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Interest rate pass-through to the rates of core deposits – a new perspective

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Non-technical summary

Research question

Most empirical studies regarding interest rate pass-through do not find a stable long-run relationship between capital market rates and variable bank rates on core deposits. Incomplete transmission raises questions about the deeper issue of the interest rate variability of these refinancing instruments as opposed to rate fixation of other types of instruments.

Contribution

The paper introduces a model of the deposit market in which risk-averse bank owners strive to smooth distributable profits over time. The market is characterized by illiquid bank assets paying fixed rates, by indefinite deposit commitment and by depositors who are insensitive to short-term fluctuations of the deposit rate. In this environment, the deposit rate will follow an exactly defined path which depends on the return on the loan portfolio. This relationship is consistent with the replicating portfolio approach many banks apply for funds transfer pricing and which can be used as a benchmark for interest rate pass-through models.

Results

This paper will show that the variability of interest rates on core deposits allows banks to stabilize their margins even if lending rates and asset structure are fixed over time and, thus, to generate a cash flow which is largely determined by past lending decisions and the past interest rate environment. In this setting, the path of the optimal deposit rate is, in principle, independent of capital market rates. An indirect relationship is due to lending rates following market rates closely. In this case, the deposit rates can be characterized as moving averages of capital market rates of different terms, i.e. as a replicating portfolio.

Nichttechnische Zusammenfassung

Fragestellung

Die meisten empirischen Studien zur Zinsweitergabe finden keine stabile Langfristbeziehung zwischen Kapitalmarktzinsen und variablen Bankzinsen auf Sicht- und Spareinlagen. Die Unvollständigkeit der Zinsweitergabe bei zinsvariablen Produkten wirft die Frage auf, welchen grundlegenden Zweck die Variabilität der Verzinsung bestimmter Finanzierungstypen in Abgrenzung zur fixierten Verzinsung anderer Finanzinstrumente verfolgt.

Beitrag

Das Papier stellt ein Modell des Depositenmarktes vor, in dem risikoscheue Bankeigner danach streben, Gewinnausschüttungen über die Zeit glätten. Des Weiteren ist der Markt bankseitig von einer illiquiden Anlagestruktur zu fixierten Zinssätzen und seitens der Einleger von einer unbestimmten Kapitalüberlassungsdauer und Unempfindlichkeit gegenüber kurzfristigen Zinsänderungen charakterisiert. In diesem Umfeld folgt der Depositenzins einen exakt spezifizierbaren Pfad, der von der Rendite des Kreditportfolios abhängig ist. Diese Beziehung ist konsistent zu der Methode der Replikationsportfolien, die viele Banken für die interne Kalkulation verwenden, und die als Benchmark für Modelle der Zinsweitergabe herangezogen werden können.

Ergebnisse

Es wird gezeigt, dass die variable Verzinsung von Sicht- und Spareinlagen den Banken eine Margenstabilisierung über die Zeit auch dann ermöglicht, wenn Anlagenstruktur und Kreditzinsen über ihre Laufzeit fixiert sind und somit einen Zahlungsstrom generieren, der weitgehend durch vergangene Investitionsentscheidungen und vergangenen Zinskonditionen determiniert ist. Damit ist aber der Pfad der Depositenzinsen prinzipiell unabhängig von den gerade geltenden Zinsen am Kapitalmarkt. Eine indirekte Beziehung ergibt sich hingegen, wenn Kreditzinsen den Zinsen am Kapitalmarkt folgen. Dann lassen sich Depositenzinsen als gleitende Durchschnitte von Kapitalmarktzinsen verschiedener Laufzeiten, d.h. als Replikationsportfolio, charakterisieren.

Interest rate pass-through to the rates of core deposits – a new perspective[1]

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Abstract

Within a Salop framework, this paper shows that banks' profit smoothing can explain incomplete pass-through of market rates to the rates of core deposits. Using time series data of deposit and lending rates of local German banks, this paper will show that local banks pass through return variations to their depositors. To the degree to which market rates influence new business lending rates, there is, therefore, an indirect channel through which market rates affect the rate of core deposits. In the absence of capital market alternatives for core deposits, this indirect channel explains why changes in the market rate affect the rate of core deposits only slowly and fractionally.

Keywords: Interest rate pass-through

JEL-Classification: G21; E43

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1 Introduction

The empirical literature on interest rate pass-through analyzes how market interest rates affect bank rates for loans or deposits. The theoretical background is given by Rousseas' (1985) mark-up theory, which claims that bank rates br (lending rates or deposit rates) are determined by a constant (positive or negative) mark-up c over the bank's marginal cost of funds mr:

$$br = c + \beta mr,$$

where β depends on the elasticity of depositors' or borrowers' demand. In the empirical literature, marginal costs of funds are approximated by either a money market rate or another market interest rate which matches the maturity of the bank rate or shares the maximum correlation with the bank rate among a variety of different market rates (de Bondt 2005). The latter is, thus, a purely statistical approach.

The lack of economic reasoning for the choice of the benchmark rate is particularly striking for core deposits such as overnight deposits or saving accounts, because these type of deposits are non-maturing and exhibit derivative elements in the form of embedded options: The bank can arbitrarily adjust the rate for the outstanding amount of core deposits at any time, while depositors can withdraw or deposit their savings at some random point in time (including on the spot), to an essentially unlimited degree and without any price risk. Furthermore, overnight deposits feature payment functions and provide banks with stable funding and a liquidity profile, unlike market-based funding. Thus, from an economic perspective it seems that the return on a single straight bond cannot meaningfully approximate the cost of funds of core deposits.[2]

The fact that benchmark market rates in the empirical literature are chosen for merely statistical reasons can lead to equivocal results in the analyses of the interest rate pass-through. For example, while de Bondt (2005) chooses EONIA as a benchmark market rate for overnight deposits de Graeve, de Jonghe and van der Vennet (2006) choose a market rate with a maturity of 15 years. Disregarding the degree of data snooping involved in the choice of the benchmark rate, a major problem is the unclear economic interpretation of this sort of pass-through, especially in the case of structural breaks or asymmetric adjustments.[3] The potentially oversimplified procedure for selecting benchmark market rates for core deposits in the classical interest rate pass-through

[2] For an option pricing perspective on core deposits see Jarrow and van Deventer (1998) and Janosi, Jarrow and Zullo (1999).

[3] A recent survey is provided by Andries and Billon (2016).

literature might be one of the reasons why, even before the euro area financial crises, stable long-run relationships between market rates and the bank rates for household overnight deposits and saving accounts have not been observed for Germany and the euro area. The lack of cointegrating relationships contrasts with the empirical findings for the case of loan rates (new business) and time deposits (Mojon 2004, de Bondt 2005, Kwapil and Scharler 2006, Sorensen and Werner 2006, Bernhofer and van Treeck 2013, Belke, Beckmann and Verheyen 2012).

Against this backdrop, this paper provides a theoretical framework and an empirical assessment of the relationship between benchmark market rates and the rates of core deposits. The starting point is that banks face the very same problem as researchers do in determining the marginal cost of deposits in the context of funds transfer pricing. To this end, banks use a variety of models to determine the marginal cost of funds.[4] Among the internal models most frequently used, the static replicating portfolio approach seems to be one of the simplest approaches for the pricing of core deposits (Rolfes and Bannert 2001). It is used by many local banks in Germany, including savings banks (Sievi and Wegner 2005) and credit cooperatives (Aublin 2012) which have a large share in the German deposit market. Relevance, simplicity and data availability make the static replicating portfolio approach particularly interesting for the analysis of the interest rate pass-through to German banks' deposit rates. According to the static replicating portfolio approach, banks model the path of the deposit rate by the moving average return on a benchmark fixed assets portfolio (replicating portfolio) plus a constant mark-up.[5] But under which circumstances can a moving average return on bonds really be interpreted as cost of funds for deposits, why does this approach perform so well for approximating core deposit rates in Germany, and what does it tell us about the objective function of banks and market conditions in the deposit market? These questions are tackled in this paper.

The paper consists of three parts. In the first part of this paper, I shall formulate a multi-period equilibrium model of the deposit market which seeks to explain the pricing behavior of local banks. It is shown that, under suitable conditions, banks can use the variable interest rate of core deposits to smooth profits in the course of time. In the second part, the adjustment processes involved are tested empirically. The limiting effects for this sort of pass-through at the zero lower bound are discussed. The third part of the paper discusses the relationship between market rates and deposit rates, both

[4] See Oesterreichische Nationalbank and Finanzmarktaufsicht Oesterreich (2008) for a nice overview of various approaches. Regulatory guidance for funds transfer pricing in European banks is given by the Committee of European Banking Supervisors (2010).

[5] For a discussion of the static replicating portfolio approach and an exemplifying application, see Maes and Timmermans (2005).

theoretically and empirically, and provides the link between profit smoothing and the (static) replicating portfolio approach applied by many local German banks.

The theoretical model is based on a dynamic version of the Salop (1979) circular road model and capital growth theory (Hakansson and Ziemba 1995).[6] In the model, depositors demand liquidity insurance in the sense of Diamond and Dybvig (1983) and cannot substitute transaction accounts by capital market alternatives. It is argued that illiquidity of fixed rate investments and intertemporal optimization of risk-averse banks result in a pass-through of return variations to depositors if the latter can exercise embedded options in their deposit contract only infrequently due to transportation costs. For simplicity of exposition, I narrow my discussion to the special case of log-utility as proposed by capital growth theory in order to depict the general principles involved.

Using a standard error-correction framework in the empirical part, it is shown that there exists an equilibrium relationship between the return on banks' loan portfolio and the variable rates of core deposits. It is argued that the pass-through of interest rates must run from the lending business to the deposit markets as the return on the loan portfolio under fixed rates is determined by past lending decisions. Under a constant balance sheet assumption and under the assumption that lending rates are some linear function of market rates, it is shown in the third part of this paper that profit smoothing results in the static replicating portfolio approach.

This paper contributes to a large body of literature. From a monetary policy perspective, deposits rates are of interest because deposits are a major source of liquidity for banks. According to Drechsler, Savov, and Schnabl (2017), deposit spreads determine the amount of liquidity households provide to the banking sector. As spreads widen, households withdraw liquidity and banks contract lending (deposit channel of monetary policy). During the financial crises, bank funding costs as an important determinant for lending rates were identified as one factor explaining heterogeneous interest pass-through of market rates to lending rates (Illes and Lombardi 2013, Darracq Paries, Moccero, Krylova and Marchini 2014, see also von Borstel, Eickmeier and Krippner 2016). In this context, deposit rates as part of banks' funding costs have even been directly used in pass-through models (Illes, Lombardi and Mizen 2015). This paper stresses that the rates of core deposits should be treated as endogenous. The paper adds to the classical interest rate pass-through literature by introducing a selection procedure for the choice of market benchmark rates for core deposits based on economic grounds.

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[6] Spatial models such as the Salop model are widely used to capture the main features of deposit markets including product differentiation and market power of banks (e.g. Repullo (2004), Matutes and Vives (1996); see Freixas and Rochet (2008) for a comprehensive representation).

This paper also contributes to the literature which analyzes the deposit market from an industrial organization perspective. The industrial organization approach views the insensitivity of core deposit (overnight deposits or saving accounts) rates from market rates as a result of market imperfection (e.g. Hannan and Berger 1991, Neumark and Sharpe 1992, Hannan and Liang 1993, Berlin and Mester 1999). The relationship between market rates and deposit rates is established under specific assumptions regarding competition between banks and depositors' and banks' access to the capital markets (for a more detailed discussion, see Fecht and Martin 2009). In neoclassical standard models such as those introduced by Monti (1972) and Klein (1971), the relation between market rates and bank rates is established under the assumption that banks have some degree of market power in the loan and deposit markets but are price takers in the securities market. The latter assumption, together with perfect substitutability between deposits and securities markets, results in a linear relationship between deposit rates and the respective market rates, and in the independence of the loan rate from the deposit rate. Both outcomes can be relaxed by simple assumptions. Dermine (1986) shows that the introduction of risk in the model establishes a causal relationship between loan rates and deposit rates while the introduction of a deposit insurance scheme can reverse this causality. The linearity between market rates and deposit rate can be relaxed by interest rate smoothing. According to Allen and Gale (1997), long-lived financial intermediaries allow for intergenerational risk sharing by holding sufficiently large buffer stocks to absorb income shocks which cannot be diversified away. These stocks enable banks to smooth deposit rates, i.e. to disentangle deposit rates and market rates, thereby allowing depositors to smooth consumption over time. This paper shows that there exists a precisely determined relationship between market rates and deposit rates even in a situation where neither depositors nor banks have access to capital markets. This relationship is a consistent with internal models banks apply and is a consequence of profit smoothing, treated elsewhere in the literature (e.g. Kumar 1988, Guttmann, Kadan and Kandel 2010, Acharya and Lambrecht 2015, Bornemann, Kick, Memmel and Pfingsten 2012).

The remainder of the paper is structured as follows. Section 2 provides the equilibrium model of the market for core deposits. Section 3 contains a description of the data set, the econometric model and the estimation results of profit smoothing. Section 4 provides the link between market rates and the rates of core deposits. Section 5 concludes.

2 Theoretical background and intuition

2.1 The general framework

A discrete time, finite horizon model with n≥2 banks is considered. Employing the Salop (1979) model, it is assumed that banks are located symmetrically along a circle with length one such that the deposit market is divided into n local markets. Investments of banks are illiquid and cannot be liquidated prior to maturity. They yield an exogenously determined fixed rate until they are redeemed. Banks are financed entirely by individuals who demand liquidity insurance in the sense of Diamond and Dybvig (1983) and, therefore, hold deposits which allow them to withdraw money on the spot (i.e. transaction accounts). These deposits pay a variable rate to be determined by the bank. This rate is not paid out in cash but rather credited to the account.

Local banks cannot substitute these accounts by market-based funding due to prohibitively high fixed costs. There is a continuum of depositors of mass 1 distributed uniformly along the circle, each one initially having a unit of endowment which forms the depository base. Transaction accounts by their very nature are primarily used for transaction purposes. That is, individuals might follow some consumption or savings plan, investing money into, e.g., investment funds, but they do not invest systematically into the transaction account. Income generation, consumption and saving decision take place instantaneously: In each period of time, the current account balance is affected only by the interest rate credited by the banks and the realization of random deviations from individuals' income, consumption or savings plan (withdrawals). Since all depositors are marginal and equal and because the system is closed, it follows that re-allocations of deposits among banks are negligible and average out in the course of time.

When selecting a bank, individuals face transportation costs for travelling to banks along the circle. Individuals deposit their money, taking into account deposit rate and transportation costs. The latter are proportional to the distance between depositor and bank and apply whenever an individual decides to invest into a particular bank. Due to transportation costs, individuals will not optimize their allocation decision each period but rather leave their money with a particular bank k until they close their transaction account and re-open an account at another bank. For simplicity, it is assumed that the length of these intervals is exogenous and random for depositors, but the expected value of the respective distribution is large.[7]

[7] According to Kiser`s (2002) survey regarding US households' switching behavior, the median tenure at their depository institution is relatively long (10 years).

Banks use deposits to conduct maturity transformation, the latter being limited for two reasons. The first is the amount of withdrawals each period, which has to be covered by liquidity inflows from lending activities. As the amount of withdrawals is random, banks have to maintain a liquidity buffer in order to meet temporary deviations from inflows and outflows. The second limiting factor is the duration of the transaction accounts: While some depositors might turn over deposits very often, other fractions of the depository base might be turned over less frequently. Based on experience, banks can break down their depository base into different liquidity classes which are time invariant. This knowledge enables banks to match the maturities of the lending portfolio (bank assets) proportionate to the withdrawal behavior of depositors. Due to the time invariance of the liquidity classes within the transaction accounts, the maturity structure of the bank lending portfolio is roughly stable over time.

2.2 Individuals, banks and equilibrium

Optimization of individuals and the demand function

The idea that individuals do not optimize their allocation decision each period but rather leave their money with a particular bank k for some time is implemented by taking recourse to capital growth theory. Individuals exhibit isoelastic risk preferences in the form of logarithmic utility (Hakansson and Ziemba 1995, also Santomero 1984). Deposits are risk-free due to the existence of a deposit guarantee scheme. As individuals do withdraw initial endowment and accrued interest at some random point in time T, it follows that individuals choose a particular bank k so as to maximize expected utility of their deposit value at the end of the investment horizon

$$\max_k E\left(lnW_T\right) \tag{2.1}$$

where W_t denotes the value of the transaction account at time t. At date t=0 equation (2.1) can be expressed as

$$\begin{aligned} &\max_k E\left[ln\left\{\left(W_0-\alpha\pi\right)\prod_{t=1}^{T}\left(1+i_{k;t}\right)\mathrm{e}_\mathrm{t}\right\}\right] \\ &=\max_k\left[ln\left(1-\alpha\pi\right)+E\sum_{t=1}^{T}ln\left(1+i_{k;t}\right)+E\sum_{t=1}^{T}ln\mathrm{e}_t\right] \end{aligned} \tag{2.2}$$

with initial endowment normalised to W_0=1, where π is the distance to be travelled to bank k, α is the price per unit to be travelled, e_t is the random withdrawal behavior of the depositors with $E\left(\mathrm{e}_t\right)=1$ and $i_{k;t}$ denotes the deposit rate at time t offered by bank k.

If the investment horizon is independent of the deposit rate distribution, and if a finite expected value of the logarithmized deposit rate exists, that is,

$$\theta_k = E\, ln\left(1+i_{k;t}\right) < \infty \tag{2.3}$$

and if finite $\mathrm{E}\left(\mathrm{T}\right) < \infty$ exists, the objective function simplifies to

$$\max_k \left[ln\left(1-\alpha\pi\right) + \theta_k E(T) \right] \tag{2.4}$$

For a wide range of stochastic processes $\{R_k\}_t = \{1 + i_k\}_t$, including stationary ergodic processes (Algoet and Cover (1988)) and notably Markov chains (Hakansson and Ziemba (1995)) the following convergence result holds:

$$\mathrm{G}_{\mathrm{k};[0;\mathrm{T}]} \xrightarrow{\text{a.s.}} \theta_{\mathrm{k}} \tag{2.5}$$

where $G_{k;[0;T]} = \prod_{t=1}^{T}\left(1 + i_{k;t}\right)^{\frac{1}{T}}$. That is, when selecting a bank, maximization of logarithmic utility is the same as choosing a bank by taking into account transportation costs and geometric mean returns only. This convergence result, thus, features the basic idea of the model that depositors will not immediately switch bank accounts if one bank offers a higher deposit rate than another bank at some point in time, but depositors will start to switch if the difference in deposit rates (net transportation costs) persists for a longer time period.

To derive the demand function, it is assumed that n-1 banks set mean return θ while bank k has to choose θ_k according to its maximization strategy. Thus, bank k has only two competitors, namely the two adjacent banks k-1 and k+1 on the circle. Demand for deposits of bank k is derived by acknowledging that any depositor located at distance π from bank k and $\left(\frac{1}{n} - \pi\right)$ from, say, bank k-1 will be indifferent regarding the deposit of savings with bank k or bank k-1 if expected utility of wealth at the end of the investment horizon is equal among the two banks, that is, if

$$Eln\left[\left(1-\alpha\pi\right)\prod_{t=1}^{T}\mathrm{R}_{k;t}\mathrm{e}_{\mathrm{t}}\right] = \mathrm{E}ln\left[\left(1-\alpha\left(\frac{1}{n}-\pi\right)\right)\prod_{t=1}^{T}\mathrm{R}_{t}\mathrm{e}_{\mathrm{t}}\right] \tag{2.6}$$

Algebraic rearrangements end up in (see appendix B)

$$\pi = \pi\left(\theta;\theta_k\right) = \frac{\left(1-e^{E(T)(\theta-\theta_k)}\right)+\frac{1}{n}\alpha e^{E(T)(\theta-\theta_k)}}{\alpha\left(1+e^{E(T)(\theta-\theta_k)}\right)} = \frac{1-\left(1+\frac{1}{n}\alpha\right)e^{E(T)(\theta-\theta_k)}}{\alpha\left(1+e^{E(T)(\theta-\theta_k)}\right)} \quad (2.7)$$

As π is the distance to be travelled between the marginal depositor and bank k, for date t=0 and pair $(\theta;\theta_k)$ the function π gives also the market area or the fraction of total deposit volume attracted by bank k with respect to bank k-1 for given E(T), n, and α. Because of the symmetry on the circle, deposit demand D_k of bank k also has to take into account the market area between bank k and bank k+1. Thus, by symmetry, deposit demand of bank k is given by

$$D_k = D_k\left(\theta;\theta_k\right) = 2\,\pi\left(\theta;\theta_k\right) \quad (2.8)$$

A consequence of (2.7) is that changes in the deposit rate affect deposit demand only indirectly via their impact on geometric mean returns. Therefore, (2.7) captures the fact that deposit demand is inelastic to movements of the deposit rate in the short run $(R_t; R_{k;t})$ but elastic to the development of the deposit rate in the long run $(\theta;\theta_k)$.[8]

Optimization of banks

Distributable profits c_t of banks are immediately disbursed to their owners.[9] Bank owners maximize expected utility[10]

$$\max_{\{c_t\}} \mathrm{E}\sum_{t=0}^{T-1}\beta^t u[c_t] + \beta^T u[D_T] \quad (2.9)$$

with discount factor $\beta_k \in [0;1]$ and deposit volume D_t.

Distributable profits are given by $c_t = (\mu_t + \varsigma_t - w - i_t)D_t$. The return on banks' loan portfolio consists of two components. The first component is μ_t, which is generated from the outstanding amount of loans built up in the periods ahead. It is entirely determined by past lending decisions and the past interest environment. The second component ς_t denotes temporary return fluctuations generated from new loans

[8] For a study of depositor behavior, see Kiser (2002).
[9] In the following, subscript k is dropped for convenience.
[10] The discussion of whether to maximize utility or profits is summarized by Santomero (1984).

contracted in period t. In period t+1, there is new business generating return ς_{t+1} and the loans contracted in period t migrate into the loan stock. Financial intermediation generates constant costs w for keeping up infrastructure in each period. For convenience, we rewrite (2.9) as $c_t = (M_t + \varsigma_t - w - R_t) D_t$, where $M_t = 1 + \mu_t$.

Expected utility is maximized keeping in mind that the initial value of the depository base of a particular bank k is a function of geometric mean returns, i.e.

$$D_0 = D(\theta; \theta_k) \tag{2.10}$$

The evolution of deposits under random withdrawal behavior is a multiplicative and not an additive process. Under compound interest and random withdrawals ε_t banks' transition equation is determined by

$$D_{t+1} = D_t R_t \varepsilon_t \tag{2.11}$$

with $E(\varepsilon_t) = 1$. Thus, the maximization problem is about finding the optimal path $\{R\}_t$ given the optimal choice for $D(\theta; \theta_k)$. Under logarithmic utility the value function reads

$$J = \max_{\theta_k, \vec{R}} E\left\{ \sum_{t=0}^{T-1} \beta^t \left[ln(M_t + \varsigma_t - w - R_t) + \ln D_t \right] + \sum_{t=0}^{T-1} \beta^t \varepsilon_t + \beta^T \ln D_T \right\} \tag{2.12}$$

s.t.(2.5), (2.10) and (2.11). Iterating backwards on the transition equation yields

$$J = \max_{\theta_k, \vec{R}} E\left\{ \left(\sum_{t=0}^{T-1} \beta^t \right) lnD(\theta; \theta_k) + \sum_{t=0}^{T-1} \beta^t ln(M_t + \varsigma_t - w - R_t) + \sum_{t=0}^{T-1} \ln R_{k,t} \left(\sum_{j=t}^{T-1} \beta^{j+1} \right) + \sum_{t=0}^{T-1} \beta^t \varepsilon_t + \beta^T \ln D_T \right\} \tag{2.13}$$

The easiest way to tackle this optimization problem is to make use of the envelope relation

$$\frac{\partial J}{\partial \theta_k} = \left. \frac{\partial L(\theta_k, \vec{R})}{\partial \theta_k} \right|_{\vec{R} = \vec{R}^{opt}} \tag{2.14}$$

for constrained optimization problems. The Lagrangian thus reads

$$L = \left(\sum_{t=0}^{T-1} \beta^t \right) lnD\left(\theta;\theta_k\right) + E\sum_{t=0}^{T-1} \beta^t ln\left(M_t + \varsigma_t - w - R_t\right)$$
$$+\sum_{t=0}^{T-1} \ln R_{k,t} \left(\sum_{j=t}^{T-1} \beta^{j+1} \right) + \sum_{t=0}^{T-1} \beta^t + \beta^T \ln D_T - \lambda \left(\frac{1}{T-1} \sum_{t=0}^{T-1} lnR_{k,t} - ln\theta_k \right) \tag{2.15}$$

The envelope theorem gives the following first order condition

$$\frac{dL}{d\theta_k} = \frac{D'}{D}\left(\sum\nolimits_{t=0}^{T-1} \beta^t\right) - \frac{\lambda}{(T-1)\theta_k} = 0$$
$$\Leftrightarrow \theta_k^{opt} = \frac{(T-1)D\lambda}{D'\left(\sum\nolimits_{t=0}^{T-1} \beta^t\right)} \tag{2.16}$$

Given θ_k^{opt} the optimal path $\{i\}_t$ can be determined by backward induction. Appendix B shows the optimal deposit rate to be

$$R_{k,T-i}^{opt} = \frac{\left(M_t + \varsigma_t - w\right)}{1 + \left(\sum_{t=0}^{i} \beta^t + \lambda \right)^{-1}} \tag{2.17}$$

Equilibrium and the development of the bank margin

Symmetric Nash equilibrium in the Salop framework is obtained by employing the demand function (2.8) in bank k's maximized objective function (2.16) and replacing θ_k with the equilibrium rate θ.

The derivative of (2.8) w.r.t. θ_k in equilibrium, i.e. at point $\theta = \theta_k$, is given by

$$D' = E(T)\left(\frac{3}{2n} - \frac{1}{\alpha} \right) \tag{2.18}$$

The depository base for each bank in equilibrium is simply given by

$$D = \frac{1}{n} \tag{2.19}$$

Employing (2.18) and (2.19) in (2.16) gives the equilibrium geometric mean rate

$$\theta_k^{opt} = \frac{\lambda}{\left(\frac{3}{2}-\frac{n}{\alpha}\right)\left(\sum_{t=0}^{T-1}\beta^t\right)}\frac{T-1}{E(T)} \tag{2.20}$$

Substituting λ in (2.17) by (2.20) gives the equilibrium path of the deposit rate for date T-i

$$R_{T-i}^{opt} = \frac{M_{T-i}+\varsigma_{T-i}-w}{1+\left(\sum_{t=0}^{i}\beta^t-\theta_k^{opt}\frac{E(T)}{T-1}\left(\frac{3}{2}-\frac{n}{\alpha}\right)\left(\sum_{t=0}^{T-1}\beta^t\right)\right)^{-1}} \tag{2.21}$$

For long-lived banks, $T\to\infty$, and $i\to\infty$, we have

$$\sum_{t=0}^{i}\beta^t-\theta_k^{opt}\frac{E(T)}{T-1}\left(\frac{3}{2}-\frac{n}{\alpha}\right)\left(\sum_{t=0}^{T-1}\beta^t\right)\to\frac{1}{1-\beta} \tag{2.22}$$

Defining the constant $\alpha=(2-\beta)^{-1}$ we can, thus, rewrite (2.21) as

$$i_t^{opt} = \alpha\mu_t+\alpha\varsigma_t-c \tag{2.23}$$

with $c=\frac{1+w}{2-\beta}-1$. Therefore, the interest credited to the depositors' account is a fraction of the return on the loan portfolio minus a constant, which depends on fixed costs w and time preference β.

3 Empirical modelling

3.1 Empirical model

In an econometric model, the adjustment process of the deposit rate will depend on the structure of the random fluctuations ς_t. For example, if ς_t is white noise, any fluctuation of the return is immediately reflected in the deposit rate. More generally, if we consider

$$\varsigma_t = \gamma\varsigma_{t-1}+\varepsilon_t \tag{3.1}$$

to be a stationary process, then (2.23) can be rewritten as an ADL model (see Hassler and Wolters 2006) of the form

$$i_t^{opt} = \alpha\mu_t + \gamma i_{t-1} - \chi\mu_{t-1} + u_t - \vartheta \qquad (3.2)$$

with $\chi = \alpha\gamma$ and $\vartheta = (1+\gamma)c$ and $u_t = \alpha\varepsilon_t$. Equation (3.2) can further be rewritten as

$$\Delta i_t = \vartheta + \delta\left(i_{t-1} - \beta\mu_{t-1}\right) + \alpha\Delta\mu_t + \omega_t \qquad (3.3)$$

where $\delta = \alpha - \chi$, and $\beta = \frac{\gamma - 1}{\alpha - \chi}$ and Δ denotes the differences operator. For more lags, we could express (3.3) more generally as

$$\Delta i_t = \vartheta + \delta\left(i_{t-1} - \beta\mu_{t-1}\right) + \sum_{k=1}^{K}\gamma_k \Delta i_{t-k} + \sum_{j=0}^{J}\alpha_j \Delta\mu_{t-j} + \omega_t \qquad (3.4)$$

Equivalently, we can rearrange (3.4) as

$$\Delta\mu_t = \vartheta_2 + \delta_2\left(i_{t-1} - \beta\mu_{t-1}\right) + \sum_{k=0}^{K}\gamma_{k,2} \Delta i_{t-k} + \sum_{j=1}^{J}\alpha_{j,2} \Delta\mu_{t-j} + u_t \qquad (3.5)$$

where $\vartheta_2 = -\frac{\vartheta}{\alpha_0}$, $\delta_2 = -\frac{\delta}{\alpha_0}$, $\gamma_{k,2} = -\frac{\gamma_k}{\alpha_0}$, $\alpha_{j,2} = -\frac{\alpha_j}{\alpha_0}$ and $u_t = -\frac{\omega_t}{\alpha_0}$.

We thus have the bivariate system

$$\left| \begin{array}{l} \Delta i_t = \vartheta_1 + \delta_1\left(i_{t-1} - \beta\mu_{t-1}\right) + \sum_{k=1}^{K}\gamma_{k,1} \Delta i_{t-k} + \sum_{j=0}^{J}\alpha_{j,1} \Delta\mu_{t-j} + \omega_t \\ \Delta\mu_t = \vartheta_2 + \delta_2\left(i_{t-1} - \beta\mu_{t-1}\right) + \sum_{k=0}^{K}\gamma_{k,2} \Delta i_{t-k} + \sum_{j=1}^{J}\alpha_{j,2} \Delta\mu_{t-j} + u_t \end{array} \right| \qquad (3.6)$$

The system is stable, if i_t and μ_t are cointegrated, that is, if both series are I(1) and if $\left(i_t - \beta\mu_t\right)$ is stationary. Of particular relevance are the parameters δ_1, δ_2, and β. Under cointegration, there exists a long-run relationship between the deposit rate and the return on the loan portfolio. The vector $\left(i_{t-1} - \beta\mu_{t-1}\right)$ gives the residuals from a level regression i_t on μ_t. The long-run coefficient β can be interpreted as the total share of return fluctuations which is passed through to depositors in the long run. The coefficients δ_1 and δ_2 can be interpreted as short-term adjustment coefficients, measuring the share of deviation from the long-run equilibrium, $(i_{t-1} - \beta\mu_{t-1})$, which is offset within a single period (speed of adjustment).

As the rate of return on the loan portfolio under fixed rates is largely determined by the past, it cannot be influenced directly by adjustments of the variable deposit rate in the present. Thus, if withdrawal behavior is stable over time, then δ_2 is equal to zero while δ_1 must be significantly positive. In this case, only the upper equation of (3.6) is of interest. The interpretation is, that banks use the variability in the rate of deposits to partly offset shocks to the return on their loan portfolio and, thus, to smooth the stream of distributable profits over time. Still, the bivariate system captures an important and more subtle relationship between the variable deposit rate and the return on the loan portfolio under fixed lending rates: If withdrawal behavior of depositors is not stable over time, the maturity structure of the liabilities changes. In order to match the flow of liquidity, these changes must gradually feed into a new maturity structure of the loan portfolio, thereby affecting the rate of return generated from the loan portfolio. That is, under changing withdrawal behavior there is a feedback loop between the deposit rate and the return on the loan portfolio. In this case, we would expect slow adjustments of the return on the loan portfolio after temporary disequilibria, implying small but non-zero values for δ_2. However, this effect converges to zero to the extent withdrawal behavior can be considered as stable, which is one of the underlying assumptions of the Salop model.

3.2 Data

Theoretical results imply a specific smoothing of distributable profits according to P&L statements over time. This is achieved by proper interest-rate adjustments of the variable rate of deposits which form the complete liability side of the balance sheet. Thus, the model is based on the following conceptual relation:

$$P = \sum_{t=1}^{12} \left(\sum_{i=1}^{n} x_{it}\mu_{it} - \sum_{j=1}^{m} y_{jt} i_{jt} \right) = \sum_{t=1}^{12} \left(\frac{\sum_{i=1}^{n} x_{it}\mu_{it}}{\sum_{i=1}^{n} x_{it}} - \frac{\sum_{j=1}^{m} y_{jt} i_{jt}}{\sum_{j=1}^{m} y_{jt}} \right) TA_t = \left(\bar{\mu} - \bar{i}\right)\overline{TA} \qquad (3.7)$$

where P denotes the annual profit, x_i and μ_i denote outstanding amount and interest rate for loan type i; y_j and i_j denote outstanding amount and interest rate for deposit type j. TA denotes total assets and $\left(\bar{\mu} - \bar{i}\right)$ is the volume-weighted average interest margin per lending unit in the respective year.

Testing hypothesis of theoretical results is based on data according to the monthly balance sheet statistics (BSI), the monthly MFI interest rate statistics (MIR), and the statistics of German banks' profit and loss accounts, all of them provided by the Deutsche Bundesbank. BSI contains balance sheet information of German MFIs based on the recognition and measurement criteria according to German GAAP. It is a complete survey of German financial institutions and provides information on balance sheet items regarding, inter alia, the counterpart sector and maturity brackets.[11] MIR is based on a sample of about 250 German monetary financial institutions covering around 80% of German banks' balance sheet total as reported in BSI. It contains volume-weighted average effective interest rates for outstanding amounts and new business of borrowing and lending activities starting from January 2003. In particular, it entails information about the interest rates on loans to the private nonfinancial sector and on deposits of households and nonfinancial corporations.[12] The statistics of German banks' profit and loss accounts is based on the (yearly) P&L statements according to German GAAP, collected by the Deutsche Bundesbank.

In principle, interest income and interest expense (P&L statements) should be deductible from interest rates (MIR) on loans and deposits (BSI). In practice, interest income and interest expense according to P&L statements often differ from the estimated values according to BSI and MIR. This is because the loan portfolio covered by MIR does not capture the total loan portfolio of a bank. Neither does it encompass borrowing to and lending from financial institutions, other financial intermediaries or the public sector, and nor does it cover other sources of interest income and interest expense stemming from capital market activities, e.g., coupon payments for bonds. Smoothing distributable profits in the course of time is, therefore, not automatically the same as smoothing the interest margin according to MIR. In particular, banks cannot reasonably smooth profits by adjusting variable deposit rates if core deposits paying a variable rate account for only a small fraction of banks' balance sheets. For this reason only banks with a strong depository base should be considered in the sample. Equivalently, the average loan rate on outstanding amounts according to MIR statistics can only proxy for the interest income per lending unit if the business model of the respective bank is predominantly captured by MIR statistics. Therefore, the sample of German banks should be chosen accordingly.

[11] More information can be found at the ECB Statistical Data Warehouse.

[12] Appendix A shows the content of MIR in more detail. More detailed information can be found in the internet (as of September 14, 2016) under URL: http://www.bundesbank.de/Navigation/DE/VeroeffentlichungenStatistische_Sonderveroeffentlichungen/statistische_sonderveroeffentlichungen.html.

Banks are often categorized by the range of services they offer. Universal banks offer a wide range of banking services while specialized banks focus on specific activities. Along their legal status, German universal banks are most frequently broken down into three pillars: savings banks, credit cooperatives, and credit banks. The latter can be further divided into big banks, foreign branches, and regional banks. Deposit markets in Germany are typically regional, as is the business model of savings banks, credit cooperatives and regional banks.[13] As it is shown, it is these banks whose business model is most adequately described by the Salop model and whose asset and liability side is covered best by MIR statistics. Only these types of banks are considered in the following and they will be referred to as "local banks". Due to their reliance on local deposit markets in Germany, sample banks were not negatively affected by the financial crises (Deutsche Bundesbank 2009).

According to MIR statistics, transaction accounts in the sense of the Salop model are best approximated by households' overnight deposits. Therefore, the interest rate of households' overnight deposits according to MIR is used as an approximation of the variable deposit rate. However, overnight deposits do not make for the total of households' deposits paying a variable rate. Saving accounts also have a certain transaction component as withdrawals on the spot are possible for depositors, albeit limited regarding their cumulative volume for each month. Saving accounts are also non-maturing and they pay a variable rate to be set by the bank. In order to capture a more meaningful proportion of banks' liabilities, the volume-weighted interest rate for core deposits (composite variable deposit rate for households' overnight deposits and saving accounts) is considered as a second approximation of banks' variable deposit rate.

To show the relevance of local banks in these deposit market segments, the next table provides information regarding the coverage by local banks.

[13] Savings banks as well as credit cooperatives each are organized, share a common appearance and take recourse to common resources provided by head organizations. Regional banks are independent private banks. A more detailed description of the German banking sector is given by Hartmann-Wendels, Pfingsten and Weber (2014).

Table 1: Market size and market share of sample banks*

	All banks according to BSI	Local banks according to BSI	Local banks according to MIR
Overnight deposits (households)	662.5	539.0	299.4
Saving accounts	600.5	521.3	208.1
Sum (= core deposits)	1262.5	1061.3	507.5
Total liabilities	7595.1	2465.6	1257.3

* Outstanding amounts in billion EUR. Monthly averages during the sample period (2003.01 - 2016.02).

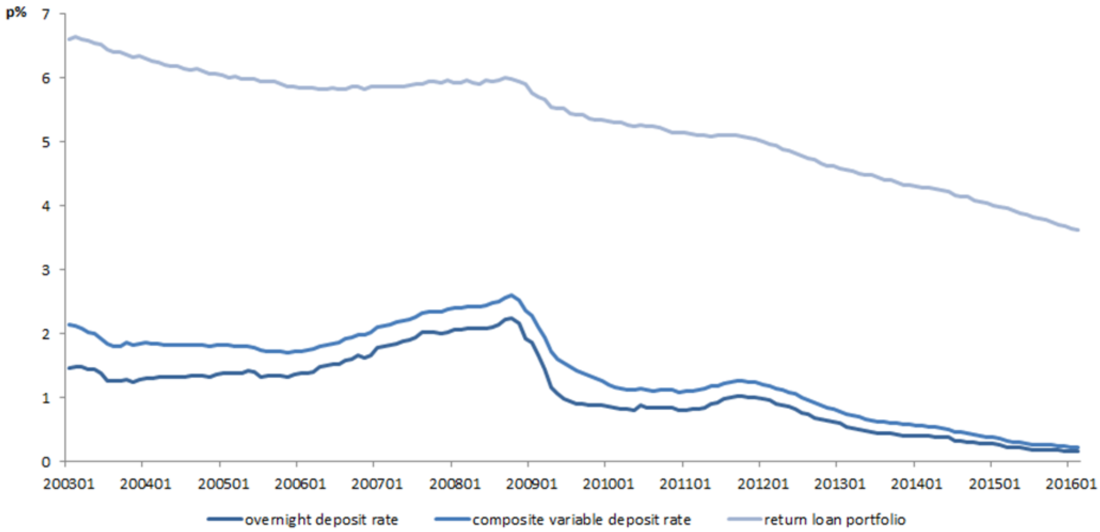
Table 1 shows that local banks have a market share of around one third regarding total assets during the sample period. But the market segment of core deposits in Germany is largely run by local banks (market share: 84%). The MIR sample of local banks covers almost 50% of this volume, distributed among 178 banks.

To assess the adequacy of MIR statistics for the approximation of bank returns we have to compare yearly interest income as reported in banks' financial statements (P&L statements)[14] with interest income as computed by (3.7) using (monthly) BSI and MIR statistics. It turns out that the computed interest income accounts for around 90% of the reported interest income according to P&L statements, implying that MIR statistics essentially capture the business model of the respective banks. Due to this high coverage, the volume-weighted average interest rate for the outstanding amount of lending activities according to MIR statistics provides a reasonably good approximation of the rate of return on the loan portfolio generated by the respective banks.

The bivariate system in (3.6) is estimated for the overnight deposit rate and the return on the loan portfolio (model A) and for the composite variable deposit rate and the return on the loan portfolio (model B), respectively. The next diagram displays the bank rates under consideration.

[14] Total interest income from lending and money market transactions less interest paid.

Figure 1: Overnight deposit rate, composite variable deposit rate and the return on the loan portfolio for local banks according to MIR statistics



3.3 Model checking and regression results

As interest rate series typically exhibit unit roots, stability of the system (3.6) is given if the respective deposit rate and the return on the loan portfolio are cointegrated. To test whether the variables are I(1) the modified Dickey-Fuller t-test (DF-GLS) proposed by Elliott, Rothenberg, and Stock (1996) is performed. It turns out that the null hypothesis of a unit root cannot be rejected.[15] To test for cointegration, the Johansen (1988, 1991) trace and maximum eigenvalue tests are applied. The coefficient on the respective deposit rate is normalized to unity while the lag order of the proposed error correction model is determined according to the information criteria AIC, SBIC and HQIC and the final prediction error (FPE). It turns out that for the case of overnight deposits as well as for the composite variable deposit rate, a model with four lags is preferable according to all of those criteria. Appendix D shows that the null hypotheses of no cointegration can be rejected for model A and model B, indicating some long-run equilibrium relationship between the return on the loan portfolio and the two deposit rates under consideration. The following table shows the long-run parameter β and the short-term adjustment coefficients δ_1 *and* δ_2 for the bivariate system (3.6), estimated according to Johansen's (1995) maximum likelihood method.

[15] See appendix C for estimation results.

Table 2: Overnight deposit rate and return on the loan portfolio vs. composite variable deposit rate and return on the loan portfolio: Short-term adjustment and long-run parameter

	model A		model B	
Dependent Variable	overnight deposit rate	return loan portfolio	composite variable deposit rate	return loan portfolio
Long-run parameter β	0.475***		0.633***	
	(0.583)		(0.068)	
Short-term adjustment δ	0.056***	0.003	0.03***	0.006
	(0.012)	(0.006)	(0.009)	(0.00624)
R-sq	0.54	.71	0.67	0.72

Note: Results based on equation (3.6) using a sample 2003.01-2016.02; standard errors in parentheses; ***p<0.01, **p<0.05, *p<0.1

Model A reveals a significant short-term adjustment coefficient for the overnight deposit rate but not for the return on the loan portfolio, which implies that deviations from the long-run relationship are primarily dissipated by adjustments in the overnight deposit rate. This result fits to the predictions of the Salop model which states that banks use the variable overnight deposit rate to smooth the stream of profits. It also reflects the reality in Germany that the return on the loan stock is determined by the past and cannot be adjusted on the spot.[16] In the long run, the overnight deposit rate absorbs almost 50% of changes in the return on the loan portfolio. Keeping in mind that households' overnight deposits have been supplying around 40% of total funding of the respective banks in the sample horizon and acknowledging the fact that most of the other funding instruments of the respective banks are medium- and long-term by nature, banks seem to be able to pass through a reasonable degree of income shocks to depositors. To illustrate this latter point in more detail, we turn to model B which captures the interest rate of saving accounts and overnight deposits jointly as a composite variable deposit rate for core deposits, in order to achieve a larger coverage of balance sheet's total liabilities. It turns out that the long-run parameter increases to more than 0.63, indicating that almost two thirds of changes in the return on the loan portfolio are passed through to depositors on average. This is expectable as the same return proxy is used in model A and in model B albeit overnight deposits can only finance a smaller fraction of the underlying loan portfolio than the total of core deposits, both in terms of volume and in terms of duration. As in model A, the short-term

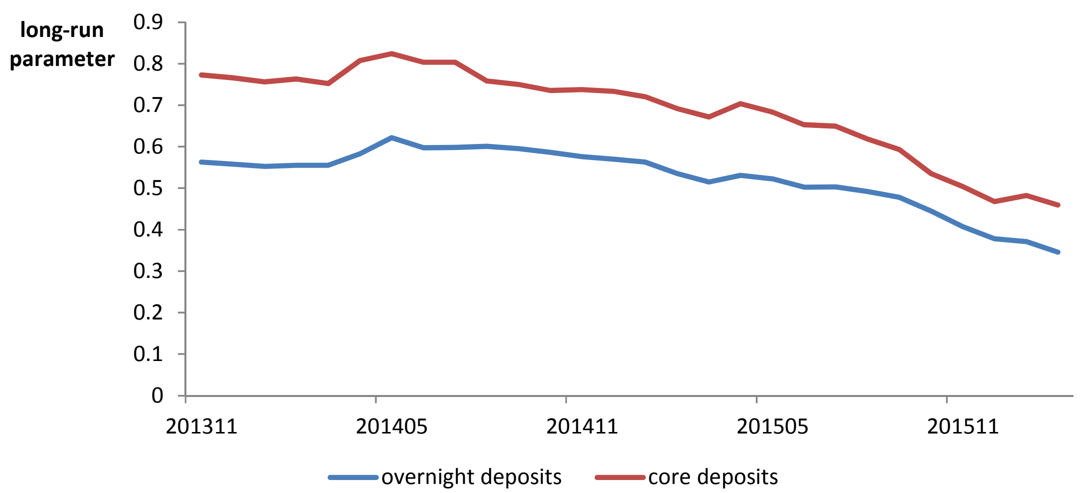
[16] An alternative explanation would be along the lines of the expectation theory: In perfect markets, the (annualized) long-term rate for new business lending should equal the (annualized) compound short-term rates. Therefore, the bank return for the outstanding amount of loans (which is a moving average lending rate of mostly long-term credit) should equal the (lagged) moving average deposit rates (short-term by nature). However, this argument seems to be hypothetical, given the fact that deposit markets are highly imperfect and, acknowledging that an equilibrium relationship between market rates and the rates of core deposits is documented in virtually no publication; the converse holds for lending rates.

adjustment coefficient in model B is not significant for the return on the loan portfolio, the converse being true for the composite variable rate of core deposits. Thus, as in model A we conclude that deviations from the equilibrium relationship are adjusted by changes in the deposit rate.

3.4 Excursus: The pass-through of return variations to core deposits in the low interest environment

Figure 1 shows that the overnight deposit rate and the composite variable deposit rate are more and more converging to the zero lower bound. This might hamper the ability of banks to pass through changes in the return on their loan stock to their depositors. A limiting effect of the zero lower bound on the pass-through can be illustrated by rolling regressions of equation (3.6). Figure 2 shows the long-run pass-through coefficient for household overnight deposits (model A) and the total of core deposits (model B), based on a rolling regression with window size n = 130.

Figure 2: Development of the long-run pass-through coefficient β for household overnight deposits and the total of core deposits



At the beginning of the sample period, sample banks were able to pass through almost 80 per cent of the changes in the rate of return on their loan portfolio to their core depositors on average. This share has diminished to less than 46 per cent, which implies that banks are less able to smooth the stream of distributable profits by simply adjusting their variable deposit rates at the zero lower bound. Another implication is that the margins of depository institutions might shrink at the zero lower bound, provided that the respective banks cannot compensate for decreasing rates of return on their loan portfolio elsewhere. This raises further questions with respect to bank business models and financial stability implications which are beyond the scope of this paper.

4 The rates of core deposits and market rates

4.1 The conceptual relationship between market rates and the rates of core deposits

According to the model, market rates will feed into deposit rates only indirectly via their impact on loan rates (new business). With every new loan contract, the return on the total loan portfolio changes and so does the rate on deposits. Therefore, the relationship between market rates and the rate of deposits depends on the interest rate sensitivity of the return on the loan portfolio and, thus, on the sensitivity of each single loan rate to changes in the market rate.

The constant balance sheet assumption implies that for every old loan which has been redeemed a new loan is granted. The redeemed loan drops out of the loan portfolio and the new loan enters in. If we assume for simplicity, that the bank grants one standard loan at each point in time t with a maturity of K years, paying a fixed rate $r_{K;t}$ to be determined at t, then the return on the loan portfolio μ_t is simply the moving average return over all the single loans in the portfolio, where the length of the moving average is determined by K:

$$\mu_t = \frac{1}{K}\sum_{k=0}^{K} r_{K;t-k}\,, \tag{4.1}$$

If the loan rate $r_{K;t}$ (new business) is pegged to the market rate $mr_{K;t}$, we have

$$\mu_t = \frac{1}{K}\sum_{k=0}^{K} mr_{K;t-k} \tag{4.2}$$

In the case where $r_{K;t}$ is a function of $mr_{K;t}$, μ_t will be the moving average return over this function. More specifically, if $mr_{K;t}$ and $r_{K;t}$ are both I(1) and cointegrated ($r_{K;t} = c + bmr_{K;t}$), we have

$$\mu_t = \frac{1}{K}\sum_{k=0}^{K} (c + bmr_{K;t-k}) = c + b\overline{mr_{K;t}} \tag{4.3}$$

where $\overline{mr_{K;t}}$ is the moving average interest rate of a bond with maturity K.

Due to differences in depositors' withdrawal behavior there is maturity segmentation in core deposits. That is, core deposits do not finance a certain loan type with fixed maturity K but rather a loan portfolio with different loan maturities $K_1, K_2, .., K_n$.

Having volume weights ψ_i, with $0 < \psi_i < 1$ and $\sum_i \psi = 1$, the return on the loan portfolio is given by a weighting function over moving averages of diverse market rates:

$$\mu_t = \sum_i \psi_i \left(c + b\overline{mr_{K_i;t}} \right)_t = c + b \sum_i \psi_i \overline{mr_{K_i;t}}, \tag{4.4}$$

This expression allows us to directly relate market rates to the deposit rates by simply plugging (4.4) into the pass-through model (3.6). As the deposit rate of local banks according to the model cannot affect market rates, we have the single equation model

$$\Delta i_t = \vartheta + \delta \left(i_{t-1} - d + \theta \left(\sum_i \psi_i \overline{mr_{K_i;t-1}} \right) \right) + \sum_{k=1}^{K} \gamma_k \Delta i_{t-k} + \sum_{j=0}^{J} \xi_j \Delta \left(\sum_i \psi_i \overline{mr_{K_i;t-j}} \right) + \omega_t \tag{4.5}$$

where $d = \beta c$, and $\theta = \beta b$, and $\xi_j = \alpha_j b$. Thus, cointegrating relationships between rates of different loan types (new business) and respective market rates translate into a cointegrating relationship between core deposit rates and volume-weighted moving averages of the respective market rates. In equilibrium, the deposit rate equals the volume-weighted moving averages of the market rates plus a constant. The portfolio of bonds paying the market rates can be interpreted as a replicating portfolio. As the weighting function is time invariant, the pricing formula is called static replicating portfolio approach.

The most important economic implication of equation (4.5) is that there is a precisely defined relationship between deposit rates and market rates, even if neither depositors nor banks have access to capital markets. This relationship is based on averages of the respective market rates. The only condition is that there is a cointegrating relationship between loan rates (new business) and market rates, for example according to markup theory. Due to the weighting function and the moving averages, the pass-through of market rates to the rates of core deposits is necessarily slow and incomplete.

4.2 An empirical application for household overnight deposits

In the following it is tested empirically whether equilibrium relationships between household overnight deposit rates and the specified averages of market rates do exist. To this end, replicating portfolios have to be constructed on the basis of a set of market rates and a set of volume weights. In order to restrict the number of possible portfolios, moving averages of length K for market rates with maturity K are calculated for Euribor (3 months), and German government bond rates with maturities of 1-4 years, 6 years, 8 years and 10 years. To further reduce the number of possible replicating portfolios, the set of admissible weights ψ_i is restricted to (0, 0.2, 0.4, 0.6, 0.8 or 1). With 6 possible weights and 8 different rates, n=792 static replicating portfolios can be calculated.[17] The null of no cointegration between the yield of each replicating portfolio h $(\mu_{h,t})$ and the overnight deposit rate i_t is tested with the ECM t-ratio test proposed by Banerjee, Dolado and Mestre (1998). To apply the ECM t-ratio test the single equation model (4.5) is rewritten as

$$\Delta i_t = \alpha + \delta i_{t-1} + \beta \mu_{h;t-1} + \sum_{k=1}^{K} \gamma_k \Delta i_{t-k} + \sum_{j=0}^{J} \xi_j \Delta \mu_{h;t-j} + \omega_t \qquad (4.6)$$

where $\alpha = \vartheta - \delta d$, and $\beta = \delta\theta$. The ECM test for the null of no cointegration is based on the t-ratio for δ.[18] It turns out that out of the 792 permutations 214 cointegrating relations can be found at a 1% significance level and 465 cointegrating relationships at a 5% level. In the following the relevance of the replicating portfolios is demonstrated. As mentioned earlier, many banks model the overnight deposit rate as the return on a replicating portfolio plus some constant. This is the same as minimizing the variance between the overnight deposit rate and the portfolio return. Among the 465 portfolios with cointegrating relationships to the overnight deposit rate at a 5% significance level, we choose the optimal portfolio according to the function $f = \min \text{var}(\mu_{h;t} - i_t)$, for h = 1,2,..,465. The optimal portfolio weights are given below.

Table 3: Weights of the optimal replicating portfolio

Market rate	Euribor	1 year	2 year	3 year	4 year	6 year	8 year	10 year
Weight	0.2	0	0	0	0	0	0.8	0

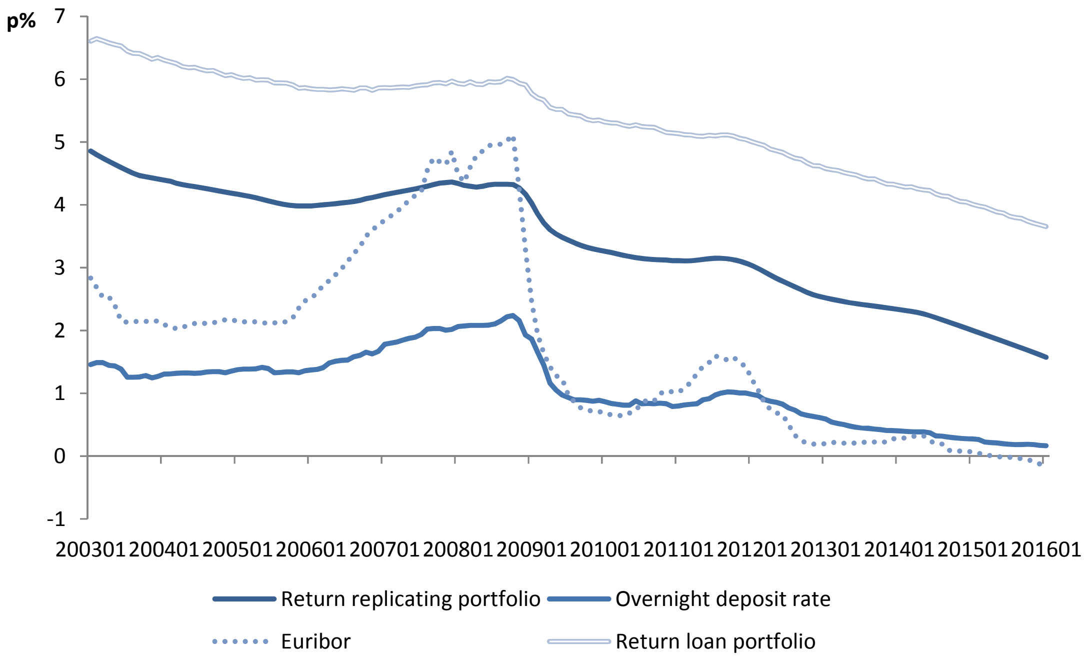
The duration of this portfolio is around 6.4 years which is within the range of benchmark rates for overnight deposits presented in the literature. This duration seems large for overnight deposits. Still, the portfolio structure mirrors maturity segmentation and reflects the fact that the maturity structure of overnight deposits is medium-term on average. The following diagram shows the overnight deposit rate, the rate of return on

[17] Other portfolio weights and market rates are, of course, possible. It turns out that using a different set of maturities or even more maturities and more granular weights ψ_i does not significantly enlarge the space of return trajectories of the replicating portfolios.

[18] Critical values are tabulated in Banerjee et al. (1998). The model is estimated by OLS; lags are added until the null hypothesis of white noise for the error term ω_t cannot be rejected according to the Q-test proposed by Ljung and Box (1978) at a 5% confidence level.

the replicating portfolio, the return on the loan portfolio and, for informational purposes, the Euribor (3 months).

Figure 3: Returns of the bank loan portfolio, and the replicating portfolio, the overnight deposit rate and the Euribor (3 months)



The economic substance behind the replicating portfolio approach can be summarized as follows: Core deposits provide banks with reiterated inflows and outflows of liquidity. The bulk of liquidity can be segmented according to maturity brackets. By setting the variable interest rate as volume-weighted moving averages of the respective market rates, banks reflect maturity segmentation and the permanence of the liquidity provided by core deposits. This very price setting behavior is equivalent to profit smoothing, if the balance sheet is roughly constant and if there is some stable long-run relationship between market rates and loan rates (new business).

5 Concluding remarks

This paper proposes a Salop model of banks which perform maturity transformation and invest liquidity provided by core deposits into illiquid investment opportunities paying a fixed rate. Bank owners' risk aversion results in profit smoothing and, thus, in the pass-through of return variations to depositors. This behavior works because core deposits are special: They are not substitutable by capital market alternatives, they pay a variable rate and depositors do not make use of withdrawal options embedded in their deposit contracts, as long as banks pay competitive rates on average. Empirically, equilibrium relationships between returns of the loan portfolio and variable deposit rates can be found. There also exists a solution space for equilibrium relationships between variable deposit rates and market rates, if the latter are considered as moving average yields of German government bonds along different maturities (replicating portfolio). This finding fits to the Salop model if single market rates and the respective new business lending rates are cointegrated.

From a monetary policy perspective, this paper points to the possibility that the pass-through of market rates to bank lending rates might not necessarily always work through the funding costs of banks, at least not in bank-based financial systems with strong local banks financed by deposits. Market imperfections in the deposit market, together with banks' profit smoothing behavior, might generate paths of the deposit rates which do not only differ substantially from any market rate. Depending on market characteristics and elasticity of demand, profit smoothing might also implement feedback mechanisms between deposit rates and past and current lending rates. Thus, models of the interest rate pass-through for core deposits which rely on single market rates might oversimplify the relation between core deposit rates and market rates and, more specifically, might not always be an adequate representation for every banking system.

In this context, it has to be kept in mind that predictions of the theoretical model presented here as well as the sample of banks in the empirical part are designed to match the reality of local banks with limited access to capital markets. Large banks which are funded primarily by the capital market might not need or not be able to pass through return variations to their depositor base because the latter might simply be too small in terms of balance sheet size. For example, investment trusts with a small depository base might adjust variable rates of overnight deposits to some market rate and steer margins by using interest rate derivatives. As a consequence, the predictions presented in this paper should not be translated into aggregate data on a one-by-one basis, as they depend on the structure of the respective banking system. However, given the poor performance of standard pass-through models in explaining the pass-through to core deposits in the euro area before and after the financial crises, the benchmark selection procedure as described in this paper might be of relevance to the German banking sector – and elsewhere.

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Appendix

A. Available interest rates for outstanding amounts according to MIR Statistics

Lending activities		Borrowing activities	
Maturity up to one year	Housing*	Overnight deposits	Households*
	Consumption and other*		Non-financials**
	Loans to non-financials**		
Maturity between one and five years	Housing*	Saving accounts redeemable upon notice (up to three months)	Households* and non-financials**
	Consumption and other*		
	Loans to non-financials**		
Maturity above five years	Housing*	Saving accounts redeemable upon notice (above three months)	Households* and non-financials
	Consumption and other*		
	Loans to non-financials**		
		Maturity up to two years	Households*
			Non-financials**
		Maturity above two years	Households*
			Non-financials**

* Loans to households including own-account workers and non-profit institutions serving households; ** loans to non-financial corporations. Counterparts located in EMU.

B. Algebraic rearrangements

Derivation of equation (2.7)

$$Eln\left[(1-\alpha\pi)\prod_{t=1}^{T}\mathrm{R}_{k;t}\mathrm{e}_{\mathrm{t}}\right]=\mathrm{E}ln\left[\left(1-\alpha\left(\frac{1}{n}-\pi\right)\right)\prod_{t=1}^{T}\mathrm{R}_{t}\mathrm{e}_{\mathrm{t}}\right]$$

$$\Leftrightarrow Eln\left[(1-\alpha\pi)\prod_{t=1}^{T}\mathrm{R}_{k;t}\right]=\mathrm{E}ln\left[\left(1-\alpha\left(\frac{1}{n}-\pi\right)\right)\prod_{t=1}^{T}\mathrm{R}_{t}\right]$$

$$\Leftrightarrow (1-\alpha\pi)e^{Eln\left(\prod_{t=1}^{T}\mathrm{R}_{k;t}\right)}=\left(1-\alpha\left(\frac{1}{n}-\pi\right)\right)e^{Eln\left(\prod_{t=1}^{T}\mathrm{R}_{t}\right)}$$

$$\Leftrightarrow (1-\alpha\pi)e^{E(T)\theta_k}=\left(1-\alpha\left(\frac{1}{n}-\pi\right)\right)e^{E(T)\theta}$$

$$\Leftrightarrow (1-\alpha\pi) = \left(1-\alpha\left(\frac{1}{n}-\pi\right)\right)e^{E(T)(\theta-\theta_k)}$$

$$\Leftrightarrow \pi = \pi(\theta;\theta_k) = \frac{\left(1-e^{E(T)(\theta-\theta_k)}\right)+\frac{1}{n}\alpha e^{E(T)(\theta-\theta_k)}}{\alpha\left(1+e^{E(T)(\theta-\theta_k)}\right)} = \frac{1-\left(1+\frac{1}{n}\alpha\right)e^{E(T)(\theta-\theta_k)}}{\alpha\left(1+e^{E(T)(\theta-\theta_k)}\right)}$$

Derivation of equation (2.17)

The value function J forT-1 for each bank k is given by

$$\mathrm{J}_{\mathrm{T}-1} = \max_{\mathrm{i}_{\mathrm{T}-1}}\left(ln\left(\mathrm{M}_{\mathrm{t}}+\varsigma_{\mathrm{t}}-\mathrm{w}-\mathrm{R}_t\right)+\ln \mathrm{D}_{\mathrm{T}-1}+\beta\mathrm{E}\ln \mathrm{D}_{\mathrm{T}}\right) \text{-} \lambda\left(\sum_{\mathrm{t}=0}^{\mathrm{T}-1} lnR_{\mathrm{k,t}} \text{-} (T\text{-}1)ln\theta^{\mathrm{opt}}\right)$$

Substituting D_{T} by the transition equation (2.11) we get

$$\mathrm{J}_{\mathrm{T}-1} = \max_{\mathrm{R}_{\mathrm{T}-1}}\left(ln\left(\mathrm{M}_{\mathrm{t}}+\varsigma_{\mathrm{t}}-\mathrm{w}-\mathrm{R}_t\right)+\ln \mathrm{D}_{\mathrm{T}-1}+\beta\ln\left(\mathrm{D}_{\mathrm{T}-1}\mathrm{R}_{\mathrm{T}-1}\varepsilon_{\mathrm{T}-1}\right)\right) \text{-} \lambda\left(\sum_{\mathrm{t}=0}^{\mathrm{T}-1} lnR_{\mathrm{t}} \text{-} (\mathrm{T}-1)\, ln\theta^{\mathrm{opt}}\right)$$

First order condition then reads

$$\frac{\partial \mathrm{J}_{\mathrm{T}-1}}{\partial \mathrm{R}_{\mathrm{T}-1}} = -\frac{1}{\mathrm{M}_{\mathrm{T}-1}+\varsigma_{\mathrm{T}-1}-\mathrm{w}-\mathrm{R}_{\mathrm{T}-1}}+\beta\frac{1}{\mathrm{R}_{\mathrm{T}-1}}-\lambda\frac{1}{\mathrm{R}_{\mathrm{T}-1}} = 0$$

$$\Leftrightarrow \mathrm{R}^{\mathrm{opt}}{}_{\mathrm{T}-1} = \frac{\mathrm{M}_{\mathrm{T}-1}+\varsigma_{\mathrm{T}-1}-\mathrm{w}}{1+(\beta+\lambda)^{-1}}$$

Plugging results into $\mathrm{J}_{\mathrm{T}-1}$, we obtain

$$\mathrm{J}_{\mathrm{T}-1} = \ln\left(\mathrm{M}_{\mathrm{T}-1}+\varsigma_{\mathrm{T}-1}-\mathrm{w}-\frac{\mathrm{M}_{\mathrm{T}-1}+\varsigma_{\mathrm{t}}-\mathrm{w}}{1+(\beta+\lambda)^{-1}}\right)+\ln \mathrm{D}_{\mathrm{T}-1}+\beta\ln\left(\mathrm{D}_{\mathrm{T}-1}\frac{\mathrm{M}_{\mathrm{T}-1}+\varsigma_{\mathrm{t}}-\mathrm{w}}{1+(\beta+\lambda)^{-1}}\varepsilon_{\mathrm{T}-1}\right)$$
$$\text{-}\lambda\left(\sum_{\mathrm{t}=0}^{\mathrm{T}-2} lnR_{\mathrm{k,t}}+\frac{1}{\mathrm{T}-1}\frac{\mu+\mu_{\mathrm{T}-1}}{1+(\beta+\lambda)^{-1}} \text{-} (\mathrm{T}-1)\, ln\theta_{\mathrm{k}}{}^{\mathrm{opt}}\right)$$

In period T-2 we then have

$$\mathrm{J}_{\mathrm{T}-2} = \max_{\mathrm{R}_{\mathrm{T}-2}}\left(ln\left(\mathrm{M}_{\mathrm{T}-2}+\varsigma_{\mathrm{T}-2}-\mathrm{w}-\mathrm{R}_{\mathrm{T}-2}\right)+\ln \mathrm{D}_{\mathrm{T}-2}+\beta\mathrm{E}\mathrm{J}_{\mathrm{T}-1}\right) \text{-} \lambda\left(\sum_{\mathrm{t}=0}^{\mathrm{T}-1} lnR_{\mathrm{t}} \text{-} (\mathrm{T}-1)\, ln\theta^{\mathrm{opt}}\right)$$

First order condition reads

$$\frac{\partial J_{T-2}}{\partial R_{T-2}} = -\frac{1}{M_{T-2} + \varsigma_{T-2} - w - R_{T-2}} + \left(\beta + \beta^2\right)\frac{1}{R_{T-2}} - \lambda \frac{1}{R_{T-2}} = 0$$

$$\Leftrightarrow R_{T-2}{}^{opt} = \frac{M_{T-2} + \varsigma_{T-2} - w}{1 + \left(\beta + \beta^2 + \lambda\right)^{-1}}$$

Therefore, we have

$$R_{T-i}{}^{opt} = \frac{M_{T-i} + \varsigma_{T-i} - w}{1 + \left(\sum_{t=0}^{i} \beta^t + \lambda\right)^{-1}}$$

C. Modified Dickey-Fuller t test for a unit root

			Critical Values		
	Lags chosen by SIC	DF-GLS Test Statistic	1%	5%	10%
Return Loan Portfolio	12	-1.972	-3.51	-2.809	-2.532
Overnight Deposit Rate	2	-1.965	-3.51	-2.954	-2.665
Composite Variable Deposit Rate	1	-1.884	-3.51	-2.964	-2.674

Observations: 144, null hypothesis: variable follows a random walk with drift

			Critical Values		
	Lags chosen by SIC	DF-GLS Test Statistic	1%	5%	10%
Return Loan Portfolio	4	2.291	-2.592	-2.041	-1.731
Overnight Deposit Rate	2	-0.692	-2.592	-2.055	-1.744
Composite Variable Deposit Rate	1	0.151	-2.592	-2.061	-1.749

Observations: 144, null hypothesis: variable follows a random walk without drift

D. Johansen tests for cointegration

Overnight deposit rate and return on the loan portfolio

	Trace Statistic			Max. Eigenvalue Statistic		
		Critical Values			Critical Values	
Max. Cointegration Rank	Statistic*	5%	1%	Statistic**	5%	1%
0	29.19	15.41	20.04	28.83	14.07	18.63
1	0.25	3.76	6.65	0.25	3.76	6.65

* H0: no more than r cointegrating relations; HA: strictly larger than r cointegrating relations

** H0: r cointegrating relations; HA: r+1 cointegrating relations; observations: 154, lags = 4

Composite variable deposit rate and return on the loan portfolio

	Trace Statistic			Max. Eigenvalue Statistic		
		Critical Values			Critical Values	
Max. Cointegration Rank	Statistic*	5%	1%	Statistic**	5%	1%
0	21.14	15.41	20.04	20.85	14.07	18.63
1	0.24	3.76	6.65	0.24	3.76	6.65

* H0: no more than r cointegrating relations; HA: strictly larger than r cointegrating relations

** H0: r cointegrating relations; HA: r+1 cointegrating relations; observations: 154, lags = 4