Business Cycles in Space

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[1] The views expressed are those of the authors and should not be interpreted as reflecting the views of the Bank of Canada, the Deutsche Bundesbank, the Eurosystem, or their staffs.

Motivation

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- Does not look at business cycles; usually no shocks in the model.
- NEG models are usually not forward looking.
- Dynamic NEG models usually contain just a few regions.
    - Difficult to track spatial dynamics.
    - Non-atomistic regions make optimal policy very difficult.
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We attempt to bring two literatures together:

- Propose new approach to building macro models with spatial heterogeneity.
- Study spatial effects of business cycles.

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- Firms physically closer to frontier firms catch up quicker (Griffith, Redding & Simpson 2009, Comin, Dmitriev & Rossi-Hansberg 2012, Cardamone 2017).
- Physical distance more important then economic distance in cross-country spillover of TFP growth (Glass, Kenjegalieva & Paez-Farrell 2013).

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Fortunately, spatial correlation enhances tractability of heterogeneous agent models.

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Individual shocks generate aggregate volatility:

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- Role for redistribution across space.
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Many interesting questions, for example:

- How do regional asymmetries effect transmission of monetary policy?
- How do local shocks affect internal migration / housing markets / labour markets across physical space?
- Who are the regional winners/losers to policy programmes or other macro/local shocks?

▸ Example [another paper]

Contributions

1. We propose a new approach to building macro models featuring spatial heterogeneity.
   - Our approach is quite general:
     - Space need not be physical space.
       E.g it could be the space of product-categories or labour skill levels.
     - The geometry of space is flexible.
       E.g. it could be a plane, torus, sphere or network.
   - Correlated shocks actually help computation:
     - Continuity means relatively few grid points are needed.
   - We provide general conditions for the existence of spatially correlated shock processes.

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    - Correlated shocks actually help computation:
        - Continuity means relatively few grid points are needed.
    - We provide general conditions for the existence of spatially correlated shock processes.

2. We develop a spatial DSGE model with the key ingredients from the NEG literature.
    - Strong agglomeration forces will lead to persistent movements in population and capital.

An example spatially correlated shock process

Suppose space is 1-dimensional over the interval $[0, 1]$.

One shock process over this space is the Orstein-Uhlenbeck process:

- Continuous time (space) analogue to Gaussian AR(1) process.
- Characterised by covariance structure:

$$\operatorname{cov}\left(\varepsilon_{x}, \varepsilon_{\tilde{x}}\right) = \exp\left(-\zeta|x - \tilde{x}|\right)$$

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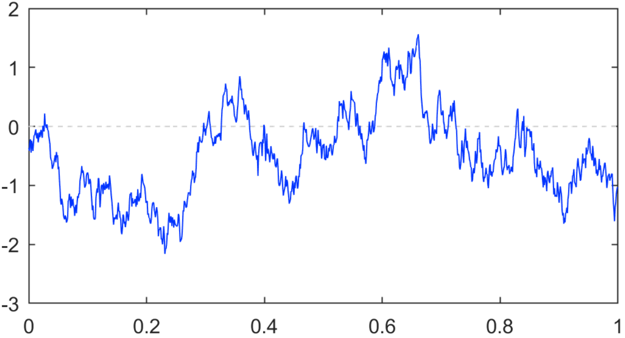
E.g. $\zeta = 8$:



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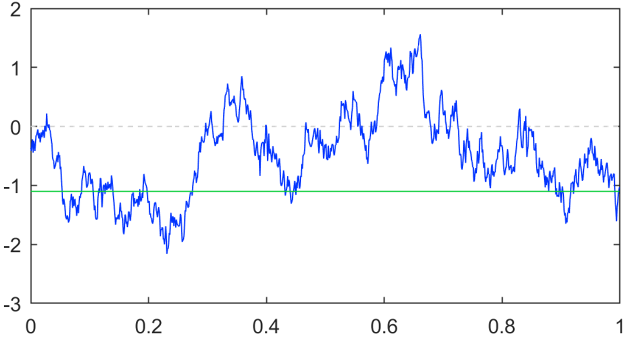

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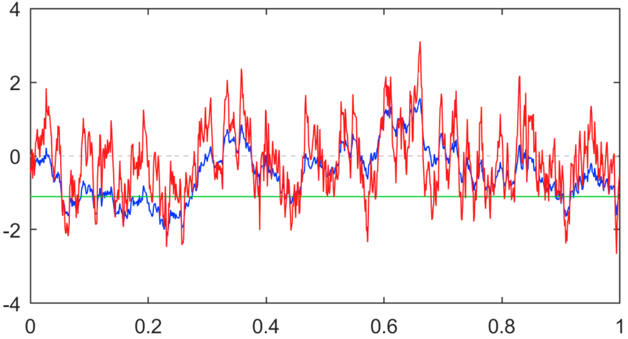
E.g. $\zeta = 8$, $\zeta = 0$ and $\zeta = 100$:



General modelling approach I

1. Define the geometry of the relevant space: plane, circle, torus, network, etc.
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    - E.g., suppose space is a circle, indexed by $x \in X$.
2. Define the model: objectives, markets, frictions and spatial shock.
    - There will be conditions at each location x giving decisions and state evolution.
    - There will be some aggregate / market conditions.
    - Example spatial stochastic process:

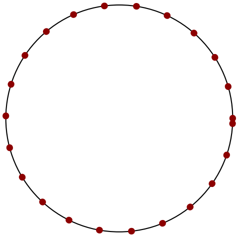
$$a_{x,t} = \rho a_{x,t-1} + \sigma \varepsilon_{x,t}$$
$$\operatorname{cov}\left(\varepsilon_{x,t}, \varepsilon_{\tilde{x},t}\right) = s\left(\zeta, d\left(x, \tilde{x}\right)\right)$$

- ζ will control spatial correlation.
- s and d must fulfil certain conditions to produce a valid process. See paper.
- Often $s(\zeta, d) = \exp\left(-\zeta d\right)$.
- Note, with $a_t \equiv \int_0^1 a_{x,t}\,\mathrm{d}x$ and $\varepsilon_t \equiv \int_0^1 \varepsilon_{x,t}\,\mathrm{d}x$, we have:

$$a_t = \rho a_{t-1} + \sigma \varepsilon_t$$

General modelling approach II

3. Choose grid geometry, e.g. $N = 100$ evenly spaced points.
    - Note on accuracy:
        - Bounded variation of shock implies error from using the trapezium rule decays at $1/N^2$ at the slowest.
        - For sufficiently smooth functions the rate is k^{-N} for some $k > 1$ (thanks to periodicity).
        - Compare with $1/\sqrt{N}$, with Monte Carlo used in e.g. Krusell-Smith.



- Approximate outcomes at locations between nodes via linear interpolation.
- Approximate integrals over space via the trapezium rule.
- E.g. market clearing conditions of the form: $0 = \int_0^1 B_{x,t}\,\mathrm{d}x$ become $0 = \sum_{n=1}^{N} B_{x_n,t}$.

General modelling approach II

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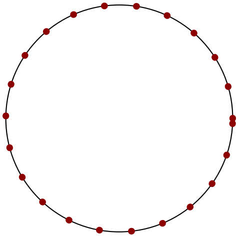


4. Solve via perturbation. e.g., with Dynare.
    - We provide a toolkit to help define spatially correlated shocks: https://github.com/tholden/DynareTransformationEngine

Model Overview

- RBC model + standard new economic geography features (following Krugman 1991).
- Key features:
  - Population movement.
  - Competing land usage: farming and residential.
  - Non-tradeable raw goods (production services).
  - Two types of final good: agricultural products and manufactured products.
  - Differentiated intermediate manufactured goods subject to iceberg trade costs.
  - Firm entry à la Bilbiie, Ghironi & Melitz (2012).

Agglomeration mechanism

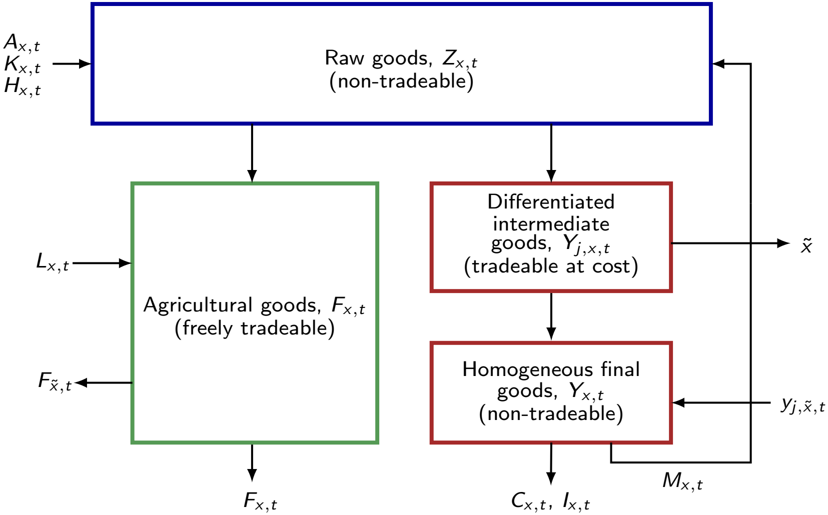
The key mechanism works as follows:

- Productivity shock at x increases wages there.
- People move to x for higher wages.
- The increased demand leads to firm entry.
- More products on sale implies increased productivity due to the taste for variety.
- This feeds back to higher wages, more migration, more firm entry etc.
- Nearby locations also benefit as iceberg transport costs mean the increased demand from x is concentrated in its neighbourhood.

Households and firms over space

- Set of points in space $x \in X$, normalised so $\int_X \mathrm{d}x = 1$.
- Distance between any $x, \tilde{x} \in X$ given by $d(x, \tilde{x})$.
- Firms, capital and population have a density over space.
- Land is uniform over space.
- Representative household at each location, each household is part of a representative family.
- Representative family maximises a utilitarian social welfare function.
  - Equivalent to complete markets.

Productive sectors at $x \in X$



Production at $x \in X$

▸ Skip details

- Raw good: $Z_{x,t} = \left[K_{x,t-1}^{\alpha}\left(A_{x,t}H_{x,t}\right)^{1-\alpha}\right]^{1-\kappa} M_{x,t}^{\kappa}$
  - Non-tradeable.
  - Sold at $\mathcal{P}_{x,t}$.

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- Manufactured good: $Y_{j,x,t} = Z_{j,x,t}$.
  - Firm entry cost ϕ_t units of raw good, exit rate δ_t.
  - Tradeable subject to iceberg costs (increasing in distance, τ_t controls rate).
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Capital holding companies at $x \in X$

- Capital law of motion:

$$K_{x,t} = (1 - \delta_K) K_{x,t-1} + \left[1 - \Phi \left(\frac{I_{x,t}}{I_{x,t-1}} \right) I_{x,t} \right]$$

- Capital rented out at $\mathcal{R}_{K,x,t}$ per unit to firms at x.
- Standard CEE assumptions about investment adjustment costs, $\Phi(\cdot)$.
- Location specific capital stocks and adjustment costs make it particularly hard to move capital between locations.

Households and the representative family

Family head maximizes:

▸ FOCs

$$\mathbb{E}_t \sum_{s=0}^{\infty} \left[\prod_{k=1}^{s} \beta_{t+k-1} \right] \int_X N_{x,t+s-1} \frac{U_{x,t+s}^{1-\varsigma}}{1-\varsigma} \, \mathrm{d}x,$$

s.t.

$$\int \left(P_{x,t} C_{x,t} + E_{x,t} \right) \mathrm{d}x + B_t = \int \left(\mathcal{R}_{L,x,t} L_{x,t} + W_{x,t} H_{x,t} \right) \mathrm{d}x + R_{t-1} B_{t-1} + T_t$$

where:

$$U_{x,t} = \left(\frac{C_{x,t}}{N_{x,t-1}} \right)^{\theta_C} \left(\frac{E_{x,t}}{N_{x,t-1}} \right)^{\theta_F} \left(\frac{1 - L_{x,t}}{N_{x,t-1}} \right)^{\theta_L} \cdots$$

$$N_{t-1} \equiv \int_X N_{\tilde{x},t-1} \, \mathrm{d}\tilde{x}, \qquad \mathcal{D}_{x,t} \equiv \int_X d\left(x, \tilde{x}\right) \mathcal{N}_{x,\tilde{x},t} \, \mathrm{d}\tilde{x},$$

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Market clearing

$$B_t = 0$$

$$Y_{x,t} = C_{x,t} + I_{x,t} + M_{x,t}$$

$$Z_{x,t} = Z_{F,x,t} + \phi_t \left[J_{x,t} - (1 - \delta_J) J_{x,t-1} \right] + \int_0^{J_{x,t}} Z_{j,x,t} \, \mathrm{d}j$$

$$\int E_{x,t} \, \mathrm{d}x = \int F_{x,t} \, \mathrm{d}x$$

Stochastic processes

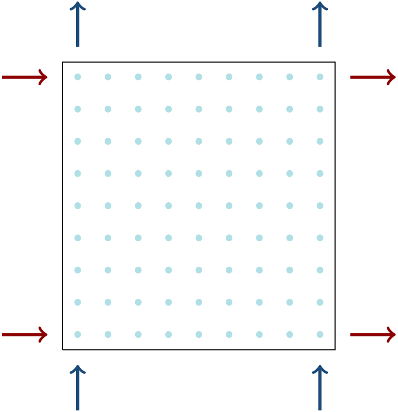
Technology:

$$A_{x,t} = A_t^P A_{x,t}^T$$
$$A_t^P = G_{A,t} A_{t-1}^P$$
$$\log G_{A,t} = \left(1 - \rho_{G_A}\right) \log G_A + \rho_{G_A} \log G_{A,t-1} + \sigma_{G_A} \varepsilon_{G_A,t}$$
$$\log A_{x,t}^T = \rho_A \log A_{x,t-1}^T + \sigma_A \varepsilon_{A,x,t}$$

Similar AR(1) processes for $G_{N,t}$, τ_t, ϕ_t and β_t.

Choice of space

- We choose a torus since this has a uniform steady state:



- Distance metric and continuous stochastic process:

▸ ACF

$$d\left([x_1, x_2], [\tilde{x}_1, \tilde{x}_2]\right) = \sqrt{\left(\min\left\{|x_1 - \tilde{x}_1|, 1 - |x_1 - \tilde{x}_1|\right\}\right)^2 + \left(\min\left\{|x_2 - \tilde{x}_2|, 1 - |x_2 - \tilde{x}_2|\right\}\right)^2}$$

$$\operatorname{cov}\left(\varepsilon_{A,x,t}, \varepsilon_{A,\tilde{x},t}\right) = s\left(\zeta, d\left(x, \tilde{x}\right)\right)$$

Shock calibration

- We have experimented with various specifications for calibrating the spatial persistence of the shock, using quarterly regional wages (1983-2016) as a proxy for productivity.

- Our results are robust across all the specifications we have tried including:
  - both state level and county level wages,
  - both fixed effect, dynamic factor and state space approaches to removing common variation over time,
  - both in differences and filtered.

- In all cases we estimate $\Sigma_{i,j} = \sigma_i \sigma_j \exp\left[-\zeta d_{i,j}\right]$, where $d_{i,j}$ is the geodesic distance between region centroids, normalized so that the maximum distance is $\sqrt{2}$.

- We estimate $\zeta \approx 7$ and set $\zeta = 8$ as our model also generates endogenous spatial persistence.

Selected calibration

- U.S. evidence suggests that the average home buyer stays in their house for around 13 years (NAHB/DHUD).
  - Calibrate ψ_3 to hit a proportion of $\frac{1}{12.5 \times 4} = \frac{1}{50}$ household members moving each quarter.
- About 75% U.S. land is in broadly agricultural usage (USDA).
  - Set $\theta_L = \frac{1-0.75}{0.75} \gamma \theta_F$.
- Spending on food in U.S. is around 20% of personal consumption expenditure excluding housing (BEA).
  - Set $\theta_F = \frac{1}{4} \theta_C$.
- U.S. population density is $41.5/\text{km}^2$, but ranges between $2.33/\text{km}^2$ for Wyoming and $470/\text{km}^2$ for New Jersey.
  - Correspond to absolute log ratios to the whole U.S. of 2.88 and 2.43 respectively.
  - Set $\Omega = 3$ to allow for such dispersion.

Remaining parameters

- Set $\theta_H = \theta_C$, and $\psi_1 = \psi_2 = \frac{\theta_F}{2}$, so one remaining degree of freedom in preference share parameters.
- We set θ_N to generate a high degree of persistence in population movements, while ensuring stability of the symmetric steady-state.
  - A more careful calibration will be in future versions.
- All parameters:

$$\alpha = 0.3, \gamma = 0.5, \kappa = 0.5, \nu = 2, \varsigma = 1.5, \zeta = 8, \lambda = 0.1, \delta_j = 0.01, \delta_k = 0.03,$$

$$\Gamma = 1, \Omega = 3, \Phi^{''}(1) = 4,$$

$$\theta_C = \theta_H = 0.2618, \theta_F = \frac{\theta_C}{4}, \theta_L = 0.0109, \theta_N = 0.3338,$$

$$\psi_1 = \psi_2 = \frac{\theta_F}{2}, \psi_3 = 0.007,$$

$$G_A = 1.005, G_N = 1.0025, \tau = 1, \phi = 1, \beta = 0.99,$$

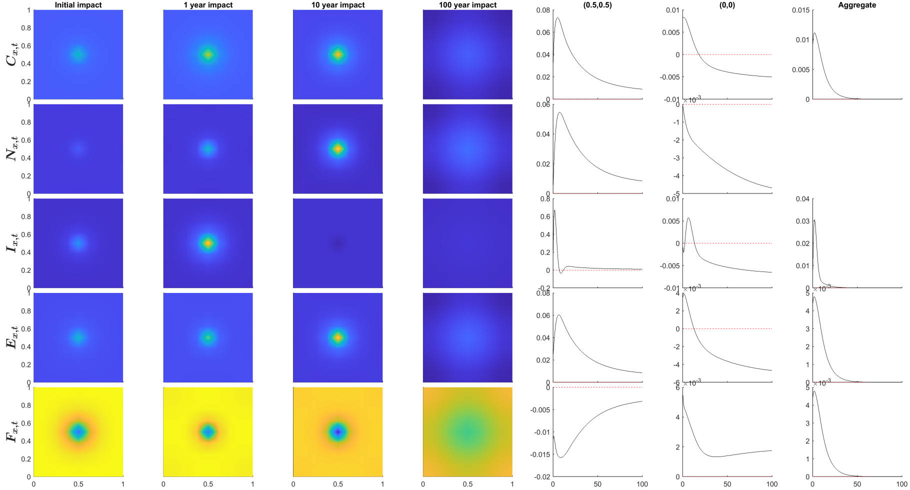
$$\rho_A = 0.9, \rho_{G_A} = 0.8, \rho_{G_N} = 0.5, \rho_\tau = 0.95, \rho_\phi = 0.95, \rho_\beta = 0.95,$$

$$\sigma_A = \sigma_{G_A} = \sigma_{G_N} = \sigma_\tau = \sigma_\phi = \sigma_\beta = 0.001.$$

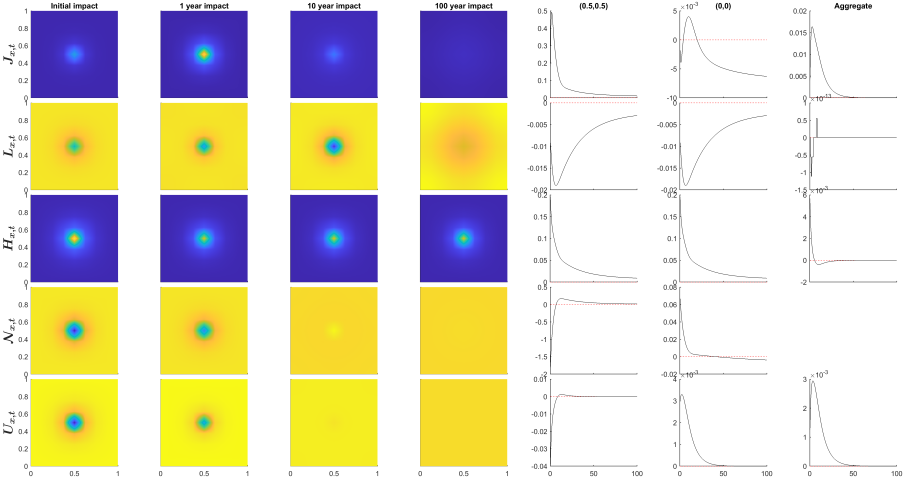
Numerical Simulations

- Use a 9×9 square grid.
- IRF simulations:
  - 1% spatial productivity shock.
  - Focus on shock centred on the point $\left(\frac{1}{2}, \frac{1}{2}\right)$.
- 10,000 year stochastic simulation (video).

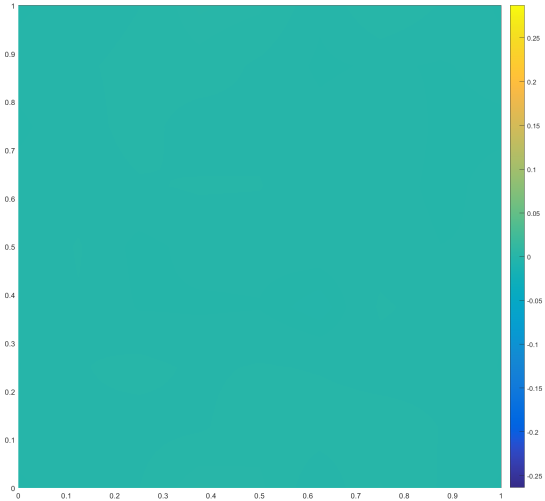
IRF to spatial productivity shock I



IRF to spatial productivity shock II



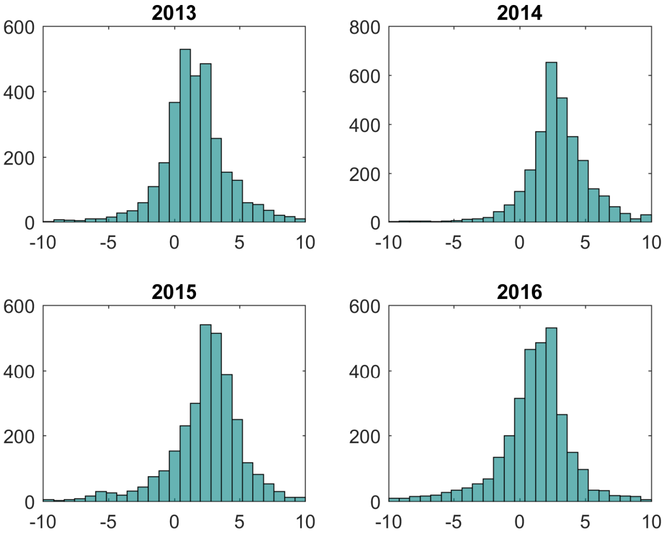
1000 years of population movements (Alternative link)



Conclusions

- This paper has presented a new approach to building heterogeneous agent models in which heterogeneity is across space.
    - Wide range of possible applications (not just physical space).
- Presented a DSGE model featuring key components of the new economic geography literature.
    - Including firm entry and strong agglomeration forces.
    - Model generates very persistent movements in population.
    - Leads to the birth and death of cities.
- Lots of plans for future work and extensions.
    - Comments appreciated!

US county-level average weekly wage oty % change (BLS)



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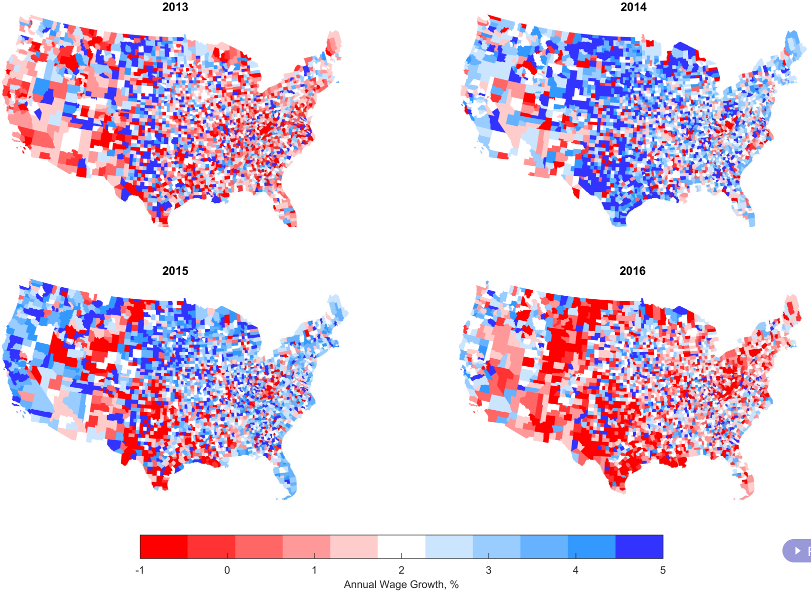
US county-level average weekly wage oty % change (BLS)



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New economic geography

Starts with Krugman (1991). See Krugman (1998) and Redding (2013) for reviews.

Branches of the existing literature:

- Stochastic, forward-looking, but few locations, e.g., two-bloc model:
  - E.g., Caselli & Coleman II (2001).
- Stationary equilibria, or purely backward looking decisions, with discreet space:
  - E.g., Michaels, Rauch & Redding (2012), Nagy (2016) and Eckert & Peters (2017).
- Continuous space, dynamic but backward-looking:
  - E.g., Desmet & Rossi-Hansberg (2014) and Desmet, Nagy & Rossi-Hansberg (2015).
- Some dynamic stochastic models in continuous space but with restrictive assumptions:
  - E.g., Quah (2002), Brito (2004), Duranton (2007), Rossi-Hansberg & Wright (2007) and Boucekkine, Camacho & Zou (2009).

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Auto-covariance function

We recommend (and use) the following auto-covariance function on a circle (identified with $[0,1]$) or torus (identified with $[0,1] \times [0,1]$):

$$\operatorname{cov}\left(\varepsilon_x, \varepsilon_{\tilde{x}}\right) = s\left(\zeta, d\left(x, \tilde{x}\right)\right)$$

where:

$$s\left(\zeta, d\right) = \frac{\exp\left(-\zeta d + \zeta \bar{d}\right) + \exp\left(\zeta d - \zeta \bar{d}\right)}{\exp\left(\zeta \bar{d}\right) + \exp\left(-\zeta \bar{d}\right)}$$

and:

$$\bar{d} \equiv \sup_{x, \tilde{x} \in X} d\left(x, \tilde{x}\right)$$

is the maximum distance between points.

The reasons for this choice are made clear in the paper.

▸ Return

For example... [To be examined fully in another paper]

The shale oil and gas "revolution" in the US provides a natural experiment to look at the effects of regional shocks on broader outcomes.

- Even with assorted controls, “high oil (gas) growth” (ERS-USDA) counties experienced 43.3ppts (25.4ppts) higher wage growth than other counties between 2000 and 2011.
- Over the period, population growth was significantly above average in such counties.
- Looking at the Bakken Shale Play area in North Dakota, up to 2012 we see sharp increases in population and wage growth not just in the shale counties, but also in neighbouring ones.
- After 2012, there is a corresponding decline.

▸ Return

Household first order conditions I

- Euler equation:

$$1 = \mathbb{E}_t \left[\Xi_{t,t+1} R_t \right]$$

- where:

$$\Xi_{t,t+1} \equiv \beta_t \frac{N_{x,t} E_{x,t} U_{x,t+1}^{1-\varsigma}}{N_{x,t-1} E_{x,t+1} U_{x,t}^{1-\varsigma}}$$

- and:

$$\frac{E_{x,t}}{N_{x,t-1} U_{x,t}^{1-\varsigma}} = \frac{E_{\tilde{x},t}}{N_{\tilde{x},t-1} U_{\tilde{x},t}^{1-\varsigma}}$$

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Household first order conditions II

- Consumption:

$$\theta_C E_{x,t} = \theta_F P_{x,t} C_{x,t}$$

- Land:

$$\theta_L E_{x,t} = \theta_F \mathcal{R}_{L,x,t} \left(1 - L_{x,t}\right)$$

- Labour:

$$\theta_H \left(\frac{H_{x,t}}{N_{x,t-1}}\right)^{\nu} = \theta_F \frac{N_{x,t-1}}{E_{x,t}} W_{x,t} \left(\frac{1}{1+\nu}\Gamma^{1+\nu} - \frac{1}{1+\nu}\left(\frac{H_{x,t}}{N_{x,t-1}}\right)^{1+\nu}\right)$$

Return

Household first order conditions III

- Population:

$$\mu_{N,x,t} = \beta_t \mathbb{E}_t \left[\begin{array}{c} \mu_{N,x,t+1} G_{N,t+1} + U_{x,t+1}^{1-\varsigma} \\ + (1-\varsigma)\, U_{x,t+1}^{1-\varsigma} \left[\begin{array}{c} \theta_H \dfrac{\left(\frac{H_{x,t+1}}{N_{x,t}}\right)^{1+\nu}}{\frac{1}{1+\nu}\Gamma^{1+\nu} - \frac{1}{1+\nu}\left(\frac{H_{x,t+1}}{N_{x,t}}\right)^{1+\nu}} - \theta_N \dfrac{\log\left(\frac{N_{x,t}}{N_t}\right)}{\frac{1}{2}\Omega^2 - \frac{1}{2}\left(\log\left(\frac{N_{x,t}}{N_t}\right)\right)^2} \\ + \psi_1 \frac{\mathcal{N}_{x,t+1}}{N_{x,t} - \mathcal{N}_{x,t+1}} - (\theta_C + \theta_F + \theta_L + \psi_3) \end{array} \right] \end{array} \right]$$

- Migration:

$$\mu_{N,x,t} = \mu_{N,\tilde{x},t} + (1-\varsigma)\, N_{x,t-1} U_{x,t}^{1-\varsigma} \left[\psi_3 \frac{N_{\tilde{x},t-1}}{N_{t-1}\mathcal{N}_{x,\tilde{x},t}} - \psi_1 \frac{1}{N_{x,t-1} - \mathcal{N}_{x,t}} - \psi_2 \frac{d(x,\tilde{x})\,\mathcal{N}_{x,t} - \mathcal{D}_{x,t}}{\bar{d}\mathcal{N}_{x,t}^2 - \mathcal{N}_{x,t}\mathcal{D}_{x,t}} \right]$$

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References I

Acemoglu, D., Carvalho, V. M., Ozdaglar, A. & Tahbaz-salehi, A. (2012), 'The Network Origins of Aggregate Fluctuations', *Econometrica* **80**(5), 1977–2016.

Bilbiie, F. O., Ghironi, F. & Melitz, M. J. (2012), 'Endogenous Entry, Product Variety, and Business Cycles', *Journal of Political Economy* **120**(2), 304–345.

Boucekkine, R., Camacho, C. & Zou, B. (2009), 'Bridging The Gap Between Growth Theory And The New Economic Geography: The Spatial Ramsey Model', *Macroeconomic Dynamics* **13**(01), 20–45.

Brito, P. (2004), The Dynamics of Growth and Distribution in a Spatially Heterogeneous World, Working Papers Department of Economics 2004/14, ISEG - Lisbon School of Economics and Management, Department of Economics, Universidade de Lisboa.

Cardamone, P. (2017), 'A Spatial Analysis of the R&D-Productivity Nexus at Firm Level', *Growth and Change* **48**(3), 313–335.

Caselli, F. & Coleman II, W. J. (2001), 'The u.s. structural transformation and regional convergence: A reinterpretation', *Journal of Political Economy* **109**(3), 584–616.

References II

Comin, D., Dmitriev, M. & Rossi-Hansberg, E. (2012), The Spatial Diffusion of Technology, No. 18534, NBER Working Paper Series, Cambridge, MA.

Desmet, K., Nagy, D. & Rossi-Hansberg, E. (2015), ‘The geography of development: Evaluating migration restrictions and coastal flooding’.

Desmet, K. & Rossi-Hansberg, E. (2014), ‘Spatial development’, **104**(4), 1211–43.

Duranton, G. (2007), ‘Urban Evolutions: The Fast, the Slow, and the Still’, *American Economic Review* **97**(1), 197–221.

Eckert, F. & Peters, M. (2017), ‘Spatial structural change and agricultural productivity’.

Gabaix, X. (2011), ‘The Granular Origins of Aggregate Fluctuations’, *Econometrica* **79**(3), 733–772.

Glass, A., Kenjegalieva, K. & Paez-Farrell, J. (2013), ‘Productivity growth decomposition using a spatial autoregressive frontier model’, *Economics Letters* **119**, 291–295.

Griffith, R., Redding, S. & Simpson, H. (2009), ‘Technological catch-up and geographic proximity’, *Journal of Regional Science* **49**(4), 689–720.

References III

Krugman, P. (1991), 'Increasing Returns and Economic Geography', *Journal of Political Economy* **99**(3), 483–499.

Krugman, P. (1998), 'What's new about the new economic geography?', *Oxford Review of Economic Policy* **14**(2), 7–17.

Michaels, G., Rauch, F. & Redding, S. J. (2012), 'Urbanization and Structural Transformation', *The Quarterly Journal of Economics* **127**(2), 535–586.

Nagy, D. (2016), 'City location and economic development'.

Quah, D. (2002), 'Spatial Agglomeration Dynamics', *American Economic Review* **92**(2), 247–252.

Redding, S. J. (2013), Economic Geography: A Review of the Theoretical and Empirical Literature, *in* R. E. Falvey, D. Greenaway, U. Kreickemeier & D. Bernhofen, eds, 'Palgrave handbook of international trade', 1 edn, Palgrave Macmillan UK, chapter 16, pp. 497–531.

Rossi-Hansberg, E. & Wright, M. L. J. (2007), 'Urban Structure and Growth', *Review of Economic Studies* **74**(2), 597–624.