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Equilibrium asset pricing in directed networks

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Non-technical summary

Research Question

The notion of an economy as a network of more or less tightly linked units has received considerable attention in the finance and economics literature. Links in a network usually have a direction, i.e., it makes a difference whether a link goes from node i to node j or the other way around. In this paper, we study whether directedness in a network at the cash flow level has implications for asset prices. To this end, we introduce a general equilibrium asset pricing model, in which negative cash flow shocks in some industries can increase the probability of *subsequent* cash flow shocks in other industries. The direction and the magnitude of this "timing of shocks" characterize the network in our model.

Contribution

The innovative combination of self and mutually exciting jump processes with recursive preferences allows for the integration of directed networks into a tractable equilibrium asset pricing model. Cash flow shocks propagate with a time lag, but, of course, equilibrium prices react immediately to any shock in the economy since markets are efficient. It is this instantaneous reaction of prices to cash flow shocks propagating slowly over time that is at the heart of our equilibrium model. Our results indicate that it is necessary to decompose expected returns into their constituents in order to understand the implications of directed cash flow shock propagation.

Results

We introduce the variable "shock propagation capacity" (*spc*) to measure directedness. Industries with a high *spc* are by definition those industries whose shocks substantially increase the risk of subsequent shocks throughout the economy. Based on a series expansion of the closed-form solution of our model, we analyze the impact of *spc* on the main equilibrium asset pricing quantities. Specifically, we prove the following two cross-sectional statements: (i) Cash flow shocks in industries with high *spc* command a high market price of risk. (ii) The response of an industry's price to its own cash flow shocks is less pronounced for industries with higher *spc*. Importantly, however, when it comes to expected excess returns, these two effects work in opposite directions, so that the overall impact of *spc* on risk premia depends on the tradeoff between them. To illustrate our theoretical findings, we estimate an empirical network from industry cash flows and find support for these predictions.

Nichttechnische Zusammenfassung

Fragestellung

Die Vorstellung der Volkswirtschaft als eines Netzwerks von verbundenen Einheiten hat in der wirtschaftswissenschaftlichen Literatur zuletzt viel Beachtung erfahren. Verbindungen in einem Netzwerk haben normalerweise eine Richtung, d.h. es macht einen Unterschied, ob eine Verbindung vom Knoten i zum Knoten j verläuft oder umgekehrt. In diesem Papier untersuchen wir die Fragestellung, ob die Richtung von Schocks in einem Netzwerk auf Cash-Flow-Ebene Auswirkungen auf Wertpapierpreise hat. Zu diesem Zweck führen wir ein Asset-Pricing-Modell ein, in dem negative Cash-Flow-Schocks in einigen Branchen die Wahrscheinlichkeit von Cash-Flow-Schocks in anderen Branchen erhöhen können. Die Richtung und die Größe dieser "Abfolge von Schocks" legen das Netzwerk fest.

Beitrag

Die innovative Kombination von sich gegenseitig verstärkenden Sprungprozessen mit rekursiven Präferenzen ermöglicht die Untersuchung von gerichteten Netzwerken in einem handhabbaren Gleichgewichts-Asset-Pricing-Modell. Cash-Flow-Schocks verbreiten sich mit einer zeitlichen Verzögerung, aber natürlich reagieren Wertpapierpreise im Gleichgewicht sofort auf jeden Schock in der Volkswirtschaft, da die Märkte effizient sind. Diese unmittelbare Reaktion der Preise auf Cash-Flow-Schocks bildet das Kernstück unseres Gleichgewichtsmodells. Unsere Ergebnisse zeigen, dass es notwendig ist, Risikoprämien in ihre Bestandteile zu zerlegen, um die Implikationen gerichteter Cash-Flow-Schocks zu verstehen.

Ergebnisse

Wir führen die Variable "shock propagation capacity" (*spc*) ein, um das Ausmaß gerichteter Schocks zu messen. Branchen mit einer hohen *spc* sind demnach Branchen, deren Schocks das Risiko von Folgeschocks in der gesamten Volkswirtschaft erheblich erhöhen. Basierend auf einer Reihenentwicklung der geschlossenen Lösung unseres Modells beweisen wir die folgenden zwei Querschnittsaussagen: (i) Cash-Flow-Schocks in Branchen mit hoher *spc* haben einen hohen Marktpreis des Risikos. (ii) Die Preisreaktion eines Wertpapiers auf seine eigenen Cash-Flow-Schocks ist für Branchen mit einer höheren *spc* weniger ausgeprägt. Bezüglich Risikoprämien wirken diese beiden Effekte jedoch in entgegengesetzte Richtungen, so dass der Einfluss von *spc* auf Risikoprämien vom Trade-off zwischen ihnen abhängt. Um unsere theoretischen Ergebnisse zu illustrieren, schätzen wir ein empirisches Netzwerk aus Branchen-Cash-Flows und finden Evidenz für diese Hypothesen.

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Equilibrium Asset Pricing in Directed Networks*

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Abstract

Directed links in cash flow networks affect the cross-section of price exposures and market prices of risk in equilibrium. In an asset pricing model featuring mutually exciting jumps, we measure directedness through an asset's shock propagation capacity (*spc*). In the model, we prove: (i) Cash flow shocks of high *spc* assets command high market prices of risk, (ii) the price reaction of an asset to its own cash flow shocks is less pronounced for high *spc* assets. To illustrate our theoretical findings, we estimate an empirical network from industry cash flows and find support for these predictions.

Keywords: Directed cash flow networks, directed shocks, mutually exciting processes, recursive preferences

JEL classification: G01, G12, D85.

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The notion of an economy as a network of more or less tightly linked units has received considerable attention in the finance and economics literature. Links in a network usually have a direction, i.e., it makes a difference whether a link goes from node i to node j or the other way around. In this paper, we document that directedness in a network at the cash flow level is of first-order importance for asset prices. We propose a general equilibrium asset pricing model, in which negative cash flow shocks in some industries can increase the probability of *subsequent* cash flow shocks in other industries.[1] The direction and the magnitude of this "timing of shocks" characterize the network in our model and we introduce the variable "shock propagation capacity" (*spc*) that measures this directedness. Industries with a high *spc* are by definition those industries whose shocks substantially increase the risk of subsequent shocks throughout the economy. Based on a series expansion of the closed-form solution of our model, we analyze the impact of *spc* on the main equilibrium asset pricing quantities. Specifically, we prove the following two cross-sectional statements: (i) Cash flow shocks in industries with high *spc* command a high market price of risk. (ii) The response of an industry's price to its own cash flow shocks is less pronounced for industries with higher *spc*. Finally, when it comes to expected excess returns, these two effects work in opposite directions, so that the overall impact of *spc* on risk premia depends on the tradeoff between them.

The intuition behind statement (i) is as follows. High *spc* industries have more links or stronger links to other industries, relative to their low *spc* counterparts. Hence, shocks originating from a high *spc* industry have a more pronounced impact on the rest of the economy. They increase the aggregate risk of subsequent shocks by a larger amount, hence they are more systematic and carry a higher market price of risk.

Statement (ii) builds on the general intuition that price-to-cash flow ratios throughout the economy decrease in response to any cash flow shock that increases the aggregate risk. However, we document that industry i's price reaction to a shock to industry j's cash flow is the result of a tradeoff between two opposing forces: (1) a price decline through direct spillover of shocks from j to i, i.e., because industry i's cash flows are riskier after the initial shock to industry j, and (2) an equilibrium hedge effect. The more shocks to industry j spill over to other industries $k \neq i$, the more "attractive" (in relative terms) will industry i be after the initial shock to j. This latter effect is always positive, irrespective of the representative investor's preference parameters, and is larger, the larger the *spc* of asset j. In particular, if a shock to a high *spc* industry increases the probability of subsequent shocks only in other, low *spc* industries, the equity of the "originating" industry itself serves as a hedging device against the risk of further propagation of cash flow shocks throughout the economy. The positive price-to-cash flow ratio reaction due to the hedge effect (2) dampens the price decline due to the direct effect (1). In particular when it comes to shocks in their own cash flows, high *spc* industries thus have a less negative price reaction than their low *spc* counterparts.

Our stylized consumption-based equilibrium asset pricing model features an arbitrary number of industries whose cash flows are linked via self and mutually exciting jump processes, and a representative investor with recursive preferences. An initial negative cash flow shock of industry i increases the *probability* of future cash flow shocks to connected industries $j \neq i$ (and potentially also to i itself), but it is unknown when (and if at all)

[1] We will use the term "industry" to refer to a node in the network throughout the paper. Of course, nodes can also represent individual firms, countries, or any other economic unit.

these shocks will *materialize*. The network thus manifests itself only indirectly via the dynamics of jump intensities as state variables, but not directly through contemporaneous shocks to the levels of several cash flows. Aggregate consumption is driven by all individual jumps, but a given jump affects the cash flow of only one industry at a time. The investor's preference for early resolution of uncertainty, i.e., the fact that she cares about the risk associated with future values of the state variables, implies that the price-to-cash flow ratios of all assets will react to a jump in any individual cash flow, and it is the structure of the network which determines the sign and the magnitude of these reactions.

We choose this model for the following three reasons. First, mutually exciting processes naturally feature directedness, with a shock going from i to j, but not necessarily vice versa. Second, the model belongs to the exponentially affine class for which there is a well-developed solution theory, and thus it remains tractable with at least semi-closed form expressions for all equilibrium quantities. A series expansion allows us to rewrite the market prices of jump risk and jump exposures as functions of *spc* for *arbitrary directed networks*. Third, the crucial model feature that cash flow shocks to one node in the network affect other nodes only with a certain time lag has been documented empirically. In a recent paper, Carvalho, Nirei, Saito, and Tahbaz-Salehi (2016) provide rich empirical evidence for such a delayed propagation of cash flow shocks at the firm level in a natural experiment setting around the nuclear incident of Fukushima in 2011. They summarize the intuition behind their result as follows: "When faced with a supply-chain disruption, individual firms are unable to find suitable alternatives in order to completely insulate themselves from the shock (at least in the short run). This is consistent with an emerging literature [...] that emphasizes the importance of search frictions and relation-specific investments along supply chains."[2] However, even though the cash flow shocks propagate with a time lag, equilibrium prices react immediately to any shock in the economy since markets are efficient. It is precisely this instantaneous reaction of prices to cash flow shocks propagating slowly over time that is at the heart of our equilibrium model.

We close the paper with an empirical illustration of our findings. Since we propose a consumption-based asset pricing model, industry cash flow data are the quantity to be modeled in this exercise.[3] We estimate an empirical cash flow network by applying the generalized variance decomposition method suggested by Diebold and Yilmaz (2014) to the earnings time series of 14 NAICS industries. Given *spc* for these industries, we regress Sharpe ratios (as a proxy for the market prices of risk), return volatilities (as a proxy for price exposures), and average excess returns of value-weighted industry portfolios on this measure. In line with the model, we find in cross-sectional regressions positive coefficients for Sharpe ratios, negative coefficients for return volatilities, and insignificant coefficients for average excess returns.

As an example, consider the two industries "Manufacturing" and "Construction."[4]

[2]See Carvalho et al. (2016), p.34.

[3]We restrain from using input-output production data to construct our cash flow network. While it is intuitive to assume that a firm or industry which is *central* in the production input-output network is also *central* in the cash flow network, it is not clear at all whether a similar relation also holds with respect to *direction*. Empirically, Carvalho et al. (2016) document that cash flow shocks can propagate both upstream and downstream along the supply chain. Consequently, directed links at the cash flow level cannot necessarily be traced back to links of the same direction at the production level.

[4]The *spc* values presented in this example are taken from the network using a forecast horizon of $H = 2$ quarters in Table 1. The descriptive statistics for the 14 value-weighted industry portfolio returns

Manufacturing stocks have a much higher Sharpe ratio than construction stocks (22.36 vs. 15.59 percentage points monthly) and a much lower return volatility (4.70 vs. 7.73 percentage points monthly). We argue that these differences reflect differences in the *spc* of the two industries: Empirically, earnings shocks in the manufacturing industry explain the variance of subsequent earnings in all other industries to a larger extent than earnings shocks in the construction industry do. Hence, manufacturing has a much higher *spc* than construction (0.57 vs. 0.15). This implies that cash flow shocks in the manufacturing industry have a pronounced potential to spread out across the economy and trigger subsequent cash flow shocks in other industries, whereas cash flow shocks in the construction industry do so to a much smaller extent.

Through a simulation exercise, we document that the cash flow network obtained from earnings data via the Diebold and Yilmaz (2014) methodology delivers a reasonable counterpart for the jump intensity network in our model. More precisely, we take the empirically estimated cash flow network to be the intensity network in our model, then simulate cash flow time series, and again apply the Diebold and Yilmaz (2014) methodology to the simulated data. The *spc* obtained from the simulated data is closely related to the *spc* from the empirically estimated network, with rank correlations of around 0.75 or higher. Moreover, regression coefficients for Sharpe ratios and return volatilities in the simulated data have the same signs as their empirical counterparts.

Our paper is linked to several strands of literature. First, there are papers studying the asset pricing implications of networks at the production level. Herskovic (2018) extends the input-output framework of Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) to a time-varying network and highlights the role of sparsity and concentration of an entire network for capturing aggregate risk. Gofman, Segal, and Wu (2018) determine a firm's vertical position in the supply chain and calculate a top-minus-bottom spread which they explain in a production economy with layer-specific capital. In an international context, Richmond (2016) relies on Katz centrality and finds that more central countries have lower interest rates and currency risk premia. The purely empirical papers of Ahern (2013) and Aobdia, Caskey, and Ozel (2014) link equity returns to trade flows between industries. However, none of these papers focus on the impact of directedness, which is the key aspect we emphasize.[5] Second, Buraschi and Tebaldi (2018) model cash flows via jumps which are not mutually exciting. However, their focus is not on directed links, but on systemic risk in banking networks. A third strand of literature analyzes networks estimated from return data. An example for such papers is Diebold and Yilmaz (2014). Many papers dealing with the measurement of systemic risk also follow this route, e.g., Billio, Getmansky, Lo, and Pelizzon (2012) and Demirer, Diebold, Liu, and Yilmaz (2017). The main difference between these papers and ours is that we model the underlying fundamentals, i.e., cash flows.

Finally, Aït-Sahalia, Cacho-Diaz, and Laeven (2015) are the first to discuss the role of mutually exciting jumps in finance applications. The methodological framework of our

are shown in Table 2.

[5]There is also a strand of literature on production or supply chain networks in economics, however, they do not focus on the asset pricing implications of network structures. Examples include, among others, Long and Plosser (1983), Acemoglu et al. (2012), Carvalho and Voigtländer (2015), Wu (2015), Acemoglu, Akcigit, and Kerr (2016), Carvalho et al. (2016), Barrot and Sauvagnat (2016), Wu (2016), Acemoglu, Ozdaglar, and Tahbaz-Salehi (2017), Ozdagli and Weber (2017), and Tascherau-Dumouchel (2018). Carvalho (2014) provides an excellent review of this literature.

model goes back to the paper by Eraker and Shaliastovich (2008). Besides, there is an increasing literature about consumption-based asset pricing models with stochastic jump intensities in the endowment process. For instance, Wachter (2013) and Gabaix (2012) analyze asset pricing puzzles like the equity premium or the excess volatility puzzle in an economy with a stochastic intensity for rare consumption disasters, but do so in a model with only one endowment stream which obviously does not lend itself to any network applications.[6]

1 Model

1.1 Fundamental dynamics

We assume a Lucas endowment economy. Log aggregate consumption $y \equiv \ln Y$ follows

$$dy_t \;=\; \mu\, dt + \sum_{j=1}^{n} K_j\, dN_{j,t},$$

where μ is the constant drift rate and the N_j $(j = 1, \ldots, n)$ are self and mutually exciting jump processes with constant jump sizes $K_j < 0$.[7] Their stochastic jump intensities $\ell_{j,t}$ have dynamics

$$d\ell_{j,t} \;=\; \kappa_j \left(\bar{\ell}_j - \ell_{j,t}\right) dt + \sum_{i=1}^{n} \beta_{j,i}\, dN_{i,t}. \tag{1}$$

The coefficients $\beta_{j,i}$ represent discrete changes in ℓ_j induced by a jump in N_i. The parameters $\beta_{j,i}$, collected in what we call the "beta matrix" or the connectivity matrix, completely determine the structure of a given network.[8] To preclude negative intensities we assume $\beta_{j,i} \geq 0$ for all pairs (j, i).

There are n industries in the economy, indexed by i, with the following dynamics for log cash flows y_i:

$$dy_{i,t} \;=\; \mu_i\, dt + L_i\, dN_{i,t} \qquad (i = 1, \ldots, n). \tag{2}$$

We do not link aggregate consumption to the sum of cash flows, but model cash flows as claims on certain risk factors in the consumption process. This is similar to the assump-

[6] This framework is extended to a two-sector economy with jump intensities driven by correlated Brownian motions in Tsai and Wachter (2016) and towards CDS pricing in Seo and Wachter (2018). Benzoni, Collin-Dufresne, Goldstein, and Helwege (2015) analyze defaultable bonds subject to contagion risk in a general equilibrium model. Nowotny (2011) investigates a one-sector economy with consumption following a self exciting process. Branger, Kraft, and Meinerding (2014) show that self exciting processes can endogenously evolve in a framework with learning about latent disaster intensities. A comprehensive summary of the disaster risk literature is provided by Tsai and Wachter (2015).

[7] We do not include diffusion terms in the dynamics of aggregate consumption for parsimony. One could of course generalize the model to incorporate additional types of diffusive risk premia, e.g., by making the expected consumption growth rate time-varying, as long as the framework remains affine.

[8] Our network is weighted in the sense that the links between nodes are represented by (positive) real numbers, not just by the binary 0-1 information whether two nodes are linked or not.

tions underlying the pricing of dividend claims in models like Bansal and Yaron (2004) or Backus, Chernov, and Martin (2011).

Equations (1) and (2) formalize how the beta matrix gives rise to a dynamic shock propagation mechanism by which negative shocks to one cash flow stream can spread across the economy. With $\beta_{j,i} > 0$, a downward jump in cash flow i immediately increases the jump intensity of cash flow j by the amount $\beta_{j,i}$. Once the increased intensity ℓ_j indeed leads to a jump in cash flow j and there is a nonzero coefficient $\beta_{k,j}$, the initial shock is passed on to asset k and can in this way be propagated through the whole network. Note that our specification is general in the sense that it also allows for "feedback loops", i.e., depending on the structure of the network, an initial shock to node i can, after a number of intermediate steps, eventually reach node i itself again. Nevertheless, each jump only affects one cash flow directly, so that network connectivity is captured exclusively via linkages in the dynamics of the state variables, not at the cash flow level itself.

Mutually exciting jumps provide certainly not the only, but a very lean and reduced-form modeling tool to capture exactly the above intuition. An initial cash flow shock in industry i increases the *probability* of future cash flow shocks to a connected industry $j \neq i$ (and potentially also firm i itself), but it is unknown when (and if at all) these shocks will *materialize*. Stated differently, a cash flow shock of one firm changes the conditional distribution of future cash flows of other firms, but does not necessarily affect the level of these cash flows instantaneously. The structure of the jump processes in our model thus differs in a time series and in a cross-sectional dimension from, for instance, contemporaneous jumps in many assets. Representing this time dimension of shock propagation alternatively by, e.g., a discrete-time vector autoregressive model would lead to the problem that the sum of AR(1) processes is not necessarily an AR(1) process itself (see Granger and Morris (1976)).

As stated above, our specification ensures that the vector $X = (y, \ell_1, \ldots, \ell_n, y_1, \ldots, y_n)'$ follows an affine jump process.[9] The joint process (N, ℓ) is Markov. In all applications of the model, we assume $\kappa_i > \beta_{i,i}$ for $i = 1, \ldots, n$, so that the vector of intensities ℓ is stationary.[10]

In the following analyses, we often refer to one particular measure for the directedness of cash flow shocks. The *shock propagation capacity*, *spc*, of asset i is defined as the respective column sum of the beta matrix without the diagonal entry:[11]

$$spc_i = \sum_{\substack{j=1 \\ j \neq i}}^{n} \beta_{j,i}. \tag{3}$$

This measure has been proposed by, e.g., Jackson (2008) and Diebold and Yilmaz (2014) and represents the total strength of the network links going from node i to all other nodes in the network. In the framework of our model, the higher the *spc* of a given node, the more a shock to its cash flow increases the jump intensities of other nodes.[12]

[9] See Appendix A for details.

[10] See, e.g., Aït-Sahalia et al. (2015) for details about mutually exciting processes, in particular, concerning conditions for stationarity.

[11] Disregarding the diagonal entry is standard practice in the literature, see Diebold and Yilmaz (2014).

[12] Although we call *spc* a measure of "directedness", it can of course also be applied in an undirected network, i.e., in a network where the connectivity matrix is symmetric.

1.2 Market prices of jump risk

Our economy is populated by a representative agent with an infinite planning horizon. We assume that the agent has recursive preferences so that the risk generated by state variables (in this case the intensities ℓ_i) will be priced in equilibrium.

The derivation of the model solution closely follows Eraker and Shaliastovich (2008).[13] They show that the continuous-time dynamics of the pricing kernel M can be written as

$$d \ln M_t = -\delta\,\theta\,dt - (1-\theta)\;d\ln R_t - \frac{\theta}{\psi}\,dy_t,$$

where δ is the subjective time preference rate, γ is the coefficient of relative risk aversion, ψ is the elasticity of intertemporal substitution (EIS), and $\theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}}$. We assume that the representative agent has a preference for early resolution of uncertainty, implying $\gamma > \frac{1}{\psi}$ and thus $\theta < 1$.

The return on the consumption claim R_t satisfies the following continuous-time version of the Euler equation

$$0 = \frac{1}{dt}\,\mathbb{E}_t\left[\frac{d\left(e^{\ln M_t + \ln R_t}\right)}{e^{\ln M_t + \ln R_t}}\right],$$

and follows from the dynamics of the log wealth-consumption ratio v and aggregate consumption. To compute R_t, we use the Campbell-Shiller log-linear approximation $d\ln R_t = k_{v,0}\,dt + k_{v,1}\,dv_t - (1-k_{v,1})\;v_t\,dt + dy_t$ with linearizing constants $k_{v,0}$ and $0 < k_{v,1} < 1$. Employing the usual affine guess for the log wealth-consumption ratio v_t, i.e., assuming $v_t = A + B'\,\ell_t$ with $B = (B_1, \ldots, B_n)'$ and $\ell_t = (\ell_{1,t}, \ldots, \ell_{n,t})'$, we can solve numerically for the coefficients A and B as well as for the linearizing constants.

The dynamics of the pricing kernel are

$$\frac{dM_t}{M_t} = -r_t\,dt - \sum_{i=1}^{n} \mathrm{MPJR}_i\,(dN_{i,t} - \ell_{i,t}dt),$$

where r_t is the equilibrium risk-free rate and MPJR_i is the market price of risk for the jump process N_i. These in general negative market prices of jump risk are given as

$$\mathrm{MPJR}_i = 1 - \exp\left\{-\gamma\,K_i + k_{v,1}\;(\theta-1)\;\left[\sum_{j=1}^{n} B_j\,\beta_{j,i}\right]\right\}, \tag{4}$$

with $k_{v,1} = \frac{e^{\bar{v}}}{1+e^{\bar{v}}}$, where $e^{\bar{v}}$ is the steady-state wealth-consumption ratio. The exponential term is a product of two factors. The first one, $\exp\{-\gamma\,K_i\}$, represents the compensation for the immediate shock caused by the jump in cash flow i. Since $K_i < 0$ these market prices of jump risk are in general negative. The second one with the remaining exponents is the compensation for the risk caused by variations in the state variables and is one of the key features of our model. It depends on the impact of the intensities ℓ_i on the equilibrium wealth-consumption ratio, represented by the components of the vector B.

[13] Details are presented in Appendix A.

The coefficients in B depend on the structure of the network, and they are in general not equal across all $j = 1, \ldots, n$. Therefore, we cannot immediately formulate the market prices of risk as functions of network measures such as *spc* just from Equation (4). To obtain predictions for how the structure of the network affects the market prices of risk, we derive the following proposition through a first-order approximation.[14]

Proposition 1. *Assume that* $\kappa_1 = \ldots = \kappa_n = \kappa$ *and* $K_1 = \ldots = K_n = K$*. Then, the market price of jump risk* $MPJR_i$ *satisfies*

$$MPJR_i \quad = \quad 1 - \exp\left\{\mathcal{A} + \mathcal{B} \cdot (\beta_{i,i} + spc_i) + O\left(\beta^2\right)\right\},$$

where the coefficients $\mathcal{A}$ *and* $\mathcal{B}$ *are given by*

$$\begin{aligned} \mathcal{A} &= -\gamma K \\ \mathcal{B} &= \frac{(\theta - 1)\ (1 - \exp\{K\ (1-\gamma)\})}{\theta\left[(1-\kappa) - \frac{1}{k_{v,1}}\right]} \end{aligned}$$

and $O(\beta^2)$ *denotes polynomial terms of order 2 or higher in the coefficients of the network matrix. Defining the first-order approximation*

$$MPJR_i^{**} \quad \equiv \quad 1 - \exp\{\mathcal{A} + \mathcal{B} \cdot (\beta_{i,i} + spc_i)\} \tag{5}$$

and assuming $\gamma > 1$, $\theta < 0$, $0 < \kappa < 1$, *and* $K < 0$, *we obtain the following results:*

(1) $\mathcal{A} > 0$ *and* $\mathcal{B} > 0$.
(2) *If* $spc_i > spc_j$, *then* $|MPJR_i^{**}| > \left|MPJR_j^{**}\right|$.

Proof: See Appendix B.1.

The second exponential factor on the right-hand side of (5) is one of the key features of our model. The *spc* of an asset is the main driver of the equilibrium market price of risk. The proposition states that the market prices of risk for jumps associated with high *spc* assets are larger (in absolute terms) than those of low *spc* assets (note that $\mathcal{A}$ and $\mathcal{B}$ do not depend on i).[15]

The economic intuition behind this key result is the following. By definition, high *spc* industries have more links or stronger links to other industries, relative to their low *spc* counterparts. Hence, cash flow shocks originating from a high *spc* industry have a more pronounced impact on the rest of the economy, i.e. they increase the aggregate risk of subsequent shocks by a larger amount. In models with stochastic cash flow jump

[14] A different strategy to obtain closed-form solutions for equilibrium quantities as functions of network measures is to focus on special cases in which the connectivity matrix is very sparse. For instance, in Online Appendix A, we derive such closed-form solutions *without approximations* in so-called star (or core-periphery) networks.

[15] In Appendix B.3, we analyze the quality of the first-order approximation in Proposition 1 by regressing the approximate solution (5) on the exact solution (4) for one of the empirical networks estimated in Section 2. The R^2 of this regression is 0.98, the ordering of the assets, and the signs of the market prices of risk are all preserved. The slope of the regression line is 0.25, implying that the higher-order terms omitted in the approximation are quantitatively sizable, but do not change any of our model results qualitatively.

intensities and recursive preferences, the wealth-consumption ratio is generally decreasing in the aggregate jump risk.[16] The wealth-consumption ratio in our economy thus reacts more negatively to cash flow shocks of high *spc* assets. These shocks are thus more "systematic" and carry a higher (i.e. more negative) market price of risk in equilibrium.

The proposition explicates that a necessary condition for this key result is that $\mathcal{B} > 0$, and this condition is satisfied under some mild preference parameter restrictions like $\theta < 0$, which implies $\psi > 1$ (if $\gamma > 1$). In this situation, the intertemporal substitution effect dominates the income effect, so that the investor wants to consume more and save less in bad times with high jump intensities. The proposition also shows that in the special case of CRRA utility ($\theta = 1$, implying $\mathcal{B} = 0$), the second term in Equation (5) vanishes, implying that state variable risk is not priced and that the market prices of risk do not depend on the network structure. Finally, MPJR_i is the larger, the larger the impact of jumps in asset i on aggregate consumption, as measured by K.

1.3 Jump exposures

In analogy to the return on the consumption claim, the returns $R_{i,t}$ on the individual cash flow claims satisfy the continuous-time Euler equations

$$0 \quad = \quad \frac{1}{dt}\,\mathbb{E}_t\left[\frac{d\left(e^{\ln M_t + \ln R_{i,t}}\right)}{e^{\ln M_t + \ln R_{i,t}}}\right].$$

To compute the expected excess return on asset i, we proceed as in the case of the consumption claim, i.e., we employ an affine guess for the log price-to-cash flow ratio of asset i, $v_{i,t} = A_i + C_i'\ell_t$ with $C_i = (C_{i,1}, \ldots, C_{i,n})'$, and use the Campbell-Shiller approximation $d\ln R_{i,t} = k_{i,0}\,dt + k_{i,1}\,dv_{i,t} - (1 - k_{i,1})\;v_{i,t}\,dt + dy_{i,t}$ with linearization constants $k_{i,0}$ and $0 < k_{i,1} < 1$. Again, we solve for the coefficients A_i and $C_{i,j}$ ($j = 1, \ldots, n$) as well as for the linearization constants $k_{i,0}$ and $k_{i,1}$ numerically.

The return on the i-th individual cash flow claim is then given by

$$dR_{i,t} \quad = \quad \ldots dt + \sum_{j=1}^{n} \text{JEXP}_{i,j}\,dN_{j,t},$$

with the jump exposures

$$\text{JEXP}_{i,j} \quad = \quad \begin{cases} \exp\left(L_i + k_{i,1}\sum_{k=1}^{n} C_{i,k}\,\beta_{k,i}\right) - 1 & \text{for } j = i \\ \exp\left(k_{i,1}\sum_{k=1}^{n} C_{i,k}\,\beta_{k,j}\right) - 1 & \text{for } j \neq i. \end{cases} \tag{6}$$

The exponential term in the exposure of asset i to jumps in its own cash flow, $\text{JEXP}_{i,i}$, has two components. First, there is the price change due to the immediate cash flow shock, represented via the jump size L_i. By assumption this component is only present in the exposure of asset i to jumps in its own cash flow i because y_i is exclusively affected by N_i, i.e., jumps in other intensities do not have a direct impact on the cash flow y_i. The second term is a special feature of models with recursive utility and captures the effect of a shock in cash flow j on asset i's price-to-cash flow ratio. For $j \neq i$, the exposure

[16] This has been shown, e.g., by Wachter (2013).

$\mathrm{JEXP}_{i,j}$ only consists of this valuation ratio effect.

In Equation (6), the coefficients $C_{i,k}$ depend on the network structure. Since they will not coincide for all $k = 1, \ldots, n$ in general, we cannot simply factor out *spc* in Equation (6). Therefore, we again apply a first-order approximation allowing us to formulate the following proposition.[17]

Proposition 2. *Assume that $\kappa_1 = \ldots = \kappa_n = \kappa$ and $K_1 = \ldots = K_n = K$. Then, the jump exposures $JEXP_{i,j}$ of asset i against shocks to cash flow j satisfy the equation*

$$JEXP_{i,j} = \begin{cases} \exp\left\{\mathcal{C}_i \cdot \sum_{k=1,k\neq i}^{n} \beta_{k,j} + \mathcal{D}_i \cdot \beta_{i,j} + O\left(\beta^2\right)\right\} - 1 & \text{for } j \neq i \\ \exp\left\{L + \mathcal{C}_i \cdot \sum_{k=1,k\neq i}^{n} \beta_{k,i} + \mathcal{D}_i \cdot \beta_{i,i} + O\left(\beta^2\right)\right\} - 1 & \text{for } j = i, \end{cases}$$

where the coefficients $\mathcal{C}_i$ and $\mathcal{D}_i$ are given by

$$\mathcal{C}_i = \frac{1 - \exp\{-K\,\gamma\} - \frac{\theta-1}{\theta}\left[1 - \exp\{K\,(1-\gamma)\}\right]}{1 - \kappa - \frac{1}{k_{i,1}}}$$

$$\mathcal{D}_i = \frac{1 - \exp\{L - K\,\gamma\} - \frac{\theta-1}{\theta}\left[1 - \exp\{K\,(1-\gamma)\}\right]}{1 - \kappa - \frac{1}{k_{i,1}}}$$

and $O(\beta^2)$ denotes polynomial terms of order 2 or higher in the network coefficients. Defining the first-order approximation

$$JEXP_{i,j}^{**} := \begin{cases} \exp\left\{\mathcal{C}_i \cdot \sum_{k=1,k\neq i}^{n} \beta_{k,j} + \mathcal{D}_i \cdot \beta_{i,j}\right\} - 1 & \text{for } j \neq i \\ \exp\left\{L + \mathcal{C}_i \cdot \sum_{k=1,k\neq i}^{n} \beta_{k,i} + \mathcal{D}_i \cdot \beta_{i,i}\right\} - 1 & \text{for } j = i \end{cases} \tag{7}$$

and assuming $\gamma > 1$, $0 < \kappa < 1$, and $-\log(2) < K < 0$, we obtain $\mathcal{C}_i > 0$ for all i. Additionally assuming $\theta < 0$, we obtain

(1) $\mathcal{D}_i < 0$, *and* $\mathcal{D}_i - \mathcal{C}_i < 0$ *for all* i.
(2) *If* $JEXP_{i,i}^{**}$, $JEXP_{j,j}^{**} < 0$, $k_{i,1} = k_{j,1}$, *and* $spc_i > spc_j$, *then* $\left|JEXP_{i,i}^{**}\right| < \left|JEXP_{j,j}^{**}\right|$.

Proof: See Appendix B.2.[18]

For $j \neq i$, the expression for $\mathrm{JEXP}_{i,j}^{**}$ comprises two terms. The quantity $\mathcal{D}_i\,\beta_{i,j}$ describes a price effect through direct spillover of shocks from j to i. A jump in asset j increases the jump intensity of asset i by $\beta_{i,j}$. Since $\mathcal{D}_i < 0$, the reaction of the price-dividend ratio of i due to this direct effect, $\exp\{\mathcal{D}_i\,\beta_{i,j}\} - 1$, is negative.

The term $\mathcal{C}_i \cdot \sum_{k=1,k\neq i}^{n} \beta_{k,j}$ represents an equilibrium "hedge effect". A jump in asset j increases the jump intensities of (some or all) other assets $k \neq i$, and the price of asset

[17] In Online Appendix A, we show that qualitatively similar closed-form solutions for the return volatility can be obtained *without approximations* in so-called star (or core-periphery) networks.

[18] In Appendix B.3, we analyze the quality of the first-order approximation in Proposition 2 by regressing the approximate solution on the exact solution of Equation (6) for one of the networks estimated in Section 2. The R^2 of this regression is 0.98, the ordering of the assets, and the signs of the jump exposures are all preserved. The slope of the regression line is 0.77, implying that the higher-order terms omitted in the approximation are quantitatively sizeable, but do not change any of our results qualitatively.

i *increases* through this mechanism, since $\mathcal{C}_i > 0$. This hedge effect is always positive, irrespective of the preference parameter θ. Intuitively, the hedge effect makes assets which are not directly affected by a jump in asset j's cash flow *relatively* more attractive. For $j \neq i$, we can rewrite

$$\mathrm{JEXP}^{**}_{i,j} \;=\; \exp\left\{\mathcal{C}_i \cdot \mathrm{spc}_j + \mathcal{C}_i \cdot \beta_{j,j} + (\mathcal{D}_i - \mathcal{C}_i)\,\beta_{i,j}\right\} - 1, \tag{8}$$

which implies that the hedge effect is larger for shocks originating from high *spc* assets than from low *spc* assets.

For $j = i$, this positive hedge effect reduces the negative cash flow effect of a jump in i on the price of asset i itself, represented by $\exp\{L\} - 1$. Again, we can write

$$\mathrm{JEXP}^{**}_{i,i} \;=\; \exp\left\{L + \mathcal{C}_i \cdot \mathrm{spc}_i + \mathcal{D}_i \cdot \beta_{i,i}\right\} - 1,$$

i.e., the hedge effect is more pronounced for a high *spc* asset than for a low *spc* asset.

The ultimate sign of $\mathrm{JEXP}^{**}_{i,j}$ depends on the trade-off between the hedge effect, $\mathcal{C}_i \cdot \sum_{k=1,k\neq i}^{n} \beta_{k,j}$, and the direct price effect, $\mathcal{D}_i \cdot \beta_{i,j}$, and thus on the network structure. Despite the fact that the hedge effect is positive for any network structure and any preference parameter θ, the choice of preferences is still very important for the overall properties of the model. For $L > K\,\gamma$, CRRA preferences ($\theta = 1$) will lead to all cross-exposures $\mathrm{JEXP}^{**}_{i,j} > 0$ being positive, so that here the hedging effect massively outweighs the direct negative effect. With recursive preferences, on the other hand, there will also be pairs of assets with $\mathrm{JEXP}^{**}_{i,j} < 0$, i.e., there will be cases when the hedging effect is not strong enough to dominate the direct negative effect. For $i = j$, the exposure $\mathrm{JEXP}^{**}_{i,i}$ comprises a third component, L, and if this parameter is chosen strongly negative, then $\mathrm{JEXP}^{**}_{i,i}$ will be negative.

1.4 Expected excess returns

Finally, the local expected excess return of asset i can be written as

$$\frac{1}{dt}\,\mathbb{E}_t\left[dR_{i,t}\right] - r_t \;=\; \sum_{j=1}^{n} \ell_{j,t}\,\mathrm{MPJR}_j\,\mathrm{JEXP}_{i,j}, \tag{9}$$

i.e., the risk premium of asset i is given by the sum of the products of jump intensity, market price of risk, and jump exposure over all n jump components.

Although the expected excess return depends on all market prices and all exposures, the summand $\mathrm{MPJR}_i\,\mathrm{JEXP}_{i,i}$ is usually the largest in this sum because the exposure $\mathrm{JEXP}_{i,i}$ also comprises the direct cash flow effect captured by the cash flow jump size L, as shown in Equation (7). Propositions 1 and 2 show that MPJR_i is higher for high *spc* assets than for low *spc* assets, whereas the relation is the other way around for $\mathrm{JEXP}_{i,i}$. Thus, we cannot obtain unambiguous cross-sectional predictions regarding the impact of *spc* on expected excess returns.

For this insight, recursive preferences are crucial. With CRRA utility, the market price of risk on all jumps would be identical, and all cross exposures would be positive for $L > K\,\gamma$. So the trade-off outlined above does not exist and high *spc* assets earn larger expected excess returns than low *spc* assets in a CRRA economy.

2 Empirical illustration

2.1 Data on industry earnings

In the following, we perform several illustrative exercises to document empirically the theoretical channels outlined above. We start by constructing quarterly time series of industry earnings following Irvine and Pontiff (2009). The sample comprises all firms in the CRSP/Compustat merged (CCM) fundamentals quarterly database from 1966-Q2 to 2014-Q4. In principle, the data is available from 1964 onwards, but before 1966-Q2 not all industries are represented in the sample. We work with firms' earnings per share (item EPSPXQ) and require a firm to have at least four consecutive data entries to be included in our sample. Following the procedure outlined in Irvine and Pontiff (2009), we winsorize the EPS data.

Based on the NAICS code dictionary from the Bureau of Economic Analysis, we sort in each quarter firms into 15 industry portfolios as in Menzly and Ozbas (2010). Following Aobdia et al. (2014), and Menzly and Ozbas (2010), we exclude the government sector. We multiply firms' earnings per share by the number of shares to obtain total earnings, sum up the total earnings across all firms in a given industry, and divide by the number of firms in that industry to account for variation over time. We then calculate log earnings growth rates for each industry.[19] We adjust each time series for seasonality using the method proposed in Hamilton (2018).[20] Eventually, we end up with a time series of 49 log earnings growth rates for each of the 14 industries, i.e., 686 quarterly observations in total.

2.2 Measurement of directed cash flow links

Having constructed quarterly industry earnings time series allows us to estimate the directed earnings network following the procedure proposed by Diebold and Yilmaz (2014).[21] The first step is to estimate a 14-dimensional VAR(1) process based on our earnings growth time series:

$$\begin{bmatrix} z_{1,t} \\ \vdots \\ z_{14,t} \end{bmatrix} = \begin{bmatrix} \phi_1 \\ \vdots \\ \phi_{14} \end{bmatrix} + \begin{bmatrix} \phi_{1,1} & \cdots & \phi_{1,14} \\ \vdots & \ddots & \vdots \\ \phi_{14,1} & \cdots & \phi_{14,14} \end{bmatrix} \begin{bmatrix} z_{1,t-1} \\ \vdots \\ z_{14,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \vdots \\ \varepsilon_{14,t} \end{bmatrix}.$$

From the coefficient matrix ϕ and the covariance matrix of the shocks ε, we compute generalized variance decompositions of quarterly earnings with a forecast horizon of $H = 1, 2, 3, 4$ quarters. We denote the fraction of H-quarter forecast error variance of industry i's earnings explained by shocks in industry j's earnings by $d^H_{i,j}$. This gives us a 14×14 matrix $(d^H_{i,j})_{i,j=1,\ldots,14}$, which Diebold and Yilmaz (2014) refer to as the *connectedness table*. In the following, this matrix serves as our empirical network matrix at the cash flow level.

[19] We follow Lochstoer and Tetlock (2018) and winsorize log earnings growth rates at $\log(0.01)$ when earnings growth rates are below -0.99.

[20] Running a Dickey and Fuller (1979) test on each resulting time series, we can reject the null hypothesis of a unit root at the 1% significance level.

[21] We thank Francis Diebold and Kamil Yilmaz for sharing their code.

There is no clear guidance towards the optimal choice of the forecast horizon H. As documented by Diebold and Yilmaz (2014), a very short horizon produces noisy estimates, but the estimates stabilize with longer horizons. Diebold and Yilmaz (2014), thus, choose $H = 12$ days for their daily stock return data. We find similar, albeit weaker, effects of the forecast horizon in our estimation and, therefore, report results for $H = 1, 2, 3, 4$ quarters in the following.

Figure 1 presents graphical illustrations of the estimated networks, where the arrowheads mark *outgoing* links. This helps to visually identify the industries with high *spc* in the graphs. From the graphs, one can see the similarity of the networks for the different forecast horizons of $H = 1, \ldots, 4$ quarters.

From the empirical network matrix, we compute the shock propagation capacity spc_j^H for industry j and horizon H analogous to Equation (3) as

$$spc_j^H = \sum_{\substack{i=1 \\ i \neq j}}^{N} d_{i,j}^H.$$

Diebold and Yilmaz (2014) call this measure *total directional connectedness to others from* j. Table 1 provides the *spc*'s of the 14 industries. Most importantly, one can see that there is a significant cross-sectional dispersion in *spc* at all horizons, so that this variable indeed has the potential to explain cross-sectional variation in asset pricing moments. In terms of important industries with high values for *spc*, manufacturing, wholesale trade, and utilities are at the top of the list for $H = 1$, and this ranking is stable across the four horizons. In contrast, cash flow shocks to construction and agriculture, forestry, fishing, and hunting seem to be less important for the rest of the economy.

2.3 Cross-sectional performance of shock propagation capacity

Having estimated the network of cash flow linkages, we now illustrate the performance of *spc* in a cross-sectional asset pricing exercise. The data is from the CRSP securities monthly database and covers exactly the sample used for the cash flow network estimation. We assign firms to industry portfolios based on their NAICS code and form value-weighted industry portfolios accordingly.

For each industry portfolio, we calculate three variables over the whole sample, which serve as dependent variables in our regressions. The average excess return of an industry portfolio is the mean of the difference between its log return and the log three-month Treasury bill return. Return volatilities are calculated as the standard deviations of log returns. Sharpe ratios are computed as average excess returns divided by return volatilities. The numbers are shown in Table 2.

The Sharpe ratio of an industry serves as a proxy for the market price of risk for cash flow shocks of the respective industry, MPJR, because these market prices of risk are not observable empirically. Recall from Equation (9) that the expected excess return on asset i is given as

$$\frac{1}{dt}\mathbb{E}\left[dR_i\right] - r = \sum_{j=1}^{n} \ell_j \, \mathrm{JEXP}_{i,j} \, \mathrm{MPJR}_j.$$

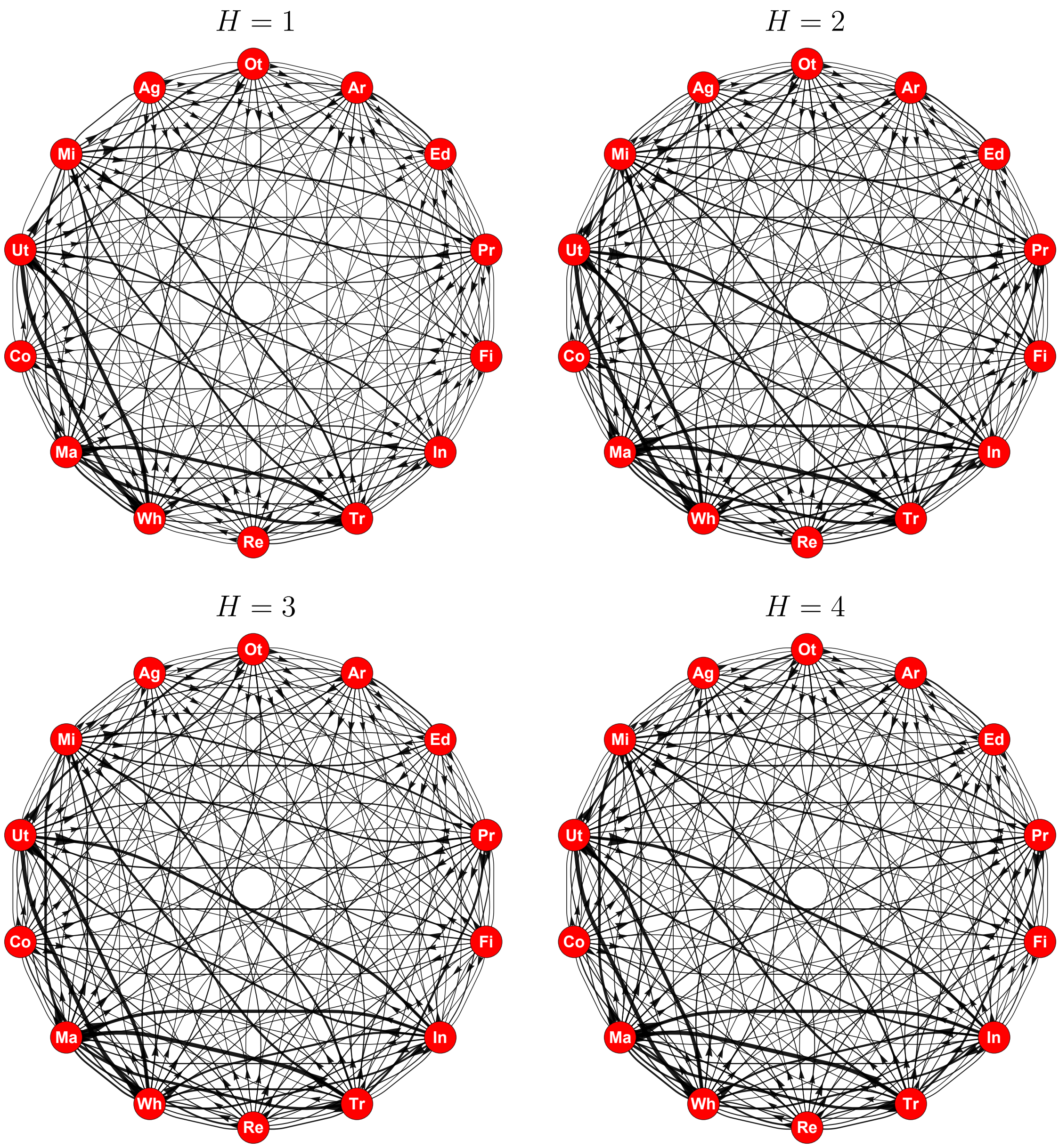

Figure 1
Empirical cash flow networks for different forecast horizons H

The pictures show the empirical cash flow networks obtained from Diebold and Yilmaz (2014) H-quarter generalized variance decompositions for $H = 1, \ldots, 4$, applied to quarterly log industry earnings growth rates over the sample from 1966-Q2 to 2014-Q4, i.e., 686 observations in total. Arrowheads mark *outgoing* links. The thickness of a link corresponds to the size of the respective entry. Diagonal entries of the connectivity matrix are disregarded. The industries are listed in Appendix C.

Industry	Forecast horizon			
	$H = 1$	$H = 2$	$H = 3$	$H = 4$
Agriculture, forestry, fishing, hunting	0.0560	0.1012	0.1061	0.1068
Mining	0.2895	0.3147	0.3237	0.3247
Utilities	0.3637	0.4185	0.4189	0.4186
Construction	0.0599	0.1548	0.1679	0.1720
Manufacturing	0.3584	0.5740	0.6330	0.6482
Wholesale trade	0.4326	0.5272	0.5273	0.5271
Retail trade	0.0782	0.2277	0.2450	0.2497
Transportation and warehousing	0.3175	0.3679	0.3744	0.3753
Information	0.1560	0.2300	0.2517	0.2554
Finance, insurance, real estate, ...	0.0448	0.0906	0.0998	0.1027
Professional and business services	0.1584	0.2861	0.3003	0.3022
Educational services, health care, ...	0.0780	0.1384	0.1445	0.1452
Arts, entertainment, accommodation, ...	0.1473	0.1388	0.1383	0.1382
Other services	0.0980	0.1375	0.1414	0.1415
Mean	0.1884	0.2648	0.2766	0.2791
Standard deviation	0.1350	0.1573	0.1643	0.1663

Table 1
Shock propagation capacities

The table shows the shock propagation capacity (*spc*) for 14 industries. *spc* is obtained from Diebold and Yilmaz (2014) H-quarter generalized variance decompositions for $H = 1, 2, 3, 4$ applied to log industry earnings growth rates over the sample from 1966-Q2 to 2014-Q4, i.e., 686 observations in total. *spc* is calculated according to Equation (3). Graphical representations of the networks are shown in Figure 1.

Industry	Sharpe ratio	Return volatility	Average excess return
Agriculture, forestry, fishing, hunting	0.1609	0.0635	0.0102
Mining	0.1351	0.0665	0.0090
Utilities	0.1459	0.0409	0.0060
Construction	0.1559	0.0773	0.0121
Manufacturing	0.2236	0.0470	0.0105
Wholesale trade	0.2070	0.0509	0.0105
Retail trade	0.1853	0.0557	0.0103
Transportation and warehousing	0.1784	0.0565	0.0101
Information	0.1888	0.0507	0.0096
Finance, insurance, real estate, ...	0.1699	0.0557	0.0095
Professional and business services	0.1563	0.0546	0.0085
Educational services, health care, ...	0.1693	0.0765	0.0130
Arts, entertainment, accomodation,...	0.1842	0.0679	0.0125
Other services	0.1362	0.0691	0.0094

Table 2

Descriptive statistics for industry portfolio returns

The table presents descriptive statistics for the returns of 14 value-weighted industry portfolios. The average excess return of an industry portfolio is the mean of the difference between its log return and the log three-month Treasury bill return. Return volatilities are calculated as the standard deviations of log returns. Sharpe ratios are computed as average excess returns divided by return volatilities. The data is from the CRSP securities monthly database and covers the sample from April 1966 to December 2014.

The i-th summand is by the far the largest on the right-hand side, since $\text{JEXP}_{i,i}$ is the only exposure containing the direct cash flow effect represented by the jump size L. The expected excess return of an asset is thus mostly driven by the response of its price and of the pricing kernel to *its own cash flow shocks*. Therefore we use the Sharpe ratio of asset i as a proxy for MPJR_i and the return volatility as a proxy for $\text{JEXP}_{i,i}$.

Table 3 reports the main results from this empirical exercise. Each of the three panels shows four univariate cross-sectional regressions, where the explanatory variables are the industry shock propagation capacities, determined using the empirical procedure outlined above, with forecast horizons of $H = 1, 2, 3, 4$ quarters. The dependent variables are return volatilities, Sharpe ratios, and average excess returns.[22]

First, the coefficients in the Sharpe ratio regressions are positive and significant for $H = 2, 3, 4$, and the R^2's are large. This is also in line with Proposition 1, that shocks to the cash flows of high *spc* industries carry a large market price of risk, which manifests itself in high Sharpe ratios for these industries.

Second, the coefficients in the return volatility regressions are all negative and significant at the 1% level, and the adjusted R^2's are high for all forecasting horizons. This negative connection is in line with Proposition 2 which states that high *spc* assets have smaller jump exposures that translate into lower return volatilities.

Third, the coefficients in the average excess return regressions are insignificant for all horizons. These results are also in line with our theoretical findings. The effects of *spc* on the market price of jump risk and on price exposures have opposite signs, so that the overall effect of *spc* on expected excess returns cannot be uniquely determined in general within the model. Hence, the role of directedness in equilibrium can only be assessed appropriately when the two opposing effects described above are disentangled.

Finally, the coefficients in the Sharpe ratio and return volatility regressions are not only statistically, but also economically significant. For $H = 2$, the standard deviation of *spc* is around 0.16. Thus, with a coefficient for *spc* in the Sharpe ratio regression of around 8.03, a one-standard-deviation difference in *spc* leads to a difference in Sharpe ratios of roughly $8.03 \cdot 0.16 \approx 1.27$ percentage points monthly. Similarly, a one-standard-deviation difference in *spc* gives rise to a difference in return volatilities of about $-4.67 \cdot 0.16 \approx -0.74$ percentage points per month.

2.4 Empirical *spc* versus model-generated *spc*

The empirics above rely on generalized variance decompositions of cash flows to estimate the structure of the underlying network, whereas the model features connectivity in a network at the jump intensity level. We now show that the connectivity and directedness information from the empirically estimated cash flow network is indeed a close representation of the underlying intensity network.

To this end, we perform the following simulation exercise for each forecast horizon $H = 1, 2, 3, 4$. We plug the empirically estimated connectivity matrix (for the cash flows) as the beta matrix (for the jump intensities) into our model which we then multiply with $\frac{1}{2}$ to make sure that the stationarity condition (A.7) holds. The remaining model parameters are taken from Table 4. Then we simulate 10,000 years of cash flows with monthly increments and run the procedure suggested by Diebold and Yilmaz (2014) on

[22] Although one of the three regressions is redundant, we report all three for the sake of completeness.

const.	$H=1$	$H=2$	$H=3$	$H=4$	$\bar{R}^2$
			Sharpe ratios		
15.9158***	6.3940				0.0404
[20.46]	[1.24]				
14.9951***		8.0267**			0.1811
[15.78]		[2.10]			
14.8668***			8.1489**		0.2143
[15.91]			[2.37]		
14.8382***				8.1778**	0.2236
[16.03]				[2.46]	
			Return volatilities		
6.8792***	-4.9428***				0.3208
[16.73]	[-3.16]				
7.1838***		-4.6676***			0.4057
[16.87]		[-4.05]			
7.1831***			-4.4660***		0.4055
[17.30]			[-4.13]		
7.1776***				-4.4060***	0.4039
[17.44]				[-4.14]	
			Average excess returns		
1.0999***	-0.4888				0.0695
[16.06]	[-1.38]				
1.1052***		-0.3677			0.0340
[13.64]		[-1.22]			
1.0994***			-0.3311		0.0206
[13.59]			[-1.17]		
1.0972***				-0.3203	0.0162
[13.59]				[-1.15]	

Table 3
Cross-sectional regressions on *spc*

The table reports the results of cross-sectional regressions of Sharpe ratios, return volatilities, and average excess returns of the 14 industry portfolios on their shock propagation capacity (*spc*). Returns within a portfolio are value-weighted. To obtain *spc*, we perform Diebold and Yilmaz (2014) H-quarter generalized variance decompositions for $H = 1, 2, 3, 4$ and calculate *spc* as given in Equation (3). Numbers in square brackets denote t-stats adjusted for cross-sectional heteroskedasticity. Statistical significance at the 1%, 5%, and 10% level is indicated by ***, **, and *, respectively.

Investors		
Relative risk aversion	γ	10
Intertemporal elasticity of substitution	ψ	1.5
Subjective discount rate	δ	0.02
Aggregate consumption		
Expected growth rate of log aggregate consumption	μ	0.02
Jump size of log aggregate consumption	$K_1 = \ldots = K_{14}$	-0.004
Industry cash flows		
Expected growth rates of log cash flows	$\mu_1 = \ldots = \mu_{14}$	0.02
Jump sizes of log cash flows	$L_1 = \ldots = L_{14}$	-0.04
Stochastic jump intensities		
Mean reversion speeds	$\kappa_1 = \ldots = \kappa_{14}$	0.85
Mean reversion levels	$\bar{\ell}_1 = \ldots = \bar{\ell}_{14}$	0.05

Table 4
Model Parameters

The table reports the parametrization of our equilibrium model. The beta matrix is determined empirically using the approach described in Section 2.2.

simulated log cash flow growth rates, exactly as we do with the empirical data, resulting in an estimate for the network matrix based on simulated data cash flows. From this, we compute the *spc* values for the different industries and compare them to the corresponding values based on the empirical network matrix that we had plugged into the model initially.

Table 5 presents correlations between the two *spc* vectors. One can see that the two network matrices are very similar with respect to the *spc* values they generate, with correlations of 0.75 or higher. Furthermore, it is especially relevant in the context of our empirical analysis that sorting industries on *spc* delivers roughly the same ordering for cash flow-based and intensity-based network matrices.

2.5 Regressions in model-generated data

In Section 2.4, we show that applying the Diebold and Yilmaz (2014) estimation method to simulated data, preserves the ordering of industries with respect to *spc*. As a final step and to further corroborate that the empirical procedure is in line with the intuition behind the theoretical model, we now analyze whether the regression results from Section 2.3 also carry over to model-generated data.

We start from the simulated path for $H = 3$ over a period of 10,000 years with monthly increments from Section 2.4.[23] Using these 14 industry cash flow time series, we compute *spc* by applying the Diebold and Yilmaz (2014) methodology exactly as in

[23]For the sake of brevity, we report the results for this sample path only. The results using the paths for $H = 1, 2, 4$ are qualitatively similar.

	Forecast horizon			
	$H = 1$	$H = 2$	$H = 3$	$H = 4$
correlation	0.85	0.81	0.87	0.66
rank correlation	0.82	0.85	0.90	0.75

Table 5
Empirical *spc* versus model-generated *spc*

The table reports correlations and rank correlations between empirically estimated and model-generated shock propagation capacities. The model-generated values are obtained from simulated data using the empirically estimated network matrix as an input. The procedure is described in detail in Section 2.4.

the data. Again the forecast horizons are $H = 1, 2, 3, 4$ quarters. Unconditional Sharpe ratios, return volatilities, and average excess returns are computed from the simulated monthly return time series exactly like their empirical counterparts in Section 2.3. Table 6 reports these results. As one can see, the analyses based on simulated and empirical data produce qualitatively similar results for Sharpe ratios (positive coefficients) and for return volatilities (negative coefficients for *spc*).

The coefficients for all regressions are much larger in Table 6 than in Table 3. The reason is that the values for *spc* are smaller in model-generated than in empirical data. The diagonal entries of the beta matrix are by definition not included when we compute *spc* according to Equation (3). So a comparably smaller value for *spc* in the model-generated data shows that self excitation, represented by the diagonal elements of the beta matrix, is more pronounced in model-generated than in empirical data. In the real world, shocks are also spread via potentially diffusive channels (which are not present in our model for the sake of parsimony) and this can increase the relative size of the shocks passed on to other industries, making the diagonal elements of the empirical network matrix smaller and the off-diagonal elements, and thus also *spc*, larger.

With the given parameters, the model produces only weakly significant coefficients in the regressions for unconditional average excess returns for $H = 2, 3, 4$, whereas in our empirical analysis we basically found no impact of *spc* on risk premia. However, given our discussion concerning the two opposing directions in which *spc* impacts exposures (negatively) and market prices of risk (positively) in the model, the results for average excess returns in Table 6 could simply mean that the positive effect of *spc* on the market prices of risk weakly dominates in the simulated economy, whereas the two effects more or less seem to offset each other in the empirical data.

In summary, the analysis generates results which are overall in line with our results from Propositions 1 and 2. Hardly surprising, however, our very stylized model does not match the unconditional volatility of the U.S. stock market (reflecting the well-documented excess volatility puzzle), as indicated by the low values for the constants in the return volatility regressions in Table 6. In principle, it would be possible to include additional features, but this would unnecessarily complicate the solution of the model and shift the focus away from the clear theoretical results derived above.[24]

[24]One way to generate stock return volatilities in the model that are closer to their empirical coun-

const.	$H = 1$	$H = 2$	$H = 3$	$H = 4$	$\bar{R}^2$
			Unconditional Sharpe ratios		
0.0026***	4.9670				-0.0705
[1.45]	[0.45]				
0.0010***		6.6106***			0.0522
[0.73]		[2.97]			
0.0010***			6.6073***		0.0527
[0.72]			[2.98]		
0.0010***				6.6073***	0.0527
[0.72]				[2.98]	
			Unconditional return volatilities		
0.9091***	-904.7605***				0.2432
[17.98]	[-2.84]				
0.9629***		-533.6300***			0.5922
[25.64]		[-4.96]			
0.9631***			-532.1669***		0.5917
[25.59]			[-4.95]		
0.9631***				-532.1604***	0.5917
[25.59]				[-4.95]	
			Unconditional average excess returns		
0.0023***	0.7408				-0.0828
[1.68]	[0.09]				
0.0012***		3.2697*			-0.0183
[1.14]		[1.78]			
0.0012***			3.2711*		-0.0179
[1.13]			[1.78]		
0.0012***				3.2712*	-0.0179
[1.13]				[1.78]	

Table 6

Cross-sectional regressions on *spc* in model-generated data

The table reports the results from cross-sectional regressions of model-generated Sharpe ratios, return volatilities, and average excess returns of 14 assets on their shock propagation capacity (*spc*). As beta matrix, we use the empirical network determined in Section 2.2 for a forecast horizon of $H = 3$ quarters. The remaining parameters are given in Table 4. Given the model solution, we run a Monte Carlo simulation over 10,000 years with monthly time increments. From the simulated data, we compute Sharpe ratios, return volatilities, and average excess returns. To obtain *spc*, we apply Diebold and Yilmaz (2014) H-quarter generalized variance decompositions to simulated log cash flow growth rates for $H = 1, 2, 3, 4$ and calculate *spc* as in Equation (3). Statistical significance at the 1%, 5%, and 10% level is indicated by ***, **, and *, respectively.

2.6 Eigenvector centrality

The existing literature on network linkages and cross-sectional asset pricing features a different approximation for the relative importance of a node in a network, namely eigenvector centrality.[25] Let the network matrix β be diagonalized as follows:

$$\beta \;=\; S \cdot \mathrm{diag}\left(\phi_1, \, \ldots, \, \phi_n\right) \cdot S^{-1}, \tag{10}$$

where the ϕ_i's are the eigenvalues, ordered by absolute size, the columns of S are the eigenvectors of β, and the rows of S^{-1} are the eigenvectors of the transposed matrix β' (usually all normalized to have unit length). The eigenvector centrality of node i is defined as the i-th entry of the first column vector in S (the so-called principal eigenvector), i.e, $evc_i = S_{i,1}$.[26] Loosely speaking, a node has a high eigenvector centrality when it is linked to many other nodes, to other central nodes, or both.

Both *evc* and *spc* are approximations, condensing the entire network matrix into one value per node. Although *evc* can also be viewed as a "directed measure", in the sense that it changes when the network matrix is transposed, we view *spc* as the more natural quantity when it comes to capturing directedness in the context of our model for the following reason. An approximation of our equilibrium asset pricing results using *evc* would combine the principal eigenvectors of both β and β', hence mixing up the impact of incoming and outgoing links. To see this, let evc_i denote the eigenvector centrality of node i, and let evc'_i denote the eigenvector centrality of node i based on the transposed matrix β'. Defining the approximation β^{***} of β via

$$\beta^{***} \;:=\; S \cdot \mathrm{diag}\left(\phi_1, 0, \ldots, 0\right) \cdot S^{-1},$$

i.e., replacing all non-principal eigenvalues by 0, one can easily show that

$$\beta^{***} \;=\; \phi_1 \cdot \begin{pmatrix} \mathrm{evc}_1\, \mathrm{evc}'_1 & \ldots & \mathrm{evc}_1\, \mathrm{evc}'_n \\ \vdots & \ddots & \vdots \\ \mathrm{evc}_n\, \mathrm{evc}'_1 & \ldots & \mathrm{evc}_n\, \mathrm{evc}'_n \end{pmatrix}.$$

spc, on the other hand, offers a straightforward interpretation of directedness, given the additive structure of our model with mutually exciting processes.

Table 7 reports the values of *evc* for the 14 industries. The cross-sectional dispersion in *evc* is similar to *spc*. Tables 8, 9, and 10 present the results of regressions analogous to those shown in Table 3, but now with *evc* as additional regressor. For the bivariate regressions, we orthogonalize *evc* with respect to *spc* to quantify the *additional* explanatory power of this measure beyond *spc*.[27]

The new regressions yield several interesting findings. The univariate regressions for

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terparts would be to introduce persistent diffusion processes representing, e.g., stochastic volatility of consumption growth.

[25] This concept was introduced by Bonacich (1972b,a) and has been applied, e.g., by Demirer et al. (2017) and Walden (2018).

[26] Further technical details about the construction of eigenvector centrality are given in Online Appendix C.

[27] In Online Appendix D, we also compare *spc* to a symmetrified version of eigenvector centrality that has been proposed in the literature recently.

Industry	Forecast horizon			
	$H = 1$	$H = 2$	$H = 3$	$H = 4$
Agriculture, forestry, fishing, hunting	0.0581	0.0689	0.0707	0.0713
Mining	0.2680	0.2869	0.2877	0.2870
Utilities	0.4577	0.3808	0.3773	0.3769
Construction	0.0847	0.0888	0.0928	0.0938
Manufacturing	0.4083	0.3425	0.3311	0.3297
Wholesale trade	0.4859	0.4038	0.4089	0.4097
Retail trade	0.0774	0.1680	0.1688	0.1687
Transportation and warehousing	0.3575	0.4039	0.3989	0.3981
Information	0.2623	0.4355	0.4365	0.4366
Finance, insurance, real estate, ...	0.0415	0.0629	0.0652	0.0655
Professional and business services	0.1878	0.1950	0.2102	0.2126
Educational services, health care, ...	0.0757	0.0802	0.0788	0.0784
Arts, entertainment, accommodation, ...	0.1583	0.1341	0.1316	0.1310
Other services	0.1870	0.1796	0.1852	0.1863
Mean	0.2222	0.2308	0.2317	0.2318
Standard deviation	0.1542	0.1399	0.1382	0.1380

Table 7
Eigenvector centrality

The table reports eigenvector centrality (*evc*) for the 14 industries in our sample. The network measure is obtained from Diebold and Yilmaz (2014) H-quarter generalized variance decompositions for $H = 1, 2, 3, 4$ applied to log industry earnings growth rates over the sample from 1966-Q2 to 2014-Q4, i.e., 686 observations in total. *evc* is calculated according to Equation (10). Graphical representations of the networks are shown in Figure 1.

	$H=1$				$H=2$		
const.	*spc*	*evc*	$\bar{R}^2$	const.	*spc*	*evc*	$\bar{R}^2$
6.9909***		-4.6955***	0.3923	7.1819***		-5.3478***	0.4246
[15.87]		[-3.46]		[16.03]		[-3.59]	
6.8792***	-4.9428***	-8.3586	0.3571	7.1838***	-4.6676***	-3.1963*	0.4247
[15.68]	[-3.32]	[-1.52]		[17.32]	[-4.12]	[-1.95]	
	$H=3$				$H=4$		
const.	*spc*	*evc*	$\bar{R}^2$	const.	*spc*	*evc*	$\bar{R}^2$
7.2001***		-5.4049***	0.4233	7.2033***		-5.4157***	0.4235
[15.98]		[-3.60]		[15.97]		[-3.60]	
7.1831***	-4.4660***	-3.2327**	0.4310	7.1776***	-4.4060***	-3.2633**	0.4328
[17.77]	[-4.33]	[-2.01]		[17.96]	[-4.40]	[-2.03]	

Table 8

Cross-sectional regressions of return volatilities on *spc* and *evc*

The table reports the results from cross-sectional regressions of the return volatilities of the 14 industry portfolios on their shock propagation capacity (*spc*) and eigenvector centrality (*evc*). Within the portfolios, returns are value-weighted. Both network measures are obtained from Diebold and Yilmaz (2014) H-quarter generalized variance decompositions for $H = 1, 2, 3, 4$ (from upper left to lower right panel), applied to log industry earnings growth rates over the sample from 1966-Q2 to 2014-Q4, i.e., 686 observations in total. *evc* is defined in Equation (10), *spc* in Equation (3). In bivariate regressions, we orthogonalize *evc* with respect to *spc*. Numbers in square brackets denote t-stats adjusted for cross-sectional heteroskedasticity. Statistical significance at the 1%, 5%, and 10% level is indicated by ***, **, and *, respectively.

$H = 1$				$H = 2$			
const.	*spc*	*evc*	$\bar{R}^2$	const.	*spc*	*evc*	$\bar{R}^2$
15.9409***		5.3110	0.0280	15.7070***		6.1260	0.0385
[19.64]		[1.17]		[22.02]		[1.64]	
15.9158***	6.3940	-1.9904	-0.0459	14.9951***	8.0267**	-2.8222	0.1171
[20.85]	[1.25]	[-0.09]		[16.40]	[2.25]	[-0.37]	
$H = 3$				$H = 4$			
const.	*spc*	*evc*	$\bar{R}^2$	const.	*spc*	*evc*	$\bar{R}^2$
15.7510***		5.9117	0.0275	15.7557***		5.8880	0.0262
[22.07]		[1.61]		[22.04]		[1.60]	
14.8668***	8.1489***	-3.9154	0.1642	14.8382***	8.1778***	-4.0006	0.1759
[17.16]	[2.70]	[-0.54]		[17.38]	[2.83]	[-0.57]	

Table 9

Cross-sectional regressions of Sharpe ratios on *spc* and *evc*

The table reports the results from cross-sectional regressions of the Sharpe ratios of the 14 industry portfolios on their shock propagation capacity (*spc*) and eigenvector centrality (*evc*). Within the portfolios, returns are value-weighted. Both network measures are obtained from Diebold and Yilmaz (2014) H-quarter generalized variance decompositions for $H = 1, 2, 3, 4$ (from upper left to lower right panel), applied to log industry earnings growth rates over the sample from 1966-Q2 to 2014-Q4, i.e., 686 observations in total. *evc* is defined in Equation (10), *spc* in Equation (3). In bivariate regressions, we orthogonalize *evc* with respect to *spc*. Numbers in square brackets denote t-stats adjusted for cross-sectional heteroskedasticity. Statistical significance at the 1%, 5%, and 10% level is indicated by ***, **, and *, respectively.

$H=1$				$H=2$			
const.	*spc*	*evc*	$\bar{R}^2$	const.	*spc*	*evc*	$\bar{R}^2$
1.1202***		-0.5057	0.1300	1.1394***		-0.5702*	0.1399
[15.25]		[-1.55]		[14.80]		[-1.86]	
1.0999***	-0.4888	-1.5207	0.1103	1.1052***	-0.3677	-0.6554*	0.0649
[16.84]	[-1.51]	[-1.62]		[14.98]	[-1.41]	[-1.81]	
$H=3$				$H=4$			
const.	*spc*	*evc*	$\bar{R}^2$	const.	*spc*	*evc*	$\bar{R}^2$
1.1450***		-0.5919*	0.1516	1.1458***		-0.5951*	0.1533
[14.76]		[-1.90]		[14.75]		[-1.91]	
1.0994***	-0.3311	-0.7147*	0.0819	1.0972***	-0.3203	-0.7237**	0.0848
[15.43]	[-1.43]	[-1.94]		[15.57]	[-1.42]	[-1.96]	

Table 10

Cross-sectional regressions of average excess returns on *spc* and *evc*

The table reports the results from cross-sectional regressions of the average excess returns of the 14 industry portfolios on their shock propagation capacity (*spc*) and eigenvector centrality (*evc*). Within the portfolios, returns are value-weighted. Both network measures are obtained from Diebold and Yilmaz (2014) H-quarter generalized variance decompositions for $H = 1, 2, 3, 4$ (from upper left to lower right panel), applied to log industry earnings growth rates over the sample from 1966-Q2 to 2014-Q4, i.e., 686 observations in total. *evc* is defined in Equation (10), *spc* in Equation (3). In bivariate regressions, we orthogonalize *evc* with respect to *spc*. Numbers in square brackets denote t-stats adjusted for cross-sectional heteroskedasticity. Statistical significance at the 1%, 5%, and 10% level is indicated by ***, **, and *, respectively.

Sharpe ratios in Table 9 show that *evc* has no explanatory power, while *spc* remains robustly significant across all horizons. In the return volatility regressions in Table 8, *evc* can explain the cross-section of return volatilities for industry portfolios. When combined with *spc*, however, the orthogonalized version of *evc* is insignificant for $H = 1$, but it seems to have explanatory power beyond *spc* for $H = 2, 3, 4$. In the bivariate regressions, the coefficient for *spc* is significant for all horizons. Finally, in the regressions for average excess returns in Table 10, *evc* yields negative and significant coefficients at the 10% level, while *spc* is not significant both in the univariate and in the bivariate regressions. Overall, we conclude that our theoretically motivated measure of directedness *spc* indeed contains additional information above and beyond the information captured by centrality measures like *evc*.

3 Conclusion

Networks have received considerable attention in the finance and economics literature. In this paper, we analyze the implications of directed links in cash flows networks for equilibrium returns. Our analysis is motivated by Carvalho et al. (2016) who provide rich empirical evidence for a delayed propagation of cash flow shocks, both at the firm and at the industry level, in a natural experiment setting around the nuclear incident of Fukushima in 2011. We model this delayed propagation with mutually exciting processes which naturally feature directedness and capture the intuition that cash flow shocks to one node in the network affect other nodes only with a certain time lag.

In our general equilibrium model we combine these self and mutually exciting jump processes for cash flows with a representative investor with recursive preferences. We prove the following cross-sectional statements for arbitrary directed networks: (i) Cash flow shocks in industries with high shock propagation capacity (*spc*) have a high market price of risk. (ii) The response of the price-to-cash flow ratio of an industry to its own cash flow shocks is less pronounced for industries with higher *spc*. Importantly, when it comes to expected excess returns, the effects of *spc* on market prices of risk and on exposures work in opposite directions, so that the overall impact of *spc* on risk premia depends on the tradeoff between these two forces.

We close the paper with an empirical illustration of our theoretical findings, where we estimate an empirical network from industry cash flows by applying the Diebold and Yilmaz (2014) generalized variance decomposition methodology. In line with the model, we find that high *spc* industries have lower return volatilities and higher Sharpe ratios than their low *spc* counterparts. Regression coefficients for average excess returns are, however, insignificant. Finally, we obtain regression results for return volatilities and Sharpe ratios in simulated data from the model which are qualitatively similar to their empirical counterparts.

To sum up, the innovative combination of self and mutually exciting jump processes with recursive preferences allows for the integration of directed networks into a tractable equilibrium asset pricing model. Our results indicate that it is necessary to decompose equilibrium asset prices and returns into their constituents in order to understand the implications of directed cash flow shock propagation.

APPENDIX

A. Model solution

To solve for the equilibrium we apply the approach proposed in Eraker and Shaliastovich (2008). The vector $X \equiv (y, \ell_1, \ldots, \ell_n, y_1, \ldots, y_n)'$ follows the affine jump process

$$dX_t \quad = \quad \mu\left(X_t\right)\, dt + \xi_t\, dN_t,$$

where we use the following notation:

- $\mu(X_t) = \mathcal{M} + \mathcal{K}\, X_t$
  with $\mathcal{M} = \begin{pmatrix} \mu \\ \kappa_1\, \bar{\ell}_1 \\ \vdots \\ \kappa_n\, \bar{\ell}_n \\ \mu_1 \\ \vdots \\ \mu_n \end{pmatrix}$ and $\mathcal{K} = \begin{pmatrix} 0 & 0 & \ldots & 0 & \ldots & 0 \\ 0 & -\kappa_1 & \ldots & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & -\kappa_n & \ldots & 0 \\ 0 & 0 & \ldots & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 0 & \ldots & 0 \end{pmatrix}$,
- $\ell_t = l_0 + l_1\, X_t$
  with $l_0 = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}$ and $l_1 = \begin{pmatrix} 0 & 1 & \ldots & 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 1 & 0 & \ldots & 0 \end{pmatrix}$,
- $\xi_t = \begin{pmatrix} \xi_{1,t}, & \ldots, & \xi_{n,t} \end{pmatrix} = \begin{pmatrix} K_1 & \ldots & K_n \\ \beta_{1,1} & \ldots & \beta_{1,n} \\ \vdots & \ddots & \vdots \\ \beta_{n,1} & \ldots & \beta_{n,n} \\ L_1 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & L_n \end{pmatrix}$.

The jump transform $\varrho\left(u\right) \equiv \mathbb{E}\left[\left(e^{u'\xi_{1,t}}, \ldots, e^{u'\xi_{n,t}}\right)\right]'$ is in our case simply equal to $\left(e^{u'\xi_{1,t}}, \ldots, e^{u'\xi_{n,t}}\right)'$, since the jump sizes are all constant.

We define the selection vectors δ_y, $\delta_{\ell_{i,t}}$ $(i = 1, \ldots, n)$, and $\delta_{y,i}$ $(i = 1, \ldots, n)$ implicitly via $dy_t = \delta_y'\, dX_t$, $d\ell_{i,t} = \delta_{\ell,i}'\, dX_t$, and $dy_{i,t} = \delta_{y,i}'\, dX_t$.

The continuous-time version of the Euler equation can be written as

$$0 \quad = \quad \frac{1}{dt}\, \mathbb{E}_t\left[\frac{d\left(e^{\ln M_t + \ln R_t}\right)}{e^{\ln M_t + \ln R_t}}\right], \qquad \text{(A.1)}$$

where R is the return on the claim to aggregate consumption. The logarithm of the pricing kernel has the dynamics

$$d\ln M_t \quad = \quad -\delta\, \theta\, dt - (1-\theta)\; d\ln R_t - \frac{\theta}{\psi}\, dy_t.$$

We apply the usual affine conjecture for the log wealth-consumption ratio

$$\begin{aligned} v_t &= A + (0, B_1, \ldots, B_n, 0, \ldots, 0) \; X_t \\ &= A + (B_1, \ldots, B_n) \, \ell_t, \end{aligned}$$

and use the Campbell-Shiller approximation for the return on the consumption claim

$$d \ln R_t = k_{v,0} \, dt + k_{v,1} \, dv_t - (1 - k_{v,1}) \; v_t \, dt + dy_t.$$

Combining the Campbell-Shiller approximation, the affine guess for v_t, and the dynamics of the log pricing kernel, we get

$$\begin{aligned} \frac{d\left(e^{\ln M_t + \ln R_t}\right)}{e^{\ln M_t + \ln R_t}} &= \left\{-\delta \, \theta + \theta \, k_{v,0} - \theta \; (1 - k_{v,1}) \; \left(A + B' \, X_t\right) + \chi_y' \; (\mathcal{M} + \mathcal{K} \, X_t)\right\} \, dt \\ &\quad + \left\{e^{\chi_y' \, \xi_t} - \mathbb{1}\right\} \, dN_t, \end{aligned} \tag{A.2}$$

where

$$\begin{aligned} \chi_y &= \theta \left[\left(1 - \frac{1}{\psi}\right) \, \delta_y + k_{v,1} \, B\right] \\ &= \left(-\theta \, \left(\frac{1}{\psi} - 1\right), \theta \, k_{v,1} \, B_1, \ldots, \theta \, k_{v,1} \, B_n, 0, \ldots, 0\right)', \end{aligned}$$

and where $\mathbb{1}$ is a vector of ones with length n. We plug expression (A.2) into the Euler equation (A.1) to get a system of equations for A and B:

$$0 = \theta \; [-\delta + k_{v,0} - (1 - k_{v,1}) \; A] + \mathcal{M}' \, \chi_y + l_0' \; [\varrho \, (\chi_y) - \mathbb{1}] \tag{A.3}$$

$$0 = \mathcal{K}' \, \chi_y - \theta \; (1 - k_{v,1}) \; B + l_1' \; [\varrho \, (\chi_y) - \mathbb{1}] \, . \tag{A.4}$$

We have two additional equations for the loglinearization constants $k_{v,0}$ and $k_{v,1}$:

$$0 = -k_{v,0} - \ln k_{v,1} + (1 - k_{v,1}) \; \left[A + B' \, \mu_X\right] \tag{A.5}$$

$$0 = A + B' \, \mu_X - \ln \, (k_{v,1}) + \ln \, (1 - k_{v,1}) \, , \tag{A.6}$$

where μ_X is a vector with i-th component $\mathbb{E}\,[X_i]$ if that expectation is finite and 0 otherwise. Due to the presence of the mutually exciting jump terms, the long-run means $\bar{\bar{\ell}}_i$, i.e., the unconditional expectations, are not equal to the respective mean reversion levels $\bar{\ell}_i$, as it would be the case, e.g., for a standard square-root process. According to Aït-Sahalia et al. (2015), the $\bar{\bar{\ell}}_i$ are the solution to the following system of equations:

$$\bar{\bar{\ell}}_i = \frac{\kappa_i \, \bar{\ell}_i + \sum_{j \neq i} \beta_{i,j} \, \bar{\bar{\ell}}_j}{\kappa_i - \beta_{i,i}} \qquad (i = 1, \ldots, n). \tag{A.7}$$

We assume $\kappa_i > \beta_{i,i}$ for $i = 1, \ldots, n$ to ensure that all the $\bar{\bar{\ell}}_i$ are positive.

We solve the four equations (A.3), (A.4), (A.5), and (A.6) via an iterative procedure. We initialize $k_{v,1}$ by setting it equal to δ, then compute $k_{v,0}$, A, and B. Given these we then compute $k_{v,1}$ again and iterate forward until the system converges.

The pricing kernel has dynamics

$$\frac{dM_t}{M_t} = -r_t \, dt - [\mathbb{1} - \varrho \, (-\lambda)]' \; (dN_t - \ell_t dt)$$

with

$$\begin{aligned} \lambda &= \gamma\,\delta_y + (1-\theta)\; k_{v,1}\, B \\ &= (\gamma, (1-\theta)\; k_{v,1}\, B_1, \ldots, (1-\theta)\; k_{v,1}\, B_n, 0, \ldots, 0)', \end{aligned}$$

so that we can immediately read off the risk-free rate and the market prices of risk. The risk-free rate is given as

$$r_t = \Phi_0 + \Phi_1'\, X_t$$

with

$$\Phi_0 = \theta\,\delta + (\theta-1)\;\left[\ln k_{v,1} + (k_{v,1}-1)\; B'\,\mu_X\right] + \mathcal{M}'\,\lambda - l_0'\;\left[\varrho\left(-\lambda\right) - \mathbb{1}\right]$$

and

$$\Phi_1 = (1-\theta)\;(k_{v,1}-1)\; B + \mathcal{K}'\,\lambda - l_1'\;\left[\varrho\left(-\lambda\right) - \mathbb{1}\right].$$

The market prices of jump risk are given as

$$\begin{aligned} \begin{pmatrix} \mathrm{MPJR}_1 \\ \vdots \\ \mathrm{MPJR}_n \end{pmatrix} &= \left[\mathbb{1} - \varrho\left(-\lambda\right)\right] \\ &= \begin{pmatrix} 1 - \exp\left(-\gamma\, K_1 + k_{v,1}\;(\theta-1)\;[B_1\,\beta_{1,1} + \ldots + B_n\,\beta_{n,1}]\right) \\ \vdots \\ 1 - \exp\left(-\gamma\, K_n + k_{v,1}\;(\theta-1)\;[B_1\,\beta_{1,n} + \ldots + B_n\,\beta_{n,n}]\right) \end{pmatrix}. \end{aligned}$$

The return on the consumption claim is given by

$$dR_t = \{\ldots\}\; dt + \{\varrho\left(\delta_y + k_{v,1}\, B\right) - \mathbb{1}\}\; dN_t$$

with jump exposures

$$\begin{pmatrix} \mathrm{JEXP}_{y,1} \\ \vdots \\ \mathrm{JEXP}_{y,n} \end{pmatrix} = \varrho\left(\delta_y + k_{v,1}\, B\right) - \mathbb{1},$$

where

$$\mathrm{JEXP}_{y,i} = \exp\left[K_1 + k_{v,1}\;(B_1\,\beta_{1,1} + \ldots + B_n\,\beta_{n,1})\right] - 1$$

for $i = 1, \ldots, n$.

To obtain the expected excess returns on the cash flow claims, we follow the same approach as for the consumption claim. The continuous-time Euler equation again reads

$$0 = \frac{1}{dt}\,\mathbb{E}_t\left[\frac{d\left(e^{\ln M_t + \ln R_{i,t}}\right)}{e^{\ln M_t + \ln R_{i,t}}}\right].$$

Applying the Campbell-Shiller approximation

$$d\ln R_{i,t} = k_{i,0}\, dt + k_{i,1}\, dv_{i,t} - (1-k_{i,1})\; v_{i,t}\, dt + dy_{i,t}$$

and the usual affine guess for the log price-to cash flow ratio

$$\begin{aligned} v_{i,t} &= A_i + (0, C_{i,1}, \ldots, C_{i,n}, 0, \ldots, 0)\; X_t \\ &= A_i + (C_{i,1}, \ldots, C_{i,n})\; \ell_t, \end{aligned}$$

we arrive at

$$
\begin{aligned}
\frac{d\left(e^{\ln M_t+\ln R_{i,t}}\right)}{e^{\ln M_t+\ln R_{i,t}}} &= \Big\{-\delta\,\theta-(1-\theta)\,\big[k_{v,0}-(1-k_{v,1})\,\big(A+B'\,X_t\big)\big]+k_{i,0} \\
&\quad -(1-k_{i,1})\,\big[A_i+C_i'\,X_t\big]+\chi_{y,i}'\,(\mathcal{M}+\mathcal{K}\,X_t)\Big\}\,dt \\
&\quad +\Big\{e^{\chi_{y,i}'\,\xi_t}-\mathbb{1}\Big\}\,dN_t, \qquad \text{(A.8)}
\end{aligned}
$$

where $\chi_{y,i}=k_{i,1}\,C_i+\delta_{y,i}-\lambda$. Plugging (A.8) into the Euler equation yields a system of equations for the coefficients A_i and C_i:

$$
\begin{aligned}
0 &= -\theta\,\delta+(1-\theta)\,\big[\ln k_{v,1}-(1-k_{v,1})\,B'\,\mu_X\big]-\ln k_{i,1}+(1-k_{i,1})\,C_i'\,\mu_X \\
&\quad +\mathcal{M}'\,\chi_{y,i}+l_0'\,\left[\varrho\left(\chi_{y,i}\right)-\mathbb{1}\right] \qquad \text{(A.9)} \\
0 &= \mathcal{K}'\,\chi_{y,i}+(1-\theta)\,(1-k_{v,1})\,B-(1-k_{i,1})\,C_i+l_1'\,\left[\varrho\left(\chi_{y,i}\right)-\mathbb{1}\right]. \qquad \text{(A.10)}
\end{aligned}
$$

The two additional equations for the log-linearization constants $k_{i,0}$ and $k_{i,1}$ are

$$
\begin{aligned}
0 &= -k_{i,0}-\ln k_{i,1}+(1-k_{i,1})\,\big(A_i+C_i'\,\mu_X\big) \qquad \text{(A.11)} \\
0 &= A_i+C_i'\,\mu_X-\ln k_{i,1}+\ln\left(1-k_{i,1}\right). \qquad \text{(A.12)}
\end{aligned}
$$

The return of the individual cash flow claim i is then given by

$$
dR_{i,t} = \{\ldots\}\,dt+\left\{\varrho\left(\delta_{y,i}+k_{i,1}\,C_i\right)-\mathbb{1}\right\}\,dN_t
$$

so that the jump exposure of the return is thus given by

$$
\begin{aligned}
\begin{pmatrix} \mathrm{JEXP}_{i,1} \\ \vdots \\ \mathrm{JEXP}_{i,i} \\ \vdots \\ \mathrm{JEXP}_{i,n} \end{pmatrix} &= \left[\varrho\left(\delta_{y,i}+k_{i,1}\,C_i\right)-\mathbb{1}\right] \\
&= \begin{pmatrix} \exp\left(k_{i,1}\,\left[C_{i,1}\,\beta_{1,1}+\ldots+C_{i,n}\,\beta_{n,1}\right]\right)-1 \\ \vdots \\ \exp\left(L_i+k_{i,1}\,\left[C_{i,1}\,\beta_{1,i}+\ldots+C_{i,n}\,\beta_{n,i}\right]\right)-1 \\ \vdots \\ \exp\left(k_{i,1}\,\left[C_{i,1}\,\beta_{1,n}+\ldots+C_{i,n}\,\beta_{n,n}\right]\right)-1 \end{pmatrix}.
\end{aligned}
$$

The expected return on the claim to cash flow i can then be written as

$$
\begin{aligned}
\frac{1}{dt}\,\mathbb{E}_t\left[dR_{i,t}\right] &= -\ln k_{i,1}+(1-k_{i,1})\,C_i'\,(\mu_X-X_t)+\left[\delta_i+k_{i,1}\,C_i\right]'\,(\mathcal{M}+\mathcal{K}\,X_t) \\
&\quad +\left[\varrho\left(\delta_{y,i}+k_{i,1}\,C_i\right)-\mathbb{1}\right]\,(l_0+l_1\,X_t)\,.
\end{aligned}
$$

The expected excess return is given by

$$
\frac{1}{dt}\,\mathbb{E}_t\left[dR_{i,t}\right]-r_t = (l_0+l_1\,X_t)'\,\left[\varrho\left(\chi_{y,i}+\lambda\right)+\varrho\left(-\lambda\right)-\varrho\left(\chi_{y,i}\right)-\mathbb{1}\right]
$$

which can be represented as

$$\frac{1}{dt}\,\mathbb{E}_t\left[dR_{i,t}\right] - r_t \;=\; \sum_{j=1}^{n} \ell_{j,t}\,\mathrm{MPJR}_j\,\mathrm{JEXP}_{i,j}.$$

B. Approximation for general network structures

B.1. Market prices of jump risk

B.1.1. First approximation step

Rewriting Equation (A.4) for $\kappa_1 = \ldots = \kappa_n = \kappa$ and $K_1 = \ldots = K_n = K$ gives the following system of equations

$$\begin{aligned} 0 &= B_1\,\theta\,\left[k_{v,1}\,(1-\kappa)-1\right] + \exp\left\{K\,(1-\gamma) + \theta\,k_{v,1}\,\left(B_1\,\beta_{1,1} + \ldots + B_n\,\beta_{n,1}\right)\right\} - 1 \\ &\vdots \\ 0 &= B_n\,\theta\,\left[k_{v,1}\,(1-\kappa)-1\right] + \exp\left\{K\,(1-\gamma) + \theta\,k_{v,1}\,\left(B_1\,\beta_{1,n} + \ldots + B_n\,\beta_{n,n}\right)\right\} - 1 \end{aligned}$$

and translating this into matrix notation yields

$$\mathbb{1} \;=\; \theta\,\left[k_{v,1}\,(1-\kappa)-1\right]\,B + \exp\left\{K\,(1-\gamma)\right\}\,\exp\left\{\theta\,k_{v,1}\,\beta'\,B\right\},$$

where now and in the following, the "exp" operator, applied to a vector, stands for element-wise application of the "exp" operator to the vector.

Next, we apply the approximation $\exp(x) = 1 + x + O\left(x^2\right)$ and solve for B:

$$B \;=\; \left(I_{n\times n} + \frac{\exp\left\{K\,(1-\gamma)\right\}}{1-\kappa-\frac{1}{k_{v,1}}}\,\beta'\right)^{-1} \frac{1}{\theta\,\left[k_{v,1}\,(1-\kappa)-1\right]}\,\left[\mathbb{1} - \exp\left\{K\,(1-\gamma)\right\}\right] + O\left(\beta^2\right) \qquad \text{(B.1)}$$

where $I_{n\times n}$ denotes an $n \times n$ identity matrix and $\frac{\exp\{K\,(1-\gamma)\}}{1-\kappa-\frac{1}{k_{v,1}}} < 0$ since $\frac{1}{k_{v,1}} > 1-\kappa$ (due to $\frac{1}{k_{v,1}} = \frac{1+e^{\bar{v}}}{e^{\bar{v}}} > 1 > 1-\kappa$ for $0 < \kappa < 1$).

To conclude the first approximation step, we define

$$B^* \;=\; \left(I_{n\times n} + \frac{\exp\left\{K\,(1-\gamma)\right\}}{1-\kappa-\frac{1}{k_{v,1}}}\,\beta'\right)^{-1} \frac{1}{\theta\,\left[k_{v,1}\,(1-\kappa)-1\right]}\,\left[\mathbb{1} - \exp\left\{K\,(1-\gamma)\right\}\right]. \qquad \text{(B.2)}$$

B.1.2. Second approximation step

Since the inverse term in Equation (B.1) has the structure of a Leontief inverse, $(I - A)^{-1} = I + A^1 + A^2 + \ldots$, we rewrite (B.1) as:

$$\begin{aligned} B &= \left[I_{n\times n} - \frac{\exp\{K\,(1-\gamma)\}}{1-\kappa-\frac{1}{k_{v,1}}}\,\beta' - \left(\frac{\exp\{K\,(1-\gamma)\}}{1-\kappa-\frac{1}{k_{v,1}}}\,\beta'\right)^2 - \ldots \right] \frac{1}{\theta\,[k_{v,1}\,(1-\kappa)-1]} \\ &\quad \times [\mathbb{1} - \exp\{K\,(1-\gamma)\}] + O\left(\beta^2\right) \\ &= \left(I_{n\times n} - \frac{\exp\{K\,(1-\gamma)\}}{1-\kappa-\frac{1}{k_{v,1}}}\,\beta' \right) \frac{1}{\theta\,[k_{v,1}\,(1-\kappa)-1]}\,[\mathbb{1} - \exp\{K\,(1-\gamma)\}] \\ &\quad + O\left(\beta^2\right) \end{aligned} \tag{B.3}$$

To conclude the second approximation step, we define

$$B^{**} = \left(I_{n\times n} - \frac{\exp\{K\,(1-\gamma)\}}{1-\kappa-\frac{1}{k_{v,1}}}\,\beta' \right) \frac{1}{\theta\,[k_{v,1}\,(1-\kappa)-1]}\,[\mathbb{1} - \exp\{K\,(1-\gamma)\}]\,. \tag{B.4}$$

Plugging (B.3) into the market price of risk from Equation (4) and rewriting this in matrix notation yields:

$$\begin{aligned} \text{MPJR} &= 1 - \exp\left\{ -\gamma\,K + \frac{k_{v,1}\,(\theta-1)}{\theta\,[k_{v,1}\,(1-\kappa)-1]}\,\left[\beta'\,[\mathbb{1} - \exp\{K\,(1-\gamma)\}] + O\left(\beta^2\right)\right] \right\} \\ &= 1 - \exp\left\{ -\gamma\,K + \frac{(\theta-1)\,(1-\exp\{K\,(1-\gamma)\})}{\theta\,\left[(1-\kappa)-\frac{1}{k_{v,1}}\right]}\,(\beta_{\text{diag}} + \text{spc}) + O\left(\beta^2\right) \right\} \\ &= 1 - \exp\left\{ \mathcal{A} + \mathcal{B}\,(\beta_{\text{diag}} + \text{spc}) + O\left(\beta^2\right) \right\} \end{aligned}$$

with $\mathcal{A}$ and $\mathcal{B}$ given in Proposition 1. Thus we define

$$\text{MPJR}^{**} = 1 - \exp\{\mathcal{A} + \mathcal{B}\,(\beta_{\text{diag}} + \text{spc})\}\,. \tag{B.5}$$

For $\gamma > 1$, $\theta < 0$, $0 < \kappa < 1$, and $K < 0$, we have $\mathcal{A} > 0$ and $\mathcal{B} > 0$ since $\frac{1}{k_{v,1}} > 1 - \kappa$.

B.2. Jump exposures

B.2.1. First approximation step

Rewriting Equation (A.10) for $\kappa_1 = \ldots = \kappa_n = \kappa$ and $K_1 = \ldots = K_n = K$ gives a system of equations for each i, exemplified in the following for $i = 1$:

$$\begin{aligned} 0 &= B_1\,(k_{v,1}-1)\,(\theta-1) + C_{1,1}\,(k_{1,1}-1) - \kappa\,[B_1\,k_{v,1}\,(\theta-1) + C_{1,1}\,k_{1,1}] \\ &\quad + \exp\{L - K\,\gamma + \beta_{1,1}\,[B_1\,k_{v,1}\,(\theta-1) + C_{1,1}\,k_{1,1}] + \ldots + \beta_{n,1}\,[B_n\,k_{v,1}\,(\theta-1) + C_{1,n}\,k_{1,1}]\} - 1 \\ &\;\;\vdots \\ 0 &= B_n\,(k_{v,1}-1)\,(\theta-1) + C_{1,n}\,(k_{1,1}-1) - \kappa\,[B_n\,k_{v,1}\,(\theta-1) + C_{1,n}\,k_{1,1}] \\ &\quad + \exp\{-K\,\gamma + \beta_{1,n}\,[B_1\,k_{v,1}\,(\theta-1) + C_{1,1}\,k_{1,1}] + \ldots + \beta_{n,n}\,[B_n\,k_{v,1}\,(\theta-1) + C_{1,n}\,k_{1,1}]\} - 1. \end{aligned}$$

Collecting terms and introducing matrix notation yields the following system for each i:

$$\begin{aligned}\mathbb{1} &= B\,(\theta-1)\,[k_{v,1}\,(1-\kappa)-1] + C_i\,[k_{i,1}\,(1-\kappa)-1] \\ &\quad + [\exp\{-K\,\gamma\}\,\mathbb{1} + (\exp\{L-K\,\gamma\} - \exp\{-K\,\gamma\})\,I_{n\times 1,i}] \bullet \exp\left\{k_{v,1}\,(\theta-1)\,\beta'\,B + k_{i,1}\,\beta'\,C_i\right\},\end{aligned}$$

where now and in the following, $\bullet$ represents element-wise multiplication of the vectors. $I_{n\times 1,i}$ is an $n\times 1$ vector with the i-th entry equal to 1 and zeros otherwise.

Again, we employ $\exp(x) = 1 + x + O\left(x^2\right)$ and solve for C_i:

$$\begin{aligned}C_i &= \left(I + \frac{\exp\{-K\,\gamma\}\,\mathbb{1} + (\exp\{L-K\,\gamma\} - \exp\{-K\,\gamma\})\,I_{n\times 1,i}}{1-\kappa-\frac{1}{k_{i,1}}} \bullet \beta'\right)^{-1} \frac{1}{k_{i,1}\,(1-\kappa)-1} \\ &\quad \times \Bigg[\mathbb{1} - (\theta-1)\,[k_{v,1}\,(1-\kappa)-1]\,B \\ &\quad - [\exp\{-K\,\gamma\}\,\mathbb{1} + (\exp\{L-K\,\gamma\} - \exp\{-K\,\gamma\})\,I_{n\times 1,i}] \bullet k_{v,1}\,(\theta-1)\,\beta'\,B \\ &\quad - \exp\{-K\,\gamma\}\,\mathbb{1} - (\exp\{L-K\,\gamma\} - \exp\{-K\,\gamma\})\,I_{n\times 1,i}\Bigg] + O\left(\beta^2\right), \qquad \text{(B.6)}\end{aligned}$$

where $\frac{\exp\{-K\,\gamma\}\,\mathbb{1} + (\exp\{L-K\,\gamma\} - \exp\{-K\,\gamma\})\,I_{n\times 1,i}}{1-\kappa-\frac{1}{k_{i,1}}} < 0$ since $\frac{1}{k_{i,1}} > 1-\kappa$ (due to $\frac{1}{k_{i,1}} = \frac{1+e^{\bar{v}_i}}{e^{\bar{v}_i}} > 1 > 1-\kappa$ for $0 < \kappa < 1$).

To conclude the first approximation step, we define

$$\begin{aligned}C_i^* &= \left(I + \frac{\exp\{-K\,\gamma\}\,\mathbb{1} + (\exp\{L-K\,\gamma\} - \exp\{-K\,\gamma\})\,I_{n\times 1,i}}{1-\kappa-\frac{1}{k_{i,1}}} \bullet \beta'\right)^{-1} \frac{1}{k_{i,1}\,(1-\kappa)-1} \\ &\quad \times \Bigg[\mathbb{1} - (\theta-1)\,[k_{v,1}\,(1-\kappa)-1]\,B \\ &\quad - [\exp\{-K\,\gamma\}\,\mathbb{1} + (\exp\{L-K\,\gamma\} - \exp\{-K\,\gamma\})\,I_{n\times 1,i}] \bullet k_{v,1}\,(\theta-1)\,\beta'\,B \\ &\quad - \exp\{-K\,\gamma\}\,\mathbb{1} - (\exp\{L-K\,\gamma\} - \exp\{-K\,\gamma\})\,I_{n\times 1,i}\Bigg]. \qquad \text{(B.7)}\end{aligned}$$

B.2.2. Second approximation step

Again the inverse term in Equation (B.6) has the structure of a Leontief inverse, and we rewrite (B.6) as:

$$\begin{aligned}
C_i &= \Bigg[I_{n\times n} - \frac{\exp\{-K\,\gamma\}\ \mathbb{1} + (\exp\{L - K\,\gamma\} - \exp\{-K\,\gamma\})\ I_{n\times 1,i}}{1-\kappa-\frac{1}{k_{i,1}}} \bullet \beta' \\
&\quad - \left(\frac{\exp\{-K\,\gamma\}\ \mathbb{1} + (\exp\{L - K\,\gamma\} - \exp\{-K\,\gamma\})\ I_{n\times 1,i}}{1-\kappa-\frac{1}{k_{i,1}}} \bullet \beta' \right)^2 - \ldots \Bigg] \frac{1}{k_{i,1}\ (1-\kappa)-1} \\
&\quad \times \Big[\mathbb{1} - (\theta-1)\ [k_{v,1}\ (1-\kappa)-1]\ B \\
&\quad - [\exp\{-K\,\gamma\}\ \mathbb{1} + (\exp\{L-K\,\gamma\} - \exp\{-K\,\gamma\})\ I_{n\times 1,i}] \bullet k_{v,1}\ (\theta-1)\ \beta'\,B \\
&\quad - \exp\{-K\,\gamma\}\ \mathbb{1} - (\exp\{L-K\,\gamma\} - \exp\{-K\,\gamma\})\ I_{n\times 1,i} \Big] + O\left(\beta^2\right) \\
&= \left(I_{n\times n} - \frac{\exp\{-K\,\gamma\}\ \mathbb{1} + (\exp\{L - K\,\gamma\} - \exp\{-K\,\gamma\})\ I_{n\times 1,i}}{1-\kappa-\frac{1}{k_{i,1}}} \bullet \beta' \right) \frac{1}{k_{i,1}\ (1-\kappa)-1} \\
&\quad \times \Big[\mathbb{1} - (\theta-1)\ [k_{v,1}\ (1-\kappa)-1]\ B \\
&\quad - [\exp\{-K\,\gamma\}\ \mathbb{1} + (\exp\{L-K\,\gamma\} - \exp\{-K\,\gamma\})\ I_{n\times 1,i}] \bullet k_{v,1}\ (\theta-1)\ \beta'\,B \\
&\quad - \exp\{-K\,\gamma\}\ \mathbb{1} - (\exp\{L-K\,\gamma\} - \exp\{-K\,\gamma\})\ I_{n\times 1,i} \Big] + O\left(\beta^2\right). \qquad \text{(B.8)}
\end{aligned}$$

To conclude the second approximation step, we define

$$\begin{aligned}
C_i^{**} &= \left(I_{n\times n} - \frac{\exp\{-K\,\gamma\}\ \mathbb{1} + (\exp\{L - K\,\gamma\} - \exp\{-K\,\gamma\})\ I_{n\times 1,i}}{1-\kappa-\frac{1}{k_{i,1}}} \bullet \beta' \right) \frac{1}{k_{i,1}\ (1-\kappa)-1} \\
&\quad \times \Big[\mathbb{1} - (\theta-1)\ [k_{v,1}\ (1-\kappa)-1]\ B \\
&\quad - [\exp\{-K\,\gamma\}\ \mathbb{1} + (\exp\{L-K\,\gamma\} - \exp\{-K\,\gamma\})\ I_{n\times 1,i}] \bullet k_{v,1}\ (\theta-1)\ \beta'\,B \\
&\quad - \exp\{-K\,\gamma\}\ \mathbb{1} - (\exp\{L-K\,\gamma\} - \exp\{-K\,\gamma\})\ I_{n\times 1,i} \Big] \qquad \text{(B.9)}
\end{aligned}$$

Plugging (B.8) into the jump exposures from Equation (6) and rewriting them in matrix notation yields:

$$\begin{aligned}
\text{JEXP}_i &= \exp\Bigg\{ L\, I_{n\times 1,i} + \frac{1 - \frac{\theta-1}{\theta}\ (1-\exp\{K\ (1-\gamma)\}) - \exp\{-K\,\gamma\}}{1-\kappa-\frac{1}{k_{i,1}}}\ \beta'\,\mathbb{1} \\
&\quad - \frac{\exp\{-K\,\gamma\}\ (\exp\{L\}-1)}{1-\kappa-\frac{1}{k_{i,1}}}\ \beta'\, I_{n\times 1,i} + O\left(\beta^2\right) \Bigg\} - 1.
\end{aligned}$$

Breaking this expression down into the jump exposures $\mathrm{JEXP}_{i,j}$ yields:

$$\begin{aligned}\mathrm{JEXP}_{i,j} &= \begin{cases} \exp\left\{\mathcal{C}_i \cdot \sum_{k=1,k\neq i}^{n} \beta_{k,j} + \mathcal{D}_i \cdot \beta_{i,j} + O\left(\beta^2\right)\right\} - 1 & \text{for } j \neq i \\ \exp\left\{L + \mathcal{C}_i \cdot \sum_{k=1,k\neq i}^{n} \beta_{k,i} + \mathcal{D}_i \cdot \beta_{i,i} + O\left(\beta^2\right)\right\} - 1 & \text{for } j = i \end{cases} \\ &= \begin{cases} \exp\left\{\mathcal{C}_i \cdot \mathrm{spc}_j + \mathcal{C}_i \cdot \beta_{j,j} + \left(\mathcal{D}_i - \mathcal{C}_i\right)\, \beta_{i,j} + O\left(\beta^2\right)\right\} - 1 & \text{for } j \neq i \\ \exp\left\{L + \mathcal{C}_i \cdot \mathrm{spc}_i + \mathcal{D}_i \cdot \beta_{i,i} + O\left(\beta^2\right)\right\} - 1 & \text{for } j = i \end{cases}\end{aligned}$$

where

$$\begin{aligned}\mathcal{C}_i &= \frac{1 - \frac{\theta-1}{\theta}\,\left[1 - \exp\left\{K\,(1-\gamma)\right\}\right] - \exp\left\{-K\,\gamma\right\}}{1 - \kappa - \frac{1}{k_{i,1}}} \\ \mathcal{D}_i &= \frac{1 - \frac{\theta-1}{\theta}\,\left[1 - \exp\left\{K\,(1-\gamma)\right\} - \exp\left\{L - K\,\gamma\right\}\right]}{1 - \kappa - \frac{1}{k_{i,1}}} \\ \mathcal{D}_i - \mathcal{C}_i &= \frac{\exp\left\{-K\,\gamma\right\}\,\left(1 - \exp\left\{L\right\}\right)}{1 - \kappa - \frac{1}{k_{i,1}}}.\end{aligned}$$

Note that $\frac{1}{k_{i,1}} > 1-\kappa$ (see above). For $\gamma > 1$, $0 < \kappa < 1$, and $-\log(2) < K < 0$, we have $\mathcal{C}_i > 0$. Additionally assuming $\theta < 0$, we obtain $\mathcal{D}_i < 0$, and $\mathcal{D}_i - \mathcal{C}_i < 0$.

Proof that $\mathcal{C}_\mathbf{i} > 0$: We rewrite $\mathcal{C}_i$ as follows:

$$\begin{aligned}\mathcal{C}_i &= \frac{1 - \frac{\theta-1}{\theta}\,\left[1 - \exp\left\{K\,(1-\gamma)\right\}\right] - \exp\left\{-K\,\gamma\right\}}{1 - \kappa - \frac{1}{k_{i,1}}} \\ &= \frac{\exp\left\{-K\,\gamma\right\}\,\left[\frac{1}{\theta}\,\left(\exp\left\{K\,\gamma\right\} - 1\right) + \exp\left\{K\right\} - 1\right]}{1 - \kappa - \frac{1}{k_{i,1}}}\end{aligned}$$

Here, we have $1-\kappa-\frac{1}{k_{i,1}} < 0$ by assumption (since $0 < \kappa < 1$). Moreover, we have $\exp\left\{-K\,\gamma\right\} > 0$ and $\frac{1}{\theta}\,\left(\exp\left\{K\,\gamma\right\} - 1\right) + \exp\left\{K\right\} - 1 < 0$.

To see the last inequality, define

$$\begin{aligned}f\left(K\right) &= \exp\left\{K\,\gamma\right\} - 1 - \left(\exp\left\{K\right\} + \gamma\right)\,\left(\exp\left\{K\right\} - 1\right) \\ &= \exp\left\{K\,\gamma\right\} - 1 - \exp\left\{2\,K\right\} - \gamma\,\exp\left\{K\right\} + \exp\left\{K\right\} + \gamma\end{aligned}$$

Then $f\left(0\right) = 0$ and

$$\begin{aligned}f'\left(K\right) &= \gamma\,\exp\left\{K\,\gamma\right\} - 2\,\exp\left\{2\,K\right\} - \gamma\,\exp\left\{K\right\} + \exp\left\{K\right\} \\ &= \gamma\,\left(\exp\left\{K\,\gamma\right\} - \exp\left\{K\right\}\right) + \exp\left\{K\right\} - 2\,\exp\left\{2\,K\right\}\end{aligned}$$

If $\gamma > 1$ and $-\ln\left(2\right) < K < 0$, then $f'\left(K\right) < 0$ which implies $f\left(K\right) > 0$. In particular,

$$\frac{\exp\left\{K\,\gamma\right\} - 1}{\exp\left\{K\right\} - 1} < \exp\left\{K\right\} + \gamma < -\theta$$

from where the statement then follows. Altogether, we thus get $\mathcal{C}_i > 0$.

Proof that $\mathcal{D}_\mathbf{i} < \mathbf{0}$: We rewrite $\mathcal{D}_i$ as follows:

$$\begin{aligned}\mathcal{D}_i &= \frac{1 - \frac{\theta-1}{\theta}\,[1 - \exp\{K\,(1-\gamma)\}] - \exp\{L - K\,\gamma\}}{1 - \kappa - \frac{1}{k_{i,1}}} \\ &= \frac{\frac{1}{\theta} + \exp\{-K\,\gamma\}\,\left[\left(1-\frac{1}{\theta}\right)\exp\{K\} - \exp\{L\}\right]}{1 - \kappa - \frac{1}{k_{i,1}}}\end{aligned}$$

Again, we have $1 - \kappa - \frac{1}{k_{i,1}} < 0$. Moreover, we have

$$\begin{aligned} & \frac{1}{\theta} + \exp\{-K\,\gamma\}\left[\left(1-\frac{1}{\theta}\right)\exp\{K\} - \exp\{L\}\right] > 0 \\ \Leftrightarrow\quad & \exp\{-K\,\gamma\}\left[\left(1-\frac{1}{\theta}\right)\exp\{K\} - \exp\{L\}\right] > -\frac{1}{\theta} \\ \Leftrightarrow\quad & \left(1-\frac{1}{\theta}\right)\exp\{K\} + \frac{1}{\theta}\exp\{K\,\gamma\} - \exp\{L\} > 0 \\ \Leftrightarrow\quad & (\exp\{K\} - \exp\{L\}) + \frac{1}{\theta}\,(\exp\{K\,\gamma\} - \exp\{K\}) > 0 \end{aligned}$$

which is true if $L < K$, $\gamma > 1$ and $\theta < 0$. This completes the proof.

B.3. Approximation quality

In this section, we assess the quality of the first-order approximations derived in the Propositions 1 and 2 using the empirical network for $H = 3$, i.e., the network which is preferred according to Table 5.

As explained in Appendix B.1, MPJR** is based on two approximation steps, B^* and B^{**}. The left part of Figure 2 shows the result of the first approximation step graphically by plotting B^* against the exact solution B of Equation (4). The middle part shows similar results for the second approximation step, B^{**} given in Equation (B.4). Finally, the right part of Figure 2 depicts the full approximation of the market prices of risk MPJR** against the exact MPJR.

Regressing B^* (or B^{**}, resp.) on B yields the following parameter estimates, t-stats, R^2, and correlations:

$$\begin{aligned} B_i^* &= \underset{(-16.2)}{-0.0003} + \underset{(189.3)}{0.8794}\; B_i + u_i, \qquad R^2 = 0.9998, \quad \text{Corr} = 0.9999 \\ B_i^{**} &= \underset{(-49.5)}{-0.0015} + \underset{(33.1)}{0.2461}\; B_i + u_i, \qquad R^2 = 0.9848, \quad \text{Corr} = 0.9923. \end{aligned}$$

Performing a similar regression of MPJR** on MPJR gives:

$$\text{MPJR}_i^{**} = \underset{(-49.1)}{-0.0383} + \underset{(32.8)}{0.2512}\;\text{MPJR}_i + u_i, \qquad R^2 = 0.9847, \quad \text{Corr} = 0.9923.$$

Altogether, we see from the figures that the first approximation step hardly affects the B coefficients at all. The second approximation step (approximating the Leontief inverse) changes all coefficients quantitatively, but not qualitatively. The ordering of the coefficients is preserved, the sign is preserved, the correlation between approximated and exact coefficients is 99%. Only the size and the dispersion is reduced.

Similarly, JEXP** is based on two approximation steps, C^* and C^{**}, and its approximation

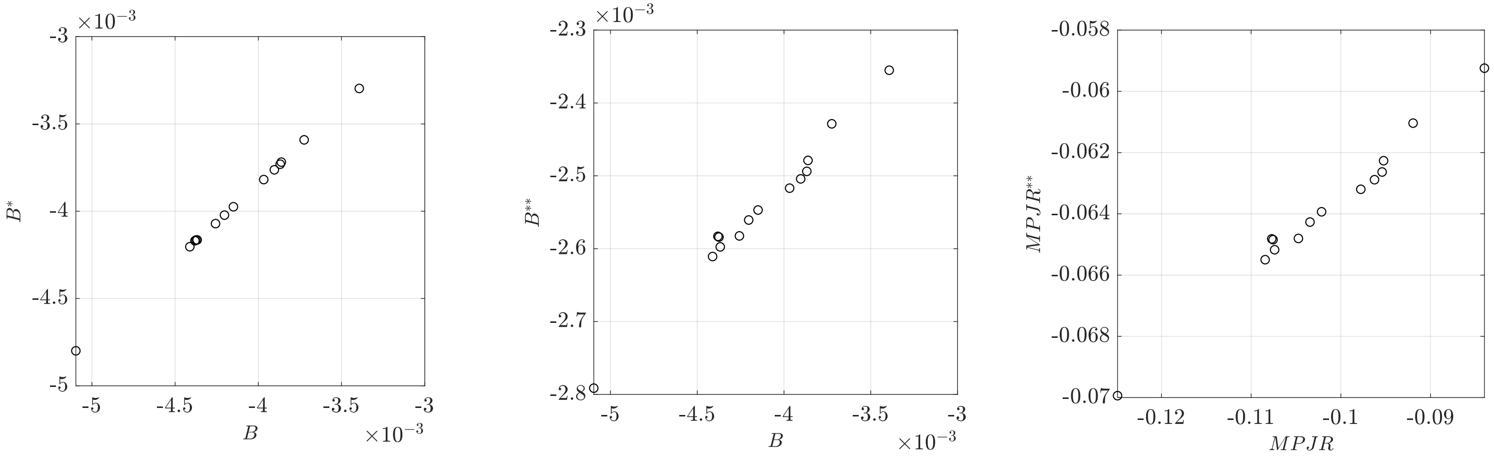

Figure 2
Approximation quality of B^*, B^{**}, and MPJR**

The graph on the left (in the middle) plot the coefficients B^* (B^{**}) as a function of the coefficients B. The graph on the right plots the approximation of the market price of jump risk as stated in Proposition 1, i.e., MPJR**, as a function of the market prices of risk MPJR for which the B coefficients have been computed numerically. The coefficients B^* and B^{**} are defined in Appendix B.1. The regression results are discussed in Appendix B.3. We use the empirical network determined in Section 2.2 for a forecast horizon of $H = 3$ quarters as the beta matrix. The remaining parameters are given in Table 4.

quality is shown in Figures 3. The corresponding regressions yield

$$C_i^* = \underset{(-51.9)}{-0.0054} + \underset{(326.1)}{0.9601} C_i + u_i, \qquad R^2 = 0.9960, \quad \text{Corr} = 0.9980.$$

$$C_i^{**} = \underset{(-23.2)}{-0.0036} + \underset{(53.4)}{0.7675} C_i + u, \qquad R^2 = 0.9869, \quad \text{Corr} = 0.9934.$$

$$\text{JEXP}_i^{**} = \underset{(-10.7)}{-0.0010} + \underset{(53.5)}{0.7673} \text{JEXP}_i + u_i, \qquad R^2 = 0.9883, \quad \text{Corr} = 0.9942.$$

Again, we see that the second approximation step is more severe than the first one. However, it does not change the coefficients qualitatively. The ordering of the coefficients as well as the sign are preserved and the correlation between approximated and exact coefficients is 99%.

C. Industries

We use the industry codes in the Industry Economic Accounts provided by the Bureau of Economic Analysis (BEA) at the sector level.[28] These are based on the North American Industry Classification System (NAICS) code structure and contain 15 groups of industries. Following Aobdia et al. (2014) and Menzly and Ozbas (2010), we exclude the government sector. We refer to the 14 industries in our network graphs in Figure 1 as:

1. Ag: Agriculture, forestry, fishing, and hunting;
2. Mi: Mining;
3. Ut: Utilities;
4. Co: Construction;
5. Ma: Manufacturing;
6. Wh: Wholesale trade;
7. Re: Retail trade;
8. Tr: Transportation and warehousing;
9. In: Information;
10. Fi: Finance, insurance, real estate, rental, and leasing;
11. Pr: Professional and business services;
12. Ed: Educational services, health care, and social assistance;
13. Ar: Arts, entertainment, recreation, accommodation, and food services;
14. Ot: Other services, except government.

[28] Available at the BEA homepage (`https://bea.gov/industry/io_annual.htm`).

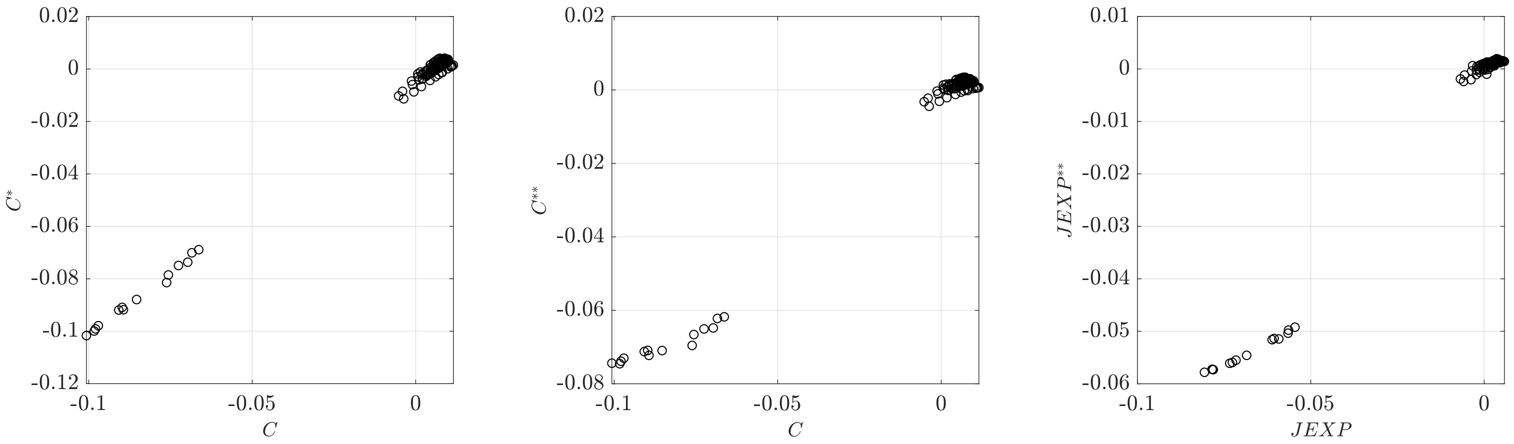

Figure 3

Approximation quality of C^{*}, C^{**}, and JEXP**

The graph on the left (in the middle) plots the coefficients C^{*} (C^{**}) as a function of the coefficients C. The graph on the right plots the approximation of the jump exposures as stated in Proposition 2, i.e., JEXP**, as a function of the jump exposures JEXP for which the C coefficients have been computed numerically. The coefficients C^{*} and C^{**} are defined in Appendix B.2. The regression results are discussed in Appendix B.3. We use the empirical network determined in Section 2.2 for a forecast horizon of $H = 3$ quarters as the beta matrix. The remaining parameters are given in Table 4.

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