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Uncertainty about QE effects when an interest rate peg is anticipated

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Non-technical summary

Research question

How can parameter uncertainty be taken into account in the evaluation of quantitative easing (QE) with accompanying forward guidance (FG)? Studies that assess the macroeconomic effects of QE with accompanying FG typically provide only point estimates. The uncertainty about parameter estimates is usually ignored. A potential reason is the two-step-approach that is generally taken in the literature when QE and FG, technically implemented by an anticipated temporary interest rate peg, are considered. In a first step, a model capable of predicting real effects of a QE programme and with a standard feedback rule for the policy rate is estimated. In a second step, given the estimated parameters, scenario analyses for QE with an anticipated interest rate peg are carried out. The uncertainty about the parameter estimates is usually neglected going from the first to the second step.

Contribution

We use prior and posterior predictive analysis to evaluate parameter uncertainty in the analysis of the Eurosystem's PSPP and FG. Prior-posterior predictive analysis provides an easy-to-implement way to evaluate parameter uncertainty in the context of a policy scenario for QE which involves solving a non-linear DSGE model while simultaneously considering a credibly announced interest rate peg.

Results

The uncertainty about the effects of the PSPP is considerable and it increases substantially when the PSPP is accompanied by FG modelled as a temporary interest rate peg. The Calvo parameters, i.e. the probabilities of being able to reset prices and wages, are the most important factors driving uncertainty about inflation. In contrast, variations in the financial friction parameter have little impact on inflation outcomes and hence uncertainty.

Nichttechnische Zusammenfassung

Fragestellung

Wie kann bei der Evaluation von QE-Programmen, die mit forward guidance (FG) einhergehen, Parameterunsicherheit berücksichtigt werden? Schätzungen hinsichtlich der makroökonomischen Effekte von QE und FG werden üblicherweise in Form einer Punktprognose veröffentlicht, d.h. die Parameterunsicherheit wird ausgeblendet. Ein möglicher Grund hierfür ist, dass die Evaluation von QE-Effekten bei gleichzeitiger FG – die mit Hilfe eines glaubhaft angekündigten temporären Zinspegs modelliert wird – üblicherweise in zwei Schritten erfolgt: Im ersten Schritt werden die Parameter eines Modells, das in der Lage ist, reale Effekte eines QE-Programms abzubilden und das durch eine übliche Feedback-Regel für den geldpolitischen Zins gekennzeichnet ist, geschätzt. Im zweiten Schritt wird auf der Grundlage der so geschätzten Parameter eine QE-Szenarienanalyse mit Zinspeg durchgeführt. Beim Übergang vom ersten zum zweiten Schritt wird die Parameterunsicherheit jedoch üblicherweise vernachlässigt.

Beitrag

Wir nutzen die sogenannte *prior/posterior predictive analysis*, um die Bedeutung von Parameterunsicherheit am Beispiel der Analyse des Staatsanleihenkaufprogramms des Eurosystems (PSPP) darzustellen. Dabei illustrieren wir die Implementierung dieses Verfahrens im Rahmen eines realistischen QE-Politikszenarios, in dem ein nichtlineares DSGE-Modell gelöst und gleichzeitig ein von der Notenbank angekündigter Zinspeg von den Agenten der Ökonomie berücksichtigt wird.

Ergebnisse

Die Unsicherheit über die makroökonomischen Effekte des PSPP ist beträchtlich und sie nimmt nochmals zu, wenn das PSPP mit FG, modelliert als glaubhaft angekündigtem Zinspeg, kombiniert wird. Den größten Einfluss auf die Unsicherheit der Inflationseffekte von QE haben die Calvo-Parameter, welche die Wahrscheinlichkeit abbilden, in einer gegebenen Periode Löhne und Preise anpassen zu können. Im Gegensatz dazu haben Veränderungen des Parameters für die finanziellen Friktionen wenig Einfluss auf den Inflationsverlauf und somit auf die Unsicherheit.

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Uncertainty about QE effects when an interest rate peg is anticipated*

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April 10, 2018

Abstract

After hitting the lower bound on interest rates, the Eurosystem engaged in a public sector purchase programme (PSPP) and forward guidance (FG). We use prior and posterior predictive analysis to evaluate the importance of parameter uncertainty in an analysis of these policies. We model FG as an anticipated temporary interest rate peg. The degree of parameter uncertainty is considerable and increasing in the length of FG. The probability of being able to reset prices and wages is the most important factor driving uncertainty about inflation. In contrast, variations in financial intermediaries' net worth adjustment costs have little impact on inflation outcomes.

Keywords: Prior/posterior predictive analysis, anticipated interest rate peg, parameter uncertainty, euro area, QE, PSPP, forward guidance puzzle

JEL Classification: C53, E32, E52

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1 Introduction

In recent years, many central banks have resorted to quantitative easing policies (QE) to provide economic stimulus at the effective lower interest rate bound. Typically, QE has been accompanied by a special type of forward guidance (FG) where central banks provide (additional) stimulus by communicating that policy rates will be kept constant for an extended period of time. Hence, there are two policies influencing the economy at the same time.

Most studies that evaluate the macroeconomic effects of QE programmes – such as the Federal Reserve's large scale asset purchase programmes (LSAPs) or the Eurosystem's public sector purchase programme (PSPP) – in tandem with FG provide only point estimates of these effects (see, inter alia, Gertler and Karadi, 2013; Andrade, Breckenfelder, Fiore, Karadi and Tristani, 2016; Sahuc, 2016; Kühl, 2016; Carlstrom, Fuerst and Paustian, 2017; Hohberger, Priftis and Vogel, 2017). One potential reason for this observation is the two-step-approach that is usually taken in the literature. In a first step, a model capable of generating real effects of QE and with a standard feedback rule for the policy rate is estimated (typically via Bayesian estimation of the underlying DSGE model). In a second step, given these estimates for the structural parameters, scenario analyses of QE in tandem with FG – technically implemented by an anticipated interest rate peg – are carried out.[1] The uncertainty surrounding the parameter estimates is usually neglected going from the first to the second step, as standard software tools to illustrate parameter uncertainty in such scenario analyses are not readily available.[2]

[1] In the following, we use the terms FG and anticipated interest rate peg interchangeably.

[2] There are three possible sources of uncertainty in this type of analysis. First, *across* model uncertainty – implying that the choice of the model for evaluating QE has an impact on the

In this paper, we provide an easy-to-implement way to analyse parameter uncertainty for scenario analyses involving QE in tandem with FG. In particular, after estimating parameters of (a linear version of) a model with Bayesian methods, we conduct prior and posterior predictive analysis to use the information about the parameter estimation obtained in the first step for the subsequent policy scenario analysis in the second step. We thus follow Leeper, Traum and Walker (2015) and Suh and Walker (2016) who use prior-posterior predictive analysis to asses specific quantitative properties of DSGE models. These authors implement predictive analysis for linearised models, which are solved and simulated using standard (first-order) perturbation techniques. We show that it is possible to exploit this technique in order to describe parameter uncertainty in the evaluation of QE accompanied by FG. The scenario analysis of QE in tandem with FG in our case is based on a non-linear DSGE model.

The model we use for our analysis is the one in Carlstrom et al. (2017) (henceforth CFP). We estimate the model with European data and subsequently simulate time paths for inflation in response to the launch of a QE programme for four different policy scenarios: In the first scenario, a QE programme that mimics the Eurosystem's PSPP affects the economy. In the second, third, and fourth scenario, the same QE programme again affects the economy but is additionally accompanied by one, two, and three years of FG, respectively. Our simulated time

simulation results – which is often illustrated by a simple comparison of median estimates of a large number of different studies (see, for instance, the meta-studies provided by International Monetary Fund, 2013; Deutsche Bundesbank, 2016). Second, *within* model uncertainty (i.e. parameter uncertainty) implying that uncertainty about estimates for the structural parameters of the underlying model will also translate into uncertainty about the estimated macroeconomic effects for a given purchase programme (the case we deal with in this paper). Third, data uncertainty (i.e. uncertainty about the information content present in collected data).

paths imply a distribution of the inflation response to a QE programme which highlights the relevance of parameter uncertainty in the assessment of QE. The prior-predictive analysis produces distributions for inflation based on the prior distributions of the estimated model parameters, while the posterior-predictive analysis is based on the distributions of the estimated parameters after having confronted the model with the data. This allows us to assess what effect on the distribution of the inflation response the data assign to the PSPP and FG within our model framework. It is easily possible to conduct the same type of analysis also with respect to other macroeconomic variables. We confine ourselves to the analysis of the inflation effects, though, to keep the analysis focused.

In principle, there are other methods of illustrating uncertainty in the evaluation of QE programmes. However, they are not applicable to the scenario we have in mind, namely the evaluation of a QE programme accompanied by FG, implemented as an anticipated interest rate peg. For example, Bayesian impulse response functions principally allow for an evaluation of parameter uncertainty when analysing QE (at least if the QE shock is part of the estimation), but they are not readily available if there is a temporary anticipated interest rate peg. Another example is the computation of conditional forecasts which principally allows for the illustration of uncertainty in the analysis of QE in combination with an interest rate peg. This method, however, involves implementing the peg by finding a series of shocks that imply a pre-specified path for the interest rate (based on the state space solution of the underlying DSGE model). These shocks are by definition unexpected, that is, agents are not able to anticipate the temporary interest rate peg – which contradicts the very idea of FG. If agents are supposed to anticipate the temporary nominal interest rate peg, this necessitates implementing the peg via anticipated (or news) shocks as in Kühl (2016) or solving and simulating the model by, for instance, assuming perfect foresight as in Sahuc (2016), neither of whom considers parameter uncertainty.

In this paper, we take the perfect foresight approach to implement the anticipated temporary interest rate peg and contribute to the literature by illustrating parameter uncertainty employing prior and posterior predictive analysis, and identifying the individual contributions of the different parameters to this uncertainty. We find that the uncertainty about the effects of the PSPP is considerable and amplified when the PSPP is accompanied by FG. The Calvo parameters – which govern the probability, in a given period, of being able to reset prices and wages – have, for a given length of FG, the biggest influence in determining the uncertainty about the inflation effects of a QE shock. The estimated financial friction parameter – which is of crucial importance in the model for QE to have real effects (see Carlstrom et al., 2017) – is of less importance regarding parameter uncertainty.

We organise our paper as follows. The next section presents the model. Section 3 provides details of the model's estimation and calibration. Section 4 describes how QE and FG are implemented. Section 5 describes the prior-posterior predictive analysis we conduct and shows the resulting simulations of inflation for our different policy scenarios. It also contains an analysis about which parameters are most important for our results. Section 6 concludes.

2 Model

In the standard New Keynesian model, asset purchases are neutral (so-called Wallace neutrality), in that they do not have an effect on real economic activity and inflation (Eggertsson and Woodford, 2003). To simulate the effects of QE and to compute the corresponding uncertainty of these effects, we therefore rely on a DSGE model developed by CFP which features funding constraints and market segmentation such that the Wallace neutrality does not apply. More precisely, in this model both households and financial intermediaries (henceforth FIs) face financial constraints. The bond market is segmented in that only FIs can purchase long-term debt instruments. These include public (i.e. government) and private (i.e. investment) bonds. From the perspective of the FIs, these are perfect substitutes and, hence, yield the same returns. However, the ability of the FIs to adjust their liability position is limited by two constraints. First, they are *leverage-constrained* because the amount of deposits they can attract is constrained by their net worth (due to a hold-up problem). Second, FIs face net worth adjustment costs. Households need to finance their investments by way of issuing (long-term) investment bonds and, thus, face a funding restriction with respect to their investments (a so-called *loan-in-advance constraint*). The purchase of government bonds increases the FIs' demand for investment bonds since the liability side of the FIs balance sheet cannot adjust easily due to the aforementioned constraints. This in turn alleviates the households' loan-in-advance-constraint.

Otherwise, the model exhibits familiar New Keynesian features. That is, it comprises households that consume, save in (short-term) deposits and supply labour, a standard production sector with monopolistic competition in intermediate good production, price and wage rigidities (see Erceg, Henderson and Levin, 2000) as well as price and wage indexation (see Christiano, Eichenbaum and Evans, 2005). If the central bank abstains from forward guidance, it follows a standard Taylor rule with some degree of interest rate smoothing. The non-linear model equations are summarised in Table 1, and a complete derivation of the model is delegated to Appendix A.

Table 1: Nonlinear model equations

Model equations:	
HH cons. decision	$\Lambda_t = \frac{\mathrm{b}_t}{C_t - hC_{t-1}} - E_t \frac{\beta h \mathrm{b}_{t+1}}{C_{t+1} - hC_t}$
Euler equation	$\Lambda_t = E_t \beta \frac{\Lambda_{t+1}}{\Pi_{t+1}} R_t^d$
Wage curve (WC)	${w_t}^{1+\varepsilon_w \eta} = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{X_t^{wn}}{X_t^{wd}}$
WC nominator	$X_t^{wn} = \lambda_{w,t} b_t \chi w_t^{\varepsilon_w(1+\eta)} H_t^{1+\eta} + E_t \left\{ \theta_w \beta \Pi_{t+1}^{\varepsilon_w(1+\eta)} \Pi_t^{-\iota_w \varepsilon_w (1+\eta)} X_{t+1}^{wn} \right\}$
WC denominator	$X_t^{wd} = \Lambda_t w_t^{\varepsilon_w} H_t + \theta_w \beta \Pi_t^{-\iota_w(\varepsilon_w - 1)} \Pi_{t+1}^{(\varepsilon_w - 1)} E_t \left\{ X_{t+1}^{wd} \right\}$
Wages law of motion	${w_t}^{1-\varepsilon_w} = (1-\theta_w)\left(w_t^*\right)^{1-\varepsilon_w} + \theta_w \left(\frac{\Pi_{t-1}^{\iota_w} w_{t-1}}{\Pi_t} \right)^{1-\varepsilon_w}$
HH decision capital	$\Lambda_t M_t P_t^k = E_t \beta \Lambda_{t+1} \left[R_{t+1}^k + M_{t+1} P_{t+1}^k (1-\delta) \right]$
HH decision inv. bonds	$\Lambda_t M_t Q_t = E_t \frac{\beta \Lambda_{t+1}(1+\kappa Q_{t+1} \mathrm{M}_{t+1})}{\Pi_{t+1}}$
Welfare	$V_t^h = b_t \left\{ \ln (C_t - hC_{t-1}) - D_t^w B \frac{H_t^{1+\eta}}{1+\eta} \right\} + \beta E_t V_{t+1}^h$
Price of capital	$R_t^k = mc_t \mathrm{MPK}_t$
Real wages	$w_t = mc_t \mathrm{MPL}_t$
Phillips curve (PC)	$\Pi_t^* = \frac{\varepsilon_p}{\varepsilon_p - 1} \frac{X_t^{pn}}{X_t^{pd}} \Pi_t$
PC nominator	$X_t^{pn} = Y_t \lambda_{p,t} mc_t(i) + E_t \left\{ \theta_p \frac{\beta \Lambda_{t+1}}{\Lambda_t} \Pi_t^{-\iota_p \varepsilon_p} \Pi_{t+1}^{\varepsilon_p} X_{t+1}^{pn} \right\}$
PC denominator	$X_t^{pd} = Y_t + E_t \left\{ \theta_p \frac{\beta \Lambda_{t+1}}{\Lambda_t} \Pi_t^{\iota_p(1-\varepsilon_p)} \Pi_{t+1}^{\varepsilon_p - 1} X_{t+1}^{pd} \right\}$
Infl. law of motion	$(\Pi_t)^{1-\varepsilon_p} = (1-\theta_p)(\Pi_t^*)^{1-\varepsilon_p} + \theta_p \left(\Pi_{t-1}^{\iota_p}\right)^{1-\varepsilon_p}$
Price dispersion	$D_t^p = \Pi_t^{\varepsilon_p} \left[(1-\theta_p) \Pi_t^{*-\varepsilon_p} + \theta_p \left(\Pi_{t-1}^{\iota_p}\right)^{-\varepsilon_p} D_{pt-1} \right]$
Wage dispersion	$D_t^w = \theta_w \left(\frac{\Pi_t}{\Pi_{t-1}^{\iota_w}} \right)^{\varepsilon_w} \left(\frac{w_t}{w_{t-1}} \right)^{\varepsilon_w} D_{wt-1} + (1-\theta_w) \left(\frac{w_t^*}{w_t} \right)^{-\varepsilon_w}$
Resource constraint	$Y_t = C_t + I_t$
Production function	$Y_t = A_t K_t^\alpha H_t^{1-\alpha} / D_t^p$
Firm's capital decision	$K_t = (1-\delta) K_{t-1} + \mu \left(1 - \psi_I \left(\frac{1}{2}\right) \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right) I_t$
Investment decision	$P_t^k \mu_t \left\{ 1 - S\left(\frac{I_t}{I_{t-1}} \right) - S'\left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right\} =$
	$1 - \beta P_{t+1}^k \frac{\Lambda_{t+1}}{\Lambda_t} \mu_{t+1} \left\{ -S'\left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right\}$
FI's balance sheet	$\bar{B}_t + \bar{F}_t = N_t + L_t$

Table 1: continued

Model equations:	
Leverage Ratio	$L_t = \frac{E_t \frac{\Lambda_{t+1}}{\Pi_{t+1}}}{\left[E_t \frac{\Lambda_{t+1}}{\Pi_{t+1}} + (\Phi_t - 1) E_t \frac{\Lambda_{t+1}}{\Pi_{t+1}} \frac{R^L_{t+1}}{R^d_t}\right]}$
Loan in advance constraint	$P^k_t I_t = \bar{F}_t - \kappa \frac{\bar{F}_t}{\Pi_t} \frac{Q_t}{Q_{t-1}}$
FI's net worth decision	$\Lambda_t \left[1 + f(N_t) + N_t f'(N_t)\right] = E_t \Lambda_{t+1} \beta \zeta \frac{P_t}{P_{t+1}} \left[\left(R^L_{t+1} - R^d_t\right) L_t + R^d_t\right]$
Long-term interest rate	$R^L_t = \frac{(1+\kappa Q_t)}{Q_{t-1}}$
Yield to maturity	$R^{10}_t = Q_t^{-1} + \kappa$
Marginal prod. of capital	$MPK_t = \alpha A_t K_{t-1}(i)^{\alpha-1} H_t(i)^{1-\alpha}$
Marginal prod. of labour	$MPL_t = (1-\alpha) A_t K_{t-1}(i)^{\alpha} H_t(i)^{-\alpha}$
Taylor rule	$R_t = (R_{t-1})^{\rho} \left(R_{ss} \Pi_t^{\tau_\Pi} \left(\frac{Y_t}{Y_{t-1}}\right)^{\tau_y}\right)^{1-\rho} \varepsilon^R_t$

Notes: b_t = discount factor shock, C_t = consumption, Λ_t = Lagrange multiplier, Π_t = inflation, R_t = nominal interest rate, w_t = real wage, X^{wn}_t = & X^{wd}_t = auxiliary variables for wage curve, X^{pn}_t = & X^{pd}_t = auxiliary variables for Phillips curve, MPL_t = marginal product of labour, MPK_t = marginal product of capital, R^L_t = Long-term rate, R^{10}_t = yield to maturity, I_t = Investment, P^k_t = price of investment, $\bar{F}_t$ = investment bonds, $\bar{B}_t$ = government bonds, Q_t = price of bond, H_t = labour, A_t = technology shock, N_t = net worth, L_t = leverage, D^p_t = price dispersion, D^w_t = wage dispersion, K_t = capital, mc_t = marginal costs, μ_t = investment shock, Φ_t = financial shock, $\lambda_{w,t}$ = wage markup shock, $\lambda_{p,t}$ = price markup shock, Y_t = output.

3 Estimation and Calibration

As is common in the literature, we calibrate a subset of structural parameters to ensure identification. For the calibration, we rely on CFP. Table 2 gives the values for the calibrated parameters. β is set to 0.99, yielding a steady state annual real interest rate of 4%. The labour income share α is set to 0.33 and the capital depreciation rate per year to 10%, implying $\delta = 0.025$. A 20% mark-up in both prices and wages is assumed, leading to $\epsilon_p = \epsilon_w = 5$. A leverage ratio of 6 leads to $\zeta = 0.9854$. The other structural parameters are estimated using Bayesian methods. For the estimation, we linearise the model around the steady state. We use eight observables for the euro area: real per capita output growth, real per capita investment growth, gross inflation, employment growth, real wage

Table 2: Calibrated parameters

Parameters	Description	Value
β	Household discount factor	0.99
ψ_I	Investment adjustment cost	2
κ	Coupon payment	0.975
L_{ss}	Steady state leverage	6
ϵ_p	Elasticity of substitution (goods)	5
ϵ_w	Elasticity of substitution (labour)	5
α	Capital share	0.33
δ	Depreciation rate	0.025

growth, the first difference of the short- and long-term interest rate, and real bank net worth growth. Data on bank net worth are taken from the European Central Bank's MFI Balance Sheet Items Statistics. All the other variables are taken from the Area-wide Model database.[3] A description of the data is provided in Appendix B. All variables are demeaned. Since we have only seven structural shocks in the model, we add a measurement error to the observations equation for bank net worth in order to avoid stochastic singularity.[4] The sample period is from 1998Q1 to 2013Q4.

Table 3: Prior and posterior distributions of structural parameters

		Prior distribution			Posterior distribution			
Param.	Description	Dist.	Mean	St. Dev.	Median	Mean	HPD inf	HPD sup
h	Habit formation	Beta	0.5000	0.2000	0.8642	0.8635	0.8193	0.9074
η	Labor disutility	Gamma	2.0000	0.5000	1.8101	1.8496	1.1055	2.5857
ι_p	Price indexation	Beta	0.6000	0.1000	0.5261	0.5263	0.3658	0.6890
ι_w	Wage indexation	Beta	0.6000	0.1000	0.3761	0.3786	0.2573	0.4991
θ_p	Price rigidity	Beta	0.7000	0.1000	0.8144	0.8139	0.7567	0.8676
θ_w	Wage rigidity	Beta	0.7000	0.1000	0.8211	0.8194	0.7641	0.8726
ρ	Interest rate smoothing	Beta	0.7500	0.1000	0.7409	0.7390	0.6850	0.7947
τ_{pi}	Inflation coeff. in TR	Normal	1.5000	0.1000	1.5912	1.5919	1.4333	1.7482
τ_y	Output growth coeff. in TR	Normal	0.5000	0.1000	0.5725	0.5723	0.4163	0.7270
ψ_N	Net worth adjustm. costs	Gamma	3.0000	1.0000	6.7634	6.8273	4.9522	8.7945

Notes: Results based on 4 chains with 500,000 draws each. HPD inf and HPD sup denote the lower and upper bound, respectively, of the 90% highest posterior density interval.

[3]We make use of the 14th update of the Area-wide Model (AWM) database from September 2014); see `http://www.eabcn.org/sites/default/files/fck_uploads/awm_database_update_14.pdf`.

[4]For the description of the shock processes see Appendix A.

Table 4: Prior and posterior distributions of parameters in shock processes

		Prior distribution			Posterior distribution			
Param.	Description	Dist.	Mean	St. Dev.	Median	Mean	HPD inf	HPD sup
ρ_A	AR(1), productivity	Beta	0.6000	0.2000	0.9719	0.9662	0.9342	0.9976
ρ_Φ	AR(1), financial	Beta	0.6000	0.2000	0.6628	0.6608	0.5719	0.7527
ρ_μ	AR(1), investment	Beta	0.6000	0.2000	0.8370	0.8347	0.7713	0.8980
ρ_{λ_W}	AR(1), wage mark-up	Beta	0.6000	0.2000	0.1741	0.1868	0.0420	0.3260
ρ_{λ_P}	AR(1), price mark-up	Beta	0.6000	0.2000	0.4542	0.4484	0.2351	0.6620
ρ_d	AR(1), discount factor	Beta	0.6000	0.2000	0.4945	0.4930	0.3297	0.6585
ρ_R	AR(1), monetary	Beta	0.6000	0.2000	0.5030	0.4993	0.3636	0.6366
ϵ_A	SE, productivity	Invgam	0.0100	1.0000	0.0056	0.0056	0.0048	0.0064
ϵ_Φ	SE, financial	Invgam	0.0500	1.0000	0.1882	0.1913	0.1419	0.2394
ϵ_μ	SE, investment	Invgam	0.5000	1.0000	0.0881	0.0887	0.0740	0.1028
ϵ_{λ_W}	SE, wage mark-up	Invgam	0.1000	1.0000	0.5742	0.6359	0.2132	1.0417
ϵ_{λ_P}	SE, price mark-up	Invgam	0.1000	1.0000	0.0528	0.0608	0.0240	0.0954
ϵ_d	SE, discount factor	Invgam	0.1000	1.0000	0.0300	0.0314	0.0206	0.0416
ϵ_R	SE, monetary	Invgam	0.0100	1.0000	0.0033	0.0033	0.0028	0.0038
ϵ_{NW}	SE, M.E. bank net worth	Invgam	0.0013	1.0000	0.0147	0.0148	0.0126	0.0171

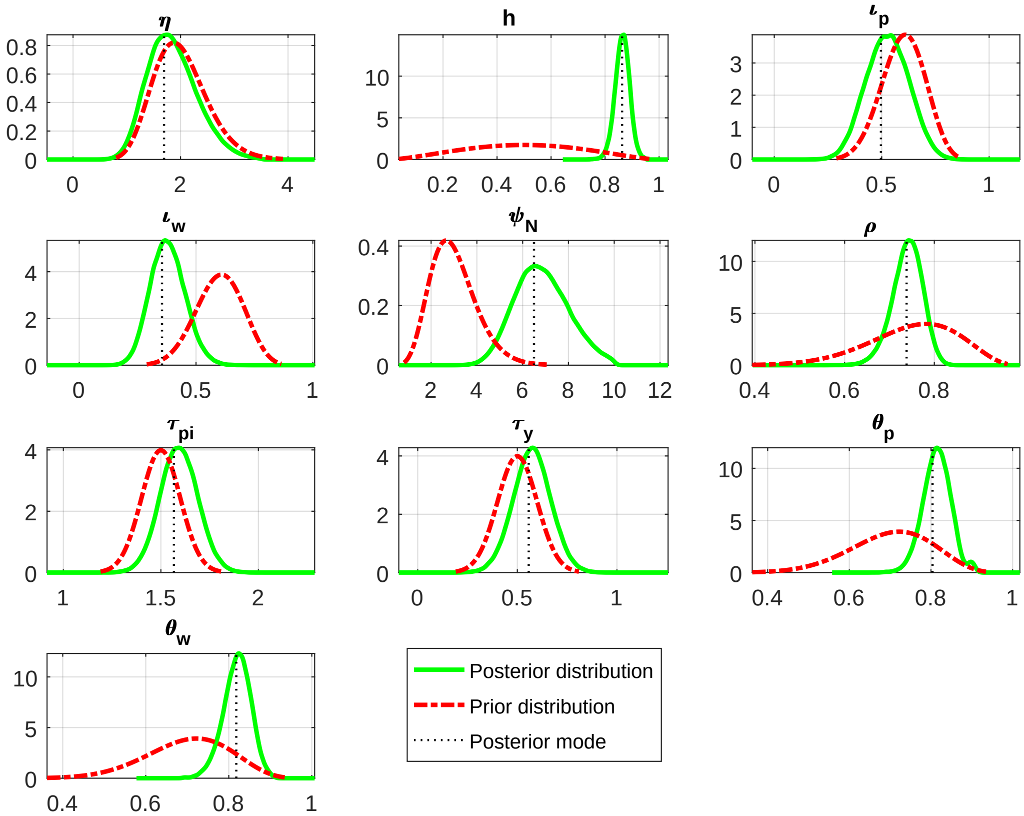
Notes: Results based on 4 chains with 500,000 draws each. HPD inf and HPD sup denote the lower and upper bound, respectively, of the 90% highest posterior density interval.

The choice of the prior distributions of the structural parameters to be estimated correspond largely to those in CFP and Christiano, Motto and Rostagno (2010). In general, we use the Beta distribution for parameters between zero and one. For the Taylor rule parameters we use the normal distribution, which is typically used for unbounded parameters. For the financial sector parameter ψ_N, which governs the importance of net worth adjustment costs, we use a gamma distribution with mean 3 and standard deviation 1. The left parts of Tables 3 and 4 display the prior distributions of the estimated parameters.

Given the prior distributions of the parameters, we draw posterior distributions using the Metropolis-Hastings algorithm. We run four chains, each with 500,000 draws.[5] The right parts of Tables 3 and 4 report the posterior median, the posterior mean, and the lower and upper bounds of the 90% highest posterior density

[5]We use Dynare 4.5.4 for the estimation of the model, see Adjemian, Bastani, Juillard, Karamé, Mihoubi, Perendia, Pfeifer, Ratto and Villemot (2011).

Figure 1: Prior and posterior distribution of structural parameters



Note: Dashed-dotted lines are prior distributions, solid lines are posterior distributions, and the vertical dotted lines are the posterior modes.

interval of the estimated parameters obtained by the Metropolis-Hastings algorithm. Convergence statistics proposed by Brooks and Gelman (1998) as well as trace plots for the estimated structural parameters are presented in Appendix D. The posterior means of the habit formation parameter (0.86), the price rigidity parameter (0.81), and the price indexation parameter (0.53) are estimated to be somewhat higher than in CFP. The posterior means of the wage rigidity (0.82), wage indexation (0.38), and labour disutility (1.85) parameter are estimated to be somewhat lower than in CFP. The posterior means of the Taylor rule parameters are in line with commonly observed values in the literature. The most noticeable difference between our estimation result and CFP's is the posterior distribution for the net worth adjustment cost parameter ψ_N. Our posterior mean for this parameter (6.82) is vastly higher than the one in CFP (0.79). This could have several reasons: First, net worth elasticity could be different in Europe compared to the USA. Second, our sample ends in 2013Q4 and thus includes data of the financial crisis. Third, we use data on bank net worth to better identify the net worth adjustment cost parameter. On average, financial frictions could thus be more severe in our sample than in CFP's sample which ends in 2008Q4. Figure 1 shows the prior and posterior distributions of the structural parameters as well as their posterior modes.

4 Implementation of QE and FG

We implement the PSPP of the Eurosystem using an AR(2) process for the real market value of long-term bonds on the balance sheets of the FIs:

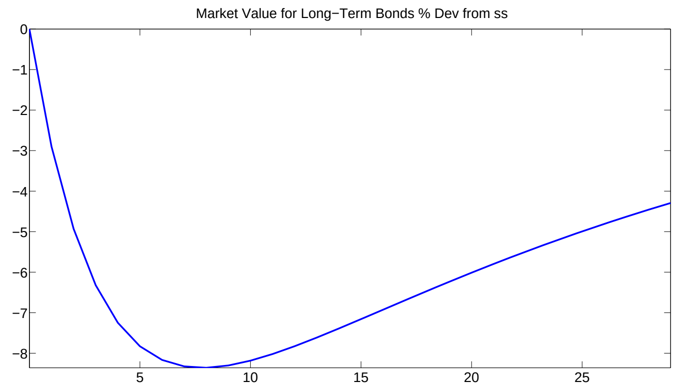
$$\bar{B}_t = \bar{B}_{ss}^{(1-\bar{\rho}_1+\bar{\rho}_2)} \left(\bar{B}_{t-1}\right)^{\bar{\rho}_1} \left(\bar{B}_{t-2}\right)^{-\bar{\rho}_2} \varepsilon_t^{\bar{B}}. \tag{1}$$

Once triggered, the entire path of the PSPP is taken into account (and, thus, known) by every agent of the model. The shock, $\varepsilon_t^{\bar{B}}$, is calibrated to mimic monthly purchases of €60 billion euro from March 2015 until September 2016, as announced by the Governing Council in January 2015.[6] This approximately implies an about

[6]This corresponds to the APP with monthly purchase volumes as announced in January 2015 (ignoring the increase to €80 bn in March 2016 and the extension until end 2017).

8% decrease in the real market value of long-term bonds (which is the same shock size that was implemented in Sahuc, 2016), as shown in Figure 2.[7]

Figure 2: Total value of long-term bonds held by the public



Note: The solid line represents the evolution of $\bar{B}$ (i.e. the market value for long-term bonds) as percent deviation from steady-state over 25 quarters.

We implement the anticipated interest rate peg by a sequence of anticipated shocks, ε_t^{TR}, which consists of binary dummy variables:

$$R_t = \varepsilon_t^{TR}\left(R_{ss}\right) + \left(1-\varepsilon_t^{TR}\right)\left(R_{t-1}\right)^{\rho}\left(R_{ss}\Pi_t^{\tau_\Pi}\left(\frac{Y_t}{Y_{t-1}}\right)^{\tau_y}\right)^{1-\rho}. \tag{2}$$

Thus, to implement one year of FG, ε_t^{TR} is set to one for four consecutive periods.

5 Prior-posterior predictive analysis

Based on our estimation, we use prior and posterior predictive analysis to generate the respective distributions of inflation responses for different policy scenarios. In the *first scenario* we do not constrain the interest rate path and describe the

[7]The AR(2) coefficients $\bar{\rho}_1$ and $\bar{\rho}_2$ are equal to 1.8 and 0.81, respectively.

model's range of predictions in response to QE. The unconstrained nature of the interest rate in this scenario implies that it increases in response to the inflationary stimulus of the QE programme according to the Taylor rule. In the *second*, *third* and *fourth scenario*, we analyse the model's range of predictions if the QE programme is accompanied by a one, two, and three year lasting anticipated interest rate peg, respectively. After the peg, the interest rate varies according to the estimated Taylor rule.

We implement *prior* predictive analysis in the following way. Given the DSGE model outlined in section 2, we posit prior densities for the model's structural parameters, where we restrict our analysis to the parameter subspace that implies a unique rational expectations equilibrium. We then simulate 100.000 sets of draws for the structural parameters based on the prior distributions specified in Table 3.[8] Then, based on these draws, we calculate perfect foresight simulations for each policy scenario. In this way, the prior predictive analysis produces a distribution of inflation effects for each policy scenario and, thus, a range of possible model-implied responses to the policy impact before confronting the model with actual data.

We use *posterior* predictive analysis to analyse the model-implied responses after having confronted the model with our data set. The posterior means/medians shown in Table 3 are based on the posterior distributions, which have been generated using the Metropolis-Hastings algorithm during the estimation of the model. We simulate 500.000 sets of draws for the posterior distributions. Based on these

[8] To generate these draws we make use of the Global Sensitivity Analysis toolbox developed by Ratto (2008).

draws we calculate 100.000 perfect foresight simulations for each policy scenario.[9] In this way, we produce a range of possible inflation responses for the four policy scenarios after the information content in the actual data has been exploited.[10]

5.1 Baseline results

Our first set of results, shown in Figure 3, refers to the policy scenario in which the central bank implements a QE programme without an interest rate peg. Consequently, in response to inflationary pressures the central bank increases its policy rate (not shown) according to the Taylor rule specified in (2), with $\varepsilon_t^{TR} = 0 \; \forall t$.

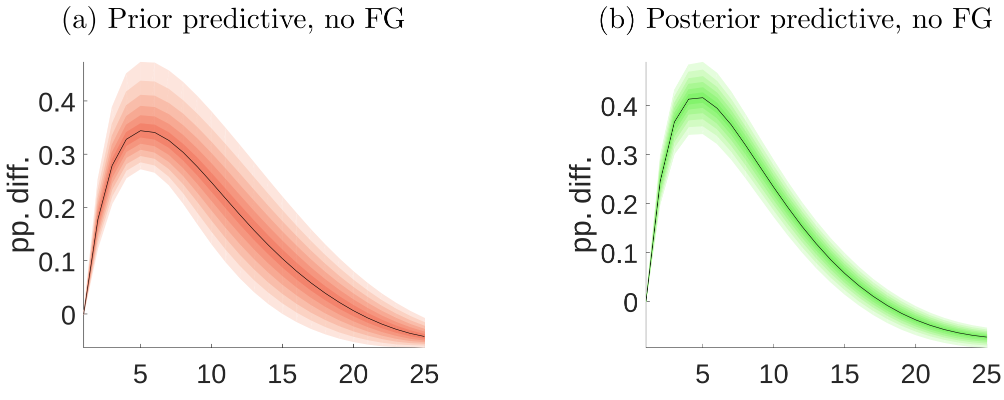
The left-hand panel (red fan charts) shows the distribution of the inflation path for the prior predictive analysis. The black line represents the median inflation path. The simulations result in a median peak response of inflation of 0.34 percentage points (pp) five quarters after the implementation of the QE programme. The uncertainty of the macroeconomic effects is highlighted by the 66 percent interval of the overall distribution of the simulations. The different shadings represent $5.\bar{5}\%$ percentiles. At the peak response, the interval exhibits a range of 0.20 pp for the effect of the PSPP on inflation. Thus, a priori, the model is able to generate notable effects of the PSPP, but the parameter uncertainty of the effects is pronounced.

The right-hand panel (green fan charts) in Figure 3 shows the distribution of

[9] To be precise, we take the last 100.000 out of the 500.000 sets to make sure that we only use posterior draws from the MCMC simulations that have converged.

[10] Note that when we analyse the inflation effects of QE in combination with FG, some draws (which still deliver unique rational expectations equilibria) imply sign switches in the simulations of the endogenous variables. This phenomenon has become known as the *reversal puzzle* (see Carlstrom, Fuerst and Paustian, 2015; Gerke, Giesen, Kienzler and Tenhofen, 2017). Here, we follow Lindé, Smets and Wouters (2016) and constrain our analysis to those cases in which the reversal puzzle does not arise (for both the prior and the posterior predictive analysis).

Figure 3: Inflation effects in response to a QE shock, prior-posterior predictive analysis



Note: The figure shows the 66 percent interval around the median response to the QE shock. The separated areas (from weak to strongly shaded) reflect 5.$\bar{5}$% percentiles. The simulations are carried out under the assumption of perfect foresight.

inflation paths for the posterior predictive analysis. The median peak response of inflation is 0.42 pp, and at the peak response, the interval exhibits a range of 0.15 pp. Hence, after confronting the model with the data, it suggests a higher median effect of the PSPP on inflation. Furthermore, as the distributional mass is more centred around the median, the posterior distributions imply a lower degree of parameter uncertainty about these effects. This may, however, not come as a surprise as the prior distributions were chosen to be fairly broad and accordingly the posterior distributions tend to be narrower (see Figure 1), leading to a sizeable share of the distributional mass to be close to the median response.

Figure 4 shows the simulation results for the scenarios in which QE is accompanied by FG. Rows one, two, and three correspond to one, two, and three years of FG, respectively. The left-hand panels (red fan charts) show the results of the prior predictive analyses, while the right-hand panels (green fan charts) show the results for the posterior predictive analyses. Looking at the prior predictive analysis for a QE programme accompanied by one year of FG (upper left graph), the median response increases strongly compared to the scenario without FG. The median peak effect is 2.62 pp, approximately seven times larger than the median peak effect without FG. In addition, a large share of the distributional mass now implies responses which are multitudes larger than in the scenario without FG. At the peak, the interval exhibits a range of 6.52 pp.

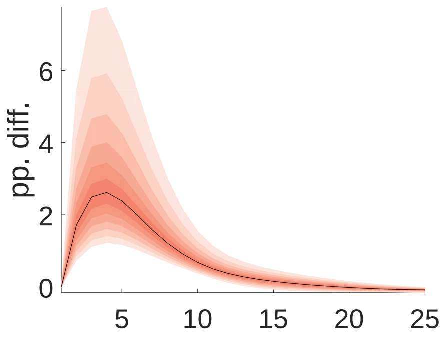
The observation that even rather short periods of FG produce such large effects of inflation in New-Keynesian-type models such as those seen in Figure 4 seems unrealistic and has become known as the FG puzzle in the literature (see Del Negro, Giannoni and Patterson, 2015). Hence, here we replicate the expected result that the median peak inflation response increases strongly with FG and accordingly the distribution of the inflation paths widens.[11]

The upper right graph in Figure 4 shows the posterior predictive analysis for a QE programme accompanied by one year of FG. It shows that the median peak response of inflation increases (to 0.81 pp) and the range of the distribution at the peak widens (to 0.48 pp) compared to the posterior predictive analysis without FG, as expected. However, this increase in inflation responses going from zero to one year of FG is not nearly as sharp as in the prior predictive analysis. It thus seems that the FG puzzle is less evident for the posterior predictive analysis. The difference between prior and posterior predictive analysis, however, loses importance for longer durations of FG, although the prior predictive analysis continues to deliver higher median peak responses of inflation as well as wider distributions

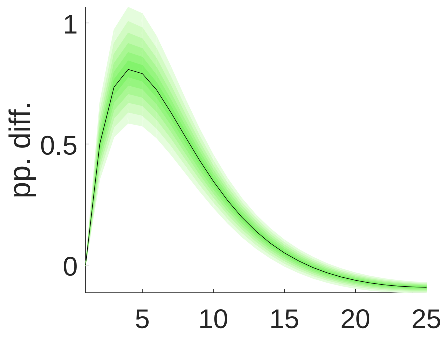
[11] Several approaches have been developed in the literature to cope with the FG puzzle, such as those in McKay, Nakamura and Steinsson (2016), Angeletos and Lian (2017), or Gabaix (2016).

Figure 4: Inflation effects in response to a QE shock and different durations of FG, prior-posterior predictive analysis

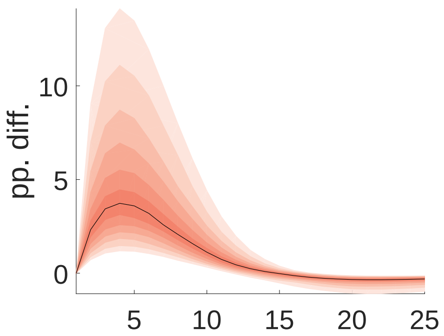
(a) Prior predictive, 1 year FG



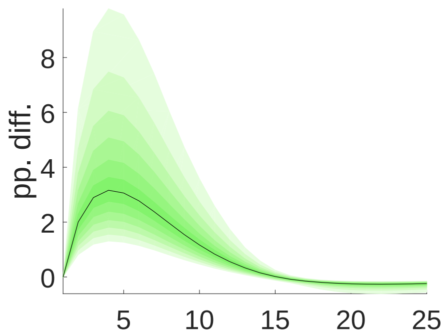
(b) Posterior predictive, 1 year FG



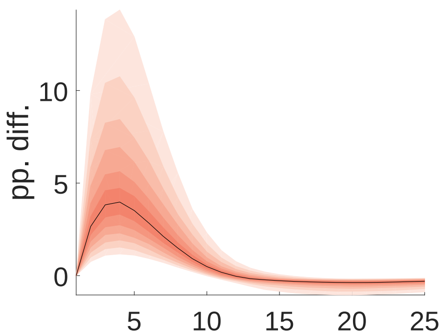
(c) Prior predictive, 2 years FG



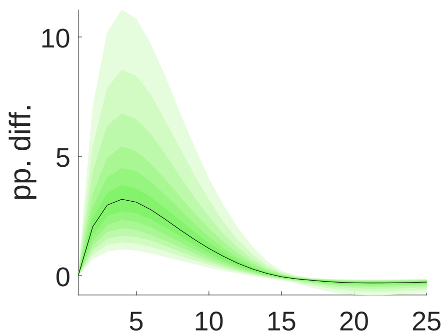
(d) Posterior predictive, 2 years FG



(e) Prior predictive, 3 years FG



(f) Posterior predictive, 3 years FG



Note: The figure shows the 66 percent interval around the median response to the QE shock accompanied by one year of FG. The separated areas (from weak to strongly shaded) reflect 5.$\bar{5}$% percentiles. The simulations are carried out under the assumption of perfect foresight.

of the inflation path than the posterior predictive analysis. This is evident in the second and third row of Figure 4 where prior and posterior predictive analyses for the inflation response for two and three years, respectively, of FG are shown. In these cases, both the median peak responses (3.73 pp and 3.98 pp for two and three years, respectively, of FG) and the ranges of the distribution at the peak (12.93 pp and 13.19 pp for two and three years, respectively, of FG) for the prior predictive analysis are higher than the median peak responses (3.16 pp and 3.20 pp for two and three years, respectively, of FG) and the ranges of the distribution at the peak (8.47 pp and 10.03 pp for two and three years, respectively, of FG) for the posterior predictive analysis but the difference in both cases is less pronounced than in the case of one year of FG. That is, for longer durations of FG, the FG puzzle is clearly evident also in the posterior predictive analysis. In any case, the uncertainty surrounding the effects of the QE programme accompanied by FG on inflation, in general, is substantial.

Comparing Figure 3(a) with Figures 4(a), 4(c) and 4(e), and Figure 3(b) with Figures 4(b), 4(d) and 4(f), it is obvious that the parametrisation of the model cannot explain the upward shift in the median path of inflation and the much wider distribution of the inflation path – it is the same within the left and right columns of Figures 3 and 4 – but can only be explained by the peg. That said, it is nevertheless useful to examine the role of the parameters for two reasons: (i) Since in the presented figures all parameters vary across draws, it is not clear what role each single parameter plays for the distribution, and thus parameter uncertainty, of the inflation response within each figure, that is, given a duration of FG and given the type of predictive analysis (prior or posterior). (ii) Figures 4(a) and 4(b) show that there is a pronounced difference in the median inflation response as well as the width of the distribution across the type of predictive analysis. Since this difference cannot be explained by the duration of FG – it is the same across the two figures – it must be the case that the parametrisation drives this result but it is not clear which parameters contribute most to this observation. Both issues (i) and (ii) can be examined within the framework of predictive analysis. We deal with issue (i) in Subsection 5.2 and with issue (ii) in Subsection 5.3.

5.2 The role of individual parameters for uncertainty

Following Leeper, Traum and Walker (2017), we derive a measure indicating which parameters are most important for our results. In particular, we ask to which extent each estimated parameter contributes to uncertainty for a given interest rate peg, within each predictive analysis. To this end, we proceed as follows. For each draw $j \in [1, J]$ of the parameters, $\vartheta^j = \left[\vartheta_1^j \ldots \vartheta_i^j \ldots \vartheta_n^j\right]'$, we simulate the model and extract the peak response of inflation to the QE shock from these simulations. In this way, we obtain a distribution of peak effects. Then, for each draw we simulate the model again using a new parameter vector $\vartheta^{i,j} = \left[\vartheta_1^j \ldots \tilde{\vartheta}_i \ldots \vartheta_n^j\right]'$, where $\tilde{\vartheta}_i$ denotes the i^{th} parameter that is now fixed at the median for each draw j. The only difference between ϑ^j and $\vartheta^{i,j}$ is thus the change from ϑ_i^j to $\tilde{\vartheta}_i$. We again extract the peak response of inflation to the QE shock from these new simulations and obtain a new distribution of peak effects. We do this for all estimated structural parameters of the model $i = 1, ..., n$. We then calculate the root mean squared error $RMSE_i = \sqrt{\frac{\sum_{j=1}^{J}(peak^j - peak_i^j)^2}{J}}$ for each of the parameters, where $peak_i^j$ denotes the peak effect of inflation for draw j when the ith parameter is fixed. The parameter with the highest RMSE has the biggest impact on the variation of the peak response of inflation to a QE shock. We calculate the RMSEs both for the prior and the posterior predictive analyses for zero, one, two, and three years of FG.

Table 5: RMSEs for peak response of inflation for the prior predictive analyses when a given parameter is fixed at its prior median and different durations of FG

(a) 0 years FG

η	1.188
h	0.308
ρ	0.306
τ_π	0.292
θ_w	0.279
θ_p	0.205
ψ_N	0.138
ι_w	0.080
τ_y	0.069
ι_p	0.030

(b) 1 year FG

θ_p	9.248
θ_w	8.408
τ_π	7.289
τ_y	5.902
h	5.666
ρ	5.620
ι_p	2.525
ψ_N	2.334
ι_w	2.316
η	1.587

(c) 2 years FG

θ_w	14.743
θ_p	12.188
τ_y	8.199
ι_w	6.919
ι_p	6.050
ρ	5.591
h	5.066
τ_π	5.050
η	3.378
ψ_N	0.554

(d) 3 years FG

θ_w	66.273
τ_y	16.768
ρ	14.671
ι_p	13.186
ι_w	8.437
τ_π	7.027
θ_p	6.559
h	4.194
η	3.280
ψ_N	0.922

Table 6: RMSEs for peak response of inflation for the posterior predictive analyses when a given parameter is fixed at its posterior median and different durations of FG

(a) 0 years FG

θ_w	0.065
θ_p	0.042
ρ	0.042
τ_π	0.028
h	0.028
τ_y	0.018
ι_w	0.013
η	0.011
ι_p	0.008
ψ_N	0.002

(b) 1 year FG

θ_w	0.214
θ_p	0.169
τ_y	0.100
τ_π	0.092
h	0.068
ρ	0.052
ι_w	0.041
ι_p	0.017
η	0.016
ψ_N	0.004

(c) 2 years FG

θ_w	8.242
θ_p	7.052
h	6.371
τ_y	6.013
ι_p	4.487
ι_w	4.307
η	2.275
ρ	2.006
τ_π	1.736
ψ_N	0.055

(d) 3 years FG

θ_w	8.665
h	6.324
θ_p	6.012
τ_y	5.518
ι_w	4.983
ι_p	4.557
τ_π	3.404
η	2.944
ρ	1.960
ψ_N	0.160

Table 5 shows the RMSEs for the peak effect of inflation for the prior predictive analyses when a given parameter is fixed at its prior median and different durations of FG. The parameters are sorted in descending order according to their RMSEs. Although there is variation in the ranking for different durations of FG, the broad picture is that the Calvo parameters θ_p and θ_w which govern the probability of resetting prices and wages, respectively, play the most important role in determining the uncertainty about the peak response of inflation to a QE shock. In contrast, ψ_N, the parameter governing net worth adjustment costs, has comparably little influence on the uncertainty about the peak inflation response to a QE shock. The latter observation is noteworthy since ψ_N greater than zero is a prerequisite for QE to have real effects in the model. A similar picture emerges for the peak effect of inflation for the posterior predictive analyses when one parameter is fixed at its posterior median, shown in Table 6. Across all durations of FG, the Calvo parameters have a strong influence on the peak effect of inflation and hence on the uncertainty about the effect, while, again, ψ_N has comparably little impact.

The result that the financial friction parameter is of minor importance for the outcomes of the policy scenarios is in line with other studies analysing the role of financial frictions for the dynamics of macroeconomic variables. For example, Suh and Walker (2016) find that the New Keynesian model with financial frictions is not capable of reproducing the empirically observed dynamics between financial and non-financial variables. That said, it is not obvious what the reason is for this observation. For example, it could be due to the model structure per se or to estimating the linear version of the model.[12] However, our approach to analyse parameter uncertainty is in no way dependent on the specific model structure or the estimation of a linear version of the model.

[12] Although we estimate the linear version of the model, our simulations are based on the fully non-linear version of the model with different durations of interest rate pegs.

5.3 Evaluating differences between prior and posterior predictive analysis

Figures 4(a) and 4(b) show that there is a marked difference in the median inflation response as well as the width of the distribution between the prior and the posterior predictive analysis of the inflation response for one year of FG.[13] Since this difference cannot be explained by the duration of FG this suggests that the strength of the FG puzzle depends on the parametrisation. Put differently, the estimation carries the model into parameter regions where the FG puzzle is not as apparent as in those parameter regions implied by prior distributions for one year of FG. To evaluate the relative influence of each single parameter on this result, we again apply the procedure described in subsection 5.2 but vary the experiment in one important aspect: Instead of fixing a given parameter to the median of the posterior distribution when conducting the posterior predictive analysis, we now fix a given parameter to the median of the prior distribution when conducting the posterior predictive analysis. We then again compare, draw by draw, the peak of the inflation response thus obtained with the peak of the inflation response obtained when all parameters are varied by computing the RMSEs for the different parameters. In this way, we are able to determine the relative influence of the parameters on the difference between the results of the prior and the posterior predictive analysis for one year of FG, that is, on producing the FG puzzle.

Table 7 shows the RMSEs for the posterior predictive analysis in the case of one year FG when a given parameter is fixed at its prior median, sorted in descending

[13] As described in section 5.1, this difference is also apparent for two and three years of FG, albeit to a less pronounced degree.

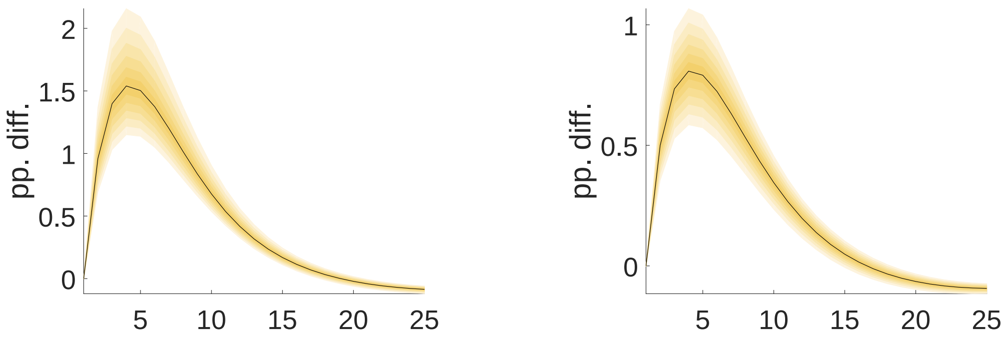
Table 7: RMSEs for peak response of inflation for the posterior predictive analysis when a given parameter is fixed at its prior median and one year of FG

θ_w	1.114
h	0.643
θ_p	0.638
τ_{pi}	0.154
ι_w	0.141
τ_y	0.119
ρ	0.060
ψ_N	0.028
ι_p	0.019
η	0.017

order according to their RMSEs. We observe that θ_w has the highest RMSE and thus the strongest influence on the different results across the prior and the posterior predictive analysis for one year of FG, whereas η has the lowest influence. ψ_N, the parameter governing net worth adjustment costs, has, once again, only little influence. We visualise this result with Figure 5 that shows the distribution of the inflation response to a QE shock produced by a posterior predictive analysis when θ_w (Figure 5(a)) and η (Figure 5(b)) are fixed at their prior median. The change of η in the posterior predictive analysis to its prior median in each draw of parameters essentially produces the same picture as in Figure 4(b), that is, the FG puzzle is apparent only very moderately. To be precise, the median peak of the inflation response changes minimally from 0.8083 pp in Figure 4(b) to 0.8079 pp in Figure 5(b), and the range of the distribution at the peak changes only to a small degree, from 0.4792 pp in Figure 4(b) to 0.4817 pp in Figure 5(b). In contrast, the change of θ_w in the posterior predictive analysis to its prior median in each draw of parameters produces a much higher median peak response of inflation of 1.5396 pp (an increase of around 90%) and a much wider range of the distribution at the

Figure 5: Inflation response to QE shock for one year FG, posterior predictive analysis with θ_w (left) and η (right) fixed at prior median.

(a) θ_w (b) η



Note: The figure shows the 66 percent interval around the median response to the QE shock accompanied by one year of FG. The separated areas (from weak to strongly shaded) reflect $5.\bar{5}$% percentiles. The simulations are carried out under the assumption of perfect foresight.

peak, namely 1.0074 pp. That is, the FG puzzle is much more pronounced and the result bears closer resemblance to the result of the prior predictive analysis for one year of FG in Figure 4(a).

Note that the ranking of the RMSEs in Table 7 is different from the ranking of RMSEs in Table 6(b) which shows the RMSEs for the posterior predictive analysis when a given parameter is fixed at its posterior mean in the case of one year of FG. This holds in general for all durations of FG. That is, it matters where a given parameter is fixed in the framework of this type of predictive analysis, and the decision of where to fix a given parameter depends on the question the researcher seeks to answer.

6 Conclusion

We use prior and posterior predictive analysis to examine the extent of parameter uncertainty when analysing the macroeconomic effects of the Eurosystem's PSPP and FG. Prior and posterior predictive analysis provides a way of carrying over the information about parameter uncertainty gained in the estimation of a model to a policy scenario analysis involving the solution of a non-linear model (in our case under the assumption of perfect foresight) with an anticipated interest rate peg, and allows for identifying the driving parameters behind this uncertainty.

We find that the uncertainty about the effects of the PSPP is considerable and increases when it is accompanied by FG. The Calvo parameters – which govern the probability, in a given period, of being able to reset prices and wages – have the biggest influence in determining the uncertainty about the inflation effects of a QE shock while the financial friction in the form of net worth adjustment costs is much less important.

In general, the method of using predictive analysis for illustrating parameter uncertainty in policy scenarios is very flexible. In particular, its use does not depend on a specific model structure, on estimating a linear or non-linear version of a model, or on solving and simulating the model in a linear or non-linear setup.

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Appendix A Model derivation

In our analysis we employ the model of Carlstrom et al. (2017).

A.1 Households and bond market structure

A.1.1 households' intertemporal consumption decision

Households maximise their intertemporal utility:

$$E_t \sum_{s=0}^{\infty} \beta^s b_{t+s} \left\{ \ln\left(C_{t+s} - hC_{t+s-1}\right) - B \frac{H_{t+s}^{1+\eta}(j)}{1+\eta} \right\},$$

where C_t is consumption, h is habit formation, $H_t(j)$ is the individual labour input from household j, and b_t is a shock to the discount factor. Lifetime utility would evaluate to:

$$V_t^h = b_t \left\{ \ln\left(C_t - hC_{t-1}\right) - D_t^w B \frac{H_t^{1+\eta}}{1+\eta} \right\} + \beta E_t V_{t+1}^h$$

The law of motion for capital is:

$$K_t \leq (1-\delta) K_{t-1} + I_t$$

Based on the households' nominal liability,

$$F_{t-1} = CI_{t-1} + \kappa CI_{t-2} + \kappa^2 CI_{t-3} + ...,$$

one can show that $CI_t = (F_t - \kappa F_{t-1})$, where CI_t is the number of bonds newly issued, and F_t is the households' nominal liability on new issues. New investments must be financed by issuing sufficient long term investment bonds which are purchased by the FI. Perpetual bonds are used with cash flows of $1, \kappa, \kappa^2$, etc.

The loan in advance constraint can be written as:

$$P_t^k I_t \leq \frac{Q_t\left(F_t - \kappa F_{t-1}\right)}{P_t} \left(= \frac{Q_t C I_t}{P_t}\right),$$

where Q_t is the time-t price of a new issue, P_t is the price level and P_t^k is the real price of capital. Moreover, the usual budget constraint is given by:

$$\overbrace{C_t + \underbrace{\frac{D_t}{P_t}}_{\substack{\text{HH real}\\ \text{deposits}}} + P_t^k I_t + \underbrace{\frac{F_{t-1}}{P_t}}_{\substack{\text{HH real liability}\\ \text{on past issues}}}}^{\text{Expenditure Side}} \leq W_t H_t + R_t^k K_t - T_t + \frac{D_{t-1}}{P_t} R_{t-1}^d + \underbrace{\frac{Q_t\left(F_t - \kappa F_{t-1}\right)}{P_t}}_{\substack{\text{HH newly issued real}\\ \text{investment bonds}}} + div_t$$

A.1.2 Households' Lagrangian

The corresponding Lagrangian maximising household utility is:

$$\mathcal{L} = E_t \sum_{s=0}^{\infty} \beta^s \left[\begin{array}{l} b_{t+s} \left\{ \ln\left(C_{t+s} - hC_{t+s-1}\right) - B \frac{H_{t+s}^{1+\eta}(j)}{1+\eta} \right\} \\ -\Lambda_{t+s} \left(\begin{array}{l} C_{t+s} + \frac{D_{t+s}}{P_{t+s}} + P_{t+s}^k I_{t+s} + \frac{F_{t+s-1}}{P_{t+s}} - W_{t+s} H_{t+s} - R_{t+s}^k K_{t+s} + T_{t+s} \\ -\frac{D_{t+s-1}}{P_{t+s}} R_{t+s-1}^d - \frac{Q_{t+s}(F_{t+s} - \kappa F_{t+s-1})}{P_{t+s}} - div_{t+s} \end{array} \right) \\ -\Lambda_{t+s}^K \left(K_{t+s} - (1-\delta) K_{t+s-1} - I_{t+s}\right) \\ \underbrace{- \vartheta_{t+s} \left(P_{t+s}^k I_{t+s} - \frac{Q_{t+s}\left(F_{t+s} - \kappa F_{t+s-1}\right)}{P_{t+s}} \right)}_{\text{Loan in advance constraint}} \end{array} \right]$$

The first-order conditions evaluate to:

$$\frac{\partial \mathcal{L}}{\partial C_t} : \Lambda_t = \frac{b_t}{C_t - hC_{t-1}} - E_t \frac{\beta h b_{t+1}}{C_{t+1} - hC_t}$$

$$\frac{\partial \mathcal{L}}{\partial D_t} : \Lambda_t = E_t \beta \frac{\Lambda_{t+1}}{\Pi_{t+1}} R_t^d \quad \text{with} \quad \Pi_{t+1} = \frac{P_{t+1}}{P_t}$$

$$\frac{\partial \mathcal{L}}{\partial I_t} : \Lambda_t^K = \vartheta_t P_t^k + \Lambda_t P_t^k = (\vartheta_t + \Lambda_t) P_t^k = M_t \Lambda_t P_t^k$$

$$\frac{\partial \mathcal{L}}{\partial F_t} : \Lambda_t M_t Q_t = E_t \frac{\beta \Lambda_{t+1} (1 + \kappa Q_{t+1} \mathrm{M}_{t+1})}{\Pi_{t+1}},$$

with $\quad \mathrm{M}_t = 1 + \frac{\vartheta_t}{\Lambda_t} \quad$ or $\quad \Lambda_t \mathrm{M}_t = \Lambda_t + \vartheta_t$.

$$\frac{\partial \mathcal{L}}{\partial K_t} : \Lambda_t M_t P_t^k = E_t \beta \Lambda_{t+1} \left[R_{t+1}^k + M_{t+1} P_{t+1}^k (1 - \delta) \right],$$

A.1.3 Financial intermediaries

The FI choose dividends div_t and their net worth N_t to maximise the value function:

$$V_t = E_t \sum_{s=0}^{\infty} (\beta \zeta)^s \Lambda_{t+s} div_{t+s}$$

where ζ is a parameter for additional impatience using the basic household kernel for discounting.

This maximisation is subject to the budget constraint which represents the law of motion for net worth, with

$$R_{t+1}^L \equiv \left(\frac{\overbrace{1}^{Coupon} + \overbrace{\kappa Q_{t+1}}^{\text{t + 1 Principal/face value of issues from t}}}{\underbrace{Q_t}_{\text{Market Price}}} \right)$$

$$div_t + N_t \underbrace{\left[1+f\left(N_t\right)\right]}_{\substack{\text{Diminishing net worth} \\ \text{by adjustment costs}}} \leq \underbrace{\frac{P_{t-1}}{P_t}\left[\underbrace{\left(R_t^L - R_{t-1}^d\right)L_{t-1}}_{\substack{\text{Earnings from leveraged net} \\ \text{worth: lending - deposits}}} + \underbrace{R_{t-1}^d}_{\substack{\text{For own net worth} \\ \text{no interest on deposit} \\ \text{has to be paid}}}\right]N_{t-1}}_{\text{Profit FI(Change in net worth)}}$$

The net worth adjustment costs which limit the ability of the FI to adjust their portfolio deviating from its steady state are:

$$f\left(N_t\right) \equiv \frac{\psi_n}{2}\left(\frac{N_t - N_{ss}}{N_{ss}}\right)^2$$

The according Lagrangian becomes:

$$\mathcal{L} = E_t \sum_{s=0}^{\infty} \left(\beta\zeta\right)^s \left[\Lambda_{t+s} div_{t+s} - \Lambda_{t+s}^N \left\{ \begin{array}{l} div_{t+s} + N_{t+s}\left[1+f\left(N_{t+s}\right)\right] - \\ \frac{P_{t+s-1}}{P_{t+s}}\left[\left(R_{t+s}^L - R_{t+s-1}^d\right)L_{t+s-1} + R_{t+s-1}^d\right]N_{t+s-1} \end{array} \right\}\right]$$

This yields the following first-order conditions:

$$\frac{\partial\mathcal{L}}{\partial div_t} : \Lambda_{t+s} = \Lambda_{t+s}^N$$

$$\frac{\partial\mathcal{L}}{\partial N_t} : \Lambda_t\left[1+f\left(N_t\right)+N_t f'\left(N_t\right)\right] = E_t\Lambda_{t+1}\beta\zeta\frac{P_t}{P_{t+1}}\left[\left(R_{t+1}^L - R_t^d\right)L_t + R_t^d\right]$$

The FIs are subject to a simple hold-up problem which limits their ability to attract deposits. When they choose to default they can seize a fraction μ_t from the household deposits. The incentive constraint for the FI not to default, because their income is greater than the assets they can keep in default, is:

$$\underbrace{E_t V_{t+1}}_{\substack{\text{Expected}\\ \text{future income}}} \geq \underbrace{\mu_t L_t N_t}_{\substack{\text{Fraction of their}\\ \text{balance sheet}}} \underbrace{E_t \Lambda_{t+1} \frac{P_t}{P_{t+1}} R^L_{t+1}}_{\substack{\text{Times next periods consumption}\\ \text{value plus earnings from lending}}}$$

The model can be calibrated for it to be binding. By choosing the fraction of assets the FI can keep in case of default to be

$$\mu_t = \Phi_t \left[1 + \frac{1}{N_t} E_t \left(\frac{g_{t+1}}{X_{t+1}} \right) \right],$$

with Φ_t an exogenous stochastic process that represents exogenous changes in the financial friction. It follows an AR(1) process:

$$\Phi_t = (1 - \rho_\Phi)\, \Phi_{ss} + \rho_\Phi \Phi_{t-1} + \varepsilon_{\Phi,t}.$$

Choosing this fraction ensures that leverage is a function independent of net worth. Hence, the FIs take leverage as given and we can aggregate the firms as they are just scaled equivalents. g_t is a function of current and forecasted market spreads z_t independent of N_{t-1}. Confirming the leverage equation, it follows:

$$E_t \frac{P_t}{P_{t+1}} \Lambda_{t+1} \left[\left(\frac{R^L_{t+1}}{R^d_t} - 1 \right) L_t + 1 \right] = \Phi_t L_t E_t \Lambda_{t+1} \frac{P_t}{P_{t+1}} \frac{R^L_{t+1}}{R^d_t}$$

$$\Leftrightarrow L_t = \frac{E_t \frac{\Lambda_{t+1}}{\Pi_{t+1}}}{\left[E_t \frac{\Lambda_{t+1}}{\Pi_{t+1}} + (\Phi_t - 1)\, E_t \frac{\Lambda_{t+1}}{\Pi_{t+1}} \frac{R^L_{t+1}}{R^d_t} \right]}$$

Using the derivation

$$\frac{\partial L_t}{\partial R^L_{t+1}} = \frac{-\frac{(\Phi_t - 1)}{R^d_t}}{\left[1 + (\Phi_t - 1) \frac{R^L_{t+1}}{R^d_t} \right]^2} \geq 0 \quad \text{for} \quad \Phi_t < 1,$$

this can be simplified to

$$L_t = \frac{1}{\left[1 + (\Phi_t - 1)\, E_t \frac{R^L_{t+1}}{R^d_t}\right]}.$$

Regarding the balance sheet of the FI and its composition, leveraged net worth is divided into holdings of long term government bonds and investment bonds:

$$N_t L_t = \overline{B}_t + \overline{F}_t,$$

with $\overline{B}_t \equiv Q_t \frac{B_t}{P_t}$ and $\overline{F}_t \equiv Q^I_t \frac{F_t}{P_t}$.

The time-t asset value of current and past issues of investment is:

$$Q_t F_t = Q_t CI_t + \kappa Q_t \left[CI_{t-1} + \kappa CI_{t-2} + \kappa^2 CI_{t-3}\right],$$

where the time-t price of the perpetuity issued in t-1 is κQ_t.

A.1.4 Term premium and price of capital mark-up

Rewriting the log-linearised version of the households' first-order condition with respect to K_t yields:

$$\lambda_t + p^k_t + m_t = E_t \left\{\lambda_{t+1} + [1 - \beta(1-\delta)]\, r^k_{t+1} + \beta(1-\delta)\left(p^k_{t+1} + m_{t+1}\right)\right\}$$

From the log-linearised version of the households first-order condition with respect to D_t, we know that $E_t\lambda_{t+1} - \lambda_t = E_t\pi_{t+1} - r_t$, and hence

$$p^k_t + m_t = E_t \left\{[1 - \beta(1-\delta)]\, r^k_{t+1} - (r_t - \pi_{t+1}) + \beta(1-\delta)\left(p^k_{t+1} + m_{t+1}\right)\right\}.$$

Iterative substitution then yields the mark-up character of m_t on the price of capital p_t^k:

$$p_t^k + m_t = E_t \sum_{j=0}^{\infty} [\beta(1-\delta)]^j \left\{ [1-\beta(1-\delta)] r_{t+j+1}^k - (r_{t+j} - \pi_{t+j+1}) \right\}$$

Similarly, one can show that iterative substitution can also be applied to the log-linearised form of the households first-order condition with respect to F_t, which then can be written as:

$$m_t = E_t \sum_{j=0}^{\infty} [\beta\kappa]^j \{\beta\kappa q_{t+j+1} - q_{t+j} - r_{t+j}\}.$$

And since $r_{t+1}^L = \frac{\kappa q_{t+1}}{R^L} - q_t = \frac{\beta\zeta}{\Pi}\kappa q_{t+1} - q_t \approx \beta\kappa q_{t+1} - q_t$, this can be written as the discounted sum of future loan to deposit spreads:

$$m_t \approx E_t \sum_{j=0}^{\infty} [\beta\kappa]^j \left\{ r_{t+j+1}^L - r_{t+j} \right\} = E_t \sum_{j=0}^{\infty} [\beta\kappa]^j \Xi_{t+j}$$

$$\Xi_{t+j} \equiv \beta\kappa q_{t+j+1}^i - q_{t+j}^i - r_{t+j} \approx r_{t+j+1}^L - r_{t+j}$$

A.2 Labour agencies

Perfectly competitive labour agencies combine differentiated labour inputs into a homogenous labour composite H_t according to the technology:

$$H_t = \left[\int_0^1 H_t(j)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dj \right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}}$$

where $\varepsilon_w \geq 1$ is the elasticity of substitution between different varieties of labour. The labour agencies purchase labour $H_t(j)$ at a nominal wage $W_t(j)$. Profit maximisation (i.e. cost minimisation) leads to the following problem:

$$\min_{H_t(j)} \int_0^1 W_t(j) H_t(j) dj$$

subject to (at least obtaining a bundle H_t):

$$\left[\int_0^1 H_t(j)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dj \right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}} \geq H_t$$

The corresponding Lagrangian is:

$$\mathcal{L} = \int_0^1 W_t(j) H_t(j) dj - \psi_t \left\{ \left[\int_0^1 H_t(j)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dj \right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}} - H_t \right\}$$

$$\frac{\partial \mathcal{L}}{\partial H_t(j)} : W_t(j) = \psi_t \left[\int_0^1 H_t(j)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dj \right]^{\frac{1}{\varepsilon_w - 1}} H_t(j)^{-\frac{1}{\varepsilon_w}}$$

$$\Leftrightarrow H_t(j) = \left(\frac{W_t(j)}{\psi_t} \right)^{-\varepsilon_w} H_t$$

Using the definition of H_t leads to:

$$H_t = \left[\int_0^1 \left(\left(\frac{W_t(j)}{\psi_t} \right)^{-\varepsilon_w} H_t \right)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dj \right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}}$$

$$\Leftrightarrow 1 = \left(\frac{1}{\psi_t}\right)^{-\varepsilon_w} \left[\int_0^1 W_t(j)^{1-\varepsilon_w} dj\right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}}$$

$$\Leftrightarrow \psi_t = \left[\int_0^1 W_t(j)^{1-\varepsilon_w} dj\right]^{\frac{1}{1-\varepsilon_w}} \equiv W_t$$

Plugging this into the demand function results in:

$$H_t(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\varepsilon_w} H_t$$

A.2.1 Optimal wage

Households are monopolistic suppliers of differentiated labour inputs $H_t(j)$ and set wages on a staggered basis (à la Calvo). In each period, the probability of resetting the wage is $(1 - \theta_w)$, while with the complementary probability (θ_w) the wage is automatically increased following the indexing rule:

$$W_t(j) = \Pi_{t-1}^{\iota_w} W_{t-1}(j)$$

The problem for a household j who can reset its wage at time t is:

$$\max_{W_t(j)} E_t \sum_{s=0}^{\infty} \theta_w^s \beta^s \left\{ \underbrace{-B\frac{H_{t+s}(j)^{1+\psi}}{1+\psi}}_{\text{Disutiliy of labour at t + s}} \underbrace{b_{t+s}}_{\substack{\text{discount}\\ \text{factor shock}}} \underbrace{\lambda_{w,t+s}}_{\substack{\text{markup}\\ \text{factor}}} \underbrace{+\Lambda_{t+s} \underbrace{\frac{W_t(j)}{P_{t+s}} H_{t+s}(j)}_{\text{real wage income at t + s}}}_{\text{Utility consequence of this income}} \right\}$$

The maximisation problem follows as:

$$\max_{W_t(j)} \Omega_t = E_t \sum_{s=0}^{\infty} \theta_w^s \beta^s \left\{ \begin{array}{l} -\lambda_{w,t+s} b_{t+s} \frac{B}{1+\psi} \left(\left(\frac{W_t(j) \left(\prod\limits_{k=1}^{s} \Pi_{t+k-1}^{\iota w} \right)}{W_{t+s}} \right)^{-\varepsilon_w} H_{t+s} \right)^{1+\psi} \\ +\Lambda_{t+s} \frac{W_t(j) \left(\prod\limits_{k=1}^{s} \Pi_{t+k-1}^{\iota w} \right)}{P_{t+s}} \left(\frac{W_t(j) \left(\prod\limits_{k=1}^{s} \Pi_{t+k-1}^{\iota w} \right)}{W_{t+s}} \right)^{-\varepsilon_w} H_{t+s} \end{array} \right\}$$

This can be rewritten in the following way:

$$\left\{ \begin{array}{l} -\lambda_{w,t+s} b_{t+s} \frac{B}{1+\psi} \left(\left(\frac{W_t(j) \left(\prod\limits_{k=1}^{s} \Pi_{t+k-1}^{\iota w} \right)}{W_{t+s}} \right)^{-\varepsilon_w} H_{t+s} \right)^{1+\psi} \\ +\Lambda_{t+s} \frac{W_t(j) \left(\prod\limits_{k=1}^{s} \Pi_{t+k-1}^{\iota w} \right)}{P_{t+s}} \left(\frac{W_t(j) \left(\prod\limits_{k=1}^{s} \Pi_{t+k-1}^{\iota w} \right)}{W_{t+s}} \right)^{-\varepsilon_w} H_{t+s} \end{array} \right\}$$

$$= E_t \sum_{s=0}^{\infty} \theta_w^s \beta^s \left\{ \begin{array}{l} -\lambda_{w,t+s} b_{t+s} \frac{B}{1+\psi} W_t(j)^{-\varepsilon_w(1+\psi)} \left(\left(\frac{\left(\prod\limits_{k=1}^{s} \Pi_{t+k-1}^{\iota w} \right)}{W_{t+s}} \right)^{-\varepsilon_w} H_{t+s} \right)^{1+\psi} \\ +\Lambda_{t+s} W_t(j)^{1-\varepsilon_w} \frac{\left(\prod\limits_{k=1}^{s} \Pi_{t+k-1}^{\iota w} \right)^{1-\varepsilon_w}}{P_{t+s}} W_{t+s}^{\varepsilon_w} H_{t+s} \end{array} \right\}$$

$$\frac{\partial \Omega_t}{\partial W_t(j)} : E_t \sum_{s=0}^{\infty} \theta_w^s \beta^s \left\{ \Lambda_{t+s} \left(1 - \varepsilon_w\right) W_t(j)^{-\varepsilon_w} \frac{1}{P_{t+s}} \left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota w} \right)^{1-\varepsilon_w} W_{t+s}^{\varepsilon_w} H_{t+s} \right\}$$

$$= E_t \sum_{s=0}^{\infty} \theta_w^s \beta^s \left\{ \lambda_{w,t+s} b_{t+s} B \left(-\varepsilon_w\right) W_t(j)^{-\varepsilon_w(1+\psi)-1} \left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota w} \right)^{-\varepsilon_w(1+\psi)} W_{t+s}^{\varepsilon_w(1+\psi)} H_{t+s}^{1+\psi} \right\}$$

$$\Leftrightarrow W_t(j)^{1+\varepsilon_w\psi} E_t \sum_{s=0}^{\infty} \theta_w^s \beta^s \left\{ \Lambda_{t+s} \frac{1}{P_{t+s}} \left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_w} \right)^{1-\varepsilon_w} W_{t+s}^{\varepsilon_w} H_{t+s} \right\}$$

$$= \frac{\varepsilon_w}{\varepsilon_w - 1} E_t \sum_{s=0}^{\infty} \theta_w^s \beta^s \left\{ \lambda_{w,t+s} b_{t+s} B \left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_w} \right)^{-\varepsilon_w(1+\psi)} W_{t+s}^{\varepsilon_w(1+\psi)} H_{t+s}^{1+\psi} \right\}$$

$$\Leftrightarrow W_t(j)^{1+\varepsilon_w\psi} = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{E_t \sum_{s=0}^{\infty} \theta_w^s \beta^s \left\{ \lambda_{w,t+s} b_{t+s} B \left[\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_w} \right)^{-\varepsilon_w} W_{t+s}^{\varepsilon_w} H_{t+s} \right]^{1+\psi} \right\}}{E_t \sum_{s=0}^{\infty} \theta_w^s \beta^s \left\{ \Lambda_{t+s} \frac{1}{P_{t+s}} \left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_w} \right)^{1-\varepsilon_w} W_{t+s}^{\varepsilon_w} H_{t+s} \right\}}$$

Rewritten in terms of real wages $\left(w_t = \frac{W_t}{P_t} \right)$:

$$W_t(j)^{1+\varepsilon_w\psi} \frac{1}{P_t^{1+\varepsilon_w\psi}} = \frac{\varepsilon_w}{\varepsilon_w - 1} \underbrace{\frac{1}{P_t^{1+\varepsilon_w\psi}}}_{=\frac{1}{P_t^{1-\varepsilon_w+\varepsilon_w(1+\psi)}}} \frac{E_t \sum_{s=0}^{\infty} \theta_w^s \beta^s \left\{ \lambda_{w,t+s} b_{t+s} B \left[\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_w} \right)^{-\varepsilon_w} W_{t+s}^{\varepsilon_w} H_{t+s} \right]^{1+\psi} \right\}}{E_t \sum_{s=0}^{\infty} \theta_w^s \beta^s \left\{ \Lambda_{t+s} \frac{1}{P_{t+s}} \left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_w} \right)^{1-\varepsilon_w} W_{t+s}^{\varepsilon_w} H_{t+s} \right\}}$$

$$\Leftrightarrow w_t(j)^{1+\varepsilon_w\psi} = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{E_t \sum_{s=0}^{\infty} \theta_w^s \beta^s \left\{ \lambda_{w,t+s} b_{t+s} B \left[\left(\frac{\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_w}}{\prod_{k=1}^{s} \Pi_{t+k}} \right)^{-\varepsilon_w} w_{t+s}^{\varepsilon_w} H_{t+s} \right]^{1+\psi} \right\}}{E_t \sum_{s=0}^{\infty} \theta_w^s \beta^s \left\{ \Lambda_{t+s} \left(\frac{\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_w}}{\prod_{k=1}^{s} \Pi_{t+k}} \right)^{1-\varepsilon_w} w_{t+s}^{\varepsilon_w} H_{t+s} \right\}} = {w_t}^{1+\varepsilon_w\psi}$$

All agents choose the same $w_t(j)$ as derived in the labour agencies first-order condition with respect to $H_t(j)$. Letting the numerator be X_t^{wn} and the denominator X_t^{wd}, then this equation can be rewritten as:

$${w_t}^{1+\varepsilon_w\psi} = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{X_t^{wn}}{X_t^{wd}},$$

where the numerator is:

$$X_t^{wn} =$$

$$\lambda_{w,t} b_t B w_t^{\varepsilon_w(1+\psi)} H_t^{1+\psi} + E_t \left\{ \theta_w \beta \underbrace{E_{t+1} \left[\begin{array}{l} \sum_{s=1}^{\infty} \theta_w^{s-1} \beta^{s-1} \lambda_{w,t+s} b_{t+s} B \\ \left(\prod_{k=1}^{s} \Pi_{t+k} \right)^{\varepsilon_w(1+\psi)} \left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_w} \right)^{-\varepsilon_w(1+\psi)} w_{t+s}^{\varepsilon_w(1+\psi)} H_{t+s}^{1+\psi} \end{array} \right]}_{= X_{t+1}^{wn} \Pi_{t+1}^{\varepsilon_w(1+\psi)} \Pi_t^{-\iota_w \varepsilon_w(1+\psi)}} \right\}$$

and the denominator:

$$X_t^{wd} = \Lambda_t w_t^{\varepsilon_w} H_t + E_t \left\{ \theta_w \beta \underbrace{E_{t+1} \left[\sum_{s=1}^{\infty} \theta_w^{s-1} \beta^{s-1} \Lambda_{t+s} \left(\frac{\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_w}}{\prod_{k=1}^{s} \Pi_{t+k}} \right)^{1-\varepsilon_w} w_{t+s}^{\varepsilon_w} H_{t+s} \right]}_{= X_{t+1}^{wd} \Pi_t^{-\iota_w(\varepsilon_w - 1)} \Pi_t^{(\varepsilon_w - 1)}} \right\}$$

The equation for $w_t(i) = w_t^*$ can be written in the following way:

$$(w_t^*)^{1+\varepsilon_w \psi} = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{X_t^{wn}}{X_t^{wd}}$$

The law of motion for wages then is:

$$W_t^{1-\varepsilon_w} = (1 - \theta_w) (W_t^*)^{1-\varepsilon_w} + \theta_w \left(\Pi_{t-1}^{\iota_w} W_{t-1} \right)^{1-\varepsilon_w}$$

$$\Leftrightarrow w_t^{1-\varepsilon_w} = (1 - \theta_w) (w_t^*)^{1-\varepsilon_w} + \theta_w \left(\frac{\Pi_{t-1}^{\iota_w} w_{t-1}}{\Pi_t} \right)^{1-\varepsilon_w}$$

A.2.2 Wage dispersion

From the demand for differentiated labour, we have differentiated labour supply from household j:

$$H_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\varepsilon_w} H_t$$

Taking the integral over households on both sides, we have:

$$\underbrace{\textstyle\int_0^1 H_t(j)dj}_{H_{ht}} = H_t \underbrace{\textstyle\int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\varepsilon_w} dj}_{D_{wt}} = H_t D_{wt}$$

Now regarding the evolution of D_{wt}, the period-t wage dispersion is:

$$D_{wt} = W_t^{\varepsilon_w}\left[\theta_w\left(\Pi_{t-1}^{\iota_w}\right)^{-\varepsilon_w}\frac{D_{wt-1}}{W_{t-1}^{\varepsilon_w}} + (1-\theta_w)(W_t{}^*)^{-\varepsilon_w}\right]$$

$$\Leftrightarrow D_{wt} = \theta_w\left(\Pi_{t-1}^{\iota_w}\right)^{-\varepsilon_w}\left(\frac{W_t}{W_{t-1}}\right)^{\varepsilon_w} D_{wt-1} + (1-\theta_w)\left(\frac{W_t{}^*}{W_t}\right)^{-\varepsilon_w}$$

$$\Leftrightarrow D_{wt} = \theta_w\left(\frac{\Pi_t}{\Pi_{t-1}^{\iota_w}}\right)^{\varepsilon_w}\left(\frac{w_t}{w_{t-1}}\right)^{\varepsilon_w} D_{wt-1} + (1-\theta_w)\left(\frac{w_t{}^*}{w_t}\right)^{-\varepsilon_w}$$

From the evolution of the aggregate wage index, we have:

$$W_t{}^{1-\varepsilon_w} = (1-\theta_w)\left(W_t^*\right)^{1-\varepsilon_w} + \theta_w\left(\Pi_{t-1}^{\iota_w} W_{t-1}\right)^{1-\varepsilon_w} \Leftrightarrow \left(\frac{W_t^*}{W_t}\right)^{-\varepsilon_w} = \left[\frac{1-\theta_w\left(\Pi_{t-1}^{\iota_w}\frac{W_{t-1}}{W_t}\right)^{1-\varepsilon_w}}{1-\theta_w}\right]^{\frac{-\varepsilon_w}{1-\varepsilon_w}}$$

Substituting this into the evolution of wage dispersion yields:

$$D_{wt} = \theta_w\left(\Pi_{t-1}^{\iota_w}\right)^{-\varepsilon_w}\left(\frac{W_t}{W_{t-1}}\right)^{\varepsilon_w} D_{wt-1} + (1-\theta_w)^{\frac{1}{1-\varepsilon_w}}\left[1-\theta_w\left(\Pi_{t-1}^{\iota_w}\frac{W_{t-1}}{W_t}\right)^{1-\varepsilon_w}\right]^{\frac{\varepsilon_w}{\varepsilon_w-1}}$$

Finally, rewriting this in terms of real wages:

$$D_{wt} = \theta_w \left(\Pi_{t-1}^{\iota_w}\right)^{-\varepsilon_w} \left(\frac{w_t}{w_{t-1}}\Pi_t\right)^{\varepsilon_w} D_{wt-1} + (1-\theta_w)^{\frac{1}{1-\varepsilon_w}} \left[1 - \theta_w \left(\Pi_{t-1}^{\iota_w} \frac{w_{t-1}}{w_t \Pi_t}\right)^{1-\varepsilon_w}\right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}}$$

A.3 Goods market

A.3.1 Final goods producers

Perfectly competitive final goods producers combine differentiated intermediate goods $Y_t(i)$ into a homogeneous good Y_t according to the technology:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} di\right]^{\frac{\varepsilon_p}{\varepsilon_p - 1}}$$

The final goods producers buy the intermediate goods on the market, package Y_t, and resell it to consumers. These firms maximise profits in a perfectly competitive environment. Their optimisation problem (cost minimisation) is:

$$\min_{Y_t(i)} \int_0^1 P_t(i) Y_t(i) di$$

subject to (at least obtaining a bundle Y_t):

$$\left[\int_0^1 Y_t(i)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} di\right]^{\frac{\varepsilon_p}{\varepsilon_p - 1}} \geq Y_t$$

Thus, the Lagrangian is:

$$\mathcal{L} = \int_0^1 P_t(i) Y_t(i) di - \Psi_t \left(\left[\int_0^1 Y_t(i)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} di\right]^{\frac{\varepsilon_p}{\varepsilon_p - 1}} - Y_t\right)$$

The first order condition w.r.t. $Y_t(i)$ is:

$$\frac{\partial \mathcal{L}}{\partial Y_t(i)} = P_t(i) - \Psi_t \left(\frac{\varepsilon_p}{\varepsilon_p - 1} \left[\int_0^1 Y_t(i)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} di \right]^{\frac{\varepsilon_p}{\varepsilon_p - 1} - 1} \frac{\varepsilon_p - 1}{\varepsilon_p} Y_t(i)^{\frac{\varepsilon_p - 1}{\varepsilon_p} - 1} \right) = 0$$

$$\Leftrightarrow P_t(i) - \Psi_t \left(\underbrace{\left[\int_0^1 Y_t(i)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} di \right]^{\frac{1}{\varepsilon_p - 1}}}_{Y_t^{\frac{1}{\varepsilon_p}}} Y_t(i)^{-\frac{1}{\varepsilon_p}} \right) = 0$$

$$\Leftrightarrow Y_t(i) = \left(\frac{P_t(i)}{\Psi_t} \right)^{-\varepsilon_p} Y_t,$$

which is the demand function.

Using the definition of Y_t leads to:

$$Y_t = \left[\int_0^1 \left(\left(\frac{P_t(i)}{\Psi_t} \right)^{-\varepsilon_p} Y_t \right)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} di \right]^{\frac{\varepsilon_p}{\varepsilon_p - 1}}$$

$$\Leftrightarrow \Psi_t = \left[\int_0^1 P_t(i)^{1-\varepsilon_p} di \right]^{\frac{1}{1-\varepsilon_p}} \equiv P_t$$

Plugging this into the demand function results in:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon_p} Y_t$$

A.3.2 Intermediate goods producers

A continuum of monopolistically competitive firms combines capital K_{t-1} and labour H_t to produce intermediate goods according to a standard Cobb-Douglas technology.

The production function is given by:

$$Y_t(i) = A_t K_{t-1}(i)^{\alpha} H_t(i)^{1-\alpha}$$

The firms minimise their cost

$$\min\left\{\frac{W_t}{P_t}H_t(i)+R_t^kK_{t-1}(i)\right\}$$

subject to their production function, such that the corresponding Lagrangian reads:

$$\mathcal{L}\;=\;\frac{W_t}{P_t}H_t(i)+R_t^kK_{t-1}(i)+\nu_t(i)\left[Y_t(i)-A_tK_{t-1}(i)^\alpha H_t(i)^{1-\alpha}\right]$$

Thus, the firms choose labour and capital as follows:

$$\frac{\partial\,\mathcal{L}_t}{\partial\,H_t\,(i)}=\frac{W_t}{P_t}-\nu_t\,(i)\underbrace{(1-\alpha)\,A_tK_{t-1}(i)^\alpha H_t(i)^{-\alpha}}_{\mathrm{MPL}(i)_t}=0$$

$$\frac{\partial\,\mathcal{L}_t}{\partial\,K_{t-1}\,(i)}\;=\;R_t^k-\nu_t\,(i)\underbrace{\alpha A_tK_{t-1}(i)^{\alpha-1}H_t(i)^{1-\alpha}}_{MPK(i)_t}=0$$

As intermediate result we get the marginal product of labour (MPL) and capital (MPK), respectively. Solving the derivative w.r.t. K_{t-1} for $\nu_t(i)$ and putting the corresponding equation into the derivative w.r.t. L_t yields:

$$\frac{K_{t-1}(i)}{H_t(i)}=\frac{\alpha}{(1-\alpha)}\frac{W_t}{P_tR_t^k}$$

Real marginal costs are derived as the shadow price of production $\nu_t(i)$. From the derivative w.r.t. H_t we have:

$$\nu_t(i)=\frac{1}{(1-\alpha)\,A_t}\left(\frac{K_{t-1}(i)}{H_t(i)}\right)^{-\alpha}\frac{W_t}{P_t}$$

Then plugging in the optimal capital-labour ratio from above, we get:

$$\nu_t(i) = \alpha^{-\alpha} (1-\alpha)^{-(1-\alpha)} \frac{\left(\frac{W_t}{P_t}\right)^{1-\alpha} \left(R_t^k\right)^{\alpha}}{A_t} = mc_t(i) = \frac{MC_t(i)}{P_t}$$

A.3.3 Optimal price setting

The intermediate goods producers set prices based on Calvo contracts. In each period firms adjust their prices with probability $(1-\theta_p)$ independently form previous adjustments. However, we depart from Calvo in the following way: For those firms that cannot adjust their prices in a given period, prices will be reset according to the following indexation rule:

$$P_t(i) = \Pi_{t-1}^{\iota_p} P_{t-1}(i),$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$ is gross inflation.

The firms that adjust their prices face the following problem:

$$\max_{P_t(i)} \Omega_t = E_t \sum_{s=0}^{\infty} \theta_p^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_t} \left[\frac{P_t(i) \left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_p} \right)}{P_{t+s}} Y_{t+s}(i) - \frac{W_{t+s}}{P_{t+s}} H_{t+s}(i) - R_{t+s}^k K_{t-1+s}(i) \right],$$

with demand given by:

$$Y_{t+s}(i) = \left(\frac{P_t(i) \left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_p} \right)}{P_{t+s}} \right)^{-\varepsilon_p} Y_{t+s}.$$

The optimisation problem is:

$$\max_{P_t(i)} \Omega_t = E_t \sum_{s=0}^{\infty} \theta_p^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_t} \left[\frac{P_t(i) \left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_p} \right)}{P_{t+s}} - \lambda_{p,t+s} mc_{t+s}(i) \right] Y_{t+s}(i)$$

Plugged in aggregate demand:

$$E_t \sum_{s=0}^{\infty} \theta_p^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_t} \left[\frac{P_t(i) \left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_p} \right)}{P_{t+s}} - \lambda_{p,t+s} mc_{t+s}(i) \right] \left(\frac{P_t(i) \left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_p} \right)}{P_{t+s}} \right)^{-\varepsilon_p} Y_{t+s}$$
$$= E_t \sum_{s=0}^{\infty} \theta_p^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_t} \left[P_t(i)^{1-\varepsilon_p} \left(\frac{\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_p} \right)}{P_{t+s}} \right)^{1-\varepsilon_p} - \lambda_{p,t+s} mc_{t+s}(i) P_t(i)^{-\varepsilon_p} \left(\frac{\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_p} \right)}{P_{t+s}} \right)^{-\varepsilon_p} \right] Y_{t+s}$$

and taking the derivative w.r.t. $P_t(i)$ - this leads to:

$$E_t \sum_{s=0}^{\infty} \theta_p^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_t} \left(\frac{\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_p} \right)}{P_{t+s}} \right)^{-\varepsilon_p} Y_{t+s} \left[(1-\varepsilon_p) \left(\frac{\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_p} \right)}{P_{t+s}} \right) \right]$$

$$= E_t \sum_{s=0}^{\infty} \theta_p^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_t} \left(\frac{\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_p} \right)}{P_{t+s}} \right)^{-\varepsilon_p} Y_{t+s} \left[\lambda_{p,t+s} mc_{t+s}(i) \left(-\varepsilon_p\right) P_t(i)^{-1} \right]$$

$$\Leftrightarrow P_t(i) = \left(\frac{\varepsilon_p}{\varepsilon_p - 1}\right) \frac{E_t \sum\limits_{s=0}^{\infty} \theta_p^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_t} \left(\frac{\left(\prod\limits_{k=1}^{s} \Pi_{t+k-1}^{\iota p}\right)}{P_{t+s}}\right)^{-\varepsilon_p} Y_{t+s} \lambda_{p,t+s} mc_{t+s}(i)}{E_t \sum\limits_{s=0}^{\infty} \theta_p^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_t} \left(\frac{\left(\prod\limits_{k=1}^{s} \Pi_{t+k-1}^{\iota p}\right)}{P_{t+s}}\right)^{-\varepsilon_p} Y_{t+s} \frac{\left(\prod\limits_{k=1}^{s} \Pi_{t+k-1}^{\iota p}\right)}{P_{t+s}}}$$

And since $P_{t+s} = P_t \prod\limits_{k=1}^{s} \Pi_{t+k}$:

$$P_t(i) = P_t \left(\frac{\varepsilon_p}{\varepsilon_p - 1}\right) \left(\frac{P_t}{P_t}\right)^{-\varepsilon_p} \frac{E_t \sum\limits_{s=0}^{\infty} \theta_p^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_t} \left(\frac{\left(\prod\limits_{k=1}^{s} \Pi_{t+k-1}^{\iota p}\right)}{\prod\limits_{k=1}^{s} \Pi_{t+k}}\right)^{-\varepsilon_p} Y_{t+s} \lambda_{p,t+s} mc_{t+s}(i)}{E_t \sum\limits_{s=0}^{\infty} \theta_p^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_t} \left(\frac{\left(\prod\limits_{k=1}^{s} \Pi_{t+k-1}^{\iota p}\right)}{\prod\limits_{k=1}^{s} \Pi_{t+k}}\right)^{-\varepsilon_p} Y_{t+s} \frac{\left(\prod\limits_{k=1}^{s} \Pi_{t+k-1}^{\iota p}\right)}{\prod\limits_{k=1}^{s} \Pi_{t+k}}}$$

$$\Leftrightarrow \underbrace{\frac{P_t(i)}{P_{t-1}}}_{=\Pi_t^*} \underbrace{\frac{P_{t-1}}{P_t}}_{=\Pi_t^{-1}} = \left(\frac{\varepsilon_p}{\varepsilon_p - 1}\right) \frac{E_t \sum\limits_{s=0}^{\infty} \theta_p^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_t} \left(\frac{\left(\prod\limits_{k=1}^{s} \Pi_{t+k-1}^{\iota p}\right)}{\prod\limits_{k=1}^{s} \Pi_{t+k}}\right)^{-\varepsilon_p} Y_{t+s} \lambda_{p,t+s} mc_{t+s}(i)}{E_t \sum\limits_{s=0}^{\infty} \theta_p^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_t} \left(\frac{\left(\prod\limits_{k=1}^{s} \Pi_{t+k-1}^{\iota p}\right)}{\prod\limits_{k=1}^{s} \Pi_{t+k}}\right)^{-\varepsilon_p} Y_{t+s} \frac{\left(\prod\limits_{k=1}^{s} \Pi_{t+k-1}^{\iota p}\right)}{\prod\limits_{k=1}^{s} \Pi_{t+k}}}$$

$$\Leftrightarrow \Pi_t^* = \left(\frac{\varepsilon_p}{\varepsilon_p - 1}\right) \frac{E_t \sum\limits_{s=0}^{\infty} \theta_p^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_t} \left(\frac{\left(\prod\limits_{k=1}^{s} \Pi_{t+k-1}^{\iota p}\right)}{\prod\limits_{k=1}^{s} \Pi_{t+k}}\right)^{-\varepsilon_p} Y_{t+s} \lambda_{p,t+s} mc_{t+s}(i)}{E_t \sum\limits_{s=0}^{\infty} \theta_p^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_t} \left(\frac{\left(\prod\limits_{k=1}^{s} \Pi_{t+k-1}^{\iota p}\right)}{\prod\limits_{k=1}^{s} \Pi_{t+k}}\right)^{1-\varepsilon_p} Y_{t+s}} \Pi_t$$

Each of the parts of this equation can be defined as follows:

$$X_t^{pd} = E_t \sum_{s=0}^{\infty} \theta_p^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_t} \left(\frac{\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_p} \right)}{\prod_{k=1}^{s} \Pi_{t+k}} \right)^{1-\varepsilon_p} Y_{t+s},$$

$$X_t^{pn} = E_t \sum_{s=0}^{\infty} \theta_p^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_t} \left(\frac{\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_p} \right)}{\prod_{k=1}^{s} \Pi_{t+k}} \right)^{-\varepsilon_p} Y_{t+s} \lambda_{p,t+s} mc_{t+s}(i),$$

where, regarding X_t^{pd}:

$$X_t^{pd} = E_t \sum_{s=0}^{\infty} \theta_p^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_t} \left(\frac{\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_p} \right)}{\prod_{k=1}^{s} \Pi_{t+k}} \right)^{1-\varepsilon_p} Y_{t+s}$$

$$\Leftrightarrow X_t^{pd} = Y_t + E_t \left\{ \theta_p \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left(\frac{\Pi_t^{\iota_p}}{\Pi_{t+1}} \right)^{1-\varepsilon_p} Y_{t+1} + \sum_{s=2}^{\infty} \theta_p^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_t} \left(\frac{\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_p} \right)}{\prod_{k=1}^{s} \Pi_{t+k}} \right)^{1-\varepsilon_p} Y_{t+s} \right\}$$

$$\Leftrightarrow X_t^{pd} = Y_t + E_t \left\{ \theta_p \frac{\beta \Lambda_{t+1}}{\Lambda_t} \underbrace{E_{t+1} \left[\sum_{s=1}^{\infty} \theta_p^{s-1} \frac{\beta^{s-1} \Lambda_{t+s}}{\Lambda_{t+1}} \left(\frac{\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_p} \right)}{\prod_{k=1}^{s} \Pi_{t+k}} \right)^{1-\varepsilon_p} Y_{t+s} \right]}_{=\Pi_t^{\iota_p (1-\varepsilon_p)} \Pi_{t+1}^{\varepsilon_p - 1} X_{t+1}^{pd}} \right\}$$

$$\Leftrightarrow X_t^{pd} = Y_t + E_t \left\{ \theta_p \frac{\beta \Lambda_{t+1}}{\Lambda_t} \Pi_t^{\iota_p (1-\varepsilon_p)} \Pi_{t+1}^{\varepsilon_p - 1} X_{t+1}^{pd} \right\}$$

and, considering X_t^{pn}:

$$X_t^{pn} = E_t \sum_{s=0}^{\infty} \theta_p^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_t} \left(\frac{\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_p} \right)}{\prod_{k=1}^{s} \Pi_{t+k}} \right)^{-\varepsilon_p} Y_{t+s} \lambda_{p,t+s} mc_{t+s}(i)$$

$$\Leftrightarrow X_t^{pn} = Y_t \lambda_{p,t} mc_t(i) + E_t \left\{ \begin{array}{l} \theta_p \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left(\frac{\Pi_t^{\iota_p}}{\Pi_{t+1}} \right)^{-\varepsilon_p} Y_{t+1} \lambda_{p,t+1} mc_{t+1}(i) \\ + \sum_{s=2}^{\infty} \theta_p^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_t} \left(\frac{\left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_p} \right)}{\prod_{k=1}^{s} \Pi_{t+k}} \right)^{-\varepsilon_p} Y_{t+s} \lambda_{p,t+s} mc_{t+s}(i) \end{array} \right\}$$

$$\Leftrightarrow X_t^{pn} = Y_t \lambda_{p,t} mc_t(i) + E_t \left\{ \theta_p \frac{\beta \Lambda_{t+1}}{\Lambda_t} \Pi_t^{-\iota_p \varepsilon_p} \Pi_{t+1}^{\varepsilon_p} X_{t+1}^{pn} \right\}$$

Thus, we can write the equation for Π_t^* in the following way:

$$\Pi_t^* = \frac{\varepsilon_p}{\varepsilon_p - 1} \frac{X_t^{pn}}{X_t^{pd}} \Pi_t$$

The law of motion for prices then is:

$$P_t^{1-\varepsilon_p} = (1-\theta_p)\left(P_t^*\right)^{1-\varepsilon_p} + \theta_p \left(\Pi_{t-1}^{\iota_p} P_{t-1}\right)^{1-\varepsilon_p}$$

$$\Leftrightarrow \left(\Pi_t\right)^{1-\varepsilon_p} = (1-\theta_p)\left(\Pi_t^*\right)^{1-\varepsilon_p} + \theta_p \left(\Pi_{t-1}^{\iota_p}\right)^{1-\varepsilon_p}$$

A.3.4 Price dispersion

From the demand for differentiated goods, we have:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon_p} Y_t$$

Taking the integral on both sides, it follows:

$$\underbrace{\textstyle\int_0^1 Y_t(i) di}_{Y_{ht}} = Y_t \underbrace{\textstyle\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon_p} di}_{D_{pt}}$$

Regarding the evolution of D_{pt}, the period-t price dispersion is:

$$D_{pt} = P_t^{\varepsilon_p} \left[\theta_p \left(\Pi_{t-1}^{\iota_p}\right)^{-\varepsilon_p} \frac{D_{pt-1}}{P_{t-1}^{\varepsilon_p}} + (1-\theta_p)(P_t{}^*)^{-\varepsilon_p}\right]$$

$$\Leftrightarrow D_{pt} = \Pi_t^{\varepsilon_p} \left[(1-\theta_p)\Pi_t{}^{*-\varepsilon_p} + \theta_p \left(\Pi_{t-1}^{\iota_p}\right)^{-\varepsilon_p} D_{pt-1}\right)$$

From the evolution of the aggregate price index, we have:

$$P_t{}^{1-\varepsilon_p} = (1-\theta_p)\left(P_t^*\right)^{1-\varepsilon_p} + \theta_p \left(\Pi_{t-1}^{\iota_p} P_{t-1}\right)^{1-\varepsilon_p}$$

$$\Leftrightarrow \left(\frac{P_t^*}{P_t}\right)^{-\varepsilon_p} = \left[\frac{1-\theta_p\left(\frac{\Pi_{t-1}^{\iota_p}}{\Pi_t}\right)^{1-\varepsilon_p}}{1-\theta_p}\right]^{\frac{\varepsilon_p}{\varepsilon_p-1}}$$

Substituting this into the evolution of price dispersion yields:

$$D_{pt} = \theta_p\left(\Pi_{t-1}^{\iota_p}\right)^{-\varepsilon_p}\Pi_t^{\varepsilon_p}D_{pt-1} + (1-\theta_p)^{\frac{1}{1-\varepsilon_p}}\left[1-\theta_p\left(\frac{\Pi_{t-1}^{\iota_p}}{\Pi_t}\right)^{1-\varepsilon_p}\right]^{\frac{\varepsilon_p}{\varepsilon_p-1}}$$

A.3.5 Capital producers

The profits of the capital producers can be defined as follows:

$$\underbrace{P_t^k\mu_t\left[1-S\left(\frac{I_t}{I_{t-1}}\right)\right]I_t}_{\text{Income}} - \underbrace{I_t}_{\text{Costs}}$$

The profit maximisation of the capital producers without constraint is described by:

$$\mathcal{L} = E_t\sum_{s=0}^{\infty}\beta^s\Lambda_{t+s}\left[P_{t+s}^k\mu_{t+s}\left[1-S\left(\frac{I_{t+s}}{I_{t+s-1}}\right)\right]I_{t+s}) - I_{t+s}\right]$$

$$\frac{\partial\mathcal{L}}{\partial I_t} : P_t^k\mu_t\left\{1-S\left(\frac{I_t}{I_{t-1}}\right)-S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right\} = 1-\beta P_{t+1}^k\frac{\Lambda_{t+1}}{\Lambda_t}\mu_{t+1}\left\{-S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right\}$$

A.4 Government policies

When the central bank does not peg the interest rate, it follows a standard Taylor rule:

$$\ln(R_t) = (1-\rho)\ln(R) + \rho\ln(R_{t-1}) + (1-\rho)\left(\tau_\pi(\pi_t - \pi) + \tau_y(y_t - y_{t-1})\right) + \varepsilon_t^r$$

QE policies are implemented via the AR(2) process:

$$\bar{B} = (\bar{B}_{ss})^{(1-\rho_{B_1}+\rho_{B_2})} * (\bar{B}_{t-1})^{(\rho_{B_1})} * (\bar{B}_{t-2})^{(-\rho_{B_2})} * \epsilon_B$$

A.5 Resource constraints and exogenous shock processes

The resource constraint evaluates to:

$$Y_t = C_t + I_t.$$

In addition to the equilibrium conditions, the model comprises seven exogenous processes.

1. Technology shock: $A_t = (1-\rho_a) * log(A_{ss}) + \rho_a * A_{t-1} + \epsilon_{A,t}$.
2. Financial shock: $\Phi_t = (1-\rho_\phi) * log(\Phi_{ss}) + \rho_{phi} * (\Phi_{t-1}) + \epsilon_{\Phi,t}$.
3. Investment shock: $\mu_t = (1-\rho_\mu) * log(\mu_{ss}) + \rho_\mu * (\mu_{t-1}) + \epsilon_{\mu,t}$.
4. Wage markup shock: $\lambda_{w,t} = (1-\rho_{\lambda^w}) * log(\lambda_{w,ss}) + \rho_{\lambda^w} * (\lambda_{w,t-1}) + \epsilon_{\lambda_{w,t}}$.
5. Price markup shock: $\lambda_{p,t} = (1-\rho_{\lambda_p}) * log(\lambda_{p,ss}) + \rho_{\lambda_p} * (\lambda_{p,t-1}) + \epsilon_{\lambda_{p,t}}$.
6. Discount factor shock: $b_t = (1-\rho_b) * log(b_{ss}) + \rho_b * (b_{t-1}) + \epsilon_{b,t}$.
7. Monetary policy residual: $R_t^\epsilon = (1-\rho_m) * log(R_{ss}^\epsilon) + \rho_m * R_{t-1}^\epsilon + \epsilon_{R,t}$;

Appendix B Data

Definition of observables

Real per capita output growth: $\frac{(YER/LFN)-(YER(-1)/LFN(-1))}{(YER(-1)/LFN(-1))}$

Real per capita investment growth: $\frac{(ITR/LFN)-(ITR(-1)/LFN(-1))}{(ITR(-1)/LFN(-1))}$

Gross inflation: $1 + \frac{HICPSA-HICPSA(-1)}{HICPSA(-1)}$

Employment growth: $\frac{LNN-LNN(-1)}{LNN(-1)}$

Real wage growth: $\frac{(WRN/HICPSA)-(WRN(-1)/HICPSA(-1))}{(WRN(-1)/HICPSA(-1))}$

First difference of short-term interset rate: $STN - STN(-1)$

First difference of long-term interest rate: $LTN - LTN(-1)$

Real bank net worth growth: $\frac{(NWB/HICPSA)-(NWB(-1)/HICPSA(-1))}{(NWB(-1)/HICPSA(-1))}$

Data description

All seasonal data are seasonally adjusted.

YER: Real GDP. Millions of ECU/euro corrected with reference year 1995. Source: Area-wide Model (AWM) database.

LFN: Labor force (persons). Source: AWM database.

ITR: Gross investment. Source: AWM database.

HICPSA: Overall Harmonised Index of Consumer Prices. Base year 1996=100. Source: AWM database.

LNN: Total employment (persons). Source: AWM database.

WRN: Nominal wage rate per head. Source: AWM database.

STN: Nominal net short-term interest rate in percent. Source: AWM database.

LTN: Nominal net long-term interest rate in percent. Source: AWM database.

NWB: Nominal capital and reserves of euro area monetary financial Institutions (excluding eurosystem) in millions of euro. Source: European Central Bank, MFI Balance Sheet Items Statistics.

Appendix C Predictive analysis

In this section, we describe how we implement the prior/posterior predictive analysis. The same approach is also taken in Suh and Walker (2016) and Leeper et al. (2015).

The prior predictive analysis we carry out entails the following steps:

1. Given the model, M outlined in Appendix A, and its corresponding structural parameters, θ, we posit a prior density, $p(\theta|M)$, specifying the range of values for the parameters and the associated probabilities. The parameters are drawn independently and $\tilde{p}\left(\theta|M\right)$ denotes the product of the marginal parameter distributions. We define an indicator function, $\mathcal{I}\left\{\theta \in \Theta_D\right\}$, that is assumed to take on the value *one* if θ generates a determinate solution of the model M and *zero* if it is not part of the subspace of parameters Θ_D that delivers a unique rational expectations equilibrium. The joint prior distribution is defined as $p\left(\theta|M\right) = c^{-1}\tilde{p}\left(\theta|M\right)\mathcal{I}\left\{\theta \in \Theta_D\right\}$, where $c = \int_{\theta\in\Theta}\tilde{p}\left(\theta|M\right)d\theta$ describes a scaling factor that ensures that the prior density integrates to one.

2. The model generates predictive distributions for its variables, y_T, using $p\left(y_T|M\right) = \int_{\theta\in\Theta} p\left(y_T|\theta, M\right)p\left(\theta|M\right)d\theta$, where $p\left(y_T|\theta, M\right) = L\left(y_T|\theta, M\right)$ denotes the likelihood of the data.

3. For any vector of interest, ω, the predictive distribution can be used to produce $p\left(\omega|y_T, \theta, M\right)$. Our statistics of interest are the response of inflation after a QE shock in periods zero to 25 after the shock, as well as the peak response of inflation.

Implementation in Dynare: First, we sample draws for the structural parameters from the prespecified corresponding prior distributions. This is done using the global sensitivity analysis (GSA) toolbox, which is part of Dynare. Then, we solve the model under perfect foresight for each draw, simulate the path for inflation in response to a QE shock and extract the peak of the inflation response. Having these statistics at hand for every draw, we can calculate their median as well as their dispersion.

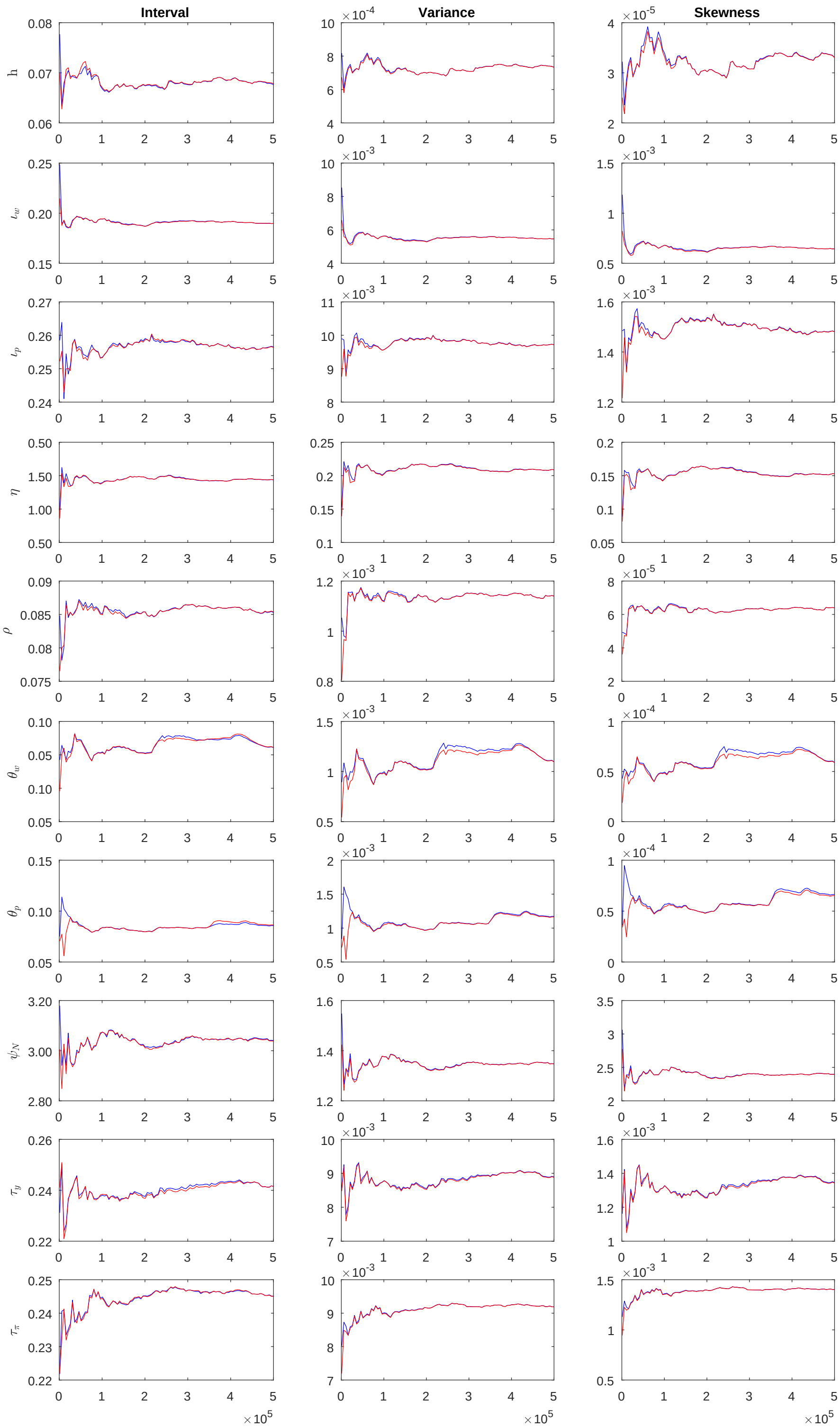
We implement the posterior predictive analysis in the same way, except that we sample draws for the structural parameters from the posterior distributions obtained by the Metropolis Hastings algorithm.

Appendix D Estimation

D.1 MCMC diagnostics

Figure 6 shows the univariate convergence statistics for all estimated parameters h, ι_w, ι_p, η, ρ, θ_w, θ_p, ψ_N, τ_π and τ_y. The statistics include three different measures: the measure presented in the column 'intervall' is based on an interval statistic constructed around the parameter mean (i.e. the 80% HPD intervall). The measures shown in the columns variance and skewness are based on a statistic that depicts squared and cubed deviations from the parameter mean. The blue line depicts theses measures based on all draws (i.e. draws from all Markov chains together - between chain measure) and the red line is based on the draws from the individual chains (within a chain measure). Each row in Figure 6 corresponds to one parameter.

Figure 6: MCMC univariate convergence diagnostics for all estimated structural parameters

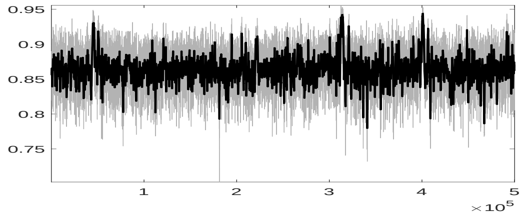


D.2 Trace plots

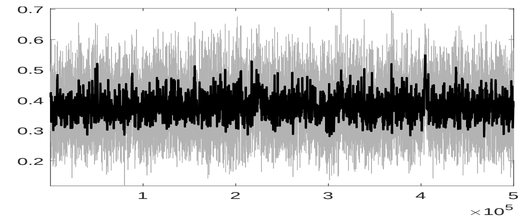
To show that the draws for the estimated structural parameters which we use for the posterior predictive analysis are not trending or display otherwise strange behavior, we also present trace plots for all structural parameters in Figure 7. Since we rely on the last 100,000 draws from the *first* Markov chain, we correspondingly present the trace plots for each of the parameters base on the first Markov chain which contains 500,000 draws in total.

Figure 7: Trace plots for estimated parameters based on first Markov chain

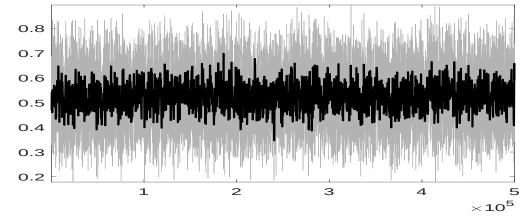
(a) h



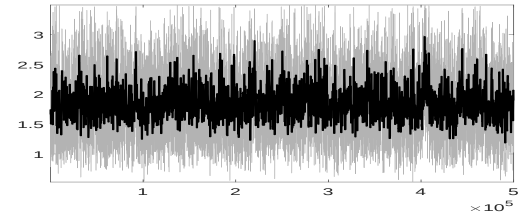
(b) ι_w



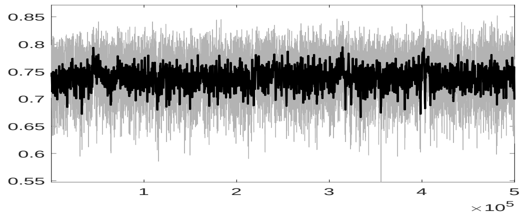
(c) ι_p



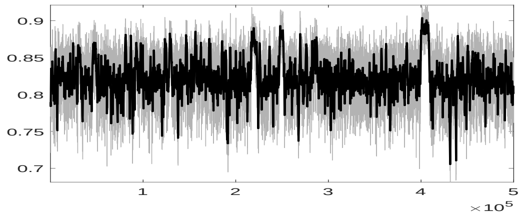
(d) η



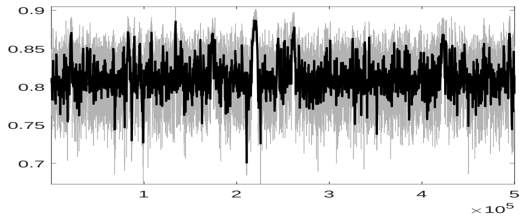
(e) ρ



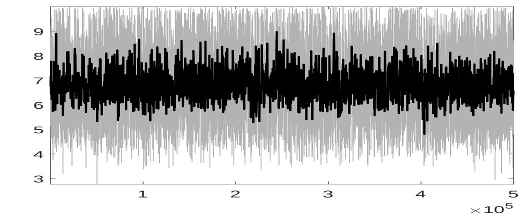
(f) θ_w



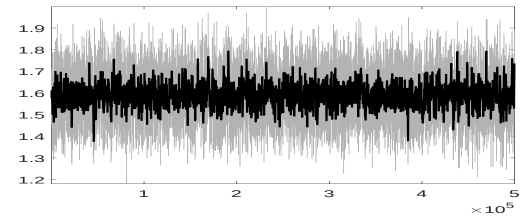
(g) θ_p



(h) ψ_N



(i) τ_π



(j) τ_y