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# Stochastic frontier analysis by means of maximum likelihood and the method of moments

Andreas Behr
(University of Münster)

Sebastian Tente
(University of Münster)

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Deutsche Bundesbank, Wilhelm-Epstein-Strasse 14, 60431 Frankfurt am Main,
Postfach 10 06 02, 60006 Frankfurt am Main

Tel +49 69 9566-0
Telex within Germany 41227, telex from abroad 414431

Please address all orders in writing to: Deutsche Bundesbank,
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# Stochastic frontier analysis by means of maximum likelihood and the method of moments

Andreas Behr*
Institute for Econometrics
University of Münster

Sebastian Tente
Institute for Econometrics
University of Münster

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**Abstract**

The stochastic frontier analysis (Aigner et al., 1977, Meeusen and van de Broeck, 1977) is widely used to estimate individual efficiency scores. The basic idea lies in the introduction of an additive error term consisting of a noise and an inefficiency term. Most often the assumption of a half-normal distributed inefficiency term is applied, but other distributions are also discussed in relevant literature. The natural estimation method seems to be Maximum Likelihood (ML) estimation because of the parametric assumptions. But simulation results obtained for the half normal model indicate that a method of moments approach (MOM) (Olson et al., 1980) is superior for small and medium sized samples in combination with inefficiency not strongly dominating noise (Coelli, 1995). In this paper we provide detailed simulation results comparing the two estimation approaches for both the half-normal and the exponential approach to inefficiency.
Based on the simulation results we obtain decision rules for the choice of the superior estimation approach. Both estimation methods, ML and MOM, are applied to a sample of German commercial banks based on the Bankscope database for estimation of cost efficiency scores.

**Keywords:** stochastic frontier, Maximum Likelihood, Method of moments, Bank efficiency

**Jel-Classification:** C13, D24

*Corresponding author: Am Stadtgraben 9, 48143 Münster, Germany, andreas.behr@wiwi.uni-muenster.de, fax: 0049-251-22012, phone: 0049-251-22954

# Non-technical summary

Parametrical analyses of efficiency are based on the estimation of a frontier function (Aigner et al., 1977, Meeusen and van de Broeck, 1977). In the context of technical efficiency, it is about a production function indicating the maximum attainable output given the particular inputs. Any lower performances can be traced back to random noise – beyond the managers' control – as well as inefficiency.

The assumption of a certain inefficiency distribution as well as a normal noise distribution usually suggests the use of the maximum likelihood approach. In doing so, the log-likelihood function has to be maximised by numerical procedures.

A two-step method of moments approach turns out to be a noteworthy robust alternative: In a first step the parameters are estimated using ordinary least squares (OLS), disregarding the inefficiency term. In a second step the biased intercept is corrected by means of the moments of the OLS residuals.

This paper expands the simulation studies by Olson et al. (1980) and Coelli (1995) for the normal-half normal model and additionally compares the performance of MOM vs. ML for the normal-exponential model.

Large sample sizes and distinct sample inefficiency reveals the superiority of the ML-approach over MOM in terms of a smaller estimation error of efficiency scores. With respect to small sample sizes and/or a small ratio of inefficiency to noise, the method of moment estimation is favorable.

On the basis of the simulation results, parametrical decision rules are derived: They indicate the relative superiority of either ML- or MOM-estimation subject to sample size and inefficiency to noise-ratio.

Both ML- and MOM-techniques are applied to the estimation of a cost frontier for German commercial banks and to obtain bank-specific inefficiencies.

# Nichttechnische Zusammenfassung

Parametrischen Effizienzanalysen (*Stochastic Frontier Analysis*, SFA) liegt die Schätzung einer *Frontier*-Funktion zugrunde (Aigner et al., 1977, Meeusen and van de Broeck, 1977). Dabei handelt es sich im Untersuchungsrahmen technischer Effizienz um eine Art Produktionsfunktion, die für jedes Unternehmen den maximalen Output bei gegebenen Inputs abbildet. Abweichungen, bzw. Untererfüllung des Idealwertes können sowohl zurückgeführt werden auf zufällige Störungen jenseits der unternehmerischen Einflußspäre, sowie auf Ineffizienz.
Üblicherweise wird aufgrund der parametrischen Verteilungsannahmen die Maximum-Likelihood-Methode (ML) verwendet. Die logarithmierte Likelihoodfunktion muß dabei mittels numerischer Verfahren maximiert werden.
Ein deutlich robusteres Verfahren ist eine zweistufige Momentenschätzung (MOM): In einem ersten Schritt wird das Modell unter Vernachlässigung des Ineffizienzterms mit Hilfe der Methode der Kleinsten Quadrate (KQ) geschätzt. Im zweiten Schritt wird die Verzerrung des *Intercept* der KQ-Schätzung mit Hilfe der aus den KQ-Residuen geschätzten Momente korrigiert.
In der vorliegenden Arbeit werden die Simulationsstudien zur Vorteilhaftigkeit von Maximum-Likelihood- bzw. Momentenschätzung unter veränderlichen Rahmenbedingungen von Olson et al. (1980) und Coelli (1995) für das Normal-Halbnormal-Modell deutlich erweitert und zudem das Normal-Exponential-Modell analysiert. Es zeigt sich, dass für große Stichproben mit ausgeprägten Ineffizienzen ML dem Momentenansatz in Bezug auf die Präzision der mittleren geschätzten Effizienzen überlegen ist. Für kleine Stichproben und/oder geringe Ineffizienz in Relation zu zufälligen Störungen hingegen zeigt sich das Momentenverfahren überlegen.
Auf Basis der Simulationsergebnisse lassen sich parametrische Entscheidungsregeln für die Wahl des überlegenen Schätzverfahrens in Abhängigkeit von Stichprobengröße und Ausmaß der relativen Ineffizienz ableiten. Als Anwendung wird mit Hilfe der beiden diskutierten Verfahren eine *cost frontier* für deutsche Geschäftsbanken geschätzt und bankindividuelle Ineffizienzen ermittelt.

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# 1 Introduction

The stochastic frontier analysis is widely used to estimate individual efficiency scores. The stochastic frontier analysis (SFA) is based on the pioneering work of Aigner et al. (1977) and Meeusen and van de Broeck (1977). Kumbhakar and Lovell (2003) provide a comprehensive overview. The basic idea is the introduction of an additive error term consisting of a noise and an inefficiency term. For the error as well as the inefficiency term distributional assumptions are made. Most often the half normal assumption is applied, but the exponential, truncated normal and gamma cases are also discussed in specific literature. While the two-parameter distributions – the truncated normal and the gamma – potentially increase the flexibility of the model, in practical applications problems of identification seem to outweigh the potential gains for either distribution (Greene, 1997, p. 103 f.), (Ritter and Simar, 1997b,a).

The natural estimation method seems to be Maximum Likelihood (ML) estimation because of the parametric assumptions. But simulation results obtained for the normal-half normal model indicate that a method of moments approach (MOM) (Olson et al., 1980) is superior for small and medium sized samples in combination with inefficiency not strongly dominating noise (Coelli, 1995).

In this paper we provide detailed simulation results comparing the two estimation approaches for both the half-normal and the exponential approach to inefficiency. Furthermore, we compare the sensitivity of the estimation approaches towards misspecification. Our simulations extend those of Coelli (1995) and Olson et al. (1980) for the normal-half normal model as to sample size and comprise also the exponential model. The extensive simulation results allow formulation of rules of thumb for deciding on the estimation approach for normally and exponentially distributed inefficiency terms.

The paper is composed as follows: In section 2 we discuss the underlying concept of efficiency and the different approaches to detect inefficiency scores. In section 3 we lay down the results of our extensive simulation studies, especially the suggestions obtained for the choice of the competing estimation approaches in the form of parametrical rules of thumb. Section 4 introduces our field of application by first describing the well-established procedures to obtain efficiency scores for banking institutions in the relevant literature. Next we demonstrate an exemplifying application to a Bankscope dataset for German banks. Section 5 contains the conclusion.

# 2 Efficiency and the stochastic frontier models

In this section we briefly describe the concept of efficiency and stochastic frontier models based on the half-normal distribution for the inefficiency term, and alternatively the exponential model.

## 2.1 The concept of output-based efficiency

Farrell (1957) introduced the idea of an empirical approach to efficiency by the firm specific quotient of observed production $y_i$ to optimal production $y_i^*$. In conformity with microeconomic theory, production processes are technical relations of employed inputs to maximum attainable output. So when assuming cross sectional data for $n$ units indexed by $i$ $(i = 1, ..., n)$ using $K$ $(k = 1, ..., K)$ different inputs contained in the input vector $x_i$ to produce a single output $y_i$, we can formulate Farrell's idea of technical efficiency:

$$TE_i = \frac{y_i}{y_i^*} = \frac{y_i}{g(x_i;\beta)} \in [0,1]$$

with $g(x_i;\beta)$ as a deterministic production function. It is the aim of the stochastic frontier approach to estimate the underlying technology constituting the production possibilities of a set of firms. We allow a parametric form for the output including stochastic terms

$$y_i = g(x_i;\beta) \cdot e^{v_i} \cdot e^{-u_i}$$

which in logs is

$$\log(y_i) = \log\left(g(x_i;\beta)\right) + v_i - u_i$$

and $v_i$ is considered as a normal error $v_i \sim N(\mu_v;\sigma_v^2)$ and $u_i$ is positive representing inefficiency.

In the following we assume a simple Cobb-Douglas production function

$$g(x_i;\beta) = e^{\beta_0} \prod_{k=1}^{K} x_{ik}^{\beta_k}$$

which in logs is

$$\log\left[g(x_i;\beta)\right] = \beta_0 + \sum_{k=1}^{K} \beta_k \log(x_{ik})$$

So the output model is given by

$$\log(y_i) = \beta_0 + \sum_{k=1}^{K} \beta_k \log(x_{ik}) + v_i - u_i$$

This leads to firm-specific efficiency scores in the Cobb-Douglas case

$$TE_i = \frac{g(x_i;\beta) \cdot e^{v_i} \cdot e^{-u_i}}{g(x_i;\beta) \cdot e^{v_i}} = e^{-u_i}$$

## 2.2 The normal-half normal model

The component $u_i$ is assumed to be positive representing production inefficiency. Most often $u_i$ is assumed to be half-sided normal

$$u_i \overset{iid}{\sim} N^+(0, \sigma_u^2)$$

The density of $u$ is given as

$$f(u) = \frac{2}{\sigma_u \sqrt{2\pi}} \exp\left(-\frac{u^2}{2\sigma_u^2}\right)$$

with the moments

$$E(u) = \frac{\sqrt{2}}{\sqrt{\pi}}\sigma_u \qquad \text{and} \qquad V(u) = \left(\frac{\pi - 2}{\pi}\right)\sigma_u^2$$

### 2.2.1 The ML approach for the normal-half normal model

Assuming independence of the error terms $v$ und $u$ the joint density function results as the product of individual density functions

$$f(u, v) = f(u) \cdot f(v) = \frac{2}{\sigma_u \sigma_v 2\pi} \exp\left(-\frac{u^2}{2\sigma_u^2} - \frac{v^2}{2\sigma_v^2}\right)$$

To obtain the density of the composed error term $\varepsilon = v - u$, we first obtain the joint density $f(u, \varepsilon)$. Integration over $u$ results in

$$f(\varepsilon) = \int_0^\infty f(u, \varepsilon) du = \frac{2}{\sigma}\phi\left(\varepsilon\sigma^{-1}\right)\left[1 - \Phi\left(\varepsilon\lambda\sigma^{-1}\right)\right]$$

where $\sigma^2 = \sigma_v^2 + \sigma_u^2$ and $\lambda = \sigma_u/\sigma_v$.

The density distribution of $\varepsilon$ is asymmetric and characterized by

$$E(\varepsilon) = E(v - u) = E(-u) = -\frac{\sqrt{2}}{\sqrt{\pi}}\sigma_u$$

The variance of $\varepsilon$ is given by

$$V(\varepsilon) = \sigma_\varepsilon^2 = V(u) + V(v) = \left(\frac{\pi - 2}{\pi}\right)\sigma_u^2 + \sigma_v^2$$

The log-likelihood is given by

$$\ln L(\varepsilon|\lambda, \sigma^2) = n \ln\left(\frac{\sqrt{2}}{\sqrt{\pi}}\right) + n \ln\left(\frac{1}{\sigma}\right) + \sum_{i=1}^{n} \ln\left[1 - \Phi\left(\varepsilon_i\lambda\sigma^{-1}\right)\right] - \frac{1}{2\sigma^2}\sum_{i=1}^{n}\varepsilon_i^2$$

using $\varepsilon_i = \log y_i - \beta \log x_i$.

Having obtained the estimates $\hat{\beta}$, $\hat{\sigma}^2 = \hat{\sigma}_u^2 + \hat{\sigma}_v^2$ and $\hat{\lambda} = \hat{\sigma}_u/\hat{\sigma}_v$, the estimates of the variance components can be recovered:

$$\hat{\sigma}_v^2 = \frac{\hat{\sigma}^2}{1 + \hat{\lambda}^2} \qquad \text{and} \qquad \hat{\sigma}_u^2 = \hat{\sigma}^2 - \frac{\hat{\sigma}^2}{1 + \hat{\lambda}^2}$$

### 2.2.2 The MOM approach to the normal-half normal model

Estimating the production model using OLS results in consistent estimates of the slope parameters $\beta_1, ..., \beta_K$, but biased estimate of the intercept $\beta_0$. As the assumed $E(\varepsilon) = 0$ in OLS, the Bias is $E(\varepsilon) = -E(u) = -\sigma_u\sqrt{\frac{2}{\pi}}$. In the method of moments approach obtained OLS residuals are used to estimate the central moments $m_2$ and $m_3$ in order to shift the regression line.

$$m_2 = \frac{1}{n}\sum_{i=1}^{n}(\varepsilon_i - \bar{\varepsilon}_i)^2 \qquad \text{and} \qquad m_3 = \frac{1}{n}\sum_{i=1}^{n}(\varepsilon_i - \bar{\varepsilon}_i)^3$$

which correspond to the moment equations (Greene, 1997)

$$m_2 = \frac{\pi - 2}{\pi}\sigma_u^2 + \sigma_v^2 \qquad \text{and} \qquad m_3 = \sqrt{\frac{2}{\pi}}\left(1 - \frac{4}{\pi}\right)\sigma_u^3$$

Solving for the variance components results in

$$\sigma_u^2 = \left(\frac{m_3}{\sqrt{\frac{2}{\pi}}\left(1 - \frac{4}{\pi}\right)}\right)^{\frac{2}{3}} \qquad \text{and} \qquad \sigma_v^2 = m_2 - \frac{\pi - 2}{\pi}\sigma_u^2$$

The biased OLS estimate of the intercept $\hat{\beta}_0^{OLS}$ can be adjusted on the basis of the estimate of the standard deviation of the inefficiency term

$$\hat{\beta}_0^{MOM} = \hat{\beta}_0^{OLS} + E(\hat{u}) = \hat{\beta}_0^{OLS} + \hat{\sigma}_u\sqrt{\frac{2}{\pi}}$$

Although the MOM-Estimator is easy to calculate, even without numerical optimization, Olson et al. (1980) note two types of errors occuring when either $m_3$ is positive (Type I error) or $m_2 \leq ((\pi - 2)/\pi)\sigma_u^2$ (Type II error). A Type I error is likely to occur when $\sigma_u$ is small ($\lambda \to 0$). This immediately leads to the estimation of a negative variance $\hat{\sigma}_u$ and prevents further calculations. In the latter case, a Type II error does not prohibit the estimation of $\beta_0^{MOM}$, but causes implausible values of $\hat{\lambda} \to \pm\infty$.

### 2.2.3 Estimates of individual inefficiencies

As it is impossible to obtain for each individual firm $i$ estimates for $u_i$ and $v_i$, the inefficiency ratio $TE_i$ is obtained as the exponential conditional expectation of $-u$ given the composed error term $\varepsilon$:

$$\widehat{TE}_i = e^{E(-u_i|\varepsilon_i)}$$

The conditional density of $u$ given $\varepsilon$ is

$$f(u|\varepsilon) = \frac{f(u,\varepsilon)}{f(\varepsilon)} = \frac{1}{\sigma^*\sqrt{2\pi}}\exp\left(-\frac{(u - \mu^*)^2}{2\sigma^{*2}}\right)\left[1 - \Phi\left(-\frac{\mu^*}{\sigma^*}\right)\right]^{-1}$$

Hence, the distribution of $u$ conditional on $\varepsilon$ is $N^{+}(\mu^{*}, \sigma^{*})$, where

$$\mu^{*} = -\frac{\varepsilon\sigma_u^2}{\sigma^2} = -\varepsilon\gamma$$
$$\sigma^{*2} = \frac{\sigma_u^2\sigma_v^2}{\sigma^2} = \sigma^2\frac{\sigma_u^2(\sigma^2 - \sigma_u^2)}{\sigma^2\sigma^2} = \sigma^2\gamma(1-\gamma)$$

using $\gamma = \sigma_u^2/\sigma^2$, the fraction of the variance of the inefficiency to the total variance.
Having obtained the distribution of $u|\varepsilon$, the expected value $E(u|\varepsilon)$ can be used as point estimators for $u_i$ (Jondrow et al., 1982):[1]

$$\hat{u}_i = E(u|\varepsilon) = \left(\frac{\sigma\lambda}{1+\lambda^2}\right)\left(z_i + \frac{\phi(z_i)}{\Phi(z_i)}\right)$$
$$z_i = \frac{-\varepsilon_i\lambda}{\sigma}$$

## 2.3 The exponential model

The component $u_i$ is assumed to follow the exponential distribution with density given in alternate parameterization $\theta = 1/\sigma_u$ as

$$f(u) = \begin{cases} \frac{1}{\sigma_u}\exp\left(-\frac{u}{\sigma_u}\right) & u \geq 0 \\ 0 & u < 0 \end{cases}$$

The moments are

$$E(u) = \sigma_u \qquad \text{and} \qquad V(u) = \sigma_u^2$$

### 2.3.1 The ML approach for the normal-exponential model

Assuming independence of the error terms $v$ und $u$ the joint density results in the product of individual density functions

$$f(u,v) = f(u)f(v) = \frac{2}{\sigma_u\sigma_v 2\pi}\exp\left(-\frac{u}{\sigma_u} - \frac{v^2}{2\sigma_v^2}\right)$$

[1]Instead of obtaining firm-specific efficiencies from $\exp[-E(u|\varepsilon)]$, Battese and Coelli (1988) propose the alternative estimator:

$$\hat{TE}_i = E(\exp(-u_i)|\varepsilon_i) = \left[\Phi\left(\frac{u_i^*}{\sigma_*} - \sigma_*\right)/\Phi\left(\frac{u_i^*}{\sigma_*}\right)\right]\exp\left(\frac{\sigma_*^2}{2} - u_i*\right)$$

where $u_i^* = -(\log y_i - x\prime\beta)\sigma_u^2/\sigma^2$ and $\sigma_*^2 = \sigma_v^2\sigma_u^2/\sigma^2$. Note that in general $\exp[-E(u|\varepsilon)] \neq E(\exp(-u_i)|\varepsilon_i)$. Furthermore, both estimators are unbiased, but inconsistent estimators because $Var(\hat{u}_i) \neq 0$ for $N \rightarrow \infty$.

To obtain the density of the composed error term $\varepsilon = v - u$, we first obtain the joint density $f(u, \varepsilon)$ and integrate out $u$ from the joint density

$$f(\varepsilon) = \int_0^\infty f(u, \varepsilon) du = \frac{1}{\sigma_u} \Phi \left( -\frac{\varepsilon}{\sigma_v} - \frac{\sigma_v}{\sigma_u} \right) \exp \left( \frac{\varepsilon}{\sigma_u} + \frac{1}{2} \frac{\sigma_v^2}{\sigma_u^2} \right)$$

The density distribution of $\varepsilon$ is asymmetric and characterized by

$$E(\varepsilon) = E(v - u) = E(-u) = -\sigma_u$$

The variance of $\varepsilon$ is given by

$$V(\varepsilon) = \sigma_\varepsilon^2 = V(u) + V(v) = \sigma_u^2 + \sigma_v^2$$

Assuming independence across subjects $i$, the likelihood is the product of individual densities $f(\varepsilon)$:

$$L(\log y | \sigma_u^2, \sigma_v^2) = \frac{1}{\sigma_u^n} \exp \left( \frac{1}{2} \frac{\sigma_v^2}{\sigma_u^2} \right)^n \prod_{i=1}^n \left[ \Phi \left( -\frac{\varepsilon_i}{\sigma_v} - \frac{\sigma_v}{\sigma_u} \right) \exp \left( \frac{\varepsilon_i}{\sigma_u} \right) \right]$$

The log-likelihood is given by

$$\begin{aligned} \ln L(\log y | \beta, \sigma_u^2, \sigma_v^2) = & -n \log(\sigma_u) + n \frac{1}{2} \frac{\sigma_v^2}{\sigma_u^2} \\ & + \sum_{i=1}^n \left[ \log \Phi \left( -\frac{\log y_i - \log x_i' \beta}{\sigma_v} - \frac{\sigma_v}{\sigma_u} \right) + \frac{\log y_i - \log x_i' \beta}{\sigma_u} \right] \end{aligned}$$

### 2.3.2 The MOM approach to the normal-exponential model

Just as in the method of moments approach for the normal-half normal model discussed above, OLS residuals are used to estimate the central moments $m_2$ and $m_3$, which correspond to (Greene, 1997)

$$m_2 = \sigma_u^2 + \sigma_v^2 \qquad \text{and} \qquad m_3 = -2\sigma_u^3$$

Solving for the variance components results in

$$\sigma_u^2 = \left( -\frac{m_3}{2} \right)^{\frac{2}{3}} \qquad \text{and} \qquad \sigma_v^2 = m_2 - \sigma_u^2$$

Analogous to the half-normal case, a Type I error occurs when $m_3 < 0$, and a Type II error when $m_2 < \sigma_u^2$ (virtually impossible). The biased OLS estimate of the intercept $\hat{\beta}_0^{OLS}$ can be adjusted based on the estimate of the standard deviation (equal to mean value) of the inefficiency term

$$\hat{\beta}_0^{MOM} = \hat{\beta}_0^{OLS} + \hat{\sigma}_u$$

#### 2.3.3 Estimates of individual inefficiencies

As the conditional distribution $f(u|\varepsilon)$ is distributed as $N^+(\tilde{\mu}, \sigma_v^2)$ and given by

$$f(u|\varepsilon) = \frac{f(u,\varepsilon)}{f(\varepsilon)} = \frac{\exp[-(u-\tilde{\mu}^2)/2\sigma^2]}{\sqrt{2\pi}\sigma_v\Phi(-\tilde{\mu}/\sigma_v)}$$

with

$$\tilde{\mu} = -\varepsilon - \frac{\sigma_v^2}{\sigma_u}$$

the expected value of inefficiency $u$ given estimated residual $\varepsilon$ in the normal-exponential model can be taken as (Kumbhakar and Lovell, 2003):

$$E\left[u_i|\varepsilon_i\right] = \tilde{\mu_i} + \sigma_v \left[\frac{\phi(-\tilde{\mu_i}/\sigma_v)}{\Phi(\tilde{\mu_i}/\sigma_v)}\right]$$

# 3 Simulation

We apply the two estimation approaches outlined above to obtain estimates of individual efficiencies $\widehat{TE}_i = \exp[E(-u_i|\varepsilon_i)]$. To assess the performance of the efficiency score estimation we calculate the mean square and mean average error

$$mse = \frac{1}{n}\sum_{i=1}^{n}\left(\widehat{TE}_i - TE_i\right)^2 \qquad mae = \frac{1}{n}\sum_{i=1}^{n}|\widehat{TE}_i - TE_i|$$

between true and estimated efficiency scores.

In the simulation study we assess the relative performance of the $ML$ and $MOM$ estimators for different $n$. Additionally, we analyse the effect of $\lambda$, the relation of inefficiency variance to variance of the normal noise, on the appropriate choice of estimation method.

## 3.1 Simulation design

In this section we analyze the estimation of individual inefficiency scores by means of Monte Carlo simulations based on $m = 2000$ replications using a standard simulation setting:

$$y_i = 1 + x_1 + x_2 + v_i - u_i$$

with $v_i \sim N(0, \sigma_v)$ and $u_i \sim N(0, \sigma_u)$ or $u_i \sim Exp(\sigma_u)$; $\sigma_u = \{0.283, 0.447, 0.526, 0.566, 0.587, 0.600, 0.614, 0.620\}$ and $\sigma_v = \sqrt{0.4 - \sigma_u^2}$. The inputs $x_1, x_2$ were drawn independently from a uniform distribution on the interval $(0, 1)$. Sample sizes are $n = \{25, 50, 75, 100, 150, 250, 500, 1000\}$. The computations were performed by means of the R Environment[2].

[2]Version 2.7.1; www.r-project.org.

To assess the robustness of the half normal and the exponential models towards misspecification, we add two misspecification scenarios. In scenario M1 we (falsly) apply the half normal model on data generated under the exponential assumption. Conversely in scenario M2 we (falsly) estimate the exponential model despite inefficiency terms $u_i$ in fact drawn from the half normal distribution.

## 3.2 Simulation results in comparison

### 3.2.1 The normal-half normal case

| n | 25 | 50 | 75 | 100 | 150 | 250 | 500 | 1000 |
|---|---|---|---|---|---|---|---|---|
| $\lambda$ | method of moments | | | | | | | |
| 0.5 | 0.1801 | 0.1742 | 0.1727 | 0.1686 | 0.1637 | 0.1568 | 0.1503 | 0.1426 |
| 1.0 | 0.1964 | 0.1877 | 0.1780 | 0.1758 | 0.1685 | 0.1590 | 0.1459 | 0.1348 |
| 1.5 | 0.1879 | 0.1697 | 0.1584 | 0.1521 | 0.1390 | 0.1298 | 0.1214 | 0.1183 |
| 2.0 | 0.1752 | 0.1499 | 0.1368 | 0.1267 | 0.1188 | 0.1113 | 0.1073 | 0.1054 |
| 2.5 | 0.1649 | 0.1308 | 0.1184 | 0.1092 | 0.1028 | 0.0981 | 0.0952 | 0.0935 |
| 3.0 | 0.1531 | 0.1185 | 0.1049 | 0.0991 | 0.0931 | 0.0884 | 0.0853 | 0.0838 |
| 4.0 | 0.1375 | 0.1028 | 0.0896 | 0.0841 | 0.0787 | 0.0741 | 0.0708 | 0.0686 |
| 5.0 | 0.1253 | 0.0938 | 0.0808 | 0.0751 | 0.0697 | 0.0643 | 0.0604 | 0.0582 |
| $\lambda$ | maximum likelihood | | | | | | | |
| 0.5 | 0.2440 | 0.2114 | 0.1962 | 0.1883 | 0.1774 | 0.1627 | 0.1496 | 0.1362 |
| 1.0 | 0.2460 | 0.2123 | 0.1935 | 0.1883 | 0.1743 | 0.1584 | 0.1442 | 0.1330 |
| 1.5 | 0.2172 | 0.1809 | 0.1645 | 0.1553 | 0.1395 | 0.1294 | 0.1212 | 0.1181 |
| 2.0 | 0.1898 | 0.1535 | 0.1374 | 0.1259 | 0.1177 | 0.1101 | 0.1067 | 0.1052 |
| 2.5 | 0.1724 | 0.1299 | 0.1152 | 0.1057 | 0.1000 | 0.0961 | 0.0944 | 0.0932 |
| 3.0 | 0.1523 | 0.1141 | 0.0997 | 0.0939 | 0.0891 | 0.0857 | 0.0838 | 0.0833 |
| 4.0 | 0.1290 | 0.0925 | 0.0804 | 0.0762 | 0.0726 | 0.0701 | 0.0687 | 0.0679 |
| 5.0 | 0.1118 | 0.0788 | 0.0693 | 0.0653 | 0.0618 | 0.0598 | 0.0579 | 0.0576 |
| $\lambda$ | advantage | | | | | | | |
| 0.5 | MOM | MOM | MOM | MOM | MOM | MOM | MLE | MLE |
| 1.0 | MOM | MOM | MOM | MOM | MOM | MLE | MLE | MLE |
| 1.5 | MOM | MOM | MOM | MOM | MOM | MLE | MLE | MLE |
| 2.0 | MOM | MOM | MOM | MLE | MLE | MLE | MLE | MLE |
| 2.5 | MOM | MLE | MLE | MLE | MLE | MLE | MLE | MLE |
| 3.0 | MLE | MLE | MLE | MLE | MLE | MLE | MLE | MLE |
| 4.0 | MLE | MLE | MLE | MLE | MLE | MLE | MLE | MLE |
| 5.0 | MLE | MLE | MLE | MLE | MLE | MLE | MLE | MLE |

Table 1: Mean Average Error normal-halfnormal approach

To assess the quality of the estimation of the inefficiency terms, we calculate mean absolute differences between estimated and true efficiency scores ($mae$, corresponding mean squared deviations $mse$ are reportet in the appendix.) for all sample sizes $n = (25, 50, 75, 100, 150, 250, 500, 1000)$ and $\lambda = (0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 4.0, 5.0)$. Table 1 shows the results subdivided into three parts: The first block gives the $mae$ in the MOM-case, the second block in the ML-case and the third block indicates the superior estimation method in terms of a smaller error. Our findings confirm the results of Coelli (1995) and Olson et al. (1980). Due to improved computer capacity in the last decade, we were in a position to perform more extensive computations. Based on these extended simulation results including larger sample sizes and more simulation runs, we confirm their main findings: MOM-estimation is found strongly superior for rather small $n$ and small $\lambda$. This is in conformity with intuition, because a small $\lambda$ implies negligible inefficiency $\sigma_u$ in comparison with dominating $\sigma_v$, which comes close to the classical OLS assumption. In this case, OLS provides the best linear unbiased estimators for $\beta_1, ..., \beta_k$, while $\beta_0$ is just slightly biased. ML-estimation should be preferred for larger $n$ and larger $\lambda$. But a look at the simulation results reveals only the smallest differences in performance for larger $n$, rendering the choice of estimation methods rather unimportant.

In general, the mean average error between estimated and true efficiency scores decreases with increasing sample sizes as well as with increasing $\lambda$. We find for small sample sizes $n = 25$ and $\lambda = 0.5$ a mean average deviation about three times the size compared to the case $n = 1000$ and $\lambda = 5.0$ for the MOM approach. In case of ML-estimation, which is found considerably inferior for small $n$, mean average deviations for small $n, \lambda$-combinations are about four times the value obtained for large $n, \lambda$.

### 3.2.2 The normal-exponential case

The results of the normal-exponential model are illustrated in table 2. The findings of the normal-half normal and the normal-exponential model resemble each other. Again small sample sizes and a small variance ratio $\lambda$ strongly suggest application of MOM-estimation. But obviously, we observe more $n, \lambda$-combinations for which ML-estimation is superior.

Again, the mean average deviation between estimated and true efficiency scores decreases with increasing sample sizes as well as with increasing $\lambda$ similarly found for the half normal model. We also find for small sample sizes $n = 25$ and $\lambda = 0.5$ a mean average deviation about three times the size compared to $n = 1000$ and $\lambda = 5.0$. Just as in the normal-half normal case MOM strongly outperforms ML-estimation for very small $\lambda$, while the preferability of ML is based on very small performance differences only.

| n | 25 | 50 | 75 | 100 | 150 | 250 | 500 | 1000 |
|---|---|---|---|---|---|---|---|---|
| $\lambda$ | method of moments | | | | | | | |
| 0.5 | 0.1630 | 0.1598 | 0.1566 | 0.1535 | 0.1509 | 0.1443 | 0.1378 | 0.1324 |
| 1.0 | 0.1806 | 0.1622 | 0.1503 | 0.1453 | 0.1412 | 0.1370 | 0.1349 | 0.1338 |
| 1.5 | 0.1669 | 0.1422 | 0.1341 | 0.1291 | 0.1250 | 0.1217 | 0.1195 | 0.1180 |
| 2.0 | 0.1551 | 0.1305 | 0.1209 | 0.1169 | 0.1125 | 0.1086 | 0.1053 | 0.1032 |
| 2.5 | 0.1489 | 0.1197 | 0.1118 | 0.1075 | 0.1022 | 0.0977 | 0.0938 | 0.0913 |
| 3.0 | 0.1419 | 0.1137 | 0.1051 | 0.1002 | 0.0950 | 0.0894 | 0.0850 | 0.0820 |
| 4.0 | 0.1355 | 0.1069 | 0.0972 | 0.0909 | 0.0843 | 0.0779 | 0.0722 | 0.0683 |
| 5.0 | 0.1355 | 0.1023 | 0.0930 | 0.0857 | 0.0781 | 0.0709 | 0.0640 | 0.0594 |
| $\lambda$ | maximum likelihood | | | | | | | |
| 0.5 | 0.1888 | 0.1729 | 0.1655 | 0.1616 | 0.1557 | 0.1470 | 0.1385 | 0.1326 |
| 1.0 | 0.2006 | 0.1695 | 0.1539 | 0.1471 | 0.1410 | 0.1366 | 0.1343 | 0.1336 |
| 1.5 | 0.1729 | 0.1408 | 0.1303 | 0.1246 | 0.1210 | 0.1188 | 0.1175 | 0.1170 |
| 2.0 | 0.1534 | 0.1211 | 0.1111 | 0.1078 | 0.1047 | 0.1030 | 0.1017 | 0.1012 |
| 2.5 | 0.1376 | 0.1057 | 0.0976 | 0.0945 | 0.0920 | 0.0901 | 0.0888 | 0.0883 |
| 3.0 | 0.1234 | 0.0948 | 0.0878 | 0.0845 | 0.0820 | 0.0800 | 0.0785 | 0.0780 |
| 4.0 | 0.1076 | 0.0793 | 0.0728 | 0.0698 | 0.0666 | 0.0648 | 0.0636 | 0.0630 |
| 5.0 | 0.0980 | 0.0694 | 0.0630 | 0.0598 | 0.0566 | 0.0547 | 0.0534 | 0.0527 |
| $\lambda$ | advantage | | | | | | | |
| 0.5 | MOM | MOM | MOM | MOM | MOM | MOM | MOM | MOM |
| 1.0 | MOM | MOM | MOM | MOM | MLE | MLE | MLE | MLE |
| 1.5 | MOM | MLE | MLE | MLE | MLE | MLE | MLE | MLE |
| 2.0 | MLE | MLE | MLE | MLE | MLE | MLE | MLE | MLE |
| 2.5 | MLE | MLE | MLE | MLE | MLE | MLE | MLE | MLE |
| 3.0 | MLE | MLE | MLE | MLE | MLE | MLE | MLE | MLE |
| 4.0 | MLE | MLE | MLE | MLE | MLE | MLE | MLE | MLE |
| 5.0 | MLE | MLE | MLE | MLE | MLE | MLE | MLE | MLE |

Table 2: Mean Average Error normal-exponential approach

### 3.2.3 Misspecification scenarios

As it is an unfeasible task to determine any real inefficiency distribution across an industry, we can basically assume an underlying misspecification in every applied Stochastic Frontier Analysis. To exemplify the impacts of misspecification against the background of MOM vs. ML-estimation, we interchanged the data generating processes of the normal-exponential and normal-halfnormal case. So table 9 given in the appendix shows the $mae$ of an exponential model estimated as halfnormal (misspecification scenario $M1$), and table 10 in the appendix obtains the results of a halfnormal model estimated as exponential (misspecification $M2$).
The findings are straightforward: The exponential model $M1$ shows the predominant advantage of ML-estimation, even more clearly than in the correctly specified case. Obviously, the $mae$ of MOM-estimation improves with increasing $n$ or $\lambda$, but does not decrease in jointly larger $n, \lambda$-combinations. The indications in the halfnormal model $M2$ suggest slight advantages of MOM-estimation facing an overall lower error.

### 3.2.4 Rules of thumb

To summarize the particular advantages of ML- or MOM-estimation, we estimated multiple linear regressions based on tables 1 and 2 for the normal and exponential cases, respectively:

$$y_k = \beta_0 + \beta_{1k} \cdot \lambda + \beta_{2k} \cdot n + \mu_k$$

with

$$y = \begin{cases} 0 & \text{if MOM has smaller error} \\ 1 & \text{if MLE has smaller error} \end{cases}$$

and $\mu_k$ as normal error in all $k$ possible combinations of sample size $n$ and $\lambda$. Predicted values $\hat{y}_k > 0.5$ imply an advantage of ML- over MOM-estimation. Figure 1 illustrates the separating line between both approaches in the half normal and exponential case. The corresponding parameter estimates are shown in table 3.

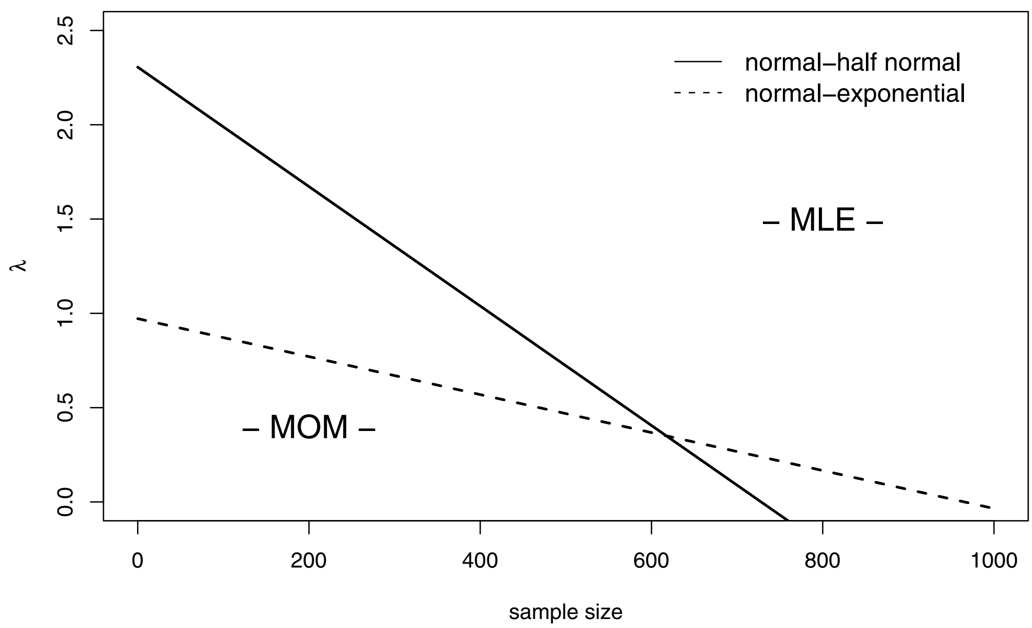


Figure 1: Rules of thumb

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| normal-half normal | | | | |
| $Intercept$ | 0.0602 | 0.0900 | 0.67 | 0.5062 |
| $\lambda$ | 0.1908 | 0.0292 | 6.53 | 0.0000 |
| $n$ | 0.0006 | 0.0001 | 4.52 | 0.0000 |
| normal-exponential | | | | |
| $Intercept$ | 0.3338 | 0.0874 | 3.82 | 0.0003 |
| $\lambda$ | 0.1710 | 0.0284 | 6.02 | 0.0000 |
| $n$ | 0.0002 | 0.0001 | 1.33 | 0.1897 |

Table 3: Parameter estimates rules of thumb

# 4 An application to German banks

## 4.1 Methodological issues

### 4.1.1 Inputs and Outputs

We review the most relevant literature covering bank efficiency estimation by means of stochastic frontier analysis. One of the most cited articles in recent literature is Mester (1996). She laid out the main features of current efficiency analyses via frontier cost functions. Resorting to cost functions instead of production functions (*technologies*) has several advantages.

Beside the methodical problems discussed above, we find another key question of empirical bank efficiency estimation in modelling inputs and outputs of the production process. As Girardone et al. (2004) state: *While the multiproduct nature of the banking firm is widely recognized, there is still no agreement as to the explicit definition and measurement of banks' inputs and outputs.* So it is common practice to operationalise bank production according to the fundamental idea of the Intermediation Approach proposed by Sealey and Lindley (1977). They had in mind a multistage production process of the financial firm, using capital, labour and material to acquire customer deposits in a first step. Lending these funds in a (virtual) second step to deficit spending units and issuing securities and other earning assets involve in general an interest profit. So financial production for intermediation purposes is about adding value to deposits.

Obviously, the use of multiple outputs does not apply to the single-output production functions described above. But, referring to Duality Theory[3], one can prove under certain regularity conditions[4] the equivalence of indirect cost functions $tc = tc(\mathbf{y}, \mathbf{c})$ and the underlying techno-

[3]Cp. Beatti and Taylor (1985), chapter 6.

[4]In particular, linear homogeneity and weak concavity in input prices if the implicit production technology is

logy $F(\mathbf{y}, \mathbf{x}) = 0$ with $tc$ total operating costs, $y'$ a vector of outputs, and $c'$ a vector of prices of the inputs $x'$. Estimating restricted stochastic cost frontiers is virtually the same as production frontiers, as the lower stochastic frontier of the 'data cloud' is simply defined by turning the sign of $u_i$, using the symmetry of $v_i$:

$$\begin{aligned} \log(tc_i) &= \log\left(g(y_i, c_i); \beta)\right) + v_i + u_i \quad |v_i \text{ symmetric} \\ \Leftrightarrow -\log(tc_i) &= -\log\left(g(y_i, c_i); \beta)\right) + v_i - u_i \end{aligned}$$

In the context of cost functions efficiency is definded in terms of cost efficiency $CE$ instead of technical efficiency $TE$. An advantage over consideration of 'pure' technologies is the possibility to evaluate scale economies, i.e. the existence of decreasing average costs in conjunction with financial firm growth.

Thus, Mester proposes a basic cost function with outputs being real estate loans, commercial/industrial and other loans as well as loans to individuals. Inputs, input-prices respectively, are prices of labour, physical capital and deposits (borrowed money). Furthermore, she included two bank quality proxies: The average volume of nonperforming loans and the average volume of equity capital. Because of a highly homogeneous dataset no further specific regional/economic distinction had to be drawn. Bos and Kool (2006) investigate small cooperative banks in the Netherlands, so they also can confine themselves to just control for a bank-specific solvency measure provided by Rabobank Netherlands added to the cost function. The authors' interpretation of the Intermediation approach imply inputs such as public relations, labour, physical capital and financial capital. Outputs are retail loans, wholesale loans, mortgages and provisions. This is a slight modification of the intermediation idea, but Lang and Welzel (1996) go even further in modelling the outputs short-term and long-term loans to non-banks, loans to banks, bonds, cash and real estate investments, fees and commissions, and revenue from sales of commodities. Obviously, some studies justify more or less distinctive alterations of the value-adding idea by Sealey and Lindley. On the other hand, authors like Altunbas et al. (2000), Perera et al. (2007), Girardone et al. (2004) are able to adopt the classical idea of banking intermediation. Table 11 in the appendix shows the different approaches at a glance.

Based on this discussion we put forward the opinion that an adequate image of banking activity in the course of a simulation study can be deduced by the basic intermediation approach, without adding any control variables.

So we accessed a Bankscope[5] cross section dataset of $n = 56$ German commercial banks with

strictly quasi-convex.

[5]Bureau van Dijk, www.bvdep.com.

positive and feasible values for all variables we are interested in. The annual statements of account refer to the end of 2005.

In line with the above-mentioned literature we assumed banks to minimize total operating costs, and set up a basic cost function with outputs $y_1$ interbank loans, $y_2$ commercial loans and $y_3$ securities. Inputs are $x_1$ fixed assets, $x_2$ number of employees and $x_3$ borrowed funds (deposits). Input prices $c_k$ can be approximated by the ratio of the costs of the inputs $x_k$ to the amount of the particular input. In the case of $x_1$, $x_2$ we obtain percentaged values, while $c_3$ is average cost per employee per year. Table 4 shows the descriptive statistics of bank size in terms of total assets $ta$, inputs $\mathbf{x}$, input prices $\mathbf{c}$ and outputs $\mathbf{y}$.

| Variable | Description | Mean | St.Dev. | Median |
|---|---|---|---|---|
| $ta$ | Total assets (BEUR) | 3894.079 | 9043.181 | 1017.800 |
| $tc$ | Total operating costs (BEUR) | 246.205 | 615.106 | 58.150 |
| $y_1$ | Interbank loans (BEUR) | 614.568 | 1374.506 | 145.650 |
| $y_2$ | Commercial loans (BEUR) | 2646.856 | 7734.673 | 374.150 |
| $y_3$ | Securities (BEUR) | 179.694 | 561.050 | 10.200 |
| $x_1$ | Fixed assets (BEUR) | 54.552 | 264.650 | 5.950 |
| $x_2$ | Employees | 645.39 | 1888.22 | 142.50 |
| $x_3$ | Borrowed funds (BEUR) | 2107.203 | 5420.554 | 486.800 |
| $c_1$ | Cost of fixed assets (% depreciation) | 0.160 | 0.089 | 0.141 |
| $c_2$ | Cost of labour (TEUR/employee) | 80.671 | 64.654 | 65.621 |
| $c_3$ | Price of funds (% interest expenses) | 0.049 | 0.058 | 0.031 |

Table 4: Descriptive statistics of inputs, outputs, prices and bank size

### 4.1.2 Shape of the cost function

As for the formal issues, one can state that most authors apply a 'regular' translog cost frontier. In most cases the translog form offers an appropriate balance between flexibility (in price and output elasticities), parameters to estimate and global fit. The exceptions among the reviewed literature are Altunbas et al. (2000), Altunbas and Chakravarty (2001), Girardone et al. (2004), and Weill (2004), using a Fourier Flexible form with additional trigonometric terms. Otherwise, Fitzpatrick and McQuinn (2005) had to restrict themselves to a simple Cobb-Douglas form due to an insufficient number of observations.

As we, too, work on a small-sized dataset ($n = 56$), we encountered severe multicollinearity in the flexible translog form. So we fall back to a simple log linear Cobb-Douglas cost function (Kumbhakar and Tsionas, 2008). To ensure linear homogeneity in input prices $tc(y, k \cdot c) =$

$k^1 \cdot tc(y,c)$ with $k > 0$, we normalize total costs and input prices by the price of labour $c_2$ (Lang and Welzel, 1996).

$$\log tc(y,c) = \beta_0 + \sum_{i=1}^{3} \beta_i \log y_i + \sum_{j=1}^{3} \gamma_j \log c_j + v + u \qquad \text{s.t.} \quad \sum_{i=j}^{3} \gamma_j = 1$$

The homogeneity-constrained cost frontier results in:

$$\log \frac{tc(y,c)}{c_2} = \beta_0 + \sum_{i=1}^{3} \beta_i \log y_i + \sum_{j=1,3} \gamma_j \log \frac{c_j}{c_2} + v + u$$

### 4.1.3 Inefficiency distribution

In the course of the simulation part, we applied the half normal and exponential assumption of inefficiency distribution. A closer look at the relevant literature reveals another 'truncated' approach (Bos and Kool, 2006, Battese et al., 2000, Fitzpatrick and McQuinn, 2005): $u_i \sim N^+(\mu, \sigma_u)$ with $\mu \geq 0$. Referring to Greene (1990), Weill (2004) is the only one using a gamma-distributed inefficiency term. But the selection of an adequate distribution of $u_i$ does not have to be overvalued, as Greene (1990) reportet extremely high rank correlations in the efficieny estimates between half normal, truncated, gamma and exponential model. So obviously there is no need to make use of two-parameter distributions.

## 4.2 Empirical evidence

To our knowledge there is not a single bank efficiency study applying the method of moments estimator we discussed. Conventionally, authors prefer Maximum Likelihood Estimation with the Jondrow et al. (1982) $\exp[-E(u|\varepsilon)]$ estimator mentioned above.

As our sample is rather small-sized, we expect the method of moments approach to deliver highly reliable efficiency scores. Table 5 shows the results of all scenarios in discussion. Our 'rules of thumb'-indicator gives rather ambivalent recommendations: Values greater than $0.5$ point at MLE application. But especially in the normal-exponential case we are facing values $\approx 0.5$. So we should reckon with equivalent results in both MOM and ML-estimation. In fact, the correlation table 6 confirms a $\rho(\widehat{CE}_{MOM}, \widehat{CE}_{MLE}) \approx 0.99$. Mean efficiencies $\frac{1}{n}\sum_i \widehat{CE}_i$ differ only between varying inefficiency distribution assumptions. By the way, just like Greene (1990), we can still report extemely high correlations $\rho > 0.95$ between the half normal and exponential approach to inefficiency.

We observe slight differences in the estimated output and price elasticities (note that $\gamma_2 = 1 - \gamma_1 - \gamma_3$). The overall scale economies are calculated as $\epsilon_c = \left(\sum_i \frac{\partial \log tc(y,c)}{\partial \log \beta_i}\right)^{-1}$. As $\epsilon_c > 1$ in all cases, the results hint at increasing returns to scale in the German banking industry.

| | normal-half normal | | normal-exponential | |
|---|---|---|---|---|
| | MLE | MOM | MLE | MOM |
| $Intercept$ | 0.760 | 1.526 | 2.096 | 1.819 |
| $y_1$ | 0.107 | 0.157 | 0.117 | 0.157 |
| $y_2$ | 0.543 | 0.505 | 0.552 | 0.505 |
| $y_3$ | 0.055 | 0.060 | 0.050 | 0.060 |
| $c_1$ | 0.208 | 0.233 | 0.254 | 0.233 |
| $c_3$ | 0.289 | 0.388 | 0.418 | 0.388 |
| $\lambda$ | 2.696 | 2.804 | 1.158 | 0.933 |
| $\sigma_v$ | 0.326 | 0.325 | 0.416 | 0.467 |
| $\sigma_u$ | 0.880 | 0.913 | 0.481 | 0.436 |
| mean $\widehat{CE}$ | 0.537 | 0.531 | 0.655 | 0.673 |
| rule of thumb | 0.608 | 0.629 | 0.541 | 0.503 |

Table 5: Estimation results, all cases

| | nhn-mle | nhn-mom | exp-mle | exp-mom |
|---|---|---|---|---|
| nhn-mle | 1.000 | | | |
| nhn-mom | 0.987 | 1.000 | | |
| exp-mle | 0.957 | 0.952 | 1.000 | |
| exp-mom | 0.951 | 0.952 | 0.988 | 1.000 |

Table 6: Correlation table, cost efficiencies

# 5 Conclusions

We put forward the MOM-approach to stochastic frontiers in bank efficiency analysis. An extensive simulation study confirmed the findings of Coelli (1995) and Olson et al. (1980): Rules of thumb suggest that the MOM-estimation of parametrical frontiers assuming a half normal inefficiency distribution can be favourable in terms of $mse(\widehat{TE}, TE)$ and $mae(\widehat{TE}, TE)$ if the sample size is medium scale ($\leq 700$ observations) and inefficiency does not strongly dominate noise ($\lambda < 2$), i.e. the bias of $\beta_0^{OLS}$ is small.

So we propose that method of moment estimation should be considered an alternative to maximum likelihood estimation. We do so especially in cases focusing on a small number of homogeneous banks (Fitzpatrick and McQuinn, 2005). Applying MOM-estimation additionally to the ML-procedure even in larger samples could shed new light on the significance of the findings.

Another practical advantage of MOM-estimation is obvious: Whenever Newton-like numerical optimization is unavailable or fails due to awkward data structure, MOM provides a robust and easy to implement loophole. A simple two-step procedure (OLS-fitting and bias correction based on estimated residuals) is available within every statistical environment.

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# 6 Appendix

## 6.1 Derivatives of the log-likelihood: half-normal

$$\ln L(y|\beta,\lambda,\sigma^2) = n\ln\left(\frac{\sqrt{2}}{\sqrt{\pi}}\right) + n\ln\left(\sigma^{-1}\right) + \sum_{i=1}^{n}\ln\left[1-\Phi\left(\left[y_i - x_i'\beta\right]\lambda\sigma^{-1}\right)\right] - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(y_i - x_i'\beta)^2$$

The derivatives are given by

$$\frac{\partial \ln L}{\partial \beta} = -\frac{n}{\sigma^2}\sum_{i=1}^{n}(y_i - x_i'\beta)x_i + \frac{\lambda}{\sigma}\sum_{i=1}^{n}\frac{\phi_i^*}{(1-F_i^*)}x_i$$

$$\begin{aligned}\frac{\partial \ln L}{\partial \sigma^2} &= -\frac{n}{2\sigma^2} + \frac{1}{2\sigma^4}\sum_{i=1}^{n}(y_i - x_i'\beta)^2 \\ &+ \frac{1}{2\sigma^3}\sum_{i=1}^{n}\frac{\phi_i^*}{(1-\Phi_i^*)}(y_i - x_i'\beta)\end{aligned}$$

$$\frac{\partial \ln L}{\partial \lambda} = -\frac{1}{\sigma} + \frac{1}{2\sigma^4}\sum_{i=1}^{n}\frac{\phi_i^*}{(1-\Phi_i^*)}(y_i - x_i'\beta)$$

where

$$\begin{aligned}\phi_i^* &= \phi([\ln y_i - x_i'\beta]\,\lambda\sigma^{-1}) \\ \Phi_i^* &= \Phi([\ln y_i - x_i'\beta]\,\lambda\sigma^{-1})\end{aligned}$$

## 6.2 Derivatives of the log-likelihood: exponential

$$\ln L(y|\beta,\sigma_u^2,\sigma_v^2) = -n\ln(\sigma_u) + n\frac{1}{2}\frac{\sigma_v^2}{\sigma_u^2} + \sum_{i=1}^{n}\ln\Phi\left(-\frac{1}{\sigma_v}(y_i - x_i'\beta) - \frac{\sigma_v}{\sigma_u}\right) + \sum_{i=1}^{n}\frac{1}{\sigma_u}(y_i - x_i'\beta)$$

$$\frac{d}{d\beta}\sum_{i=1}^{n}\ln\Phi\left(-\frac{1}{\sigma_v}(y_i - x_i'\beta) - \frac{\sigma_v}{\sigma_u}\right)$$

$$\frac{\partial \ln L}{\partial \beta} = \sum_{i=1}^{n}\frac{x_i'/\sigma_v\phi\left(-\frac{1}{\sigma_v}(y_i - x_i'\beta) - \frac{\sigma_v}{\sigma_u}\right)}{\Phi\left(-\frac{1}{\sigma_v}(y_i - x_i'\beta) - \frac{\sigma_v}{\sigma_u}\right)} - \sum_{i=1}^{n}x_i'/\sigma_u$$

$$\frac{\partial \ln L}{\partial \sigma_u} = -\frac{n}{\sigma_u} - n\frac{\sigma_v^2}{\sigma_u^3} + \sum_{i=1}^{n} \frac{\sigma_v \phi\left(-\frac{1}{\sigma_v}(y_i - x_i'\beta) - \frac{\sigma_v}{\sigma_u}\right)}{\sigma_u^2 \Phi\left(-\frac{1}{\sigma_v}(y_i - x_i'\beta) - \frac{\sigma_v}{\sigma_u}\right)} - \sum_{i=1}^{n} \frac{1}{\sigma_u^2}(y_i - x_i'\beta)$$

$$\frac{\partial \ln L}{\partial \sigma_v} = n\frac{\sigma_v}{\sigma_u^2} + \sum_{i=1}^{n} \left(\sigma_v^{-2}(y_i - x_i'\beta) - \frac{1}{\sigma_u}\right) \frac{\phi\left(-\frac{1}{\sigma_v}(y_i - x_i'\beta) - \frac{\sigma_v}{\sigma_u}\right)}{\Phi\left(-\frac{1}{\sigma_v}(y_i - x_i'\beta) - \frac{\sigma_v}{\sigma_u}\right)}$$

## 6.3 Tables

| n | 25 | 50 | 75 | 100 | 150 | 250 | 500 | 1000 |
|---|---|---|---|---|---|---|---|---|
| $\lambda$ | method of moments | | | | | | | |
| 0.5 | 0.0516 | 0.0475 | 0.0464 | 0.0441 | 0.0416 | 0.0383 | 0.0351 | 0.0318 |
| 1.0 | 0.0612 | 0.0565 | 0.0513 | 0.0502 | 0.0464 | 0.0412 | 0.0343 | 0.0284 |
| 1.5 | 0.0588 | 0.0485 | 0.0425 | 0.0390 | 0.0319 | 0.0269 | 0.0226 | 0.0212 |
| 2.0 | 0.0531 | 0.0391 | 0.0319 | 0.0269 | 0.0228 | 0.0192 | 0.0177 | 0.0171 |
| 2.5 | 0.0480 | 0.0299 | 0.0235 | 0.0192 | 0.0166 | 0.0150 | 0.0142 | 0.0137 |
| 3.0 | 0.0424 | 0.0246 | 0.0181 | 0.0158 | 0.0136 | 0.0123 | 0.0115 | 0.0111 |
| 4.0 | 0.0345 | 0.0182 | 0.0131 | 0.0112 | 0.0098 | 0.0087 | 0.0080 | 0.0076 |
| 5.0 | 0.0291 | 0.0150 | 0.0106 | 0.0090 | 0.0077 | 0.0066 | 0.0059 | 0.0055 |
| $\lambda$ | maximum likelihood | | | | | | | |
| 0.5 | 0.1370 | 0.0928 | 0.0746 | 0.0639 | 0.0546 | 0.0455 | 0.0359 | 0.0294 |
| 1.0 | 0.1191 | 0.0800 | 0.0667 | 0.0614 | 0.0520 | 0.0414 | 0.0333 | 0.0274 |
| 1.5 | 0.0891 | 0.0578 | 0.0471 | 0.0417 | 0.0323 | 0.0268 | 0.0225 | 0.0211 |
| 2.0 | 0.0654 | 0.0426 | 0.0331 | 0.0269 | 0.0226 | 0.0188 | 0.0176 | 0.0171 |
| 2.5 | 0.0571 | 0.0308 | 0.0229 | 0.0184 | 0.0159 | 0.0145 | 0.0140 | 0.0136 |
| 3.0 | 0.0464 | 0.0240 | 0.0171 | 0.0145 | 0.0127 | 0.0116 | 0.0111 | 0.0110 |
| 4.0 | 0.0348 | 0.0159 | 0.0110 | 0.0095 | 0.0085 | 0.0079 | 0.0076 | 0.0075 |
| 5.0 | 0.0274 | 0.0118 | 0.0082 | 0.0071 | 0.0062 | 0.0059 | 0.0055 | 0.0054 |
| $\lambda$ | advantage | | | | | | | |
| 0.5 | MOM | MOM | MOM | MOM | MOM | MOM | MOM | MLE |
| 1.0 | MOM | MOM | MOM | MOM | MOM | MOM | MLE | MLE |
| 1.5 | MOM | MOM | MOM | MOM | MOM | MLE | MLE | MLE |
| 2.0 | MOM | MOM | MOM | MOM | MLE | MLE | MLE | MOM |
| 2.5 | MOM | MOM | MLE | MLE | MLE | MLE | MLE | MLE |
| 3.0 | MOM | MLE | MLE | MLE | MLE | MLE | MLE | MLE |
| 4.0 | MOM | MLE | MLE | MLE | MLE | MLE | MLE | MLE |
| 5.0 | MLE | MLE | MLE | MLE | MLE | MLE | MLE | MLE |

Table 7: Mean Square Error normal-halfnormal approach

| n | 25 | 50 | 75 | 100 | 150 | 250 | 500 | 1000 |
|---|---|---|---|---|---|---|---|---|
| $\lambda$ | method of moments | | | | | | | |
| 0.5 | 0.0461 | 0.0441 | 0.0421 | 0.0403 | 0.0387 | 0.0348 | 0.0307 | 0.0274 |
| 1.0 | 0.0566 | 0.0443 | 0.0368 | 0.0339 | 0.0313 | 0.0288 | 0.0278 | 0.0272 |
| 1.5 | 0.0478 | 0.0331 | 0.0287 | 0.0262 | 0.0245 | 0.0232 | 0.0224 | 0.0218 |
| 2.0 | 0.0407 | 0.0274 | 0.0233 | 0.0216 | 0.0201 | 0.0188 | 0.0178 | 0.0172 |
| 2.5 | 0.0372 | 0.0229 | 0.0198 | 0.0184 | 0.0167 | 0.0154 | 0.0143 | 0.0136 |
| 3.0 | 0.0338 | 0.0207 | 0.0176 | 0.0160 | 0.0146 | 0.0129 | 0.0119 | 0.0111 |
| 4.0 | 0.0308 | 0.0182 | 0.0150 | 0.0132 | 0.0115 | 0.0099 | 0.0087 | 0.0078 |
| 5.0 | 0.0305 | 0.0168 | 0.0138 | 0.0116 | 0.0098 | 0.0082 | 0.0068 | 0.0059 |
| $\lambda$ | maximum likelihood | | | | | | | |
| 0.5 | 0.0612 | 0.0515 | 0.0469 | 0.0446 | 0.0413 | 0.0363 | 0.0311 | 0.0275 |
| 1.0 | 0.0692 | 0.0490 | 0.0393 | 0.0353 | 0.0314 | 0.0288 | 0.0275 | 0.0272 |
| 1.5 | 0.0535 | 0.0339 | 0.0279 | 0.0249 | 0.0232 | 0.0222 | 0.0217 | 0.0215 |
| 2.0 | 0.0434 | 0.0250 | 0.0204 | 0.0189 | 0.0177 | 0.0171 | 0.0167 | 0.0165 |
| 2.5 | 0.0358 | 0.0193 | 0.0159 | 0.0148 | 0.0139 | 0.0134 | 0.0130 | 0.0128 |
| 3.0 | 0.0297 | 0.0156 | 0.0130 | 0.0120 | 0.0113 | 0.0107 | 0.0103 | 0.0102 |
| 4.0 | 0.0239 | 0.0111 | 0.0091 | 0.0084 | 0.0076 | 0.0072 | 0.0069 | 0.0068 |
| 5.0 | 0.0206 | 0.0088 | 0.0069 | 0.0062 | 0.0055 | 0.0052 | 0.0049 | 0.0048 |
| $\lambda$ | advantage | | | | | | | |
| 0.5 | MOM | MOM | MOM | MOM | MOM | MOM | MOM | MOM |
| 1.0 | MOM | MOM | MOM | MOM | MOM | MOM | MLE | MOM |
| 1.5 | MOM | MOM | MLE | MLE | MLE | MLE | MLE | MLE |
| 2.0 | MOM | MLE | MLE | MLE | MLE | MLE | MLE | MLE |
| 2.5 | MLE | MLE | MLE | MLE | MLE | MLE | MLE | MLE |
| 3.0 | MLE | MLE | MLE | MLE | MLE | MLE | MLE | MLE |
| 4.0 | MLE | MLE | MLE | MLE | MLE | MLE | MLE | MLE |
| 5.0 | MLE | MLE | MLE | MLE | MLE | MLE | MLE | MLE |

Table 8: Mean Square Error normal-exponential approach

| n | 25 | 50 | 75 | 100 | 150 | 250 | 500 | 1000 |
|---|---|---|---|---|---|---|---|---|
| $\lambda$ | method of moments | | | | | | | |
| 0.5 | 0.2035 | 0.2022 | 0.1992 | 0.1963 | 0.1933 | 0.1874 | 0.1850 | 0.1801 |
| 1.0 | 0.2100 | 0.2013 | 0.1931 | 0.1923 | 0.1938 | 0.1934 | 0.1965 | 0.1969 |
| 1.5 | 0.1896 | 0.1817 | 0.1841 | 0.1845 | 0.1895 | 0.1931 | 0.1967 | 0.1982 |
| 2.0 | 0.1748 | 0.1745 | 0.1775 | 0.1787 | 0.1858 | 0.1910 | 0.1953 | 0.1966 |
| 2.5 | 0.1680 | 0.1685 | 0.1734 | 0.1775 | 0.1830 | 0.1871 | 0.1935 | 0.1945 |
| 3.0 | 0.1639 | 0.1670 | 0.1724 | 0.1746 | 0.1821 | 0.1875 | 0.1945 | 0.1975 |
| 4.0 | 0.1554 | 0.1622 | 0.1665 | 0.1744 | 0.1829 | 0.1894 | 0.1969 | 0.1995 |
| 5.0 | 0.1532 | 0.1592 | 0.1660 | 0.1703 | 0.1816 | 0.1906 | 0.1980 | 0.2011 |
| $\lambda$ | maximum likelihood | | | | | | | |
| 0.5 | 0.2537 | 0.2280 | 0.2139 | 0.2078 | 0.1934 | 0.1846 | 0.1785 | 0.1756 |
| 1.0 | 0.2291 | 0.1960 | 0.1840 | 0.1800 | 0.1763 | 0.1736 | 0.1738 | 0.1736 |
| 1.5 | 0.1834 | 0.1614 | 0.1555 | 0.1536 | 0.1513 | 0.1508 | 0.1503 | 0.1502 |
| 2.0 | 0.1533 | 0.1381 | 0.1343 | 0.1317 | 0.1309 | 0.1303 | 0.1297 | 0.1295 |
| 2.5 | 0.1321 | 0.1197 | 0.1170 | 0.1165 | 0.1142 | 0.1136 | 0.1128 | 0.1124 |
| 3.0 | 0.1176 | 0.1058 | 0.1049 | 0.1032 | 0.1018 | 0.1005 | 0.0997 | 0.0995 |
| 4.0 | 0.0964 | 0.0861 | 0.0841 | 0.0841 | 0.0825 | 0.0815 | 0.0808 | 0.0804 |
| 5.0 | 0.0858 | 0.0724 | 0.0713 | 0.0707 | 0.0700 | 0.0689 | 0.0679 | 0.0674 |
| $\lambda$ | advantage | | | | | | | |
| 0.5 | MOM | MOM | MOM | MOM | MOM | MLE | MLE | MLE |
| 1.0 | MOM | MLE | MLE | MLE | MLE | MLE | MLE | MLE |
| 1.5 | MLE | MLE | MLE | MLE | MLE | MLE | MLE | MLE |
| 2.0 | MLE | MLE | MLE | MLE | MLE | MLE | MLE | MLE |
| 2.5 | MLE | MLE | MLE | MLE | MLE | MLE | MLE | MLE |
| 3.0 | MLE | MLE | MLE | MLE | MLE | MLE | MLE | MLE |
| 4.0 | MLE | MLE | MLE | MLE | MLE | MLE | MLE | MLE |
| 5.0 | MLE | MLE | MLE | MLE | MLE | MLE | MLE | MLE |

Table 9: Mean Average Error misspecification 1

| n | 25 | 50 | 75 | 100 | 150 | 250 | 500 | 1000 |
|---|---|---|---|---|---|---|---|---|
| $\lambda$ | method of moments | | | | | | | |
| 0.5 | 0.1451 | 0.1409 | 0.1382 | 0.1377 | 0.1351 | 0.1331 | 0.1300 | 0.1281 |
| 1.0 | 0.1884 | 0.1813 | 0.1742 | 0.1728 | 0.1690 | 0.1638 | 0.1559 | 0.1479 |
| 1.5 | 0.1964 | 0.1823 | 0.1739 | 0.1698 | 0.1608 | 0.1538 | 0.1474 | 0.1450 |
| 2.0 | 0.1962 | 0.1758 | 0.1664 | 0.1587 | 0.1525 | 0.1466 | 0.1433 | 0.1412 |
| 2.5 | 0.1935 | 0.1677 | 0.1590 | 0.1510 | 0.1454 | 0.1423 | 0.1392 | 0.1379 |
| 3.0 | 0.1902 | 0.1628 | 0.1514 | 0.1472 | 0.1434 | 0.1391 | 0.1361 | 0.1352 |
| 4.0 | 0.1829 | 0.1571 | 0.1475 | 0.1430 | 0.1389 | 0.1355 | 0.1327 | 0.1320 |
| 5.0 | 0.1773 | 0.1537 | 0.1445 | 0.1419 | 0.1367 | 0.1342 | 0.1316 | 0.1304 |
| $\lambda$ | maximum likelihood | | | | | | | |
| 0.5 | 0.1688 | 0.1548 | 0.1488 | 0.1469 | 0.1425 | 0.1392 | 0.1351 | 0.1322 |
| 1.0 | 0.2117 | 0.1959 | 0.1865 | 0.1837 | 0.1780 | 0.1706 | 0.1612 | 0.1510 |
| 1.5 | 0.2124 | 0.1921 | 0.1798 | 0.1749 | 0.1644 | 0.1554 | 0.1472 | 0.1440 |
| 2.0 | 0.2044 | 0.1781 | 0.1666 | 0.1554 | 0.1485 | 0.1410 | 0.1363 | 0.1338 |
| 2.5 | 0.1954 | 0.1612 | 0.1511 | 0.1408 | 0.1322 | 0.1289 | 0.1252 | 0.1235 |
| 3.0 | 0.1838 | 0.1489 | 0.1348 | 0.1284 | 0.1234 | 0.1180 | 0.1152 | 0.1140 |
| 4.0 | 0.1669 | 0.1312 | 0.1168 | 0.1107 | 0.1056 | 0.1022 | 0.0994 | 0.0982 |
| 5.0 | 0.1525 | 0.1168 | 0.1036 | 0.0982 | 0.0932 | 0.0903 | 0.0877 | 0.0862 |
| $\lambda$ | advantage | | | | | | | |
| 0.5 | MOM | MOM | MOM | MOM | MOM | MOM | MOM | MOM |
| 1.0 | MOM | MOM | MOM | MOM | MOM | MOM | MOM | MOM |
| 1.5 | MOM | MOM | MOM | MOM | MOM | MOM | MLE | MLE |
| 2.0 | MOM | MOM | MOM | MLE | MLE | MLE | MLE | MLE |
| 2.5 | MOM | MLE | MLE | MLE | MLE | MLE | MLE | MLE |
| 3.0 | MLE | MLE | MLE | MLE | MLE | MLE | MLE | MLE |
| 4.0 | MLE | MLE | MLE | MLE | MLE | MLE | MLE | MLE |
| 5.0 | MLE | MLE | MLE | MLE | MLE | MLE | MLE | MLE |

Table 10: Mean Average Error misspecification 2

| Inputs | Outputs | Reference |
|---|---|---|
| Altunbas et al. (2000) | | |
| labour, funds, physical capital | total loans, securities, off-balance sheet items (contingent liabilities, acceptances, guarantees | Intermediation (varied) |
| Altunbas and Chakravarty (2001) | | |
| labour, total funds, physical capital | all types of loans, total aggregate securities, off-balance sheet activities | Intermediation (varied) |
| Battese et al. (2000) | | |
| public loans, guarantees, deposits, number of branches, value of inventories | costs of labour use | Input-requirement model |
| Bos and Kool (2006) | | |
| public relations, labour, housing, physical capital | retail loans, wholesale loans, mortgages, provisions | Intermediation (varied) |
| Ferrier and Lovell (1990) | | |
| employees, occupancy costs, materials | demand deposit accounts, time deposit accounts, real estate loans, real estate loans, installment loans, commercial loans | Production |
| Fitzpatrick and McQuinn (2005) | | |
| labour, physical capital, financial capital | consumer/commercial/other loans, non-interest revenue | Intermediation (varied) |
| Girardone et al. (2004) | | |
| employees, total customer deposits, total fixed assets | total customer loans, other earning assets | Intermediation |
| Lang and Welzel (1996) | | |
| employees, fixed assets, deposits | short-term and long-term loans to non-banks, loans to banks, bonds/cash/real estate investments, fees and commissions, revenue from sales of commodities | Intermediation (varied) |
| Mester (1996) | | |
| labour, physical capital, deposits | real estate loans, commercial/industrial/government/... loans, loans to individuals | Intermediation |
| Perera et al. (2007) | | |
| funds, labour, capital | net total loans, other earning assets | Intermediation |
| Weill (2004) | | |
| labour, physical capital, borrowed funds | loans, investment assets | Intermediation |

Table 11: Input and output-modelling in selected bank efficiency studies

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