The natural rate of interest

The concept of the natural rate of interest has been attracting increasing public interest in recent years. For some, the finding that there has been a persistent fall in long-term interest rates is proof of a decline in the equilibrium interest rate and of a simultaneous trend towards persistently low rates of growth (secular stagnation theory). Moreover, from a monetary policy perspective, the natural rate of interest is often seen as the key benchmark for adopting an accommodative or restrictive monetary policy stance. The latter presupposes that it can also be determined reliably.

The natural rate of interest – defined by Knut Wicksell as the "rate of interest on loans which is neutral in respect to commodity prices" – is not directly measurable, however. Many of the methods of calculating the natural rate of interest do, in fact, point to a decline in a large number of developed economies since around the 1980s, reaching an exceptionally low level since the financial crisis. Even so, the estimated level of the natural rate of interest varies greatly depending on the method used and can often only be estimated with very wide uncertainty bands. The considerable degree of estimation uncertainty suggests that a robust monetary policy strategy should not place too much emphasis on specific measures for the level of the natural interest rate.

In addition to the model uncertainties in calculation, conceptual differences are also responsible for heterogeneous results, with the two dimensions of time and risk being of particular importance. The time dimension plays a role because in quantification methods, which take a short-term equilibrium as a basis, the natural rate of interest is much more volatile and is currently lower than in quantifications assuming a more long-term equilibrium. The risk dimension is not taken into account in any of the current quantification approaches for the natural rate of interest since, as a general rule, they use secure bond yields as a starting point, even though, with regard to the real economy, a risky return on capital would be a more appropriate indicator. Considering such measures for the return on equity or total capital, however, a protracted decline in yields cannot be ascertained, say, for Germany; the reduced yields on risk-free securities and corporate bonds stand in marked contrast to this.

Among the explanatory factors for sustained lower yields, there is, in view of this, more to suggest an increase in demand for lower-risk assets and less to support the secular stagnation theory, as this should also be reflected in the measures for the return on capital.

Introduction

Natural rate of interest as a benchmark for a goods market equilibrium with stable prices

The phenomenon of low interest rates is evident in Germany, like in most euro area countries, in Japan and in many other developed economies in the case of government bond yields, as it is in interest rates on savings deposits and mortgage loans. Low interest rates are often attributed to the action taken by central banks. However, this way of seeing things neglects the fact that central banks – in terms of their monetary policy decisions – are just one of many factors that determine long-term yields. This is because, even in an imagined world without a central bank, a rate of interest would materialise. Its level would result, among other things, from households' propensity to consume or save, enterprises' propensity to invest and the propensity of economic agents to assume risks or to convert assets into liquidity without complications. In such a world, the greater the propensity to save and the lower the inclination to invest, the lower the interest rate would be, for example. Starting from this kind of thought experiment, the question arises as to whether such an interest rate is relevant and quantifiable in a world like ours, too, with money and central banks. About 120 years ago, the Swedish economist Knut Wicksell developed the concept of the natural rate of interest, which he summarised as follows: "There is a certain rate of interest on loans which is neutral in respect to commodity prices, and tends neither to raise nor to lower them. This is necessarily the same as the rate of interest which would be determined by supply and demand if no use were made of money and all lending were effected in the form of capital goods. It comes to much the same thing as to describe it as the current value of the natural rate of interest on capital."[1]

This natural rate of interest thus has two key features. On the one hand, it is linked to a goods market equilibrium and, on the other hand, to price stability. Measuring this natural rate of interest and understanding how it arises is of particular interest for monetary policymakers. This is because central banks can influence the short-term real interest rate relative to its equilibrium value by changing the short-term nominal interest rates, thereby influencing the real economy and inflation developments.[2] If the key interest rate less the expected inflation rate is below the natural interest rate, it may be expected that households will use the opportunity of loans at relatively favourable rates to expand consumption; enterprises thereupon invest more, produce above potential in the goods market and raise their prices, which leads to an increase in the rate of inflation. If, on the other hand, the key interest rate less expected inflation is higher than the natural interest rate, capacity underutilisation and falling inflation would be expected.

It is therefore not surprising that analysing the natural rate of interest is of particular importance for central banks. Since this rate cannot be observed, however, models and estimation methods have to be relied on. The following section provides an overview of common methods of measuring the natural rate of interest and their results. The final section assesses these findings.

Quantification

Necessary assumptions about time dimension and risk

The natural rate of interest cannot be measured as a theoretical construct; rather, it requires certain assumptions about the relationship between measurable variables and the natural rate of interest in order for it to be quantifiable. In this respect, two dimensions prove to be especially important: time and risk. In the case of the time dimension, a short-term interest rate, such as a three-month interest rate, is usually considered, which means that

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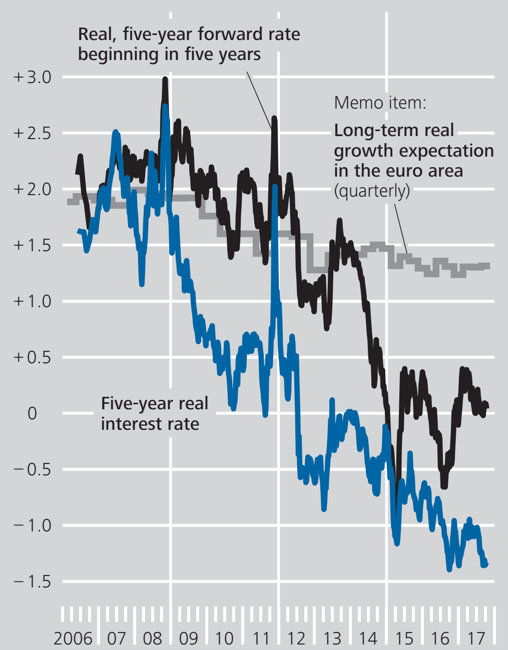
1 K Wicksell (1898, translated 1936), Interest and prices. A study of the causes regulating the value of money. London, Macmillan & Co.

2 See A Weber, A Worms and W Lemke (2008), How useful is the concept of the natural real rate of interest for monetary policy?, Cambridge Journal of Economics 32, pp 49-63.

forward premiums can be neglected. The natural rate of interest can be analysed in the long term when all the adjustment processes have been completed, or in the medium term when a large part of the adjustment processes can be completed, or in the short term when – driven by a possible large number of adjustment processes – it shows a very high variability. With regard to the risk dimension, the original concept of lending real capital is actually geared to a risky interest rate. However, this perspective has been lost in virtually all of the relevant model approaches, as they are, in fact, based on a rate of interest that is as secure as can be. With this shortened perspective, however, important information risks being lost, as can be demonstrated. Put in simple terms, the commonly used methods for determining the natural rate of interest can be divided into two groups: first, methods which draw conclusions about the natural rate of inflation from financial market data on real returns, and, second, methods which emphasise more strongly economic transmission mechanisms and which, above all, do not attempt to determine the equilibrium interest rate independently of the economy's growth potential.

Real yield on inflation-linked government bonds in the euro area*

% pa, weekly averages



Source: Consensus Economics and Bundesbank calculations. * Calculated on the basis of French bonds, which are indexed on the Harmonised Index of Consumer Prices excluding tobacco for the euro area as a whole.
Deutsche Bundesbank

Quantifications on the basis of bond yields

Secure bond yields showing downward trend since around 1980

Owing to their very low default risk premiums, sovereign bond yields of fiscally sound countries are a good starting point for determining a virtually risk-free real, natural rate of interest. Nominal interest rates on government bonds can be broken down as follows (according to the Fisher equation):

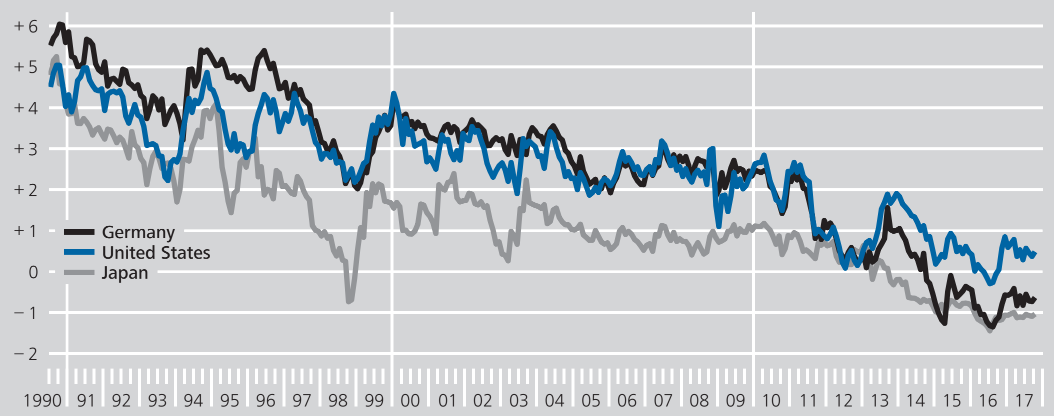
$$y^n = r^n + E^n(\pi) + RP^n(\pi) + liq^n.$$

The nominal safe interest rate y^n with a maturity of n years is roughly equal to the sum of the safe real interest rate r^n, the inflation expectation $E^n(\pi)$ and risk premiums for inflation uncertainty $RP^n(\pi)$ as well as a premium for the differences in liquidity liq^n between real and nominal bonds. Bonds whose coupons and repayments grow with inflation make it possible to directly determine a safe real interest rate r^n which can be achieved over a specific period of time while maintaining purchasing power. From models of a term structure for real returns, this can be used to determine different medium and long-term real interest rates of differing maturities as indicators of equilibrium real interest rates for the euro area (see chart above).[3] Against the backdrop of growth theory considerations, a long-term equilibrium interest rate should not be noticeably below an

3 The market for French paper is the most important market for inflation-linked bonds in the euro area. Indexation is on the basis of the European Harmonised Index of Consumer Prices excluding tobacco. Eight bonds are outstanding at present, from which a seasonally adjusted real term structure is determined using the Nelson-Siegel model.

Long-term real interest rate in various currency areas*

%, monthly data



Source: Thomson Reuters, Consensus Economics and Bundesbank calculations. * Five-year forward interest rate commencing in five years, real interest rates on the basis of nominal interest rates less concurrent inflation expectations.
Deutsche Bundesbank

economy's potential growth rate over an extended period of time.[4]

The five-year real interest rates fell below 1% in 2009 and have been on a declining trajectory since then, although the decline has been at a more moderate pace of late. They currently stand at below -1%. This measure still includes short-term adjustment effects from the economy and monetary policy, however. Closer to the concept of a medium to long-term natural real rate of interest are real forward rates, which do not contain any short-term developments. Until 2014, such long-term real rates (5y5y forward rates) were close to the long-term Consensus survey data on long-term growth expectations, which cover more or less the same horizon. The real forward rates have fallen considerably since then and have been hovering around zero since 2015. By contrast, long-term growth expectations fell much less sharply during the observation period. In particular, the sharp decline in the real forward return since 2014 is not found in the growth surveys.[5]

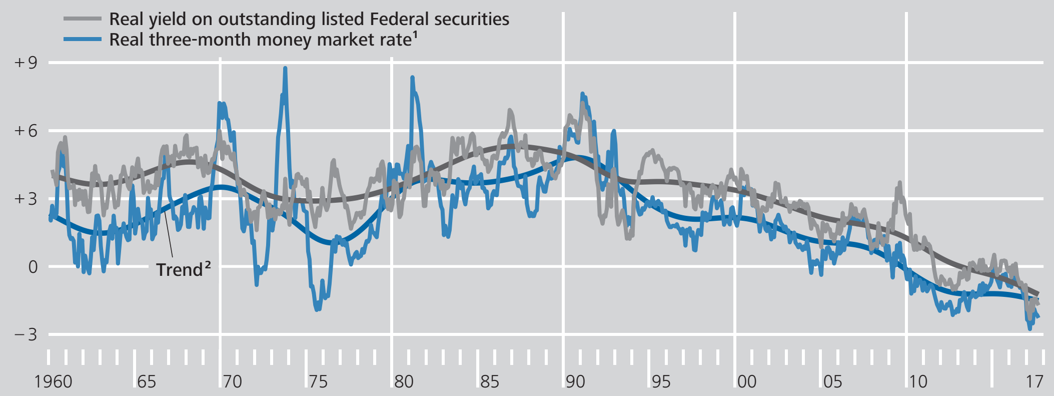
In this connection, the debate that has arisen about the Eurosystem's asset purchase programme since 2014 may have raised the premiums for scarcity and liquidity, as a result of which the real rate shown in the chart is squeezed by non-standard monetary policy measures, and the neutral real interest rate is underestimated.

The markets for inflation-linked bonds in the euro area and elsewhere are a rather new segment of the capital market and therefore do not allow a long historical analysis. That said, for making such a historical comparison, it is possible to calculate a real return from classical, nominal bond yields using survey-based inflation expectations. The chart above shows real interest rates five years ahead for Germany, the United States and Japan.[6] In line with developments in real yields on inflation-linked bonds, the chart on page 29 shows a steady decline across all the currency areas under analysis.

4 The necessary assumptions include, in particular, a utility function with constant relative elasticity of intertemporal substitution – so that the real interest rate moves one-for-one with real growth expectations – and that fluctuations in time preference and the real risk premium do not dominate the dynamics of real growth expectations; see, for example, J Y Campbell, A W Lo and A C MacKinlay (1997), The Econometrics of Financial Markets, Princeton University Press, p 309.
5 The deviation can also occur as a result of distortions in the formation of expectations in the survey data. Traditional patterns, such as a delayed adjustment of the survey expectations to new developments, are not very plausible, however, owing to the long duration of the deviation.
6 In the absence of a long history of surveys on the European Economic and Monetary Union, Germany is taken as a proxy for the euro area.

Ex ante real interest rates* based on Federal securities and money market rates

% pa, monthly data



Source: Federal Statistical Office and Bundesbank calculations. * Calculated using monthly averages, adjusted for inflation expectations. **1** Determined on the basis of the three-month money market rates reported by Frankfurt banks and the three-month Eonia swap. **2** Smoothed using HP filter (λ = 129 600).

Deutsche Bundesbank

This decline began well before the launch of the euro, however. The marked decrease in the long-term real interest rates from 6% in the early 1990s to figures between 2% and 3.5% in the first decade of EMU was accompanied by a halving of long-term growth expectations from 3% to around 1.5%. Furthermore, along with the increase in savings due to demographic change, the reduced level of uncertainty since around the mid-1980s (caused by the "Great Moderation") probably contributed to a decline in the risk premium and thus to falling real interest rates during this period.[7]

The acceleration of this decline in the wake of the financial and sovereign debt crisis is particularly pronounced in the euro area. Although the long-term prospects for growth on the two sides of the Atlantic did not veer apart markedly, it is apparent that the real rates of interest have been diverging since 2013. Their exceptionally low figure in Germany is probably the consequence of a particular shortage of German bonds brought about by the Eurosystem's asset purchase programme, which lowers yields.

If an even longer period is to be covered, it is necessary – in the absence of survey data with a sufficiently long history – to approximate inflation expectations using a time series approach. On the basis of a small number of assumptions and only minor requirements regarding the available data, the approach applied by Hamilton et al (2016) is used here to derive inflation expectations from the actual inflation rate.[8] The chart above shows real interest rates calculated in this way (and additionally smoothed) using German and European data. For the period between 1960 and around 1990, these reveal a roughly stable long-term real interest rate at around 4%. Following German reunification, a steady downward trend can be seen, as above, with particularly marked declines recorded around 2009 as well as around 2016.

Irrespective of the method applied to calculate equilibrium interest rates from financial market data, it is evident that real yields in major currency areas are currently at an all-time low. However, the deviations over the past few years of the real interest rate variables from growth expectations based on surveys suggest

7 See also Deutsche Bundesbank, Globalisation and monetary policy, Monthly Report, October 2007, pp 15-33.
8 See J Hamilton, E Harris, J Hatzius and K West (2016), The equilibrium real funds rate: past, present and future, IMF Economic Review 64 (4), pp 660-707. With regard to the ex ante real interest rate, in line with the Fisher equation, as in the approach employing survey expectations, the expected inflation rate is deducted from a nominal interest rate. The inflation expectation is autoregressively estimated on the basis of the actual inflation rate.

that a cautious interpretation is called for, since risk assessments and liquidity can also push down real interest rates derived from financial market prices to a level below the neutral real interest rate as defined by Wicksell.

Quantification on the basis of assumptions about the relationship between interest rates and output

Approaches using risk-free bond yields with potential growth estimates also indicate a downward trend

While the approaches so far have used only interest rate and inflation data, numerous other approaches investigate the natural rate of interest, together with other economically relevant variables, such as potential output or the natural rate of unemployment. These approaches encompass a large number of dynamic stochastic general equilibrium (DSGE) models, in which the natural rate of interest is defined as a short-term equilibrium real interest rate that appears in the model if prices and wages are fully flexible.[9] This means, for example, that any impact of monetary policy on the natural rate of interest is, by definition, ruled out. Since, according to this definition of the natural rate of interest used in DSGE models, numerous additional unexpected developments in the real economy and on the financial market can nevertheless shift the natural rate of interest in one direction or the other, the natural rate of interest calculated in this manner is, as a rule, very volatile. With regard to the aforementioned time dimension, it is therefore assumed that a multitude of more short-term fluctuations are also reflected in the natural rate of interest calculated in this way.

Aside from DSGE models, this quantification method also includes vector autoregressive time series models, such as those used by Lubik and Matthes (2015) and Kiley (2015).[10] In these, data on interest rates and output are estimated jointly under certain assumptions. One very prominent example is the method of Laubach and Williams (2003), which has now been used for a large number of countries (see the box on page 33 et seq).[11]

Quantification on the basis of returns on capital

Mixed trends in risk-prone returns on capital

Notwithstanding all the differences between the various approaches, the estimates have nonetheless so far presented a clear picture: The natural rate of interest determined on the basis of default-free securities appears to have fallen since the 1980s or later across all the observed time horizons and irrespective of the method used, with the lowest point being found either around 2009 (ie at the height of the financial crisis) or, depending on the model, in the recent past. Differences in the time perspective appear to impact solely on the absolute level of the natural rate of interest and its volatility.

So far, however, the risk dimension has been left aside – in the literature on the natural interest rate it has become common to use government bond yields as a yardstick and thus, as a general rule, securities which have a low risk of default and high liquidity. This seems both

9 In this connection, see J Andrés, J David López-Salido and E Nelson (2009), Money and the natural rate of interest: Structural estimates for the United States and the euro area, Journal of Economic Dynamics and Control 33, pp 758-776; R Barsky, A Justiniano and L Melosi (2014), The natural rate of interest and its usefulness for monetary policy, American Economics Review 104, pp 37-43; M Del Negro, D Giannone, M P Giannoni and A Tambalotzi (2017), Safety, Liquidity, and the Natural Rate of Interest, Federal Reserve Bank of New York Staff Reports 812; V Cúrdia, A Ferrero, G C Ng and A Tambalotti (2015), Has US monetary policy tracked the efficient interest rate?, Journal of Monetary Economics 70, pp 72-83; as well as M Goldby, L Laureys and K Reinold (2015), An estimate of the UK's natural rate of interest, weblog article on Bank Underground, the weblog of Bank of England staff, 11 August 2015.
10 See T A Lubik and C Matthes (2015), Calculating the natural rate of interest: a comparison of two alternative approaches, Economic Brief 15-10, Federal Reserve Bank of Richmond; M T Kiley (2015), What can the data tell us about the equilibrium real interest rate?, Finance and Economics Discussion Series 2015-077, Board of Governors of the Federal Reserve System.
11 See T Laubach and J C Williams (2003), Measuring the natural rate of interest, Review of Economics and Statistics 84, pp 1063-1070.

Estimating a medium-term equilibrium interest rate for the euro area using the Holston, Laubach and Williams (2017) approach

The broad decline in interest rates observed in recent years raises the question of whether this is merely a temporary phenomenon – that is, mainly a reflection of the crisis and the accommodative monetary policy response that will go into reverse once conditions return to normal – or whether the downward movement is, in fact, an indication of profound structural change, suggesting that low interest rates are here to stay. Any attempt to answer this question would therefore need to investigate empirically whether the observed path of interest rates should be interpreted primarily as a response to transitory cyclical fluctuations, or whether rates are instead being driven by factors which are shifting the long-term steady state of the macroeconomy and which change over time at no more than a glacial pace.[1] Evidence of the latter would suggest that the "natural rate of interest" – that is, the interest rate that would prevail once all cyclical fluctuations have subsided – has fallen.

One method based on this idea of determining the natural rate of interest empirically, and which has been discussed by leading central bankers of late,[2] is the approach formulated by Holston, Laubach and Williams.[3] This approach is based on a theoretical standard model which describes economic and inflation dynamics by means of two functions. The first is an aggregate demand function – in other words, the relationship between the output gap (ie the difference between actual and potential output) and the interest rate gap – the second, a Phillips curve – that is, the relationship between inflation and the output gap. For the purposes of this approach, the interest rate gap is the difference between the short-term *ex ante* real interest rate and the unobserved natural rate of interest: if these two rates are identical, the output gap can be said to be closed and inflation remains static. Specifically, where the inflation expectations of economic agents are assumed to be adaptive, that is to say, backward-looking, these two equations will be as follows:

$$\tilde{y}_t = a_{y,1}\tilde{y}_{t-1} + a_{y,2}\tilde{y}_{t-2} + \frac{a_r}{2}\sum_{j=1}^{2}(r_{t-j} - r^*_{t-j}) + \varepsilon_{\tilde{y}t}, \quad (1)$$

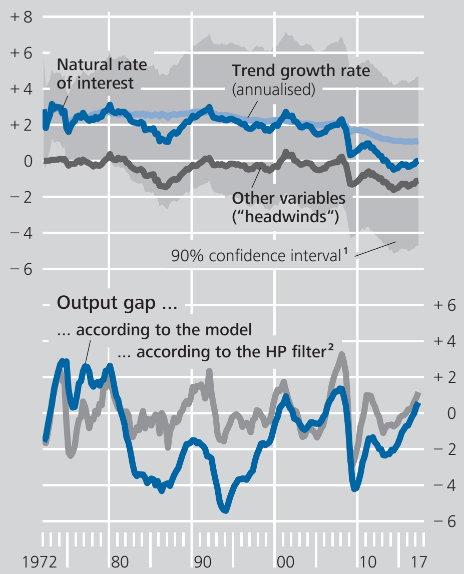
$$\pi_t = b_\pi \pi_{t-1} + (1-b_\pi)\pi_{t-2,4} + b_y \tilde{y}_{t-j} + \varepsilon_{\pi,t}, \quad (2)$$

where $\tilde{y}_t = 100(y_t - y^*_t)$ represents the output gap, y_t and y^*_t are the logarithms of actual output and natural output, r_t and r^*_t

1 For more on this topic, see also the points discussed in R C M Beyer and V Wieland (2016), Schätzung des mittelfristigen Gleichgewichtszinses in den Vereinigten Staaten, Deutschland und dem Euroraum mit der Laubach-Williams-Methode, IMFS Working Paper Series No 100.
2 See, for example, J Yellen (2017), The Economic Outlook and the Conduct of Monetary Policy, remarks at Stanford Institute for Economic Policy Research, Stanford University, 19 January 2017; and V Constâncio (2016), The challenge of low real interest rates for monetary policy, lecture at Utrecht School of Economics, 15 June 2016.
3 For more on this topic, see K Holston, T Laubach and J C Williams (2017), Measuring the Natural Rate of Interest: International Trends and Determinants, Journal of International Economics 108, Supplement 1, pp 59-75, based on T Laubach and J C Williams (2003), Measuring the natural rate of interest, Review of Economics and Statistics 84, pp 1063-1070. Variations on this model framework used to determine the natural rate of interest in the euro area can also be found in J S Mésonnier and J-P Renne (2007), A time-varying "natural" rate of interest for the euro area, European Economic Review 51, pp 1768-1784; J Garnier and B R Wilhelmsen (2009), The natural rate of interest and the output gap in the euro area: a joint estimation, Empirical Economics 36, pp 297-319; and S Fries, J-S Mésonnier, S Mouabbi and J-P Renne (2016), National natural rates of interest and the single monetary policy in the euro area, Banque de France Working Paper No 611.

Natural rate of interest in the euro area based on the Holston, Laubach and Williams approach

%, quarterly data



Source: Bundesbank calculations based on the Holston, Laubach and Williams (2017) approach. **1** The interval only depicts the filter uncertainty, not the additional uncertainty with respect to the parameters. **2** Smoothing parameter λ = 1,600.

Deutsche Bundesbank

denote the real short-term interest rate[4] and the natural rate of interest, π_t denotes consumer price inflation and $\pi_{t\text{-}2,4}$ the average of its second and fourth lags. $\varepsilon_{\tilde{y}t}$ and $\varepsilon_{\pi,t}$ capture transitory shocks to the output gap and inflation.[5]

In this model, the natural rate of interest, based on classical growth theory, is sensitive to the rate of (trend) growth in potential output, g_t, and also to a number of economically unobservable variables (dubbed "headwinds" in the literature), which are collated to form a single variable that is assumed to be highly persistent, namely z_t, and which influence the correlation between output and the interest gap over an extended period of time.[6] Both the rate of trend growth and the headwinds variable are configured as unit root processes – that is to say, the following holds true for the natural rate of interest:

$$r_t^* = g_t + z_t. \quad (3)$$

While the projection horizon for the natural rate of interest computed in this model estimation is not defined outright, the modelling strategy and the way the variable and shock processes are defined suggest it will in any case offer a medium to longer-term perspective.[7]

The adjacent chart shows which path the natural rate of interest for the euro area – derived from the model estimations of Holston, Laubach and Williams – has followed since 1972.[8] It is evident from the chart that the natural rate of interest has experienced a trend decline over the past three decades. An extended spell at between 2% and 3% at the beginning of the observation period gave way to a four-year downward trend beginning in 1982 to somewhere in the region of 1%. After that, the natural rate of interest recovered until 1990 and then fluctuated around the 2%

4 The short-term real interest rate is the three-month Euribor rate less the four-quarter moving average of past inflation as a proxy for inflation expectations. For more detailed information on the data sources, see also footnote 8.
5 To prevent short-term fluctuations in both aggregate demand and inflation dynamics from being captured in the computed natural rate of interest, transitory exogenous shocks are accounted for in the aggregate demand and supply function.
6 Classical growth theory states that the long-term equilibrium rate of interest is computed as $r = \frac{1}{\sigma} g_c + \Theta$, where σ denotes the intertemporal elasticity of substitution in consumption, g_c the growth rate of pro capita consumption, and Θ the rate of households' time preference.
7 "Our definition takes a 'longer-run' perspective, in that it refers to the level of real interest rates expected to prevail, say, five to ten years in the future, after the economy has emerged from any cyclical fluctuations and is expanding at its trend rate." From T Laubach and J C Williams (2016), Measuring the natural rate of interest redux, Business Economics 41, pp 57-67. This inferred horizon also matches inter alia the five-year, five-year-forward interest rate expectations derived from a term structure model, the level and path of which correlate quite distinctly with the natural rate of interest estimated here. See Deutsche Bundesbank, Monetary policy indicators at the lower bound based on term structure models, Monthly Report, September 2017, p 21ff.
8 The model is estimated at a quarterly frequency using synthetic data for the euro area prior to 1999. Data are taken from the Area Wide Model (AWM) database. Further details can be found in K Holston, T Laubach and J C Williams (2017), op cit.

mark. The impact of the financial and economic crisis sent it from roughly 2% to barely above zero within the space of just a year. Recovering briefly between 2010 and 2012, the natural rate of interest continued its decline after the sovereign debt crisis set in, moving into negative territory in mid-2013 and hitting a low of -0.5% at the beginning of 2014. Since then, it returned to just above 0%, where it has remained since mid-2017.

The chart clearly shows that volatility in the natural rate of interest is being driven chiefly by the headwinds – the unobservable forces which the model cannot directly interpret in economic terms. Trend growth in potential output, on the other hand, remained very stable throughout the observation period up until the beginning of the 1990s, in a reflection of the modelling assumptions, before a gradual decline set in which accelerated significantly as the financial and economic crisis took hold. The annualised rate of trend growth stood at 1.1% in the first half of 2017.

Overall, the results and the forces driving the natural rate of interest are fraught with a very high degree of estimation uncertainty, however. Even minor tweaks to the technical assumptions made concerning the time series characteristics of the individual model variables, to the estimation method, or to the datasets used produce economically significant changes in the level and path of the natural rate of interest.[9] In essence, the results are so sensitive because of the multitude of unobserved variables. These include the natural rate of interest with its two components – the rate of trend growth and the headwinds – as well as potential output itself. The paths these variables follow depend in part on identifying and separating transitory and permanent shocks, which can vary substantially for even the smallest changes in parameters.[10]

At the same time, the path of the natural rate of interest is driven directly by the model assumptions. Above all, the assumption concerning backward-looking inflation expectations in the Phillips curve leads to a high rate of variability in the natural rate of interest during spells of substantial inflation volatility. As a case in point, the period of disinflation in the 1980s shows that the model can only provide a sufficiently robust explanation of the path of inflation given a backward-looking formation of expectations if it is accompanied by a falling and negative output gap. At a given real rate of interest, this would automatically imply a falling natural rate of interest in the model used here, which materialises, in turn, in a decline in the unobservable variables. Furthermore, the negative output gap observed during these years contrasts in part with other frequently used statistical filtering techniques such as the Hodrick-Prescott filter used to determine potential output and thus also the output gap.

Having gained prominence in both academic circles and the economic policy debate, the model outlined in this box has been extended and modified along various dimensions. Modifications include adjustments to the assumed time series characteristics of the factors driving the natural rate of interest, and to the estimation method, which can impact on the level and volatility of the natural rate of interest.[11] Ex-

9 See R C M Beyer and V Wieland (2017), Instability, imprecision and inconsistent use of equilibrium real interest rate estimates, IMFS Working Paper Series No 110.
10 In the model, the natural rate of interest, potential output, the rate of trend growth, and the headwinds are all unobservable variables that need to be filtered out using the estimation method.
11 See, for example, M T Kiley (2015), What can the data tell us about the equilibrium real interest rate?, Finance and Economics Discussion Series 2015-077, Board of Governors of the Federal Reserve System; K F Lewis and F Vazques-Grande (2017), Measuring the natural rate of interest: alternative specifications, Finance and Economics Discussion Series 2017-059, Board of Governors of the Federal Reserve System; and A Pescatori and J Turunen (2016), Lower for longer: neutral rates in the US, IMF Economic Review 64, pp 708-731.

tensions based on the above model have added variables to the description of the path followed by the natural rate of interest and the output gap, including variables which relate to the financial cycle, the risk appetite of financial market agents, and a possible binding lower bound, and can likewise have a bearing on the path of the natural rate of interest.[12] Finally, the model framework presented here for estimating the natural rate of interest is closely linked to trend/cycle models, and to dynamic stochastic general equilibrium (DSGE) models which can likewise assist in identifying and visualising the (medium-term) path of the natural rate of interest.[13] The overall verdict of the different analyses is that the natural rate of interest has experienced a trend decline, though the different approaches disagree as to its level and volatility. This holds true for the euro area as well as for other currency areas like the United States. Note, however, that results are always fraught with a high degree of estimation uncertainty.

12 See M Juselius, C Borio, P Disyatat and M Drehmann (2016), Monetary policy, the financial cycle and ultra-low interest rates, BIS Working Paper No 569; A Pescatori and J Turunen (2015), op cit; C S Hakkio and A L Smith (2017), Bond premiums and the natural rate of interest, Federal Reserve Bank of Kansas City, Economic Review, pp 5-40; A Cukierman (2016), Reflections on the natural rate of interest, its measurement, monetary policy and the zero lower bound, CEPR Discussion Paper No 11467.
13 See Del Negro et al (2017), op cit; Cúrdia et al (2015), op cit; Barsky et al (2014), op cit; A Gerali and S Neri (2017), Natural rates across the Atlantic, Working Paper No 1140, Banca d'Italia; B K Johannsen and E Mertens (2016), A Time Series Model of Interest Rates With the Effective Lower Bound, Finance and Economics Discussion Series 2016-033, Board of Governors of the Federal Reserve System; R K Crump, S Eusepi and E Moench (2017), The term structure of expectations and bond yields, Federal Reserve Bank of New York Staff Reports No 775; and T A Lubik and C Matthes (2015), Calculating the natural rate of interest: a comparison of two alternative approaches, Economic Brief 15-10, Federal Reserve Bank of Richmond.

understandable and reasonable if the natural rate of interest is to be regarded as a reference point for the key interest rate to be chosen by the central bank, the most immediate impact of which is on money market rates, which are likewise deemed very safe. This means that the natural rate of interest and (the inflation-adjusted) key interest rate are compared in a manner that is equivalent in terms of risk. At the same time, however, monetary policymakers should not focus exclusively on a secure interest rate. Precisely in view of their relevance to inflationary pressure due to the real economy, it is also sensible not to lose sight completely of interest rates at further stages of the monetary policy transmission process. In this respect, differing measures for returns on capital are an appropriate indicator.

Return on borrowed capital on downward trajectory, while return on total capital displays a slightly upward trend

A study of various measures of returns on capital reveals a slower decline for returns on borrowed funds, especially in the case of corporate bonds with a high credit rating, but no decline in the given various measures of the return on capital and the return on equity (see the box on page 37 et seq).

Assessment

Need to determine causes before making an assessment

For various reasons, the natural rate of interest has been attracting greater interest of late. For monetary policymakers, the natural interest rate is in many cases a key concept for gauging the degree of expansion of their monetary policy. To this end, a reliable and robust analysis of the causes of the level of and change in the natural rate of interest is required. Surveying selected methods of determining the natural rate of interest shows, however, that quantified conclusions about the level of this variable are subject to considerable uncertainty. Above and beyond the customary model uncertainties, differences are also due to differing concepts of the natural interest rate: In particular, the time dimension, that is to say short-term equilibrium

Developments in the real return on capital in Germany

Measuring the real return on capital – the ratio of earnings to capital input – requires a number of assumptions about the group of entities under observation as well as a definition of both earnings and capital. Since each of these assumptions can be subjected to critical scrutiny, the present text pursues an approach which is as broad as possible in order to obtain a robust outcome. In line with the thrust of the main text, this is less a matter of a precise determination of the level of the return on capital than of its trend over time. For this purpose, six approaches are explored below, sorted by the type of capital ranging from borrowed capital and total capital to equity capital.[1]

Effective interest rates on bank loans to non-financial corporations

In view of the fact that enterprises in the corporate sector in Germany and the euro area are predominantly financed by loans from the banking sector,[2] lending rates are an obvious metric.[3] The chart on the left below shows the interest rate for bank loans to non-financial corporations, which account for nearly two-thirds of gross value added in Germany, adjusted for inflation expectations over comparable maturities and on a 12-month average, broken down by original maturity. There are two distinct phases of the decline in interest rates, from 2009 to 2012 and from 2015 onwards.

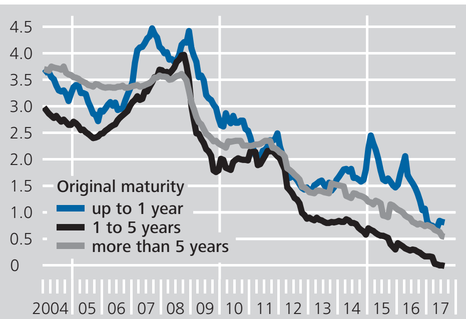
Returns on corporate bonds

Along with bank loans, corporate bonds represent the most important form of debt financing. The chart on the right below documents the remuneration of German corporate bonds over the past 60 years adjusted for autoregressive estimated inflation expectations over a medium-term maturity. The chart at the top of the next page shows

1 Tax aspects are not considered separately in this connection – as a general rule, the return before tax on earnings is assumed as a basis.
2 See H Uhlig and F De Fiore (2011), Bank finance versus bond finance, Journal of Money, Credit and Banking, 43(7), pp 1399-1421.
3 See K Wicksell (1898, trans. 1936), Interest and prices. A study of the causes regulating the value of money. London, Macmillan & Co.

Real effective interest rates on bank loans to non-financial corporations*

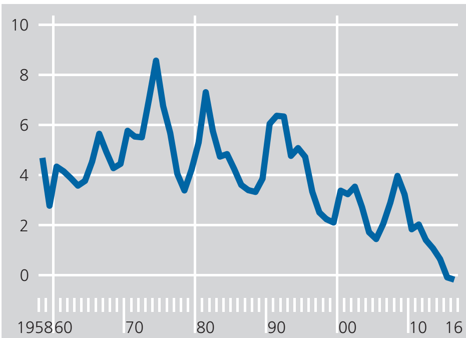
% pa, month-end levels



Sources: Consensus Economics and Bundesbank calculations. * Levels, by original maturity, real interest rates based on nominal interest rates less concurrent inflation expectations; 12-month averages.
Deutsche Bundesbank

Real return on outstanding domestic corporate bonds*

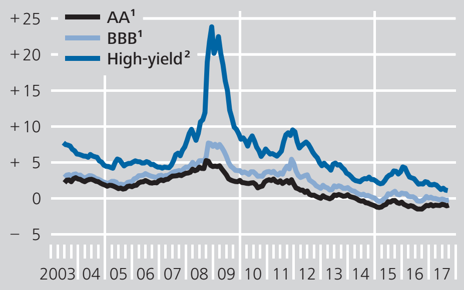
% pa



Sources: Federal Statistical Office and Bundesbank calculations. * Domestic bonds of enterprises (non-MFIs) less autoregressive estimated inflation expectations of medium-term maturity.
Deutsche Bundesbank

Real return on outstanding euro-denominated corporate bonds by rating class

% pa



Sources: Merrill Lynch, Consensus Economics, Thomson Reuters and Bundesbank calculations. **1** iBoxx indices with residual maturity of seven to ten years. **2** Merrill Lynch index across all maturities.
Deutsche Bundesbank

European corporate bonds, broken down by class of risk. Here, too, it is apparent that there has been a declining trend since the mid-1980s; yield spreads between sovereign and corporate bonds are stable or shrinking.

Net return on the aggregate capital stock

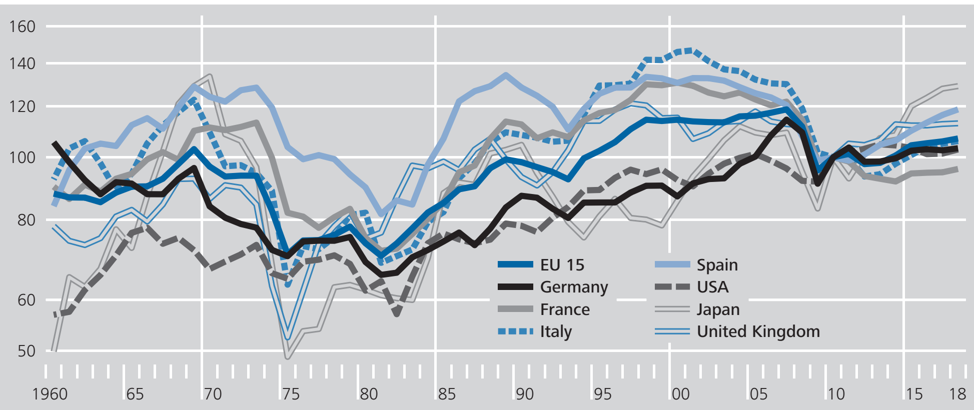
Besides the return on borrowed capital, it is possible to calculate various measures for the remuneration of total capital, which, in addition to borrowed funds, also includes equity capital. One possible broad definition of the return on capital encompassing the economy as a whole would be the ratio of the share of net national income due to capital as a factor of production and the net capital stock, as used by the European Commission. The time series dating back to 1960 for the countries considered here as well as the EU 15 show, amidst quite a large range of fluctuation, a general upward trend interrupted by recession-related troughs, including around the years 2009 and 2013 (see the chart below).

Return on the productive capital of non-financial corporations

More specifically tailored to the needs of enterprises and, in particular, their productive capital are measures from the national accounts for the non-financial corporations sector. Here, it is possible to calculate the return on the entire productive capital using the ratio of the operating surplus to the productive capital stock, with the productive capital stock being approximated across all fixed assets. Items suitable for use as narrower measures of the productive capital stock are tangible fixed assets (all assets ex-

Net return on the aggregate capital stock for selected countries

2010 = 100, log. scale



Source: European Commission.
Deutsche Bundesbank

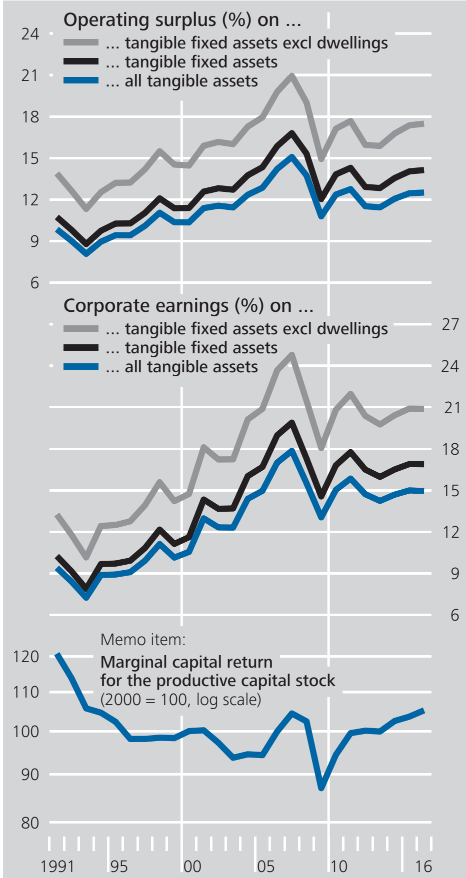
cluding intellectual property) as well as fixed assets excluding dwellings.

The operating surplus is the term used to describe the remuneration for all investors – equity and debt alike – and is therefore well suited to calculating the return on total assets.[4] Deducting interest payments to the creditors from this (and adding interest and dividends received) gives the corporate earnings and, with these, a measure of the return on capital geared to the equity providers.

As may be seen from the adjacent chart, all of the proposed measures of the return on total assets since 1991 show no declining trend. And even the sharp drop in risk-free interest rates and interest rates on corporate bonds in the wake of the financial crisis cannot be discerned for these measures. In the early 1990s, the level lies between 7% and 14%; two decades later, it is between 14% and 21%, with significant declines in the years 1993, 2009 and 2013.

However, it is the average returns on capital which are shown; economic decisions do not, as a general rule, depend on the average return, but on the marginal return – in other words, the expected return on every additional euro invested. The average return is thus no more than a rough approximation of the marginal return,[5] although the marginal return on capital for the productive capital stock,[6] which is also shown as a memo item, does not display a significantly different profile following a catching-up process from the mid-1990s onwards due to German reunification.

Average return on non-financial corporations' capital based on the national accounts*



Sources: Federal Statistical Office and Bundesbank calculations. * Based on different definitions of net earnings and net fixed assets.
Deutsche Bundesbank

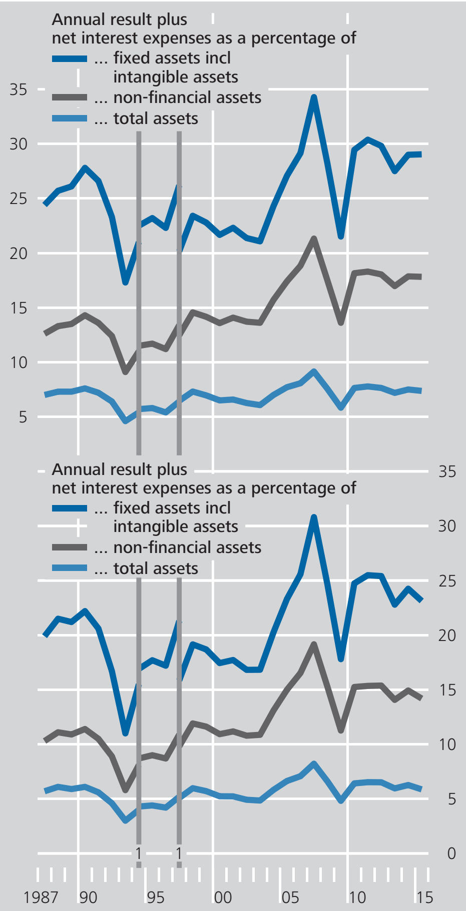
Return on total capital on the basis of annual financial statement data

The pool of data on the annual financial statements gives the Bundesbank access to information on the balance sheets and profit and loss accounts of more than 100,000 enterprises. Data extrapolated

4 The operating surplus is essentially nominal output less intermediate consumption, depreciation, and compensation of employees – in other words, the remuneration of labour as a factor of production.
5 Under the simplifying assumption of a Cobb-Douglas production function, the average return tracks the marginal return. Concerning the reasons for the two types of returns drifting apart, see J Poterba (1998), The rate of return to corporate capital and factor shares: new estimates using revised national income accounts and capital stock data, Carnegie-Rochester Conference Series on Public Policy 48, pp 211-246, especially p 224.
6 Calculated on the basis of a theoretical model with a CES production function.

Average returns on corporate capital from financial statements*

%



Source: Bundesbank calculations. * According to different definitions of earnings (before taxes on earnings) and productive capital stock. 1 Reference dataset changed.
Deutsche Bundesbank

from this cover over 90% of the turnover of the non-financial corporations sector. These data are extremely helpful not only owing to their breadth of coverage and the time period but also because they allow a sectoral breakdown for the purposes at hand. For internal consistency, when using this dataset the annual result before taxes on income and, additionally, the annual result before taxes on income plus interest expenses and less interest earnings are used as the numerator for calculating the return on capital. As the denominator, tangible assets including intangible fixed assets are used along with non-financial assets, which additionally include inventories, and total assets, which also encompass financial assets.[7] As in the case of national accounts data, in the adjacent chart it is possible to identify a moderately rising trend as well as two slumps in the years 2009 and 2013. In contrast to the national accounts data, however, the return levels are higher on average, although the definitions of capital tend to be broader than in the measures based on national accounts metrics. For the best comparable measure – the annual result before taxes on income plus interest expenses less interest earnings as a percentage of tangible and intangible fixed assets from the annual accounts data pool, as well as, from the national accounts, the operating surplus as a percentage of all assets – the return levels based on the annual financial accounts data are more than twice as high. This is probably due, among other things, to incomplete coverage of all the economic sectors in the annual accounts data pool, as well as differences in the respective accounting rules.

Earnings yield of German DAX 30 companies

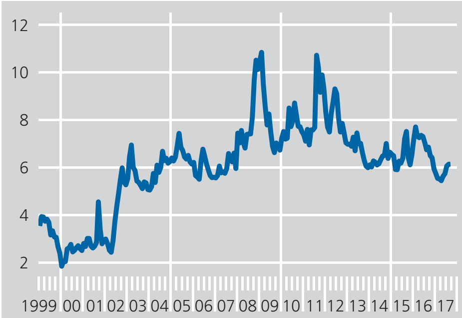
Finally, for listed enterprises the earnings yield can be calculated as the ratio of the expected profits 12 months ahead to the market value of the enterprises. In contrast to the previous measures, this rate of return is forward-looking and thus provides a basis for deciding on a possible, marginal additional investment. On the other hand, the earnings yield captures only equity capital. As shown by the chart on page 41, the earnings yield displays no clear-cut trend and fluctuates between 4% and 11%, with identifiable significant declines in the wake

7 These include cash and bank balances, short and long-term receivables, securities, participating interests and deferred income.

of the 2009 financial crisis and the 2013 European sovereign debt crisis. By contrast, the equity risk premium, ie the excess return that investors expect with given dividend expectations and given a risk-free alternative vehicle for the risk of an equity investment, shows a trend rise over the available period. This does not necessarily mean that shares have become more risky or less attractive but perhaps only that secure forms of investment have become harder to come by.[8]

Real earnings yield of DAX-listed enterprises*

%, month-end levels



Sources: Thomson Reuters I/B/E/S, Consensus Economics and Bundesbank calculations. * Earnings expectations 12 months ahead in relation to the price index, adjusted for expected inflation.

Deutsche Bundesbank

Conclusion

It is apparent that, in the wake of declining government bond yields, bank lending rates for enterprises and bond yields of private issuers, especially those with a high credit rating, have also been declining more and more, too – as is entirely consistent with arbitrage between government bonds and safe corporate bonds. Measures of the return on total capital and the return on equity, on the other hand, display no downward trend. This picture is also repeated for the United States, where the real return on capital – with the exception of troughs in 2001 and 2009 – has been showing an almost continuous upward trend since the 1990s.[9] And even looking at the real return on capital over a period of 200 years does not necessarily imply that how it has developed over the past 30 years has ushered in a new, unprecedented era.[10] From this perspective, there is much to suggest that a change in risk behaviour is the major factor behind the marked difference between the returns on borrowed funds and those on equity financing. Investors in equity capital as a volatile residual income class are not experiencing any sustained fall in returns, whereas the return on fixed interest income streams has declined noticeably.

8 On the design and the identification of the earnings yield and the equity risk premium, see Deutsche Bundesbank, Stock market valuations – theoretical basics and enhancing the metrics, Monthly Report, April 2016, pp 15ff.
9 See, for example, P Gomme, B Ravikumar and P Rupert (2011), The return to capital and the business cycle, Review of Economic Dynamics, 14(2), pp 262-278; updated and republished as (2015), Secular stagnation and returns on capital, St. Louis Economic Synopsis No 19/2015.
10 See T Piketty (2014), Capital in the twenty-first century. Translated by Arthur Goldhammer, The Belknap Press of Harvard University Press, Cambridge, Massachusetts, especially Figures 6.3 and 6.4.

or medium to long-term equilibrium, and the inclusion of risk elements, do indeed result in very different assessments. From a monetary policy perspective, there are therefore some arguments for not giving too much weight to the concept of a natural rate of interest and for regarding it instead as one of a large number of indicators relevant to monetary policy, while remaining aware of its limitations.

The natural rate of interest has recently been advanced as an indicator of a sustained dramatic deterioration in the long-term outlook for growth (secular stagnation theory). In the academic debate, four explanatory factors have emerged as possible causes of a secular downward trend in the natural interest rate: a stronger propensity to save, a reduced inclination to invest, policy measures and, lastly, a change in risk aversion.

Indications of demand for less risky assets as a key driver

The finding presented in this article indicating an almost consistently high return on equity and total capital appears especially compatible with an explanation based more on a higher demand for less risky assets and less so with theories about an increased propensity to save or a reduced inclination to invest since, in principle, both should be evident in the returns on borrowed capital as well as on equity capital.[12] Seen in that light, the finding given here does not corroborate the secular stagnation theory. Nevertheless, a study of the deeper-lying causes of a virtually constant return on equity with, at the same time, lower rates of interest on borrowed funds still merits further analyses.

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12 Many research papers also find that the risk factor plays a crucial role in the natural rate of interest. These papers include M T Kiley (2015), op cit; regarding the specific form of risk aversion in premiums on safe investments and high liquidity in M del Negro, D Giannone, M P Giannoni and A Tambaloti (2017), Safety, Liquidity, and the Natural Rate of Interest, Federal Reserve Bank of New York Staff Reports 812; in a structural model for the United States and the euro area: Gerali and Neri (2017), Natural rates across the Atlantic, Banca d'Italia Working Paper No 1140. An approach that explicitly also considers a consistently high return on assets is chosen by R Caballero, E Farhi and P-O Gourinchas (2017), Rents, Technical Change, and Risk Premia. Accounting for Secular trends in Interest Rates, Returns on Capital, Earnings Yields, and Factor Shares, American Economic Review: Papers & Proceedings 107(5), pp 614-620. One possible explanation for the divergence of returns on equity and on capital from safe returns based on a shortage of safe-haven assets is presented by R Caballero, E Farhi and P-O Gourinchas (2017), The Safe Asset Shortage Conundrum, Journal of Economic Perspectives 31(3), pp 29-46.