DEUTSCHE BUNDESBANK
EUROSYSTEM

Discussion Paper

Deutsche Bundesbank
No 34/2017

A severity function approach to scenario selection

Frieder Mokinski

Deutsche Bundesbank, Wilhelm-Epstein-Straße 14, 60431 Frankfurt am Main,
Postfach 10 06 02, 60006 Frankfurt am Main

Tel +49 69 9566-0

Please address all orders in writing to: Deutsche Bundesbank,
Press and Public Relations Division, at the above address or via fax +49 69 9566-3077

Internet http://www.bundesbank.de

ISBN 978–3–95729–408–1 (Printversion)
ISBN 978–3–95729–409–8 (Internetversion)

Non-technical summary

Research Question

Scenarios for banking sector stress tests are expected to be severe yet plausible. A scenario is severe if it puts the banking sector under serious stress, and it is plausible if there is a non-negligible probability that it materializes. How can we come up with such a scenario?

Contribution

The discussion paper presents an approach to scenario selection. The "severity function approach" (abbreviated below as SFA) needs two inputs: 1) a probabilistic forecasting model to assess the plausibility of alternative scenarios, and 2) a severity function to approximate how stressful a scenario is. The SFA then finds the scenario with the highest severity among a set of equally plausible scenarios and thus operationalizes the concept of "severe yet plausible". The key challenge of the SFA is to come up with a good severity function: Such a function should approximate the stress test impact of alternative scenarios.

Results

The present paper uses the SFA to find a stress test scenario for the German banking sector. This scenario implies a deep recession throughout the first two years of its four-year horizon and a flat yield curve. It can be expected to raise banks' credit risk and to lower their net interest income, thus putting serious pressure on banks' income.

Nichttechnische Zusammenfassung

Fragestellung

Szenarien für Finanzsektor-Stresstests sollten schwerwiegend und gleichzeitig plausibel sein. Ein Szenario ist schwerwiegend, wenn es den Finanzsektor vor ernsthafte Schwierigkeiten stellt, und es ist plausibel, wenn es eine nicht vernachlässigbare Eintrittswahrscheinlichkeit aufweist. Wie lassen sich derartige Szenarien generieren?

Beitrag

Dieser Artikel stellt einen Ansatz zur Generierung von Stresstest-Szenarien vor. Der sogenannte "Severity Function Approach" (kurz SFA) benötigt 1) ein Prognosemodel, das hilft die Plausibilität unterschiedlicher Szenarien zu beurteilen, und 2) eine "Severity Funktion", um abzuschätzen, wie nachteilig sich ein Szenario auf den Finanzsektor auswirken würde. Der SFA wählt auf dieser Basis das Szenario mit dem höchsten Wert der Severity-Funktion, das gleichzeitig eine vorgegebene Eintrittswahrscheinlichkeit nicht unterschreitet. Der Ansatz operationalisiert folglich die Idee, dass Szenarien schwerwiegend und gleichzeitig plausibel sein sollten. Die entscheidende Herausforderung des SFA ist die Wahl einer geeigneten Severity-Funktion.

Ergebnisse

Im Artikel wird der SFA eingesetzt, um ein Stresstest-Szenario für den deutschen Bankensektor zu generieren. Das vorgestellte Szenario beschreibt eine Rezession, die sich über die ersten zwei Jahre des vierjährigen Szenariohorizonts erstreckt, und ein Abflachen der Zinsstrukturkurve. Das Szenario stellt eine aus Sicht des Bankensektors schwerwiegende Verschlechterung des makrofinanziellen Umfelds dar: Durch die Rezession steigen die Kreditausfälle und die flache Zinsstrukturkurve schmälert das Zinsergebnis der Banken.

A Severity Function Approach to Scenario Selection*

Frieder Mokinski
Deutsche Bundesbank

Abstract

The severity function approach (abbreviated SFA) is a method of selecting adverse scenarios from a multivariate density. It requires the scenario user (e.g. an agency that runs banking sector stress tests) to specify a "severity function", which maps candidate scenarios into a scalar severity metric. The higher the value of this metric, the more harmful a scenario is. In selecting a scenario the SFA proceeds as follows: First, it isolates a set of equally severe scenario candidates. This set is determined by the condition that more severe scenarios only occur with some user-specified probability. Second, from this set it selects the candidate with the highest probability density, i.e. the most plausible scenario. The approach hence operationalizes the mantra that "scenarios should be severe yet plausible".

Keywords: Stress Testing, Conditional Forecasting, Density Forecasting, Time series, Bayesian VAR, Simulation

JEL classification: C11, C32, C53, C61, G01, G32.

*Contact address: Wilhelm-Epstein-Straße 14, 60431 Frankfurt am Main. Phone: +49 (0) 69 9566 2927. E-Mail: frieder.mokinski@bundesbank.de. I am grateful to Jörg Breitung, Sandra Eickmeier, Malte Knüppel, Fabian Krüger, Christoph Memmel, Esteban Prieto, Peter Raupach, Markus Roth, Harald Uhlig and Edgar Vogel for their helpful comments. Any errors are my sole responsibility.

1 Introduction

Central banks and other institutions such as the IMF regularly apply stress tests to assess the resilience of the banking sector. Such stress tests consider a range of questions, such as by how much an adverse shock would reduce capital buffers, the credit supply or the provision of services to other agents in the economy; whether there is evidence of increasing banking sector resilience over time; or to what extent higher capital buffers - for just a few selected banks or the whole sector - would reduce the level of stress induced by an adverse scenario. With regard to methodology, stress tests come in two varieties, taking either a bottom-up or top-down approach. In a bottom-up stress test, the stress testing agency provides a scenario and banks report their impact estimates. By contrast, in top-down stress tests, the stress testing agency uses its own models to estimate the impact of a scenario on the banking sector.[1] Whether the approach is bottom-up or top-down, stress tests require scenarios. Such scenarios are multi-period paths for the risk factors of the stress test. In typical banking sector stress tests, these risk factors are headline macroeconomic and financial figures such as different interest rates or bond yields (important drivers of banks' net interest income), measures of economic activity (important for credit risk), and metrics for stock market valuations (important for trading income). Stress scenarios are commonly required to be "severe yet plausible" (see, for example, Borio, Drehmann, and Tsatsaronis, 2014). In this context, "severe" means that the scenario should be stressful for the banking sector, and "plausible" means that the scenario should materialize with a non-negligible probability.

In this article, I present a method of finding adverse scenarios for stress tests, called the "severity function approach" (SFA). The SFA requires two inputs: a severity function and a probabilistic forecasting model. The severity function's purpose is to map each scenario into a scalar measure of its severity, and the probabilistic forecasting model's purpose is to provide a metric of scenario plausibility (through its predictive density function). The SFA then chooses the most plausible scenario from among all the scenarios for which the severity function value exceeds a given hurdle. In doing this, the hurdle value is chosen indirectly through the parameter α. $\alpha = 0.99$, for example, means that the hurdle is such that, according to the predictive density, 99% of all scenarios have a lower severity function value, i.e. are less severe. By choosing a higher value for α, a more extreme but also less plausible scenario is obtained. The SFA can be seen as an operationalization of the requirement that stress scenarios should be "severe yet plausible": it maximizes plausibility (as measured by the probability density of the scenario) subject to a minimum severity side condition (that less severe scenarios occur with probability α).

The SFA is closely related to a (financial) portfolio risk metric known as "Maximum Loss", which has been proposed by Studer (1997) and refined by Breuer, Jandacka, Rheinberger, and Summer (2009) and Breuer and Csiszár (2013). Maximum Loss assumes that portfolio losses are driven by stochastic risk factors. The risk metric is then defined as the worst case loss subject to the condition that these risk factors lie inside a specified "trust region". In the case of an elliptical risk factor distribution, this trust region is

[1] A well-known bottom-up stress test is the EBA stress test (see http://www.eba.europa.eu/risk-analysis-and-data/eu-wide-stress-testing). The Bank of England's "Risk Assessment Model of System Institutions" (RAMSI) is an example of a top-down stress test (see Aikman, Alessandri, Eklund, Gai, Kapadia, Martin, Mora, Sterne, and Willison, 2009).

the closed subset of risk factor realizations that have Mahalanobis distance from the unconditional mean which is smaller than a given hurdle value. As a by-product, the Maximum Loss approach gives the corresponding realization of the risk factors, i.e. a risk factor scenario. Replacing the portfolio loss concept with the severity function used in this paper, it turns out that the two approaches do similar things: In a region that is bounded by a plausibility constraint, they both search for the most harmful scenario. Indeed, analytically, the two approaches are identical as may be seen, for example, by comparing the analytical results in Studer (1997, p. 43) and section 2.1 of this paper. The difference between the two approaches thus lies in the reference function: whereas Maximum Loss considers the value of a portfolio at a specified point in time, this paper uses the more general concept of a severity function. This severity function is usually user-specified rather than an (estimated) model of the determinants of a portfolio. Additionally, the severity function will typically depend on the realizations of risk factors across a number of periods rather than just one specified horizon as in the case of portfolio losses.

An alternative approach to obtaining adverse scenarios is conditional forecasting (see Waggoner and Zha (1999) and Camba-Mendez (2012) for the method, and Baumeister and Kilian (2014) for an application). The corresponding conditional predictive density is the distribution of the forecast path under the specified conditions. These conditions can refer to the observed variables and to unobserved (structural) shocks. Examples of questions that conditional forecasting can answer are:

- How would a monetary policy that holds the short-term interest rate constant at 0.0 percent for the next 8 quarters affect the economic system?
- How would an oil supply shock that reduces the global flow volume by 10 percent for a full quarter affect the global economy?

More generally, conditional forecasting is applicable if we have an idea of what could happen and we want to know the way in which this would affect the other variables in our empirical model. By contrast, the SFA assumes that we can quantify the severity of a scenario. It then searches in a plausibility-constrained set of candidates for the most severe scenario.

The paper is organized as follows: Section 2 introduces the SFA, while section 3 presents an application of the method. This empirical application also contrasts the SFA and conditional forecasting. Moreover, it considers a synthesis of both, where the SFA is applied to a conditional predictive density. This application suggests that the SFA is a very useful approach for finding a scenario that is severe yet plausible, especially when combined with conditional forecasting.

2 Methodology

First of all, it is important to clarify the term 'scenario': A scenario is a forecast path for the multivariate random variable y_t (dimension $k \times 1,\ k \geq 1$) covering the periods $T+1$ to $T+h$. Let $\widehat{y}_t$ denote the scenario forecast of y_t, and let $\widehat{\mathbf{Y}} := \begin{bmatrix} \widehat{y}'_{T+1} & \dots & \widehat{y}'_{T+h} \end{bmatrix}'$ collect the full scenario path. Analogously, $\mathbf{Y}_{T+h} := \begin{bmatrix} y'_{T+1} & \dots & y'_{T+h} \end{bmatrix}'$ collects the corresponding random variables.[2]

The basic inputs to the severity function approach are a predictive density function $f_Y(\mathbf{Y}_{T+h})$, and a scalar severity function $s(\mathbf{Y}_{T+h})$ that measures the extent to which a forecast path is adverse. The SFA scenario at severity level α is then defined as

$$\widehat{\mathbf{Y}}_\alpha \;:= \text{argmax}_{\widehat{\mathbf{Y}}}\; f_Y(\widehat{\mathbf{Y}}) \text{ s.t. } \Pr\left[s(\mathbf{Y}_{T+h}) < s(\widehat{\mathbf{Y}})\right] = \alpha \tag{1}$$

Thus, $\widehat{\mathbf{Y}}_\alpha$ is the scenario with maximal probability density (plausibility) such that the probability of observing more severe scenarios is α (severity). Put differently, among the $(1-\alpha) \times 100\%$ of the most severe scenarios, $\widehat{\mathbf{Y}}_\alpha$ is the scenario with maximal probability density.[3] Section 2.1 considers the special case of a linear severity function coupled with a multivariate normal predictive density. In this special case, the optimization problem of equation (1) can be solved analytically.

The first input to the optimization problem, the predictive density function $f_Y(\mathbf{Y}_{T+h})$, will typically come from a time series model or an estimated DSGE model. If the scenario is supposed to satisfy one or more hard conditions on the path of the risk factor, then it is possible to use a **conditional** forecast density (for details on conditional forecasting see, for example, Waggoner and Zha (1999) and Camba-Mendez (2012)). Such conditions can refer to observable variables or to unobserved structural shocks (provided the model is structurally identified). In the following application, I use a Bayesian VAR to produce both a standard (unconditional) predictive density and a conditional predictive density that satisfies a hard condition on the path of real GDP growth.

The second input to the optimization problem, the severity function $s(\mathbf{Y}_{T+h})$, measures the extent to which a scenario is adverse. Suppose, for example, we have four risk factors, an interest rate, real GDP growth, an unemployment rate, and a broad based stock market index, and we search for a scenario that spans 12 quarters. In this setup, the severity function will have 48 (i.e. 4×12) arguments that it condenses into a single scalar metric of severity. If the severity function specification is linear in the elements of the forecast path, then this amounts to 48 slope parameters. So, how do we choose these parameters? I can think of three approaches and I will elaborate on these in sections 2.2 and 2.3: (i) guesstimation, i.e. by guessing the parameters; (ii) empirical estimation of the functional relationship between the risk factors and a severity metric; (iii) simulation-based estimation of the functional relationship based on test runs of the stress test. Estimation of the severity function requires that we decide on a concept of severity and a corresponding proxy variable that is either empirically observable (case ii) or can be constructed from stress test output (case iii). This proxy variable will then be the left-hand side variable of a regression of the severity metric on the risk factors. Suppose, conceptually, we decided to look at the aggregate level of stress in the banking sector. We could then, for example, attempt to construct a proxy variable that measures how many banks are either close to

[2]Note that, below, depending on the context, y_t and Y_t will be used to denote either random variables or realizations.

[3]Whether equation (1) has a unique solution $\widehat{\mathbf{Y}}_\alpha$ depends on the predictive density f_Y and the severity function $s(\cdot)$. To give one example, suppose $\mathbf{Y} \sim \mathrm{N}(\mu, \Sigma)$ and $s(\mathbf{Y}) = (\mathbf{Y}-\mu)'\Sigma^{-1}(\mathbf{Y}-\mu)$, i.e. we use the Mahalanobis distance as the severity function. In this case, there is no unique maximum because the side condition of equation (1) determines a set of candidates, which all have the same probability density.

undercutting or actually fall short of their regulatory capital ratios. Alternatively, taking a narrower perspective on severity, we could focus on (a) specific component(s) of banks' income, for example, on credit impairments, or taking a broader perspective, we could consider the decline in aggregate credit supply to the real economy as a severity metric. Evidently, alternative severity concepts and proxy variables will produce different severity function specifications, and, in turn, will deliver different stress test scenarios. If we decide to guesstimate the severity function (case i), it still makes sense to think conceptually about the measurement of severity, as this facilitates coming up with a sensible severity function specification.

2.1 Special case: linear severity function, multivariate normal predictive density

Below I consider the special case of a linear severity function coupled with a multivariate normal predictive density, for which the maximization problem in equation (1) can be solved analytically. Specifically, suppose

1. the predictive density function $f_Y(\mathbf{Y}_{T+h})$ is multivariate normal with $\mathrm{E}[\mathbf{Y}_{T+h}] = \mu$ and $\mathrm{V}[\mathbf{Y}_{T+h}] = \Sigma$,
2. and the severity function is linear with $s(\mathbf{Y}_{T+h}) = \mathbf{Y}'_{T+h}\beta$,

then appendix A.1 shows

$$(\widehat{\mathbf{Y}}_\alpha - \mu) = \frac{\Phi^{-1}(\alpha)}{(\beta'\Sigma\beta)^{0.5}}\Sigma\beta. \tag{2}$$

Interpretation: $(\widehat{\mathbf{Y}}_\alpha - \mu)$, i.e. the extent by which the scenario deviates from the mean μ of the predictive density, depends

1. on the severity level α via the term $\Phi^{-1}(\alpha)$, i.e. the greater α is, the higher this deviation will be; and
2. on both the variance of the predictive density (Σ) and the slope of the severity function (β) via the product $\Sigma\beta$:
   - Consider β first and assume $\Sigma = I$ for the ease of exposition. We then have $(\widehat{\mathbf{Y}}_\alpha - \mu) = \theta \times \beta$, where $\theta := \frac{\Phi^{-1}(\alpha)}{(\beta'\Sigma\beta)^{0.5}}$ is a positive scalar. Thus, the sign and size of each element of β determines directly by how much and in which direction the corresponding element of $\widehat{\mathbf{Y}}_\alpha$ deviates from its mean μ.
   - In the case of a non-diagonal Σ, the deviations from μ additionally depend on covariances. For example, suppose $\mathbf{Y}_{T+h}$ has two elements with covariance matrix

     $$\Sigma = \begin{bmatrix} 1 & -.5 \\ -.5 & 1 \end{bmatrix}$$

     and suppose the slope of the severity function is $\beta = \begin{bmatrix} 1 & .1 \end{bmatrix}'$, i.e. the two elements of $\mathbf{Y}_{T+h}$ correlate negatively and the severity function increases in both elements. Imputing for Σ and β and expanding, we obtain $(\widehat{\mathbf{Y}}_\alpha - \mu) =$

$\theta \times \begin{bmatrix} .95 & -.4 \end{bmatrix}'$, meaning that the second element of $\widehat{\mathbf{Y}}_\alpha$ deviates **negatively** from its mean. This is surprising, as the previous bullet point suggested that - for a diagonal Σ - the deviation from the mean should be positive for both elements of $\mathbf{Y}_{T+h}$ due to the positive sign of both elements of β. The reason for this surprising result is twofold: First, the negative correlation among the two elements of $\mathbf{Y}_{T+h}$ means that there is as greater likelihood of observing both elements deviating from their mean in opposite directions than of observing them deviating in the same direction. Second, the severity function favors positive deviations from the mean much more heavily for the first element than for the second element of $\mathbf{Y}_{T+h}$. In terms of the SFA, it is therefore optimal to let the first element, which is more important for severity, deviate strongly from its mean in the direction of higher severity and to add plausibility (to the strong deviation of the first element) by letting the less important second element deviate somewhat in the direction of lower severity.

Finally, note that the scalar term $(\beta'\Sigma\beta)^{0.5}$ is a mere normalizing constant.

2.2 Severity function guesstimation

The idea of the guesstimation approach is that the scenario user specifies the severity function based on her preferences about the scenario. Section 2.2.1 starts by introducing a particularly parsimonious parameterization of the linear severity function. Thereafter, in sections 2.2.2 and 2.2.3, it gives advice on choosing the elements of this parameterization in a way that is compatible with the scenario user's preferences.

2.2.1 A parsimonious parameterization of the severity function

In the case of a linear severity function $s(\mathbf{Y}_{T+h}) = \mathbf{Y}'_{T+h}\beta$, specifying the severity function amounts to choosing β. Instead of specifying each element of β separately, i.e. a total of $k \times h$ free parameters, this section uses the following, more parsimonious parametrization with only $k + h$ free parameters

$$\beta = \underbrace{\begin{bmatrix} w_1 & \dots & w_h \end{bmatrix}'}_{h=\text{forecast-horizon}} \otimes \underbrace{\begin{bmatrix} b_1 & \dots & b_k \end{bmatrix}'}_{k=\#\text{elements}(y_t)}. \quad (3)$$

By imputing this parameterization into the linear severity function, we obtain $s(\mathbf{Y}_{T+h}) = \sum_{i=1}^{h} \sum_{j=1}^{k} w_i \cdot b_j \cdot y_{T+i}(j)$, where $y_{T+i}(j)$ is the jth element of y_{T+i}. In this parameterization, $w_1, \dots, w_h$ govern how much weight the severity function gives to each forecast horizon, and $b_1, \dots, b_k$ govern the behavior of the severity function across the k scenario variables. *Example:* We can set $w_1 = 1, w_2 = 0.01, w_3 = \dots = w_h = 0$ to express that only forecast horizon 1 and, to a lesser extent, also forecast horizon 2 matter for the severity function. And we can set $b_1 = 1, b_2 = -0.25, b_3 = \dots = b_k = 0$ to express that the severity function increases in the value of the first variable in y_t, decreases with a smaller slope in the second variable, and does not depend on the other variables. Imputing these values into the severity function gives $s(\mathbf{Y}_{T+h}) = y_{T+1}(1) - 0.25y_{T+1}(2) + 0.01y_{T+2}(1) - 0.0025y_{T+2}(2)$.

2.2.2 Choice of $b_1, \ldots, b_k$

The table below shows how certain "scenario preferences" can be modeled using the proposed parametrization in equation (3). If the scenario user wants to combine several

scenario user wishes to express that scenario severity increases	choice of b
...as variable j obtains higher (lower) values	$b_j = 1 \ (= -1)$
...as variable j rises relative to variable j^*	$b_j = -b_{j^*} = 1$
...as variable j and variable j^* rise (fall) jointly	$b_j = b_{j^*} = 1 \ (= -1)$

severity function properties as specified in the table above, she can aggregate these properties in the following way: $b = \sum_{l=1}^{L} v_l \cdot b^l$, where L is the number of properties to be combined, v_l is the relative weight given to property l, and b^l is a $k \times 1$ vector that represents the lth property.

Let me clarify the approach with an example. Suppose the scenario variables are $y_t = \begin{bmatrix} i3m & i10y & rgdp \end{bmatrix}$, i.e. a short-run interest rate, a long-run interest rate and the level of real GDP. Suppose that according to the scenario users evaluation, the following three properties characterize a severe scenario: (1) a flattening or inversion of the yield curve, (2) a rise in the overall level of interest rates, and (3) low real GDP growth. Let us first specify a severity function for each of these three properties separately:

1. $\begin{bmatrix} b^1_{i3m} & b^1_{i10y} & b^1_{rgdp} \end{bmatrix}' = \begin{bmatrix} 1 & -1 & 0 \end{bmatrix}'$ provokes inversion of the yield curve
2. $\begin{bmatrix} b^2_{i3m} & b^2_{i10y} & b^2_{rgdp} \end{bmatrix}' = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}'$ provokes a high interest rate level
3. $\begin{bmatrix} b^3_{i3m} & b^3_{i10y} & b^3_{rgdp} \end{bmatrix}' = \begin{bmatrix} 0 & 0 & -1 \end{bmatrix}'$ provokes low real GDP growth

Additionally, assume that we choose the relative weights of the three properties as $v_1 = 1,\ v_2 = .5,\ v_3 = .5$ to reflect the fact that a flattening of the yield curve is of first-order importance, whereas a high overall level of interest rates and low real GDP growth are of second-order importance. By aggregation, we obtain

$$\begin{bmatrix} b_{i3m} & b_{i10y} & b_{rgdp} \end{bmatrix}' = \begin{bmatrix} 1.5 & -.5 & -.5 \end{bmatrix}'. \tag{4}$$

2.2.3 Choice of $w_1, \ldots, w_h$

In choosing the horizon weights $w_1, \ldots, w_h$, it should be kept in mind that the variance of the predictive density tends to rise with the forecast horizon, i.e. elements of Σ that refer to more distant forecast horizons will typically be greater.[4] Thus, equation (2), which can be restated as $(\widehat{\mathbf{Y}}_\alpha - \mu) = \varrho \Sigma \beta$ (where ϱ is a scalar), implies that the extent to which the scenario path $\widehat{\mathbf{Y}}_\alpha$ deviates from the mean of the predictive density μ will tend to increase with the forecast horizon.

One way of counteracting this tendency is to let the elements of β decay at a sufficiently rapid pace. Suppose the scenario user has a preference for a scenario in which "shocks"

[4] There are two reasons for this: unknown shocks that accumulate and the impact of estimation uncertainty.

(i.e. deviations from the mean μ of the predictive density) occur relatively early. In this case, the elements of β have to decline at a pace that is somewhat faster than the pace at which the variance of the predictive density rises. A parsimonious parametrization is $w_{\tilde{h}} = \tilde{h}^{-\kappa}$, where $\kappa > 0$ governs how quickly the elements of the slope vector β (see equation 3) decrease with the forecast horizon. If the preference for early shocks is very strong, a possible refinement is $w_{\tilde{h}} = \tilde{h}^{-\kappa} \times 1_{(\tilde{h} \leq h^*)}$, where $1_{(\tilde{h} \leq h^*)} = 1$ if $\tilde{h} \leq h^*$ and $1_{(\tilde{h} \leq h^*)} = 0$ otherwise, i.e. anything that happens after period $T + h^*$ does not matter for the severity function.

2.3 Severity function estimation

As outlined in section 2, estimation can either be based on empirical data or on test runs of the stress test, i.e. simulation-based estimation. Suppose we consider a linear specification of the severity function, i.e. $s(\mathbf{Y}_{T+h}) = \mathbf{Y}'_{T+h}\beta$, where β could either be left unrestricted or be replaced with the parsimonious parameterization of section 2.2.1. The empirical estimation approach would then draw on time series data on the risk factors collected in $\mathbf{Y}_{T+h}$ and on an observed severity metric $s^o(\mathbf{Y}_{T+h})$ (see section 2 for suggestions) to estimate the functional relationship between the risk factors and the severity metric, i.e. the elements of β. The simulation-based estimation approach would instead rely on a sufficiently large number of simulated scenarios, for each of which a severity metric would be obtained from test runs of the stress test.

3 Empirical Application

Below I use the severity function approach to find a stress test scenario for the German banking sector. This scenario will have a 16-quarter horizon and refers to the following risk factors: a 3-month interbank rate (*i3m*), the current yields on German government bonds with two and ten years to maturity (*i2y*, *i10y*), the gross domestic product in real terms (*rgdp*), an unemployment rate (*unemp*), a consumer price index (*cpi*), and a broad-based stock index (*cdax*). Table 1 collects more details on the data, how they are used in the empirical model, and on the estimation sample. Briefly, the next steps are

1. Present the econometric method used to obtain a predictive density for $\mathbf{Y}_{2016:Q1+16} = \begin{bmatrix} y_{2016:Q2} & \dots & y_{2020:Q1} \end{bmatrix}'$ - a Bayesian VAR (Vector Auto-Regression);

2. Turn to the structural identification of the model. The paper will use the structural VAR (SVAR) representation to back out a path of structural (i.e. economic) shocks for any scenario. This step is **not** necessary in an application of the severity function approach. The upside of having a structurally identified model is that we can use the corresponding structural shock series to construct a scenario narrative;

3. Specify the severity function;

4. Present the SFA-based scenarios and contrast them with an alternative scenario generated using conditional forecasting methodology;

Table 1: Data, Transformations and Estimation Sample

Variable (x_t)	Seasonally and calendar adjusted?	Transformation used in VAR (y_t)	Original data frequency[1]	Description
i3m	no	x_t	monthly	3-month EURIBOR
i2y	no	x_t	monthly	Estimated current yield on German government bonds with two years to maturity
i10y	no	x_t	monthly	Current yield on German government bonds with 9–10 years to maturity
rgdp	yes	$\ln(x_t)$	quarterly	German real GDP (chain-linked index)
unemp	yes	x_t	monthly	German unemployment rate (section 16 Social Security Code III)
cpi	yes	$\ln(x_t)$	monthly	Consumer price index
cdax	no	$\ln(x_t)$	monthly	CDAX price index

Additional information: The estimation sample extends from 1992:Q1 to 2016:Q1. The data have been downloaded from http://www.bundesbank.de/SiteGlobals/Forms/Suche_Statistik/EN/Statistiksuche_Text_Formular.html using the following mnemonics: BBK01.SU0316; BBK01.WZ9810; BBK01.WX3950; BBNZ1.Q.DE.Y.H.0000.A; BBDL1.M.DE.Y.UNE.UBA000.A0000.A01.D00.0.R00.A; BBDP1.M.DE.Y.VPI.C.A00000.I10.L; BBK01.WU001A

[1] Quarterly series were obtained from monthly data by averaging.

Let me first turn to the **method employed to obtain a predictive density** for the scenario path. Given its great success in forecasting applications, I opt for a VAR estimated using Bayesian methods with a natural conjugate variant of the well-known Litterman Prior.[5] The estimated VAR regression equation reads

$$y_t = c + \sum_{i=1}^{4} B_i y_{t-i} + \varepsilon_t, \tag{5}$$

where $y_t = \begin{bmatrix} i3m_t & i2y_t & i10y_t & \ln(rgdp_t) & unemp_t & \ln(cpi_t) & \ln(cdax_t) \end{bmatrix}'$ and $\varepsilon_t \sim \mathrm{N}\,(\,0\,,\,\Sigma\,)$. The Bayesian approach has the additional advantage (relative to frequentist approaches) of providing predictive densities that incorporate parameter estimation uncertainty. Details can be found in Appendix A.2.

In the next step, **structural identification**, I recover the SVAR representation of the VAR from its reduced-form (5). The structural representation reads

$$A_0 y_t = a + \sum_{i=1}^{4} A_i y_{t-i} + u_t, \tag{6}$$

where $a = A_0 \times c$, $A_i = A_0 \times B_i$, $i = 1, \ldots, 4$ and $u_t = A_0 \times \varepsilon_t$ with $\mathrm{V}\,[u_t] = I_k$. Unlike the reduced-form representation (5), the structural representation has a matrix A_0 which spells out the contemporaneous relationships between the variables in y_t and a vector u_t of orthogonal "structural" (economic) shocks. Knowledge of A_0 suffices to map between the two representations. This implies that, given the residuals from the reduced-form regression ε_t, we can recover the structural shocks as $u_t = A_0 \times \varepsilon_t$ and use them to construct a scenario narrative. To obtain the SVAR representation, I use an approach based on sign restrictions on the structural impulse response functions.[6] Specifically, I use the restrictions outlined in table 3, to identify shocks to aggregate supply (AS), aggregate demand (AD) and to monetary policy (MP). Impulse response functions and decompositions of the forecast error variance can be found in Appendix A.3. Note that the SFA does not require structural identification, but that the structurally identified model provides some interesting options: First, as outlined above, its structural shocks can provide the basis for a scenario narrative; see section 3 for an example. Second, it facilitates generating scenarios that condition on certain structural shocks. We could, for example, consider a conditional predictive density that assumes a demand shock of the same magnitude as the biggest shock observed over the last 20 years. Third, we could use the structural representation to specify a severity function that loads on structural shocks. In this way we could come up with a more economically motivated SFA scenario.

The **severity function specification** closely resembles the one presented in section 2.1, i.e. severity falls in the term spread, increases with the interest rate level and decreases with real GDP. Similar to equation (4), I choose

$$\begin{bmatrix} b_{i3m} & b_{i2y} & b_{i10y} & b_{rgdp} & b_{unemp} & b_{cpi} & b_{cdax} \end{bmatrix}' = \begin{bmatrix} 1.5 & 0 & -.5 & -.5 & 0 & 0 & 0 \end{bmatrix}'. \tag{7}$$

[5]See Doan, Litterman, and Sims (1984) for the original approach and Banbura, Giannone, and Reichlin (2010) for the variant used in this paper

[6]For details, see Canova and De Nicolo (2002) and Uhlig (2005)

The severity function specification is completed by defining $w_{\tilde{h}} = \tilde{h}^{-2}$ i.e. the slope elements decay at a quadratic rate.

Table 2 presents the baseline forecast and **three alternative scenarios**, which are also depicted in figure 1. The annual forecasts found in the table have been constructed by averaging the corresponding quarterly level forecasts. The table considers the following scenarios / forecasts:

1. *Baseline Forecast* is the mean of the unconditional predictive density. This forecast path implies a moderate continued decline in the short-term interest rate (2019-forecast : $-.55$), which is expected to take place at a relative constant term spread of roughly 60 basis points, along with subdued real GDP growth, a continued but mild trend towards lower unemployment (2019-forecast : 5.83%), CPI inflation below the ECB target, and a stable level of the broad-based stock price index.

2. *Conditional Forecast* is the mean of the conditional predictive distribution under the assumption that real GDP eight quarters ahead, i.e. in 2018:Q1, will be six percent below its level in the baseline forecast.[7] It is thus a deep recession scenario by assumption.[8]

   In this scenario, the German economy experiences a deep recession in 2017 and 2018, followed by a partial recovery in 2019. During the recession, the short-term interest rate dives deep into negative territory, whereas the term spread expands continuously (2018 forecast : 2.00%), inflation is close to zero, and the stock price index initally dips by more than 20% but recovers to its initial level towards the end of the scenario path.

   Is this an adverse scenario for the banking sector? On the one hand, subdued economic activity raises credit risks (as default rates tend to be counter cyclical). On the other hand, the steep yield curve can be expected to boost net interest income, which for most banks is the single most important component of income.[9] These two counteracting channels suggest that the scenario only implies a moderate stress level for banks.

3. *SFA-Scenario I* uses the severity function specification outlined above in conjunction with the **unconditional** predictive density to find the corresponding scenario at the $\alpha = .99$ severity level.

   In this scenario, the yield curve inverts, which is unsurprising given the severity function specification (see equation 7). At the same time, the path for real activity (*rgdp, unemp*) is somewhat more expansive than in the baseline, and inflation is a bit higher.

[7]Note that no assumptions are made on the level of real GDP at horizons from one to seven quarters or beyond the eight-quarter horizon.

[8]Conditional forecasting considers the conditional distribution of a forecast path subject either to restrictions on individual elements of the forecast path or, provided that the underlying forecast model is structurally identified, on implicit shocks. Details on the approach used in this paper can be found in Camba-Mendez (2012).

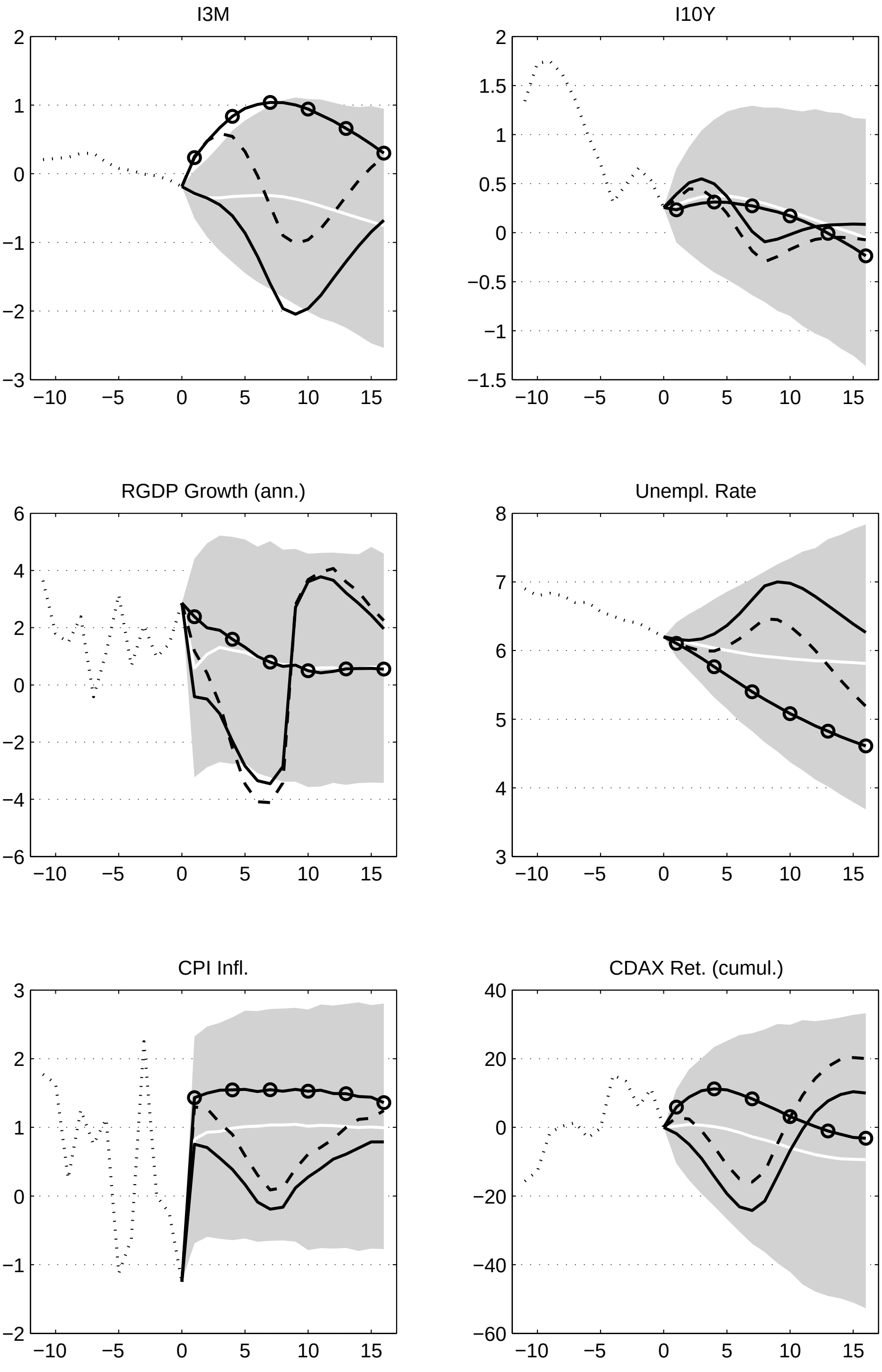
[9]To understand why, note that most banks supply long-term loans, which are financed by short-term debt, i.e. they engage in maturity transformation. Naturally, the income from maturity transformation rises with long-term interest rates and falls with short-term interest rates.

Table 2: Scenarios

Year	*I3M*	*I10Y*	*RGDP*	*UNEMP*	*CPI*	*CDAX*
2015	-0.02	0.50	107.75	6.40	106.88	520.28
			Baseline Forecast (Levels)			
2016	-0.30	0.30	109.26	6.15	106.97	469.96
2017	-0.33	0.36	110.40	6.01	107.99	476.10
2018	-0.38	0.24	111.23	5.91	109.07	469.80
2019	-0.55	0.07	111.94	5.83	110.17	466.84
			Conditional Forecast ($RGDP_{(T+8)} = .94 \times \mathrm{E}\left[RGDP_{(T)}\right]$)			
2016	-0.34 (−0.03)	0.40 (+0.09)	108.66 (−0.54%)	6.19 (+0.04)	106.89 (−0.08%)	449.64 (−4.32%)
2017	-1.10 (−0.77)	0.18 (−0.17)	106.52 (−3.52%)	6.52 (+0.51)	107.17 (−0.76%)	398.17 (−16.37%)
2018	-1.90 (−1.52)	-0.10 (−0.34)	105.69 (−4.97%)	7.02 (+1.11)	107.08 (−1.82%)	460.32 (−2.02%)
2019	-1.09 (−0.54)	0.03 (−0.04)	109.29 (−2.37%)	6.62 (+0.79)	107.52 (−2.40%)	566.71 (+21.39%)
			SFA-Scenario I (based on unconditional forecast density)			
2016	0.27 (+0.57)	0.27 (−0.04)	109.75 (+0.45%)	6.07 (−0.08)	107.18 (+0.19%)	492.65 (+4.83%)
2017	0.90 (+1.22)	0.30 (−0.05)	111.33 (+0.84%)	5.65 (−0.37)	108.73 (+0.69%)	510.39 (+7.20%)
2018	0.90 (+1.29)	0.21 (−0.03)	112.14 (+0.82%)	5.21 (−0.70)	110.32 (+1.14%)	486.82 (+3.62%)
2019	0.60 (+1.15)	0.01 (−0.07)	112.74 (+0.71%)	4.87 (−0.96)	111.90 (+1.57%)	464.40 (−0.52%)
			SFA-Scenario (based on conditional forecast density)			
2016	0.27 (+0.57)	0.38 (+0.08)	109.11 (−0.13%)	6.11 (−0.04)	107.10 (+0.12%)	470.60 (+0.14%)
2017	0.04 (+0.37)	0.09 (−0.26)	106.93 (−3.14%)	6.23 (+0.21)	107.85 (−0.13%)	419.28 (−11.93%)
2018	-0.91 (−0.52)	-0.18 (−0.42)	105.81 (−4.87%)	6.50 (+0.59)	108.06 (−0.93%)	473.28 (+0.74%)
2019	-0.11 (+0.44)	-0.01 (−0.08)	109.78 (−1.93%)	5.80 (−0.03)	108.83 (−1.21%)	574.63 (+23.09%)

Notes: Baseline forecast = mean of unconditional predictive density; Value in parentheses: % or PP (percentage point) deviation of scenario from the Baseline Forecast

Figure 1: Scenarios



Legend: [grey box] = area spanned by the 10% and 90% quantile of the unconditional predictive density; [white line] = unconditional forecast; [black line] = conditional forecast; [circle] = SFA-scenario I; [dashed line] = SFA-scenario II

Note: forecast horizon is plotted on the horizontal axis; forecast horizon 0 is the last data point used for forecasting, i.e. 2016:Q1.

From a banking-sector perspective, the yield curve inversion puts serious pressure on net interest income, especially since the term spreads stays low for a relatively long time. Nonetheless, the scenario is not very adverse because the path for real activity is somewhat too expansive, thus putting little pressure on credit risk.

This deficiency could either be cured by specifying a severity function that puts more weight on a decline in real GDP or, alternatively, by using a different predictive density, which puts more weight on dampened real activity. *SFA-Scenario II* demonstrates the second option.

4. *SFA-Scenario II* uses the **conditional** predictive density from the second scenario (*Conditional Forecast*), which imposes that real GDP declines by six percent over the following eight quarters, in conjunction with the same severity function and severity level used in the third scenario (*SFA-Scenario I*).

   As a banking sector stress test scenario, *SFA-Scenario II* is the best choice among the three candidates: It combines a relatively flat term structure with a (temporary) serious recession. At the same time, the short-term interest rate does not dive too deeply into negative territory, thus making the scenario economically more plausible than SFA-Scenario I.

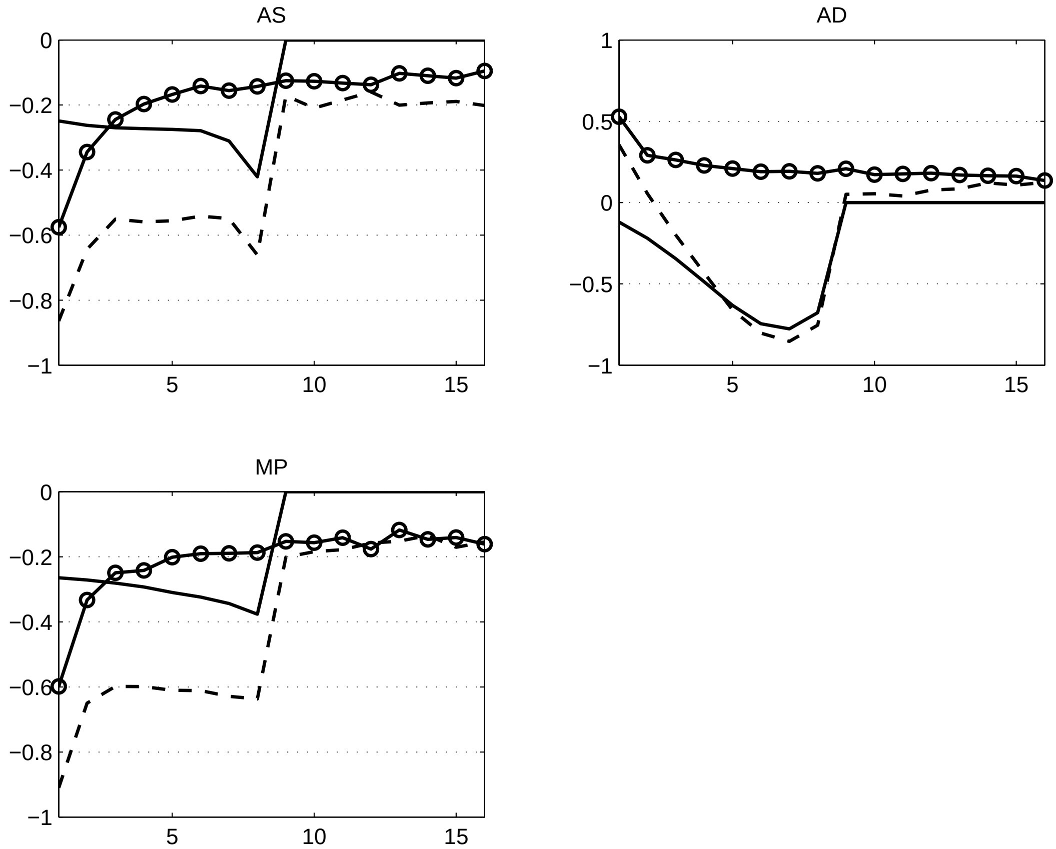
Figure 2 presents series of the identified structural shocks for the three scenarios *Conditional Forecast, SFA-Scenario I and SFA-Scenario II.* The shock series for *Conditional Forecast* and *SFA-Scenario II* both tell similar stories: A series of adverse supply shocks (e.g. an unexpected rise in commodity prices), combined with adverse demand shocks (e.g. low export demand from other euro area countries, EMEs or the USA) and a monetary policy that turns out to be more restrictive than expected (for example, as it runs out of instruments to further loosen the monetary policy stance). SFA-Scenario I, which has a more expansive path for real activity, differs most markedly with respect to the series of the aggregate demand shock, which turns out to be mildly expansive at all horizons.[10]

4 Concluding remarks

This paper presents a method - the Severity Function Approach (SFA) - of finding suitable adverse scenarios for stress testing applications. The SFA searches for the scenario with maximal plausibility subject to a constraint on the severity of the scenario. This constraint is expressed in terms of the probability of more severe scenarios. The paper shows that the optimization problem has an analytical solution if the severity function is linear and the predictive density is multivariate normal, and it gives advice on alternative approaches to coming up with a suitable severity function. In turn, the paper applies the SFA in order to find a scenario for macro-prudential stress testing of the German banking sector. This application shows that the SFA is a valuable alternative and/or complement to conditional forecasting, which is a well-known and widespread approach in scenario analysis.

[10]Note that I have only identified three shock in my seven-variable SVAR. This implies that the shock series presented here do not "tell the full story", i.e. the remaining four unidentified shocks also matter for the scenario, i.e. they are non-zero. As we cannot attach any economic meaning to these shocks, they are not presented here.

Figure 2: Corresponding structural shocks



Legend: [—] = conditional forecast; [○] = SFA-scenario I; [– –] = SFA-scenario II

Note: The forecast horizon is plotted on the horizontal axis; The first depicted point refers to the one-quarter horizon, i.e. to 2016:Q2; To obtain the depicted series, I draw repeatedly from the posterior distribution of the parameters of the **structural** VAR (see Arias et al., 2014). For each single draw, I recover the series of structural shocks that would have led to the respective scenario path of the observables. The depicted structural shocks are obtained by averaging across these draws.

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A Appendix

A.1 SFA scenario in the special case of section 2.1

First, note that the general optimization problem of equation (1) can be restated in terms of the natural logarithm of the predictive density:

$$\widehat{\mathbf{Y}}_\alpha = \text{argmax}_{\widehat{\mathbf{Y}}} \ \ln f_Y(\widehat{\mathbf{Y}}) \text{ s.t. } \Pr\left[s(\mathbf{Y}_{t+h}) > s(\widehat{\mathbf{Y}})\right] = \alpha$$

Next, set up the corresponding Lagrangian:

$$L(\widehat{\mathbf{Y}}, \lambda) = -\frac{1}{2}\left[k \cdot h \cdot \ln 2\pi + \ln|\Sigma| + \left(\widehat{\mathbf{Y}} - \mu\right)' \Sigma^{-1} \left(\widehat{\mathbf{Y}} - \mu\right)\right] - \lambda \left(\alpha - 1 + \Phi\left[\frac{a'\left(\widehat{\mathbf{Y}} - \mu\right)}{\sqrt{a'\Sigma a}}\right]\right),$$

where I have (i) imputed the functional form of the multivariate normal predictive density (remember: $\mathbf{Y}_{t+h} \sim \mathrm{N}\,(\,\mu\,,\,\sigma\,))$ and (ii) made use of the fact that $s(\mathbf{Y}_{t+h}) = a'\mathbf{Y}_{t+h} \sim \mathrm{N}\,(\,a'\mu\,,\,a'\Sigma a\,)$. The solution for $\widehat{\mathbf{Y}}_\alpha$ in equation (2) can now be obtained by the standard procedure, i.e. setting up first-order conditions, solving for the Lagrange multiplier λ, imputing this solution and finally solving for $\widehat{\mathbf{Y}}$.

A.2 Bayesian estimation of the reduced-form VAR & its predictive density

Bayesian estimation of the reduced-form VAR loosely follows Banbura et al. (2010) and Giannone, Lenza, and Primiceri (2015), who present a natural-conjugate variant of the well-known Litterman prior (see Doan et al., 1984).

To facilitate presentation of the prior, consider the following **matrix representation** of the reduced-form **VAR** in equation (5):

$$Y = XB + E,$$

where $Y = \begin{bmatrix} y_1 & \ldots & y_T \end{bmatrix}'$, $B = \begin{bmatrix} c & B_1 & \ldots & B_p \end{bmatrix}'$, $X = \begin{bmatrix} x_1 & \ldots & x_T \end{bmatrix}'$, $x_t = \begin{bmatrix} 1 & y'_{t-1} & \cdots & y'_{t-p} \end{bmatrix}'$, $E = \begin{bmatrix} \varepsilon_1 & \ldots & \varepsilon_T \end{bmatrix}'$ and $\varepsilon_t \sim \mathrm{N}\,(\,0\,,\,\Sigma\,)$.
The **prior** belongs to the natural conjugate normal-inverse Wishart family with

$$\Sigma \sim IW(\underline{\Sigma}, k+2), \tag{A.1}$$
$$vec(B)|\Sigma \sim \mathrm{N}\,(\,vec(\underline{B})\,,\,\Sigma \otimes \underline{\Omega}\,)\,, \tag{A.2}$$

implying the **posterior** distribution

$$\Sigma|Y, X \sim IW(\underline{\Sigma} + \hat{E}'\hat{E} + (\hat{B} - \underline{B})'\Omega^{-1}(\hat{B} - \underline{B}), T + k + 2), \tag{A.3}$$
$$vec(B)|\Sigma, Y, X \sim \mathrm{N}\left(vec(\hat{B})\,,\,\Sigma \otimes \left(X'X + \underline{\Omega}^{-1}\right)^{-1}\right), \tag{A.4}$$

where $\hat{B} = (X'X + \Omega^{-1})^{-1}(X'Y + \Omega^{-1}\underline{B})$ and $\hat{E} = Y - X\hat{B}$.
Below, I outline the **parameterization of the prior**, i.e. of the elements of $\underline{\Sigma}$, $\underline{B}$ and $\underline{\Omega}$:

- $\underline{\Sigma} = diag(\underline{\sigma}_1^2 \ \dots \ \underline{\sigma}_k^2)$, where $\underline{\sigma}_i^2$ is the residual variance of a univariate $AR(p)$ autoregression with a constant for the ith element of y_t[11]
- $\underline{B} = \begin{bmatrix} 0_{(1\times k)} & I_k & 0_{(k\times k)} & \cdots & 0_{(k\times k)} \end{bmatrix}'$
- $\underline{\Omega} = diag\left(\nu^{-2}, \ \lambda^2 \begin{bmatrix}1, & 1/2^2, & \dots, & 1/p^2\end{bmatrix} \otimes \begin{bmatrix}1/\underline{\sigma}_1^2, & \dots, & 1/\underline{\sigma}_k^2\end{bmatrix}\right)$, with $\nu \to 0$[12]

This prior is diffuse about the vector of intercepts c (as $\kappa \to 0$), but informative about the matrices of slope parameters $\{B_1, \dots, B_p\}$. The prior mean of the slope $\underline{B}$ expresses the belief that each variable is generated from a univariate random walk process. In the specification of the prior variances in equation (A.2), the hyperparameter λ governs the overall tightness of the prior for $B_1, \dots, B_p$: If $\lambda = 0$, the prior expresses that we are absolutely certain that the true data-generating process is a set of univariate random walks. If, by contrast, $\lambda \to \infty$, the prior becomes diffuse. In my application, I choose $\lambda = .1$, which is somewhat more informative than suggested by Carriero, Clark, and Marcellino (2015), who find an optimal value of λ close to .2. My choice is motivated by two factors: the relatively short estimation sample and the relatively large number of parameters induced by a model with $p = 4$ lags. The series $\begin{bmatrix}1, & 1/2^2, & \dots, & 1/p^2\end{bmatrix}$ in the parameterization of $\underline{\Omega}$ implies that the prior gets tighter, the greater the lag we consider. It thus reflects the belief that more distant lags play a minor role. Finally, in $\Sigma \otimes \underline{\Omega}$, the terms Σ and $\begin{bmatrix}1/\underline{\sigma}_1^2, & \dots, & 1/\underline{\sigma}_k^2\end{bmatrix}$ (see parameterization of $\underline{\Omega}$) result in a prior that accommodates differences in the scale and variability of the different variables.

It is well known that for horizons greater than one period the **predictive density** of the Bayesian VAR described above is not available in closed form. Simulated draws can, however, be obtained by drawing a sequence of Σ and B (from equations A.3-A.4) and shocks (remembering $\varepsilon_t \sim \mathrm{N}(0, \Sigma)$) and then assembling the implied draw of $\mathbf{Y}_{T+h}$ (see Carriero et al., 2015). In my application, for each draw of a total of 1,000 draws from the posterior of Σ and B, I draw ten paths of shocks (i.e. of $\varepsilon_{T+1}, \dots, \varepsilon_{T+h}$) and thus arrive at a total of 10,000 draws from the predictive density. To be able to apply the results of Appendix A.1 for the SFA, I assume that the predictive density comes from a multivariate normal distribution with mean and variance given by the respective statistics of the simulated draws. Note that the true predictive density is non-Gaussian. The approximation of the true predictive density could potentially be improved by mixture-type distributions. Focusing on univariate forecast distributions, Krüger, Lerch, Thorarinsdottir, and Gneiting (2016) show that a mixture-type approximation to a Bayesian forecast distribution outperforms the Gaussian approximation on theoretical and empirical grounds. Empirical evidence by Warne, Coenen, and Christoffel (2017) suggests that similar results may apply in the multivariate case; however, the difference between the mixture versus Gaussian approximations seems rather small in their case.[13]

[11] $diag(x)$ generates a diagonal matrix with the vector x on its main diagonal (and zeros everywhere else)

[12] In my application, I use $\nu = .01$

[13] I wish to thank a referee for making this point.

A.3 Structural identification

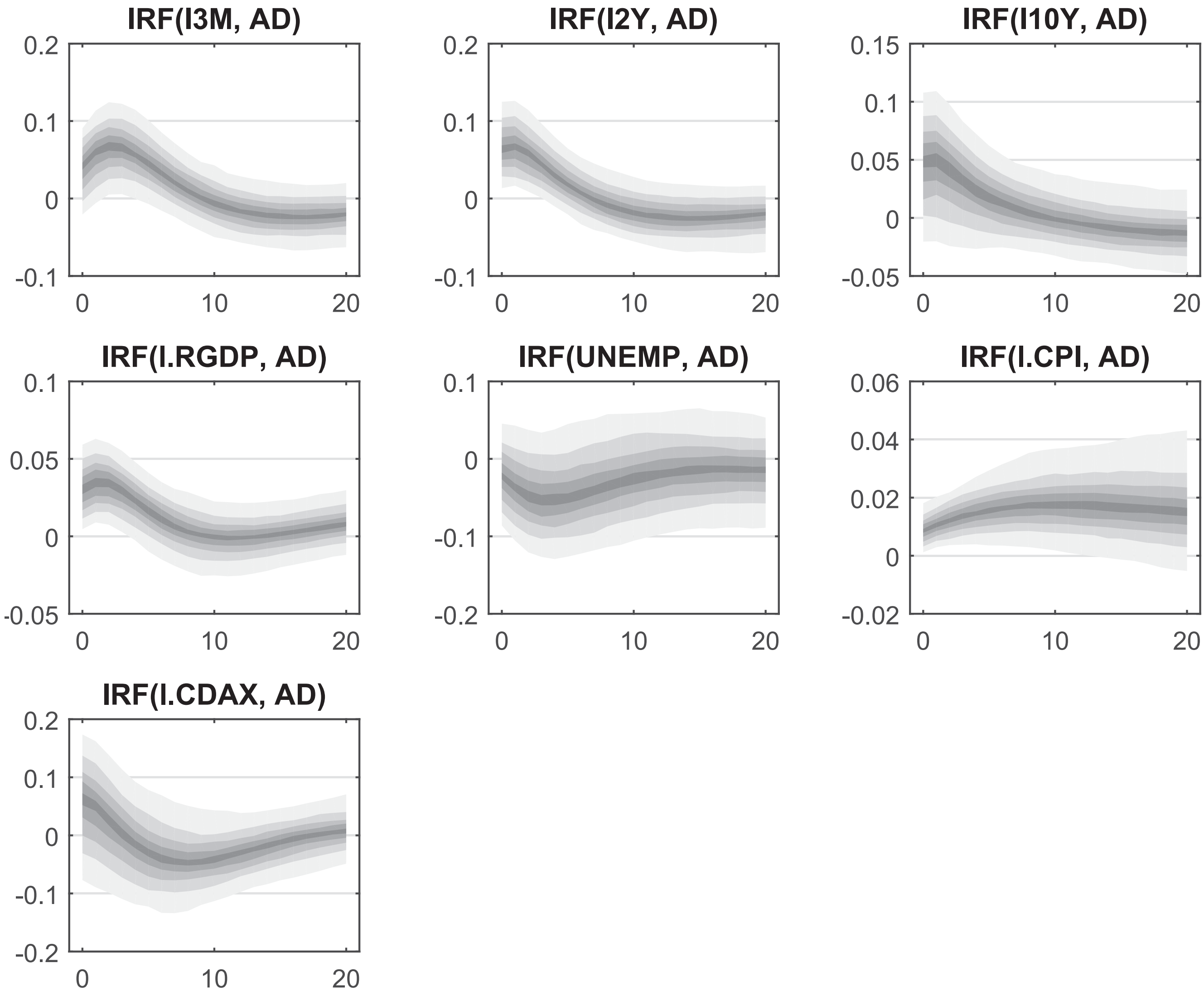
The structural identification of the VAR (i.e. of the parameters of equation 6) uses the algorithm outlined in Arias et al. (2014) with the sign restrictions specified in table 3. Figures 3, 4, and 5 show the impulse responses to an aggregate demand, an aggregate supply and a monetary policy shock, respectively. Figure 6 shows the aggregated forecast error variance decompositions for the three shocks, which - as measured by its median - ranges for most variables between 35% and 50%. This number indicates that beyond the three identified shocks, there are other major drivers of unexpected variations in the seven endogenous variables. This fact should be kept in mind when predicted shock series are used as a basis for a scenario narrative.

Table 3: Sign Restrictions

	i3m	i2y	i10y	ln(rgdp)	unemp	ln(cpi)	ln(cdax)
AS	-	-		+		-	
AD		+		+		+	
MP	-	-		+		+	

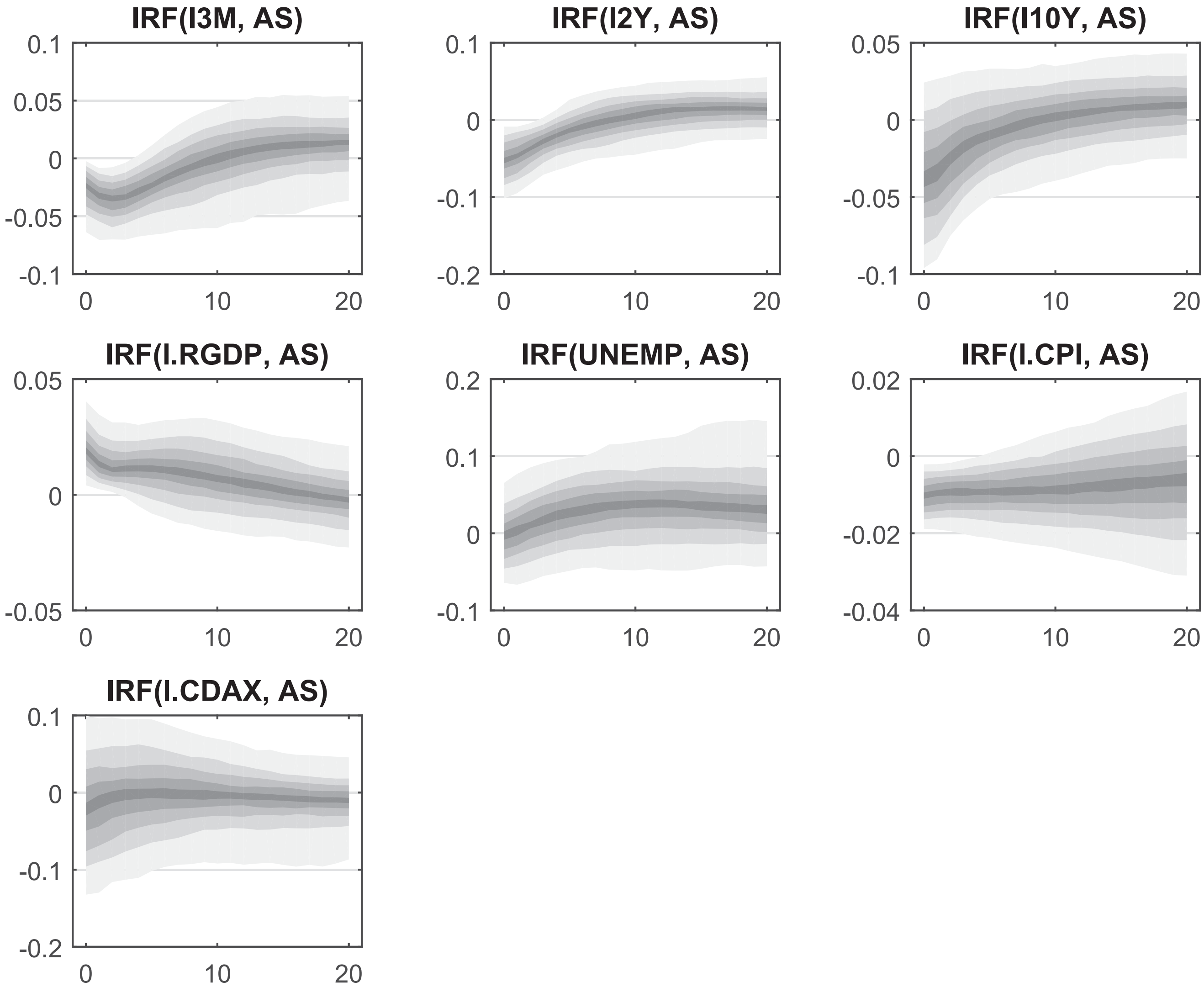
These restrictions are common choices, resembling, for example, those used by Eickmeier et al. (2009). All sign restrictions are imposed contemporaneously and for the following two quarters. Note that the identification scheme for the MP shock is problematic: Since 1999, the euro area has had a single currency and a single monetary policy under the responsibility of the European Central Bank (ECB). Given this new monetary policy regime, it is hard to justify an identification scheme in which policy choices depend only on German economic data, but not on the economic data of other Euro area countries. The monetary policy shock that I identify can therefore be interpreted, at best, as monetary policy choices that are unexpected from the perspective of German economic data alone. My reason for deciding against identifying the MP shock based on euro area variables is that I am interested in a solid scenario narrative with structural shocks that are strong drivers of the scenario variables of interest.

Figure 3: Aggregate demand shock - impulse responses



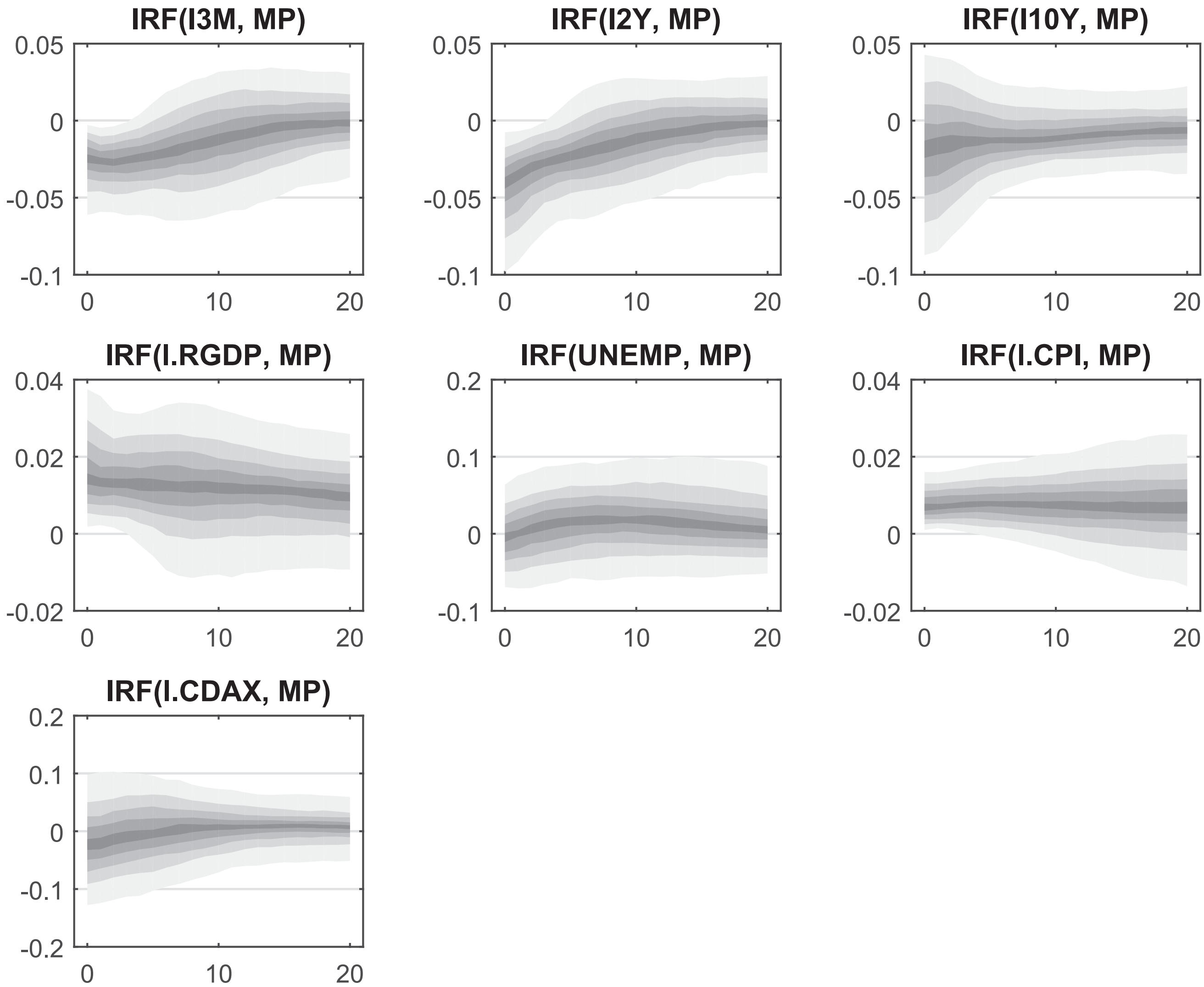
Notes: shaded areas delimited by 5%, 15%, ..., 95% quantile.

Figure 4: Aggregate supply shock - impulse responses



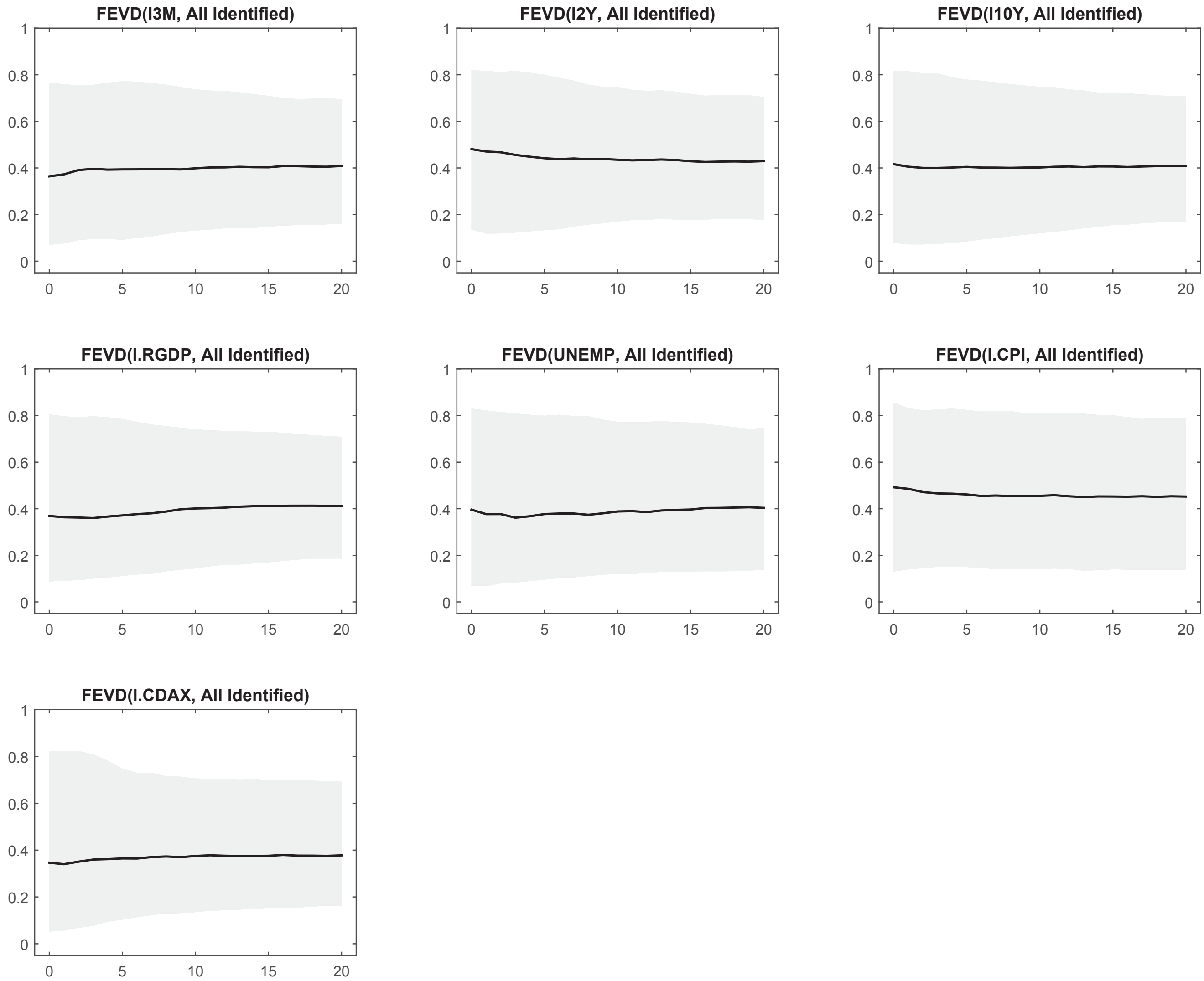
Notes: shaded areas delimited by 5%, 15%, . . . , 95% quantile.

Figure 5: Monetary policy shock - impulse responses



Notes: shaded areas delimited by 5%, 15%, ..., 95% quantile.

Figure 6: AD, AS & MP shock - aggregated forecast error variance decomposition



Notes: shaded area delimited by 16% and 84% quantile; solid line is median forecast error variance decomposition.