

## **Asset correlations and credit portfolio risk – an empirical analysis**

Klaus Düllmann

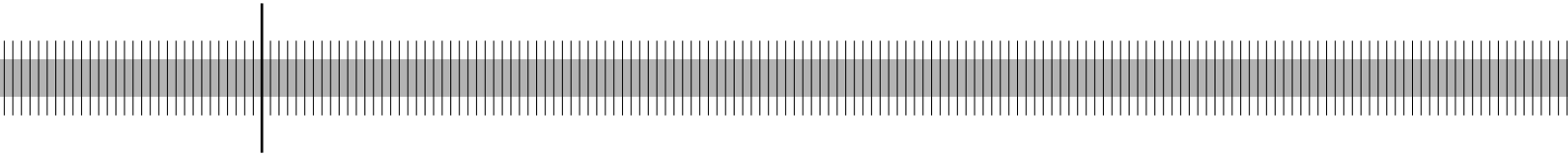
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# Asset Correlations and Credit Portfolio Risk – An Empirical Analysis

## Abstract

In credit risk modelling, the correlation of unobservable asset returns is a crucial component for the measurement of portfolio risk. In this paper, we estimate asset correlations from monthly time series of Moody's KMV asset values for around 2,000 European firms from 1996 to 2004. We compare correlation and value-at-risk (VaR) estimates in a one-factor or market model and a multi-factor or sector model. Our main finding is a complex interaction of credit risk correlations and default probabilities affecting total credit portfolio risk. Differentiation between industry sectors when using the sector model instead of the market model has only a secondary effect on credit portfolio risk, at least for the underlying credit portfolio. Averaging firm-dependent asset correlations on a sector level can, however, cause a substantial underestimation of the VaR in a portfolio with heterogeneous borrower size. This result holds for the market as well as the sector model. Furthermore, the VaR of the IRB model is more stable over time than the VaR of the market model and the sector model, while its distance from the other two models fluctuates over time.

**Keywords:** Asset correlations, sector concentration, credit portfolio risk

**JEL Classification:** G 21, C 15

## Non-Technical Summary

The correlations between two firms' asset-value returns, commonly referred to as *asset correlation*, are a key factor in measuring the credit risk of loan portfolios. Since asset values are not directly observable, we employ time series of asset values of European firms which are based on the Moody's KMV model. A descriptive analysis of these asset correlations and correlations with industry sector indices is a first contribution of this paper. We observe a considerable fluctuation of asset correlations which suggests further research on their stability over time. The second contribution is a comprehensive analysis how asset correlations as input parameters into a credit risk model affect the *value-at-risk* which is a measure of credit risk for a portfolio. We observe that borrower-dependent asset correlations produces a substantially higher value-at-risk than median asset correlations computed on a sector level. We attribute this finding mainly to the empirical fact that asset correlations tend to increase with borrower size, which means that sector averages understate the correlation effect. We conclude that the way asset correlations are used in the credit risk model also has a substantial impact on the risk assessment of a portfolio. This methodological challenge adds to the empirical challenge of estimating asset correlations reliably. Furthermore, our results suggest that the regulatory capital charge of the internal ratings-based approach of Basel II is less volatile over time than value-at-risk in the other credit risk models in our study.

## Nichttechnische Zusammenfassung

Korrelationen zwischen den Unternehmenswertänderungen zweier Firmen, sogenannte *Asset-Korrelationen*, sind ein Schlüsselfaktor bei der Messung von Ausfallrisiken in Kreditportfolios. Da Unternehmenswerte nicht direkt beobachtbar sind, verwenden wir zur Bestimmung von Asset-Korrelationen europäischer Unternehmen Zeitreihen von Unternehmenswerten, die auf dem Modell von Moody's KMV basieren. Der erste Forschungsbeitrag des Diskussionspapiers umfasst eine deskriptive Analyse dieser Asset-Korrelationen sowie der Korrelationen von Unternehmenswertänderungen mit Industriesektorenindices. Der zweite Beitrag ist eine umfassende Analyse, in welcher Weise Asset-Korrelationen als Eingangsgrößen in Kreditrisikomodelle den *Value-at-Risk* als Maß für das Kreditrisiko eines Portfolios beeinflussen. Wir beobachten, dass kreditnehmerabhängige Asset-Korrelationen zu einem erheblich höheren Value-at-Risk führen als die Verwendung von Medianen von Asset-Korrelationen, die auf Sektorebene ermittelt werden. Dieses Ergebnis führen wir vor allem auf den empirisch beobachtbaren Anstieg der Asset-Korrelationen mit der Größe des Kreditnehmers zurück, auf Grund dessen Sektormittelwerte den Korrelationseffekt unterschätzen. Dies läßt darauf schließen, dass die Art und Weise, wie Asset-Korrelationen in einem Kreditrisikomodell berücksichtigt werden, eine erhebliche Bedeutung für die Risikobewertung des Portfolios hat. Dieses methodische Modellierungsproblem ergibt sich zusätzlich zu den bestehenden empirischen Schwierigkeiten, Asset-Korrelationen zuverlässig schätzen zu können. Unsere Untersuchungsergebnisse legen ferner nahe, dass sich die regulatorischen Kapitalanforderungen in dem auf internen Ratings basierenden Ansatz von Basel II im Zeitablauf weniger volatil verhalten als der Value-at-Risk in den weiteren untersuchten Kreditrisikomodellen.



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# 1. Introduction

As interactions between credit instruments are a cornerstone of modeling credit risk on a portfolio level, researchers have increasingly turned their attention to the estimation of asset correlations. The development of the model-based internal ratings-based (IRB) approach in Basel II strengthened this focus and has motivated banks to further develop their own internal risk models. These models provide the basis for banks to estimate their own economic capital requirements, in which correlation modeling plays a central role.

In parallel, market activity in instruments which allow market participants to directly trade credit risk correlations is growing strongly. In particular, the market for collateralized debt obligations (CDOs) has witnessed strong growth and increasing depth.<sup>1</sup> Both developments demonstrate the importance of modelling credit risk correlation from a practitioner's perspective, too.

A major problem in estimating credit correlations is the paucity of data. The literature offers two main methodologies for estimating credit risk correlations.<sup>2</sup> First, they can be estimated from default rates or rating migrations; however, this approach is made difficult by the scarcity of joint default or migration events. The second frequently used approach is to extend structural credit risk models in the spirit of Merton (1974) from a univariate to a multivariate framework in order to allow for default dependence among different sets of individual firms. Practitioners frequently use equity correlations as proxies for asset correlations. However, the performance of this method may be limited because stock prices can be affected by factors unrelated to credit risk.

The purpose of this paper is to estimate asset correlations based on asset values from the Moody's KMV (MKMV) model and to apply them in a credit value-at-risk (VaR) analysis. The main contributions to the literature are the direct use of model-based asset values for correlation estimation, an analysis of their time dynamics and their application in a portfolio model for credit risk. Compared with alternative methods for correlation estimation, our approach has a key advantage in that it exploits the full structure and performance of the univariate MKMV firm value model while at

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<sup>1</sup>See e.g. the discussion in chapter 6 in BIS (2005).

<sup>2</sup>For a discussion see, for example, chapter 10 in Duffie and Singleton (2003).

the same time being relatively tractable. The MKMV methodology is commonly used by banks and academics to measure credit risk of listed firms.<sup>3</sup>

Our MKMV sample resembles a loan book consisting of 2,000 European corporates. We focus on the variation of asset correlations across time and across industry sectors and compare the impact of the use of individual asset correlations and sector-specific asset correlations on credit portfolio risk. Sector-specific asset correlations also allow us to address the impact of sector concentration which is defined as the risk arising from an unbalanced distribution across industry sectors or geographical regions.

We proceed in two steps. In the first step, we compare asset correlation estimates based on two structurally very similar standard credit risk models.

1. A *single factor / “market” model*, in which correlation is modelled by a single common risk factor defined as the returns of the aggregate portfolio of all firms in the sample.
2. A *multi-factor / “sector” model*, in which the systematic risk factors are linked to industry sectors. We further differentiate between correlations of firms in the same sector (*intra-sector correlations*) and correlations across sectors involving the correlation between pairs of sector indices (*inter-sector correlations*).

In the second step, we use the asset correlation estimates to calculate the Expected Loss (EL) and Value-at-Risk (VaR).

In the existing literature, our study is closely linked to Lopez (2004), who analyzed the empirical relationship between the probability of default (PD), firm size and asset correlation as obtained in the proprietary MKMV correlation model. Lopez (2004) observed that the average asset correlation is a decreasing function of PD and an increasing function of asset size.<sup>4</sup> We offer three extensions to Lopez (2004). First, we estimate time series of asset correlations. Second, we investigate potential sector-specific differences of asset correlations. Finally, we analyze in detail the implications of asset correlations for the economic capital required to cover default risk. In contrast to Lopez (2004)’s analysis, which is based on international data, ours is based on European data.

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<sup>3</sup>See, for example, Berndt et al. (2005)

<sup>4</sup>Other related empirical studies on credit correlation are Dietsch and Petey (2002) and Das et al. (2004). The effect of portfolio dependencies on credit portfolio risk has been explored by Duellmann and Scheule (2003) for Germany.

Our main finding is a complex interaction of credit risk correlations and PDs which affects total credit portfolio risk; this has important implications for both banks' internal credit risk modelling processes and banking supervision.

We first find substantial time variation in asset correlations both for the market model and the sector model. This suggests that asset correlation estimates should be regularly validated. For example, the median inferred asset correlation in the market model ranges from 4% to 16% during our sample period from 1996 to 2004. For the sector model, the inferred intra-sector asset correlations are only about 2 percentage points higher than the inferred asset correlations in the market model and exhibit a similar time pattern. Upturns in the stock market tend to increase asset correlations which tend to decrease in stock market downturns.

Second, we find that the modelling of individual asset correlations has a strong impact on VaR for credit portfolios of heterogeneous borrower size, suggesting that the omission of individual dependencies can substantially reduce the VaR estimate. The reason is that large firms tend to exhibit higher correlations than smaller firms and thereby substantially add to portfolio risk. For banks' internal purposes, the use of sector-specific asset correlations has to be chosen carefully in order not to neglect this risk. Compared with using individual instead of sector-dependent asset correlations, replacing a multi-sector model by a single-factor model has a much weaker impact on VaR estimates.

Third, the VaR of the IRB model is more stable over time than the VaR of the market model and the sector model. This result is due mainly to the smoothing effect of the negative dependency of asset correlations on PD which is hard-wired into the IRB model. It is encouraging with respect to the discussion on the procyclicality of the IRB model vs. internal models. From a regulatory perspective, it is also important that the distance of the IRB model from the other models in terms of VaR fluctuates over time. Economic capital may exceed the Basel II IRB minimum capital in periods of high asset correlations.

The paper is organized as follows: Section 2 describes our sample and its properties. In Section 3, we outline the correlation estimation for the two models and the empirical results. Section 4 presents a detailed dynamic analysis of the portfolio's credit risk in which the risk measure VaR is determined for a hypothetical portfolio. In Section 5, we summarize our results and draw some conclusions for modelling the risk of a portfolio of credit-risky assets.

## 2. Data

Our sample is based on data from MKMV's Credit Monitor. For a comprehensive set of listed firms, this database contains the asset value, the asset volatility, the market value of equity, the book value of liabilities and the expected default frequency (EDF)<sup>5</sup>, which measures the probability that the firm value will fall below a pre-defined default threshold within a year. By construction, the EDF is bounded from below at 0.02% and from above at 20%.

The basis for the MKMV model is the structural modelling approach introduced by Merton (1974), but the proprietary MKMV methodology contains a number of refinements and modifications such as the use of a large database of observed defaults. In structural models, the likelihood of a firm's default is linked to firm-specific structural variables, namely the market value of a firm's assets and its total debt. The key input parameters in this methodology are the volatility of the asset value and a measure of the firm's leverage. MKMV empirically estimates the asset value and its volatility from the time series of stock prices and balance sheet data. According to the empirical analysis in Arora et al. (2005), the MKMV approach shows a good forecasting performance for default risk.

Our sample contains monthly time series of asset values from January 1996 to February 2004.<sup>6</sup> The initial dataset comprises 7,119 European firms with publicly traded equity<sup>7</sup> or a total of 532,836 observations but needed to be adjusted as described in the following.

In order to control for different currencies, we transform all asset values into euro based on monthly exchange rates.<sup>8</sup> As all major frictions in European large currency markets occurred before 1996, large exchange rate fluctuations should not affect our asset correlation estimates.

Our analysis focuses only on non-financial firms, since financial institutions typically

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<sup>5</sup>EDF is a trademark of MKMV. Further information about the MKMV methodology can be found in Crouhy et al. (2000).

<sup>6</sup>Further information about the dataset can be found in Marcelo and Scheicher (2004).

<sup>7</sup>The MKMV eligibility criteria are the availability of market data on a firm's equity and financial statement data, a minimum market capitalization of USD 100m, and that a firm is not only traded in a stock market outside the company's domicile.

<sup>8</sup>Before 1998, we use the exchange rates for Deutsche Mark (DEM).

have a different credit risk profile.<sup>9</sup> Additionally, we exclude all firms in the sector “Other” due to the small number of observations in this sector.

We calculate firms’ times series of asset returns as first differences of log asset values which posed two challenges: Firstly, how to cope with data errors and, secondly, how to treat missing values.

In order to remove outliers due to data errors, we eliminate the upper and the lower 1% tails of the overall asset return distribution for the pooled sample.<sup>10</sup>

Around three-quarters of the firms exhibit missing values in their time series of asset values. One reason is the increasing number of firms in the sample over time even though various firms also leave the data sample.<sup>11</sup> To strike a balance between a better coverage of the sample and the need for data consistency, we use time series without missing values for each 24-month time window for our correlation and portfolio risk analysis. In cases where there are no more than three missing entries between two observed asset values, we replace missing observations with the last observation before the gap in order to extend our sample coverage.<sup>12</sup>

The firms are assigned individually to six industry sectors defined by MKMV. These industry sectors are Basic and Construction Industry (BasCon), Consumer Cyclical (ConCy), Consumer Non-Cyclical (ConNC), Capital Goods (Cap), Energy and Utilities (EnU) and Telecommunication and Media (Tel). Table 1 shows the industry sector distribution in the edited sample. The edited sample contains a total of

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<sup>9</sup>MKMV adjusted their model for financial firms because of their high leverage, in particular with respect to the default threshold. By excluding the financial sector, we avoid comparability problems across sectors.

<sup>10</sup>The first percentile of the (monthly) asset returns is -30.3% and the 99th percentile is +48.5%. We also analyzed the impact of symmetrically cutting 0%, 1%, 2%, 3%, 4% and 5% off the asset returns at each end of their distribution. While cutting off 1% had a strong impact on the asset correlation estimates, the additional cut-off values did not cause substantial changes in the asset correlation estimates.

<sup>11</sup>The dataset grows continuously from initially 3,204 firms in January 1996 to 4,424 firms in March 1999, before jumping to 6,397 firms one month later. From April 1999 on, the sample size varies between 6,250 and 6,444 firms. The highest number of firms per point in time (6,444) is still considerably lower than the total number of firms in the MKMV sample (7,119) since firms enter and leave the dataset over time and there are missing values in the database.

<sup>12</sup>We checked the impact of these adjustments by comparing the estimates of the asset correlations with the estimates from the original time series which contain the gaps and we found no significant differences.

1,988 firms. The sector distribution in the edited sample does not vary substantially from the sector distribution in the original sample except for the Tel sector, as many Tel firms enter the data sample only during later periods. Industry sectors with more than 20% of total observations are Consumer Cyclical (32%) and Basic and Construction (24%), followed by Consumer Non-Cyclicals, Technology, Media and Telecommunication, Capital Goods and Energy and Utilities, with portfolio shares between 5% and 17%. The three largest countries are UK, France and Germany with a total sample portion of almost 60%.

## 2.1. Descriptive Statistics of the Data Sample

After the adjustment procedures described in the previous section, the dataset still consists of 1,988 European firms with 147,112 monthly observations. Table 1 shows the (equally weighted) mean values of three major MKMV parameters, namely the asset value, the equity value, the EDF (Expected Default Frequency) as well as the asset returns and equity returns for individual sectors and the total sample. The asset (equity) return is defined as the monthly log return of the firm's asset values (the market value of a firm's equity).

Based on averages, we observe the highest EDFs in the Telecommunication and Media sector, followed by the Consumer Cyclical sector. Firms in the Energy and Utilities sector exhibit the lowest EDF with an average of 0.53%. The largest firms are concentrated in this sector with an average asset value of 12.8 billion euros, while firms in the Capital Goods sector exhibit only an average asset size of 1 billion euros. The mean asset returns are of relatively similar size in all sectors, ranging from 0.58% in the Cap sector to 1.03% in the Tel sector.

## 3. Asset Correlation Estimation

In this section, we estimate asset correlations with a market model and a sector model. For the construction of the sector indices and the market index we use all asset return time series of the edited sample at the corresponding point in time. In contrast, for the correlation estimation we use only time series without missing

**Table 1**  
**Descriptive Statistics of the Sample After Adjustments**

This table shows descriptive statistics of listed European non-financial companies between January 1996 and February 2004. The asset value and the equity value are measured in million euros. BasCon refers to Basic and Construction Industry, ConCy to Consumer Cyclical, ConNC to Consumer Non-Cyclical, Cap to Capital goods, EnU to Energy and Utilities and Tel to Telecommunication and Media.

	BasCon	ConCy	ConNC	Cap	EnU	Tel	Total Sample
Number of Firms	475	633	336	219	102	223	1,988
EDF (mean)	1.73%	1.84%	1.13%	1.71%	0.53%	2.15%	1.65%
Asset Value (mean)	1,680.5	2,014.3	3,773.3	1,002.3	12,852.7	6,878.4	3,226.9
Equity Value (mean)	811.7	912.3	2,427.7	547.6	7,508.0	4,559.4	1,885.5
Asset Return (mean)	0.59%	0.77%	0.74%	0.58%	0.94%	1.03%	0.74%
Equity Return (mean)	1.07%	1.19%	1.09%	1.26%	1.20%	1.50%	1.19%

observations in the corresponding time interval.<sup>13</sup>

For both the sector and the market model we analyze the time variation of asset correlations by means of 74 overlapping 24-month time windows, starting from February 1996 to January 1998 and ending with the period from April 2002 to February 2004.

### 3.1. Asset Correlations in the Market Model

For the market model, we calculate the asset correlations as the squared sample correlations (*“market correlations”*) between the time series of monthly log returns of the individual firms and the log returns of the market portfolio. The market portfolio comprises the asset value-weighted sample of all firms for which asset values for the return calculation are available. The large number of firms in the market portfolio prevents single firms from having a substantial impact on the market index, which has been verified by robustness checks. Figure 1 shows the time series of selected quantiles of the cross-section of market correlations, namely the 25th, median, 75th, 95th and the maximum.

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<sup>13</sup>Robustness checks have shown, however, that the results are robust against this selection, in particular against extrapolating missing values up to 12 observation dates.

**Figure 1. Selected Quantiles of Asset Correlations in the Market Model**

This figure shows selected quantiles of asset correlations, defined as squared sample correlations between equity returns and returns of the market index. They are based on the total sample from January 1998 until February 2004. Q25 (q50, q75, q95) correspond to the 25th (median, 75th, 95th) percentile and “max” to the highest market correlation at a specific point in time. The names of months refer to the end of 24-month time windows.

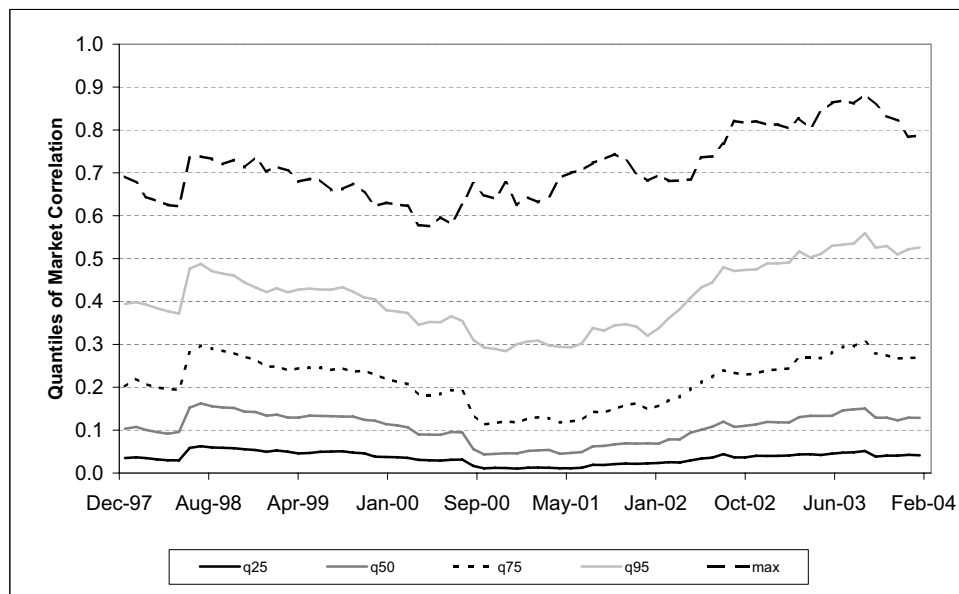




Figure 1 reveals a substantial volatility of all quantiles of the market correlations over the period from January 1998 to February 2004. Moreover, all quantiles follow a similar pattern over time. The median market correlation (displayed by means of the dashed line) varies between 4% and 16% during the observation period. The highest correlations are recorded for the 24-month periods ending in summer 1998<sup>14</sup> and fall 2003. The overall cross-sectional median market correlation is 10.2%. The evolution of the 25th quantile (q25) and the 75th quantile (q75) of the market correlations is in line with the evolution of the median market correlation, with the q25-values roughly equalling one third of the median market correlations and the q75-values roughly equal two times the medians. The highest quantiles substantially exceed the median market correlations and vary between the 45th (q95) and the 75th (max), respectively.<sup>15</sup>

In Figure 1 we observe the lowest market correlations for the period from the beginning of 2001 to mid-2002 and in early 1998. Hence, during the period of major turbulence in the equity markets from March 2000 on, co-movement of asset values is relatively low. This finding cannot be attributed to changes in the sample composition since the market correlations of the firms entering or leaving the sample were not systematically different from the rest.

Furthermore, we evaluated whether the low correlations were caused by the boom in the equity markets until mid-2000 followed by a sharp decline afterwards, which increased the idiosyncratic component in the movement of firms' asset values. In a sector-specific analysis (for more details see the next section) we find that the decline in the market correlations occurred for all industry sectors except the Tel sector, where the inferred asset correlations remained stable at that time and even started to rise from the beginning of 2001 on (see Figure 3). The values displayed for mid-2001 refer to the period from mid-1999 till mid-2001, which covers both an upturn and a downturn in the equity markets accompanied by high equity volatility and thus also high asset value volatility. The upturn in the equity markets covered a period of more than two years and basically affected all firms similarly sooner or later, leading to higher market correlations. In contrast, the downturn in 2001 affected some sectors immediately (namely Tel sector firms), but other sectors only

<sup>14</sup>The dates refer to time windows. Hence, June 1998, for example, refers to the time interval from July 1996 to June 1998.

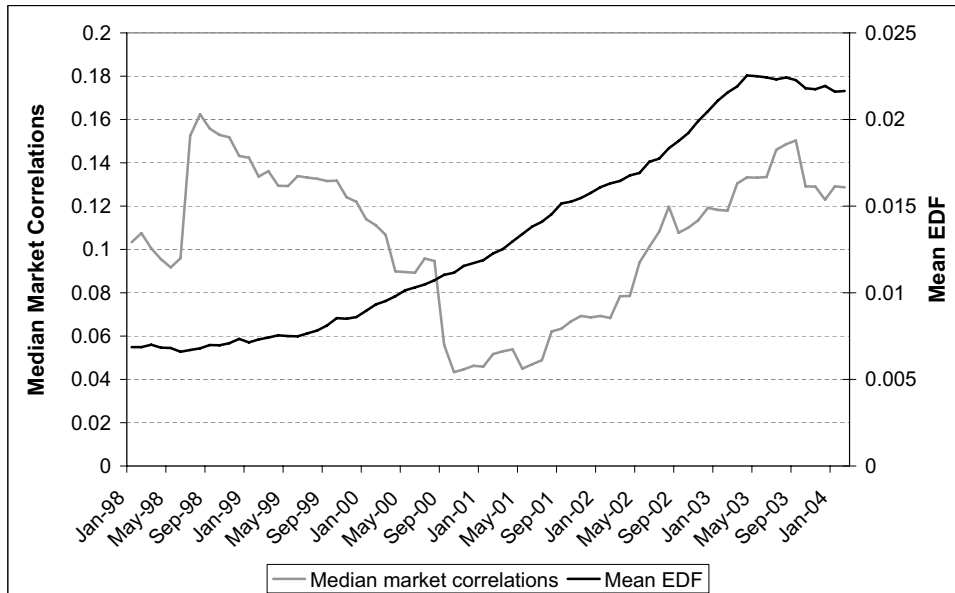
<sup>15</sup>It is important to note that the high quantiles of the market correlations are substantially higher than the asset correlations applied in the IRB risk weight functions of Basel II.

with a time lag (for example the consumer non-cyclical sector), thereby resulting in lower market correlations overall. Thus, firms' asset values were affected by stock market upturns and downturns differently. In the subsequent period of increasing stock prices, on average, correlations showed again a positive trend until the end of the sample period in 2004.

In order to explore the effects of univariate credit risk variation on asset correlations, we evaluate the corresponding time series movement of the mean EDF.<sup>16</sup> The mean EDF represents the cross-sectional arithmetic average of the firms' mean EDF in the corresponding time period. As Figure 2 shows, the co-movement of the median market correlation and the mean EDF is rather weak, at least for the first period until mid-2001. The variation of the market correlations shows the cyclical pattern described above, whereas the EDF exhibits a more or less continuous increase until the beginning of 2003.

**Figure 2. Median Asset Correlations and the Average EDF**

This figure shows the evolution of the median of market correlations and the mean EDFs of the total sample from January 1998 until February 2004. The months refer to the end of 24-month time windows.



<sup>16</sup>Given that the EDF is bounded from below and above, we use the mean EDF for a comparison with median correlations.

### 3.2. Asset Correlations in the Sector Model

For the sector model, we first compute sector-by-sector asset correlations from the squared correlations between the log returns of the individual firms and the corresponding sector index. This procedure is analogous to the market model. Then we differentiate between intra-sector and inter-sector correlations, which are both determined at a sector-aggregate level. The *intra-sector asset correlations* are defined as the median of the (individual) asset correlations in every sector (below also referred to as *sector correlations*). The *inter-sector correlations* are calculated as the sample correlations between the time series of two sector-index returns. Therefore, asset correlations in the sector model fundamentally differ from correlations in the market model in the sense that they are always aggregated at sector level, whereas the market model contains individual pairwise correlations (with the market index).

**Figure 3. Evolution of Median Sector Correlations**

This figure shows sector-by-sector the evolution of the median sector correlations and the mean EDFs of the total sample from January 1998 until February 2004. BasCon refers to Basic and Construction Industry, ConCy to Consumer Cyclical, ConNC to Consumer Non-Cyclical, Cap to Capital Goods, EnU to Energy and Utilities and Tel to Telecommunication and Media. The months refer to the end of 24-month time windows.

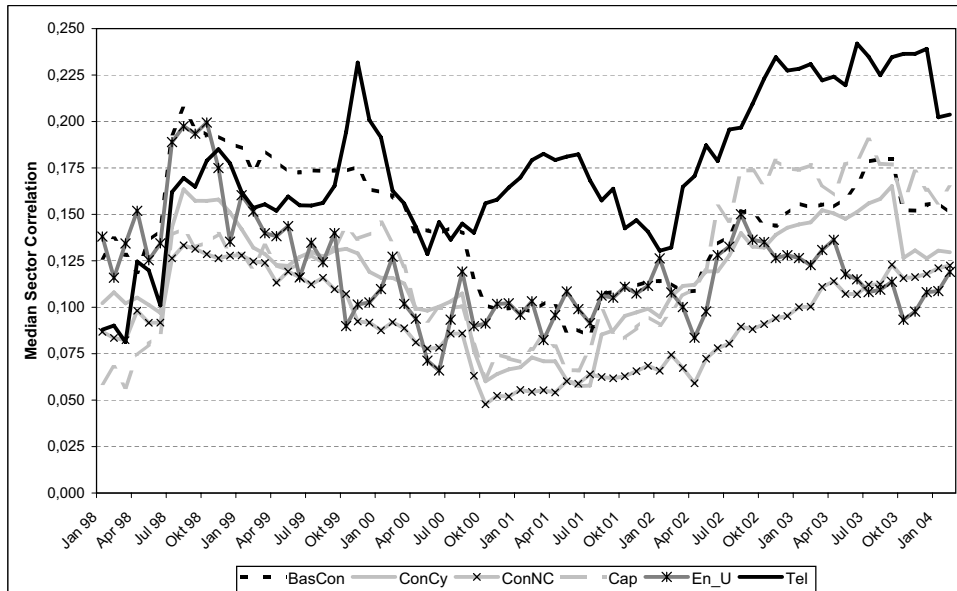


Figure 3 shows the evolution of the median sector correlations for the six industry sectors over time. The dates refer to the ends of the two-year time windows. From Figure 3 we find that the fluctuations of the median sector correlations over time are substantial and exhibit a similar pattern to the correlations in the market model. Again, the lowest correlations occur in early 1998 and from 2000 to mid-2002. The median sector correlations range from 4.8% (in the ConCy sector) to 24.2% (in the Tel sector). The overall median of the sector correlations is 12.3%, which is approximately 2 percentage points higher than the corresponding value in the market model. A co-movement of the median sector correlations except for the Telecommunication and Media (Tel) sector is plausible due to the different patterns of stock prices in the latter sector.<sup>17</sup> Finally, we observe the highest volatility of the median sector correlation in the Tel sector and the lowest for the ConNC sector. We conclude that the differences in asset correlations across sectors are relatively moderate given large potential differences between the sectors.

Next, we compare median intra-sector correlations with the EDF. As an example, we plot the time series of median asset correlations and the EDFs for the BasCon sector. The time patterns for the other sectors are similar (except for the Tel sector). The time variation of sector correlations differs from the dynamics of the mean sector EDFs, particularly due to differences in the first part of the observation period. This finding is consistent with the results for the market model.

To analyze inter-sector correlations, we estimate the correlation between the sector indices. As an example, Table 2 shows the correlation matrix for all sector index pairs in the first 24-month time interval, i.e. from February 1996 to January 1998. We observe that the correlation ranges from 95% between the Basic and Construction sector and the Consumer Cyclical sector to 72% between the Consumer Non-Cyclical and the Capital Goods sector (or between the Capital Goods sector and the Telecommunication and Media sector).

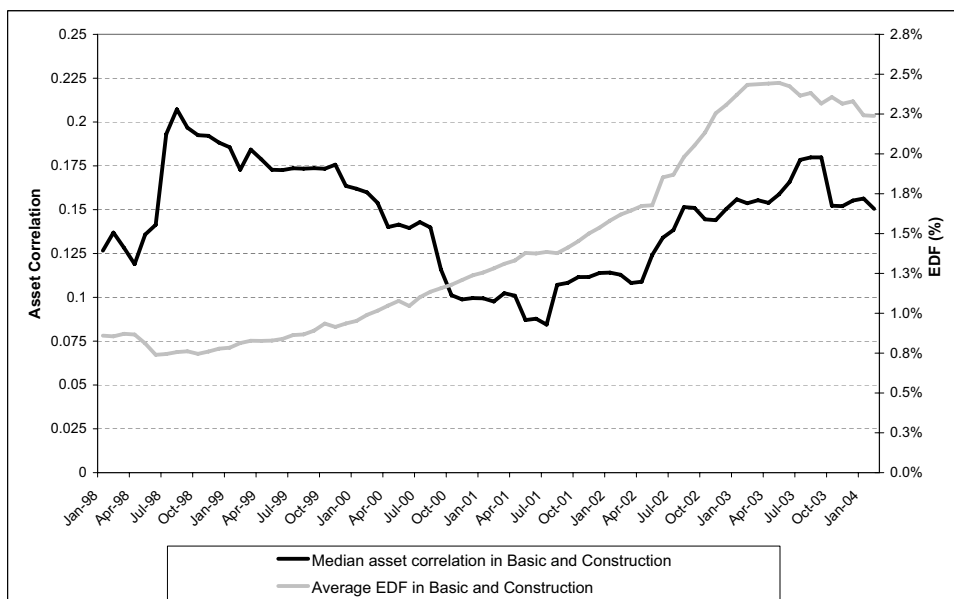
Figure 5 shows the time series of the correlations between the sector indices for all industry sectors and the Tel sector for 1998 to 2004, as the correlation time series for this set of index pairs exhibit the highest volatilities. The graph shows that there is considerable movement in the correlation for most index pairs over time.

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<sup>17</sup>An important issue in this context is that industries react with a different time lag to cyclical changes, which may in this case enforce the difference between the Tel sector and the other sectors; the Tel sector is very sensitive, while other sectors tend to be less sensitive to cyclical changes.

**Figure 4. Median Intra-Sector Asset Correlations and Mean EDFs for the Basic and Construction Industry**

This figure shows the median of all intra-sector correlations and the mean EDFs of the industry sector “Basic and Construction Industry” from January 1998 until February 2004. The names of months refer to the end of 24-month time windows.



**Table 2****Correlation Between Sector Indices from February 1996 to January 1998**

BasCon refers to Basic and Construction Industry, ConCy to Consumer Cyclical, ConNC to Consumer Non-Cyclical, Cap to Capital Goods, EnU to Energy and Utilities and Tel to Telecommunication and Media.

	BasCon	ConCy	ConNC	Cap	EnU	Tel
BasCon	<b>1</b>	0.95	0.84	0.77	0.78	0.85
ConCy	0.95	<b>1</b>	0.92	0.80	0.88	0.94
ConNC	0.84	0.92	<b>1</b>	0.72	0.84	0.94
Cap	0.77	0.80	0.72	<b>1</b>	0.75	0.72
EnU	0.78	0.88	0.84	0.75	<b>1</b>	0.85
Tel	0.85	0.94	0.94	0.72	0.85	<b>1</b>

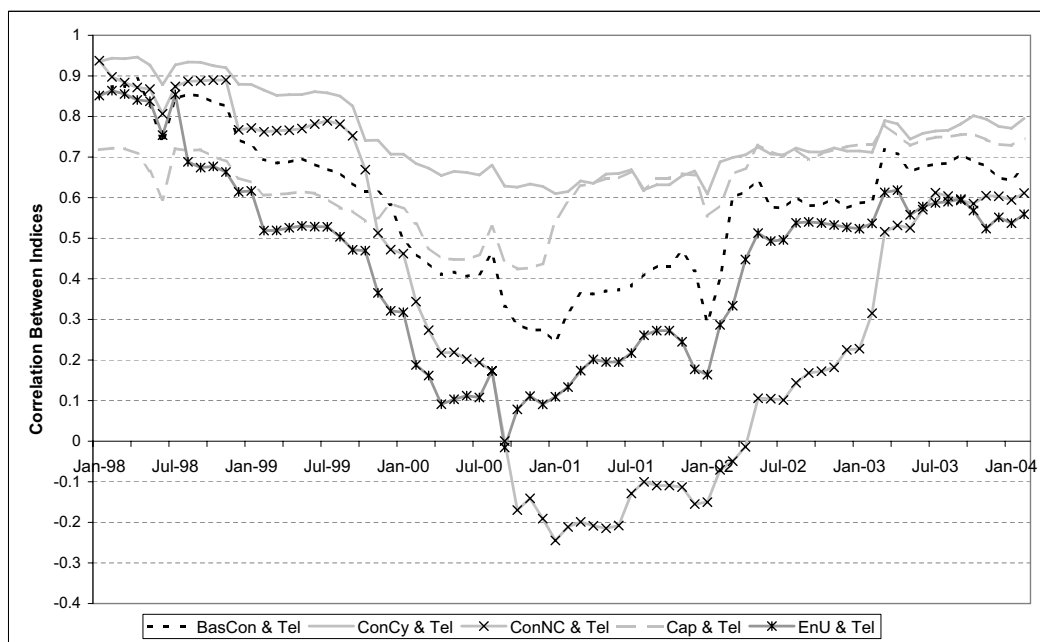
Similar to the case of the market and sector correlations, we observe the lowest correlations from the end of 2000 to the first half of 2002. For the beginning and the end of the observation period, we find that the inter-sector correlations are all on a relatively high level of between 0.5 up to 0.9 for all index pairs. This general tendency applies also to the inter-sector correlations not included in Figure 5. The largest fluctuation occurs between the Telecommunication and Media sector and the Consumer Non-Cyclical sector, indicating that the stock market turbulence affected the Telecommunication and Media sector differently than, for example, the consumer sector. We also observe that correlations between the sector pairs even become negative over certain time periods. The second highest fluctuation occurs in the case of the Consumer Non-Cyclical sector and the Telecommunication and Media sector, which also exhibits a different cyclical pattern. For the other three index pairs the volatility of the correlations is far less pronounced, but still considerable.

### 3.3. Discussion

Overall, our estimates of the level of asset correlations are consistent with results in the previous literature. For instance, Lopez (2004) documents an average asset correlation of 12.5% for a large international MKMV sample consisting mainly of US firms, which is relatively close to the median asset correlations in the market model (10.1%) and the median intra-sector asset correlations in the sector model (12.3%).

**Figure 5. Correlations Between Selected Pairs of Sector Indices**

This figure shows the time series of the correlations between the sector indices for all industry sectors with the sector “Telecommunication and Media”. BasCon refers to Basic and Construction Industry, ConCy to Consumer Cyclical, ConNC to Consumer Non-Cyclical, Cap to Capital goods, EnU to Energy and Utilities and Tel to Telecommunication and Media. The names of months from January 1998 until February 2004 refer to the end of 24-month time windows.



Furthermore, the strong variation of all correlations demonstrates that estimating credit portfolio risk requires time-dependent asset correlations in order to fully reflect the statistical properties of risk dynamics. Our finding of time-dependent asset correlations is closely related to similar observations for the co-movement of stock returns.<sup>18</sup> Upturns in the stock market tend to increase asset correlations which tend to decrease in stock market downturns.

Moreover, we observe that sector correlations and market correlations follow a similar pattern and that sector correlations tend to be only moderately higher than market correlations. A potential factor in our result is the composition of our sample, which contains the largest European firms. These large corporates are more strongly correlated with the macroeconomic cycle than smaller firms and often operate in

<sup>18</sup>See, for example, Bollerslev et al. (1988), Longin and Solnik (1995) and Ang and Chen (2002).

several sectors. This result is in line with Duellmann and Scheule (2003) and Lopez (2004), who found that asset correlations increase with firm size. It may not hold true for SME portfolios, where firms are generally more concentrated in a single sector.

A caveat in our analysis is that we don't know how the definition of industry sectors has affected our estimates of intra-sector asset correlations. However, the definitions of sectors and the methods for assigning borrowers to sectors have so far received relatively little attention in the literature.

Finally, a comparison of the dynamics of asset correlations and EDFs has shown that there are relatively few commonalities between the two variables in both correlation models. This observation supports previous empirical work, which stresses the necessity of considering asset correlations and PDs as separate determinants of credit portfolio risk.<sup>19</sup>

## 4. The Impact of Asset Correlations on Credit Portfolio Risk

### 4.1. Model Framework and Simulation Methodology

In this section, we study the impact of asset correlations on credit portfolio VaR in the market and sector model. Based on the asset correlations calculated above, we compute the evolution of the VaR risk measure for 74 overlapping time windows, each spanning 24 months. We restrict our portfolio for every time window to those firms with no missing values, namely for which asset correlation, EDF and total liabilities as a proxy of exposure size are available. Under this restriction, the portfolio size stays relatively stable over time at around 1,600 exposures and is smaller than the overall number of data sets which have no missing asset correlations in at least one time interval (1,988) but may still have missing EDFs or total liabilities.

We assume that all borrowers can be uniquely assigned to individual business sectors. Let  $N$  denote the total number of borrowers or loans in the portfolio,  $S$  the number

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<sup>19</sup>See Duellmann and Scheule (2003).



of sectors and  $s : \{1, \dots, N\} \rightarrow \{1, \dots, S\}$  a mapping which assigns every borrower uniquely to its sector. The relative exposure of borrower  $i$  in the portfolio is denoted by  $w_i$  and defined by the ratio of its book value of liabilities  $LBS_i$  and the aggregate of the book values of all borrowers in the credit portfolio:

$$w_i = \frac{LBS_i}{\sum_{i=1}^N LBS_i}. \quad (1)$$

The definition of  $w_i$  is inspired by the interpretation that the portfolio comprises all European listed non-financial firms under the hypothetical assumption that their liabilities are all bank debt. Even if bank loans are only one component of debt financing, our results are robust as long as the relative share of other financing sources is roughly equally distributed among firms. The book values of liabilities are extracted from the MKMV database.

In our setup, credit risk is defined as the loss arising from a default event which is consistent with the traditional book-value approach to loan portfolio management. Hence, migration risk is not captured by our analysis.

In the sector model, the dependence structure between borrower defaults is driven by sector-dependent systematic risk factors which are usually correlated. As each risk factor is uniquely assigned to a different sector, the number of sectors and factors are equal. The unobservable, normalized asset return  $X_i$  of borrower  $i$  in sector  $s(i)$  is given for  $i \in \{1, \dots, N\}$  by

$$X_i = r_{s(i)} Y_{s(i)} + \sqrt{1 - r_{s(i)}^2} \zeta_i. \quad (2)$$

The disturbance terms  $\zeta_i$  are independent Gaussian (i. e. standard normally) distributed. The systematic risk factors  $Y_{s(i)}$  are assumed to be linearly independent and follow a joint normal distribution with mean zero and correlation matrix  $\Omega = \{\omega_{\nu, \nu'}\}_{\nu, \nu'=1, \dots, S}$ .

The asset correlation for each pair of borrowers  $i$  and  $j$  is then given by

$$cor(X_i, X_j) = r_{s(i)} r_{s(j)} \omega_{s(i), s(j)}. \quad (3)$$

In the sector model, we assume that  $r_{s(i)}$  is shared by all borrowers in the same sector. It is estimated by the square root of the median of all intra-sector asset correlations in the sample. The inter-sector correlations  $\omega_{s(i), s(j)}$  are estimated by the sample correlation of index returns for the  $i$ -th and  $j$ -th sector.

Let  $\psi_i$  denote the loss severity, which we assume to be known when default occurs.<sup>20</sup> Although several studies have presented tentative empirical evidence of systematic risk in the loss severity<sup>21</sup>, we assume in the following that  $\psi_i$  is subject only to idiosyncratic risk, which is sufficiently diversified so that we can replace  $\psi_i$  by its expected value in the VaR calculations. We assume a value of 0.45, which is the value set by supervisors for senior corporate exposures in the Basel II foundation IRB approach.

The portfolio loss  $L$  in the sector model is given by

$$L = \sum_{i=1}^N w_i \psi_i 1_{\{X_i \leq \Phi^{-1}(p_i)\}} \quad (4)$$

with  $X_i$  defined by (2) and  $\Phi^{-1}$  the inverse of the cumulative distribution function of the standard normal distribution.

The VaR for a given confidence level  $q$  is obtained by sampling the loss distribution, given by (2) and (4). We set the confidence level  $q = 99.9\%$  and perform 500,000 simulation runs for each VaR calculation.

In the market model, portfolio losses are still described by (2) and (4) but  $Y_{s(i)}$  is now the same for all sectors, i. e.  $Y = Y_{s(i)}$  for  $i \in \{1, \dots, N\}$ . Furthermore, the coefficient  $r_{s(i)}$  of the systematic factor in (2) depends on the firm and is estimated by the sample correlation of its asset returns and the market index returns. Since we allow for heterogenous PDs and pairwise asset correlations, we have to rely again on Monte Carlo simulations for the VaR calculation.

As a benchmark for the single-factor model, we calculate the VaR under the asset correlation assumptions of the IRB risk weight functions for corporate exposures in Basel II. Whereas the risk weight functions contain only the unexpected loss component, we focus on VaR including expected loss and unexpected loss, which is given by

$$VaR_{99.9\%}^{IRB} = \sum_{i=1}^N w_i \psi_i \Phi \left( \frac{\Phi^{-1}(p_i) + \sqrt{\rho(p_i)} \Phi^{-1}(1-q)}{\sqrt{1-\rho(p_i)}} \right) \quad (5)$$

and

$$\rho(p_i) = 0.24 - 0.12 (1 - e^{-50 p_i}). \quad (6)$$

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<sup>20</sup>By this simplifying assumption we remain agnostic that the loss severity is not certain when the default event occurs but rather the result of a possibly lengthy recovery process.

<sup>21</sup>See, for example, Altman et al. (2002) and Düllmann and Trapp (2004).

We calculate the VaR for the previously described portfolio for the 74 time intervals. For the individual firms, the main inputs are their sector affiliation, individual or sector-specific asset correlations, individual EDFs and individual liabilities. Table 3 below summarizes the input parameters for the two alternative market model specifications, the sector model and the IRB model. The second market model uses sector-specific median correlations (labelled as *Market model (sec. corr.)*) instead of borrower-dependent correlations. It resembles the market model, as systematic risk is driven by a single factor while correlations are sector-dependent as in the sector model. Therefore, this model can help to explain differences between the market and the sector model by disentangling the impact of the number of factors from the use of sector-dependent instead of borrower-dependent correlations.

**Table 3**  
**Input Data for the Market Model, the Sector Model and the IRB Model for VaR Calculation**

Model	EDFs, Liabilities	Asset Correlations	No. Factors
Market model	Individual	Individual	1
Market model (sec. corr.)	Individual	Sector-dependent	1
Sector model	Individual	Sector-dependent	6
Basel II IRB model	Individual	PD dependent	1

## 4.2. Analysis of Credit Portfolio Risk

In order to compare the VaR for the different models and to assess its time variation, descriptive statistics of the time series of VaR for the market model, the sector model and the IRB model are shown in Table 4.

The highest median and mean VaR are observed for the market model (with borrower-dependent market correlations). The differences in mean and median between this model and the other three models are by far larger than the differences between the three remaining models. This result holds even more strongly if the models are compared on the basis of the VaR maximum over time or if the standard deviation of VaR is considered. The latter is twice as high for the market model as for the other three models.

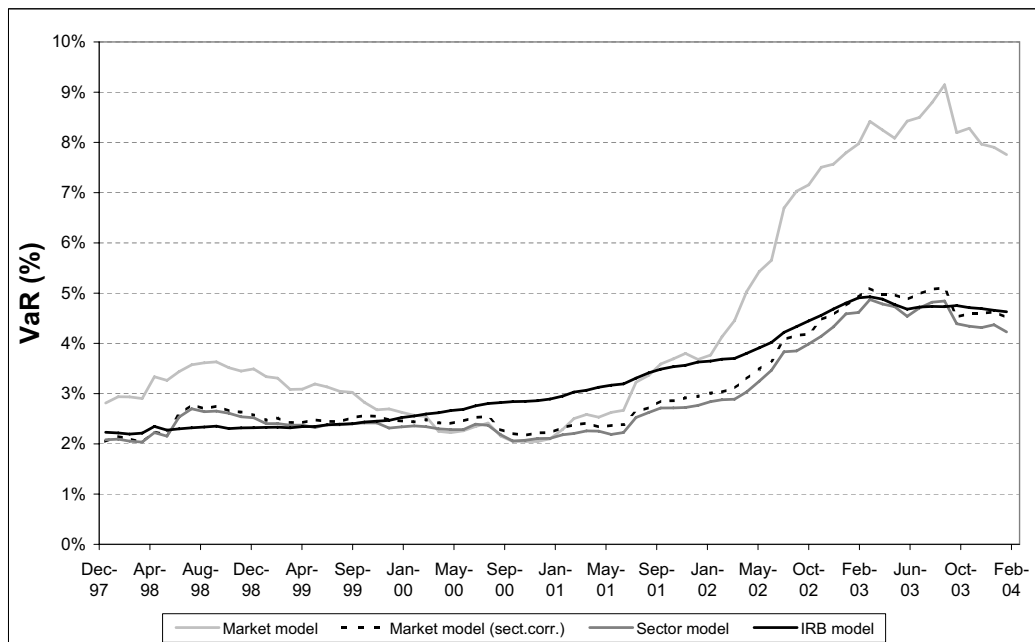
Figure 6 visualizes the evolution of the VaR for the four models, which varies over

**Table 4**  
**Descriptive VaR Statistics for the Market Model, the Sector Model and the IRB Model for Portfolios with Heterogenous Exposure Size**

Model	Market Model	Market Model	Sector Model	IRB Model
Correlation depends on ...	Borrower	Sector	Sector	PD
Max	9.15%	5.10%	4.87%	4.93%
Mean	4.36%	3.10%	2.95%	3.28%
Median	3.35%	2.58%	2.52%	2.92%
Standard Deviation	2.26%	1.00%	0.93%	0.95%

**Figure 6. Dynamic Credit VaR Analysis for Portfolios with Heterogenous Exposure Size**

This figure shows the evolution of the VaR for the market model, the sector model and the IRB model, which are specified as shown in Table 4. The “Market model (sec. corr.)” is a market model with median sector-specific asset correlations. Exposure size is heterogenous, depending on each firm’s liabilities. The names of months from January 1998 until February 2004 refer to the end of 24-month time windows.



the sample period between around 2% and 10% and tends to increase over time. The fluctuation is highest for the market model, as already indicated in Table 4 by the higher standard deviation. A most striking observation in Figure 6 is that the

difference in level between the market model and the other three models is mainly caused by the period between January 2002 and the end of the observation period in February 2004, a period in which the EDFs are also elevated. The difference in level confirms the aggregate results of Table 4. In the following analysis we address three main questions:

1. What are the main drivers of the increase in the VaR estimates over time?
2. Why are the VaR estimates of the market model substantially different from the other three models which are much more in sync?
3. How can we explain the similarity but also the smoother evolution of the VaR from the IRB model compared with the sector model?

Turning to the first question, we study the evolution over time of EDFs, asset correlations and the name concentration of the portfolio in order to identify drivers of the VaR increase over time. Figure 7 shows the relationship between the VaR, the average EDF, the median market correlation for the market model and the Herfindahl-Hirschman Index (HHI). Market correlation is defined as the correlation between the asset returns and the returns of the market factor or, equivalently, the square root of the asset correlation in a single-factor model.

Examining Figures 6 and 7 together, we find that the VaR movement is driven both by EDFs and market correlations. Contrary to the EDFs for which the median is substantially higher, the mean and median of the market correlations are close to each other, which suggests a much more symmetric cross-sectional distribution. As the average market correlation does not reach new peaks during the strong increase in the VaR after January 2002, we conclude that the asset correlation can explain the peaks only in combination with the higher level of EDFs.

As highlighted by several authors, credit concentrations may play a material role for credit portfolio risk.<sup>22</sup> The stability of the Herfindahl-Hirschman Index (HHI)<sup>23</sup> over time in Figure 7 together with its low level of around one percent on average suggest, however, that the VaR increase is not driven by higher name concentration.

Figure 8 shows the number of borrowers in each sector over time. Since the distribution of borrowers over sectors remains stable over time, credit concentrations in

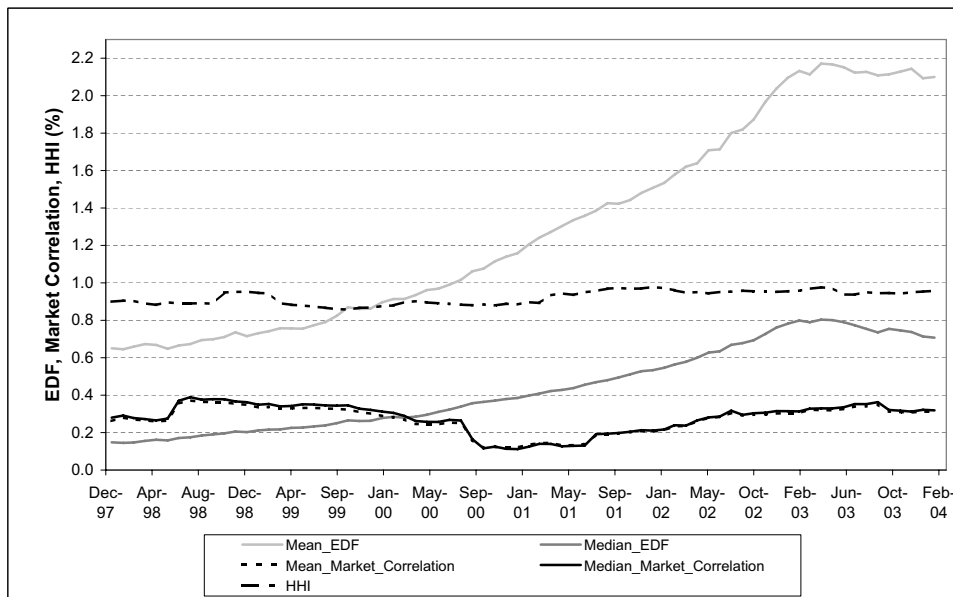
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<sup>22</sup>See, for example, Gordy and Luetkebohmert (2007) or Duellmann and Masschelein (2006).

<sup>23</sup>The HHI is calculated as the sum of squared relative exposures.

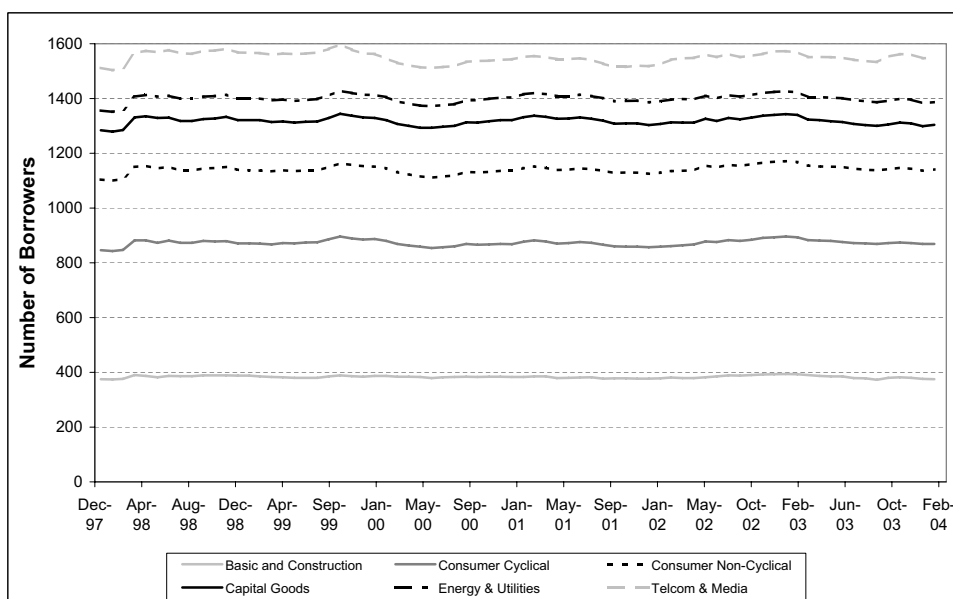
**Figure 7. Evolution of Asset Correlations, EDF and Name Concentration for the Market Model**

This figure shows the mean and median EDF, the median correlation with the market factor and the Herfindahl-Hirschman Index (HHI). The names of months from January 1998 until February 2004 refer to the end of 24-month time windows.



**Figure 8. Sector Decomposition of the Sample of Firms Over Time**

This figure shows the portfolio share of exposures in six industry sectors in absolute numbers over the sample period from January 1998 until February 2004.



industry sectors cannot explain the increase in VaR either. In summary, the joint increase in the level of EDFs and correlations appears to be the main driver of the higher VaR results after January 2002.

The second question which arises both from Table 4 and Figure 6 is why the VaR results for the market model are so different from those of the other models. This question is particularly important as borrower-dependent asset correlations are typically not available for non-listed companies. In order to answer this question, we proceed in two steps. Firstly, we discuss the differences between the market model (with borrower-dependent asset correlations) and the two models with sector-dependent correlations. Secondly, we study differences between the latter two models, i. e. the market model with sector-dependent correlations and the sector model.

According to Table 4, with borrower-dependent correlations the market model produces a mean (median) VaR which is 40% (30%) higher than with sector-dependent correlations. This result could be driven by the empirically well-established fact that larger firms which (by construction) are represented by larger exposures in our portfolio tend to have on average higher asset correlations.<sup>24</sup> As a consequence, averaging correlations as done in the models with sector-dependent correlations could underestimate risk. Table 5 provides summary statistics of time series of two non-linear correlation measures, the Spearman rank correlation coefficient and Kendall's tau. Both measures are employed at each observation date to determine a possible correlation between exposure size and market correlation.

The numbers in Table 5 show a positive non-linear correlation between exposure size and market correlations. Whether it is strong enough to explain the difference between the models remains, however, at this point an open issue.

Another factor that drives the VaR estimates in the market model may be sample noise in the asset correlations. Given the relatively short time series of asset returns, it is not unreasonable to expect that estimation noise leads to more dispersed asset correlations than they truly are. Since the functional relationship between asset correlations and VaR is highly non-linear, this dispersion may well inflate the VaR. In the case of sector-dependent asset correlations, this noise is reduced by taking cross-section averages. Following this reasoning, the consequence should be lower VaR estimates. This second explanation would imply that the VaR estimates of

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<sup>24</sup>See, for example, Dietsch and Petey (2002) or Lopez (2004).

**Table 5**  
**Summary Statistics of Correlation Coefficients for Exposure Size and Market Correlation**

This table shows Spearman Rank Correlation Coefficient and Kendall's Tau. The sample consists of time series of 74 monthly observations of exposure size and the MKMV probability of default (EDF) from January 1998 until February 2004 .

Statistic	Spearman	Kendall
Max	0.38	0.27
Mean	0.27	0.18
Median	0.27	0.18
Min	0.14	0.09
Standard Deviation	0.08	0.05

the market model (with borrower-dependent correlations) are inflated. The first explanation instead suggests the opposite, namely that portfolio risk is correctly measured by this model but underestimated if borrower-dependent correlations are not accounted for.

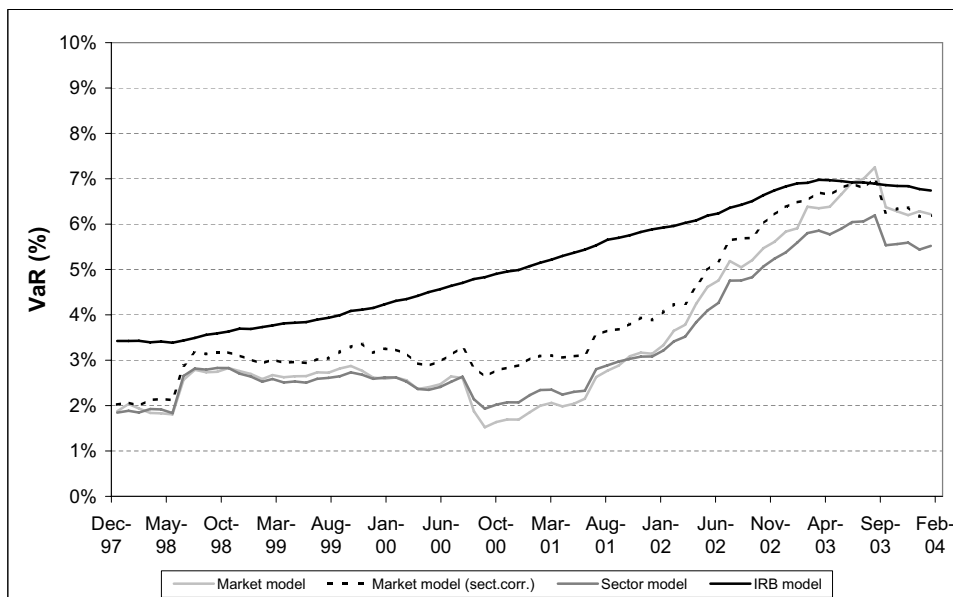
In order to explore which of the two explanations is more important, we rerun the portfolio risk analysis with a portfolio that is homogenous in terms of the size of single exposures. More specifically, it is the same portfolio but with every exposure size set to one currency unit. If the correlation between borrower size and asset correlation drives the VaR estimates, then we expect that the VaR estimates should no longer be higher than in the models with sectoral correlation averages. If, however, the estimation noise is the more important driver, then we expect the VaR estimates to still be substantially higher if borrower-dependent correlations are used. The results of the portfolio with uniform exposure size are given by Table 6 and Figure 9.

Comparing Figure 9 with Figure 6, we find that the substantial difference after January 2002 between the VaR of the market model and the other three models disappears. The IRB model now produces the highest VaR estimates, followed by the market model with sector-dependent correlations. An exception occurs in September 2003, which is the only month in which the market model produces a higher VaR than all three other models, even though the difference is small. It is



**Figure 9. Dynamic Credit VaR for Portfolios with Homogenous Exposure Size**

This figure shows the evolution of the VaR for the market model, the sector model and the IRB model which are specified as shown in Table 4. The “Market model (sec. corr.)” is a market model with median sector-specific asset correlations. Exposure size is homogeneous. The names of months from January 1998 until February 2004 refer to the end of 24-months time windows.



**Table 6**  
**Descriptive VaR Statistics for the Market Model, the Sector Model and the IRB Model for Homogenous Exposure Size**

Model	Market Model	Market Model	Sector Model	IRB Model
Correlation depends on ...	Borrower	Sector	Sector	PD
Max	7.25%	7.02%	6.19%	6.98%
Mean	3.51%	4.00%	3.35%	5.14%
Median	2.74%	3.18%	2.69%	5.03%
Standard deviation	1.69%	1.52%	1.38%	1.26%

an indication that simulation noise may indeed play a role in explaining high VaR values of the market model, but it is obviously only a secondary role compared with the impact of the correlation between borrower size and PD.

With the exception of the market model, the mean and median VaR are higher in Table 9 than the corresponding values in Table 6. We attribute this result to the negative correlation between borrower size and PD: Since large borrowers exhibit, on average, lower PDs, VaR values should be higher in a portfolio with homogenous borrower size in which this effect no longer applies. Table 7 gives sample statistics of correlation coefficients, measuring the dependency between PD and exposure (or borrower) size.

**Table 7**  
**Summary Statistics of Correlation Coefficients for Exposure Size and Probability of Default (EDF)**

This table shows the Pearson Correlation Coefficient, Spearman Rank Correlation Coefficient and Kendall's Tau. The sample consists of time series of 74 monthly observations of exposure size and the MKMV probability of default (EDF) from January 1998 until February 2004 .

Statistic	Pearson	Spearman	Kendall
Max	-0.05	-0.23	-0.16
Mean	-0.08	-0.29	-0.20
Median	-0.09	-0.30	-0.21
Min	-0.10	-0.33	-0.22
Standard Deviation	0.02	0.03	0.02

In summary, comparing the simulation results for two portfolios which are homogeneous and heterogeneous in terms of borrower size suggests that a positive correlation between borrower size and correlation with the systematic risk factor explains the higher VaR estimates in Figure 6 for the market model. This effect also dominates the VaR impact of a negative correlation between size and PD, which works in the opposite direction. This finding has important implications for credit risk modeling, as it suggests that the VaR of a credit portfolio can easily be underestimated if the positive dependence between the correlation (with the systematic risk factor) and the borrower size is not accounted for.

The VaR results for the market model with sector-dependent correlations and the sector model are quite similar. This becomes immediately clear not only from the aggregate statistics in Table 4 but also from the strong co-movement and similar level in Figures 6 and 9. The relative difference in VaR in the case of heterogeneous portfolios, for example, is only 5% for the mean and less than 2% for the median. The number of factors, therefore, which is the only difference between these two models, appears to play a relatively minor role. This finding for the impact of the number of factors is consistent with recent results obtained by Tarashev and Haibin (2007).

Finally, we return to the third question: why VaR, as measured by the IRB model, is overall in the same range as in the case of the sector model. A potential reason is offered by the way how the IRB model was originally calibrated. As its calibration was carried out using standard industry portfolio models, which structurally resemble the sector model, it comes as no surprise that the IRB model produces overall similar results, at least for a typical portfolio with heterogeneous exposure size.

Figure 6 also shows that the VaR of the IRB model increases more smoothly over time. At the peaks of the VaR cycle, both market models produce higher VaR estimates. This may be mainly due to the negative PD dependency of the asset correlations in the IRB model. Since higher PDs – or EDFs in this case – *ceteris paribus* reduce the asset correlation, the steepness of VaR as a function of the EDF is substantially reduced. An important reason why a PD dependency of the asset correlation was originally introduced in the Basel II formula – apart from empirical evidence – was the desire to reduce procyclical effects. The evolution of the VaR in the IRB model in Figure 6 demonstrates – at least for the underlying portfolio – that the IRB model can indeed substantially reduce the fluctuation over time compared

with traditional portfolio factor models.

## 5. Summary and Conclusions

In this paper, we estimate asset correlations from the time series of asset returns, based on the MKMV model, and we analyze their impact on the aggregate credit risk of a hypothetical loan portfolio. This portfolio comprises a large sample of listed European non-financial firms. Our sample covers eight years with monthly observations. We compare the time-varying individual correlation estimates in a market model and sector-specific estimates in a sector model and analyze their impact on the economic capital required for credit portfolio risk.

Overall, our analysis of asset correlations for both models reveals a level in line with previous studies such as Lopez (2004). We find that the median of the asset correlations in the sector model (12.3%) is only moderately higher than in the market model (10.1%). This result seems to be due in part to the fact that our sample contains very large firms, which cannot always be uniquely assigned to a single industry sector. Therefore, a considerable number of the firms in our sample are affected by the cyclical developments in several industries at the same time. The relatively small number of 6 sectors may also play a role since it suggests a considerable heterogeneity inside a sector which may reduce correlations. Moreover, we find substantial fluctuations in asset correlations and that it is material to consider time-varying asset correlations when estimating credit portfolio risk.

We also find that, across sectors, the inferred asset correlations exhibit a similar pattern, with the Telecom sector as the only main exception. Furthermore, a comparison of the evolution of asset correlations and EDFs reveals few similarities. For our findings, a caveat is that a finer sector classification may lead to more precise estimates, but this robustness check is not feasible due to data constraints.

The relatively minor differences between the inferred asset correlations in the market and the sector model motivate the use of a sector model in which asset correlations are only needed as averages at a sector level. These lower data requirements in terms of asset correlations can greatly simplify the implementation of the model in practice. Accordingly, we carry out a portfolio analysis with borrower-dependent

asset correlations for the market model and sector-specific correlations for the sector model. We also apply the Basel II IRB model. We find that the VaR fluctuates substantially over time for the market and the sector model. Furthermore, we find that the variation is driven by both changes in the EDFs and the asset correlations.

Simulation results for a portfolio that is heterogenous in terms of borrower size (which is set equal to exposure size) reveals that the VaR of the market model (with borrower-dependent correlations) is substantially higher than for the sector model. As this distance in VaR disappears for the homogenous portfolio, a positive correlation between borrower size and correlation with the systematic risk factor emerges as the reason for the higher VaR estimates of the market model for the heterogenous portfolio. This effect also dominates the VaR impact of a negative correlation between size and PD, which works in the opposite direction. This finding has important implications for credit risk modelling as it suggests that it is desirable to use accurate, borrower-dependent asset correlations as inputs to the model whenever available. In the case of the studied heterogenous portfolio, it would have been more appropriate to apply a single-factor model with borrower-dependent correlations than a multi-factor model with sector-dependent correlations. Since the reason, i. e. the negative dependency of correlation on borrower size, disappears for the homogenous portfolio, it can be argued that this result loses applicability if the portfolio becomes very fine-grained and differences in size between exposures become negligible. If borrower size and exposure size are not perfectly matched as in our example portfolios, then the result will also be diluted to some extent.

Comparing the market model and the sector model with the IRB model, we find that the VaR of the IRB model is more stable over time, which is due mainly to the smoothing effect of the hard-wired negative dependency of asset correlations on PD. This result is encouraging with respect to the discussion on procyclicality of the IRB model vs. internal models. A comparison in levels produces diverse results. For the (arguably more realistic) portfolio with heterogenous exposure size, the IRB model matches the models with sector-dependent correlations reasonably well in terms of VaR. In the case of a more fine-grained portfolio with homogenous exposure size, it produces overall substantially more conservative risk estimates, although at the peaks of credit risk in the observation period the distance from the other models effectively disappears.

From our analysis, several issues require further research. A finer sector classification

may reveal a higher level of risk-sensitivity in the sector model. As the sector model is substantially easier to handle than the market model and the availability of firm-specific correlations is problematic in everyday banking practice, the sector model has practical appeal. In this case, however, banks should consider the use of sufficiently high asset correlations to avoid underestimating credit portfolio risk. Asset correlations taken from high quantiles may also be useful for stress-testing purposes, for example.

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