The role of volatility patterns in financial markets

A broad range of stress indicators is drawn on for the ongoing monitoring of financial markets. Volatility developments in the prices and yields of assets play a special role in this context owing to their dual nature as both a stress factor and a stress indicator. With respect to financial stability, it is important to differentiate between a level of volatility that reflects fundamentally justified price movements under normal market conditions and detrimental excess volatility. The dynamic evolution of the dispersion of asset price movements, in turn, is characterised by particular patterns which will be illustrated using stock market volatilities as examples. One such pattern is an alternation between phases of high and low volatility. This phenomenon must be taken into account in the formation of expectations regarding the future dispersion of share prices. The following empirical analysis comes to the conclusion that volatility patterns and other stress indicators as at end-August 2005 do not indicate any immediately pending problems. However, this does not lessen the need to keep financial market stability at the focus of attention.

Volatility in the financial markets – both a stress factor and a stress indicator

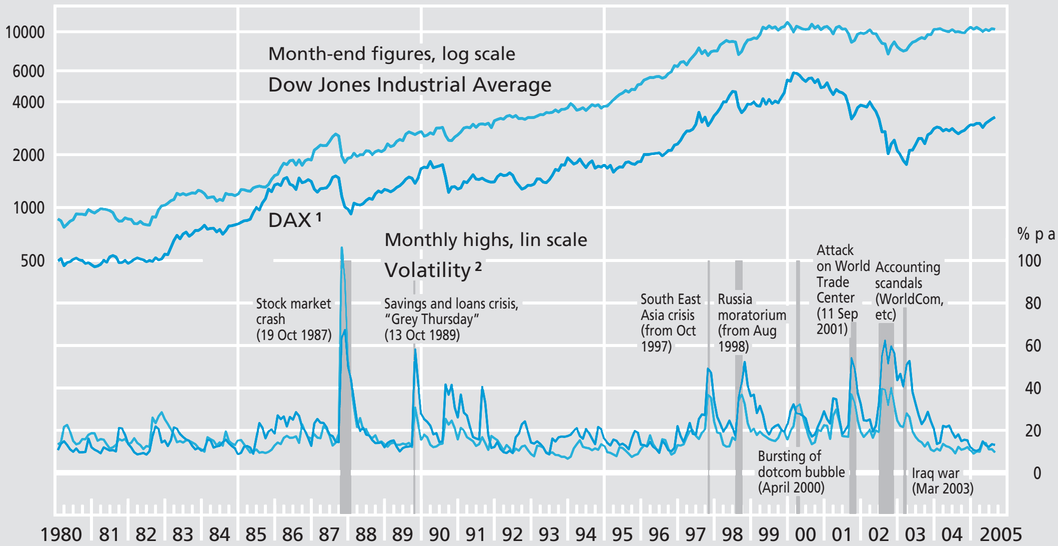
The term volatility describes the extent to which asset prices fluctuate over a given period. It is often expressed in terms of the standard deviation of the changes of logarithmic asset prices. However, as a rule, volatility changes over time. Alternatively, therefore, volatility is often computed using an exponential weighting scheme, with more recent observations receiving a higher weighting in the calculation than observations of the more distant past.[1]

Volatility as a measure of risk

From an investor's point of view, volatility often serves as a proxy for uncertainty. In technical terms, volatility is a symmetric measure that increases both during periods of marked positive price changes as well as during periods of sharply declining prices. However, particularly the latter are perceived[2] by investors as being stressful,[3] whereas, – other things being equal – strong upswings in prices are less likely to cause concern.[4] Since for risk-averse investors the risk of seeing lower valuations of their investment positions when volatility increases is regarded as outweighing the chances of possible rising valuations, they will demand to be compensated for taking on such risks. This is one of the key concepts of portfolio theory. Hence strong market-wide[5] price movements – particularly if they occur unexpectedly and are not hedged – can lead to problems concerning liquidity and creditworthiness, either directly or indirectly by an altered perception on the part of (potential) counterparties. If several or for the financial system particularly relevant market participants are negatively affected at the same time by sharp, unexpected market movements, this may lead to disruptions in the various functions of the financial system (such as payment settlement, risk transfer and risk assessment, credit and liquidity allocation). This may in turn spill over to the real economy and price developments as capital suppliers will become more aware of potential information deficits. Their risk awareness may increase. Capital demanders, too, may take an increasingly cautious view when assessing investment opportunities. Greater uncertainty and negative changes in households' assets may also lead to a deterioration in consumer confidence. The extent

1 For details see page 65 et seq and the box on pages 66-67.
2 Financial institutions attempt to assess the vulnerability of their portfolios to certain pre-defined events by using value at risk models and stress tests. Such events might include price movements on the equity markets, exchange rate and interest rate movements etc. See Deutsche Bundesbank, Stress tests at German banks – methods and results, *Monthly Report,* October 2004, pp 75-84 and Deutsche Bundesbank, Stress testing the German banking system, *Monthly Report,* December 2003, pp 53-61.
3 In line with Illing and Liu (2003), financial market stress can be described as the pressure which is exerted on economic agents by uncertainty and changing expectations concerning losses on the financial markets. The level of financial market stress ultimately depends on the degree of vulnerability of the financial system and the scale of the shock. In this context, the degree of stress varies with the size of the expected loss, risk, uncertainty regarding future losses as well as the attitude towards risk of the financial market participants. A crisis can thus be defined as a phase of extreme financial market stress. See M Illing and Y Liu (2003), An Index of Financial Stress for Canada, Bank of Canada Working Paper 2003-14.
4 From the perspective of the financial market supervisory authorities, such price upswings are interesting all the same: frequent and sharp price surges may be a sign of euphoric exaggeration and may lead to the formation of bubbles and hence precede and exacerbate stress situations which arise from subsequent price corrections.
5 Precisely in phases of very sharp share price movements, price changes of individual shares regularly show a high degree of correlation. The diversification potential attained through equity diversification cannot provide a safeguard against such market-wide developments.

Share prices and volatility



Sources: Bloomberg, Deutsche Börse AG and Bundesbank calculations. — 1 End of 1987 = 1000; from 1988 price index. — 2 Volatility calculated using an exponentially weighted model: the update formula is used to obtain for each trading day the volatility as the root of the sum of the variance of the preceding day (weighted by a factor of 0.94) and the squared current value of the yield (weighted by a factor of 0.06). The annualised figure was obtained by multiplying by the factor root of 250. See J C Hull, Options, Futures & Other Derivatives, Prentice Hall, Upper Saddle River, 2000.

Deutsche Bundesbank

of such spill-over effects hinges largely on the respective financial market environment: portfolios which are strongly leveraged through derivatives and credit financing, the widespread pursuit of stop-loss and trend-following strategies as well as liquidity crunches and low transparency all amplify volatility and heighten the risk of systemic instability.[6]

Volatility and stress situations

Fluctuations in the volatility of asset yields are, however, not only a stress factor; volatility patterns can also serve financial market observers as stress indicators. A look back at the past shows that sharp upswings in volatility may occur as an accompanying feature of particular stress situations in the financial markets. This is illustrated below using the example of the equity markets in Germany and in the United States. Extreme volatility upswings in the German share price index (DAX) and the Dow Jones Industrial Average have occurred regularly during financial market episodes which can be considered particularly stressful[7] and can be associated with events which placed a particular strain on the financial markets.

When looking at the historical development of share price volatility on the equity market (see chart), the volatility spike in the fourth quarter of 1987 is particularly striking. On 19 October 1987, the Dow Jones lost over

6 See IMF, Financial Asset Price Volatility: A Source of Instability?, in: *Global Financial Stability Report*, September 2003, pp 62-88, specifically p 62.
7 In the chart above, the spikes of each index were classified as extreme if they ranked above the respective 97.5% quantile.

20% of its value. There were many reasons for this slump, which was the largest loss ever to be recorded on a single day on the US market: waning confidence in the sustainability of the large US current account deficits, fears of recession, a speculative price bubble and new trading techniques (portfolio hedging through automatic sales) all contributed to the slide of share prices. At the same time, against the backdrop of a very critical debate on the consequences of highly leveraged buy-outs, the US Congress discussed legislative changes aimed at making hostile takeovers by way of such buy-outs[8] much more difficult. This further clouded the outlook for share prices. The volatility of the Dow Jones peaked on 21 October at a level of 106.6% per year, compared with an average of 14.9% per year since 1980.

Changes in asset prices and the market process

Continuous changes in asset prices and yields are, of course, a natural feature of financial markets in response to the constantly changing relationship between supply and demand. The unceasing stream of new information leads to reassessments of future income from assets and to a constant adjustment of supply and demand as well as of the prices and yields of the assets in question. Changes in exogenous data and a potential change in their evaluation mean there will always be a certain amount of asset price dispersion which represents an equilibrium adjustment to new data and assessments. Hence, not every fluctuation in the rate of price changes poses a risk to financial market stability.

Volatility that mirrors the fundamentals is a hallmark of efficient financial markets. It merely reflects the intensity of change in the underlying fundamentals and the resulting assessment uncertainty as to future developments. However, if a major negative shock on the equity market occurs, this, together with the resulting (direct) increase in volatility, even if it is in line with the fundamentals, creates stress in the financial system. In such a scenario, even a destabilisation of the financial system is possible. This is compounded by the fact that, following a sharp increase, the volatility often tends to exhibit considerable persistence or even rise further. Distinct patterns of volatility clusters, which accompany abrupt and possibly self-reinforcing share price processes, may be a reflection of market inefficiencies. While a definitive judgement can often only be made ex post, central banks and supervisory authorities also have to conduct an ex ante or contemporaneous assessment with respect to potential undesirable developments in the context of monitoring financial markets.[9] This diagnosis requires appropriate indicators as well as procedures for their analytical evaluation. As regards the latter, experience has to be relied on to a considerable extent.

8 A leveraged buy-out (LBO) denotes the acquisition of an enterprise that is financed to a large extent by borrowed capital. In this way, the equity capital used for this purpose is leveraged, which may enable investors to achieve a correspondingly high return on equity.
9 With respect to financial stability aspects, a very low level of volatility (over an extended period of time) can also give cause for concern if it, for example, encourages investors to underestimate the risks of possible valuation corrections when making investment decisions or to overrate return considerations to the detriment of risk aspects. See IMF, Global Financial Market Developments, in: *Global Financial Stability Report*, April 2005, pp 8-61, specifically p 8.

Volatility and its dynamics – modelling concepts

Sharp share price swings often occur in clusters ...

In the past, extreme share price slumps were frequently followed by phases of increased price fluctuations on the equity market (ripple effect – see chart opposite). Furthermore, above and beyond these extreme episodes, sharp price swings appear to exhibit a tendency to cluster. Major movements of the DAX and the S&P 500 have often been followed by marked countermovements and phases of increased price volatility. Also, negative tail events, defined here as days when the share price index falls by more than 3%, seem to be concentrated during certain periods (see the box "Volatility and negative extreme events illustrated by the DAX and the Dow Jones Industrial Average" on page 64).

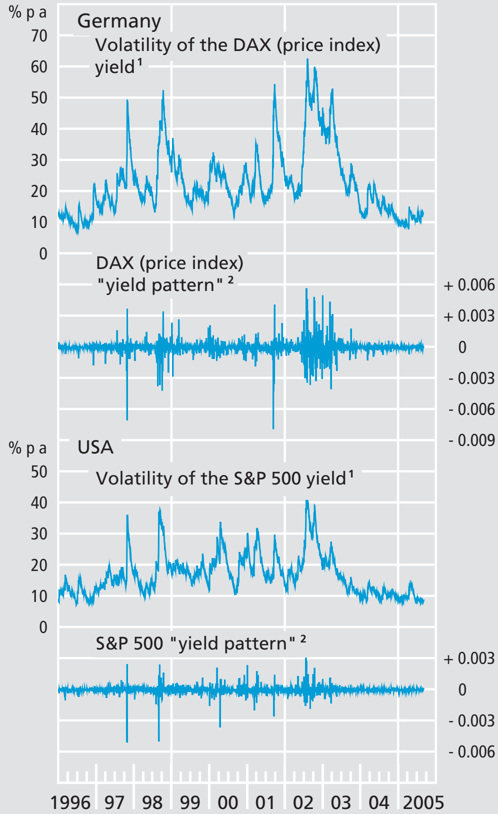
... and reflect heightened nervousness

One explanation for such phenomena might be that periods of marked price fluctuations are solely due to a relatively high frequency of new information which changes the markets' assessment. Hence, volatility would then simply be the result of share price fluctuations in line with the fundamentals. But this is rather implausible. It is more likely that in such cases volatility often also reflected heightened nervousness in the markets. New information was thus also differently received.

Hence asset price changes which generate market tensions might trigger a change in agents' risk assessment and also raise the degree of risk aversion, for example owing to a narrowing of the room for manoeuvre following the initial stress event. As market-wide

Volatility and "yield pattern" on the stock markets in Germany and the USA

Daily data



Source: Bloomberg and Bundesbank calculations. — **1** Volatility calculated using an exponentially weighted model. — **2** Squared yields calculated retaining the plus or minus sign of the yield (calculated as the differences of the logarithmic daily share price index levels). The squaring of the yield serves to accentuate the phases of high yield volatility.

Deutsche Bundesbank

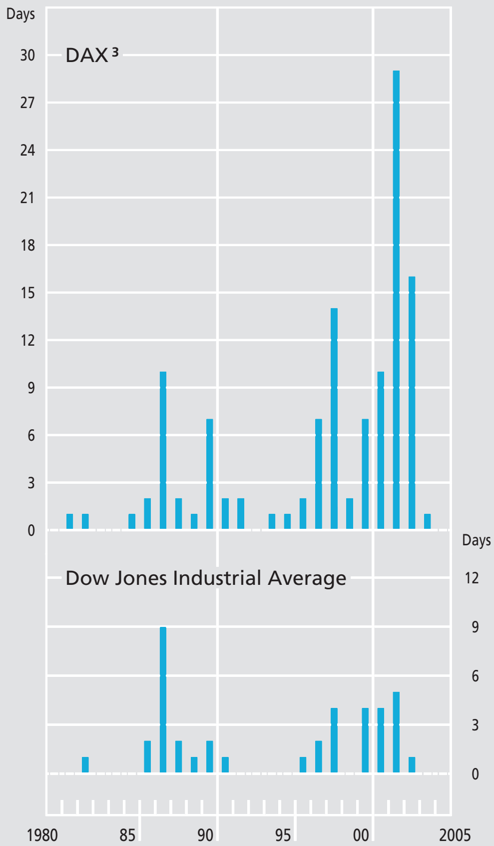
movements have greater stress potential than changes in individual share prices, the former could become the focus of investors' attention. In the perception of stock market investors, the prospects of different enterprises would then be adjusted by the same yardstick, ie in the light of general market developments. Such a macro-market convergence of investor behaviour would lead to a dearth of counterparties, and large share price

Volatility and negative extreme events illustrated by the DAX and the Dow Jones Industrial Average

Periods of extreme volatility are associated with very sharp changes in prices or yields. For investors, a particularly severe depreciation of the assets in their portfolio can be stressful. If, for example, one defines a negative extreme event as a slump in the stock market index of more than 3% compared with the previous day,[1] the number of such days per annum differs markedly over time for the DAX and the Dow Jones. However, both show pronounced spikes in the number of negative extreme events in 1987, 1998 and 2002. These years are associated with stock market crises and protracted phases of greatly increased volatility. The crises were triggered by "Black Monday" in October 1987, the Russian crisis and ensuing problems of the LTCM hedge fund in 1998 and the accounting scandals in 2002 (eg Enron, Worldcom etc.).

Consequently, extreme events often represent "exceptional movements" triggered by unique circumstances. Therefore, attention is instead generally focussed on volatility in terms of average observed fluctuations when attempting to derive expectations regarding future fluctuations. Nevertheless, these are inextricably linked to extreme events as protracted phases of especially high volatility are often heralded by negative extreme events.

Negative extreme events [2]



1 See also, for example, IMF (2003), "Financial Asset Price Volatility: A Source of Instability?", in *Global Financial Stability Report*, September, pp 62-88. — 2 Negative extreme events are defined as a slump of more than 3% in the stock market index compared with the previous day's value. The chart shows the number of days per annum when a negative extreme event occurred. Last revised end-August 2005. — 3 Price index since 1988.

Deutsche Bundesbank

swings across the entire market would be the natural consequence.

High sensitivity to new information and the subsequent correction of perceived exaggerations may lead to further share price swings. In this way volatility sometimes develops a self-reinforcing momentum of its own which must be taken into account in the process of modelling and when interpreting volatility patterns.

Deriving expected volatility

The historical volatility of a time series in the sense of average realised fluctuations (per defined time unit)[10] is traditionally calculated as an empirical standard deviation.[11] A simple way of forming expectations concerning future volatility – particularly for a short-term expectation horizon – would be to form stationary expectations. In this case, at a point in time t-1, one would expect a volatility for t which corresponds to the historical volatility measured at t-1.

If, for instance, equity market yields are analysed over a longer period of time, it emerges that phases of varying fluctuation intensity can frequently be observed (see chart on page 63). These phenomena of, first, a non-constant variance, ie one which changes over time (heteroscedasticity), and second, its pattern of clusters (conditional heteroscedasticity) are relevant for financial market stability because they mean that sharp share price swings generally occur in clusters. Hence it can be seen that an historical standard deviation (volatility) of the equity market yield calculated with a moving window of a given sample length[12] does not remain constant, but varies. If, however, volatility fluctuates over time, it makes sense to derive expectations about future volatility using a dynamic approach.

The simplest approach is to first extend the traditional method by applying a weighting scheme which takes greater account of more recent observations when calculating historically realised volatility. By contrast, a calculation of the simple standard deviation means that observations made in the more distant past are weighted just as strongly as the more recent observations.[13] Alternatively, therefore, the exponentially weighted historical volatility is often calculated by financial market practitioners. This is computed as the root of the average of past fluctuations with exponentially declining weights for observations of the more distant past. By using an updat-

10 The calculation of the historical volatility of a time series depends on the data frequency and the selected time unit respectively. Thus one can calculate five-minute, daily, or monthly volatilities, in which the choice of data frequency should depend on market liquidity, among other things. As a rule, although high- frequency data result in more precise estimates, it is better to select longer data intervals in the case of less liquid markets. See S-H Poon and C W J Granger (2005), Practical Issues in Forecasting Volatility, *Financial Analysts Journal*, Vol 61 (1), pp 45-56.

11 For reasons of comparability, volatilities are subsequently often annualised. To this end, the standard deviation calculated at a certain observation frequency is multiplied by the root of the number of possible observations per year. As an approximation, the annualised historical volatility can be calculated by multiplying the volatility per business day by a factor of $\sqrt{250}$ (see also European Central Bank, *Monthly Bulletin,* May 2000, box 2: Recent trends in the volatility of stock price indices). The calculation assumes that there are 250 business days per year. However, as the individual observations are unlikely to be independent drawings of identically distributed random variables, this annualisation is naturally prone to potential biases.

12 See box on pp 66-67.

13 When using a moving window this applies as long as these observations are still within the window. After that, however, they are factored into the calculation with a weight of zero.

A comparison of different measures of volatility using the daily DAX yield as an example

In this example, let the DAX yield be defined as the change in the natural logarithm (log) of the DAX compared with the previous day's value, ie $r_t = log\,(DAX_t) - log\,(DAX_{t-1})$.

At time t-1, let observations (realised values) for the time series be available back to time t-M, ie $r_{t-1}, r_{t-2}, \ldots r_{t-M}$. For this sample of length M and with end-time t-1, the unbiased (sample) variance is defined as

$$\sigma^2_{t-1} = \frac{1}{M-1}\sum_{i=1}^{M}(r_{t-i} - \overline{r}_{t-1})^2, \text{with } \overline{r}_{t-1} = \frac{1}{M}\sum_{i=1}^{M} r_{t-i}$$

as the sample mean.

The corresponding "historical volatility" at time t-1 (the simple standard deviation of the sample) is calculated as the square root of the variance. For a longer time series of past observations, the sample variance may also be calculated at different points in time using a moving window of length M, ie using sections of length M of the full sample. Start and end-dates of the sample sections in this case are correspondingly repositioned at each step, ie these dates are each moved step-by-step and unit-by-unit to the current boundary.

As the average yield remains close to zero over time, the formula for the sample variance can be approximately calculated as the weighted average of the previous squared yield values

$$\sigma^2_{t-1} = \sum_{i=1}^{M}\alpha_i r^2_{t-i} \text{ with } \alpha_i = \frac{1}{M},\ i = 1, \ldots, M$$

as constant weights.

When using exponentially declining weights, ie $\alpha_{i+1} = \lambda\,\alpha_i$, with $0 < \lambda < 1$, this is known as an exponentially weighted (moving) average volatility model.[1] Given the variance value at time t-2 and the squared yield at time t-1, the subsequent variance calculation can be simplified further to the update formula $\sigma^2_{t-1} = \lambda\sigma^2_{t-2} + (1-\lambda)\,r^2_{t-1}$.[2]

For these approaches to modelling historical volatility, the simplest way to form an expectation concerning the volatility value for the next period t is to assume that it is equal to the value calculated at time t-1.[3]

The "expectations" derived under such a simple approach display a pattern over time that looks very similar to the one-step volatility expectations obtained from time series models for both yields and the dynamic development of the variance of the unsystematic component (the error process) in the yield equation. In a GARCH(1,1) model, for example, it is assumed that the error process variance expected for t at time t-1[4] depends on a constant[5] ω, the variance expected for t-1 at t-2 (σ^2_{t-1}) and the squared value of the error process at t-1 (ε^2_{t-1}): $\sigma^2_t = \omega + \alpha\varepsilon^2_{t-1} + \beta\sigma^2_{t-1}$. Therefore, this for-

1 See, for example, J C Hull (2000), Options, Futures & Other Derivatives, Prentice Hall, Upper Saddle River, p 370 et seq. — **2** For the daily DAX yield data used here a value of λ =0.94 was chosen, a value that is frequently used in applied financial market analyses for daily data (see, for example, IMF (2003), Financial Asset Price Volatility: A Source of Instability?, in: *Global Financial Stability Report*, September, pp 62-88). — **3** Hull (2000) avoids this distinction in his notation by deriving expectations directly from past values. — **4** In addition, there is the standard assumption that the error process ε will have an average of zero. — **5** This makes it possible to take account of a long-run average level in the variance. — **6** In this case, the conditional variance process obtained from estimating the GARCH(1,1)-M model for the DAX (price index) yields for the period 1990 to the end of August 2005 was

$$\sigma^2_t = \underset{(0.0000004)}{0.000001} + \underset{(0.00861)}{0.07683}\,\varepsilon^2_{t-1} + \underset{(0.00852)}{091873}\,\sigma^2_{t-1}$$

Deutsche Bundesbank

mula also "updates" the variance (forecast) based on "new" information.

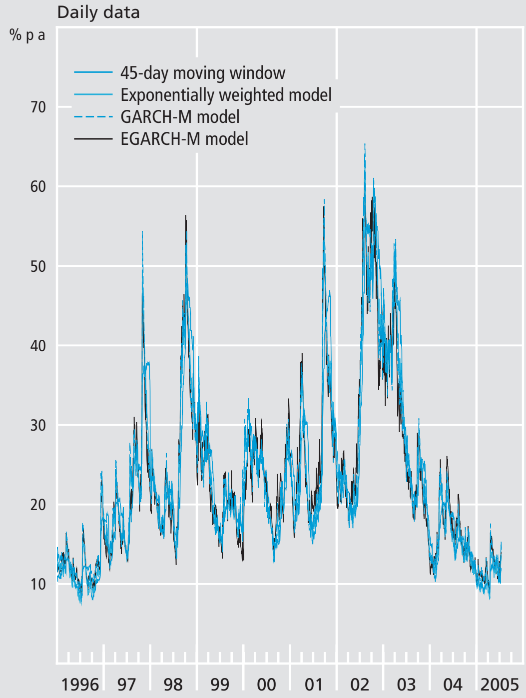
By contrast, the EGARCH(1,1) model assumes that the development of the logarithmic conditional variance can be captured by

$$log(\sigma_t^2) = \omega + \alpha\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| + \gamma\frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta\,log(\sigma_{t-1}^2).$$

For $\gamma \neq 0$ positively and negatively scaled residuals from the preceding period have differing effects on the logarithmic conditional variance (asymmetry), where for $\gamma \prec 0$ the absolute effect of negatively scaled residuals on the conditional variance is greater (leverage).

The adjacent chart shows the volatility expectations for the DAX yield derived for the various dynamic approaches.[6] The model volatility forecasts were converted to annual percentage figures by multiplying them by a factor of $\sqrt{250} \cdot 100$ There is considerable similarity between the four time series graphs. This can be explained by the fact that the short-term yields fluctuate more or less unsystematically around zero and are therefore primarily dominated by an error process. Furthermore, when estimating the simple GARCH model the coefficients for the volatility equation are relatively close to those values that are implicitly used as constraints in the update formula of the exponentially weighted volatility model.[7] The approximate match

Forecasts of volatility in the DAX (price index) yield:
A comparison of models



with the volatility series obtained from the traditional formula using a moving window in this case stems from the use of a relatively short observation window. The use of a longer window (ie a larger value for M) in each step would, by contrast, result in greater smoothing of the volatility pattern and, thus, to larger deviations from the time series patterns obtained from the other approaches.

(with standard errors in parenthesis). For the EGARCH (1,1)-M approach, the corresponding estimation led to

$$log(\sigma_t^2) = \underset{(0.02561)}{-0.20106} + \underset{(0.01334)}{0.13087}\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| \underset{(0.00813)}{-0.05301}\frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \underset{(0.00236)}{0.98875}\,log(\sigma_{t-1}^2).$$

This suggests a certain leverage effect. Both estimations were performed in EViews under the assumption of a t-distribution for the error process . Compared to the simpler GARCH method with 12 iterations, the EGARCH estimation took far longer to converge (168 iterations). The coefficient of the conditional standard deviation in the yield equation is significant in both cases (0.055 for the GARCH model and 0.037 for the EGARCH model). Data source: Bloomberg. — 7 Although some of the constraints set in the simple update formula are slightly breached, this breach is of little consequence.

ing formula, the volatility to be determined at t can be approximately calculated quite simply by using the value obtained for the previous period (ie for t-1) and the fluctuation realised in t (see the box on pages 66-67). Empirically optimal weights for the respective observation period can be determined through estimation; the most widespread applications, however, mostly refrain from such estimations in favour of using ad hoc specified values.

Particular observable volatility patterns must, however, also be taken into account in estimation approaches. In this context, the category of the ARCH and GARCH (Generalised Autoregressive Conditional Heteroscedasticity) approaches[14] is one possibility of modelling a dynamic development of both the underlying time series and the variance of the disturbance process using past data (see box on pages 66-67). The process which describes the GARCH variance can be used to establish a time series for the volatility expected for the respective following period, taking account of the conditional heteroscedasticity.[15] Moreover, by extending the equation for the yield process, the GARCH-M approach makes it possible, for instance, to take into account a possible systemic feedback effect of the expected volatility – and hence the expected risk of an investment – on the expected yield.

Further generalisations of the GARCH approach primarily concern the modelling of the conditional variance process.[16] The exponential GARCH model (EGARCH) introduced by Nelson, for instance, allows one to take into account potential asymmetric effects of residuals and leverage effects of negative residuals on the process of the logarithmic conditional variance.[17]

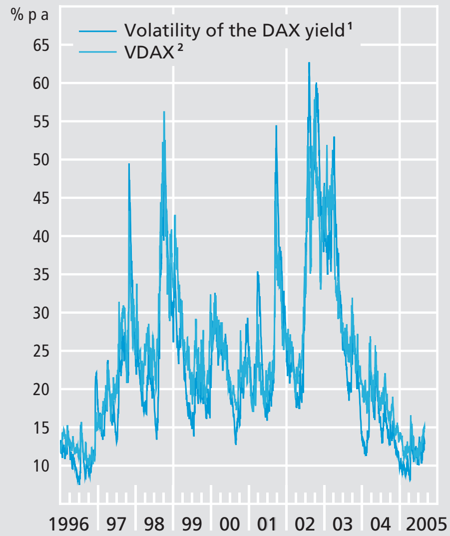
By contrast, the measure of implied volatility is a concept which stems from the models for pricing options – or conditional claims. For a given volatility, such a pricing model (for example, the Black-Scholes models)[18] can be used to derive the fair price of an option on an underlying on the basis of certain assumptions. By comparison, if the data observable on the market – the option price at time t, the strike price of the option and the price of the underlying in t – are taken as the starting point, the pricing model can be "inverted". This gives the volatility at which the realised

14 Mentioned at the beginning of this article was the phenomenon of clustering in the level of volatility, which has key implications for financial stability. In other words, volatility changes over time: this is called heteroscedasticity. This was first taken into account by Engle (1982) in the autoregressive conditional heteroscedastic model and then generalised by T Bollerslev (1986) in the GARCH model, where the prefix G stands for "Generalised". Under the ARCH approach, variance is explained solely by its past pattern. The GARCH approach adds the estimates of variance made for preceding periods. See R F Engle (1982), Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation, in: Econometrica, Vol 50 number 4, pp 987-1007, and T Bollerslev (1986), Generalized Autoregressive Conditional Heteroskedasticity, in: Journal of Econometrics, April, Vol 31 number 3, pp. 307-327.
15 It can be shown that the calculation of variance using an exponential weighting can be viewed as a special case of a simple GARCH model. See J C Hull (2000), Options, Futures & Other Derivatives, Prentice Hall, Upper Saddle River.
16 For the various models see, for example, J Y Campbell, A W Lo and A C MacKinlay (1997), The Econometrics of Financial Markets, Princeton University Press, Princeton.
17 See also D B Nelson (1991), Conditional Heteroskedasticity in Asset Returns: A New Approach, in: Econometrica, Vol 59, pp 347-370.
18 See F Black and M Scholes (1973), The Pricing of Options and Corporate Liabilities, in: Journal of Political Economy, Vol 81, pp 637-659 and F Black (1976), The Pricing of Commodity Contracts, in: Journal of Financial Economics, Vol 3, March 1976, pp 167-179.

option price corresponds precisely to its "fair" model-based value.[19] This measure is thus obtained implicitly from the method used to price options. Moreover, owing to the market price orientation, this volatility can be seen as the average price dispersion which is expected by the market.[20] Furthermore, it is customary to derive implied measures of volatility also at the aggregated level for broad market indices. For a number of years, volatility indices have also been available for selected financial market indices which should make it possible to trade volatility directly, for example via derivatives. These indices include the VDAX which is calculated on the basis of options, as well as the VIX as a measure of dispersion of the S&P 500 share price index in the US market.[21] For the above example of the DAX, there is a clear correspondence between developments in the realised volatility of the daily changes of the logarithmic DAX value (ie the DAX yield) and the volatility index VDAX. This illustrates again that in the case of a short expectation horizon, there is generally a relatively high degree of congruency in the changes of the volatility expectations derived from the various approaches.

Volatility of the DAX yield and VDAX

Daily data



Source: Bloomberg and Bundesbank calculations. — **1** Historical volatility of the DAX (total return index) yield calculated using an exponentially weighted model. — **2** See online documentation of Deutsche Börse AG.

Deutsche Bundesbank

Volatility as a measure of nervousness – a comparison with additional indicators

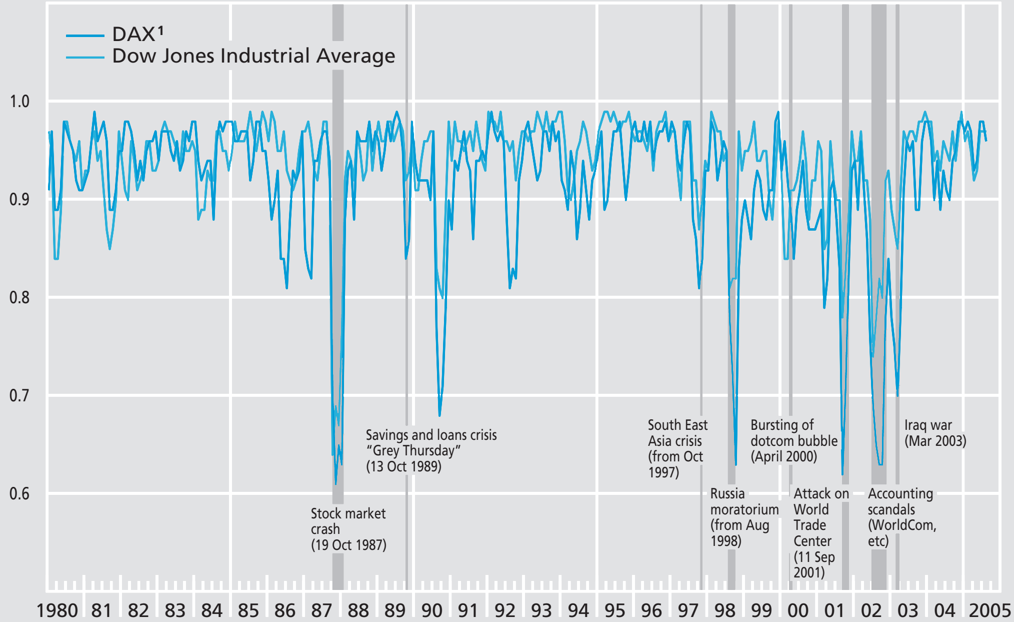
Volatility spikes and adverse price movements

One indicator of adverse price movements on the equity market which is occasionally used in the literature expresses the level of a share price index as a percentage of the maximum which it reached in a previous reference

19 See also Deutsche Bundesbank, The information content of derivatives for monetary policy, *Monthly Report,* November 1995, pp 17 ff. For information on the theoretical contradiction that the derivation of such a model occurs under the assumption of a level of volatility which is constant over time, see, for example, J Y Campbell, A W Lo and A C MacKinlay (1997), p 378.
20 For information on the particularities of implied volatility measures, such as "volatility smiles" when differentiating by strike prices, see, for example, Hull (2000), p 435 ff. To take into account potential asymmetries, the concept of the implied risk-neutral density function attempts to ascertain from options under certain simplifying assumptions an overall "distribution" of market expectations and not just a measure of the expected average dispersion. See, for example, Deutsche Bundesbank, Instruments used to analyse market expectations: risk-neutral density functions, *Monthly Report,* October 2001, pp 31-47 and A M Malz (1997), Estimating the Probability Distribution of the Future Exchange Rate from Option Prices, in: The Journal of Derivatives, Vol 5 (2), pp 18-36.
21 For details, in particular the differences between the new and the old volatility indices, the DAX-related VDAX and the VDAX-New on the German Stock Exchange and the S&P 500-related VIX and the S&P 100-related VIX-OLD on the Chicago Board Options Exchange, see Deutsche Börse's online documentation (vdax_guide.pdf at http://deutsche-boerse.com) and that of the Chicago Board Options Exchange (http://www.cboe.com/micro/vix/vixwhite.pdf).

CMAX Index* – indicator of stock market slumps

Monthly lows



Source: Bloomberg and Bundesbank calculations. — * The daily CMAX Index is calculated as the index level in relation to the maximum level in the past 60 trading days. — **1** From 1988 price index.

Deutsche Bundesbank

period (CMAX).[22] As a result, price slumps are associated with a particularly low value. As the reference period includes the respective current value, the indicator can reach a maximum level of 1. The chart above shows the development of this ratio for the DAX and Dow Jones (with a reference period of 60 trading days) since January 1980. In contrast to indicators such as the CMAX, (historical) volatility is, technically speaking, a symmetric measure. However, in its role as a stress indicator, it should react in phases of falling asset prices in particular.

Indeed, the phases of exceptionally high volatility shaded in grey in the chart were mostly associated with situations in which the level of both the DAX and the Dow Jones fell considerably.[23] Not always were phases of increased volatility started off by price slumps in the observed markets, though. When the bubble burst in the IT and communications technology sectors (ITC bubble) in the spring of 2000, initial shifts took place from growth to value stocks, thereby fuelling a rise in the Dow Jones Industrial Average and the DAX.

22 See, for example, S A Patel and A Sarkar (1998), Crises in Developed and Emerging Stock Markets, in: Financial Analysts Journal, November/December 1998, pp 50-61.
23 The leverage hypothesis and the volatility feedback hypothesis serve as explanations for such a joint occurrence of volatility and falling share prices. The leverage hypothesis states that in the event of a falling stock market value, the percentage share of equity capital falls. As the equity capital bears the entire enterprise risk, the volatility of the equity capital should consequently increase. By contrast, according to the volatility feedback hypothesis, volatility shocks lead to share price declines. See J Y Campbell, A W Lo and A C MacKinlay (1997).

High degree of parallel movement with other indicators

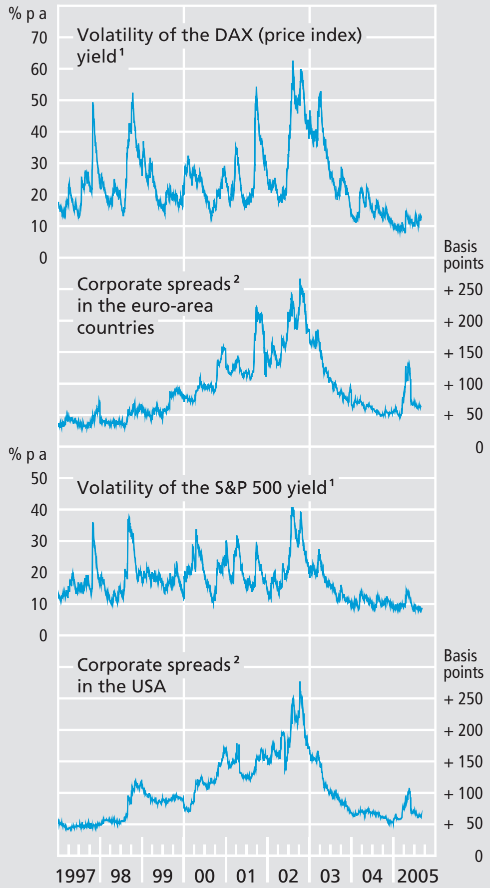
The fact that increased volatility may be a sign of nervous financial markets is also underscored by a comparison of volatility with further measures which are widely used as stress indicators.

For example, an increase in the yield premia on risky bonds and opposite developments on the equity and bond markets are seen as a possible expression of shifts in investor portfolios to relatively safe securities. The quest for safety is viewed as a sign of increased pessimism and a lower willingness to take on risk.[24]

The chart opposite shows a clear correspondence between the volatility of asset price changes on the equity market and differences between the risk premia of riskier and less risky corporate bonds. Moreover, rising volatility has been frequently associated with a growing divergence between the price movements of government bonds and share indices (see chart on page 72). However, the fact that the indicators of the quest for safety in the financial markets do not exactly match those of the stress indicator "volatility" can be seen, *inter alia*, in the developments of the past few months. Thus yield spreads on the bond market started to widen in March of this year, although from the second half of May they began to narrow again somewhat.[25] In addition, from the middle of last year, opposite movements have repeatedly been observed between government bonds and equity valuations. The volatility patterns, however, showed no significant upswings up to end-August 2005. Despite a slight increase, equity market volatility in the USA

Stock market volatility and corporate spreads

Daily data



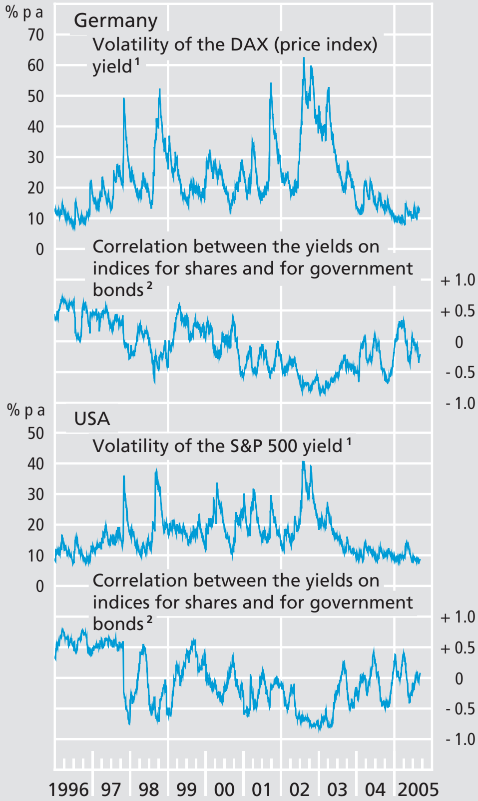
Sources: Bloomberg, Merrill Lynch and Bundesbank calculations. — **1** Volatility calculated using an exponentially weighted model. — **2** Corporate spreads calculated as the difference between Merrill Lynch option-adjusted spreads (OAS) of BBB and AAA-rated corporate bond indices.

Deutsche Bundesbank

24 Tarashev et al use option price developments and historical volatility to derive an index of investors' risk aversion. They, too, find that periods of high risk aversion tend to coincide with periods of high volatility on the equity markets. See N Tarashev, D Tsatsaronis and D Karampatos, Investors' attitude towards risk: what can we learn from options, in: BIS Quarterly Review, June 2003, pp 63-72.
25 Thus in mid-July 2005, the relevant Merrill Lynch corporate spreads for BBB-rated over AAA-rated corporate bonds were only 11 (4) basis points above the level recorded in the euro area (in the USA) on 15 March 2005 (before GM's profit warning of 16 March 2005), whereas on 17 May they had still been 78 (45) basis points higher.

Stock market volatility and the correlation between yields on share and government bond indices

Daily data



Source: Bloomberg and Bundesbank calculations. — **1** Volatility calculated using an exponentially weighted model. — **2** Yields of total return bond indices (government bonds with a maturity of seven to ten years) calculated as the differences in the logarithmic index values. Correlation was calculated using a 45 day moving window.

Deutsche Bundesbank

and in Germany is still at a very low level by historical standards. All in all, the stress level at the moment appears to be relatively low.

However, it should be borne in mind that spread movements and equity-government bond correlations are not only indicators of the quest for safety. Portfolio shifts between government bonds and equities are just as much the result of a search for yield as a search for safety; widening spreads could, for example, be triggered to a considerable extent by changes in the assessment of default risk in line with the fundamentals.

Hence the various indicators would lead to slightly differing assessments of the stress perceived on the financial markets during certain phases (albeit not at the current end). When it comes to assessing financial market stress, therefore, volatility patterns are indicators which should be evaluated in the overall context, ie together with additional information and instruments. Undisputedly, however, volatility patterns are of key importance for assessing the situation in financial markets. This stems from their dual nature as a stress factor and an indicator which is available on a timely basis.

In its Global Financial Stability Report of April this year, the IMF suggested that the markets might be exhibiting an exaggerated level of confidence.[26] This assessment seems plausible in the light of the relatively low level of volatility seen, for example, on the equity markets in the USA and Germany up to the end of August 2005. Moreover, a look at the past shows that financial market phases of low volatility may be followed by sharp share price movements and that episodes of extreme volatility have generally been associated with "stress" in the financial markets. Furthermore, no final judgment can be made as to the extent to which market dynamics in

26 The Bank for International Settlements' risk appetite index accordingly indicates a relatively high level of risk appetite in the credit markets at the current end (start of Q2 2005). See BIS, 75th Annual Report 2005, section VI: Financial Markets.

the past few years have been shaped by special factors, such as the excess liquidity observed in the markets or the growing importance of hedge funds. This can impair the indicative quality of conventional stress indicators. It is therefore important in the context of gauging the danger of future financial market stress to identify such "special factors" and to take due account of them when assessing the development of financial market stress indicators on the one hand and financial market stability on the other. Ideally, such an analysis should ultimately serve to identify potential crisis constellations at an early stage – preferably before they materialise.