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Assessing the uncertainty in central banks' inflation outlooks

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Non-technical summary

Research Question

Many central banks publish their macroeconomic forecasts together with measures of the uncertainty surrounding these forecasts. In principle, these uncertainty measures are liable to change with every new forecast. If, for instance, the current macroeconomic environment is assessed as being more uncertain than when the previous forecast was made, the measure of forecast uncertainty reported with the current forecast may be higher as a result. Similarly, surveys of professional forecasters often ask participants to state the level of current forecast uncertainty. However, recent research indicates that these values can be subject to systematical biases. Furthermore, the changes in the reported forecast uncertainty observed over time do not appear to be correlated with the changes in the size of the corresponding forecast errors observed ex post. In our paper, we investigate whether the forecast uncertainty published by central banks for their inflation forecasts suffers from similar problems.

Contribution

We examine the quality of inflation forecast uncertainty data provided by several central banks using the same methods as those employed to analyse survey-based measures of forecast uncertainty. We focus on data of the Bank of England, the Banco Central do Brasil (the Brazilian central bank), the Magyar Nemzeti Bank (the Hungarian central bank) and the Sveriges Riksbank (the Swedish central bank) because they are particularly suited, on account of their nature and scope, for this purpose. Additional methods we apply in our paper enable us to draw more in-depth conclusions about the quality of the measures published by the central banks.

Results

Much like the survey-based measures of forecast uncertainty, we find that the central bank data we examine in our paper exhibit certain biases. The statistical evidence for these biases is not particularly strong, however. Our paper also shows that a change in forecast uncertainty reported by central banks tends to be a reliable indicator of a corresponding change in the size of future forecast errors. We conclude that the inflation forecast uncertainty reported by central banks appears to be more reliable than that recorded by surveys of professional forecasters.

Nichttechnische Zusammenfassung

Fragestellung

Viele Zentralbanken machen bei ihren makroökonomischen Prognosen auch Angaben über die mit diesen Prognosen verbundene Unsicherheit. Grundsätzlich können sich diese Angaben mit jeder neuen Prognose ändern. Wird beispielsweise das aktuelle makroökonomische Umfeld im Vergleich zum vorherigen Prognosezeitpunkt als unsicherer eingeschätzt, so kann bei der aktuellen Prognose eine entsprechend höhere Prognoseunsicherheit ausgewiesen werden. Auch bei Umfragen unter professionellen Wirtschaftsprognostikerinnen und -prognostikern wird häufig die Höhe der aktuellen Prognoseunsicherheit abgefragt. Jüngere Forschungsergebnisse deuten allerdings darauf hin, dass diese Werte nicht selten systematische Verzerrungen aufweisen. Darüber hinaus scheinen die im Zeitverlauf beobachteten Veränderungen der angegebenen Prognoseunsicherheit in keinem Zusammenhang zu den Veränderungen des Ausmaßes der entsprechenden, im Nachhinein beobachteten Prognosefehler zu stehen. In unserer Arbeit untersuchen wir, ob die von Zentralbanken für die Inflation veröffentlichte Prognoseunsicherheit mit ähnlichen Problemen behaftet ist.

Beitrag

Wir überprüfen die Güte von Daten zur Inflationsprognoseunsicherheit verschiedener Zentralbanken mit denselben Methoden, welche auch zur Analyse von umfragebasierten Angaben zur Prognoseunsicherheit verwendet werden. Wir konzentrieren uns dabei auf die Daten der Zentralbanken des Vereinigten Königreichs, Brasiliens, Ungarns und Schwedens, da sie sich in Art und Umfang in besonderer Weise für diesen Zweck eignen. Zusätzliche Verfahren, die in unserer Arbeit benutzt werden, erlauben tiefer gehende Aufschlüsse über die Qualität der von den Zentralbanken veröffentlichten Angaben.

Ergebnisse

Ähnlich wie umfragebasierte Angaben zur Prognoseunsicherheit weisen auch die untersuchten Daten der Zentralbanken gewisse Verzerrungsmuster auf. Statistisch sind diese Verzerrungen jedoch kaum belegbar. Zudem zeigt sich, dass eine Veränderung der angegebenen Prognoseunsicherheit bei Zentralbanken oft in verlässlicher Weise auf eine entsprechende Veränderung des Ausmaßes künftiger Prognosefehler schließen lässt. Die von Zentralbanken für die Inflation angegebene Prognoseunsicherheit scheint also eine verlässlichere Information darzustellen als jene, die in Umfragen unter professionellen Wirtschaftsprognostikerinnen und -prognostikern erfasst wird.

Bundesbank Discussion Paper No 56/2018

Assessing the Uncertainty in Central Banks' Inflation Outlooks*

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Abstract

Recent research has found that macroeconomic survey forecasts of uncertainty exhibit several deficiencies, such as horizon-dependent biases and lower accuracy than simple unconditional uncertainty forecasts. We examine the inflation uncertainty forecasts from the Bank of England, the Banco Central do Brasil, the Magyar Nemzeti Bank and the Sveriges Riksbank to assess whether central banks' uncertainty forecasts might be subject to similar problems. We find that, while most central banks' uncertainty forecasts also tend to be underconfident at short horizons and overconfident at longer horizons, they are mostly not significantly biased. Moreover, they tend to be at least as precise as unconditional uncertainty forecasts from two different approaches.

Keywords: Density Forecasts, Fan Charts, Forecast Optimality, Forecast Accuracy

JEL classification: C13, C32, C53.

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1 Introduction

Economic research concerning the measurement of macroeconomic uncertainty has increased substantially since the Great Recession. The recent surge led to the construction of uncertainty indices like in Baker, Bloom, and Davis (2016), Rossi and Sekhposyan (2015) and Jurado, Ludvigson, and Ng (2015), where the latter two actually rely on measures of forecast uncertainty. Central banks already reported measures of forecast uncertainty long before the Great Recession, but their motivation for doing so is regarded as different from the recent motivation in academia.

While academics mainly appear to be interested in the effects caused by changes in uncertainty, like in Bloom (2009), several central banks have mainly intended to communicate that future realizations should not be expected to coincide exactly with the central banks' point forecasts. Central banks like the European Central Bank or the Deutsche Bundesbank resort to reporting unconditional measures of forecast uncertainty for this purpose. Others, however, have tried to provide additional information by publishing conditional measures, in line with the idea of Jurado et al. (2015) who suggest relating macroeconomic uncertainty to conditional forecast uncertainty, i.e. to the conditional volatility of the unforecastable component of macroeconomic time series. Our study focuses on members of the latter group of central banks, namely the Bank of England (BoE), the Banco Central do Brasil (BCB, the Brazilian central bank), the Magyar Nemzeti Bank (MNB, the Hungarian central bank), and the Sveriges Riksbank (SR, the Swedish central bank). In fact, these central banks have issued density forecasts for inflation which are displayed as fan charts. Broadly speaking, a central bank's uncertainty forecast corresponds to the width of its fan chart.

Point forecasts of central banks are often regarded as mean forecasts. Especially the central banks' point forecasts for inflation frequently turn out to outperform all competing forecasts in terms of mean-squared forecast errors, as documented, for instance, in Groen, Kapetanios, and Price (2009) and Faust and Wright (2009). Yet if the central bank does not have a quadratic loss function, these results must be interpreted with caution, because then, in general, point forecasts do not represent the mean of the forecast density. In fact, empirical studies often find that several characteristics of central bank forecasts are suggestive of more complicated loss functions, featuring asymmetry, state dependence, or time-variation, like in Capistrán (2008), Wang and Lee (2014), or Patton and Timmermann (2007). Given the existence of many potential loss functions, it might actually be preferable to issue (and to evaluate) density forecasts. Each user can then infer the point forecast corresponding to her loss function from this density forecast, and the evaluation can rely either on the entire density or only on elements of interest. The evaluation of density forecasts issued by central banks has mostly focused on the BoE with important contributions by Clements (2004), Wallis (2004), and Mitchell and Hall (2005). The asymmetry incorporated in central banks' density forecasts is studied in Knüppel and Schultefrankenfeld (2012). However, the dispersion of central banks' forecast densities, i.e. uncertainty forecasts, has not been investigated explicitly yet. But, interestingly, when the BoE's density forecasts fail to pass tests for correct specification, this is often explained by the excessive width of its fan charts, i.e. its excessively large uncertainty forecasts, as done in Clements (2004), Wallis (2004), and Dowd (2007).

Conditional mean forecasts are typically evaluated with respect to their optimality such as their bias, for instance, and their accuracy. In this work, we do the same for the conditional uncertainty forecasts. With respect to bias, we investigate whether the ex-ante uncertainty, i.e. the uncertainty surrounding a central bank's mean forecast which is predicted by the central bank, coincides, on average, with the ex-post uncertainty, i.e. the size of the realized forecast errors of this mean forecast. The terms 'ex-ante' and 'ex-post uncertainty' were coined by Clements (2014) in the context of macroeconomic survey forecasts. He discovers that, for shorter horizons, survey participants tend to issue variance forecasts, i.e. forecasts for the uncertainty surrounding their mean forecasts which are, on average, larger than the squared realized forecast errors of their mean forecasts. Put differently, for the short term, survey participants are underconfident, i.e. they issue upward-biased uncertainty forecasts. However, at longer horizons, the participants are overconfident, that is, they produce downward-biased uncertainty forecasts. In the words of Clements (2014), at shorter (longer) horizons, the participants' ex-ante uncertainty is larger (smaller) than the ex-post uncertainty. Since central banks publish multi-horizon forecasts, we can investigate whether their uncertainty forecasts are subject to a similar horizon-dependent pattern, and we study the optimality of these forecasts using Mincer and Zarnowitz (1969) regressions.

Moreover, we compare the forecast accuracy of the central banks' ex-ante uncertainty to the accuracy that would be obtained using two very simple benchmarks. One benchmark forecast determines ex-ante uncertainty only based on the average size of past forecast errors. The choice of this benchmark is motivated by the fact that it represents the unconditional measure of ex-ante uncertainty used by central banks like the European Central Bank or the Deutsche Bundesbank. What is more, Clements (2018) documents this benchmark's superior accuracy when comparing it to the conditional measures of ex-ante uncertainty provided by survey participants, especially for inflation. Thus, if this benchmark's superiority could also be observed with respect to central banks' conditional uncertainty forecasts, it might be recommendable to replace them with unconditional forecasts, not least because the production of the latter does not require any modeling efforts. However, it does require the availability of a sufficiently large number of past forecast errors. Therefore, we also investigate the accuracy of simple model-based unconditional measures of ex-ante uncertainty as an alternative.

The remainder of this paper is structured as follows. In Section 2, we explain how the uncertainty forecasts will be evaluated. Section 3 contains a description of the data. Section 4 shows the results of the empirical evaluations. Section 5 concludes.

2 Evaluating Forecast Uncertainty

2.1 Forecast Optimality

The object to be evaluated in this study is the forecast for the squared error of a corresponding conditional h-step-ahead mean forecast. The expected squared error is given by

$$\sigma^2_{t+h|t} = E\left[\hat{e}^2_{t+h|t} \,|\, \mathcal{I}_t\right] \\ = E\left[\left(y_{t+h} - \hat{y}_{t+h|t}\right)^2 \,|\, \mathcal{I}_t\right],$$

where $\mathcal{I}_t$ is the information set of the forecaster in period t and y_{t+h} denotes the value of the target variable in period $t+h$. The variable $\hat{y}_{t+h|t}$ is the corresponding conditional mean forecast made in period t which coincides with $E\left[y_{t+h} \,|\, \mathcal{I}_t\right]$ only in the case of mean forecast optimality. The variable $\hat{e}_{t+h|t}$ is the forecast error of the forecast $\hat{y}_{t+h|t}$. It should be stressed that $\sigma^2_{t+h|t}$ depends on the mean forecast $\hat{y}_{t+h|t}$, implying that all evaluations of forecasts for $\sigma^2_{t+h|t}$ are conditional on $\hat{y}_{t+h|t}$.[1]

The forecast for $\sigma^2_{t+h|t}$ is denoted by $\hat{\sigma}^2_{t+h|t}$. In Clements (2014), $\hat{\sigma}^2_{t+h|t}$ is labelled ex-ante uncertainty, while $\sigma^2_{t+h|t}$ is referred to as ex-post forecast uncertainty because it cannot be assessed before y_{t+h} is observed. If, for instance, the forecast density is normal, the ex-ante uncertainty determines the width of the interval $\pm 1.96\hat{\sigma}_{t+h|t}$ within which the forecaster expects the forecast error $\hat{e}_{t+h|t}$ to lie with a probability of 0.95. Independently of the distribution of the forecast density, $\hat{\sigma}^2_{t+h|t}$ is the forecaster's prediction for the expected squared forecast error $E\left[\hat{e}^2_{t+h|t} \,|\, \mathcal{I}_t\right]$. The squared forecast error $\hat{e}^2_{t+h|t}$ itself is a noisy measure for the unobservable ex-post forecast uncertainty $\sigma^2_{t+h|t}$, with their relationship being given by $\hat{e}^2_{t+h|t} = \sigma^2_{t+h|t} + v_{t+h}$ with $E\left[v_{t+h} \,|\, \mathcal{I}_t\right] = 0$.[2]

If the uncertainty forecast $\hat{\sigma}^2_{t+h|t}$ is optimal, ex-ante and ex-post forecast uncertainty

[1] More on this follows in Subsection 2.3. It might be worth noting that we do not assume mean forecast optimality in our uncertainty forecast evaluations.

[2] It would be more exact to write $v_{h,t+h}$ instead of v_{t+h}, because, for instance, $v_{1,t+1} \neq v_{2,t+1}$ in general. Yet, for notational convenience, we suppress the dependence of residuals and parameters on h in what follows.

coincide, so it holds that

$$\hat{\sigma}^2_{t+h|t} = \sigma^2_{t+h|t}. \tag{1}$$

Since $\sigma^2_{t+h|t}$ is not observable, tests for uncertainty forecast optimality rely on the noisy measure for ex-post forecast uncertainty $\hat{e}^2_{t+h|t}$. For the unconditional expectations of ex-ante uncertainty $\hat{\sigma}^2_{t+h|t}$ and $\hat{e}^2_{t+h|t}$, uncertainty forecast optimality implies that

$$\begin{aligned} E\left[\hat{\sigma}^2_{t+h|t}\right] &= E\left[\sigma^2_{t+h|t}\right] \\ &= E\left[E\left[\hat{e}^2_{t+h|t}|\mathcal{I}_t\right]\right] \\ &= E\left[\hat{e}^2_{t+h|t}\right] \end{aligned} \tag{2}$$

holds.

Condition (2) can be used to test for bias of the uncertainty forecasts. Following Clements (2014), we test the null hypothesis $c = 1$ in the regression equation

$$\frac{\hat{e}^2_{t+h|t}}{\hat{\sigma}^2_{t+h|t}} = c + u_{t+h}, \tag{3}$$

where u_{t+h} denotes the error term.[3] This test will be referred to as the Clements test.

Moreover, we test condition (1) employing Mincer and Zarnowitz (1969) regressions, i.e. by testing the joint null hypothesis $c = 0, b = 1$ in the regression equation

$$\hat{e}^2_{t+h|t} = c + b\hat{\sigma}^2_{t+h|t} + u_{t+h}. \tag{4}$$

Since $\hat{e}^2_{t+h|t}$ can be strongly affected by outliers, as discussed, for example, in Violante and Laurent (2012), we also test the null hypothesis $b \leq 0$ in the regression equation

$$|\hat{e}_{t+h|t}| = c + b\hat{\sigma}_{t+h|t} + u_{t+h} \tag{5}$$

[3] In fact, condition (2) suggests using the regression equation $\hat{e}^2_{t+h|t} - \hat{\sigma}^2_{t+h|t} = c + u_{t+h}$ and testing for $c = 0$. However, we decide to follow Clements (2014) here in order to ensure comparability. We also used the test for $c = 0$ and found that the empirical results are broadly in line with the test based on condition (2).

in order to verify whether there is a significantly positive comovement between the (square root of the) uncertainty forecasts and the size of the corresponding forecast errors.

2.2 Measuring Forecast Accuracy

Evaluating the accuracy of a forecast requires the specification of a corresponding loss function. In the context of density forecasts, these loss functions are typically labeled scoring rules. While the accuracy of mean forecasts can be assessed without further information from the underlying forecast densities, this is not possible with variance forecasts. However, they can be evaluated jointly with the corresponding mean forecasts, as explained in Gneiting (2011).[4] Clements (2018) assumes that the forecast density is normal, with the mean forecast and the variance forecast being its parameters.[5] Then he uses the logarithmic score to evaluate forecast accuracy. We avoid the assumption of a normal distribution by employing the score proposed by Dawid and Sebastiani (1999). This proper score, which is discussed in Gneiting and Raftery (2007) and used, for instance, in Knüppel and Krüger (2017), evaluates mean and variance forecasts jointly. It is given by

$$\mathrm{DSS}_{t+h|t} = \log \hat{\sigma}^2_{t+h|t} + \frac{\hat{e}^2_{t+h|t}}{\hat{\sigma}^2_{t+h|t}}. \tag{6}$$

If the underlying forecast density is normal, $\mathrm{DSS}_{t+h|t}$ is equivalent to the logarithmic score.[6] The smaller the score, the more accurate is the forecast.

It might be interesting to note that, given the mean forecast and, thus, given $\hat{e}^2_{t+h|t}$, the score described by equation (6) is minimized if $\hat{\sigma}^2_{t+h|t} = \hat{e}^2_{t+h|t}$. If this condition were fulfilled in every period, the Mincer-Zarnowitz regressions (4) would yield a constant equal to zero, a slope coefficient equal to one, and an R^2 equal to one. The same parameter

[4]In the words of Gneiting (2011), the variance is not 'elicitable' as a forecast object, because there is no loss function such that the correct forecast of the variance is the unique minimizer of the expected loss. However, the pair (mean, variance) is jointly elicitable. For the concept of higher-order elicitability, see Fissler and Ziegel (2016).

[5]If the forecasters provide insufficient information in order to fit normal distributions, Clements (2018) resorts to triangular distributions as proposed by Engelberg, Manski, and Williams (2009).

[6]Therefore, we arrive at the same results that would be obtained with the approach of Clements (2018), but without the need to make a distributional assumption.

values and a lower R^2 would be attained with the optimal feasible conditional uncertainty forecast $\hat{\sigma}^2_{t+h|t} = \sigma^2_{t+h|t}$. If one were not allowed to make conditional uncertainty forecasts, but only to forecast a single value $\hat{\sigma}^2_h$ for all periods, i.e. to make unconditional uncertainty forecasts only, it would be optimal to use the average of $\hat{e}^2_{t+h|t}$ over the N evaluation periods. With a very small amount of noise added to this uncertainty forecast, the Mincer-Zarnowitz regressions would yield a constant close to one, a slope coefficient close to zero, but R^2 close to zero. In all situations, the uncertainty forecasts would pass the Clements test of equation (3), but with the conditional forecast $\hat{\sigma}^2_{t+h|t} = \sigma^2_{t+h|t}$, one would obtain a lower and, thus, better score than with the unconditional forecast $\hat{\sigma}^2_h$.

2.3 Benchmark Forecasts Based on Unconditional Uncertainty

While $\text{DSS}_{t+h|t}$ evaluates the mean and the variance forecast jointly, different variance forecasts can be ranked according to their realized scores if the same mean forecast is used in each instance. This case will be given below, because we will assess the different variance forecasts in connection with the same conditional mean forecast $\hat{y}_{t+h|t}$ of the central bank. The usual Diebold and Mariano (1995) test can be used to test for equal accuracy of two competing forecasts, where $\text{DSS}_{t+h|t}$ is the loss associated with the forecast given by $\left(\hat{y}_{t+h|t}, \hat{\sigma}^2_{i,t+h|t}\right)$, and i denotes the variance forecast used.

In addition to the central banks' own variance forecasts $\hat{\sigma}^2_{t+h|t}$, we consider two alternative uncertainty forecasts. Both approaches yield unconditional forecasts, because both implicitly assume that all parameters which determine the forecast uncertainty are constant over time. Formally, this assumption means that $\sigma^2_{t+h|t} = \sigma^2_h$ holds. In the absence of estimation uncertainty, the unconditional uncertainty forecasts would therefore also be constant over time, i.e. $\hat{\sigma}^2_{t+h|t} = \hat{\sigma}^2_h$ would hold. However, variations in the uncertainty forecasts of both approaches will be observed because both approaches use rolling windows, i.e. time-varying finite samples for estimating σ^2_h. The accuracy obtained with these unconditional uncertainty forecasts will be compared to the accuracy of the conditional uncertainty forecasts of the central banks. The same type of accuracy comparison, yet with forecasts by survey participants instead of central banks, is performed by Clements (2018).

One approach uses the simple Bayesian autoregressive (AR) model

$$y_t = c + \sum_{i=1}^{p} \theta_i y_{t-i} + u_t, \tag{7}$$

with u_t iid $N(0, \sigma_u^2)$ and uninformative priors, with a uniform prior on $c, \theta_1, \theta_2, \ldots, \theta_p$ and a Jeffrey's prior on σ_u^2. With y_t denoting the last available observation, samples from the joint predictive distribution of $\left(y_{t+1|t}, y_{t+2|t}, \ldots, y_{t+H|t}\right)$ can be generated employing the algorithm described in Karlsson (2013). We use the variance

$$\hat{\sigma}^2_{\text{BAR},t+h|t} = Var\left[y_{t+h|t}\right] \tag{8}$$

as the uncertainty forecast of the model to be compared to the central bank's uncertainty forecast $\hat{\sigma}^2_{t+h|t}$. Each time the model is estimated, the lag length p will be determined by the BIC criterion, with the largest value of p considered being equal to 8.

This approach will be referred to as the BAR approach. As mentioned above, when calculating $\text{DSS}_{t+h|t}$ for the variance forecast $\hat{\sigma}^2_{\text{BAR},t+h|t}$, we use the mean forecast $\hat{y}_{t+h|t}$ of the central bank to calculate the forecast errors $\hat{e}_{t+h|t}$ in equation (6) because we only intend to evaluate different approaches for generating uncertainty forecasts. Thus, the mean forecasts obtained from the BAR approach are not employed in the evaluation exercise.

The second approach is directly related to the way many central banks assess their unconditional forecast uncertainty, namely by using past forecast errors. For example, the BoE estimates its unconditional forecast uncertainty based on its forecast errors from the preceding 40 quarters, as stated in Wallis (2004).[7] This estimate serves as an initial assessment of forecast uncertainty, which is then modified according to current conditions.

[7] Similarly, the Federal Open Market Committee (FOMC) employs the errors of different forecasts made over the previous 20 years for its assessment of unconditional forecast uncertainty, as described in Reifschneider and Tulip (2017).

Clements (2018) uses forecast errors from the preceding 50 quarters in his analysis. We also determine the unconditional forecast uncertainty based on past forecast errors, i.e. by using the simple average of the past squared errors

$$\hat{\sigma}^2_{\mathrm{PFE},t+h|t} = \frac{1}{n}\sum_{i=0}^{n-1} \hat{e}^2_{t-i|t-h-i}, \tag{9}$$

where the mean forecasts of the central bank are used to calculate the forecast errors $\hat{e}_{t+h|t}$. This approach will be referred to as the PFE approach.

While the PFE approach has the advantage of yielding — in the absence of structural breaks — the correct unconditional uncertainty forecasts asymptotically, it requires a sufficiently large sample of past forecast errors. As for the BAR approach, it is likely to produce excessively large uncertainty forecasts especially at short horizons, because it ignores important information used by the central bank in the production of its mean forecasts, but it only requires the availability of a sufficiently large sample of past observations of the target variable.

We employ rolling estimation windows for both approaches. However, given that the forecast error samples of the SR and the MNB are very small, the PFE approach is only applied to the samples of the BoE and the BCB. Although both of our approaches are designed to yield unconditional uncertainty forecasts, they might capture low-frequency movements or structural breaks in forecast uncertainty due to the use of using rolling windows. Yet compared to conditional approaches, they would be expected to be relatively slow in doing so.

Since there is no general consensus on the ideal size of the estimation window for $\hat{\sigma}^2_{\mathrm{PFE},t+h|t}$, we determine n based on empirical results. To be more precise, we choose n such that the score proposed by Dawid and Sebastiani (1999) becomes reasonably small in the samples of the BoE and the BCB. Addressing this issue is interesting in its own right, because it might provide guidance for central banks which have to choose n as well.

While it would be possible to choose different values of n for different horizons h, such an approach might be difficult to communicate in practice. Moreover, empirically optimal horizon-dependent values of n could be unreliable due to the small sample sizes. More robust results can be expected by choosing a single n for all horizons. The question then arises how to weight the results for different horizons in order to choose a single value of n for all of them. Instead of using an ad-hoc weighting scheme, we resort to the Dawid and Sebastiani (1999) score for multivariate forecasts which is given by

$$\mathbf{DSS}_{t+H|t} = \log|\hat{\boldsymbol{\Sigma}}_{t+H|t}| + \hat{\mathbf{e}}'_{t+H}\hat{\boldsymbol{\Sigma}}^{-1}_{t+H|t}\hat{\mathbf{e}}_{t+H}, \tag{10}$$

with

$$\hat{\mathbf{e}}_t = \left(\hat{e}_{t|t-1}, \hat{e}_{t|t-2}, \ldots, \hat{e}_{t|t-H}\right)'$$

and

$$\hat{\boldsymbol{\Sigma}}_{t+H|t} = \frac{1}{n}\sum_{i=0}^{n-1}\left(\hat{\mathbf{e}}_{t-i}\,\hat{\mathbf{e}}'_{t-i}\right).$$

The fact that this approach also uses forecasts of covariances, which are not of interest in the uncertainty forecasts published by central banks, is a consequence of the desire to find a single value of n for all forecast horizons $h = 1, 2, ..., H$.[8]

3 Data

3.1 Central Bank Forecasts

As mentioned above, we use inflation forecasts from four central banks that are producing conditional density forecasts. The BoE, the MNB and the SR use the two-piece normal (TPN) distribution for their forecasts, of which one can calculate the mean and the

[8]Due to the relatively small samples we study, the value of n is going to be determined based on a sample that will overlap with the sample to be used for the evaluation of the uncertainty forecast accuracy. Therefore, it will not be a true out-of-sample evaluation, and the comparison will be biased against the central banks' uncertainty forecasts. However, the choice of n turns out to be of limited importance unless n is set to excessively small values, thereby yielding relatively inaccurate uncertainty forecasts. Moreover, we are going to choose the same value of n for the BoE and the BCB in order to limit the potential for bias against the central banks' uncertainty forecasts.

variance.[9] Yet, while the BoE publishes all parameters characterizing its forecast densities, MNB and SR only provide the mode and quantiles corresponding to the 30, 50 and 70% and 50, 75, and 90% prediction intervals, respectively, of their TPN distributions. The BCB, in turn, publishes the forecast median and quantiles corresponding to the 10, 30 and 50% prediction intervals from its usually symmetric forecast distributions. Hence, we have to back out the required parameters, where to the BCB data, a normal distribution is fitted using a least squares criterion. For the MNB and the SR data, TPN distributions are fitted using a likelihood ratio criterion as described in García and Manzanares (2007).[10]

The BoE and the BCB both have a fairly long history of publishing fan charts, with the samples under study range from 1998Q1 and 1999Q4, respectively, to 2016Q4, yielding up to 76 (BoE) and 69 (BCB) mean and variance forecasts for inflation, depending on the forecast horizon used. MNB and SR have both discontinued their usage of the TPN distribution and changed their approaches to quantifying forecast uncertainty. For the MNB, we have 34 quarterly forecasts ranging from 2002Q4 to 2011Q2.[11] The SR sample ranges from 1999Q4 to 2006Q3 and includes 27 quarterly forecasts.[12]

From the BoE and the BCB, we use inflation forecasts made conditional on assuming that short-term interest rates will follow market expectations. While both central banks also publish forecasts using a constant-rates assumption, the MNB employed only the constant-rates approach during the time period under consideration. The SR dropped the constant-rates approach as of June 2005 and used market expectations before entirely changing the forecasting methodology after the end of our sample.[13] To consider samples

[9] We use BoE forecast data as provided under https://www.bankofengland.co.uk/inflation-report/inflation-reports. BCB forecast data is taken directly from the bank's inflation report documents, available at http://www.bcb.gov.br/?INFLAREPORTFV. The MNB fan chart data can be retrieved in *"Charts and Data Series Chapter"* files under https://www.mnb.hu/en/publications/reports/inflation-report/quarterly-report-on-inflation-from-1998-march-2014. The SR provides *"numerical data on which the diagrams are based"* under http://archive.riksbank.se/en/Web-archive/Published/Published-from-the-Riksbank/Monetary-policy/Monetary-Policy-Report/.

[10] Knüppel and Schultefrankenfeld (2012, 2017) provide descriptions of the details for these procedures.

[11] The MNB forecast data from 2008Q4 were not made publicly available.

[12] Originally, the SR issues monthly forecasts, from which we select forecasts in accordance with the publication dates of the bank's inflation reports. As the publication frequency of the reports was changed from three to four per year as of 2001, there are only three inflation forecast dates in 2000.

[13] We gain two more observations for the BCB sample by filling in constant-rates forecasts for 2002Q4

as long as possible, we focus on maximum forecast horizons of $H = 8$ quarters ahead for the BoE, $H = 5$ for the BCB, $H = 6$ for the MNB and $H = 7$ for the SR.[14] Since the nowcast, i.e. the forecast for $h = 0$, is also considered, we have $H + 1$ horizons to evaluate for each central bank.

Our evaluations of the uncertainty forecasts implicitly assume that the typical conditioning assumptions about the future interest rate path made in central bank forecasts have negligible effects on forecast uncertainty. Concerning the conditioning of forecasts on interest rates as expected by financial markets, this assumption is in line with the evaluations that are conducted by the BoE itself, as done by Elder, Kapetanios, Taylor, and Yates (2005) or the bank's Independent Evaluation Office in 2015.[15] Given that the choice between the market-rates assumption and the assumption of constant interest rates has no detectable consequences for the mean-squared errors of mean forecasts and the logarithmic scores of density forecasts for inflation, as documented in Knüppel and Schultefrankenfeld (2017), we are confident that neither of the two typical conditioning assumptions is relevant for the results of our evaluations. In fact, for the BoE and the BCB, we also evaluated the constant-rates forecasts and obtained results virtually identical to those reported below.[16]

3.2 Inflation Figures

The inflation figures we use as realizations to construct forecast errors of the central bank mean forecasts and for the BAR approach are quarterly series of year-on-year changes in the price indexes that these central banks actually target. The BoE moved from an inflation target of 2.5% RPIX inflation towards targeting 2.0% CPI inflation and the data used goes back to 1988Q1, which means that a rolling estimation sample of

and 2003Q1, because the market-rates scenarios are missing for these periods.

[14]In 2004, the BoE started to forecast up to 12 quarters out.

[15]The report on the BoE's forecasting performance is available under https://www.bankofengland.co.uk/independent-evaluation-office/forecasting-evaluation-november-2015.

[16]The conjecture concerning the irrelevance of the interest rate assumption for our evaluations is further supported by the fact that, although the respective mean forecasts differ marginally, the BoE provides identical uncertainty parameters for its constant-rates and its market-rates inflation forecasts.

40 quarters is available for the BAR approach, and the uncertainty forecasts obtained cover the BoE's entire forecast sample period described above.[17] The BCB adopted a flexible inflation target of the IPCA index, the Brazilian national consumer price index equivalent, in the late 1990s, to bring down huge inflation rates from the period before, in which IPCA inflation as of 1996Q1 was still running at roughly 22%.[18] Using IPCA inflation figures from 1996Q1 onwards, utilizing 28 quarters for the rolling estimation sample appears to provide a reasonable trade-off between a decent window size and a sufficiently long evaluation period. However, the evaluation sample does not cover the full sample of available BCB forecasts. Hungary also experienced high inflation rates in the 1990s, ranging between 10 and 30%. We use CPI data from 1992Q4 onwards and a rolling estimation sample of 40 quarters to produce variance forecasts covering the entire MNB sample period. For Sweden, which comprises relatively more moderate CPI inflation rates of up to 10% around 1990, the rolling estimation sample corresponds to 36 quarterly observations, and the model's variance forecasts cover the complete sample of the SR's forecasts outlined above.[19]

3.3 Properties of Inflation Forecasts and Figures

In Figures 2, 3, 4, and 5, we show the uncertainty forecasts of the respective central bank, the uncertainty forecasts obtained from the BAR approach, the mean forecast errors of the central bank, and, if available, the uncertainty forecasts obtained from the PFE approach.

In Figure 2 we see that the BoE's forecast errors in the second half of the sample tend to be larger than in the first half. Broadly in line with this development, the uncertainty forecasts of the BoE increase substantially from about 2008 to 2010 and remain on their elevated levels afterwards. The BAR approach and the PFE approach also pick up the

[17] The RPIX is the UK retail price index excluding mortgage payments. Since December 2003, the BoE has targeted the consumer price index CPI.

[18] IPCA stands for Índice Nacional de Preços ao Consumidor Amplo.

[19] Analogously to the SR's forecasts, the BAR approach for the SR sample is based on monthly year-on-year inflation rates from 1988M01 to 2006M12. Results are converted to quarterly format selecting the forecasts in accordance with the publication dates of the SR's inflation reports.

increase in forecast uncertainty, albeit a bit later than the BoE does.

The BCB's forecast errors displayed in Figure 3 are extremely large around the year 2003, while they tend to be small especially from 2006 to 2014. The uncertainty forecasts of the BCB are relatively large until 2007, when a pronounced decrease occurs, and the forecasts continue to be small afterwards. Like in the case of the BoE, the uncertainty forecasts obtained from the BAR approach and the PFE approach experience a similarly large change several periods after those of the central bank.

For the MNB's forecast errors in Figure 4, there are no protracted episodes where forecast errors are particularly small or large. After a period with practically unchanged uncertainty forecasts by the MNB from 2004 to 2008, these forecasts increase in 2009 and basically remain unchanged until the end of the sample. The BAR approach gives large uncertainty forecasts for a few observations at the beginning of the sample, but they quickly decrease.

Figure 5 indicates that the SR's forecast errors do not experience large changes in their magnitudes. The SR's uncertainty forecasts only exhibit small variations, and the same holds for the uncertainty forecasts from the BAR approach.

To sum up, large changes in the magnitudes of forecast errors and uncertainty forecasts are only observed in the larger samples for the BoE and the BCB. While for the BoE, the smaller samples used for the BAR and the PFE approach also cover the periods where pronounced changes in the size of the forecast errors occur, this does not hold for the corresponding smaller samples used in the case of the BCB.

An interesting observation not completely evident from the figures is that the uncertainty forecasts of all central banks virtually never decrease as the horizon increases. Thus, the conventional random walk stochastic volatility model might be an appropriate way to describe time-varying inflation uncertainty as perceived by central banks. In contrast to that, stochastic volatility following stationary AR processes or conventional GARCH processes would eventually lead to a decrease of uncertainty for larger horizons.[20]

[20]Of course, it might also be that central banks have stationary but very persistent processes of time-varying inflation uncertainty in mind. However, even for horizons up to 12 quarters ahead as published

Figure 6 plots the average value of $\hat{e}^2_{t+h|t}$ (average ex-post uncertainty) against the average value of $\hat{\sigma}^2_{t+h|t}$ (average ex-ante uncertainty) for each sample, each uncertainty-forecast approach, and each horizon. Values above the 45 degree line indicate overconfident uncertainty forecasts, i.e. forecasts subject to a downward bias, whereas values below the line correspond to underconfident uncertainty forecasts, i.e. forecasts subject to an upward bias.

With the exception of the MNB, the uncertainty forecasts by central banks yield results remarkably close to the 45 degree line in all samples considered. The MNB turns out to be noticeably overconfident for all horizons except $h = 0$. By contrast, the BAR approach produces pronouncedly underconfident uncertainty forecasts in all cases except for the BoE's sample. In the latter case, it behaves similarly to the BoE's uncertainty forecasts, being marginally underconfident for larger forecast horizons. Finally, the PFE approach yields virtually unbiased uncertainty forecasts for the BoE's sample, but pronouncedly underconfident forecasts for the BCB's sample.

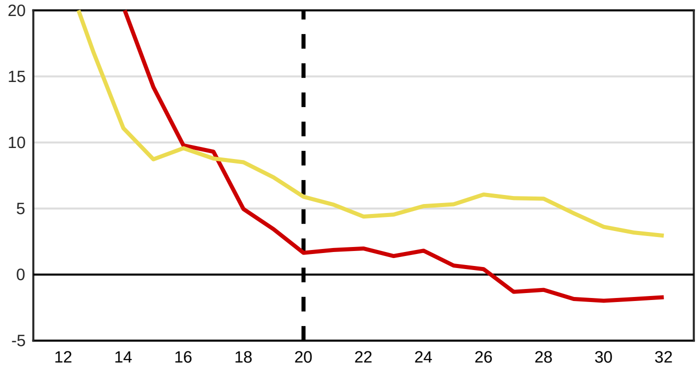
4 Results

4.1 General Setup

The length n of the rolling estimation window for the PFE approach is determined based on the Dawid and Sebastiani (1999) score for multivariate forecasts, as mentioned above. We split the balanced samples of forecast errors of the BoE and the BCB such that the rolling window can cover up to 32 quarters. While $n = 12, 13, \ldots, 32$ values are used for the estimation of $\hat{\mathbf{\Sigma}}_{t+H|t}$, the remaining 28 values are used for the evaluation in the case of the BoE. In the case of the BCB, the latter number equals 27.

by the BoE since 2004, uncertainty forecasts never decrease when the horizon increases. Interestingly, the conventional random walk stochastic volatility model also tends to outperform its competitors when making density forecasts of macroeconomic time series for the USA for up to 8 quarters ahead, as shown by Clark and Ravazzolo (2015).

Figure 1: Dawid-Sebastiani score and window length



Notes: The lines plot the average Dawid-Sebastiani score $\mathbf{DSS}_{t+H|t}$ for multivariate forecasts on the Y axis over an expanding window size of $n = 12, 13, \ldots, 32$ on the X axis. The red solid line is based on Bank of England past forecast errors for $h = 0, 1, \ldots, 8$, and the yellow solid line is based on Banco Central do Brasil past forecast errors for $h = 0, 1, \ldots, 5$. Values larger than 20 are not displayed.

The average scores obtained are displayed in Figure 1. For small values of n, the scores are often very large, and they tend to decrease with n. The best results occur for $n \geq 30$, but these values would leave us with very small samples for the tests to be conducted. Smaller values of n with relatively low scores are, for instance, given by $n = 22$ for the BCB and by $n = 20$ for the BoE. We choose to set $n = 20$ in the PFE approach, corresponding to a window size of 5 years of quarterly data.[21]

In the following tests of forecast optimality and equal forecast accuracy, we use a significance level of 0.05. For the uncertainty forecasts of the BoE and the BCB, we consider two different samples for each horizon. The larger sample corresponds to the evaluation sample of the BAR approach, and the smaller sample to the evaluation sample of the PFE approach.[22]

[21] As mentioned above, the BoE employs a larger sample of 10 years of past quarterly forecast errors for its assessment of unconditional forecast uncertainty. Our results suggest that it would be reasonable to use samples larger than about 7 years if the evaluation sample size were of no concern.

[22] It might be worth noting that the latter sample sizes are larger than the evaluation samples used to determine n, because only 20 instead of up to 32 quarters are employed for the rolling-window estimation, and because unbalanced samples are used for the horizon-specific tests. Since these additional observations were not considered when making the choice of n, the comparisons with the central banks' forecasts become less biased against those.

4.2 Forecast Optimality — Clements Tests

The results of the Clements tests documented in Table 1 indicate that the BoE is overconfident with respect to forecast uncertainty for most horizons. However, the downward biases in its uncertainty forecasts are mostly insignificant, with the exception of $h = 5$ in the sample of the PFE approach. The sign of the bias is remarkable insofar as the BoE was found to provide overly wide fan charts, i.e. to be underconfident in the earlier contributions by Clements (2004), Mitchell and Hall (2005), Wallis (2004), and Dowd (2007) mentioned above. With the larger sample in this study, only for the nowcast can a significant upward bias of the uncertainty forecasts be found. The latter result also holds for the BAR approach. Given that this approach determines forecast uncertainty without taking current-quarter information into account, this result is not too surprising. For horizons $h \geq 1$, the BAR approach produces downward-biased uncertainty forecasts. This bias is more pronounced than that observed for the BoE's uncertainty forecasts, and it is significant for all $h \geq 3$. While the PFE approach produces more strongly downward-biased uncertainty forecasts than the BoE for some horizons, none of its biases are significantly different from zero.

When the Clements test is applied to the BCB samples, the results in Table 2 show that the BCB was overconfident for all horizons but $h = 1$. The upward bias in the uncertainty forecasts for $h = 1$ is significant in both samples under study, while for all other horizons, the biases are insignificant in both sample sizes considered. The BAR approach yields underconfident uncertainty forecasts for $h \leq 2$ and overconfident forecasts for the larger horizons. The upward bias is significant for $h = 0$ and $h = 1$. The PFE approach produces overconfident forecasts for all horizons except $h = 1$, but the biases are not very large and always insignificant. Additional full-sample results for the BCB's uncertainty forecasts displayed in Table 9 turn out to be quantitatively similar to those obtained in the smaller samples, but the biases are insignificant for all horizons.

It is interesting to compare the test results for the BCB with the results displayed in Figure 3. While the plots for the BAR approach and the PFE approach suggest that both approaches produce highly underconfident uncertainty forecasts for all horizons, the estimation results often indicate the opposite. This is due to the fact that for the plots, squared errors and uncertainty forecasts themselves are averaged, whereas the regressions use the average of the ratio of these quantities. While, on average, the uncertainty forecasts are larger than the squared errors, there are a few observations where large forecast errors coincide with relatively small uncertainty forecasts. This often causes the ratio of these quantities to exceed a value of one.

Concerning the results for the MNB displayed in Table 3, the average value of the ratio $\hat{e}^2_{t+h|t}/\hat{\sigma}^2_{t+h|t}$ is very large for the horizons $h \geq 1$, often exceeding 3. However, these strong downward forecast biases turn out to be insignificant which might partly be due to the small sample size. By contrast, the BAR approach yields underconfident uncertainty forecasts for all horizons, and this bias is significant for $h \leq 2$.

Finally, concerning the results of the SR sample displayed in Table 4, the SR's uncertainty forecasts are overconfident for $h > 0$, but these downward biases are insignificant. Like in the case of the BoE, the uncertainty nowcast is underconfident, and its upward bias is significant. The BAR approach yields underconfident uncertainty forecasts for all horizons except $h = 7$, and the upward bias is significant for $h = 0, 3, 4$.

To sum up, underconfidence of central banks is only observed in some cases for shorter horizons, whereas for $h \geq 2$, their uncertainty forecasts are virtually always overconfident. By and large, this corresponds to the findings of Clements (2014) for macroeconomic survey forecasts. However, in contrast to the results for survey participants, the overconfidence of the central banks' uncertainty forecasts is almost never significant. The BAR approach tends to produce underconfident uncertainty forecasts especially at smaller horizons, and its forecasts are significantly biased in several cases. Finally, while the forecasts of the PFE approach are mostly overconfident, their biases are never significant.

4.3 Forecast Optimality — Mincer-Zarnowitz Regressions

For the BoE samples, the results of the Mincer-Zarnowitz regressions, i.e. of the tests for forecast optimality, and of the related tests for positive comovement of absolute forecast errors and uncertainty forecasts measured by the standard deviation can be found in Table 5. In the larger sample, which forms the basis for the results in the upper half, forecast optimality of the BoE's uncertainty forecasts cannot be rejected for $h \geq 3$ according to the Mincer-Zarnowitz regressions. Moreover, for these horizons, neither the constant nor the slope coefficient appears to be significantly different from its optimal value of zero and one, respectively. These test results are remarkably good, given that the sample under study is relatively large and, therefore, the power of the tests can be expected to be large as well.

The optimality of the uncertainty forecasts of the BAR approach is rejected for all horizons, and the slope coefficients are close to zero and always significantly different from one.

A significantly positive comovement between absolute forecast errors and uncertainty forecasts is found for the BoE for $h \geq 2$. Thus, if the BoE widens its fan charts, this tends to be a reliable indicator for larger future forecast errors. For the BAR approach, the slope coefficients are positive except for $h = 0$, but significance is observed for $h \geq 7$ only. Hence, due to the use of a rolling estimation window, the BAR approach probably captures some low-frequency movements of forecast uncertainty, but the comovement of its uncertainty forecasts with absolute forecast errors is relatively low.

In the smaller sample, for the BoE's uncertainty forecasts, optimality cannot be rejected for five horizons, namely for $h = 3, 4, 5, 6, 8$. With the PFE approach, forecast optimality can be rejected for all horizons except $h = 0$. A significantly positive comovement between absolute forecast errors and the BoE's uncertainty forecasts is observed for $h = 5, 6, 7$, whereas with the PFE approach, almost all coefficients are very close to zero. Thus, one may conclude that in the smaller sample the unconditional forecast uncertainty as measured by the PFE approach does not capture movements in forecast uncertainty, and that the BoE considerably improves upon such unconditional forecasts by modifying them according to current conditions.

Concerning the BCB samples, most uncertainty forecasts considered perform worse than for the BoE samples. As shown in Table 6, forecast optimality is rejected for all approaches and all horizons except for the BCB's uncertainty forecasts for $h = 0$. Moreover, in the larger sample, the BCB and the BAR approach produce uncertainty forecasts which are significantly positively related to the corresponding absolute forecast errors only for $h = 1$. For most other horizons, the corresponding slope coefficients are negative or close to zero. In the smaller sample of the PFE approach, the results for the slope coefficients are similar. The only significantly positive coefficient estimate is again observed for $h = 1$.

For the MNB sample, the results in Table 7 are similar to those for the BCB samples. Forecast optimality is rejected in all cases except for the MNB uncertainty forecast for $h = 2, 3, 6$. However, all three slope coefficients are strongly negative. Therefore, the inability to reject optimality in these cases might at least partly be due to the small sample size and, consequently, the low power of the tests. With respect to the regressions of the absolute forecast errors on uncertainty forecasts, negative slope coefficients are obtained with the MNB forecasts for all $h \geq 1$. With the BAR approach, most slope coefficients are positive and relatively small. For $h = 0$ and $h = 6$, they turn out to be significantly positive.

Finally, concerning the results for the SR samples shown in Table 8, there are several cases where optimality cannot be rejected, but these might again be due to the small sample size. While for the SR's uncertainty forecasts, optimality cannot be rejected for $h = 1$, $h = 3$, and $h \geq 5$, only the slope coefficients for $h = 0$ and $h = 3$ are positive. With the BAR approach, optimality is rejected for all horizons. Concerning the absolute forecast errors, the slope coefficients are significantly positive only for $h = 0$ with the SR forecasts and for $h = 6$ with the BAR approach. While all but two slope coefficients of the SR's forecasts are negative, the forecasts of the BAR approach yield four negative slope coefficients.

On the whole, these results suggest that it is difficult to forecast changes in uncertainty with the approaches used by the central banks under study. Otherwise, the slope coefficients obtained with the central banks' uncertainty forecasts should be clearly closer to one (in the Mincer-Zarnowitz regressions) and significantly positive (in the regressions with absolute errors) much more often than those obtained with the unconditional approaches. However, one should also bear in mind that most of these comparisons are based on relatively small samples. If central banks were able to forecast short-run changes in forecast uncertainty very well, this ability should be detected by the tests even in such small samples. Yet if the ability of central banks is mainly restricted to forecasting long-run changes in forecast uncertainty, this can only be observed when larger samples are used.

In fact, the results for the BoE in the larger sample suggest that low-frequency movements in uncertainty are captured very well by its forecasts.[23] This conjecture is not only based on the test results described for the BoE, but also on the impressions gained from Figure 2. Interestingly, the same conjecture applies to the BCB when considering the full-sample results of its uncertainty forecasts displayed in Table 9. Notably, forecast optimality cannot be rejected for any horizon. Moreover, all slope coefficients are positive, although this result is almost never significant. Figure 3 also indicates that the BCB reasonably aligned its uncertainty forecasts to the strong, long-lasting reduction in uncertainty. While this alignment only took place many quarters after the reduction occurred, it still preceded the corresponding changes in the unconditional approaches by several quarters. Thus, the joint results from the smaller and larger samples considered in our study suggest the encompassing conclusion that central banks are able to detect long-run changes in forecast uncertainty, potentially with a certain delay, and to incorporate them into their uncertainty forecasts in an adequate manner, while they are not able to produce reliable forecasts of short-run movements in forecast uncertainty.

[23] If the same were true for high-frequency movements, the results in the smaller sample should display more evidence in favor of optimality.

4.4 Forecast Accuracy

The results of the Diebold and Mariano (1995) test for equal predictive accuracy of the central banks' forecasts and the forecasts based on either the BAR approach or the PFE approach are displayed in Table 10. Concerning the comparisons with the BAR approach, the BoE produces more accurate uncertainty forecasts for all $h \geq 1$. For $h = 3, 4, 5, 6, 7$, the differences in accuracy are significant. A similar picture emerges for the BCB. Compared to the BAR approach, its uncertainty forecasts are more accurate for all horizons, with the differences for $h = 1, 2, 3, 4$ being significant. The MNB produces significantly more precise uncertainty forecasts than the BAR approach for $h = 0$. For all larger horizons, the BAR approach gives better results, but the differences are insignificant. Finally, the SR's uncertainty forecasts are more accurate than those of the BAR approach for all horizons except $h = 1, 5, 7$, but all differences are insignificant.

Considering all 30 comparisons, the central banks' uncertainty forecasts perform better than the BAR approach in 20 cases. Moreover, in 10 of these cases, their uncertainty forecasts are significantly better than those of the BAR approach, whereas the opposite is never observed.

When the BoE's uncertainty forecasts are compared to those of the PFE approach, only the forecasts for $h = 0$ and $h = 4$ from the PFE approach are more accurate than their BoE counterparts. Except for $h = 0$, all differences are significant. The BCB's forecasts are more accurate than those of the PFE approach for $h \geq 1$, and these differences are significant for $h \geq 3$. Thus, for 12 out of 15 cases, the central banks' uncertainty forecasts are more accurate than those of the PFE approach. While for 3 of these 12 cases, the differences in accuracy are significant, a significantly more precise PFE forecast is observed in one case.

By and large, central banks' uncertainty forecasts tend to outperform unconditional uncertainty forecasts from simple autoregressions in terms of accuracy.[24] Although central

[24]More sophisticated models such as those considered by Clements and Galváo (2017) might, of course, lead to different results.

banks also appear to deliver better uncertainty forecasts than the unconditional uncertainty forecasts from the PFE approach, i.e. forecasts derived from a rolling window of past forecast errors, it should be noted that the statistical evidence for the latter statement is not particularly strong.

The results observed may be related to the presence of different uncertainty regimes in the samples under study. For example, it might be possible that unconditional approaches give better uncertainty forecasts in normal times, whereas the conditional uncertainty forecasts of central banks might be more accurate in turbulent times. In order to test this hypothesis, we investigate the conditional predictive accuracy of the different forecast approaches as proposed by Giacomini and White (2006). The conditioning information we consider is the monthly VIX (the volatility index published by the Chicago Board Options Exchange) and a monthly measure of oil price volatility.[25] For both of these economic uncertainty measures, we always use the values in the month before the forecasts are made.

First of all, it might be interesting to investigate whether these uncertainty measures are correlated with the uncertainty forecasts of central banks. As shown in Table 11, both measures actually exhibit moderately negative correlations with the BoE's uncertainty forecasts. This result suggests that none of these measures is considered to be an important driver of inflation forecast uncertainty by the BoE. The same statement can be made for the BCB, where all correlations are close to zero. On the other hand, the correlations of the VIX with the MNB's and the SR's uncertainty forecasts mostly range from 0.4 to 0.6. Concerning oil price volatility, the correlation only reaches around 0.2 for the MNB, but mostly varies between 0.6 and 0.8 for the SR. Of course, these differences between the central banks might at least partly be caused by the different sample periods.

While there are rather diverse relationships in place between the central banks' uncertainty forecasts and the uncertainty measures, the test results for equal conditional predictive accuracy shown in Tables 12 and 13 are always very similar to the previous

[25]We employ the variance of the daily percentage price changes within a month to measure this volatility, using the Brent crude oil price in US dollars.

test results for unconditional predictive accuracy. The signs of most test statistics do not change when using the conditional version of the test. The changes in the magnitude of the test statistics lead to a few changes concerning the significance of differences for the BoE's and the BCB's forecasts. For instance, with both uncertainty measures, the difference between the BCB's and the BAR approach forecast becomes insignificant for $h = 4$, while the difference between the BCB's and the PFE approach forecast becomes significant for $h = 2$. However, we cannot detect any pattern in the test results which would allow us to draw conclusions about the behavior of uncertainty forecast accuracy of central banks with respect to the state of economic uncertainty.

5 Conclusion and Outlook

The conditional inflation uncertainty forecasts of the Bank of England, the Banco Central do Brasil, the Magyar Nemzeti Bank, and the Sveriges Riksbank are analyzed with respect to forecast optimality and accuracy. While we can reject forecast optimality in several cases based on Mincer-Zarnowitz regressions, the central banks' uncertainty forecasts hardly suffer from significant biases. However, there is a tendency for these forecasts to be underconfident at short horizons and overconfident at longer horizons, which is broadly consistent with findings for survey participants reported in Clements (2014). Often, we are not able to find a significantly positive comovement between uncertainty forecasts and the size of forecast errors. Given the samples under consideration, our results suggest that short-run fluctuations in forecast uncertainty are not successfully detected by the central banks under study, while this tends to happen when low-frequency movements in forecast uncertainty take place.

From the two alternative approaches considered for producing unconditional uncertainty forecasts based on a rolling estimation window, the BAR approach tends to perform worse than the central banks' conditional uncertainty forecasts. It often yields significantly biased uncertainty forecasts. Although its uncertainty forecasts are some-times preferable to the central banks' forecasts with respect to their comovement with the size of the forecast errors, by and large, its forecast accuracy turns out to be inferior. The PFE approach, which can only be applied to the BoE's and the BCB's samples, never produces significantly biased uncertainty forecasts. However, as might be expected, there is hardly any comovement of its uncertainty forecasts with the size of the forecast errors. The forecast accuracy of the PFE approach is mostly lower than the forecast accuracy of the BoE and the BCB, but only a few of these differences are significant. On the whole, we can conclude that the superiority of unconditional forecasts over survey forecasts as described in Clements (2018) does not carry over to central banks' uncertainty forecasts.

It remains to be investigated whether alternative conditional uncertainty forecasts could outperform the central banks' uncertainty forecasts. Since the PFE approach yields very good uncertainty forecasts with respect to bias, but suffers from the lack of comovement between its uncertainty forecasts and the size of forecast errors, the approach by Clark, McCracken, and Mertens (2017) appears to be promising. This approach produces conditional uncertainty forecasts based on past forecast errors and delivers good results for survey forecasts. An application to central bank forecasts is awaited.

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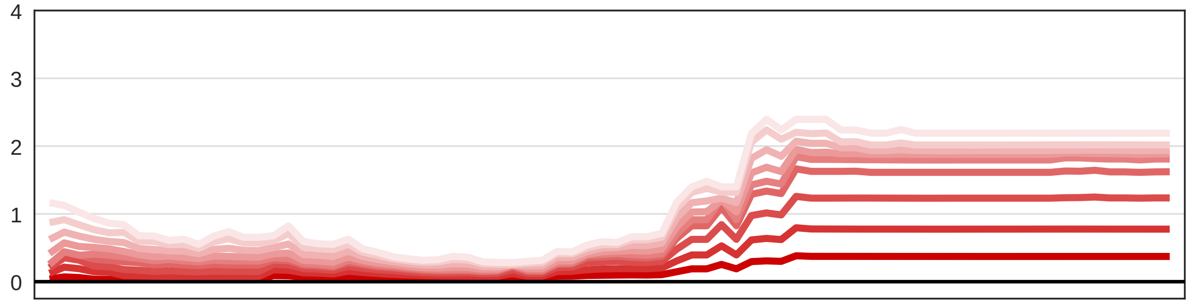
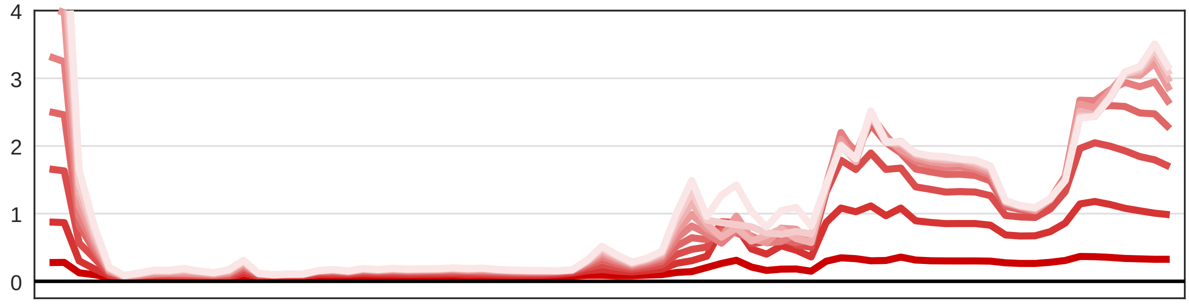
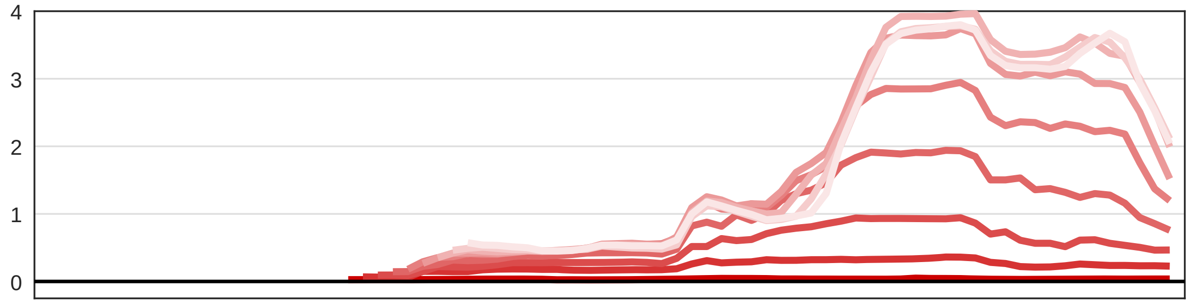
Figure 2: Bank of England time series plots



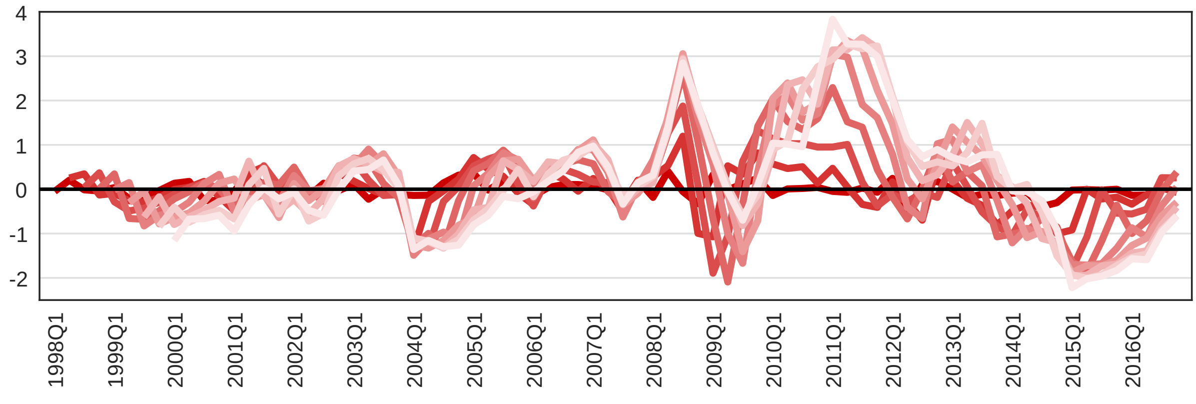
BoE variance forecasts for $h = 0, 1, \ldots, 8$



BAR approach variance forecasts for $h = 0, 1, \ldots, 8$



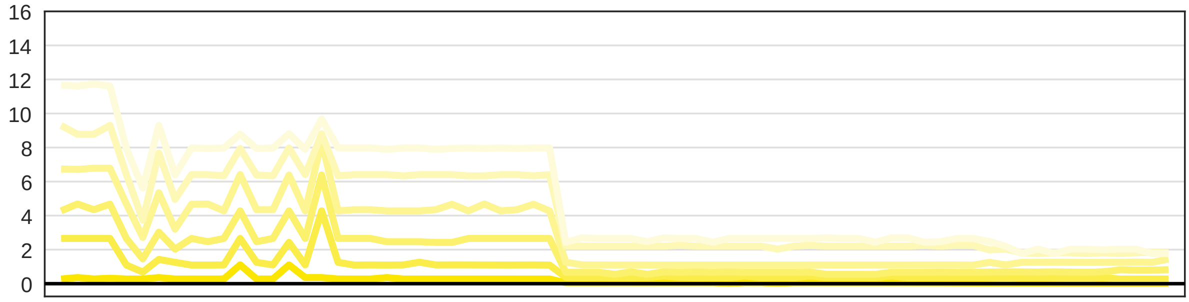
PFE approach variance forecasts for $h = 0, 1, \ldots, 8$



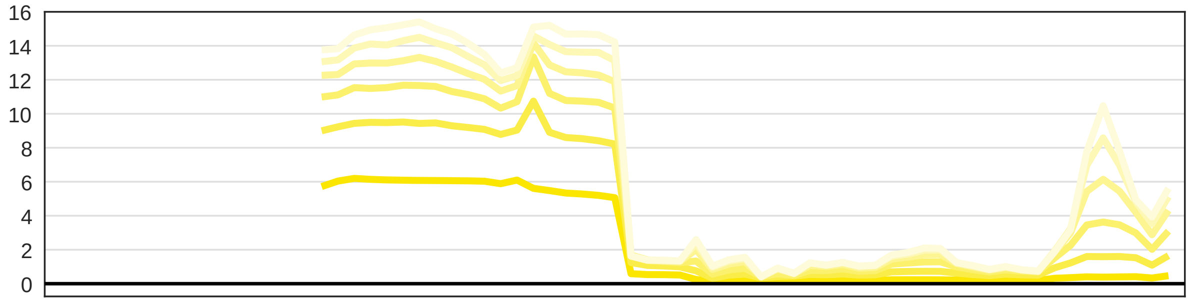
BoE forecast errors of the mean forecast for $h = 0, 1, \ldots, 8$

Notes: Each plot shows series for forecast horizons $h = 0, 1, \ldots, 8$, with the color fading as the horizon extends. The top panel plots the time series of the Bank of England inflation variance forecasts. The second panel plots the variance forecasts obtained from the BAR approach, using UK RPIX/CPI quarterly year-on-year inflation figures and a rolling window of $n = 40$. The third panel plots the variance forecasts obtained from the PFE approach, using historical squared forecast errors of the BoE's mean forecast and a rolling window of $n = 20$. The bottom panel plots the historical forecast errors of the BoE's inflation mean forecasts.

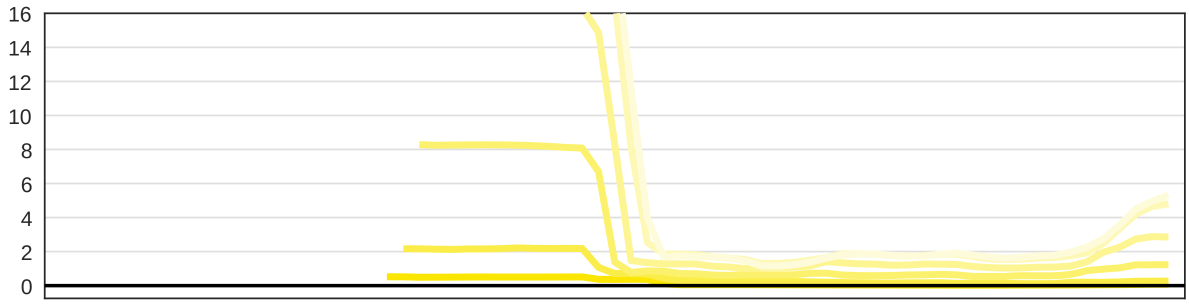
Figure 3: Banco Central do Brasil time series plots



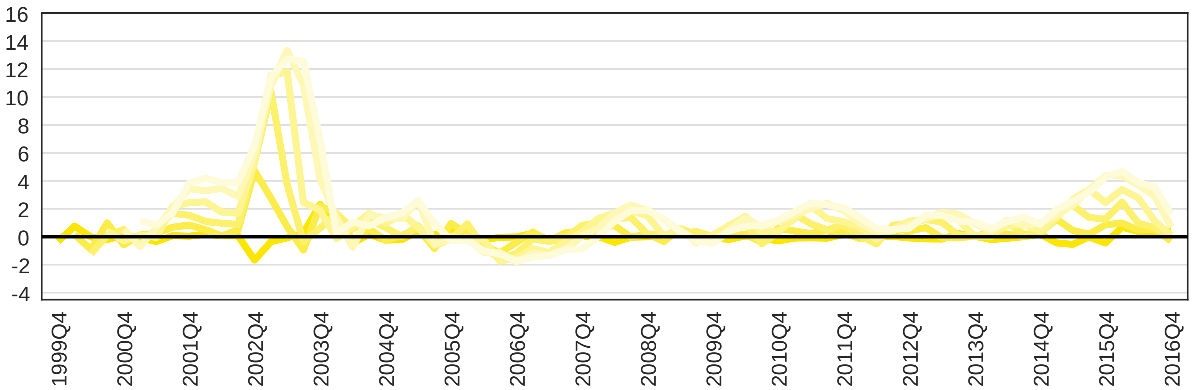
BCB variance forecasts for $h = 0, 1, \ldots, 5$



BAR approach variance forecasts for $h = 0, 1, \ldots, 5$



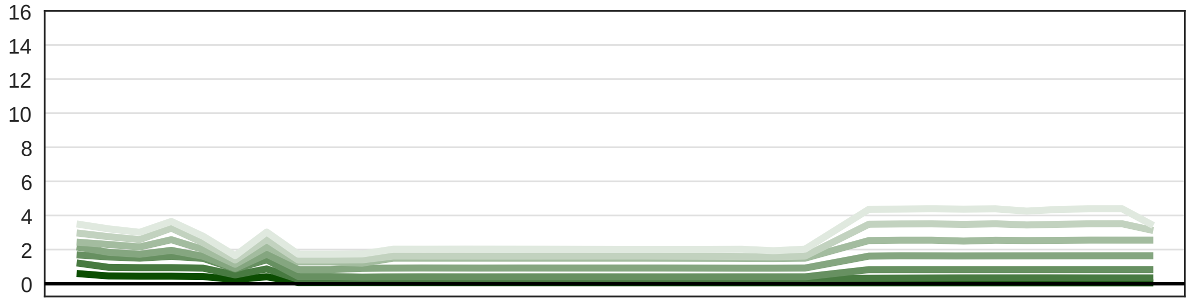
PFE approach variance forecasts for $h = 0, 1, \ldots, 5$



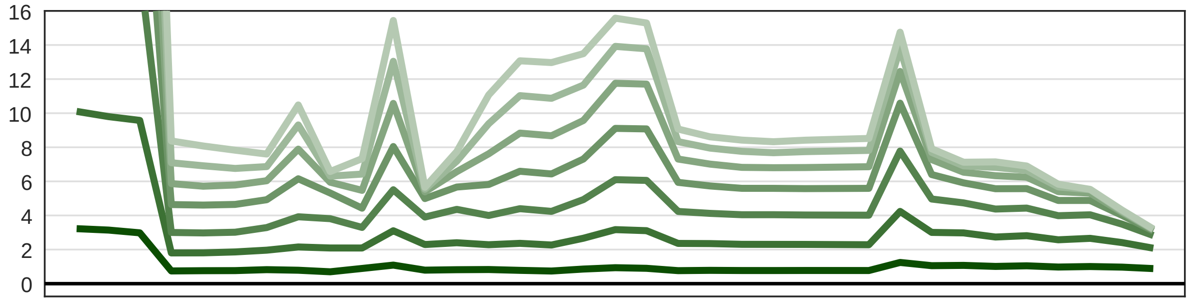
BCB forecast errors of the mean forecast for $h = 0, 1, \ldots, 5$

Notes: Each plot shows series for forecast horizons $h = 0, 1, \ldots, 5$, with the color fading as the horizon extends. The top panel plots the time series of the Banco Central do Brasil inflation variance forecasts. The second panel plots the variance forecasts obtained from the BAR approach, using Brazilian IPCA quarterly year-on-year inflation figures and a rolling window of $n = 28$. The third panel plots the variance forecasts obtained from the PFE approach, using historical squared forecast errors of the BCB's mean forecast and a rolling window of $n = 20$. The bottom panel plots the historical forecast errors of the BCB's inflation mean forecasts.

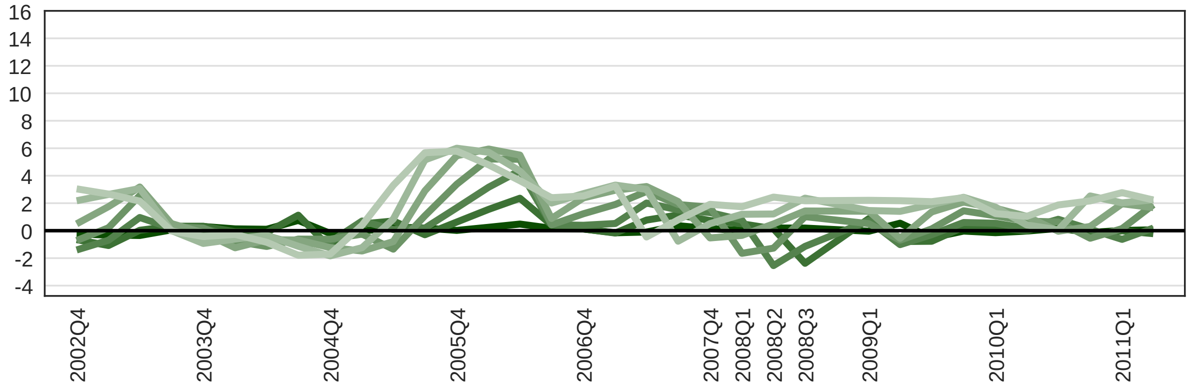
Figure 4: Magyar Nemzeti Bank time series plots



MNB variance forecasts for $h = 0, 1, \ldots, 6$



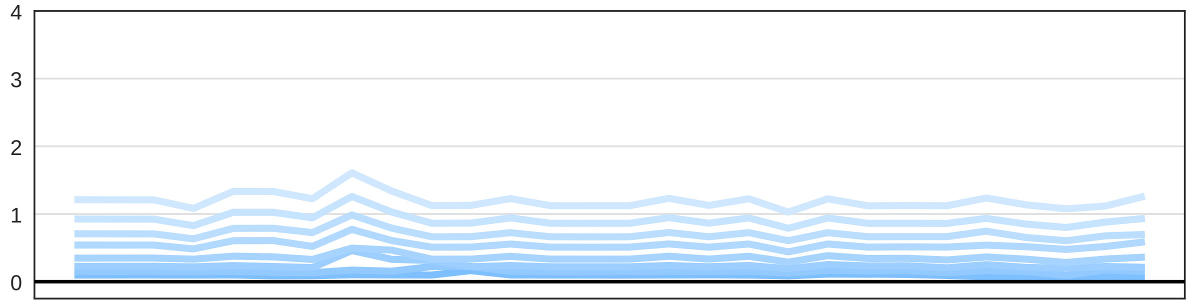
BAR approach variance forecasts for $h = 0, 1, \ldots, 6$



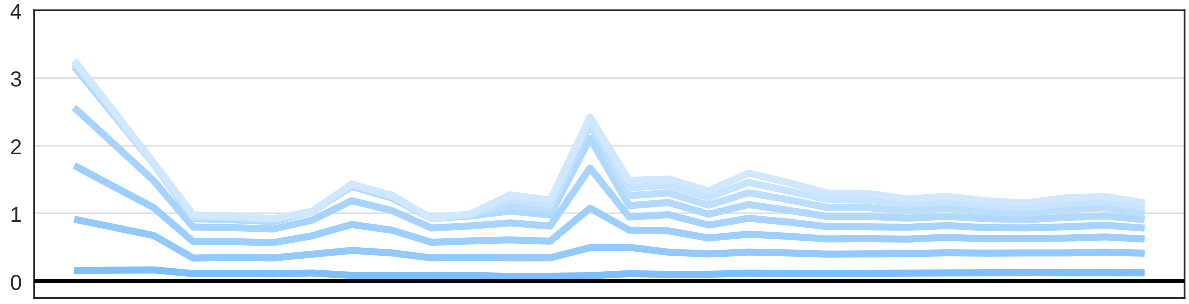
MNB forecast errors of the mean forecast for $h = 0, 1, \ldots, 6$

Notes: Each plot shows series for forecast horizons $h = 0, 1, \ldots, 6$, with the color fading as the horizon extends. The top panel plots the time series of the Magyar Nemzeti Bank inflation variance forecasts. The second panel plots the variance forecasts obtained from the BAR approach, using Hungarian CPI quarterly year-on-year inflation figures and a rolling window of $n = 40$. The bottom panel plots the historical forecast errors of the MNB's inflation mean forecasts. Forecast data for 2008Q4 is not provided.

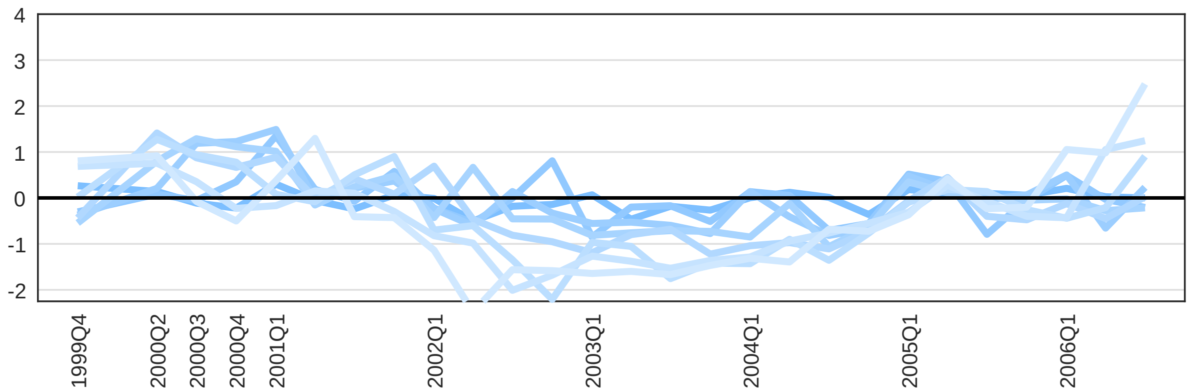
Figure 5: Sveriges Riksbank time series plots



SR variance forecasts for $h = 0, 1, \ldots, 7$



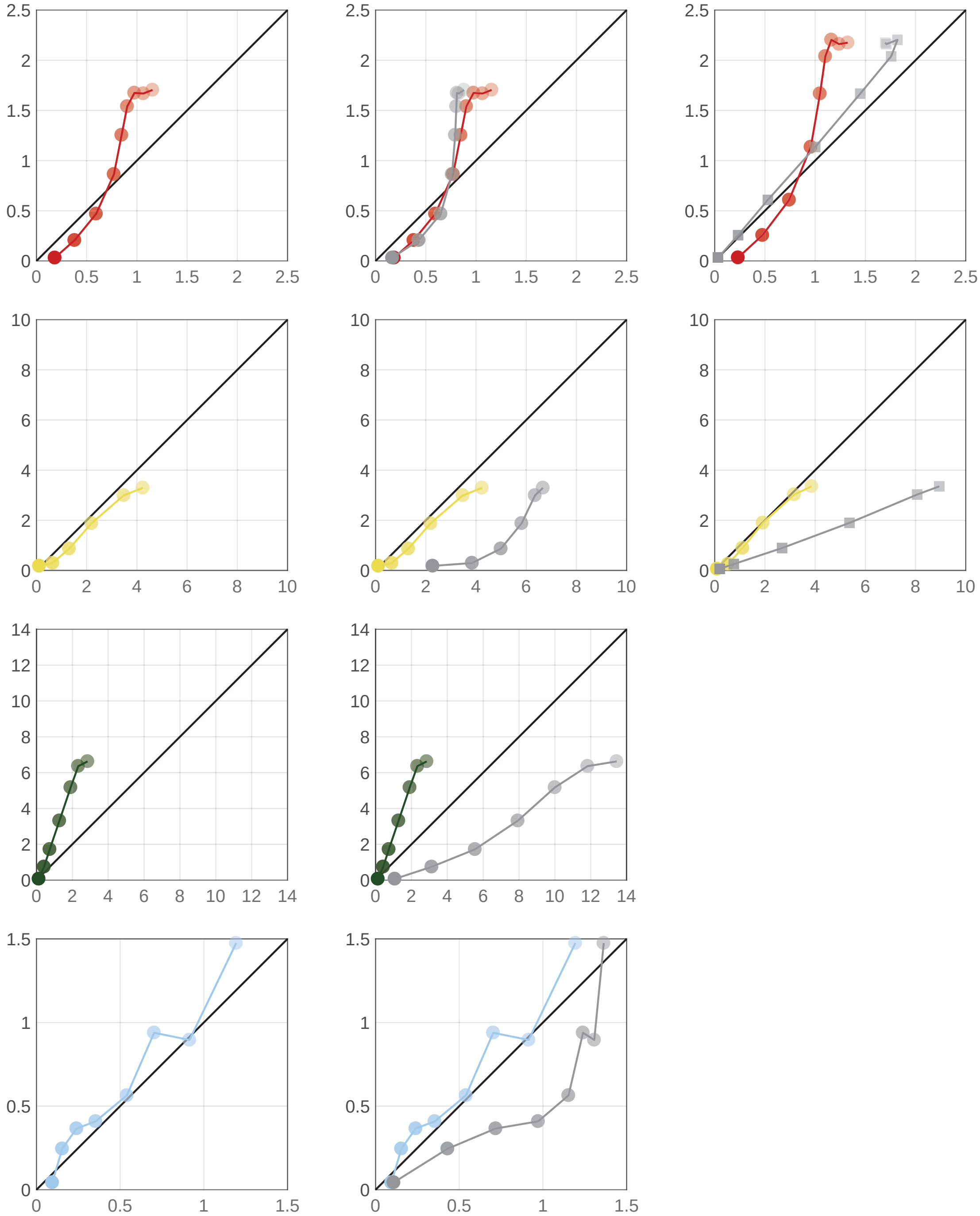
BAR approach variance forecasts for $h = 0, 1, \ldots, 7$



SR forecast errors of the mean forecast for $h = 0, 1, \ldots, 7$

Notes: Each plot shows series for forecast horizons $h = 0, 1, \ldots, 7$, with the color fading as the horizon extends. Before 2001, only three inflation reports were published per year. The top panel plots the time series of the Sveriges Riksbank inflation forecast variances. The second panel plots the variance forecasts obtained from the BAR approach, using Swedish CPI quarterly year-on-year inflation figures and a rolling window of $n = 36$. The bottom panel plots the SR's historical forecast errors of the inflation mean forecasts.

Figure 6: Comparing ex-ante and ex-post forecast uncertainty



Notes: Panels from top row to bottom row: Bank of England, Banco Central do Brasil, Magyar Nemzeti Bank, Sveriges Riksbank. Each marker represents a specific forecast horizon h. For $h = 0$, the marker is closest to the origin, with the colors fading as the forecast horizon extends. First column: The X axis plots the central banks' average ex-ante forecast uncertainty, represented by the horizon-wise time series average of the inflation forecast variance $\hat{\sigma}^2_{t+h|t}$ against the corresponding ex-post forecast uncertainty, represented by the horizon-wise time series average of the squared forecast errors $\hat{e}^2_{t+h|t}$ on the Y axis (markers: red, yellow, green, blue dots, respectively). The second column adds the average forecast uncertainty based on $\hat{\sigma}^2_{\mathrm{BAR},t+h|t}$, on the X axis against the average of $\hat{e}^2_{t+h|t}$ on the Y axis (marker: grey dots). The third column repeats column 1 using a shorter sample to match the rolling window of the PFE approach (MNB, SR: not available) and adds the average forecast uncertainty based on $\hat{\sigma}^2_{\mathrm{PFE},t+h|t}$ on the X axis against the average of $\hat{e}^2_{t+h|t}$ on the Y axis (marker: grey squares).

Table 1: Bank of England: Comparing ex-ante and ex-post uncertainty

h	0	1	2	3	4	5	6	7	8
	Regression equation: $\hat{e}^2_{t+h\|t}/\hat{\sigma}^2_{t+h\|t} = c + u_{t+h}$								
c	0.43	0.96	1.20	1.55	1.86	1.96	1.83	1.66	1.64
	(0.12)	(0.27)	(0.35)	(0.43)	(0.49)	(0.53)	(0.51)	(0.49)	(0.52)
$p(c=1)$	**0.00**	0.88	0.57	0.20	0.08	0.08	0.10	0.18	0.22
	Regression equation: $\hat{e}^2_{t+h\|t}/\hat{\sigma}^2_{\text{BAR},t+h\|t} = c + u_{t+h}$								
c	0.33	1.01	1.56	2.35	3.08	3.25	3.40	3.59	3.62
	(0.07)	(0.28)	(0.45)	(0.68)	(0.81)	(0.88)	(0.94)	(1.08)	(1.19)
$p(c=1)$	**0.00**	0.98	0.22	**0.05**	**0.01**	**0.01**	**0.01**	**0.02**	**0.03**
N	76	75	74	73	72	71	70	69	68
	Regression equation: $\hat{e}^2_{t+h\|t}/\hat{\sigma}^2_{t+h\|t} = c + u_{t+h}$ (with smaller PFE approach sample size)								
c	0.29	1.07	1.51	1.83	2.11	2.40	2.33	2.22	2.27
	(0.10)	(0.35)	(0.48)	(0.52)	(0.62)	(0.69)	(0.70)	(0.72)	(0.74)
$p(c=1)$	**0.00**	0.84	0.30	0.11	0.08	**0.05**	0.06	0.10	0.09
	Regression equation: $\hat{e}^2_{t+h\|t}/\hat{\sigma}^2_{\text{PFE},t+h\|t} = c + u_{t+h}$								
c	1.02	1.60	1.90	1.76	1.71	2.06	2.38	2.53	2.56
	(0.23)	(0.59)	(0.67)	(0.51)	(0.57)	(0.72)	(0.92)	(0.99)	(0.97)
$p(c=1)$	0.93	0.31	0.18	0.14	0.22	0.15	0.14	0.13	0.11
N	56	54	52	50	48	46	44	42	40

Notes: Andrews (1991) HAC standard errors using a quadratic spectral kernel and automatic bandwidth determination in parentheses. Figures for $p(\bullet)$ denote the p-value of the respective hypothesis given in parentheses. Bold figures imply statistical significance at the 5% level. N denotes the number of observations.

Table 2: Banco Central do Brasil: Comparing ex-ante and ex-post uncertainty

h	0	1	2	3	4	5	6	7	8
	Regression equation: $\hat{e}^2_{t+h\|t}/\hat{\sigma}^2_{t+h\|t} = c + u_{t+h}$								
c	2.22	0.53	1.09	1.37	1.37	1.35			
	(0.89)	(0.12)	(0.35)	(0.54)	(0.70)	(0.77)			
$p(c = 1)$	0.17	**0.00**	0.80	0.50	0.60	0.65			
	Regression equation: $\hat{e}^2_{t+h\|t}/\hat{\sigma}^2_{\text{BAR},t+h\|t} = c + u_{t+h}$								
c	0.15	0.21	0.66	1.30	2.12	2.54			
	(0.05)	(0.07)	(0.25)	(0.60)	(1.09)	(1.47)			
$p(c = 1)$	**0.00**	**0.00**	0.18	0.62	0.31	0.30			
N	53	52	51	50	49	48			
	Regression equation: $\hat{e}^2_{t+h\|t}/\hat{\sigma}^2_{t+h\|t} = c + u_{t+h}$ (with smaller PFE approach sample size)								
c	2.02	0.57	1.20	1.56	1.59	1.60			
	(0.99)	(0.13)	(0.37)	(0.58)	(0.77)	(0.87)			
$p(c = 1)$	0.31	**0.00**	0.59	0.34	0.45	0.49			
	Regression equation: $\hat{e}^2_{t+h\|t}/\hat{\sigma}^2_{\text{PFE},t+h\|t} = c + u_{t+h}$								
c	1.15	0.91	1.05	1.42	1.70	1.92			
	(0.49)	(0.33)	(0.43)	(0.72)	(0.88)	(0.96)			
$p(c = 1)$	0.76	0.80	0.91	0.56	0.43	0.34			
N	49	47	45	43	41	39			

Notes: Andrews (1991) HAC standard errors using a quadratic spectral kernel and automatic bandwidth determination in parentheses. Figures for $p(\bullet)$ denote the p-value of the respective hypothesis given in parentheses. Bold figures imply statistical significance at the 5% level. N denotes the number of observations.

Table 3: Magyar Nemzeti Bank: Comparing ex-ante and ex-post uncertainty

h	0	1	2	3	4	5	6	7	8
	Regression equation: $\hat{e}^2_{t+h\|t}/\hat{\sigma}^2_{t+h\|t} = c + u_{t+h}$								
c	1.05	2.49	3.95	3.37	3.24	3.48	2.85		
	(0.35)	(0.87)	(1.96)	(1.84)	(1.72)	(1.60)	(1.09)		
$p(c=1)$	0.89	0.10	0.14	0.21	0.20	0.13	0.10		
	Regression equation: $\hat{e}^2_{t+h\|t}/\hat{\sigma}^2_{\text{BAR},t+h\|t} = c + u_{t+h}$								
c	0.07	0.30	0.38	0.52	0.67	0.77	0.77		
	(0.02)	(0.11)	(0.18)	(0.26)	(0.31)	(0.32)	(0.29)		
$p(c=1)$	**0.00**	**0.00**	**0.00**	0.07	0.29	0.48	0.43		
N	34	34	34	34	34	34	34		

Notes: Andrews (1991) HAC standard errors using a quadratic spectral kernel and automatic bandwidth determination in parentheses. Figures for $p(\bullet)$ denote the p-value of the respective hypothesis given in parentheses. Bold figures imply statistical significance at the 5% level. N denotes the number of observations.

Table 4: Sveriges Riksbank: Comparing ex-ante and ex-post uncertainty

h	0	1	2	3	4	5	6	7	8
	Regression equation: $\hat{e}^2_{t+h\|t}/\hat{\sigma}^2_{t+h\|t} = c + u_{t+h}$								
c	0.42	1.66	1.61	1.15	1.07	1.36	1.00	1.26	
	(0.08)	(0.46)	(0.57)	(0.34)	(0.28)	(0.46)	(0.34)	(0.37)	
$p(c=1)$	**0.00**	0.16	0.30	0.66	0.80	0.45	1.00	0.49	
	Regression equation: $\hat{e}^2_{t+h\|t}/\hat{\sigma}^2_{\text{BAR},t+h\|t} = c + u_{t+h}$								
c	0.43	0.64	0.59	0.46	0.50	0.80	0.70	1.16	
	(0.12)	(0.18)	(0.23)	(0.14)	(0.12)	(0.24)	(0.27)	(0.34)	
$p(c=1)$	**0.00**	0.06	0.08	**0.00**	**0.00**	0.42	0.29	0.64	
N	27	27	27	27	27	27	27	27	

Notes: Andrews (1991) HAC standard errors using a quadratic spectral kernel and automatic bandwidth determination in parentheses. Figures for $p(\bullet)$ denote the p-value of the respective hypothesis given in parentheses. Bold figures imply statistical significance at the 5% level. N denotes the number of observations.

Table 5: Bank of England: Assessing forecast optimality by Mincer-Zarnowitz and related regressions

h	0	1	2	3	4	5	6	7	8
	Regression equation: $\hat{e}^2_{t+h\|t} = c + b\hat{\sigma}^2_{t+h\|t} + u_{t+h}$								
c	0.03	0.18	0.33	0.52	0.60	0.56	0.39	0.34	0.66
	(0.01)	(0.07)	(0.17)	(0.29)	(0.37)	(0.44)	(0.45)	(0.46)	(0.60)
b	-0.01	0.07	0.23	0.45	0.77	1.09	1.32	1.25	0.90
	(0.04)	(0.12)	(0.20)	(0.30)	(0.43)	(0.52)	(0.59)	(0.47)	(0.45)
$p(c = 0, b = 1)$	**0.00**	**0.00**	**0.00**	0.13	0.27	0.37	0.41	0.50	0.50
	Regression equation: $\hat{e}^2_{t+h\|t} = c + b\hat{\sigma}^2_{\text{BAR},t+h\|t} + u_{t+h}$								
c	0.03	0.21	0.50	0.94	1.35	1.58	1.65	1.56	1.43
	(0.01)	(0.08)	(0.18)	(0.35)	(0.60)	(0.78)	(0.83)	(0.74)	(0.58)
b	-0.01	0.00	-0.05	-0.10	-0.12	-0.05	0.03	0.13	0.31
	(0.06)	(0.09)	(0.13)	(0.18)	(0.25)	(0.30)	(0.30)	(0.27)	(0.32)
$p(c = 0, b = 1)$	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.01**	**0.01**	**0.01**
	Regression equation: $\|\hat{e}_{t+h\|t}\| = c + b\hat{\sigma}_{t+h\|t} + u_{t+h}$								
c	0.16	0.25	0.26	0.31	0.26	0.20	0.11	0.04	0.14
	(0.03)	(0.08)	(0.14)	(0.20)	(0.24)	(0.24)	(0.23)	(0.20)	(0.26)
b	-0.06	0.18	0.36	0.50	0.73	0.86	0.94	0.97	0.82
	(0.08)	(0.12)	(0.20)	(0.27)	(0.29)	(0.30)	(0.31)	(0.26)	(0.26)
$p(b \leq 0)$	0.79	0.08	**0.04**	**0.03**	**0.01**	**0.00**	**0.00**	**0.00**	**0.00**
	Regression equation: $\|\hat{e}_{t+h\|t}\| = c + b\hat{\sigma}_{\text{BAR},t+h\|t} + u_{t+h}$								
c	0.17	0.29	0.44	0.64	0.77	0.78	0.76	0.68	0.59
	(0.03)	(0.08)	(0.13)	(0.21)	(0.26)	(0.27)	(0.25)	(0.21)	(0.15)
b	-0.08	0.11	0.10	0.09	0.14	0.23	0.28	0.37	0.48
	(0.10)	(0.11)	(0.16)	(0.21)	(0.22)	(0.23)	(0.23)	(0.22)	(0.22)
$p(b \leq 0)$	0.80	0.17	0.27	0.33	0.27	0.16	0.11	**0.05**	**0.02**
N	76	75	74	73	72	71	70	69	68
	Regression equation: $\hat{e}^2_{t+h\|t} = c + b\hat{\sigma}^2_{t+h\|t} + u_{t+h}$ (with smaller PFE approach sample size)								
c	0.04	0.32	0.68	1.02	1.22	1.42	1.34	1.46	2.21
	(0.01)	(0.13)	(0.32)	(0.54)	(0.67)	(0.68)	(0.62)	(0.63)	(1.06)
b	-0.02	-0.11	-0.07	0.12	0.42	0.63	0.84	0.73	0.23
	(0.05)	(0.18)	(0.30)	(0.40)	(0.51)	(0.56)	(0.62)	(0.56)	(0.64)
$p(c = 0, b = 1)$	**0.00**	**0.00**	**0.00**	0.10	0.20	0.11	0.07	**0.05**	0.11
	Regression equation: $\hat{e}^2_{t+h\|t} = c + b\hat{\sigma}^2_{\text{PFE},t+h\|t} + u_{t+h}$								
c	0.06	0.40	0.79	1.28	1.92	2.63	3.00	2.93	2.94
	(0.03)	(0.16)	(0.33)	(0.61)	(1.02)	(1.38)	(1.54)	(1.36)	(1.17)
b	-0.80	-0.61	-0.30	-0.13	-0.16	-0.26	-0.32	-0.27	-0.23
	(0.81)	(0.52)	(0.45)	(0.43)	(0.42)	(0.43)	(0.45)	(0.44)	(0.33)
$p(c = 0, b = 1)$	0.08	**0.01**	**0.02**	**0.04**	**0.01**	**0.00**	**0.00**	**0.00**	**0.00**
	Regression equation: $\|\hat{e}_{t+h\|t}\| = c + b\hat{\sigma}_{t+h\|t} + u_{t+h}$ (with smaller PFE approach sample size)								
c	0.18	0.39	0.57	0.65	0.59	0.62	0.49	0.51	0.80
	(0.04)	(0.12)	(0.21)	(0.28)	(0.31)	(0.28)	(0.25)	(0.25)	(0.37)
b	-0.09	0.01	0.07	0.23	0.49	0.57	0.70	0.66	0.40
	(0.10)	(0.16)	(0.24)	(0.30)	(0.32)	(0.32)	(0.33)	(0.32)	(0.36)
$p(b \leq 0)$	0.83	0.47	0.38	0.22	0.07	**0.04**	**0.02**	**0.02**	0.14
	Regression equation: $\|\hat{e}_{t+h\|t}\| = c + b\hat{\sigma}_{\text{PFE},t+h\|t} + u_{t+h}$								
c	0.30	0.39	0.67	0.82	0.92	1.15	1.24	1.26	1.28
	(0.14)	(0.16)	(0.29)	(0.38)	(0.50)	(0.56)	(0.59)	(0.56)	(0.46)
b	-0.91	0.01	-0.05	0.04	0.13	0.04	-0.01	-0.01	-0.02
	(0.75)	(0.29)	(0.39)	(0.37)	(0.35)	(0.33)	(0.35)	(0.35)	(0.30)
$p(b \leq 0)$	0.88	0.49	0.55	0.45	0.35	0.46	0.51	0.51	0.52
N	56	54	52	50	48	46	44	42	40

Notes: Andrews (1991) HAC standard errors using a quadratic spectral kernel and automatic bandwidth determination in parentheses. Figures for $p(\bullet)$ denote the p-value of the respective hypothesis given in parentheses. Bold figures imply statistical significance at the 5% level. N denotes the number of observations.

Table 6: Banco Central do Brasil: Assessing forecast optimality by Mincer-Zarnowitz and related regressions

h	0	1	2	3	4	5	6	7	8
	Regression equation: $\hat{e}^2_{t+h\|t} = c + b\hat{\sigma}^2_{t+h\|t} + u_{t+h}$								
c	-0.06	-0.03	1.04	2.33	4.90	5.45			
	(0.12)	(0.09)	(0.35)	(1.07)	(2.46)	(2.95)			
b	2.14	0.52	-0.13	-0.21	-0.55	-0.51			
	(1.94)	(0.11)	(0.12)	(0.22)	(0.41)	(0.40)			
$p(c=0, b=1)$	0.80	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**			
	Regression equation: $\hat{e}^2_{t+h\|t} = c + b\hat{\sigma}^2_{\text{BAR},t+h\|t} + u_{t+h}$								
c	0.05	0.19	0.95	2.28	3.94	4.69			
	(0.03)	(0.06)	(0.33)	(1.08)	(2.04)	(2.38)			
b	0.06	0.03	-0.02	-0.07	-0.15	-0.21			
	(0.05)	(0.02)	(0.03)	(0.08)	(0.15)	(0.17)			
$p(c=0, b=1)$	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**			
	Regression equation: $\|\hat{e}_{t+h\|t}\| = c + b\hat{\sigma}_{t+h\|t} + u_{t+h}$								
c	0.06	0.00	0.85	1.32	2.38	2.52			
	(0.14)	(0.15)	(0.27)	(0.46)	(0.88)	(1.03)			
b	0.65	0.57	-0.11	-0.17	-0.55	-0.52			
	(0.60)	(0.18)	(0.18)	(0.23)	(0.37)	(0.40)			
$p(b \leq 0)$	0.14	**0.00**	0.73	0.77	0.93	0.90			
	Regression equation: $\|\hat{e}_{t+h\|t}\| = c + b\hat{\sigma}_{\text{BAR},t+h\|t} + u_{t+h}$								
c	0.14	0.28	0.74	1.23	1.73	2.00			
	(0.06)	(0.08)	(0.20)	(0.37)	(0.53)	(0.63)			
b	0.10	0.09	-0.00	-0.07	-0.15	-0.23			
	(0.08)	(0.04)	(0.07)	(0.10)	(0.14)	(0.16)			
$p(b \leq 0)$	0.12	**0.02**	0.51	0.75	0.87	0.92			
N	53	52	51	50	49	48			
	Regression equation: $\hat{e}^2_{t+h\|t} = c + b\hat{\sigma}^2_{t+h\|t} + u_{t+h}$ (with smaller PFE approach sample size)								
c	0.07	0.12	1.05	2.47	5.24	6.02			
	(0.03)	(0.10)	(0.41)	(1.12)	(2.63)	(3.21)			
b	-0.07	0.27	-0.13	-0.25	-0.69	-0.73			
	(0.12)	(0.17)	(0.20)	(0.24)	(0.50)	(0.54)			
$p(c=0, b=1)$	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**			
	Regression equation: $\hat{e}^2_{t+h\|t} = c + b\hat{\sigma}^2_{\text{PFE},t+h\|t} + u_{t+h}$								
c	0.07	0.20	0.94	2.27	3.94	4.61			
	(0.03)	(0.07)	(0.36)	(1.04)	(1.94)	(2.24)			
b	-0.04	0.09	-0.01	-0.05	-0.08	-0.11			
	(0.06)	(0.06)	(0.05)	(0.07)	(0.08)	(0.08)			
$p(c=0, b=1)$	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**			
	Regression equation: $\|\hat{e}_{t+h\|t}\| = c + b\hat{\sigma}_{t+h\|t} + u_{t+h}$ (with smaller PFE approach sample size)								
c	0.21	0.11	0.84	1.34	2.56	2.96			
	(0.06)	(0.17)	(0.32)	(0.46)	(0.99)	(1.25)			
b	-0.05	0.44	-0.09	-0.16	-0.67	-0.80			
	(0.18)	(0.22)	(0.26)	(0.22)	(0.46)	(0.55)			
$p(b \leq 0)$	0.61	**0.03**	0.64	0.77	0.92	0.92			
	Regression equation: $\|\hat{e}_{t+h\|t}\| = c + b\hat{\sigma}_{\text{PFE},t+h\|t} + u_{t+h}$								
c	0.21	0.27	0.71	1.21	1.74	2.01			
	(0.06)	(0.10)	(0.23)	(0.39)	(0.52)	(0.57)			
b	-0.03	0.19	0.03	-0.04	-0.12	-0.19			
	(0.10)	(0.10)	(0.10)	(0.10)	(0.11)	(0.11)			
$p(b \leq 0)$	0.61	**0.03**	0.40	0.65	0.86	0.96			
N	49	47	45	43	41	39			

Notes: Andrews (1991) HAC standard errors using a quadratic spectral kernel and automatic bandwidth determination in parentheses. Figures for $p(\bullet)$ denote the p-value of the respective hypothesis given in parentheses. Bold figures imply statistical significance at the 5% level. N denotes the number of observations.

Table 7: Magyar Nemzeti Bank: Assessing forecast optimality by Mincer-Zarnowitz and related regressions

h	0	1	2	3	4	5	6	7	8
	Regression equation: $\hat{e}^2_{t+h\|t} = c + b\hat{\sigma}^2_{t+h\|t} + u_{t+h}$								
c	0.07	1.14	3.46	9.43	14.23	13.09	11.53		
	(0.03)	(0.43)	(1.60)	(4.85)	(7.52)	(5.86)	(5.23)		
b	-0.02	-0.93	-2.36	-4.77	-4.75	-2.89	-1.72		
	(0.07)	(0.56)	(1.38)	(2.82)	(2.92)	(1.70)	(1.22)		
$p(c = 0, b = 1)$	**0.00**	**0.00**	0.06	0.10	**0.04**	**0.04**	0.10		
	Regression equation: $\hat{e}^2_{t+h\|t} = c + b\hat{\sigma}^2_{\mathrm{BAR},t+h\|t} + u_{t+h}$								
c	0.05	0.87	1.88	3.44	5.07	6.01	6.45		
	(0.03)	(0.36)	(1.08)	(2.16)	(3.19)	(3.45)	(2.93)		
b	0.02	-0.04	-0.03	-0.02	0.01	0.03	0.01		
	(0.02)	(-0.04)	(-0.03)	(-0.02)	(0.01)	(0.03)	(0.01)		
$p(c = 0, b = 1)$	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**		
	Regression equation: $\|\hat{e}_{t+h\|t}\| = c + b\hat{\sigma}_{t+h\|t} + u_{t+h}$								
c	0.18	1.12	2.09	4.23	4.74	4.01	3.20		
	(0.06)	(0.34)	(0.69)	(1.59)	(2.27)	(1.81)	(1.51)		
b	0.07	-0.77	-1.38	-2.59	-2.21	-1.31	-0.61		
	(0.12)	(0.48)	(0.72)	(1.23)	(1.40)	(0.97)	(0.72)		
$p(b \leq 0)$	0.28	0.94	0.97	0.98	0.94	0.91	0.80		
	Regression equation: $\|\hat{e}_{t+h\|t}\| = c + b\hat{\sigma}_{\mathrm{BAR},t+h\|t} + u_{t+h}$								
c	0.06	0.69	0.71	1.22	1.38	1.44	1.81		
	(0.08)	(0.29)	(0.41)	(0.62)	(0.75)	(0.75)	(0.55)		
b	0.14	-0.03	0.10	0.04	0.11	0.19	0.11		
	(0.06)	(0.12)	(0.10)	(0.16)	(0.15)	(0.12)	(0.06)		
$p(b \leq 0)$	**0.01**	0.61	0.16	0.40	0.24	0.06	**0.03**		
N	34	34	34	34	34	34	34		

Notes: Andrews (1991) HAC standard errors using a quadratic spectral kernel and automatic bandwidth determination in parentheses. Figures for $p(\bullet)$ denote the p-value of the respective hypothesis given in parentheses. Bold figures imply statistical significance at the 5% level. N denotes the number of observations.

Table 8: Sveriges Riksbank: Assessing forecast optimality by Mincer-Zarnowitz and related regressions

h	0	1	2	3	4	5	6	7	8
	Regression equation: $\hat{e}^2_{t+h\|t} = c + b\hat{\sigma}^2_{t+h\|t} + u_{t+h}$								
c	-0.13	0.45	0.70	0.12	1.38	1.84	2.51	4.39	
	(0.06)	(0.30)	(0.30)	(0.74)	(0.55)	(1.39)	(1.59)	(2.86)	
b	1.84	-1.30	-1.38	0.81	-1.50	-1.29	-1.77	-2.44	
	(0.59)	(1.64)	(0.78)	(2.01)	(0.84)	(1.79)	(1.58)	(2.26)	
$p(c = 0, b = 1)$	**0.00**	0.33	**0.00**	0.90	**0.00**	0.42	0.19	0.32	
	Regression equation: $\hat{e}^2_{t+h\|t} = c + b\hat{\sigma}^2_{\text{BAR},t+h\|t} + u_{t+h}$								
c	0.08	0.52	0.73	0.41	0.35	1.04	0.65	1.53	
	(0.05)	(0.21)	(0.38)	(0.28)	(0.27)	(0.57)	(0.38)	(0.77)	
b	-0.33	-0.63	-0.51	-0.00	0.18	-0.08	0.19	-0.04	
	(0.39)	(0.39)	(0.36)	(0.17)	(0.19)	(0.26)	(0.20)	(0.38)	
$p(c = 0, b = 1)$	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.03**	
	Regression equation: $\|\hat{e}_{t+h\|t}\| = c + b\hat{\sigma}_{t+h\|t} + u_{t+h}$								
c	-0.47	0.57	1.04	-0.48	1.86	1.77	3.87	4.13	
	(0.20)	(0.48)	(0.31)	(1.49)	(0.87)	(1.44)	(1.93)	(2.46)	
b	2.06	-0.48	-1.14	1.70	-1.65	-1.17	-3.24	-2.84	
	(0.64)	(1.17)	(0.52)	(2.43)	(1.11)	(1.62)	(1.95)	(2.14)	
$p(b \leq 0)$	**0.00**	0.66	0.98	0.25	0.93	0.76	0.95	0.90	
	Regression equation: $\|\hat{e}_{t+h\|t}\| = c + b\hat{\sigma}_{\text{BAR},t+h\|t} + u_{t+h}$								
c	0.28	0.87	1.00	0.30	0.28	0.85	0.11	0.60	
	(0.18)	(0.30)	(0.52)	(0.45)	(0.37)	(0.65)	(0.29)	(0.56)	
b	-0.35	-0.76	-0.62	0.23	0.35	-0.06	0.59	0.38	
	(0.55)	(0.42)	(0.54)	(0.38)	(0.33)	(0.53)	(0.30)	(0.43)	
$p(b \leq 0)$	0.73	0.96	0.87	0.27	0.15	0.54	**0.03**	0.20	
N	27	27	27	27	27	27	27	27	

Notes: Andrews (1991) HAC standard errors using a quadratic spectral kernel and automatic bandwidth determination in parentheses. Figures for $p(\bullet)$ denote the p-value of the respective hypothesis given in parentheses. Bold figures imply statistical significance at the 5% level. N denotes the number of observations.

Table 9: Banco Central do Brasil: Additional full sample results

h	0	1	2	3	4	5	6	7	8
	Regression equation: $\hat{e}^2_{t+h\|t}/\hat{\sigma}^2_{t+h\|t} = c + u_{t+h}$								
c	1.90	0.73	1.53	2.06	2.25	2.22			
	(0.69)	(0.22)	(0.57)	(0.84)	(0.94)	(0.95)			
$p(c = 1)$	0.20	0.22	0.35	0.21	0.19	0.20			
	Regression equation: $\hat{e}^2_{t+h\|t} = c + b\hat{\sigma}^2_{t+h\|t} + u_{t+h}$								
c	0.12	-0.37	-1.02	-0.52	-0.33	1.28			
	(0.06)	(0.51)	(2.26)	(2.98)	(4.28)	(4.19)			
b	0.43	1.34	2.35	2.41	2.42	2.00			
	(0.40)	(0.96)	(2.36)	(2.28)	(1.88)	(1.35)			
$p(c = 0, b = 1)$	0.08	0.32	0.72	0.39	0.48	0.54			
	Regression equation: $\|\hat{e}_{t+h\|t}\| = c + b\hat{\sigma}_{t+h\|t} + u_{t+h}$								
c	0.17	-0.21	0.24	0.56	0.63	0.73			
	(0.06)	(0.28)	(0.58)	(0.85)	(1.09)	(1.11)			
b	0.26	0.90	0.64	0.60	0.71	0.69			
	(0.20)	(0.41)	(0.62)	(0.72)	(0.67)	(0.61)			
$p(b \leq 0)$	0.10	**0.02**	0.15	0.21	0.15	0.13			
N	69	68	67	66	65	64			

Notes: Andrews (1991) HAC standard errors using a quadratic spectral kernel and automatic bandwidth determination in parentheses. Figures for $p(\bullet)$ denote the p-value of the respective hypothesis given in parentheses. Bold figures imply statistical significance at the 5% level. N denotes the number of observations.

Table 10: Testing for unconditional predictive accuracy

h	0	1	2	3	4	5	6	7	8
	Bank of England's VS. BAR approach uncertainty forecasts								
DM(BoE, BAR)	0.47	-0.97	-1.80	-2.15	-2.60	-2.34	-2.45	-2.40	-1.86
p(DM = 0)	0.64	0.33	0.07	**0.03**	**0.01**	**0.02**	**0.01**	**0.02**	0.06
N	76	75	74	73	72	71	70	69	68
	Bank of England's VS. PFE approach uncertainty forecasts								
DM(BoE, PFE)	3.46	-0.13	-0.79	-0.29	0.46	-0.16	-1.13	-1.30	-1.31
p(DM = 0)	**0.00**	0.90	0.43	0.77	0.64	0.88	0.26	0.20	0.19
N	56	54	52	50	48	46	44	42	40
	Banco Central do Brasil's VS. BAR approach uncertainty forecasts								
DM(BCB, BAR)	-0.50	-3.07	-2.34	-2.37	-2.12	-1.85			
p(DM = 0)	0.62	**0.00**	**0.02**	**0.02**	**0.03**	0.06			
N	53	52	51	50	49	48			
	Banco Central do Brasil's VS. PFE approach uncertainty forecasts								
DM(BCB, PFE)	0.74	-1.18	-1.95	-2.62	-2.14	-2.11			
p(DM = 0)	0.46	0.24	0.05	**0.01**	**0.03**	**0.03**			
N	49	47	45	43	41	39			
	Magyar Nemzeti Bank's VS. BAR approach uncertainty forecasts								
DM(MNB, BAR)	-4.41	0.25	0.95	0.79	0.85	1.05	0.95		
p(DM = 0)	**0.00**	0.80	0.34	0.43	0.40	0.29	0.34		
N	34	34	34	34	34	34	34		
	Sveriges Riksbank's VS. BAR approach uncertainty forecasts								
DM(SR, BAR)	-1.92	0.04	-0.18	-1.59	-1.30	0.22	-0.18	0.09	
p(DM = 0)	0.05	0.96	0.86	0.11	0.19	0.83	0.86	0.92	
N	27	27	27	27	27	27	27	27	

Notes: The unconditional predictive accuracy of central banks' uncertainty forecasts is evaluated using the test procedure of Diebold and Mariano (1995), where a forecast's loss is measured by the score $DSS_{t+h|t}$ as proposed by Dawid and Sebastiani (1999). Values of DM represent the Diebold-Mariano test statistic, a negative sign implies that the central bank's uncertainty forecast scores better than the alternative, and $p(\bullet)$ denotes the p-value of the hypothesis given in parentheses. Bold figures imply statistical significance at the 5% level. N denotes the number of observations.

Table 11: Correlations of central bank variance forecasts with potential measures of economic uncertainty

h	0	1	2	3	4	5	6	7	8
	Bank of England								
VIX	-0.18	-0.17	-0.17	-0.16	-0.16	-0.14	-0.11	-0.08	-0.08
oilpvol	-0.19	-0.18	-0.18	-0.17	-0.17	-0.16	-0.15	-0.13	-0.13
	Banco Central Do Brasil								
VIX	0.11	0.07	0.01	0.00	0.01	0.01			
oilpvol	-0.01	0.04	0.03	0.03	0.03	0.04			
	Magyar Nemzeti Bank								
VIX	0.23	0.30	0.43	0.57	0.57	0.58	0.59		
oilpvol	-0.06	-0.04	0.05	0.12	0.16	0.20	0.22		
	Sveriges Riksbank								
VIX	0.29	0.10	0.44	0.45	0.45	0.51	0.52	0.49	
oilpvol	0.11	0.18	0.82	0.78	0.66	0.66	0.66	0.65	

Notes: ‘VIX’ denotes the monthly Chicago Board Options Exchange Volatility Index, ‘oilpvol’ denotes the variance of the daily oil price changes in percent within a month, using the Brent crude oil price in US-Dollars. Data is selected as to precede the publication dates of the respective central bank’s forecast by a month.

Table 12: Testing for conditional predictive accuracy - VIX

h	0	1	2	3	4	5	6	7	8
	Bank of England's VS. BAR approach uncertainty forecasts								
GW(BoE, BAR)	0.27	-1.01	-1.66	-2.09	-2.49	-2.16	-2.21	-2.52	-2.00
$p(\text{GW} = 0)$	0.79	0.31	0.10	**0.04**	**0.01**	**0.03**	**0.03**	**0.01**	**0.05**
N	76	75	74	73	72	71	70	69	68
	Bank of England's VS. PFE approach uncertainty forecasts								
GW(BoE, PFE)	3.40	-0.26	-0.88	-0.55	0.32	-0.31	-1.36	-1.41	-1.39
$p(\text{GW} = 0)$	**0.00**	0.80	0.38	0.58	0.75	0.76	0.18	0.16	0.16
N	56	54	52	50	48	46	44	42	40
	Banco Central do Brasil's VS. BAR approach uncertainty forecasts								
GW(BCB, BAR)	-0.43	-3.38	-2.68	-2.24	-1.91	-1.88			
$p(\text{GW} = 0)$	0.67	**0.00**	**0.01**	**0.03**	0.06	0.06			
N	53	52	51	50	49	48			
	Banco Central do Brasil's VS. PFE approach uncertainty forecasts								
GW(BCB, PFE)	0.50	-1.16	-2.65	-3.14	-2.40	-2.21			
$p(\text{GW} = 0)$	0.62	0.25	**0.01**	**0.00**	**0.02**	**0.03**			
N	49	47	45	43	41	39			
	Magyar Nemzeti Bank's VS. BAR approach uncertainty forecasts								
GW(MNB, BAR)	-3.96	0.04	0.68	0.35	0.42	0.75	0.65		
$p(\text{GW} = 0)$	**0.00**	0.97	0.50	0.72	0.68	0.46	0.52		
N	34	34	34	34	34	34	34		
	Sveriges Riksbank's VS. BAR approach uncertainty forecasts								
GW(SR, BAR)	-1.58	0.16	-0.14	-1.61	-0.92	0.49	0.18	0.45	
$p(\text{GW} = 0)$	0.11	0.88	0.89	0.11	0.36	0.63	0.86	0.65	
N	27	27	27	27	27	27	27	27	

Notes: The conditional predictive accuracy of central banks' uncertainty forecasts is evaluated using the test procedure of Giacomini and White (2006), where a forecast's loss is measured by the score $DSS_{t+h|t}$ as proposed by Dawid and Sebastiani (1999). Values of GW represent the Giacomoni-White test statistic, a negative sign implies that the central bank's uncertainty forecast scores better than the alternative, and $p(\bullet)$ denotes the p-value of the hypothesis given in parentheses. Bold figures imply statistical significance at the 5% level. N denotes the number of observations.

Table 13: Testing for conditional predictive accuracy - Oil price volatility

h	0	1	2	3	4	5	6	7	8
	Bank of England's VS. BAR approach uncertainty forecasts								
GW(BoE, BAR)	-0.32	-1.29	-1.54	-2.05	-2.36	-2.28	-2.34	-2.26	-1.96
$p(\text{GW} = 0)$	0.75	0.20	0.12	**0.04**	**0.02**	**0.02**	**0.02**	**0.02**	0.05
N	76	75	74	73	72	71	70	69	68
	Bank of England's VS. PFE approach uncertainty forecasts								
GW(BoE, PFE)	2.19	-0.10	-0.51	-0.15	1.10	0.83	-0.98	-1.22	-1.23
$p(\text{GW} = 0)$	**0.03**	0.92	0.61	0.88	0.27	0.41	0.33	0.22	0.22
N	56	54	52	50	48	46	44	42	40
	Banco Central do Brasil's VS. BAR approach uncertainty forecasts								
GW(BCB, BAR)	0.23	-3.52	-2.62	-2.63	-1.51	-1.46			
$p(\text{GW} = 0)$	0.82	**0.00**	**0.01**	**0.01**	0.13	0.15			
N	53	52	51	50	49	48			
	Banco Central do Brasil's VS. PFE approach uncertainty forecasts								
GW(BCB, PFE)	0.68	-2.28	-2.43	-2.94	-2.19	-1.74			
$p(\text{GW} = 0)$	0.50	**0.02**	**0.02**	**0.00**	**0.03**	0.08			
N	49	47	45	43	41	39			
	Magyar Nemzeti Bank's VS. BAR approach uncertainty forecasts								
GW(MNB, BAR)	-2.71	0.11	0.23	0.14	0.50	0.78	0.76		
$p(\text{GW} = 0)$	**0.01**	0.91	0.82	0.89	0.62	0.44	0.44		
N	34	34	34	34	34	34	34		
	Sveriges Riksbank's VS. BAR approach uncertainty forecasts								
GW(SR, BAR)	-1.22	-0.00	0.21	-0.77	-1.36	-0.07	-0.85	0.40	
$p(\text{GW} = 0)$	0.22	1.00	0.83	0.44	0.17	0.94	0.39	0.69	
N	27	27	27	27	27	27	27	27	

Notes: The conditional predictive accuracy of central banks' uncertainty forecasts is evaluated using the test procedure of Giacomini and White (2006), where a forecast's loss is measured by the score $DSS_{t+h|t}$ as proposed by Dawid and Sebastiani (1999). Values of GW represent the Giacomoni-White test statistic, a negative sign implies that the central bank's uncertainty forecast scores better than the alternative, and $p(\bullet)$ denotes the p-value of the hypothesis given in parentheses. Bold figures imply statistical significance at the 5% level. N denotes the number of observations.