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Sector concentration in loan portfolios and economic capital

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Abstract:

The purpose of this paper is to measure the potential impact of business-sector concentration on economic capital for loan portfolios and to explore a tractable model for its measurement. The empirical part evaluates the increase in economic capital in a multi-factor asset value model for portfolios with increasing sector concentration. The sector composition is based on credit information from the German central credit register. Finding that business sector concentration can substantially increase economic capital, the theoretical part of the paper explores whether this risk can be measured by a tractable model that avoids Monte Carlo simulations. We analyze a simplified version of the analytic value-at-risk approximation developed by Pykhtin (2004), which only requires risk parameters on a sector level. Sensitivity analyses with various input parameters show that the analytic approximation formulae perform well in approximating economic capital for portfolios which are homogeneous on a sector level in terms of PD and exposure size. Furthermore, we explore the robustness of our results for portfolios which are heterogeneous in terms of these two characteristics. We find that low granularity ceteris paribus causes the analytic approximation formulae to underestimate economic capital, whereas heterogeneity in individual PDs causes overestimation. Indicative results imply that in typical credit portfolios, PD heterogeneity will at least compensate for the granularity effect. This suggests that the analytic approximations estimate economic capital reasonably well and/or err on the conservative side.

Keywords: sector concentration risk, economic capital

JEL-Classification: G18, G21, C1

Non Technical Summary

An unbalanced exposure distribution of a loan portfolio, either across regional or business sectors, generally increases the associated credit risk. If credit risk is measured by a single systematic risk factor, sector concentration is usually not accounted for. The purpose of this paper is twofold. The empirical part measures the potential impact of business-sector concentration on the economic capital (or unexpected loss) of several loan portfolios. The sector composition of these portfolios is based on information from the German central credit register (Millionenkreditmeldewesen) on the sector composition of real bank portfolios. In this way it is ensured that our results are representative of real banks.

The model used in the empirical part requires Monte Carlo simulations for the calculation of economic capital, which can be noisy and time-consuming for the high-confidence levels typically used for the calculation of economic capital for credit risk. Therefore, in the theoretical part of the paper we explore a simpler, more tractable model for measuring portfolio risk which has a closed-form solution for economic capital and only requires input parameters, in particular exposure size and default probability, on a sector level. The model assumptions of PD homogeneity in every sector and fully diversified idiosyncratic credit risk are indeed not met by real credit portfolios. Indicative results nevertheless suggest that the analytic approximation still estimates economic capital reasonably well for typical credit portfolios and/or errs on the conservative side.

Nicht technische Zusammenfassung

Forderungskonzentrationen in einem Kreditportfolio, entweder in bestimmten geografischen Regionen oder in Industrie- bzw. Dienstleistungssektoren, erhöhen im Allgemeinen das Portfoliorisiko. Wird das Kreditrisiko mit einem Ein-Faktor-Modell gemessen, bleiben Sektorkonzentrationen üblicherweise unberücksichtigt. Das Diskussionspapier gliedert sich in einen empirischen und einen theoretischen Teil. Im empirischen Teil wird die mögliche Auswirkung von Branchenkonzentrationen auf das ökonomische Kapital (oder den unerwarteten Verlust) von ausgewählten Kreditportfolios bestimmt. Die Sektorverteilung dieser Portfolios beruht auf Informationen über die Sektorverteilung realer Bankportfolios aus dem Millionenkreditmeldewesen. Diese Datenbasis soll sicherstellen, dass unsere Ergebnisse repräsentativ für reale Banken sind.

Das im empirischen Teil verwendete Modell erfordert Monte-Carlo-Simulationen, welche bei den hohen Konfidenzniveaus, die typischerweise für die Bestimmung des ökonomischen Kapitals für Kreditrisiken verwendet werden, verrauscht und zeitaufwändig sein können. Aus diesem Grund untersuchen wir im theoretischen Teil ein einfacheres Modell zur Bestimmung des Portfoliorisikos, welches eine geschlossene Näherungsformel für das ökonomische Kapital liefert und nur Eingangsparameter, insbesondere Forderungshöhe und Ausfallwahrscheinlichkeit, auf Sektorebene verlangt. Zwar sind die in diesem Modell getroffenen Annahmen homogener Ausfallwahrscheinlichkeiten innerhalb der Sektoren und eines vollständig diversifizierten firmenspezifischen Risikos in realen Kreditportfolios so nicht erfüllt. Die Ergebnisse unserer Untersuchungen deuten aber gleichwohl an, dass die Näherungsformel auch in typischen Kreditportfolios noch relativ gute bzw. konservative Näherungswerte für das ökonomische Kapital liefert.

Contents

Sector Concentration in Loan Portfolios and Economic Capital[1]

1. Introduction

The failure of not recognizing diversification within banks' credit portfolios was a key criticism of the 1988 Basel Accord. The minimum regulatory capital requirements (Pillar 1) even in the internal ratings based (IRB) approach of the Basel Framework of June 2004, however, still do not differentiate between portfolios with different grades of diversification. Recognizing that banks' portfolios can exhibit credit risk concentrations, Basel II stipulates that this risk be addressed in the supervisory review process (Pillar 2), thus creating a need for an appropriate methodology to measure this risk.

Concentration risk in banks' credit portfolios arises either from an excessive exposure to certain names (often referred to as *name concentration* or *coarse granularity*) or from an excessive exposure to a single sector or to several highly correlated sectors (i.e. *sector concentration*). In the past, financial regulation and previous research have focused mainly on the first aspect of concentration risk.[2] Therefore, in this paper our focus is on sector concentration risk, although granularity is also analyzed. Sectors are defined in the following as business sectors. Sectors defined by geographical regions are not considered in this paper but our methodology would still be applicable in that case.

The critical role credit risk concentration has played in past bank failures has been documented in the literature.[3] Therefore, the importance of prudently managing sectoral concentration risk in banks' credit portfolios is generally well recognized. However, existing literature does not provide much guidance on how to measure sectoral concentration risk. Consequently, whether particular levels of concentration need to be translated into an additional capital buffer remains an open question.

This paper contributes to the literature in the following ways. First, in the empirical part, we measure economic capital in a CreditMetrics-type multi-factor model and evaluate how important the increase in economic capital is in a sequence of portfolios with increasing sector

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[1] For their comments we thank, Alexandre Adam, Marc Carey, Marc De Ceuster, Michael Gordy, Thilo Liebig, Janet Mitchell, Joël Petey, Peter Raupach, Andrea Resti, participants in the Research Task Force of the Basel Committee on Banking Supervision as well as participants at the 2006 AFSE conference on "Recent Developments in Financial Economics", the 2006 European Banking Symposium, the 6th Annual Bank Research Conference (FDIC/JFSR), the 2006 C.R.E.D.I.T. Conference "Risks in Small Business Lending", the 13th Annual Meeting of the German Finance Association (DGF), and the 2006 Annual Meeting of the Financial Management Association. We thank Eva Lütkebohmert, Christian Schmieder and Björn Wehlert for their invaluable support in compiling the German credit register data and Julien Demuynck and Jesus Saurina for providing us with data from the French and the Spanish credit registers.

The views expressed here are our own and do not necessarily reflect those of the Deutsche Bundesbank or the National Bank of Belgium.

[2] See EU Directive 93/6/EEC, Joint Forum (1993) and Gordy (2003).

[3] See, for example, BCBS (2004a).

concentration. The analysis is based on portfolios which were constructed from German central credit register data on 2224 banks. The benchmark portfolio reflects the average business-sector distribution of the banking system as some of the more concentrated portfolios mirror sector concentrations observed in individual banks. Information on business-sector concentration of banks is not publicly available, thus central credit registers represent unique sources of data on sector concentrations in existing banks. Our emphasis on empirically observable sector concentrations is therefore an important contribution.

We find that economic capital can substantially increase with sector concentration. Its increase from a credit portfolio representing the average sector distribution of the German banking system to a portfolio that is concentrated in a single sector can be as high as 50%.

Second, in the theoretical part we evaluate the accuracy of an analytic approximation for value-at-risk (VaR) and economic capital (*EC*) which was proposed in Pykhtin (2004) and offers a tractable closed-form solution for the measurement of concentration risk. *EC* is defined as the difference between the VaR and the expected loss of a credit portfolio. We have applied a simplified version of the Pykhtin model which further reduces the computational burden by requiring the input parameters exposure size and probability of default (PD) only on a sector level. Such a methodology could be useful for risk managers and supervisors in search of robust, fit-for-purpose tools to measure sector concentration in a bank's loan portfolio. The model allows banks and supervisors to approximate economic capital for loan portfolios without running computationally intensive Monte Carlo simulations.

The methodological framework of the Pykhtin model builds on earlier work by Gordy (2003) and Wilde (2001) on granularity adjustments in the asymptotic single risk factor (ASRF) model. Whereas the granularity adjustment deals with an unbalanced exposure distribution across names, the Pykhtin model offers a treatment for an unbalanced distribution across (correlated) sectors. *EC* is given in closed form as the sum of the *EC* in a single risk factor model (in which the correlation with the single systematic risk factor depends on the sector) and a multi-factor adjustment term. We explore the approximation performance both before the multi-factor adjustment is applied and afterwards which means we consider two approximation formulae.

We find that for portfolios with highly granular sectors and homogeneous PDs in every sector, both analytic approximation formulae perform extremely well. Moreover, the multi-factor adjustment term is relatively small, so that *EC* in the single risk factor model is already close to the true *EC* values obtained by simulations. Our results hold for portfolios with different levels of sector concentration, a different number of sectors as well as under various weights of the sector factors (i.e. factor weights sometimes referred to in literature as factor loadings) and various assumptions about factor correlations. Furthermore, we explore the accuracy of our model when we relax the assumptions that the portfolio is infinitely granular within each sector and that all exposures in the same sector have the same PD. We find that the model cp underestimates *EC* in cases of low granularity, whereas it cp overestimates *EC* in the presence of heterogeneity in individual PDs, in particular if creditworthiness increases with exposure size. The resulting errors in *EC* from both effects together were less than 10% in the cases under study. Which of the two effects prevails depends on the specific input parameters. The results seem to suggest, however, that for representative credit portfolios, the effect of PD heterogeneity is likely to be stronger than the effect of granularity. This implies that the analytic approximations err on the conservative side.

To our knowledge there is only one recent empirical paper that considers the impact of sector concentration risk on economic capital. Burton et al (2005) simulate the distribution of portfolio credit losses for a number of real US syndicated loan portfolios. They find that, although name concentration can meaningfully increase *EC* for smaller portfolios (which are defined as portfolios with exposures of less than US$10 billion), sector concentration risk is the main contributor to *EC* for portfolios of all sizes.

Two other models that measure concentration risk in a tractable model are presented by Garcia Cespedes et al (2005) and Düllmann (2006). Garcia Cespedes et al (2005) developed an adjustment to the single risk factor model in the form of a scaling factor to the economic capital required by the ASRF model. This "diversification factor" is an approximately linear function of a Hirschmann-Herfindahl index, calculated from the aggregated sector exposures. This model, however, does not allow for different asset correlations across sectors. Contrary to the approach in our paper, it cannot distinguish between a portfolio which is highly concentrated towards a sector with a high correlation with other sectors, and another portfolio which is equally highly concentrated, but towards a sector which is only weakly correlated with other sectors. Düllmann (2006) extends Moody's Binomial Expansion Technique by introducing default infection into the hypothetical portfolio on which the real portfolio is mapped in order to retain a simple solution for VaR. Unlike the Pykhtin model, the models developed by Garcia Cespedes et al and Düllmann require the calibration of parameters using Monte Carlo simulations.

The paper is organized as follows. In Section 2 we present the default-mode version of the well-established multi-factor CreditMetrics model which serves as a benchmark. Furthermore, we discuss the simplified version of the Pykhtin model.

The empirical part of our paper comprises Sections 3 and 4. The credit portfolios on which the empirical analyses are based are described in Section 3. In Section 4 we explore the impact of sector concentration on *EC* by gradually increasing sector concentration, starting from a benchmark portfolio.

In the theoretical part, which comprises Sections 5 to 7, we evaluate the performance of Pykhtin's (2004) analytic approximation for economic capital by comparison with *EC* estimates from Monte Carlo simulations. Section 5 focuses on highly granular portfolios which are homogeneous on a sector level and, in particular, on the sensitivity of the results to the number of risk factors and correlation figures. Section 6 deals with portfolios characterized by lower granularity and Section 7 introduces PD heterogeneity on an exposure level. Section 8 summarizes and concludes.

2. Measuring concentration risk in a multi-factor model

2.1. General framework

We assume that every loan in a portfolio can be assigned to a different borrower, so that the number of exposures or loans equals the number of borrowers. Each borrower i can uniquely be assigned to a single specific sector. In practice, (large) firms often comprise business lines from different industry sectors. However, we make this assumption here for practical and presentational purposes. Let M denote the number of borrowers or loans in the portfolio, M_s the number of borrowers in sector s, S the number of sectors and w_{si} the weight of the exposure of borrower i in sector s relative to the total portfolio exposure.

The general framework is a multi-factor default-mode Merton-type model.[4] The dependence structure between borrower defaults is driven by sector-dependent systematic risk factors which are usually correlated. Each risk factor can be uniquely assigned to a different sector, so that the number of sectors and the number of factors are the same. Credit risk occurs only as a default event at the end of a one-year horizon, which is consistent with traditional book-value accounting. The unobservable, normalized asset return X_{si} of the i-th borrower in sector s triggers the default event if it crosses the default barrier γ_{si} . The corresponding unconditional default probability p_{si} is defined as

$$p_{si} = P\left(X_{si} \leq \gamma_{si}\right).$$

The latent variable X_{si} follows a factor model and can be written as a linear function of an industry sector risk factor Y_s and an idiosyncratic risk factor ε_{si} :

$$\text{(1a)} \qquad X_{si} = r_s Y_s + \sqrt{1-r_s^2}\,\varepsilon_{si}$$

where $s \in \{1,...,S\}$ and $i \in \{1,...,M_s\}$. The higher the value of the sector-dependent factor weight r_s , the more sensitive the asset returns of firm i in sector s are to the sector factor. The disturbance term ε_{si} follows a standard normal distribution. The assumed weight on the idiosyncratic risk guarantees that X_{si} has a standard normal distribution$_i$.

[4] See also Gupton et al (1997), Gordy (2000), and Bluhm et al (2003) for more detailed information on this type of models. The origin of these models can be found in the seminal work by Merton (1974).

The correlations between the systematic sector risk factors Y_s and Y_t are denoted by ρ_{st} and are often referred to as factor correlations. The sector factors can be expressed as a linear combination of independent, standard normally distributed factors $Z_1,...,Z_S$.

$$\text{(1b)} \qquad Y_s = \sum_{t=1}^{S} \alpha_{st} Z_t \ \text{ with } \sum_{t=1}^{S} \alpha_{st}^2 = 1 \ \text{ for } s \in \{1,...,S\}.$$

The matrix $(\alpha_{st})_{1 \le s,t \le S}$ is obtained from a Cholesky decomposition of the factor correlation matrix. The asset correlation ω_{st} for each pair of borrowers in sectors s and t, respectively, can be shown to be given by

$$\text{(2)} \qquad \omega_{st} = r_s \, r_t \, \rho_{st} = r_s \, r_t \sum_{n=1}^{S} \alpha_{sn} \alpha_{tn} .$$

Dependencies between borrowers arise only from their affiliation with the industry sector and from the correlations between the systematic sector factors. The intra-sector asset correlation for each pair of borrowers is simply the factor weight r_s^2 squared.

If a firm defaults, the amount of loss depends on the stochastic loss severity ψ_{si} whose realization is assumed to be known at the time of default. The credit losses of the whole portfolio are given by

$$\text{(3)} \qquad L = \sum_{s=1}^{S} \sum_{i=1}^{M_s} w_{si} \psi_{si} 1_{\{X_{si} \le N^{-1}(p_{si})\}} .$$

where $1_{\{\cdot\}}$ gives the indicator function.

We assume the same expected loss severity $\mu = E[\psi_{si}]$ for all borrowers and that all idiosyncratic risk in loss severities is diversified away in the portfolio.[5]

In summary, the model needs the following input parameters:

- relative exposure size w_{si} and default probability p_{si} of the i-th borrower in sector s
- the factor correlation matrix and
- the sector-dependent factor weight r_s

2.2. The CreditMetrics default-mode model

To obtain the loss distribution, CreditMetrics applies Monte Carlo simulations by generating asset returns and counting the default events. In each simulation run the portfolio loss is determined from equation (3). For each exposure, the asset returns for the corresponding borrower are generated according to equations (1a/b) and compared with the default threshold,

[5] The models analyzed in this paper can also be extended to incorporate idiosyncratic risk in loss severities, if required.

which can be determined given the borrower's default probability. If the realized value of the asset return falls below the threshold γ_{si}, the borrower is in default. The portfolio loss of a simulation run is calculated by adding up the incurred losses from the defaulted borrowers. The number of simulation runs in our analyses is typically 200,000. Portfolio losses obtained in each simulation run are then sorted to form the distribution of portfolio losses, from which *EC* can be calculated as the difference between the *q*-quantile of this loss distribution (i.e., the VaR) and the expected loss. Since it is obtained by simulation, we refer to it in the following as EC_{sim}.

2.3. Analytic *EC* approximation

In this section, we describe an analytical approximation to the VaR in the framework of a multi-factor model. We use a simplified version of the model developed by Pykhtin (2004). The model approximates the VaR in a multi-factor model by adding a "multi-factor" adjustment term to the VaR in a single factor model in which the correlation of the firm's asset returns with the single factor depends on the firm's sector. The main advantage of this model is its tractability, since it does not require Monte Carlo simulations. Furthermore, we have simplified the model in such a way that it only requires exposure size and PD on a sector level instead of an individual borrower level. The factor correlation matrix and the factor weights are still needed as in the CreditMetrics model.

On the basis of the work by Gouriéroux et al (2000) and Martin and Wilde (2002), we can approximate the portfolio loss *L* (see equation 3) by a perturbed loss variable $L_\eta = L^* + U \cdot \eta$, where L^* is a random variable constructed such that the *q*-quantile of its distribution given in closed form is close to the *q*-quantile of the distribution of *L*. *U* is defined as the perturbation $L - L^*$ and η is its scaling parameter. L^* depends on the default probability $\hat{p}(Y^*)$, conditional on a single systematic risk factor Y^*:

$$(4) \qquad L^* = \mu \sum_{s=1}^{S} w_s \hat{p}_s\left(Y^*\right) \text{ with } \hat{p}_s(Y^*) = N\left(\frac{N^{-1}(p_s) - c_s Y^*}{\sqrt{1-c_s^2}}\right)$$

where c_s is the correlation between the systematic risk factor Y^* and the asset returns of the firms in sector *s*. In oder to relate relate L^* to *L*, Y^* finally needs to be related to the risk factors $Z_1, \dots, Z_S$ in the original model. If Y^* in (4) is replaced by the realization corresponding to the *q*-quantile $t_q\left(Y^*\right)$, then L^* equals the VaR $t_q\left(L^*\right)$ for a confidence level *q* in the asymptotic single risk factor (ASRF) model with infinitely granular sectors. Note that this single risk factor model differs from the well-known ASRF model in that the asset correlation c_s is determined by the sector to which the borrower belongs. To avoid confusion, we call this model the "ASRF* model", reserving the term "ASRF model" for the model with uniform asset correlations.

The q-quantile of the loss distribution, $t_q(L)$ can then be approximated by $t_q\left(L_\eta\right)$, or as the sum of the VaR in the ASRF model $t_q(L^*)$ and a multi-factor adjustment Δt_q. This multi-factor adjustment can be determined from a Taylor series expansion of $t_q(L_\eta)$. The first-order effect

$$\left.\frac{dt_q(L_\eta)}{d\eta}\right|_{\eta=0} = E\left[U \middle| L^* = t_q(L^*)\right]$$

vanishes because we require that L^* for all portfolio compositions equals the expected loss conditional on Y^*, that is $L^* = E\left[L \mid Y^*\right]$. By keeping terms up to quadratic order and neglecting higher-order terms, we can approximate the portfolio loss quantile $t_q(L)$ as follows:[6]

$$(5) \qquad t_q(L) \approx t_q(L^*) + \underbrace{\left.\frac{1}{2}\frac{d^2 t_q(L_\eta)}{d\eta^2}\right|_{\eta=0}}_{\Delta t_q}$$

The first summand in (5) denotes the VaR $t_q\left(L^*\right)$ in the single risk factor model. The second summand denotes the multi-factor adjustment, Δt_q, which can be calculated according to Pykhtin (2004) by

$$(6) \qquad \Delta t_q = -\frac{1}{2l'(y)}\left[v'(y) - v(y)\left(\frac{l''(y)}{l'(y)} + y\right)\right]\Bigg|_{y=N^{-1}(1-q)}$$

where $l'(y)$ and $l''(y)$ denote, respectively, the first and second derivative of the portfolio loss function given by equation (4) and setting $Y^* = y$. $v(y)$ gives the variance of L conditional on $Y^* = y$. Its first derivative is $v'(y)$. The details and the inputs of these equations are presented in Appendix B.

The link between L and L^* is achieved by restricting Y^* to the space of linear mappings of the risk factors $Z_1,\ldots,Z_S$:

$$Y^* = \sum_{s=1}^{S} b_s Z_s \,.$$

[6] See Pykhtin (2004) for proofs.

The correlations between the industry risk factors Y_s and the systematic risk factor Y^* are denoted by ρ_s^*. These are used to calculate the (also sector-dependent) correlations in the ASRF* model using the following mapping function for $s \in \{1,...,S\}$:

$$(7)\ c_s = r_s \rho_s^* \text{ where } \rho_s^* = \sum_{t=1}^{S} \alpha_{st} b_t .$$

Defining c_s for $s \in \{1,...,S\}$ by (7) ensures that the required equality $L^* = E\left[L \mid Y^*\right]$ holds for any portfolio composition.

There is no unique solution to determine the coefficients $b_1,...,b_S$. In the following, we will use the approach in Pykhtin (2004), which is briefly summarized in Appendix C.

3. Portfolio composition

3.1. Data set and definition of sectors

Our analyses are based on loan portfolios which reflect characteristics of real bank portfolios obtained from European credit register data. Our benchmark portfolio represents the overall sector concentration of the German banking system and was constructed by aggregating the exposure values of loan portfolios of 2224 German banks in September 2004. The sample includes branches of foreign banks located in Germany. Credit exposures to foreign borrowers, however, are excluded. We deem this to be a reasonable approximation of a portfolio characterized by a degree of diversification which banks can on average achieve given that it represents the aggregate relative sector exposures of the national banking system. In principle, we could also have created a more diversified portfolio in the sense of having a lower VaR. However, such a portfolio would be specific to the credit risk model used and would not be obtainable for all banks.

All credit institutions in Germany are required by the German Banking Act (*Kreditwesengesetz*) to report quarterly exposure amounts of those borrowers whose indebtedness to them amounts to €1.5 million or more at any time during the three calendar months preceding the reporting date. In addition, banks report national codes that are compatible with the NACE classification scheme and indicate the economic activity of the borrower and his country of residence. Banks are required to aggregate individual borrowers for regulatory reporting purposes to *borrower units* which are linked, for example, by equity holdings and constitute an entity sharing roughly the same risk. The aggregation of exposures on a business sector level was carried out on the basis of borrower units. If borrowers in the same unit belong to different sectors, the dominating exposure amount determines the final sector allocation. Therefore, the credit register includes not only exposures above €1.5 million, but also smaller exposures to individual borrowers belonging to a borrower unit that exceeds this exposure limit. This characteristic substantially increases its coverage of the credit market. The industry classification chosen by CreditMetrics is the Global Industry Classification Standard (GICS), which was jointly launched by Standard & Poor's and Morgan Stanley Capital International (MSCI) in 1999. The classification scheme was developed to establish a global standard for categorizing firms into sectors and industries according to their principal business activities. It comprises 24 industries grouped into 10 broad sectors.[7] GICS further divides these groups into industries and sub-industries. However, the latter more detailed schemes are not used by vendor models. In the following, we use the broad sector classification scheme. Because some of the industry groups that form the broad "Industrial" sector are very heterogeneous, we decided to split this sector into three industry groups: Capital Goods (including Construction), Commercial Services and Supplies, and Transportation.[8]

Credit register datasets, however, use the NACE industry classification system, which is quite different from the GICS system. In order to use the information from the credit register, we mapped[9] the NACE codes onto the GICS codes. Similar mappings are used by other vendor models, such as S&P's Portfolio Risk Tracker. We have excluded exposures to the financial sector (sector G) which comprises exposures to Banks (G1), Diversified Financials (G2), Insurance Companies (G3) and Real Estate (G4) because of the specificities of this sector. Exposures to the real estate sector are heavily biased as a large number of them are exposures to borrowers that are related to the public sector. Since we could not differentiate between private and public enterprises in the real estate sector, we have excluded this sector from the following analyses. We have also disregarded exposures to households since there is no representative stock index for them. This is a typical limitation of models relying on stock price returns for the estimation of asset correlations. In sum, we distinguish between 11 sectors, which can be considered as broadly representing the Basel II asset classes Corporate and SMEs.

3.2. Comparison with French, Belgian and Spanish banking systems

A rough comparison of the relative share of the sector decomposition between the aggregated German, French, Belgian and Spanish banking systems shows that the numbers are similar.[10] The only noticeable difference is the greater share of the Capital Goods sector (33%) and the

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7 See Table 12 in Appendix A, which shows the broad sectors and the more detailed industry groups.

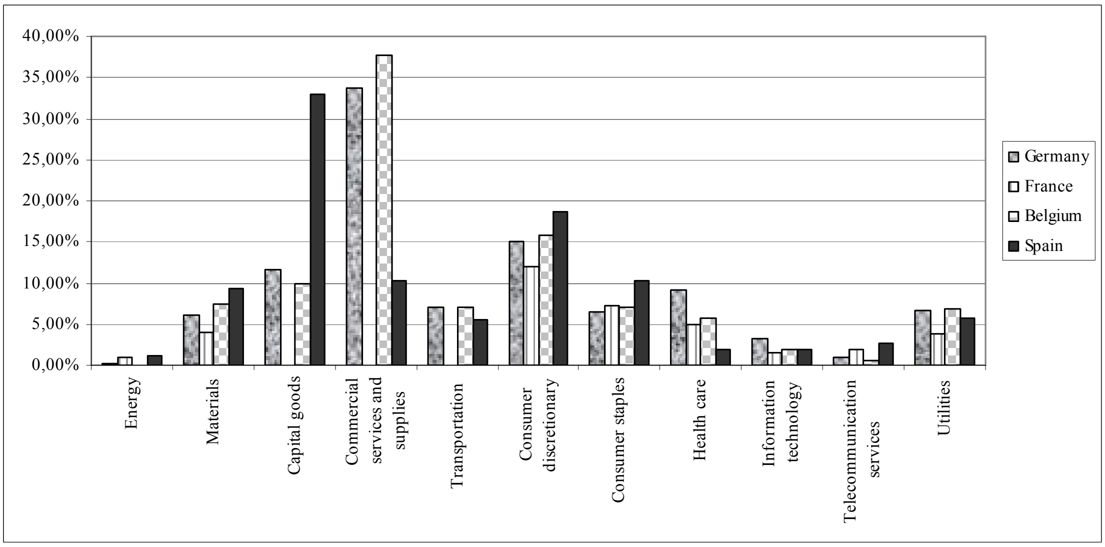
8 Unreported simulations have shown that results are not affected by using the more detailed classification scheme.

9 See Table 13 in Appendix A for the mapping.

10 The exact figures are provided by Table 14 in Appendix A.

smaller share of the Commercial Services and Supplies sector in Spain compared to Germany and Belgium. In general, however, the average sector concentrations are very similar across the four countries, which suggests that our results are to a large extent transferable.

Figure 1: Comparison of average sector concentration for Germany, Spain, Belgium, and France (*)



(*) A breakdown of Industrial sector C into the three categories Capital Goods, Commercial Services and Supplies, and Transportation is not available for France. The sector shares of the aggregated sector C, however, are quite similar for all four countries.

3.3. Description of the benchmark portfolio

The sectoral distribution of exposures in the benchmark portfolio, which is shown in Table 1, represents the sectoral distribution of aggregate exposures in the German banking system. The degree of concentration in this reference portfolio is purely national and driven by the firms' sector composition because we do not consider the impact of regional or country factors in our analysis. It is not uncommon for banks to use a more detailed sector classification scheme. We consider it more conservative to use a relatively broad sector classification scheme rather than a very detailed one. In a broad sector classification scheme, a larger proportion of exposures is attached to one sector. Therefore, correlations between exposures of the same sector (intra-sector correlations), which are typically greater than the correlations between exposures of different sectors (inter-sector correlations), will play a larger role.

In order to focus on the impact of sector concentration, we assume an otherwise homogeneous portfolio by requiring that all other characteristics of the portfolio are uniform across sectors. We assume a total portfolio volume of €6 million that consists of 6,000 exposures of equal size which implies a highly granular portfolio in which each exposure represents only 0.017% of the total portfolio exposure. Every borrower has a probability of default (PD) of 2% and every exposure is to a different borrower, thus circumventing the need to consider multiple exposure defaults. We set a uniform expected loss severity or loss given default (LGD) of 45%, which is the corresponding supervisory value for a senior unsecured loan in the Foundation IRB approach of the Basel II framework.[11] In the CreditMetrics approach, industry weights can be assigned to each borrower according to its participation. Here, we assume that every firm is exposed to only one sector as its main activity. Furthermore, we assume banks do not reduce exposure to certain sectors by purchasing credit protection.

Table 1: Composition of the benchmark portfolio (using the GICS sector classification scheme)

	Total exposure	Number of exposures	% exposure
A: Energy	11,000	11	0.18%
B: Materials	361,000	361	6.01%
C1: Capital Goods	692,000	692	11.53%
C2: Commercial Services and Supplies	2,020,000	2,020	33.69%
C3: Transportation	429,000	429	7.14%
D: Consumer Discretionary	898,000	898	14.97%
E: Consumer Staples	389,000	389	6.48%
F: Health Care	545,000	545	9.09%
H: Information Technology	192,000	192	3.20%
I: Telecommunication Services	63,000	63	1.04%
J: Utilities	400,000	400	6.67%
Total	6,000,000	6,000	

3.4. Sequence of portfolios with increasing sector concentration

In order to measure the impact on *EC* of more concentrated portfolios than the benchmark portfolio, we construct a sequence of six portfolios, each with increased sector concentration relative to the previous one. To this end, we gradually increase sector concentration in our benchmark portfolio by using the following algorithm. In each step we remove x exposures from all sectors and add them to a previously selected sector. This procedure is repeated until a single-sector portfolio, which is the portfolio with the highest possible concentration, is obtained. The sector which receives x exposures at every step and also the amount x that is transferred to this sector are determined in such a way that some of the generated portfolios reflect a degree of sector concentration that is actually observable in real banks.[12]

Table 2 shows a sequence of seven portfolios in the order of increasing sector concentration. The increase in sector concentration is also reflected in the Herfindahl-Hirschmann Index (HHI),[13] given in the last row which is calculated at sector level. Portfolio 1 has been constructed from the benchmark portfolio by re-allocating one third of each sector exposure to the sector Capital Goods. The even more concentrated portfolios 2, 3, 4 and 5 have been created by repeated application of this rule. Portfolios 2 and 5 are similar to portfolios of

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11 See BCBS (2004b).

12 This procedure for generating a sequence of portfolios with increasing sector concentration is by no means unique. Results however are not sensitive to alternative rules of portfolio generation.

13 See Hirschmann (1964).

existing banks[14] insofar as the sector with the largest exposure size has a similar share of the total portfolio. Furthermore, the HHI is similar to what is observed in real-world portfolios. Finally, we created portfolio 6 with the highest degree of concentration as a one-sector portfolio by shifting all exposures to the Capital Goods sector.

Table 2: Sequence of portfolios with increasing sector concentration

	Benchmark portfolio	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4	Portfolio 5	Portfolio 6
A: Energy	0%	0%	0%	0%	0%	0%	0%
B: Materials	6%	4%	3%	2%	2%	1%	0%
C1: Capital Goods	12%	41%	56%	71%	78%	82%	100%
C2: Commercial Services & Supplies	34%	22%	17%	11%	8%	7%	0%
C3: Transportation	7%	5%	4%	2%	2%	1%	0%
D: Consumer Discretionary	15%	10%	7%	5%	4%	3%	0%
E: Consumer Staples	6%	4%	3%	2%	2%	1%	0%
F: Health Care	9%	6%	5%	3%	2%	2%	0%
H: Information Technology	3%	2%	2%	1%	1%	1%	0%
I: Telecommunication Services	1%	1%	1%	0%	0%	0%	0%
J: Utilities	7%	4%	3%	2%	2%	1%	0%
HHI	17.6	24.1	35.2	51.5	61.7	68.4	1

3.5. Intra and inter-sectoral correlations

Given that asset correlations are usually not observable, we have followed market practice in using sample correlations of stock index returns for their estimation. Table 3 shows the correlation matrix of the log-returns of those MSCI EMU industry indices which correspond to the 11 sectors. The sector factor correlations are based on weekly return data covering the period from November 2003 to November 2004. Sectors that are highly correlated with other sectors (i.e. sectors that have an average inter-sector correlation of greater than 65%) are Materials (B), Capital Goods (C1), Transportation (C3) and Consumer Discretionary (D). Sectors that are moderately correlated with other sectors, i.e. sectors that have an average inter-sector correlation of between 45% and 65%, are Commercial Services and Supplies (C2), Consumer Staples (E) and Telecommunication (I). Sectors that are the least correlated with other sectors, i.e. sectors that have an average inter-sector correlation of less than 45%, are Energy (A) and Health Care (F). The relative order of these sectors is broadly in line with results reported in other empirical papers.[15] The heterogeneity between Capital Goods, Commercial Services and Supplies, and Transportation is confirmed by noticeable differences in correlations. The intra-sector correlations and/or inter-sector correlations between exposures

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14 Confidentiality requires those banks with a high sector concentration remain anonymous.

15 See, for example, De Servigny and Renault (2001), FitchRatings (2004) and Moody's (2004). It is difficult to compare the absolute inter-sector correlation values as different papers report different types of correlations. De Servigny and Renault (2001) report inter-sector default correlation values, FitchRatings (2004) reports inter-sector equity correlations while Moody's (2004) provides correlation estimates inferred from co-movements in ratings and asset correlation estimates. Furthermore, the different papers distinguish between a different number of sectors.

are obtained by multiplying the sector correlations of Table 3 with the sector-dependent factor weights, see equation (2).

Table 3: Correlation matrix based on MSCI EMU industry indices (based on weekly log return data covering the Nov 2003 - Nov 2004 period; in percent)

	A	B	C1	C2	C3	D	E	F	H	I	J
A: Energy	100	50	42	34	45	46	57	34	10	31	69
B: Materials		100	87	61	75	84	62	30	56	73	66
C1:Capital Goods			100	67	83	92	65	32	69	82	66
C2:Commercial Svs & Supplies				100	58	68	40	8	50	60	37
C3:Transportation					100	83	68	27	58	77	67
D: Consumer discretionary						100	76	21	69	81	66
E: Consumer staples							100	33	46	56	66
F: Health Care								100	15	24	46
H: Information Technology									100	75	42
I: Telecommunication Services										100	62
J: Utilities											100

More difficult than the estimation of sector correlations is the determination of the factor weights, which determine the intra-sector asset correlations. We do not use the formula provided in CreditMetrics to compute the factor weights as recent research has suggested that this formula does not fit the German data very well.[16] Instead, we assume a unique factor weight for all exposures and calibrate the value of the factor weight to match the corresponding IRB regulatory capital charge. More precisely, we determine a factor weight $r_s=0.50$ for all sectors $s \in \{1,...,S\}$ such that the economic capital EC_{sim} of the benchmark portfolio equals the IRB capital charge for corporate exposures, assuming a default probability of 2%, an LGD of 45% and a maturity of one year.

Setting the sector factor weight to 0.5 is slightly more conservative than empirical results for German companies suggest. The average of all the correlation entries in the factor correlation matrix is 0.59, which implies by evoking equation (2) an average asset correlation of 0.14 between exposures. Empirical evidence[17] has shown that German SMEs typically have an average asset correlation of 0.09, which suggests $r_s = 0.39$. Large firms, however, are typically more exposed to systematic risk than SMEs and therefore usually have higher asset correlation values.[18]

Equation (2) implies that intra-sector asset correlations are thus fixed at 25%. Inter-sector asset correlations can be calculated by multiplying the factor weights of both sectors by the inter-sector factor correlation. The lowest correlation between the Energy sector index and the Information Technology sector index of 10% translates into an inter-sector asset correlation of 2.5%. The highest correlation occurs between the Commercial Services and Supplies and the

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[16] See Hahnenstein (2004) for a detailed analysis.

[17] See Hahnenstein (2004).

[18] See, for example, Lopez (2004) for empirical evidence of this relation for the US.

Consumer Discretionary sector indices. At 92%, it translates into an inter-sector asset correlation of 23%.

4. Impact of sector concentration on economic capital

In this section we analyze the impact of increasing sector concentration on economic capital, which is defined as the difference between the 99.9% percentile of the loss distribution and the expected loss. The results are given in Table 4. We observe for the corporate portfolios that economic capital increases from the benchmark portfolio to portfolio 2 by 20%. Economic capital for the concentrated portfolio 5 increases by a substantial 37% relative to the benchmark portfolio. These results demonstrate the importance of taking sector concentration into account when calculating *EC*.

Typically, the corporate portfolio comprises only a fraction of the total loan portfolio (which also contains loans to sovereigns, other banks and private retail clients). Although the increase in sector concentration may have a significant impact on the *EC* of the corporate credit portfolio, it may have a much smaller impact in terms of a bank's total credit portfolio. For a meaningful comparison, we assume that the corporate credit portfolio comprises 30% of the total credit portfolio and that the banks need to hold capital amounting to 8% of their total portfolio. By assuming that there are no diversification benefits between corporate exposures and the bank's other assets, the *EC* of the total portfolio can be determined as the sum of the *EC* for the corporate exposures and the *EC* for the remaining exposures.

Table 4: Impact of sector concentration on economic capital (EC_{sim}) for the sequence of corporate portfolios and for the sequence of total portfolios(*) of a bank (in percent)

	Benchmark portfolio	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4	Portfolio 5	Portfolio 6
Corporate portfolio	7.8	8.8	9.5	10.1	10.3	10.7	11.7
Total portfolio	8.0	8.2	8.5	8.7	8.8	8.9	9.2

(*) Total portfolio includes 30% corporate credit and 70% other credit (to retail, sovereign,…)

The results for the total portfolios of the bank are also shown in Table 4. As expected, the impact of an increase in sector concentration is much less severe when looking at the *EC* for the total portfolio. Economic capital for portfolio 5, for example, increases by about 16% relative to the benchmark portfolio instead of 37% if only the corporate portfolio is taken into account.

In order to verify how robust our results are to the input parameters, we carried out the following three robustness checks (RC1 - RC3):

- a lower uniform PD of 0.5% instead of 2% for all sectors (RC1),
- a different factor correlation matrix (See Table 15, Appendix A) representing the correlation matrix with the highest average annual correlation over the period between 1997 and 2005 (RC2) and

- a uniform intra-sector asset correlation of 15% and a uniform inter-sector asset correlation of 6% (RC3), which are values used by Moody's for the risk analysis of synthetic CDOs.[19]

The results of the three robustness checks are summarized in Table 5. Although the absolute level of *EC* varies between these robustness checks, the relative increase in *EC* compared with the benchmark portfolio is similar to previous results in this section. For Moody's correlation assumptions in RC3, the increase in *EC* is stronger than for the other robustness checks. This can be explained by the larger difference between intra-sector and inter-sector correlations, which is justified by the higher number of sectors they use, and which leads to a stronger *EC* increase when the portfolio becomes more and more concentrated in a single sector. We conclude that the observed substantial relative increase in *EC* due to the introduction of sector concentration is robust against realistic variation of the input parameters. Furthermore, this increase in *EC* may be even greater, depending on the underlying dependence structure.

Table 5: EC for the benchmark portfolio and its relative increase for the more concentrated portfolios 1 - 6 (in percent of total exposure)

Portfolio	**Using "Initial rule"**	**RC1: PD=0.5%**	**RC2: Higher correlation**	**RC3: Moody's**
	EC			
Benchmark portfolio	7.8	3.3	8.7	4.0
	Proportional change of EC in %			
Portfolio 1	+13	+12	+6	+6
Portfolio 2	+20	+21	+13	+18
Portfolio 3	+30	+29	+22	+39
Portfolio 4	+35	+37	+24	+46
Portfolio 5	+36	+42	+24	+51
Portfolio 6	+49	+52	+33	+77

5. Evaluation of the EC approximations for sector-dependent PDs and high granularity

The purpose of this section is to use the model by Pykhtin to calculate *EC* and to compare these *EC* approximations with the *EC* obtained from simulations. In this section we assume first homogeneity within each sector and second a highly granular exposure distribution in each sector. Because of these two assumptions of our simplified model, the results can be understood as an upper bound in terms of approximation quality. We further test the accuracy of the *EC* approximations by varying the sector distributions, the factor correlations, the factor weights, the number of factors and the sector PD. Portfolios of coarser granularity and heterogeneous PDs on an exposure level are studied in Sections 6 and 7.

[19] See Fu et al (2004).

We again assume a confidence level q of 99.9% and employ the following three risk measures (where $EL = \mu \sum_{s=1}^{S} \sum_{i=1}^{M_s} w_{si} p_{si}$):

- economic capital in the ASRF* model, which is defined as $EC^* = t_{99.9\%}\left(L^*\right) - EL$
- economic capital based on the multi-factor adjustment,

  $EC_{MFA} = t_{99.9\%}\left(L^*\right) + \Delta t_{99.9\%} - EL$
- economic capital based on Monte Carlo (MC) simulations, EC_{sim}

Firstly, we present results for the benchmark portfolio and for the more concentrated portfolios 1 - 6 in Table 6. The model parameters are the same as in Section 4.

Table 6: Comparison of EC^*, EC_{MFA} and EC_{sim} for different exposure distributions across sectors with increasing sector concentration given a default probability of 2% (in percent of total exposure)

Portfolio	EC^*	EC_{MFA}	EC_{sim}	**Relative error(*) of** EC_{MFA}
Benchmark portfolio	7.8	7.9	7.8	1.3%
Portfolio 1	8.7	8.8	8.8	0.0%
Portfolio 2	9.4	9.4	9.5	-1.1%
Portfolio 3	10.1	10.1	10.1	0.0%
Portfolio 4	10.5	10.5	10.3	1.9%
Portfolio 5	10.7	10.7	10.7	0.0%
Portfolio 6	11.6	11.6	11.7	-0.9%

(*) The relative error is defined as the relative difference between EC_{sim} and EC_{MFA}.

The *EC* figures for the benchmark portfolio in Table 6 show that EC^* and EC_{MFA} provide extremely accurate proxies for EC_{sim}. This result suggests that in the given examples the calculation of EC^* may, in practice, be sufficiently accurate for certain risk-management purposes. The four *EC* estimates for the more highly concentrated portfolios 1 - 6 indicate that economic capital increases as expected, but that our results for the approximation performance of EC^* and EC_{MFA} still hold. According to Table 6, relative errors of EC_{MFA} are in a relatively small range between 0.0% and 1.9%.

Secondly, we check whether our results differ when we vary the underlying correlation structure. To this end, we calculate in Table 7 the three risk measures for different factor correlation matrices. More specifically, we assume homogeneous factor correlation matrices in which the entries (outside the main diagonal) vary between 0 and 1 in increments of 0.2. The last case, in which all factor correlations are equal to one, corresponds to the case of a single-factor model.

Table 7: Comparison of EC^*, EC_{MFA} and EC_{sim} for different factor correlations ρ, given a default probability of 2% (in percent of total exposure)

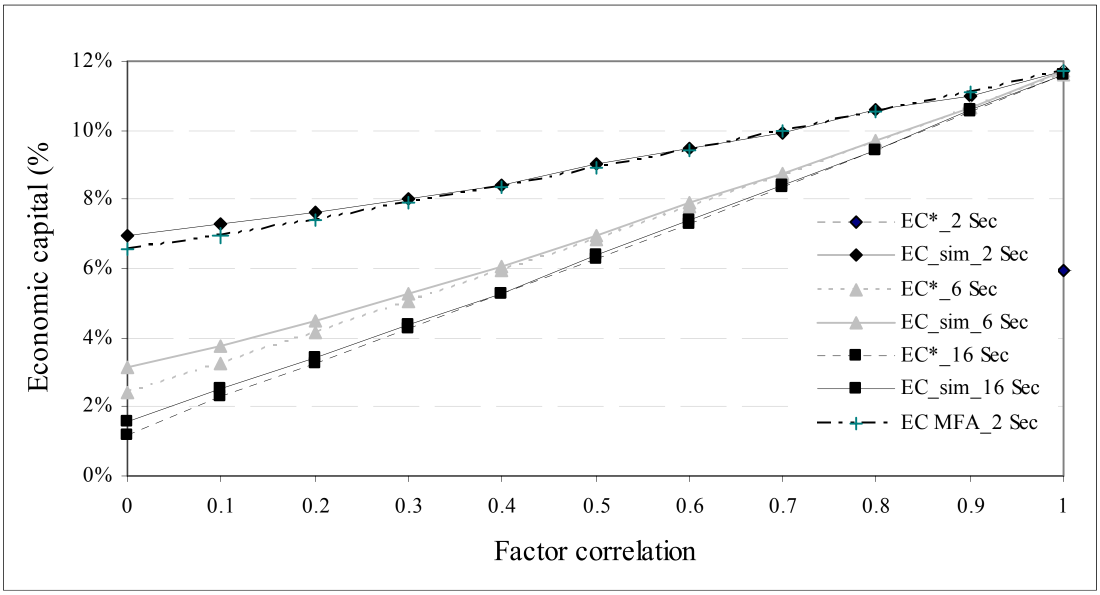
Factor correlation ρ	EC^*	EC_{MFA}	EC_{sim}	**Relative error of** EC_{MFA}
0.0	3.3	3.9	4.0	-2.5%
0.2	4.5	4.9	5.0	-2.0%
0.4	6.1	6.3	6.3	0.0%
0.6	7.9	7.8	8.0	-2.5%
0.8	9.7	9.7	9.9	-2.0%
1.0	11.6	11.6	11.7	-0.8%

Table 7 shows EC_{sim} and its proxies EC^* and EC_{MFA} for increasing factor correlations. As expected, economic capital increases with increasing factor correlations, since a higher factor correlation reduces the diversification potential by shifting probability mass to the tail of the loss distribution. The highest relative error of EC_{MFA} of all factor correlations considered is 2.5% which still reveals a good approximation performance. With increasing factor correlations the multi-factor model approaches the structure of a one-factor model for which EC^* and EC_{MFA} coincide. In all cases EC^* is relatively close to EC_{MFA}. Therefore, our earlier results concerning the good approximation performance of EC^* and EC_{MFA} also hold under different factor correlation assumptions.

Thirdly, we vary the value of the factor weight *r* from 0.2 to 0.8. There is a strong increase in *EC* with the factor weight but this does not affect the approximation quality, neither of EC^* nor of EC_{MFA}.

Fourthly, we explore how the results depend on the number of factors. For this purpose we vary the number of factors from 2 to 16. Figure 2 shows how EC^*, EC_{MFA} and EC_{sim} depend on the number of sectors and the factor correlation. EC_{MFA} is only plotted for 2 sectors because its values are indistinguishable from EC_{sim} for 6 and for 16 sectors.

Figure 2: Economic capital (EC^* , EC_{MFA} and EC_{sim}) for different factor correlation values for 2, 6 and 16 sectors (in percent of total exposure)



For a given number of sectors, *EC* increases in Figure 2 with factor correlation as expected. If the factor correlation approaches one, then *EC* values coincide, irrespective of the number of sectors. The reason is that in the limiting case of a factor correlation equal to one, the model collapses to a single-factor model.

For a factor correlation of 0.6, which is also the average of the entries in the correlation matrix in Table 3, and also for higher factor correlations, the relative approximation error is below 1% for EC_{MFA} and below 2% for EC^*. Therefore, the previous results showing a good approximation performance of EC^* and an even better one for EC_{MFA} are found to be robust with respect to the number of sectors, at least for realistic factor correlations.

Figure 2 also shows that EC^* and EC_{sim} generally decrease when the number of sectors increases for given asset correlation values. This result can be explained by risk reduction through diversification across sectors.

Fifthly, we test whether our results for the approximation performance of EC^* and EC_{MFA} are sensitive to PD heterogeneity on a *sector level.* For this purpose we employ the scaled default rates for sectors from Table 8.

Table 8: Average historical default rates (1990-2004; before and after scaling to an exposure-weighted expected average default rate of 2% for the benchmark portfolio; in percent)

Sector	**Unscaled default rate**	**Scaled default rate**
A: Energy	1.5	1.0
B: Materials	2.8	1.9
C1: Capital Goods	2.9	2.0
C2: Commercial Services and Supplies	3.7	2.5
C3: Transportation	2.9	2.0
D: Consumer Discretionary	3.2	2.2
E: Consumer Staples	3.5	2.4
F: Health Care	1.6	1.1
H: Information Technology	2.4	1.6
I: Telecommunication Services	3.6	2.4
J: Utilities	0.6	0.4

Source: own calculation, based on S&P (2004)

The historical default rates in Table 8 are, on average, higher than the value of 2% which is used for the PDs in the case of homogeneous PDs for all sectors. In order to isolate the effect of PD heterogeneity between sectors, we scale the historical default rate, p_s^{hist}, for every sector s as follows,

$$(8) \qquad p_s^{scaled} = p_s^{hist} \frac{0.02}{\sum_{s=1}^{S} w_s \cdot p_s^{hist}}.$$

In this way, we ensure that the weighted average PD of the benchmark portfolio stays at 2% even in the case of PD heterogeneity across sectors.

The results for EC_{sim} and the two analytical approximations of *EC*, using the scaled historical default rates as PD estimates, are given in Table 9.

Table 9: Comparison of EC^*, EC_{MFA} and EC_{sim} (in percent of total exposure), based on sector-dependent default probabilities, estimated from historical default rates

Portfolio	EC^*	EC_{MFA}	EC_{sim}	**Relative error of** EC_{MFA}
Benchmark portfolio	8.0	8.0	8.0	0.0%
Portfolio 1	8.8	8.9	8.8	1.1%
Portfolio 2	9.4	9.4	9.5	-1.1%
Portfolio 3	10.1	10.1	10.1	0.0%
Portfolio 4	10.5	10.5	10.4	1.0%
Portfolio 5	10.7	10.7	10.7	0.0%
Portfolio 6	11.6	11.6	11.7	-0.9%

For all risk measures the results in Table 9 are relatively close to those in Table 6. The more concentrated the exposures are in one sector, the smaller the difference to Table 6 becomes.

This is explained by the fact that the sector PDs are calibrated to an average value of 2%, which is also the PD used for Table 6. The approximation quality of EC^* and EC_{MFA} is similar to Table 6. We conclude that, in qualitative terms, the results obtained for a uniform PD also hold for heterogeneous sector-dependent PDs.

6. Evaluation of the EC approximations for sector-dependent PDs and low granularity

Simulation results in the previous section, which reveal a reasonably good approximation quality for EC^* and EC_{MFA}, were obtained conditional on a uniform PD in every sector and highly granular portfolios. However, portfolios of small banks, in particular, are less granular. In the following we explore the impact of lower granularity. From the set of seven portfolios, only the benchmark portfolio and portfolio 6 are considered as they have the lowest and the highest sector concentration. The impact of granularity is considered for the following two cases.

In the first case, characterized by a portfolio of representative granularity, the distribution of exposure size was selected from a sample of typical small, regional German banks to reflect an average granularity in terms of the HHI. The purpose is to measure the impact of granularity for an exposure distribution that is representative for real banks. However, since the exposure distribution is based on central credit register data, only larger exposures are captured[20] in the underlying data set with the consequence that this exposure distribution is less granular than what we can expect for real bank portfolios. The HHI of the portfolio, measured on single-exposure level, is 0.0067 compared with an HHI of 0.001 for the highly granular portfolios used in the previous section. Descriptive statistics on exposure size of the new portfolio are shown in Table 16 in Appendix D. Unfortunately, the borrower-specific data on exposure size contain no sector information.[21] The allocation of exposures to sectors was achieved by randomly drawing exposures from the data set under the constraint that the generated distribution of exposures across sectors mirrors the sectoral distribution of the benchmark portfolio. To control for any sampling bias in the results we repeated this random assignment thereby creating several portfolios. These portfolios have the same sector distribution but vary in the distribution of individual exposure size in each sector. With these portfolios we verified the robustness of the results in this and the following section.

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[20] See section 3.1 for more information on the characteristics of exposures included in the German central credit register.

[21] The reason for this missing information is that we do not use credit register data directly but a matched sample of credit register data and a second database which provides us also with individual borrower PDs not included in the credit register but required for the analysis of PD heterogeneity in section 7.

In the second case, characterized by *low granularity*, we consider the highest individual exposure shares that are admissible under the EU large exposure rules.[22] In this way, we obtain an upper limit for the potential impact of granularity. According to the EU rules, an exposure is considered "large" if its amount requires 10% or more of regulatory capital. Banks are generally not allowed to have an exposure that requires at least 25% of regulatory capital. Furthermore, the sum of all large exposures must not require more than 8 times the regulatory capital.[23]

We assume that a bank's regulatory capital is 8% of its total loan volume. For a total portfolio value of 6,000 currency units, banks are required to hold 480 currency units in capital. Each large exposure requires a minimum amount of capital of 48 currency units and a maximum amount of 120 currency units. The total sum of all large exposures must not exceed 3,840 currency units. With these restrictions, the least granular admissible exposure distribution of our portfolio consists of

- 3840/120 = 32 loans of 120 currency units
- 2160/47 = 45 loan exposures of 47 currency units (just below the large exposure limit of 48) and
- a remaining single exposure of 45 currency units

The HHI of this portfolio, measured on a single-exposure level, is 0.015. Since this portfolio is charaterized by relatively coarse granularity, its HHI is considerably higher than for the portfolio with representative granularity. While keeping the average sector concentration of the portfolio constant, we increase the granularity of the portfolio to reflect the exposure size distribution of this least granular portfolio. More details of this portfolio can be found in Table 17, Appendix D.

Economic capital from simulations, EC_{sim}, and the analytic proxies EC^* and EC_{MFA} are given in Table 10.

Table 10: Comparison of EC^*, EC_{MFA} and EC_{sim} (in percent of total exposure) for portfolios with representative and low granularity using sector-dependent default probabilities

Portfolio	**Granularity**	EC^*	EC_{MFA}	EC_{sim}	**Relative error of** EC_{MFA}
Benchmark portfolio	representative	8.0	8.0	8.6	-7%
	low	8.0	8.0	9.3	-14%
Single sector portfolio	low	11.6	11.6	12.7	-8%

22 See Directive 93/6/EEC of 15 March 1993 on the capital adequacy of investment firms and credit institutions.

23 The last two restrictions may be breached with permission of the German Federal Financial Supervisory Authority (BaFin), in which case the excess must be fully backed by capital.

The EC_{sim} value of 9.3% for the low granular benchmark portfolio is 1.3 percentage points (or 14% in relative terms) higher than for the highly granular benchmark portfolio in Table 9. This difference appears to be substantial, but we have to consider that the granularity of the portfolio in Table 10 is very low since it reflects the lowest granularity permissible under European bank regulation. EC_{sim} for the single sector portfolio 6 in Table 10 is higher than for the benchmark portfolio, which is consistent with earlier reported results.

The EC_{sim} value of 8.6% for the benchmark portfolio with representative granularity is relatively close to the value of 9.3% for the portfolio with low granularity, at least if compared with EC_{sim} of 8.0% for the infinitely granular benchmark portfolio in Table 9. One reason is that some exposures in the portfolio with representative granularity technically violate the large exposure rules.[24] Therefore, as mentioned before, the portfolio of "representative" granularity should still be regarded as conservative in terms of granularity.

For the purpose of this analysis, the approximation errors of the *EC* proxies, EC^* and EC_{MFA}, are more important than the level of *EC*. Both *EC* proxies are based on the assumption of infinite granularity in each sector, while the EC_{sim} calculations take granularity into account. We find that EC^* and EC_{MFA} can substantially underestimate *EC* by up to 14%, in particular for portfolios with coarse granularity .

7. Evaluation of EC approximations for heterogeneous sectors

So far we have only considered sector-dependent PDs, which means PD variation on a sector level, but not on the exposure level. In the following we explore the impact of heterogeneous PDs inside a sector together with the impact of granularity. For the benchmark portfolio of representative granularity analyzed in the previous section, we also have individual borrower PDs which were computed from a logit model based on firms' accounting data. In order to apply the logit model, borrower information from the central credit register on exposure size had to be matched with a balance sheet database, also maintained by the Deutsche Bundesbank.[25] Using empirical data on exposure size and PD automatically captures a potential dependence between these two characteristics.

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24 This can be explained either by special BaFin approval or, most likely, by data limitations given that our credit register data do not contain loans below €1.5 million. The latter implies that their sum is lower than the total portfolio exposure of the data-providing real bank and, therefore, our relative exposure weights are biased upwards. In other words, it is well possible that the large exposure limit is breached for our portfolio, although the limit is still met by the data-providing bank.

25 More details on the database and the logit model that was used to determine the PDs can be found in Krüger et al. (2005).

In order to guarantee comparability with previous results, we apply the same scaling procedure as in Section 6 to ensure that the exposure-weighted average PD in each sector is the same as the corresponding scaled default rate given inTable 8. Information on this PD distribution is given in Table 18, Appendix D.

The portfolio with the lowest granularity admissible under the EU large exposure rules is an artificially generated portfolio, so that we have no PD information for single exposures. Therefore, we randomly assign PDs from an empirical aggregate PD distribution based on the same balance sheet database, but this time aggregated over a sample of banks. The empirical PD distribution is given in Table 20 and information on the PD distribution of the low granular portfolio is provided in Table 19, Appendix D.[26]

The results for PD heterogeneity in every sector are given in Table 11. The reduction of EC_{sim} compared to Table 10, which occurs for both portfolios, is due to the PD heterogeneity on the exposure level. This impact of PD heterogeneity has also been noted by Hanson et al (2005) and can be explained by the concavity of the dependence of *EC* on PD.

Table 11: Comparison of EC^*, EC_{MFA} and EC_{sim} for portfolios with heterogeneous sectors (in percent of total exposure)

Portfolio	**Granularity**	EC^*	EC_{MFA}	EC_{sim}	**Relative error of** EC_{MFA}
Benchmark portfolio	representative	8.0	8.0	7.7	+4%
	low	8.0	8.0	8.5	-6%
Single sector portfolio	low	11.6	11.6	10.8	+8%

Since EC^* and EC_{MFA} do not account for PD heterogeneity on the exposure level, these values stay unchanged from Table 10 while EC_{sim} decreases. As a consequence, the underestimation by using EC^* and EC_{MFA} instead of EC_{sim} is reduced relative to Table 10, or even reversed to an overestimation of *EC*. This is confirmed by the approximation error in the last column of Table 11, which is lower when using heterogeneous PDs compared to the case of sector-dependent PDs in Table 10.

For the single-sector portfolio and the benchmark portfolio with representative granularity, the approximation errors of the *EC* proxies are positive, implying that the effect of PD heterogeneity is stronger than the granularity effect, measured relative to the highly granular portfolio with homogeneous sector PDs. As a consequence, the *EC* proxies provide conservative estimates. Comparing the conservativeness of the single-factor portfolio and the

[26] Since a negative correlation between exposure size and PD emerged as a stylized fact in recent empirical literature (See, for example, Dietsch and Petey (2002) or Lopez (2004)), we also considered the case that the PDs are perfectly ordered in terms of decreasing exposure size. We found that our results are robust in this case.

benchmark portfolio in Table 11, we observe that the degree of overestimation halves from +8% to +4%. This suggests further robustness checks, in particular for portfolios with a higher number of sectors.

In summary, the approximation errors for all portfolios considered vary between -6% and +8%. The results of Table 10 and Table 11 taken together demonstrate that the effect of PD heterogeneity counterbalances the effect of granularity. In general it is not possible to determine which of the two opposing effects dominates. For the portfolio with a representative granularity in Table 11, the effect of granularity is arguably weaker, which suggests that for portfolios of "average granularity" in real banks, PD heterogeneity would tend to overcompensate the granularity effect and EC^* and EC_{MFA} would provide conservative estimates.

Further empirical work is warranted to confirm this indicative result.

Our analysis has shown that PD heterogeneity on the exposure level improves the performance of the analytic *EC* approximations relative to the situation of a granular portfolio with (only) sector-dependent PDs. The reason is that PD heterogeneity reduces the underestimation of *EC* that is caused by the granularity of the portfolio. This effect is even stronger if larger exposures or firms have lower PDs than smaller ones. Furthermore, PD heterogeneity appears not to affect the relative difference between EC_{MFA} and EC^*.

8. Summary and conclusions

The minimum capital requirements for credit risk in the IRB approach of Basel II implicitly assume that credit portfolios of banks are well diversified across business sectors. Potential concentration risk in certain business sectors is covered by Pillar 2 of the Basel II Framework, which comprises the supervisory review process.[27] To what extent the regulatory minimum capital requirements can understate economic capital is an empirical question. In this paper we provide a tentative answer by using data from the German central credit register. Credit risk is measured by economic capital in a multi-factor asset value model and determined by Monte Carlo simulations.

In order to measure the impact of concentration risk on economic capital, we start in the empirical part with a benchmark portfolio that reflects the aggregate exposure distribution across sectors of the German banking system. Since the exposure distributions across business sectors are similar in Belgium, France and Spain, we expect that our main results also hold for other European countries. Starting with the benchmark portfolio, we have successively increased sector concentration, considering degrees of sector concentration which are observable in real banks. The most concentrated portfolio contained exposures only to a single sector. Compared with the benchmark portfolio, economic capital for the concentrated portfolios can increase by almost 37% and by 50% in the case of a one-sector portfolio. We have subjected our results to various robustness checks. We find that the increase in economic capital may be even greater, contingent on the dependence structure. This result clearly underlines the necessity to take inter-sector dependency into account for the measurement of credit risk.

Since concentration in business sectors can substantially increase economic capital, a tractable and robust calculation method for economic capital which avoids the use of computationally burdensome Monte Carlo simulations is desirable. For this purpose the theoretical part evaluates the accuracy of a model developed by Pykhtin (2004), which provides an analytical approximation of economic capital in a multi-factor framework. We have applied a simplified, more tractable version of the model which requires only sector-aggregates of exposure size, PD and expected loss severity. The dependence structure is captured by the correlation matrix of the original multi-factor model. Furthermore, we have evaluated the extent to which EC^*, as the first of two components in the analytic approximation of economic capital, already provides a reasonable proxy of economic capital. EC^* refers to the economic capital for a single-factor model in which the sector-dependent asset correlations are defined by mapping the richer correlation structure of the multi-factor model. The benchmark for the approximation quality is always the economic capital figure of the original multi-factor model which is obtained from MC simulations.

We have shown that the analytic approximation formulae perform very well for portfolios with highly granular and homogeneous sectors. This result holds for portfolios with different sector concentrations and for various factor weights and correlation assumptions. Furthermore, we have found that EC^* is relatively close to the simulation-based economic capital for most of the realistic input parameter tupels considered.

Finally, we explore the robustness of our results against the violation of two critical model assumptions, namely infinite granularity in every sector and sector-dependent PDs. We find that coarser granularity and PD heterogeneity (on the single exposure level) have counterbalancing effects on the performance of the analytic approximations for economic capital. Coarser granularity induces the analytic approximation formulae to have a downward bias which increases to 14% in extreme cases of portfolios with the lowest granularity permissible by EU large exposure rules, depending on the sector structure of the portfolio.

Replacing sector-dependent PDs by heterogeneous PDs on the individual exposure level reduces economic capital, but does not affect the analytic approximations. As a consequence, the downward bias decreases. The relative error of the analytic approximation, measured

[27] See BCBS (2004b), paragraphs 770-777.

relative to the simulation-based economic capital figure, lies in a range between –6% and +8%, dependent on the exposure distribution across sectors and the number of factors. In summary, we find that heterogeneity in individual PDs and low granularity partly balance each other in their impact on the performance of the analytic approximations. Which effect prevails depends on the specific input parameters. Indicative results suggest that in representative credit portfolios, PD heterogeneity will at least compensate for the granularity effect, which suggests that the analytic formulae approximate economic capital reasonably well and err on the conservative side.

In the cases studied, it is possible to use the analytic economic capital approximations of the simplified Pykhtin model without sacrificing much accuracy. This is an important result as it suggests, pending further robustness checks, that supervisors and banks can reasonably well approximate their economic capital for their credit portfolio by a relatively simple formula and without running computationally burdensome Monte Carlo simulations.

Further research seems to be warranted, particularly in further advancing Pykhtin's methodology in a direction which improves its approximation accuracy while staying parsimonious in terms of data requirements. This could be achieved, for example, by exploring alternative ways to map the correlation matrix of the multi-factor model into sector-dependent asset correlations.

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Appendix A

Table 12: GICS Classification Scheme: Broad Sector and Industry Groups

A: Energy	A1: Energy
B: Materials	B1: Materials
C: Industrial	C1: Capital goods C2: Commercial Services and Supplies C3: Transportation
D: Consumer Discretionary	D1: Automobiles and Components D2: Consumer Durables and Apparel D3: Hotels, Restaurants and Leisure D4: Media D5: Retailing
E: Consumer Staples	E1: Food and Drug Retailing E2: Food, Beverage and Tobacco E3: Household and Personal Products
F: Health Care	F1: Health Care Equipment and Services F2: Pharmaceuticals and Biotechnology
G: Financials	G1: Banks G2: Diversified Financials G3: Insurance G4: Real estate
H: Information Technology	H1: Software and Services H2: Technology Hardware & Equipment H3: Semiconductors & Semiconductor Equipment
I: Telecommunication Services	I1: Telecommunication Services
J: Utilities	J1: Utilities

Table 13: Mapping NACE codes to GICS codes

2 (or more) -digit code	Description	Mapped to GICS
1	Agriculture and hunting	E
2	Forestry	B
5	Fishing	E
10	Coal mining	B
11	Crude petroleum and natural gas extraction	A
12	Mining of uranium and thorium	B
13	Mining of metal ores	B
14	Other mining and quarrying	B
15	Food and beverages manufacturing	E
16	Tobacco manufacturing	E
17	Textile manufacturing	D
18	Textile products manufacturing	D
19	Leather and leather products manufacturing	D
20	Wood products	D
21	Pulp, paper and paper products	B
22	Publishing and printing	C2
23	Manufacture of coke, refined petroleum products and nuclear fuel	A

24 (excl 244)	Chemicals and chemical products manufacturing	B
244	Pharmaceuticals	F
25	Rubber and plastic manufacturing	D
26	Other non-metallic mineral products	B
27	Basic metals manufacturing	B
28	Fabricated metal manufacturing	B
29	Machinery and equipment manufacturing	C1
30	Office machinery and computers manufacturing	H
31	Electrical machinery manufacturing	H
32	TV and communication equipment manufacturing	H
33	Medical and optical instruments manufacturing	F
34	Car manufacturing	D
35	Other transport equipment manufacturing	D
36	Furniture manufacturing	D
37	Recycling	J
40	Gas and electricity supply	J
41	Water supply	J
45	Construction	C1
50	Car sales, maintenance and repairs	D
51	Wholesale trade	C2
52 (excl 5211, 522,523)	Retail trade	D
522, 523	Consumer staples	E
55	Hotels and restaurants	D
60	Land transport	C3
61	Water transport	C3
62	Air transport	C3
63	Transport supporting activities and travel agencies	C3
64	Post and telecommunication	I
65	Financial institutions	G1
66	Insurance	G3
67	Support to financial institutions	G1
70	Real estate	G4
71	Machinery and equipment leasing manufacturing	C1
72	Computer and related activities	H
85	Health care and social work	F
90	Sewage and refuse disposal	J
96	Residential property management	G4

Table 14: Comparison of sector concentrations, aggregated exposure values over banks in Germany, France, Belgium and Spain

Sector	Germany	France	Belgium	Spain
A1: Energy	0.18%	0.88%	0.05%	1,05%
B1: Materials	6.01%	3.97%	7.45%	9,34%
C: Industrial[28]	52.36%	63.82%	54.77%	48,53%
C1: Capital Goods	11.53%		9.89%	32,90%
C2: Commercial Services and Supplies	33.69%		37.74%	10,20%
C3: Transportation	7.14%		7.14%	5,43%
D: Consumer Discretionary	14.97%	11.91%	15.77%	18,60%
E: Consumer Staples	6.48%	7.21%	7.05%	10,20%
F: Health Care	9.09%	5.00%	5.64%	1,85%
H1: Software and Services	3.20%	1.47%	1.86%	1,99%
I1: Telecommunication Services	1.04%	1.91%	0.54%	2,67%
J1: Utilities	6.67%	3.82%	6.87%	5,77%

Table 15: Correlation matrix based on MSCI EMU industry indices (based on weekly log return data covering the Nov 2002 - Nov 2003 period; in percentages).

	A	**B**	**C1**	**C2**	**C3**	**D**	**E**	**F**	**H**	**I**	**J**
A: Energy	100	62	66	43	62	67	78	70	50	47	72
B: Materials		100	91	78	77	85	73	69	74	68	69
C1:Capital Goods			100	76	80	92	74	68	81	72	75
C2:Commercial Svs & Supplies				100	66	81	58	53	71	58	52
C3:Transportation					100	78	68	59	70	65	64
D: Consumer discretionary						100	71	66	86	72	70
E: Consumer staples							100	75	62	60	70
F: Health Care								100	55	44	70
H: Information Technology									100	69	58
I: Telecommunication Services										100	67
J: Utilities											100

[28] Aggregate of C1, C2 and C3 only used for comparison with French data. Not used in the analysis.

Appendix B

The multi-factor adjustment Δt_q can be calculated according to Pykhtin (2004) as follows:

$$\text{(A1)} \qquad \Delta t_q = -\frac{1}{2l'(y)}\left[v'(y) - v(y)\left(\frac{l''(y)}{l'(y)} + y \right)\right]\Bigg|_{y=N^{-1}(1-q)}$$

where y denotes the single systematic risk factor.

The first and second derivatives of the loss distribution function in a one-factor model are

$$\text{(A2)} \qquad \begin{aligned} l'(y) &= \mu \sum_{s=1}^{S} w_s \hat{p}'_s(y) \\ l''(y) &= \mu \sum_{s=1}^{S} w_s \hat{p}''_s(y) \end{aligned}$$

where $\hat{p}'_s(y)$ and $\hat{p}''_s(y)$ are, respectively, the first and the second derivatives of the conditional probability of default.

$$\text{(A3)} \qquad \begin{aligned} \hat{p}'_s(y) &= -\frac{c_s}{\sqrt{1-c_s^2}} N'\left(\frac{N^{-1}(p_s) - c_s y}{\sqrt{1-c_s^2}} \right) \\ \hat{p}''_s(y) &= -\frac{c_s}{\sqrt{1-c_s^2}} \frac{N^{-1}(p_s) - c_s y}{\sqrt{1-c_s^2}} N'\left(\frac{N^{-1}(p_s) - c_s y}{\sqrt{1-c_s^2}} \right). \end{aligned}$$

The factor weight in the ASRF* model is denoted by c_s which can be written as $c_s = r_s \rho_s^*$ where ρ_s^* denotes the correlation between the composite sector factor Y_s and the systematic factor Y^* in the ASRF* model.

For the conditional variance $v(y)$ and its first derivative hold

$$\begin{aligned} v(y) &= \mu^2 \sum_{s=1}^{S}\sum_{t=1}^{S} w_s w_t \left[N_2\left(N^{-1}(\hat{p}_s(y)), N^{-1}(\hat{p}_t(y)), \omega_{st}^Y \right) - \hat{p}_s(y)\hat{p}_t(y) \right] \\ &\quad + \mu^2 \sum_{s=1}^{S} w_s^2 \left[\hat{p}_s(y) - N_2\left(N^{-1}(\hat{p}_s(y)), N^{-1}(\hat{p}_s(y)), \omega_{ss}^Y \right) \right] \end{aligned}$$

$$\begin{aligned} v'(y) &= 2\mu^2 \sum_{s=1}^{S}\sum_{t=1}^{S} w_s w_t \hat{p}'_s(y) \left[N\left(\frac{N^{-1}(\hat{p}_t(y)) - \omega_{st}^Y \hat{p}_s(y)}{\sqrt{1-\left(\omega_{st}^Y\right)^2}} \right) - \hat{p}_t(y) \right] \\ &\quad + \mu^2 \sum_{s=1}^{S} w_s^2 \hat{p}'_s(y) \left[1 - 2 \cdot N\left(\sqrt{\frac{1-\omega_{ss}^Y}{1+\omega_{ss}^Y}} N^{-1}(\hat{p}_s(y)) \right) \right] \end{aligned}$$

where $N_2(\ )$ denotes the cumulative distribution function of the bivariate-normal distribution and ω_{st}^{Y} has the meaning of a conditional asset correlation for two exposures in sectors *t* and *s*, conditional on Y^*. This conditional asset correlation can be written as

$$\omega_{st}^{Y} = \frac{\omega_{st} - c_s c_t}{\sqrt{\left(1 - c_s^2\right)\left(1 - c_t^2\right)}} .$$

Appendix C

In Pykhtin (2004) the coefficients $b_1,...,b_S$ are obtained by maximizing the correlation between Y^* and the risk factors $Y_1, ..., Y_S$ which leads to the following optimization problem:

$$\max_{b_1,...,b_S} \sum_{s=1}^{S} \theta_s \sum_{t=1}^{S} \alpha_{s,t} b_t \ .$$

subject to $\sum_{s=1}^{S} b_s^2 = 1$. The solution of this problem is given by

$$b_t = \sum_{s=1}^{S} \frac{\theta_s}{\lambda} \alpha_{st} \ .$$

λ is the Lagrange multiplier chosen to satisfy the constraint. Again, there is no unique solution for θ_s. We follow Pykhtin who reported good results when defining

$$\theta_s = \mu \ w_s N\left(\frac{N^{-1}(p_s) + r_s N^{-1}(q)}{\sqrt{1 - r_s^2}} \right).$$

Appendix D

Table 16: Descriptive statistics of exposure distribution of a portfolio of 11 sectors, representative in terms of granularity

Sector	Exposure No.	Minimum	25% percentile	Median	75% percentile	Maximum
1	1	1	NA	1.2	NA	1.2
2	36	0	1.3	4.7	8.9	43.2
3	69	0	2.2	6.1	12.8	127.6
4	203	0	1.7	5.3	10.5	152.3
5	43	0.1	2.1	5.4	10.5	60.0
6	90	0	1.4	5.1	9.3	112.2
7	39	0.1	1.3	4.9	10.0	42.2
8	55	0.2	1.8	4.8	11.3	74.2
9	19	0.1	0.7	3.6	5.8	22.0
10	6	0.2	0.6	3.0	7.8	8.5
11	40	0.0	1.3	5.8	11.6	68.8

Table 17: Descriptive statistics of exposure distribution of a low granular portfolio of 11 sectors

Sector	Exposure No.	Minimum	25% percentile	Median	75% percentile	Maximum
1	1	11	NA	11	NA	11
2	8	32	47	47	47	47
3	6	92	120	120	120	120
4	17	100	120	120	120	120
5	10	6	47	47	47	47
6	8	58	120	120	120	120
7	9	13	47	47	47	47
8	9	33	47	47	47	120
9	5	4	47	47	47	47
10	2	16	16	31.5	47	47
11	9	24	47	47	47	47

Table 18: Scaled PD distribution of a portfolio of 11 sectors, representative in terms of granularity

Sector	Exposure No.	Minimum	25% percentile	Median	75% percentile	Maximum
1	1	1.0%	NA	1.0%	NA	1.0%
2	36	0.0%	0.3%	0.6%	1.2%	7.4%
3	69	0.1%	0.9%	1.6%	2.9%	21.8%
4	203	0.0%	0.9%	1.7%	3.4%	15.5%
5	43	0.0%	0.8%	1.8%	2.7%	6.0%
6	90	0.1%	1.1%	2.2%	3.6%	14.0%
7	39	0.0%	0.9%	1.6%	3.3%	11.1%
8	55	0.0%	0.5%	1.0%	2.0%	5.6%
9	19	0.0%	0.4%	1.1%	1.6%	4.1%
10	6	1.0%	2.2%	3.4%	4.1%	5.9%
11	40	0.0%	0.2%	0.3%	0.6%	2.2%

Table 19: Scaled PD distribution of a low granular portfolio of 11 sector

Sector	Exposure No.	Minimum	25% percentile	Median	75% percentile	Maximum
1	1	1.0%	NA	1.0%	NA	1.0%
2	8	0.3%	1.3%	1.3%	1.3%	4.2%
3	6	0.4%	1.5%	1.5%	1.5%	5.1%
4	17	0.1%	1.8%	1.8%	1.8%	5.9%
5	10	0.1%	1.5%	1.5%	1.5%	4.9%
6	8	0.4%	1.4%	1.4%	1.4%	4.7%
7	9	0.1%	1.7%	1.7%	1.7%	5.8%
8	9	0.1%	0.7%	0.7%	0.7%	2.4%
9	5	0.3%	1.2%	1.2%	1.2%	3.9%
10	2	2.4%	2.4%	2.4%	2.4%	2.4%
11	9	0%	0.3%	0.3%	0.3%	1%

Table 20: Quality distribution of German firms in the Bundesbank database

Rating grade	AAA	AA	A	BBB	BB	B
Share in percent	2	6	11	55	24	2
PD in percent	0.01	0.02	0.07	0.26	0.87	3.27

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