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Interest-rate pegs, central bank asset purchases and the reversal puzzle

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Non-technical Summary

Research question

Against the backdrop of the financial crisis, central banks implemented quantitative easing (QE) and forward guidance (FG) policies. Over the past few years, a consensus has emerged that such measures lead to expansionary macroeconomic effects. However, it is also well known that New Keynesian models can give rise to puzzles if the nominal interest rate is constrained by the zero lower bound or a temporary peg (due to a FG policy). In this study, we focus on the *reversal puzzle* (see Carlstrom et al., 2015): If the central bank implements QE in combination with FG, the effects on output and inflation increase with the length of FG up to some critical duration; if this critical duration is exceeded, the model predicts recession and deflation.

Contribution

Our contribution is threefold. First, we illustrate the relevance of the reversal puzzle in a plausible policy scenario by analyzing the macroeconomic effects of QE in combination with FG of variable duration. Second, we derive the analytical solution of the model and show that two ingredients are necessary, though not sufficient, for the puzzle to occur: First, the solution of the model needs to feature complex eigenvalues, and second, monetary policies' FG needs to be perfectly anticipated by the agents. Third, we formulate two possibilities for overcoming sign switches in the model's simulations.

Results

We find that the reversal puzzle is very tenacious and that it is intimately related to agents' expectations in the economy. This is illustrated by two different modifications to our analysis, which both lead to the disappearance of the reversal puzzle. First, we show that a deviation from the assumption of perfect foresight (i.e. if we, instead, solve the model under certainty equivalence) eliminates the puzzle. Second, if we retain the assumption of perfect foresight but manage agents' inflation expectations with a price-level-targeting strategy, the reversal puzzle is also absent. Both modifications and its implications for the reversal puzzle are explained using the solution of the model.

Nichttechnische Zusammenfassung

Fragestellung

Zahlreiche Notenbanken haben auf die jüngste Krise mit Wertpapierankaufprogrammen (QE) reagiert und dabei Orientierungen über die zukünftige Ausrichtung der Geldpolitik gegeben ("forward guidance" - FG). In überwiegender Übereinstimmung haben diese Maßnahmen expansiv gewirkt. Es ist jedoch auch bekannt, dass das Neukeynesianische Modell mitunter Anomalien produziert, und zwar wenn der Politikzins temporär restringiert wird. Wir befassen uns mit einer solchen Anomalie, dem sogenannten *reversal puzzle* (Carlstrom et al., 2015): Wenn die Zentralbank ein Kaufprogramm in Kombination mit FG implementiert, steigen zunächst die makroökonomischen Effekte mit der Länge von FG bis zu einer kritischen Dauer an; wird diese kritische Dauer überschritten, kehren sich die Effekte um, und das Modell prognostiziert eine Rezession und Deflation.

Beitrag

Dieses Papier trägt dreifach zur Literatur bei. Zunächst veranschaulichen wir die Relevanz des *reversal puzzle* in einem plausiblen Politikszenario. Dabei analysieren wir die Effekte von QE in Kombination mit einem Zinspeg von unterschiedlicher Dauer. Daran anschließend nutzen wir die analytische Lösung des Modells und zeigen, dass zwei Bedingungen für das Auftreten des genannten Phänomens notwendig (aber nicht hinreichend) sind: Erstens, die Lösung des Modells muss komplexe Eigenwerte enthalten, und zweitens, der Zinspeg muss von den Agenten vollkommen antizipiert werden. Abschließend diskutieren wir zwei Möglichkeiten, wie das *revesal puzzle* vermieden werden kann.

Ergebnisse

Wie unsere Ergebnisse zeigen, kann das *reversal puzzle* sehr hartnäckig sein und hängt eng mit der Erwartungsbildung der Akteure in der Ökonomie zusammen. Wir illustrieren dies mit Hilfe zweier Modifikationen. Im Kontext der ersten Modifikation zeigen wir, dass das *reversal puzzle* ausbleibt, wenn wir die Annahme vollkommener Voraussicht aufgeben (d.h., wenn die Agenten den Zinspeg nicht antizipieren). Im Rahmen der zweiten Modifikation behalten wir zwar die Annahme perfekter Voraussicht bei, ändern die Inflationserwartungen der Agenten in dem Sinn, als wir nach Ende des Zinspegs eine Strategie der Preisniveausteuerung unterstellen. Auch dann tritt das *reversal puzzle* nicht auf. Beide Modifikationen und ihre Implikationen für das *reversal puzzle* werden anhand der analytischen Lösung des Modells erläutert.

BUNDESBANK DISCUSSION PAPER NO 21/2017

Interest-rate pegs, central bank asset purchases and the reversal puzzle*

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Abstract

We analyze the macroeconomic implications of a transient interest-rate peg in combination with a QE program in a non-linear medium-scale DSGE model. In this context, we re-examine what has become known as the reversal puzzle (Carlstrom, Fuerst and Paustian, 2015) and provide an analytical explanation for its appearance. We show that the puzzle is intimately related with agents' expectations. If, for instance, agents do not anticipate the peg, the reversal does not appear. The same is true if agents' inflation expectations are influenced by a monetary authority which follows a price-level-targeting rule instead of a standard Taylor rule. In this case, sign reversals do not occur even for very long durations of pegged nominal interest rates.

Keywords: Unconventional Monetary Policy, Interest-Rate Peg, Perfect Foresight, Reversal Puzzle, Price-Level Targeting

JEL Classification: E32, E44, E52, E61

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1 Introduction

In the aftermath of the financial crisis, major central banks adopted a series of unconventional monetary policy measures both to restore the functioning of the monetary transmission mechanism and to provide further accommodation in a low-inflation environment. In particular, sovereign bond purchase programs, often referred to as quantitative easing (QE), and forward guidance were implemented by major central banks as policy responses to anemic growth and low inflation. Such unconventional measures were adopted as alternative instruments when central banks had reached the effective lower bound on nominal short-term interest rates. While there is no consensus yet as to how far a given purchase program can stimulate output and inflation, empirical models and (suitably modified) New Keynesian models alike support the view that QE and forward guidance are expansionary at the zero lower bound (see, for example, Carlstrom, Fuerst and Paustian, 2017; Gertler and Karadi, 2013; Chen, Cúrdia and Ferrero, 2012, and the references therein).

However, recently Cochrane (2015), García-Schmidt and Woodford (2015), and Carlstrom et al. (2015) among others have illustrated that the standard New Keynesian model can give rise to puzzles if the policy rate is kept constant for some time, typically at the zero lower bound.[1] Prominent among these is the forward guidance puzzle. In this phenomenon, forward guidance (i.e., an announced intention of the central bank to keep the nominal interest rate constant for a given period of time) implies huge expansionary and inflationary effects in the canonical

[1] Unlike the papers by Cochrane (2015) and García-Schmidt and Woodford (2015) our paper is not about multiplicity and equilibrium selection. For a discussion of multiplicity in the context of the zero lower bound, see also Holden (2017).

New Keynesian model (e.g., Del Negro, Giannoni and Patterson, 2015). A related effect, the *reversal puzzle*, has been described recently by Carlstrom et al. (2015). According to the nature of this puzzle, pegging the interest rate for a sufficiently long period of time in the standard New Keynesian model can give rise to counterintuitive sign reversals in the path of the endogenous variables. The effect of the interest-rate peg can switch from highly expansionary to highly contractionary even for small changes in the length of the interest-rate peg.

In this paper, we illustrate that the reversal puzzle can be a tenacious problem in the context of analyzing the effects of a QE program[2] and make two suggestions about how to overcome it. For our analysis, we jointly analyze a temporary, fully anticipated interest-rate peg in combination with a QE program in the model of Carlstrom et al. (2017). In the absence of the peg, the model predicts the orthodox view, that is, an increase in output and inflation in response to the launch of a QE program. If, however, the central bank keeps the policy rate constant for some time, the responses of the model's variables can switch their sign: Output and inflation first increase with the duration of the peg and tend to explode as the duration of the peg approaches some critical value (after four quarters in our analysis). But if the duration of the peg exceeds this critical value, the model predicts a counterintuitive sign reversal (i.e., the reversal puzzle) or, put differently, a sizeable deflation instead of inflation. Furthermore, if we continue to hold the interest rate fixed for an even longer period of time, the sign of the model's predictions changes once again. Thus, the qualitative response of output and inflation oscillates as the duration of the peg expands into the far future. We

[2]For the peg to produce this puzzle, some shock must occur. We choose a QE shock to design a plausible policy scenario.

provide intuition for this puzzling result and show analytically that the reversal critically hinges on two things: First, the model needs to feature endogenous state variables, which imply complex eigenvalues in the solution of the system, and second, the assumption that the interest-rate peg is perfectly anticipated. The necessity of complex eigenvalues in the model's solution was already pointed out by Carlstrom et al. (2015). In particular, they find, based on the canonical 3-equation DNK model with inflation indexation, that a necessary and sufficient condition for the existence of the reversal puzzle is that the explosive eigenvalues become complex-valued. However, in contrast to Carlstrom et al. (2015), we show that if we use a richer model to describe a plausible policy scenario the existence of complex eigenvalues is merely a necessary but no longer a sufficient condition. We further show that the presence of the reversal puzzle is very robust with respect to the calibration of the model.

The analytical solution of the model suggests that the expectations of the agents in the economy are key for the appearance of the reversal puzzle. We illustrate this finding by discussing two different modifications to our analysis. First, if agents take the nominal interest-rate peg into account only contemporaneously, but expect the peg to be absent in the future (no anticipation), i.e., if they operate as under certainty equivalence, sign reversals disappear irrespective of the duration of the peg. Thus, if we deviate from the assumption of perfect anticipation of the interest-rate peg and, instead, assume that agents' expectations do not incorporate forward guidance into their decisions (i.e., the future development of the nominal interest-rate peg), the reversal puzzle does not occur. Second, if we retain the assumption that agents perfectly anticipate the future interest rate path but change agents' expectations by assuming that monetary policy is going to follow a different monetary policy rule, namely price-level targeting instead of a Taylor rule, the reversal puzzle vanishes, too.

These two suggestions for overcoming the reversal puzzle therefore underscore the fact that the appearance of the reversal puzzle is intimately related to agents' expectations. If people do not anticipate the interest-rate peg, it is merely a series of unanticipated shocks and therefore the reversal does not appear. But why is price-level targeting effective at overcoming the reversal? If monetary policy follows a price-level-targeting rule, the central bank aims at stabilizing the aggregate price level around a predetermined path. Thus, if there is a shock that pushes the price level away from the target-price path, future inflation will be required to adjust in a way to bring the price path back to target. Put differently, past deviations from target are corrected under price-level targeting. If monetary policy is credible, the forward-looking rational agent thus expects a complete correction of past deviations. This mitigates the expansionary effect of the anticipated peg and QE from the very beginning. A bit more technically, the response of inflation to a series of forward guidance shocks will neither grow exponentially and reach an asymptote nor will it reverse its sign. As a consequence, sign reversals do not occur even for very long durations of pegged nominal interest rates (in our example for much more than 15 years).

The importance of forward-lookingness and therefore inflation expectations for the explanation of the reversal puzzle has already been emphasized by Carlstrom et al. (2015), who analyzed the reversal puzzle in the context of a standard New Keynesian model.[3] They argue that, in particular, inflation indexation is an important precondition for the sign switch to arise: If there is a backward-looking element in the Phillips curve, which is due to indexation, there is "...nothing to anchor the terminal level of inflation"[4] at the end of the peg.[5] By contrast, in their model variant without indexation, the level of inflation after the peg does not depend on the previous pegged period (the same is true for output) and therefore the Taylor rule (that kicks in after the peg) imposes a terminal condition on the inflation rate.[6]

To overcome the reversal puzzle Carlstrom et al. (2015) suggest assuming sticky information instead of Calvo pricing (see also Kiley, 2016). The key difference between the sticky-price and the sticky-information framework is that under sticky prices (and assuming indexation) the Phillips curve states that current inflation depends on both lagged inflation and expectations of tomorrow's inflation, whereas under sticky information the Phillips curve states that only past expectations of the current marginal costs and inflation matter for inflation today. In this sense the firms are much less forward-looking in the sticky information framework. As a consequence, there are no inflation reversals in their sticky-information model.[7]

[3]Laséen and Svensson (2011) and Lindé, Smets and Wouters (2016) also mention the appearance of sign switches when simulating their models in combination with a transient interest-rate peg. However, both studies do not elaborate on the source of the puzzle.

[4]See Carlstrom et al. (2015), p. 234.

[5]If the Phillips curve entails lagged inflation (which adds an endogenous state variable to the system), the terminal level of inflation is no longer exogenous. Put differently, there is nothing to pin down the terminal level of inflation at the end of the peg. Consequently, if initial inflation depends on terminal inflation, but terminal inflation depends on initial inflation, the counterintuitive reversal may arise.

[6]Solving the model backward gives a monotonic (exponential) path for inflation (and output).

[7]Several other authors relate the existence of the forward guidance puzzle to the important role of expectations and provide alternative approaches to deal with it. Notable examples include Farhi and Werning (2017) who depart from the assumption of rational expectations and adopt some form of bounded rationality, and Angeletos and Lian (2016) who introduce an incom-

We organize our paper as follows. The next section presents the model we use for the analysis and its main characteristics. Section 3 explains how we implement the QE program and the transient interest-rate peg. Section 4 presents a short description of the solution method. Section 5 provides our first set of simulation results. In subsection 5.1, we analyze the effects of a QE program without an interest-rate peg to illustrate the transmission of the QE program: The model is consistent with the prevailing view that QE generates a positive response of inflation and output. Subsection 5.2 then illustrates the effects of QE in combination with an interest-rate peg of variable duration and shows that the model produces a reversal puzzle if the policy rate is kept constant for a sufficiently long period of time. We then derive the solution of the model and show why reversals appear. Subsection 5.3 documents that the reversal puzzle is hard to get rid off since it remains for a very wide range of calibrations of the model. In section 6 we present the aforementioned two modifications to our analysis, which – independently of each other – help to overcome the reversal puzzle. In subsection 6.1 we deviate from the assumption of perfect foresight and assume that agents do not anticipate the peg at all. Subsequently, in subsection 6.2, we retain the assumption of perfect foresight, but change the way monetary policy is conducted from a standard Taylor rule to a price-level-targeting rule. Section 7 concludes.

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plete information setup into the New-Keynesian model, thereby relaxing the routinely applied assumption of common knowledge.

2 A model with financial intermediaries

To assess the effects of a government bond purchase program and to be able to describe under what conditions an interest-rate peg can give rise to a reversal, we use the fully non-linear version of an estimated New Keynesian model with financial intermediaries (henceforth FIs) which was recently developed by Carlstrom et al. (2017). The model differs from "standard" DSGE models (see, for instance, Fernández-Villaverde and Rubio-Ramírez, 2006; Christiano, Eichenbaum and Evans, 2005; Smets and Wouters, 2007) in the following ways.

Bond market: One key feature leading to the effectiveness of bond purchases in this model is that the financial market is assumed to be segmented. In particular, it is assumed that only FIs can purchase long-term bonds in the financial market. There are two types of long-term bonds available: investment bonds, F_t, issued by households and government bonds, B_t, issued by the government. Following Woodford (2001), long-term investment and long-term government bonds are modeled as perpetuities, which pay exponentially decaying coupons $1, \kappa, \kappa^2, \ldots$. Perpetuities issued at time t pay their first coupon in $t+1$, as shown in table 1.

Table 1: Bond pricing

	t	t+1	t+2	t+3	t+4
Bond Price	Q_t				
Coupon		1	κ	κ^2	...

The corresponding price of the perpetuity is given by $Q_t = 1/(R_t^{10} - \kappa)$, implying that the gross yield to maturity (i.e., the discount rate that makes the present value of the bond's stream of promised cash payments equal to its price) is given by $R_t^{10} = Q_t^{-1} + \kappa$. The duration is calibrated to be ten years (40 quarters), i.e., $40 = (1-\kappa)^{(-1)}$, which implies $\kappa = 0.975$.

Financial intermediaries: In addition to the assumption of a segmented financial market, the model features financial intermediaries which trade in the financial market. These FIs use accumulated net worth, N_t, and (short-term) deposits, D_t, to finance purchases of long-term (investment and government) bonds. From the perspective of the FIs, long-term government bonds and long-term investment bonds are perfect substitutes. Their balance sheet – in real terms – is given by

$$\bar{B}_t + \bar{F}_t = \frac{D_t}{P_t} + N_t = L_t N_t, \tag{1}$$

where L_t denotes leverage of the FIs and $\bar{B}_t \equiv \frac{B_t}{P_t} Q_t$ and $\bar{F}_t \equiv \frac{F_t}{P_t} Q_t$ denote the real market values of government and investment bonds, respectively.

The ability of FIs to adjust their liability position (for instance, in response to financial market shocks) is assumed to be limited by two constraints. First, FIs face quadratic adjustment cost in net worth accumulation; second, FIs are leverage-constrained, because they face what is known as a "hold-up" problem. This hold-up problem emerges because FIs can choose to default on obligations to depositors. If FIs choose to default, depositors can, by assumption, only seize a share $(1-\mu_t)$ of the FIs assets. The remaining share will be kept by FIs. However, the share of seized assets is assumed to depend on the level of net worth of the FIs as well as on other state variables. In particular, higher net worth makes the hold-up problem less severe.

To make sure that the FIs will never default on their obligations to depositors, the value of their expected profits needs to be greater than (or at least equal to) the value of the share of assets that can be seized in the event of a default. Thus, the *binding* time-t incentive compatibility constraint is given by

$$E_t \frac{P_t}{P_{t+1}} \Lambda_{t+1} \left[\left(R_{t+1}^L - R_t^d \right) L_t + R_t^d \right] N_t = \mu_t L_t N_t E_t \Lambda_{t+1} \frac{P_t}{P_{t+1}} R_{t+1}^L, \tag{2}$$

where P_t denotes the price level at time t, R_t^L and R_t^d denote the long-term and the deposit rates, and Λ_{t+1} describes the Lagrangian multiplier associated with the household's optimal consumption decision. In equation (2), the left-hand side describes the time-t+1 expected profits (measured in consumption equivalents) and the right-hand side denotes the value of the share of assets (also measured in consumption equivalents) that the FIs will seize in the case in which they choose to default.

Given these two constraints (i.e., the net worth adjustment cost and the incentive constraint), the FIs maximize their value function:

$$V_t = E_t \sum_{s=0}^{\infty} (\beta \zeta)^s \Lambda_{t+s} \mathrm{div}_{t+s}, \tag{3}$$

where ζ is an additional impatience parameter of the FIs and div_{t+s} denotes expected future dividends, subject to their budget constraint:

$$\mathrm{div}_t + N_t \left[1 + f(N_t) \right] \leq \frac{P_{t-1}}{P_t} \left[\left(R_t^L - R_{t-1}^d \right) L_{t-1} + R_{t-1}^d \right] N_{t-1}, \tag{4}$$

which states that dividends and adjustments of the stock of net worth (diminished by adjustment cost) need to be financed by generated profits. In this budget constraint, $R^L_{t+1} \equiv \left(\frac{1+\kappa Q_{t+1}}{Q_t}\right)$ and $f(N_t) \equiv \frac{\psi_n}{2}\left(\frac{N_t - N_{ss}}{N_{ss}}\right)^2$ denotes the net worth adjustment cost.[8]

Households: Households are assumed to maximize their utility

$$E_t \sum_{s=0}^{\infty} \beta^s \exp(rn_{t+s}) \left\{ \ln(C_{t+s} - hC_{t+s-1}) - \chi \frac{H^{1+\eta}_{t+s}(j)}{1+\eta} \right\}, \tag{5}$$

where β is the discount factor, η is the inverse elasticity of labor supply, χ is the relative utility weight of labor, C_t is consumption, h is the degree of habit formation, $H_t(j)$ is the labor input of household j, and $\exp(rn_t)$ is a discount factor shock which follows an AR(1) process. Households take into account the following budget constraint:

$$\begin{aligned} C_t + \frac{D_t}{P_t} + P^k_t I_t + \frac{F_{t-1}}{P_t} \leq \\ \frac{W_t(j)}{P_t} H_t(j) + R^k_t K_{t-1} - T_t + \frac{D_{t-1}}{P_t} R_{t-1} + \frac{Q_t(F_t - \kappa F_{t-1})}{P_t} + \mathrm{div}_t, \end{aligned} \tag{6}$$

where D_t denotes short-term deposits, K_t is the physical capital stock, $W_t(j)$ denotes the nominal wage, $R^d_t = R_t$ is the nominal interest rate on deposits,[9] R^k_t is the real rental rate of capital, T_t is lump-sum taxes, and div_t is the dividend flow

[8] The right-hand side of equation (4) can be expanded into two components: $R^L_t L_{t-1} N_{t-1} - R^d_{t-1}(L_{t-1}N_{t-1} - N_{t-1})$, where the first component, $R^L_t L_{t-1} N_{t-1}$, describes the FI's income from credit operations and the second component, $R^d_{t-1}(L_{t-1}N_{t-1} - N_{t-1})$, denotes the cost associated with its deposits.

[9] Since T-bills are used to implement the short-term policy rate, R_t, and T-bills are perfect substitutes with deposits, the deposit rate equals the policy rate.

from financial intermediaries. In addition, households take into account the law of motion for physical capital:

$$K_t \leq (1-\delta)K_{t-1} + I_t, \tag{7}$$

and, in particular, a loan-in-advance constraint, which describes that capital investment of households needs to be financed by the issuance of long-term investment bonds:

$$P_t^k I_t \leq \frac{Q_t\left(F_t - \kappa F_{t-1}\right)}{P_t} = \frac{Q_t CI_t}{P_t}, \tag{8}$$

where P_t^k is the real price of capital, I_t denotes investment in physical capital, F_{t-1} denotes household's nominal liabilities at time t, $CI_t = (F_t - \kappa F_{t-1})$ is the time-t issuance of new investment bonds, and Q_t denotes the time-t price of newly issued bonds. As described below, it is mainly through this loan-in-advance constraint that QE, via its impact on bond prices, is going to have real effects.

The rest of the model is standard in the sense that it exhibits the typical New Keynesian features. As in Erceg, Henderson and Levin (2000), households are monopolistic suppliers of differentiated labor inputs $H_t(j)$. They set wages on a staggered basis (à la Calvo). In each period, the probability of resetting the wage is $(1-\theta_w)$, while with the complementary probability the wage is automatically increased following an indexing rule, $W_t(j) = \Pi_{t-1}^{\iota_w} W_{t-1}(j)$. The problem for household j who can reset its wage at time t is

$$\max_{W_t(j)} E_t \sum_{s=0}^{\infty} \theta_w^s \beta^s \left\{ -\chi \frac{H_{t+s}^{1+\eta}(j)}{1+\eta} \exp(rn_{t+s}) \lambda_{w,t+s} + \Lambda_{t+s} \frac{W_t(j)}{P_{t+s}} H_{t+s}(j) \right\}, \tag{9}$$

where the first term describes the disutility of labor, which is subject to a discount factor shock and a markup shock, $\lambda_{w,t+s}$. The second term describes the real wage income, which is multiplied by the household's marginal utility of consumption, Λ_{t+s}.

Goods market: Final goods producers combine differentiated intermediate goods, $Y_t(i)$, into a homogeneous good, Y_t, according to the technology:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} di \right]^{\frac{\varepsilon_p}{\varepsilon_p - 1}}. \tag{10}$$

The final goods producers buy the intermediate goods on the market, package Y_t, and resell it to consumers. These firms maximize profits in a perfectly competitive environment.

A continuum of monopolistically competitive firms combine capital K_{t-1} and labor H_t to produce intermediate goods according to a standard Cobb-Douglas technology:

$$Y_t(i) = A_t K_{t-1}(i)^{\alpha} H_t(i)^{1-\alpha}, \tag{11}$$

where A_t denotes a technology shock which follows an AR(1) process. These firms maximize profits subject to their production function. The intermediate goods producers set prices based on Calvo contracts. In each period, firms adjust their prices with probability $(1 - \theta_p)$. For those firms that cannot adjust their prices in a given period, prices will be reset according to the following indexation rule: $P_t(i) = \Pi_{t-1}^{\iota_p} P_{t-1}(i)$. Thus, the corresponding maximization problem for the intermediate goods producers is

$$\max_{P_t(i)} \Omega_t = E_t \sum_{s=0}^{\infty} \theta_p^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_t} \left[\frac{P_t(i) \left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_p} \right)}{P_{t+s}} - \lambda_{p,t+s} mc_{t+s}(i) \right] Y_{t+s}(i), \quad (12)$$

where $\lambda_{p,t+s}$ is an AR(1) price mark-up shock, Π_t is gross inflation, ι_p denotes the degree of price indexation, and mc_t denotes real marginal cost.

Capital goods producers transform I_t units of final goods into $\varphi_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t$ units of new capital goods. The function $S(\cdot)$ captures the presence of adjustment cost in investment – i.e., $S\left(\frac{I_t}{I_{t-1}}\right) \equiv \frac{\psi_i}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2$ – and φ_t represents an AR(1) investment-specific shock. The time-t profit of capital producers is given by

$$P_t^k \varphi_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t - I_t. \quad (13)$$

Government policies: Fiscal policy is passive, i.e., lump-sum taxes move to support interest payments on government debt while there are no other government expenditures. The central bank follows a standard Taylor rule:

$$R_t = (R_{t-1})^{\rho} \left(R_{ss} \Pi_t^{\tau_\Pi} \left(\frac{Y_t}{Y_{t-1}} \right)^{\tau_y} \right)^{1-\rho}. \quad (14)$$

Equilibrium conditions: In total, the non-linear model features 30 equilibrium conditions, 7 AR(1) processes, and 8 exogenous shocks (in addition to the shocks for the AR(1) processes, the shock ε_t^{TR} controls the exogenous transient interest-rate peg).[10]

The corresponding endogenous variables are gathered in the vectors Υ_{1t} and Υ_{2t}, and the exogenous variables in the vector υ_t:

$$\begin{aligned}\Upsilon_{1t} = \big\{&\Lambda_t, C_t, I_t, H_t, M_t, R_t^k, V_t, R_t, R_t^L, R_t^{10}, Q_t, \bar{F}_t, N_t, L_t, mc_t, ...\\ &\mathrm{MPK}_t, \mathrm{MPL}_t, P_t^k, Y_t, D_t^p, K_t, \Pi_t, \Pi_t^\star, X_t^{pn}, X_t^{pd}, X_t^{wn}, X_t^{wd}, w_t^\star, w_t, D_t^w\big\}\end{aligned}$$

$$\begin{aligned}\Upsilon_{2t} &= \big\{\bar{B}_t, A_t, rn_t, \Phi_t, \mu_t, \lambda_{p,t+s}, \lambda_{w,t+s}\big\}\\ \upsilon_t &= \left\{\varepsilon_t^{TR}, \varepsilon_t^{\bar{B}}, \varepsilon_t^{A}, \varepsilon_t^{rn}, \varepsilon_t^{\Phi}, \varepsilon_t^{\mu}, \varepsilon_t^{\lambda_p}, \varepsilon_t^{\lambda_w}\right\}.\end{aligned}$$

While we incorporate all of the shocks to obtain empirically plausible estimates for the model's structural parameters (see table 2 in appendix B), we set most of the shocks to zero when we analyze the reversal puzzle. Specifically, we then only consider the shocks to the two policy rules, i.e., $\upsilon_t = \left\{\varepsilon_t^{TR}, \varepsilon_t^{\bar{B}}.\right\}$. The first is needed to implement an interest-rate peg, the latter to trigger the QE program, which is described formally in the next section.

[10]The individual equations of the non-linear equilibrium conditions are presented in Appendix A.

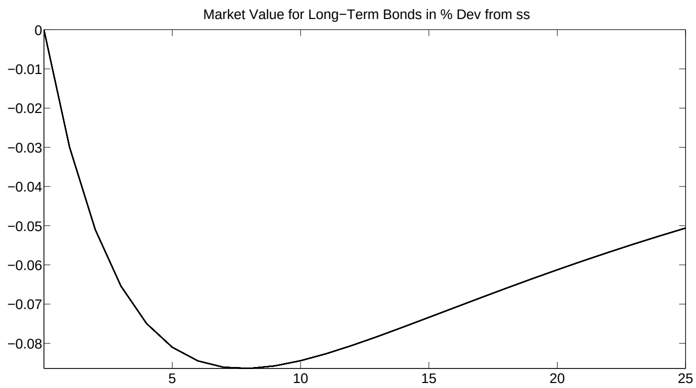
3 Modeling QE and an interest-rate peg

We assume that the government controls the supply of long-term bonds independently of macroeconomic conditions. As in Carlstrom et al. (2017), a QE program is implemented by a persistent AR(2) process for the real market value of long-term bonds available to the financial intermediaries:

$$\bar{B}_t = \bar{B}_{ss}^{(1-\bar{\rho}_1+\bar{\rho}_2)} \left(\bar{B}_{t-1}\right)^{\bar{\rho}_1} \left(\bar{B}_{t-2}\right)^{-\bar{\rho}_2} \varepsilon_t^{\bar{B}} \, . \tag{15}$$

This assumption is useful for two reasons: First, the AR(2) process is part of the model's equilibrium conditions and therefore taken into account by every agent. Thus, agents perfectly anticipate the path of the outstanding stock (value) of government bonds in the economy once a QE program has been started. Second, the (inverse) hump shape implied by an AR(2) process is well suited to representing a plausible QE program: During the phase of purchases, the total value of outstanding bonds held by the public (i.e., excluding the central bank) declines, while it returns only gradually to the steady state after the purchases stop eventually – in our case after 6 quarters. Technically, the QE program is triggered by a *single* shock, i.e., $\varepsilon_t^{\bar{B}} < 1$, that occurs in the first period of the model simulation (see figure 1).

Figure 1: Total value of long-term bonds held by the public



Note: The solid line represents the evolution of $\bar{B}_t$ (i.e., the market value of long-term bonds) in percentage deviation from the steady state over 25 quarters.

The interest-rate peg is implemented via a *sequence* of shocks, which consists of binary dummy variables, $\varepsilon_t^{TR} \in \{0, 1\}$.[11] These are set to one for periods of pegged nominal rates and zero otherwise:

$$R_t = \varepsilon_t^{TR} (R_{ss}) + \left(1 - \varepsilon_t^{TR}\right) (R_{t-1})^{\rho} \left(R_{ss} \Pi_t^{\tau_\Pi} \left(\frac{Y_t}{Y_{t-1}} \right)^{\tau_y} \right)^{1-\rho}. \tag{16}$$

4 Solution method

We conduct our analysis by solving the non-linear model under perfect foresight, i.e., the agents in the economy anticipate future shocks perfectly. The model

[11] One could probably implement the transient interest-rate peg via a non-differentiable function (i.e., a min- or max-operator). However, this would render the peg endogenous with regard to its duration. Implementing the peg via the dummy approach allows us to determine the duration of the peg in a completely exogenous way.

equations can be cast into the following general form:

$$E_t\left[\Gamma\left(\Upsilon_{t+1},\Upsilon_t,\Upsilon_{t-1},\upsilon_t\right)\right]=0, \tag{17}$$

where E_t denotes the expectation operator, Γ is a non-linear function, Υ_t summarizes all endogenous variables of the system, and υ_t includes the two exogenous variables, $\varepsilon_t^{\bar{B}}$ and ε_t^{TR}. As mentioned, knowledge of the latter shock implies that the agents in the economy also fully anticipate the interest rate path of monetary policy. Thus, *every* component of υ_t is perfectly known and we can proceed as if the economy were purely deterministic. That is, we can skip E_t from equation (17) and stack the set of equilibrium equations (for each of the T periods of the simulation horizon) as follows:

$$\begin{aligned}\Gamma\left(\Upsilon_2,\Upsilon_1,\Upsilon_0,\upsilon_1\right)&=0\\ &\vdots\\ &\vdots\\ \Gamma\left(\Upsilon_{T+1},\Upsilon_T,\Upsilon_{T-1},\upsilon_T\right)&=0.\end{aligned} \tag{18}$$

We solve the system of stacked equations via Newton's method as described in Adjemian and Juillard (2014).[12] The simulations are initialized at the steady state – such that Υ_0 is given – and will return to it within our pre-specified simulation horizon,[13] i.e., the initial as well as the terminal state (Υ_{T+1}) of the simulations

[12]We implement this procedure using Dynare 4.4.3., which offers several algorithms to solve models under the assumption of perfect foresight. The appearance of the reversal puzzle is not a feature of the specific algorithm we use to obtain the subsequent results.

[13]We set $T = 500$ to ensure a successful transition between initial and terminal state.

will be the steady state.

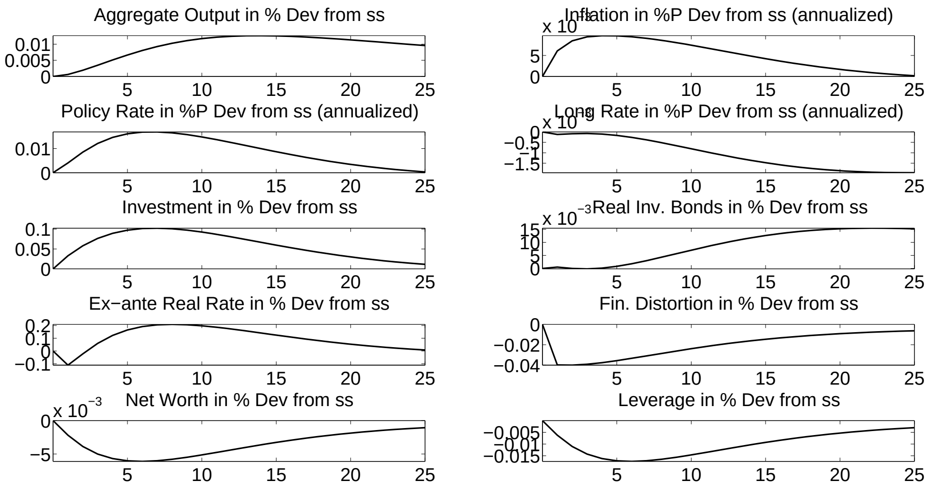
5 The reversal puzzle

5.1 The benchmark case: QE without interest-rate peg

We first consider a QE shock without an accompanying transient interest-rate peg to describe the transmission of QE in this model. While it is not the primary focus of our analysis, it shows that the model is perfectly able to reproduce the conventional result: QE is expansionary and inflationary.

The exogenous process describing the bond purchase program is triggered by a single shock in the first period of the simulation. As monetary policy follows a prototypical Taylor rule, ε_t^{TR} is zero for all t.

Figure 2: Simulation of a QE shock without accompanying transient interest-rate peg



Note: The figure shows responses of (quarterly) output, investment, real investment bonds, net worth, and the financial distortion in percent deviations from the steady state. Inflation, the real interest rate, leverage, as well as the short- and long-term rates are measured in (annualized) percentage point deviations from the steady state.

The transmission can be described as follows. The decreasing supply of long-term government bonds available for FIs implies an upward pressure on its price and, correspondingly, lowers its yield to maturity – moderately but persistently (the yield to maturity is given by $R_t^{10} = 1/Q_t + \kappa$). The term premium, too, decreases (which in the present model is essentially the distortion that is related to the loan-in-advance constraint).[14] The decrease in available bonds leads to a reduction in banks' net worth and leverage. Thus, the purchase of bonds shortens the FIs' balance sheet, but net worth mobility is limited due to the portfolio

[14]Accordingly, the QE program *reduces* the distortion that is due to market segmentation.

adjustment cost.[15] Correspondingly, FIs demand for investment bonds increases (portfolio adjustment). Since investment bonds and government bonds are perfect substitutes, the price of investment bonds also rises. Therefore, the households' loan-in-advance constraint is relaxed and, as a result, investment demand increases. Higher investment demand, in turn, increases aggregate output and so does the inflation rate. In short, QE is *expansionary* and *inflationary* in the present model and, as a response, monetary policy increases its policy rate if it follows a Taylor rule.

5.2 Sign reversals: QE with interest rate peg

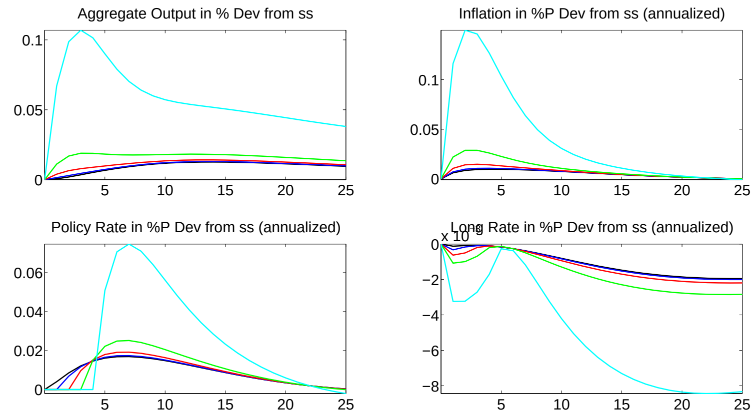
To illustrate the reversal puzzle that is closely related to the interest-rate peg, we now assume that the central bank does not follow a Taylor rule but instead keeps the short-term nominal interest rate unchanged for a pre-announced period of time, P, alongside its QE program. Arguably, this scenario bears major relevance since a QE program is typically introduced when the policy rate cannot be lowered any further. Furthermore, central banks typically do not want to offset the desired expansionary effect of a QE program by increasing the policy rate.

Figure 3 presents the results for output, inflation, and the short- as well as long-term rates. Panel (a) shows simulated time paths when the policy rate is kept constant for up to four periods, i.e., $\{P \in \mathbb{Z} \mid 0 \leq P \leq 4\}$, panel (b) presents the corresponding outcomes for $\{P \in \mathbb{Z} \mid 5 \leq P \leq 10\}$, and panel (c) shows results for $\{P \in \mathbb{Z} \mid 19 \leq P \leq 25\}$.

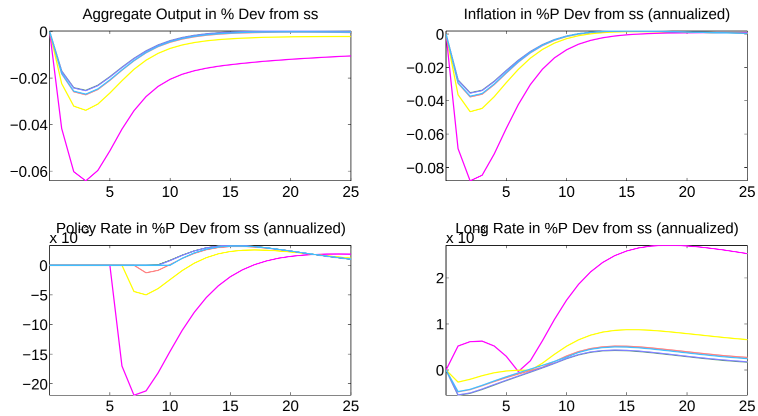
[15] Note: since net worth mobility is limited, arbitrage opportunities are not eliminated immediately.

Figure 3: Simulation results of a QE shock in combination with an interest-rate peg of variable duration under perfect foresight

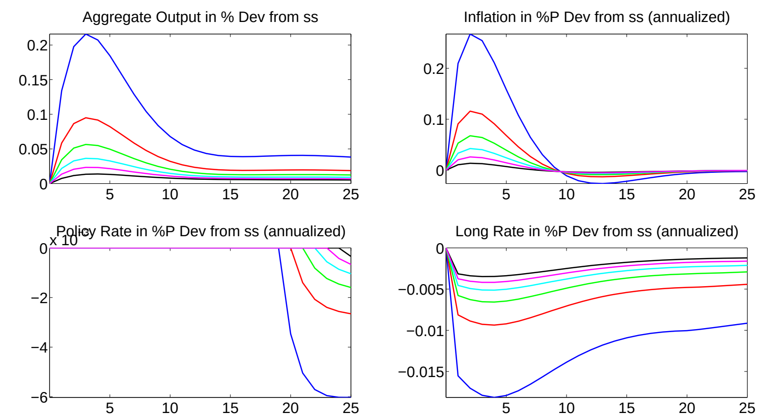
(a) Zero to four periods of pegged rates



(b) Five to ten periods of pegged rates



(c) 19 to 25 periods of pegged rates



Note: The figure shows simulation results for output, inflation, short-term interest rate, and long-term interest rate based on a QE shock in combination with an interest-rate peg of duration P. The period zero denotes the steady-state. The QE shock, together with the first forward guidance shock, occurs in period one. Panel (a) shows results for $0 \leq P \leq 4$, panel (b) shows results for $5 \leq P \leq 10$, and panel (c) shows results for $19 \leq P \leq 25$.

For up to three periods of pegged interest rates, the QE program leads to a comparatively modest increase in inflation and output. For a duration of four quarters, however, the corresponding responses increase markedly. The peak effect of output for an additional period of pegged rates (i.e., $P = 4$ compared to $P = 3$) is about four times larger.

If we increase the duration of pegged rates further, i.e., $5 \leq P \leq 10$ (see panel (b)), the simulated time series reverse their sign, that is, inflation and output decrease considerably after the inception of QE. Put differently, for $5 \leq P \leq 10$ the model predicts a severe recession in response to a QE program. Yet, if we increase the duration of the interest-rate peg even further (see panel (c)), the sign of the simulated time series switches once again, predicting a rather large expansionary effect. Thus, in response to a QE shock in combination with an interest-rate peg, the model variables oscillate with the duration of the peg.

How can such a remarkable reversal be explained? As explained in our introduction, Carlstrom et al. (2015) argue that in the context of a standard 3-equation New Keynesian model, it is, in particular, inflation indexation that is an important precondition for the sign switch to arise. This is because, if the Phillips curve entails lagged inflation (which adds an endogenous state variable to the system), then the terminal level of inflation is no longer exogenous. There is nothing to pin down the terminal level of inflation at the end of the peg. Consequently, if initial inflation depends on terminal inflation, but terminal inflation depends on initial inflation, a counterintuitive reversal may arise. To gain a deeper understanding of why reversals may occur, we complement the analysis by Carlstrom et al. (2015). In particular, we first take a closer look at the forward solution of the sub-system of the difference equations. Later on, we complete the discussion by elaborating on the backward solution.

We consider the linearized version of the model, which we can write in the following general form:

$$\Gamma_0 Y_t = \Gamma_1 Y_{t-1} + \Phi \varepsilon_t + \Psi \eta_t. \tag{19}$$

Y_t denotes the endogenous variables, ε_t describes the fundamental shocks (for instance, the QE shock or the shock governing the interest-rate peg), and η_t indicates the forecast errors. Following Sims (2001), we apply the QZ decomposition:

$$Q^{'} \Lambda Z^{'} = \Gamma_0 \tag{20}$$

$$Q^{'} \Omega Z^{'} = \Gamma_1. \tag{21}$$

As a result, we can rewrite equation (19) such that

$$Q^{'} \Lambda \underbrace{Z^{'} Y_t}_{\omega_t} = Q^{'} \Omega \underbrace{Z^{'} Y_{t-1}}_{\omega_{t-1}} + \Phi \varepsilon_t + \Psi \eta_t. \tag{22}$$

Premultiplying by Q and redefining $Z^{'} Y_t \equiv w_t$ implies

$$\Lambda w_t = \Omega w_{t-1} + Q \Phi \varepsilon_t + Q \Psi \eta_t. \tag{23}$$

Partitioning equation (23) into explosive and nonexplosive parts yields

$$\begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ 0 & \Lambda_{22} \end{bmatrix} \begin{bmatrix} w_{1,t} \\ w_{2,t} \end{bmatrix} = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ 0 & \Omega_{22} \end{bmatrix} \begin{bmatrix} w_{1,t-1} \\ w_{2,t-1} \end{bmatrix} + \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} \left(\Phi \varepsilon_t + \Psi \eta_t \right), \qquad (24)$$

where the second equation, i.e., the one containing the unstable eigenvalues, can separately be written as

$$\Lambda_{22} w_{2,t} = \Omega_{22} w_{2,t-1} + Q_2 \left(\Phi \varepsilon_t + \Psi \eta_t \right). \qquad (25)$$

Multiply equation (25) by Ω_{22}^{-1} to obtain

$$\Omega_{22}^{-1} \Lambda_{22} w_{2,t} = \Omega_{22}^{-1} \Omega_{22} w_{2,t-1} + \Omega_{22}^{-1} Q_2 \left(\Phi \varepsilon_t + \Psi \eta_t \right). \qquad (26)$$

Rewrite this expression as

$$J w_{2,t} = w_{2,t-1} + \Omega_{22}^{-1} Q_2 \left(\Phi \varepsilon_t + \Psi \eta_t \right). \qquad (27)$$

In this expression, $J \equiv \Omega_{22}^{-1} \Lambda_{22}$ collects the ratios (i.e., the generalized eigenvalues) of the diagonal elements of Λ and Ω. Thus, J contains the generalized eigenvalues on its diagonal (i.e., when α_{jj} denotes the diagonal elements of matrix Λ and δ_{jj} denotes the diagonal elements of matrix Ω, then the generalized eigenvalues on the diagonal of matrix J are the ratios of these diagonal elements of Λ and Ω), such that

$$J = \begin{bmatrix} \frac{\alpha_{11}}{\delta_{11}} & & & * \\ & \frac{\alpha_{22}}{\delta_{22}} & & \\ & & \ddots & \\ * & & & \frac{\alpha_{jj}}{\delta_{jj}} \end{bmatrix}. \tag{28}$$

Shifting equation (27) one period forward and solving for $w_{2,t}$, we finally obtain

$$w_{2,t} = Jw_{2,t+1} - \Omega_{22}^{-1} Q_2 \left(\Phi \varepsilon_{t+1} + \Psi \eta_{t+1} \right). \tag{29}$$

Iterating forward yields:

$$w_{2,t} = -\sum_{n=1}^{\infty} J^{n-1} \Omega_{22}^{-1} Q_2 \left(\Phi \varepsilon_{t+n} + \Psi \eta_{t+n} \right). \tag{30}$$

Here, it is assumed that $\lim_{n\to\infty} J^n w_{2,t+n} = 0$. Since equation (30) contains future fundamental shocks and forecast errors, taking expectations leads to

$$w_{2,t} = -E_t \left\{ \sum_{n=1}^{\infty} J^{n-1} \Omega_{22}^{-1} Q_2 \Phi \varepsilon_{t+n} \right\}. \tag{31}$$

Since under perfect foresight the transient interest-rate peg is perfectly known to the agents in the economy, $E_t\left[\varepsilon_{t+n}\right]$ will be *nonzero* for the time period the central bank actually fixes the policy rate (i.e., $P > 0$) and zero afterwards. If some of the diagonal elements of J turn out to be complex, they can be written in polar form. Define the complex diagonal elements $\frac{\alpha_{jj}}{\delta_{jj}} = z_{jj}$. Then, we can write $z_{jj} = a + bi$ or in polar form $z_{jj} = r\left(\cos\phi + i \sin\phi\right)$.[16] If now – because of known nonzero future

[16] a describes the real part of a complex eigenvalue and bi describes the imaginary part. While

ε_{t+n} – also powers of J enter the solution for $w_{2,t}$, we can write (by de Moivre's formula):

$$z_{jj}^k = r^k (\cos k\phi + i \sin k\phi), \quad \text{for } k = 0, ..., P-1. \tag{32}$$

Hence, the forward solution of the system involves trigonometric functions, which depend on the length P of a given interest-rate peg. Thus, the longer the central bank keeps the policy rate fixed (i.e., the bigger P is), the farther we "move" along the trigonometric functions contained on the diagonal elements of matrix J. As a consequence, with an increasing duration of pegged policy rates, the simulations (which we presented in figure 3) first approach an asymptote (i.e., the effect of an additional period of pegged policy rates grows exponentially) and afterwards the simulations switch their sign before they reach another asymptote and switch their sign again, and so on.

While the reversal completely vanishes if one shuts down inflation indexation in the context of the 3-equation DNK model, as indicated by Carlstrom et al. (2015), this should not be expected for the more elaborate medium-sized model presented here, since this model contains several other backward-looking elements (i.e., endogenous state variables like capital, wages, net worth, etc.). And it is the interplay of backward and forward-looking elements in the model that can give rise to complex-valued eigenvalues and, thus, sign switches in the model's simulations.

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a and b are real numbers describing a pair of numerical Cartesian coordinates, r and ϕ denote the corresponding polar coordinates (i.e., distance and angle).

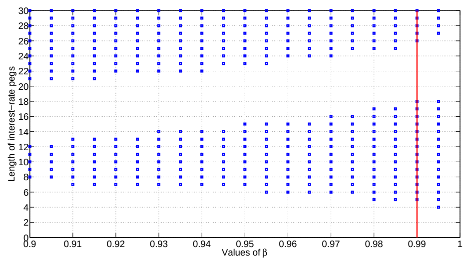
5.3 A sensitivity analysis

One might conjecture that the results thus far depend very much on the specific parameterization of the model. In this section, we document that this is not the case. To this end, we conduct an extensive grid search over the model's structural parameters and illustrate for which duration of the anticipated interest-rate peg reversals in the initial response of inflation occur. Specifically, we vary each parameter one-by-one, holding the other parameters constant at their benchmark values, to document that the reversal does not arise only for a very specific paramterization of the model.

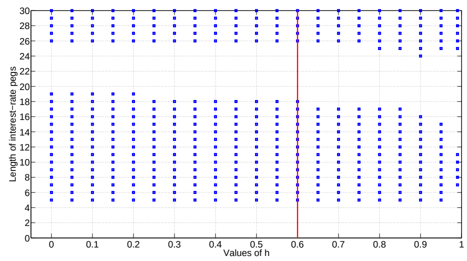
The household sector: Figure 4 shows that simply reducing the forward-lookingness of the agents in the economy by decreasing the discount parameter β (see subpanel (a) in the upper left), or increasing the backward-lookingness of the agents by increasing the consumption habit parameter, does not prevent reversals from occuring. We only observe that the duration of the peg that is required for the reversal to appear changes. For instance, if households discount future consumption more heavily (a smaller value for β), the peg has to be a few quarters longer in order for the reversal to appear (this is consistent with Carlstrom et al. (2015) who document that the implementation of a *discounted Euler equation*, along the lines of McKay, Nakamura and Steinsson (2016), does not resolve the reversal puzzle). In addition, variations in the parameters determining the investment decision of the households (i.e., the capital depreciation rate δ and the capital adjustment costs ψ_i shown in the subpanels (c) and (d)) do not prevent sign switches.

Figure 4: Duration of nominal interest-rate peg for which the reversal puzzle occurs for different values of the household sector's structural parameters

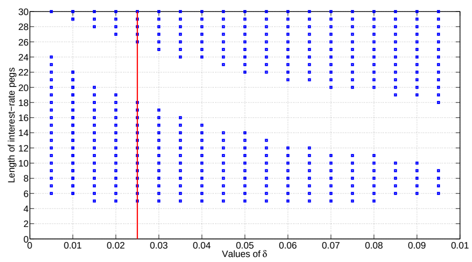
(a) Discount parameter β



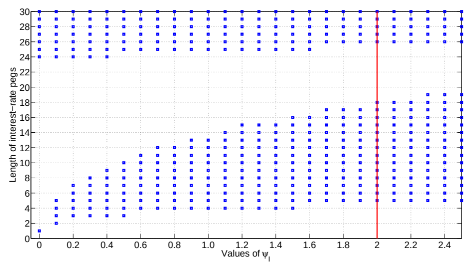
(b) Habit parameter h



(c) Capital depreciation δ



(d) Capital adjustment cost ψ_i

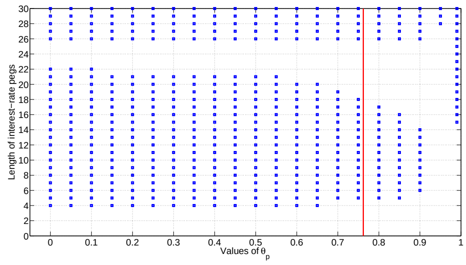


Note: The figure shows simulations for increasing durations of a pegged policy rate of up to thirty quarters for different parameter values. The blue points indicate a different sign compared to the scenario without interest-rate peg. The red line marks the calibrated value for the respective parameters, which were used to carry out the analysis in subsections 5.1 and 5.2.

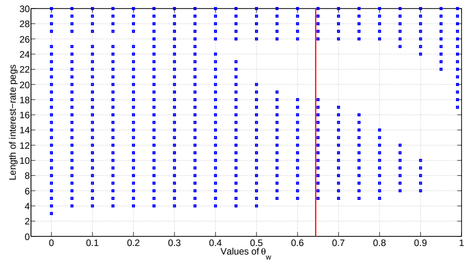
The firm sector: Figure 5 presents results from our grid search over the parameter values which drive the behavior of the price and wage setters (i.e., the Calvo parameters for prices, θ_p, and wages, θ_w, as well as the parameters for price and wage indexation, ι_p and ι_w). Once again, we observe that the required duration of the interest-rate peg in order for the reversal to appear, varies with different parameter values. However, we observe that if firms behave in a less forward-looking manner (i.e., for Calvo parameters for prices and wages > 0.9), the peg has to be a few years longer in order for the reversal to appear.

Figure 5: Duration of nominal interest-rate peg for which the reversal puzzle occurs for different values of the firm sector's structural parameters

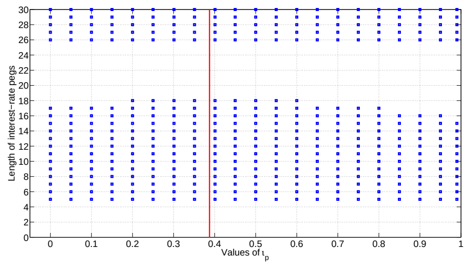
(a) Calvo parameter: prices θ_p



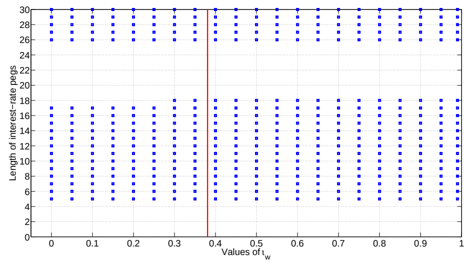
(b) Calvo parameter: wages θ_w



(c) Price indexation ι_p



(d) Wage indexation ι_w



Note: The figure shows simulations for increasing durations of a pegged policy rate of up to thirty quarters. Again, the blue points indicate a different sign compared to the scenario without interest-rate peg. The red line marks the calibrated value for the respective parameters, which were used to carry out the analysis in subsections 5.1 and 5.2.

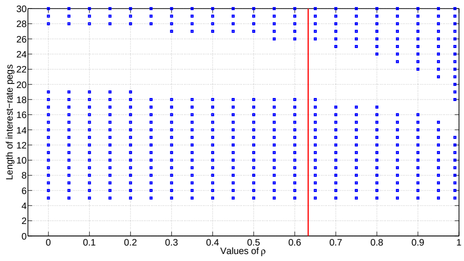
It should be noted that even if we shut down price and wage indexation jointly (i.e., $\iota_p = 0$ and $\iota_w = 0$) and re-run all the grids for all structural parameter values, sign reversals still occur. Thus, beyond indexation, there remain elements in the present model that produce sign switches.

Monetary policy: Figure 6, finally, presents the results from a grid search over the Taylor-rule coefficients, which become active, of course, only after the peg has ended.[17] As before, the occurrence of sign switches in the simulations does not depend on individual parameter values. Only the length of the peg, for which the reversal occurs, is affected by changes of the parameters. In particular, a more aggressive inflation stabilization (i.e., a higher coefficient τ_π) requires a longer duration of the interest-rate peg in order for the reversal to occur. Introducing history dependence by means of interest rate smoothing does not prevent reversals from occurring, either; see upper left panel in figure 6.

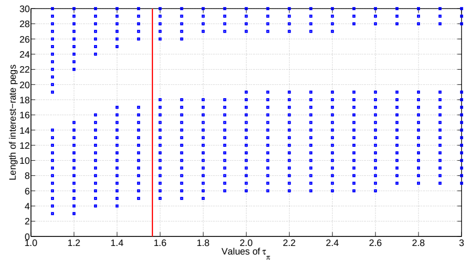
[17] Note that the agents perfectly anticipate the duration of forward guidance. Thus, they are perfectly aware of the point in time at which the Taylor rule is in place again.

Figure 6: Duration of nominal interest-rate peg for which the reversal puzzle occurs for different values of the Taylor-rule parameters

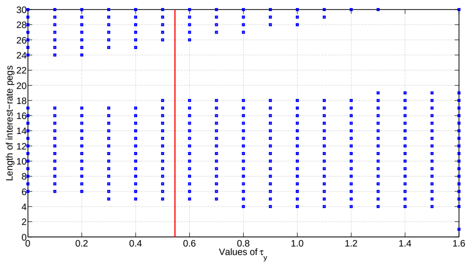
(a) Interest rate smoothing ρ



(b) MP reaction coefficient on inflation τ_π



(c) MP reaction coefficient on output τ_y



Note: The figure shows simulations for increasing durations of a pegged policy rate of up to thirty quarters. Once more, the blue points indicate a different sign compared to the scenario without interest-rate peg. The red line marks the calibrated value for the respective parameters, which were used to carry out the analysis in subsections 5.1 and 5.2.

6 Overcoming the reversal puzzle

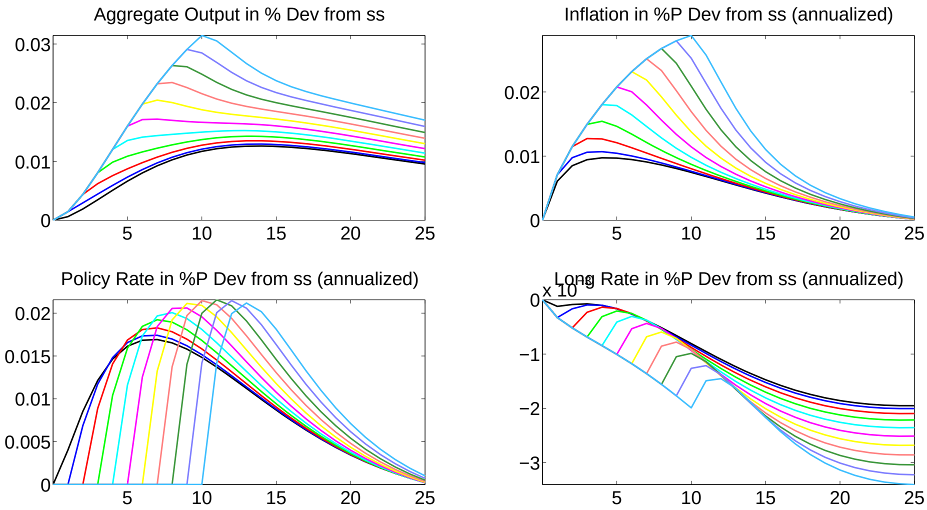
6.1 No anticipation

Thus far, we have illustrated that the reversal puzzle is a very robust feature of the simulation set-up. Due to the presence of complex-valued eigenvalues, the model's dynamics switch sign, depending on the duration of a temporary interest-rate peg.

However, the complex eigenvalues on the main diagonal of the matrix J only imply a sign switch in the model simulations if the agents *anticipate* the interest-rate peg (i.e., $E_t [\varepsilon_{t+n}] \neq 0$). Thus, an obvious modification is to assume that agents do not anticipate the interest-rate peg. Instead, they are surprised each period that the interest rate is kept constant.

To examine such a scenario, we use a procedure that has recently also been employed by Arias, Erceg and Trabandt (2016), Christiano, Eichenbaum and Trabandt (2015), and Adjemian and Juillard (2013). In contrast to the perfect foresight approach, the endogenous variables are now computed by running a deterministic simulation for each period of the simulation horizon with the previous period as an initial condition for the next period and the steady state as terminal condition. In each period, agents now expect that the exogenous shocks will be zero for all future periods, i.e., they assume $E_t (\varepsilon_{t+n}) = 0$. Less technically, they expect that monetary policy does not peg its policy rate for an extended period of time P. Thus, at every step of the simulation (i.e., in each period), future shocks will not be anticipated, which implies that certainty equivalence applies. Figure 7 presents the corresponding simulation results.

Figure 7: Simulation results of a QE shock in combination with an unanticipated interest-rate peg of variable duration



Note: The figure shows simulation results for output, inflation, short-term interest rate, and long-term interest rate, based on a QE shock in combination with an unanticipated interest-rate peg of duration P, with $0 \leq P \leq 10$. The period zero denotes the steady state. The QE shock, together with the first forward guidance shock, occurs in period one. The simulations are carried out based on the assumption that both price and wage indexation are present (i.e., $\iota_p = 0.3876$ and $\iota_w = 0.3807$).

In the absence of anticipation, the dynamics of aggregate output and inflation neither explode nor reverse their sign, even if we extend the duration of the peg considerably. To see why this is the case, consider equation (31), which we show here again for convenience:

$$w_{2,t} = -E_t \left\{ \sum_{n=1}^{\infty} J^{n-1} \Omega_{22}^{-1} Q_2 \Phi \varepsilon_{t+n} \right\}. \tag{33}$$

Recall: we solve the non-linear model for each period of the entire simulation horizon. In each period, the model is solved under the assumption that $\varepsilon_{t+n} = 0$ for all $n > 0$. Thus, now the solution for $w_{2,t}$ does not depend on the matrix J anymore, such that the corresponding complex eigenvalues are no longer relevant. As a consequence, the simulated time paths of the model will not move along the trigonometric functions resulting from the complex elements on the main diagonal of matrix J. Thus, the explosive complex eigenvalues cannot induce explosive or cyclical effects in the solution of $w_{2,t}$. The model-implied dynamics following a QE shock and a corresponding implementation of a transient interest-rate peg therefore deliver orthodox results.

6.2 Price-level targeting

While the anticipation of the interest-rate peg and therefore the expectations of the agents about future shocks are key to understanding the reversal puzzle, it is somewhat unrealistic, or at least unattractive, to assume, as in the last section, that agents are not able to take into account announced monetary policy measures like forward guidance. We now present a way of overcoming the puzzle without imposing such a strong assumption.

An effective way of making a noteworthy change in what agents expect is to assume that monetary policy follows a price-level-targeting rule after the nominal interest-rate peg. Indeed, price-level targeting is known to be able to induce stabilization effects, even if the interest rate is kept constant. This is because price-level targeting implies history dependence of monetary policy, which in turn strengthens the *expectations channel*.

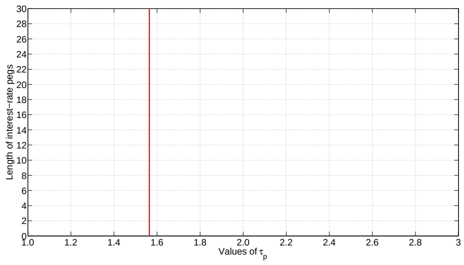
To illustrate the consequences of such a change in the monetary policy strategy, we therefore substitute the Taylor rule in Equation (16) with the following price-level targeting rule:

$$R_t = \varepsilon_t^{TR}\left(R_{ss}\right) + \left(1 - \varepsilon_t^{TR}\right)\left(R_{t-1}\right)^{\rho}\left(R_{ss}P_t^{\tau_p}\left(\frac{Y_t}{Y_{t-1}}\right)^{\tau_y}\right)^{1-\rho}, \tag{34}$$

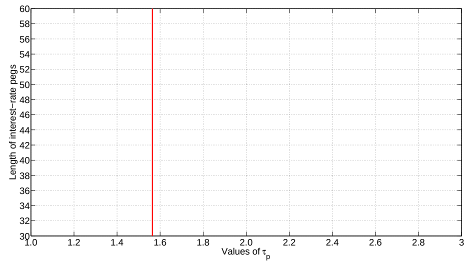
where P_t describes the price level, and τ_p the response coefficient of monetary policy to deviations of the price level from its target (which is equal to one in our analysis).

Figure 8: Duration of nominal interest-rate peg for which inflation reversals occur for different values of τ_p

(a) Duration of pegged rates: one to thirty quarters



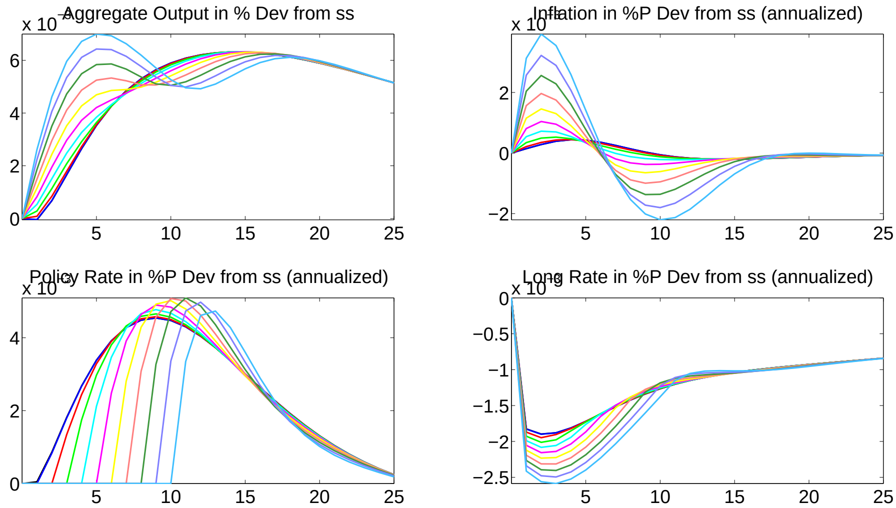
(b) Duration of pegged rates: thirty to sixty quarters



Note: The figure shows simulations for increasing duration of a pegged policy rate. As above, the blue points indicate a different sign compared to the scenario without interest-rate peg. The red line marks the calibrated value for τ_p.

Figure 8 shows the results from a corresponding grid search analysis. The 'empty' figure highlights that if the monetary authority switches to a price-level-targeting rule, the reversal puzzle completely vanishes, even for much longer durations of pegged policy rates, i.e., for more than fifteen years. Why is this the case? The main difference between inflation targeting and price-level targeting is that, under the former strategy, shocks to the price level are treated as *bygones*. Thus, a policy that aims at keeping the future inflation rate close to a given target value in the medium term, will not correct for past shocks. Instead, under price-level targeting, the central bank commits itself to correcting deviations of the price level from its target. Thus, if there is a shock that pushes the price level away from the target price path, future inflation will be required to adjust in a way to bring the price path back to target. As a result, past deviations from target are corrected under price-level targeting. If monetary policy is credible, the forward-looking rational agents are going to expect a complete correction of past deviations. This, in turn, dampens agents' expectations about output and inflation deviations and therefore mitigates the expansionary effect of the anticipated nominal interest-rate peg together with the expansionary QE program on output and inflation from the very beginning. Due to the muted output and inflation responses, sign reversals do not occur even for very long durations of pegged nominal interest rates. Put differently, the response of inflation to, for instance, a series of forward guidance shocks (i.e., shocks governing the nominal interest-rate peg, ε_t^{TR}) will neither grow exponentially and reach an asymptote nor will it reverse its sign.

Figure 9: Macroeconomic implications of nominal interest-rate peg under price-level targeting



Note: The figure shows simulations for output, inflation, and the short- as well as long-term interest rates, for an increasing duration of a pegged policy rate (i.e., zero to ten periods). The period zero denotes the steady state. The QE shock, together with the first forward guidance shock, occurs in period one.

Figure 9 presents the simulated time paths for output, inflation, and the short- as well as long-term interest rate. The model implies plausible dynamics under reasonable policy scenarios, and even more importantly, the simulations do not change dramatically for only minor changes in the policy scenario.

It is important to note that, with the price-level-targeting specification, the complex eigenvalues, showing up in the solution of the system, do not vanish (see also table 3 in appendix C). Put differently, the existence of complex eigenvalues is thus merely a necessary but not a sufficient condition for the reversal to appear. Intuitively, to observe a reversal, the complex eigenvalues of the forward and backward solution have to 'interact' in a very specific way. To understand why these complex eigenvalues do not induce cyclical effects in the case of price-level targeting – even though there are *anticipated* future shocks hitting the system – we have to take a look at the solution of the system of difference equations as a whole.

Having already solved for $w_{2,t}$ in equation (31), the final step is to solve for $w_{1,t}$. Following Sims (2001) again, we consider the unique solution of the system[18] in terms of $w_t = [w_{1,t}, w_{2,t}]'$, which is formulated below as

$$\begin{aligned}\begin{bmatrix}\Lambda_{11} & \Lambda_{12}-\Xi\Lambda_{22}\\ 0 & I\end{bmatrix}\begin{bmatrix}w_{1,t}\\ w_{2,t}\end{bmatrix} &= \begin{bmatrix}\Omega_{11} & \Omega_{12}-\Xi\Omega_{22}\\ 0 & 0\end{bmatrix}\begin{bmatrix}w_{1,t-1}\\ w_{2,t-1}\end{bmatrix}+\begin{bmatrix}Q_1-\Xi Q_2\\ 0\end{bmatrix}(\Phi\varepsilon_t)\\ &- E_t\begin{bmatrix}0\\ \sum_{n=1}^{\infty} J^{n-1}\Omega_{22}^{-1}Q_2\Phi\varepsilon_{t+n}\end{bmatrix},\end{aligned}$$

where the matrix Ξ describes the relationship between fundamental and expectational errors, i.e., $Q_1\Psi = \Xi Q_2\Psi$.

Rearranging and taking the inverse formula for partitioned matrices into account

[18]As derived in equation (43) in Sims (2001).

yields:

$$
\begin{aligned}
\begin{bmatrix} w_{1,t} \\ w_{2,t} \end{bmatrix} &= \begin{bmatrix} \Lambda_{11}^{-1} & -\Lambda_{11}^{-1}\left(\Lambda_{12}-\Xi\Lambda_{22}\right) \\ 0 & I \end{bmatrix} \begin{bmatrix} \Omega_{11} & \Omega_{12}-\Xi\Omega_{22} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} w_{1,t-1} \\ w_{2,t-1} \end{bmatrix} \\
&+ \begin{bmatrix} \Lambda_{11}^{-1} & -\Lambda_{11}^{-1}\left(\Lambda_{12}-\Xi\Lambda_{22}\right) \\ 0 & I \end{bmatrix} \begin{bmatrix} Q_1-\Xi Q_2 \\ 0 \end{bmatrix} \left(\Phi\varepsilon_t\right) \\
&- \begin{bmatrix} \Lambda_{11}^{-1} & -\Lambda_{11}^{-1}\left(\Lambda_{12}-\Xi\Lambda_{22}\right) \\ 0 & I \end{bmatrix} \begin{bmatrix} 0 \\ \sum_{n=1}^{\infty} J^{n-1}\Omega_{22}^{-1}Q_2\Phi \end{bmatrix} E_t\varepsilon_{t+n}.
\end{aligned}
$$

Thus, the solution for $w_{1,t}$ is also affected by nonzero future ε_{t+n} via the matrix J

$$
w_{1,t} = \ldots - \Lambda_{11}^{-1}\left(\Lambda_{12}-\Xi\Lambda_{22}\right)\left(\sum_{n=1}^{\infty} J^{n-1}\Omega_{22}^{-1}Q_2\Phi\right) E_t\varepsilon_{t+n}. \tag{35}
$$

In contrast to the forward solution (i.e., the solution for $w_{2,t}$), the matrices Λ_{11}, Λ_{12}, and Ξ now enter the expression for the solution of the stable part of the system (i.e., the solution for $w_{1,t}$). This implies that these matrices may govern the whole system in such a way that the complex eigenvalues contained in both the stable and the unstable parts may 'cancel' each other out in the system. If this is the case, anticipated future shocks will no longer induce cyclical effects in the simulation of the model.

To reiterate, given the assumption of perfect foresight, the presence of complex eigenvalues in the solution of the system is only a necessary but not a sufficient condition for the reversal puzzle to appear. It seems to be the *balance* between stable and unstable complex elements in the solution of the model that matters.

This reinforces the suggestion by Carlstrom et al. (2015), who argue that the source of the reversal puzzle is a feedback between forward and backward-looking factors of the underlying model. To us, this explanation is mirrored in the complex elements of the solution of the underlying system of difference equations.

7 Conclusion

The reversal puzzle is one of several peculiarities (like the forward guidance puzzle, paradox of toil etc.) that the New Keynesian model can produce when the nominal interest rate is constrained due to the zero lower bound or a temporary interest rate peg. In this study, we demonstrate that the reversal puzzle is a relevant and tenacious phenomenon in a medium-scale DSGE model (with many endogenous state variables) by analyzing a plausible policy scenario in which the model economy is affected by a QE program and forward guidance.

We provide results from extensive numerical research suggesting that the reversal puzzle appears for a wide range of different parameterizations of the model under investigation. In particular, we show that the reversal puzzle is not just the outcome of an irrelevant or negligible parameter constellation. It is rather an empirically relevant part of the overall parameter space that delivers inflation reversals for plausible durations of forward guidance.

We discuss two modifications of our analysis to overcome the reversal puzzle which shed light on the important role of agents' expectations in delivering the puzzle. The first modification is to assume that agents in the model economy act under certainty equivalence, i.e., they take the nominal interest-rate peg into account only contemporaneously, but expect the peg to be absent in the future. The second modification is to assume a price-level targeting rule instead of a Taylor rule while keeping the assumption of perfect foresight. Both modifications suggest the conclusion that the reversal puzzle disappears when expectations do not respond strongly to future shocks – in the first modification because the agents do not take into account future shocks at all (and, correspondingly, do not adjust their expectations), and in the second modification because the agents expect the central bank to fully correct for the shocks that make the price level deviate from its target path (which implies that agents correspondingly adjust their inflation expectations from the very beginning).

The modifications we present to overcome the reversal puzzle illustrate the mechanisms and the determinants of the puzzle, contributing to a better understanding of the phenomenon as such. However, in a sense, they are not practical solutions: Having a specific (policy) analysis in mind, there might be good reasons not to assume certainty equivalence or a central bank targeting the price level, not least because these elements might be undesirable from an empirical point of view. Developing practical solutions in this sense might be a promising avenue for future research.

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A Non-linear equilibrium conditions

Below, we show the non-linear equilibrium conditions of the model (without corresponding AR(1) processes) outlined in section 2. They largely correspond to Appendix A in Carlstrom et al. (2017).

$$\Lambda_t = \frac{\mathrm{rn}_t}{C_t - hC_{t-1}} - E_t \frac{\beta h \mathrm{rn}_{t+1}}{C_{t+1} - hC_t} \tag{A.1}$$

$$\Lambda_t = E_t \beta \frac{\Lambda_{t+1}}{\Pi_{t+1}} R_t^d \tag{A.2}$$

$${w_t}^{1+\varepsilon_w \eta} = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{X_t^{wn}}{X_t^{wd}} \tag{A.3}$$

$$X_t^{wn} = \lambda_{w,t} b_t \chi w_t^{\varepsilon_w (1+\eta)} H_t^{1+\eta} + E_t \left\{ \theta_w \beta \Pi_{t+1}^{\varepsilon_w (1+\eta)} \Pi_t^{-\iota_w \varepsilon_w (1+\eta)} X_{t+1}^{wn} \right\} \tag{A.4}$$

$$X_t^{wd} = \Lambda_t w_t^{\varepsilon_w} H_t + \theta_w \beta \Pi_t^{-\iota_w (\varepsilon_w - 1)} \Pi_{t+1}^{(\varepsilon_w - 1)} E_t \left\{ X_{t+1}^{wd} \right\} \tag{A.5}$$

$${w_t}^{1-\varepsilon_w} = (1 - \theta_w) \left(w_t^* \right)^{1-\varepsilon_w} + \theta_w \left(\frac{\Pi_{t-1}^{\iota_w} w_{t-1}}{\Pi_t} \right)^{1-\varepsilon_w} \tag{A.6}$$

$$\Lambda_t M_t P_t^k = E_t \beta \Lambda_{t+1} \left[R_{t+1}^k + M_{t+1} P_{t+1}^k (1 - \delta) \right] \tag{A.7}$$

$$\Lambda_t M_t Q_t = E_t \frac{\beta \Lambda_{t+1} \left(1 + \kappa Q_{t+1} \mathrm{M}_{t+1} \right)}{\Pi_{t+1}} \tag{A.8}$$

$$V_t^h = b_t \left\{ \ln \left(C_t - hC_{t-1} \right) - D_t^w \chi \frac{H_t^{1+\eta}}{1+\eta} \right\} + \beta E_t V_{t+1}^h \tag{A.9}$$

$$R_t^k = mc_t \mathrm{MPK}_t \tag{A.10}$$

$$w_t = mc_t \mathrm{MPL}_t \tag{A.11}$$

$$\Pi_t^* = \frac{\varepsilon_p}{\varepsilon_p - 1} \frac{X_t^{pn}}{X_t^{pd}} \Pi_t \tag{A.12}$$

$$X_t^{pn} = Y_t \lambda_{p,t} mc_t(i) + E_t \left\{ \theta_p \frac{\beta \Lambda_{t+1}}{\Lambda_t} \Pi_t^{-\iota_p \varepsilon_p} \Pi_{t+1}^{\varepsilon_p} X_{t+1}^{pn} \right\} \tag{A.13}$$

$$X_t^{pd} = Y_t + E_t \left\{ \theta_p \frac{\beta \Lambda_{t+1}}{\Lambda_t} \Pi_t^{\iota_p (1-\varepsilon_p)} \Pi_{t+1}^{\varepsilon_p - 1} X_{t+1}^{pd} \right\} \tag{A.14}$$

$$\left(\Pi_t \right)^{1-\varepsilon_p} = (1 - \theta_p) \left(\Pi_t^* \right)^{1-\varepsilon_p} + \theta_p \left(\Pi_{t-1}^{\iota_p} \right)^{1-\varepsilon_p} \tag{A.15}$$

$$D_t^p = \Pi_t^{\varepsilon_p}\left[(1-\theta_p)\Pi_t^{*-\varepsilon_p} + \theta_p\left(\Pi_{t-1}^{\iota_p}\right)^{-\varepsilon_p} D_{pt-1}\right] \tag{A.16}$$

$$D_t^w = \theta_w \left(\frac{\Pi_t}{\Pi_{t-1}^{\iota_w}}\right)^{\varepsilon_w} \left(\frac{w_t}{w_{t-1}}\right)^{\varepsilon_w} D_{wt-1} + (1-\theta_w)\left(\frac{w_t^*}{w_t}\right)^{-\varepsilon_w} \tag{A.17}$$

$$Y_t = C_t + I_t \tag{A.18}$$

$$Y_t = A_t K_t^{\alpha} H_t^{1-\alpha} / D_{pt} \tag{A.19}$$

$$K_t = (1-\delta)\, K_{t-1} + \varphi \left(1 - \psi_I \left(\frac{1}{2}\right)\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) I_t \tag{A.20}$$

$$P_t^k \varphi_t \left\{1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right\} =$$

$$1 - \beta P_{t+1}^k \frac{\Lambda_{t+1}}{\Lambda_t} \varphi_{t+1} \left\{-S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right\} \tag{A.21}$$

$$\bar{B}_t + \bar{F}_t = N_t + L_t \tag{A.22}$$

$$L_t = \frac{E_t \frac{\Lambda_{t+1}}{\Pi_{t+1}}}{\left[E_t \frac{\Lambda_{t+1}}{\Pi_{t+1}} + (\Phi_t - 1)\, E_t \frac{\Lambda_{t+1}}{\Pi_{t+1}} \frac{R_{t+1}^L}{R_t^d}\right]} \tag{A.23}$$

$$P_t^k I_t = \bar{F}_t - \kappa \frac{\bar{F}_t}{\Pi_t} \frac{Q_t}{Q_{t-1}} \tag{A.24}$$

$$\Lambda_t \left[1 + f\left(N_t\right) + N_t f'\left(N_t\right)\right] = E_t \Lambda_{t+1} \beta \zeta \frac{P_t}{P_{t+1}} \left[\left(R_{t+1}^L - R_t^d\right) L_t + R_t^d\right] \tag{A.25}$$

$$R_t^L = \frac{(1 + \kappa Q_t)}{Q_{t-1}} \tag{A.26}$$

$$R_t^{10} = Q_t^{-1} + \kappa \tag{A.27}$$

$$MPK_t = \alpha A_t K_{t-1}(i)^{\alpha-1} H_t(i)^{1-\alpha} \tag{A.28}$$

$$MPL_t = (1-\alpha)\, A_t K_{t-1}(i)^{\alpha} H_t(i)^{-\alpha} \tag{A.29}$$

$$R_t = \varepsilon_t^{TR}\left(R_{ss}\right) + \left(1 - \varepsilon_t^{TR}\right)\left(R_{t-1}\right)^{\rho} \left(R_{ss} \Pi_t^{\tau_\Pi} \left(\frac{Y_t}{Y_{t-1}}\right)^{\tau_y}\right)^{1-\rho} \tag{A.30}$$

B Steady-state & parameter values

Table 2: Parameter values

β	Discount factor	0.99
α	Capital share	0.33
δ	Depreciation rate	0.025
h	Habit parameter	0.6
η	Inverse elasticity of labor supply	2
ϵ_p	Elasticity of substitution implying steady-state price markup	5
ϵ_w	Elasticity of substitution implying steady-state wage markup	5
ι_p	Price indexation	0.3876
ι_w	Wage indexation	0.3807
θ_p	Calvo Parameter (Prices)	0.7621
θ_w	Calvo Parameter (Wages)	0.6443
ρ	Interest rate smoothing	0.6327
τ_Π	MP Inflation coefficient	1.5643
τ_y	MP Output coefficient	0.5459
ψ_i	Investment adjustment cost	2
ψ_n	Portfolio adjustment cost	0.79
κ	Long-term bond coupon	0.975
Π_{ss}	Steady-state gross inflation	1
L_{ss}	Steady-state leverage level	6
$\frac{\bar{B}_{ss}}{\bar{B}_{ss}+\bar{F}_{ss}}$	Ratio of government securities to total FI assets	0.35

C Eigenvalues under inflation targeting and price-level targeting

Table 3: Eigenvalues of the system

Inflation targeting			Price-level targeting		
Modulus	Real	Imaginary	Modulus	Real	Imaginary
—	—	—	9.433e-17	-9.433e-17	0
2.382e-17	2.382e-17	0	2.217e-16	2.217e-16	0
4.499e-16	-4.499e-16	0	0.3947	0.3947	0
0.3676	0.3676	0	0.4391	0.4391	0
0.5936	0.5936	0	0.6443	0.6443	0
0.6409	0.6339	0.09459	0.6799	0.6799	0
0.6409	0.6339	-0.09459	0.7382	0.7382	0
0.6443	0.6443	0	0.7424	0.6879	0.2793
0.7382	0.7382	0	0.7424	0.6879	-0.2793
0.7621	0.7621	0	0.7621	0.7621	0
0.8717	0.8685	0.07484	0.8588	0.8564	0.06393
0.8717	0.8685	-0.07484	0.8588	0.8564	-0.06393
0.9618	0.9618	0	0.9618	0.9618	0
0.9797	0.9797	0	0.9818	0.9818	0
1.01	1.01	0	1.01	1.01	0
1.036	1.036	0	1.036	1.036	0
1.069	1.069	0.03547	1.048	1.048	0
1.069	1.069	-0.03547	1.161	1.161	0
1.171	1.171	0	1.296	1.214	0.4539
1.225	1.185	0.3098	1.296	1.214	-0.4539
1.225	1.185	-0.3098	1.306	1.306	0
1.325	1.325	0	1.325	1.325	0
1.35	1.35	0	1.402	1.402	0
1.568	1.568	0	1.568	1.568	0
1.594	1.594	0	1.638	1.638	0
Inf	Inf	0	Inf	Inf	0
Inf	Inf	0	Inf	Inf	0
Inf	Inf	0	Inf	Inf	0
Inf	-Inf	0	Inf	-Inf	0
Inf	-Inf	0	Inf	-Inf	0