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Incorporating prediction and estimation risk in point-in-time credit portfolio models

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Abstract

In this paper we focus on the analysis of the effect of prediction and estimation risk on the loss distribution, risk measures and economic capital. When variables for the determination of probability of default and loss distribution have to be predicted because they are not available at the time the prediction is made, the prediction is prone to errors. The model parameters for the estimation of probability of default or asset correlation are not available, and usually have to be estimated using historical data. The incorporation of prediction and estimation risk generally leads to broader loss distributions and therefore to rising values of risk parameters such as Value at Risk or Expected Shortfall. The level of economic capital required may be strongly underestimated if prediction and estimation risk are ignored.

Keywords: probability of default, PD, credit risk, default correlation, asset correlation, point in time, value at risk, estimation risk, credit portfolio models, credit risk management

JEL Codes: C1, G21

Non-technical Summary

The present paper studies the impact of the incorporation of estimation and prediction risk in credit portfolio models. It particularly addresses potential changes in the loss distribution and the risk variables, such as Value at Risk (VaR) or Expected Shortfall, and in the economic capital. Generally speaking, the input parameters of a credit portfolio model, such as probability of default (PD) and default correlations, must be estimated from empirical data. The resulting estimated values are prone to uncertainty and errors and are generally inconsistent with the "true" values. Moreover, the PD models often contain predictions for risk factors, eg macroeconomic factors, which can contain prediction errors. This article develops methods to incorporate estimation and prediction errors adequately in loss distribution simulations. A data set of the Deutsche Bundesbank is used to demonstrate the practical implementation. This set comprises the master, financial accounting and default data of some 60,000 West German firms for the years 1989 to 2003. The key results of the study are as follows.

1. The inclusion of estimation and prediction risk generally leads to a broader loss distribution and to an (in some cases distinct) increase in VaR and Expected Shortfall.

2. Required economic capital may be strongly underestimated if prediction and estimation risk are ignored.

3. A relatively long data history and sizeable portfolios are advantageous as, in those cases, risk parameters can be estimated more precisely, thereby reducing, *ceteris paribus*, the estimation risk.

4. For the risk parameter loss given default (LGD), which is assumed to be a constant here, estimation and prediction risk need to be incorporated as well.

Nichttechnische Zusammenfassung

Der vorliegende Beitrag untersucht die Auswirkungen der Einbeziehung von Schätz- und Prognoserisiken in Kreditportfoliomodellen. Insbesondere geht es dabei um mögliche Veränderungen der Schadensverteilung sowie von Risikokennzahlen wie Value at Risk oder Expected Shortfall und des ökonomischen Eigenkapitals. In der Regel müssen die Inputparameter eines Kreditportfoliomodells wie z.B. Ausfallwahrscheinlichkeiten und -korrelationen aus empirischen Daten geschätzt werden. Die resultierenden Schätzwerte sind mit Schätzfehlern behaftet und stimmen im Allgemeinen nicht mit den „wahren" Werten überein. Ferner enthalten die Modelle für Ausfallwahrscheinlichkeiten häufig Prognosen für Risikofaktoren, z.B. makroökonomische Faktoren, die ebenfalls fehlerbehaftet sein können. In diesem Artikel werden Verfahren entwickelt, wie Schätz- und Prognosefehler bei der Simulation der Schadensverteilung adäquat berücksichtigt werden können. Die praktische Umsetzung wird anhand eines Datensatzes der Deutschen Bundesbank demonstriert. Dieser umfasst Stamm-, Bilanz- und Ausfalldaten von bis zu 60.000 westdeutschen Unternehmen für die Jahre 1989 bis 2003. Die zentralen Ergebnisse der Studie sind:

1. Die Einbeziehung von Schätz- und Prognoserisiken führt im Allgemeinen zu breiteren Verlustverteilungen und zu einer (in einigen Fällen deutlichen) Erhöhung der Risikokennzahlen Value at Risk bzw. Expected Shortfall.

2. Werden Schätz- und Prognoserisiken ignoriert, kann dies eine deutliche Unterschätzung des ökonomisch notwendigen Eigenkapitals zur Folge haben.

3. Längere Datenhistorien und größere Portfolien sind von Vorteil, da in diesen Fällen die Risikoparameter genauer geschätzt werden können, wodurch ceteris paribus das Schätzrisiko verringert wird.

4. Für den Risikoparameter LGD, der hier als konstant angenommen wird, sind ebenfalls Schätz- bzw. Prognoserisiken zu berücksichtigen.

Content

Incorporating Prediction and Estimation Risk in Point-in-Time Credit Portfolio Models

1 Introduction

In recent years, financial researchers and banks have focused increasingly on credit risk measurement and management. The new capital adequacy framework drawn up by the Basel Committee on Banking Supervision (BCBS) decisively fostered this development. Under this framework, credit institutions are permitted to use internal estimates of the relevant determinants of obligors' credit risk, such as the probability of default (PD), loss given default (LGD) and exposure at default (EAD). Default correlations represent another key credit risk variable when assessing a credit portfolio. Modelling default correlations and correlations in the changes in credit ratings also poses a great challenge, and further research needs to be done on this topic.

A number of models exist to capture and quantify credit portfolio risk. Of these, banks most commonly use the CreditMetrics, CreditRisk+, Moody's KMV and CreditPortfolioView models. At first glance, these models have different structures and loss distribution results often vary substantially from model to model. This may be attributed above all to the different distribution assumptions and the diverse procedures used to parametrise the input parameters. However, it can be shown (see Koyluoglu and Hickman (1998), Gordy (2000) and Hamerle and Rösch (2005)) that the distribution moments of default rates in all three models can be transformed into each other and that the predicted loss distributions then no longer differ significantly, provided that the specification of input parameters is comparable.

Prediction and estimation risk, which have hardly been analysed in connection with credit risk models up to now (see, however, Knapp (2002) and Löffler (2003)), represent a further source of uncertainty.

When using a credit risk model, all input parameters must generally be estimated or predicted. For example, the approximate PD of obligors in a particular rating class may be identified via the default rate (or average default rates). Alternatively, a statistical default model that delivers individual PD predictions may be specified. All estimates and predictions are uncertain and are prone to errors. This is also true for the other risk parameters, eg default correlations, LGD etc. This paper examines the adequate integration of estimation and prediction risks into credit portfolio models.

To simultaneously assess the PD and default correlations, statistical default models containing systematic and unsystematic risk components are defined as factor models. The systematic risk component is split into an observable and an unobserved part. The observable part is captured by means of macroeconomic risk factors, the unobserved part is modelled by means of random time effects. In addition, the models also contain obligor-specific information, for example, in the form of a rating score. The unknown model parameters are estimated using the maximum likelihood method. Maximum likelihood theory also helps to provide the variances and covariances of the parameter estimates that are required to factor in estimation and prediction risk to determine the loss distribution. As this analysis does not focus on the risk parameters EAD and LGD, they are specified deterministically. In a next step, the loss distributions are simulated. Several model specifications are compared including and excluding estimation and prediction risk.

The remainder of the paper is organised as follows:

Section 2 describes methods for estimating obligors' PD and loss distribution and integrating estimation and prediction risk. Section 3 presents the data set of the Deutsche Bundesbank used for the analysis. We present the four estimation and prediction models for PD that were used for the evaluation and the empirical model results. The simulation approach applied to determine the predicted loss distribution is also depicted in this section. This section closes with the analysis of the empirical loss distribution and an interpretation of the models and their results and conclusions. Section 4 summarises the results of this paper and concludes with an outlook.

2 Model approach

2.1 Simultaneous modelling of default probabilities and correlations

The states "default" and "non-default" of obligor i in period t – in most applications one year –are modelled using the indicator variable D_{it}, ie

$$D_{it} = \begin{cases} 1 & \text{borrower } i \text{ defaults in period } t \\ 0 & \text{otherwise} \end{cases}$$

$i \in N_t$, $t = 1,...,T$.

N_t denotes the "risk set" consisting of all obligors who are not defaulted at the beginning of period t.

Alternatively, the default event may be triggered in the model when a metric variable R_{it} ($i \in N_t$, $t = 1,...,T$) falls below a prescribed threshold at a particular point in time (in the observed period) t. The seminal works of Merton (1974, 1977) and Black and Scholes (1973) laid the fundament for this approach.

The firm value model relies on a level of company information that is rarely available in practice. Accurate information about the capital structure must be available, and it must be possible to determine the assumed tradable firm value of the company at all times. However, this is usually not the case for small and medium-sized enterprises, which account for the lion's share of banks' portfolios. Therefore, the random variable R_{it} ($i \in N_t$, $t = 1,...,T$) which triggers the default event is assumed to be latent and unobservable. Only the default event itself can be observed. By analogy to the firm value model, the default event is assumed to occur when R_{it} falls below a threshold c_{it}, ie

$$R_{it} < c_{it} \Leftrightarrow D_{it} = 1. \tag{1}$$

As in the case of index or factor models used to determine capital market risk, we assume that default risk can be split into systematic and unsystematic risk components.

The systematic risk components F_t ($t = 1,...,T$) apply to all firms in a particular risk segment (eg a sector) during period t and hence cannot be diversified. Unsystematic risks U_{it} ($i \in N_t$, $t = 1,...,T$) only affect a single firm.

A simple single-factor model is assumed for random variable R_{it}, which triggers the default event, in equation (2), ie

$$R_{it} = \sqrt{\rho}\ F_t + \sqrt{1-\rho}\ U_{it} \tag{2}$$

($i \in N_t$, $t = 1,...,T$), where $F_t \sim N(0,1)$ represents standard normally distributed, systematic risk components, ie components that have an impact on all firms at a specified time and thus are not diversifiable. Unsystematic firm-specific (and hence diversifiable) risk drivers U_{it} are also assumed to have a standard normal distribution.

Moreover, the unsystematic risks of different firms are assumed to be independent of each other and of the systematic risk factors F_t.

Equation (2) represents the CreditMetrics (default mode) model. It also corresponds to the specification of the Basel II model. The correlation of the threshold values R_{it} and R_{jt} of two companies is given by ρ. By analogy to the firm value model, these variables may be interpreted as normalised returns of the firm value. Therefore ρ is frequently referred to as the asset correlation. It is assumed that the value of ρ is always the same within a risk segment, for example, a sector, whereas another single-factor model with a different value of ρ may apply within a different sector.[1]

Together with equation (1) and given threshold c_{it}, we obtain the conditional PD given the systematic risk factor F_t

$$\lambda_{it}(F_t) = P(R_{it} < c_{it}) = P\left(U_{it} < \frac{c_{it} - \sqrt{\rho}\ F_t}{\sqrt{1-\rho}} \right) = \Phi\left(\frac{c_{it} - \sqrt{\rho}\ F_t}{\sqrt{1-\rho}} \right). \tag{3}$$

For a given realisation f_t of the systematic factor of a risk segment, only unsystematic risk, which is assumed to be independent, is effective. Accordingly, joint default by two obligors in period t is conditionally independent with the respective (conditional) probabilities (3).

[1] By contrast, the Basel II model assumes one single-factor model for the entire credit portfolio.

The determination of PDs according to (3) is implicitly contingent on the additional condition that the obligor has not defaulted in the preceding periods. Thus, the probabilities in (3) may also be seen as "time-discrete hazard rates" (see eg Hamerle and Tutz (1989)). The models examined here are simple, time-discrete versions of the intensity-based models (see, for example, Duffie and Singleton (1999) and Jarrow and Turnbull (1995)) frequently used in risk neutral valuation of defaultable bonds or credit derivatives. Intensity-based models are formulated in continuous time and represent generalisations of time-discrete hazard rate models.

The unconditional PD is obtained as the expected value of the distribution of F_t, ie

$$\lambda_{it} = E(\lambda_{it}(F_t)) = \int_{-\infty}^{\infty} \lambda_{it}(f_t)\varphi(f_t)df_t, \tag{4}$$

where $\varphi(z)$ indicates the density function of the standard normal distribution.

In the unconditional approach, the defaults occurring during a period are correlated. It follows that

$$\rho(D_{it}, D_{jt}) = \frac{\lambda_{ijt} - \lambda_{it}\lambda_{jt}}{\sqrt{\lambda_{it}(1-\lambda_{it})}\,\sqrt{\lambda_{jt}(1-\lambda_{jt})}}, \tag{5}$$

where

$$\lambda_{ijt} = \int_{-\infty}^{\infty} \lambda_{it}(f_t)\lambda_{jt}(f_t)\,\varphi(f_t)\,df_t = \Phi_2\left(c_{it}, c_{jt}, \rho\right)$$

is the joint PD of obligors i and j in period t. $\Phi_2\left(z_1, z_2, \rho\right)$ denotes the distribution function of the two-dimensional standard normal distribution with the correlation parameter ρ. The default correlations between pairs of obligors generally differ, as the default thresholds may be obligor-specific.

Examining the joint defaults of N_t obligors in a given risk segment assuming a given value of f_t in period t, we obtain

$$P(D_{1t} = d_{1t},..., D_{N_t t} = d_{N_t t} | f_t) = \prod_{i \in N_t} \lambda_{it}(f_t)^{d_{it}} [1 - \lambda_{it}(f_t)]^{(1-d_{it})} \tag{6}$$

where $d_{it} \in \{0,1\}$.

Finally, the unconditional joint probability of default in period t is

$$P\left(D_{1t} = d_{1t},..., D_{N_t t} = d_{N_t t}\right) = \int_{-\infty}^{\infty} \prod_{i \in N_t} \lambda_{it}(f_t)^{d_{it}} [1 - \lambda_{it}(f_t)]^{(1-d_{it})} \varphi(f_t) df_t \ . \tag{7}$$

2.2 Dynamic obligor-specific modelling

In a next step, obligor-specific and systematic risk factors which are observable in dynamic modelling are integrated into the modelling approach.

The systematic risk components are split into an observable and an unobserved part. The observable components capture changes in the macroeconomic environment, specifically cyclical developments. Macroeconomic indicators such as interest rates, the unemployment rate, the GDP growth rate, new orders or the external value of the euro are the key variables. Furthermore, it has been determined that the major macroeconomic risk drivers affect PD with a delay of about one or two years; see also Rösch (2003) and Hamerle, Liebig and Rösch (2003). Consequently, modelling and prediction of PD may be based on the known values of these risk factors.

This paper chooses a different method to demonstrate how prediction risk can be accounted for. Predictions are based on a multi-factor model.[2] Using this approach, we model sectoral insolvency rates as dependent on the macroeconomic factors and cyclical indicators described above. As a rule, the macroeconomic factors chosen have an impact with a delay of one to two years.

A very simplified model is estimated in this paper. We consider only a single sector. In a linear regression model, the transformed[3] insolvency rate

[2] See Knapp and Hamerle (1999).

[3] This transformation is often used in econometrics and ensures predicted insolvency rates to be between 0 and 1.

$$y_t = \ln \frac{1 - z_{t,MFM}}{z_{t,MFM}}$$

($z_{t,MFM}$: insolvency rate of year t in the construction sector[4]) can be estimated as a function of public sector gross fixed investment (construction investment) in the preceding year and the construction sector's business climate index in the preceding year.[5]

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 z_{t-1,investment} + \alpha_3 z_{t\text{-}1,climate} + \varepsilon_t \,. \tag{8}$$

The estimation results for model (8) are presented in section 3.2.

On the basis of model (8), predictions $\hat{y}_t$ of the transformed insolvency rate can be made first. Next, by means of inverse transformation, predictions $\hat{z}_{t,MFM}$ of the current period t may be estimated and then integrated into the estimate for λ_{it}. The corresponding regression parameter in the PD model is denoted by γ, see equations (9) and (10), respectively. Additionally, some unobserved systematic risk remains in the model which is captured by the random effect variable F_t. This risk may be caused by contemporary systematic risk factors whose values are unknown at the time the rating or the PD prediction is calculated.

Obligor-specific risk factors may enter in the form of rating information, for instance, rating classes or individual rating scores. The size, legal form or age of the firm may represent additional factors. The information is assumed to have been recorded at time $t-1$ and is summarised in vector $\boldsymbol{x}_{it-1}$. The corresponding parameter vector is denoted by $\boldsymbol{\beta}$.

Hence, individual, time-dependent default thresholds c_{it} are determined for each obligor by means of dynamic individual modelling according to

$$c_{it} = \beta_0 + \boldsymbol{\beta}' \boldsymbol{x}_{it-1} + \gamma\, \hat{z}_{t,MFM} \,. \tag{9}$$

[4] The abbreviation MFM denotes the multi-factor model used to predict the insolvency rate.

[5] Several macroeconomic variables, eg business climate index, unemployment rate and interst rate were included in the model. (8) shows the best model for the estimation and prediction of the insolvency rate in the construction sector.

Taking (3) into account, probit specification leads to

$$\lambda_{it}(f_t) = \Phi\left(\frac{\beta_0 + \boldsymbol{\beta}' \boldsymbol{x}_{it-1} + \gamma\, \hat{z}_{t,MFM} - \sqrt{\rho}\, f_t}{\sqrt{1-\rho}} \right). \tag{10}$$

2.3 Parameter estimation

A central assumption in modelling the default distribution of a given number N_t of obligors in a segment (eg a sector) during time *t* is that unsystematic risk is independent. For given realisations f_t of the unobserved systematic risk components as well as of $\boldsymbol{x}_{it-1}$ and $\hat{z}_{t,MFM}$ the following relationship holds

$$P\left(D_{1t} = d_{1t}, ..., D_{N_t t} = d_{N_t t} \middle| f_t\right) = \prod_{i \in N_t} \lambda_{it}(f_t)^{d_{it}} \left[1 - \lambda_{it}(f_t)\right]^{(1-d_{it})} \tag{11}$$

where $d_{it} \in \{0,1\}$ and $\lambda_{it}(f_t)$ are given by probit specification (10).

As f_t is unobserved, (11) cannot be used to estimate the parameters by the maximum likelihood method. Instead, the unconditional joint probabilities of default must be used for estimation. According to (7), for given values of $\boldsymbol{x}_{it-1}$ and $\hat{z}_{t,MFM}$ it follows that

$$P\left(D_{1t} = d_{1t}, ..., D_{N_t t} = d_{N_t t}\right) = \int_{-\infty}^{\infty} \prod_{i \in N_t} \lambda_{it}(f_t)^{d_{it}} \left[1 - \lambda_{it}(f_t)\right]^{(1-d_{it})} \varphi(f_t) df_t\ .$$

If all periods $t = 1, ..., T$ are taken into account, we obtain for the log likelihood function of the overall model

$$\begin{aligned} & l(\beta_0, \boldsymbol{\beta}, \gamma, \rho) \\ & = \sum_{t=1}^{T} \ln \left\{ \int_{-\infty}^{\infty} \prod_{i \in N_t} \lambda_{it}(f_t)^{d_{it}} \left[1 - \lambda_{it}(f_t)\right]^{(1-d_{it})} \varphi(f_t)\, df_t \right\} \end{aligned} \tag{12}$$

where $\lambda_{it}(f_t)$ is given by (10).

The integral on the right-hand side of (12) can be evaluated using numerical methods such as the Gauss-Hermite procedure.

Variances and covariances of the parameter estimates are required to take account of the estimation risk. These estimates can be derived from the theory of maximum likelihood estimation. While the result is only an asymptotical formula, it may serve as a very useful proxy considering the large samples usually available in credit risk modelling.

2.4 Predicting the loss distribution

Once the unknown parameters have been estimated for the given model specification, these estimates can be used to calculate predictions. If the estimation period covers T periods, the predictions refer to period $T+1$. The values $\boldsymbol{x}_{iT}$ and $\hat{z}_{T+1,MFM}$ are then used in the dynamic individual approach. This allows us to obtain the individual predictions of PD.

$$\hat{\lambda}_{iT+1} = \hat{\lambda}\left(\boldsymbol{x}_{iT}, \hat{z}_{T+1,MFM}\right).$$

For credit risk models the predicted loss distribution for the next period $T+1$ (for example, one year) must be calculated. The loss distribution may then be used to calculate risk measures such as expected loss, unexpected loss, Value at Risk and Expected Shortfall.

This paper assumes a loss (rate) given default of 45% at all times, as the analysis does not focus on this risk parameter.[6] Moreover, various EAD values are specified. These values are based on a German bank's actual loan portfolio. The values span a range of 21.94 to 340.31 monetary units, with an average of 101.89 monetary units. The total exposure comes to 60,930.27 monetary units.

As the predicted loss in the next period $T+1$, we obtain

$$L_{T+1} = \sum_{i \in N_{T+1}} D_{iT+1} EAD_{iT+1} LGD_{iT+1},$$

where N_{T+1} indicates the number of obligors in the portfolio at the beginning of period $T+1$.

[6] The value of 45% is chosen in line with the LGD value assigned under the foundation internal ratings based (FIRB) approach applicable to senior claims on corporates, sovereigns and banks not secured by recognised collateral corporate, bank and sovereign senior unsecured exposures (see Basel Committee on Banking Supervision (2004), paragraph 287).

Calculating a "relative" predicted loss expressed as a percentage of the total exposure, we obtain

$$L^*_{T+1} = \frac{L_{T+1}}{\sum\limits_{i \in N_{T+1}} EAD_{iT+1}} .$$

Since EAD and LGD are deterministic, the joint distribution (7) for the predicted defaults is the basis for calculating the predicted loss distribution. This distribution is above all based on the parameter estimates to predict $\hat{\lambda}_{iT+1}$ as well as the parameter estimates for the distribution of the unobserved systematic risk component F_t. We obtain

$$\begin{aligned} &\hat{P}\left(D_{1T+1} = d_{1T+1}, \ldots, D_{N_{T+1}T+1} = d_{N_{T+1}T+1}\right) \\ &= \int\limits_{-\infty}^{\infty} \prod_{i \in N_t} \hat{\lambda}_{iT+1}\left(f_{T+1}\right)^{d_{iT+1}} \left[1 - \hat{\lambda}_{iT+1}\left(f_{T+1}\right)\right]^{(1-d_{iT+1})} \varphi\left(f_{T+1}\right) df_{T+1} . \end{aligned} \tag{13}$$

In the case of dynamic individual modelling, simulations generally have to be used to predict the distribution, as $2^{N_{T+1}}$ different probabilities would have to be calculated to obtain the precise predicted distribution of loss L_{T+1}.

2.5 Incorporating prediction risk

The values $\hat{z}_{T+1,MFM}$ which are entered into the predicted loss distribution are forecasts. These forecasts are uncertain and contain prediction errors, meaning that the actual insolvency rate of the next year may deviate from the predicted rate. To account for the uncertainty, new predictions are produced in each simulation run using model (8) by first generating the error term ε_{T+1} randomly and then using the result to calculate the (realised) prediction $z_{T+1,MFM}$.[7] The procedure for simulating the predicted loss distribution is described in detail in section 3.3.

The inclusion of prediction risk described above has the same impact as an increase in the variance of the random time effect F_t and hence the same impact as an increase in the asset correlation.

[7] In CreditPortfolioView a comparable procedure is used (see Wilson (1997a, b)).

2.6 Incorporating estimation risk

Incorporating estimation risk into the calculation of the predicted loss distribution of credit portfolio models is an important extension to the model. Jorion (1996) pointed out the issue of integrating estimation risk in the context of measuring market risk. Very little literature is available on integrating estimation risk in credit risk assessment. Löffler (2003) provides some examples of the possible effects of estimation risk on the risk measurement of loan portfolios. Knapp (2002) also provides examples of how to take estimation risk into account.[8]

The parameter estimates contained in (13) are point estimates which, in turn, may be subject to estimation errors. Especially if the portfolios are small or the data history is short, or both, these errors may well be substantial. As a consequence, the values of risk measures such as Value at Risk or Expected Shortfall must be considered as estimates themselves and may deviate more or less substantially from true values. In such cases, confidence intervals containing the true values may be specified. This paper specifies the distributions of Value at Risk and Expected Shortfall by means of simulations. The upper and lower bounds of the confidence intervals can then be derived.

The joint probability distribution of the maximum likelihood estimates of the model parameters is needed to simulate the distribution of the risk measures. It follows from the theory of maximum likelihood estimation that the respective maximum likelihood estimates are asymptotically normally distributed. In addition, the covariance matrix can be estimated.[9] The resulting (approximative) multivariate normal distribution may be used for random sampling of different realisations of parameter estimations. In a next step, a complete loss distribution is simulated for every realised combination. The simulation procedure is described in detail in section 3.3.

[8] See Knapp (2002), p 178 ff.
[9] See, for example, Fahrmeier, Hamerle and Tutz (1996), section 2.3.

3 Empirical results

3.1 The data

Empirical evaluations are based on a data set of the Deutsche Bundesbank. This data set comprises the master, financial accounting and default data of up to 60,000 West German firms from 1989 to 2003. Insolvency as defined by the Insolvency Code (Insolvenzordnung) in effect from 1 January 1999 was taken as the default criterion.[10] Prior to 1999 the default criterion for West Germany was insolvency or over indebtedness as outlined in the bankruptcy code.[11] In this paper our analyses focus on the construction sector.

As the data is limited to companies that submitted bills to the Deutsche Bundesbank for rediscounting, both the sectoral and the period insolvency rates of the original data are lower than the insolvency rate of West German enterprises as a whole from 1989 to 2003. Therefore, the insolvency rate of the original data set was adjusted to the sectoral insolvency rate in West Germany.[12] Figure 1 in section 3.2.1 shows the insolvency rate of the construction sector in West Germany.

3.2 Models

3.2.1 Obligor-specific and macroeconomic risk factors

We used the model presented in equation (10) in section 2.2. Only a single obligor-specific risk factor is used in this paper, ie a rating score, specified as $x_{i,t,Score}$. This score was developed on the basis of the available data and incorporates a variety of key balance sheet indicators.[13] Lower scores indicate a higher creditworthiness.

The estimation results of the multi-factor model for the construction sector are shown in Table 1.

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[10] The default criterion was defined as D_{it} in section 2.1.

[11] See Scheule (2003), p 113.

[12] For additional reasons for this calibration, see Scheule (2003), p 115 f.

[13] The score for the construction sector includes the following key indicators: the ratio of bank liabilities to assets, the ratio of liabilities arising from goods and services (trade creditors) to net turnover, the ratio of staff costs to net turnover and the ratio of the profit or loss on ordinary activities to operational income. See also Falke (2005).

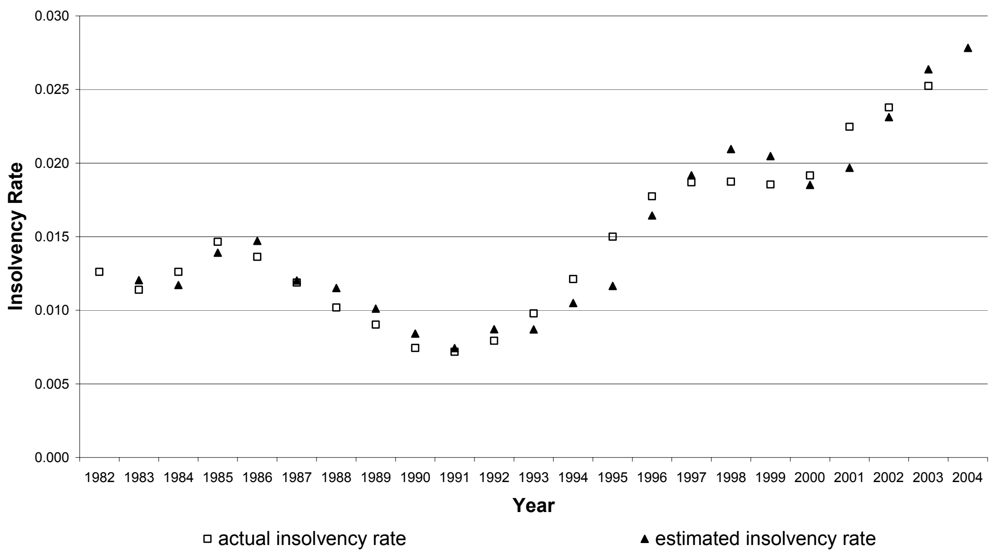
Table 1: Results for model (8)

Parameter	Estimate	Standard error	p-value
α_0	0.199695	0.334006	0.5578
α_1	0.511316	0.190627	0.0157
α_2	0.121410	0.041478	0.0094
α_3	0.008513	0.004747	0.0907

Thus, if last year's insolvency rate or public sector gross fixed capital formation (construction investment) or the construction sector's business climate index rise, the insolvency rate of the construction sector declines, respectively.[14] These relationships are significant at the 10% level. The estimated value of R^2 is 88.8%, and the estimated error variance of the regression model (8) is $\hat{\sigma}_{\varepsilon}^2 = 0.0125$.

The prediction of the annual insolvency rate of the construction sector (West Germany) for 2004 is given by $\hat{z}_{2004,MFM} = 2.78\%$. A real-fit-diagram of actual and estimated insolvency rates is shown in Figure 1.

Figure 1: Actual and estimated insolvency rates in the construction sector



[14] This negative relation results from the transformation of the insolvency rate in the model.

3.2.2 Model specifications

The following models are estimated:

Model 1 (neither prediction nor estimation risk included)

In addition to the constant, this model contains the obligor-specific score[15] of the previous year, $x_{i,t-1,Score}$, the sectoral prediction $\hat{z}_{t,MFM}$ of the multi-factor model and a random time effect, f_t. For a given value of f_t we obtain:

$$\lambda_{it}(f_t) = \Phi\left(\frac{\beta_0 + \beta_{Score} x_{i,t-1,Score} + \gamma\, \hat{z}_{t,MFM} - \sqrt{\rho}\, f_t}{\sqrt{1-\rho}}\right), \qquad (15)$$

where $\Phi(z)$ is the distribution function of the standard normal distribution.

Model 2 (prediction risk included)

In addition to model 1 this model allows for prediction uncertainty of the insolvency rate. The procedure is described in more detail in section 3.3.

Model 3 (estimation risk included)

This model extends model 1 by taking into account the estimation risk. To this end, the joint distribution of parameter estimates $\hat{\beta}_0$, $\hat{\beta}_{Score}$, $\hat{\gamma}$ and $\hat{\rho}$ is used. The procedure is described in more detail in section 3.3.

Model 4 (both prediction and estimation risk included)

Both prediction uncertainty and estimation risk are taken into account in model 4. For more details see section 3.3.

[15] This score was modelled according to Falke (2005).

3.3 Simulation of the loss distribution

A Monte Carlo simulation is used to determine the predicted loss distribution. The loss distribution for 2004 is predicted for 598 non defaulted construction companies represented in the data set of the Deutsche Bundesbank for 2003.

We first describe the most complex situation below, in which both prediction and estimation risk are taken into account. Exclusion of these risk types simplifies the procedure accordingly.

Estimation uncertainty is captured by taking into account the joint distribution of the parameter estimates $\hat{\beta}_0$, $\hat{\beta}_{Score}$, $\hat{\gamma}$ and $\hat{\rho}$. In accordance with the maximum likelihood theory, a multivariate normal distribution is assumed where the covariance matrix is estimated by means of the maximum likelihood estimates.[16] The vector of the maximum likelihood estimates is entered as the mean vector. A realisation of estimates of β_0, β_{Score}, γ and ρ is sampled from this distribution in an outer loop. Next, the complete loss distribution for this realisation is generated, taking into account the prediction risk. We proceed as follows:

- Step 1: A random realisation f_{T+1} of the systematic risk factor is drawn from a standard normal distribution.

- Step 2: Using the values of $x_{i,T,Score}$ and $\hat{z}_{T+1,MFM}$ for 2003 and 2004, respectively, we compute the predictions of the conditional PD $\hat{\lambda}_{iT+1}(f_{T+1})$, $i \in N_{T+1}$, according to the specifications of models 1 and 2 and according to the parameter estimates obtained from the outer loop.

- The prediction $\hat{z}_{T+1,MFM} = 0.0278$ is used in models 1 and 3. Models 2 and 4, which incorporate prediction risk, are sampled from a distribution of the prediction value $\hat{z}_{T+1,MFM}$. To this end, the transformed insolvency rate y_{T+1} is predicted by sampling the distribution of the error term ε_{T+1} ($\sim N(0;0.0125)$). Using the inverse transformation, we obtain the realisation of the predicted insolvency rate $\hat{z}_{T+1,MFM}$.

[16] The calculation of the covariance matrix estimate is performed with NLMIXED procedure using SAS statistical software.

- Step 3: According to the conditional independence, N_{T+1} Bernoulli events are generated with these conditional PDs. Taking into account the relevant EADs and an LGD of 45% the portfolio loss realisation is calculated.

- Steps 1 through 3 are repeated 10,000 times, and the loss distribution of the portfolio is calculated.

The simulation program then returns to the beginning of the outer loop and generates a further realisation of the parameter estimates. For the new realisation, a complete (conditional) loss distribution is simulated, and so forth. The cycle is repeated 2,000 times.

For each of these 2,000 loss distributions risk measures such as VaR or Expected Shortfall may be calculated in order to generate estimates of the probability distributions for these risk measures. Using the distribution of the risk measures appropriate confidence intervals may be calculated.

On the other hand, the (conditional) loss distributions may be aggregated into a single unconditional loss distribution.

If no estimation risk is taken into account, ie in model 1 and 2, the outer loop of the simulation is not applied.

3.4 Simulation results and interpretation

First, the loss distributions and some of their indicators are provided for all four models, with the risk parameters given as percentage shares of total exposure.

Table 3: Loss distribution risk parameters for models 1 through 4

Portfolio loss	**Model 1 (excluding prediction risk, excluding estimation risk)**	**Model 2 (including prediction risk, excluding estimation risk)**	**Model 3 (including estimation risk, excluding prediction risk)**	**Model 4 (including prediction risk, including estimation risk)**
Expected loss	0.98128%	0.99664%	0.98891%	1.00266%
99% quantile	1.78260%	1.91625%	1.84433%	2.00234%
99.9% quantile	2.09167%	2.31395%	2.18248%	2.45691%
VaR (99%)	0.80132%	0.91960%	0.85541%	0.99968%
VaR (99.9%)	1.11040%	1.31730%	1.19356%	1.45426%
Expected Shortfall (99%)	1.92113%	2.09323%	1.99356%	2.20157%
Standard deviation	0.30888%	0.34408%	0.32581%	0.36303%

The loss distributions of models 1 through 4 may be represented graphically as follows:

Figure 2: Loss distribution model 1 (excluding prediction and estimation risk)

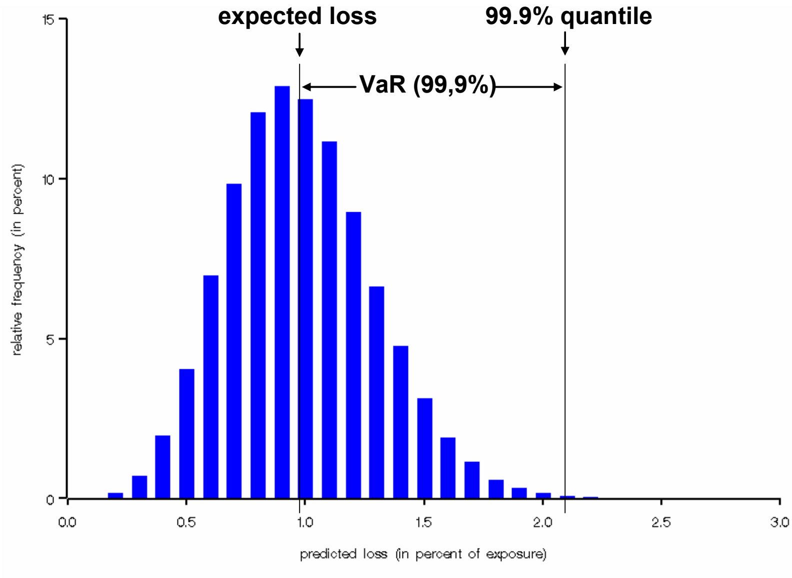


Figure 3: Loss distribution model 2 (including prediction risk, excluding estimation risk)

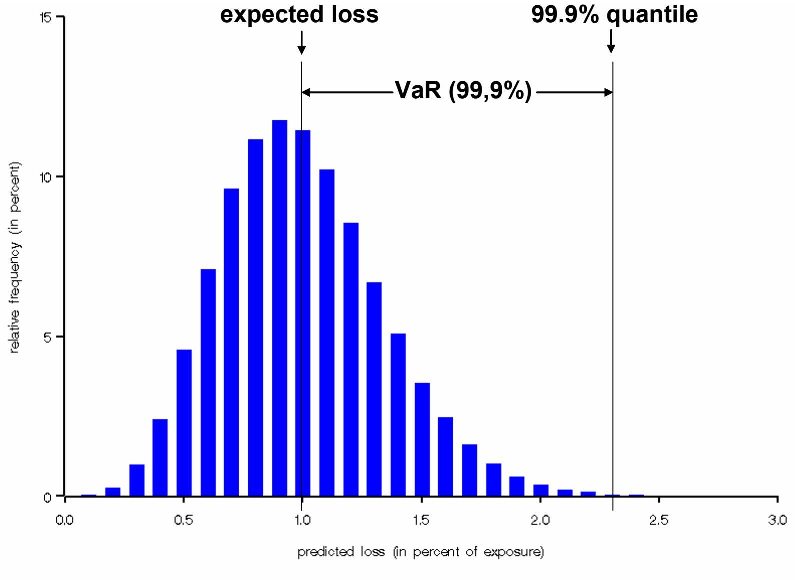


Figure 4: Loss distribution model 3 (including estimation risk, excluding prediction risk)

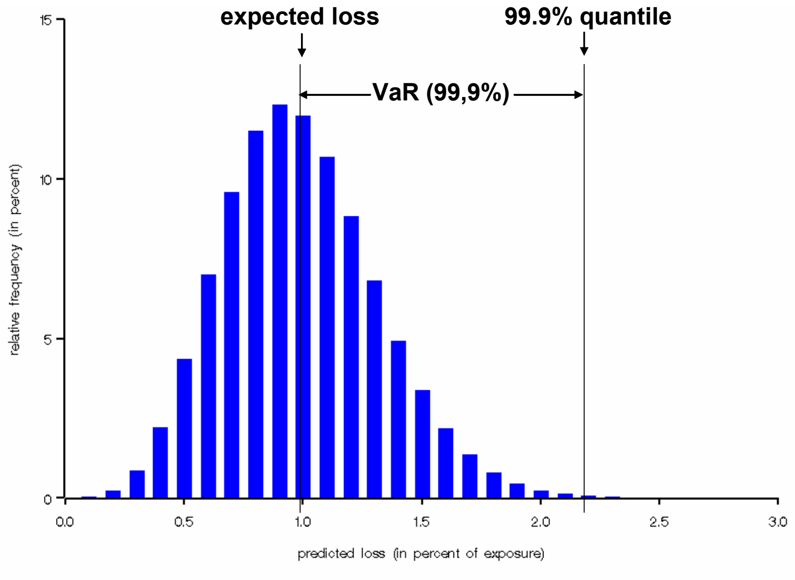
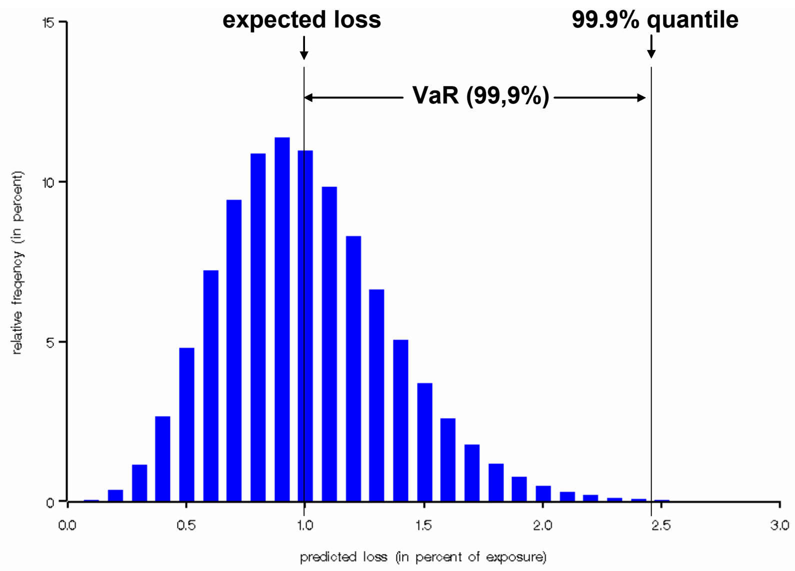


Figure 5: Loss distribution model 4 (including prediction and estimation risk)



A comparison of models 1 and 2 shows that accounting for prediction uncertainty causes expected loss to rise. This result follows from Jensen's inequality, as the convex course of the lower part of the probit transformation curve indicates. For the same reason, expected loss rises slightly in models 3 and 4.

Figures 2 through 5 and table 3 illustrate that the inclusion of prediction and estimation uncertainty leads to “broader” loss distributions. Accordingly, the relevant VaR values (the difference between the respective loss distribution quantile and expected loss) and the Expected Shortfall value increase, in some cases considerably. In particular, the VaR (99.9%) rises by about 7% if estimation risk is included and by 18% if prediction risk is included. If prediction and estimation risk are captured, it rises by some 30% against the benchmark model (which excludes both prediction and estimation risk).[17]

Finally, we consider the distributions of the risk parameters VaR (99%) and Expected Shortfall (99%). The empirical distributions can be calculated from the 2,000 realised conditional loss distributions. Table 4 shows selected indicators of both distributions, and Figures 6 and 7 contain the corresponding graphical representations.

[17] The comparison for models 3 and 4 is based on the unconditional loss distribution data. Taking into account the risk measure distributions results in different figures (see the explanation below).

Table 4: Selected VaR (99%) and Expected Shortfall (99%) distribution parameters

	Model 4 VaR (99%)	**Model 4 Expected Shortfall (99%)**
Mean	0.92736%	2.10129%
Median	0.92136%	2.08965%
90% quantile	1.04651%	2.37637%
95% quantile	1.08711%	2.46289%
99% quantile	1.18271%	2.68772%

Figure 6: VaR distribution (99%) for model 4 (including prediction and estimation risk)

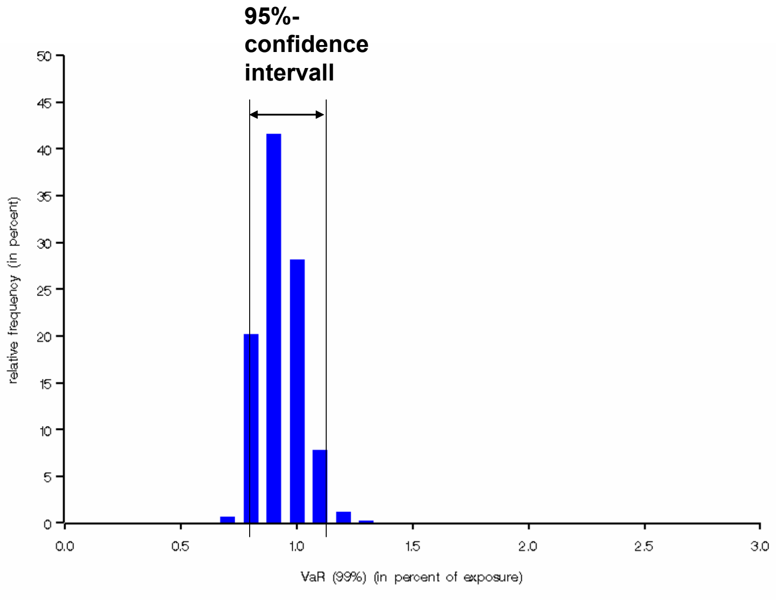
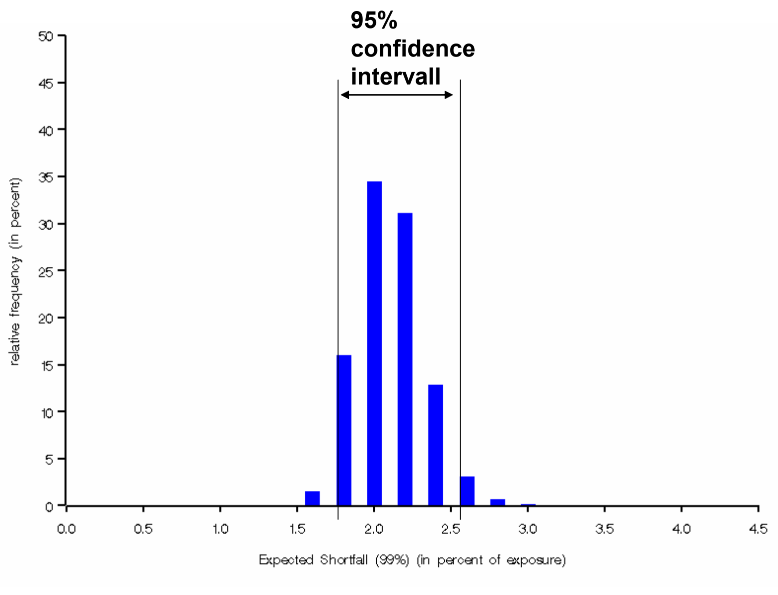


Figure 7: Expected Shortfall distribution (99%) for model 4 (including prediction and estimation risk)



Both the VaR distribution and the Expected Shortfall distribution are considerably right skewed. A 95% confidence interval for the VaR is [0.77109%; 1.11950%] for model 4, a 95% confidence interval for the Expected Shortfall for the same model is [1.72509%; 2.56071%]. Both distributions provide useful information if additional estimation risks are taken into account when calculating capital requirements. If a conservative approach is chosen, the upper bound of the confidence interval, for example, may serve as a risk measure. If the values of VaR (99%) of model 1 and model 2 (0.80132% and 0.91960%) are compared with the upper bound of the 95% confidence interval of the value of VaR (99%) in model 4 (1.11950%), the capital requirements geared to the VaR would rise accordingly by 39% or by 22%.

The Expected Shortfall (99%) also rises, taking into account the estimation risk. If the upper bound of the 95% confidence interval is chosen, Expected Shortfall rises by 33% compared to Expected Shortfall in model 1 and by 22% compared to model 2.

The impact of estimation risk on the loss distribution depends on the quality of the estimates. Better estimates of the model parameters – ie lower variances of the parameter estimates– lead to a smaller impact on the loss distribution.

4 Conclusions

This paper analyses the effects of prediction and estimation risk on the loss distribution of credit portfolio models. The empirical analysis is based on a data set from the Deutsche Bundesbank containing default data of West German firms in the construction sector.

A statistical default model is estimated to predict individual probabilities of default (PDs) of the firms. The model is an individual, dynamic extension of the Basel II probit specification. In addition to a rating score based on key balance sheet indicators, the model takes macroeconomic information into account. The prediction of the insolvency rate in the construction sector serves as a macroeconomic risk factor. We use a multi-factor model based on the lagged values of sectoral insolvency rate, public sector construction investment and the business climate index of the construction sector to predict the insolvency rate of the construction sector.

The unknown parameters of the statistical default model are estimated with the maximum likelihood method. As the distribution of the estimates can be derived from the maximum likelihood theory, the estimation risk can be integrated into the simulation of the predicted loss distribution. The distribution of the insolvency rates can be determined from the estimation of the multi-factor model. EAD is specified on the basis of a bank's actual loan portfolio, and LGD is set at 45% at all times.

It turns out that the credit risk of loan portfolios is substantially underestimated if prediction and estimation risk are not taken into account. If these additional sources of risk are included, risk measures such as VaR or Expected Shortfall increase noticeably, in turn increasing economic capital requirements.

Prediction and estimation risk will gain more and more importance in the future, particularly in the analysis of credit portfolio models with a time horizon of several years.

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