Banks' internal risk management models and their prudential recognition

The Basle Market Risk Paper,[1] which amended the Basle Capital Accord of 1988, requires internationally operating banks to apply capital charges to their market price risks[2] as well as to their counterparty risks as from December 31, 1997. One reason is the sharply growing importance of off-balance-sheet business; in Germany alone, the trading volume has risen by more than thirty times since 1986. To measure these risks, and also to calculate the required capital charge, under certain conditions the institutions are permitted to use internal risk management models as an alternative to the standardised methods prescribed by banking supervisors.

With the entry into force of the amended version of Principle I on October 1, 1998, German banking supervisors have adopted these regulations on the basis of the relevant amendments to the EU Capital Adequacy Directive and have expanded them to cover all German financial services and credit institutions.

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1 Amendment to the Capital Accord to Incorporate Market Risks, Basle Committee on Banking Supervision, January 1996. — **2** The market price risk or market risk consists of a general market risk and a specific market risk. The former denotes the risk that general market fluctuations can lead to price changes and thus to changes in the value of the financial products (securities, derivatives et al.). In the specific risk, however, the causes of price changes are issuer-related.

Banking business and risk management strategies undergoing change

Market price risks gaining in importance

Banks have always had to be aware of the risks of their business and to develop strategies to quantify those risks as much as possible and to keep them within reasonable limits.

Although counterparty risk has traditionally played a prominent role in banking business, other risks have also become more important as business structures have changed and new market and business methods have emerged. Against the background of financial markets becoming more globalised and volatile, trade in financial products has increased considerably. More attention must therefore be devoted to market price risks involved in trading activity, in particular foreign exchange risk, interest rate risk, equity price risk and commodity risk.

Increased complexity of business ...

In addition, the complexity of the transactions and the methods used has increased markedly owing to the use of derivatives. The increasing computerisation, without which these transactions could not be handled, has also played a role.

... and a different environment ...

The changed underlying conditions have also caused management risks in the broadest sense (computer system risks, staffing risks – such as the fluctuation of whole teams of staff from the trading sector, including settlement – and the like) to become more important; those risks must therefore be covered nowadays in the context of controlling measures taken by the institutions.

Besides, the globalisation of financial markets has led to a further intensification of competition for capital and to a reduction in the profit margin. This trend is forcing banks which seek to actively participate in trading in financial products to develop controlling and management methods that ensure a quick reaction to market and product changes and which presuppose knowledge and understanding of the risks. After all, banks are interested in shifting the available capital to the most profitable transactions in order to raise shareholder value, while taking account of the risks involved.[3] This makes it necessary to calculate the position and the risk accurately and quickly in order to ascertain what is called economic capital.[4]

... require new controlling and management methods

Traditional methods of measuring risks (such as the calculation of open positions based on nominal values) which take neither the risk profile of individual transactions or entire portfolios nor the link between individual risks into account are only partly suited to effectively managing the trade book and can at most provide certain background information. In light of the changed underlying conditions, therefore, it is necessary to supplement the previously applied methods.

3 For these purposes, management instruments such as RAROC (Risk Adjusted Return on Capital) have been developed, with which a ratio can be established between risk and return. For example, it would be possible to state for two trading desks which of the two has generated the higher return, the given risk being identical.
4 The name often given to the capital which, in the bank's own estimation, must be available to cover the risks incurred.

Standardised prudential measurement methods

Standardised methods contain only coarse distinctions

When quantifying the risks, which is relevant for setting the minimum capital requirement, banking supervisors have traditionally prescribed relatively simple methods. The methods are oriented to the individual risk categories (counterparty risk, individual market price risks), yet the distinctions they draw are only coarse. On the basis of nominal or market values, the prescribed weighting ratios are used to calculate the capital charges. The weights assume the existence of average diversified portfolios, i.e. they take insufficient account, in particular, of the risk of individual transactions or the risk correlation with other business. Therefore, these methods inevitably lead to overestimations or underestimations of the risks of individual transactions or component parts of the portfolio. Furthermore, portfolio effects, i.e. the offsetting of matching positions, cannot be adequately represented using the standardised methods. The prudential capital calculated in this manner is therefore not necessarily equivalent to the economic capital that a bank calculates using more precise internal models. However, it acts as a buffer that in practice has proved to be adequate.

Avoiding duplication of calculation

Standardised prudential methods cannot be used in practice by large institutions to manage their business owing to the previously described inadequacies and to the greater and greater requirements being placed on methods of risk measurement. Such banks, therefore, must also simultaneously calculate the risks for themselves using their own much

more precise methods to the extent that the complexity of their transactions increases, which leads to high administrative costs.

Therefore, when developing new market risk regulations, bank supervisors recognised that much more complex methods of measuring and managing market price risks have been devised in academe and the business world, methods which can also be accepted as a basis for calculating prudential capital. Since capital requirements are usually lower when based on internal risk management models than when standardised prudential methods are applied, an incentive is being created for the banks to develop suitable methods for controlling the risk in the trading book.

Features of banks' internal risk management models

Value at risk (VaR) takes account of holding period, confidence level and reference period

In risk management models, mathematical and statistical approaches, which use stochastic methods of calculating the level of the value at risk (VaR), are applied. When calculating the VaR, account is taken of the fact that the bank cannot immediately close or sell loss-generating risk positions to make adjustments for disadvantageous market changes but, due to low market liquidity, for instance, needs a certain period of time (known as the holding period). Furthermore, on the basis of historic market price fluctuations, an estimate can be made of the maximum loss that is likely to occur at a given probability (confidence level). For instance, a VaR of DM 1 million for a trading portfolio, given a reference period of 250 trading days,

a confidence level of 99% and a holding period of 1 day, means that based on market data over the past 250 days there is a 99% chance that the loss on the trading portfolio will not exceed DM 1 million by the next day. A change in the aforementioned basic assumptions of the model calculation can lead to considerable deviations in the value at risk.

The VaR can generally be calculated for individual transactions, certain groups of transactions, various local or regional components of the portfolio, individual risk areas such as interest rate risk, equity price risk or foreign exchange risk, or for all market price risks.

Methods of risk measurement can be chosen

The value at risk can be calculated on the basis of various methodical approaches, with the choice of methods also being determined by, among other things, the composition and size of the portfolio to be calculated, the availability of market data and the memory and speed of the DP systems being used. The most common methods are:

– Historic simulation

– Variance covariance approach

– Monte Carlo simulation.

More precise explanations of the individual methods can be found in the Annex at the end of this article.

The prudential suitability of banks' internal risk management models

Quantitative and qualitative criteria

The basic prerequisite for prudential recognition of internal risk management models is that they are integral parts of a bank's risk management. This means that the models are used not only to calculate prudential capital requirements but also for the bank's internal purposes. Since the use of different parameters for calculation (e.g. holding period of one day or 10 days) can lead to considerable variations in risk amounts, it was necessary, in order to ensure a minimum capital for market price risks and to create comparable conditions of competition, to set quantitative and qualitative requirements as a condition for permitting the internal models to be used for banking supervision purposes.

Conditions for the suitability of models

These include an assumed minimum holding period of ten days, since loss-incurring market risk positions can in some cases not immediately be closed or sold, therefore causing losses to possibly cumulate. In detail, this effect is dependent on the liquidity of the markets in question and the size of the position to be closed.

Quantitative requirements

A further condition is the observance of a 99% confidence level. That means that there is only a 1% probability that the losses will exceed the VaR calculated on the basis of the historic rate or price trend.

Finally, an observation period of at least one year (250 trading days) is intended to ensure

when the model parameters derived from historic market data are used, such as sensitivities and correlations[5], that a sufficiently long period is taken as a basis in order to run the model calculation on the basis of stable and representative data. The question of what series of historic data is best suited to forecasting risks cannot be answered wholesale because the answer depends on the respective portfolio.

Qualitative requirements

Qualitative requirements are designed to assure that the model is adequately enshrined in the organisational environment of the institution, i.e. that the trading activity is managed by the model.

Stress tests

Stress tests as additional information

An important quality requirement is the running of stress tests. Crisis scenarios are used to simulate the changes that would occur in the value of a portfolio caused by extreme market situations or market price changes.

In an ideal case, these special calculations should incorporate those factors that are relevant to the appropriate portfolio. The extreme change in these factors can highlight extraordinary potential losses in the trading book.[6] The stress tests say nothing about the likelihood of crisis scenarios occurring. Since the VaR is based on "normal" market conditions, stress tests are designed to give the bank's management a further medium of information from which it can be seen what losses could be incurred in a worst-case scenario in unfavourable market conditions. The results of the stress tests are to be taken into

Qualitative requirements
– Soundly organised working and operational procedures
– Risk controlling which is independent of trading activity
– Sufficient documentation of the model
– Ongoing internal reviews of the models
– Conducting of stress tests
– Limits used should be dependent on the value at risk ascertained using the models
– Audit by the internal audit department at least once a year
– Bank management should be kept informed daily
– Data sets should be updated at least every three months
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account when judging the suitability of the limits based on the bank's internal model.

In banking practice, in most cases past crisis scenarios are reconstructed, such as the stock market crash of 1987, the EMS crisis of 1992 or the interest rate crash of 1994. The stress tests are supposed to offset the methodical weak point of internal risk management models, which, among other things, is inherent in the fact that the future-oriented risk estimation is based on historic data of a fixed observation period.

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5 See Annex for an explanation of these terms.
6 In unusual cases, major losses may occur if market prices remain unchanged over a certain period of time (e.g. for positions in long straddles).

Backtesting

The accuracy of a risk management model used in the risk forecast is a further important criterion for recognition by bank supervisors. To be able to judge this, the institutions must compare the VaR forecast on the previous day with the actually incurred losses (back-testing). If the model often "generates" inaccurate forecasts within a certain period (i.e. the actually incurred loss is greater than the calculated VaR), the forecasting accuracy of the model is assessed as follows: the more inaccurate forecasts are found in the previous 250 trading days, the more negative the assessment given the model by banking supervisors. Up to four inaccurate estimates are considered acceptable, whereas the factor with which the VaR must be multiplied to calculate the capital charge (see below) increases if the number of inaccurate forecasts exceeds the limit.

What is called the "traffic light approach" (see table on page 71) shows the gradation of this amplifying factor.

Calculation of the prudential capital requirements

The values at risk for market price risks are to be calculated daily and form the basis for the own funds an institution must have. The VaR, even if calculated using the most sophisticated mathematical and statistical methods, is merely a good guide for the potential risk, since the forecast value is calculated on the basis of assumptions that can only approximate reality. Therefore, the values at risk calculated are multiplied by a factor of at least three. In addition, account is taken of the fact that this is the first time institutions have been allowed to calculate their prudential capital requirements using their own methods. That means the banks and the supervisors are relatively inexperienced in the use of this new approach.

The level of the multiplication factor has been a worldwide bone of contention for a long time. Now the volatility swings that have been appearing time and again have shown just how important this factor is. Since then this factor is no longer the subject of debate.

In the case of methodical and organisational weakness still considered acceptable, banking supervisors may increase the multiplication factor to four. In addition, in the event of not-quite-satisfactory backtesting results, the factor may be increased by up to 1 more point (see page 71).

Recognition of risk management models in Germany by banking supervisors

The use of internal risk management models to calculate prudential capital requirements must be approved by German supervisors. The models are subjected to an intensive on-site examination, at the request of the institutions, with regard to compliance with quantitative and qualitative requirements, including stress tests and backtesting. The tests are performed by model teams made up of staff members of the Federal Banking Supervisory

Basle "Traffic Light Approach"		
Zone	Number of exceptions (actual loss > VaR)	Multiplication factor for back-testing
Green zone	0	0.00
	1	0.00
	2	0.00
	3	0.00
	4	0.00
Yellow zone	5	0.40
	6	0.50
	7	0.65
	8	0.75
	9	0.85
Red zone	10 or more	1.00

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quirements as of December 31, 1997 calculated using internal methods. The examinations have been continued for 14 institutions over the course of 1998 in light of the entry into force of the amended Principle 1 on October 1, 1998. Other institutions will probably likewise convert their internal risk management methods, which means they can also be used to calculate prudential own funds in the foreseeable future. This is to be welcomed by banking supervisors, since that involves an improvement of internal risk management and, consequently, a positive trend by institutions from an organisational point of view.

... will evolve into an ongoing process of examinations

Since the trading activities and the market parameters, as well as the organisation, of a bank are constantly in flux, which has a direct impact on the use of internal risk management models, the conditions for approving the models must be reexamined whenever material changes occur. Even if adjustments are not made to the models, after a certain period follow-up checks are a must in order to ensure that the models are being applied correctly. This will lead to an ongoing process of examinations and a continuous dialogue between supervisors and the bank, whereby the specific situation of the institutions must be taken into consideration. Thus the institutions also have the possibility of taking into account indications of still-existing weaknesses in their risk controlling at an early stage already, which is likely to tend to entail

Office and the Deutsche Bundesbank. By examining internal risk management models using their own staff, German banking supervisors are extending their direct on-site supervisory activities.

... pose a new specialist challenge

The examination and approval of internal models, owing to their complexity and to the need to judge every single detail of the models, present a new specialist challenge to supervisors. The supervisory authorities have reacted by hiring specialists and by instituting comprehensive measures to train the staff deployed for these tasks.

The examinations begun in 1997 ...

The first such examinations took place as early as August 1997, in order to give what are called the "Basle banks"[7] the possibility of calculating and publishing the capital re-

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[7] In Germany, 15 internationally operating institutions are known as "Basle banks" because they have undertaken to comply with the Basle capital standard.

stabilitsation effects for the financial system as a whole, too.

Trend towards a modelling of risks

Optimising credit risk management with new methods

The overall changed environment in the banking sector is forcing banks to optimise their risk-related capital management, i.e. their economic capital. Now that capital management has reached a certain stage of maturity for the market price risks with the help of new market instruments (particularly the various forms of derivatives) as well as with mathematical models, new techniques are also entering the arena for the main risk block of banks, credit risks. They include:

– increasing the efficiency of the global deployment of collateral;

– expanding netting agreements to balance-sheet transactions;

– new methods of securitising claims, and

– developing credit derivatives.

Important methodical and organisational issues still open

The most important condition for the optimal use of these instruments is the exact quantification of the individual credit risks and particularly in the context of a portfolio. Traditional measuring methods must therefore be replaced or complemented by mathematical methods which are, in principle, comparable with those in the market risk sector. Although such models are to some degree already being used by individual internationally operating institutions to measure trading-book

credit risk, important methodical and organisational issues have not generally been clarified yet. There is still some question, for instance, about what data or parameters (e.g. rating by rating agencies or by the bank's own credit department) can be used as a basis. The question of checking the forecast accuracy of the models (backtesting) and the requirements to be placed on stress tests have not been satisfactorily clarified, either. In addition, the rather great time and effort in terms of EDP and the organisational inclusion of credit risk models in daily risk controlling pose a completely new challenge to the institutions.

In almost all banks, therefore, the practical implementation of the strategies has not gone beyond the development stage, which means it is not yet possible to assume the existence of general market standards. However, they are necessary to a certain degree as a prerequisite for supervisors to recognise credit risk models.

The same goes for other risks, such as liquidity risks or operational risks, for which, in connection with the ascertaining and optimisation of economic capital, in practice various attempts have likewise already been made to develop a more precise method of quantification using model calculations.

Models promote risk culture yet are not crystal balls

Supplementing traditional management instruments with risk models is to be welcomed from a prudential standpoint; however, the increase in knowledge that they make possible should not be overestimated. It is true that the complex mathematical and statistical

methods of these models actually provide a better basis for understanding and estimating risks, yet models are first and foremost abstractions of reality. Their results are therefore always to be seen from this angle and must not be treated as gospel but must instead be critically reviewed time and again. Models make it possible to analyse more precisely the relatively opaque structure of risk factors, their convoluted interrelationships and their interdependencies. They provide information for the risk-oriented management of a bank, yet do not remove the need for conscious decisions by those responsible. A model is not a crystal ball which can forecast a crisis like the ones in Asia or Russia, but it can serve well to estimate the range of the effect of such events on a bank's ability to bear risks. To that extent, models – despite their inherent limitations – make valuable contributions to promoting risk awareness and to the development of a pronounced risk culture.

In the Annex to this article on page 74, the modelling of market price risks will be dealt with in more detail – also using a simple example.

Annex

Methods of determining value at risk

This annex, using a sample portfolio as a starting point, is intended to show how the value at risk (VaR) is determined, using various methods (variance covariance approach, historic simulation and the Monte Carlo simulation). Then, the individual findings will be contrasted and explained.

Determinants of the portfolio risk

Portfolio analysis and risk factors ...

The starting point for risk calculation is always an analysis of the portfolio structure and a determination of the risk factors responsible for changes in the value of the portfolio. The value of a one-year US $ investment, for instance, depends on the risk factors of the interest rate for one-year US $ deposits and the US $/DM exchange rate.

... in the interest rate sector

When selecting the risk factors in the interest rate sector, there is generally the problem that basically the interest rates are to be seen as risk factors for every possible maturity, i.e. for the entire yield curve. In order to limit the number of risk factors, however, usually only interest rates for standardised maturities ("time buckets") are used as risk factors. The occurring cash flows are then allocated to these interest rates. This process is also called mapping. This is the only way to capture the important risk factors, on the one hand, while keeping the number of volatilities and correlations to be estimated within a manageable scale, on the other.

... as well as in the sector of equities and foreign exchange

In the equities sector, the individual equities can be cited as risk factors. However, it is also possible, for instance, to compile them according to national markets and to use the relevant index (in Germany, for example, the German Stock Index, or DAX) as a risk fac-

tor. In the foreign exchange sector, basically every individual pair of currencies constitutes a risk factor.

Sensitivities

The degree to which a position is dependent on a risk factor is expressed by sensitivity. The sensitivity of a position describes how much in terms of Deutsche Mark the value of a position changes if the risk factor increases by one unit. Simple examples of sensitivity measures are the delta[8] of an option or the present value of a basis point (PVBP) of a bond.[9]

Volatilities

In order to ascertain the risk of a position, the question of how sharply the risk factor can change over time must be answered. This is expressed through the volatility[10] of the risk factor. The higher the volatility of the risk factor, the higher the risk involved in the position, since there is greater uncertainty surrounding the future change in value.

Since portfolios regularly contain a large number of positions with varying risk factors, it is not enough to describe the behaviour of individual risk

8 The delta of an option measures the change in the value of an option when the price of the underlying rises by one Deutsche Mark. This is thus a measure of sensitivity of the individual option. The delta of a portfolio made up of several options corresponds to the sum of the deltas of the individual options.
9 The present value of a basis point (PVBP) of an interest rate instrument shows the absolute change in the value of the interest rate instrument given a parallel shift of the yield curve by one basis point. Analogously to the delta, the PVBP of a portfolio consisting of interest rate instruments corresponds to the sum of the PVBPs of the individual interest rate instruments.
10 In the simplest case, the volatility of a risk factor corresponds to the standard deviation of its changes. Reference can be made here to both the absolute and relative changes (risk factor yields). In some cases, logarithmic factors of change are also normally used. Modified methods assign a greater weight to more current data than to those data going further back into the past. More complex methods of estimating the volatility are GARCH (general autoregressive conditional heteroscedasticity) approaches which can also take account of temporal changes in volatility.

factors in an isolated manner. Rather, it is additionally necessary to describe the joint behaviour of the risk factors, i.e. the correlations.

Correlations

An example will be cited here to underline the significance of correlations. In practice, the zero-coupon yields for nine years and ten years show a large degree of correlation, i.e. their trends are relatively parallel over time. If, for example, a long position exists in a nine-year zero bond and a short position in a ten-year zero bond, a large portion of the risk is offset, since losses incurred in one position are largely offset by gains from the other position.

The sample portfolio

Sample portfolio

The sample portfolio is compiled as follows:

5	DAX call options
	(1 DAX point = DM 1),
	Strike 6,500,
	Date of maturity July 1, 1999
DM 100,000	Zero-coupon bond
	Date of maturity July 1, 2007
US $ 5,000	Spot position

The change in the value of the sample portfolio depends on the risk categories of equity risk, interest rate risk and foreign exchange risk. The risk factors selected were the DAX index, the US $/DM exchange rate, and for the reference date of the calculation, July 1, 1998, the nine-year DM zero-coupon yield. The status referring to July 1, 1998, the volatilities and the correlations of the risk factors as well as the sensitivities of the portfolio to the risk factors, can be seen in the table on this page.[11]

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[11] For the calculation of the volatilities and the correlations, the last 250 trading days were used as an observation period, with all observations being weighted equally.

Volatilities and correlations of the risk factors and sensitivities of the portfolio to risk factors as of July 1, 1998

Risk factors	Status on the reference date
DAX index:	5,906.85 points
Nine-year DM zero-coupon yield:	5.04 %
US $/DM exchange rate	DM 1.8190

One-day volatilities of the risk factors	
DAX index:	95.1 points
Nine-year DM zero-coupon yield:	3.86 BP
US $/DM exchange rate	DM 0.01055

Correlations between the risk factors

	DAX	US $	Nine-year zero-coupon yield
DAX	1	0.1849	– 0.0534
US $	0.1849	1	– 0.1448
Nine-year zero-coupon yield	– 0.0534	– 0.1448	1

The portfolio's sensitivities to the risk factors

DAX index:	DM 2.265/point [1]
Nine-year DM zero-coupon yield:	– DM 55.0421/BP [2]
US $/DM exchange rate	5,000

[1] The delta of the option is DM 0.453/point. To ascertain the sensitivity of the portfolio, this value is to be multiplied by the number of options in the portfolio. — [2] The PVBP of the zero-coupon bond is – DM 0.0550421/BP.

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Variance covariance approach

Linear changes in value

The most common approach to calculating risk is the variance covariance approach. It is assumed here that the relationship between the change in the value of the portfolio and the risk factor yields is linear, i.e. changes in the value of the portfolio can be calculated linearly from the changes in the risk factors.[12]

Assumption of the multivariate normal distribution

For the change in the risk factors, the variance covariance matrix assumes a multivariate normal distribution.[13] To determine the distribution, it then suffices to calculate the volatilities and correlations[14] of the risk factors. Further, the sensitivities of the positions to the individual risk factors are needed.

The VaR of an individual position is calculated according to the following formula:

$$VaR = |\, 2{,}33^{[15]} \cdot sensitivity \cdot volatility^{[16]} |$$

For this sample portfolio, the individual values at risk are as follows:

$$VaR_{DAX} = |\, 2.33 \cdot DM\ 2.265/point \cdot 95.1\ points\,| = DM\ 501.89$$

$$VaR_{US\ \$} = |\, 2.33 \cdot 5.000 \cdot DM\ 0.01055\,| = DM\ 122{,}91$$

$$VaR_{bond} = |\, 2.33 \cdot DM\ (-55{,}0421)/BP \cdot 3.86\ BP\,| = DM\ 495{,}04$$

Adding the values at risk of the individual positions yields the following sum:

VaR without a correlation ...

$$VaR_{DAX} + VaR_{US\ \$} + VaR_{bond} = DM\ 1{,}119.84$$

When taking into account the correlation and diversification effects[17], the VaR for the portfolio, calculated according to the following formula:

$$VaR = 2.33\sqrt{\vec{x}^{T}\,C\vec{x}}$$

$$where\ \vec{x} = \begin{bmatrix} \sigma_1\ \Delta_1 \\ \sigma_2\ \Delta_2 \\ \vdots \\ \sigma_n\ \Delta_n \end{bmatrix}$$

σ_i = absolute volatility of the risk factor i

Δ_i = sensitivity to the risk factor i

C = correlation matrix

is DM 760.93. The VaR at the portfolio level is thus DM 358.91, or around 32 %, lower than the sum of the individual VaR values.

... VaR with correlation

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12 This assumption leads to a neglect of second-order risks (gamma risks) and risks of a higher order. Particularly in the case of large option positions, that may lead to unsatisfactory results.
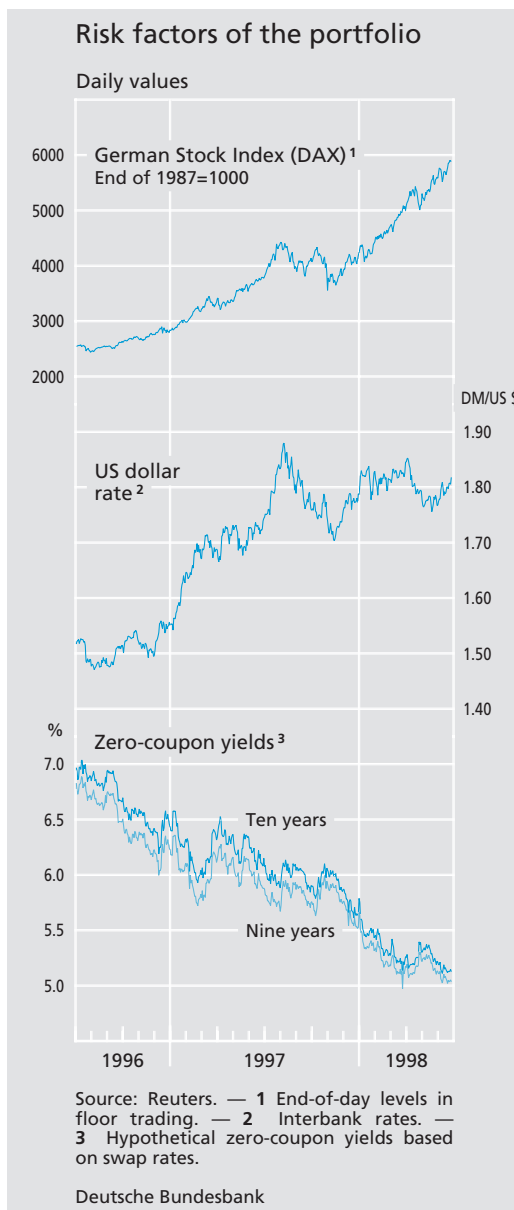
13 This means that the frequency of the occurrence of extreme changes in the risk factors (fat tails) is underestimated.

14 The correlations are described using correlation coefficients which are put together to form a correlation matrix. Here, the correlation coefficient describes the extent of the linear connection between two risk factors.

15 The factor of 2.33 is for scaling to a prudentially required forecast interval with a confidence level of 99 %. This value is based on an assumption of normal distribution, since the 1% quantile of a normally distributed random variable with the expected value of 0 and the variance of σ^2 is $-\,2.33\ \sigma^2$.

16 In the example, the VaR is supposed to be calculated for a holding period of one day. For that reason, the one day volatility is to be applied here.

17 Diversification effects are the result of the simultaneous occurrence of losses from several positions being less probable then an isolated occurrence, and of losses in one position being able to be offset by profits/gains in other positions.

Risk factors of the portfolio

Daily values

German Stock Index (DAX)[1]
End of 1987=1000

US dollar rate[2]

Zero-coupon yields[3]

Ten years

Nine years

Source: Reuters. — **1** End-of-day levels in floor trading. — **2** Interbank rates. — **3** Hypothetical zero-coupon yields based on swap rates.

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Historical simulation

Simulation of historical changes in the risk factors ...

In the historical simulation, the distribution of the future portfolio value changes is estimated by applying the historical changes in the risk factors to their current level. The historical simulation is made up of individual simulation steps. For each step a change in all risk factors is simulated, with the change in the risk factors corresponding to a historically observed change in the risk factors[18].

After each simulation step, the whole portfolio is revalued,[19] and the portfolio value change is then calculated. The VaR is then the result of the 1% quantile of the distribution of the simulated portfolio value changes. Given a historical observation period of 250 trading days, and thus 250 simulated portfolio value changes, the VaR thus corresponds to the third-worst[20] value.

Arranged by size, the following simulated changes in value would be the result for the sample portfolio:

− 999.15; − 963.09; − 860.04; − 840.42; − 784.38; − 687.96; − 640.47; − 563.18; − 552.20 ...

The VaR is thus DM 860.04.

... results in simulated changes in value

Monte Carlo simulation

In the Monte Carlo simulation, the distribution of the portfolio value changes is estimated by simulating the changes in the risk factors using a random number generator.
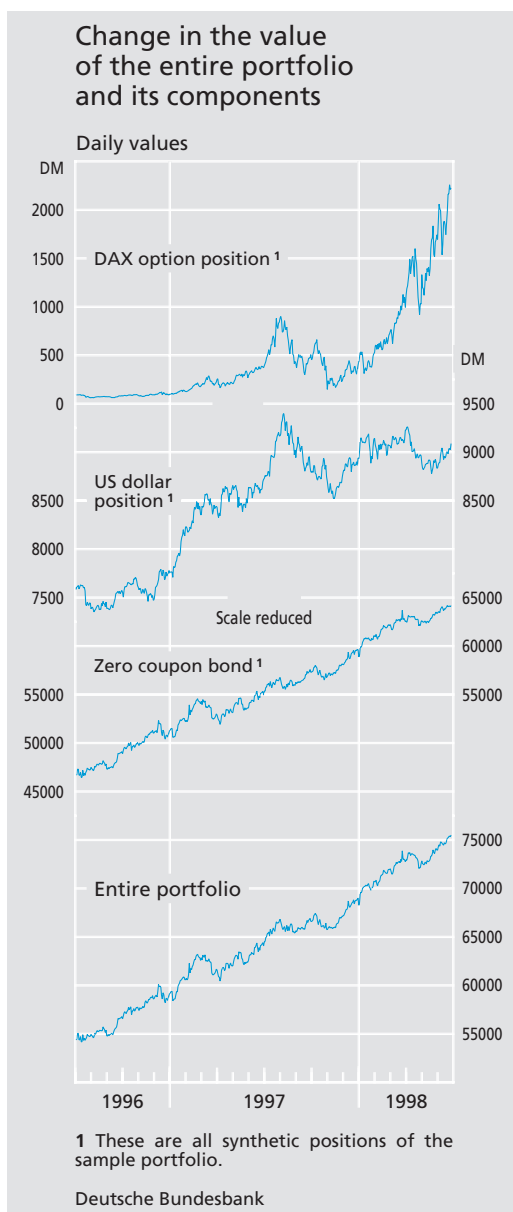
Simulation with a random number generator ...

Firstly, as for the variance covariance approach, the volatilities and correlations of the risk factors are calculated.

18 Here, the volatilities of the risk factors and the correlations between the risk factors are implicitly taken into consideration.
19 Numerically speaking, it may be easier to approximate the portfolio value change depending on the changes in the risk factors using a Taylor series. In order to capture non-linear risks, it will then be necessary to include also the terms in the Taylor series having an order higher than 1.
20 Theoretically the "two-and-a-half-worst" value should be taken as a basis. However, since this value does not exist, one must fall back on the third-worst value. In that case, the losses would exceed the VaR in exactly two out of 250 cases (0.8 %). If, for example, the fourth-worst value were taken as a basis, then the losses would exceed the VaR in three out of 250 cases (1.2 %). That, however, would contravene the envisaged confidence level of 99 %.

Change in the value of the entire portfolio and its components

Daily values

1 These are all synthetic positions of the sample portfolio.

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changes in the portfolio value and amounts to, for instance, DM 735.67. For a large number of simulation steps, the distribution generated in this manner approaches the true distribution of the changes in the portfolio value.[22]

Comparison of methods and backtesting

In this section, the daily changes in value of the sample portfolio are compared with the values at risk calculated according to the various methods (backtesting) and explained. The calculations are only meant as examples to illustrate the point. They also do not permit wholesale statements on what methods tend to lead to a lower or higher capital charge. Questions of that type can as a rule only be answered for a specific portfolio.

Actual change in values versus forecast changes in values

Risk and performance coefficients were calculated for a period comprising 250 trading days and ending on July 1, 1998. The number of days for which values at risk were calculated was chosen in such a manner that it is possible to run exactly one backtest in accordance with banking supervisory requirements.[23]
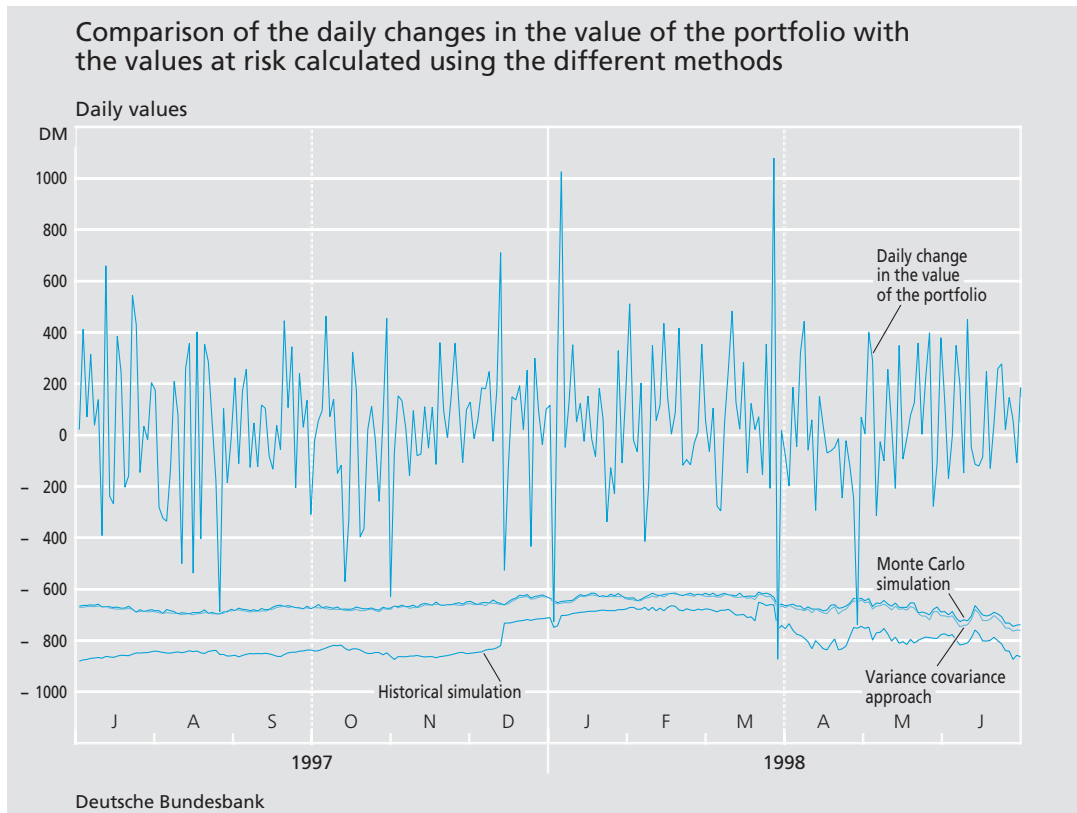
The risk factors selected, on which the value of the portfolio during the period under review depends,

Trend in value of the risk factors and ...

... results in simulated portfolio value changes

The Monte Carlo simulation is made up of individual simulation steps. For each step a change in all risk factors is simulated using a random number generator; here, for example, a multivariate normal distribution with the previously calculated volatilities and correlations is assumed. After every simulation step, the entire portfolio is revalued[21] and the change in the value of the portfolio is calculated. The value at risk is then the result of the 1% quantile of the distribution of the simulated

21 Numerically speaking, it may be easier here, too, to approximate the portfolio change depending on the changes in the risk factors using a Taylor series. In order to capture non-linear risks, it will of course be necessary here, too, to include the terms of the Taylor series having an order higher than 1.
22 For this calculation, 80,000 simulations were run per valuation day.
23 The Basle Market Risk Paper and Principle I (as amended) demand that the forecast accuracy of the model be ascertained through a daily comparison of the value at risk calculated using the risk model on the basis of a holding period of one working day with the change in value of the financial instruments included in the model calculation. To this end, the number of exceptions in the last 250 trading days in which the losses incurred by the portfolio exceed the value at risk is to be taken as a basis.

Comparison of the daily changes in the value of the portfolio with the values at risk calculated using the different methods

Daily values

Deutsche Bundesbank

were the DAX index, the US $/DM exchange rate, and the DM zero coupon yields for nine years and ten years. The temporal trend in the risk factors used can be derived from the chart on page 77.[24]

Here, a high historical correlation only exists for nine-year and ten-year DM zero-coupon yields.

... change in the value of the portfolio positions and of the entire portfolio

The change in the value of the individual positions over time is the result of the financial instruments being directly dependent on the risk factors.[25] The chart on page 78 shows both the change in the value of the individual positions and of the entire portfolio for the last 500 days. Here, only the last 250 values are relevant for the purpose of back-testing.

Backtesting ...

The chart on this page contrasts the daily changes in value of the sample portfolio with the values at

risk calculated for those days using the various methods. If a one-day loss exceeds the predicted VaR, for the purposes of backtesting this is an exception as defined by Principle I (as amended). Here, at a confidence level of 99 % and 250 observations, an average of 2.5 exceptions are expected. However, only six or more exceptions are significant for the hypothesis that the forecast accuracy of the model is insufficient. Two or three ex-

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24 Since a historical observation period of one year, or more precisely, 250 trading days, is to be used as a basis for the calculation of risk, and the calculation is run for risk coefficients for 251 days, data series with a length of 501 trading days will be necessary for the risk factors.
25 The risk factor for the zero-coupon bond maturing on July 1, 2007 is, on July 1, 1997, the ten-year zero-coupon yield, and on July 1, 1998 the nine-year zero-coupon yield. For the periods in between, there is a dependency on both risk factors, since in mapping payments with a broken maturity are split into two payments with a whole-year maturity (here: 9 and 10 years). For the DAX option, for reasons of simplicity, the volatility and the short-term interest rate are assumed to be constant.

ceptions, as noticed in the calculations, will quite probably be unproblematical.

When comparing the VaR time series calculated using the three methods, one notices, for one thing, that very similar results are gained using the variance covariance approach and the Monte Carlo simulation. For another, the results of the historical simulation are almost all visibly higher than the results of the other two methods.

The reason for the similarity of the VaR time series calculated using the variance covariance approach and the Monte Carlo simulation is the similarity of the models used. In both cases, the same multivariate normal distribution of the relative changes in risk factors was assumed. The only difference is the inclusion of non-linear risks in the Monte Carlo simulation[26], whereas the variance covariance approach is exclusively based on a linear approximation of the changes in value of the portfolio. Here, the VaR calculated using the Monte Carlo simulation is lower on average, since there exists both a positive convexity for the bond and a positive gamma for the option.[27]

Owing to the fact that the option position in the sample portfolio tends to be small, and that the convexity of the bond has only an extremely minor effect, there are only very slight differences in the performance between both methods, of around 1% to 4%. However, in the case of major option positions, these non-linear risks do play an important role, though.

The reason why the VaR time series calculated using historical simulation is higher throughout than the time series calculated using the other methods is that when using the historical simulation, there is no explicit assumption regarding the form of the distribution of the changes in the risk factors (e.g. multivariate normal distribution). Thus, the historical simulation implicitly also takes account of the fat tails of the distribution of the risk factor yields.[28] This effect is particularly strong for the selected sample portfolio since the zero-coupon bond makes up a major portion of the portfolio and the fat tails are particularly pronounced in the interest rate sector.

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26 Non-linear risks are entered into the Monte Carlo simulation, as well as the historical simulation, by taking into account the second-order term of the Taylor series when approximating the changes in portfolio value in relation to the changes in the risk factors.
27 For the investor, positive convexity or positive gamma means that in the case of a loss, the losses are lower, and in the case of a profit, the profits are higher, than in a purely linear approximation. In principle, long positions in bonds have a positive convexity and long positions in standard options a positive gamma.
28 In principle, however, it is possible to take account of fat tails (see footnote 13) in the Monte Carlo simulation, too.