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Modelling dynamic portfolio risk using risk drivers of elliptical processes

Rafael Schmidt
(Universität zu Köln)

Christian Schmieder
(Deutsche Bundesbank)

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Deutsche Bundesbank, Wilhelm-Epstein-Strasse 14, 60431 Frankfurt am Main,
Postfach 10 06 02, 60006 Frankfurt am Main

Tel +49 69 9566-1
Telex within Germany 41227, telex from abroad 414431

Please address all orders in writing to: Deutsche Bundesbank,
Press and Public Relations Division, at the above address or via fax +49 69 9566-3077

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Abstract

The situation of a limited availability of historical data is frequently encountered in portfolio risk estimation, especially in credit risk estimation. This makes it, for example, difficult to find temporal structures with statistical significance in the data on the single asset level. By contrast, there is often a broader availability of cross-sectional data, i.e., a large number of assets in the portfolio. This paper proposes a stochastic dynamic model which takes this situation into account. The modelling framework is based on multivariate elliptical processes which model portfolio risk via sub-portfolio specific volatility indices called portfolio risk drivers. The dynamics of the risk drivers are modelled by multiplicative error models (MEM) - as introduced by Engle (2002) - or by traditional ARMA models. The model is calibrated to Moody's KMV Credit Monitor asset returns (also known as firm-value returns) given on a monthly basis for 756 listed European companies at 115 time points from 1996 to 2005. This database is used by financial institutions to assess the credit quality of firms. The proposed risk drivers capture the volatility structure of asset returns in different industry sectors. A characteristic temporal structure of the risk drivers, cyclical as well as a seasonal, is found across all industry sectors. In addition, each risk driver exhibits idiosyncratic developments. We also identify correlations between the risk drivers and selected macroeconomic variables. These findings may improve the estimation of risk measures such as the (portfolio) Value at Risk. The proposed methods are general and can be applied to any series of multivariate asset or equity returns in finance and insurance.

Key words: Portfolio risk modelling, Elliptical processes, Credit risk, multiplicative error model, volatility clustering, Moody's KMV Credit Monitor database.

JEL classification: C51, C16, C13

Non-technical summary

Over the past years, the availability of data for financial analysis in general and portfolio risk analysis in particular has substantially improved. This situation enables the use of more sophisticated methods for portfolio management and risk analysis and has attracted many scholars from the industry, academia, and banking supervision.

The present research project proposes a multidimensional stochastic dynamic model that identifies portfolio risk drivers via volatilities in two dimensions, over time and across industry sectors. The identification and the need of modelling volatility dynamics in financial data goes (at least) back to the research by the Nobel laureate Robert Engle and has gained importance during the last two decades. The volatility is often referred to as the key driver of risk in a financial portfolio, and many performance measures or risk measures express the amount of risk via the volatility.

The model is applied to market-based credit risk data (monthly asset returns also known as firm-value returns) covering a limited period of time but comprising a large number of assets. The proposed method is particularly useful in the situation of a broad availability of cross-sectional data, i.e., a large number of assets in the portfolio. The data are obtained from the Moody's KMV Credit Monitor database.

It is shown that the model is able to identify volatility patterns that remain otherwise hidden on the single-firm level. In this way, insights into the underlying factors which drive portfolio risk are possible. A characteristic temporal structure of the risk drivers, cyclical as well as a seasonal, is found across all industry sectors. In addition, each risk driver exhibits idiosyncratic developments. We also identify correlations between the risk drivers and selected macroeconomic variables. The findings may be used for the improvement and validation of Value at Risk estimates. The proposed methods are general and can be applied to any series of multivariate asset or equity returns in finance.

Nicht-technische Zusammenfassung

In den letzten Jahren hat sich die Verfügbarkeit von Mikrodaten für finanzwirtschaftliche Untersuchungen, speziell im Bereich des Portfoliomanagements, deutlich verbessert. Im Zuge dessen ist das Interesse von Praktikern und Wissenschaftlern an komplexen stochastischen Methoden zur Anwendung im Bereich des Portfoliomanagements gewachsen. Nach wie vor besteht jedoch im Bereich der Kreditrisikoanalyse Nachholbedarf bei der Modellierung und Validierung von Portfoliorisiken.

Die Autoren entwickeln ein mehrdimensionales stochastisches Modell, um dynamische Volatilitätsstrukturen zu identifizieren, welche das Portfoliorisiko treiben. Die Identifizierung und Modellierung von Volatilitäten in Finanzmarktzeitreihen spielt nicht zuletzt seit den bahnbrechenden Forschungsarbeiten des Nobelpreisträgers Robert Engle eine grundlegende Rolle für die Analyse von Portfoliorisiken. Die Volatilität wird oft als der wichtigste Treiber des Portfoliorisikos bezeichnet und eine Vielzahl von bekannten Risiko- und Performancemaßen beschreibt das Risiko explizit anhand der Volatilität.

Das vorgeschlagene stochastische Modell wird auf eine Zeitreihe von Kreditrisikodaten (monatliche Asset Renditen bzw. so genannte Firmwert Renditen) der Moody's KMV Credit Monitor Datenbank angewendet. Dieser Datensatz besitzt zwar eine beschränkte Historie, umfaßt jedoch eine große Anzahl von Firmen. Das stochastische Modell ist dabei besonders für diese Art von Datensituation geeignet.

Es zeigt sich, dass mit Hilfe des Modells Volatilitätsstrukturen erkannt werden können, die ansonsten auf Einzelfirmenebene verborgen blieben. Hierbei sind besonders konjunkturbedingte Strukturen, unterjährige Saisonalitäten und branchenspezifische Charakteristiken von Interesse. Weiterhin werden Korrelationen zwischen den Risikotreibern und verschiedenen makroökonomischen Indikatoren festgestellt. Die Ergebnisse können verwendet werden, um die Modellierung und Schätzung von Portfoliorisiken zu verbessern. Darüber hinaus eignet sich die Methode generell auch für andere finanzwirtschaftliche Fragestellungen, im Rahmen derer Portfoliorisiken modelliert und analysiert werden.

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Modelling dynamic portfolio risk using risk drivers of elliptical processes[1]

1 Introduction

The motivation of this paper is based on a common situation in portfolio risk modelling, in particular credit risk modelling: time series data are sparse, while cross-sectional data are broadly available.[2] The limitation of historical data may have various reasons, e.g., a limited collection or observation horizon, or a structural break due to a change in the collection methodology. Systematic data collection in financial institutions principally started during the last decade when sophisticated IT and database systems have been established. The database used in this study - Moody's KMV Credit Monitor (in short: MKMV) database - is one of the most valuable sources for credit risk modelling and credit risk analysis. The database comprises credit risk relevant data of listed companies, such as credit exposures, expected default frequencies (EDFs), asset values or asset returns. Amongst others, this database has been used to calibrate the Basel II one-factor credit risk model, see Basel Committee on Banking Supervision (2006). MKMV asset returns are frequently used by financial institutions to estimate the asset correlations in structural credit risk models, cf. Lopez (2004) and Berndt et al. (2005). The data history of this database goes back to the beginning of the 1990s, but exhibits a structural break around the year 1995/1996. Thus for consistency reasons, the shorter time interval - from 1996 to 2005 - comprising only 115 monthly time observations for 756 listed European companies is considered. We propose a high-dimensional stochastic model which takes this data situation into account and is capable of capturing the temporal structure of portfolio risk via so-called risk drivers.

These risk drivers model the behavior of the sub-portfolio specific volatilities on an aggregated level. The identification and the need of modelling volatility dynamics in financial data goes (at least) back to the seminal work by Engle (1982) and Bollerslev (1986) and has gained importance during the last two decades.[3] The volatility is often referred to as the key driver of risk in a financial portfolio, and many performance measures or risk measures - such as the Sharpe ratio or the Value at Risk (VaR) in a Gaussian model - express the amount of risk via the volatility, cf. Jorion (2006). The general multivariate stochastic process, proposed in this paper, models the volatility using so-called risk drivers. These risk drivers enter the model as multiplicative factors and can be retrieved from the data. The stochastic process is termed *elliptical process* if it includes one single risk driver or *generalized elliptical process* if it includes multiple risk drivers, which are for example industry-specific. The risk drivers deliver information about the volatility in the portfolio which is otherwise not visible on a disaggregated single obligor level. This information is

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[2]By cross-sectional we refer to the number of assets in the portfolio.

[3]For an overview see Alexander (2001) and references therein.

then incorporated into the portfolio risk estimation. Furthermore, risk managers may use the risk drivers as indicators of portfolio risk attributed to the volatility over time. An advantage of the proposed model is its fast random number generation. The model can be seen as a time-dynamic extension of the time-static model considered in Bingham et al. (2003).

The dynamics of the portfolio risk drivers are modelled by so-called multiplicative error models (MEM), as introduced by Engle (2002), and extended and applied by Chou (2005) and Engle and Gallo (2006). These dynamics capture the amount of volatility clustering inherent in the data via a GARCH-type structure. Alternative models such as nonlinear regression or trend models are also considered. A limiting argument even justifies the usage of traditional ARMA models for large portfolios.

Concerning the estimation of the multivariate dependence structure of the model, we suggest a simple correlation estimator which is based on ranks and is therefore robust. The estimator is called Blomqvist's beta and is a measure of non-linear dependence that is solely determined by the copula of the multivariate distribution. A functional relationship between Blomqvist's beta and the dispersion matrix of the generalized elliptical process shows that this measure of dependence is (asymptotically) consistent. In order to reduce the number of (correlation) parameters, we utilize the correlation structure of a one-factor model which is also the building block of the IRB portfolio model of Basel II, see Basel Committee on Banking Supervision (2006).

The set of competing multivariate volatility models can be divided into three categories. The first category consists of multivariate GARCH models and related types. Most prominent members are the DVEC(p,q) models (Bollerslev et al. 1988), the matrix-diagonal models (Ding 1994, Bollerslev et al. 1994), the BEKK models (Engle and Kroner 1995), the CCC model (Bollerslev 1990) or the PGARCH model (Alexander 1998). Without further restrictions, all these models estimate the volatility dynamics on the level of the univariate margins, however, for the (short horizon) risk data considered in this paper, no significant volatility structures can be found on this level. A common drawback of the unrestricted models is that the number of parameters to be estimated increases fast with increasing dimension and, thus, increases the forecast uncertainty - see also Gouriéroux (1997) for an overview. We also mention EWMA models which are applied in practice, e.g., in the RiskMetrics proposal; see also Foster and Nelson (1996). In the univariate context, EWMA models correspond to IGARCH models. The second category are stochastic volatility models which model the volatility as a latent (unobserved) random source, see e.g. Tsay (2002) and references therein. They differ from our approach as we actually retrieve the risk drivers from the data. The third category of volatility models refers to the direct modelling of (univariate) portfolio returns as in McNeil and Frey (2000). This approach is useful in terms of dimensionality reduction and it certainly captures relevant volatility characteristics of the underlying assets. However, it ignores multivariate aspects which are, for example, necessary for portfolio allocation or the risk analysis of sub-portfolios.

In the empirical study, the model is calibrated to Moody's KMV Credit Monitor asset returns (also known as firm-value returns) for 756 listed European companies observed at 115 monthly time points from 1996 to 2005. A main finding is that the temporal structure of the volatility is statistically significant on the risk driver level only, whereas it is insignificant on the level of the single assets due to the limited data history. Furthermore, we observe temporal structures of the risk drivers which are similar across all industry sectors both in a seasonal and a cyclical context, with each sector's risk driver also exhibiting idiosyncratic developments. We also find empirical correlations with selected macroeconomic variables.

These findings may be used to improve the quality of risk measurement via time-dynamic VaR models.

The paper is organized as follows. Section 2 introduces elliptical processes and the corresponding risk drivers. We start with a single risk driver in Section 2.1. Particular emphasis is placed on the interpretation of the risk driver and related examples of multivariate distributions. Section 2.2 models the dynamics of the risk driver using a multiplicative error model (MEM) and provides possible extensions. Thereafter, Section 2.3 generalizes the models to multiple risk drivers and the subsequent sections elaborate its estimation and the corresponding random number generation. In Section 3 the model is applied to credit risk analysis. In particular, we calibrate the model to MKMV asset return data - described in Section 3.1 - and extract and model the risk drivers in Section 3.2. The following sections examine the empirical correlation of the risk drivers with selected macroeconomic variable and indicate their usage for portfolio VaR estimation. Section 4 concludes.

2 Elliptical processes

2.1 Elliptical processes with single risk driver

Let T be some countable index set representing time, i.e. set $T = \{\dots, -1, 0, 1, \dots\}$. A d-dimensional stochastic process $\mathbf{X} = (\mathbf{X}_t)_{t\in T}$ is called *elliptically contoured process* (in short: *elliptical process*) if its margins $\mathbf{X}_t$, for fixed $t \in T$, have the stochastic representation:

$$\mathbf{X}_t \stackrel{d}{=} \mathbf{m} + R_t A' \mathbf{U}_t, \tag{1}$$

where $\mathbf{m}$ is a d-dimensional *location vector* and $A'A = \Sigma$ is a symmetric positive-definite $d \times d$ *dispersion matrix*. The d-dimensional random vector $\mathbf{U}_t$ is uniformly distributed on the $(d-1)$-dimensional unit sphere $\mathbb{S}^{d-1} := \{x \in \mathbb{R}^d : ||x|| = 1\}$, where $||\cdot||$ denotes the Euclidean norm, and $R_t \geq 0$ is a one-dimensional random variable. The collection of random variables $\{R_t \mid t \in T\}$ is stochastically independent of $\{\mathbf{U}_t \mid t \in T\}$. Thus $\mathbf{X}_t$, for fixed $t \in T$, possesses an elliptically contoured distribution, which is typically defined via a density function having a quadratic form as argument; for a review see Fang et al. (1990).

The random variable R_t, for fixed t, describes the radial part of $\mathbf{X}_t$ if Σ equals the identity matrix I and if the location vector $\mathbf{m} = \mathbf{0}$. In that case, $\mathbf{U}_t$ denotes the *angle vector*, since a realization of $\mathbf{U}_t$ corresponds to the angle of $\mathbf{X}_t$ (measured on the unit sphere). In particular, the following relationship holds

$$R_t \stackrel{d}{=} ||(A')^{-1}(\mathbf{X}_t - \mathbf{m})|| \qquad \text{and} \qquad \mathbf{U}_t \stackrel{d}{=} \frac{(A')^{-1}(\mathbf{X}_t - \mathbf{m})}{||(A')^{-1}(\mathbf{X}_t - \mathbf{m})||}. \tag{2}$$

If $E(R_t^2) < \infty$, then the matrix $c_t\Sigma$ corresponds to the variance-covariance matrix of $\mathbf{X}_t$ with scaling factor $c_t = E(R_t^2)/d > 0$. Thus, the variance-covariance matrix depends on the distribution of R_t which may be non-stationary.

Interpretation of R_t. Consider a portfolio comprising d assets, and let $\mathbf{X}$ describe the randomness of the d-dimensional asset returns. The process $R = (R_t)_{t\in T}$ is called the *risk driver* of $\mathbf{X}$ since it determines the degree of the overall volatility of $\mathbf{X}$ over time. More precisely, R is the random source which equally contributes to the volatility of each single-asset return and thus represents a driver of the overall volatility structure in the portfolio. Using R, one may model different temporal structures of the volatility, for example, *volatility clustering* - observed in many financial data - or *seasonal volatility structures* - found e.g. in high-frequency assets returns. In Section 3, we demonstrate that volatility clustering and seasonal volatilities are present in the KMV asset-return series. The main motivation of considering the risk driver R comes from formula (2), which implies that - except for the estimation error of A and $\mathbf{m}$ - the distribution of R can directly be retrieved from the observations of $\mathbf{X}$.

The collection of $\{\mathbf{U}_t \mid t \in T\}$ is assumed to be mutually independent. This assumption appears to be reasonable in the present setting of few temporal observations but broad availability of cross-sectional data (thus, $\mathbf{U}_t$ is high dimensional). Besides, multivariate statistical tests for temporal correlation of the $\mathbf{U}_t$ will have low power in this setting. Alternatively, $\mathbf{U}_t$ could be modelled as a random walk on the unit-sphere, cf. Bingham (1972).

Examples. Elliptical processes are constructed by choosing different risk drivers R. In portfolio risk modelling, the tail behavior of the distribution of $\mathbf{X}$ is usually a key factor during the model-selection process. Heavy tails assign a higher probability to the (joint) occurrence of extreme events - such as extremely negative asset returns. In the following, we specify three distributions of R_t which yield either light tails, semi-heavy tails or heavy tails of $\mathbf{X}$. The temporal specification of R is left to the next section.
i) **Heavy tails.** Let R_t^2/ν_1 be F-distributed with ν_1 and ν_2 degrees of freedom, thus, R_t has density

$$f_R(x) = \frac{\nu_2^{\nu_2/2}}{B(\nu_1/2, \nu_2/2)} \frac{2x^{\nu_1-1}}{(\nu_2 + x^2)^{(\nu_1+\nu_2)/2}}, \qquad x > 0, \nu_1, \nu_2 > 0$$

with beta-function B. The tail decay of f_R (at infinity) is that of a power law, i.e. $f_R(x) \sim ax^{-b}, b > 0$, as $x \to \infty$. The tail decay of the univariate margins of $\mathbf{X}_t$ possesses the same size, see Prop. 3.4 in Schmidt (2002). If $\mathbf{X}_t$ is a d-dimensional random vector, the particular choice $\nu_1 = d$ yields a d-dimensional *Student's t-distribution* with ν_2 degrees of freedom for $\mathbf{X}_t$.
ii) **Semi-heavy tails.** Let R_t possess the Bessel-type density

$$f_R(x) = c\frac{x^{\nu-1}}{(1+x^2)^{\nu/4-\lambda/2}} K_{\lambda-\nu/2}(\alpha\sqrt{1+x^2}),$$
$$\text{with } c = \frac{\alpha^{\nu/2} 2^{1-d/2}}{\Gamma(\nu/2) K_\lambda(\alpha)}, \ x > 0, \lambda \in \mathbb{R}, \nu, \alpha > 0,$$

where K_λ denotes the modified Bessel-function of the third kind with index λ (see Magnus et al. (1966), pp. 65). The tail decay of f_R (at infinity) is exponential of order one, i.e. $f_R(x) \sim ax^b \exp(-cx), c > 0$, as $x \to \infty$, see Abramowitz and Stegun (1964), p.364, for the asymptotic expansion of K_λ for large arguments. The tail decay of the univariate margins of $\mathbf{X}_t$ is of the same size. If $\mathbf{X}_t$ is a d-dimensional random vector, the choice $\nu = d$ yields a d-dimensional *generalized hyperbolic distribution* for $\mathbf{X}_t$. We also refer to Barndorff-Nielsen and Blæsild (1981) who discuss semi-heavy tails of the univariate generalized hyperbolic distribution.
iii) **Light tails.** Let R_t be χ-distributed with ν degrees of freedom, having density

$$f_R(x) = \frac{1}{2^{(\nu/2-1)}\Gamma(\nu/2)} e^{-x^2/2} x^{\nu-1}, \qquad x > 0, \nu > 0.$$

Note that R_t^2 is χ^2-distributed with ν degrees of freedom. The tail decay of f_R (at infinity) is exponential of order two, i.e. $f_R(x) \sim ax^b \exp(-cx^2), c > 0$, as $x \to \infty$. The tail decay of the univariate margins of $\mathbf{X}_t$ possesses the same size. If $\mathbf{X}_t$ is a d-dimensional random vector, then the particular choice $\nu = d$ yields a *multivariate normal distribution* for $\mathbf{X}_t$.

2.2 Risk-driver dynamics

The risk driver R is a nonnegative valued stochastic process. Once the location $\mathbf{m}$ and the dispersion Σ are estimated (cf. Section 2.4), R can be extracted using formula (2). The temporal structure of R can be of any type, e.g., it may include deterministic trends, seasonal components, autoregressive components as well as volatility clustering. The following model is useful if $\mathbf{X}$ exhibits volatility clustering; it takes the nonnegativity of R into account.

MEM dynamics. The risk driver R is decomposed into a conditionally deterministic scale factor - evolving according to a GARCH-type equation - and a positive innovation term. This type of model is known as multiplicative error model (MEM) and has been introduced in Engle (2002), see also Engle and Gallo (2006).

Let $\mathcal{F}_t = \sigma\{R_s,\ s \le t\}$ denote the information of the process R up to time t. Then R_t takes the form

$$R_t = \mu_t \varepsilon_t \qquad \text{with } \mu_t \in \mathcal{F}_{t-1},\ \varepsilon_t \perp \mathcal{F}_{t-1}, \text{ and } \varepsilon_t \ge 0. \tag{3}$$

The $\{\varepsilon_t\}$ are independent and identically distributed (iid) with unit mean and variance σ^2, and the evolution of μ_t depends on an unknown parameter vector θ, i.e. $\mu_t = \mu_t(\theta)$. These conditions imply that $E(R_t \mid \mathcal{F}_{t-1}) = \mu_t$ and $Var(R_t \mid \mathcal{F}_{t-1}) = \sigma^2\mu_t^2$. The evolution of μ_t is modelled by some GARCH-type structure, which may also include asymmetric effects. For example, consider the GARCH(p,q)-type structure

$$\mu_t = \omega + \sum_{i=1}^{p} \alpha_i R_{t-i} + \sum_{j=1}^{q} \beta_j \mu_{t-j}, \qquad p, q \in \mathbb{N},\ \omega > 0, \alpha_i, \beta_j \ge 0\ \forall i, j. \tag{4}$$

The unconditional mean of R_t is then given by $E(R_t) = \omega/(1 - \sum \alpha_i - \sum \beta_j)$ for all $t \in T$. The choice of the distribution of ε_t essentially determines the (un)conditional distribution of $\mathbf{X}_t$. The example distributions stated in Section 2.1 yield a variety of possible distributions for ε_t. Initially, one should concentrate on R_t being conditionally χ-distributed or R_t^2/d being conditionally F-distributed, which yield a d-dimensional normal or Student's t-distribution for $\mathbf{X}_t|\mathcal{F}_{t-1}$, respectively.

Boosting the dimension. Let $\mathbf{X}_t$ be d-dimensional and $R_t^{(d)}$ be the related risk driver, indexed by dimension d. Suppose that - conditional on $\mathcal{F}_{t-1}$ - the risk driver $R_t^{(d)}$ is χ-distributed with d degrees of freedom. In case the dimension d is very large, which means that the number of assets in the portfolio is very large, the following Fisher approximation eases the statistical estimation. Note that the MEM model is not yet implemented in statistical packages. The Fisher approximation yields

$$R_t^{(d)} - \sqrt{d - 1/2} \xrightarrow{d} R_t^{(\infty)} \sim N(0, 1/2) \quad \text{as } d \to \infty.$$

Thus for large portfolios, the risk driver can be approximated by a non-centered normal distribution, whose negative values occur with negligible likelihood. A rule of thumb for a sufficiently good approximation is $d \geq 40$, see e.g. Severo and Zelen (1960) for empirical results and alternative approximations. An advantage of the approximation is that the innovations $\{\varepsilon_t\}$ - in the MEM model centered by $\sqrt{d - 1/2}$ - need not to be nonnegative anymore. Hence, traditional ARMA models represent a possible alternative for the risk driver dynamics. Our empirical study shows that for large portfolios the MEM model and the ARMA model yield similar results.

If - conditional on $\mathcal{F}_{t-1}$ - the risk driver $(R_t^{(d)})^2/d$ is F-distributed with $\nu_1 = d$ and ν_2 degrees of freedom, then the following approximation holds

$$\frac{R_t^{(d)}}{\sqrt{2}} \overset{d}{\simeq} \tilde{R}_t^{(d)} \quad \text{for large } d,$$

where $\tilde{R}_t^{(d)}$ has a noncentral Student's t-distribution with ν_2 degrees of freedom and centrality parameter $\sqrt{d - 1/2}$.

2.3 Generalized elliptical processes with multiple risk drivers

The elliptical process defined so far is driven by a single risk driver. This process is applicable if the d asset returns are equally distributed - except for a different dispersion or location. However, if we consider a portfolio consisting of assets which belong to different industries or geographical regions, the assumption of equally distributed returns may be violated. We therefore define generalized elliptical processes, which allow for different risk drivers in different sub-portfolios and which include elliptical processes as a special case.

A d-dimensional stochastic process $\mathbf{X} = (\mathbf{X}_t)_{t \in T}$ is called *generalized elliptical process* (with k sectors) if its margins $\mathbf{X}_t$, for fixed $t \in T$, have the following stochastic representation:

$$\mathbf{X}_t \overset{d}{=} \mathbf{m} + (R_{t,1}^* \mathbf{V}_{t,1}, R_{t,2}^* \mathbf{V}_{t,2}, \ldots, R_{t,k}^* \mathbf{V}_{t,k})' \tag{5}$$

with random vectors $\mathbf{V}_{t,1} = (V_{t,1}, \ldots, V_{t,j_1})$, $\mathbf{V}_{t,2} = (V_{t,j_1+1}, \ldots, V_{t,j_2})$, $\ldots$, $\mathbf{V}_{t,k} = (V_{t,j_{k-1}+1}, \ldots, V_{t,d})$ such that $\mathbf{V}_t = (\mathbf{V}_{t,1}, \ldots, \mathbf{V}_{t,k})' = A'\mathbf{U}_t$ and $\mathbf{U}_t$ is uniformly distributed on the $(d-1)$-dimensional unit sphere $\mathbb{S}^{d-1}$. In formula (5), the vector of vectors is understood as a d-dimensional vector, which is a slight abuse of notation. The collection of $\{\mathbf{V}_t \mid t \in T\}$ is assumed to be mutually independent. Moreover, the random variables $\{R_{t,i}^* \mid t \in T, i = 1, \ldots, k\}$ are stochastically independent of $\{\mathbf{V}_t \mid t \in T\}$. The temporal and contemporaneous (across the sectors i) dependence structure between the risk drivers $R_{t,i}^*$ can be of any type. For example, the contemporeneous dependence structure may take the form

$$R_{t,i}^* = f_i(R_t), \ i = 1, \ldots, k, \tag{6}$$

for some nonnegative increasing functions f_i and random variable R_t; similar to the model by Daul et al. (2003). The interpretation of this model is that the risk drivers $R_{t,i}^*$ are completely correlated across the sectors, but there impact per sector is of different magnitude. This approach allows, e.g., to model a different tail distribution per sectors.

Given the sector $i \in \{1, \ldots, k\}$, the following holds:

$$||(B_i')^{-1}(\mathbf{X}_{t,i} - \mathbf{m}_i)|| \overset{d}{=} ||R_{t,i}^* \mathbf{V}_{t,i}|| \overset{d}{=} R_{t,i}, \qquad i = 1, \ldots, k, \tag{7}$$

where $B_i'B_i = \Sigma^{(ii)}$, $\Sigma^{(ii)}$ is the i-th partition-matrix of Σ, $\mathbf{X}_{t,i} = (X_{t,j_{i-1}+1}, \ldots, X_{t,j_i})'$, and $\mathbf{m}_i$ is the i-th partition-vector of $\mathbf{m}$, corresponding to sector i. Further, $R_{t,i}$ denotes the radial variable or risk driver of the elliptical process $\mathbf{X}_{t,i} = (X_{t,j_{i-1}+1}, \ldots, X_{t,j_i})$. The relationship between $R_{t,i}$ and $R^*_{t,i}$ is

$$R_{t,i} \stackrel{d}{=} B_{t,i} \cdot R^*_{t,i}, \tag{8}$$

where $(B^2_{t,1}, \ldots, B^2_{t,k}) \sim D_k(j_1/2, (j_2 - j_1)/2, \ldots, (j_k - j_{k-1})/2)$ is Dirichlet distributed. Moreover, the collection $\{B_{t,i}\}$ is stochastically independent of $\{R^*_{t,i}\}$. The proof of formula (8) is analogue to the proof of Theorem 2.6 in Fang et al. (1990), p. 33; see also Chapter 1.4 in this reference.

2.4 Model Estimation

A *two-stage estimation* is utilized in order to estimate the distribution of $\mathbf{X}$. First, we estimate the (time invariant) parameters $\mathbf{m}$ and Σ, and, second, we identify the distribution of the risk drivers $R_{t,i},\ i = 1, \ldots, k$.

2.4.1 Dispersion and location

In the situation of only few temporal observations but a broad availability of cross-sectional data, the large number of parameters in the dispersion matrix Σ would yield an over-specification of the model. One way to reduce the number of parameters is the consideration of factor models, which is frequently done in portfolio risk modelling, see e.g. the internal model proposed by the Basel Committee on Banking Supervision (2006). Let us first assume that $\mathbf{X}$ is an elliptical process with single risk driver R, representing the asset returns of a portfolio with k sectors (or sub-portfolios). A simple one-factor model is

$$(X_{t,j} - m_j)/\sqrt{\Sigma_{jj}} \stackrel{d}{=} \omega_j Z_t + \sqrt{1 - \omega_j^2}\,\varepsilon_{t,j} \quad \text{for} \quad j = 1, \ldots, d,\ t \in T, \tag{9}$$

where the ω_j are *equal* if X_j belongs to the same sector. Here, we assume that the $(d+1)$-dimensional vector $(Z_t, \varepsilon_{t,1}, \ldots, \varepsilon_{t,d})'$ has unit dispersion I, zero location, and belongs to the same family of elliptical distributions as $\mathbf{X}_t$. The correlation entries of the corresponding dispersion matrix Σ take the form $\rho_{ij} = \Sigma_{ij}/\sqrt{\Sigma_{ii}\Sigma_{jj}} = \omega_i\omega_j$ if $i \neq j$. Note that this parameter reduction implies that the ρ_{ij} coincide if i and j, respectively, belong to the same sector. Estimators of ρ_{ij} - within this factor model - have been discussed in the literature, see e.g. Gordy (2000) and references therein. They are either based on ML-procedures or Pearson's sample covariance. However, if $\mathbf{X}$ is a generalized elliptical process with multiple risk drivers, these estimators may not be suitable as they are not necessarily (asymptotically) consistent. An example is given in table 1, where we estimate ρ_{ij} of a generalized elliptical process by Pearson's sample correlation.

Because of these findings, we provide an alternative estimator for the correlation parameters, which is (asymptotically) consistent. First, we make the following observation: Let $X_{t,i}$ and $X_{t,j}$ be the i-th and j-th margin of $\mathbf{X}_t$ belonging to a generalized elliptical process $\mathbf{X}$. If $P\left(X_{t,j} = \tilde{x}_{t,j}\right) = 0$ for all $j = 1, \ldots, d$, then

$$\begin{aligned} P\left(X_{t,i} < \tilde{x}_{t,i}, X_{t,j} < \tilde{x}_{t,j}\right) &= P\left(R^*_{t,k_i} V_{t,i} < 0, R^*_{t,k_j} V_{t,j} < 0\right) \\ &= P\left(V_{t,i} < 0, V_{t,j} < 0\right) \\ &= P\left(Z_{t,i} < 0, Z_{t,j} < 0\right), \end{aligned} \tag{10}$$

where $\tilde{x}_{t,i} = m_i$ is the median of $X_{t,i}$ and $\mathbf{Z}_t \sim N_d(\mathbf{0}, \Sigma)$. For the last equality, we utilized the observations of example iii) in Section 2.1. Thus, the orthant probabilities of $(X_{t,i}, X_{t,j})'$ are invariant with respect to the risk driver R. However, there exists a well-known relationship between the orthant probabilities of a d-dimensional normal distribution and the correlation parameters $\rho_{ij} = \Sigma_{ij}/\sqrt{\Sigma_{ii}\Sigma_{jj}}$:

$$4P\left(X_{t,i} < \tilde{x}_{t,i}, X_{t,j} < \tilde{x}_{t,j}\right) - 1 = 4P\left(Z_{t,i} < 0, Z_{t,j} < 0\right) - 1 = 2\arcsin(\rho_{ij})/\pi. \qquad (11)$$

The left-hand side of equation (11) corresponds to the population version of Blomqvist's beta - denoted by β_{ij} - which is a rank-based dependence measure introduced in Blomqvist (1950). The sample version of Blomqvist's β_{ij} between the i-th and j-th margin of $\mathbf{X}$ is defined by

$$\hat{\beta}_{ij} = \frac{2}{n}\sum_{t=1}^{n}\left(\mathbf{1}_{\{\hat{U}_{t,i}^{(n)} \le 1/2,\, \hat{U}_{t,j}^{(n)} \le 1/2\}} + \mathbf{1}_{\{\hat{U}_{t,i}^{(n)} > 1/2,\, \hat{U}_{t,j}^{(n)} > 1/2\}}\right) - 1,$$

where $\hat{U}_{t,i}^{(n)} = \frac{1}{n}(\text{rank of } X_{t,i} \text{ in } X_{1,i}, \ldots, X_{n,i})$; for related results on asymptotic normality and efficiency we refer to Schmid and Schmidt (2006). Thus for a generalized elliptical process, an asymptotically consistent and robust estimator of ρ_{ij} is given by

$$\hat{\rho}_{ij} = \sin(\pi\hat{\beta}_{ij}/2). \qquad (12)$$

The correlation parameters of $\mathbf{X}$ within one sector and between two sectors are then derived as the average of the $\hat{\rho}_{ij}$ which belong to the one sector and the two sectors, respectively. The positive definiteness of the resulting dispersion matrix - if it is not already given - can be obtained by using techniques proposed e.g. in Rousseeuw and Molenberghs (1993).

Table 1 illustrates the magnitude of the bias if ρ_{ij} is estimated using Pearson's sample correlation. Though the bias is usually small it may become large if one marginal distribution is light tailed while another marginal distribution is heavy tailed.

The location vector $\mathbf{m}$ and the dispersion parameters Σ_{jj}, respectively, are e.g. estimated by the sample median and the (trimmed) sample variance; these estimators are consistent if the risk drivers are ergodic. Further parameter restrictions could be imposed on the (volatility) parameters Σ_{jj}. We set $\hat{\Sigma}_{ij} := \hat{\rho}_{ij}\sqrt{\hat{\Sigma}_{ii}\hat{\Sigma}_{jj}}$ if $i \neq j$.

Approximate realizations of the risk drivers $R_{t,i}$ are now obtained using formula (7). In particular,

$$\hat{R}_{t,i} := ||(\hat{B}_i')^{-1}\big(\mathbf{X}_{t,i} - \hat{\mathbf{m}}_i\big)||, \qquad i = 1, \ldots, k \qquad (13)$$

where $\hat{B}_i'\hat{B}_i = \hat{\Sigma}^{(ii)}$. Suitable stochastic processes may now be identified for the time series $\hat{R}_i = (\hat{R}_{t,i})_{t\in T}$. The estimation of an MEM model - given in Section 2.2 - is discussed next.

2.4.2 The risk driver

Let $(R_t)_{t=1,\ldots,n}$ denote the (approximate) observations of the risk driver R. In order to ease the presentation, we assume that R is the (single) risk driver of a d-dimensional elliptical process. Let R evolve according to the MEM model given in formula (3). Suppose that $R_t|\mathcal{F}_{t-1}$ is χ-distributed with $\nu > 0$ degrees of freedom yielding a d-dimensional normal distribution for $\mathbf{X}_t|\mathcal{F}_{t-1}$ if $\nu = d$. This choice is closely related to the error distribution considered in Engle and Gallo (2006). These authors adopt a *Gamma*-distribution for

Table 1: Estimated correlation parameter ρ using Blomqvist's $\hat{\beta}$ as in formula (12) - this estimator is denoted by $\hat{\rho}_B$ - or using Pearson's sample correlation $\hat{\rho}_P$. The underlying data have been generated from a bivariate generalized elliptical process - as given in formula (5) - with $X_{t,1}$ (and $X_{t,2}$) being t-distributed random variables with $\alpha_1 = 7$ (and α_2) degrees of freedom - which are independently drawn across time. The sample length is one million.

original parameter	α_2	2.1	3	4	5	6	7
$\rho = 0.8$	$\hat{\rho}_B$	0.801	0.800	0.798	0.799	0.800	0.800
	$\hat{\rho}_P$	0.577	0.742	0.787	0.797	0.800	0.800
$\rho = 0.5$	$\hat{\rho}_B$	0.500	0.501	0.501	0.499	0.500	0.500
	$\hat{\rho}_P$	0.320	0.460	0.492	0.496	0.500	0.500
$\rho = 0.2$	$\hat{\rho}_B$	0.199	0.199	0.200	0.198	0.199	0.202
	$\hat{\rho}_P$	0.106	0.184	0.196	0.200	0.201	0.199

Note that Pearson's correlation ρ is not well defined for $\alpha_2 \leq 2$.

the error term, i.e. $\varepsilon_t|\mathcal{F}_{t-1} \sim Gamma(\phi, \phi),\ \phi > 0$. Note that the χ^2-distribution - not the χ-distribution - is a special case of the *Gamma*-distribution. Since our primary focus is rather on multivariate modelling, we adopt the χ-distribution for R which yields the multivariate normal distribution as a special case for $\mathbf{X}$. More precisely, we assume that

$$c \cdot \varepsilon_t \mid \mathcal{F}_{t-1} \sim \chi(\nu) \qquad \Longrightarrow \qquad R_t \mid \mathcal{F}_{t-1} \sim (\mu_t/c) \cdot \chi(\nu),\ \nu > 0, \tag{14}$$

with $c = \sqrt{2}\Gamma\{(\nu+1)/2\}/\Gamma(\nu/2)$. The scaling of ε_t by c ensures the identifiability of the model, i.e. $E(\varepsilon_t|\mathcal{F}_{t-1}) = 1$. Note that for $\nu = d$, $\mathbf{X}_t|\mathcal{F}_{t-1}$ possesses a multivariate normal distributions.

Under assumption (14), the contribution of a generic observation r_t to the log-likelihood function ℓ_t is

$$\ell_t = \ln L_t = \left(1 - \frac{\nu}{2}\right)\ln 2 - \ln\Gamma\left(\frac{\nu}{2}\right) + \nu \ln c - \nu \ln \mu_t + (\nu - 1)\ln r_t - \left(\frac{c}{\mu_t}\right)^2 \frac{r_t^2}{2}.$$

Using this formula, one can calculate the contribution of r_t to the score, the Hessian, and the first order conditions for the ML estimation of the MEM model.

In case the densities of $R_t|\mathcal{F}_{t-1},\ t \in T$, belong to the same exponential family

$$f(r_t|\mathcal{F}_{t-1}) = \exp[\nu\{r_t\vartheta_t - b(\vartheta_t)\} + d(r_t, \nu)],$$

the MEM model is a member of the family of Generalized Linear Autoregressive Moving Average (GLARMA) models as pointed by Cipollini et al. (2006); for more background on this family we refer to Benjamin et al. (2003) and references therein.

2.5 Random number generation

An advantage of generalized elliptical processes is their feasible simulation even in very high dimensions. This is because the simulation reduces more or less to the simulation of the risk drivers and, thus, eases the *curse of dimensionality.* A simulation algorithm for generating paths of generalized elliptical processes is given next. Assume that the location vector $\mathbf{m}$ and the dispersion matrix Σ are known (or estimated, respectively). Note that the generation of sample paths from a single risk driver is a univariate problem. In this case, the distribution of the risk drivers R and R^* in formula (8) coincides. The case of multiple risk drivers is more involved. For the time being assume that the dynamics of the risk drivers R_i and R_i^* - as specified in (8) - are given as well as the related generation algorithms.

Algorithm of generating pseudo-random paths of generalized elliptical processes:

Step 1 Calculate $\Sigma = A'A$, e.g., using Cholesky decomposition.

Step 2 Sample a path of length n from $(R^*_{t,1}, \dots, R^*_{t,k})$.

Step 3 Sample d times n independent random numbers $Z_{t,1}, \dots, Z_{t,d}$, $t = 1, \dots, n$, from a univariate standard-normal distribution $N(0, 1)$.

Step 4 Set $\mathbf{Z}_t = (Z_{t,1}, \dots, Z_{t,d})$ for $t = 1, \dots, n$.

Step 5 Set $\boldsymbol{U}_t = ||\mathbf{Z}_t||^{-1} \cdot \mathbf{Z}_t$ for $t = 1, \dots, n$.

Step 6 Set

$$\mathbf{V}_t = (\mathbf{V}_{t,1}, \dots, \mathbf{V}_{t,k})' = A'\mathbf{U}_t$$

with $\mathbf{V}_{t,1} = (V_{t,1}, \dots, V_{t,j_1})$, $\mathbf{V}_{t,2} = (V_{t,j_1+1}, \dots, V_{t,j_2})$, $\dots$, $\mathbf{V}_{t,k} = (V_{t,j_{k-1}+1}, \dots, V_{t,d})$. The partition corresponds to the sector partition.

Step 7 Return $\mathbf{X}_t = \mathbf{m} + (R^*_{t,1}\mathbf{V}_{t,1}, R^*_{t,2}\mathbf{V}_{t,2}, \dots, R^*_{t,k}\mathbf{V}_{t,k})'$.

In the case of multiple risk drivers $R_{t,i}$, one needs to derive the (conditional) distribution of $R^*_{t,i}$. First, the distribution of $(R_{t,1}, \dots, R_{t,k})$ is identified using the estimation procedure elaborated in Section 2.4.1. Thus, given the information $\mathcal{F}_{t-1}$, the distributions of $R_{t,i}$ and $B_{t,i}$ in formula (8) are known. Taking the logarithm on both sides of formula (8) shows that the extraction of the distribution of $R^*_{t,i}$ is a deconvolution problem which is typically considered in signal and image processing. The distribution of $R^*_{t,i}$ may either be retrieved by explicit or numerical deconvolution, see Haykin (2000) for more background and related references.

3 Analyzing and modelling portfolio credit risk

In the present section, the above theoretical framework of generalized elliptical processes is applied to the risk analysis and risk modelling of a credit portfolio.

3.1 Data description

The empirical analysis is based on data from Moody's KMV Credit Monitor (in short: MKMV) and covers the period from February 1996 to August 2005. MKMV utilizes a structural Merton-type credit risk model (Merton 1974) that has been refined using empirical evidence and is commonly used among practitioners in order to assess a firm's creditworthiness. MKMV provides, for example, so-called *expected default frequencies* (in short: EDFs) which refer to the firms' probability of default and, thus, to its creditworthiness.[4] The EDF is calculated as the likelihood that the firm's asset value falls below a given default threshold. The original Merton model (Merton 1974) treats the firm's equity as a call option on the firms asset value. These asset values form the basis of the forthcoming analysis. We start with an analysis of the time dynamics of the related risk drivers. Afterwards, these risk-driver dynamics are used for credit Value at Risk (VaR) estimation.

The data set comprises of credit risk data of 756 European non-financial firms with publicly traded equity, amongst others, it comprises asset values, asset returns, EDFs and exposures (the firms' total liabilities).[5] From February 1996 to August 2005, there are 115 monthly observations available. We have particularly chosen this time horizon since the MKMV methodology was adjusted by the end of 1995. Thus, a consideration of asset values before and after this structural break may cause inconsistencies. The chosen time period covers up- and downturns in the financial markets e.g. induced by the Asian crises (around 1997/98) and the September 11, 2001 event. The firms are assigned individually to six industry sectors as defined by MKMV. These industry sectors are Basic and Construction Industry (BasCon), Consumer Cyclical (ConCy), Consumer Non-Cyclical (ConNC), Capital goods (Cap), Energy and Utilities (EnU) and Telecommunication and Media (Tel). Descriptive statistics of the database are shown in table 2.[6]

Table 2: **Descriptive statistics of the data set**

Asset values are measured in million euros and corresponding log returns are calculated on a monthly basis. The total sample values are averaged over all firms in the sample. BasCon refers to Basic and Construction Industry, ConCy to Consumer Cyclical, ConNC to Consumer Non-Cyclical, Cap to Capital goods, EnU to Energy and Utilities and Tel to Telecommunication and Media.

Industry	1 BasCon	2 ConCy	3 ConNC	4 Cap	5 EnU	6 Tel	Total sample
February 1996 to August 2005							
Number of firms	200	236	135	90	40	55	756
Annual EDF (mean)	0.98%	1.27%	0.59%	0.81%	0.33%	1.06%	0.95%
Asset value (mean)	2,414	3,477	5,187	1,932	6,761	11,350	4,064
Log asset return (mean)	0.62%	0.61%	0.6%	0.63%	0.72%	0.57%	0.61%

The largest industry sectors are ConCy and BasCon comprising 31% and 26% of the total number of firms in the portfolio. The second largest sectors are ConNC and Cap with 18% and 12% of the total portfolio. The third group consists of Tel and EnU with a portfolio size of 7% and 4%. It is shown below that the sector size has an impact on the level of the risk driver. The mean EDFs of the sectors range from 0.33% in the EnU sector to 1.27% in the ConCy sector. The largest firms in the sample are contained in Tel, which exhibit on average a five times higher asset value than Cap firms (11.4bn Euros vis-a-vis 1.9bn Euros). The average asset returns - over the time horizon February 1996 to August 2005 -

[4]For further information about the MKMV methodology see, for example, Crouhy et al. (2000) and references therein.

[5]We assume that the distribution of the firms' total liabilities represents the exposure distribution of a hypothetic credit portfolio.

[6]The raw MKMV database has been modified in two aspects. First, all asset returns have been transformed into Euro currency; Before 1998, Deutsche Mark (DEM) has been used as a reference currency. Second, only time series without missing or erroneous asset values, EDFs and exposures are considered.

are around 0.62% in all industry sectors.

3.2 Dynamics of the risk drivers

We start with analyzing the time dynamics of the risk drivers.

3.2.1 Comparison between different industry sectors

Figure 1 compares the time evolution of the risk drivers $R_{t,i}$ - by utilizing formula (13) - for various pairs of industry sectors, namely the risk driver for the Basic and Construction sector (BasCon) together with the risk driver from a sector with
i) large sector size (industry sector 2: ConCy),
ii) middle sector size (industry sector 4: Cap), and
iii) small sector size (industry sector 6: Tel).

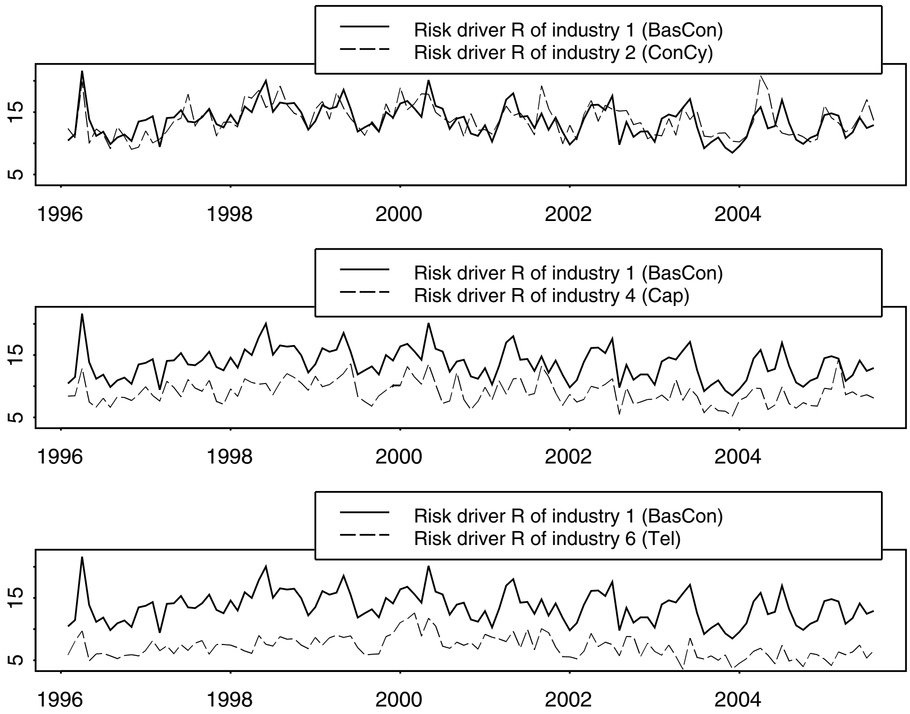

Figure 1: Risk-driver dynamics of MKMV asset returns for industries 1, 2, 4, and 6.

We observe a positive correlation between the sector size and the level of the risk driver, for example, the risk driver exhibits a higher nominal level for the BasCon sector compared to the Tel sector. This outcome is expected, as it implies that the total risk in a sub-portfolio increases with the number of firms. Second, we find that the risk drivers tend to exhibit peaks at 12-month time intervals, particularly for the BasCon and for the Cap sector. This can be explained as follows. The MKMV asset values are based on the market value of the firms' equity and the book-value of the firms' liabilities. In most cases, MKMV updates the balance sheet data of European firms once every 12 months. For example, for the BasCon this usually happens in April, May or June. Thus, the volatility of the asset values tends to increase around this time and is causally related to the development of the sector-specific risk drivers in these months. The update of the firms' liabilities is particularly important for asset-rich firms, such as firms in the BasCon sector, and for firms which exhibit substantial balance sheet reorganization.

Moreover, all risk drivers show a characteristic long-term pattern or cyclical trend, which becomes apparent in figure 2, where we use a different scaling. The level of the risk drivers increases until 2000, followed by a decrease of the same magnitude until the end of the observation period. This pattern is particularly pronounced for the Tel sector; by contrast, there is a less characteristic 12-month seasonality. The reason is that most firms in the Tel sector have undergone a dynamic development and a substantial increase of their equity basis during the observation period, and the volatility of their equity value has already been on a high level. This implies that the balance sheet updates had a lower impact on the risk driver in the Tel sector. By contrast, the EnU sector shows a modest overall cyclical behavior in the risk driver.

In summary: The risk drivers of all industries show temporal dependencies which are also confirmed by the autocorrelation functions given in figure 3. We point out that a main motivation of considering risk drivers was the finding of those autocorrelations on the aggregate risk-driver level. By contrast, no significant autocorrelations of the asset returns and the squared asset return were found on the single (disaggregated) firm level, cf. figure 4.

3.2.2 MEM versus ARMA model

Following our exposition in Section 2.2, we estimate an MEM and an ARMA model for the risk driver in each industry sector and compare both. The ARMA model is motivated by the large number of firms in each sector and the findings of Section 2.2. We identify the simplest MEM model by considering a GARCH(1,0)-type structure in equation (4), and compare it to the corresponding ARMA model, namely the AR(1) model. The innovation terms are assumed to be normally distributed, which is justified below. The respective models are denoted by MEM(lag1) and AR(lag1). Additionally we fit two more autoregressive structures: in the first case we regress on lag 12 (in short: AR(lag12)) and in the second case we regress on lags 1 and 12 (in short: AR(lag1; lag12)). The motivation for the latter two models is the observed 12-month seasonality of the risk drivers, described in the previous section. The estimated parameters (except the intercept) are provided in table 3. The residuals of the estimated MEM(lag1) and AR(lag1) model are shown in figures 5 and 6.

From table 3 and figures 5 and 6 we conclude:

i) The estimated parameters (or loadings) of the MEM(lag1) and the AR(lag1) model are close to each other. Moreover, the QQ-plots of the corresponding residuals possess a very similar structure. In particular they show that the normal innovations are a reasonable choice for the MEM and the AR model in this setting. The forecast quality of the AR(lag1) model is illustrated in figure 7. We remark that the QQ-plots of the (original) risk-driver realizations are highly skewed, which again justifies the usage of MEM or AR models.

ii) One reason that the results of the MEM and the AR model are close to each other is the large number of firms in each industry sector and the limiting argument given in Section 2.2.

iii) The industry sectors 1 and 4 show a large AR-parameter (or loading) at lag 12 which is in line with the findings in figure 3. See also the related discussion in Section 3.2.1.

Table 3: Parameter estimates (except the intercept) of the risk driver following an MEM or ARMA model per industry sector.

	MEM(lag1)	AR(lag1)	AR(lag1; lag12)	AR(lag12)
Industry 1 (BasCon)	.448	.455	.424 ; .255	.375
Industry 2 (ConCy)	.390	.383	.422 ; .058	.130
Industry 3 (ConNC)	.347	.348	.378 ; .102	.157
Industry 4 (Cap)	.358	.347	.323 ; .346	.386
Industry 5 (EnU)	.260	.251	.229 ; .104	.127
Industry 6 (Tel)	.510	.530	.491 ; .158	.328

All parameters are statistically significantly different from zero at a confidence level of 1%.

3.3 Risk drivers vis-à-vis macroeconomic influences

We analyze possible correlations between the sector-specific risk drivers and selected macroeconomic variables.[7] German macroeconomic variables are utilized as the majority of firms in the data sample are based in Germany. We consider the following six macroeconomic variables: The seasonally adjusted unemployment rate,[8] the gross domestic product (GDP), an index for the industry production (Ind_Product), the money-market rates for three-month funds (InterestRate), the development of order bookings in the industry (OrderBookings) and the inflation rate (CPI).[9] The (sample) correlations between the macroeconomic variables and the risk drivers are presented in table 4. The three macroeconomic variables which exhibit the highest correlation with the risk drivers are the interest rate, the unemployment rate and the GDP. For those variables, the highest correlation is found for the Tel sector, the ConNC sector and the Cap sector. The results for the Tel sector (industry 6) confirm the fact that the business of Tel firms is affected by the cyclical behavior of the economy. This effect is particularly revealed by the strong correlation with the interest rate and the unemployment rate. The ConNC sector (industry 3) is known to be sensitive towards changes of consumer price levels, which is reflected in the correlation with both the CPI and the interest rate. Also the correlation with the unemployment rate - which has an impact on the consumption behavior - is in line with our expectations.

The fact that the interest rate exhibits the highest correlation with all risk drivers demonstrates the importance of this economic variable for monetary policy. For the unemployment rate, the correlations are negative, which implies that a higher unemployment rate comes along with a lower level of the risk driver. The GDP shows a moderate correlation with the risk drivers ranging from 7% to 10%. Figures 8 and 9 illustrate the co-movement between the risk drivers and the interest rate and unemployment rate, respectively. In sum, the previous results show that the cyclical behavior of the economy has an impact on the (temporal) development of the risk drivers in most industries.

[7] Statistical influences of macroeconomic variables on credit risk have been investigated by some authors, see e.g. Allen and Saunders (2003) and references therein.

[8] The unemployment rate and the GDP are used with a lag of six months in order to incorporate the time lag where the cyclical effects become evident.

[9] Robustness studies show that the corresponding macroeconomic variables of France and the UK produce similar results.

Table 4: Sample correlation of selected macroeconomic variables with the industry-specific risk drivers. 'Unemploy' refers to the unemployment rate, 'GDP' to the gross domestic product, 'Ind_Product' to the industry production, 'InterestRate' to the three-month money market rate, 'OrderBookings' to the index of order bookings in the industry, and 'CPI' to the development of the price level.

Risk driver	Unemploy	GDP	Ind_Product	InterestRate	OrderBookings	CPI
Industry1 (BasCon)	-16.7%	8.8%	1.0%	21.9%	1.3%	-4.5%
Industry2 (ConCy)	-22.3%	7.6%	15.5%	17.7%	-9.6%	0.6%
Industry3 (ConNC)	-23.9%	10.4%	0.6%	34.7%	-12.7%	12.3%
Industry4 (Cap)	-14.5%	9.7%	7.4%	24.0%	-12.0%	0.8%
Industry5 (EnU)	-13.3%	7.4%	8.3%	17.2%	-12.8%	5.0%
Industry6 (Tel)	-47.1%	7.0%	-0.4%	51.7%	1.5%	4.1%

The correlations for Unemploy, GDP, and InterestRate are all significantly different from zero at 1% level.

3.4 Dynamic VaR estimation in portfolios

The Value at Risk (VaR) of a portfolio - comprising d assets - is the current standard risk measure in practice and in the regulatory framework (Basel Committee on Banking Supervision 2006). In this framework, the dynamic VaR of a credit portfolio is calculated from the portfolio loss distribution L given by

$$L_t = \sum_{j=1}^{d} w_{t,j}\psi_{t,j}\mathbf{1}_{\{X_{t,j}\leq K_j\}}, \qquad t \in T, \tag{15}$$

where $w_{t,j}$ denotes the relative exposure of obligor j at time $t \in T$ which is defined as the ratio of the book value of liabilities of obligor j with respect to the aggregated book value of liabilities in the portfolio; obtained from the MKMV database. A justification for the latter definition is given in Duellmann et al. (2006), p.15. Further, $\psi_{t,j}$ refers to the loss severity at default, which we assume to be constant at 45%, K_j is the obligor-specific default threshold, and $\mathbf{X} = (\mathbf{X}_t)_{t\in T}$ denotes the process of asset returns. The loss severity of 45% corresponds to the value defined in the IRB approach (Basel Committee on Banking Supervision 2006) for corporate exposures. Assume that the asset-return vector evolves according to a generalized elliptical process where the innovations of the MEM model are χ-distributed. The seasonal components of the risk drivers are modelled by splines having 3 degrees of freedom. The VaR at some confidence level is then obtained by sampling from L - as stated in formula (15) - and using the algorithm given in Section 2.5. The results are presented in figure 10 for each industry sector. As expected, the dynamic VaR is largely determined by the temporal structure of the risk drivers. The characteristic shape of the portfolio VaR across all industries in figure 10 is mainly caused by the higher stock market and asset volatilities around the year 2000, induced by the European stock market rally during this time. This shape is particularly pronounced for the Telecommunication and Media industry (Tel), cf. also Section 3.2.1 where we analyze the related risk driver.

Analytical formulas for the VaR may also be obtained in special cases. For example, assume that the time dynamics of the return vector of d stock prices follows an elliptical process $\mathbf{X} = (\mathbf{X}_t)_{t\in T}$. Then the VaR of the corresponding portfolio can be expressed in closed form since the margins of the process are elliptically contoured. The corresponding formulas are e.g. developed in Bingham et al. (2003). More precisely,

$$\mathrm{VaR}_t^\alpha = w_t'\mathbf{m} - h_t(\alpha)\sqrt{w_t'\Sigma w_t}, \tag{16}$$

where w_t denotes the d-dimensional vector of portfolio weights and $\alpha > 0$ is the confidence level. Further, $h_t(\alpha)$ corresponds to the $1 - 2\alpha$ quantile of the positive random variable $R_t B_t$, where B_t^2 is $Beta(1/2, (d-1)/2)$ distributed and the collection $\{B_t \mid t \in T\}$ is independent of the risk driver $R = (R_t)_{t \in T}$. For more details regarding the derivation, we refer to the last mentioned reference. Under the assumption of mutual independence of $\{\mathbf{U}_t \mid t \in T\}$ - as given in (1) - the set $\{B_t \mid t \in T\}$ is also mutually independent. This implies that the temporal dependence structure is determined by the risk driver R. Thus, utilizing the temporal structure of the risk drivers may improve the forecast and estimation quality of the portfolio VaR. A related empirical study for MKMV Credit Monitor data is currently in progress and will be presented in a forthcoming paper.

4 Conclusion

Multivariate generalized elliptical processes are proposed for modelling the time dynamics of asset returns in the situation of few temporal observations but a broad availability of cross-sectional data. Risk drivers - which describe the overall volatility structure - are introduced and their relationships to some multivariate distributions are established. The time dynamics of the risk drivers is either modelled using multiplicative error models (MEM) or non-linear regression or smoothing techniques. A limiting argument also justifies the usage of ARMA processes in certain situations. The model is applied to the risk analysis of a credit portfolio which consists of firms included in the MKMV Credit Monitor database. It is shown that the portfolio's Value at Risk is largely determined by the time dynamics of the risk drivers. The proposed methods are general and can be applied to any series of multivariate asset or equity returns in finance and insurance.

Our main empirical findings are significant temporal structures of the volatility of MKMV asset-return data on the (aggregated) risk-driver level. These temporal structures - on the risk-driver level - show similar patterns across all considered industries both in a seasonal and a cyclical context, with each industry's risk driver also exhibiting idiosyncratic developments. Further, correlations between the risk drivers and various macroeconomic variables are identified, which are particularly high for the Telecom sector.

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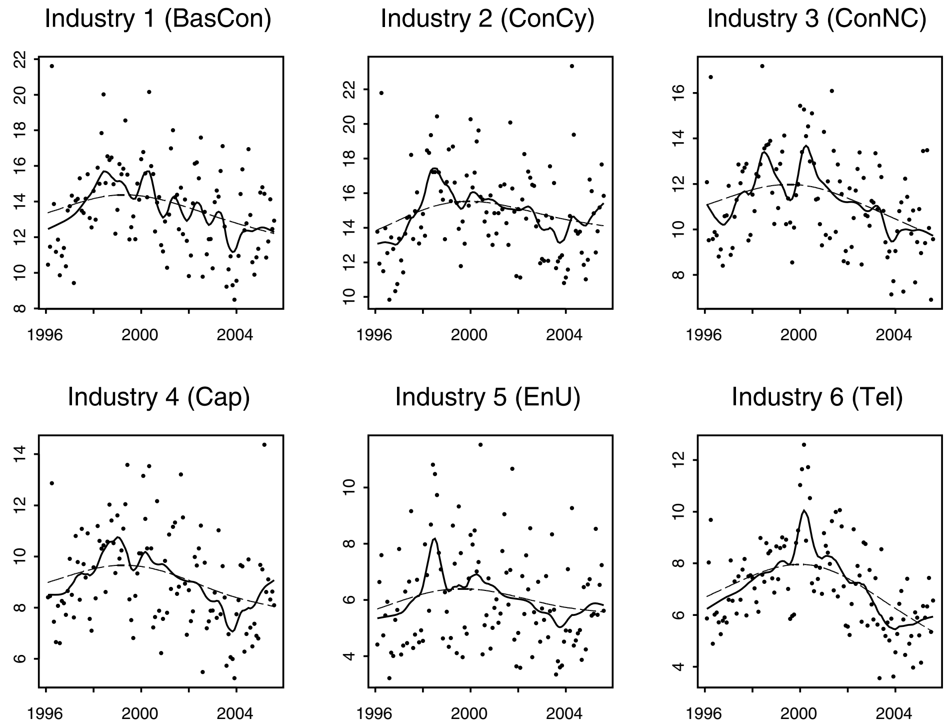

Figure 2: Risk-driver dynamics - for each industry sector given in table 2 over the time horizon from February 1996 to August 2005 - along with the Friedman's super span-smoother (solid line) and a spline having 3 degrees of freedom (dashed line).

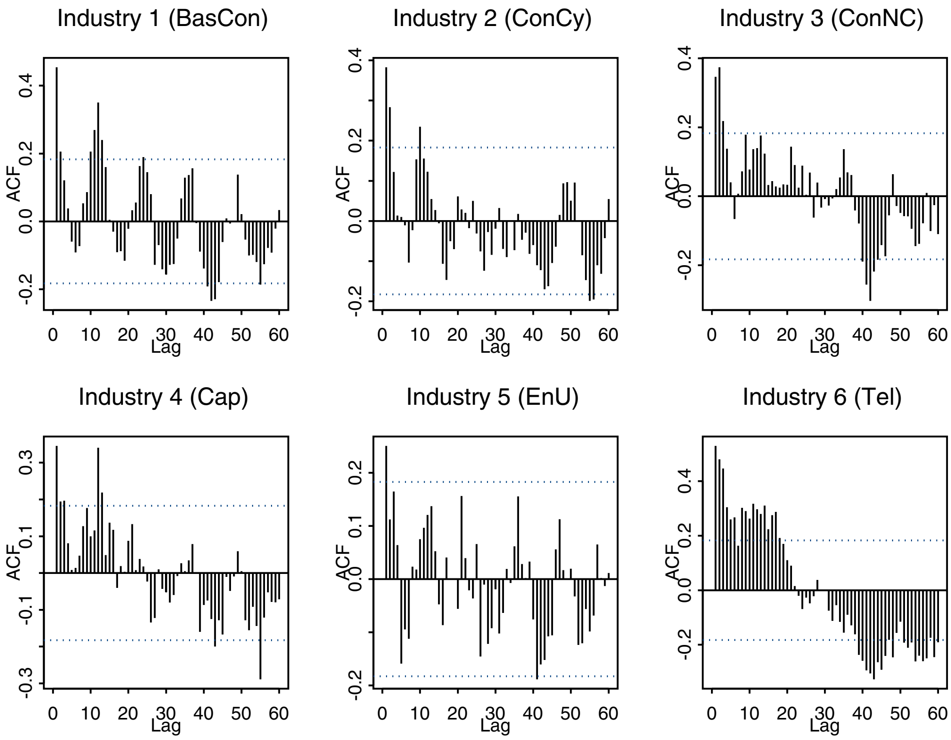

Figure 3: Autocorrelation function of the risk driver - for each industry sector given in table 2 over the time horizon from February 1996 to August 2005. The dotted line corresponds to the 5% confidence level.

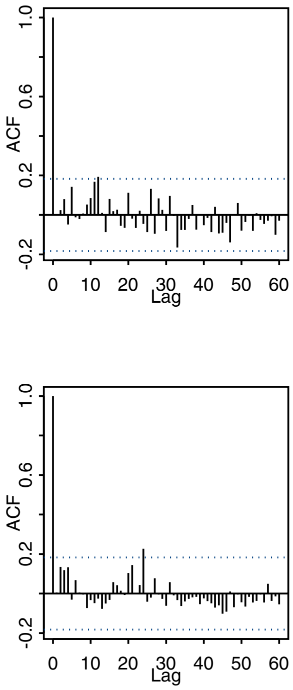

Figure 4: Autocorrelation function of the squared asset-return series of six randomly chosen firms in the MKMV Credit Monitor database over the time horizon from February 1996 to August 2005. The dotted line corresponds to the 5% confidence level.

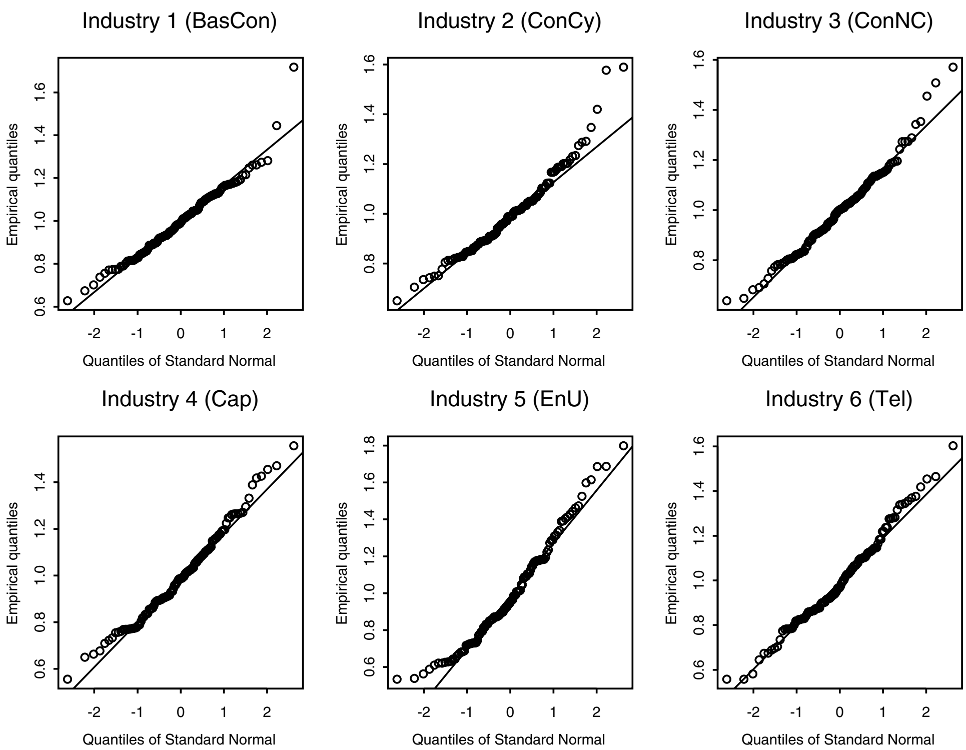

Figure 5: QQ-plot of the estimated MEM(lag1) residuals of the risk driver - for each industry sector given in table 2 over the time horizon from February 1996 to August 2005.

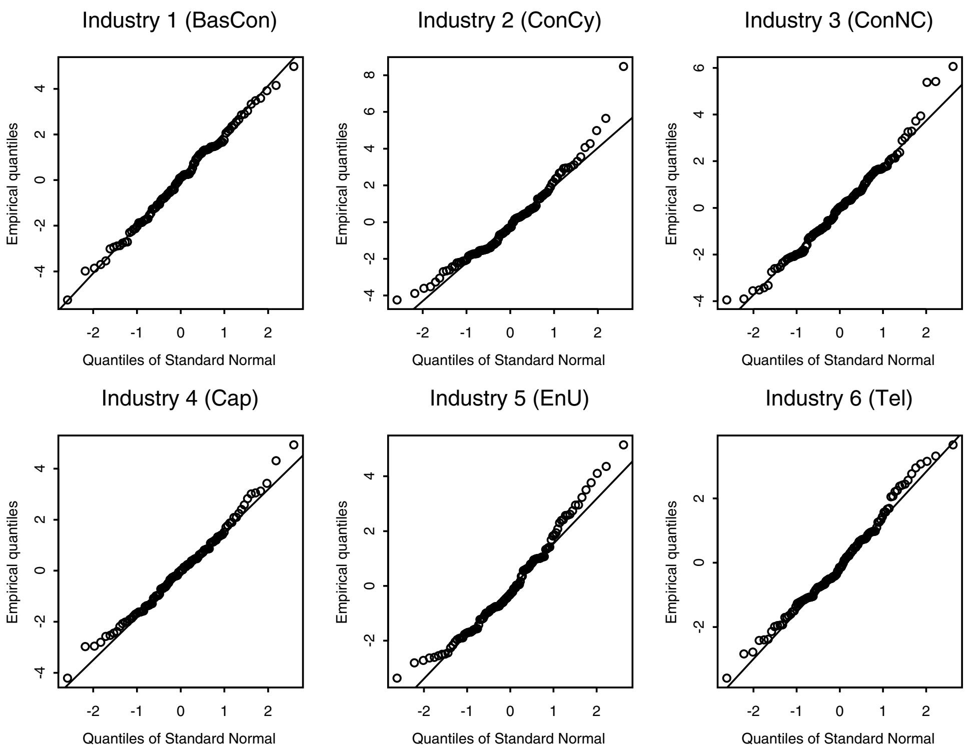

Figure 6: QQ-plot of the estimated AR(lag1) residuals of the risk driver - for each industry sector given in table 2 over the time horizon from February 1996 to August 2005.

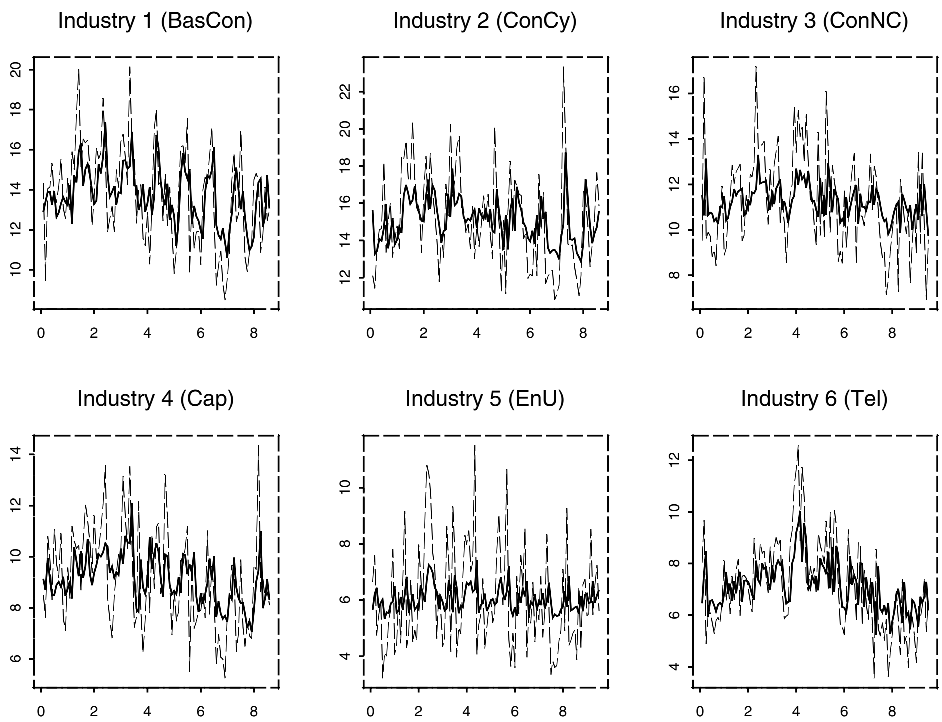

Figure 7: One-step forecast (solid line) of the fitted AR(lag1) model together with the risk driver (broken line) - for each industry sector given in table 2 over the time horizon from February 1996 to August 2005.

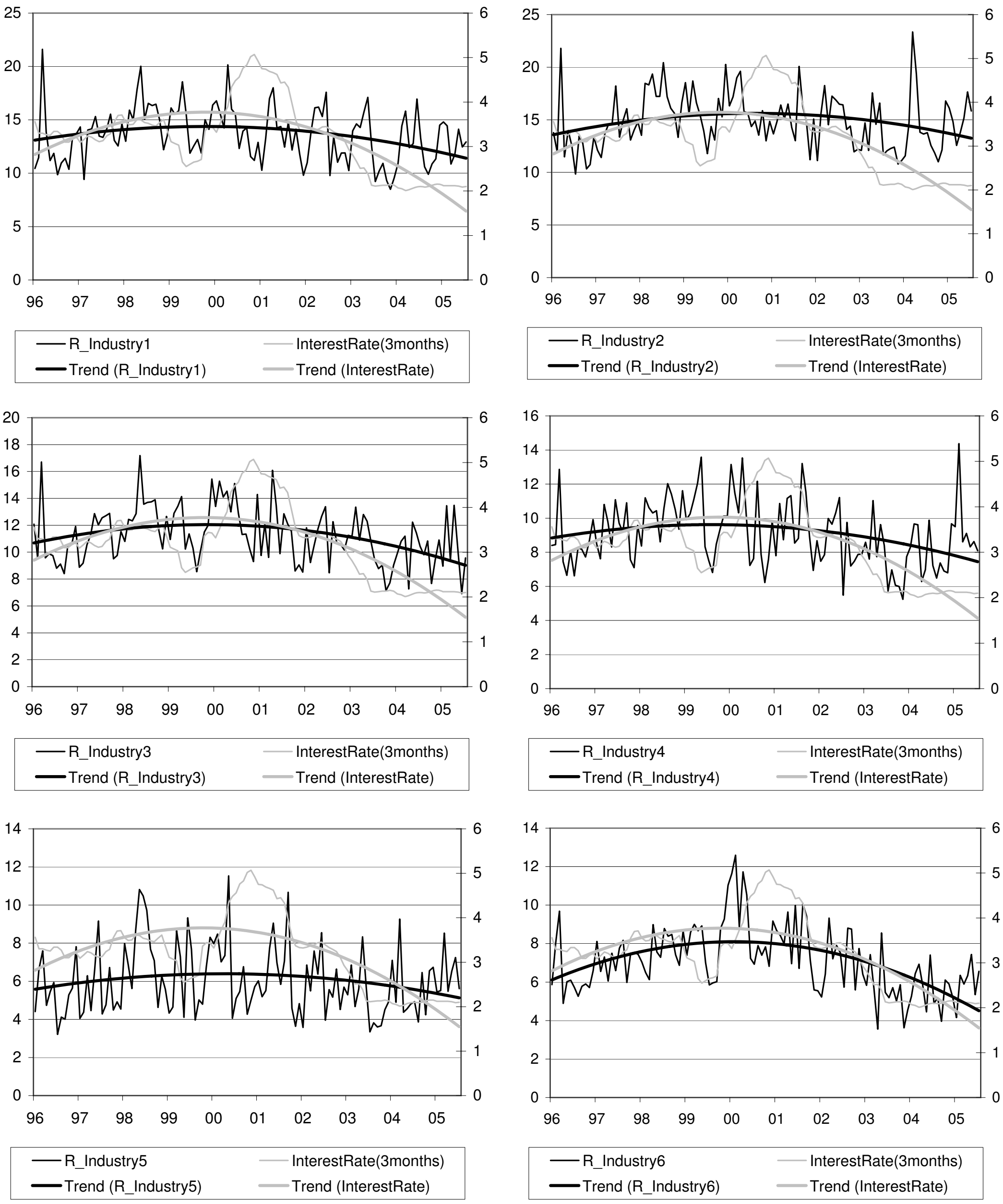

Figure 8: Risk-driver dynamics versus interest rate (three-month money market rate) over the time horizon from February 1996 to August 2005 for industry 1 (BasCon), industry 2 (ConCy), industry 3 (ConNC), industry 4 (Cap), industry 5 (EnU), and industry 6 (Tel) given in table 2.

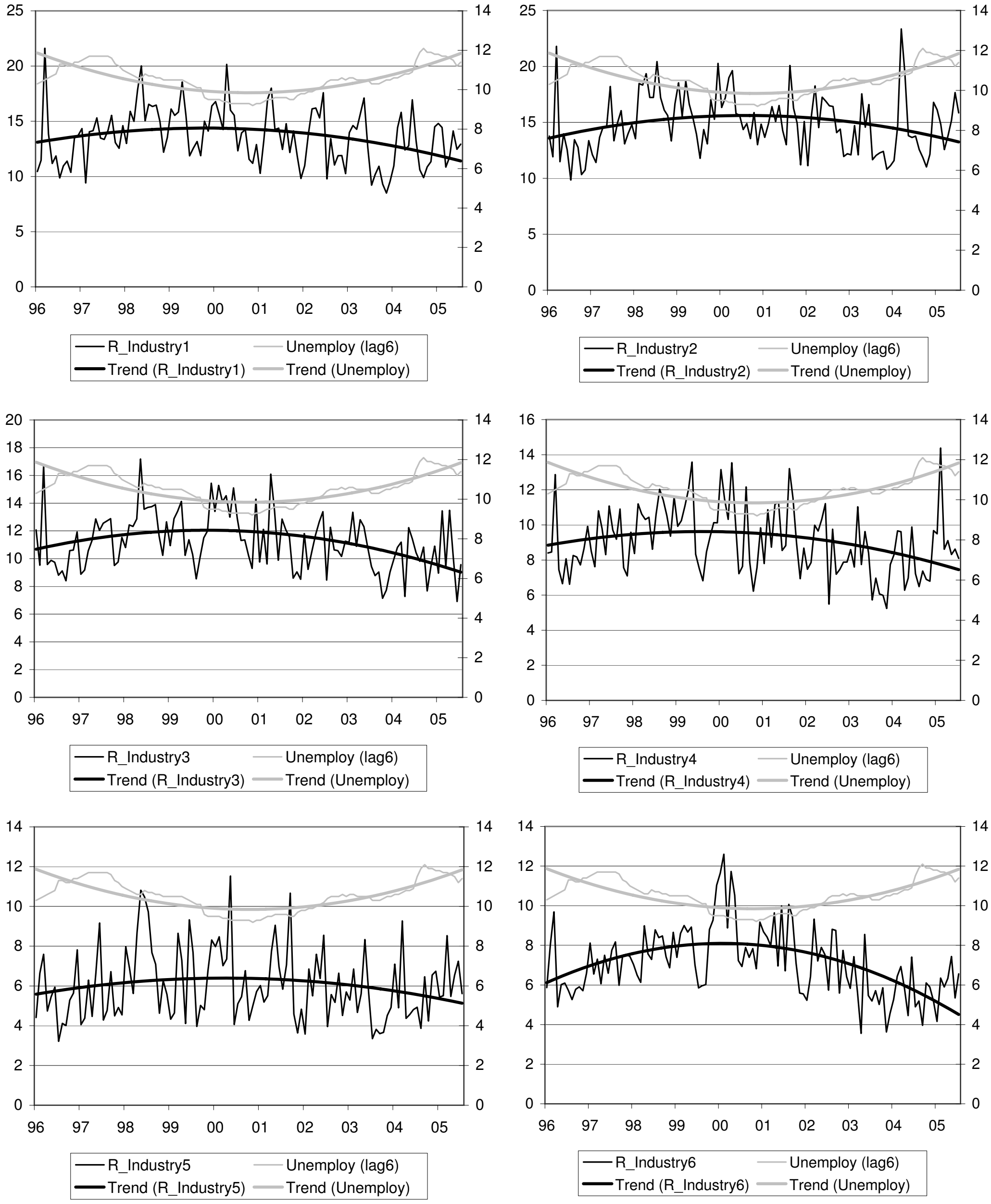

Figure 9: Risk-driver dynamics versus unemployment rate (6-month lag) over the time horizon from February 1996 to August 2005 for industry 1 (BasCon), industry 2 (ConCy), industry 3 (ConNC), industry 4 (Cap), industry 5 (EnU), and industry 6 (Tel) given in table 2.

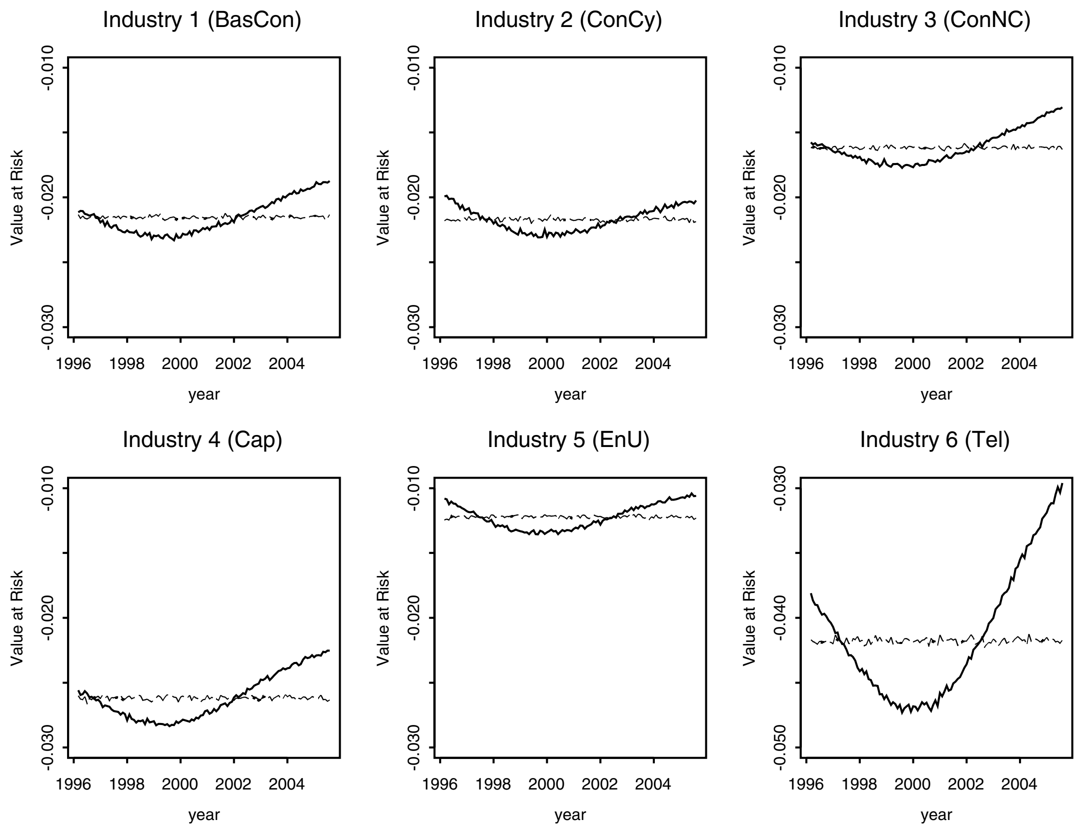

Figure 10: Dynamic Value at Risk estimates (at 5% confidence level) for industry-specific credit portfolios comprising MKMV listed firms - as given in table 2 - over the time horizon from February 1996 to August 2005. The estimation is based on 50,000 random numbers generated from the portfolio loss distribution - see formula (15) - as described in Section 3.4 (solid line), cf. also figure 2. The dashed line shows the VaR estimation under the assumption of independent and identically distributed risk drivers, i.e. if no temporal dependence structure is assumed.

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