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## Safe but fragile: information acquisition, sponsor support and shadow bank runs

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# Non-technical summary


### Research Question

Shadow banks operate outside the perimeter of traditional banking regulation, even though they are typically created and operated ('sponsored') by regulated financial institutions. A key feature in shadow banks' design is the provision of contingent liquidity support by their sponsoring institutions. Such sponsor support intends to lower individual shadow banks' susceptibility to investor runs and may help them avoid losses from selling assets at discounted prices. This leads to the question adressed by the present paper within a theoretical model: How do market and funding liquidity depend on shadow banks' aggregate choice of liquidity sources (liquidity support *versus* asset sales)?

### Contribution

In the model, shadow banks can cover funding shocks by either selling assets in a pooling market or by resorting to sponsors' outside liquidity line. This choice creates incentives for shadow banks to acquire private information about the true quality of their assets in order to avoid selling good assets at a discount. In aggregate, this behavior can create endogenous adverse selection.

### Results

The equilibrium of the model is characterized by self-fulfilling endogenous adverse selection that may induce a market freeze and precipitate a panic-driven investor run. The equilibrium is inefficient and exhibits substantial welfare losses. We compare different policies that can be used to restore market and funding liquidity: debt or asset purchases by the central bank prevent inefficient dry-ups and improve welfare; liquidity injections to shadow banks' sponsors, however, may backfire and exacerbate the underlying adverse selection frictions.

# Nichttechnische Zusammenfassung

## Fragestellung

Schattenbanken bewegen sich außerhalb des Einflussbereichs traditioneller Bankenregulierung und der Zentralbank, obwohl sie üblicherweise von regulierten Finanzinstituten gegründet und gemanagt werden ('sponsoring'). Eine plötzliche Kündigung kurzlaufender Investorengelder führt zu Verlusten der Schattenbanken, wenn diese ihren Liquiditätsbedarf nur durch Notverkäufe auf illiquiden Wertpapiermärkten decken können. Um dies zu vermeiden, stellen Sponsoren den Schattenbanken Kreditlinien zur Verfügung. Mithin können Schattenbanken zur Deckung ihres Liquiditätsbedarfs zwischen Liquiditätslinie und Wertpapierverkäufen wählen. Dies führt zu der in der vorliegenden Arbeit anhand eines theoretischen Modells untersuchten Fragestellung: Wie hängen Markt- und Finanzierungsliquidität von der (aggregierten) Wahl der Liquiditätsquellen ab?

## Beitrag

Im Modell können Schattenbanken Liquiditätsbedarfe entweder durch Wertpapierverkäufe in einem Pooling-Markt oder durch Rückgriff auf eine Kreditlinie ihres Sponsors decken. Diese Wahlmöglichkeit schafft Anreize für die einzelne Schattenbank, private Informationen über die Qualität ihrer Aktiva zu erwerben. Dadurch vermeidet sie, hochwertige Aktiva zu niedrigen Preisen zu verkaufen. Im Aggregat führt dieses Verhalten zu selbst-erfüllender endogener adverser Selektion.

## Ergebnisse

Im Gleichgewicht des Modells kann durch diese endogene adverse Selektion ein Preisverfall im Wertpapiermarkt und ein Investoren-Run entstehen. Das Gleichgewicht ist ineffizient und weist deutliche Wohlfahrtsverlusten auf. Es werden mithin verschiedene Politikmaßnahmen untersucht, die Markt- und Finanzierungsliquidität verbessern und Wohlfahrtsverluste mindern können. Direkte Ankäufe von Schuldtiteln oder Wertpapieren durch die Zentralbank können Wohlfahrtsverluste reduzieren. Direkte Liquiditätshilfen für die Sponsoren dagegen verstärken unter Umständen die adverse Selektion und die Wohlfahrtsverluste.

Deutsche Bundesbank Discussion Paper No 15/2018

# Safe but Fragile: Information Acquisition, Sponsor Support and Shadow Bank Runs*

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**Abstract**

This paper proposes a theory of shadow bank runs in the presence of sponsor liquidity support. We show that liquidity lines designed to insulate shadow banks from market and funding liquidity risk can be destabilizing, as they provide them with incentives to acquire private information about their assets' type. This can lead to inefficient market liquidity dry-ups caused by self-fulfilling fears of adverse selection. By lowering asset prices, information acquisition also reduces shadow banks' equity value and may spur inefficient investor runs. We compare different policies that can be used to boost market and funding liquidity. While debt purchases prevent inefficient dry-ups, liquidity injections may backfire by exacerbating adverse selection frictions.

**Keywords:** Information Acquisition, Adverse Selection, Bank Runs, Global Games

**JEL classification:** D82, G01, G20

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# 1 Introduction

The strong growth of market-based finance over the past decades has given rise to a number of new types of financial intermediaries. These so-called shadow banks operate outside the perimeter of traditional banking regulation and the public financial safety net, even though they are typically created and operated by regulated financial institutions.[1] A key feature in shadow banks' design is the provision of credit guarantees or liquidity lines by their sponsoring institutions. Such sponsor support is intended to lower individual shadow banks' susceptibility to investor runs by shielding them from deteriorations in market liquidity conditions. In particular, liquidity lines – by providing shadow banks facing sudden funding withdrawals with a contingent liquidity source – may help them avoid losses that would otherwise accrue from selling assets at discounted prices.[2] The present paper challenges this view by arguing that the provision of such liquidity support can, in fact, be detrimental to aggregate market and funding liquidity conditions. We show this by developing a theory of asset market freezes and investor runs based on shadow banks' ability to acquire private information about their assets.

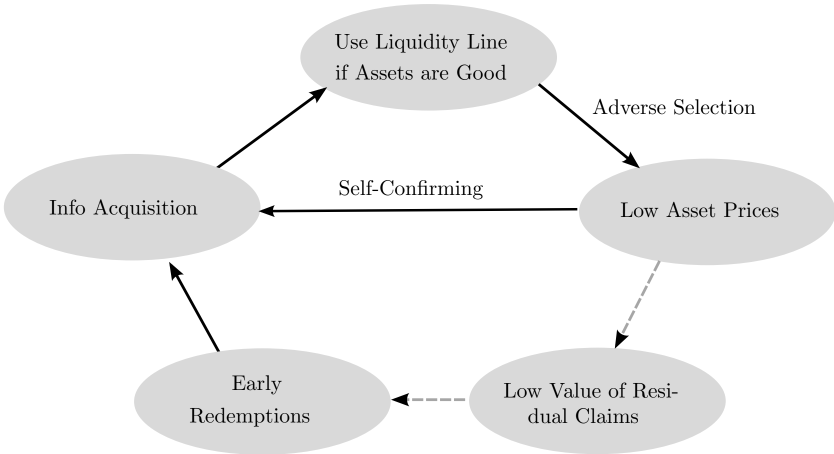
Contrary to the conventional view outlined above, we argue that market liquidity conditions are not *independent* of shadow banks' choice of liquidity sources since this choice depends on the quality of assets on their balance sheets. Building on Akerlof (1970)'s key insight that asymmetric information can impede trade, we show that access to pre-committed liquidity lines gives shadow banks an incentive to acquire private information about their assets in order to avoid selling good assets at a discount. This can lead to market freezes spurred by *endogenous* adverse selection which may, in turn, precipitate panic-driven investor runs. In general, our model shows that agents' access to "informationally insensitive" liquidity sources (whose costs are unaffected by the agents' private information) can lead liquidity to dry-up in competitive asset markets that would otherwise be immune to information-related trading frictions. Thus, rather than making the shadow banking sector safer, the presence of liquidity lines may in fact be a *source* of financial fragility by opening the door to market and funding liquidity dry-ups driven by self-fulfilling fears of adverse selection in asset markets.

**Overview of the Model.** Our results are based on a three-date model with three types of risk-neutral agents: shadow banks (e.g. off-balance sheet conduits or mutual funds), wholesale investors, and deep-pocketed asset market traders. Shadow banks (henceforth referred to as "funds" for short) enter the economy with long-term assets, partly financed by redeemable liabilities. These liabilities are held by wholesale investors that can choose to redeem them before funds' long-term assets mature. Funds' assets differ in terms of their payoff at maturity: some pay out a large cash flow (*good type*),

[1] See Pozsar, Adrian, Ashcraft, and Boesky (2010) for an overview. A large variety of different types of non-bank financial institutions can be subsumed under the term 'shadow banks', ranging from money market funds and other open-ended mutual funds that provide funding to off–balance sheet vehicles like SIVs, ABCP conduits or hedge funds.

[2] See Brady, Anadu, and Cooper (2012) for a discussion of various cases of sponsor support during the 2007-09 crisis.

Figure 1: Model Mechanism




while others pay out a small cash flow (*bad type*). Although funds initially do not know their assets' type, they can expend resources to privately learn the type. To obtain the liquidity needed to meet short-term redemptions, funds can then either sell assets in a competitive secondary market or tap a costly liquidity line provided by an (outside) sponsoring institution.[3]

The value of information in this environment stems from funds' ability to hold on to good assets by resorting to their liquidity lines rather than selling them at a discount.[4] Information acquisition by funds also generates an externality, as it induces an adverse selection problem in secondary markets that impedes the provision of market liquidity (i.e. lowers asset prices). This leads to a feedback from market prices to information acquisition, as lower prices reduce funds' opportunity costs of using their liquidity lines in case they have good assets.

The key contribution of our paper is to show that this feedback can generate self-fulfilling market liquidity dry-ups. To illustrate the underlying mechanism, suppose a fund faces redemptions and believes that other funds have acquired information (*cf.* the solid lines of Figure 1). If informed funds with good assets opt to finance redemptions using their liquidity lines, the relative share of bad assets in the secondary market increases and asset prices fall. This "lemons discount" raises the value from witholding good assets from the market and, *a fortiori*, the gain from acquiring information. The mere belief that others acquire information thus increases the private surplus from information acquisition, spurring self-fulfilling market freezes driven by *endogenous* adverse selection.

[3]Off-balance sheet conduits and MMFs had extensive recourse to the balance sheets of their sponsors. This included "liquidity enhacements," or private liquidity lines through which sponsoring institutions could repurchase performing assets if conduits failed to roll over maturing liabilities.

[4]As in Hirshleifer (1971), information acquisition has no social value in our model as its only serves to redistribute rents across funds.

This information-induced dry-up in market liquidity can also raise investors' incentives to redeem their claims early and amplify funds' funding liquidity risk. Funds that sell assets in order to meet early redemptions have to sell increasingly large quantities as prices fall. Early redemptions in this case dilute the claims held by investors at maturity and may lead to self-fulfilling investor runs if asset prices are sufficiently low (*cf.* the dashed line in Figure 1). The increased funding risk then raises funds' incentives to acquire information, which further pushes down prices and sparks even more redemptions.

Importantly, financial fragility in our model results from strategic complementarities in funds' information acquisition decisions. In particular, the coordination problem among investors precipitating redemption runs *only* emerges if endogenous information acquisition by funds leads assets to trade at a discount in secondary markets. When secondary markets are liquid, investors' claims at maturity are unaffected by the volume of early redemptions, and their redemption decisions are purely driven by their idiosyncratic liquidity shocks. This distinguishes our paper from standard bank run models *à la* Diamond and Dybvig (1983), where fragility stems from strategic complementarities in creditors' withdrawal decisions that arise due to the *exogenous* discount banks incur when assets are liquidated prematurely.

The strategic complementarities characterizing funds' information acquisition and investors' redemption decisions can lead to multiple Pareto-ranked equilibria. Equilibria *without* information acquisition are characterized by high secondary market prices and low funding risk. These equilibria Pareto-dominate equilibria *with* information acquisition that are characterized by low market prices and high funding liquidity risk. The coordination failure leading funds to acquire information can therefore generate significant welfare losses due to inefficient liquidity dry-ups. In order to select a unique equilibrium and study the effects of different types of policy interventions, we employ global game techniques by adapting the methodology of Goldstein (2005). This is done by introducing a macroeconomic state that affects the riskiness of assets' cash flows.

We show that depending on the parameters of the model, two different regimes can arise: a *weak dependence* and *strong dependence* regime. In the former, information acquisition by funds leads to a drop in asset prices that may spur panic-driven investor runs. However, no reverse feedback exists and market liquidity risk is unaffected by the volume of early redemptions. In the latter regime, market and funding liquidity mutually reinforce each other. Market liquidity dry-ups in this case are always accompanied by investor runs, and funding liquidity risk "spills over" and raises funds' incentives to acquire information about their assets.

Finally, we analyze a number of policy measures that can be used to mitigate these inefficient liquidity dry-ups. Inspired by measures adopted by central banks during the 2007-09 financial crisis to shore up liquidity in securitized asset markets, we focus on three specific policy interventions: asset purchases, outright debt purchases, and liquidity injections to funds' sponsoring institutions. First, asset purchases reduce both market and funding liquidity risk by lowering funds' information rents and boosting their residual equity value at maturity. However, as this policy does not completely eliminate funds' incentives to acquire information, it sometimes requires the policymaker to incur losses. Second, the policymaker can implement the efficient allocation, and prevent both inefficient market freezes and investor runs, using outright debt purchases. In effect, by completely shielding funds from funding liquidity risk, debt purchases eliminate the coordination failure leading funds to acquire information. Third, we show that liquidity injections to sponsoring institutions may backfire. Insofar as this policy reduces sponsors' opportunity cost of providing liquidity lines, it may exacerbate the adverse selection problem that causes market liquidity to dry up in the first place.

**Relation to the Literature.** Our paper highlights the fragility of financial institutions that heavily rely on market-based liquidity provision and sponsor support to manage their funding risk. It relates to a recent literature studying the origins and consequences of sponsor support for off-balance sheet vehicles and shadow banks. Ordonez (2016) and Segura (2017) focus on the reputational and signaling effects of sponsors' support decisions. In contrast to their papers, our model highlights the fragility of the shadow banking sector that arises from the interaction between market and funding liquidity engendered by sponsors' support decisions. Similar to Parlatore (2016), our paper emphasizes the detrimental effects of strategic complementarities arising from the interaction between support decisions and market prices. Complementarities in her model arise because fire-sales by funds without support raise their default risk, thereby reducing sponsors' incentives to offer support and increasing the need to fire-sell assets. In our model, funds face complementarities in information acquisition because sponsor support allows them to retain good assets. on their balance sheet. This leads to endogenous adverse selection in asset markets and can induce a mutual amplification between market and funding risk.

The source of financial fragility in our paper makes it conceptually different from the "classical" bank run literature - e.g. Diamond and Dybvig (1983), Rochet and Vives (2004), and Goldstein and Pauzner (2005) - where, as mentioned above, banks' fragility arises from the nature of demandable deposit contracts in the presence of exogenous liquidation discounts. Panic-driven runs only emerge in our framework because information acquisition leads asset prices to fall, thereby eroding the value of residual claims. In this sense, our paper builds on the literature studying how adverse selection can lead to self-fulfilling market freezes, including Eisfeldt (2004), Plantin (2009), Malherbe (2014) and Heider, Hoerova, and Holthausen (2015). However, contrary to these papers that treat asymmetric information as a primitive, adverse selection frictions emerge endogenously in our model due to funds strategic information acquisition behavior. Bolton, Santos, and Scheinkman (2011) also study how liquidity crises unfold over time in the presence of exogenous asymmetric information. Similar to our model, a key motive for not selling assets in their model is the option value from retaining good assets when asset prices are low due to adverse selection frictions. In contrast to our paper, they do not explore the consequences of this motive for the emergence of endogenous adverse selection and abstract from the interaction between market and funding liquidity.

Our paper also relates to a growing literature on information acquisition in financial markets. Gorton and Ordonez (2014), building on Dang, Gorton, and Holmström (2015), study how information acquisition amplifies aggregate shocks to collateral values. The value of information in their model corresponds to an information rent that accrues to creditors from liquidating bad collateral at a pooling price.[5] Importantly, the feedback between market prices and information acquisition implied by this information rent induces strategic substitutability (rather than strategic complementarity) in information production. Hence, the self-fulfilling liquidity dry-ups that are the focus of our paper cannot arise in Gorton and Ordonez (2014)'s framework. Other recent papers studying strategic complementarities in information acquisition include Fishman and Parker (2015) and Bolton, Santos, and Scheinkman (2016). The source of strategic complementarities is conceptually different from the one studied here, however, as it operates through the rents informed investors extract when buying (rather than selling) assets.[6] These papers also do not consider the interaction between market and funding liquidity risk that underlies our model of investor runs.

The mutual amplification of market and funding liquidity risk in our model links our paper to another literature studying the destabilizing effect of margins (Brunnermeier and Pedersen, 2009; Biais, Heider, and Hoerova, 2015; Kuong, 2015). In these papers, market illiquidity can amplify firm deleveraging due to a fire sale externality. This "margin channel", however, differs substantially from our "information acquisition channel" in both its empirical and policy implications. First, fire sales resulting from funding constraints lead prices to decline when firm deleveraging becomes excessive. In contrast, in our model prices decline because some firms (namely those that know they hold good assets) opt *not* to sell their assets in secondary markets. Thus, while the "margin channel" suggests that low asset prices should be associated with high trading volumes, our "information acquisition channel" does not.[7] Second, but related to the first point, fire sales caused by funding constraints emerge due to a lack of overall liquidity in the economy. Liquidity injections that relax funding constraints therefore dampen price declines caused by fire sales. Again, this contrasts with our framework, where liquidity injections may exacerbate market illiquidity by reinforcing adverse selection frictions.

Finally, our paper draws from the large literature on global games that interprets liquidity dry-ups as the result of a coordination failure; e.g. Morris and Shin (2003, 2004a,b). Compared to this literature, our model studies a new channel of coordination

[5] In a related model studying information acquisition by sellers (rather than buyers), Dang, Gorton, and Holmström (2013) shows that the value of information is the minimum of either the information rent from selling a low payoff security at a high price, or the gain from not selling a high payoff security at a low price. Firms' surplus from information acquisition in our model is similar to the latter. While Dang et al. (2013) focus on optimal security design, we study the feedback between information acquisition and market prices.

[6] Feijer (2015) also studies strategic complementarities in information acquisition in a model with contracting frictions caused by a risk-shifting problem. The feedback mechanism in his model is distinct from the one here, as it operates through initial borrowing costs rather than secondary market prices. He also does not consider the interaction between market and funding liquidity risk.

[7] This "double whammy" (as Tirole (2011) refers to it) of declining prices and trading volumes fits well with observed price and trading movements in securitized asset markets during the 2007-08 financial crisis.

failure that explicitly ties market liquidity risk to an adverse selection problem caused by firms' information acquisition behavior. Our paper differs from most global game models as it features strategic complementarities within and across two groups of agents. Methodologically, our analysis is closely related to the twin crisis model of Goldstein (2005) who first extended global game techniques to a setting with two types of agents and a common fundamental.

The remainder of the paper is organized as follows. In Section 2, we treat the amount of early redemptions as *given* and characterize funds' information acquisition decisions and the resulting unique equilibrium in terms of the macroeconomic state. This serves to illustrate the model's main mechanism. In Section 3, we endogenize the redemption decisions of investors and derive the joint equilibrium between funds and investors. We characterize the different regimes and the welfare properties. Implications for policy are subsequently discussed in Section 4, while Section 5 concludes. All proofs are relegated to the Appendix.

# 2 Information Acquisition and Market Liquidity

## 2.1 Model Basics

We consider an economy populated by a continuum of risk-neutral financial institutions, indexed by $j \in [0,1]$, that operate for three dates $t \in \{0,1,2\}$. We think of these institutions as non-bank entities like off-balance sheet conduits (e.g. structured investment vehicles), hedge funds, or money market funds. For simplicity, we henceforth refer to the institutions in our model as funds.

**Assets.** Each fund enters the economy at date $t = 0$ holding one unit of a perfectly divisible long-term asset that pays out at date $t = 2$. The asset's payoff at maturity consists of two parts: (i) a risky component $\tilde{X}(\theta)$, and (ii) a non-marketable control rent $Q > 0$. The risky component $\tilde{X}(\theta)$ has the following payoff structure:

$$\tilde{X}(\theta) = \begin{cases} X(\theta) & \text{with probability } \pi \\ \theta X(\theta) & \text{with probability } 1-\pi \end{cases}$$

When the realized payoff is $X(\theta)$, the asset is said to be of a *good* type; otherwise it is said to be of a *bad* type. The realization of the assets' type is assumed to be i.i.d. across funds. The parameter $\theta \in \Theta \subset [0,1]$ is a macroeconomic state that affects assets' returns in a mean-preserving spread sense: i.e. $\mathbf{E}[\tilde{X}(\theta)] = F$ for all $\theta \in \Theta$ such that $\frac{\mathrm{d}}{\mathrm{d}\theta}X(\theta) < 0$ and $\frac{\mathrm{d}}{\mathrm{d}\theta}\theta X(\theta) > 0$.[8] The macroeconomic state $\theta$ can be interpreted as a measure of uncertainty affecting funds' assets, with high (low) values of $\theta$ indicating a low (high) degree of macroeconomic uncertainty.

In addition to the risky component, each fund obtains a non-marketable control rent $Q > 0$ per unit of asset under management at $t = 2$. This control rent can be interpreted

[8] More explicitly, $X(\theta) = \frac{F}{\pi+(1-\pi)\theta}$ such that $\frac{\mathrm{d}}{\mathrm{d}\theta}X(\theta) = -\frac{(1-\pi)F}{(\pi+(1-\pi)\theta)^2}$ and $\frac{\mathrm{d}}{\mathrm{d}\theta}\theta X(\theta) = \frac{\pi F}{(\pi+(1-\pi)\theta)^2}$.

as additional value created by funds if assets remain on their balance sheet, e.g. due to funds' superior asset management capabilities, or benefits from regulatory arbitrage. Hence, while the *ex ante* book value of the asset is given by $F$, from the funds' perspective the *ex ante* expected value of assets held until maturity is equal to $F + Q$.

**Information Structure.** The macroeconomic state $\theta$ is drawn at date $t = 0$ from a uniform distribution over $\Theta$.[9] The realization of $\theta$ becomes common knowledge before the market opens at date $t = 1$.

At date $t = 0$, each fund receives a noisy private signal about the macroeconomic state:

$$\theta_j = \theta + \epsilon_j$$

where $\epsilon_j$ is i.i.d. across funds and drawn from a uniform distribution over $[-\epsilon, \epsilon]$. In addition to observing this noisy signal about $\theta$, funds can acquire private information about the type of their asset at a fixed cost $\psi > 0$ at $t = 0$. For simplicity, we assume that by acquiring information funds perfectly observe whether their asset is good or bad.[10] We denote by $\Omega_j \in \{n, g, b\}$ fund $j$'s information set conditional on not acquiring information ($n$), or acquiring information and verifying the asset's type to be good ($g$) or bad ($b$). Correspondingly,

$$\mathbf{E}[\tilde{X}(\theta)|\Omega_j, \theta_j] \in \{F, \mathbf{E}(X(\theta)|\theta_j), \mathbf{E}(\theta X(\theta)|\theta_j)\}$$

denotes fund $j$'s beliefs at $t = 0$ about its asset's type given its information set.

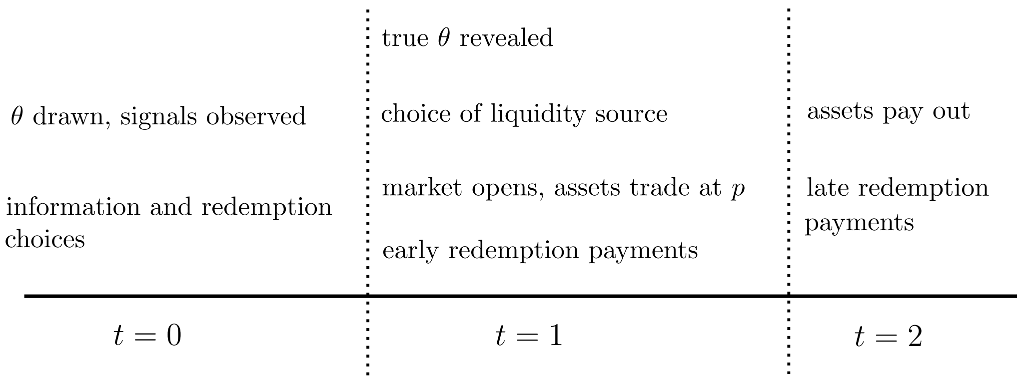
**Liabilities.** Each fund is financed by a distinct unit mass of investors. A fraction $(1 - \alpha)$ of each fund's liabilities are irredeemable, e.g. long-term debt or equity shares held by passive investors. The remaining fraction $\alpha$ is redeemable at date $t = 1$. More specifically, we build on Liu and Mello (2011) and model the redemption process as follows: investors must notify their fund about their redemption decision at $t = 0$; their claims are then priced at the current marketable book value of the fund, $F$, and disbursed to investors at $t = 1$. For the moment, we assume that an exogenous share $\lambda \in [0, 1]$ of redeemable liabilities are withdrawn and need to be repaid at date $t = 1$. We endogenize investors' redemption decisions and study the resulting feedback between market and funding liquidity risk in Section 3.

**Liquidity Sources.** The balance sheet structure described above implies that funds are subject to a standard liquidity mismatch problem: while the long-term asset does not pay out until $t = 2$, funds must finance redemptions of $\alpha\lambda F$ at $t = 1$. We assume that funds can obtain the liquidity needed to meet early redemptions in one of two ways. First, each fund has access to a private liquidity line from which cash can be drawn

[9] The restriction to a uniform distribution is without loss of generality; any other continuous distribution with finite support $\Theta$ could be assumed.

[10] Our results would not be altered if we assumed that funds could only observe a noisy signal about the idiosyncratic state of their assets.

Figure 2: Sequence of Events




down at a unit cost of $\kappa > 1$. Alternatively, funds can sell their assets on a competitive secondary market in $t = 1$ at price $p$. The buyers in the secondary market are large in number, deep-pocketed and risk-neutral and stand ready to purchase assets at their expected value at $t = 1$, i.e. $p = \mathbf{E}[\tilde{X}(\theta)|\theta]$. Since the control rent is non-marketable, funds selling assets to meet early redemptions must necessarily forego the additional payoff $Q$ per unit of asset sold. This can also be interpreted as a fixed fire-sale discount incurred by funds when deleveraging. The sequence of events is summarized in Figure 2.

## 2.2 Liquidity Sources and Asset Prices

**Liquidity Lines *vs.* Asset Sales.** At date $t = 1$, funds choose between the two liquidity sources in order to maximize their expected equity value, given their information set $\Omega_j$ and the realized macroeconomic state $\theta$. Denote by $V^{LL}_{\Omega_j}$ and $V^{AS}_{\Omega_j}$ the equity value of a fund with information set $\Omega_j$ that uses liquidity lines $(LL)$ or asset sales $(AS)$ to obtain liquidity. The expected equity value of a fund that obtains $\ell_j \geq \alpha\lambda F$ units of liquidity at $t = 1$ by selling assets is given by

$$\mathbf{E}[V^{AS}_{\Omega_j}(\ell_j)|\theta] = \mathbf{E}\left[\max\left\{\left(\tilde{X}(\theta) + Q\right)\left(1 - \frac{\ell_j}{p}\right) + (\ell_j - \alpha\lambda F), 0\right\}\middle|\Omega_j, \theta\right] \tag{1}$$

Similarly, the value of a fund choosing to obtain liquidity *via* its liquidity line equals

$$\mathbf{E}[V^{LL}_{\Omega_j}(\ell_j)|\theta] = \mathbf{E}\left[\max\left\{\tilde{X}(\theta) + Q - \kappa\ell_j + (\ell_j - \alpha\lambda F), 0\right\}\middle|\Omega_j, \theta\right] \tag{2}$$

Since $\kappa > 1$, funds meeting early redemptions using their liquidity lines will never choose to obtain more liquidity than that needed to meet early redemptions. The choice of $\ell_j$ for funds selling assets, however, depends on their information set and the size of $Q$. In what follows, we assume that the non-marketable control rent always exceeds the information rent that informed funds with bad assets could obtain by selling their entire portfolio at a price above their assets' true value. This ensures that informed funds will never choose to sell more assets than what is needed to meet their liquidity needs.

**Assumption 1.** *Let* $\underline{\theta} \equiv \min\{\Theta\}$*, then the non-transferable control rent* $Q$ *is such that*

$$Q > F - \underline{\theta} X(\underline{\theta})$$

Equations (1) and (2) imply that funds' preference between liquidity lines and asset sales depends on the market price ($p$) and the cost of liquidity lines ($\kappa$). To fix funds' preference ordering over liquidity sources given their information set $\Omega_j$, we impose the following restrictions on the set of macroeconomic states $\Theta$:

**Assumption 2.** *The lower and upper bounds of* $\Theta$*,* $\min\{\Theta\} \equiv \underline{\theta}$ *and* $\max\{\Theta\} \equiv \overline{\theta}$*, are such that*

$$\frac{F + Q}{\underline{\theta} X(\underline{\theta})} < \kappa < \frac{X(\overline{\theta}) + Q}{F}$$

*where the parameters* $\pi$ *and* $Q$ *are such that* $\Theta \neq \emptyset$*.*

The upper bound on $\theta$ corresponds to a standard "lemons condition." It implies that even if assets trade at their *ex ante* expected value ($p = F$), informed funds holding good assets always prefer to meet redemptions by tapping their liquidity lines. Effectively, this inequality implies that the cash flow of a good asset is sufficiently large to compensate funds for the cost of the liquidity line, *regardless* of the price at which assets trade in secondary markets. The lower bound on $\theta$, on the other hand, implies that even if assets trade at the lowest possible price ($p = \underline{\theta} X(\underline{\theta})$), uninformed funds prefer to meet redemptions by selling assets in the absence of default.[11]

In what follows, we assume that the fraction of irredeemable liabilities is sufficiently large such that funds never become illiquid even if they face full redemptions at $t = 1$, i.e. $\lambda = 1$. In particular, funds never default if the following inequality is satisfied:

$$\alpha F < \min\left\{\frac{\underline{\theta} X(\underline{\theta}) + Q}{\kappa}, \underline{\theta} X(\underline{\theta})\right\} = \frac{F}{\kappa}$$

which implies that the face value of funds' redeemable liabilities is strictly less than the liquidation value of funds' with bad assets regardless of whether they obtain liquidity through asset sales or *via* their liquidity lines.[12] This no-default assumption allows us to abstract from gambling incentives driven by funds' limited liability constraint.[13]

[11] The lower bound on $\theta$ is a technical assumption needed to guarantee the existence of an equilibrium in funds' information acquisition game.

[12] The simplification of the right-hand-side of the inequality follows from Assumptions 1 and 2.

[13] When this assumption is violated, uninformed funds could prefer to resort to the liquidity line (instead of selling assets) and gamble that their asset turns out to be good. Note that potential losses from this gambling behaviour would have to be borne by the sponsor whenever funds defaulted on their liquidity line. Given funds' limited liability constraint, investors would be wiped-out despite funds having received interim support. This, however, would be difficult to square with available evidence on sponsor

**Assumption 3.** *The fraction of redeemable liabilities is such that* $\alpha < 1/\kappa$.

**Lemma 1.** *Given Assumptions 1-3, informed funds with good assets strictly prefer the liquidity line, while informed funds with bad assets and uninformed funds strictly prefer asset sales.*

**Asset Price.** Buyers that purchase assets in the secondary market must break even. Since the macroeconomic state becomes common knowledge before the markets open at $t = 1$, buyers' expectations are conditioned on the realized value of $\theta$. Their participation constraint is therefore given by

$$p \leq \mathbf{E}[\tilde{X}|\theta] = X(\theta)(\tau + (1-\tau)\theta)$$

where $\tau \in [0, 1]$ denotes the fraction of good assets supplied to the market. Competition in the market ensures that this inequality binds in equilibrium.

The price at which assets trade depends on buyers' beliefs about the share of good assets supplied to the market. Given Lemma 1, only uninformed and informed bad funds supply their assets to the market. Hence, whenever some funds acquire information, the share of good assets traded in the secondary market will be strictly less than the share of good assets in the economy: i.e. $\tau < \pi$. We assume that trading in the secondary market is anonymous, so that market participants cannot infer assets' state based on the quantity fund $j$ supplies to the market.[14] Letting $\sigma \in [0, 1]$ denote the share of funds acquiring information, the fraction of good assets traded in the market is then equal to

$$\tau(\sigma) = \frac{\pi(1-\sigma)}{1-\pi\sigma}$$

and the market price can be rewritten as

$$p(\sigma, \theta) = F - (\pi - \tau(\sigma))(1-\theta)X(\theta) \tag{3}$$

By acquiring information, funds induce an asymmetric information friction in secondary markets, as informed funds with good assets withhold these from the market. The resulting adverse selection problem leads assets to trade at a discount compared to their *ex ante* book value. Importantly, this discount is strictly decreasing in the fraction of informed funds since the share of good assets traded in the market falls as more funds acquire information, i.e. $\tau'(\sigma) < 0$. Moreover, since the value of bad assets rises when the macroeconomic state improves, the price also increases in the macroeconomic state $\theta$.

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support. For example, Brady et al. (2012) show that although a substantial number of MMFs could have 'broken the buck' had they not received support dring the 2007-08 crisis, there are no instances where funds defaulted *after* they received support. Similarly, Gorton (2010) documents that structured investment vehicles that received sponsor support did not default.

[14] This assumption rules out the possibility of funds using their liquidity lines to signal their type to potential buyers. For an analysis of the signaling effects of liquidity lines see Segura (2017).

**Lemma 2.** *The secondary market price is strictly decreasing in the fraction of informed funds:* $p_\sigma(\sigma, \theta) < 0$*, and is strictly increasing in the macroeconomic state:* $p_\theta(\sigma, \theta) > 0$*.*

**Discussion of Modelling Environment.** The assumption that liquidity lines are costly in our model is key, and can be justified for a number of reasons. For example, providing funds with liquidity may require sponsoring institutions to pass on valuable investment opportunities for which they must be compensated. Funds may nonetheless prefer to use liquidity lines rather than maintaining cash balances since liquidity lines allow funds to avoid paying the liquidity premium implied by holding liquid assets in states of the world where they do not face a liquidity shortfall (Acharya, Almeida, and Campello, 2013). Moreover, while cash balances constitute a sunk cost for funds if these are stored before liquidity shocks are realized, the costs of contingent liquidity lines are not sunk but rather are only incurred if funds choose to draw them down. These costs should therefore affect funds' choice of liquidity source when financing early redemptions.

Funds' choice of liquidity source in our model can be more broadly interpreted as a choice between: (i) deleveraging or (ii) borrowing funds from their sponsor at a fixed interest rate. We restrict attention to these two liquidity sources insofar as we consider our model relevant for understanding environments where the issuance of new (debt or equity) securities by funds is not feasible. As in Gorton and Pennacchi (1990), risk-less debt in our model has the advantage of being an informationally-insensitive security and consequently not subject to adverse selection discounts. Consequently, if funds could costlessly issue new securities, they would all optimally choose to issue risk-less debt. Our model assumes that the only way in which funds can obtain such informationally-insensitive financing is by tapping their costly liquidity lines, and shows that funds' choice of liquidity source in this case fundamentally depends on their private information.[15] Liquidity lines also differ from new debt issuances as their cost (i.e. the interest charged on funds drawn from the line) are contracted upon before the realization of liquidity shocks and therefore do not react to contemporaneous market information.

In principle our model could also be applied to commercial banks or depository institutions. However, a number of our modeling assumptions make us inclined to interpret the funds in our model as shadow banks or off-balance sheet vehicles. First, compared to commercial banks who can access retail deposit markets and are subject to minimum reserve and liquidity requirements, non-bank financial institutions rely to a much stronger degree on market-based liquidity management. The assumption that uninformed funds prefer deleveraging to tapping their liquidity line (*cf.* Assumption 2) is consistent with the fact that off-balance sheet vehicles often relied on so-called "dynamic liquidity management" strategies to manage their funding risk, meaning that they regularly sold assets in order to obtain liquidity notwithstanding the recourse to their sponsors' balance sheets (Covitz, Liang, and Suarez, 2013). Second, credit intermediation activities of depository institutions are enhanced by official sector guarantees (e.g. deposit insurance schemes or access to central bank discount windows). In constrast, the activities of shadow banks

[15]Note that deleveraging is also costly as it requires funds to forgo the non-marketable control rent, $Q$.

lie outside the perimeter of banking regulation and are only indirectly enhanced through private agreements with third parties, e.g. their sponsoring institutions (Pozsar et al., 2010).

### 2.3 Equilibrium: Information Acquisition

**Surplus from Information Acquisition.** Given Lemma 1, if a fund does not acquire information, it prefers to meet early redemptions by selling assets. The expected equity value of an uninformed fund at $t = 0$ therefore equals $\mathbf{E}[V_n^{AS}(\alpha\lambda F)|\theta_j]$. If a fund acquires information, then with probability $\pi$ the asset is verified to be good and with probability $1-\pi$ it is verified to be bad. Again, by Lemma 1, funds with good assets always prefer to use their liquidity lines, while bad funds opt to sell assets. The expected surplus from acquiring information at $t = 0$, given signal $\theta_j$, is then equal to

$$\mathbf{E}[S(\sigma, \theta; \lambda)|\theta_j] \equiv \mathbf{E}\left[\pi V_h^{LL}(\alpha\lambda F) + (1-\pi)V_l^{AS}(\alpha\lambda F) - V_n^{AS}(\alpha\lambda F)\middle|\, \theta_j\right]$$

where the subindexes $\{n, h, l\}$ indicate fund $j$'s information set $\Omega_j$ at $t = 0$. Using equations (1) and (2), this function can be rewritten as

$$\mathbf{E}[S(\sigma, \theta; \lambda)|\theta_j] = \mathbf{E}\left[\pi\left(\frac{X(\theta) + Q}{p(\sigma, \theta)} - \kappa\right)\alpha\lambda F\middle|\, \theta_j\right] \tag{4}$$

The expected surplus from acquiring information can be interpreted as the option value from holding good assets rather than selling them at the pooling price. In particular, informed funds with good assets benefit from using their liquidity lines rather than trading in the market as they only forego $\kappa$ units of output tomorrow for one unit of liquidity today, compared to $X(\theta) + Q$ units of output tomorrow for $p(\sigma, \theta)$ units of liquidity today. The upper bound on $\overline{\theta}$ (Assumption 2) guarantees that this difference is always positive.

As shown by Lemma 2, the market price declines as more funds become informed due to adverse selection. Importantly, lower prices reduce the opportunity cost of using liquidity lines and thereby increase the value from acquiring information (*cf.* equation (4)). This feedback between the value of information and the market price generates *strategic complementarities* in information acquisition: i.e. for any private signal $\theta_j \in \Theta$, fund $j's$ surplus from acquiring information is increasing in the fraction of funds acquiring information, $\sigma$.[16] In addition, because the price increases whereas the value of good assets declines in the macroeconomic state, the surplus from information strictly decreases in $\theta$.

[16]The control rent $Q$ is important for generating the monotonicity of the surplus function in $\sigma$. The control rent effectively eliminates the incentives of funds with bad assets to sell off their portfolio at a premium $p - \theta X(\theta)$ and obtain a 'lemon rent'. This allows us to isolate the complementarities in information acquisition created by the 'option rent' from withholding good assets. The option rent exists also in the presence of a lemon rent, e.g. when $Q = 0$. However, a lemon rent could overturn the complementarities and lead funds' decisions to become strategic substitutes instead. The consequences of such a lemon rent are already studied extensively in the literature, e.g. by Gorton and Ordonez (2014) or Malherbe (2014).

**Lemma 3.** *The surplus from information acquisition is strictly increasing in the fraction of informed funds:* $S_\sigma(\sigma, \theta; \lambda) > 0$*, and is strictly decreasing in the macroeconomic state:* $S_\theta(\sigma, \theta; \lambda) < 0$*.*

In equilibrium, funds choose to acquire information if and only if their expected net surplus from doing so is positive: i.e. whenever $\mathbf{E}[S(\sigma; \theta; \lambda)|\theta_j] - \psi > 0$. In what follows, we impose the following restriction on the relationship between funds' information acquisition costs and the set of macroeconomic states $\Theta$:

**Assumption 4.** *There exist* $\underline{\theta}_F \in \Theta$ *and* $\overline{\theta}_F \in \Theta$*, such that the costs for acquiring information satisfy*

$$S(1, \overline{\theta}_F; \lambda) < \psi < S(0, \underline{\theta}_F; \lambda)$$

Assumption 4 implies that whenever the state is above (below) the bound $\overline{\theta}_F$ ($\underline{\theta}_F$), the net surplus from information acquisition is strictly negative (positive), no matter what strategies the other funds choose. Moreover, it further implies that there are signals above (below) which it becomes a dominant action to refrain from (engage in) information acquisition.

**Monotone Strategies.** Funds choose whether or not to acquire information based on their signal of the macroeconomic state $\theta_j$, their beliefs regarding other funds' information acquisition decisions and the expected secondary market price. In equilibrium, the realized share of informed funds and the resulting market price must be consistent with funds' initially held beliefs.

In the absence of fundamental uncertainty – i.e. in an environment where the realization of $\theta$ would already be common knowledge at $t = 0$ – the economy would exhibit multiple equilibria for intermediate values of $\theta \in [\underline{\theta}_F, \overline{\theta}_F]$ due to the strategic complementarities characterizing funds' information acquisition decision. In one equilibrium, funds, expecting market prices to be high, refrain from information acquisition and market liquidity provision is undistorted by asymmetric information frictions. In a second equilibrium, funds, expecting market liquidity to dry-up, acquire information and precipitate an adverse selection problem by withholding good assets from secondary markets. The noisiness of funds' signals and the breaking of common knowledge about the macroeconomic state allows to isolate a unique equilibrium (Morris and Shin, 2003).

A strategy for fund $j$ is defined as a mapping $\sigma_j : \Theta \to [0, 1]$ which specifies for each signal $\theta_j \in \Theta$ the probability with which fund $j$ chooses to acquire information about the idiosyncratic state of its asset. A strategy is *monotone*, summarized by a critical threshold $\theta^*_{j,\epsilon}$, whenever the fund acquires information with probability one if $\theta_j < \theta^*_{j,\epsilon}$ and otherwise refrains from information acquisition. A symmetric monotone strategy is a monotone strategy where all funds use the same critical threshold $\theta^*_{F,\epsilon}$. As we show below, restricting attention to symmetric monotone strategies is without loss of generality.

**Unique Monotone Equilibrium.** By the law of large numbers and using the assumptions of uniformly distributed states and signals, the share of funds acquiring information given a symmetric monotone strategy summarized by $\theta^*_{F,\epsilon}$ is equal to

$$\sigma(\theta^*_{F,\epsilon}, \theta) = \mathbf{Pr}(\theta_j < \theta^*_{F,\epsilon}|\theta) = G\left(\frac{\theta^*_{F,\epsilon} - \theta + \epsilon}{2\epsilon}\right) \tag{5}$$

where $G(x) = \min\{\max\{x, 0\}, 1\}$.

In equilibrium, the threshold value $\theta^*_{F,\epsilon}$ must be such that a fund that observes the signal $\theta_j = \theta^*_{F,\epsilon}$ is just indifferent between acquiring information or not, given that other funds use the monotone strategy around $\theta^*_{F,\epsilon}$. Given the uniform prior assumption, the posterior belief about $\theta$ for a fund receiving signal $\theta^*_{F,\epsilon}$ is uniform over $[\theta^*_{F,\epsilon} - \epsilon, \theta^*_{F,\epsilon} + \epsilon]$. The threshold $\theta^*_{F,\epsilon}$ must therefore solve

$$\mathbf{E}[S(\sigma(\theta^*_{F,\epsilon}, \theta), \theta; \lambda)|\theta^*_{F,\epsilon}] = \frac{1}{2\epsilon}\int_{\theta^*_{F,\epsilon}-\epsilon}^{\theta^*_{F,\epsilon}+\epsilon} S(\sigma(\theta^*_{F,\epsilon}, \theta), \theta; \lambda)\mathrm{d}\theta = \psi \tag{6}$$

Changing the variable of integration using the definition of the share of informed funds given by equation (5), the latter condition can be written as

$$\int_0^1 S(\sigma, \theta(\theta^*_{F,\epsilon}, \sigma); \lambda)\mathrm{d}\sigma = \psi \tag{7}$$

where $\theta(\theta^*_{F,\epsilon}, \sigma) = \theta^*_{F,\epsilon} - \epsilon\left(2G^{-1}(\sigma) - 1\right)$ and $G^{-1}(\sigma) = \inf\{x|G(x) \geq \sigma\}$.

**Proposition 1. (Unique Monotone Equilibrium)**

1. *There exists a unique monotone equilibrium where funds acquire information if and only if $\theta_j < \theta^*_{F,\epsilon}$, where $\theta^*_{F,\epsilon} \in (\underline{\theta}_F, \overline{\theta}_F)$.*

2. *There are no other equilibria in non-monotone strategies.*

**Equilibrium Properties.** The information acquisition decisions of funds in equilibrium imply that market liquidity dries up for states below $\theta^*_{F,\epsilon} + \epsilon$. This critical state, and therefore the prevalence of market illiquidity, depends crucially on the marketability and the expected cash flow of the assets, as measured by $F$ and $Q$.

**Corollary 1.** *The threshold $\theta^*_{F,\epsilon}$ below which funds acquire information is (i) strictly increasing in the control rent $Q$: $\partial\theta^*_{F,\epsilon}/\partial Q > 0$; and (ii) increasing or decreasing in the expected cash flow of the asset: $\partial\theta^*_{F,\epsilon}/\partial F \gtreqless 0$.*

Increases in the control rent, $Q$, reflect deteriorations in the marketability of assets and raise the surplus from information acquisition, thereby exacerbating the coordination problem among funds. Put differently, as the marketability of assets decreases, secondary asset markets become more prone to endogenous adverse selection frictions.

While changes to the control rent have an unambiguous effect on the threshold, increases in the expected cash flows are associated with two opposing effects: a negative *price effect* and a positive *redemption effect*. The price effect implies that funds have to sell less assets to meet a given amount of redemptions as cash flows increase. This lowers the surplus from information acquisition and tends to lower the threshold below which funds acquire information. Larger expected cash flows, however, also imply that investors are entitled to a larger claim if they withdraw early. This redemption effect raises the surplus from information acquisition and tends to push up the threshold below which funds acquire information. Whether the price or the redemption effect dominates depends, among other things, on the magnitude of the costs of liquidity lines. If $\kappa$ is sufficiently large, the negative price effect dominates and higher expected cash flows reduce the set of states where the market dries up due to adverse selection frictions.

Another important feature of the equilibrium is that the set of states where market liquidity dries up due to adverse selection is increasing in the fraction of early redemptions, $\lambda$. Similar to the redemption effect mentioned above, a larger share of early redemptions increases the surplus from acquiring information and, for fixed information acquisition costs, leads funds to acquire information about the type of their assets for a larger range of signals.

**Corollary 2.** *The threshold $\theta^*_{F,\epsilon}$ below which funds acquire information is strictly increasing in the fraction of early withdrawals: $\partial\theta^*_{F,\epsilon}/\partial\lambda > 0$.*

This last result raises the issue of how funds' funding liquidity risk is affected by market liquidity conditions. The next section shows how the endogenous adverse selection, stemming from funds acquiring private information about their assets' type, can lead market and funding illiquidity to become mutually reinforcing. This feedback arises because investors' incentives to redeem their claims early may rise if they expect asset prices to decline, as this erodes the residual equity value of funds covering redemptions *via* asset sales.

# 3 Market Illiquidity and Redemption Risk

## 3.1 Investors' Redemption Decisions

**Active Investors.** We follow Diamond and Dybvig (1983) and Liu and Mello (2011) and assume that active investors are subject to idiosyncratic liquidity shocks that affect their valuation for $t = 2$ consumption. In particular, we assume that each active investor faces a liquidity shock with probability $\mu$, implying that a total share $\mu \in (0, 1)$ become *impatient* and always redeem their claims at $t = 1$. The remaining share $(1 - \mu)$ of active investors are *patient*: they face no urgent liquidity need, but may nonetheless redeem their claim early if the payoff from doing so exceeds the expected value of their claim at maturity.[17] Investor types are private information, implying that funds cannot condition redemption payments on whether an investor is patient or impatient.

Patient investors, like funds, receive noisy signals about the macroeconomic state $\theta$ at date $t = 0$. These signals have the same structure as the signals received by funds:

[17] While our results require the mass of impatient investors to be strictly positive, $\mu$ can be arbitrarily small. That is, all our results hold even in the limiting case where $\mu \to 0$.

$\theta_i = \theta + \epsilon_i$, where $i$ indexes patient investors and $\epsilon_i$ is i.i.d. across investors and funds, drawn from a uniform distribution over $[-\epsilon, \epsilon]$. Based on their signals, patient investors form beliefs about funds' expected equity value at maturity, taking the information acquisition behaviour of funds, the resulting market price and the redemption decisions of other patient investors as given.

**Surplus from Early Redemption.** The total share of active investors redeeming their shares at $t = 1$ is given by $\lambda \in [\mu, 1]$. The value of a claim at maturity equals the *pro-rata* share of a fund's equity value at date $t = 2$, denoted by $D_2(\lambda, \theta; \sigma)$. A patient investor who observes signal $\theta_i$ expects the value of claims redeemed at $t = 2$ to be

$$\mathbf{E}\left[D_2(\lambda, \theta; \sigma) | \theta_i\right] = \mathbf{E}\left[\left.\frac{\sigma\pi V_h^{LL}(\alpha\lambda F) + \sigma(1-\pi)V_l^{AS}(\alpha\lambda F) + (1-\sigma)V_n^{AS}(\alpha\lambda F)}{1-\alpha\lambda}\right| \theta_i\right]$$

Using equations (1) and (2), this expression can be rewritten as follows

$$\mathbf{E}\left[D_2(\lambda, \theta; \sigma) | \theta_i\right] = \mathbf{E}\left[\left.\frac{1}{1-\alpha\lambda}\left((F+Q)\left(1 - \frac{\alpha\lambda F}{p(\sigma, \theta)}\right) + \sigma S(\sigma, \theta; \lambda)\right)\right| \theta_i\right]$$

The expected equity value of an investor's claim at maturity consists of two parts: (i) the residual equity value of funds' portfolios after assets have been sold to cover early redemptions; and (ii) the information rents accruing to informed funds. A patient investor prefers early redemption if and only funds' *ex ante* book value, $F$, exceeds funds' expected *per capita* equity value at maturity given the signal $\theta_i$. That is,

$$\mathbf{E}[W(\lambda, \theta; \sigma)|\theta_i] \equiv F - \mathbf{E}\left[D_2(\lambda, \theta; \sigma) | \theta_i\right] \geq 0 \tag{8}$$

Note that for all values of $\theta$ and $\lambda$, $W(\lambda, \theta; 0) = -Q < 0$: i.e. in the absence of information acquisition by funds, patient investors would never choose to redeem their claims early because of the control rent that they can earn if assets remain on funds' balance sheets. If $\sigma > 0$, however, asset prices fall below funds' *ex ante* book value, i.e. $p(\sigma, \theta) < F$. A fund that sells assets to cover early redemptions in this case must liquidate more than one unit of asset per claim redeemed at $t = 1$. This erodes the residual value of the funds' portfolio and dilutes the claims of patient investors that hold out until maturity. This effect is counteracted by the fact that a larger share of early redemptions raises the information rents accruing to informed funds: i.e. $S_\lambda(\sigma, \theta; \lambda) > 0$. However, the former effect always dominates the latter, leading the residual equity value of funds to decline as the fraction of early redemptions rises. In other words, investors' redemption decisions are *strategic complements* whenever $\sigma > 0$. Moreover, since asset prices are increasing in the macroeconomic state, the surplus from early redemption unambiguously decreases in $\theta$.

**Lemma 4.** *Investors' surplus from early redemption is increasing in the share of early redemptions and the fraction of informed funds:* $W_\lambda(\lambda, \theta; \sigma) \geq 0$ *and* $W_\sigma(\lambda, \theta; \sigma) > 0$*, and strictly decreasing in the macroeconomic state:* $W_\theta(\lambda, \theta; \sigma) < 0$*.*

For $\sigma > 0$, there exist realizations of the macroeconomic state such that patient investors consider it always dominant to redeem early, and states where they consider it dominant to stay invested in the fund until maturity. These regions are bounded (from above and below, respectively), and these bounds are implicitly defined by the following conditions[18]

$$W(\mu, \underline{\theta}_I; \sigma) = 0 \quad \text{and} \quad W(1, \overline{\theta}_I; \sigma) = 0$$

As for funds' information acquisition decision, patient investors must choose whether or not to redeem their claims early based on their signal of the macroeconomic state $\theta_i$ and their expectations regarding other investors' redemption decisions and the secondary market price. A strategy for investor $i$ is then defined as a mapping $\lambda_i : \Theta \to [0, 1]$ which specifies for each signal $\theta_i \in \Theta$ a probability with which a patient investor $i$ redeems his claim early. As before, we restrict attention to symmetric monotone strategies summarized by a critical signal $\theta^*_{I,\epsilon}$ whereby investors always redeem their claims early with probability one if $\theta_i < \theta^*_{I,\epsilon}$, and never redeem otherwise.

As for the information acquisition game characterized above, the law of large numbers and the assumption of uniformly distributed states and signals implies that the share of investors redeeming their claim at $t = 1$ given the symmetric monotone strategy around $\theta^*_{I,\epsilon}$ is equal to

$$\lambda(\theta^*_{I,\epsilon}, \theta) = \mu + (1 - \mu)G\left(\frac{\theta^*_{I,\epsilon} - \theta + \epsilon}{2\epsilon}\right)$$

For fixed values of $\sigma > 0$, the equilibrium threshold is determined by the signal realization $\theta_i$ that makes investors just indifferent between redeeming their claims at $t = 1$ or $t = 2$. That is, for a given $\sigma$, $\theta^*_{I,\epsilon}$ solves

$$\mathbf{E}[W(\lambda(\theta^*_{I,\epsilon}, \theta), \theta; \sigma)|\theta^*_{I,\epsilon}] = \frac{1}{2\epsilon}\int_{\theta^*_{I,\epsilon}-\epsilon}^{\theta^*_{I,\epsilon}+\epsilon} \big(F - D_2(\lambda(\theta^*_{I,\epsilon}, \theta), \theta; \sigma)\big)\, d\theta = 0$$

### 3.2 Information Acquisition and Redemption Equilibrium

The joint monotone equilibrium between funds and patient investors is characterized by critical values $\{\theta^{**}_{F,\epsilon}, \theta^{**}_{I,\epsilon}\}$ such that funds acquire information if and only if $\theta_j < \theta^{**}_{F,\epsilon}$ and patient investors redeem early if and only if $\theta_i < \theta^{**}_{I,\epsilon}$. The equilibrium thresholds $\theta^{**}_{F,\epsilon}$ and $\theta^{**}_{I,\epsilon}$ simultaneously solve the following two indifference conditions:

$$\mathbf{E}[\,S(\sigma, \lambda, \theta)|\,\theta^{**}_{F,\epsilon}] \;=\; \int_0^1 S\left(\sigma, \mu + (1-\mu)G\left(G^{-1}(\sigma) + \frac{\theta^{**}_{I,\epsilon} - \theta^{**}_{F,\epsilon}}{2\epsilon}\right), \theta(\theta^{**}_{F,\epsilon}, \sigma)\right) \mathrm{d}\sigma \;=\; \psi \tag{9}$$

[18] Note that since $W(\lambda, 0, \theta) < 0$ for all $\lambda \in [\mu, 1]$ and $\theta \in \Theta$, it must be that $\lim_{\sigma\to 0} \underline{\theta}_I = \overline{\theta}_I = \underline{\theta}$.

and

$$\mathbf{E}[W(\lambda,\sigma,\theta)|\,\theta^{**}_{I,\epsilon}] \;=\; \int_{\mu}^{1} W\left(\lambda, G\left(G^{-1}\left(\frac{\lambda-\mu}{1-\mu}\right)+\frac{\theta^{**}_{F,\epsilon}-\theta^{**}_{I,\epsilon}}{2\epsilon}\right), \theta(\theta^{**}_{I,\epsilon},\lambda)\right)\mathrm{d}\lambda \;=\; 0 \tag{10}$$

where $\theta(\theta^{**}_{I,\epsilon},\lambda)=\theta^{**}_{I,\epsilon}-\epsilon\left(2G^{-1}\left(\frac{\lambda-\mu}{1-\mu}\right)-1\right)$.

The thresholds defined by conditions (9) and (10) are bounded from above and from below. These bounds are determined by funds' and investors' expected surplus under "extreme beliefs." For funds, they correspond to realizations of the macroeconomic state such that the net expected surplus from information acquisition is equal to zero if funds believe no (all) patient investors redeem their claims early. Formally,

$$\theta^{*}_{F,\epsilon}(\mu):\;\mathbf{E}\left[S(\sigma(\theta^{*}_{F,\epsilon},\theta),\mu,\theta)\,\middle|\,\theta^{*}_{F,\epsilon}\right]=\psi \quad\text{and}\quad \theta^{*}_{F,\epsilon}(1):\;\mathbf{E}\left[S(\sigma(\theta^{*}_{F,\epsilon},\theta),1,\theta)\,\middle|\,\theta^{*}_{F,\epsilon}\right]=\psi$$

Similarly, for investors, they correspond to realizations of $\theta$ such that the expected surplus from early redemption is equal to zero if investors believe no (all) funds acquire information

$$\theta^{*}_{I,\epsilon}(0)=\underline{\theta},\quad\text{and}\quad \theta^{*}_{I,\epsilon}(1):\;\mathbf{E}[W(\lambda(\theta^{*}_{I,\epsilon},\theta),1,\theta)|\,\theta^{*}_{I,\epsilon}]=0$$

Notice that patient investors never redeem their claims early if they expect funds to refrain from information acquisition, regardless of the realization of the macroeconomic state. However, this never arises in equilibrium since it is always dominate for funds to acquire information for sufficiently small realizations of $\theta$ as there is always a positive mass $\mu$ of impatient investors that redeem their shares early. Broadly speaking, funds' decision to acquire information *induces* a coordination problem among investors and precipitates redemption runs whenever the macroeconomic state is sufficiently low.

**Proposition 2. (Joint Monotone Equilibrium)**

1. *There exists a unique equilibrium in monotone strategies where equilibrium thresholds are such that* $\theta^{**}_{I,\epsilon}\le\theta^{**}_{F,\epsilon}$ *with* $\theta^{**}_{I,\epsilon}\in(\underline{\theta},\theta^{*}_{I,\epsilon}(1)]$.
2. *There are no other equilibria in non-monotone strategies.*

An interesting implication of Proposition 2 is that redemption runs only arise in cases where market liquidity dries up due to adverse selection. However, secondary market freezes precipitated by funds' decision to acquire private information about their assets need not always result in investor runs. In other words, while funding illiquidity implies market illiquidity, the converse need not be true. Our model thereby complements the classical bank run literature, e.g. Diamond and Dybvig (1983) and Goldstein and Pauzner (2005), where banks selling assets to meet early withdrawals face an exogenous fire sale discount. More specifically, our model proposes a channel through which such fire sale discounts endogenously emerge due to funds' strategic incentives to acquire information about their assets, and shows that while market illiquidity is a necessary condition for runs to arise, it is not always sufficient.

### 3.3 Global Game Solution

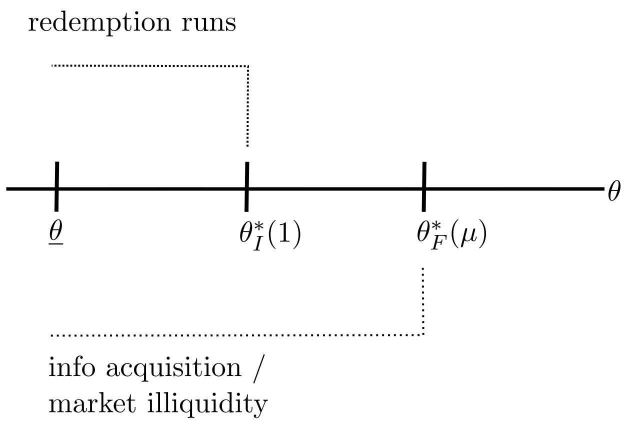
To facilitate the characterization of the equilibrium, we focus on the limiting case where agents' private signals about the macroeconomic state become arbitrarily precise, i.e. $\epsilon \to 0$. In this case, the equilibrium behavior of agents becomes degenerate around the realized state and the equilibrium outcome depends on the ordering of the bounds $\theta^*_{I,0}(1)$ and $\theta^*_{F,0}(\mu)$. Following the terminology of Goldstein (2005), we distinguish between a *weak dependence* and a *strong dependence* regime.

**Proposition 3.** *The equilibrium thresholds $\theta^{**}_{F,\epsilon}$ and $\theta^{**}_{I,\epsilon}$ for vanishing noise, $\epsilon \to 0$, are such that:*

1. *Weak dependence: $\theta^{**}_{I,0} \to \theta^*_{I,0}(1)$ and $\theta^{**}_{F,0} \to \theta^*_{F,0}(\mu)$ if and only if $\theta^*_{I,0}(1) < \theta^*_{F,0}(\mu)$.*
2. *Strong dependence: $\theta^{**}_{I,0} \to \theta^{**}_{F,0} \in [\theta^*_{F,0}(\mu), \theta^*_{I,0}(1)]$ if and only if $\theta^*_{F,0}(\mu) < \theta^*_{I,0}(1)$.*

In the *weak dependence* regime, funds' equilibrium threshold is at its lower bound, $\theta^*_{F,0}(\mu)$. Information acquisition triggers a run by patient investors who redeem their claims for realizations of the macroeconomic state below $\theta^*_{I,0}(1)$. However, for states $\theta \in (\theta^*_{I,0}(1), \theta^*_{F,0}(\mu))$, patient investors abstain from redemptions despite the asset price having collapsed due to the relatively low degree of macroeconomic uncertainty. Hence, while information acquisition by funds triggers redemptions in the weak dependence case, the coordination failure among patient investors does not exert an additional feedback on funds' decisions to acquire information (see Figure 3).

Figure 3: Weak Dependence Regime



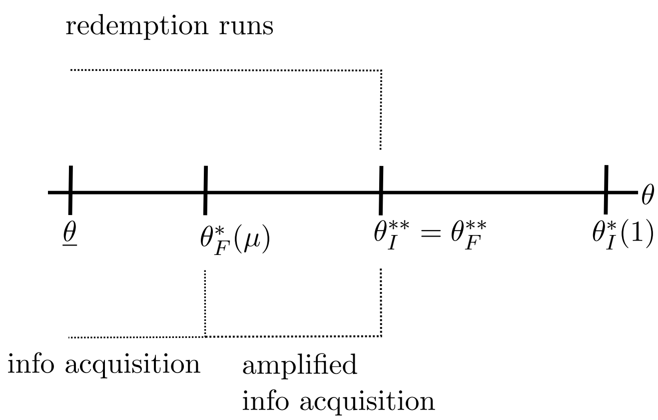

Things are different in the *strong dependence* regime, where funds' and investors' thresholds converge such that $\theta^{**}_{I,0} \to \theta^{**}_{F,0} \in [\theta^*_{F,0}(\mu), \theta^*_{I,0}(1)]$. In this case, redemption risk and market illiquidity coincide and reinforce each other. Market liquidity dry-ups due to adverse selection are now always accompanied by sudden redemptions by patient investors. The higher funding liquidity risk increases funds' incentives to acquire private information about their assets, raising the likelihood of asset market freezes caused by adverse selection. Thus, in the strong dependence case, the coordination failure among investors "spills over" and amplifies the coordination failure among funds, engendering a destabilizing feedback between redemption risk and market illiquidity (see Figure 4).

**Weak versus Strong Dependence.** Whether the weak or the strong dependence regime obtains depends crucially on the characteristics of funds' assets, such as assets' expected cash flow or the control rent. Proposition 3 implies that the *strong dependence* regime obtains if and only if $\theta^*_{F,0}(\mu) < \theta^*_{I,0}(1)$.

**Corollary 3.** *The economy is more susceptible to the strong dependence regime if: (i) the control rent $Q$ is small; and (ii) the price effect dominates the redemption effect and expected cash flows $F$ are large.*

If the marketability of the assets is high, the coordination problem among the funds is less severe. As a consequence, the threshold $\theta^*_{F,0}(\mu)$ is small. At the same time, if $Q$ is low, investors do not gain much if they roll over, implying that $\theta^*_{I,0}$ must be relatively large. This makes the economy more susceptible to a destabilizing feedback between market and funding illiquidity: if the degree coordination frictions within funds is muted, they react relatively more to the coordination problem among investors. Similarly, for a given share of early withdrawals, the negative price effect implies that larger cash flows make funds less prone to acquire information and thereby diminish the coordination problem among funds and lower the threshold $\theta^*_{F,0}$. As higher expected cash flows increase investors' incentives to withdraw early, whenever expected cash flows are sufficiently large, the strong dependence regime and the mutual amplification between market and funding liquidity obtains.

Figure 4: Strong Dependence Regime



### 3.4 Welfare

**Efficient Thresholds.** When defining the relevant welfare benchmark, we restrict attention to allocations that maximize aggregate utility from consumption. The problem faced by the social planner consists of choosing thresholds $\{\theta_F^{sp}, \theta_I^{sp}\} \in \Theta^2$ that maximize the expected payments made to investors.

**Definition 1.** *Given thresholds $\theta_F^{sp}$ and $\theta_I^{sp}$, and associated values $\sigma = \sigma(\theta_F^{sp}, \theta)$ and $\lambda = \lambda(\theta_I^{sp}, \theta)$, investors' expected utility from consumption is*

$$\mathcal{U}(\sigma, \lambda; \theta) = \mathbf{E}_0 \left[\alpha\lambda F + (1 - \alpha\lambda) D_2(\sigma, \lambda; \theta)\right]$$

Using funds' value functions (1) and (2), we can rewrite the welfare function as follows

$$\mathcal{U}(\sigma, \lambda; \theta) = \mathbf{E}_0 \left[F + Q - \alpha\lambda F \left(\sigma\pi(\kappa - 1) + (1 - \sigma\pi)\frac{Q}{p(\sigma, \theta)}\right)\right]$$

Even without taking into account the cost of information acquisition $\psi$, social welfare when $\sigma > 0$ is always strictly less than when $\sigma = 0$. This is because information acquisition leads funds with good assets to meet early redemptions using their liquidity lines rather than selling assets. While informed good funds avoid the early liquidation of their assets, and thereby do not forgo the additional control rent $Q$, Assumption 2 implies that the cost of liquidity lines $\kappa$ is sufficiently large such that aggregate consumption decreases as more funds become informed.[19] In other words, the unrealized gains from trade that result from informed good funds using their liquidity line always exceed the foregone control rent from selling assets. Information acquisition is thus unambiguously inefficient in this economy, as it serves only a private *rent-seeking* purpose.

In the absence of market liquidity risk, it is never socially (nor privately) optimal for patient investors to redeem their shares early since funds' expected equity value at maturity exceeds their *ex ante* book value, $F$. As funds not acquiring information always opt to meet redemptions by selling assets at their fair value, early redemptions in this case only lead to a destruction of funds' equity value due to the foregone control rent, $Q$.

**Proposition 4.** *The Pareto efficient thresholds are such that $\theta_F^{sp} = \underline{\theta}$ and $\theta_I^{sp} = \underline{\theta}$.*

**Inefficiency of the Market Equilibrium.** The inefficiency regarding funds' information decisions results from the collapse in market liquidity when funds acquire private information about the type of their asset. The externality distorting funds' incentives operates through changes in the market price, $p$. In particular, individual funds that acquire information and withhold good assets from the market do not internalize how their behavior affects other funds' option value from holding on to good assets.

The inefficiency regarding investors' redemption decisions arises because market liquidity risk induces a coordination failure among patient investors which leads to excessive

[19] As mentioned above, the cost of liquidity lines may stem from the fact that sponsors must forgo positive net present value investment opportunities in order to disburse liquidity to their funds at $t = 1$.

redemptions in equilibrium. This coordination failure operates through the residual equity value of funds. More specifically, individual investors that demand early redemption do not internalize how their redemption decision affects funds' residual equity value, and thereby the payment obtained by investors that redeem their shares at maturity. Importantly, the coordination failure distorting investors' redemption decisions is induced by adverse selection in secondary markets: i.e. funding liquidity risk is a *consequence* of funds' private rent-seeking incentives. Absent information acquisition by funds, inefficient redemption runs would never obtain in equilibrium.

# 4 Policy Implications

Our model allows to evaluate the efficacy of different policy measures aimed at minimizing the risk of market and funding liquidity dry-ups. We focus attention on three specific policies: (i) liquidity injections that reduce the cost of private liquidity lines, (ii) asset purchase programs that place a floor on the price at which assets trade, and (iii) outright purchases of debt securities. The focus on these three measures is motivated by the policies that central banks and policy makers implemented during the financial crisis 2007-09 in order to shore up asset and funding markets. For example, beginning in August 2007, the US Federal Reserve (Fed) adopted a number of policy measures to shore up wholesale funding markets including the ABCP market. At first, "conventional" liquidity injections were implemented *via* a lowering of central bank discount rates and short-term repurchase transactions.[20] These liquidity injections, however, failed to stop the precipitous fall in outstanding ABCP. They also did not prevent the run on money market funds that followed in the wake of Lehman Brother's bankruptcy. Subsequently, in the fall of 2008, the US Treasury Department announced that it would temporarily guarantee all assets held by money market funds. While this succeeded in stopping the run on money market funds, it failed to prop up the further collapsing ABCP market. This led the Fed to provide large amounts of non-recourse loans to commercial banks in order for them to purchase ABCP from money market funds. A few weeks later, the Fed also began purchasing commercial paper directly from issuers.[21] These policy measures specifically targeting the ABCP market were also accompanied by outright purchases of asset-backed securities.[22]

[20] In the euro area, the ECB injected 95 € billion into overnight lending markets on August 9, 2007. Over the following days, the Fed followed suit and injected $62 billion. On September 18, 2007 the Fed supplemented these measures by launching the Term Auction Facility (TAF) which conducted longer-term repurchase transactions totaling $100 billion (Kacperczyk and Schnabl, 2010).

[21] The non-recourse loans were administered by the Boston Fed's liquidity facility (AMLF) and purchased roughly $150 billion worth of commercial paper in its first two week of activity. Outright debt purchases were carried out by the Commercial Paper Funding Facility (CPFF) which purchased over $300 billion worth of commercial paper. Through these two facilities, the Fed ended up holding about 25% of outstanding commercial paper by the end of 2008 (Kacperczyk and Schnabl, 2010).

[22] The Fed extended non-recourse loans to buyers of both newly issued ABSs and legacy MBSs through its Term Asset-Backed Securities Loan facility (TALF) .

**Liquidity Injections.** We begin by assessing the effect of liquidity injections, e.g. lowering interest rates that reduce the cost of funds' liquidity lines. Maintaining the bounds on $\kappa$ implied by Assumption 2, such a policy has an ambiguous effect on market and funding liquidity risk. Liquidity injections have a (direct) negative effect on market liquidity insofar as they decrease funds' opportunity cost of tapping their liquidity lines. This increases funds' incentives to acquire private information about their assets, thereby amplifying the adverse selection problem in secondary markets. The resulting fall in asset prices increases investors' incentives to redeem their claims early. Concomitantly, however, liquidity injections that lower the cost of liquidity lines increase the residual equity value of informed funds holding good assets, and thus decrease investors' incentives to redeem early.[23] This second channel implies a (indirect) positive effect on market liquidity, as fewer early redemptions lower funds' surplus from acquiring information.

**Corollary 4.** *Liquidity injections that lower the cost of liquidity lines $\kappa$ can either increase or decrease market and funding liquidity risk: $\frac{d\theta^{**}_{F,\epsilon}}{d\kappa} \gtrless 0$ and $\frac{d\theta^{**}_{I,\epsilon}}{d\kappa} \gtrless 0$.*

**Asset Purchases.** Next, we consider the effect of a government commitment to purchase assets at a reservation price $q(\theta) > \theta X(\theta)$ for all $\theta \in \Theta$. By placing a floor on asset prices, this policy reduces funds' incentives to acquire information by lowering the option value from withholding good assets from the market. It also reduces investors' incentives to redeem their claims early by raising funds' residual equity value. Even though the floor on asset prices reduces the private surplus from information acquisition, it does not fully eliminate market liquidity risk since funds find it strictly dominant to acquire information for sufficiently small values of $\theta$.[24] Thus, any price guarantee $q(\theta) > \theta X(\theta)$ requires the government to buy bad assets at a price above their fundamental value in some states.

**Corollary 5.** *Asset price guarantees that place a floor on $p$ decrease market liquidity risk and decrease funding liquidity risk. The expected cost from purchasing assets at price $q(\theta) > \theta X(\theta)$ is equal to:*

$$C^{\mathcal{AP}} = (1-\pi) \int_{\underline{\theta}}^{\max\{\underline{\theta}^q_F(\mu), \theta^q_{I,0}\}} \alpha \left(\mu + (1-\mu)\mathbb{1}_{\theta < \theta^q_{I,0}}\right) F \left(1 - \frac{\theta X(\theta)}{q(\theta)}\right) \mathrm{d}\theta \ > 0$$

[23] The destabilizing effect of liquidity lines has also been pointed out by He and Xiong (2012). In their dynamic debt run model, liquidity lines amplify creditors' incentives to run when asset volatility is high because banks' fundamentals deteriorate while they obtain funds through their liquidity lines. This effect does not arise in our static framework. Instead, cheaper liquidity lines amplify funding withdrawals due to their effect on funds' information acquisition incentives, and thereby the market value of funds' assets.

[24] Formally, given some reservation price $q(\theta) > \theta X(\theta)$, the lower dominance region of funds' information acquisition game is given by

$$\underline{\theta}^q_F(\mu): \quad \int_0^1 \pi \left( \frac{X(\underline{\theta}^q_F(\mu)) + Q}{\max\{q(\underline{\theta}^q_F(\mu)), p(\sigma, \underline{\theta}^q_F(\mu))\}} - \kappa \right) \alpha \mu F \mathrm{d}\sigma = \psi$$

**Outright Debt Purchases.** Finally, we consider the effect of outright purchases of debt securities, such as those conducted by the Federal Reserve under the CPFF.[25] In the context of our model, this can be thought of as lowering the fraction of redeemable claims. By committing to purchase claims *at par* at $t = 1$, the government effectively protects funds from funding liquidity risk. In so doing, it lowers funds' incentives to acquire information. Debt purchases also reduce investors' incentives to redeem early by raising market liquidity, implying that the government ultimately only needs to purchases claims held by impatient investors.

**Corollary 6.** *Debt purchases that lower the fraction of redeemable claims* $\alpha$ *decrease market and funding liquidity risk:* $\frac{d\theta^{**}_{F,\epsilon}}{d\alpha} > 0$ *and* $\frac{d\theta^{**}_{I,\epsilon}}{d\alpha} > 0$. *A commitment to buy all redeemable shares implements the efficient allocation.*

If its purchases are unbounded, the government can completely eliminate market liquidity risk by ensuring that no fund acquires information in equilibrium. Debt purchases can therefore be used to implement the efficient allocation described above. Importantly, this policy does not require the government to purchase the totality of funds' outstanding claims, as the absence of market liquidity risk reduces investors' incentives to redeem early. Debt purchases thus mostly operate *via* an announcement effect that reduces the coordination failure among funds and investors. This arises because the commitment to purchase claims *at par* if they are not rolled over effectively eliminates the maturity mismatch on funds' balance sheet. If claims held by the government are treated the same as those held by private investors, such a policy also never requires the government to incur a loss. While the government has to step in and absorb outstanding claims held by impatient investors at $t = 1$, it is always paid back in full at $t = 2$ when assets mature.

## 5 Conclusion

This paper proposes a model of (shadow) bank runs based on a feedback between information acquisition and market liquidity. The value of information arises from the option of holding on to good assets by covering redemptions using private liquidity lines rather than selling assets. This generates an adverse selection problem in secondary markets which reduces market liquidity. Falling prices, in turn, erode shadow banks' equity value and raise investors' incentives to redeem their claims. This can amplify funding withdrawals and cause market and funding illiquidity to become mutually reinforcing.

An implication of our paper is that shadow banks' access to private liquidity lines may have contributed to the fragility of the shadow banking sector during the 2007-09 crisis. Another possible factor behind the run on off-balance sheet vehicles at the time may have been investors' fears about the soundness of vehicles' sponsors, and thus the credibility of their guarantees. Our model shows that, even in the absence of these commitment problems, liquidity lines may have been inherently destabilizing by giving

[25] Under the CPFF (Commercial Paper Funding Facility) the Federal Reserve provided funding to specially created limited liability company that then bought highly rated unsecured commercial paper or ABCP with short maturities, e.g. three-month, directly from issuers.

shadow banks incentives to acquire private information about the quality of assets on their balance sheet.

Broadly speaking, our paper builds on Gorton (2010)'s idea that the run on the shadow banking sector during the 2007-09 financial crisis was caused by a sudden regime switch whereby "informationally insensitive" securities suddenly became "informationally sensitive." A key contribution of our model is to show that such regimes can be sustained by self-fulfilling beliefs about shadow banks' information acquisition behavior. It thereby provides a new framework studying the interaction between information acquisition, market liquidity and funding risk that helps explain the fragility of the shadow banking sector. Although our modelling assumptions make us inclined to think of the funds in our model as shadow banking arrangements, the model also sheds some light on more general market-based financial intermediation where fluctuations in the value of intermediaries' assets and liabilities are closely tied to changes in market prices. From this perspective, it highlights the fragility of financial institutions holding complex and opaque securities that rely on a mix of market-based liquidity and third-party support to manage their funding risk.

# Appendix

## A1 Proofs

*Proof of Lemma 1.* Before proving the lemma, we show that Assumptions 1 - 3 are not mutually exclusive, i.e. there exist non-empty intervals $\Pi(\kappa)$ and $\mathcal{Q}(\kappa)$ such that any $\pi \in \Pi(\kappa)$ and $Q \in \mathcal{Q}(\kappa)$ satisfy Assumptions 1 and 2. To see this, observe first that we can use Assumption 2 to solve for a largest lower and a smallest upper bound on $\Theta$:

$$\underline{\theta} = \frac{\pi(F+Q)}{\kappa F - (1-\pi)(F+Q)} \quad \text{and} \quad \overline{\theta} = \frac{F - \pi(\kappa F - Q)}{(1-\pi)(\kappa F - Q)}$$

$\Pi(\kappa)$ and $\mathcal{Q}(\kappa)$ must be such that $0 \leq \underline{\theta} < \overline{\theta} \leq 1$. Note first that $\underline{\theta} \geq 0$ and $\overline{\theta} \leq 1$ require $Q \leq (\kappa - 1)F$. Second,

$$\underline{\theta} < \overline{\theta} \;\Leftrightarrow\; (\kappa-1)F((\kappa+1)\pi - 1) < Q((\kappa+1)\pi - 1)$$

Hence, whenever $\pi > (\kappa+1)^{-1}$, the latter implies $Q \geq (\kappa-1)F$ in contradiction to $\underline{\theta} \geq 0$ and $\overline{\theta} \leq 1$. Therefore, $\pi < (\kappa+1)^{-1}$. Finally, substitung the explicit form for $X(\theta) = (\pi + (1-\pi)\theta)^{-1}F$ and the above expression for $\underline{\theta}$ into Assumption 1 and solving for $Q$ yields $Q > \frac{(\kappa-1)F}{\kappa+1}$. Summarizing, any combination of $\pi$ and $Q$ from $\Pi(\kappa) = (0, (\kappa+1)^{-1})$ and $\mathcal{Q}(\kappa) = (\frac{(\kappa-1)F}{\kappa+1}, (\kappa-1)F)$ satisfies Assumptions 1 and 2.

To prove the Lemma, note that for all $\Omega_j$, we have that $\ell_j^* = \alpha\mu F$ regardless of whether funds use assets sales or the liquidity line to meet redemptions. This follows because for all $\Omega_j$ and $\theta$ we have that $\frac{d}{d\ell_j}\mathbf{E}[V_{\Omega_j}^{LL}(\ell_j)|\theta] < 0$ since $\kappa > 1$ and $\frac{d}{d\ell_j}\mathbf{E}[V_{\Omega_j}^{AS}(\ell_j)|\theta] < 0$ due to Assumption 1.

For informed funds such that $\Omega_j \in \{g, b\}$ notice that $\mathbf{E}[V_{\Omega_j}^{LL}(\alpha\mu F)|\theta] \gtrless \mathbf{E}[V_{\Omega_j}^{AS}(\alpha\mu F)|\theta]$ implies

$$\max\left\{(\mathbf{E}[\tilde{X}(\theta)|\Omega_j,\theta] + Q) - \alpha\mu F\kappa, 0\right\} \gtrless (\mathbf{E}[\tilde{X}(\theta)|\Omega_j,\theta] + Q)\max\left\{1 - \frac{\alpha\mu F}{p}, 0\right\}, \quad \forall\theta \in \Theta$$

Similarly, for uninformed funds such that $\Omega_j \in \{n\}$ notice that $\mathbf{E}[V_n^{LL}(\alpha\mu F)|\theta] \gtrless \mathbf{E}[V_n^{AS}(\alpha\mu F)|\theta]$ implies

$$\mathbf{E}_0\left[\max\left\{(\tilde{X}(\theta) + Q) - \alpha\mu F\kappa, 0\right\}\bigg|\theta\right] \gtrless \mathbf{E}_0\left[(\tilde{X}(\theta) + Q)\max\left\{1 - \frac{\alpha\mu F}{p}, 0\right\}\bigg|\theta\right], \quad \forall\theta \in \Theta$$

From Assumption 2, it follows that informed funds holding a good asset prefer the liquidity line while informed funds holding a bad asset and uninformed funds prefer asset sales for all $\alpha\mu F < \min\{(\theta X(\theta) + Q)/\kappa, \theta X(\theta)\}$. Substituting the lower bound for $Q$ implied by Assumption 1, this inequality implies that $\alpha\mu < 1/\kappa$, which must hold due to Assumption 3. □

*Proof of Proposition 1.* We prove the proposition by showing that funds' information acquisition game satisfies all the properties of Proposition 2.2 in Morris and Shin (2003).

The required properties are:

1. *Action Monotonicity:* $S(\sigma, \theta; \lambda)$ is increasing in $\sigma$.
2. *State Monotonicity:* $S(\sigma, \theta; \lambda)$ is decreasing in $\theta$.
3. *Continuity:* $S(\sigma, \theta; \lambda)$ is continuous in both $\sigma$ and $\theta$.
4. *Finite Expectations of Signals:* The distribution of $\epsilon_j$ is integrable.
5. *Uniform Limit Dominance:* There exists $\underline{\theta}_F \in \Theta$, $\overline{\theta}_F \in \Theta$ and such that: (i) $S(\sigma, \theta; \lambda) > \psi$ for all $\sigma \in [0, 1]$ and $\theta \leq \underline{\theta}_F$; and (ii) $S(\sigma, \theta; \lambda) < \psi$ for all $\sigma \in [0, 1]$ and $\theta \geq \underline{\theta}$.
6. *Strict Laplacian State Monotonicity:* There exists a unique $\theta_F^*$ solving $\int_0^1 S(\sigma, \theta_F^*; \lambda)d\sigma = \psi$.

Properties 1 and 2 are implied by Lemma 3. Properties 3 and 4 follow from the definition of the surplus function and the uniform distribution of signals, respectively. Property 5 is implied by Assumption 4. Finally, Property 6 follows from the fact that $\int_0^1 S(\sigma, \underline{\theta}_F; \lambda)d\sigma > \psi$, $\int_0^1 S(\sigma, \overline{\theta}_F; \lambda)d\sigma < \psi$ and

$\int_0^1 S_\theta(\sigma,\theta;\lambda)d\theta < 0$ for all $\lambda > 0$. It follows that there exists a unique monotone equilibrium and that there are no other equilibria in non-monotone strategies. □

*Proof of Corollary 1.* Rewrite the equilibrium condition (7) as

$$A(\theta^*_{F,\epsilon}, F, Q, \lambda) \equiv \alpha\lambda\pi F \int_0^1 \left( \frac{X(\theta(\theta^*_{F,\epsilon},\sigma)) + Q}{p(\sigma, \theta(\theta^*_{F,\epsilon},\sigma))} - \kappa \right) \mathrm{d}\sigma - \psi = 0$$

From the proof of Proposition 1, $A_{\theta^*_{F,\epsilon}} < 0$. Thus, by the implicit function theorem, for $\tau \in \{F, Q, \lambda\}$,

$$\operatorname{sign}\left\{ \frac{\mathrm{d}\theta^*_{F,\epsilon}}{\mathrm{d}\tau} \right\} = \operatorname{sign}\{A_\tau(\theta^*_{F,\epsilon}, F, Q, \lambda)\}$$

Thus, $A_Q(\theta^*_{F,\epsilon}, F, Q, \tau) = \alpha\lambda\pi F \int_0^1 \left( \frac{1}{p(\sigma,\theta(\theta^*_{F,\epsilon},\sigma))} \right) \mathrm{d}\sigma > 0$, implying that increases in $Q$ increase the threshold.

However,

$$A_F(\theta^*_{F,\epsilon}, F, Q, \tau) = \underbrace{\alpha\lambda\pi \int_0^1 \left( \frac{X(\theta(\theta^*_{F,\epsilon},\sigma)) + Q}{p(\sigma, \theta(\theta^*_{F,\epsilon},\sigma)} - \kappa \right) \mathrm{d}\sigma}_{\text{redemption effect } (+)} \underbrace{- \alpha\lambda\pi \int_0^1 \left( \frac{Q}{p(\sigma, \theta(\theta^*_{F,\epsilon},\sigma))} \right) \mathrm{d}\sigma}_{\text{price effect } (-)} \gtreqless 0$$

The latter can be rewritten as

$$A_F(\theta^*_{F,\epsilon}, F, Q, \tau) = \alpha\lambda\pi \int_0^1 \left( \frac{X(\theta(\theta^*_{F,\epsilon},\sigma))}{p(\sigma, \theta(\theta^*_{F,\epsilon},\sigma))} - \kappa \right) \mathrm{d}\sigma \gtreqless 0$$

which is negative, i.e. the negative price effect dominates, if $\kappa$ becomes sufficiently large. □

*Proof of Corollary 2.* By the same argument as in the previous proof, the threshold $\theta^*_{F,\epsilon}$ is strictly increasing in the share of withdrawals since $A_\lambda(\theta^*_{F,\epsilon}, F, Q, \lambda) = \alpha\pi F \int_0^1 \left( \frac{X(\theta(\theta^*_{F,\epsilon},\sigma)) + Q}{p(\sigma,\theta(\theta^*_{F,\epsilon},\sigma))} - \kappa \right) \mathrm{d}\sigma > 0$. □

*Proof of Lemma 4.* We begin by showing that $W_\lambda(\lambda, \theta; \sigma) > 0$ for all $\sigma > 0$. Differentiating the surplus from early redemption with respect to $\lambda$ yields

$$W_\lambda(\lambda, \theta; \sigma) \propto \mathbf{E}[(F + Q)(F - p(\sigma, \theta)) - \sigma\pi(X + Q - \kappa p(\sigma, \theta))F|\theta_i]$$

Notice that from Assumption 2 and the fact that $p(\sigma, \theta) \geq \theta X(\theta)$, it must be that $\kappa p(\sigma, \theta) \geq (F + Q)$. Hence, we need to show that

$$\mathbf{E}[(F + Q)(F - p(\sigma, \theta)) - \sigma\pi(X - F)F|\theta_i] \geq 0$$

Using the definition of $p(\sigma, \theta)$ and $F$, this condition can be rewritten as follows

$$\mathbf{E}[(F + Q)(\pi - \tau(\sigma))(1 - \theta)X(\theta) - \sigma\pi(1 - \pi)(1 - \theta)X(\theta)F|\theta_i] \geq 0$$

Substituting in for $\tau(\sigma)$ and rearranging, this inequality implies

$$\frac{F + Q}{1 - \pi\sigma} \geq F$$

which is always satisfied and holds strictly for all $\sigma > 0$. Next, we show that $W_\sigma(\lambda, \theta; \sigma) > 0$. Differentiating the surplus from early redemption with respect to $\sigma$ yields

$$W_\sigma(\lambda, \theta; \sigma) \propto \mathbf{E}\left[ -((F + Q) - \sigma\pi(X(\theta) + Q)) \frac{p_\sigma(\sigma, \theta)}{p(\sigma, \theta)} - \pi(X(\theta) + Q - \kappa p(\sigma, \theta)) \middle| \theta_i \right]$$

Substituting in for $F$, this condition can be rewritten as

$$W_\sigma(\lambda,\theta;\sigma) \propto \mathbf{E}\left[-(\pi(1-\sigma)X(\theta)+(1-\pi)\theta X(\theta)+(1-\sigma\pi)Q)\frac{p_\sigma(\sigma,\theta)}{p(\sigma,\theta)} - \pi(X(\theta)+Q-\kappa p(\sigma,\theta))\bigg|\theta_i\right]$$

Using the definition of $p(\sigma,\theta)$, this expression can again be rewritten as

$$W_\sigma(\lambda,\theta;\sigma) \propto \mathbf{E}\left[\frac{\pi(1-\pi)}{1-\pi\sigma}(1-\theta)X(\theta)-(1-\sigma\pi)Q\frac{p_\sigma(\sigma,\theta)}{p(\sigma,\theta)} - \pi(X(\theta)+Q-\kappa p(\sigma,\theta))\bigg|\theta_i\right]$$

As before, notice that we must have $\kappa p(\sigma,\theta) \geq (F+Q)$. We therefore need to show that

$$\mathbf{E}\left[\frac{\pi(1-\pi)}{1-\pi\sigma}(1-\theta)X(\theta)-(1-\sigma\pi)Q\frac{p_\sigma(\sigma,\theta)}{p(\sigma,\theta)} - \pi(1-\pi)(1-\theta)X(\theta)\bigg|\theta_i\right] > 0$$

Simplifying this condition, we obtain the following inequality

$$\mathbf{E}\left[\pi(1-\pi)(1-\theta)X(\theta)\left(\frac{1}{1-\pi\sigma}-1\right)-(1-\sigma\pi)Q\frac{p_\sigma(\sigma,\theta)}{p(\sigma,\theta)}\bigg|\theta_i\right] > 0$$

which is always satisfied since $p_\sigma(\sigma,\theta) < 0$. □

*Proof of Proposition 2. (i) Unique monotone equilibrium:* We show that there exists a unique montone equilibrium where thresholds are such that $\theta_I^{**} \leq \theta_F^{**}$.

Suppose that funds and investors use monotone strategies around $\theta_F^{**}$ and $\theta_I^{**}$. From the proof of Proposition 1, we know that for a fixed value of $\theta_{I,\epsilon}^{**}$ (and hence a fixed value of $\lambda \geq \mu$) there exists a unique threshold $\theta_{F,\epsilon}^{**}(\theta_{I,\epsilon}^{**})$ that solves condition (9). Note that the optimal information acquisition threshold solving equation (9) is weakly increasing in $\theta_{I,\epsilon}^{**}$ with slope given by

$$\frac{\mathrm{d}\theta_{F,\epsilon}^{**}}{\mathrm{d}\theta_{I,\epsilon}^{**}} = \frac{(1-\mu)\int_0^1 S_\lambda(\cdot)\,d\sigma}{(1-\mu)\int_0^1 S_\lambda(\cdot)\,d\sigma - 2\epsilon\int_0^1 S_\theta(\cdot)d\sigma} < 1$$

where the condition follows from application of the implicit function theorem and the fact that $S_\lambda(\cdot) > 0$ and $S_\theta(\cdot) < 0$.

Substituting condition (9) into condition (10) yields

$$H(\theta_{I,\epsilon}^{**}) \equiv \int_\mu^1 W\left(\lambda, G\left(\frac{\lambda-\mu}{1-\mu}+\frac{\theta_{F,\epsilon}^{**}(\theta_{I,\epsilon}^{**})-\theta_{I,\epsilon}^{**}}{2\epsilon}\right), \theta(\theta_{I,\epsilon}^{**},\lambda)\right)d\lambda$$

Notice that

$$H(\theta_{I,\epsilon}^{*}(0)) = \int_\mu^1 W\left(\lambda, G\left(\frac{\lambda-\mu}{1-\mu}+\frac{\theta_{F,\epsilon}^{**}(\theta_{I,\epsilon}^{*}(0))-\theta_{I,\epsilon}^{*}(0)}{2\epsilon}\right), \theta(\theta_{I,\epsilon}^{*}(0),\lambda)\right)d\lambda > 0$$

since $\theta_{F,\epsilon}^{*}(\mu) > \underline{\theta}$ for all $\mu > 0$, $\underline{\theta}_I(0) = \underline{\theta}$ and $W_\sigma(\cdot) > 0$. Furthermore, we also have that

$$H(\theta_{I,\epsilon}^{*}(1)) = \int_\mu^1 W\left(\lambda, G\left(\frac{\lambda-\mu}{1-\mu}+\frac{\theta_{F,\epsilon}^{**}(\theta_{I,\epsilon}^{*}(1))-\theta_{I,\epsilon}^{*}(1)}{2\epsilon}\right), \theta(\theta_{I,\epsilon}^{*}(1),\lambda)\right)d\lambda \leq 0$$

where the condition follows from the fact that $G(\cdot) \in [0,1]$ and $W_\sigma(\cdot) > 0$. Hence, by application of the intermediate value theorem, the function $H(\theta_{I,\epsilon}^{**})$ must intersect the $x$-axis at least once for values of $\theta_{I,\epsilon}^{**} \in (\theta_{F,\epsilon}^{*}(\mu), \theta_{I,\epsilon}^{*}(1)]$. Since $\frac{\mathrm{d}\theta_{F,\epsilon}^{**}}{\mathrm{d}\theta_{I,\epsilon}^{**}} < 1$, it follows that

$$H'(\theta_{I,\epsilon}^{**}) = \frac{1}{2\epsilon}\int_\mu^1 W_\sigma(\cdot)\left(\frac{\mathrm{d}\theta_{F,\epsilon}^{**}}{\mathrm{d}\theta_{I,\epsilon}^{**}}-1\right)d\lambda + \int_\mu^1 W_\theta(\cdot)d\lambda < 0$$

since $W_\theta(\cdot) < 0$, implying that there exists a unique value $\theta_{I,\epsilon}^{**} \in (\theta_{F,\epsilon}^{*}(\mu), \theta_{I,\epsilon}^{*}(1)]$ that solves $H(\theta_{I,\epsilon}^{**}) = 0$.

Finally, we show that $\theta^{**}_{I,\epsilon} \leq \theta^{**}_{F,\epsilon}$. By application of the implicit function theorem we have that

$$\frac{d\theta^{**}_{I,\epsilon}}{d\theta^{**}_{F,\epsilon}} = \frac{\int_\mu^1 W_\sigma(\cdot)d\lambda}{\int_\mu^1 W_\sigma(\cdot)d\lambda - 2\epsilon\int_\mu^1 W_\theta(\cdot)} < 1$$

Since $\theta^*_{I,\epsilon}(0) = \underline{\theta}$ and $\theta^*_{F,\epsilon}(\mu) > \underline{\theta}$, the unique fixed point must be such that $\theta^{**}_{I,\epsilon} \leq \theta^{**}_{F,\epsilon}$.

*(ii) No other non-monotone equilibria:* The argument closely follows the argument in Goldstein (2005). Towards a contradiction, suppose that an alternative non-monotone equilibrium exists where funds acquire information for some signals $\theta_j > \theta^{**}_{F,\epsilon}$ and where patient investors redeem early for some signals $\theta_i > \theta^{**}_{I,\epsilon}$. By the existence of dominance regions there exist bounds $\theta^N_F$ and $\theta^N_I$ such that funds do not acquire information for $\theta_j > \theta^N_F$ and investors never redeem for $\theta_i > \theta^N_I$. Let $\sigma_N$ and $\lambda_N$ denote the fractions of funds who acquire information and investors who run in this non-monotone equilibrium. They satisfy

$$\sigma_N(\theta) \leq G\left(\frac{\theta^N_F - \theta + \epsilon}{2\epsilon}\right) \quad \text{and} \quad \lambda_N(\theta) \leq \mu + (1-\mu)G\left(\frac{\theta^N_I - \theta + \epsilon}{2\epsilon}\right)$$

A fund whose type is just $\theta_j = \theta^N_F$ must be indifferent between acquiring and not acquiring information:

$$\frac{1}{2\epsilon}\int_{\theta^N_F-\epsilon}^{\theta^N_F+\epsilon} S(\sigma_N(\theta), \lambda_N(\theta), \theta)\mathrm{d}\theta - \quad = 0$$

Since the surplus from information acquisition is increasing in $\sigma_N$ and $\lambda_N$, it follows that

$$\frac{1}{2\epsilon}\int_{\theta^N_F-\epsilon}^{\theta^N_F+\epsilon} S\left(G\left(\frac{\theta^N_F - \theta + \epsilon}{2\epsilon}\right), G\left(\frac{\theta^N_I - \theta + \epsilon}{2\epsilon}\right), \theta\right)\mathrm{d}\theta - \psi \geq 0$$

Changing variables of integration yields,

$$\int_0^1 S\left(\sigma, G\left(G^{-1}(\sigma) + \frac{\theta^N_I - \theta^N_F}{2\epsilon}\right), \theta(\theta^N_F, \sigma)\right)\mathrm{d}\sigma - \psi \geq 0$$

Comparing this to equation (9) in the text implies

$$\int_0^1 \left[S\left(\sigma, G\left(G^{-1}(\sigma) + \frac{\theta^N_I - \theta^N_F}{2\epsilon}\right), \theta(\theta^N_F, \sigma)\right) - S\left(\sigma, \mu + (1-\mu)G\left(G^{-1}(\sigma) + \frac{\theta^{**}_{I,\epsilon} - \theta^{**}_{F,\epsilon}}{2\epsilon}\right), \theta(\theta^{**}_{F,\epsilon}, \sigma)\right)\right]\mathrm{d}\sigma \geq 0$$

But since $\theta^N_F > \theta^{**}_{,F\epsilon}$ (by assumption) and the surplus function is decreasing in $\theta$ the latter can only hold if

$$\theta^N_I - \theta^N_F \;>\; \theta^{**}_{I,\epsilon} - \theta^{**}_{F,\epsilon} \tag{A1}$$

Repeating this line of reasoning for the expected surplus from early redemption implies

$$\theta^N_F - \theta^N_I \;>\; \theta^{**}_{F,\epsilon} - \theta^{**}_{I,\epsilon} \tag{A2}$$

(A2) obviously contradicts (A1), implying that funds will never acquire information at types above $\theta^{**}_{F,\epsilon}$ and investors will never redeem early at types above $\theta^{**}_{I,\epsilon}$. A symmetric argument establishes that agents will not switch at types below $\theta^{**}_{F,\epsilon}$ and $\theta^{**}_{I,\epsilon}$. Thus, a non-monotone equilibrium cannot exist. □

*Proof of Proposition 3.* We first prove that $\theta^{**}_{F,\epsilon} \overset{\epsilon\to 0}{\to} \theta^*_{F,0}(\mu)$ and $\theta^{**}_{I,\epsilon} \overset{\epsilon\to 0}{\to} \theta^*_{I,0}(1)$ if and only if $\theta^*_{I,0}(1) < \theta^*_{F,0}(\mu)$. Sufficiency follows by noting that when $\theta^{**}_{I,\epsilon} < \theta^{**}_{F,\epsilon}$, condition (9) implies

$$\lim_{\epsilon\to 0}\mathbf{E}[\,S(\sigma,\lambda,\theta)|\,\theta^{**}_{F,\epsilon}] = \mathbf{E}[\,S(\sigma,\mu,\theta)|\,\theta^{**}_{F,\epsilon}] \quad \Leftrightarrow \quad \lim_{\epsilon\to 0}\theta^{**}_{F,\epsilon} = \theta^*_{F,0}(\mu) \tag{A3}$$

Similarly, condition (10) implies

$$\lim_{\epsilon\to 0}\mathbf{E}[\,W(\lambda,\sigma,\theta)|\,\theta^{**}_{I,\epsilon}] = \mathbf{E}[\,W(\lambda,1,\theta)|\,\theta^{**}_{I,\epsilon}] \quad \Leftrightarrow \quad \lim_{\epsilon\to 0}\theta^{**}_{I,\epsilon} = \theta^*_{I,0}(1) \tag{A4}$$

Hence, we must have $\theta^*_{I,0}(1) < \theta^*_{F,0}(\mu)$. Necessity then follows from observing that $\theta^{**}_{I,\epsilon} \not< \theta^{**}_{F,\epsilon}$ if $\theta^*_{I,0}(1) \geq \theta^*_{F,0}(\mu)$.

Second, we show that $\theta^{**}_{I,\epsilon} \overset{\epsilon \to 0}{\to} \theta^{**}_{F,0}$ if and only if $\theta^*_{I,0}(1) \geq \theta^*_{F,0}(\mu)$. From above, by contraposition, $\theta^*_{I,0}(1) \geq \theta^*_{F,0}(\mu)$ if and only if $\theta^{**}_{I,0} \geq \theta^{**}_{F,0}$ as $\epsilon \to 0$. But since Proposition 2 implies that $\theta^{**}_{I,\epsilon} \not> \theta^{**}_{F,\epsilon}$, it must be that $\theta^{**}_{I,0} = \theta^{**}_{F,0}$.

Finally, we show that indeed $\theta^{**}_{F,\epsilon} \overset{\epsilon \to 0}{\to} \theta^{**}_{F,0} \in [\theta^*_{F,0}(\mu), \theta^*_{I,0}(1)]$ if $\theta^*_{I,0}(1) \geq \theta^*_{F,0}(\mu)$. From condition (9), we have that

$$\lim_{\epsilon \to 0} \mathbf{E}[\, S(\sigma, \lambda, \theta) | \, \theta^{**}_{F,\epsilon}] \geq \mathbf{E}[\, S(\sigma, \mu, \theta) | \, \theta^{**}_{F,\epsilon}] \quad \Leftrightarrow \quad \lim_{\epsilon \to 0} \theta^{**}_{F,\epsilon} \geq \theta^*_{F,0}(\mu)$$

where the inequality follows from $\lim_{\epsilon \to 0} G\left(G^{-1}(\sigma) + \frac{\theta^{**}_{I,\epsilon} - \theta^{**}_{F,\epsilon}}{2\epsilon}\right) \in [0,1]$, $S_\lambda(\cdot) > 0$ and $S_\theta(\cdot) < 0$. Similarly, using condition (10), we have

$$\lim_{\epsilon \to 0} \mathbf{E}[\, W(\lambda, \sigma, \theta) | \, \theta^{**}_{I,\epsilon}] \leq \lim_{\epsilon \to 0} \mathbf{E}[\, W(\lambda, 1, \theta) | \, \theta^{**}_{I,\epsilon}] \quad \Leftrightarrow \quad \lim_{\epsilon \to 0} \theta^{**}_{I,\epsilon} \leq \theta^*_{I,0}(1)$$

where the inequality follows from $\lim_{\epsilon \to 0} G\left(G^{-1}\left(\frac{\lambda - \mu}{1-\mu}\right) + \frac{\theta^{**}_{F,\epsilon} - \theta^{**}_{I,\epsilon}}{2\epsilon}\right) \in [0,1]$, $W_\sigma(\cdot) > 0$ and $W_\theta(\cdot) < 0$. Since $\theta^{**}_{I,\epsilon} \to \theta^{**}_{F,\epsilon}$ as $\epsilon \to 0$, it follows that $\theta^{**}_{F,\epsilon} \to \theta^{**}_{F,0} \in [\theta^*_{F,0}(\mu), \theta^*_{I,0}(1)]$. Clearly, this interval is empty if $\theta^*_{I,0}(1) < \theta^*_{F,0}(\mu)$.

□

*Proof of Corollary 3.* By Corollary 1, the threshold $\theta^*_{F,0}(\mu)$ is strictly increasing in $Q$ and it is strictly decreasing in $F$ if the negative price effect dominates.

Using equation (8), $\theta^*_{I,\epsilon}$ is given by the solution to

$$B(\theta^*_{I,\epsilon}, F, Q, 1) \equiv F - \int_\mu^1 D_2(\lambda, \theta(\theta^*_{I,\epsilon}, \lambda); 1) \mathrm{d}\lambda = 0$$

As this is strictly decreasing in $\theta^*_{I,\epsilon}$, we have, for $\tau \in \{F, Q\}$:

$$\mathrm{sign}\left\{ \frac{\mathrm{d}\theta^*_{I,\epsilon}(1)}{\mathrm{d}\tau} \right\} = \mathrm{sign}\{ B_\tau(\theta^*_{I,\epsilon}, F, Q, 1)\}$$

Observe that

$$B_Q(\theta^*_{F,\epsilon}, F, Q, 1) = -\int_\mu^1 \frac{1}{1 - \alpha\lambda} \left(1 - \frac{\alpha\lambda F}{p(1, \theta(\theta^*_{I,\epsilon}, \lambda))} + S_Q(1, \theta(\theta^*_{I,\epsilon}, \lambda))\right) \mathrm{d}\lambda < 0$$

since $S_Q(\cdot) > 0$ ( cf. Corollary 1). Thus, as $\theta^*_F(\mu)$ increases and $\theta^*_I(1)$ decreases in $Q$, for sufficiently small $Q$, the economy is more susceptible to the strong dependece regime.

Moreover,

$$B_F(\theta^*_{F,\epsilon}, F, Q, 1) = 1 - \int_\mu^1 \frac{1}{1 - \alpha\lambda} \left(1 - \frac{\alpha\lambda F}{p(1, \theta(\theta^*_{I,\epsilon}, \lambda))} + S_F(1, \theta(\theta^*_{I,\epsilon}, \lambda))\right) \mathrm{d}\lambda > 0$$

because $1 - \alpha\lambda F / p(\cdot) < 1 - \alpha\lambda$ and $S_F(\cdot) < 0$ (*cf.* Corollary 1).

Thus, as $\theta^*_F(\mu)$ decreases in $F$ whenever the negative price effect dominates and $\theta^*_I(1)$ increases in $F$, the economy is more susceptible to the strong dependence regime when $F$ is sufficiently large. □

*Proof of Proposition 4.* Using funds' value functions (1) and (2), aggregate utility from consumption can be written as

$$\mathcal{U}(\sigma, \lambda; \theta) = \mathbf{E}_0 \left[\alpha\lambda F + (F + Q)\left(1 - \frac{\alpha\lambda F}{p(\sigma, \theta)}\right) + \sigma\left(\pi\left(\frac{X(\theta) + Q}{p(\sigma, \theta)} - \kappa\right)\alpha\lambda F\right)\right]$$

Rearranging this condition yields

$$\mathcal{U}(\sigma,\lambda;\theta)=\mathbf{E}_0\left[F+Q-\alpha\lambda F\sigma\pi(\kappa-1)-\frac{\alpha\lambda F}{p(\sigma,\theta)}\left(F+Q-\sigma\pi(X+Q)-(1-\sigma\pi)p\right)\right]$$

Substituting the definition of $p(\sigma,\theta)$ and rearranging yields the desired condition

$$\mathcal{U}(\sigma,\lambda;\theta)=\mathbf{E}_0\left[F+Q-\alpha\lambda F\sigma\pi(\kappa-1)-\frac{\alpha\lambda F}{p(\sigma,\theta)}(1-\sigma\pi)Q\right]$$

Notice that $\mathcal{U}_\sigma(\sigma,\lambda;\theta)<0$ for all $\lambda\in[0,1]$ since, by Assumption 2, we must have

$$\kappa>\frac{\theta X(\theta)+Q}{\theta X(\theta)}$$

since $F>\theta X(\theta)$. Given the definition of $\sigma(\theta_F^{sp},\theta)$, it follows that $\theta_F^{sp}=\underline{\theta}$ and $\sigma(\underline{\theta},\theta)=0$ for all $\theta\in\Theta$.

Moreover, we have that $U_\lambda(\sigma,\lambda;\theta)<0$ for any $\sigma\in[0,1]$. Thus, given the definition of $\lambda(\theta_I^{sp},\theta)$, it follows immediately that $\theta_I^{sp}=\underline{\theta}$. □

*Proof of Corollaries 4-6.* The equilibrium thresholds solve the following system of equations

$$A(\theta_{F,\epsilon}^{**},\theta_{I,\epsilon}^{**})\equiv\int_0^1 S\left(\sigma,\mu+(1-\mu)G\left(\sigma+\frac{\theta_{I,\epsilon}^{**}-\theta_{F,\epsilon}^{**}}{2\epsilon}\right),\theta(\theta_{F,\epsilon}^{**},\sigma)\right)\mathrm{d}\sigma-\psi=0 \tag{A5}$$

$$B(\theta_{F,\epsilon}^{**},\theta_{I,\epsilon}^{**})\equiv\int_\mu^1 W\left(\lambda,G\left(\frac{\lambda-\mu}{1-\mu}+\frac{\theta_{F,\epsilon}^{**}-\theta_{I,\epsilon}^{**}}{2\epsilon}\right),\theta(\theta_{I,\epsilon}^{**},\lambda)\right)\mathrm{d}\lambda=0 \tag{A6}$$

*1. Liquidity Injections.* The Jacobian of the system of equations (A5)-(A6) is given by

$$\mathbf{J}=\begin{bmatrix}-\frac{1}{2\epsilon}(1-\mu)\int_0^1 S_\lambda(\cdot)d\sigma+\int_0^1 S_\theta(\cdot)d\sigma & \frac{1}{2\epsilon}(1-\mu)\int_0^1 S_\lambda(\cdot)d\sigma\\ \frac{1}{2\epsilon}\int_\mu^1 W_\sigma(\cdot)d\lambda & -\frac{1}{2\epsilon}\int_\mu^1 W_\sigma(\cdot)d\lambda+\int_\mu^1 W_\theta(\cdot)d\lambda\end{bmatrix}$$

and its determinant is equal to

$$|\mathbf{J}|=\int_0^1 S_\theta(\cdot)d\sigma\int_\mu^1 W_\theta(\cdot)d\lambda-\frac{1}{2\epsilon}\left((1-\mu)\int_0^1 S_\lambda(\cdot)d\sigma\int_\mu^1 W_\theta(\cdot)d\lambda+\int_\mu^1 W_\sigma(\cdot)d\lambda\int_0^1 S_\theta(\cdot)d\sigma\right)>0$$

where the inequality follows from the fact that $S_\theta(\cdot)<0$, $W_\theta(\cdot)<0$, $S_\lambda(\cdot)>0$ and $W_\sigma(\cdot)>0$. Application of the implicit function theorem implies that the derivative of the system of equations (A5)-(A6) with respect to $\kappa$ satisfies

$$\mathbf{J}\begin{bmatrix}\frac{d\theta_{F,\epsilon}^{**}}{d\kappa}\\ \frac{d\theta_{I,\epsilon}^{**}}{d\kappa}\end{bmatrix}=\begin{bmatrix}-\frac{\partial A}{\partial\kappa}\\ -\frac{\partial B}{\partial\kappa}\end{bmatrix}$$

where $\frac{\partial A}{\partial\kappa}=\int_0^1 S_\kappa(\cdot)d\sigma<0$ by the definition of $S(\sigma,\lambda,\theta)$ and $\frac{\partial B}{\partial\kappa}=\int_\mu^1 W_\kappa(\cdot)d\lambda>0$ by the definition of $D_2(\lambda,\sigma,\theta)$. By Cramer's rule, we therefore have that

$$\frac{d\theta_{F,\epsilon}^{**}}{d\kappa}=\frac{1}{|\mathbf{J}|}\begin{vmatrix}-\int_0^1 S_\kappa(\cdot)d\sigma & \frac{1}{2\epsilon}(1-\mu)\int_0^1 S_\lambda(\cdot)d\sigma\\ -\int_\mu^1 W_\kappa(\cdot)d\lambda & -\frac{1}{2\epsilon}\int_\mu^1 W_\sigma(\cdot)d\lambda+\int_\mu^1 W_\theta(\cdot)d\lambda\end{vmatrix}\gtrless 0$$

Similarly, we have that

$$\frac{d\theta_{I,\epsilon}^{**}}{d\kappa}=\frac{1}{|\mathbf{J}|}\begin{vmatrix}-\frac{1}{2\epsilon}(1-\mu)\int_0^1 S_\lambda(\cdot)d\sigma+\int_0^1 S_\theta(\cdot)d\sigma & -\int_0^1 S_\kappa(\cdot)d\sigma\\ \frac{1}{2\epsilon}\int_\mu^1 W_\sigma(\cdot)d\lambda & -\int_\mu^1 W_\kappa(\cdot)d\lambda\end{vmatrix}\gtrless 0$$

*2. Asset Purchase Programs.* Given an asset price guarantee $q(\theta)>\theta X(\theta)$, funds' surplus function

(4) implies that funds' equilibrium threshold in this case solves

$$A^{\mathcal{AP}}(\theta_{F,\epsilon}^{**},\theta_{I,\epsilon}^{**}) = \int_0^1 \pi\left(\frac{X(\theta(\theta_{F,\epsilon}^{**},\sigma))+Q}{\max\{q(\theta(\theta_{F,\epsilon}^{**},\sigma)),p(\sigma,\theta(\theta_{F,\epsilon}^{**},\sigma)\}}-\kappa\right)\alpha\lambda(\theta_{F,\epsilon}^{**},\theta_{I,\epsilon}^{**})d\sigma-\psi=0$$

Similarly, investors' surplus function (8) in this case solves

$$B^{\mathcal{AP}}(\theta_{F,\epsilon}^{**},\theta_{I,\epsilon}^{**}) = F-\int_\mu^1 D_2(\lambda,\sigma(\theta_{F,\epsilon}^{**},\theta_{I,\epsilon}^{**}),\theta(\theta_{I,\epsilon}^{**},\lambda))d\lambda=0$$

Taking the limit as $\epsilon\to 0$, we obtain

$$\lim_{\epsilon\to 0}\theta_{F,\epsilon}^{**} = \begin{cases}\underline{\theta}_F^q(\mu) & \text{if } \overline{\theta}_I^q(1)<\underline{\theta}_F^q(\mu)\\ \theta_{I,0}^q\in[\underline{\theta}_F^q(\mu),\overline{\theta}_I^q(1)] & \text{if } \overline{\theta}_I^q(1)\geq\underline{\theta}_F^q(\mu)\end{cases}$$

where

$$\underline{\theta}_F^q(\mu): \quad \int_0^1 \pi\left(\frac{X(\underline{\theta}_F^q(\mu))+Q}{\max\{q(\underline{\theta}_F^q(\mu)),p(\sigma,\underline{\theta}_F^q(\mu))\}}-\kappa\right)\alpha\mu F d\sigma=\psi$$

and

$$\overline{\theta}_I^q(1): \quad \int_\mu^1 \frac{1}{1-\alpha\lambda}\left((F+Q)\left(1-\frac{\alpha\lambda F}{\max\{q(\theta_I^q(1)),p(1,\overline{\theta}_I^q)\}}\right)+S^q(1,\lambda,\overline{\theta}_I^q(1))\right)d\lambda=F$$

and $\lim_{\epsilon\to 0}\theta_{I,\epsilon}^{**}=\overline{\theta}_I^q(1)$ or $\lim_{\epsilon\to 0}\theta_{I,\epsilon}^{**}=\theta_{I,0}^q\in[\underline{\theta}_F^q(\mu),\overline{\theta}_I^q(1)]$ depending on whether $\overline{\theta}_I^q(1)\lesseqgtr\underline{\theta}_F^q(\mu)$.

Notice that $\underline{\theta}_F^q(\mu)<\theta_{F,0}^*(\mu)$ since $q(\theta)>\theta X(\theta)$. It follows that asset price guarantees strictly decrease market liquidity risk in both *weak* and *strong dependence* regimes. By putting a lower bound on asset prices, the government also props up funds' residual equity value and thereby strictly decreases funding liquidity risk: i.e. $\overline{\theta}_I^q(1)<\theta_{I,0}^*(1)$. Also, notice that funds still acquire information for values of $\theta<\max\{\underline{\theta}_F^q(\mu),\theta_{I,0}^q\}$, implying that the government will be forced to purchase bad assets at an inflated price in those states. Given some price floor $q>\theta X(\theta)$, the expected cost of asset price guarantees equals

$$\int_{\underline{\theta}}^{\overline{\theta}}\alpha\lambda(\min\{\overline{\theta}_I^q(1),\theta_{I,0}^q\},\theta)F(1-\pi\sigma(\max\{\underline{\theta}_F^q(\mu),\theta_{I,0}^q\},\theta))\max\left\{1-\frac{p(\sigma(\max\{\underline{\theta}_F^q(\mu),\theta_{I,0}^q\},\theta))}{q(\theta)},0\right\}d\theta>0$$

which simplifies to

$$C^{\mathcal{AP}}=\int_{\underline{\theta}}^{\max\{\underline{\theta}_F^q(\mu),\theta_{I,0}^q\}}\alpha\left(\mu+(1-\mu)\mathbb{1}_{\theta<\theta_{I,0}^q}\right)F(1-\pi)\left(1-\frac{\theta X(\theta)}{q(\theta)}\right)d\theta>0$$

*3. Outright Debt Purchases.* We consider outright debt purchases that reduce the fraction of redeemable claims, $\alpha$. Differentiating the system of equations (A5)-(A6) with respect to $\alpha$, we obtain

$$\frac{\partial A}{\partial\alpha}=\int_0^1 S_\alpha(\sigma,\cdot)d\sigma>0 \quad\text{and}\quad \frac{\partial B}{\partial\alpha}=\int_\mu^1 W_\alpha(\lambda;\cdot)d\lambda>0$$

By the implicit function theorem, we then have

$$\frac{d\theta_{F,\epsilon}^{**}}{d\alpha}>0 \quad\text{and}\quad \frac{d\theta_{I,\epsilon}^{**}}{d\alpha}>0$$

so that market liquidity and funding liquidity risk are both increasing in the fraction of redeemable claims. For $\alpha=0$, the equilibrium thresholds that solve (A5)-(A6) simplify to $\theta_{F,\epsilon}^{**}=\theta_{I,\epsilon}^{**}=\underline{\theta}$. □

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