

Firms, Failures, and Fluctuations

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Joint Spring Conference 2019: Systemic Risk and the Macroeconomy
Deutsche Bundesbank – ECB

Preliminary: Handle with care!

Firm-to-Firm Linkages and Production Networks

- Modern economies organized as complex production networks between firms
 - ▶ Expenditure on intermediate goods & services in the U.S. \approx 1 GDP.

(1) High levels of persistence in firm-to-firm relationships:

- ▶ Chile: median firm retains 41% and 46% of its domestic suppliers and customers between two average years (Huneus, 2018)
- ▶ U.S.: 70% of link destructions due to one party's exit (Taschereau-Dumouchel, 2018)

(2) Firm-to-firm linkages can result in cascading failures:

- ▶ bankruptcies due to spillovers over credit linkages (Jacobson and Von Schedvin, 2015)
- ▶ the aftermath of the Great East Japan Earthquake (Carvalho et al., 2016)

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Firm-to-Firm Linkages and Production Networks

- Existence of firm-specific relationships indicates:
 - (a) relationship-specific surplus
 - (b) non-competitive division of surplus and existence of non-trivial markups

- Both features are absent from most models in the literature, which are typically better approximations for industry-level linkages
 - ▶ competitive
 - ▶ monopolistically competitive + CES (constant markups)

- Important to model explicitly and understand these features for two reasons:
 - (1) how shocks change relationship-specific surpluses and markups endogenously
 - (2) propagation of shocks, not just through competitive prices but also failures

What We Do

- A firm-level model of production networks
 - ▶ firm-specific relationships
 - ▶ market power and endogenous markups
 - ▶ endogenous bankruptcies

- Cascading failures are an important channel for the propagation and amplification of shocks.

- Today:
 - ▶ existence and uniqueness results
 - ▶ comparative statics
 - ▶ macroeconomic implications

Related Literature

- Production networks:
 - ▶ Long and Plosser (1983); Horvath (1998, 2000); Carvalho (2010); Acemoglu et al. (2012, 2017); Atalay (2017); Baqaee (2018), and many more...
- Endogenous production networks:
 - ▶ Carvalho and Voigtländer (2014); Oberfield (2018); Acemoglu and Azar (2018)
- Misallocation and markups:
 - ▶ Jones (2013), Bigio and La'O (2018), Baqaee and Farhi (2019), Liu (2018)
- Models of firm-level interactions
 - ▶ Taschereau-Dumouchel (2018); Tintelnot et al. (2018); Kikkawa et al. (2018)

Roadmap

1. Model
2. Solution concept
3. Existence and uniqueness
4. Comparative statics
5. Macroeconomic implications
 - industry-level aggregation
 - aggregate comparative statics

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Model

- An economy with $n + 1$ industries.
 - ▶ industries $\{1, \dots, n\}$ produce intermediate goods
 - ▶ industry 0 produces the final good.
- Each industry consists of two types of firms:
 - ▶ competitive fringe producing a **generic** variant of the good
 - ▶ collection of firms producing **customized** variants
- A unit mass of households
 - ▶ log utilities over the final good
 - ▶ one unit of labor supplied inelastically

Timing

- $t = 0$:
 - ▶ customized firms decide whether to operate their technologies by paying a fixed cost.

- $t = 1$:
 - ▶ active firms enter into pairwise contracts that determine price
 - ▶ commitments to deliver as many units as demanded by their customers

- $t = 2$:
 - ▶ production and consumption take place.

Generic Producers

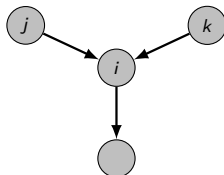
- A competitive fringe of firms i°
- constant returns to scale technology:

$$y_i^\circ = F_i(\ell_i^\circ, B_{i1}q_{i1}^\circ, \dots, B_{in}q_{in}^\circ),$$

- ▶ ℓ_i° : labor input
 - ▶ q_{ij}° : quantity of generic variants used as inputs
 - ▶ B_{ij} : productivity shock
- All inputs are gross complements (elasticity of substitution ≤ 1).

Customized Producers

- a (finite or infinite) collection of firms
- customized variants can only be used by specific firms as intermediate inputs
- formalized as an **exogenous network G**



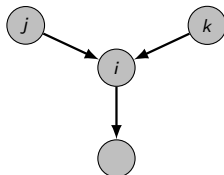
Assumption

The firm-level production network G satisfies the following:

- (i) each firm in G has at most one customer;
- (ii) each firm in G has at most one customized supplier in any given industry;

Customized Producers

- a (finite or infinite) collection of firms
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The firm-level production network \mathbf{G} satisfies the following:

- (i) each firm in \mathbf{G} has at most one customer;
- (ii) each firm in \mathbf{G} has at most one customized supplier in any given industry;

Customized Producers: Technology

- Same production function as the generics.
- Can always use generic variants as inputs

$$y_i = F_i \left(\ell_i, \{A_{ij}x_{ij} + B_{ij}x_{ij}^{\circ}\}_{(j,i) \in \mathbf{G}}, \{B_{ij}x_{ij}^{\circ}\}_{(j,i) \notin \mathbf{G}} \right).$$

Assumption

Customized variants result in higher productivities:

$$A_{ij} \geq B_{ij}$$

- For today: Leontief production technologies:

$$y_i = \min \left\{ \ell_i, \{A_{ij}x_{ij} + B_{ij}x_{ij}^{\circ}\}_{(j,i) \in \mathbf{G}^*}, \{B_{ij}x_{ij}^{\circ}\}_{(j,i) \notin \mathbf{G}^*} \right\}.$$

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Customized Producers: Fixed Operational Costs

- Operating active technology requires a **fixed cost** z_i in the units of labor at $t = 0$
- cost is sunk by the contracting stage at $t = 1$

- Induces an **endogenous** subnetwork of active firms

$$\mathbf{G}^* \subseteq \mathbf{G}$$

Consumption Good Sector

- Firms transforming various inputs into industrial aggregates:

$$x_{0i} = F_{0i} \left((x_{0i} + x_{0i}^{\circ})_{(i,0) \in \mathbf{G}} \right),$$

- Outputs then aggregated to a single consumption good:

$$y_0 = F_0(x_{01}, \dots, x_{0n})$$

- Different from customized producers:
 - ▶ no productivity difference between generic and customized inputs
 - ▶ no entry costs
 - ▶ no market power

Contracts and Terms of Trade

- Active firms can enter into pairwise contracts at $t = 1$
- Contract between $(j, i) \in \mathbf{G}^*$ specifies a price p_{ij}
- A commitment by the supplier to deliver as many units as demanded by the customer at fixed price p_{ij} .

Contracts and Terms of Trade

- Generic producers:

- ▶ price at marginal cost irrespective of customers' identity: $p_i^o = c_i^o$

- Customized producers:

- ▶ Rubinstein bargaining with random offers over infinitely many subperiods.
- ▶ supplier and customer make offers with probabilities δ_{ij} and $1 - \delta_{ij}$.
- ▶ if rejected, proceed to the next subperiod.
- ▶ both parties discount time at rate $\eta \uparrow 1$.
- ▶ if no agreement, the two parties cannot trade at $t = 2$.

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Contracts and Terms of Trade

- The bargaining powers δ_{ij} will determine equilibrium markups, markups' response to shocks, and shocks' pass-through.

- The customer has access to the outside option of using the generic variant:

$$p_{ij} \leq p_{ij}^{\circ} \frac{A_{ij}}{B_{ij}}$$

- Both parties have the outside option of walking away:
 - ▶ they reach an agreement in equilibrium only if there are positive gains from trade
 - ▶ imposes endogenous restrictions on p_{ij} as a function of other prices in the economy

Summary

- $t = 0$:
 - ▶ the network of potential relationships \mathbf{G} is realized
 - ▶ customized producers decide to operate their technologies by paying a fixed cost
 - ▶ network of active firms: $\mathbf{G}^* \subseteq \mathbf{G}$

- $t = 1$:
 - ▶ firms enter into pairwise fixed-price contracts
 - ▶ commitments to deliver as many units as demanded by their customers

$$\mathbf{p} = (p_{ij})_{(j,i) \in \mathbf{G}^*}$$
$$\mathbf{p}^\circ = (p_1^\circ, \dots, p_n^\circ, w)$$

- $t = 2$:
 - ▶ all firms make input and output decisions
 - ▶ households make consumption decisions

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Solution Concept: Production Equilibrium ($t = 2$)

Definition

Take the prices and the network \mathbf{G}^* as given. In a **production equilibrium**,

- (i) firms minimize costs while meeting their output obligations to their customers;
- (ii) the representative household maximizes her utility;
- (iii) all markets clear.

Solution Concept: Pricing Equilibrium ($t = 1$)

Definition

For given \mathbf{G}^* , a **pricing equilibrium** is collection of prices $(\mathbf{p}, \mathbf{p}^\circ)$ and quantities

- (i) the quantities in any ensuing subgame correspond to a production equilibrium;
- (ii) no generic producer i° can earn higher profits by offering a different price;
- (iii) there is no $(j, i) \in \mathbf{G}^*$ such that one party can earn higher profits by
 - ▶ renegotiating with an existing partner
 - ▶ negotiating with a new partner
 - ▶ terminating an already existing agreement

Solution Concept: Full Equilibrium ($t = 0$)

Definition

A **full equilibrium** is network \mathbf{G}^* and collections of prices & quantities such that

- (i) the quantities form production equilibria in the subgames at $t = 2$;
- (ii) the prices correspond to a pricing equilibrium in the subgames at $t = 1$;
- (iii) no customized firm has an incentive to change its decision to operate at $t = 0$:

$$\begin{aligned}\pi_i(\mathbf{G}^*) - z_i w &\geq 0 && \forall i \in \mathbf{G}^* \\ \pi_i(\mathbf{G}^* \cup \{i\}) - z_i w &< 0 && \forall i \notin \mathbf{G}^*.\end{aligned}$$

- Endogenizes the production network $\mathbf{G}^* \subseteq \mathbf{G}$
- Firms account for how their decision shapes the outcomes of the various pairwise bargaining processes, input and output prices, quantities, and household wealth.

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Production Equilibrium ($t = 2$)

Theorem

For any $(\mathbf{G}^, \mathbf{p}, \mathbf{p}^o)$, a production equilibrium exists and is unique.*

Pricing Equilibrium ($t = 1$)

Theorem

For any feasible network of active firms \mathbf{G}^* ,

(a) a pricing equilibrium $(\mathbf{p}, \mathbf{p}^\circ)$ always exists and is unique;

(b) all pairs of firms $(j, i) \in \mathbf{G}^*$ reach an agreement;

(c) vector of generic prices $\mathbf{p}^\circ = (p_1^\circ, \dots, p_n^\circ, w)$ is the solution to system of equations:

$$p_i^\circ = c_i(w, p_1^\circ / B_{i1}, \dots, p_n^\circ / B_{in});$$

(d) vector of customized prices $\mathbf{p} = (p_{ij})_{(j,i) \in \mathbf{G}^*}$ is solution to the system of equations

$$p_{ij} = \min \{ \widehat{p}_{ij}, p_j^\circ A_{ij} / B_{ij} \},$$

where \widehat{p}_{ij} is the unique solution to the equation

$$f_{ij}(\widehat{p}_{ij}) = \delta_{ij} \pi_i \frac{\partial \pi_j}{\partial p_{ij}} + (1 - \delta_{ij}) \pi_j \frac{\partial \pi_i}{\partial p_{ij}} = 0.$$

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Endogenizing Production Networks: Full Equilibrium ($t = 0$)

Theorem

Suppose all production functions are Leontief. Then,

- (a) a full equilibrium exists;*
- (b) set of full equilibria has a greatest element with respect to the set inclusion order;*
- (c) aggregate output in the greatest equilibrium is higher than all other equilibria.*

- All failures in the greatest full equilibrium are “fundamental”
- Strategic complementarities only in PE but not in GE:
 - ▶ operation of a firm that makes negative net profits may reduce the profits of others.
 - ▶ cannot use lattice theoretic results like Tarski's or Milgrom and Roberts (1994).
 - ▶ monotonicity for the set of firms that make positive profits

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Exogenous Production Networks: Bargaining Power

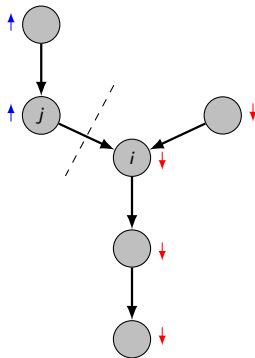
Theorem

An increase in a supplier's bargaining power vis-à-vis the customer

- (a) *increases* all upstream **markups** and *decreases* downstream and horizontal **markups**;
- (b) *increases* all upstream **profits** and *decreases* downstream and horizontal **profits**;
- (c) *increases* all upstream and downstream **prices** and *decreases* all horizontal **prices**.

Exogenous Production Networks: Bargaining Power

- Changes in markups and profits in response to increase in δ_{ij} :



Exogenous Production Networks: Production Network

Theorem

Let $\underline{\mathbf{G}}^* \subseteq \bar{\mathbf{G}}^*$ denote two feasible production networks. Then, for all $i \in \underline{\mathbf{G}}^*$,

(a)

$$\mu_i(\underline{\mathbf{G}}^*) \leq \mu_i(\bar{\mathbf{G}}^*);$$

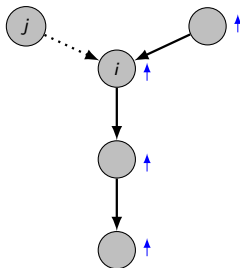
(b) if $\int_{\{j \in \bar{\mathbf{G}}^* \setminus \underline{\mathbf{G}}^*\}} (\pi_j(\bar{\mathbf{G}}^*) - wz_j) dj \geq 0$, then,

$$\pi_i(\underline{\mathbf{G}}^*) < \pi_i(\bar{\mathbf{G}}^*)$$

- Growing the set of active firms **increases profits** and **markups** of already active firms.

Exogenous Production Networks: Production Network

- Expanding the set of active firms is isomorphic to increasing the bargaining power of the already active firms.



- Strategic complementarities only in **PE** but not in **GE**:
 - ▶ holding aggregate demand constant, expanding the set of active firms increases profits (**PE**)
 - ▶ but operation of a firm that makes negative net profits may reduce aggregate demand (**GE**).

Exogenous Production Networks: Productivity Shocks

Theorem

For any $(j, i) \in \mathbf{G}^*$, an increase in productivity A_{ij}

- (a) *increases* all **markups** in the economy;
- (b) *increases* the **profits** of all firms that are downstream and horizontal to j ;
- (c) *increases* the **price** of firms that are upstream and horizontal to j and *decreases* the **price** of downstream firms.

Comparative Statics: Endogenous Production Networks

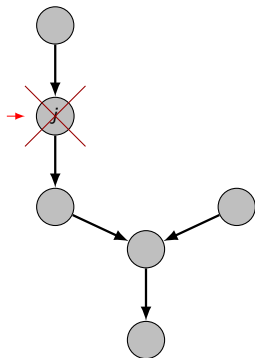
Theorem

An increase in the vector of fixed costs z

- (a) *shrinks the set of active firms in the greatest full equilibrium;*
- (b) *lowers aggregate output in the greatest full equilibrium;*
- (c) *reduces markups and profits of all remaining firms.*

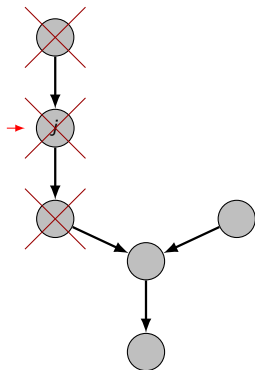
- There are complementarities in production → failure cascades
 - ▶ PE effect: direct propagation of failures over the network
 - ▶ GE effect: reduction in aggregate demand, thus lower profits for all firms

Cascading Failures



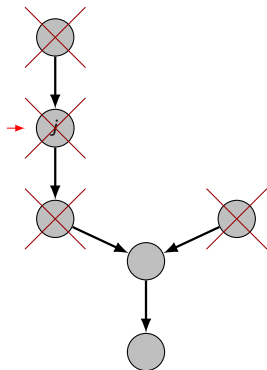
- Negative shock to j can result in upstream, downstream, and horizontal failures.
- both PE and GE effects.

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Industry-Level Aggregation

- Can the model be aggregated to the industry level?
- Baqaee and Farhi (2019):

$$\Delta \log \text{GDP} = \tilde{\lambda}' \underbrace{\Delta \log A}_{\text{change in productivity}} - \lambda' \underbrace{\Delta \log \mu}_{\text{change in markups}} - \tilde{\Lambda} \underbrace{\Delta \log \Lambda}_{\text{change in labor income share}}$$

- ▶ **Implication:** changes in labor income share and industry-level markups are sufficient statistics for measuring productivity shocks' aggregate effects.
- ▶ **Question:** can we use industry-level aggregates to obtain for shocks' macro effects, ignoring firm-level networks and failures?

Industry-Level Aggregation

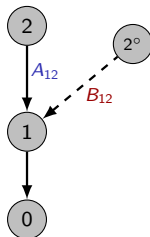
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Industry-Level Aggregation: Counterexample

- ψ_2 = fraction of active firms in industry 2.



- industry-level markups depend on the composition of firms:

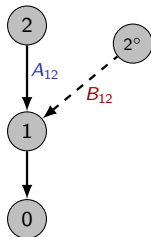
$$\mu_2^\circ = 1$$

$$\mu_2 = 1 - \delta_{12} + \delta_{12} A_{12} / B_{12}$$

$$\mu_1 = [(1 - \psi_2) / \mu_2^\circ + \psi_2 / \mu_2]^{-1}$$

Industry-Level Aggregation: Counterexample

- A proportional increase in A_{12} and B_{12} (TFP shock to industry 1).



- Industry-level aggregation holds if and only if

$$(1 - \delta_{12}) \frac{d\psi_2}{dA_{12}} = 0.$$

- With endogenous failures, industry-level variables are not sufficient statistics for the impact of industry-level shocks.

Aggregate Comparative Statics: Fixed Costs

- ψ : mass of active firms

$$\frac{d \log \text{GDP}}{dz_k} = - \frac{\frac{\partial \log \text{GDP}}{\partial z_k} \Big|_{\psi \text{ fixed}} + \sum_r \left(\frac{\partial \psi_r}{\partial z_k} \right) \left(\frac{\partial \log \text{TFP}}{\partial \psi_r} \right)}{1 + \sum_r \left(\frac{\partial \psi_r}{\partial \log y} \right) \left(\frac{\partial \log \text{TFP}}{\partial \psi_r} \right)}$$

Annotations for the equation above:

- partial equilibrium (+) points to the numerator's first term.
- cascade effect (-) points to the numerator's second term.
- aggregate demand effect (+) points to the denominator's first term.
- entry effect (-) points to the denominator's second term.

- ▶ **PE effect:** as if \mathbf{G}^* were exogenous (holding ψ constant)
- ▶ cascade effect: increase in fixed costs shrink the set of active firms
- ▶ entry/exit effect: less active firms reduces aggregate productivity
- ▶ aggregate demand: less active firms decreases final demand

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- cascade effect (-) points to the numerator's second term.
- aggregate demand effect (+) points to the denominator's first term.
- entry effect (-) points to the denominator's second term.

- ▶ PE effect: as if G^* were exogenous (holding ψ constant)
- ▶ **cascade effect**: increase in fixed costs shrink the set of active firms
- ▶ entry/exit effect: less active firms reduces aggregate productivity
- ▶ aggregate demand: less active firms decreases final demand

Aggregate Comparative Statics: Fixed Costs

- ψ : mass of active firms

$$\frac{d \log \text{GDP}}{dz_k} = - \frac{\frac{\partial \log \text{GDP}}{\partial z_k} \Big|_{\psi \text{ fixed}} + \sum_r \left(\frac{\partial \psi_r}{\partial z_k} \right) \left(\frac{\partial \log \text{TFP}}{\partial \psi_r} \right)}{1 + \sum_r \left(\frac{\partial \psi_r}{\partial \log y} \right) \left(\frac{\partial \log \text{TFP}}{\partial \psi_r} \right)}.$$

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Aggregate Comparative Statics: Productivities

$$\frac{d \log \text{GDP}}{d \log A_{ij}} = - \frac{\frac{\partial \log \text{GDP}}{\partial \log A_{ij}} \Big|_{\psi \text{ fixed}} + \sum_r \left(\frac{\partial \psi_r}{\partial \log A_{ij}} \right) \left(\frac{\partial \log \text{TFP}}{\partial \psi_r} \right)}{1 + \sum_r \left(\frac{\partial \psi_r}{\partial \log y} \right) \left(\frac{\partial \log \text{TFP}}{\partial \psi_r} \right)}.$$

partial equilibrium (+) cascade effect (?)

aggregate demand effect (+) entry effect (-)

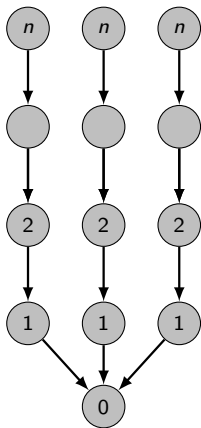
Aggregate Comparative Statics

for today:

some implications of how interactions between firm-specific relationships, markups, and failures shape aggregate output

Example 1: Failure Propagations

- Supplier has all the bargaining power $\delta_{k-1,k} = 1$.



ψ_k = fraction of active firms

Example 1: Failure Propagations

- aggregate output:

$$\text{GDP} = \frac{L - \bar{z}}{\sum_{k=1}^n (\psi_k - \psi_{k+1}) (A_{12} \dots A_{k-1,k} B_{k,k+1} \dots B_{n-1,n})^{-1}}$$

- failure cascades:

$$\psi_{k+1} = \psi_k \mathcal{H}_{k+1} \left(\frac{(1 - B_{k,k+1}/A_{k,k+1}) \text{GDP}}{A_{12} \dots A_{k-1,k} B_{k,k+1} \dots B_{n-1,n}} \right)$$

- Output in the economy with **endogenous set of active firms** relative to an economy with **exogenous set of active firms** with $\psi_k = 1$:

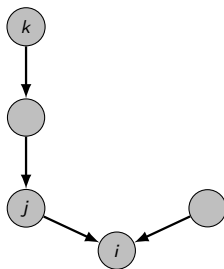
$$\lim_{A_k \rightarrow \infty} \lim_{A_1 \downarrow B_1} \frac{\text{GDP}_{\text{end}}}{\text{GDP}_{\text{exg}}} = 0.$$

Example 2: Productivity Shocks in a Non-Competitive Economy

- Leontief technologies + exogenous production network:

$$\frac{d \log \text{GDP}}{d \log A_{ij}} > 0$$

- Endogenous production network:
positive productivity shocks may **reduce** aggregate outputs



- ▶ an increase in A_{ij} increases k 's markup, but may reduce its profits and lead to its failure.

Summary and Next Steps

- A firm-level model that takes relationship-specific surplus and firm failures into account.
 - ▶ how shocks change firm-specific relationships and markups endogenously
 - ▶ propagation of shocks via failures
- Aggregated industrial-level variables (Domar weights, sectoral markups) not sufficient statistics for understanding the above.
- Different implications from to standard models with competitive pricing/constant markups
- Next steps:
 - ▶ more detailed comparative statics?
 - ▶ quantitative exercise for the various forces in a more realistic economy?
 - ▶ measuring the various terms in the data?