parameter values. Our investigation suggests that congestion effects inherent in the search framework matter more for indeterminacy than increasing returns.

Variable search effort has the opposite effect; as long as search costs are convex in search intensity, the indeterminacy region shrinks in comparison to the standard search model. This happens because of a strategic complementarity that reduces congestion: vacancy posting by firms raises labor market tightness and the job finding rate, which leads to greater search effort by the unemployed, which in turn dampens the tightening of the labor market. When search costs are instead (mildly) concave, a model solution may not exist even under the Hosios rule.

We have also shown that indeterminacy is eliminated in a framework where labor market frictions are modelled as hiring costs rather than a search process with linear vacancy posting costs.

References

Andolfatto, D. (1996, March). Business Cycles and Labor-Market Search. *American Economic Review 86*(1), 112–132.

Atolia, M., S. Chatterjee, and S. J. Turnovsky (2010, September). How misleading is linearization? Evaluating the dynamics of the neoclassical growth model. *Journal of Economic Dynamics and Control 34*(9), 1550–1571.

Barnichon, R. (2010, November). Productivity and unemployment over the business cycle. *Journal of Monetary Economics 57*(8), 1013–1025.

Benhabib, J. and R. E. Farmer (1994). Indeterminacy and increasing returns. *Journal of Economic Theory 63*(1), 19 – 41.

Benhabib, J. and R. E. Farmer (1999). Chapter 6. Indeterminacy and sunspots in macroeconomics. Volume 1 of *Handbook of Macroeconomics*, pp. 387 – 448. Elsevier.

Berentsen, A., G. Rocheteau, and S. Shi (2007, January). Friedman Meets Hosios: Efficiency in Search Models of Money. *Economic Journal 117*(516), 174–195.

Bhattacharya, J. and H. Bunzel (2003). Dynamics of the planning solution in the discrete-time textbook model of labor market search and matching. *Economics Bulletin 5*(19), 1–10.

Bils, M. and J.-O. Cho (1994). Cyclical factor utilization. *Journal of Monetary Economics 33*(2), 319–354.

Blanchard, O. and J. Galí (2010, April). Labor markets and monetary policy: A new keynesian model with unemployment. *American Economic Journal: Macroeconomics 2*(2), 1–30.

Burda, M. C., K. R. Genadek, and D. S. Hamermesh (2017, June). Non-work at work, unemployment and labor productivity. CEPR Discussion Papers 12087, C.E.P.R. Discussion Papers.

Cheron, A. and B. Decreuse (2016, 07). Matching with Phantoms. *Review of Economic Studies 84*(3), 1041–1070.

Christensen, B. J., R. Lentz, D. T. Mortensen, G. R. Neumann, and A. Werwatz (2005). On-the-job search and the wage distribution. *Journal of Labor Economics 23*(1), 31–58.

Christiano, L. J., M. S. Eichenbaum, and M. Trabandt (2016, July). Unemployment and Business Cycles. *Econometrica 84*, 1523–1569.

Christoffel, K., K. Kuester, and T. Linzert (2009, November). The role of labor markets for euro area monetary policy. *European Economic Review 53*(8), 908–936.

den Haan, W. J., G. Ramey, and J. Watson (2000). Job destruction and propagation of shocks. *The American Economic Review 90*(3), 482–498.

Dossche, M., V. Lewis, and C. Poilly (2019). Employment, hours and the welfare effects of intra-firm bargaining. *Journal of Monetary Economics 104*, 67–84.

Faberman, R. J., A. I. Mueller, A. Sahin, and G. Topa (2017, August). Job Search Behavior among the Employed and Non-Employed. NBER Working Papers 23731, National Bureau of Economic Research, Inc.

Farmer, R. E. A. (2019, May). The Indeterminacy Agenda in Macroeconomics. CEPR Discussion Papers 13745, C.E.P.R. Discussion Papers.

Farmer, R. E. A. and J.-T. Guo (1994, June). Real business cycles and the animal spirits hypothesis. *Journal of Economic Theory 63*(1), 42–72.

Fernández-Villaverde, J., F. Mandelman, Y. Yu, and F. Zanetti (2019). Search complementarities, aggregate fluctuations and fiscal policy. Unpublished manuscript, University of Pennsylvania.

Fujita, S. and G. Ramey (2009). The cyclicality of separation and job finding rates*. *International Economic Review 50*(2), 415–430.

Galí, J. and T. van Rens (2014, March). The vanishing procyclicality of labor productivity. CEPR Discussion Papers 9853, C.E.P.R. Discussion Papers.

Gertler, M. and A. Trigari (2009, February). Unemployment Fluctuations with Staggered Nash Wage Bargaining. *Journal of Political Economy 117*(1), 38–86.

Gomme, P. and D. Lkhagvasuren (2015). Worker search effort as an amplification mechanism. *Journal of Monetary Economics 75*, 106 – 122.

Hagedorn, M. and I. Manovskii (2008, September). The cyclical behavior of equilibrium unemployment and vacancies revisited. *American Economic Review 98*(4), 1692–1706.

Hall, R. E. (2005a, March). Employment fluctuations with equilibrium wage stickiness. *American Economic Review 95*(1), 50–65.

Hall, R. E. (2005b). Job loss, job finding, and unemployment in the u.s. economy over the past fifty years. *NBER Macroeconomics Annual 20*, 101–137.

Hall, R. E. and P. R. Milgrom (2008, September). The limited influence of unemployment on the wage bargain. *American Economic Review 98*(4), 1653–74.

Hall, R. E. and S. Schulhofer-Wohl (2018, January). Measuring Job-Finding Rates and Matching Efficiency with Heterogeneous Job-Seekers. *American Economic Journal: Macroeconomics 10*(1), 1–32.

Hashimzade, N. and S. Ortigueira (2005, March). Endogenous Business Cycles With Frictional Labour Markets. *Economic Journal 115*(502), 161–175.

Hertweck, M. S. (2013, October). Strategic wage bargaining, labor market volatility, and persistence. *The B.E. Journal of Macroeconomics 13*(1), 1–27.

Hobijn, B. and A. Sahin (2009, September). Job-finding and separation rates in the OECD. *Economics Letters 104*(3), 107–111.

Hosios, A. J. (1990). On the efficiency of matching and related models of search and unemployment. *Review of Economic Studies 57*(2), 279–298.

Kaas, L. (2010, October). Variable Search Intensity with Coordination Unemployment. *The B.E. Journal of Macroeconomics 10*(1), 1–33.

Keane, M. and R. Rogerson (2012, June). Micro and macro labor supply elasticities: A reassessment of conventional wisdom. *Journal of Economic Literature 50*(2), 464–76.

Krause, M. U. and T. A. Lubik (2010). Instability and indeterminacy in a simple search and matching model. *Economic Quarterly* (3Q), 259–272.

Krueger, A. B. and A. Mueller (2010, April). Job search and unemployment insurance: New evidence from time use data. *Journal of Public Economics 94*(3-4), 298–307.

Lazaryan, N. and T. A. Lubik (2018, May). Global dynamics in a search and matching model of the labor market. *Economic Theory*.

Leduc, S. and Z. Liu (2019). The weak job recovery in a macro model of search and recruiting intensity. *American Economic Journal: Macroeconomics*. Forthcoming.

Lewis, V., S. Villa, and M. Wolters (2019). Labor productivity and inflation dynamics. Manuscript, Bank of Italy, Bundesbank and University of Jena.

Ljungqvist, L. and T. J. Sargent (2017, September). The fundamental surplus. *American Economic Review 107*(9), 2630–65.

Lubik, T. A. and F. Schorfheide (2003, November). Computing sunspot equilibria in linear rational expectations models. *Journal of Economic Dynamics and Control 28*(2), 273–285.

Merz, M. (1995). Search in the labor market and the real business cycle. *Journal of Monetary Economics 36*(2), 269 – 300.

Mortensen, D. T. (2009, November). Island matching. *Journal of Economic Theory 144*(6), 2336–2353.

Mortensen, D. T. and E. Nagypál (2007). More on unemployment and vacancy fluctuations. *Review of Economic Dynamics 10*(3), 327 – 347.

Mortensen, D. T. and C. A. Pissarides (1994, July). Job creation and job destruction in the theory of unemployment. *Review of Economic Studies 61*(3), 397–415.

Ohanian, L. E. and A. Raffo (2012). Aggregate hours worked in oecd countries: New measurement and implications for business cycles. *Journal of Monetary Economics 59*(1), 40–56.

Oi, W. Y. (1962). Labor as a quasi-fixed factor. *Journal of Political Economy 70*, 538–538.

Pan, W.-F. (2019). Building sectoral job search indices for the united states. *Economics Letters 180*, 89 – 93.

Petrongolo, B. and C. A. Pissarides (2001). Looking into the black box: A survey of the matching function. *Journal of Economic literature 39*(2), 390–431.

Pissarides, C. A. (1985, September). Short-run Equilibrium Dynamics of Unemployment Vacancies, and Real Wages. *American Economic Review 75*(4), 676–690.

Pissarides, C. A. (2000, January). *Equilibrium Unemployment Theory, 2nd Edition*, Volume 1 of *MIT Press Books*. The MIT Press.

Pissarides, C. A. (2009). The unemployment volatility puzzle: Is wage stickiness the answer? *Econometrica 77*(5), 1339–1369.

Rotemberg, J. J. and L. H. Summers (1990). Inflexible prices and procyclical productivity. *Quarterly Journal of Economics 105*(4), 851–874.

Shimer, R. (2004). Search intensity. Unpublished manuscript, University of Chicago.

Shimer, R. (2005). The cyclical behavior of equilibrium unemployment and vacancies. *The American Economic Review 95*(1), 25–49.

Shimer, R. (2012, April). Reassessing the ins and outs of unemployment. *Review of Economic Dynamics 15*(2), 127–148.

Solon, G., R. Barsky, and J. A. Parker (1994). Measuring the cyclicality of real wages: How important is composition bias. *Quarterly Journal of Economics 109*(1), 1–25.

Stiglitz, J. E. (1987). Competition and the number of firms in a market: Are duopolies more competitive than atomistic markets? *Journal of Political Economy 95*(5), 1041–1061.

Wen, Y. (1998, July). Capacity utilization under increasing returns to scale. *Journal of Economic Theory 81*(1), 7–36.

Yashiv, E. (2000, December). The determinants of equilibrium unemployment. *American Economic Review 90*(5), 1297–1322.

Yashiv, E. (2007). Labor search and matching in macroeconomics. *European Economic Review 51*(8), 1859–1895.

Zanetti, F. (2006, October). Labor market frictions, indeterminacy, and interest rate rules. *Journal of Money, Credit and Banking 38*(7), 1959–1970.