EUROPEAN CENTRAL BANK

EUROSYSTEM


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European Central Bank


# Forecasting and stress testing with quantile vector autoregression

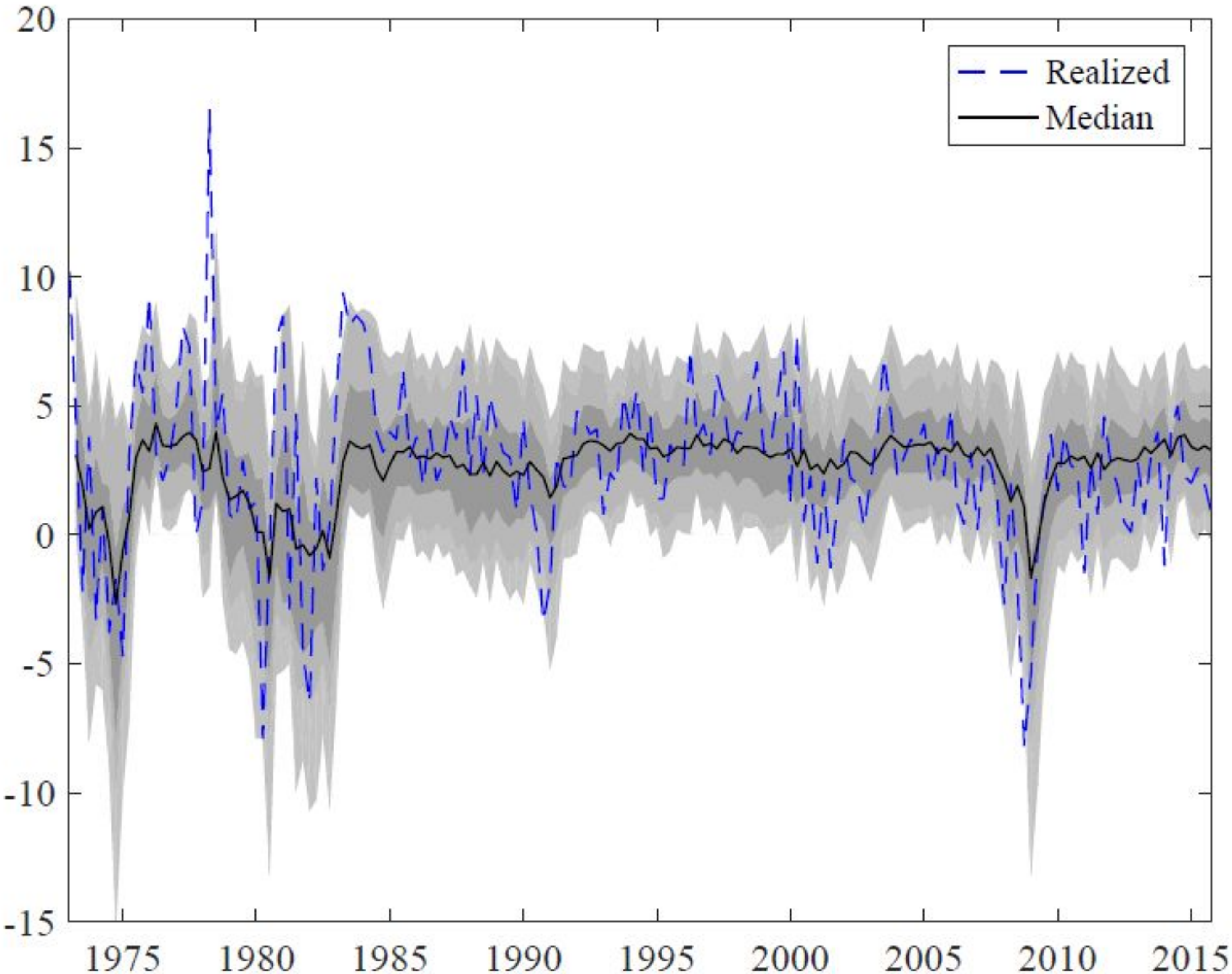
Frankfurt am Main, 15 May, 2019

Conference on Systemic Risk and the Macroeconomy

## A dynamic multivariate time series model

## Systematic influences of conditioning variables on the conditional distribution of the response

# US Growth at Risk




Source: Adrian et al., AER 2019

# Outline

**Quantile regression**

Quantile VAR – Identification and forecasting

Growth vulnerabilities in Europe

# Central bank related applications

CAViaR: Engle and Manganelli, 2004

VAR for VaR: White, Kim and Manganelli, 2015

CoVaR: Adrian and Brunnermeier, 2016

GaR: Growth at Risk (Adrian et al., 2019)

IaR: Inflation at Risk (Ghysels et al., 2018)

CaR: Capital at Risk

# Ordinary Least Squares – Mean

$$Y_{t+1} = X_t\beta + \varepsilon_{t+1} \qquad \mathrm{E}(\varepsilon_{t+1}|X_t) = 0$$

$$\equiv \mu_t \quad + \varepsilon_{t+1}$$

Minimise sum of squared residuals

$$\hat{\beta}_{OLS} = \arg\min \sum_{t=1}^{T} (Y_{t+1} - X_t\beta)^2$$

Gauss, around 1800

# Least absolute deviation – Median

$$Y_{t+1} = X_t\beta + \varepsilon_{t+1} \qquad \mathrm{P}(\varepsilon_{t+1} < 0|X_t) = 0.5$$

$$\equiv m_t \quad + \varepsilon_{t+1}$$

Minimise sum of absolute residuals

$$\hat{\beta}_{0.5} = \arg\min \sum_{t=1}^{T} |Y_{t+1} - X_t\beta|$$

Laplace (1789)

Maximum Likelihood

# Quantile regression

$$Y_{t+1} = X_t\beta + \varepsilon_{t+1} \qquad \mathrm{P}(\varepsilon_{t+1} < 0|X_t) = \theta \in (0,1)$$

$$\equiv q_t \;\; + \varepsilon_{t+1}$$

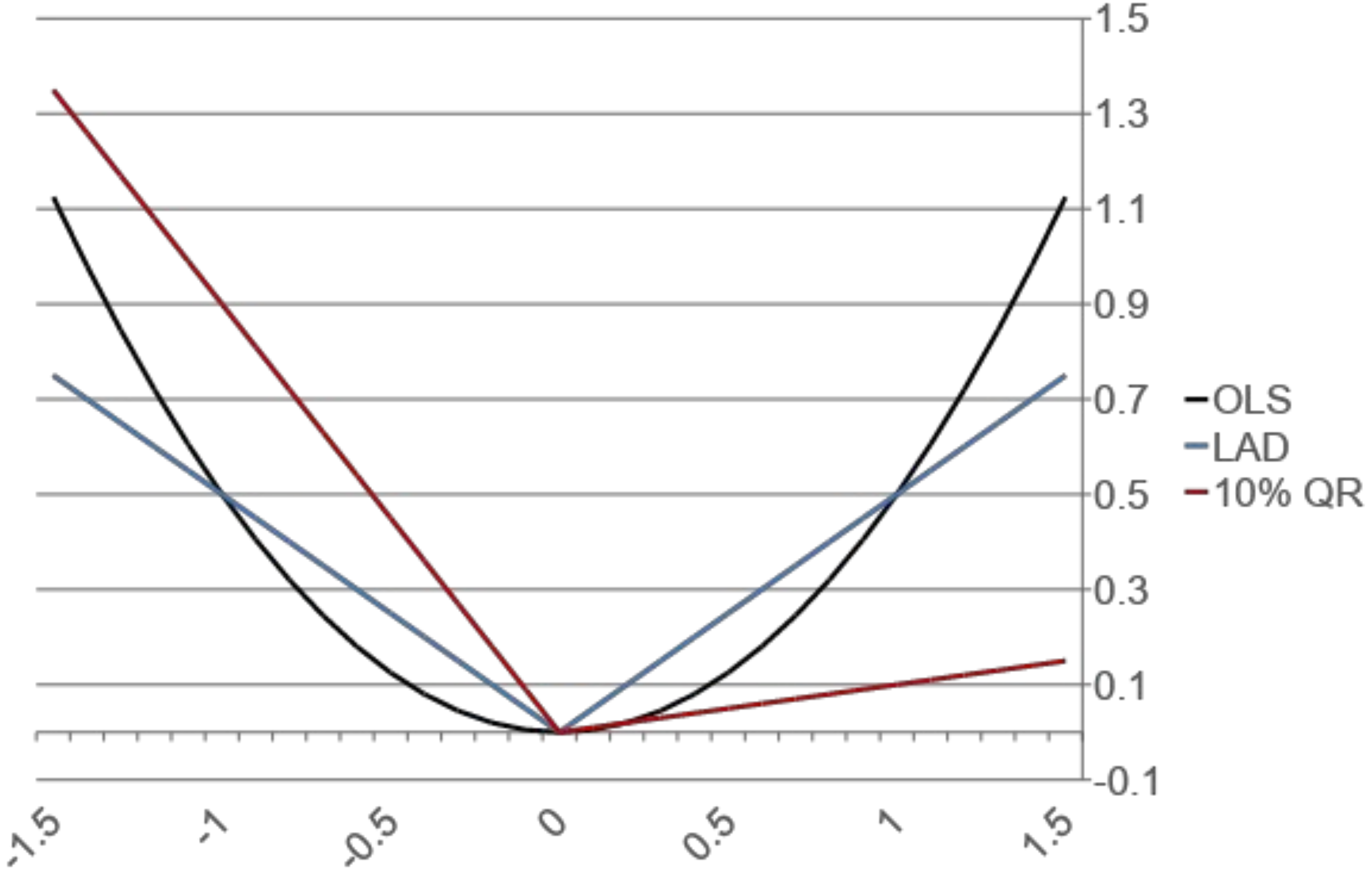
Minimise sum of weighted absolute residuals

$$\hat{\beta}_\theta = \arg\min \sum_{Y_{t+1} > X_t\beta} \theta|Y_{t+1} - X_t\beta| +$$

$$+ \sum_{Y_{t+1} < X_t\beta} (1-\theta)|Y_{t+1} - X_t\beta|$$

Koenker and Bassett (Econometrica 1978)

# Comparison of loss functions



# Outline

Quantile regression

**Quantile VAR – Identification and forecasting**

Growth vulnerabilities in Europe

# Structural VAR

$$Y_{t+1} = w + A_0 Y_{t+1} + A_1 Y_t + \varepsilon_{t+1} \qquad \varepsilon_{t+1} | \mathcal{F}_{t+1} \sim N(0, \Sigma)$$

with $A_0$ lower triangular

# Structural VAR

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$$= \nu + BY_t + (I - A_0)^{-1} \varepsilon_{t+1}$$

with $A_0$ lower triangular = Choleski decomposition

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with $A_0$ lower triangular = Choleski decomposition

$$\mu_{t+1} = \nu + B Y_{t+1}$$

$$= \nu + B \mu_t + B (I - A_0)^{-1} \varepsilon_{t+1}$$

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$$\equiv \mu_t \qquad + (I - A_0)^{-1}\varepsilon_{t+1}$$

with $A_0$ lower triangular = Choleski decomposition

$$\mu_{t+1} = \nu + BY_{t+1}$$

$$= \nu + B\mu_t + B(I - A_0)^{-1}\varepsilon_{t+1}$$

Expectation of the expectation at time *t*:

$$E(\mu_{t+1}|\mathcal{F}_t) = \nu + B\mu_t$$

# Structural quantile VAR

$$Y_{t+1} = w + A_0 Y_{t+1} + A_1 Y_t + \varepsilon_{t+1} \quad \mathrm{P}(\varepsilon_{t+1} < 0 | \mathcal{F}_{t+1}) = \theta$$

with $A_0$ lower triangular

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$$= \nu + BY_t + (I - A_0)^{-1}\varepsilon_{t+1}$$

$$\equiv q_t \qquad\qquad + (I - A_0)^{-1}\varepsilon_{t+1}$$

with $A_0$ lower triangular

$$q_{t+1} = \nu + BY_{t+1}$$

$$= \nu + Bq_t + B(I - A_0)^{-1}\varepsilon_{t+1}$$

# Forecasting with structural quantile VAR

$$Y_{t+1} = w + A_0 Y_{t+1} + A_1 Y_t + \varepsilon_{t+1} \quad \mathrm{P}(\varepsilon_{t+1} < 0 | \mathcal{F}_{t+1}) = \theta$$
$$= \nu + BY_t + (I - A_0)^{-1} \varepsilon_{t+1}$$
$$\equiv q_t \qquad\qquad + (I - A_0)^{-1} \varepsilon_{t+1}$$

with $A_0$ lower triangular

$$q_{t+1} = \nu + BY_{t+1}$$
$$= \nu + Bq_t + B(I - A_0)^{-1} \varepsilon_{t+1}$$

Quantile of the quantile at time *t*:

$$Q(q_{t+1} | \mathcal{F}_{t+1}) = \nu + Bq_t$$

# Careful with the cross section

$$(I - A_0)^{-1}\varepsilon_{t+1} = \begin{bmatrix} \varepsilon^1{}_{t+1} \\ a\varepsilon^1{}_{t+1} + \varepsilon^2{}_{t+1} \end{bmatrix}$$

# Careful with the cross section

$$(I - A_0)^{-1}\varepsilon_{t+1} = \begin{bmatrix} {\varepsilon^1}_{t+1} \\ a{\varepsilon^1}_{t+1} + {\varepsilon^2}_{t+1} \end{bmatrix}$$

$$Q((I - A_0)^{-1}\varepsilon_{t+1} | \mathcal{F}_t, {Y^1}_{t+1}) = \begin{bmatrix} {\varepsilon^1}_{t+1} \\ a{\varepsilon^1}_{t+1} \end{bmatrix}$$

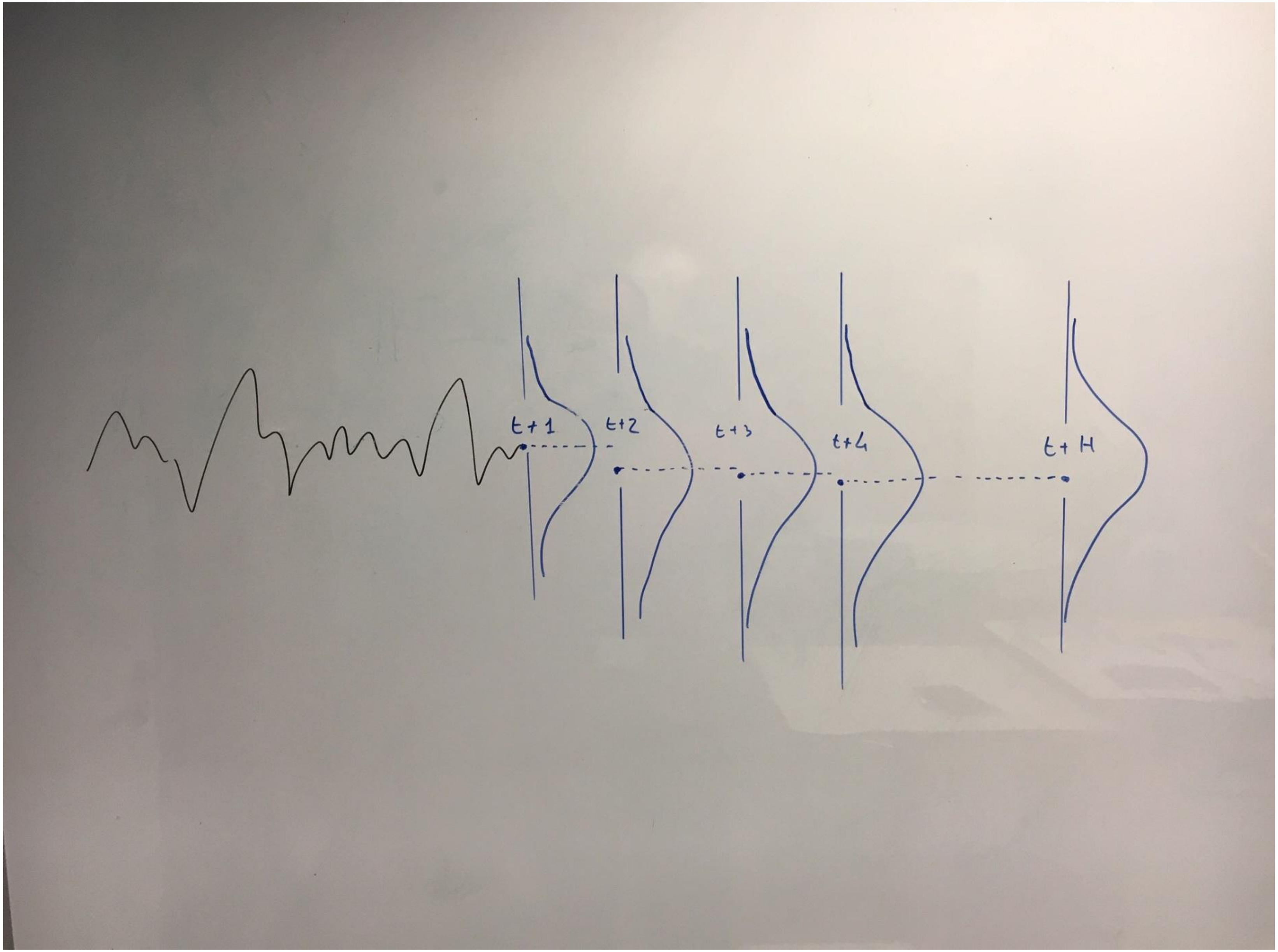
# Careful with the cross section

$$(I - A_0)^{-1}\varepsilon_{t+1} = \begin{bmatrix} \varepsilon^1{}_{t+1} \\ a\varepsilon^1{}_{t+1} + \varepsilon^2{}_{t+1} \end{bmatrix}$$

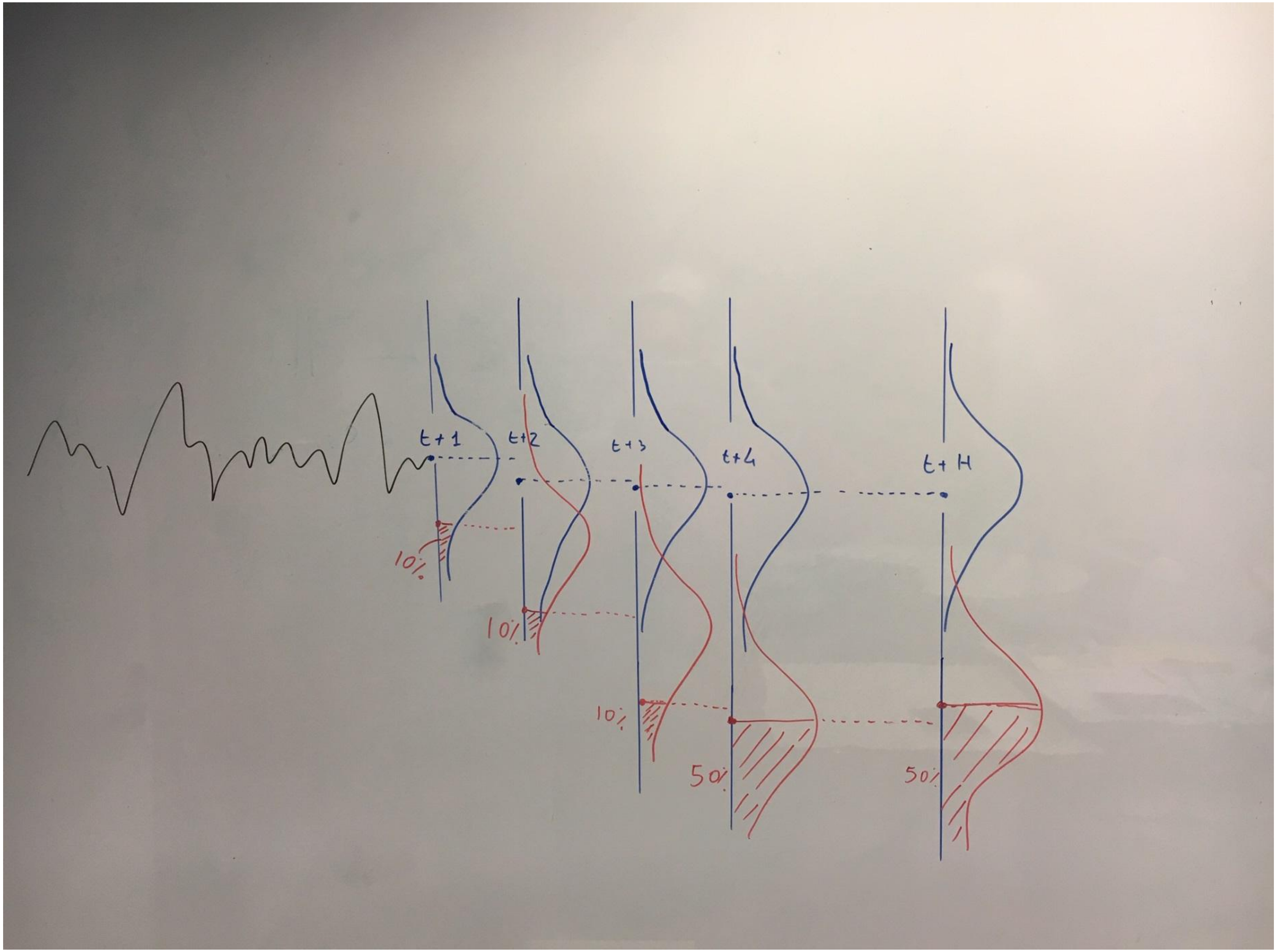
$$Q((I - A_0)^{-1}\varepsilon_{t+1} | Y_t, Y^1{}_{t+1}) = \begin{bmatrix} \varepsilon^1{}_{t+1} \\ a\varepsilon^1{}_{t+1} \end{bmatrix}$$

$$Q\left(\begin{bmatrix} \varepsilon^1{}_{t+1} \\ a\varepsilon^1{}_{t+1} \end{bmatrix} | Y_t\right) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

# Forecasting with VAR



# Forecasting with quantile VAR



# Stress testing with quantile VAR

Choose a sequence of quantile shocks and compute the corresponding quantile:

Ex.: $\{\theta_{t+1}, \ldots, \theta_{t+H}\} = \{0.10,\ 0.10,\ 0.10,\ 0.50, \ldots, 0.50\}$

$$\hat{Y}_{t+H} | \{\theta_{t+1}, \ldots, \theta_{t+H}\} = \nu + (B^{\theta_{t+1}} B^{\theta_{t+2}} \ldots B^{\theta_{t+H}}) Y_t$$

# Outline

Quantile regression

Quantile VAR – Identification and forecasting

**Growth vulnerabilities in Europe**

# Vulnerable growth in Europe

Quantile VAR:

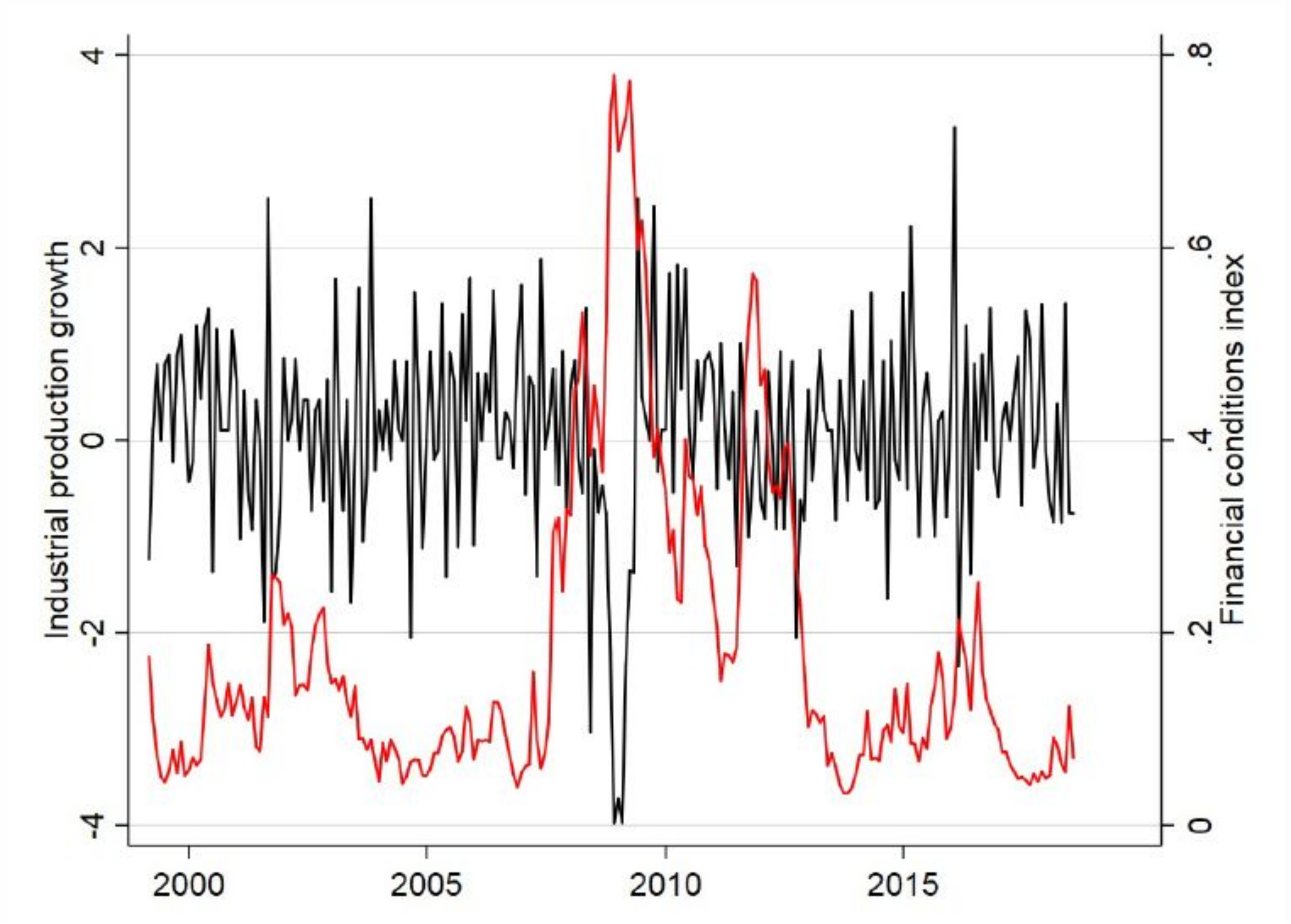
$$Y_{1,t+1} = \omega_1 + a_{11}Y_{1,t} + a_{12}Y_{2,t} + \varepsilon_{1,t+1}$$
$$Y_{2,t+1} = \omega_2 + a_0 Y_{1,t+1} + a_{21}Y_{1,t} + a_{22}Y_{2,t} + \varepsilon_{2,t+1}$$

CISS

EA IP

CISS = Composite Indicator of Systemic Stress
(Kremer et al. 2012)

# Data



Industrial production growth
Financial conditions index
4
2
0
-2
-4
.8
.6
.4
.2
0
2000
2005
2010
2015

# Important linkages

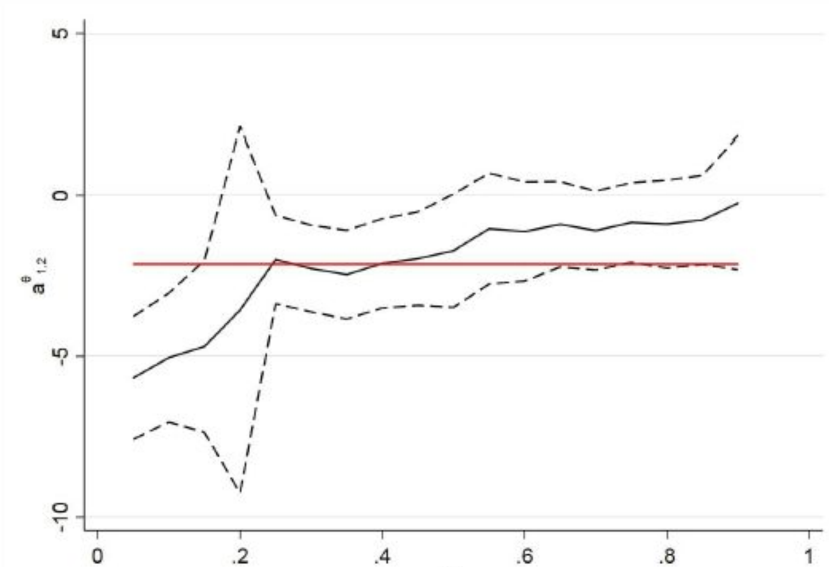
Quantile VAR:

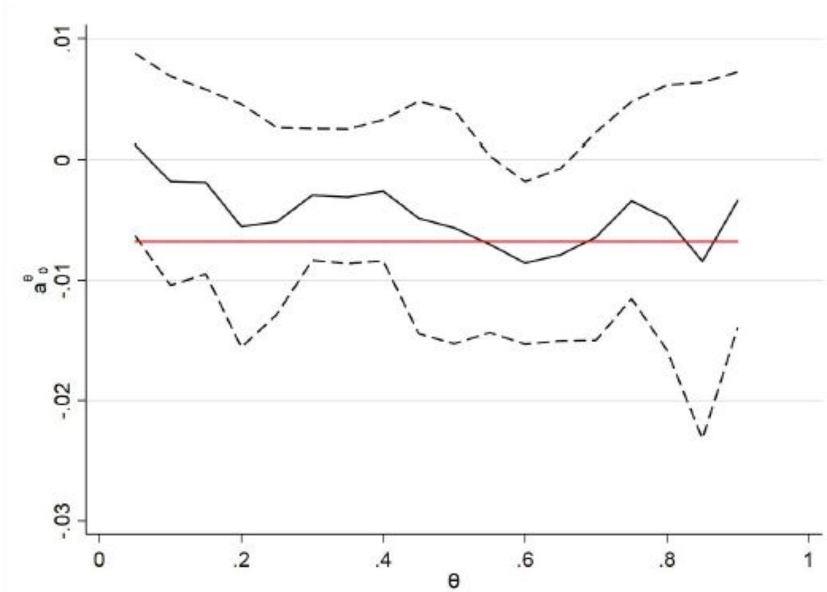
$$Y_{1,t+1} = \omega_1 \qquad\qquad\qquad\quad + a_{11}Y_{1,t} + a_{12}Y_{2,t} + \varepsilon_{1,t+1}$$
$$Y_{2,t+1} = \omega_2 + a_0 Y_{1,t+1} + a_{21}Y_{1,t} + a_{22}Y_{2,t} + \varepsilon_{2,t+1}$$

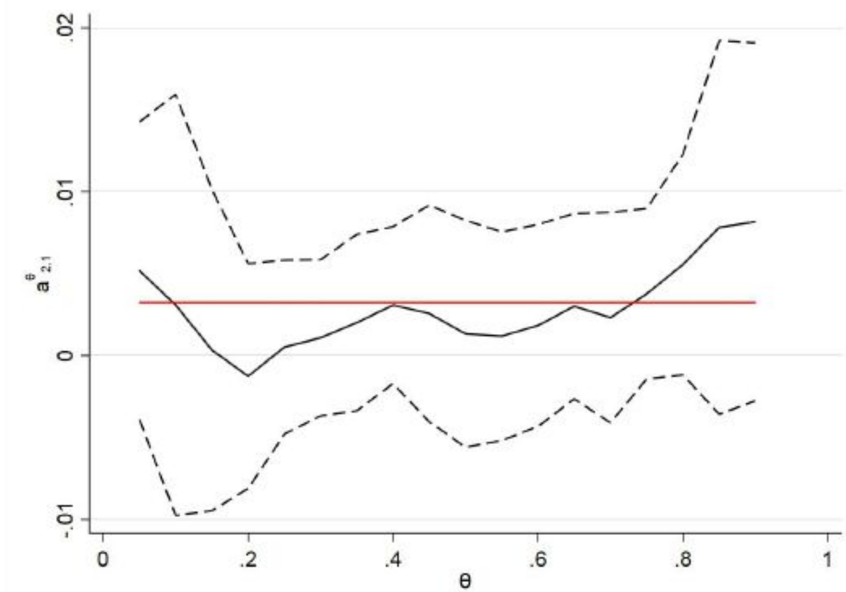
Testing the presence of linkages:

$$H_0 \colon a_{12} = a_0 = a_{21} = 0$$

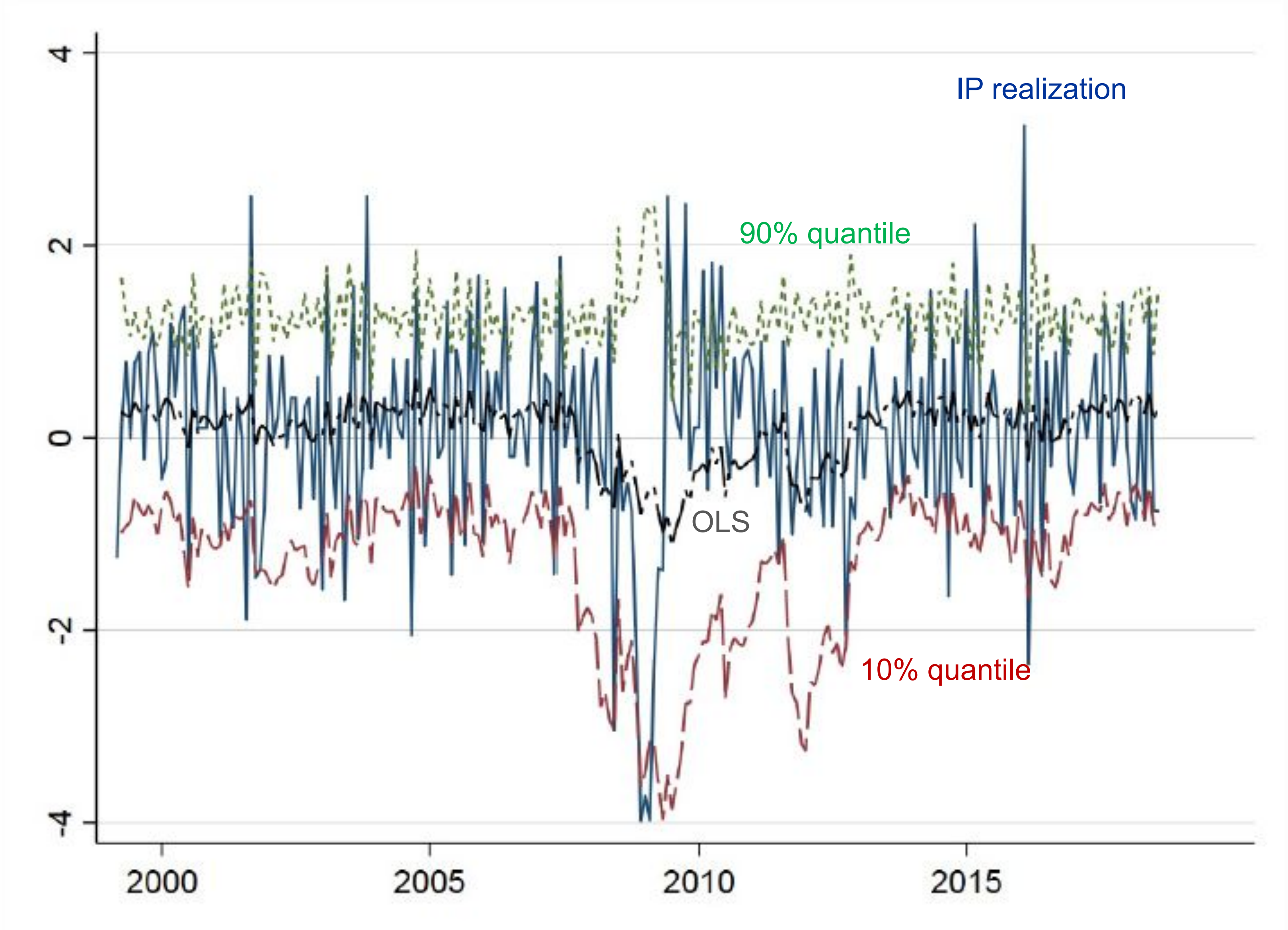
# Important linkages



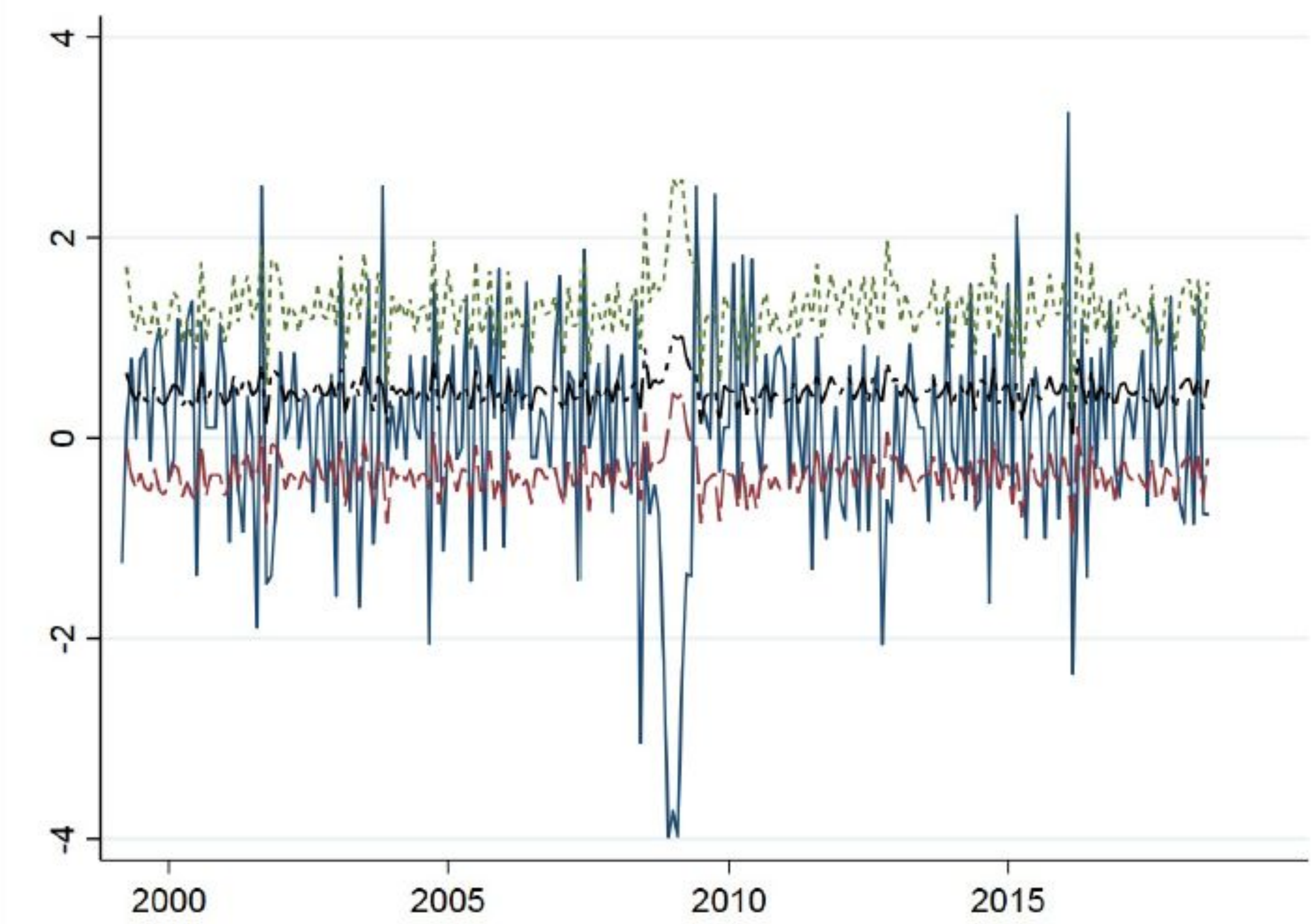



# Euro area Growth at Risk



# Euro area Growth at Risk



# Impulse response function for quantile VAR

$$Y_{t+1} = w + A_0 Y_{t+1} + A_1 Y_t + \varepsilon_{t+1} \quad \mathrm{P}(\varepsilon_{t+1} < 0 | \mathcal{F}_{t+1}) = \theta$$

$$\equiv q_t + (I - A_0)^{-1} \varepsilon_{t+1}$$

The quantile of the quantile is:

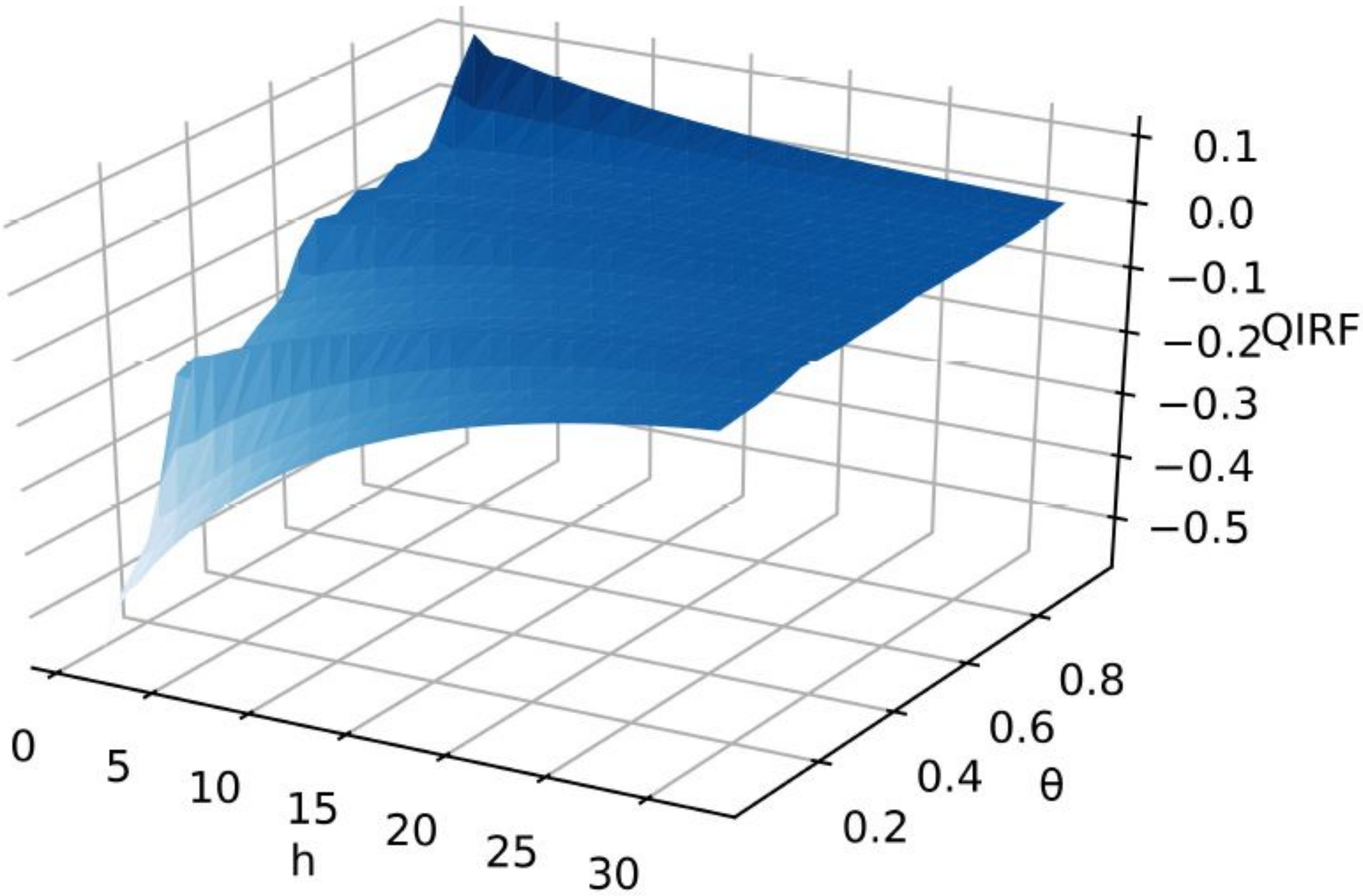
$$Q_t(\dots Q_{t+H-1}(q_{t+H})) = \nu + B^{H+1} Y_t$$

Quantile IRF:

$$\partial Q_t(\dots Q_{t+H-1}(q_{t+H})) / \partial \varepsilon_t{}' = B^{H+1} (I - A_0)^{-1}$$

# Quantile impulse response function for IP

## Shock to CISS



# Stress testing euro area growth

Quantile VAR:

$$Y_{1,t+1} = \omega_1 \qquad\qquad\qquad\qquad + a_{11}Y_{1,t} + a_{12}Y_{2,t} + \varepsilon_{1,t+1}$$

$$Y_{2,t+1} = \omega_2 + a_0 Y_{1,t+1} + a_{21}Y_{1,t} + a_{22}Y_{2,t} + \varepsilon_{2,t+1}$$

Stress scenario: sequence of quantile shocks

$$t+1: \quad \theta_{t+1} = (0.10, 0.90)$$

$$t+2: \quad \theta_{t+2} = (0.10, 0.90)$$

$$t+3: \quad \theta_{t+3} = (0.10, 0.90)$$

$$t+4: \quad \theta_{t+4} = (0.10, 0.90)$$

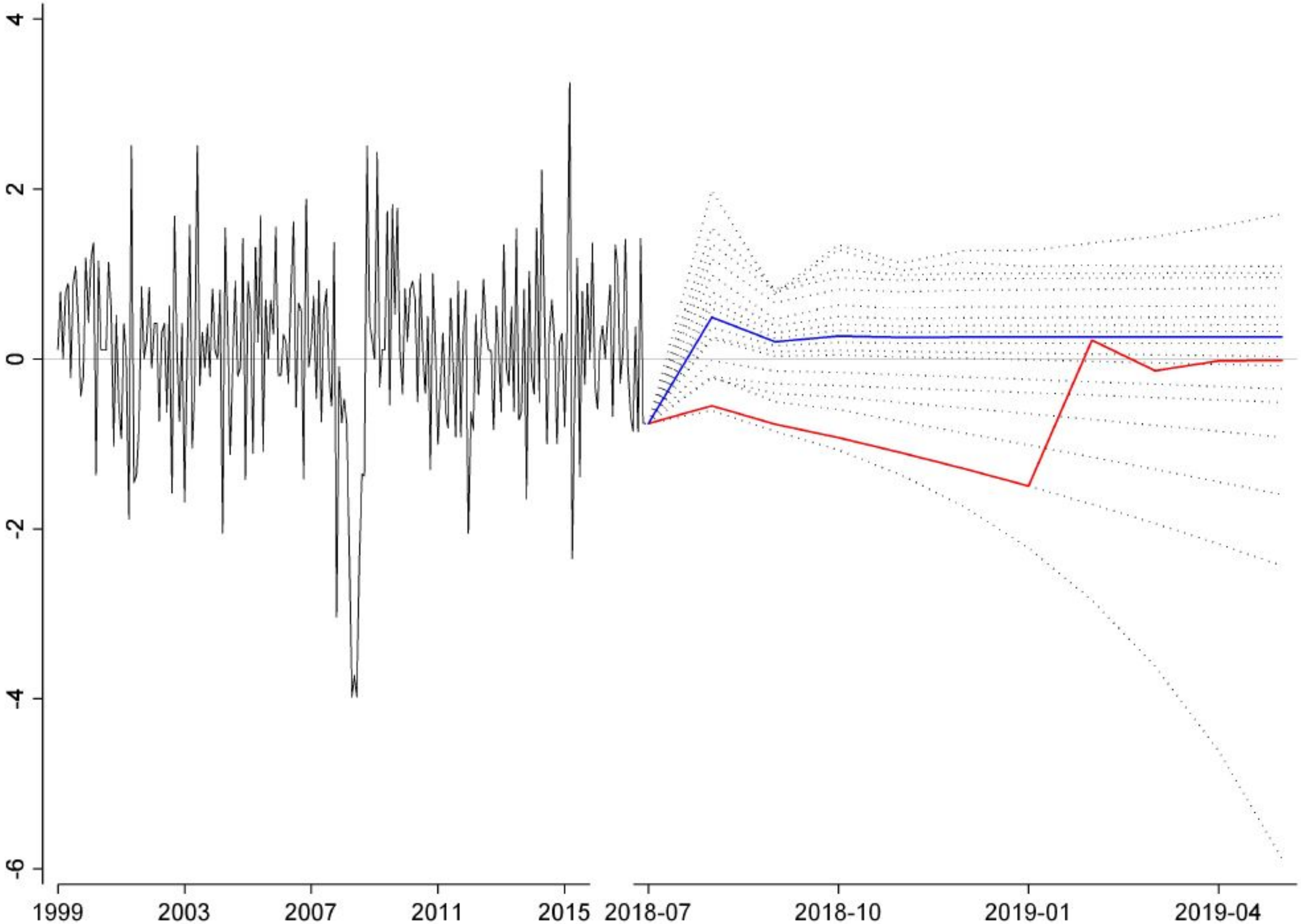
$$t+5: \quad \theta_{t+5} = (0.10, 0.90)$$

$$t+6: \quad \theta_{t+6} = (0.10, 0.90)$$

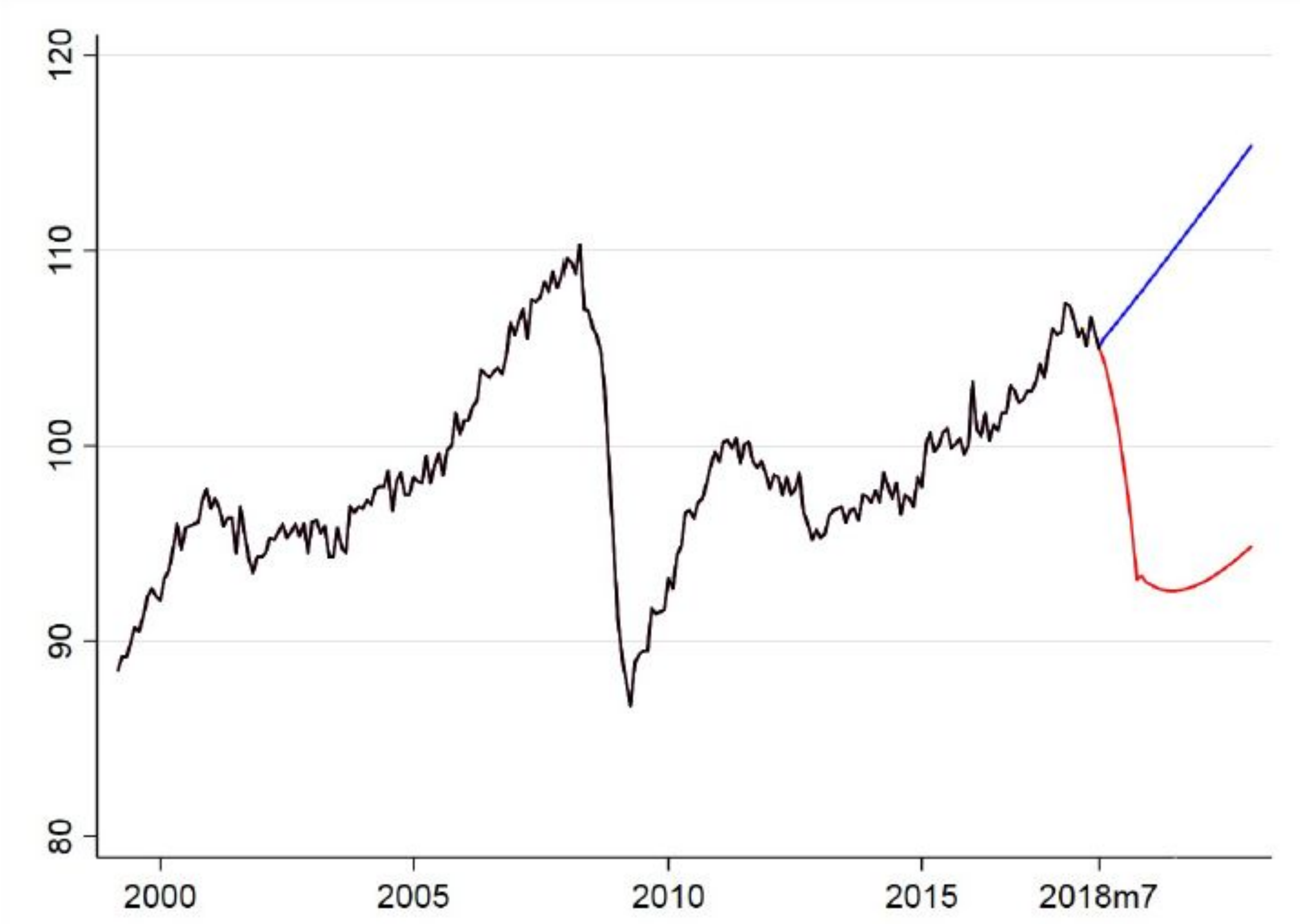
$$t+7: \quad \theta_{t+7} = (0.50, 0.50)$$

$$\hat{Y}_{t+H} | \{\theta_{t+1}, \dots, \theta_{t+H}\} = \nu + (B^{\theta_{t+1}} B^{\theta_{t+2}} \dots B^{\theta_{t+H}}) Y_t$$

# EA IP forecast under stress scenario



# EA IP forecast under stress scenario



# Conclusion

Methodological contribution:

- Introduced quantile VAR
- Showed how to do forecasting with quantile regression
- Quantile forecasting <-> Stress testing

Empirical findings:

- Strong macro-financial linkages in the euro area
- Macro-financial linkages are activated under stress
- Standard OLS VAR misses most of the action

# Will the tortoise really win in the end?

S. PORTNOY AND R. KOENKER