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Anatomy of regional price differentials: Evidence from micro price data

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Non-technical summary

Research Question

High income regions typically have higher price levels. Therefore, a meaningful comparison of regional financial well-being requires not only information on the incomes but also on the regional price levels. Knowledge of these price levels also facilitates spatial comparisons of real wages and real output. Unfortunately, the current regional price levels of the German counties and cities are not known, even though the Federal Statistical Office collects prices from all over Germany for its inflation measurement.

Contribution

From the price data collected by the Federal Statistical Office, the present study compiles a highly disaggregated regional price index for the 402 German counties and cities. Existing regional price indices of other large industrial countries are either incomplete or they are based on a much cruder subdivision of the country's territory. To fully exploit the exceptionally high quality of the German data, a novel compilation method is developed. It is also shown that simplified compilation methods would lead to less reliable results.

Results

The differences in the price levels of the 402 counties and cities of Germany are largely driven by the cost of housing and to a much lesser degree by the prices of goods and services. A region's price level affects the price levels of its neighboring regions. The overall price level in the most expensive region, Munich, is about 27 percent higher than in the cheapest region.

Nichttechnische Zusammenfassung

Fragestellung

Regionen mit hohen Einkommen besitzen normalerweise auch höhere Lebenshaltungskosten. Deshalb erfordert ein aussagekräftiger Vergleich der regionalen Einkommensverhältnisse nicht nur die regionalen Einkommensdaten, sondern auch das jeweilige regionale Preisniveau. Kenntnisse dieser Preisniveaus erlauben zudem räumliche Vergleiche der realen Löhne und der realen Produktion. Obwohl das Statistische Bundesamt für seine Inflationsmessung Preise aus ganz Deutschland heranzieht, sind die gegenwärtigen regionalen Preisniveaus der deutschen Landkreise und kreisfreien Städte unbekannt.

Beitrag

Die vorliegende Studie berechnet auf Grundlage der Preisdaten des Statistischen Bundesamtes einen regionalen Preisindex für die 402 deutschen Landkreise und kreisfreien Städte. Bereits bestehende regionale Preisindizes anderer großer Industriestaaten weisen entweder erhebliche Lücken auf oder nehmen eine nur sehr grobe regionale Unterteilung vor. Um die reichhaltigen Informationen in den Daten des Statistischen Bundesamtes optimal zu nutzen, entwickelt die vorliegende Studie eine neuartige Berechnungsmethode. Es wird auch gezeigt, dass vereinfachte Berechnungsmethoden zu weniger verlässlichen Ergebnissen führen würden.

Ergebnisse

Die Unterschiede in den regionalen Preisniveaus der 402 Landkreise und kreisfreien Städte werden vor allem von den Wohnkosten und deutlich weniger von den Preisen der Güter und Dienstleistungen bestimmt. Das Preisniveau einer Region strahlt auf die Preisniveaus der benachbarten Regionen aus. München ist die teuerste Region. Ihr Preisniveau übersteigt das der billigsten Region um 27 Prozent.

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Anatomy of Regional Price Differentials: Evidence From Micro Price Data*

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Abstract

Over the last three decades the supply of economic statistics has vastly improved. Unfortunately, statistics on regional price levels (sub-national purchasing power parities) have been exempt from this positive trend, even though they are indispensable for meaningful spatial comparisons of regional output, income, wages, productivity, standards of living, and poverty. To improve the situation, our paper demonstrates that a highly disaggregated and reliable regional price index can be compiled from data that already exist. We use the micro price data that have been collected for Germany's Consumer Price Index in May 2016. For the computation we introduce a multi-stage version of the Country-Product-Dummy method. The unique quality of our price data set allows us to depart from previous spatial price comparisons and to compare only exactly identical products. We find that the price levels of the 402 counties and cities of Germany are largely driven by the cost of housing and to a much lesser degree by the prices of goods and services. The overall price level in the most expensive region, Munich, is about 27 percent higher than in the cheapest region. Our results also reveal strong spatial autocorrelation.

Keywords: spatial price comparison, regional price index, PPP, CPD-method, hedonic regression, consumer price data.

JEL Classification: C21, C43, E31, O18, R10.

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1 Introduction

When the International Comparison Program (ICP) was created in 1968, it narrowed a gaping hole in economic statistics. The ICP's price level estimations facilitated international comparisons of *real* economic indicators such as the countries' real GDP, real growth, real per capita income, real investment, real wages, real income distributions, living standards, and poverty rates. The fact remains, however, that the regional differences within countries like India or China can be much larger than the difference between these two countries. Comparable price levels and real economic indicators are also needed on the sub-national level. For example, such information is needed for tracking the progress of regional cohesion and for the design of effective social policies. Furthermore, several economic theories can be best put to the test on the basis of *regional* real economic indicators. Examples are the urban wage premium (e.g., Glaeser and Maré, 2001; Wheeler, 2006; Yankow, 2006), the wage curve theory (e.g., Blanchflower and Oswald, 1995), and the contradictory results of Krugman (1991) and Südekum (2009) concerning the price level differentials between urban and rural regions.

Therefore, the natural extension of the ICP would be National Comparison Programs administered by the national statistical offices cooperating with the ICP. If these offices were completely free to design a data collection process for the purpose of regional price level comparisons, they would subdivide their respective country into many small rural, urban, and metropolitan regions. Then they would draw up a long list of extremely tightly defined representative products (henceforth, we use this term for goods and services) and would record each product's prices in those regions in which the product is representative. They would complement these prices by data on the regional cost of housing. Based on such an "ideal price data set" the statistical office would be able to regularly compile a regional price index for the complete country.

Even though some attempts in this direction have been undertaken, a sustainable procedure with a thoroughly regionalized data collection process has not yet been established. Official regional price comparisons are currently published by the Office of National Statistics (ONS) of the United Kingdom (e.g., Wingfield, Fenwick and Smith, 2005; ONS, 2018), by the Bureau of Economic Analysis (BEA) of the Unites States (e.g., Aten, 2017), and by the Government of Western Australia (GoWA, 2017). The latter index draws on prices from 27 major cities in Western Australia, while the BEA index utilizes the prices from 35 metropolitan and 3 urban areas in the United States. The ONS visits 21 locations across the United Kingdom. Considerable thought and resources have been devoted to the compilation of these data sets. Nevertheless, the regions are very large and inhomogeneous (e.g., Scotland is one region) and/or parts of the country are not included in the analysis (e.g., rural U.S. regions). Therefore, none of the data sets can be considered as "ideal". Notwithstanding these deficiencies, the official price indices of Western Australia, the United States, and the United Kingdom represent a highly welcome achievement that may encourage other countries to establish similar projects.

Theoretically, compiling an "ideal price data set" appears feasible, because most national statistical offices have decided to collect their Consumer Price Index (CPI) data from different regions. However, the number of sampled regions is usually too small to exploit the price data for a comprehensive interregional price comparison. The Federal Statistical Office of Germany is a notable exception. It collects its CPI data from about 400 different regions. Though not designed for the purpose of regional price comparisons, it is worldwide probably the best data source for that purpose. It contains not only the prices of all individual products, but also their precise specifications and their outlet types. Furthermore, it includes a large sample of rents along with detailed information about the characteristics of the respective flats and houses.[1]

Utilizing this unique data set as our principal data source, we are able to compile a spatial price comparison for the 402 regions (295 counties and 107 cities) of Germany. It is worldwide the first CPI based interregional price comparison that includes the complete household consumption basket for all regions of a complete major industrial country where the average regional size is below 1,000 square kilometer (the size of Scotland is 80,077 square kilometer). This is the paper's first contribution.

In interregional (and intertemporal) price comparisons it is usual practice to begin the computational procedure by assigning seemingly equivalent products to a group of comparable products (e.g., branded plain yoghurt, 125 grams). The prices of all products assigned to the same group are considered as directly comparable. If the price of plain brand A yoghurt in a supermarket located in region 1 exceeds the price of plain brand B yogurt in a discount store located in region 2, this would be taken as evidence for a higher yoghurt price level in region 1. Obviously, this evidence is weak. The higher price of brand A yoghurt could be caused by deviating brand premia or by different outlet types rather than by differences in the regional price levels. In other words, the initial grouping of products into groups of comparable products may generate tainted price data material giving rise to biased regional price indices (e.g., Silver and Heravi, 2005, p. 463; Silver, 2009, pp. 8-9). This potential contamination is particularly problematic for national statistical offices, because their interregional price indices quite likely find their way into contracts and other legal documents. As a consequence, national statistical offices are extremely reluctant to adopt any methodology that could be challenged in a legal dispute. Working with potentially contaminated price data is such a methodology.

The potential for biased regional price levels depends not only on the degree of contamination in the price material but also on the applied estimation method which, in turn, depends on the completeness of the data. In CPI data sets, very few groups of comparable products are recorded in all regions. A popular method to deal with these data gaps is the Country-Product-Dummy (CPD) approach pioneered by Summers (1973). It regresses the prices of the product groups on two sets of dummy variables. The first set represents the regions (or countries), while the second set represents the various product groups. If in the yoghurt example the yoghurt recorded in region A were of higher quality than that in region B (e.g., better brand, more appealing outlet type), a CPD regression that neglects this quality difference would overestimate the price deviation between the two regions.[2]

[1] More than 50% of the German population live in rented flats and houses. Therefore, rents are considered as an appropriate proxy for the cost in owner occupied housing.

[2] This issue is well known from the ICP 2005 where CPD regressions use average prices of product groups. Hill and Syed (2015, p. 524) convincingly demonstrate that this practice is inferior to a CPD regression that is based on individual price quotes. We fully agree with this assessment and add the recommendation that each product dummy must relate to a tightly defined product and not to a

To avoid this bias, Kokoski (1991, p. 32), Kokoski, Moulton and Zieschang (1999, p. 138), and Silver (2009, pp. 13-15) advocate a *hedonic* CPD regression that expands the set of regressors by variables that capture the qualitative characteristics of the individual products (e.g., taste, design, storage life, outlet type,...). Such an approach relies on the assumption that the impact of the qualitative characteristics on the price is identical for all regions and groups of comparable products. If this assumption is untenable, the regression equation must be further inflated by interaction terms between regional dummies and qualitative characteristics. In our own experimentation with hedonic CPD regressions we also encountered practical problems. Our CPI micro data cover the whole range of consumer products. Even though these data usually contain all the information necessary to unambiguously identify the product, this same information is often insufficient to describe the product's qualitative characteristics in a satisfactory way. As a consequence, the automation of hedonic CPD regressions turned out to be complex and prone to error.

Therefore, we introduce an alternative approach that rigorously minimizes the potential for contaminated price data and, in the context of our own comprehensive CPI data set, is easier to implement into an automated compilation process. Since we know not only the prices of the individual products but also their complementary attributes (precise specification and outlet type), we refrain from any grouping of products into groups of comparable products. Instead, we identify pairs of perfectly matching products. The complementary attributes of such a pair coincide in every respect, except for the region. This Perfect Matches Only (PMO) precept rejects all products that have been observed in only one region, because they are likely to introduce bias in the CPD regression. This bias could be avoided, only if for each basic heading a separate hedonic CPD regression was implemented that includes information on all relevant characteristics. As pointed out before, CPI data usually do not contain this information and, in view of the large number of basic headings, the associated workload would be prohibitive.

The PMO precept defines for each individual product its own vector of regional prices, while the traditional grouping approach defines such a vector for every group of comparable products. Therefore, with the PMO precept, the number of price vectors is much higher. The gaps within these vectors, however, are larger than in the grouping approach. To deal with these gaps, we embed our PMO precept into the weighted CPD approach advocated by Rao (2001, 2005), Hajargasht and Rao (2010), and Diewert (2005). We develop a multi-stage variant of this approach. It allows us to analyze our rent data by a separate full-fledged hedonic regression and to merge the resulting regional rent index with the regional price indices derived from the price data. Furthermore, this method solves an analytical problem posed by data confidentiality regulations of the Federal Statistical Office of Germany.[3] We believe that our multi-stage CPD regression based on the PMO precept represents, if not a completely new approach, an important addition to the methodologies available for interregional price comparisons. This is the second contribution of our paper.

Our work demonstrates that national statistical offices with a sufficiently regionalized CPI data collection procedure are able to produce, as a byproduct, a reliable regional price

group of seemingly very similar products.

3 The expenditure data necessary for the weighting could be incorporated into the analysis only after one stage of aggregation of the original price data.

index. The actual implementation must respect the specifics of the respective country. Our elaborate multi-stage CPD approach based on the PMO precept offers considerable flexibility and, in our view, ensures the highest possible degree of accuracy. Therefore, we advocate it as a useful reference for future interregional price comparison projects. For such projects it would be interesting to know whether simplified compilation procedures strongly influence the result. The high accuracy of our reference approach allows us to come up with a sound answer. This is our paper's third contribution.

Its final contribution is an examination of some widely held beliefs that are often based on anecdotal rather than systematic empirical evidence. For example, most economists think that in industrial countries the regional dispersion of housing costs exceeds that of prices of services and even more so that of goods. It is unknown, however, how strong the differences in the dispersion are. Furthermore, it is believed that, with a sufficient level of spatial disaggregation, the regional price levels change only gradually between neighboring regions.

The remainder of the paper is laid out as follows. Section 2 provides an overview of the available empirical studies on interregional price comparisons. Section 3 describes the data set underlying our own investigation. The applied methodology is explained in Section 4. Section 5 presents the results and Section 6 concludes.

2 Literature Review

Regional price level comparisons differ with respect to their geographical features, their data sources, and their methods for transforming these data sources into regional price levels. The geographical features include not only the country and its coverage (partial or full), but also the size and the number of regions. Table 1 provides an overview of the various studies and some of their main features.[4]

Country: Currently, official regional price indices exist only for the United Kingdom (ONS, 2018), the United States (Aten, 2017), and Western Australia (Government of Western Australia: GoWA, 2017). For several countries, however, exploratory studies exist: Australia, Brazil, China, the Czech Republic, Germany, India, Italy, Philippines, Poland, and Vietnam (see column "COUNTRY" of Table 1). Janský and Kolcunová (2017) attempt to estimate a regional price index for the complete EU28.

Coverage: Regional price level measurement requires regional information. For some regions such information may not be available. Therefore, some studies cover only parts of the country (see column "COV" of Table 1). When the complete country is covered, the regions are usually very large. In most cases, a region's data are collected from a single metropolitan area within the respective region.[5]

[4] Studies that compare the regional price levels of individual items or groups of items without transforming these results into the regions' overall price levels are not included in this survey. Examples are Hoang (2009) and Majumder, Ray and Sinha (2012) who investigate regional food prices in Vietnam and India, respectively.

[5] For example, Biggeri, Laureti and Polidoro (2017b) subdivide Italy into 19 regions where each region is represented by its most important city.

AUTHOR	COUNTRY	COV.	#REG.	SIZE	DATA	HOUS.	METHODOLOGY
Almås and Johnsen (2012)	China	partial	30	127550	household survey	yes	Engel analysis
Aten (1999)	Brazil	partial	10	major cities	CPI data	no	several
Aten and Menezes (2002)	Brazil	partial	11	major cities	household survey	no	weighted CPD
Aten (2017)	United States	full	51	25675	CPI micro data	yes	weighted CPD, then Geary-Kha.
BBSR (2009)	Germany	full	393	909	own data	yes	Laspeyres index
Biggeri, Ferrari and Zhao (2017a)	China	partial	31	269968	governmental data	yes	Eurostat-OECD
Biggeri *et al.* (2017b)	Italy	full	19	15860	CPI micro data	no	CPD
Blien, Gartner, Stüber and Wolf (2009)	Germany	full	327	761	Ströhl (1994)	no	extrapolation
Brandt and Holz (2006)	China	full	62	154790	CPI data	yes	Lowe index
Cadil, Mazouch, Musil and Kramulova (2014)	Czech Republic	full	14	5633	CPI data	yes	Eurostat-OECD
Chakrabarty, Majumder and Ray (2015)	India	partial	15	84751	household survey	no	COLI from demand system
Chakrabarty, Majumder and Ray (2018)	India	partial	30	84751	household survey	no	household CPD
Coondoo, Majumder and Ray (2004)	India	partial	4	821750	household survey	no	household CPD
Coondoo, Majumder and Chattopadhyay (2011)	India	partial	30	84751	household survey	no	Engel analysis
Deaton and Dupriez (2011)	India	partial	41	75374	household survey	no	Eurostat-OECD
	Brazil	partial	10	851600	household survey	no	Eurostat-OECD
Dikhanov (2010)	India	partial	10	217496	household survey	no	Eurostat-OECD
Dikhanov, Palanyandy and Capilit (2011)	Philippines	full	17	20202	CPI micro data	yes	CPD, then (geom.) Laspeyres
Gong and Meng (2008)	China	partial	30	278967	household survey	yes	Engel analysis
GoWA (2017)	Australia	partial	27	93699	own data	yes	Laspeyres index
Janský and Kolcunová (2017)	EU 28	full	281	15600	several other studies	partly	extrapolation

Continued on next page

AUTHOR	COUNTRY	COV.	#REG.	SIZE	DATA	HOUS.	METHODOLOGY
Kocourek, Šimanová and Šmída (2016)	Czech Republic	full	78	1011	CPI data	yes	CPD plus GEKS
Kosfeld, Eckey and Lauridsen (2008)	Germany	full	439	813	Ströhl (1994)	yes	extrapolation
Kosfeld and Eckey (2010)	Germany	full	439	813	Ströhl (1994)	yes	extrapolation
Li, Zhang and Du (2005)	China	partial	31	major cities	CPI data	yes	Fisher index
Li and Gibson (2014)	China	full	288	33323	real estate data	yes	Törnqvist index
Majumder, Ray and Sinha (2015a)	India	partial	30	84751	household survey	no	COLI from demand system
	Vietnam	full	3	110403	household survey	no	COLI from demand system
Majumder, Ray and Sinha (2015b)	India	partial	15	169502	household survey	no	several
Majumder and Ray (2017)	India	partial	30	84751	household survey	no	several
Mishra and Ray (2014)	Australia	full	7	1098857	household survey	yes	COLI from demand system
Musil, Kramulová, Čadil and Mazouch (2012)	Czech Republic	full	14	5633	CPI data	yes	Eurostat-OECD
ONS (2018)	United Kingdom	full	12	20207	own data	no	Eurostat-OECD
Rokicki and Hewings (2019)	Poland	full	66	4738	CPI data	yes	Eurostat-OECD plus extrapol.
Roos (2006a)	Germany	partial	16	22312	Ströhl (1994)	yes	extrapolation
Roos (2006b)	Germany	full	440	812	Ströhl (1994)	no	extrapolation
Ströhl (1994)	Germany	partial	51	major cities	own data	no	Laspeyres index
Waschka, Milne, Khoo, Quirey and Zhao (2003)	Australia	partial	8	major cities	CPI micro data	no	Eurostat-OECD
Wingfield *et al.* (2005)	United Kingdom	full	12	20207	own data	yes	Laspeyres index

Table 1: Main features of recent studies on regional price comparisons: country (column heading COUNTRY), coverage of country (COV.), number of regions (#REG.), average size of regions in square kilometer (SIZE), primary data source (DATA), inclusion of housing cost (HOUS.), and applied computational approach (METHODOLOGY).

Size and Number of Regions: The number of regions ranges from 3 to 440 (see column "#REG." of Table 1), while the average size of the regions ranges from 761 to 1,098,857 square kilometer (see column "SIZE" of Table 1).

Primary Data Source: None of the listed studies is based on an "ideal price data set". The studies by BBSR (2009), Kawka (2010), ONS (2018), and Ströhl (1994) are special, because they utilize price data that were collected specifically for that purpose. This is a laborious and expensive task. The collection process of the price data for BBSR (2009) and Kawka (2010) took three years. Due to cost considerations, Ströhl (1994) had to confine his analysis to 50 German cities and the ONS (2018) had to content itself with a disaggregation of Britain into 12 large regions. All other studies rely on price data that have been collected for other purposes (see column "DATA" of Table 1). Several of these studies utilize CPI data. Very few studies can draw on micro price data. In many non-OECD countries, sufficiently regionalized CPI data are not available (e.g., China, India, Vietnam), even though in such countries the regional price differences are probably much larger than in OECD countries. Therefore, researchers turned to the data provided by household expenditure surveys.

Housing: The studies also differ with respect to the range of items that are included. Most work conducted in developing countries concentrates on food items. Less than half of the studies include the cost of housing (see column "HOUS." of Table 1).

Methodology: Depending on the available data set, different computational approaches have been developed to transform the regional data into regional price levels (see column "METHODOLOGY" of Table 1). CPI data typically describe the observed market prices of a wide range of items reflecting the consumption patterns of typical households. These data are combined with the households average expenditure shares on the various items. Using this information, some studies define a "reference region" and use some standard index formula (e.g., Laspeyres, Fisher, Lowe, Törnqvist) to compute each region's price level relative to the reference region's price level. Other studies rely on variants of the GEKS index, following a recommendation by Eurostat-OECD (2012) for the computation of international purchasing power parities. A third group of studies applies some variant of the CPD method. A recent survey of the various methods can be found in Laureti and Rao (2018).

Some authors cannot draw on CPI data, but have to do with household expenditure survey data. In most of these studies a household's expenditures on some item are divided by the household's purchased quantity of that item to obtain a unit value that can be interpreted as the "implicit price" that this household pays for the item. One major problem with this approach is the variation in the item quality across households (e.g., Deaton, 1988, p. 420; McKelvey, 2011, p. 157). Suppose that rice purchased by households in region A is of higher quality than that purchased by households in region B. If region A's unit value of rice exceeds that of region B, this may reflect the difference in rice quality and should not be taken as evidence for a difference in regional rice price levels. In response to these concerns, various correction methods have been developed that compute "quality adjusted unit values". Based on these adjusted unit values and the household expenditures, some studies compute multilateral price indices (e.g., CPD, GEKS). Other studies estimate the parameters of a demand system, and from those a regional cost of living index (COLI) that compares the regional expenditures necessary to achieve a given utility level. A third group of studies exploits Engel's Law which states that a household's share of food expenditures falls as its real income increases. If two households located in different regions have identical food expenditure shares, but the nominal income of the first household exceeds that of the second household by 10%, then this implies that the price level in the first household's region is also 10% higher than in the region of the second household.

3 Data

The CPI micro data that we have the privilege of working with were provided to us by the Research Data Center (RDC) of the Federal Statistical Office and Statistical Offices of the Länder. These data are unique in several respects. First, thanks to the federal structure of Germany, its CPI compilation is based on a profoundly regionalized data collection process. Second, the price data come with detailed supplementary information revealing whether two price observations relate to exactly the same product. Third, the data set includes housing and related costs. Fourth, all prices are collected within one month. Because of the combination of these four features, the German CPI micro data come much closer to the rating of an "ideal price data set" than any of the data sets that were available to the authors of the studies listed in Table 1.

The German territory is subdivided into 402 regions (295 counties and 107 cities).[6] In each region and each month a large set of consumer price data is collected. In our analysis we use the data from May 2016. The data includes 381,983 consumer prices for goods, services, and rents that are classified into 650 categories denoted as *basic headings*. The actual collection of the price data is mostly conducted by the Statistical Offices of the Länder (Statistische Landesämter) while the Federal Statistical Office (Statistisches Bundesamt) complements the collected data by the prices of products which are known to be identical all over Germany (e.g. books and cigarettes) and by the prices of some products that require particularly careful quality adjustment procedures (e.g., cars and computers).

The German consumer price data represent a stratified sample where products are selected non-randomly within narrowly defined categories.[7] The hierarchical categorization of the products follows the United Nations' Classification of Individual Consumption by Purpose (COICOP).[8] At the highest classification level there are 12 divisions (see Table 2). Division 04 "Housing, water, electricity, gas, and other fuels" includes also rents. It turns out that rents are the most relevant data for our interregional price level comparisons. Fortunately, the information in the rent data exceeds that of goods and services. This enables us to analyze the rent data by a more sophisticated method than that applicable to the goods and services. Therefore, we split the data set into two subsets: 366,401 price data assigned to 645 basic headings and 15,582 rent data assigned to 5 basic headings.

6 The merger of two regions in November 2016 reduced this number to 401.

7 One exception are rents. Since 2016 they are collected from a stratified random sample (Goldhammer, 2016).

8 COICOP classifies consumption expenditures of private households, non-financial organizations and the state, while our consumer price data incorporate private households only.

ID	DIVISION	WEIGHT	#BH	#PRICES
01	Food and non-alcoholic beverages	12.57	161	97217
02	Alcoholic beverages, tobacco and narcotics	4.65	13	10378
03	Clothing and footwear	5.07	63	97823
04	Housing, water, electricity, gas and other fuels	32.42	36	21648
05	Furnishings, household equipment and maintenance	5.46	87	40597
06	Health	4.82	22	10394
07	Transport	15.30	53	22546
08	Communication	0.02	1	473
09	Recreation and culture	8.02	101	36942
10	Education	1.04	5	2478
11	Restaurants and hotels	4.59	43	11252
12	Miscellaneous goods and services	6.04	65	30235
		100.00	650	381983

Table 2: The 12 COICOP divisions covering household consumption expenditures and their expenditure weights (WEIGHT, measured in % and compiled in 2010), number of basic headings (#BH) and number of price observations (#PRICES). Source: RDC of the Federal Statistical Office and Statistical Offices of the Länder, Consumer Price Index, May 2016, own computations.

3.1 Price Data

For interregional price level comparisons, the prices for one and the same product must be available in multiple regions. Whether a pair of products is identical can be examined by comparing their characteristics documented in the complementary information of our price data. To each price observation we have not only the price and the region, but also several other product identifying attributes. These include the product's *amount* (e.g. the weight or quantity) and the respective *unit* of measurement (e.g. gram). The latter two variables describe the physical characteristics of the product. Furthermore, *outlet* specifies the price observation's type of store (e.g., supermarket, discount store, internet, and mail-order business), while *offer* indicates whether the price is an exceptional offer. Depending on the respective basic heading, several additional characteristics are available (e.g., brand, packaging, ...).

In contrast to the existing studies on interregional price comparisons, we do not group seemingly equivalent products into directly comparable products. Instead, we adhere to our Perfect Matches Only (PMO) precept. Table 3 presents a typical example. It shows the prices, the regions, and complementary information for the basic heading "rice". As the data are collected independently by the fourteen Statistical Offices of the Länder, different spellings occur and the reported values for characteristics such as "amount" and "unit" are often incoherent (e.g., some price collectors write 0.5 kg, others 500 g). These inconsistencies greatly complicate the identification of identical products.

In Table 4 none of the fourteen products exactly match. However, a closer look at the data reveals strong similarities between the characteristics as merely some of the spellings and units vary. Correcting and harmonizing the spellings and the units of measurement reduces the number of different products from fourteen to seven. These seven products are listed in the lines of Table 4. The columns of the table indicate the region in which the product has been observed. Since Product 7 has been observed in only one region, it

REGION	OUTLET	AMOUNT	UNIT	OFFER	CHARACTERISTICS	PRICE
A	discount store	1	kg	0	(Uncle Bens, basmati, bag)	1.69
D	discount store	0.5	kg	0	(Oryza, long grain, bag)	0.99
A	supermarket	0.5	kg	0	(Oryza, short gr., bulk)	0.98
B	discount store	1000	g	0	(Oncle bens, Basmati, bag)	1.59
E	discount store	500	g	0	(Oryza, long gr., bag)	0.97
A	supermarket	0.5	kg	1	(Oryza, l. grain, bulk)	0.79
C	supermarket	0.5	kg	0	(Oryza, short grain, bulk)	0.96
E	discount store	0.5	kg	0	(reisfit, longgrain, bag)	1.09
A	discount store	1	kg	0	(Reis-fit, med. grain, bag)	1.99
C	supermarket	500	g	0	(Uncle Ben's, basmati, bulk)	0.79
B	discount store	1	kg	0	(Reisfit, medium gr., bag)	1.89
C	discount store	1	kg	0	(Oncle Bens, Basmati, Bag)	1.89
B	supermarket	0.5	kg	0	(Uncle Ben, basm., bulk)	0.69
D	discount store	500	g	0	(Reisfit, long grain, Bag)	0.99

Table 3: Exemplary consumer price data for rice before data processing (all values fictitious).

provides no usable information for the interregional price comparison.

	A	B	C	D	E
Product 1 (discount store, 1, kg, 0, Uncle Bens, basmati, bag)	1.69	1.59	1.89	×	×
Product 2 (discount store, 1, kg, 0, Reisfit, medium grain, bag)	1.99	1.89	×	×	×
Product 3 (discount store, 0.5, kg, 0, Reisfit, long grain, bag)	×	×	×	0.99	1.09
Product 4 (discount store, 0.5, kg, 0, Oryza, long grain, bag)	×	×	×	0.99	0.97
Product 5 (supermarket, 0.5, kg, 0, Oryza, short grain, bulk)	0.98	×	0.96	×	×
Product 6 (supermarket, 0.5, kg, 0, Uncle Bens, basmati, bulk)	×	0.69	0.79	×	×
Product 7 (supermarket, 0.5, kg, 1, Oryza, long grain, bulk)	0.79	×	×	×	×

Table 4: Price matrix for rice after data processing (lines indicate products, columns indicate regions).

The data processing increases the number of perfectly matching pairs from zero to eight. This is important, because only identical products that have been observed in different regions provide unbiased information for interregional price comparisons. Before the data processing, a comparison between the five regions' price levels of rice is impossible. After the data processing, regions A, B, and C can be compared to each other, and regions D and E can be compared. However, a direct comparison of regions D or E to regions A, B, or C is still not feasible.

In our original price data set, the problem with inconsistency applies not only to the rice data, but also to the other basic headings. With 366,401 price observations, a manual correction and harmonization of the different spellings and units is infeasible. Therefore, we apply deterministic string matching algorithms for this purpose. Furthermore, we automatically convert, where possible, the units of measurement to the most frequent units within the basic heading. Our corrections reduce the number of different products by 8.46%, raising the number of estimated price levels by 14.32%. For all basic headings, the price data cover 389 of the 402 regions.

The basic heading "real property taxes" contains the taxes to be paid for constructible real property and real property with buildings. For a regional comparison of these taxes one would need a representative real property present in all regions and the taxes to be paid for this property. However, the available data do not allow for such a comparison. Therefore, we follow a different approach, also utilized by BBSR (2009, pp. 38-39). Once per year, the Statistical Offices of the Länder publish the overall property tax revenues for all 402 German regions.[9] We use the data of 2016. Assuming a similar real property structure across regions (e.g., relation of single- to double-family houses), the average tax revenue per household and region can be computed and used as a regional price index of the basic heading "real property taxes". Therefore, we replace the regional property taxes contained in our original price data by the computed averages per household and region.

Furthermore, we make use of fuel prices for diesel and gasoline that have been assembled by the German Market Transparency Unit for Fuels.[10] These fuel prices are available on municipality level for May 2016. We aggregate the fuel prices to the superior regional level, that is, to our 402 regions. For this purpose we denote a municipality's diesel price by d_s^r where the sub- and superscript indicate that municipality s is located in region r. S^r denotes the number of municipalities s in region r, pop_s^r is the population in municipality s, and

$$g_s^r = pop_s^r \Big/ \sum_{s=1}^{S^r} pop_s^r$$

is the population share of municipality s in region r. The average diesel price, d^r, in region r can be computed as a weighted arithmetic mean of the diesel prices in the region's municipalities:

$$d^r = \sum_{s=1}^{S^r} g_s^r \cdot d_s^r \quad \text{for } r = 1, \ldots, 402 \ .$$

Analogously, we compute the prices for gasoline. We replace the prices for diesel (below 60 cetan) and gasoline (95 octan) contained in our original data by the computed averages of the 402 regions, respectively.

3.2 Rent Data

The German CPI includes both rents and the cost of owner occupied housing. Roughly 54% of German houses and flats are occupied by renters (Statistisches Bundesamt, 2017, p. 161). This is one reason, why the cost of owner occupied housing is measured by the rental equivalence approach. This approach assumes that the cost of living in one's own house or flat is equivalent to the rent that would typically arise for such an accommodation.

The Federal Statistical Office groups the German rent data under five basic headings, one covering single-family houses and the other four covering different types of flats, where the criteria are the year of construction (before 1949 / since 1949) and the living space (up to 70 sqm / above 70 sqm). By this stratification, the Federal Statistical Office intends to ensure a minimum number of flats present in each category. In addition, the German rent sample is stratified by the type of landlord (private landlords, public and private housing companies) and the 96 spatial planning regions in Germany.[11] The rent data that are

9 The data are available via https://www.regionalstatistik.de/genesis/online.

10 The data were downloaded from the web portal https://creativecommons.tankerkoenig.de.

11 A planning region is a group of neighboring regions that are characterized by strong commuter

available to us cover 381 of the 402 regions. Only 315 of the 15,582 rent observations refer to the basic heading "single-family houses". In view of this sparse data base and the large difference between single-family houses and flats, we exclude the 315 observations on single-family houses from our rent data set.

The literature on the measurement of housing prices (e.g., Wabe, 1971, pp. 249-251) differentiates between house parameters (e.g., living space and quality of the flat's equipment such as its windows, floors, etc.) and locational parameters (quality of residential area). Both types of information are available in our rent data. However, due to data confidentiality reasons, we do not know the exact year of construction of a flat and the type of landlord. Furthermore, the data cover only flats in existing buildings.[12]

The summary statistics for the continuous variables of our data set are listed in Table 5. The rent is net of utilities. It is measured in € and the living space in square meter (sqm). The length of tenancy is measured in days and indicates the time interval between the date of data collection (May 2016) and the start of the tenancy.

VARIABLE	MIN	MEDIAN	MEAN	MAX	SD
rent (*rent*)	60.29	328.90	358.87	2537.26	151.11
living space (*sqm*)	11.33	62.00	64.58	242.98	18.81
length of tenancy (*len*)	1.00	2738.00	4358.56	25462.67	4503.21

Table 5: Summary statistics by variable (short names in brackets). Source: RDC of the Federal Statistical Office and Statistical Offices of the Länder, Consumer Price Index, May 2016, own computations.

Besides the living space and the length of tenancy, we have further information about each flat. These are given as categorical variables in the data. The summary statistics for the rent by the respective categories of each variable are shown in Table 6. The quality of the flat's equipment, *equ*, is classified into three levels: low, medium and high. This classification follows a BMVBS (2012) guideline, which provides a standardized evaluation catalog. The quality of the residential area, *area*, ranks the quality of the flats' surrounding area in four classes from low to very high. For both variables the average rent increases with the flat's quality. The variable *priv* indicates whether the flat is privately or publicly funded.[13] 71% of the flats have a built-in kitchen. This characteristic is captured by the variable *kit*.

The regionalized structure of the data allows us to identify the region in which the flat is located. The number of rent observations strongly varies between rural and urban regions. For example, Berlin and Munich together represent approximately 10% of all rent observations while some of the rural regions represent less than 0.1%.

For 21 regions the rent data of the Federal Statistical Office do not provide sufficient information to compute a rent level. Furthermore, these rent data cover only a

relations. A typical planning region has an economic center surrounded by a more rural area. In Germany, the planning regions are classified by the Bundesinstitut für Bau-, Stadt- und Raumforschung (BBSR).

12 The neglect of flats in newly completed buildings can be a problem for intertemporal price comparisons, but less so for interregional price comparisons.

13 Goldhammer (2016, p. 88) mentions that there is only little social housing in some federal states which explains the small fraction of publicly funded flats (approximately 10%) in the rent data.

VARIABLE	MIN	MEDIAN	MEAN	MAX	SD	%
quality of equipment (*equ*):						
(1) low	60.29	305.00	327.13	1593.22	127.37	38
(2) medium	85.00	337.75	361.86	1770.02	136.70	53
(3) high	120.59	410.00	473.60	2537.26	239.58	9
quality of residential area (*area*):						
(1) low	65.74	309.49	324.31	865.00	114.18	9
(2) medium	79.37	326.55	350.32	1493.49	132.84	48
(3) high	85.00	330.00	367.24	2511.39	161.76	38
(4) very high	138.92	370.00	440.12	1793.33	234.45	5
private housing (*priv*):						
(1) yes	60.29	326.80	359.76	2537.26	154.75	90
(2) no	89.35	342.10	351.16	986.90	114.57	10
built-in kitchen (*kit*):						
(1) yes	75.00	344.11	375.99	2537.26	156.89	71
(2) no	68.07	293.62	315.97	1577.77	125.71	29

Table 6: Rent by flat characteristics (short names in brackets) and relative frequencies of flat characteristics (in %). Source: RDC of the Federal Statistical Office and Statistical Offices of the Länder, Consumer Price Index, May 2016, own computations.

small fraction of tenant changeovers in existing buildings and no flats in newly completed buildings. Therefore, we draw on a second data source. The BBSR collects rents for flats without furnishing and with a living space between 40 and 130 sqm. The rents are net of utilities and cover tenant changeovers in existing buildings as well as flats in newly completed buildings. Furthermore, as the data is collected from internet platforms and from newspaper ads, it represents quoted rather than transactional rents. Although the quoted rents are expected to be on average higher than the corresponding transactional rents, no evidence exists that this difference varies between regions. Therefore, the quoted rents serve as an indicator for regional rent level differences and, therefore, become part of the regional rent index numbers.[14] The BBSR has provided us with an average rent per sqm in all 402 regions as of the second quarter 2016. The regional average rents range from 4.23 € per sqm to 15.61 € per sqm. The rent in the cheapest region is 39.45% below the population weighted German average rent level, while the most expensive region is 123.25% above that average.

3.3 Weighting

Our price and rent data are complemented by a two-dimensional system of expenditure weights provided by the Federal Statistical Office. The latest available system of weights is from 2010.

The first dimension of this system are the expenditure shares that a typical German consumer spends on the various basic headings. The expenditure share weights available to us are identical across regions. Moreover, the weights that we use deviate slightly from the original weights published by the Federal Statistical Office, because 16.08% of total

[14] Faller, Helbach, Vater and Braun (2009) find an overall deviation of 8% between quoted and transactional prices for purchases of flats and houses. For rents, they expect that this deviation becomes smaller.

expenditures relate to basic headings that are not included in our data set. For example, in our rent data, two of the seven basic headings listed in the original weighting scheme are missing (representing 0.98% of total expenditures). Therefore, we rescale the weights of the remaining five basic headings such that they sum to 20.99% which is the sum of the seven original expenditure shares relating to rents.[15] The same procedure we apply to the basic headings relating to our price data set, resulting in a total weight of 79.01%. In that data set, 97 basic headings are missing (representing 15.10% of total expenditures).

The expenditure weights relating to the highest classification level, denoted as divisions, are listed in Table 2. The weights reveal that private households spent 32.42% of their total expenditures on housing and related components. This category includes rents.

The second dimension of the weighting system are the outlet types. On average, discount stores (36.7%) and specialized shops (26.0%) have the largest market shares in Germany, while the market share of internet and mail-order business (8.7%) is relatively low (see Sandhop, 2012, p. 269). Other outlets are department stores (2.80%), hypermarkets (12.10%), supermarkets (12.40%), other retail (1.00%), and private and public service provider (0.30%).

For more than two thirds of the 650 basic headings we know how expenditures on a particular basic heading are divided between the eight types of outlets. As a consequence, we apply a differentiated weighting of outlet types across basic headings. For most basic headings, only some of these outlet types are relevant. Rice, for example, has been observed only in hypermarkets, supermarkets, and discount stores. Like the expenditure share weights of basic headings, also the expenditure share weights of outlet types are uniform across regions.

4 Methodology

Even though our rent data exhibit some gaps, the information in this data set is richer than in the price data set. Therefore, a hedonic regression technique can be applied to compute regional rent levels. This approach is outlined in Section 4.1.

Also our price data set exhibits gaps, because none of the products with regionally varying prices is observed in all regions. Therefore, the regional price levels cannot be computed by standard price index formulas. Instead, we estimate the price levels by a multi-stage version of the Country-Product-Dummy (CPD) method. We describe and apply the (unweighted) CPD method in Section 4.2 where we compute the regional price levels of products that belong to the same basic heading and type of outlet. A weighted variant of the CPD method is used to aggregate for each basic heading the regional price levels of different outlet types to obtain the regional price levels of the respective basic heading. This second step is described in Section 4.3.

To obtain for each region its (overall) price level, the regional rent levels from Section 4.1 and the basic headings' regional price levels from Section 4.3 must be aggregated.

[15] With a weight of 10.4%, rents also are the most important product category in Eurostat's Harmonized Index of Consumer Prices (HICP). In contrast to the weights compiled for the German CPI, owner-occupied housing is not included in the weights of the HICP.

This occurs in two steps. First, for each region separate price indices of housing, goods and services are computed (Section 4.4). Again, weighted CPD regressions are utilized for this purpose. Finally, Section 4.5 describes how these three price indices are aggregated into the region's overall price index.

4.1 Hedonic Regression of Regional Rent Levels

The information from an observation of the rent data is richer than that of the price data. This allows us to compute the regional rent levels by the hedonic regression approach. It estimates the "implicit prices" of the flats' characteristics. Knowing these implicit prices, one can compile the rent levels prevailing in different regions.

The hedonic method assumes a functional relation between the rent p_i of flat i ($i = 1, 2, \ldots, N$) and it's K characteristics q_{ki} ($k = 1, 2, \ldots, K$):

$$p_i = f(q_{1i}, \ldots, q_{Ki}) \ .$$

If a simple linear specification of the regression equation is chosen, the derivative $\partial p_i/\partial q_{ki}$ measures the implicit price of characteristic k. To estimate the hedonic regression equation, the functional relation as well as the characteristics q_{ki} need to be further specified.

In our rent data, we have 15,267 flats that are located in 381 of the 402 regions. To indicate the region of a flat, we use dummy variables, $region_i^r$ ($r = 1, \ldots, 381$), with $region_i^r = 1$, if flat i is located in region r, and $region_i^r = 0$ otherwise. Besides its region, each flat is characterized by $K = 6$ additional variables: living space (sqm_i), length of tenancy (len_i), quality of equipment (equ_i, three levels: low, medium, high), quality of the residential area ($area_i$, four levels: low, medium, high, very high), private versus social housing ($priv_i$), and existence of a built-in kitchen (kit_i).

For 643 observations, the data are incomplete. As a consequence, the number of observations available for the hedonic regression falls to $N =$ 14,624 and the number of regions to $R = 366$. For each of these regions at least three complete observations exist.

The relationship between the rent and the six characterizing variables varies across regions, in particular, between urban and rural regions. To account for this regional heterogeneity we incorporate interaction terms for the intercept. A simple Box-Cox test suggests that a logarithmic specification of the regression model is more appropriate than a fully linear or a log-linear specification. Furthermore, a linear specification would most likely suffer from heteroskedasticity. Our (unweighted) hedonic regression model has the following form:

$$\begin{aligned} \ln rent_i = \alpha + \sum_{r=1}^{366} \beta_{0r}\ region_i^r + \beta_1 \ln sqm_i + \beta_2\ priv_i + \beta_3 \ln len_i \\ + \beta_4\ priv_i\ \ln len_i + \sum_{e=1}^{2} \beta_{5e}\ equ_{ei} + \sum_{a=1}^{3} \beta_{6a}\ area_{ai} + \beta_7\ kit_i + u_i \ . \end{aligned} \tag{1}$$

The error term u_i is assumed to be normally distributed with expected value 0 and variance σ^2.[16]

The hedonic regression equation (1) with its regional dummy variables $region_i^r$ ensures that each of the 366 regions has its own intercept, $\alpha + \beta_{0r}$. To avoid perfect multicollinearity, we impose the restriction that $\sum_{r=1}^{366} \widehat{\beta}_{0r} = 0$, we drop the parameter β_{01} from the regression, and we compute its estimated value from $\widehat{\beta}_{01} = -\sum_{r=2}^{366} \widehat{\beta}_{0r}$. As a consequence, α represents the average regional intercept while the parameters β_{0r} of the 366 regions show the percentage deviation from that average. We expect that the estimated regional intercepts, $\widehat{\alpha} + \widehat{\beta}_{0r}$, are larger in high-income regions (e.g., Frankfurt or Munich) than in low-income regions (e.g., Blien *et al.*, 2009).

The elasticity β_1 indicates the percentage change of the rent in response to a 1% increase in the living space. We expect that this percentage change is strongly positive, that is, $\widehat{\beta}_1 > 0$.[17]

The dummy variable $priv_i$ has the value 0, if the landlord is from the private sector, and the value 1 otherwise. While private housing aims more or less at profit maximization, social housing tries to ensure that also low-income households can find an affordable flat. For this purpose, governments subsidize flats such that tenants pay a lower rent than on the private market. Therefore, we expect that $\widehat{\beta}_2 < 0$.

In a hedonic regression analysis of West German rent data of the German Socio-Economic Panel, Hoffmann and Kurz (2002, p. 22) showed that in private housing "rents vary inversely with the length of occupancy", but that in social housing this relationship does not hold.[18] We expect similar results, that is, $\widehat{\beta}_3 < 0$ and $\widehat{\beta}_4 > 0$.[19]

The two dummy variables equ_{ei}, $e = 1, 2$, classify the flats' quality of equipment into the three classes "low", "medium", and "high". A medium quality level is the reference. Therefore, the dummy variable equ_{1i} has the value 1, if the equipment level is low, and the value 0 otherwise. Correspondingly, $equ_{2i} = 1$, if the equipment level is high, and $equ_{i2} = 0$ otherwise. We expect that $\widehat{\beta}_{51} < 0$ and $\widehat{\beta}_{52} > 0$.

The quality of the neighborhood can be "low", "medium", "high", or "very high". These four levels are represented by the three dummy variables $area_{ai}$, $a = 1, 2, 3$, with $area_{ai} = 1$, if the flat is located in a neighborhood of quality a, and $area_{ai} = 0$ otherwise. A medium quality level is the reference. Therefore, $area_{1i} = 1$ indicates a low quality, $area_{2i} = 1$ indicates a high quality, and $area_{3i} = 1$ indicates a very high quality. As a consequence, we expect that $\widehat{\beta}_{61} < 0$ and $\widehat{\beta}_{63} > \widehat{\beta}_{62} > 0$.

16 In Appendix A.1 it is shown that the predicted rent, $\widehat{\ln rent^r}$, is not affected when in (1) instead of $\ln(rent_i)$ the endogenous variable $\ln(rent_i/sqm_i)$ is used.

17 It is conceivable that the percentage change is larger in urban areas than in rural areas, because in urban areas space is scarcer, and therefore, has a more dominant impact on the rent level than the length of tenancy, say (e.g., Tabuchi, 2001). This could be captured by including interaction terms between the living space and the region. However, we refrain from including such interaction terms to avoid overfitting, in particular for those regions with a relatively scarce number of observations.

18 A theoretical explanation for the negative relationship is given by Schlicht (1983).

19 In the international context, empirical studies show ambiguous results. Rondinelli and Veronese (2011) use rent data of the Household Consumption Expenditure budget survey and the Survey of Italian Household Income and Wealth for the years 1998 and 2006 and find strong evidence for a length of tenancy discount for Italy. In contrast, Barker (2003) shows for 102 apartment complexes from metropolitan areas in the United States "that the length-of-residence discounts are less common than discounts on the first month's rent for new tenants".

The dummy variable kit_i takes the value 1, if the flat has no built-in kitchen, and it takes the value 0 otherwise. Accordingly, we expect that $\widehat{\beta}_7 < 0$.

Although our hedonic regression equation (1) includes many important characteristics, some potentially relevant ones are missing. For example, Diewert (2013, p. 25) mentions the price determining effect of the age of the building.[20] A flat's energy consumption and rent are expected to be inversely related as a higher energy consumption implies higher heating cost for the tenant. However, these information are missing in our rent data. By contrast, information on the availability of a balcony and a garage are available. These variables, however, are pooled with information on the availability of a terrace and a parking lot, respectively. This pooling may be a reason why the variables show no significant impact on the rent.

To compute each region's rent level, we define a reference flat and compile for each region the logarithm of the rent that, according to our hedonic regression, must be paid for this reference flat.[21] Our reference flat is privately financed and it has a built-in kitchen. The quality of the equipment and the residential area are classified as medium. Additionally, we assume for the reference flat a living space of 65 sqm and a length of tenancy of 7 years. Both values nearly coincide with the respective median of all flats in the rent data. For each of the 366 regions included in the hedonic regression, we are able to compute the predicted logarithmic rent that must be paid for the reference flat. We denote this estimate by $\widehat{\ln rent^r}$.

As pointed out before, the rent data of 15 other regions were incomplete. For these regions, we merely know the rent and the size of the flats. Therefore, we do not include these regions in the hedonic regression. Instead, we calculate the region's average rent per square meter as a simple geometric mean and we multiply this number by 65, the size of the reference flat:

$$\overline{rent}^r = \prod_{i=1}^{N^r} \left(\frac{rent_i^r}{sqm_i^r} \right)^{1/N^r} \cdot 65 \quad \text{for } r = 367, \ldots, 381 \;, \tag{2}$$

where N^r is the number of observed flats in region r.[22] We combine the predicted loga-

[20] We are able to differentiate between flats built before and since 1949 in the rent data. However, this scarce differentiation showed no significant impact on the flat's rent. An explanation for this result is given by Dübel and Iden (2008, p. 20) who find for Germany some U-shaped relation between the flats' age and the paid rent: higher rents for extremely old and new buildings and a lower rent for flats with an age between these two extremes. This so-called "vintage effect" is also found by Mundt and Wagner (2017, p. 39) for Austria.

[21] Clearly, as no interaction terms between the regional dummy variables and the other variables are included in our hedonic regression (1), we could directly use the estimated regional rent levels $\widehat{\beta}_{0r}$. Our "reference flat"-approach, however, yields the same rent levels and, in addition, offers some more flexibility as shown in the following.

[22] Alternatively, we computed the relatives between the average rent levels per square meter in regions $r = 367, \ldots, 381$ and the overall average rent level per square meter of all 381 regions. The logarithm of these relatives then served as a proxy for the coefficient $\widehat{\beta}_{0r}$. In combination with the other estimated coefficients of Equation (1), we predicted a logarithmic rent level for regions $r = 367, \ldots, 381$. However, the differences to (2) were negligible so that we decided to choose the more straightforward approach as outlined above.

rithmic rents (from the hedonic regression) and the logarithms of the average rents to

$$\ln rent^r = \begin{cases} \widehat{\ln rent}^r & \text{for } r = 1, \ldots, 366 \\ \ln \overline{rent}^r & \text{for } r = 367, \ldots, 381 \end{cases} .$$

Therefore, the normalized logarithmic rent levels are

$$\widehat{\ln P^r_{\text{rent}}} = \ln rent^r - \ln rent^1, \quad \text{for } r = 1, \ldots, 381 . \tag{3}$$

All regional rent levels are combined in the vector $\widehat{\ln P_{\text{rent}}} = \left(\widehat{\ln P^1_{\text{rent}}} \ \ldots \ \widehat{\ln P^{402}_{\text{rent}}} \right)$, with 21 values missing. By definition, $\widehat{\ln P^1_{\text{rent}}} = 0$. This vector represents the five basic headings covered by the rent data set of the Federal Statistical Office.

As pointed out in Section 3.2, we received from the BBSR a complementary data set. It shows the regional logarithmic rent levels, $\ln \widetilde{rent}^r$, related to tenant changeovers in existing buildings and newly completed buildings. The normalized logarithmic rent levels

$$\ln \widetilde{P}^r_{\text{rent}} = \ln \widetilde{rent}^r - \ln \widetilde{rent}^1, \quad \text{for } r = 1, \ldots, 402 , \tag{4}$$

are combined in the vector $\ln \widetilde{P}_{\text{rent}} = \left(\ln \widetilde{P}^1_{\text{rent}} \ \ldots \ \ln \widetilde{P}^{402}_{\text{rent}} \right)$. By definition, the rent in the reference region $r = 1$ is $\ln \widetilde{P}^1_{\text{rent}} = 0$.

The rent level vectors $\widehat{\ln P_{\text{rent}}}$ and $\ln \widetilde{P}_{\text{rent}}$ complement the 645 vectors $\widehat{\ln P_b}$ that are estimated from the price data set. This estimation is described in the following sections.

4.2 Unweighted CPD Regression of Prices Relating to the Same Basic Heading and Outlet Type

Each observation of our price data set comprises the product's price, the region in which this price was recorded, and some additional characteristics. These additional characteristics allow us to identify those observations that relate to identical products. Identity of products requires not only conformable product characteristics, but also an identical outlet type (e.g., supermarket). This is our PMO precept.

Suppose, for example, that we have collected *supermarket* prices of different *rice* products i ($i = 1, \ldots, N$) in different regions r ($r = 1, \ldots, R$), but that not all of the N rice products have been observed in all R regions. The CPD method introduced by Summers (1973, p. 10-11) assumes that each observed price, $price^r_i$, can be obtained by multiplying region r's overall price level $\tilde{P}^r$ by product i's general value $\tilde{\pi}_i$, and by a log-normally distributed random variable ε^r_i:

$$price^r_i = \tilde{P}^r \tilde{\pi}_i \varepsilon^r_i . \tag{5}$$

This equation can be transformed into a linear regression model. For reasons that become clear shortly, we define

$$P^r = \tilde{P}^r / k \qquad \text{and} \qquad \pi_i = k \tilde{\pi}_i , \tag{6}$$

with k being some constant. Then, Equation (5) can be rewritten as

$$\ln price_i^r = \ln P^r + \ln \pi_i + u_i^r \ , \tag{7}$$

with $u_i^r = \ln \varepsilon_i^r \sim N(0, \sigma^2)$. For each region s, a dummy variable $region^s$ can be defined such that $region^s = 1$ when $r = s$ and $region^s = 0$ otherwise. Similarly, for every product j, a dummy variable $product_j$ can be defined such that $product_j = 1$ when $i = j$ and $product_j = 0$ otherwise. Defining $\alpha^r = \ln P^r$ and $\beta_i = \ln \pi_i$, we can express Equation (7) also in the following form:

$$\ln price_i^r = \sum_{s=1}^{R} \alpha^s \ region^s + \sum_{j=1}^{N} \beta_j \ product_j + u_i^r \ . \tag{8}$$

Equation (8) can be viewed as a linear regression model with two fixed effects, albeit one suffering from perfect multicollinearity.

We can avoid the perfect multicollinearity by specifying k such that one of the parameters α^r is exogenously fixed. As one possibility, one region can be defined as the reference region for the price levels of the other regions.[23] We choose region $r = 1$ as the reference region: $k = \tilde{P}^1$. Definition (6) then yields $P^1 = \tilde{P}^1/\tilde{P}^1 = 1$ which gives $\ln P^1 = \alpha^1 = 0$. As a consequence, $\alpha^1 region^1 = 0$ for all r $(r = 1, \ldots, R)$. Therefore, the term $\alpha^1 region^1$ can be dropped from Equation (8) and perfect multicollinearity is removed:

$$\ln price_i^r = \sum_{s=2}^{R} \alpha^s \ region^s + \sum_{j=1}^{N} \beta_j \ product_j + u_i^r \ . \tag{9}$$

Ordinary least squares (OLS) regression of the log-linear model (9) gives the $(R-1)$ coefficients $\widehat{\alpha}^2, \ldots, \widehat{\alpha}^R$. Finally, the formula $\widehat{\ln P^r} = \widehat{\alpha}^r$ yields estimates of the $(R-1)$ regional logarithmic price levels $\ln P^2, \ldots, \ln P^R$. For the reference region, $r = 1$, we know that $\ln P^1 = 0$.

In our price data we have $B = 645$ different basic headings. Before we conduct the CPD regressions described above, we split the price data set of each basic heading b $(b = 1, \ldots, B)$ into L_b price data sets each of which relates to a different outlet type l. For each of these price data sets we conduct a separate CPD regression. This distinction between outlet types is necessary, because we will show in Section 5.2 that the variation of the prices across regions is not uniform across outlet types.[24] For example, Table 4 contains a price data set related to the basic heading b = rice. Since only two different outlet types occur, one may split that price data set into one relating to the outlet type discount stores and a second one relating to the outlet type supermarket, that is, $L_{\text{rice}} = 2$.

The price matrix of Table 4, however, exhibits a peculiarity leading to a modified splitting procedure. In the terminology of the World Bank (2013, p. 98) the price matrix is "not connected" , because regions A, B, and C form one block of regions and regions D and E form a second block of regions and price comparisons between the two blocks

23 The choice of the reference region does not affect the estimated ratios $\exp(\widehat{\alpha}^r - \widehat{\alpha}^s)$.

24 Alternatively, one could do without the split and, instead, add to the regression (9) a dummy variable that controls for the outlet type, and also interaction terms that control for the dependencies between outlet types and regional dummies.

are not possible. The standard approach to deal with such price matrices is to exclude the price observations related to one of the two blocks or, even more radical, to exclude the complete basic heading. Clearly, both variants lead to a loss of valuable information. Therefore, we introduce a different approach. To extract the maximum information from Table 4, we would assign Products 1 and 2 to outlet type "discount store (regions A, B, C)" , Products 3 and 4 to outlet type "discount store (regions D, E)" , and Products 5 and 6 to outlet type "supermarket" . For each of these $L_{\text{rice}} = 3$ data sets we would conduct a separate CPD-regression.

The number of resulting outlet types, L_b, differs between the basic headings of our price data set. We conduct within each basic heading L_b separate CPD regressions. Each of them aggregates all price observations relating to the same basic heading, b, and the same outlet type, l, into a vector of $R = 402$ estimated regional logarithmic price levels: $\widehat{\ln P_{bl}} = \left(\widehat{\ln P_{bl}^1} \ \ldots \ \widehat{\ln P_{bl}^{402}}\right)$. Since no expenditure weights of individual products are available, these CPD estimations are based upon unweighted OLS regressions. Due to the gaps in our price data set, some of the $R = 402$ regional logarithmic price levels, $\ln P_{bl}^r$, cannot be estimated such that the corresponding vector, $\widehat{\ln P_{bl}}$, is incomplete.[25]

4.3 Weighted CPD Regression of Prices Relating to the Same Basic Heading

The next task is to aggregate the L_b estimated vectors $\widehat{\ln P_{bl}} = \left(\widehat{\ln P_{bl}^1} \ \ldots \ \widehat{\ln P_{bl}^{402}}\right)$ relating to basic heading b, into the basic heading's vector of estimated regional logarithmic price levels, $\widehat{\ln P_b} = \left(\widehat{\ln P_b^1} \ \ldots \ \widehat{\ln P_b^{402}}\right)$. Therefore, the product dummy variables of regression model (9), $product_j$, must be replaced by outlet dummy variables, $outlet_m$, with $outlet_m = 1$ when $l = m$ and $outlet_m = 0$ otherwise. For the aggregation of the vectors $\widehat{\ln P_{bl}}$ into the vector $\widehat{\ln P_b}$, expenditure weights relating to outlet types are available.

For example, according to our price data, rice is sold in three different outlet types, namely in hypermarkets, supermarkets, and discount stores. Accordingly, in Section 4.2 we computed three vectors $\widehat{\ln P_{bl}}$, with b = rice and l = (hypermarket, supermarket, discount store). These three vectors must be aggregated into the vector $\widehat{\ln P_b}$. Rice expenditures, however, vary across the three outlet types. Therefore, we apply a weighted variant of the CPD method that was proposed by Rao (2001, p. 15), Rao (2005, p. 575), Hajargasht and Rao (2010, p. S39), and Diewert (2005, pp. 562-563). This weighted approach aggregates the logarithmic price levels $\widehat{\ln P_{bl}}$ into $\widehat{\ln P_b}$ with respect to the relative importance of outlet type l within basic heading b.

To simplify the notation, we drop the index b from the variables, that is, we write $\ln P^r$, $\ln P_l^r$, and L instead of $\ln P_b^r$, $\ln P_{bl}^r$, and L_b. Then, the weighted regression model

[25] This might lead to situations where the reference region $r = 1$ is not available in the price data set. We then use another reference region in our CPD regression. Thus, $\ln P_{bl}^1 = 0$ does not necessarily apply.

can be stated in the following form:

$$\sqrt{w_l}\ \widehat{\ln P_l^r} = \sqrt{w_l}\ \sum_{s=2}^{R} \alpha^s\ region^s + \sqrt{w_l}\ \sum_{m=1}^{L} \gamma_m\ outlet_m +\ u_l^r\ , \tag{10}$$

where w_l is the explicit weight given to outlet type l. The weights w_l should reflect the relative importance that the researcher assigns to outlet type l in the estimation of the logarithmic price levels $\ln P^r$ relating to basic heading b.

As mentioned in Section 3.3, we know for more than two thirds of the basic headings the expenditure shares, $\tilde{w}_l$, that the consumers of basic heading b transact in outlet type l. Therefore, the impact of each outlet type l on the coefficients $\widehat{\alpha}^2, \ldots, \widehat{\alpha}^R$ should reflect these expenditure shares.[26] Only if for each outlet type, l, the same number of estimated logarithmic price levels $\widehat{\ln P_l^r}$ were available, choosing $w_l = \tilde{w}_l$ would yield the desired weighting. In our data set, however, the number of regions for which we were able to derive estimates, $\widehat{\ln P_l^r}$, differs between outlet types. These differences add to the *explicit* weights, w_l, an *implicit* weighting of outlet types. The larger an outlet type's number of available estimates, $\widehat{\ln P_l^r}$, the larger is the impact of that outlet type on the estimation of the coefficients $\widehat{\alpha}^2, \ldots, \widehat{\alpha}^R$. More specifically, problems would arise in basic headings with "internet and mail-order business", because this is the only outlet type without gaps. Therefore, it would receive an unduly strong impact on the estimation. To correct for this unwarranted implicit weighting, one can use in regression model (10) the adjusted weights $w_l = \tilde{w}_l / o_l$, where o_l is the share of observations of outlet type l within the basic heading.[27] This type of adjustment ensures that each outlet type receives the "overall weight" that it would receive, if in regression model (10) each outlet type entered with the same number of observations and simultaneously the explicit weights $w_l = \tilde{w}_l$ were applied.[28]

An OLS estimation of regression model (10) yields the coefficients $\widehat{\alpha}^2, \ldots, \widehat{\alpha}^R$. As in regression model (9), these coefficients are estimates of $\ln P^2, \ldots, \ln P^R$. Therefore, $\widehat{\ln P^r} = \widehat{\alpha}^r$.

For each basic heading b ($b = 1, \ldots, B$) we conduct a separate weighted CPD regression (10) and compute from the estimated coefficients the regional logarithmic price levels $\widehat{\ln P_b^r}$. Adding to each vector the logarithmic price level of the reference region $r = 1$, we end up with $B = 645$ different vectors $\widehat{\ln P_b} = \left(\widehat{\ln P_b^1}\ \ldots\ \widehat{\ln P_b^{402}}\right)$. Again, some of the logarithmic regional price levels, $\ln P_b^r$, cannot be estimated such that the corresponding vector, $\widehat{\ln P_b}$, is incomplete.

[26] A justification for the use of expenditure shares can be found in Rao (2005, footnote 4, p. 575).

[27] Neither the weights o_l nor the weights w_l must add up to one.

[28] For 38% of the basic headings we do not have the consumption shares $\tilde{w}_l$. In these cases we do without any explicit weighting, that is, $w_l = 1$.

4.4 Weighted CPD Regressions for Goods, Services, and Housing

In Sections 4.2 and 4.3 we compiled for each basic heading covered by the price data set a vector of estimated logarithmic regional price levels, $\widehat{\ln P_b} = \left(\widehat{\ln P_b^1} \ \ldots \ \widehat{\ln P_b^{402}}\right)$, while in Section 4.1 we derived the vector $\widehat{\ln P_{\text{rent}}} = \left(\widehat{\ln P_{\text{rent}}^1} \ \ldots \ \widehat{\ln P_{\text{rent}}^{402}}\right)$ containing the estimated logarithms of the regional rent levels. These were complemented by the vector of logarithmic regional rent levels, $\ln \widetilde{P}_{\text{rent}} = \left(\ln \widetilde{P}_{\text{rent}}^1 \ \ldots \ \ln \widetilde{P}_{\text{rent}}^{402}\right)$, which we computed from the BBSR rent data. In the following, we refer to the two rent vectors, $\widehat{\ln P_{\text{rent}}}$ and $\ln \widetilde{P}_{\text{rent}}$, by the generic term "housing".

Since most of these 647 vectors are incomplete, their further aggregation into the overall regional price level vector, $\widehat{\ln P} = \left(\widehat{\ln P^1} \ \ldots \ \widehat{\ln P^{402}}\right)$, could be conducted by another weighted CPD regression of the type (10). We merely have to replace the outlet dummy variables of regression model (10), $outlet_m$, by basic heading dummy variables, $heading_h$, with $heading_h = 1$ when $b = h$ and $heading_h = 0$ otherwise:

$$\sqrt{w_b}\, \widehat{\ln P_b^r} = \sqrt{w_b} \sum_{s=2}^{R} \alpha^s \ region^s + \sqrt{w_b} \sum_{h=1}^{H} \delta_h \ heading_h + \ u_b^r \ , \tag{11}$$

where the weight w_b is the regionally uniform expenditure share of basic heading b. Hence, we aggregate the logarithmic price levels, $\widehat{\ln P_b^r}$, with respect to the expenditure share weight of basic heading b. The logarithm of the reference region's price level is, by definition, $\ln P^1 = 0$. The other logarithmic regional price levels are computed from $\widehat{\ln P^r} = \widehat{\alpha}^r$. This yields the vector $\widehat{\ln P} = \left(\widehat{\ln P^1} \ldots \widehat{\ln P^{402}}\right)$ with $\widehat{\ln P^1} = 0$.[29]

Regression equation (11) would imply that the values of the regional dummies are independent from the basic headings. As pointed out before, however, there is a widely held belief that housing costs vary more strongly across regions than the prices of services and that the latter vary more strongly than the prices of goods. In addition, most basic headings are represented by incomplete vectors. As a consequence, a CPD regression including all 647 basic headings is prone to bias. Therefore, we refrain from such an all-encompassing CPD regression and, instead, split the 647 basic headings into three separate segments: housing (2 basic headings, weight 20.99%), services (153 basic headings, weight 25.56%), and goods (492 basic headings, weight 53.45%). For each segment we conduct a separate CPD regression of the form (11).[30] We obtain the three complete vectors

[29] Referring to the analysis of Goldberger (1968), Kennedy (1981, p. 801) points out that the expected value of the estimator $\exp(\widehat{\alpha}^r)$ is not $\exp(\alpha^r)$, but $\exp(\alpha^r + 0.5\text{var}(\widehat{\alpha}^r))$. This implies that the values of $\exp(\alpha^r)$ and, therefore, the values of P^r should be estimated by $\exp(\widehat{\alpha}^r - 0.5\widehat{\text{var}}(\widehat{\alpha}^r))$ and not by $\exp(\widehat{\alpha}^r)$. In our regression, however, we cannot estimate the variances, $\widehat{\text{var}}(\widehat{\alpha}^r)$ in a reliable way. Therefore, we have to do without this adjustment.

[30] In the weighted CPD regression (10) we used modified expenditure weights such that each outlet type's impact on the overall result is proportional to its expenditure weight. In regression (11) we do without such modifications, because they would give more weight to fragmentary basic headings the estimated price levels of which tend to be less reliable than those of basic headings for which we have many observations.

$\widehat{\ln P}_{\text{housing}}$, $\widehat{\ln P}_{\text{services}}$, and $\widehat{\ln P}_{\text{goods}}$, with 402 transitive regional price levels, respectively. Their aggregation into the multilateral system of regional price index numbers is described in the next section.

4.5 Regional Price Index: Aggregation of Goods, Services, and Housing

Since the three vectors $\widehat{\ln P}_{\text{housing}}$, $\widehat{\ln P}_{\text{services}}$, and $\widehat{\ln P}_{\text{goods}}$ are complete, we compute for each region, r, the weighted arithmetic mean of its three logarithmic index values,

$$\widehat{\ln P^r} = w_{\text{housing}} \widehat{\ln P^r}_{\text{housing}} + w_{\text{services}} \widehat{\ln P^r}_{\text{services}} + w_{\text{goods}} \widehat{\ln P^r}_{\text{goods}} \ , \tag{12}$$

where w_{housing}, w_{services}, and w_{goods} are the expenditure share weights of the three segments. In Appendix A.2 it is shown that exactly the same estimates, $\widehat{\ln P^r}$, are obtained when we apply another weighted CPD regression or the GEKS approach where the underlying bilateral price index numbers are computed as weighted Jevons indices. This is due to the fact that the three vectors of regional price levels are complete and, in addition, the expenditure share weights are regionally uniform.

We re-normalize the logarithmic price level estimates in (12) so that our final multilateral system of regional index numbers is defined by:

$$\widehat{P^r} = \exp\left(\widehat{\ln P^r} - \ln P^{\text{Ger}}\right) \tag{13}$$

with $\ln P^{\text{Ger}} = \sum_{r=1}^{402} g^r \cdot \widehat{\ln P^r}$, where the weights, g^r, are defined as region r's population share. This normalization ensures that the weighted geometric mean of the normalized regional price levels, $\widehat{P^r}$, is $P^{\text{Ger}} = 1$. Therefore, $(\widehat{P^r} - 1)$ is the percentage deviation between the price level of region r and the weighted geometric mean of all regional price levels, P^{Ger}. Furthermore, the index numbers in (13) are transitive which ensures that our multilateral system of regional index numbers is internally consistent.

5 Empirical Results

The regional rent levels (housing) are presented in Section 5.1. Summary statistics and further analysis of the estimated price levels of goods and services are provided in Section 5.2. The overall price levels of the 402 German regions are presented in Section 5.3, along with a comparison of the regional price levels of goods, services, and housing. Furthermore, we examine the spatial correlation of the overall price levels. Finally, Section 5.4 examines whether the overall regional price levels change when simplified data editing procedures are employed.

5.1 Housing

As described in Section 4.1, we estimate from the CPI rent data of the Federal Statistical Office the logarithmic rent levels of 381 regions, $\ln rent^r$ $(r = 1, \dots, 381)$. For this purpose we use the hedonic regression equation (1). The corresponding regression statistics are presented in Table 7.

Dependent variable: $\ln(rent_i)$	
Intercept	2.621 (0.031) ***
$region_i^r$ = Frankfurt	0.556 (0.043) ***
$region_i^r$ = Munich	0.511 (0.012) ***
$\vdots$	$\vdots$
$region_i^r$ = Stuttgart	0.414 (0.013) ***
$\vdots$	$\vdots$
$region_i^r$ = Hamburg	0.301 (0.041) ***
$\vdots$	$\vdots$
$region_i^r$ = Cologne	0.282 (0.015) ***
$\vdots$	$\vdots$
$region_i^r$ = Dusseldorf	0.257 (0.017) ***
$\vdots$	$\vdots$
$region_i^r$ = Berlin	0.185 (0.008) ***
$\vdots$	$\vdots$
$region_i^r$ = Wunsiedel	−0.389 (0.055) ***
$\ln(sqm_i)$	0.846 (0.007) ***
$priv_i$ = social housing	−0.254 (0.040) ***
$\ln(len_i)$	−0.046 (0.001) ***
$\ln(len_i)\, priv_i$ = social housing	0.019 (0.005) ***
equ_{1i} = low	−0.039 (0.004) ***
equ_{2i} = high	0.107 (0.006) ***
$area_{1i}$ = low	−0.042 (0.006) ***
$area_{2i}$ = high	0.052 (0.004) ***
$area_{3i}$ = very high	0.141 (0.008) ***
kit_i = no	−0.044 (0.004) ***
Number of observations = 14624	
Residual standard error = 0.183	
Adjusted R^2 = 0.748	
F-statistic: 116.9 on 375 and 14248 degrees of freedom, p-value: $< 0\%$	
Significance level: *** $< 0.1\%$, ** $< 1\%$, * $< 5\%$	

Table 7: Estimated coefficients of hedonic regression model (1) with White's (1980) heteroskedasticity-robust standard errors in brackets. Regional fixed effects for variable $region_i^r$ in descending order from highest (r = Frankfurt) to lowest (r = Wunsiedel).

The regression's adjusted R^2 is 0.75. This indicates that our hedonic regression has a high explanatory power.[31] The estimated coefficients have the expected signs and are in

[31] Hoffmann and Kurz (2002, p. 18) report values that range from 0.53 to 0.65 for multiple cross-section analysis of West German rent data of the German Socio-Economic Panel. Kholodilin and Mense (2012, p. 17) use rent data of flats located in Berlin, collected from internet ads within the period

most cases significant. Some minor exceptions exist among the regional intercept terms, $\hat{\beta}_{0r}$ (variables $region_i^r$). This might be caused by a relatively small number of observations in the respective regions.

The estimated regional intercepts for the seven most populous cities in Germany are above the national average and range from $\hat{\beta}_{0,\text{Berlin}} = 0.185$ to $\hat{\beta}_{0,\text{Frankfurt}} = 0.556$. This implies that the rent level in the most expensive region, Frankfurt am Main, is 74.23% above the unweighted average rent level of all regions included in the hedonic regression.[32] The cheapest region, Wunsiedel im Fichtelgebirge, is 32.34% below that average.

The elasticity $\hat{\beta}_1 = 0.846$ (variable sqm_i) indicates that the rent increases by 0.846% in response to a 1% increase in the size of the flat. Correspondingly, $\hat{\beta}_3 = -0.046$ (variable len_i) indicates that in private housing the rent decreases by 0.046% in response to a 1% increase in the length of tenancy. Since $\hat{\beta}_3 + \hat{\beta}_4 = -0.027$, this length of tenancy discount holds true also for social housing, albeit weaker. The remaining coefficients show the expected effects on the logarithmic rent level. A markup has to be paid for a higher quality of equipment, $\widehat{\beta}_{5e}$ (variables equ_{ei}), and neighborhood, $\widehat{\beta}_{6a}$ (variables $area_{ai}$), as well as the availability of a built-in kitchen, $\widehat{\beta}_7$ (variable kit_i).

Since the CPI rent data provided by the Federal Statistical Office represent rents that are contractually paid by tenants, we denote them as *transactional* rents. By contrast, the rents $\ln \widetilde{rent}^r$ $(r = 1, \ldots, 402)$ provided by the BBSR stem from internet and newspaper ads and relate to tenant changeovers in existing buildings and newly completed buildings. Therefore, we denote them as *quoted* rents. Figure 6 (page 37 of Appendix A.3) shows two maps of Germany. They depict indices of transactional and quoted rent levels.

Kendall's τ $(= 0.59)$ documents a high similarity between the regional rankings of quoted and transactional rent levels. This similarity is confirmed by Figure 1. It reveals that the regional variation in the (logarithmic) quoted rents exceeds the variation in the (logarithmic) transactional rents. The dashed diagonal line indicates equality between quoted and transactional rents. The figure reveals that in almost all regions the quoted rent exceeds the transactional rent. As shown by the slope of the regression line, this markup increases with the transactional rent level. Transactional rents correspond to existing rental contracts, while the quoted rents correspond to rents that are free to renegotiate. Therefore, the increasing markup may indicate that in large cities (they have the largest transactional rents) the upward trend in rent levels during 2016 is stronger than in more rural regions. In Figure 1, the seven most populous German cities exhibit particularly large markups. This reinforces our decision to also include rents related to tenant changeovers in our regional price comparison.

2011 to 2012. The goodness of fit of their hedonic regression is 0.65. Behrmann and Goldhammer (2017, p. 22) use the 2017 rents of the German CPI data for twelve of the sixteen Federal States. They report a value of 0.77.

32 For the interpretation of coefficients relating to dummy variables some care is warranted, because the endogenous variable is logarithmic. Elaborating a comment by Halvorsen and Palmquist (1980, p. 474), Kennedy (1981, p. 801) recommends to compute the adjusted coefficient

$$\widehat{\beta}^* = e^{\widehat{\beta} - 0.5\mathrm{var}(\widehat{\beta})} - 1 \ .$$

This adjusted coefficient indicates the percentage change in the rent caused by a change of the dummy variable from the value 0 to the value 1.

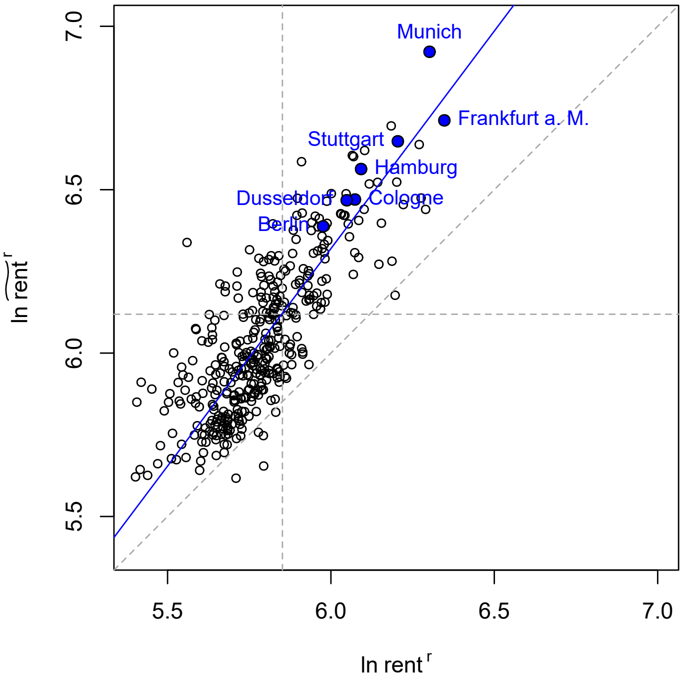

Figure 1: Transactional rents, $\ln rent^r$, for the 65 sqm reference flat (horizontal axis, in €) and quoted rents, $\ln \widetilde{rent}^r$, for a 65 sqm flat (vertical axis, in €). Population weighted averages as dashed horizontal and vertical lines, weighted least squares regression as solid blue line.

Following the weighting system of the Federal Statistical Office, we assign to rents an expenditure weight of 20.99%. We must decompose this weight into the weights assigned to the transactional rents, $\widehat{\ln P_{\text{rent}}}$, and to the quoted rents, $\ln \widetilde{P}_{\text{rent}}$. For this decomposition we draw on the average tenant changeover rate in Germany in 2016. This rate was nearly 9% (Techem, 2017). Therefore, we assign a weight of 1.89% (= 9/100 of 20.99%) to the quoted rents and a weight of 19.10% to the transactional rents. These two weights add up to 20.99%. Therefore, 79.01% is the total weight of all other basic headings. They represent the prices of all goods and services. To these we turn next.

5.2 Price Levels of Goods and Services

In our price data, we have $\sum_{b=1}^{645} L_b = 6{,}323$ independent data sets, each relating to a specific outlet type and a specific basic heading. As outlined in Section 4.2, we conduct for each of these data sets a separate unweighted CPD regression and obtain 6,323 price vectors, $\widehat{\ln \mathbf{P}_{bl}}$, with 111,540 estimated logarithmic price levels, $\widehat{\ln P_{bl}^r}$. Their distribution, separated by outlet type, is shown in Figure 2. For the purpose of this figure the logarithmic prices are re-normalized such that the average logarithmic price level of the respective vector is zero. Furthermore, we exclude the price levels of basic headings with regionally uniform prices (e.g., all price levels related to internet and mail-order business). We also ignore price vectors relating to department stores, because they represent less than 1% of all price levels.

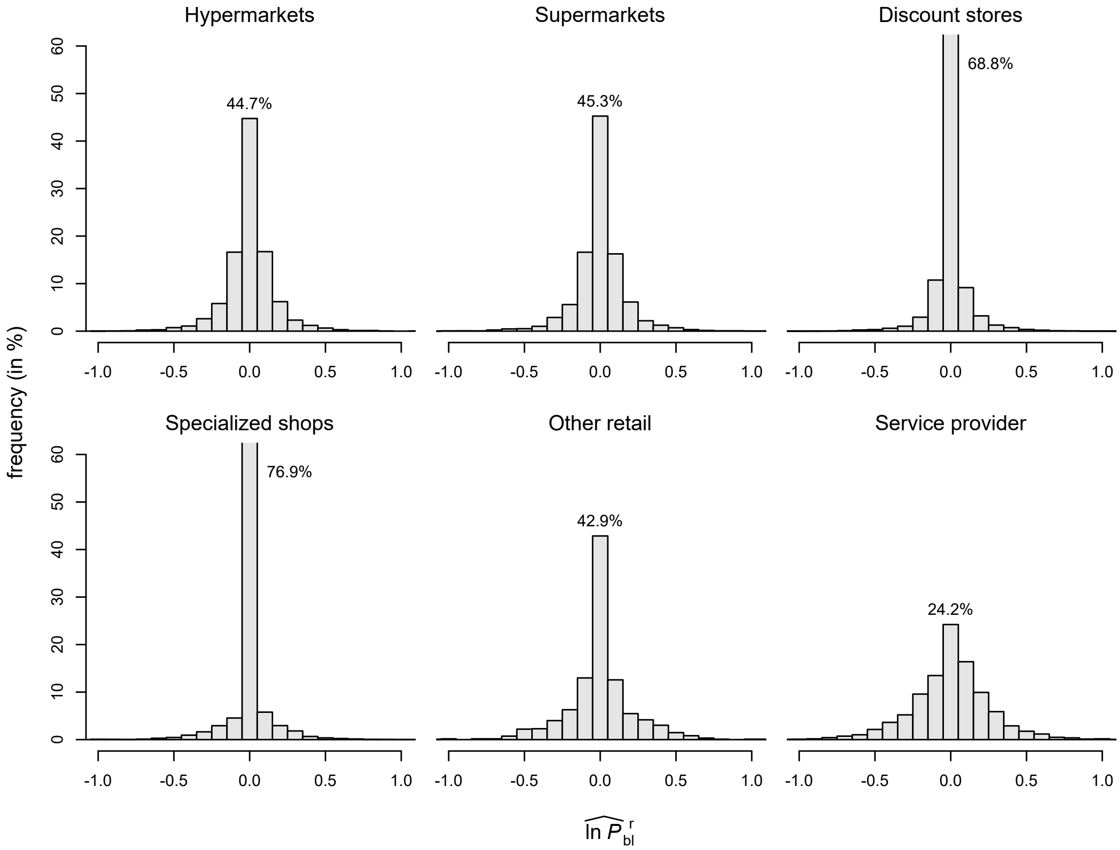

Figure 2: Distribution of estimated logarithmic price levels, $\widehat{\ln P^r_{bl}}$, by outlet type. Relative frequencies (in %) on vertical axis, classes of width 0.1 on horizontal axis.

Figure 2 shows that the majority of price levels is closely centered around zero. For specialized shops, 76.9% of the estimated price levels fall into the interval $[-0.05,\ 0.05]$, indicating deviations of less than 5% from the average price level. The corresponding number for discount stores is 68.8% while it drops to approximately 45% for hypermarkets, supermarkets, and other retail.

The smallest percentage (24.2%) arises for public and private service provider. This squares with the widely held belief that price levels tend to fluctuate stronger for services than for goods. For a deeper investigation of this belief one should compare the regional price dispersion of basic headings relating to services with those relating to goods. The compilation of these vectors was described in Section 4.3. Following that procedure, we obtain 598 vectors $\widehat{\ln P_b}$. 47 other basic headings exhibit a uniform price in all regions (e.g., books and cigarettes). Their combined weight is 12.25%. Nine other basic headings with a combined expenditure weight of 0.80% are so fragmentary that their corresponding vectors are empty.[33]

Figure 3 depicts the regional price indices of four basic headings: nuts and raisins (representing division 01: food and non-alcoholic beverages), women's shoes (division 03: clothing and footwear), cup of coffee, tea, hot chocolate (division 11: restaurants and

[33] We assign their weight proportionally to the remaining 636 basic headings of the price data set, such that the aggregated weight of these 636 basic headings is still 79.01%.

hotels), and inpatient care (division 12: miscellaneous goods and services). The regions are ordered by their quoted rent levels. For each of the 402 regions, the solid line shows the level of quoted rents, while the points represent the basic heading's price level.

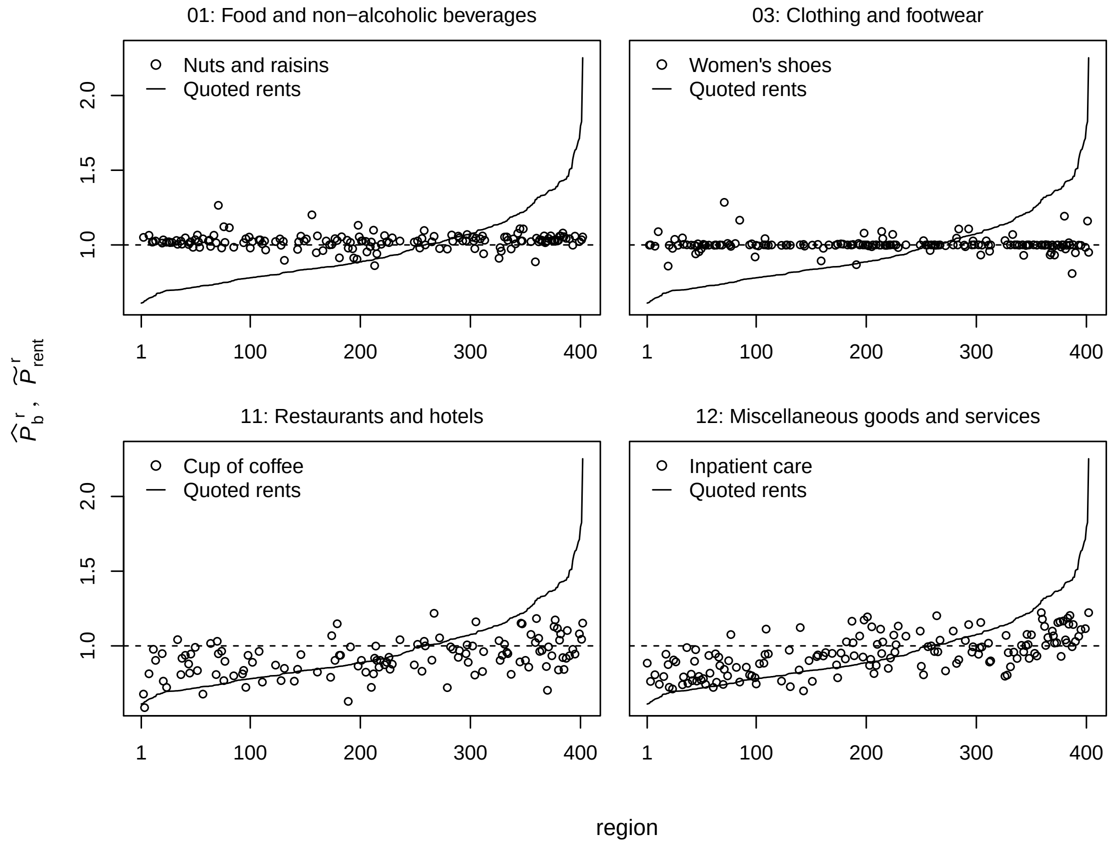

Figure 3: Estimated price levels $\widehat{P_b^r}$ for basic headings b = (nuts and raisins, women's shoes, cup of coffee, and inpatient care), ordered by quoted rent levels $\widetilde{P}^r_{\text{rent}}$ from lowest (region $r = 1$) to highest (region $r = 402$), respectively.

The figure indicates that the regional price levels of services (bottom panels of Figure 3) fluctuate more than those of goods (top panels of Figure 3). Taking into account all basic headings, this observation remains stable; the coefficient of variation for services is 0.28 and 0.12 for goods.[34]

More importantly, Figure 3 reveals that the regional price levels for the basic headings representing goods fluctuate closely around the horizontal axis, implying that they are more or less uncorrelated with the quoted rent levels (nuts and raisins: $\tau = 0.12$, women's shoes: $\tau = -0.03$). By contrast, the price levels of the basic headings representing services are positively correlated with the quoted rent levels (cup of coffee: $\tau = 0.30$, inpatient care: $\tau = 0.47$). The overall correlation between price levels of those basic headings relating to services and the quoted rent levels is $\tau = 0.13$, while it is $\tau = 0.03$ for basic headings representing goods.

[34] The classification of basic headings into goods (durables, semi-durables, non-durables) and services follows ILO, IMF, OECD, UNECE, Eurostat and The World Bank (2004, p. 465 ff.).

5.3 Overall Price Levels

As described in Sections 4.4 and 4.5, the regional price indices of the various basic headings are aggregated into the regional price indices of goods, services, and housing. Finally, these three price indices are aggregated to the overall regional price index. The latter are normalized by the population weighted average price level, $\ln P^{\text{Ger}}$. Table 8 contains summary statistics of the estimated price index numbers, $100 \cdot \widehat{P^r}$. By definition, the population weighted mean, $100 \cdot P^{\text{Ger}}$, is 100. If we omit the population weights, the (unweighted) mean drops to 98.37. This indicates that regions with larger populations tend to have higher price levels.

MIN	Q25	MEDIAN	MEAN	BASE	Q75	MAX	SD
90.40	95.33	97.92	98.37	100.00	100.67	114.90	4.09

Table 8: Summary statistics of estimated price index numbers, $100 \cdot \widehat{P^r}$, with the population weighted average as base (= 100).

The seven most populous German cities confirm this observation. The most expensive region is Munich. It's price level is 14.90% above the population weighted average. Frankfurt (= 11.50%), Stuttgart (= 9.81%), Cologne (= 7.90%), Dusseldorf (= 7.07%), Hamburg (= 6.70%), and Berlin (= 2.56%) also exhibit above-average price levels. The distribution is skewed to the right, indicating that strong deviations from the population weighted average more frequently occur in expensive regions than in inexpensive ones.

The overall price index numbers of the 402 German regions are shown in the left hand panel of Figure 4. We also decompose the overall price levels into housing (transactional and quoted rents), goods, and services. These price index numbers are shown in the other three panels of that figure.

The index numbers for goods vary only slightly. They range from 92.58 to 103.93. For services, this range enlarges to 89.07 to 121.35. By contrast, the housing index numbers show strong regional differences. They range from 63.67 to 166.01. Therefore, the overall price level is largely driven by housing.

It is well known that Luxembourg and Switzerland exhibit a particularly high cost of living. People working in the border regions of these countries try to avoid the high cost by shifting some of their expenditures to the bordering German regions, driving up the prices there. In Figure 4, this is not only visible for housing, but even for goods and services.

The left panel of Figure 4 also reveals that the high price levels found in the seven major cities spread out into their neighboring regions. Moran's $I = 0.58$ ($p < 0.01$) indicates positive spatial autocorrelation.[35] This positive spatial autocorrelation is mainly driven by housing ($I = 0.65,\ p < 0.01$) rather than by goods ($I = 0.18,\ p < 0.01$) or services ($I = 0.23,\ p < 0.01$).

[35] Moran's (1950) I measures the degree of spatial autocorrelation. We compute Moran's I based on a row-standardized approach, where each neighboring region receives a weight according to its population size.

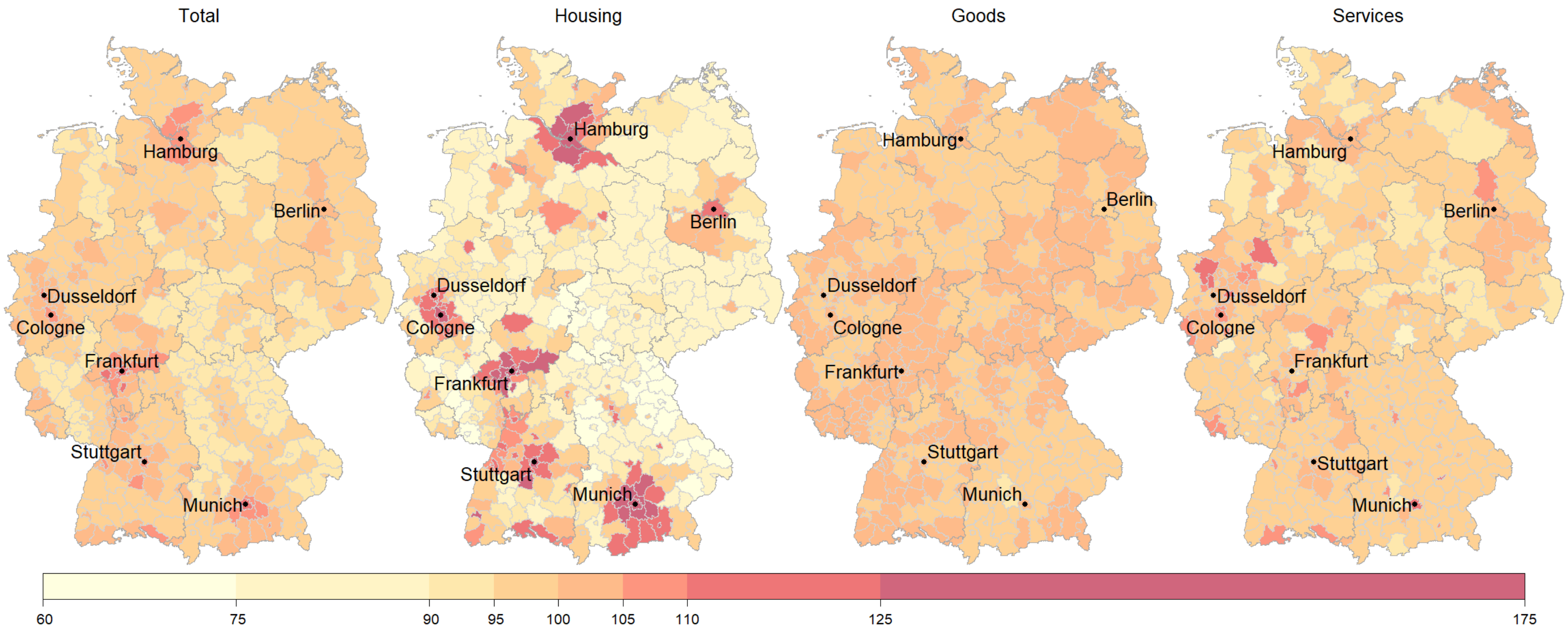

Figure 4: Regional price index $100 \cdot \widehat{P}^r$ (left panel), housing price index (left center panel), price index for goods (right center panel) and price index for services (right panel) normalized by population weighted average price level (= 100), respectively.

Figure 5 provides a more comprehensive picture of the spatial autocorrelation structure. It shows the relation between the estimated logarithmic price levels, $\widehat{\ln P^r}$, and the (local) Moran's I^r coefficients of the 402 regions. The u-shaped relation indicates positive spatial autocorrelation especially in those regions with price levels clearly above or clearly below the population weighted average, $\ln P^{\text{Ger}} = 0$, while regions with intermediate price levels exhibit very low spatial correlation. This implies that price levels change only gradually as one travels from inexpensive to expensive regions, or *vice versa*.

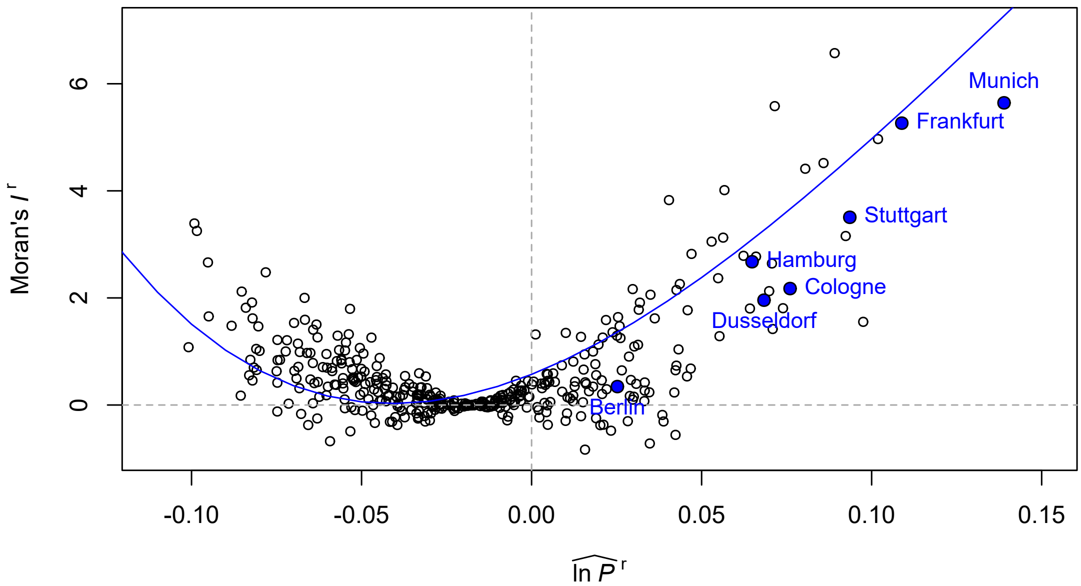

Figure 5: Estimated, logarithmic price levels, $\widehat{\ln P^r}$, (horizontal axis) and local Moran's I^r (vertical axis) of our 402 regions. Cubic least squares regression as solid blue line.

5.4 Simplified Compilation Procedures

In Section 3.1 we described the comprehensive editing of the price data. A major part of this editing is necessary to implement the PMO precept in our regional price level computations. The precept postulates that prices of products are comparable, only if the characteristics of the products coincide in every respect. Without extensive editing of the original price data few products would satisfy this condition (see our illustrative example in Tables 3 and 4).

For the compilation of the regional price levels we use a multi-stage CPD approach to ensure the highest possible accuracy. In Sections 4.2 to 4.5 we described the four stages of this approach in more detail. In the first stage, CPD regressions aggregate products relating to the same basic heading and outlet type. This yields for each basic heading several vectors of regional price levels, each vector relating to a different outlet type. If one ignored the PMO precept, the first stage could be skipped. In the second stage, the vectors relating to the same basic heading are aggregated. This yields for each basic heading a single vector of regional price levels. Finally, the vectors of the basic headings are aggregated into the price levels of housing, goods, and services and these into the overall price levels of the regions (summary statistics in Table 8).

The editing of the original price data is necessary to conduct the first stage of our multi-stage approach which, in turn, is necessary to adhere to the PMO precept. One may ask whether the resulting high degree of accuracy justifies the effort. Would a less rigorous CPD approach generate other regional price levels? Table 9 provides an answer. It presents the summary statistics of two alternative CPD approaches that weaken the PMO precept to different degrees. Both alternatives preserve stages three and four of the original multi-stage approach, but merge the first two stages into a single stage.

	MIN	Q25	MEDIAN	MEAN	BASE	Q75	MAX	SD
(i)	77.58	92.83	96.47	97.04	100.00	100.47	129.16	7.73
(ii)	79.90	94.54	97.40	98.03	100.00	101.08	115.39	4.96

Table 9: Summary statistics of estimated price index numbers, $100 \cdot \widehat{P^r}$, by degree of product definition within a basic heading: (i) none and (ii) outlet type. Population weighted average as base (= 100).

Variant (i) is the more extreme one. It treats all products within a basic heading as directly comparable, regardless of their qualitative characteristics and their outlet type. Therefore, the time consuming extensive data editing is no longer necessary. Tables 8 and 9 reveal that in Variant (i) the overall price levels of the regions fluctuate more noticeably around their population weighted average than with the PMO precept. The range of the overall price index is from 77.58 to 129.16, while the PMO precept generates price levels that range from 90.40 to 114.90. This is a considerable deviation. The correlation between the price levels of Variant (i) and the PMO based price levels is merely $\rho = 0.69$.

Somewhat more encouraging is Variant (ii). It considers only those products as directly comparable that belong to the same basic heading and are sold in the same outlet type. As in Variant (i), this eliminates the need for the extensive data editing. The range of regional price levels narrows to 79.90 and 115.39. The correlation between the price levels obtained in Variant (ii) and those derived from our PMO precept is $\rho = 0.90$.

In sum, a CPD approach that drops the product dummies and the outlet dummies and retains only the basic heading dummies and the regional dummies, generates very poor results. A CPD regression that drops only the product dummies but retains all other dummies performs far better, though a loss in accuracy remains.

6 Concluding Remarks

The main goal of this paper was to compile sub-national price levels for the 402 counties and cities in Germany. To this end, we introduced a multi-stage CPD approach that is based on the Perfect Matches Only (PMO) precept. This precept bans the assignment of seemingly similar products into groups of directly comparable products. Instead, the computation of regional price levels takes its information only from pairs of identical products. Applied to the German CPI data set, this rigorous approach ensures that the accuracy of the compilations is not impaired by artificially contaminated price information. Our study demonstrates that the regionalized structure of the German CPI data allows for the computation of an accurate regional price index. This index is also unique in its level of spatial disaggregation.

Our results reveal considerable price differentials across the 402 regions. The overall price level in the most expensive region, Munich, is about 27% higher than in the cheapest region. We find that these price differentials are mainly driven by housing. The most expensive region exceeds the cheapest one by 161%. For services the corresponding number is merely 36% and for goods 12%. We also show that the price levels of metropolitan areas tend to be higher than those of more rural areas. The seven most populous cities (Berlin, Hamburg, Munich, Cologne, Frankfurt, Stuttgart and Dusseldorf) exhibit price levels clearly above the German average. Furthermore, our findings reveal regional spillover effects. In the neighborhood of expensive regions the price levels tend to be higher than in the neighborhood of inexpensive regions and *vice versa*. This positive spatial autocorrelation can be mainly attributed to housing.

Our regional price index lays the groundwork for any investigation that requires real economic indicators at the sub-national level (e.g., income, wages, productivity, investment, and consumption). Neglecting the issue of regional price disparities produces misleading results. For example, the German Federal Government publishes a yearly report on the current status of cohesion between East and West Germany. In this report it compares the per capita gross domestic products as well as the labor productivities of the five Neue Länder (East Germany without Berlin) to the average of the ten West German States (BMWi, 2018, pp. 88-93). The report completely neglects that, on average, the price levels in the West are considerably higher than those in the East. Therefore, the numbers presented in the report overestimate the gap between the Neue Länder and West Germany. A second example is the measurement of life satisfaction. Deckers, Falk and Hannah (2016, p. 1339) demonstrate the relevance of regional price levels in this important field of research. Drawing on the regional price levels computed by Kawka (2010), they show that, for a given nominal income, life satisfaction falls by 0.1 units (satisfaction is measured on a scale from 0 to 10) as the regional price level increases by 10%. Poverty rates that neglect regional price levels can also be misleading. Such price levels are also indispensable for a regional indexation of wages, social security benefits and other contractual payments.

Our multi-stage CPD approach stands out because of its high degree of accuracy and flexibility. This ensures that it can be easily adopted to other regional price comparison projects based on CPI micro data. The results of our study show that the differentiation between outlets is of utmost importance for the reliability of a regional price index, while the implementation of the PMO precept provides further accuracy. Whether this additional gain in accuracy is worth the effort, depends on how the regional price index will be used. For a regional price index published by a national statistical institute any loss in accuracy is unacceptable, because such an index must be unassailable. For the purpose of economic research, however, a more pragmatic approach that significantly reduces the workload while maintaining a reasonable degree of accuracy might be worth considering.

Finally, it must be pointed out that the computation of regional price levels based on CPI micro data is still in its infancy. Certainly, future studies should examine alternative approaches to the compilation of regional price indices. Some of these alternatives were not realizable with our data set due to data confidentiality restrictions that prevent the linkage of our CPI micro data to "external" data sources, such as expenditure weights and BBSR rents. Notwithstanding these limitations, our study introduces a novel methodology that derives a regional price index from CPI micro data. In our view, this index is unique in terms of accuracy and regional disaggregation and, therefore, represents a useful reference for future projects in the field of regional price comparisons.

A Appendix

A.1 Dependent Variable in Logarithmic Hedonic Models

Consider the hedonic regression model

$$\ln(rent_i) = \alpha + \sum_{s=1}^{R} \beta_{0s} \; region_i^s + \beta_1 \ln(sqm_i) + \; u_i \;, \tag{14}$$

with regional dummy variables, $region_i^s$, and living space, sqm_i, as exogenous variables. To avoid perfect multicollinearity, we impose the restriction $\sum_{s=1}^{R} \widehat{\beta}_{0s} = 0$.

The estimator of the slope coefficient, $\widehat{\beta}_1$, is defined by

$$\widehat{\beta}_1 = \frac{\sum\limits_{i=1}^{N} \ln(sqm_i) \ln(rent_i) - \sum\limits_{r=1}^{R} \left(\frac{1}{N^r} \sum\limits_{i=1}^{N^r} \ln(sqm_i^r) \sum\limits_{i=1}^{N^r} \ln(rent_i^r) \right)}{\sum\limits_{i=1}^{N} \ln(sqm_i) \ln(sqm_i) - \sum\limits_{r=1}^{R} \left(\frac{1}{N^r} \sum\limits_{i=1}^{N^r} \ln(sqm_i^r) \sum\limits_{i=1}^{N^r} \ln(sqm_i^r) \right)} \;, \tag{15}$$

with N^r being the number of flats in region r, while $rent_i^r$ is the rent and sqm_i^r is the living space of flat i in region r. For simplicity, we derive the estimator for the regional intercepts, $\widehat{\alpha}_s$, albeit these are not included in equation (14).[36] The formula is given by

$$\widehat{\alpha}_s = \frac{1}{N^s} \sum_{i=1}^{N^s} \Big(\ln(rent_i^s) - \widehat{\beta}_1 \ln(sqm_i^s) \Big) \;, \tag{16}$$

so that, using basic rules for ordinary least squares regression with categorical variables,

$$\widehat{\alpha} = \frac{1}{R} \cdot \sum_{s=1}^{R} \widehat{\alpha}_s \quad \text{and} \quad \widehat{\beta}_{0s} = \widehat{\alpha}_s - \widehat{\alpha} \tag{17}$$

follows for each region $s = 1, \ldots, R$.

In case we use the logarithmic rent per sqm, $\ln(rent_i/sqm_i)$, as dependent variable, the hedonic regression model takes the following form:

$$\ln(rent_i/sqm_i) = \widetilde{\alpha} + \sum_{s=1}^{R} \widetilde{\beta}_{0s} \; region_i^s + \widetilde{\beta}_1 \ln(sqm_i) + \; u_i \;. \tag{18}$$

As the explanatory variables are unchanged, it is sufficient to substitute $\ln(rent_i)$ and

[36] The coefficient $\widehat{\alpha}_s$ is the intercept of region s in case the intercept α is removed from Equation (14).

$\ln(rent_i^r)$ in equation (15) by $\ln(rent_i/sqm_i)$ and $\ln(rent_i^r/sqm_i^r)$ in order to derive $\widehat{\widehat{\beta}}_1$. It can be easily shown that

$$\widehat{\widehat{\beta}}_1 = \widehat{\beta}_1 - 1 \ . \tag{19}$$

Using equations (16) and (19), the estimator for the regional intercept terms, $\widehat{\widehat{\alpha}}_s$, is defined by

$$\begin{aligned}
\widehat{\widehat{\alpha}}_s &= \frac{1}{N^s} \sum_{i=1}^{N^s} \left(\ln(rent_i^s/sqm_i^s) - \widehat{\widehat{\beta}}_1 \ln(sqm_i^s) \right) \\
&= \frac{1}{N^s} \sum_{i=1}^{N^s} \left(\ln(rent_i^s/sqm_i^s) - \left(\widehat{\beta}_1 - 1\right) \ln(sqm_i^s) \right) \\
&= \widehat{\alpha}_s \ ,
\end{aligned} \tag{20}$$

which implies that, using equation (17),

$$\widehat{\widehat{\alpha}} = \widehat{\alpha} \quad \text{and} \quad \widehat{\widehat{\beta}}_{0s} = \widehat{\beta}_{0s} \tag{21}$$

holds true for regions $s = 1, \ldots, R$. Equations (19) and (21) remain valid when further explanatory variables are included in the hedonic regression model. Only the underlying formulas for $\widehat{\beta}_1$ in (15) and $\widehat{\alpha}_s$ in (16) become more complex.

Interregional rent level comparisons usually estimate the regional rents for some reference flat. Let's suppose that the reference flat is located in region r, that is $s = r$, and exhibits a living space of $\bar{x}$ sqm. Using equation (14), the predicted logarithmic rent of this flat is then computed as

$$\ln\left(\widehat{rent}^r\right) = \widehat{\alpha} + \widehat{\beta}_{0r} + \widehat{\beta}_1 \ \ln(\bar{x}) \tag{22}$$

while the logarithmic rent per sqm of this flat, using equation (18), is predicted by

$$\ln\left(\widehat{rent}^r/\bar{x}\right) = \widehat{\widehat{\alpha}} + \widehat{\widehat{\beta}}_{0r} + \widehat{\widehat{\beta}}_1 \ \ln(\bar{x}) \ . \tag{23}$$

Using (19) and (21), equation (23) can be rewritten as

$$\begin{aligned}
\ln\left(\widehat{rent}^r\right) - \ln(\bar{x}) &= \widehat{\alpha} + \widehat{\beta}_{0r} + \left(\widehat{\beta}_1 - 1\right) \ \ln(\bar{x}) \\
\ln\left(\widehat{rent}^r\right) &= \widehat{\alpha} + \widehat{\beta}_{0r} + \widehat{\beta}_1 \ \ln(\bar{x})
\end{aligned} \tag{24}$$

which coincides with (22). Therefore, the predicted rent of the reference flat is independent of the choice between $\ln(rent_i^s/sqm_i^s)$ and $\ln(rent_i^s)$ as dependent variable in the hedonic regression model.

A.2 Weighted CPD and GEKS estimates with Complete Prices

Suppose that we have complete price and expenditure share information available for N products (or basic headings or outlets) in R regions. Moreover, it is assumed that the products have different expenditure share weights, but that these weights are uniform across regions. The corresponding weighted CPD regression model can be cast in general matrix notation. For that purpose we define the vector $\mathbf{y} = (\mathbf{y}_1 \dots \mathbf{y}_N)'$, with $\mathbf{y}_i = \left(\ln p_i^1 \dots \ln p_i^R\right)'$ for all products $i = 1, \dots, N$. Furthermore, the $(NR \times (R-1+N))$-matrix $\mathbf{X}$ comprises the $R-1$ dummy variables $region^r$ $(r = 2, \dots, R)$ and the N dummy variables $product_i$ $(i = 1, \dots, N)$. The weights w_i can be written into the diagonal $(N \times N)$-matrix $\widetilde{\mathbf{W}} = \operatorname{diag}(w_1 \dots w_N)$, where $\sum_{i=1}^N w_i = 1$. Defining $\widehat{\boldsymbol{\beta}}_{\text{CPD}} = (\widehat{\alpha}^2 \dots \widehat{\alpha}^R \quad \widehat{\beta}_1 \dots \widehat{\beta}_N)'$, the weighted least squares (WLS) estimator is defined by

$$\widehat{\boldsymbol{\beta}}_{\text{CPD}} = \left(\mathbf{X}'\mathbf{W}^{-1}\mathbf{X}\right)^{-1} \mathbf{X}'\mathbf{W}^{-1}\mathbf{y} \;, \tag{25}$$

where $\mathbf{W}^{-1} = \widetilde{\mathbf{W}} \otimes \mathbf{I}_R$ is a $(RN \times RN)$-matrix. With complete observations, it is easy to show that

$$\left(\mathbf{X}'\mathbf{W}^{-1}\mathbf{X}\right) = \begin{pmatrix} \mathbf{I}_{R-1} & \mathbf{J}_{(R-1)\times N}\widetilde{\mathbf{W}} \\ \widetilde{\mathbf{W}}\,\mathbf{J}_{N\times(R-1)} & R \cdot \widetilde{\mathbf{W}} \end{pmatrix} \;,$$

where $\mathbf{I}_{R-1}$ is an identity matrix of dimension $(R-1)\times(R-1)$ and $\mathbf{J}$ is a $((R-1)\times N)$-matrix consisting of elements that all have the value 1. Using computation rules on partitioned matrices, the inverse of $\left(\mathbf{X}'\mathbf{W}^{-1}\mathbf{X}\right)$ can be written as

$$\left(\mathbf{X}'\mathbf{W}^{-1}\mathbf{X}\right)^{-1} = \begin{pmatrix} \mathbf{I}_{R-1} + \mathbf{J}_{R-1} & -\mathbf{J}_{(R-1)\times N} \\ -\mathbf{J}_{N\times(R-1)} & \frac{1}{R} \cdot \left[\widetilde{\mathbf{W}}^{-1} + (R-1)\cdot \mathbf{J}_N\right] \end{pmatrix} \;. \tag{26}$$

Furthermore, it follows that

$$\left(\mathbf{X}'\mathbf{W}^{-1}\mathbf{y}\right) = \begin{pmatrix} \sum_{i=1}^N w_i \ln p_i^2 \\ \vdots \\ \sum_{i=1}^N w_i \ln p_i^R \\ w_1 \sum_{r=1}^R \ln p_1^r \\ \vdots \\ w_N \sum_{r=1}^R \ln p_N^r \end{pmatrix} \;. \tag{27}$$

Inserting (26) and (27) in (25) yields the $(R-1)$ regional price level estimates, $\widehat{\alpha}^r$, with

$$\widehat{\alpha}^r = \sum_{i=1}^N w_i \cdot \ln\left(p_i^r / p_i^1\right)$$

for all regions $r = 2, \dots, R$. The logarithmic price level of the base region $r = 1$ is, by definition, $\alpha^1 = 0$. Thus, $\widehat{\alpha}^r$ is defined as a weighted arithmetic average of the logarithmic price relatives between region r and the base region.

In general, the GEKS approach states that

$$\ln P^{1r} = \frac{1}{R} \cdot \sum_{s=1}^R \ln\left(\dot{P}^{1s} \cdot \dot{P}^{sr}\right) \;, \tag{28}$$

where $\dot{P}^{sr}$ is the bilateral price index number of region r compared to the base region s (e.g., Rao and Hajargasht, 2016, p. 416). If we compute the bilateral price index numbers as a weighted Jevons index, that is, $\dot{P}^{sr} = \prod_{i=1}^{N} \left(\frac{p_i^r}{p_i^s}\right)^{w_i}$, it can be easily shown that (28) simplifies to

$$\ln P^{1r} = \sum_{i=1}^{N} w_i \cdot \ln\left(p_i^r / p_i^1\right)$$

for regions $r = 2, \ldots, R$. Thus, the weighted CPD and GEKS-Jevons estimates for the regional price levels coincide in the case of complete prices and regionally uniform weights.

A.3 Regional Rent Index Numbers

Figure 6 shows the regional index numbers for transactional rents ($100 \cdot \widehat{P}_{\text{rent}}$, left panel) and quoted rents ($100 \cdot \tilde{P}^r_{\text{rent}}$, right panel). The index numbers stem from Equations (3) and (4). The reference region is Kassel city, respectively.[37] For transactional rents, the cheapest region lies 33% below the rent level of Kassel while the most expensive region, Frankfurt am Main, is 71% above. In comparison, the quoted rents are in minimum 39% below and in maximum 125% (Munich) above the reference region's rent level. Regions shaded in gray indicate the absence of index numbers.

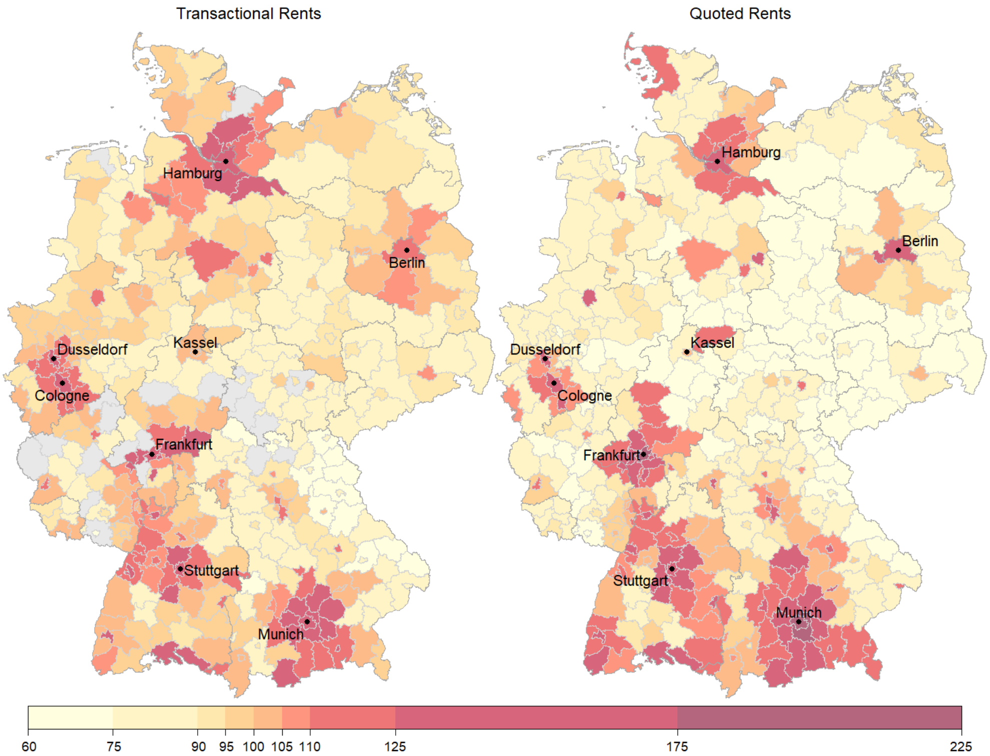

Figure 6: Regional index numbers for transactional rents (left panel) and quoted rents (right panel), normalized by the rent level of Kassel city, respectively. Gray shaded regions indicate the absence of index numbers.

[37] The choice of Kassel city as reference region is grounded on the fact that Kassel's rent level is close to the population weighted average rent level for both transactional and quoted rents. In addition, Kassel is almost the geographical center of Germany.

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