

# **Deriving the term structure of banking crisis risk with a compound option approach: the case of Kazakhstan**

Stefan Eichler

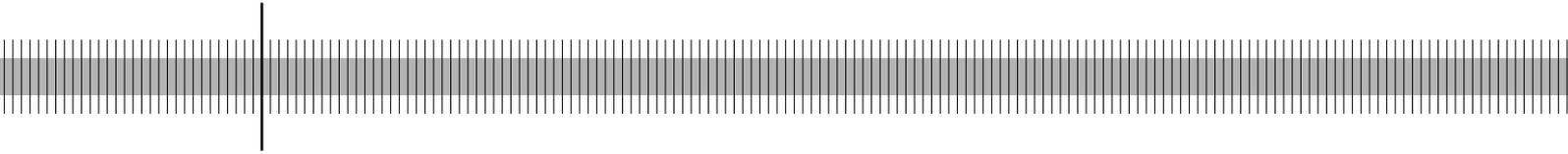
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## **Abstract**

We use a compound option-based structural credit risk model to infer a term structure of banking crisis risk from market data on bank stocks in daily frequency. Considering debt service payments with different maturities this term structure assigns a separate estimator for short- and long-term default risk to each maturity. Applying the Duan (1994) maximum likelihood approach, we find for Kazakhstan that the overall crisis probability was mainly driven by short-term risk, which increased from 25% in March 2007 to 80% in December 2008. Concurrently, the long-term default risk increased from 20% to only 25% during the same period.

**JEL classification:** G21; G17; G32; G12; G18

**Keywords:** Banking crisis; Bank default; Option pricing theory; Compound option; Liability structure

## **Non-technical summary**

Previous papers on estimating bank default risk which rely on option-based models consider only a single date when the bank's debt matures (Chan-Lau et al. (2004), Gropp et al. (2004, 2006), Chen et al. (2006)). Although within the Merton (1974) framework the debt maturity influences the probability of default (see, for example, Anderson und Sundaresan (1996), Leland (2002), and Crosbie and Bohn (2003)), separate probabilities of default for different debt maturities cannot be derived therein. We contribute to the literature by applying the Geske (1977) compound option approach to infer short-term and long-term probabilities of bank default separately. This enables us to account for the influence of the composition of debt, i.e. the ratio of short-term to long-term debt on default risk, rather than simply accounting for the total amount of debt.

We apply the Geske (1997) approach to the four leading market-traded Kazakh banks in 2007 and 2008. Using daily stock market data, we simultaneously estimate the unknown quantities of our structural model, i.e., a bank's value as well as its value's volatility and drift, which are needed to calculate the default probability. In order to do this, we use the maximum likelihood approach as developed by Duan (1994).

We find that the overall default probability of the Kazakh banks analyzed increased steadily from 40% on average at the beginning of 2007 to 90% in December 2008. Relating the default probabilities with the daily news published on Reuters.com, we find that higher (lower) default probabilities are associated with bad (good) news about the health of the Kazakh banking system or on macroeconomic development. This suggests that stock market investors take publicly available information into account when setting their expectations about bank default probabilities.

Analyzing the term structure of default probabilities, we find that the short-term probability of default increased from 25% in March 2007 to 80% in December 2008. The long-term probability of default (given no default in the short run) increased from 20% to 25% within the same period. This can be interpreted as evidence that market participants expect a banking crisis in the short-term rather than in the long-term.

## **Nichttechnische Zusammenfassung**

Bestehende Ansätze, welche optionspreisbasierte Modelle zur Ermittlung von Bankausfallwahrscheinlichkeiten nutzen, unterstellen lediglich einen einzigen Fälligkeitstermin der Bankverbindlichkeiten (Chan-Lau et al. (2004), Gropp et al. (2004, 2006), Chen et al. (2006)). Obwohl die Restlaufzeit der Gesamtverbindlichkeiten die Ausfallwahrscheinlichkeit im Modell von Merton (1974) beeinflusst (Anderson und Sundaresan (1996), Leland (2002), and Crosbie and Bohn (2003)), können mit diesem Modell keine getrennten Ausfallwahrscheinlichkeiten für verschiedene Laufzeiten abgeleitet werden. Unser Beitrag zur Literatur besteht in der Anwendung des Geske (1977)-Compound-Optionen-Ansatzes, um getrennte Schätzungen für kurz- und langfristige Ausfallwahrscheinlichkeiten zu erhalten. Dieser Ansatz ermöglicht es, den Einfluss der Fristenstruktur der Bankschulden, d.h. des Verhältnisses von kurz- zu langfristigen Verbindlichkeiten, auf die Ausfallwahrscheinlichkeit abzubilden, anstatt lediglich für die absolute Höhe der Schulden zu kontrollieren.

Wir wenden den Ansatz von Geske (1977) auf die vier führenden kasachischen Banken im Zeitraum von 2007 bis 2008 an. Basierend auf täglichen Aktienmarktdaten schätzen wir die zur Berechnung der Ausfallwahrscheinlichkeiten benötigten unbekanntenen Größen des strukturellen Modells, d.h. den Marktwert der Bank und dessen Standardabweichung und Wachstumsrate, simultan. Dabei wird der Maximum-Likelihood-Schätzansatz von Duan (1994) verwendet.

Die Gesamtausfallwahrscheinlichkeit der analysierten kasachischen Banken stieg von Werten um durchschnittlich 40% Anfang 2007 bis 90% im Dezember 2008 stetig an. Ein Vergleich mit den auf Reuters.com gemeldeten Nachrichten zeigt, dass höhere (niedrigere) Ausfallwahrscheinlichkeiten mit schlechten (guten) Meldungen über den Zustand des kasachischen Bankensystems oder mit makroökonomischen Fundamentaldaten Kasachstans einhergehen. Dieses Ergebnis legt nahe, dass Aktienmarktinvestoren öffentlich verfügbare Informationen bei der Erwartungsbildung bzgl. der Bankausfallwahrscheinlichkeiten nutzen.

Die Laufzeitstruktur der Ausfallwahrscheinlichkeiten zeigt, dass die kurzfristige Ausfallwahrscheinlichkeit der Banken von 25% im März 2007 auf 80% im Dezember 2008 gestiegen ist. Die langfristige Wahrscheinlichkeit eines Bankausfalls (gegeben, dass sich in der kurzen Frist kein Bankausfall ereignet hat) stieg im selben Zeitraum von 20% auf 25%. Offenbar erwartet der Markt demnach eine Bankenkrise eher in der kurzen als in der langen Frist.



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# Deriving the Term Structure of Banking Crisis Risk with a Compound

## Option Approach: The Case of Kazakhstan\*

### 1 Introduction

This paper derives short- and long-term probabilities of default for the four leading Kazakh banks using daily stock market data. This contributes to an emerging literature that aims to infer the probability of bank default from stock price information by applying structural credit risk models. Several important and interesting papers apply the Merton (1974) options-based approach assuming the simplest possible capital structure where the banks' liabilities become due at a single date (see, for example, Chan-Lau et al. (2004), Gropp et al. (2004, 2006), Chen et al. (2006)). Although within the Merton (1974) model the debt maturity influences the probability of default (see, for example, Anderson und Sundaresan (1996), Leland (2002), and Crosbie and Bohn (2003)), separate probabilities of default for different debt maturities cannot be derived herein. Our paper applies a model that accounts for a more complex capital structure with multiple debt service payments.<sup>1</sup> By applying the Geske (1977) compound option approach, we consider short-term and long-term liabilities separately. In this way, we account for the influence of the *composition* of debt, i.e., the ratio of short-term to long-term debt on default risk, rather than simply the *amount* of total debt. This enables us to derive a

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<sup>1</sup> The maturity structure of a firm's debt can also be viewed as a result of the shareholders' decision to maximize the firm's market value. Dangl and Zechner, for example, show that firms face a trade-off between transaction costs and the costs to reduce the firm's leverage when deciding about the optimal maturity structure of debt. While a larger fraction of short-term debt is associated with higher transaction costs for rolling over maturing debt, it allows for reducing the leverage in times when the firm experiences profitability losses.

term structure of default risk, i.e., we are able to derive short-term and long-term probabilities of bank default.

The development of structural models of default risk started with Merton (1974). For the case of corporate defaults, Merton interprets the value of equity as a call option that enables the shareholders to buy the firm at maturity where the strike price of the call equals the face value of debt. A default of the borrower occurs if the value of the firm falls below a certain threshold value (the face value of the debt).

A critical feature of the basic Merton (1974) model is the assumption that the firm's entire debt is due at a single date. This is the only point in time at which a default can occur. Black and Cox (1976) developed the first model in which a default can occur at any date up to maturity, i.e., whenever the value of the firm hits a certain threshold. This can be seen as a simple approach to model a default by missing one of several required debt service payments before the final payment is due. These threshold models nevertheless fail to model the dependency between different payments. Moreover, the threshold is also assumed to be independent of the actual payments and their term structure.<sup>2</sup>

Geske's (1977) model addresses these aspects by considering a multi-period debt payment framework applying compound option theory. In the simplest situation, it is assumed that two debt service payments are required at two different dates. Delianedis and Geske (1998) apply this approach to the case of U.S. enterprises. This paper applies it to Kazakh banks. The model can be solved by backward induction. After the first debt service payment is made, the Merton situation prevails, i.e., the equity value constitutes the value of a basic call option that enables shareholders to buy the firm at the second (final) payment date whereby the strike equals the second (final) debt payment. Before the first debt payment is made, the equity value equals the price of a compound call option – which gives the holder

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<sup>2</sup> Instead, Black and Cox (1976) assume the default threshold to be a monotonic increasing function of time, whereas, for example, Longstaff and Schwartz (1995) assume that the threshold is constant.

the right to buy the simple call option of the second period by paying the strike price at the first payment date, i.e., by making the first debt payment.

This multi-period approach models the dependency between short-term and long-term payments. This in turn distinguishes short-term probability of bank default from long-term probability of bank default. We apply the Geske (1977) approach to the four largest market-traded Kazakh banks in 2007 and 2008. Based on stock market data, we simultaneously estimate the unknown quantities of our structural model, i.e., the state variable (the value of the bank) and the parameters of its stochastic process, which are needed to calculate the default probability. Here, we apply the estimation approach developed by Duan (1994) which relies on the maximization of the likelihood function for a time series of observed market data.

The main results can be summarized as follows. The overall default probability of the Kazakh banks analyzed show a steady upward trend in the years 2007 and 2008. While stock market investors estimated the overall probability of a banking crisis to be 40% at the beginning of 2007, this probability rose to almost 90% in December 2008. By relating the default probabilities with the daily news published on Reuters.com, we find that higher (lower) default probabilities are associated with bad (good) news about the health of the Kazakh banking system or on macroeconomic quantities. This indicates that stock market investors take publicly available information into account when setting their expectations about bank default probabilities. Looking at the term structure of default probabilities, we find that the short-term probability of default increased from 25% in March 2007 to 80% in December 2008 while the long-term probability of default (given no default in the short-term) increased from 20% to 25% in the same period. This can be interpreted as evidence that market participants expect a banking crisis in the short-term rather than in the long-term.

Our approach may be interesting for supervisory agencies for several reasons. Extracting information on market expectations about the likelihood of short- and long-term

default sheds light on the potential problems banks face. This helps supervisory agencies to decide which instruments to use to rescue vulnerable banks. If, for example, the short-term probability of default rises but the long-term probability of default stays constant, supervisors may interpret this as evidence that the bank suffers from short-term problems, such as temporary financing and liquidity problems. An increase in the long-term probability of default, on the other hand, points to fundamental problems, such as bad loans produced by recessions or a downward trend in commodity prices which depresses the growth of commodity-based economies such as that of Kazakhstan.

Supervisory agencies may use these market signals when evaluating which instruments to use to rescue vulnerable banks. A rising short-term probability of bank default calls for monetary easing, such as a reduction in interest rates and/or an expansion of the money supply, to improve the short-term financing and liquidity conditions of banks. Higher long-term default probabilities indicate structural problems that can only be solved by propping up the equity base, restructuring, or – as a last resort – nationalization of banks.

Another advantage of this approach results from the nature of the data used. Traditional bank monitoring systems use low-frequency balance sheet information to signal bank distress. Our approach uses stock market data available on a daily basis. This enables supervisors to react promptly to changing fundamentals. Market data is also, by nature, forward-looking: stock prices are based on expected future cash flows while balance sheet data reflect the bank's previous health. Thus, our approach may be interesting for supervisory purposes as stock prices signal future problems of banks that may be alleviated by implementing regulatory measures.

The advantages of using stock market data rather than balance sheet information apply especially for emerging economies such as Kazakhstan. A typical obstacle for banking regulation in these countries is that supervisory agencies need time to develop efficient frameworks to interpret balance sheet information with regard to bank default risk. Our

approach, on the other hand, can be implemented without having decade-long experience of best practices in banking regulation. Another obstacle is that the accounting standards of banks may not fully meet the data requirements to implement balance sheet-based frameworks, and, thus, the availability of data may be questionable. Our market-based approach can be applied in this case since it requires minimal information, i.e., stock prices and aggregate debt figures.

The rest of the paper is organized as follows. Section 2 reviews the literature on measuring and forecasting bank distress. Section 3 explains the multi-period debt service model. Section 4 discusses the estimation procedure. Section 5 presents the empirical application to Kazakhstan. Section 6 concludes.

## **2 Literature**

Bank distress can be analyzed on the macroeconomic or on the bank-specific level. Papers that explain country-wide banking crises use a binary dependent variable that reflects whether a crisis in the banking sector occurs or not (see, for example, Demirgüç-Kunt and Detragiache (1998, 2005), and Davis and Karim (2008)). Using macroeconomic data these approaches have been applied to design early warning systems of banking crises with remarkable forecast ability.

To measure bank-specific distress, the literature relies on two types of data. One strand of the literature employs accounting data; the other uses market data. While using accounting data implies an *ex post* analysis of bank fragility, market data reflects market participants' perception of bank default risk *ex ante*.

Several papers focus on accounting data to forecast or explain bank distress. These approaches are interesting and insightful as they take the position of supervisory agencies – which use data on CAMEL variables (capital adequacy, assets quality, management quality,

earnings, and liquidity) to quantify bank distress. Männasoo and Mayes (2009) find that CAMEL indicators play an important role in explaining bank distress. Using a sample of Eastern European banks, they are able to predict eight out of sixteen distress episodes during 2002-2004. Poghosyan and Cihak (2008) find that several CAMEL variables – especially asset quality and profitability – estimate the distress of banks located in the European Union (EU) particularly well. Arena (2008) finds for a set of East Asian and Latin American banks that CAMEL indicators have a remarkable explanatory power in predicting bank failure. He also concludes that macroeconomic variables such as economic growth or real exchange rate volatility can account for the regional differences in bank distress.

The use of market data, such as stock prices or subordinated debt spreads, has become popular in the literature to measure bank distress and exhibits several advantages.<sup>3</sup> First, market data is forward-looking as market participants set prices depending on their expectations about future cash flows. Second, data on practical banking regulation is available on a daily basis while accounting data is updated only monthly or quarterly. Thus, using market data would enable supervisory agencies to react quickly to problems adversely affecting banks.

Comparisons of different sources of information confirm that stock price information generally outperforms supervisory or rating agencies' balance sheet-based assessments of bank conditions. Berger et al. (2000) conclude that stock market and bond investors predict future bank performance more precisely than supervisors – except when the supervisor has recently inspected the bank. Bongini et al. (2002) find that stock market information, accounting data, and ratings have a similar ability to assess bank fragility although stock prices respond more quickly to changing bank conditions than ratings or balance sheet information. Gropp et al. (2006) find an asymmetric forecast ability of stock prices and subordinated debt spreads when the forecast window is considered. Stock prices perform best

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<sup>3</sup> Flannery (1998) reviews the different sources of market information that can help supervisory agencies assessing bank distress.

within a forecast window of six to 18 months before a rating downgrade. Spreads can predict downgrades within a forecast window of one year or less.

Some authors use information on market-traded debt securities to infer expectations on bank distress. Evanoff and Wall (2001) show that yield spreads of subordinated debt predict changes in ratings of bank supervisors as well or better than capitalization ratios drawn from a bank's balance sheet. Deyoung et al. (2001), by contrast, find that bank examinations provide relevant information to supervisory agencies several quarters before information about the bank's condition is reflected in subordinated debt yield spreads.

The literature that uses stock market information to measure bank fragility can be grouped into two branches. The first branch uses stock prices to infer probabilities of default. The second branch uses stock market information as an independent variable in regressions which aim to improve the understanding of supervisory rating changes (see, for example, Gunther et al. (2001), Curry et al. (2003), and Distinguin et al. (2006)). A common finding of the second branch is that stock market data helps explain rating downgrades or other forms of bank distress. Krainer and Lopez (2004) find that including stock market information in their forecast framework helps predict supervisory rating changes up to four quarters in advance.

Several important papers apply option pricing theory to derive the probability of bank default from stock price information (see, for example, Chan-Lau et al. (2004), Gropp et al. (2004, 2006), Chen et al. (2006)). These approaches use the Merton (1974) model, which assumes that the equity of a bank is equivalent to a call option on the bank's assets and where the value of the debt represents the strike price. By employing information on stock prices, the bank's debt, and its maturity, the probability of default, i.e., the likelihood that the value of the assets falls short of the value of the debt at maturity, can be derived.

A convenient indicator that can be derived from the basic options-based approach is the distance to default, i.e. the difference between the value of the bank's assets and debt at maturity (Crosbie and Bohn (2003)). Chan-Lau et al. (2004) use the distance to default

measure to assess the fragility of 38 banks in 14 emerging economies. They are able to forecast rating downgrades up to nine months in advance in-sample and show that their model also performs well out of sample. Applying the distance to default measure to a sample of EU banks, Gropp et al. (2004, 2006) find that stock prices predict rating downgrades 6 to 18 months in advance. The forecast ability of stock prices is lower over the short-term. Chen et al. (2006) find that stock prices effectively forecast Estonian bank distress.

The approaches described above assume that the total debt is due at a single date. This enables them to derive a measure for the overall probability of default. We contribute to the literature by distinguishing between short-term and long-term debt. This enables us to derive short-term and long-term probabilities of default. This term structure of bank default risk may help supervisory agencies to address the bank's problems more accurately.

### **3. The Model Framework**

Our assessment of banking crisis risk is derived from the default probabilities estimated for individual banks of the country considered. These default probabilities are derived from stock market data. We consider stock prices to estimate the value of a bank's equity. The equity value is then used to derive the firm's value – which is employed to estimate the default probability. To infer the firm's value from the equity, we apply the structural credit risk model of Geske (1977) – a generalization of the Merton (1974) model – to multiple debt service payments.

#### **3.1 The Merton Model and the Black-Cox Model**

In his groundbreaking paper, Merton (1974) formulates the basic idea that a borrower defaults when the firm's value – which is described by a stochastic state variable – falls below a certain threshold. In the Merton model – where the firm has the simplest debt structure, i.e.,

the debt becomes due at one point in time – the firm defaults at the payment date if the stochastic value of the firm is lower than the amount required for the debt service payment. Based on this idea, pricing formulas can be derived to estimate the value of the firm's equity and debt at any date before the payment date.

The pricing formulas are based on the assumption that the development of the firm's value,  $W$ , over time can be described by the following Ito stochastic process:

$$dW = \mu_W W dt + \sigma_W W dZ, \quad (1)$$

where  $\mu_W$  is the constant drift rate, i.e., the expected rate of return on the firm's value,  $\sigma_W$  is the constant volatility, and  $dZ$  is a standard Gauss-Wiener process. In addition Merton (1974) makes the typical assumptions of neoclassical finance theory regarding the existence of perfect capital markets where borrowing and lending at an identical risk-less interest rate is possible, which is constant over time and equal for all maturities. Although idealized these assumptions are well-accepted and widely used in the literature concerned with the application of structural credit risk models on corporate default risk and bank defaults. It follows from Equation (1) that growth rates for equidistant time intervals  $\Delta t = T-t$  are independently identically normally distributed:

$$w_{t,T} \sim \text{i.i.n.}[(\mu_W - \frac{\sigma_W^2}{2})(T-t); \sigma_W \sqrt{T-t}]. \quad (2)$$

According to Merton, the following differential equation for the value of a contingent claim on the firm (e.g. by stocks or a debt contract),  $G$ , can be derived:

$$\frac{\partial G}{\partial W} r_s W + \frac{\partial G}{\partial t} + \frac{1}{2} \frac{\partial^2 G}{\partial W^2} \sigma_W^2 W^2 - r_s G = 0. \quad (3)$$

For the situation where the debt payment,  $B_T$ , is due at a single point in time,  $T$ , the value of the debt at any point in time,  $t$  ( $<T$ ), is derived by Merton (1974):

$$F_t = W_t - W_t \cdot N(d + \sigma_W \sqrt{T - t}) + B_T \cdot e^{-r_s(T-t)} \cdot N(d), \quad (4)$$

$$\text{where: } d = \frac{\ln(W_t / B_T) + (r_s - \sigma_W^2 / 2)(T - t)}{\sigma_W \sqrt{T - t}}.$$

$N(\dots)$  describes the value of the cumulative standard normal distribution for the argument in parentheses. In the Merton framework the value of the firm's equity can be calculated using the Black-Scholes formula (see Black and Scholes (1973)):

$$E_t = W_t \cdot N(d + \sigma_W \sqrt{T - t}) - B_T \cdot e^{-r_s(T-t)} \cdot N(d), \quad (5)$$

$$\text{where: } d = \frac{\ln(W_t / B_T) + (r_s - \sigma_W^2 / 2)(T - t)}{\sigma_W \sqrt{T - t}}.$$

Several papers use the Merton framework with a single payment date – and, thus, a single default opportunity (see introduction). In reality, the term structure of debt is typically much more complex with a multitude of debt service payments due at different dates. Another strand of the literature addresses this by considering a possible default before the payment is due. This approach dates back to Black and Cox (1976). In this framework, the borrower defaults if the state variable reaches a default threshold at any date before maturity. This default can be thought as being triggered by missing one of several debt servicing payments due within time to maturity.

The drawback of the Black and Cox model is that the default threshold is independent of the actual structure of payments – the amount of and time span between payments. Instead, the default threshold is assumed to be either constant over time (Longstaff and Schwartz, 1995) or as a monotonic increasing function of time (Black and Cox, 1976). With this approach it is not possible to consider the term structure of the borrower's debt service payments, nor is it possible to capture the dependency between different payments due at different dates.

### 3.2 The Geske Model

Geske (1977) captures the borrower's term structure of debt. His model considers several debt service payments due at different dates and provides a closed pricing formula for risky debt. The model makes the same assumptions as the Merton model except that the whole debt is due at one single date. Thus, the Merton model can be seen as a special case of the Geske model.

We explain the basic idea of the Geske model by considering a simple situation where only two debt servicing payments,  $B_1$  and  $B_2$ , have to be made (at  $T_1$  and  $T_2$ , respectively). Between the first and the second payment dates, the Merton case prevails, since no debt service payments are required after  $T_2$ . Thus, the default threshold at  $T_2$  is  $B_2$ , i.e., to avoid a default at  $T_2$ , the firm's value must be greater than or at least equal to  $B_2$ . To avoid a default at the first payment date,  $T_1$ , however, the firm's value must be greater than a threshold that is higher than the required payments,  $B_1$ , rather than equal to  $B_1$ , as explained in the following.

To derive the threshold for the first payment date  $T_1$  we consider at first the situation at the second payment date  $T_2$ . As explained above, equity can be interpreted as a hypothetical call option on the firm, if no later payments are due. The option's intrinsic value at the *second payment,  $T_2$* , date is given by  $E_{T_2} = \max(W_{T_2} - B_2; 0)$ : If the borrower defaults, the equity value is zero and the lender retains all assets. Otherwise, the shareholders receive the difference between firm's value and the debt repayment,  $W_{T_2} - B_2$ . Thus, between the first and the second payment date the Merton case prevails where the equity equals a simple call option, which gives the shareholder the right to buy the firm at the second payment date,  $T_2$ , by paying the strike price  $B_2$ . At any date  $t$  with  $T_1 \leq t \leq T_2$  This option can be valued using the Black-Scholes (1973) formula.

*At the first payment date,  $T_1$* , the shareholders are obligated to make debt service payments in the amount of  $B_1$ . In doing so, the shareholders buy the option and maintain their

claim on the firm's value. If they refuse to make the debt service payments, they lose their claim on the firm. The shareholder's right to decide whether to buy the option at  $T_1$  or not is an option, too. The right to buy a call option is called a compound (call) option.

Under which conditions the shareholders will opt to service the debt? At the first payment date  $T_1$  the shareholders will service the debt only if the value of the option is positive at  $T_{1+}$ , i.e., immediately after that payment is made ( $E_{T_{1+}} > 0$ ). Thus, the shareholders avoid a default at  $T_1$  – when the first payment is due, if the option value is higher than or at least equal to the required debt service payments:

$$E_{T_1} - B_1 \geq 0 \Rightarrow E_{T_1} \geq B_1. \quad (6)$$

To determine the threshold value at which a default occurs at  $T_1$ , the Black-Scholes formula for a call option (see Equation (5)) is inserted into (6). The threshold value,  $W_Q$ , is the value of  $W$ , which turns the resulting formula into an equation:

$$B_1 = W_Q \cdot N_1(d + \sigma_W \sqrt{T_2 - T_1}) - B_2 \cdot e^{-r_s(T_2 - T_1)} \cdot N_1(d), \quad (7)$$

where:  $d = \frac{\ln(W_Q / B_2) + (r_s - \sigma_W^2 / 2)(T_2 - T_1)}{\sigma_W \sqrt{T_2 - T_1}}$ .

If  $W_{T_1}$  is less than  $W_Q$ , the right hand side of (7) is less than  $B_1$ , i.e., the value of the option is less than the price required to buy it,  $B_1$ . In this case, the shareholders refuse to buy the option. If  $W_{T_1}$  is greater than  $W_Q$ , the value of the option exceeds its price. In this case, the shareholders buy the option, i.e., they service the debt, and no default occurs.

As the value of the equity equals a compound option at any date  $t$  before  $T_1$ , the value of the equity – which provides the option right – can be calculated using the pricing formula for a compound call option (see Geske (1979)):

$$E_t = W_t N_2(d_1 + \sigma_W \sqrt{T_1 - t}, d_2 + \sigma_W \sqrt{T_2 - t}; \rho) - B_2 e^{-r_s(T_2 - t)} N_2(d_1, d_2; \rho) - B_1 e^{-r_s(T_1 - t)} N_1(d_1) \quad (8)$$

$$\text{where: } d_1 = \frac{\ln(W_t / W_Q) + (r_s - \sigma_W^2 / 2)(T_1 - t)}{\sigma_W \sqrt{T_1 - t}},$$

$$d_2 = \frac{\ln(W_t / B_2) + (r_s - \sigma_W^2 / 2)(T_2 - t)}{\sigma_W \sqrt{T_2 - t}}, \text{ and } \rho = \sqrt{\frac{T_1 - t}{T_2 - t}}.$$

$N_1(x)$  and  $N_2(x_2, x_2, \rho)$  describe the values of the one- and two-dimensional cumulative standard normal distribution for the arguments in parentheses, respectively. The value of the default risky debt at a date  $t$  before  $T_1$  is given by:

$$F_t = W_t - W_t N_2(d_1 + \sigma_W \sqrt{T_1 - t}, d_2 + \sigma_W \sqrt{T_2 - t}; \rho) + B_2 e^{-r_s(T_2 - t)} N_2(d_1, d_2; \rho) + B_1 e^{-r_s(T_1 - t)} N_1(d_1), \quad (9)$$

### 3.3 The Default Probabilities

The default probability is the probability that the firm's value at the respective payment date will be below the threshold value. When only one debt service payment is required, the probability at date  $t$  that the borrower will default at date  $T$  is given by:

$$PoD_{t,T} = N \left( \frac{\overbrace{\ln(B_T / W_t)}^{w_{\min}} - \overbrace{(\mu_W - \sigma_W^2 / 2)(T - t)}^{\text{mean}}}{\underbrace{\sigma_W \sqrt{T - t}}_{\text{standard deviation}}} \right). \quad (10)$$

The contractual payment amount  $B_T$  is assumed to be known while the future firm value  $W_T$  is assumed to be unknown at  $t$ . Hence, the current value of  $W_t$  is used for the calculation.<sup>4</sup>

To explain Equation (10) we consider growth rates of the state variable,  $w_{t,T} = \ln(W_T / W_t)$ . The probability of default equals the probability that the realized growth rate is less than the minimum growth rate  $w_{\min} = \ln(B_T / W_t)$ , necessary to avoid a default. The growth rates are independently identically normally distributed (see Equation (2)). Hence, the

<sup>4</sup> Here we assume that we can observe the current value  $W_t$ . Later we show how this unobservable value can be estimated from observable values.

probability that the realized growth rate over  $T-t$  is less than  $w_{\min}$  – and, thus, that  $W_T$  is less than  $B_T$  – can be estimated by standardizing  $w_{\min}$  with mean and standard deviation and calculating the value of cumulative standard normal distribution for the resulting standardized growth rate as in Equation (10).

With two debt service payments due at different dates, the borrower can default at the first and at the second date. Here, three default probabilities are to distinguish: the probability of defaulting at the first date, the probability of defaulting at the second date, and the overall probability of defaulting at the first or the second date. The formula for the short-term default probability – i.e., the default probability at the first date – is similar to Equation (10) except that  $B_T$  must be replaced by the threshold value,  $W_Q$  – which determines whether a default occurs at the first payment date (see Delianedis and Geske (1998)):

$$\text{PoD}_{t,T_1} = N\left(\frac{\ln(W_Q / W_t) - (\mu_W - \sigma_W^2 / 2)(T_1 - t)}{\sigma_W \sqrt{T_1 - t}}\right). \quad (11)$$

To determine the other two default probabilities, it is helpful to remember that the probability of defaulting is the opposite of the probability of not defaulting, or the probability of surviving (PoS). Hence, the default probability can be described in terms of the probability of survival (see Delianedis and Geske (1998)):<sup>5</sup>

$$\text{PoD}_{t,T_1} = 1 - \text{PoS}_{t,T_1} = N\left(\frac{\ln(W_t / W_Q) + (\mu_W - \sigma_W^2 / 2)(T_1 - t)}{\sigma_W \sqrt{T_1 - t}}\right). \quad (12)$$

Similarly, the joint default probability is the opposite of the joint survival probability, i.e., the probability that the borrower defaults neither at  $T_1$  nor at  $T_2$ . In our model, this is the probability that the firm's value at  $T_1$  exceeds both the threshold at  $T_1$ ,  $W_Q$ , and the threshold at  $T_2$ ,  $B_2$ . This joint probability can be calculated using the two-dimensional standard normal

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<sup>5</sup> Since the standard normal distribution is symmetric with mean zero,  $1-N(x) = N(-x)$ .

distribution,  $N_2(m_1, m_2, \rho)$  (see Delianedis and Geske, 1998). Thus, the joint default probability is given by:

$$\text{PoD}_{t, T_1, T_2} = 1 - N_2(w_1, w_2; \rho), \quad (13)$$

$$\text{where: } m_1 = \frac{\ln(W_t / W_Q) + (\mu_W - \sigma_W^2 / 2)(T_1 - t)}{\sigma_W \sqrt{T_1 - t}},$$

$$m_2 = \frac{\ln(W_t / B_2) + (\mu_W - \sigma_W^2 / 2)(T_2 - t)}{\sigma_W \sqrt{T_2 - t}}, \text{ and } \rho = \sqrt{\frac{T_1 - t}{T_2 - t}}.$$

The default probability at  $T_2$  given no default at  $T_1$  is the opposite of the joint survival probability given no default at date  $T_1$  (see Delianedis and Geske (1998)):

$$P_{T_2, t} = 1 - \frac{N_2(m_1, m_2; \rho)}{N_1(m_1)}. \quad (14)$$

#### 4. Estimation of the Model's Parameters

We derive the unknown quantities, i.e. the firm's value and the parameters of its stochastic process, from the observable market values of equity using the pricing equation (8). If the actual value of the equity,  $E_t$ , and the debt service payments,  $B_i$ , the payment dates,  $T_i$ , and the risk-less interest rate,  $r_s$ , are given, the valuation equation can be used to calculate the firm's value,  $W_t$ , and volatility,  $\sigma_W$ . More precise, the pricing Equation (8) can be solved either for  $W_t$  or for  $\sigma_W$  (iteratively). However, both unobservable values must be estimated simultaneously using only one equation. This requires additional structure. We consider time series data (rather than observations from one date only) and estimate the firm value and its volatility using a maximum likelihood approach. This avoids some drawbacks of alternative approaches as explained below.

#### 4.1 Common Estimation Approaches

One approach often applied uses a second equation (see McQuown (1993) or Delianedis and Geske (1998)). Thus, two unknown quantities can be derived from two equations. For example, the following equation describes the relationship between the volatility of equity (derivative security) and the volatility of the firm, if the equity value is given by Equation (8):

$$\sigma_E = \frac{\partial E}{\partial W} \frac{W}{E} \sigma_W = N_2(d_1 + \sigma_W \sqrt{T_1 - t}, d_2 + \sigma_W \sqrt{T_2 - t}; \rho) \cdot \frac{W}{E} \sigma_W. \quad (15)$$

This relationship follows from the stochastic differential equation describing the dynamics of the equity value. This stochastic differential equation can be derived by applying Ito's lemma on a derivative of an underlying – for which the stochastic process is given by Equation (1) (see Merton (1974)):

$$dE = \left[ \frac{\partial E}{\partial W} \mu_W W + \frac{\partial E}{\partial t} + \frac{1}{2} \frac{\partial^2 E}{\partial W^2} \sigma_W^2 W^2 \right] dt + \overbrace{\frac{\partial E}{\partial W} \sigma_W W}^{\sigma_E E} dZ. \quad (16)$$

If the volatility,  $\sigma_E$ , of the derivative security could be estimated, Equations (15) and (8) could be solved for the two unknown variables,  $W_t$  and  $\sigma_W$ . In the papers mentioned above, the volatility of the derivative security (the equity value) is estimated using a time series of observed values of this security, whereby a sample estimator for the standard deviation is used:

$$\hat{\sigma}_{E,N,(\Delta t)}^* = \sqrt{\frac{1}{N} \sum_{n=0}^{N-1} (W_{t-n} - \hat{\mu}_{E,N,(\Delta t)}^*)^2}. \quad (17)$$

Here,  $\hat{\mu}_{E,N,(\Delta t)}^*$  is the common estimator for the mean of a sample with  $n$  elements. This approach implies the assumption that the volatility of the equity,  $\sigma_E$ , is constant. But from equation (15) follows that this cannot be true: since under the assumptions of the model  $\sigma_W$  is constant and the other quantities in (15) generally change over time (e.g., because the partial

derivative depends on the time to maturity and, thus, changes with declining time to maturity). So, the two-equations approach is problematic, because it's the assumptions conflict with the assumptions of the pricing model.

A second important approach applies the (extended) Kalman filter to exploit information from time series data (see, for example, Claessens and Pennacchi, 1996, or Keswani, 2000). This application requires a linear approximation of the model equation: The Kalman filter estimates unobservable quantities (e.g. the firm's value and volatility) from a time series of observable quantities (e.g. the market value of equity) – which are covered by some noise. This requires a model to connect the observable quantity (equity) with the unobservable quantity (the firm value) as described in the pricing Equation (8). The Kalman filter was originally designed for linear model equations. Applying the filter to non-linear equations (as equation (8)) requires a linear approximation, e.g. a Taylor-approximation – which may cause errors. The Kalman filter also relies on certain assumptions which have to be made in addition to the model assumptions explained in Section 3. These concern the distribution of the variables and the noise. First, the error terms – which arise if the model's equation (e.g. the pricing equation) is used to calculate the latent quantities from observable quantities – are assumed to be normally distributed and serially independent. Second, the state variable is assumed to follow an arithmetic Brownian motion. Third, the residuals of the Brownian motion are assumed to be independent from the error terms.

#### **4.2 Maximum Likelihood Estimation of the Model's Parameters**

Despite their drawbacks, the approaches explained so far provide interesting results and are widely used and well accepted in the literature. We nevertheless avoid the drawbacks of these approaches by applying the time series maximum likelihood approach proposed by Duan (1994) – who estimated the Vasicek (1977) model for the term structure of risk-free interest

rates and insurance contracts for bank deposits. This approach has rarely been applied in the literature and has never been applied to estimate the Geske (1977) model for bank assets. Recently, Duan et al. (2003) applied this approach to estimate the Merton (1974) model for corporate liabilities where the firm's value is determined by the equity value.

If the value of the volatility,  $\sigma_w$ , is known, the firm's value can be calculated by inserting the market value of equity into the pricing Equation (8). This can be done for a time series of market values of equity,  $E_{t_n}$  ( $n = 0, \dots, N$ ). We obtain a time series of the firm's value,  $W_{t_n}$  where the value of  $\sigma_w$  is arbitrary but constant over time. This follows from the assumption regarding the stochastic process (see, Equation (1)), which implies that the volatility,  $\sigma_w$ , is a constant parameter, i.e. it does not change over time.

The estimator of the volatility is chosen by maximization of a likelihood function for the observed time series. Again, we use the assumption on the stochastic process of the firm value, which implies that the growth rates of the firm's value for equidistant time intervals are independently identically normally distributed (see Equation (2)). If the growth rates of the firm's value were observable, the likelihood function which corresponds to the normal distribution would be used. Since the growth rates of the firm's value are not directly observable, but are instead derived from the observable equity values for a given volatility, the likelihood function of the observable equity values – expressed in terms of growth rates of the firm's value – is used. Assuming that the state variable follows the stochastic process described by Equation (1) and that the connection between the state variable and the equity value can be calculated using Equation (8), the log-likelihood function is given by (see Duan (1994)):

$$LLF = \sum_{n=0}^{N-1} -\ln(\sqrt{2\pi}) - \ln(\hat{\sigma}_{w(\Delta t)}) - \frac{1}{2} \left( \frac{w_{t-n} - \hat{\mu}_{w,N,(\Delta t)}}{\hat{\sigma}_{w(\Delta t)}} \right)^2 - \sum_{n=0}^{N-1} \ln \left( \frac{\partial E_{t-n}}{\partial W_{t-n}} \right) - \sum_{n=0}^{N-1} \ln W_{t-n} \quad (18)$$

In Equation (18) the firm value  $W_{t_n}$  and its observed growth rates  $w_{t_n}$ , their mean  $\mu_w$  and their standard deviation  $\sigma_w$  and the values of the partial derivative of  $E_{t_n}$  with respect to  $W_{t_n}$  are required. If pricing Equation (8) is used, the partial derivative can be calculated by:

$$\frac{\partial E_{t_n}}{\partial W_{t_n}} = N_2(d_1 + \sigma_w \sqrt{T_1 - t}, d_2 + \sigma_w \sqrt{T_2 - t}; \rho). \quad (19)$$

The standard deviation of the time series can be determined using Equation (2) and the value of the volatility parameter  $\sigma_w$ :

$$\hat{\sigma}_{w(\Delta t)} = \sigma_w \sqrt{\Delta t}. \quad (20)$$

The mean is estimated from the observed growth rates by:

$$\hat{\mu}_{w,N,(\Delta t)}^* = \frac{1}{N} \sum_{n=0}^{N-1} w_{t-n}. \quad (21)$$

To find the best estimator for the volatility, the (initially) arbitrary volatility value is iterated. For each volatility value, the corresponding time series of growth rates of the firm's value is calculated using the observed time series of the market values of equity. The necessary input data, i.e., the partial differential and the parameters, are then calculated. Finally, the value of the likelihood function (18) is determined for each volatility value. The volatility value which yields the maximum value of the likelihood function is chosen as the estimator. The corresponding time series for the firm's value,  $W_{t_n}$ , provides the estimation for the values of the firm. The estimator for the drift parameter,  $\mu_w$ , is derived from the mean estimator using Equation (2):

$$\hat{\mu}_w = \frac{\hat{\mu}_{w,N,(\Delta t)}^*}{\Delta t} + \frac{\hat{\sigma}_w^2}{2}. \quad (22)$$

Having specified the time series of the firm value and the corresponding parameters for volatility and drift, we can estimate the default probabilities as explained in Section 3.3.

## **5 Empirical Application**

This section applies the model and the estimation approach outlined in the previous sections to estimate banking crisis probabilities in Kazakhstan from 2007 to 2008. First, we estimate the individual default probabilities of all Kazakh banks for which the required input data is available. Second, we average these default probabilities to estimate the banking crisis risk for the country as a whole.

### **5.1 Input Data and Estimation Procedure**

To estimate the default probabilities of Kazakh banks, the following input data is required: the market value of equity, liabilities, their term structure, and the risk-less interest rate. This section discusses the availability of this data and the process used to specify the input parameters. The stock prices of the four Kazakh banks considered are drawn from Bloomberg. Balance sheet data is taken from Bankscope. The interest rates are obtained from Datastream. All variables are denominated in Kazakh tenge.

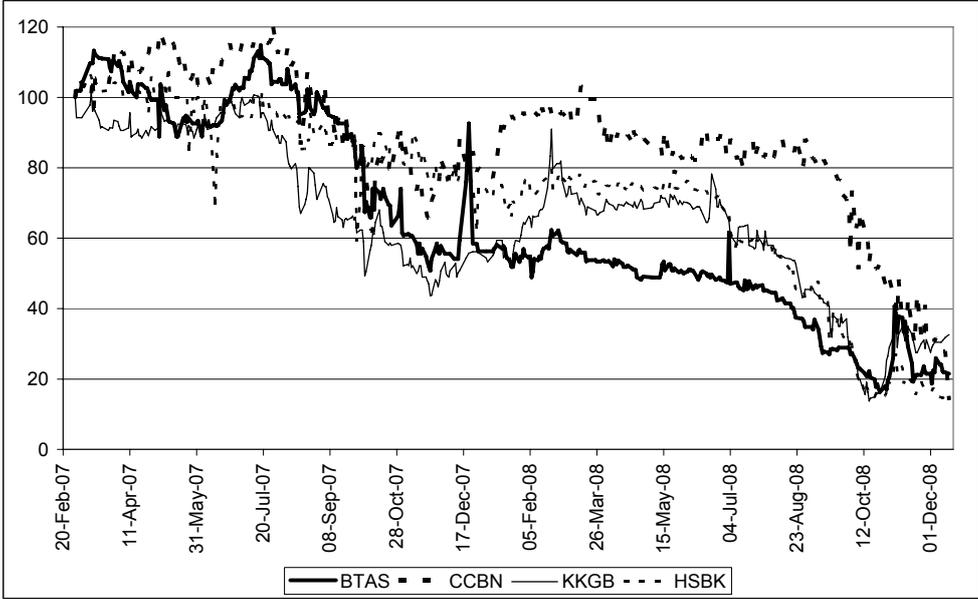
We consider the time span between March 2007 and December 2008. Data on stock prices prior to March 2007 is either unavailable or contains many missing values due to infrequent trading. From March 2007 daily updated price data is available for the four major Kazakh banks.<sup>6</sup> Other stock market-traded Kazakh banks are not considered due to insufficient liquidity in stock market trades. We calculate the market value of equity,  $E_{t_n}$ , for every observation date using the number and price of stocks.

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<sup>6</sup> Due to regulatory concerns, we do not report detailed information about the banks considered.

Figure 1 displays the development of the equity values. The value for March 1, 2007 is set to 100. For all four banks, the equity values declined considerably during the observation period, especially since the outbreak of the subprime lending crisis in the summer of 2007. At the end of 2008, equity values had declined 70-80% since March 2007 while considerable differences between banks did occur.

**Figure 1: Development of Equity Values of Kazakh Banks 2007 - 2008**



Although the Geske approach would enable us to consider every debt service payment required, such exact data on the debt service payments is not available. The data only enable us to distinguish between short- and long-term liabilities. Short-term debt includes consumer deposits and short-term bank loans. Long-term debt includes long-term borrowing from banks, subordinated debt, hybrid capital, debt securities, and derivatives. Although a more detailed consideration of debt service payments would be preferable, even distinguishing between short- and long-term debt is more advantageous than other approaches and contributes to the existing literature. Our approach improves risk assessment since it captures the influence of the term structure of liabilities on default risk. It also estimates short-term and long-term default risk, which may be influenced by different factors.

Due to different contracts and consumer portfolios, each bank's debt has its own maturity. In the application, we specify an overall or average maturity for the two categories of short-term and long-term debt. We assume that the maturity of short-term liabilities is one year.<sup>7</sup> The maturity of long-term debt is assumed to be three years on average.

Short- and long-term debt must be assigned a risk-less interest rate. Geske (1977) specifies the model with identical risk-less interest rates for all maturities. It is possible, however, to consider different rates for different time spans between the observation date and the date of maturity (see, for example, Delianedis and Geske (1998)). We identify the short-term interest rate by the yield of short-term notes provided by the National Bank of Kazakhstan. To identify the long-term interest rate, we employ data on long-term Kazakh treasury bills.

Data on interest rates is available monthly. Stock market data is provided daily. Data on the debt structure is updated only annually. In the application, we consider daily time series since we are interested in the most current assessment of default risk. Furthermore, information on the firm value is mainly reflected by stock prices, although the debt structure and interest rates also influence the estimated firm value. We consider daily time series of market values of equity inferred from stock prices. Other input data is included at the highest possible frequency. This means that the interest rate is updated every month while the debt structure is updated at the beginning of 2008. We apply the estimation approach described in the last section to the whole time series of stock market data to include the largest possible data set in the estimation. Since our approach relies on the assumption that the parameters are constant and the estimators should converge toward the true value the higher the number of observations is, we expect larger samples to obtain better estimators.

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<sup>7</sup> Papers applying the basic Merton (1974) model to forecast bank default typically assume a maturity of one year debt for the entire debt and do not distinguish between short- and long-term default probabilities.

## 5.2 Results

This section discusses the results for the estimates of banking crisis risk in Kazakhstan using our estimation approach. Since the Geske (1977) model allows us to estimate a term structure of default risk rather than a single default probability, we calculate short-term default risk and the overall default probability, i.e., the risk that the bank will default either on its short-term liabilities at  $T_1$  or on its long-term liabilities at  $T_2$ . We further estimate the (conditional) long-term risk, i.e., the probability that the bank will default on its long-term liabilities given no default at  $T_1$ . Using the estimation procedure explained in the last section we can infer an assessment of default risk for every Kazakh bank for which the required input data are available. We calculate the average of the bank-specific default probabilities to obtain an overall estimate of banking crisis risk in Kazakhstan.

Figure 2 displays the results for the Kazakh banking crisis indicator. Figures A.1 - A.4 in the Appendix display the results for the individual banks. The dots on the solid bold line represent the overall crisis probabilities at the dates on the x-axis. The dashed line displays the short-term crisis probabilities and the thin solid line the conditional long-term crisis probabilities. The default probabilities of the banking system as a whole can be interpreted as indicators of banking crisis risk and are influenced by news concerning the health of the banking sector.

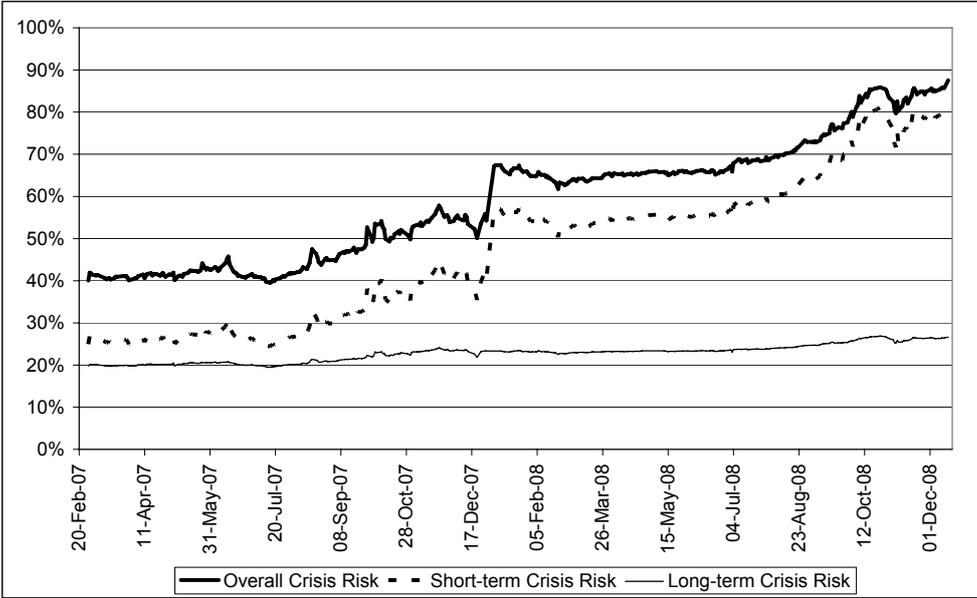
Figure (2) shows that the overall risk of a banking crisis had already reached a relatively high level – 40% – in the first half of 2007. The beginning of the international financial crisis in summer 2007 triggered a further increase of the overall crisis risk for Kazakhstan. This increase was caused by deteriorating stock prices of the banks considered (see Figure 1). Default probabilities increased over 50 percent at the end of September 2007 – just before Standard & Poor’s warned that it would downgrade Kazakh bank credit ratings.<sup>8</sup>

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<sup>8</sup> See Reuters.com, October 2, 2007: “S&P mulls downgrading Kazakhstan by Oct. 9.”

By the end of November, the overall default probability rose to almost 60%, reflecting growing uncertainties about the Kazakh banking system. In December, the overall default probability declined as the BTA Bank announced that the drain in deposits had ceased.<sup>9</sup>

**Figure 2: Indicators of Banking Crisis Risk in Kazakhstan**



At the beginning of 2008, the crisis risk jumps to around 67% as a result of new information on the liability structure – rather than stock prices (see Figure 1). This clearly shows the influence of the liability structure, i.e., the ratio of short- to long-term liabilities, on crisis risk. Our model is well suited to capture information concerning the structure of liabilities, whereas other approaches, relying, e.g., on the Merton model or on a threshold approach a la Black and Cox, can not capture such information. While the short-term risk increases, the conditional long-term risk is not heavily affected by changes in the liabilities. The increase in short-term risk also leads to an increase in the overall crisis risk.

From January to February 2008, the overall crisis probability decreased to around 63%. This reflects positive events in the Kazakh banking system, such as (Korea’s) Kookmin Bank’s announced takeover of Bank CenterCredit<sup>10</sup> or the European Bank for Reconstruction

<sup>9</sup> See Reuters.com, December 10, 2007: “Kazakh bank BTA says deposit drain over.”  
<sup>10</sup> See Reuters.com, January 28, 2008: “Kookmin to buy \$1 bln stake in Kazakh bank-report.”

and Development's (EBRD) plans to finance projects in Kazakhstan totalling USD 1 billion in 2008.<sup>11</sup>

From May to July 2008, the overall crisis probability remained fairly stable at values around 65%. In August 2008, the overall crisis probability increased to 73% as the war in Georgia raised concerns about the transit of Kazakh oil.<sup>12</sup> In September 2008, news about bank earnings in the first half of 2008 fuelled concerns about the health of the Kazakh banking system.<sup>13</sup> As credit default swaps of Kazakh banks hit record highs at the end of September, the overall crisis probability rose – reaching 78% on September 30, 2008.<sup>14</sup> In October 2008, the stock prices of Kazakh banks continued to deteriorate, leading to ever-higher overall banking crisis probabilities which increased to 86%. Severe concerns about the solvency of Kazakh banks led Moody's Investors Service to cast doubt on the future of the Kazakh banking system.<sup>15</sup> At the end of October, the Kazakh government announced a USD 5 billion bail-out plan for the banking sector<sup>16</sup>, which temporarily helped to calm the markets. It also brought the overall crisis probability down – although not below 80% – at the beginning of November. As the largest Kazakh banks agreed to the bail-out on November 11, 2008, investors were obviously disappointed by the final amount of only USD 3.47 billion as can be seen by the subsequent increase in default probabilities.<sup>17</sup>

While the overall crisis probability was heavily influenced by news about the health of the Kazakh banking system, the term structure of risk, i.e., the short-term and the long-term crisis probability, was influenced by the debt structure of the individual banks. The high

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<sup>11</sup> See Reuters.com, February 19, 2008: "EBRD to support Kazakh banks amid credit woes."

<sup>12</sup> See Reuter.com, August 11, 2008: "Caspian conflict raises energy transit worries."

<sup>13</sup> BTA's net earnings fell by 31% in the first half of 2008 (see Reuters.com, September 12, 2008: "Kazakh bank BTA's H1 earnings fall 31pct"). Halyk Savings Bank's net earnings fell by 27% in the first half of 2008 (see Reuters.com, September 15, 2008: "Kazakh Halyk Bank H1 2008 net income down").

<sup>14</sup> According to analyst Luis Eduardo Cost, the default probability of the BTA bank implied in the price of credit default swaps was around 70% at the end of September 2008 (see Reuters.com, September 30, 2008: "Kazakh banks CDS debt insurance prices hit new highs").

<sup>15</sup> See Reuters.com, October 20, 2008: "Kazakhstan to pump in \$15 bln to aid economy."

<sup>16</sup> See Reuter.com, October 28, 2008: "Kazakh govt offers banks \$5 bln in capital injection."

<sup>17</sup> See Reuter.com, November 11, 2008: "Kazakh govt, banks agree \$3.47 bln bailout package."

overall crisis probability was driven largely by the short-term risk. At the beginning of 2007, the short-term risk was approximately 25%, and the conditional long-term risk was approximately 20%. During the observation period, the long-term risk increased gradually only, reaching 25% by the end of 2008, while the short-term risk increased dramatically, reaching 80% by the end of 2008. At the end of the observation period, the short-term risk was considerably higher than the long-term risk – indicating that a banking crisis is likely to occur sooner rather than later.

## **6 Conclusion**

We apply the Geske (1977) compound option model to derive short- and long-term default probabilities for the four leading Kazakh banks based on their stock prices. Together, the probabilities provide an assessment of the overall crisis risk for the Kazakh banking sector. Market data-based risk assessment is well-suited to signal an imminent banking crisis due to its high frequency and forward-looking nature. By distinguishing between short- and long-term default risk, it is possible to determine whether short-term liquidity problems or long-term solvency problems exist.

Applying the Duan (1994) maximum likelihood approach to estimate the Geske (1977) model, we find that the overall banking crisis probability rose from 40% in February 2007 to almost 90% in December 2008. This corresponds to sharp increases in the short-term default probability from 25% to 80% and moderate increases in the long-term default probability from 20% to 25% over the same period.

These results provide useful information for Kazakh bank supervisory agencies. First, market participants expect a crisis in the Kazakh banking system in the short-term. Obviously, stock market investors do not believe that the USD 3.47 billion bail-out is sufficient to avert a crisis. This implies that the government should provide the banking sector with more financial support and should pursue a more expansionary monetary policy. The good news is that

market participants remain optimistic about Kazakh banks' growth opportunities in the long-term.

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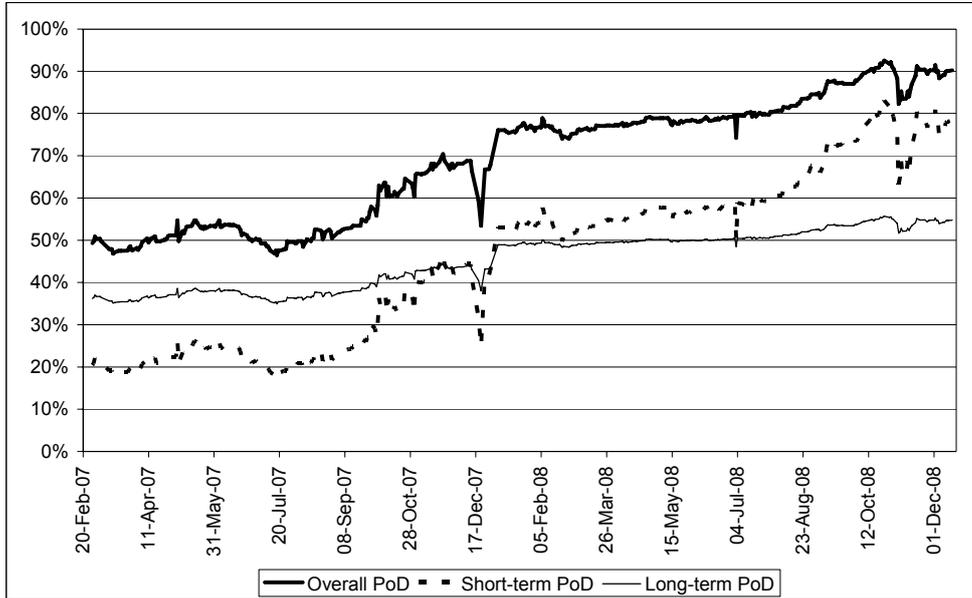
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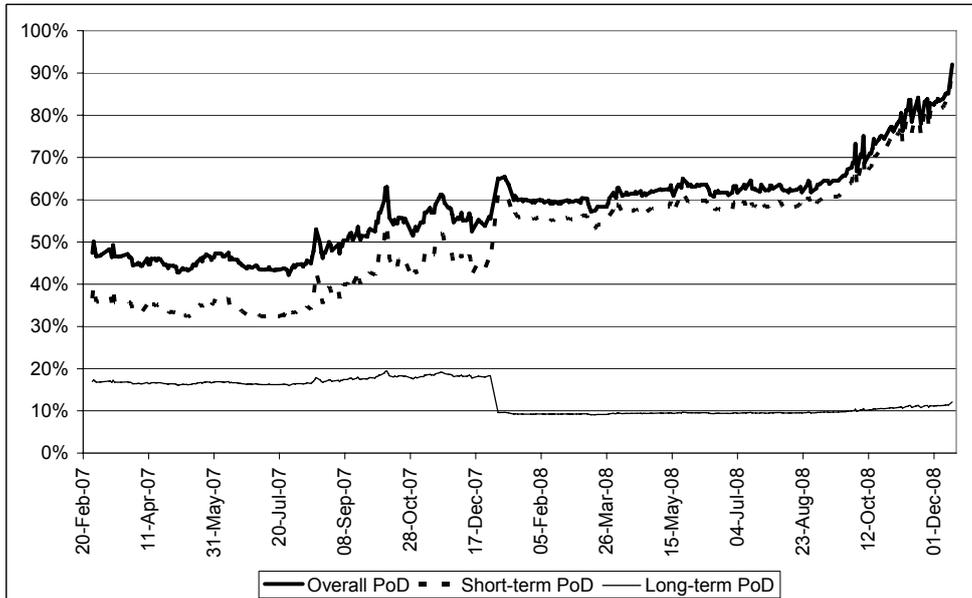
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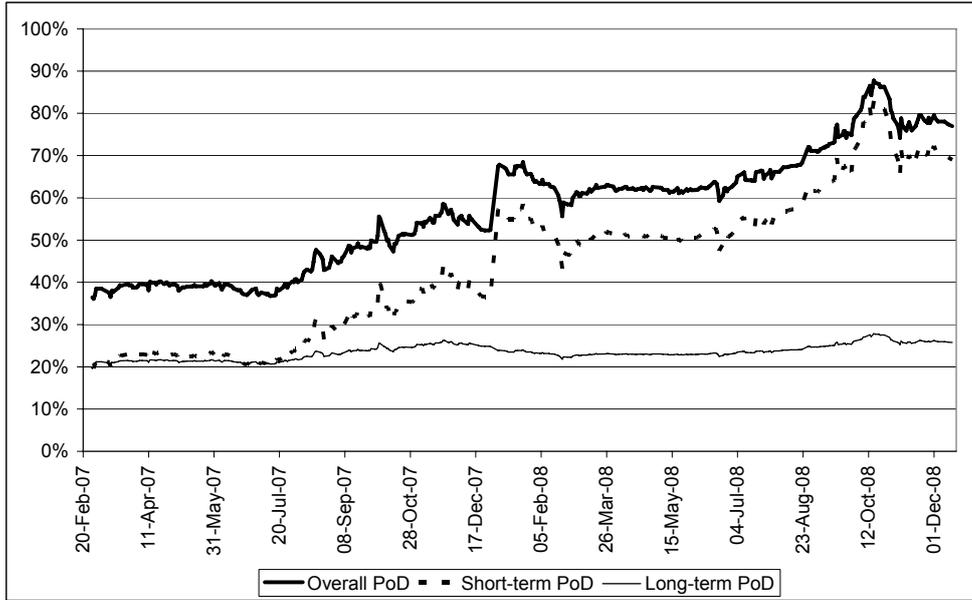
### Figure A1: Probabilities of Default (PoDs) for Bank 1



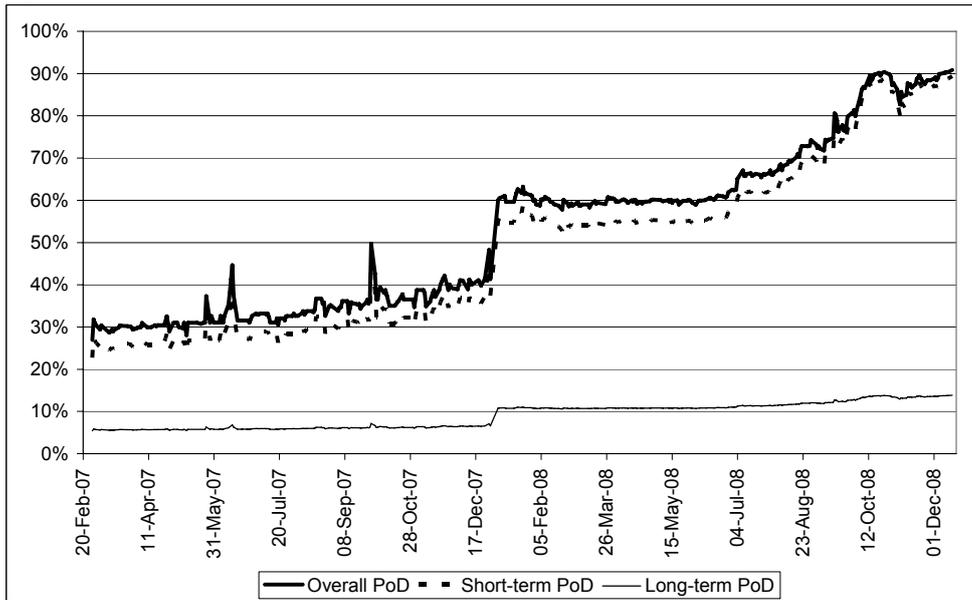
### Figure A2: Probabilities of Default (PoDs) for Bank 2



**Figure A3: Probabilities of Default (PoDs) for Bank 3**



**Figure A4: Probabilities of Default (PoDs) for Bank 4**



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