Uncertainty of macroeconomic forecasts

In 2007, the Bundesbank began publishing its forecasts on the future economic development of Germany – a procedure that an increasing number of other central banks have also adopted for their own domestic economies. In these macroeconomic forecasts, the central banks often go beyond providing mere point predictions by also giving an indication of how uncertain the predictions are themselves. The scale of a forecast's uncertainty is an important piece of information for addressees in its own right, especially if the predictions are to be used as the basis for policy decisions.

The forecast uncertainty itself, however, is also not known and has to be estimated. Compared with simple forecast models, the uncertainty in Bundesbank forecasts is relatively small and similar to that of other institutions. That said, the unexpectedly steep drop in gross domestic product observed during the recent financial crisis has led to a conspicuous rise in forecast uncertainty estimates for the future. Since then, the possibility that extreme events could occur has had a markedly higher probability than it did prior to the crisis.

Introductory statement

Economics staff at the European Central Bank and the national central banks of the euro area produce semi-annual macroeconomic projections which the Governing Council uses as a basis for monetary policy decisions. The macroeconomic forecasts on the German economy, which the Bundesbank produces in this context, have been published since December 2007.[1]

Aside from its point predictions, the Bundesbank also publishes margins of uncertainty for those variables that are of the greatest public interest, namely rates of change in the real gross domestic product (GDP) and the Harmonised Index of Consumer Prices (HICP).[2] Other central banks also release regular information on forecast uncertainty. The ECB refrains from publishing point forecasts, but instead provides ranges for the expected future rates of change in GDP, in its components and in the HICP. The width of these ranges reflects the scale of the forecast uncertainty.

Importance of information on forecast uncertainty

Information about forecast uncertainty is important in many ways with regard to the expectations and actions of participants in economic activity. The risk premiums of nominal bonds, for example, are aligned with, among other things, the scale of uncertainty associated with expected inflation rates in the future. The greater the uncertainty, the higher the risk premiums demanded by risk-averse investors. Indications of forecast uncertainty also plays an important role in many other economic decision-making processes. If central banks want to rule out future deflationary developments to a high level of probability, for example, they require information about the uncertainty of the inflation forecasts. The same is true of GDP forecasts for fiscal policy-makers if they want to avoid exceeding a certain deficit ratio, for instance. Overall, it is clear that the ability to determine the level of uncertainty in future developments can affect economic decision-making in many respects.

Causes of forecast uncertainty, concepts for their measurement and forms of representation

Forecasts are generated using statistical models – either implicitly or explicitly.[3] The forecast values are determined by the model structure and its parameters, as well as by past and future values for different variables. The latter can be sub-divided into endogenous and exogenous variables, as well as disturbance terms. Endogenous variables are described within the model while exogenous variables influence the result "from the outside", so they must first be forecast them-

1 See, for example, Deutsche Bundesbank, Outlook for the German economy: macroeconomic projections for 2008 and 2009, Monthly Report, December 2007, p 17 ff.
2 This is the seasonally and calendar-adjusted GDP. All the GDP assessments that follow also refer to the real, seasonally and calendar-adjusted GDP as well as the original HICP values.
3 For examples of forecast models at the Bundesbank see Deutsche Bundesbank, Short-term forecasting methods as instruments of business cycle analysis, Monthly Report, April 2009, p 31 ff.

selves before the models can be used to generate predictions.[4]

Various reasons for forecast uncertainty

Based on these considerations, five principal causes of forecast uncertainty can be identified: data uncertainty, parameter uncertainty, future disturbances' uncertainty (shocks), future exogenous variables' uncertainty and model uncertainty.

Data uncertainty exists if the observed variables contain measurement errors. The first publications of the national accounts, which are of great importance for the forecast, are still partly based on estimates, for example, and are often revised during the subsequent quarters. The values used for the prediction therefore frequently differ from the actual data.

Parameter uncertainty is predominantly influenced by the size of the sample available for estimating the model, ie from the length of the available time series. The longer the time series are, the lower the parameter uncertainty usually is. This is not the case, however, if parameter values change over time.

As every model is merely an approximation of reality, it also contains unpredictable disturbance terms whose realisations are unforeseeable at the time of the forecast. One such example would be a surprisingly cold winter, causing unpredictable output disruptions. The future development of model-exogenous variables that are assumed in the forecast, such as the price of oil, is also uncertain and in most cases deviates from the actual development.

Finally, there is model uncertainty, and it is uncertain whether the model used for the prediction correctly captures the economic relationships.

Measuring uncertainty using models, …

Assessing the scale of each factor's influence on forecast uncertainty is a difficult task, and quantifying model uncertainty is particularly difficult. If a model is given, however, then – conditional on that model – the effects of data and parameter uncertainty, future shocks and uncertainty regarding the future development of model-exogenous variables can be estimated in principle. To be able to take the uncertainty of the future developments of model-exogenous variables and data uncertainty into consideration, other models must first be set up to predict the model-exogenous variables and estimate the data, and then have to be integrated into the main model for estimating forecast uncertainty. The forecast uncertainty of this model can then be analysed using stochastic simulations.[5] Such simulations provide a large sample of predicted values which are seen as realisations of random variables, the distribution of which can be used to calculate measures of forecast uncertainty .

A model-based estimate of forecast uncertainty therefore requires a considerable

4 To generate inflation rate forecasts, model-endogenous variables to be considered are, for example, present and past inflation rates, as well as in larger models, the current values of wages, monetary aggregates or capacity utilisation. The price of oil or the level of VAT, for example, are likely to be treated as model-exogenous variables whose future values have to be forecast outside of the model, and in certain circumstances with the help of another model.
5 See, for example, R C Fair (2003), Bootstrapping macroeconometric models, Studies in Nonlinear Dynamics & Econometrics, Volume 7, Issue 4, Article 1.

amount of effort. Another problem is that in most cases, forecasts are not solely model-based, but also rely on information from outside the model. This includes data which are available earlier and are sampled with higher frequency than the model data. A model-based assessment of forecast accuracy can therefore, primarily in the short term, at best provide approximate results. Against this backdrop, it is not surprising that model-based estimates of forecast uncertainty are only of limited importance in practice.

... past forecast errors ...

A much simpler estimate of uncertainty can be calculated using past forecast errors. This implicitly takes account of all uncertainty sources that have contributed to the deviations between realised values and the forecast values. Using past forecast errors, measures of dispersion can be estimated and used as a yardstick for the uncertainty levels of current projections. Popular estimated measures of dispersion are the mean absolute error (MAE) of forecasts or the root mean squared error (RMSE).[6]

One problem with using this method can be that only a few forecast errors are available for the required estimates. In this case, the precision of the estimates is low.[7] Furthermore, the empirical measures of dispersion can be non-monotonic over the forecast horizons, whereas it is generally expected that forecast uncertainty increases with the forecast horizon. The empirical measures of dispersion are therefore sometimes smoothed over the horizons so as to present a plausible picture of forecast uncertainty.

... or surveys

A third solution is to make use of surveys, whereby the questions are either focussed directly on the dispersion measure of interest, or the (divergent) point predictions are evaluated for individuals or institutions. Central banks sometimes use the various estimates within their decision-making bodies in this context. An assessment of uncertainty can be made by examining to what degree the point predictions deviate from one another. Under certain circumstances, such an estimate may, however, prove unsuitable for making probability statements. If, for example, all respondents are expecting an inflation rate of around 2% in the long term, but each one of them is very unsure about their own prediction, a judgement based solely on point predictions would come to the conclusion that the uncertainty for long-term forecasts of the inflation rate is very low. In actual fact, a sizeable deviation of the long-term inflation rate from 2% is not unlikely in this case according to those interviewed. Nevertheless, the dispersion of point predictions can provide important information, especially about possible

6 The MAE is defined as

$$\text{MAE} = \tfrac{1}{N}\sum_{t=1}^{N} |u_t|,$$

where the forecast error at time t is denoted by u_t, and N is the number of available forecast errors. The RMSE, which is an estimate of the root of the expected squared forecast error, is calculated as

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{t=1}^{N} u_t^2}.$$

7 Precision of the estimates can be improved if the forecast uncertainty estimate is undertaken simultaneously for all forecast horizons. See M Knüppel (2009), Efficient estimation of forecast uncertainty based on recent forecast errors, Discussion paper by the Deutsche Bundesbank Research Centre, Series 1, No 28/2009.

changes to the forecast uncertainty over time.[8]

In addition, the aforementioned survey-based methods can be combined by surveying individuals or institutions on both their point and uncertainty predictions, or on the probabilities for specific ranges of the variable in question. An aggregated forecast uncertainty can then be calculated from this information, which takes into account the differences between the point predictions as well as the uncertainty of each respondent.

The methods stated are not mutually exclusive. An estimate based on forecast errors, for example, can generally produce a quite reliable assessment of past forecast uncertainty. If the surveys show that the respondents believe current uncertainty to be considerably higher than past uncertainty, however, the estimate can be adjusted accordingly.[9] In actual fact, several representations of forecast uncertainties are based on hybrid forms of the approaches described here.

All current forecast uncertainty estimates are ultimately based on uncertainty that has been observed in the past. This becomes especially clear when using past forecast errors, but it is also the case when using model-based methods and even, implicitly or explicitly, with survey-based methods.[10] For this reason, a reliable estimate of the current forecast uncertainty can only be achieved if the forecast uncertainty does not vary too greatly over time.

Several possible types of representation

Various alternatives are available not only when measuring but also when representing forecast uncertainty. The choice of representation depends on the forecast characteristics, the type of forecast uncertainty measurement used and the scope of the information to be conveyed. A forecast of many monthly values warrants a different type of representation than the forecast of a single annual value, for example. And a survey-based measurement of uncertainty can require a different representation than a model-based measurement.

Central banks often use fan charts of the type first published by the Bank of England in 1996. Often in these charts, a certain probability is given that is covered by each individual fan section. The charts illustrate that forecasts are uncertain and that values close to the point predictions, ie values in the middle of the chart, are more probable than those that lie further from the point predictions.[11] It also becomes clear that forecast uncertainty rises in line with the forecast horizon.

8 See P Giordani and P Söderlind (2003), Inflation forecast uncertainty, European Economic Review, 47 (6), pp 1037-1059.
9 However, there are indications that forecasters tend to overestimate forecast uncertainty. See E M Leeper (2003), An inflation reports report, Sveriges Riksbank Economic Review 2003:3, pp 94-118; K F Wallis (2004), An assessment of Bank of England and National Institute inflation forecast uncertainties, National Institute Economic Review, No 189, pp 64-71; K Dowd (2007), Too good to be true? The (in)credibility of the UK inflation fan charts, Journal of Macroeconomics, Volume 29, Issue 1, March 2007, pp 91-102.
10 The respondents' estimates, too, are aligned to uncertainties observed in the past.
11 For further explanations about fan charts see E Britton, P Fisher and J Whitley (1998), The Inflation Report Projections: Understanding the fan chart, Quarterly Bulletin of the Bank of England, February 1998, pp 30-37.

Fan charts are well-suited for representing forecast uncertainty over many consecutive periods. If the prediction uncertainty is only required for a few specific points in time, a histogram or a probability density can be shown, for example.[12]

Overview of central banks' approaches

In practice, central banks that publish forecast uncertainty measures use a wide range of estimation procedures and representation methods (see table on page 35). Partly these procedures can be identified as one of the methods presented here and partly they are a result of combinations. Measures of uncertainty based on past forecast errors play a major role for many central banks.

Central banks use various methods to measure uncertainty ...

The uncertainty estimates for ECB and ESCB staff projections are based on past forecast errors. Especially large forecast errors are seen as outliers, however, and are excluded from the sample. The mean absolute forecast errors are calculated from the remaining forecast errors. The width of the forecast ranges for the annual forecasts, which are published in a table, is twice that of the absolute error. The ECB states that the probability of a realisation within such a range is 57.5%.[13] This is in line with a normal (Gaussian) distribution of errors (corrected for outliers).

The Bundesbank's uncertainty forecasts are also based on the mean absolute error of past forecasts and relate to quarterly periods. As no distribution assumption is made for the forecast errors, as in the case of the ECB, no explicit probability statements are implied by the published fan charts. They can be derived, however, if a certain distribution is assumed, as in the case of the ECB's euro-area projection.[14]

The Sveriges Riksbank publishes fan charts whose width is determined by the root mean squared forecast error. A normal distribution is assumed for the forecast errors.[15] The Sveriges Riksbank forecasts quarterly values.

Model-based uncertainty forecasts, for example, are used by the Norges Bank (central bank of Norway). One of the reasons why past forecast errors are not used is that, in the available sample, they had been extraordinarily large due to special factors for one of the forecast variables, namely the three-month money market rate.[16] The available sample was considered too small for accurate estimates of the forecast uncertainty. It is as-

12 A histogram can be useful if the uncertainty forecast is based on surveys that focus on probabilities for specific ranges of the variable under scrutiny. A probability density is appropriate if an assumption is made about the distribution of forecast errors.
13 See European Central Bank, New procedure for constructing Eurosystem and ECB staff projection ranges, December 2009, http://www.ecb.int/pub/pdf/other/newprocedureforprojections200912en.pdf.
14 A fan chart width of twice the mean absolute error corresponds with a probability of 57.5%, for example, and a width of one mean absolute error with a probability of 31% if the forecast errors are normally distributed. If a different distribution is assumed, as a rule, additional assumptions about other distribution parameters must initially be made. The assumption of a normal distribution is often used for macroeconomic forecast errors. The large GDP forecast errors during the financial crisis, however, have shown that more caution must be exercised where this assumption is concerned.
15 See Sveriges Riksbank, Monetary Policy Report 2007:1, p 22.
16 See Norges Bank, Inflation Report, November 2005, pp 19-21.

sumed here, too, that the forecast errors are distributed normally.

At the Bank of Japan the Policy Board members' estimates are key.[17] The members provide histograms of their own forecast probabilities. The highest value (mode) in such a histogram is also the point prediction of that particular member. These histograms are then aggregated by calculating the average of the corresponding values for each range in all histograms.[18] The dispersion in the published aggregated histograms is thereby influenced by the differences in the point predictions as well as by the uncertainty of each Policy Board member. The forecasts refer to annual values.

... or combine various procedures

Many central banks combine elements of different procedures. Past forecast errors and the various estimates of the decision-makers are brought together at the US Federal Reserve and the Bank of England, for example.

For its annual forecasts, the Federal Reserve publishes the average root mean squared error of past forecasts from various institutions[19] as well as estimates of the FOMC[20]

17 The Policy Board of the Bank of Japan is responsible for monetary policy decisions.
18 See Bank of Japan, Outlook for Economic Activity and Prices, April 2008, p 9.
19 The RMSE of the Federal Reserve's own forecasts is also included. These differ only slightly from the RMSEs of forecasts produced by other public and private institutions. See P Tulip and D Reifschneider (2007), Gauging the uncertainty of the Economic Outlook from historical forecasting errors, Finance and Economics Discussion Series 2007-60, Washington: Board of Governors of the Federal Reserve System.
20 The Federal Open Market Committee (FOMC) is the policy-making body of the Federal Reserve System.

Overview of various uncertainty estimation methods and representations

Central bank	Frequency of variables	Method of estimating uncertainty 1	Representation
ECB and ESCB (staff projections)	Annually	Past forecast errors (MAE) (corrected)	Table with uncertainty ranges
Deutsche Bundesbank	Quarterly	Past forecast errors (MAE)	Fan charts
Sveriges Riksbank (Central Bank of Sweden)	Quarterly	Past forecast errors (RMSE)	Fan charts
Norges Bank (Central Bank of Norway)	Quarterly	Model-based	Fan charts
Bank of Japan	Annually	Point and uncertainty estimates by Policy Board members	Histograms
Federal Open Market Committee (FOMC, component of the Federal Reserve), individual members	Annually	Past forecast errors (RMSE)	Verbal (uncertainty greater, smaller or same in comparison to the past)
Also: All FOMC members	Annually	Differences in point predictions by FOMC members	Box-plots and histograms
Bank of England	Quarterly	Past forecast errors (RMSE) and estimates by MPC	Fan charts
Bank of Canada	Quarterly	Past forecast errors for first two forecast horizons, model-based for all other horizons	Fan charts

1 MAE: mean absolute forecast error; RMSE: root mean squared forecast error.

Deutsche Bundesbank

members on whether the current forecast uncertainty is greater, smaller or roughly the same in comparison to the past. The probability of a realisation within a range that is as wide as twice the root mean squared error (set by a FOMC member's forecast) is around 70%.[21] Furthermore, box-plots and histograms reflect the FOMC members' various point predictions.

The Bank of England initially calculates the root mean squared errors using forecast errors over the last 10 years.[22] On the basis of this value, the MPC members[23] agree on the width of the fan chart into which their estimates on the current forecast uncertainty are entered. Explicit probability statements can then be implied from the fan chart for quarterly projections and the underlying probability distributions, whose parameters are also published by the Bank of England. The Bank of England's approach has been adopted by several other institutions, in some cases with slight modifications.[24]

The Bank of Canada takes account of information from past forecast errors as well as model-based uncertainty measures. The uncertainty calculated from past forecast errors is used for the first two forecast quarters, while the uncertainty is derived from a model for additional forecast horizons, whereby the lagged effects of the forecast errors for the first two quarters are also included.[25] The reason is that the model forecast for the near future is modified relatively strongly by exogenous information, which plays a smaller role in the medium and long term. It is assumed that the forecast errors are normally distributed.

Bundesbank forecast uncertainty

The Bundesbank's forecast uncertainty calculation is based on actual errors made by the Bundesbank for Germany in the Eurosystem's macroeconomic staff projections. For these forecasts, joint estimates are made for all national central bank and ECB forecasts for the development of important exogenous variables, such as the price of oil, exchange rates and interest rates – with interest rates being of particular importance.

Bundesbank forecast based on certain interest rate paths ...

For central banks the treatment of current interest rate developments and interest rate developments expected in the future is of particular importance in their forecasts. Firstly, the short-term interest rates are the main monetary policy instrument for achieving primary monetary policy objectives. Secondly, monetary policy has a considerable effect on future interest rate expectations through changes in the current short-term interest rate and the manner in which they are communicated. In this context, the way in which the expected short-term interest rate is factored into central banks' own forecasts is extremely complex. There has been a major change in this area within the Eurosystem

21 See Board of Governors of the Federal Reserve System, Monetary Policy Report to the Congress, 15 July 2008, p 45.
22 See E Britton, P Fisher and J Whitley (1998), loc cit.
23 The Monetary Policy Committee (MPC) is the policy-making body at the Bank of England.
24 This includes the IMF as well as central banks such as the Bank of Chile and the Bank of Hungary. The Sveriges Riksbank also used the Bank of England's procedure until 2007, combining it with the Executive Board's estimates and past forecast errors.
25 See Bank of Canada, Methodology used to construct fan charts in the Monetary Policy Report, April 2009, http://www.bankofcanada.ca/en/mpr/pdf/backgrounder_fancharts.pdf.

over the past few years. Until spring 2006, the forecasts were based on the assumption that interest rates would remain constant in the future. Since then, the interest rates expected by market players (market interest rates) have been used. In both cases, the forecasts can be seen as projections conditional on a specific interest rate path. The aim of the forecast, therefore, is not to determine the best-possible unconditional estimate of future developments, but rather to find the best-possible estimate subject to the condition that interest rates follow the assumed course.[26] Forecast deviations from the actual development that result from an interest rate development other than that assumed cannot therefore merely be seen as a sign of poor forecast performance.[27]

... with consequences for forecast uncertainty

The 2006 change in the assumptions regarding interest rates has consequences for forecast uncertainty. Generally the long-term inflation forecast will roughly adopt the value aimed for by the central bank if market interest rates and the incorporated future interest rate expectations are used for the forecast and assuming the central bank has credibility with regard to achieving its objective. By contrast, if constant interest rates are assumed, the associated long-term inflation forecast can deviate more strongly from the central bank's target inflation rate and/or from the actually realised inflation rate. Correspondingly, errors in the long-term inflation forecast when using market interest rates should be smaller on average than when using constant interest rates. This is presumably less valid in the short term, as assuming unchanged interest rates for such a period can prove more often to be accurate and as interest rate changes only have a small effect on growth and prices in the short term.

Owing to the fact that this procedure was only introduced in the Eurosystem in 2006, market interest rates have only been used in eight forecasts so far. A reliable estimate of forecast uncertainty is not possible with such a small sample. The current estimate for the entire period from spring 1999 to autumn 2009 is therefore based on a total of 22 forecasts, meaning that the forecast uncertainty calculated in this way probably tends to be overstated.

To calculate the forecast errors, the corresponding realisations are also required in addition to the forecasts themselves, and their values are repeatedly revised by statistical offices. Revisions may be induced by new data or the use of new data measurement concepts. Thus, the first release of GDP data for the preceding quarter generally deviates from the next publication for the same period. Furthermore, methodological changes in data acquisition or evaluation can also lead to changes long after the publication date.

26 If the forecast variables deviate from the monetary policy target, this generally indicates a need for monetary policy action. Until 2006 this meant that the interest rates should not remain at their prevailing levels. After 2006, deviations from the monetary policy target would imply that interest rates should take a different course from that expected by market players.

27 For exchange rates, the technical assumption is that during the forecast period they should remain at the level observed at the time the forecast was generated. Thus, the forecasts are also conditioned on the corresponding exchange rate paths. The quality of the forecasts could be noticeably affected by the rule applicable for Eurosystem forecasts permitting consideration solely of fiscal policy measures for which either the legislative procedure has been completed or which have been sufficiently specified and are likely to be adopted.

Uncertainty of Bundesbank forecasts – a comparison

It is generally desirable that forecasts are as accurate and precise as possible. Therefore, the lower the level of uncertainty of a forecast, the better it is thought to be. When comparing a number of different forecasts, however, it is essential to ensure that the forecast environments are identical. This includes ensuring that the respective forecasts are based on the same information set and, hence, that the forecasts are generated at the same time. Furthermore, conditional forecasts should, as far as possible, be compared with other forecasts which are subject to the same conditions. Finally, forecasts should relate to the same or at least very similar variables. In the following, every effort has been made to ensure that these conditions are met as far as possible. Nevertheless, the Bundesbank forecasts which are conditional on a certain interest rate path are compared with unconditional forecasts.

The uncertainty of the forecasts is considered for the quarterly year-on-year growth rates of the Harmonised Index of Consumer Prices (HICP) and the real seasonally and calendar adjusted gross domestic product (GDP). The Bundesbank forecasts are compared with two simple forecast models, which often have fairly good predictive properties, however. In addition, they are compared with the Consensus forecasts in which a large number of individual forecasts are combined.[1] The accuracy of combined forecasts is generally high and superior to that of most individual forecasts. The mean absolute error (MAE) is used as the measure of uncertainty. Quarterly forecasts compiled on a semi-annual basis in the period from the second quarter of 1999 to the fourth quarter of 2009 are examined.[2]

A random walk (RW) model and a first-order autoregressive (AR(1)) model are used as simple alternative models. Where the relevant variable at time t is denoted by y_t and the forecast of the variable at time $t+h$ is denoted by $\hat{y}_{t+h}$ and $T\text{-}1$ is the last period during which the variable was observed, then the forecasts for the RW model are given by

$$\hat{y}_{T+h} = y_{T-1}$$

and the forecasts for the AR(1) model by

$$\hat{y}_{T+h} = c + \rho\hat{y}_{T+h-1} = \rho^{h+1}y_{T-1} + c\sum_{i=0}^{h}\rho^{i} \ , h= 0, 1, 2, ...$$

whereby ρ is the autoregressive coefficient and c is the constant of the AR(1) model which have to be estimated.[3] In the following comparison of forecast uncertainty, the quarter prior to the forecast being prepared is used as the time period $T\text{-}1$.[4] The forecast must also be prepared for the current period T, with the result that h takes the value "zero" for the forecast of this quarter.[5] This forecast is referred to as a zero-step forecast. Such a forecast is necessary as the current quarterly value of the relevant variables is not yet known at the time that the forecast is prepared.

The MAE of the various forecasts for the annual HICP rate can be seen in the chart on page 39. In the short term, the Bundesbank forecast performs considerably better than that of the simple models, whereas in the case of longer-term forecast horizons of up to eight quarters, the differences become smaller or even disappear.

In the short term, the Consensus forecast entails roughly the same degree of uncertainty as the

1 These forecasts are compiled and published by Consensus Economics Inc. The combination of the individual forecasts is derived by calculating the average. — **2** The realisations used to calculate the forecast errors are shown in the chart on page 40 of the main text. — **3** The AR(1) model is given by $y_t = c + \rho y_{t-1} + \varepsilon_t$, where ε_t is an error term with expectation zero. The coefficients c and ρ are estimated in the following on the basis of a rolling window of 20 quarters. — **4** This is the first quarter of the year in the case of the spring forecast and the third quarter in the case of the autumn forecast. Here, the data are used which were also used by the Bundesbank when preparing forecasts. — **5** The forecast

Bundesbank forecast. For the forecast horizons h=3 to h=6, the uncertainty of the Consensus forecast is slightly lower than that of the Bundesbank forecast.

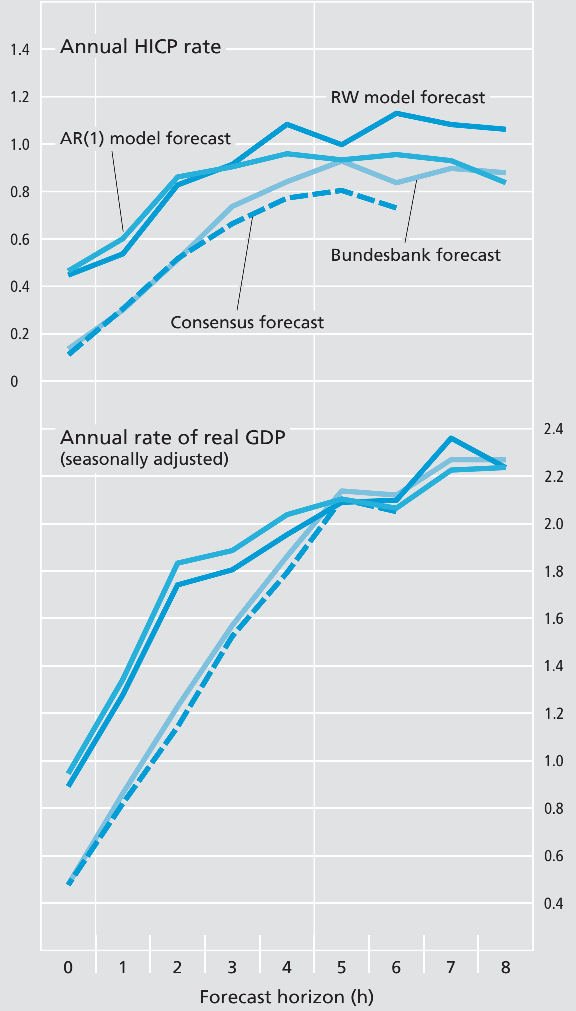
The assumption of unchanged interest rates in a large part of the sample of the Bundesbank forecasts is likely to have limited the accuracy of the Bundesbank forecasts, especially with regard to the inflation forecast and here above all in the long term. It would therefore seem plausible that the Bundesbank forecast, in relation to the other forecasts, performs better for small forecast horizons than for large forecast horizons.

Also in the case of GDP, the Bundesbank forecast is superior to the simple models in the short term, as shown in the adjacent chart, whereas the uncertainty of the Consensus forecast is very similar. For the longer term, from around five quarters, virtually no differences can be observed between the accuracy of the various forecasts.

Studies on the forecast uncertainty of other central banks sometimes yield results which deviate somewhat from those provided here for the Bundesbank. For example, studies on the forecasts of the Federal Reserve for the USA and the Bank of England for the United Kingdom show that their inflation forecasts are more accurate than those of simple models. The opposite holds true for GDP forecasts, however, where the simple models offer slight advantages across virtually all forecast horizons.[6]

Mean absolute forecast errors depending on forecast horizon

Percentage points



for the current quarter is often referred to as the "nowcast". — **6** For the USA, see J Faust and J Wright, Comparing Greenbook and reduced form forecasts using a large realtime dataset, Journal of Business & Economic Statistics, October 2009, Vol 27, No 4, pp 468–479. For the United Kingdom, see J Groen, G Kapetanios and S Price (2009), A real time evaluation of Bank of England forecasts of inflation, and growth, International Journal of Forecasting, Vol 25, pp 74–80.

Time series used for calculating forecast errors

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Methodological changes and their effects are virtually unpredictable for forecasters, so that realisations used to calculate forecast errors should be influenced by methodological changes as little as possible. Conversely, it is preferable to have as much data as possible to calculate the realisations used. Hence it is advisable to avoid using very early or very late publications. The Bundesbank calculates its forecast errors for GDP of a given quarter based on values published in the third subsequent quarter, and its forecast errors for HICP are based on values published in the third or fourth subsequent quarter.[28] The realisations calculated using this method can be seen in the chart on this page.

Compared to simple models, the Bundesbank's forecast errors in the past have been rather small. The scale of these errors roughly corresponds to what would be obtained if an average were taken of a large number of forecasts from other institutions. This means that Bundesbank forecasts perform very well with regard to their uncertainty (see the box on pages 38 and 39).

The effects of the financial crisis on forecast uncertainty and the probability of extreme results for GDP

Financial crisis illustrates problems with forecast uncertainty estimation

During the financial crisis, GDP fell sharply throughout almost the whole world. This was the case in Germany notably in the fourth quarter of 2008 and first quarter of 2009. Even though the Bundesbank had already predicted downturns for these quarters in autumn 2008, the scale of these falls was surprising, leading to sizeable errors in forecasting both by the Bundesbank and other institutions. In the fourth quarter of 2008, real GDP fell by 2.4% compared with the previous quarter and by as much as 3.5% in the first quarter of 2009.[29] In December 2008, the Bundesbank predicted a fall of 0.6% in the fourth quarter of 2008 and 0.1% in the first quarter of 2009. It was primarily because of these two forecast errors that the annual rate of change for 2009 was signifi-

28 These GDP values are taken from the Bundesbank's real-time database. Revisions owing to additional data have practically no effect on the HICP, and this variable is therefore not included in the Bundesbank's real-time database. The realisations are from internal Bundesbank sources.
29 The most extreme quarter-on-quarter rates observed between the first quarter of 1999 and the third quarter of 2008 were merely -0.5% and 1.4%.

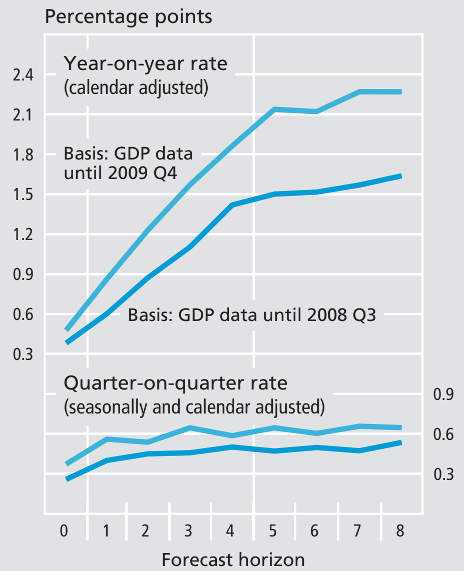
cantly overestimated in December 2008 (see the box on pages 42 and 43).

The chart on this page shows the mean absolute errors for the year-on-year and quarter-on-quarter rates of change for GDP which result from using different sample sizes. If data for German GDP until the third quarter of 2008 are used, then these errors turn out to be considerably smaller than those that result from data up to fourth quarter of 2009. The differences are mainly caused by the forecast errors for the fourth quarter of 2008 and the first quarter of 2009.

The mean absolute errors in the larger sample are between 17% and 45% higher than those in the smaller sample. The average increase across all horizons for the year-on-year rate of change is around 40% and around 30% for the quarter-on-quarter rate. It is usually assumed that expanding the sample would lead to more reliable estimates. In this instance, it would seem that the smaller sample in fact underestimated the actual GDP forecast uncertainty. This could be a result of the very steady development in growth and prices during the observed time period. However, it is also possible that the actual forecast uncertainty in the larger sample is overestimated.[30]

If this is the case, the actual forecast uncertainty – the expected absolute forecast error – should be somewhere between the values which can be seen in the chart on this page. An extreme assumption would be to treat the experience of the crisis as outliers that should not influence estimates of uncertainty in the future. This however does not seem to be justified *a priori*.

Change to forecast uncertainty* owing to financial crisis



* Forecast uncertainty estimated using mean absolute forecast errors of real GDP depending on forecast horizon.

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It is noticeable that the mean absolute errors of the quarter-on-quarter GDP rates from the larger sample show a slight zigzag course, giving the impression that forecast uncertainty at times seems to decrease as the forecast horizon increases. The error for the one-step forecast is larger than that for the two-step forecast, for example. This course is due to the forecast error for the first quarter of 2009. The quarter-on-quarter rate of -3.5% led to extraordinarily large forecast errors. To present a more plausible picture of

30 As the sample covers 11 years, an overestimation can easily be made if extreme GDP downturns such as those seen at the turn of the year 2008-2009 occur on average less than once every 11 years.

The impact of quarterly rates on the annual rate of change in gross domestic product in 2009

The technical relationship between quarterly rates and the annual rate of change of a variable provides interesting insights into the causes of the major forecast errors in 2009 in the annual rate of change in gross domestic product (GDP) which many institutions made at the end of 2008. It shows that the exact points in time at which these surprising developments occurred were particularly significant for the forecast error.

In December 2008, the Bundesbank forecast a year-on-year decline in GDP of 0.8% in 2009. At around this time, other institutions predicted a similar decline or, in some cases, a mere stagnation for this period. The Bundesbank's forecast was based on an estimated statistical carry-over[1] of -0.8% for 2009 and the assumption that GDP would stagnate in the course of 2009. In fact, the statistical carry-over was -2.2% and GDP continued to fall significantly in the first quarter, resulting in an annual rate of change of -4.9% for 2009.

The extent of the forecast error in the annual rate of change for 2009 relates closely to the forecast errors for the fourth quarter of 2008 and the first quarter of 2009. Where G_i is the annual rate of change of year i compared with the previous year and $g_{i,j}$ the quarter-on-quarter rate of quarter j from year i, G_i can be approximated using the following equation.[2]

$$G_i \approx U_i + {}^1\!/\!_4\,(4 \cdot g_{i,1} + 3 \cdot g_{i,2} + 2 \cdot g_{i,3} + g_{i,4})$$

The statistical carry-over for year i, designated U_i, which is included in this approximation, is calculated as follows.

$$U_i \approx {}^1\!/\!_4\,(g_{i-1,2} + 2 \cdot g_{i-1,3} + 3 \cdot g_{i-1,4}).$$

Thus, the quarter-on-quarter rates are factored into the calculation of the annual rates of change and of the statistical carry-over with different weights. The quarter-on-quarter rate of the first quarter of a year is the most important when calculating the annual rate of change. The most important quarter-on-quarter rate when calculating the statistical carry-over is the rate for the fourth quarter of the previous year.

In December 2008, the Bundesbank's forecast error for the current quarter of 1.8 percentage points led to an overestimation of the statistical carry-over of approximately ¾ · 1.8 percentage points = 1.35 percentage points and thus to the annual rate of change for 2009 being overestimated by the same amount. The Bundesbank's

1 Statistical carry-over is defined as the value of GDP in the fourth quarter of the previous year in relation to the quarterly average of the previous year. It thus gives the value of the annual rate of change which would ensue if GDP in the year in question remained at the level reached in the fourth quarter of the previous year. The statistical carry-over can be positive or negative. — **2** A similar approximation can be found in A J Patton and A Timmermann (2010), Predictability of output growth and inflation: A multi-horizon survey approach, un-

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3.4 percentage point forecast error at that time for the quarter-on-quarter rate of the first quarter of 2009 caused the annual rate of change for 2009 to be overestimated by roughly the same amount. Taken together, both forecast errors for the quarter-on-quarter rates gave rise to an overestimation of the annual rate of change for 2009 of some 4.75 percentage points. Since the quarter-on-quarter rates for the other quarters of 2009 were slightly underestimated on average, the final forecast error for the annual rate of change for 2009 was 4.1 percentage points.

The extent of the error in the 2008 autumn forecast of the annual rate of change for 2009 was therefore caused not only by the fact that the forecast period included two quarters of exceptionally strong GDP decline, but also by the especially high weights of these quarters in the calculation of the annual rate of change. If, for example, the quarter-on-quarter rate had amounted to -3.5% in the third quarter of 2009 instead of in the first quarter of 2009, the impact of this exceptional value on the annual rate of change for 2009 would have been halved.[3] The decline in GDP would have been smaller in annual terms and thus the forecast error for the annual rate of change would have been less pronounced.

However, the decomposition of the annual rate of change into weighted quarter-on-quarter rates also shows that, in the fourth quarter of a year, it is essentially possible to make fairly reliable forecasts of the annual rate of change for the following year. This presupposes that the forecast errors for the quarter-on-quarter rates of the current and next quarter are small. If the fourth quarter of the current year and the first quarter of the following year are predicted accurately, and if the data for the second and third quarters of the current year are known, a large part of the information needed to accurately predict the annual rate of change for the following year is already available. If these four quarter-on-quarter rates are known, the forecaster then has 62.5% of all of the information required to predict the annual rate of change.[4]

published manuscript. — **3** Nevertheless, unlike the quarter-on-quarter rate of the first quarter of 2009, the quarter-on-quarter rate for the third quarter of 2009 also affects the annual rate of change for 2010. — **4** This figure is calculated by dividing the weightings of the four quarter-on-quarter rates by the weights of all quarter-on-quarter rates used to calculate the annual rate of change.

$$\frac{1/4(1+2+3+4)}{1/4(1+2+3+4+3+2+1)} = 0.625$$

forecast uncertainty, smoothed mean absolute errors are used for the uncertainty margins which the Bundesbank gives for its forecasts.[31]

The chart on page 41 suggests that estimated forecast uncertainty – measured by mean absolute errors – has changed significantly because of the sharp GDP downturn around the turn of the year 2008-2009, but that the scale of this change is not extreme. However, even moderate changes in the dispersion measures can have a strong impact on the probability of extreme events.

Uncertainty underestimated prior to the crisis ...

If, based on knowledge of the GDP data up to the third quarter of 2008, anyone had been asked to gauge the likelihood of a GDP downturn of 2.4% in the fourth quarter of 2008 and of 3.5% in the first quarter of 2009 as well as of the Bundesbank's one and two-step forecast errors of 1.8% and 3.4% for the current and next quarter, they would have inevitably concluded in the light of past forecast errors that these events were extremely unlikely. Furthermore, an analysis of the forecast errors and quarter-on-quarter rates would have indicated that these variables were following a normal distribution. The probabilities that would have resulted on the basis of the above of the occurrence of the forecast errors and the quarter-on-quarter rates that did then actually follow are in fact extremely small.[32]

... and possibly overestimated afterwards

The larger sample of error forecasts and quarter-on-quarter rates that is now available produces considerably greater probabilities of such events. Assuming a normal distribution for all analysed variables, a future GDP change of 2.4% would now have a probability of 0.35%. A future GDP change of 3.5%, however, continues to have a very small probability of less than 0.01%.

Although, assuming a normal distribution of forecast errors, the probabilities of this massive scale of declines in macroeconomic activity now seem significantly greater than before the crisis, in absolute terms they are still very small. A 0.01% probability of a quarterly event occurring implies that this event its observed on average only once every 2,500 years. But if, as explained above, the forecast uncertainty is now presumably being overestimated, it must be asked whether the assumption of a normal distribution for forecast errors and quarter-on-quarter rates can still be justified, as this distribution allocates extremely small probabilities to extreme events. In actual fact, statistical tests would now reject the hypothesis of normal distribution.[33] Assuming a different distribution can have

31 The MAEs of the individual forecast horizons are fitted by a function in the forecast horizon function. If MAE(h) denotes the smoothed MAE of the h-step forecast, then this is determined by using the function

$$MAE(h) = c_1 + c_2 \cdot 2\left(\frac{\exp(c_3 \cdot h)}{1+\exp(c_3 \cdot h)} - \frac{1}{2}\right)$$

whose coefficients are estimated using the unsmoothed MAEs. MAE(0) therefore equals c_1, and MAE(h) approaches the value c_1+c_2 for large values of h. The speed of this approximation is determined by c_3.
32 The probability of a GDP change of 2.4% would be estimated at around one hundred thousandth of one per cent. A forecast error of 1.8% in the zero-step forecast would have a probability of around one millionth of one per cent. The probabilities of a GDP change in the order of 3.5% and a one-step forecast error of 3.4% would be more than one thousand times less likely again.
33 The tests performed take the possibility of a serial correlation in the analysed variables into consideration. See I Lobato and C Velasco (2004), A simple test of normality for time series, Econometric Theory, Vol. 20, pp 671- 689.

major consequences for the probabilities of extreme events. A t-distribution with five degrees of freedom for the quarter-on-quarter rates, for example, would imply that (based on the larger sample) a GDP change of 3.5% has a likelihood of 0.83% and therefore would occur on average about once every 30 years.[34]

Which distribution is suitable for forecasting errors in GDP changes cannot be determined unequivocally owing to the small size of the sample. In light of the forecast errors observed in connection with the financial crisis, however, it should be a distribution that allocates larger probabilities to extreme events than is the case with normal distribution.[35]

The Bundesbank's fan charts only cover ranges whose width is not more than twice the mean absolute error and so do not include extreme events. Therefore, for the ranges shown in the Bundesbank fan charts, a change in the assumed distribution would not necessarily produce a clear change in the corresponding probabilities for these ranges.[36]

The large differences between the probabilities of extreme events before and after the crisis and for various distribution assumptions illustrate the difficulty of making reliable statements about such probabilities. Despite these differences, the size of the probabilities calculated here shows that the GDP downturns and forecast errors observed during the crisis would have been regarded as virtually impossible before the crisis.

Conclusion

Macroeconomic forecasts are fraught with uncertainty for a great many reasons, but information about this uncertainty is required for many economic decisions. The scale of the uncertainty itself is uncertain, however, and can therefore only be estimated. There are various methods available for this purpose, but it is not always easy to precisely estimate the forecast uncertainty.

The estimated uncertainty of the Bundesbank's forecasts for growth and prices is, on the whole, smaller than that of simple but commonly used forecast models and is only slightly different from that of a combined forecast which is calculated using the forecasts from many other institutions. Combined forecasts are generally considered to be very accurate, so that the uncertainty of the Bundesbank's forecasts may be gauged comparably small.

The recent financial crisis led to a steep downturn in economic development, the scale of which was previously estimated to be highly unlikely. The lesson to be learned from

34 Based on the RMSE of the smaller sample, the corresponding probability would be estimated at just 0.06%. An upper limit for probabilities can also be set using Chebyshev's inequality. This upper limit for the observed quarter-on-quarter rate and the RMSE of the large sample's forecast error is 5.6%. Such a value suggests that a GDP change of 3.5% would occur once every four and a half years on average. In view of the data available, this probability seems to be too high, however.
35 A t-distribution is just one of many distributions that satisfy this criterion.
36 For the range with the width of twice the absolute error, for example, a normal distribution and a t-distribution with five degrees of freedom both result in probabilities of around 60%. For the normal distribution, the probability is somewhat below this value and somewhat above it for the t-distribution.

the financial crisis, therefore, is that the uncertainty about future economic developments is larger than was previously assumed. This is especially true for the probability of extreme unforseen changes.