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# Measuring business sector concentration by an infection model

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**Abstract**

Results from portfolio models for credit risk tell us that loan concentration in certain industry sectors can substantially increase the *value-at-risk* (VaR). The purpose of this paper is to analyze whether a tractable "infection model" can provide a meaningful estimate of the impact of concentration risk on the VaR. I apply rather parsimonious data requirements, which are comparable to those for Moody's Binomial Expansion Technique (BET) and considerably lower than for a multi-factor model.

The infection model extends the BET model by introducing default infection into the hypothetical portfolio on which the real portfolio is mapped in order to obtain a simple solution for the VaR. The infection probability is calibrated for a range of typical values of input parameters, which capture the concentration of a portfolio in industry sectors, default dependencies between exposures and their credit quality.

The accuracy of the new model is measured for test portfolios with a realistic industry-sector composition, obtained from the German central credit register. I find that a carefully calibrated infection model provides a reasonably close approximation to the VaR obtained from a multi-factor model and outperforms by far the BET model. The simulation results suggest that the calibrated infection model promises to provide a fit-for-purpose tool to measure concentration risk in business sectors that could be useful for risk managers and banking supervisors alike.

**Keywords:** asset correlation, concentration risk, credit risk, multi-factor model, value-at-risk

**JEL Classification:** G 21, C 15, C 20

**Non–technical Summary**

Concentration risk in business sectors is widely perceived to be one of the most important causes of major problems in banks. The empirical challenges in estimating asset correlations for a multi-factor model and the computational burden of calculating the value-at-risk of large credit portfolios by Monte Carlo simulations inspire research for more tractable models which pose less severe data requirements.

The "Binomial Expansion Technique", developed by Moody's constitutes a simple approach to measure risk in heterogeneous credit portfolios. As shown in this paper, it fails, however, in producing a reasonably accurate value-at-risk in the presence of material default correlations.

In this paper the infection model by Davis und Lo (2001) is applied and the parameter "infection probability" is determined in such a way that the loss distribution is calibrated to that of a multi-factor model in the adverse 99.9% quantile of the loss distribution. In this way the impact of the asset correlation on the value-at-risk is captured by the infection probability.

The calibrated model allows to determine the value-at-risk of a credit portfolio based on quite parsimonious data requirements and without the need to run Monte Carlo simulations. The only input data which are required are the following four parameters: the average default probability of a loan portfolio, sector–weighted average intra–sector and inter–sector asset correlations, and the Hirschmann-Herfindahl-Index, calculated from the aggregate sector exposures.

The evaluation of the calibrated infection model is based on relative errors in the value-at-risk for a wide range of realistic default probabilities and asset correlations. The median of these errors is only around 5% for a portfolio which reflects the aggregated sector distribution of the German banking system. In this way the infection model outperforms the "Binomial Expansion Technique" for which the corresponding median error is 34%.

**Nichttechnische Zusammenfassung**

Kreditkonzentrationen in Industriesektoren können eine wesentliche Risikoquelle für Kreditinstitute darstellen. Die Messung dieser Risiken wird erschwert durch Schwierigkeiten bei der Schätzung von *Assetkorrelationen* sowie durch den hohen Rechenaufwand, der mit der Bestimmung eines *Value–at–Risk* durch Monte Carlo–Simulationen in einem Mehrfaktorenmodell verbunden ist. Diese zwei Problemkreise motivieren die Suche nach Modellansätzen, die möglichst geringe Datenanforderungen stellen und rechentechnisch einfach umsetzbar sind.

Die *Binomial Expansion Technique* von Moody's liefert ein Beispiel für einen einfach umsetzbaren Modellansatz um das Risiko in heterogenen Kreditportfolien zu messen. Dieses Arbeitspapier zeigt, dass sich mit diesem Verfahren der *Value–at–Risk* bei korrelierten Ausfallereignissen nicht mehr hinreichend genau bestimmen läßt.

Daher wird als Alternative zu diesem Modell das Ansteckungsmodell von Davis und Lo (2001) untersucht. Der Modellparameter Ansteckungswahrscheinlichkeit wird so kalibriert, dass die zugehörige Verlustverteilung mit derjenigen eines Mehrfaktorenmodelles im 99.9%–Quantil übereinstimmt. Dadurch wird der Korrelationseinfluss auf den *Value–at–Risk* in der Ansteckungswahrscheinlichkeit berücksichtigt.

Das auf diese Weise kalibrierte Modell erlaubt die Bestimmung des *Value-at-Risk* für ein Kreditportfolio mit relativ niedrigen Datenanforderungen und ohne Rückgriff auf Monte Carlo-Simulationen. Als Eingangsdaten werden lediglich die folgenden vier Parameter benötigt: die durchschnittliche Ausfallwahrscheinlichkeit des Kreditportfolios, die Sektor-gewichteten durchschnittlichen Intra-Sektor- und Inter-Sektor-Korrelationen, sowie der Herfindahl-Hirschmann-Index, errechnet aus den aggregierten Kreditforderungen je Sektor.

Als Grundlage zur Bewertung des kalibrierten Ansteckungsmodelles dienen die prozentualen Fehler in der *Value–at–Risk*-Messung für eine breite Auswahl von Ausfallwahrscheinlichkeiten und *Assetkorrelationen.* Der Median der Messfehler beträgt ca. 5% für ein Portfolio, welches die über Industriesektoren des deutschen Bankensystems aggregierte Kreditverteilung widerspiegelt. Das Ansteckungsmodell liefert damit wesentlich genauere Werte als die *Binomial Expansion Technique*, bei welcher der Median der Messfehler bei 34% liegt.

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# 1. Introduction

The purpose of this paper is to develop a robust "tool" that sufficiently approximates the value-at-risk of loan portfolios which are concentrated in certain *sectors*. In a portfolio of credit–risky exposures, such as bank loans, *sector concentration risk* arises if the portfolio is unbalanced in exposures to certain sectors, which entails dependencies between default events. In the following the focus is on industry sectors, however, sectors can also be defined geographically, in which case the methodology still applies.

The importance of concentration risk in loan portfolios has become evident in past banking failures. Banks specializing in loans to economically weak industry sectors or less developed regions have sustained significant losses. A well–known example is the failure of the German Schmidt Bank in 2001. This bank was heavily concentrated in a less–developed region, which was close to the former East German border and characterized by a fragile and concentrated industry structure. Basel Committee on Banking Supervision (2004) lists examples from other countries which highlight the importance of prudently managing concentration risk.

The relevance of sector concentration risk is also recognized by financial supervisors, although existing supervisory rules usually only govern single-name concentration, for example, large exposure rules.[1] Supervisory concerns are revealed, for example, by the statement of the Basel Committee on Banking Supervision that "risk concentrations are arguably the single most important cause of major problems in banks." [2]

In the following it is assumed that default dependencies are sufficiently well described by (linear) default correlations. For a portfolio with given default probabilities and a given exposure distribution across names, *sector concentration* can be defined as an interim state between two extremes "perfect sector concentration" and "perfect sector diversification". Perfect sector concentration would be the (undesirable) case in which, *ceteris paribus* no change in the sector allocation of at least one exposure would increase the *value-at-risk* (VaR) of this portfolio. Conversely, perfect sector diversification would be achieved with a portfolio in which no change in the sector allocation of an individual exposure would decrease the VaR.

[1]See Council of the European Union (2005) for EU banks.

[2]See Basel Committee on Banking Supervision (2005).

Sector concentration needs to be distinguished from "single-name concentration" or *granularity* which is caused by an unbalanced distribution across single borrowers. Single-name concentration has been comprehensively discussed in the literature on *granularity adjustments* in a single risk factor model, closely linked to the development of the New Basel Accord.[3] This paper focuses, however, solely on sector concentration. For this purpose I further differentiate between *intra*–sector correlation and *inter*–sector correlation. Intuitively, intra–sector correlation should generally be the higher of the two correlations if the definition of sectors is meaningful. Depending on the model, a distinction also needs to be made between *asset correlation*, which is the correlation of the unobservable, normalized asset returns, and *default correlation*, which refers to the correlation of the default events of two borrowers. Depending on the model applied, both correlation definitions will be necessary.

A key motivation for analyzing a new approach is the recognition that the "binomial expansion technique" (BET) developed by Moody's does not provide sufficiently accurate estimates of the VaR. This is shown in Section 2 using numerical examples of VaR calculations for realistic levels of default correlations. This paper explores how the deficiencies of the BET approach can be remedied by extending the model to incorporate infectious defaults as inspired by Davis und Lo (2001). This model condenses loss dependencies, which are usually modelled by sector correlations, into an infection probability. The new model imposes rather parsimonious data requirements, which are comparable to those for the BET model and considerably lower than for a multi–factor model. Therefore, the new model is not only transparent but also tractable, in terms of both the required computational effort and the data requirements.

In spite of its relatively simple structure, the infection model presented in this paper may provide a good VaR approximation since it is calibrated exactly to the tail of the loss distribution in the multi–factor model. This approach is expected to deliver better results than a moment–matching methodology, which is also used, for example, in the BET model, and which can easily fail to capture the true tail behavior.

To verify its accuracy, I compare the VaR in the new *infection model* to the VaR

[3]The Second Consultative Paper on Basel II, published in January 2001, contained a (later abandoned) proposal for a "granularity adjustment" of the risk-weight functions of the internal ratings based (IRB) approach. See Gordy (2004) for an overview of recent academic work on this subject.

calculated in a multi–factor model setting, in which various sectors constitute the systematic factors for a set of test portfolios. These test portfolios comprise exposure distributions across sectors which reflect the aggregate sector distribution of corporate lending in the German banking system as well as sector distributions of existing banks with more concentrated portfolios. Furthermore, the portfolios differ in terms of default probabilities and asset correlations. The purpose of comparing these two models is to determine, how much accuracy is lost by applying the infection model and whether it still provides a reasonably accurate, but technically much more easily computable, VaR.

The idea of calibrating a model with a simple structure and closed-form solution to the VaR is also pursued in recent work by Cespedes et al. (2005). However, their approach comprises a single–factor model with a scaling factor which is calibrated to capture diversification across sectors. The scaling factor is defined as a function of a Herfindahl–Hirschman index[4], based on the economic capital required in each sector by a single–factor model. It does not capture individual estimates of intra–sector and inter–sector correlations which feed into the infection model. Furthermore, the infection model differentiates between the contribution of the level of the probability of default ($PD$) and the impact of the asset correlations on the VaR. The Pykhtin (2004) approach retains the multi–factor model but provides a closed-form solution for the VaR which is based on a methodology previously employed by Wilde (2001) for a VaR adjustment which accounts for single-name concentration. Pykhtin's closed-form solution, however, is more complex than the proposal in this paper and does not reduce the data requirements compared with a multi–factor model. Instead of reducing the complexity of a multi–factor model, Witt (2004) comes from the opposite direction and extends Moody's BET model to form the "correlated binomial approach". However, his proposal requires estimates of default correlations as inputs which are difficult to obtain in the real world.

If the infection model presented in this paper proves to be sufficiently accurate, it could be used as a fit–for–purpose tool for risk managers in banks or for banking supervisors. Risk managers may also find it useful as a benchmark for the results of more sophisticated internal models.

The paper is organized as follows. Section 2 describes the concept of the BET model and shows the limitations of its accuracy in calculating VaR if default events are

[4]See Hirschmann (1964).

correlated. The infection model is introduced and calibrated in Section 3. The key parameter in the calibration procedure is the degree of default infection, which is determined by minimizing the squared error in the VaR calculation over a wide range of realistic input parameter values.

In Section 4, the model performance is evaluated by applying the calibrated infection model to a range of test portfolios. The purpose of these tests is to verify if the calibrated model provides a valid VaR approximation over the entire relevant parameter space. Section 5 summarizes and concludes.

# 2. Binomial Expansion Methodology

## 2.1. Model Setup

An important area where default correlation needs to be measured is the assessment of credit risk in securitization structures such as CDOs and CLOs. Therefore, it is not surprising that external rating agencies were among the first to develop a suitable methodology for that purpose. The BET model developed by Moody's was one of the first approaches and emerged as a market standard, not least because its underlying principles are very transparent.[5]

The key idea of the BET model is to map the real portfolio into a hypothetical homogenous portfolio that consists of loans sharing the same probability of default (PD) and the same exposure volume and in which default events of all exposures are pairwise independent. The number of defaults in this hypothetical portfolio is binomially distributed so that the VaR can easily be determined. The calculation of the VaR requires only two input parameters: the PD and the modified diversity score, which is defined as the number of exposures in the hypothetical portfolio. The mapping between the two portfolios is defined by matching the first two moments of the portfolio loss distribution.

Let $A_{i,k}$ denote exposure $k$ in sector $i$ of the real portfolio, $m$ the number of sectors, $n(i)$ the number of exposures in sector $i$, and $D$ the modified diversity score. For ease of presentation it is assumed that every exposure belongs to a different borrower,

[5]See Cifuentes et al. (1996), Cifuentes und O'Connor (1996), and Cifuentes und Wilcox (1998).

eliminating the need to differentiate between exposures and borrowers. Let $A$ refer to the total exposure, which is the same for both portfolios:

$$A = \sum_{i=1}^{m} \sum_{j=1}^{n(i)} A_{i,j}. \tag{1}$$

The uniform exposure size in the hypothetical portfolio is given by $A/D$. It is assumed that exposures in the same sector share the same PD. The average PD of the exposures in the hypothetical portfolio, $\bar{p}$, is calculated by setting the mean loss of the real portfolio equal to the mean loss of the hypothetical portfolio, where $LGD$ denotes the loss given default:

$$\sum_{i=1}^{m} \sum_{j=1}^{n(i)} p_i \cdot A_{i,j} \cdot LGD = D \cdot \bar{p} \cdot \frac{A}{D} \cdot LGD. \tag{2}$$

From (2) it follows that $\bar{p}$ equals the weighted average PD of the real portfolio:

$$\bar{p} = \frac{\sum_{i=1}^{m} p_i \cdot \sum_{j=1}^{n(i)} A_{i,j}}{A}. \tag{3}$$

Let $U_{i,j}$ denote the indicator function that signals a default of exposure $j$ in sector $i$ in the original portfolio and $U_k$ a default of the $k$-th exposure in the hypothetical portfolio. Matching the variances of losses in the real and the hypothetical portfolio provides an explicit expression for the modified diversity score:

$$Var\left(\sum_{i=1}^{m} \sum_{j=1}^{n(i)} A_{i,j} \cdot LGD \cdot U_{i,j}\right) = Var\left(\frac{A}{D} \cdot LGD \cdot \sum_{k=1}^{D} U_k\right). \tag{4}$$

Note that the value of the $LGD$ parameter does not affect the result of the moment matching because this parameter cancels out in (4) and (2).

It is assumed that the pairwise default correlation $\omega_{j,l}^{i,k}$ of two exposures $k$ and $l$ in sectors $i$ and $j$ is the same within each sector $(\omega_i^{intra}, \omega_j^{intra})$ and between any two sectors $(\omega^{inter})$:

$$\text{For } i \in \{1, ..., m\} : \ \omega_{j,l}^{i,k} = \begin{cases} 1 & : \ i = j \text{ and } k = l \\ \omega_i^{intra} & : \ i = j \text{ and } k \neq l \\ \omega^{inter} & : \ i \neq j. \end{cases} \tag{5}$$

Let $p_i$ denote the default probability, which is the same for all borrowers in sector $i$. Then, from (4) it follows for the modified diversity score $D$ that

$$D = \frac{A^2 \, \bar{p} \, (1 - \bar{p})}{\sum_{i=1}^{m} \sum_{j=1}^{m} \sum_{k=1}^{n(i)} \sum_{l=1}^{n(j)} A_{i,k} \, A_{j,l} \, \omega_{j,l}^{i,k} \, \sqrt{p_i \, (1 - p_i) \, p_j \, (1 - p_j)}}. \tag{6}$$

Since the number of defaults in the hypothetical portfolio is binomially distributed, calculation of the VaR for a confidence level of 99.9% requires the 99.9% percentile of the binomial distribution with parameters $D$ and $\bar{p}$. Then, the VaR is computed by multiplying this percentile by the average exposure size, $A/D$, and the $LGD$.

$$VaR^{BET} = \frac{A}{D} \cdot LGD \cdot Bin^{-1}_{99.9\%}(D, \bar{p}). \tag{7}$$

## 2.2. Evaluation of VaR Estimates from the BET Model

In the following I evaluate the accuracy of the BET model for a VaR calculation. The "real" portfolio that is used as the benchmark comprises 1,000 credit–risky exposures. It is homogeneous in terms of the notional amount (one currency unit), the default probability (2%) and the pairwise asset correlation (either 0.0, 0.1 or 0.2). The last property implies a single–sector model.

The well–known single risk factor model is used as a benchmark. Default is triggered if the unobservable, normalized asset value $X_k$ of exposure $k$ falls below an exogenous default threshold, $\Phi^{-1}(\bar{p})$, where $\Phi(.)^{-1}$ denotes the inverse of the cumulative standard normal distribution function. The loss distribution is obtained by sampling the model equation

$$X_k = \sqrt{\rho}\, Y + \sqrt{1-\rho}\, \epsilon_k, \tag{8}$$

in which $Y$ denotes the systematic factor and $\epsilon_k$ a disturbance term which are both independent and standard–normally distributed.

Table 1 compares VaR estimates obtained from Monte Carlo simulation with the corresponding VaR estimates, based on the BET model. The number of exposures in the hypothetical portfolio is given by the diversity score and the exposure size by the ratio of the total exposure of the real portfolio divided by the diversity score. The VaR estimate in the BET model is obtained by invoking (7). For an asset correlation of 0.1, for example, the VaR estimate is calculated as $6 \cdot 1,000/63$ and rounded up to 95. The increasing asset correlation, therefore, has two consequences which affect the VaR estimate in opposite directions. It reduces the diversity score in the denominator of (7) which c. p. increases the VaR estimate. Since the inferred number of defaults in the hypothetical portfolio decreases with a lower number of

**Table 1**

**Comparison of VaR estimates from the Binomial Expansion Technique with simulation-based estimates**

The following table compares VaR estimates from Monte Carlo simulation, $VaR^{MC}$, based on a single risk factor model with VaR estimates from applying the BET model, $VaR^{BET}$. The confidence level is 99.9%. The portfolio comprises 1,000 credit–risky exposures and is homogeneous in terms of the notional amount (one currency unit), the default probability (2%) and the pairwise asset correlation (either 0.0, 0.1 or 0.2).

| Asset correlation | Hypothetical portfolio | | | Real portfolio |
|---|---|---|---|---|
| $\rho$ | Diversity score $D$ | Inferred no. of defaults | $VaR^{BET}$ | $VaR^{MC}$ |
| 0.0 | 1000 | 35 | 35 | 35 |
| 0.1 | 63 | 6 | 95 | 130 |
| 0.2 | 27 | 4 | 148 | 227 |

exposures $D$, VaR also c. p. decreases. The net effect, however, is always an increase in the VaR estimate.

In the special case of uncorrelated default events, the BET model provides the correct VaR of 35, as expected. If the asset correlation is increased to 0.1 and afterwards to 0.2, the VaR estimate obtained from the BET model also increases. However, it substantially underestimates the true VaR in both cases. For an asset correlation of 0.1, the VaR estimate from the BET model is 27% below its true value, and for an asset correlation of 0.1, the shortfall increases to 35%. These results suggest that the BET model does not provide a sufficiently accurate estimate to recommend its application as a measure for sector–concentration risk. However, they say nothing about how well the BET model performs in capturing granularity in the VaR.

# 3. The Infection Model

## 3.1. Model Setup

The key idea of the new model is to extend the BET model by introducing a new type of default dependency into the hypothetical portfolio. This initially appears to be counter–intuitive because the hypothetical model was constructed precisely in order to be tractable through the assumption that default events are independent. However, instead of mapping the real portfolio into a portfolio with independent exposures, it is mapped into a portfolio with default dependencies which are judiciously constructed in order to obtain an easy-to-calculate solution for the VaR.

The number of exposures in the hypothetical portfolio is given by the diversity score $D$, as in the BET model. Let $Z_1, ..., Z_D$ denote indicator variables signaling that an exposure in the hypothetical portfolio has defaulted.

Following the approach by Davis und Lo (2001), it is assumed that any asset in the hypothetical portfolio can either "autonomously" default or default because it is "infected" by the default of another asset. Let $U_j^{autonom}$ and $U_{j,k}^{infected}$ denote indicator functions which equal 1 if an "autonomous" default of exposure $j$ occurs or if exposure $j$ is "infected" by exposure $k$. $U_j^{autonom}$ and $U_{j,k}^{infected}$ are independent for $j, k \in \{1, ..., D\}$ and $j \neq k$. Let $Z_j$ be the indicator function that signals a default of exposure $j$, either because it has "autonomously" defaulted or because it has been infected by the default of another exposure. Let $p_j = P\left[U_j^{autonom} = 1\right]$ denote the autonomous default probability in the hypothetical portfolio and $q_{j,k} = P\left[U_{j,k}^{infected} = 1\right]$ the probability that an infection can take place, in this case triggered by a default of exposure $k$. The default indicator $Z_j$ is defined as follows:

$$Z_j = U_j^{autonom} + (1 - U_j^{autonom}) \left(1 - \prod_{k,k \neq j} \left(1 - U_k^{autonom} \cdot U_{j,k}^{infected}\right)\right). \qquad (9)$$

It is assumed that all exposures in the hypothetical portfolio share the same autonomous default probability and the same infection probability. The autonomous default probability is determined as the exposure–weighted average of the default probabilities in the real portfolio. Therefore, $p_j = \bar{p}$ and $q_j = q$ for $j \in \{1, ..., D\}$.

The probability of $\nu$ defaults in the hypothetical portfolio is then given by[6]

$$f(\nu; D, \bar{p}, q) = \binom{D}{\nu} \Big[ \bar{p}^{\nu} \, (1-\bar{p})^{D-\nu} \, (1-q)^{\nu \, (D-\nu)} +$$
$$\sum_{i=1}^{\nu-1} \binom{\nu}{i} \bar{p}^{i} \, (1-\bar{p})^{D-i} (1-(1-q)^{i})^{\nu-i} \, (1-q)^{i \, (D-\nu)} \Big].$$

Therefore, the loss distribution $F^{IM}$ for up to $N$ defaults, $0 \leq N \leq D$,[7]

$$F^{IM}\left(\left[\frac{L \cdot D}{A \cdot LGD}\right]\right) = P(L \leq \frac{N \cdot A \cdot LGD}{D}) = \sum_{\nu=0}^{N} f(\nu; D, \bar{p}, q), \tag{10}$$

depends on five parameters: the loss given default, $LGD$, the total exposure $A$, the diversity score $D$, the "autonomous" default probability $\bar{p}$, and the probability of infection $q$. Since $LGD$ enters the loss distribution as a linear factor in the same way as it enters the VaR in the BET model (see (7)), its value does not change the outcome of a comparison between the two models. Therefore, $LGD$ is set equal to 100% in the following.

Davis und Lo (2001) regard the estimation of $q$ as the "main outstanding problem". I do not attempt to estimate this parameter but instead to "calibrate" it such that it delivers roughly the same VaR as a multi–factor model.

Since $EL$ is generally easier to measure than higher moments or extreme quantiles of the loss distribution, it seems natural to calibrate $EL$ in the infection model such that it matches $EL$ in the original portfolio. In the BET framework, the $EL$ of the hypothetical portfolio corresponds with the $EL$ of the real portfolio.

$$EL^{BET} = \sum_{j=1}^{D} LGD \cdot j \cdot \frac{A}{D} \cdot \binom{D}{j} \bar{p}^{j} \, (1-\bar{p})^{(D-j)} = A \cdot LGD \cdot \bar{p}. \tag{11}$$

In the infection model the $EL$ is given by[8]

$$EL^{IM} = LGD \cdot A \cdot \left(1 - (1-\bar{p})(1-\bar{p} \cdot q)^{D-1}\right). \tag{12}$$

As long as the autonomous default probability in the infection model is defined by the average default probability $\bar{p}$ in the real portfolio, the $EL$ generally differs

[6]See theorem 1 in Davis und Lo (2001).
[7]$[x]$ denotes the rounding function which rounds $x$ down to the nearest natural number.
[8]See Davis und Lo (2001).

between both portfolios. The $EL$ coincides only in the special case in which $q$ equals zero. In this trivial case, the infection model collapses to the binomial model.

In order to match $EL$ for the infection model for positive values of $q$, it would be necessary to adjust $\bar{p}$ depending on the parameter $q$. However, in the calibration methodology $q$ is not known *ex ante* but is the parameter which is determined at the very end in order to achieve a match in the VaR of the multi-factor model and the infection model. In principle it is possible to determine $q$ and $\bar{p}$ simultaneously by requiring that the VaR and the $EL$ both be matched. However, due to numerical problems I did not pursue this approach further. Instead, I focus on matching only the VaR and accept a discrepancy in the $EL$.

## 3.2. Calibration of the Infection Model

The analysis of the BET model is carried out in two steps. The first step comprises a linear regression in order to calibrate the infection parameter $q$, based on fictive exposure distributions across sectors. This step is described in the following section. The second step, described in Section 4, comprises an evaluation of the accuracy of the calibrated model for portfolios with empirically observed exposure distributions across sectors.

For a given portfolio and a parameter set of PDs and asset correlations, an optimal value $q^*$ can, in principle, be obtained by equating the VaR in the multi–factor model with the VaR in the infection model and solving numerically for $q$. The solution $q^*$, thus obtained, achieves a match in the adverse 99.9% quantile of the loss distribution of both models. Since $q^*$ depends on the portfolio characteristics, particularly the exposure distribution across sectors and asset correlations, it is not possible to find a unique $q^*$ for all portfolios. However, intuition suggests that $q$ depends on a limited number of factors which sufficiently capture portfolio characteristics, such as the distribution of exposures across sectors, correlation of default events, and default probabilities. Therefore, in the following I apply a linear model to determine a proxy of $q^*$ for every portfolio, dependent on three systematic factors. This regression model is given by

$$\ln(q) = \beta_0 + \beta_1 \ln(HHI) + \beta_2 \ln(\bar{p}) + \beta_3 \ln(\bar{\rho}^{intra}) + \beta_4 \ln(\bar{\rho}^{inter}) + \epsilon. \qquad (13)$$

$HHI$ denotes the Hirschman-Herfindahl index, calculated as the sum of squared relative exposure shares of the sectors in the portfolio. $\bar{\rho}^{intra}$ is the weighted–average intra–sector asset correlation of all exposures, weighted by the total exposure amounts to the individual sectors, $\bar{\rho}^{inter}$ the average inter–sector asset correlation, and $\epsilon$ the idiosyncratic disturbance term. Note that here I use asset correlations as explanatory variables instead of default correlations, which are necessary to calculate the diversity score. However, the asset correlation $\rho_{i,j}$ can be transformed into the default correlation $\omega_{i,j}$ by invoking the ratio

$$\omega_{i,j} = \frac{\Phi_2\left(\Phi^{-1}(p_i), \Phi^{-1}(p_j), \rho_{i,j}\right) - p_i\, p_j}{\sqrt{p_i\,(1-p_i)\,p_j\,(1-p_j)}}. \tag{14}$$

where $\Phi_2(.)$ denotes the cumulative distribution function of the bivariate normal distribution.

The regression coefficients $\beta_0, ..., \beta_4$ in (13) are estimated by minimizing the squared differences between VaR for portfolios with different sector concentrations and various typical parameter sets of $\bar{\rho}^{intra}$, $\bar{\rho}^{inter}$ and $\bar{p}$. The optimization is carried out by a linear regression based on (13), in which $q$ is replaced by the "optimal" $q^*$ which equates the VaR in both models for the respective parameter set.

In the special case of zero inter–sector correlation, the regression function has a jump discontinuity. Therefore, I estimate the model separately under the restriction $\beta_4 = 0$.

The benchmark or "true" model is a multi–factor model in which the factors are defined by business sectors. The normalized one–period asset return of borrower $k$ in sector $i$ is defined by a single–factor model

$$X_{i,k} = r_i\, Y_i + \sqrt{1 - r_i^2}\,\zeta_{i,k} \tag{15}$$

where $\zeta_{i,k}$ denotes the idiosyncratic disturbance term. The parameter $r_i^2$ describes the correlation between a borrower's normalized, unobservable asset return process and the systematic risk factor $Y_i$. The sector–dependent systematic factor $Y_i$ can be written as a linear combination of $m$ orthogonal factors. Borrower $k$ in sector $i$ defaults if his normalized asset value process $X_{i,k}$ crosses an exogenous default barrier $\Phi^{-1}(p_{i,k})$ where $p_{i,k}$ denotes the unconditional default probability.

The value of $r_i$ is given by $r_i^2 = \rho_i^{intra}$. Given $r_i$ and $r_j$, the factor correlations $cor(Y_i, Y_j)$ can be computed from the inter–sector asset correlations $\rho_{i,j}^{inter}$ in different sectors $i$ and $j$ as follows:

$$cor(Y_i, Y_j) = \frac{\rho_{i,j}^{inter}}{r_i\, r_j}. \tag{16}$$

The VaR of the real portfolio is determined by MC simulation of the portfolio losses, $L^{MC}$, given by

$$L^{MC} = \sum_{i=1}^{m} \sum_{k=1}^{n(i)} A_{i,k}\, 1_{\{X_{i,k} \leq \Phi^{-1}(p_{i,k})\}}, \tag{17}$$

exploiting the assumption that the LGD equals 100%.

The portfolios used for calibration comprise 2000 exposures of one euro each. These fine-grained portfolios ensure that the impact of sector concentration is not limited by the granularity of the portfolios. In order to get meaningful results from the calibration exercise, I focus on a subset of the parameter space which captures the relevant range of typical values for $HHI$, $\bar{\rho}^{intra}$, $\bar{\rho}^{inter}$, and $\bar{p}$.

In terms of sector concentration I explore four portfolios with sectors of different sizes. I start with the relatively concentrated portfolio 1 that comprises only three sectors which have a relative share of 50%, 30%, and 20% of the total exposure. I proceed to portfolios 2, 3, and 4 using the following algorithm. The sector size in every new portfolio is determined by splitting every sector share of the old portfolio into two sectors which contain one-third and two-thirds respectively. The resulting composition of the portfolios is presented in Table 8 in the Appendix. The maximum of 24 sectors in portfolio 4 is inspired by the two–digit GICS classification scheme of CreditMetrics which is based on 24 industry groups.

Other vendor models, such as Moody's KMV, employ an even greater number of sectors. However, increasing the number of sectors by adjusting the sector definition without changing the portfolio will decrease concentration in individual sectors and increase correlation across sectors. Broadly speaking sector concentration risk would be transformed into overall correlation risk. Since the focus is here on sector concentration risk, higher numbers of factors are not considered. Another motivation for not considering more factors is the fact that fit–for–purpose tools such as the infection model could be most valuable for regionally focused and medium–size banks because such tools impose relatively parsimonious data requirements with a limited number of sectors.

For the calibration exercise, uniform intra–sector correlations $\bar{\rho}^{intra}$ for all sectors and also uniform inter–sector correlations $\bar{\rho}^{inter}$ are assumed. This specification is motivated by balancing parsimony and accuracy but also by a requirement of the infection model which needs correlation values on a sector–by–sector basis for the calculation of the diversity score. In cases where this information is not available to banks, supervisors may consider providing a rough estimate of an *average* asset correlation based on their own experience. This value could also be used for the calculation of the diversity score, thereby further reducing data requirements.

Defining a realistic range of typical asset correlations for the calibration exercise is not straightforward. The results of numerous empirical studies which have been carried out in recent years are quite diverse.[9] It seems fair to summarize that a generally accepted industry consensus about the range of asset correlations has not emerged so far. However, the relatively broad range (between 0.05 and 0.4) appears to cover all relevant values. The 15 pairs of asset correlations $\bar{\rho}^{intra}$ and $\bar{\rho}^{inter}$ which are used for the calibration are listed in Table 2.[10]

**Table 2**

**Correlation parameters for model calibration**

The following table lists the intra–sector asset correlations and the corresponding inter–sector correlations used to calibrate the infection probability.

| No. | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| intra–sector | 0.05 | 0.1 | 0.15 | 0.2 | 0.3 | 0.4 |
| inter–sector | 0.025 | 0.025 | 0.025 | 0.05 | 0.05 | 0.05 |
| | | 0.05 | 0.05 | 0.075 | 0.1 | 0.1 |
| | | | 0.075 | 0.1 | 0.15 | 0.15 |

Realistic PD values were taken from historical default rates observed for common rating categories, in particular 0.03%, 0.2%, 0.5%, 1%, 2%, and 5%. As common intuition tells us that corporate loans have, on average, lower PDs than retail exposures and also that systematic risk decreases with the default probability, PDs of up to 5% are considered to be most relevant for the analysis.

With 4 different sector distributions, 6 PDs and 15 pairs of intra–sector and inter–

[9]See, for example, Lopez (2002) as an example of relatively high estimates of asset correlations and Roesch (2003) who estimates relatively low values.

[10]In the special case of zero inter–sector correlation ($\beta_4 = 0$), I also consider values of 0.02 and 0.5 for the intra–sector asset correlation.

sector asset correlations, a total of 360 parameter sets are considered. For every parameter set the VaR is determined in the multi–factor model by simulation. Afterwards the optimal $q^*$ is computed which generates roughly the same VaR in the infection model. The linear regressions are carried out using the $q^*$ values as "observations" of the dependent variable. The estimates of the regression coefficients $\beta_0, ..., \beta_4$ in (13) are summarized in Table 3 for the two cases with and without inter–sector correlation. The regression coefficients are highly significant with one

**Table 3**

**Regression results**

The following table presents the results of regressing the natural logarithm of the "optimal" infection parameter $q^*$ on the Hirschman-Herfindahl index ($HHI$), the average asset correlations ($\bar{\rho}^{intra}$ and $\bar{\rho}^{inter}$), and the default probability $\bar{p}$. "***" signals significance at the 99% confidence level.

| Regressor<br>coefficient | Intercept<br>$\beta_0$ | $HHI$<br>$\beta_1$ | $\bar{p}$<br>$\beta_2$ | $\bar{\rho}^{intra}$<br>$\beta_3$ | $\bar{\rho}^{inter}$<br>$\beta_4$ |
|---|---|---|---|---|---|
| a) Zero inter–sector correlation | | | | | |
| estimate | −0.286 | 1.060*** | 0.349*** | 1.795*** | – |
| Standard error | 0.146 | 0.045 | 0.019 | 0.030 | – |
| b) With inter–sector correlation | | | | | |
| estimate | 0.813*** | 0.466*** | 0.488*** | 1.067*** | 0.688*** |
| Standard error | 0.097 | 0.022 | 0.009 | 0.036 | 0.036 |

exception. Since the coefficient of the intercept is insignificant in the case of zero inter–sector correlation, this regression model is also estimated without intercept with only marginal changes in the other coefficient estimates. The signs of the estimated coefficients are as expected in all cases. The infection probability increases, as expected, with sector concentration, measured by $HHI$, with the default probability $\bar{p}$, and also with the asset correlations $\bar{\rho}^{intra}$ and $\bar{\rho}^{inter}$.

The adjusted $R^2$ of the regression estimation is 96% without inter–sector correlation and 95% with inter–sector correlation. These results indicate that the explanatory power of the regression models is sufficient to keep the calibration error from unexplained noise in the residuals within reasonable bounds.

# 4. Evaluation of the Infection Model

When evaluating the model, the calibrated model is applied to various test portfolios in order to measure the accuracy of the VaR. For the test portfolios in the analyses the distribution of exposures across sectors was constructed from credit register data on real bank portfolios. This test is extremely important because of three different sources of error which can distort the results of the calibration when applied to real portfolios.

The first error component derives from a discrete diversity score and does not depend, therefore, directly on the calibration procedure. The diversity score has to be an integer since the loss distribution, defined by (10), is a discrete distribution. This error decreases as the number of exposures increases in the portfolio.

The second error component depends on the unexplained variation in the regression model, defined by (13). In order to reduce this error, various specifications of (13) were tried until an adjusted $R^2$ of 96% seemed satisfactory.

The third, and arguably most important, error component derives from the difference between the parameter set of the real portfolio and the parameter sets used for calibration. A realistic range of parameters was used in order to reduce this effect, yet this difference still constitutes a potential source of error, especially with respect to the sector concentration, measured by $HHI$.

To evaluate the accuracy of the model calibration, three test portfolios were used with realistic sector distributions. The first test portfolio represents the overall business–sector concentration of the German banking system, including branches of foreign banks, on German *corporate*, non–financial borrowers. I consider this to be a reasonable approximation of a balanced portfolio. This view is guided by the intuition that banks' portfolios, on average, cannot be more diversified than the average relative sector distribution of the national banking system. However, a VaR-minimizing portfolio could be better diversified and could yield a different and possibly more uniform distribution of exposures across sectors. Portfolio 1 was constructed by aggregating large corporate exposures[11] from the loan portfolios of 2224 German banks at the end of September 2004. The data were extracted from the German central credit register, which collects commercial, industrial, and

[11] See sections 13 and 14 of the German Banking Act ("Kreditwesengesetz").

consumer loans. Since the credit register reports only national industry codes which are compatible with the NACE classification scheme, the sector exposures were mapped to the Global Industry Classification Standard (GICS). For every sector the total exposure, aggregated over all banks in the sample, was scaled so that the sum of all sectors is consistent with the hypothetical total volume of EUR 6,000,000.

The calibration accuracy is subsequently measured for two more concentrated test portfolios which were constructed to resemble real bank portfolios with respect to the HHI. The sector distribution of all three portfolios is shown in Table 4. With HHI ranging from 17.6% for the first portfolio to 61.7% for the last, the portfolios cover a relatively broad range of exposure concentrations in sectors.

**Table 4**
**Sector decomposition of the test portfolios**

The following table presents the sector decomposition of test portfolios 1–3 according to the GICS classification scheme, together with the Herfindahl-Hirschman index (HHI), based on the aggregated sector exposures.

| Sector | Portfolio 1 | Portfolio 2 | Portfolio 3 |
|---|---|---|---|
| A: Energy | 0.2% | 0.1% | 0.1% |
| B: Materials | 6.0% | 4.0% | 1.5% |
| C1: Goods | 11.5% | 41.0% | 77.9% |
| C2: Commercial services | 33.7% | 22.5% | 8.4% |
| C3: Transportation | 7.14% | 4.8% | 1.8% |
| D: Consumer discretionary | 15.0% | 10.0% | 3.8 % |
| E: Consumer staples | 6.5% | 4.3% | 1.6% |
| F: Health care | 9.1% | 6.1% | 2.3% |
| H: Information technology | 3.2% | 2.1% | 0.8% |
| I: Telecommunication services | 1.1% | 0.7% | 0.3% |
| J: Utilities | 6.7% | 4.5% | 1.7% |
| HHI | 17.6% | 24.0% | 61.7% |

Little computational work is required to apply the calibrated infection model. The VaR is determined as the 99.9% adverse percentile of the loss distribution defined by (10). This calculation requires the parameter $q$ which is determined from (13) with estimates of the regression coefficients $\beta_0, ..., \beta_4$ given in Table 3 and the portfolio–dependent parameter values $HHI$, $\bar{\omega}^{intra}$, $\bar{\omega}^{inter}$, and $\bar{p}$.

When analyzing each test portfolio, the parameter $HHI$ is kept constant because it is defined by the exposure distribution across sectors which is also constant for each test portfolio. In order to increase the representativeness of the results $\bar{p}$, $\bar{\rho}^{intra}$, and $\bar{\rho}^{inter}$ are varied in each test portfolio in the same range as for the calibration. For 3 test portfolios, 6 PDs and 15 pairs of $\bar{\rho}^{intra}$ and $\bar{\rho}^{inter}$, this produces 270 VaR estimates for the multi–factor model, for the BET model, and for the infection model respectively.

Table 5 shows descriptive statistics of the VaR for portfolios 1, 2, and 3. The relatively strong increase in the $HHI$ for portfolio 3 compared with portfolios 1 and 2 is mirrored by a strong increase of 66% and 48% in the VaR.

**Table 5**

**Descriptive statistics of the VaR for portfolio 1–3**

The following table contains the median, standard deviation, and 75% quantile of the VaR over 90 tuples of default probabilities and intra–sector and inter–sector asset correlations. The VaR was determined by MC simulations. All numbers are given in percent.

| Test portfolio | 1 | 2 | 3 |
|---|---|---|---|
| Median | 5.6 | 6.3 | 9.3 |
| Standard deviation | 8.5 | 9.2 | 13.4 |
| 75% quantile | 12.6 | 14.2 | 19.4 |

Table 6 shows descriptive statistics of the infection probabilities for the three test portfolios. For each portfolio the median, standard deviation, and the 75% quantile are reported for 90 tuples of default probabilities and intra–sector and inter–sector asset correlations. The results are relatively similar for portfolio 1 and 2 but for the more concentrated portfolio 3 the level and the dispersion of the infection probabilities roughly double.

Table 9 in the Appendix provides an extraction of VaR estimates and VaR errors for selected combinations of $\bar{\rho}^{intra}$, $\bar{\rho}^{inter}$, and $\bar{p}$ for portfolio 1. Column 4 of the table contains the VaR (in percent), obtained from MC simulations, and the percentage errors of the BET model and the infection model relative to the simulation-based VaR.

**Table 6**
**Descriptive statistics of the calibrated infection probabilities for portfolio 1–3**

The following table contains the median, standard deviation, and 75% quantile of the infection probabilities over 90 tuples of default probabilities and intra–sector and inter–sector asset correlations. All numbers are given in percent.

| Test portfolio | 1 | 2 | 3 |
|---|---|---|---|
| Median | 0.17 | 0.20 | 0.41 |
| Standard deviation | 0.40 | 0.47 | 0.80 |
| 75% quantile | 0.42 | 0.53 | 0.94 |

According to Table 9, the relative error in the VaR estimate is substantially lower for the infection model than for the BET model. With few exceptions, it is below 10% and may be regarded as sufficiently small for practical purposes to measure the impact of sector concentration on the VaR with the infection model. The regions in which the infection model produces the highest relative errors in VaR are characterized by low PDs in combination with relatively high asset correlations.

Figure 1 visualizes the percentage error in the VaR for two selected values of $\bar{p}$, namely 20bp and 2%. The inter–sector correlation is set to zero. Figure 1 shows that in the infection model the relative errors increase with $\bar{\rho}^{intra}$ only for the lower PD of 20 bp. For $\bar{\rho}^{intra} > 0.15$ and $\bar{p}_i = 0.2\%$ the approximation error exceeds the 10% boundary; however, this occurs relatively rarely in the 270 considered cases. For the BET model the relative errors are considerably higher and increase with the asset correlations, as expected.

Figure 2 differs from Figure 1 in that the inter–sector correlation is set to 5% instead of zero. The results are qualitatively very similar to the case of zero inter–sector correlation. Again, there is no monotonic relation between the intra–sector correlation and the accuracy of the infection model. This suggests that the calibration of the infection model achieves its purpose of accounting for changes in the asset correlation. Only for relatively extreme combinations of low PDs and high asset correlations, not shown in Figure 2, are peaks in the relative VaR errors observed. This is different from the BET model, for which a positive relationship can overall be observed between the asset correlation and the VaR error can be observed.

**Figure 1. Percentage absolute error in the VaR estimate in the BET model and in the infection model for portfolio 1 (with zero inter–sector asset correlation)**

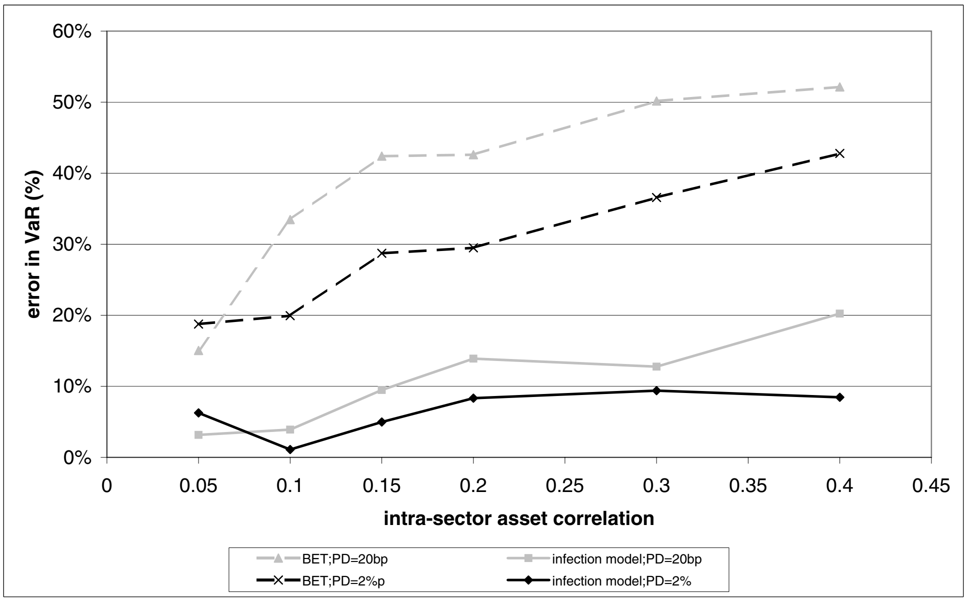


**Figure 2. Percentage absolute error in the VaR estimate in the BET model and in the infection model for portfolio 1 (with inter–sector asset correlation of 2.5% for an intra–sector correlation of 5% and 5% for higher intra–sector correlations)**

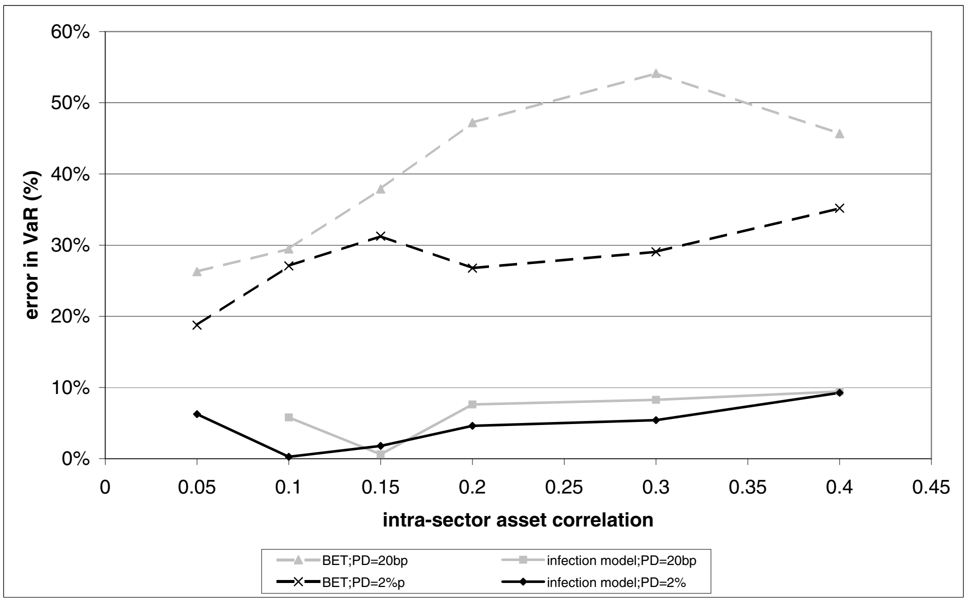

**Figure 3. Percentage absolute error in the VaR estimate in the BET model and in the infection model for portfolio 3 (with zero inter–sector asset correlation)**

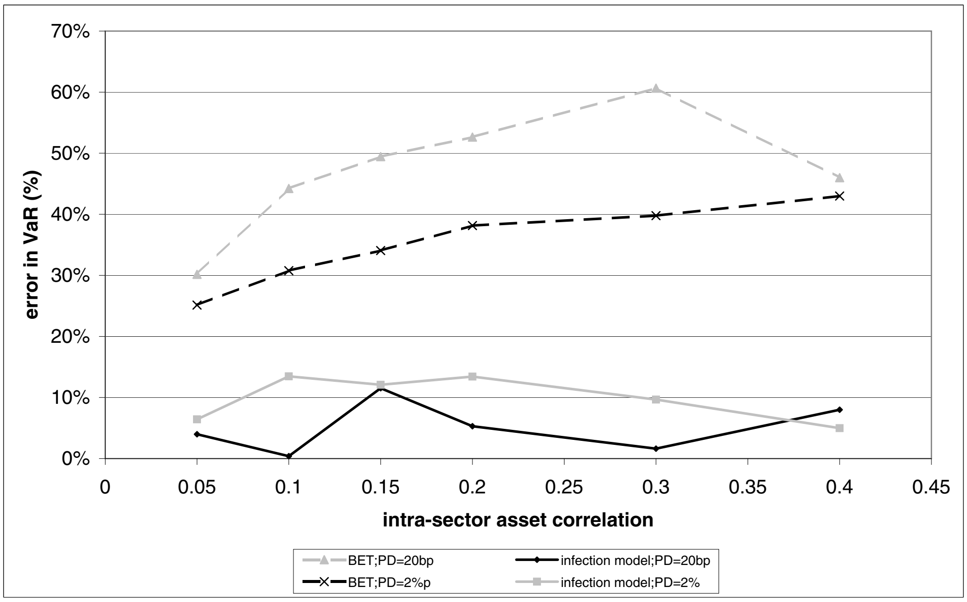


Figure 3 shows the approximation error in a heavily concentrated single–sector portfolio. Again, the relative errors of the infection model appear to be relatively robust against changes in the asset correlation.

Table 7 summarizes descriptive statistics of the absolute values of the relative VaR errors. Results are presented for both models and for all three test portfolios, calculated from 90 tuples of $\bar{\rho}^{intra}$, $\bar{\rho}^{intra}$, and $\bar{p}$ for each portfolio. The gain in accuracy by using the infection model instead of the BET model is considerable and amounts to 28.8–31.8 percentage points for the median errors. For the infection model the median error and also the other two statistics are considerably higher for portfolio 3 (which has the highest concentration) than for the other two portfolios. This may at least partly be explained by the fact that the most concentrated portfolio is out–of–sample in the sense that the highest HHI for the calibration was 38% compared with a considerably higher value of 61.7% in portfolio 3. If portfolios with such a concentrated sector distribution are relevant for practical purposes, a re–calibration can be carried out based on more concentrated portfolios. With one exception,[12],

[12]For the smallest parameter values, which means for $\bar{\rho}^{intra} = 0.05$, $\bar{\rho}^{inter} = 0$ and $\bar{p} = 0.03\%$, both models achieve roughly the same accuracy.

**Table 7**

**Accuracy of value-at-risk estimates for the three test portfolios**

The following table contains the median, standard deviation, and 75% quantile of the absolute VaR approximation errors for test portfolios 1–3. The approximation error is defined as the absolute value of the relative difference between the value–at–risk (VaR) in the multi–factor model and the benchmark model which is either the original BET model or the calibrated infection model. All numbers are given in percent.

| test portfolio | | 1 | 2 | 3 |
|---|---|---|---|---|
| BET model | Median | 34.1 | 36.7 | 39.9 |
| | Standard deviation | 10.9 | 11.6 | 10.0 |
| | 75% quantile | 42.4 | 44.7 | 47.9 |
| Infection model | Median | 5.3 | 4.9 | 9.8 |
| | Standard deviation | 5.1 | 6.0 | 9.6 |
| | 75% quantile | 8.9 | 10.4 | 17.1 |

replacing the BET model by the infection model substantially reduces the error in the VaR. It is up to the practitioner to decide whether the results are sufficiently accurate for his purposes given the "fit–for–purpose" character of the approach.

# 5. Summary and Conclusions

Results from portfolio models for credit risk tell us that exposure concentrations in certain industry sectors can substantially increase the VaR. The purpose of this paper is to analyze a tractable "infection model" that permits a meaningful estimate of the impact of concentration risk on the VaR. The required input parameters – the sector–based Herfindahl–Hirschman index, the average (intra–sector and inter–sector) asset correlations, weighted by total sector exposures, and the average default probability in each sector – can be calculated from the data which are already required to calculate a diversity score for the BET model. Therefore, the new model imposes no additional data requirements compared with the BET model and considerably fewer than a multi–factor model.

The infection model extends the Binomial Expansion Technique (BET) developed by

Moody's for the valuation of CDOs. The key idea is to introduce default infection for the hypothetical portfolio on which the real portfolio is mapped in order to obtain a simple solution for the VaR. The infection probability is calibrated for a range of typical values of the input parameters to the VaR obtained from a multi–factor model.

The accuracy of the new model and its calibration are measured for three test portfolios. The exposure distributions across sectors of these portfolios are defined based on information from the German public credit register. The first portfolio reflects the average sector distribution of the German banking system whereas the second and third portfolios are more concentrated and resemble those of real banks. The tests also consider a broad range of realistic values of default probabilities and intra–sector as well as inter–sector asset correlations. I find that a carefully calibrated model approximates the VaR obtained from a multi–factor model reasonably closely. The highest errors are observed for combinations of very low default probabilities and high asset correlations. With a reduction of roughly 30 percentage points in the median of the relative VaR approximation errors, the infection model out–performs by far the BET approach, which is dismissed for its lack of accuracy.

Future work will comprise further robustness checks, with attention given to the heterogeneity of intra–sector correlations and PDs. A comparison of the specific shape of the loss distribution implied by the infection model with alternatives is also warranted. The loss distribution can be replaced by another distribution, for example a beta–binomial distribution, and the additional parameter can be calibrated similarly to $q^*$ in the infection model.

Parsimonious data requirements constitute an important, practical advantage of the infection model. Instead of a fully specified correlation matrix, it is sufficient to provide an average intra–sector and an average inter–sector correlation as inputs. From the evaluation of the calibrated infection model, it can be concluded that the model offers a fit–for–purpose tool to measure concentration risk in business sectors. It could be useful for risk managers in banks as well as banking supervisors, and may be especially suited for application in regionally focused and medium–sized banks because of its relatively sparse data requirements.

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# Appendix

**Table 8**

**Composition of portfolios used for calibration**

The following table presents the sector decomposition of the four portfolios that were used for the calibration as a percentage. The last row contains the Herfindahl-Hirschman index (HHI) of the respective portfolio.

| Sector | Portfolio no. 1 | Portfolio no. 2 | Portfolio no. 3 | Portfolio no. 4 |
|---|---|---|---|---|
| 1 | 50 | 33.33 | 22.22 | 14.81 |
| 2 | 30 | 16.67 | 11.11 | 7.41 |
| 3 | 20 | 20 | 11.11 | 7.41 |
| 4 | | 10 | 5.56 | 3.70 |
| 5 | | 13.33 | 13.33 | 7.41 |
| 6 | | 6.67 | 6.67 | 3.70 |
| 7 | | | 6.67 | 3.70 |
| 8 | | | 3.33 | 1.85 |
| 9 | | | 8.89 | 8.89 |
| 10 | | | 4.44 | 4.44 |
| 11 | | | 4.44 | 4.44 |
| 12 | | | 2.22 | 2.22 |
| 13 | | | | 4.44 |
| 14 | | | | 2.22 |
| 15 | | | | 2.22 |
| 16 | | | | 1.11 |
| 17 | | | | 5.93 |
| 18 | | | | 2.96 |
| 19 | | | | 2.96 |
| 20 | | | | 1.48 |
| 21 | | | | 2.96 |
| 22 | | | | 1.48 |
| 23 | | | | 1.48 |
| 24 | | | | 0.74 |
| HHI | 0.38 | 0.21 | 0.18 | 0.065 |

## Table 9

### Accuracy of value-at-risk estimates for the test portfolio

The following table lists the value-at-risk (normalized for a total exposure of 100 currency units) obtained from simulation in a multi–factor model and the relative errors of the BET model and the calibrated infection model.

| $\bar{\rho}^{intra}$ | $\bar{\rho}^{inter}$ | $\bar{p}$ | Value–at–risk multi–factor | Approximation error in percent BET model | infection model |
|---|---|---|---|---|---|
| 0.05 | 0.025 | 0.0003 | 0.2 | 21.9% | 9.4% |
| 0.05 | 0.025 | 0.002 | 0.9 | 26.3% | 1.4% |
| 0.05 | 0.025 | 0.005 | 2.0 | 20.9% | 5.0% |
| 0.05 | 0.025 | 0.01 | 3.6 | 24.6% | 4.0% |
| 0.05 | 0.025 | 0.02 | 6.3 | 18.8% | 6.3% |
| 0.05 | 0.025 | 0.05 | 12.9 | 12.3% | 4.1% |
| 0.15 | 0.025 | 0.0003 | 0.4 | 39.5% | 5.9% |
| 0.15 | 0.025 | 0.002 | 1.5 | 36.9% | 5.3% |
| 0.15 | 0.025 | 0.005 | 3.1 | 33.1% | 4.4% |
| 0.15 | 0.025 | 0.01 | 5.2 | 30.0% | 3.7% |
| 0.15 | 0.025 | 0.02 | 8.6 | 26.0% | 1.4% |
| 0.15 | 0.025 | 0.05 | 16.7 | 19.4% | 2.5% |
| 0.15 | 0.05 | 0.0003 | 0.4 | 34.1% | 15.3% |
| 0.15 | 0.05 | 0.002 | 1.8 | 37.9% | 0.6% |
| 0.15 | 0.05 | 0.005 | 3.7 | 34.3% | 1.4% |
| 0.15 | 0.05 | 0.01 | 6.1 | 27.1% | 6.3% |
| 0.15 | 0.05 | 0.02 | 10.2 | 31.3% | 1.8% |
| 0.15 | 0.05 | 0.05 | 19.7 | 21.1% | 5.2% |
| 0.2 | 0.05 | 0.0003 | 0.5 | 46.0% | 8.0% |
| 0.2 | 0.05 | 0.002 | 2.1 | 47.2% | 7.6% |
| 0.2 | 0.05 | 0.005 | 4.3 | 41.1% | 5.8% |
| 0.2 | 0.05 | 0.01 | 7.0 | 33.4% | 0.0% |
| 0.2 | 0.05 | 0.02 | 11.5 | 26.8% | 4.6% |
| 0.2 | 0.05 | 0.05 | 21.4 | 23.8% | 4.7% |
| 0.3 | 0.1 | 0.0003 | 0.9 | 55.2% | 12.0% |
| 0.3 | 0.1 | 0.002 | 3.8 | 50.0% | 0.0% |
| 0.3 | 0.1 | 0.005 | 7.1 | 39.7% | 9.6% |
| 0.3 | 0.1 | 0.01 | 11.3 | 42.7% | 0.4% |
| 0.3 | 0.1 | 0.02 | 17.0 | 30.0% | 2.1% |
| 0.3 | 0.1 | 0.05 | 30.4 | 24.13% | 1.2% |

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