Discussion Paper

The optimal conduct of central bank asset purchases

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Non-technical summary

Research Question

During the course of the global financial crisis, central banks embarked on various forms of unconventional monetary policy, one of which was asset purchase programmes. In most cases, central bank asset purchases were deployed once the scope for a more accommodative monetary policy stance through interest rate cuts had been exhausted. Although the general effects of asset purchases have been discussed quite intensively, less is known about the consequences of the exact form which the asset purchase programmes take.

Contribution

The aim of this paper is to discuss the optimal conduct of unconventional monetary policies using an estimated DSGE model for the euro area. On the one hand, we evaluate various quantity-based government bond purchase programmes in terms of their time profile and the information set agents have. This is done for a realistic lower bound scenario for the policy rate, where "realistic" means that it is based upon observed (and expected) shocks. On the other hand, we discuss the optimality of asset purchases following a welfare-based approach. In this regard, we show in a lower bound case for the policy rate what optimal asset purchases would look like and how they stabilise the economy. The paper focuses *inter alia* on the interplay between forward guidance on the policy rate and asset purchases. Regarding the welfare evaluation, we first ignore the lower bound constraint on the policy rate before explicitly focusing on it.

Results

Considering *ad hoc* asset purchase programmes, we show that their macroeconomic multipliers get stronger when the lower bound on the policy rate is binding and when the purchasing path is fully communicated and anticipated by economic agents. From a more normative standpoint, interest rate policy and asset purchases feature strong strategic complementarities during both normal and crisis times. In an environment when nominal interest rates reach their effective lower bound, optimal monetary policy is to keep the policy rate low for a longer period in time and to engage in asset purchases. Our results also point to a clear sequencing of the exit strategy, first stopping the asset purchases and later raising the policy rate. In terms of macroeconomic stabilisation, optimal asset purchase strategies deliver sizeable benefits and have the potential to largely offset the costs of the lower bound on the policy rate.

Nichttechnische Zusammenfassung

Fragestellung

Im Verlauf der globalen Finanzkrise führten Zentralbanken verschiedene unkonventionelle Politikinstrumente ein, zu denen Wertpapierankaufprogramme zählten. In den meisten Fällen wurden Anleiheankaufprogramme als Ergänzung von *forward guidance* für den Leitzins genutzt, um eine weitere monetäre Akkommodation beim Erreichen der Nullzinsgrenze durchzuführen. Angesichts begrenzter Erfahrungen mit der Umsetzung von Ankaufprogrammen besteht Untersuchungsbedarf hinsichtlich der genauen Ausgestaltung des Kaufpfades und seiner Konsequenzen für die Ökonomie.

Beitrag

Das Ziel des Papiers ist es, Wertpapierankaufprogramme als unkonventionale geldpolitische Maßnahmen hinsichtlich ihrer optimalen Ausgestaltung in einem für den Euroraum geschätzten DSGE-Modell zu untersuchen. Ein Fokus liegt dabei auf der Bewertung unterschiedlicher Pfade für mengenorientierte Staatsanleihenkaufprogramme unter Berücksichtigung eines realistischen Szenarios für die Nullzinsgrenze. Darüber hinaus wird mithilfe eines wohlfahrtsmaximierenden Ansatzes ein "optimaler" Pfad für den Bestand an Staatsanleihen bei der Zentralbank hergeleitet. Dieser Ansatz erlaubt es zu zeigen, inwiefern Staatsanleihenkäufe in der Lage sind, als zusätzliches Stabilisierungsinstrument zu dienen. Dabei werden die Fälle mit und ohne Nullzinsgrenze verglichen. Für den Fall der Nullzinsgrenze ist es somit möglich, die Interaktion zwischen optimalen Wertpapierkäufen und optimalem Zinspfad zu zeigen.

Ergebnisse

Für die *ad hoc*-Wertpapierankaufprogramme zeigt sich, dass ihre Stabilisierungswirkungen an der Nullzinsgrenze umso effektiver sind, je länger diese gilt. Eine vollständige Kommunikation und Antizipation des Programmms erzielt dabei die stärksten Wirkungen. Unter normativen Gesichtspunkten können Staatsanleihenkäufe hinsichtlich der Stabilisierung der Wirtschaft an der Nullzinsgrenze die Verwendung von *forward guidance* unterstützen und dabei die Verweildauer an der Nullzinsgrenze reduzieren. Auch außerhalb der Nullzinsgrenze entfaltet ein Wertpapierankaufprogramm Stabilisierungswirkungen. Der Wohlfahrtsansatz impliziert, dass Kaufprogramme an die wirtschaftliche Entwicklung angepasst werden sollten. Im Fall der Nullzinsgrenze zeigt der hergeleitete Pfad, dass der Bestand an Staatsanleihen bei der Zentralbank zuerst reduziert werden sollte, bevor die Zinsen angehoben werden.

The optimal conduct of central bank asset purchases[*]

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Abstract

We analyse the effects of central bank government bond purchases in an estimated DSGE model for the euro area. In the model, central bank asset purchases are relevant in so far as agency costs distort banks' asset allocation between loans and bonds, and households face transaction costs when trading government bonds. Such frictions in the banking sector induce inefficient time-variation in the term premia and allow for a credit channel of central bank government bond purchases. Considering *ad hoc* asset purchase programmes like the one implemented by the ECB, we show that their macroeconomic multipliers get stronger when the lower bound on the policy rate becomes binding and when the purchasing path is fully communicated and anticipated by the agents. From a more normative standpoint, interest rate policy and asset purchases feature strong strategic complementarities during both normal and crisis times. In an environment when nominal interest rates reach their effective lower bound, optimal monetary policy is to keep the policy rate low for a longer period in time and to engage in asset purchases. Our results also point to a clear sequencing of the exit strategy, first stopping the asset purchases and later raising the policy rate. In terms of macroeconomic stabilisation, optimal asset purchase strategies deliver sizeable benefits and have the potential to largely offset the costs of the lower bound on the policy rate.

Keywords: Zero Lower Bound, Optimal Monetary Policy, Banking, Quantitative Easing, DSGE.

JEL classification: C61, E52, G11.

1 Introduction

During the course of the global financial crisis, central banks embarked on various forms of unconventional monetary policy, one of which was asset purchase programmes. In most cases, central bank asset purchases were deployed once the scope for a more accommodative monetary policy stance through interest rate cuts had been exhausted. They were adopted in conjunction with some form of forward guidance on the future path of the policy rate, far beyond standard practice in normal times. Some monetary authorities also gave some clear indications of the sequencing of the exit strategy. Although the general effects of asset purchases have been discussed quite intensively, less is known about the consequences of the form which asset purchase programmes take.

This paper aims to discuss the optimal conduct of unconventional monetary policy using an estimated DSGE model for the euro area. The objective of the paper is twofold: on the one hand, we evaluate various quantity-based government bond purchase programmes in terms of their time profile and the information set agents have. This is done for a realistic lower bound scenario for the policy rate, where "realistic" means that it is based upon observed (and expected) shocks. On the other hand, we discuss the optimality of asset purchases following a welfare-based approach. In this regard, we show in a lower bound case for the policy rate what optimal asset purchases would look like and how they stabilise the economy. The paper focuses *inter alia* on the interplay between forward guidance on the policy rate and asset purchases. Regarding the welfare evaluation we first ignore the lower bound constraint on the policy rate before explicitly focusing on it.

In the first part of the paper we consider *ad hoc* asset purchase programmes of the central bank, instead of a rule-based policy which would be conditional on the state of the economy. Although programmes recently introduced by central banks, such as the ECB's asset purchase programme, have been recalibrated along with material changes in the inflation outlook, the first implementation can be regarded as a regime shift in the conduct of policy due to their unprecedented nature. Therefore, our prime interest is directed at evaluating the unexpected announcement of a one-off purchase programme. In this regard, we follow the literature on government output multipliers (Christiano, Eichenbaum, and Rebelo, 2011a). Specifically, we analyse the macroeconomic transmission of *ad hoc* programmes in an unconstrained environment and in the presence of the lower bound on the policy rate. Such an occasionally binding constraint brings some non-linearity into the model and makes the macroeconomic multipliers of central bank asset purchases quite sensitive to the underlying crisis scenario. Furthermore, we discuss the specific modalities of the *ad hoc* programme. Altogether, our results show that central bank asset purchases are more powerful *i)* in an environment in which the policy rate has reached its effective lower bound, *ii)* the longer the duration of the lower bound period, *iii)* when, at the lower bound, the programme is fully communicated and anticipated, and *iv)* when it is complemented by forward guidance extending the lift-off date for the policy rate beyond agents' expectations formulated on the basis of how policy is conducted during normal times.

In the second part, we take a normative perspective and apply similar optimal policy concepts as proposed by Eggertsson and Woodford (2003) to derive a path for government bond purchases. Optimal paths for government bonds and the short rate are derived under commitment (similar to Adam and Billi (2006)). Optimal policy including asset purchases

exploits the strategic complementarities between the two policy instruments. They feature distinctive propagation channels and different macroeconomic stabilisation properties and should not be considered a perfect substitute. Within the confines of model validity, optimal policy in the presence of the effective lower bound on interest rate displays: *i)* a longer period of binding lower bound constraint and strong use of forward guidance, *ii)* activist asset purchase policy and *iii)* a sequencing of the exit strategy, first stopping asset purchases and later raising the policy rate. In terms of macroeconomic stabilisation, optimal asset purchase strategy delivers sizeable benefits and has the potential to largely offset the costs associated with the lower bound constraint on the policy rate.

Evidence has built up on the effectiveness of such unconventional policies in affecting financial prices, credit conditions and expenditure decisions through a variety of channels: direct effects on the price of assets in the targeted market segment (see, for example, Hancock and Passmore (2011) or Altavilla, Giannone, and Lenza (2014)), changes in expectations due to the signalling effect of the programmes (see *inter alia* Krishnamurthy and Vissing-Jorgensen (2011), Gilchrist and Zakrajšek (2013) or Joyce, Lasaosa, Stevens, and Tong (2011).), and more indirect effects *via* the portfolio decisions of banks and other financial institutions. In the euro area, which would constitute the empirical basis for this paper, the pass-through of asset purchases on sovereign yields and on broader financing costs, notably bank lending rates, appeared significant and might put the emphasis on bank-based transmission channels (see notably ECB (2015)). For this reason, we focus on government bond purchases and on their impact on the bank credit channel through portfolio rebalancing.

In our model, banking frictions affect the pricing of long-term government bonds and create a term premium. Conventional monetary policy has an impact on the economy by affecting consumption and savings decisions as in traditional models without a banking sector. It also feeds into banks' funding costs and bank asset valuation. Through these channels, conventional monetary policy influences the provision of loans to non-financial agents. In equilibrium, banks' capital structure and asset composition are determined jointly with the excess returns on loans and government bonds. The banking frictions do indeed limit arbitrage in the sovereign bond market and lead to endogenous time-variation in the term premium which might complicate macroeconomic stabilisation through conventional monetary policy. This is particularly true if the policy rate reaches its lower bound. Monetary policy can nevertheless have an impact on long-term rates via the expectation hypothesis of the term structure by communicating the future path of the policy rate (forward guidance). However, term premia are not directly affected by forward guidance, and central bank asset purchases can be used as an instrument of monetary policy to affect long-term rates if the lower bound holds for short-term rates.

More precisely, our modelling strategy consists of introducing the minimal set of frictions into established DSGE models with satisfactory empirical properties in order to account for bank portfolio decisions between sovereign holdings and loan contracts. The specification of the DSGE model is first inherited from Smets and Wouters (2007) for the non-financial blocks and the estimation strategy. We introduce a segmented banking sector à la Gerali, Neri, Sessa, and Signoretti (2010) and Darracq Pariès, Kok, and Palenzuela (2011) and allow for risky corporate debt contracts à la Bernanke, Gertler, and Gilchrist (1999) with predetermined lending rates. Finally, for the bank portfolio allocation frictions, we follow the approach of Gertler and Karadi (2013). In our model,

central bank asset purchases are relevant in so far as agency costs constrain the asset allocation of banks between loans and bonds, and households face transaction costs when trading government bonds.

The estimation of the DSGE model enriches the analysis of this paper along two dimensions. First, it enables crisis scenarios to be designed which are more realistic than the ones contemplated in the closely related literature. We would argue that the macroeconomic multipliers evaluated for the *ad hoc* central bank asset purchase programme have satisfactory empirical plausibility in this respect. Second, the estimation provides a realistic set of structural business cycle shocks for the euro area. Such shock distributions are instrumental for quantifying the stabilisation gains from the optimal conduct of policy.

Our paper is linked to the normative debate on monetary policy frameworks which has been intensified by the crisis. At the beginning of this discussion, Eggertsson and Woodford (2003) dismissed the usefulness of "pure" quantitative easing policies (i.e. policies aimed at replacing short-term assets with excess reserves) at the lower bound of interest rates provided that some appropriate form of forward guidance was implemented. Later on, Cúrdia and Woodford (2011) revisited the potential benefits of targeted asset purchases, to the extent that the financial system was significantly disrupted and the unconventional policies could deliver adequate credit easing. Our model also consists of various frictions which create wedges between risk-free interest rates and ultimate borrowing rates. These wedges are determined endogenously in the general equilibrium in our model. Our paper is also close to Ellison and Tischbirek (2014) or Jones and Kulish (2013), who provide arguments for using an active asset purchase strategy as an additional instrument in normal times and when the policy rate hits the effective lower bound. We contribute to this discussion by explicitly deriving the optimal path of asset purchases jointly with the optimal path for the policy rate. To the best of our knowledge, we are the first to discuss the optimal interplay between policy measures based upon an estimated model with an elaborated banking sector which resembles some real world features.

The remainder of this paper is organised as follows. Section 2 describes the main features of the DSGE model, highlighting the key frictions which are essential to the transmission of central bank asset purchases as well as to the empirical performance of the model. Section 3 presents the estimation of the DSGE model. Section 4 starts with a comparison of a standard and a non-standard monetary policy shock and then explores the macroeconomic multipliers of central bank asset purchases when the monetary policy rate is constrained at its effective lower bound and the central bank implements an *ad hoc* asset purchase programme. Some sensitivity analysis regarding the implementation design of the programme is also performed. Section 5 derives the optimal policy and elaborates on the desirability of combining two instruments through the cycle and in times of crisis.

2 The model

The model consists of households, goods producers, capital producers, non-financial corporations (called entrepreneurs) investing in capital projects, and banks which fund the projects of non-financial corporations. Since households cannot provide their savings directly to the real sector, banks need to intermediate these funds. Both entrepreneurs and banks are exposed to endogenous borrowing constraints. Additionally, the loan market operates under imperfect competition. Hence, financial frictions and market power in the loan market create inefficiencies in borrowing conditions. The real sector is rather standard and features staggered prices and wages.

The model is based upon Smets and Wouters (2007) regarding the real sector and combines elements in the banking sector from Gertler and Karadi (2011, 2013), Gerali et al. (2010), and Darracq Pariès et al. (2011) with elements from Christiano, Motto, and Rostagno (2014) as done similarly by Rannenberg (2016) and Kühl (2016). The model economy evolves along a balanced-growth path driven by a positive trend, γ, in the technological progress of the intermediate goods production and a positive steady state inflation rate, $\pi^\star$. In the description of the model, stock and flow variables are expressed in real and effective terms (unless stated otherwise): they are deflated by the price level and the technology-related balanced-growth path trend.

2.1 Households

The economy is populated by a continuum of heterogenous infinitely-lived households. Each household is characterised by the quality of its labour services, $h \in [0,1]$. At time t, the intertemporal utility function of a generic household h is

$$\mathcal{W}_t(h) = \mathbb{E}_t \sum_{j=0}^{\infty} \left(\beta\gamma^{1-\sigma_c}\right)^j \varepsilon_{t+j}^b \mathcal{U}\left(C_{t+j}(h) - \eta C_{t+j-1}(h)\diagup\gamma, N_{t+j}^S(h)\right)$$

with β as the time preference rate. Household h obtains utility $\mathcal{U}$ from consumption of an aggregate index $C_t(h)$, relative to an internal habit depending on its past consumption η, while receiving disutility from the supply of their homogenous labour $N_t^S(h)$. Utility also incorporates a consumption preference shock ε_t^b. The growth rate of technology is given by γ, and σ_c is the intertemporal elasticity of substitution.

Each household h maximises its intertemporal utility under the following budget constraint:

$$D_t(h) + Q_{B,t}\left[B_{H,t}(h) + \frac{1}{2}\chi_H\left(B_{H,t}(h) - \overline{B}_H\right)^2\right] + C_t(h)$$

$$= \frac{R_{D,t-1}}{\pi_t}D_{t-1}(h)\diagup\gamma + \frac{R_{G,t}}{\pi_t}Q_{B,t-1}B_{H,t-1}\diagup\gamma$$

$$+ \frac{(1-\tau_{w,t})W_t^h N_t^S(h) + A_t(h) + T_t(h)}{P_t} + \Pi_t(h)$$

where P_t is an aggregate price index, $R_{D,t}$ is the one period ahead nominal gross deposit rate, $D_t(h)$ a deposit, $R_{G,t}$ the return on government bonds, $Q_{B,t}$ the price of the govern-

ment bond, and $B_{H,t}(h)$ the quantity of government bonds with $\overline{B}_H$ as the corresponding steady state value. W_t^h is the nominal wage, $T_t(h)$ are government transfers (both expressed in effective terms), and $\tau_{w,t}$ is a time-varying labour tax. $\Pi_t(h)$ corresponds to the profits net of transfers from the various productive and financial segments owned by the households. χ_H is the households' portfolio adjustment cost. A positive value of χ_H prevents full (frictionless) arbitrage of the returns on securities by the household sector. Finally, $A_t(h)$ is a nominal stream of income (both in effective terms) coming from state contingent securities and equating marginal utility of consumption across households $h \in [0,1]$.

In equilibrium, households' choices in terms of consumption, hours and deposit holdings are identical. The first-order condition of the household problem with respect to government bond holdings is

$$\mathbb{E}_t \left[\Xi_{t,t+1} \frac{(R_{G,t+1} - R_{D,t})}{\pi_{t+1}} \right] = \chi_H \left(B_{H,t} - \overline{B}_H \right) \tag{1}$$

where $\Xi_{t,t+1}$ is the period t stochastic discount factor of the households for nominal income streams at period $t+1$.

The instantaneous household utility $\mathcal{U}$ has the following functional form

$$\mathcal{U}(X_1, X_2) = \frac{X_1^{1-\sigma_c}}{1 - \sigma_c} \exp \left(\widetilde{L} \frac{(\sigma_c - 1)}{(1 + \sigma_l)} X_2^{1+\sigma_l} \right).$$

where $\tilde{L}$ is a positive scale parameter. The first-order conditions of the household problem with respect to consumption, labour, deposit are

$$\Lambda_t = \mathcal{U}'_{1,t} - \beta \gamma^{-\sigma_c} \eta \mathbb{E}_t \mathcal{U}'_{1,t+1} \tag{2}$$

$$\Lambda_t \frac{W_t^h}{P_t} = \mathcal{U}'_{2,t} \tag{3}$$

$$\mathbb{E}_t \left[\Xi_{t,t+1} \frac{R_{D,t}}{\pi_{t+1}} \right] = 1 \tag{4}$$

where Λ_t is the Lagrange multiplier associated with the budget constraint and $\Xi_{t,t+1} = \beta \gamma^{-\sigma_c} \frac{\Lambda_{t+1}}{\Lambda_t}$ is the period t stochastic discount factor of the households for nominal income streams at period $t+1$.

2.2 Labour supply and wage setting

Intermediate goods producers make use of a labour input N_t^D produced by a segment of labour packers. Those labour packers operate in a competitive environment and aggregate a continuum of differentiated labour services $N_t(i)$, $i \in [0,1]$ using a Kimball (1995) technology.

The Kimball aggregator is defined by

$$\int_0^1 H \left(\frac{N_t(i)}{N_t^D}; \theta_w, \psi_w \right) \mathrm{d}i = 1$$

5

where we consider the following functional form:

$$
\begin{aligned}
H\left(\frac{N_t(i)}{N_t^D}\right) &= \frac{\theta_w}{(\theta_w(1+\psi_w)-1)}\left[(1+\psi_w)\frac{N_t(i)}{N_t^D}-\psi_w\right]^{\frac{\theta_w(1+\psi_w)-1}{\theta_w(1+\psi_w)}} \\
&\quad -\left[\frac{\theta_w}{(\theta_w(1+\psi_w)-1)}-1\right]
\end{aligned}
$$

This function, where the parameter ψ_w determines the curvature of the demand curve, has the advantage that it reduces to the standard Kimball aggregator under the restriction $\psi_w = 0$.

The differentiated labour services are produced by a continuum of unions which transform the homogeneous household labour supply. Each union is a monopoly supplier of a differentiated labour service and sets its wage on a staggered basis, paying households the nominal wage rate W_t^h. Every period, any union faces a constant probability $1 - \alpha_w$ of optimally adjusting its nominal wage, say $W_t^*(i)$, which will be the same for all suppliers of differentiated labour services.

We denote thereafter w_t as the aggregate real wage, expressed in effective terms, that intermediate producers pay for the labour input provided by the labour packers and w_t^* as the effective real wage claimed by reoptimising unions.

When they cannot reoptimise, wages are indexed on past inflation and steady state inflation according to the following indexation rule:

$$
W_t(i) = \gamma \left[\pi_{t-1}\right]^{\xi_w} \left[\pi^\star\right]^{1-\xi_w} W_{t-1}(i)
$$

with $\pi_t = \frac{P_t}{P_{t-1}}$ the gross rate of inflation.

Taking into account that they might not be able to choose their nominal wage optimally in the near future, $W_t^*(i)$ is chosen to maximise their intertemporal profit under the labour demand from labour packers. Unions are subject to a time-varying tax rate $\tau_{w,t}$ which is affected by an i.i.d shock defined by $1 - \tau_{w,t} = (1 - \tau_w^\star)\varepsilon_t^w$.

2.3 Banks

The banking sector is owned by the households and is segmented in various parts: bankers, retail branches and loan officers. First, bankers collect household deposits and provide funds to the retail lending branches. As in Gertler and Karadi (2011, 2013), bankers can divert funds and depositors enforce on them an incentive constraint which forces the bankers to hoard a sufficient level of net worth. This creates a financing cost wedge related to bank capital frictions. Second, retail lending branches receive funding from the bankers and allocate it to the loan officers. In the retail segment, a second wedge results from banks operating under monopolistic competition and facing nominal rigidity in their interest rate setting. In the third segment of the banking sector, loan officers extend loan contracts to entrepreneurs, which implies a third financing cost wedge related to credit risk compensation.

2.3.1 Bankers

Every period, a fraction $(1 - f)$ of households' members are workers, while a fraction fe are entrepreneurs and the remaining mass $f(1 - e)$ are bankers. Bankers face a probability ζ_b of remaining bankers over the next period and a probability $(1 - \zeta_b)$ of becoming a worker again. When a banker exits, the accumulated earnings are transferred to the respective household, while newly entering bankers receive initial funds from their household. Overall, households transfer a real amount $\Psi_{B,t}$ to new bankers for each period t. As shown later in this section, bankers' decisions are identical, so we will focus on the decision problem for a representative banker.

Bankers operate in competitive markets providing loans to retail lending branches, $L_{BE,t}$, and purchasing government securities, $B_{B,t}$, at price $Q_{B,t}$. To finance their lending activity, bankers receive deposits, D_t, from households, with a gross interest rate, $R_{D,t}$, and accumulate net worth, $NW_{B,t}$. Their balance identity, in real terms, reads

$$L_{BE,t} + Q_{B,t}B_{B,t} = D_t + NW_{B,t}. \tag{5}$$

The accumulation of the bankers' net worth from period t to period $t+1$ results from the gross interest received from the loans to the retail lending bank, the gross return on government bond holdings, $R_{G,t+1}$, the lump-sum share of profits (and losses) coming from retail lending and loan officers' activity, $\Pi^R_{B,t+1}$, per unit of each banker's net worth, minus the gross interest paid on deposits:

$$NW_{B,t+1} = \frac{R^B_{N,t+1}}{\pi_{t+1}}NW_{B,t}\big/\gamma.$$

with

$$R^B_{N,t+1} \equiv (R_{BLE,t} - R_{D,t})\,\kappa^l_{B,t} + (R_{G,t+1} - R_{D,t})\,\kappa^g_{B,t} + R_{D,t} + \Pi^R_{B,t+1} \tag{6}$$

$$\kappa^l_{B,t} \equiv \frac{L_{BE,t}}{NW_{B,t}} \text{ and } \kappa^g_{B,t} \equiv \frac{Q_{B,t}B_{B,t}}{NW_{B,t}} \tag{7}$$

Iterating this equation backward implies

$$NW_{B,t+1} = \widetilde{R}^B_{N,t+1-s,t+1}NW_{B,t+1-s}\big/\gamma^s \tag{8}$$

where $\widetilde{R}^B_{N,t+1-s,t+1} = \prod\limits_{i=0}^{s}\left\{\frac{R^B_{N,t+1-i}}{\pi_{t+1-i}}\right\}$ and $\widetilde{R}^B_{N,t+1-s,t+1-s} = 1$. The bankers' objective is to maximise their terminal net worth when exiting the industry, which occurs with probability $(1 - \zeta_b)$ each period. The value function for each banker is therefore given by

$$\mathcal{V}_{B,t} = (1 - \zeta_b)\sum_{k=0}^{\infty}(\zeta_b)^k\,\Xi_{t,t+k+1}NW_{B,t+k+1}$$

Using (8), the value function can be written recursively as follows

$$\mathcal{V}_{B,t} = (1 - \zeta_b)\,NW_{B,t}\,(\mathcal{X}_{B,t} - 1)$$

7

with

$$\mathcal{X}_{B,t} = 1 + \zeta_b \mathbb{E}_t \left[\Xi_{t,t+1} \frac{R^B_{N,t+1}}{\pi_{t+1}} \mathcal{X}_{B,t+1} \right].$$

As in Gertler and Karadi (2013), bankers can divert a fraction of their assets and transfer them without costs to the households. In this case, the depositors force the default on the intermediary and will only recover the remaining fraction of the asset. The corresponding incentive compatibility constraint is

$$\begin{align} \mathcal{V}_{B,t} &\geqslant \lambda_b \left(L_{BE,t} + \delta_b Q_{B,t} B_{B,t} \right) \tag{9} \\ &\geqslant \lambda_b \left(\kappa^l_{B,t} + \delta_b \kappa^g_{B,t} \right) NW_{B,t}. \end{align}$$

The diversion rate for private loans is λ_b and $\lambda_b \delta_b$ for government securities. Under the parameter values considered thereafter, the constraints are assumed to always bind in the vicinity of the steady state.

Given their initial net worth, the end-of-period t contracting problem for bankers consists of maximising $\mathcal{V}_{B,t}$ for the exposures to private sector loans $\kappa^l_{B,t}$ and government securities $\kappa^g_{B,t}$ subject to the incentive constraint (9):

$$\mathcal{V}_{B,t} = \max_{\{\kappa^l_{B,t}, \kappa^g_{B,t}\}} \left\{ \zeta_b \widetilde{\mathcal{X}}_{B,t} NW_{B,t} \right\} \tag{10}$$

where we denoted $\widetilde{\mathcal{X}}_{B,t} \equiv (\mathcal{X}_{B,t} - 1) \frac{(1-\zeta_b)}{\zeta_b}$ and $\widetilde{\mathcal{X}}_{B,t}$ follows

$$\widetilde{\mathcal{X}}_{B,t} = \mathbb{E}_t \left[\Xi_{t,t+1} \frac{R^B_{N,t+1}}{\pi_{t+1}} \left(\zeta_b \widetilde{\mathcal{X}}_{B,t+1} + (1-\zeta_b) \right) \right]. \tag{11}$$

Note that the stream of transfers $\Pi^R_{B,t+1+s}$ is considered exogenous by bankers in their decision problem, which implies that $\frac{\partial \Pi^R_{B,t+1+s}}{\partial \kappa^l_{B,t}} = 0$.

The first-order conditions for this problem can then be formulated as

$$\mathbb{E}_t \left[\Xi_{t,t+1} \frac{\partial R^B_{N,t+1}}{\partial \kappa^l_{B,t}} \left(\zeta_b \widetilde{\mathcal{X}}_{B,t+1} + (1-\zeta_b) \right) \Big/ \pi_{t+1} \right] = \mu_t \lambda_b \tag{12}$$

$$\mathbb{E}_t \left[\Xi_{t,t+1} \frac{\partial R^B_{N,t+1}}{\partial \kappa^g_{B,t}} \left(\zeta_b \widetilde{\mathcal{X}}_{B,t+1} + (1-\zeta_b) \right) \Big/ \pi_{t+1} \right] = \mu_t \lambda_b \delta_b \tag{13}$$

where μ_t is the Lagrange multiplier related to the incentive constraint.

Aggregating across bankers, a fraction ζ_b continues to operate into the next period while the rest exits from the industry. The new bankers are endowed with starting net worth, proportional to the assets of the old bankers. Accordingly, the aggregate dynamics of bankers' net worth is given by

$$NW_{B,t} = \zeta_b \frac{R^B_{N,t}}{\pi_t} NW_{B,t-1} \Big/ \gamma + \Psi_{B,t}. \tag{14}$$

2.3.2 Retail lending branches and loan officers

A continuum of retail lending branches indexed by j, provides differentiated loans to loan officers. The total financing needs of loan officers follow a CES aggregation of differentiated loans $L_{E,t} = \left[\int_0^1 L_{E,t}(j)^{\frac{1}{\mu_E^R}} \mathrm{d}j \right]^{\mu_E^R}$. Differentiated loans are imperfect substitutes, with the elasticity of substitution $\frac{\mu_E^R}{\mu_E^R - 1} > 1$. The corresponding average return on loans is $R_{LE} = \left[\int_0^1 R_{LE}(j)^{\frac{1}{1-\mu_E^R}} \mathrm{d}j \right]^{1-\mu_E^R}$.

Retail lending branches are monopolistic competitors which levy funds from the *bankers* and set gross nominal interest rates on a staggered basis *à la* Calvo (1983), each period facing a constant probability $1 - \xi_E^R$ of being able to reoptimise. If a retail lending branch cannot reoptimise its interest rate, the interest rate is left at its previous-period level:

$$R_{LE,t}(j) = R_{LE,t-1}(j)$$

The retail lending branch j chooses $\hat{R}_{LE,t}(j)$ to maximise its intertemporal profit

$$\mathbb{E}_t \left[\sum_{k=0}^{\infty} \left(\beta \gamma^{-\sigma_c} \xi_E^R \right)^k \Xi_{t,t+k+1} \left(\hat{R}_{LE,t}(j) L_{E,t+k}(j) - R_{BLE,t+k}(j) L_{E,t+k}(j) \right) \right]$$

where the demand from the loan officers is given by

$$L_{E,t+k}(j) = \left(\frac{\hat{R}_{LE,t}(j)}{R_{LE,t}} \right)^{-\frac{\mu_E^R}{\mu_E^R - 1}} \left(\frac{R_{LE,t}}{R_{LE,t+k}} \right)^{-\frac{\mu_E^R}{\mu_E^R - 1}} L_{LE,t+k}$$

and $R_{BLE,t}$ is the gross funding rate on the loans from the *bankers*. In the model, the staggered setting of the lending rate acts as maturity transformation in banking activity and leads to imperfect pass-through of market interest rates to bank lending rates.

Finally, loan officers operate in perfect competition. They receive one-period loans from the retail lending branches, which cost an aggregate gross nominal interest rate $R_{LE,t}$, set at the beginning of period t. They extend loan contracts to entrepreneurs which pay a state-contingent return $\widetilde{R}_{LE,t+1}$. Loan officers have no other source of funds, so that the volume of the loans they provide to the entrepreneurs equals the volume of funding they receive. Loan officers seek to maximise their discounted intertemporal flow of income so that the first-order condition of their decision problem gives

$$\mathbb{E}_t \left[\Xi_{t,t+1} \frac{\left(\widetilde{R}_{LE,t+1} - R_{LE,t} \right)}{\pi_{t+1}} \right] = 0. \tag{15}$$

Profits and losses made by retail branches and loan officers are transferred back to the bankers.

2.4 Entrepreneurs

As explained before, every period, a fraction fe of the representative household's members are entrepreneurs. Like bankers, each entrepreneur faces a probability ζ_e of remaining an entrepreneur over the next period and a probability $(1 - \zeta_e)$ of becoming a worker again. To keep of the share of entrepreneurs constant, we assume that a similar number of workers randomly become entrepreneurs. When an entrepreneur exits, their accumulated earnings are transferred to the respective household. At the same time, newly entering entrepreneurs receive initial funds from their household. Overall, households transfer a real amount $\Psi_{E,t}$ to the entrepreneurs for each period t. Finally, as it will become clear later, entrepreneurs' decisions on leverage and the lending rate are independent of their net worth and are therefore identical. Accordingly, we will highlight the decision problem of a representative entrepreneur.

At the end of the period t entrepreneurs buy the capital stock K_t from the capital producers at the real price Q_t (expressed in terms of consumption goods). They transform the capital stock into an effective capital stock $u_{t+1}K_t$ by choosing the utilisation rate u_{t+1}. The adjustment of the capacity utilisation rate entails some adjustment costs per unit of capital stock $\Gamma_u(u_{t+1})$. The cost (or benefit) Γ_u is an increasing function of capacity utilisation and is zero at steady state. The functional forms used for the adjustment costs on capacity utilisation are given by $\Gamma_u(u_t) = \frac{\overline{r_K}}{\varphi}(\exp[\varphi(u_t - 1)] - 1)$ with $\overline{r_K}$ as the steady-state rental rate for capital and φ as a scaling parameter.

The effective capital stock can then be rented out to intermediate goods producers at a nominal rental rate of $r_{K,t+1}$. Finally, by the end of period $t+1$, entrepreneurs sell back the depreciated capital stock $(1-\delta)K_t$ to capital producer at price Q_{t+1}. The gross nominal rate of return on capital across from period t to $t+1$ is therefore given by

$$R_{KK,t+1} \equiv \pi_{t+1}\frac{r_{K,t+1}u_{t+1} - \Gamma_u(u_{t+1}) + (1-\delta)Q_{t+1}}{Q_t}. \tag{16}$$

where π_{t+1} is the inflation rate.

Each entrepreneur's return on capital is subject to a multiplicative idiosyncratic shock $\omega_{e,t}$. These shocks are independent and identically distributed across time and across entrepreneurs. $\omega_{e,t}$ follows a lognormal CDF $F_e(\omega_{e,t})$, with mean 1 and variance $\sigma_{e,t}$ which is assumed to be time-varying. By the law of large numbers, the average across entrepreneurs (denoted with the operator $\widetilde{E}$) for the expected return on capital is given by $\widetilde{E}[\mathbb{E}_t(\omega_{e,t+1}R_{KK,t+1})] = \mathbb{E}_t\left(\int_0^\infty \omega_{e,t+1}\mathrm{d}F_{e,t}(\omega)R_{KK,t+1}\right) = \mathbb{E}_t(R_{KK,t+1})$.

The entrepreneur's choice of capacity utilisation is independent of the idiosyncratic shock and implies that

$$r_{K,t} = \Gamma'_u(u_t). \tag{17}$$

Entrepreneurs finance their purchase of capital stock with their net worth $NW_{E,t}$ and a one-period loan $L_{E,t}$ (expressed in real terms, deflated by the consumer price index) from the commercial lending branches:

$$Q_tK_t = NW_{E,t} + L_{E,t}. \tag{18}$$

In the tradition of costly-state-verification frameworks, lenders cannot observe the realisation of the idiosyncratic shock unless they pay a monitoring cost μ_e per unit of

assets that can be transferred to the bank in case of default. We constrain the set of lending contracts available to entrepreneurs. They can only use debt contracts in which the lending rate $R_{LLE,t}$ is predetermined at the previous time period.

Default will occur when the entrepreneurial income that can be generated by the lender falls short of the agreed repayment of the loan. At period $t+1$, once aggregate shocks are realised, this will happen for draws of the idiosyncratic shock below a certain threshold $\overline{\omega}_{e,t}$, given by

$$\overline{\omega}_{e,t+1}\chi_e R_{KK,t+1}\kappa_{e,t} = R_{LLE,t}\left(\kappa_{e,t}-1\right) \tag{19}$$

where $R_{LLE,t}$ is the nominal lending rate determined at period t and $\kappa_{e,t}$ is the corporate leverage defined as

$$\kappa_{e,t} = \frac{Q_t K_t}{NW_{E,t}.} \tag{20}$$

χ_e represents the share of the entrepreneur's assets (gross of capital return) that banks can recover in case of default. When banks take over the entrepreneur's assets, they have to pay the monitoring costs.

The ex post return to the lender on the loan contract, denoted $\widetilde{R}_{LE,t}$, can then be expressed as

$$\widetilde{R}_{LE,t} = G_e(\overline{\omega}_{e,t})\chi_e R_{KK,t}\frac{\kappa_{e,t-1}}{\kappa_{e,t-1}-1} \tag{21}$$

where

$$G_e(\overline{\omega}) = (1 - F_e(\overline{\omega}))\overline{\omega} + (1-\mu_e)\int_0^{\overline{\omega}}\omega dF_e(\omega).$$

We assume that entrepreneurs are myopic and the end-of-period t contracting problem for entrepreneurs consists in maximising the next-period return on net worth for the lending rate and leverage:

$$\max_{\{R_{LLE,t},\kappa_{e,t}\}}\mathbb{E}_t\left[(1-\chi_e\Gamma_e(\overline{\omega}_{e,t+1}))R_{KK,t+1}\kappa_{e,t}\right]$$

subject to the participation constraint of the lender (15), the equation (19) for the default threshold $\overline{\omega}_{e,t+1}$, and with

$$\Gamma_e(\overline{\omega}) = (1 - F_e(\overline{\omega}))\overline{\omega} + \int_0^{\overline{\omega}}\omega dF_e(\omega).$$

After some manipulations, the first-order conditions for the lending rate and the leverage lead to

$$\mathbb{E}_t\left[(1-\chi_e\Gamma_e(\overline{\omega}_{e,t+1}))R_{KK,t+1}\kappa_{e,t}\right] = \frac{\mathbb{E}_t\left[\chi_e\Gamma_e'(\overline{\omega}_{e,t+1})\right]}{\mathbb{E}_t\left[\Xi_{t,t+1}G_e'(\overline{\omega}_{e,t+1})\right]}\mathbb{E}_t\left[\Xi_{t,t+1}\right]R_{LE,t} \tag{22}$$

where

$$\Gamma_e'(\overline{\omega}) = (1 - F_e(\overline{\omega})) \text{ and } G_e'(\overline{\omega}) = (1 - F_e(\overline{\omega})) - \mu_e\overline{\omega}dF_e(\overline{\omega}).$$

As anticipated at the beginning of the section, the solution to the problem shows that all entrepreneurs choose the same leverage and lending rate. Moreover, the features of the contracting problem imply that the *ex post* return to the lender $\widetilde{R}_{LE,t}$ will differ from

the *ex ante* return $R_{LE,t-1}$. Log-linearising equation (22) and the participation constraint (15), one can show that innovations in the *ex post* return are notably driven by innovations in $R_{KK,t}$.

Finally, aggregating across entrepreneurs, a fraction ζ_e continues operating into the next period while the rest exits from the industry. The new entrepreneurs are endowed with starting net worth, proportional to the assets of the old entrepreneurs. Accordingly, the aggregate dynamics of entrepreneurs' net worth is given by

$$NW_{E,t} = \zeta_e \left(1 - \chi_e \Gamma_e(\overline{\omega}_{e,t}) \right) \frac{R_{KK,t}}{\pi_{t-1}} \kappa_{e,t-1} NW_{E,t-1} \diagup \gamma + \Psi_{E,t}. \tag{23}$$

In the estimation, we also introduce a shock on the net worth of entrepreneurs which can be rationalised either as time-varying transfers to new entrepreneurs $\Psi_{E,t}$, or as a multiplicative shock on the survival probability of entrepreneurs, $\varepsilon_t^{\zeta_e}$.

2.5 Capital producers

Using investment goods, a segment of perfectly competitive firms, owned by households, produce a stock of fixed capital. At the beginning of period t, those firms buy back the depreciated capital stocks $(1-\delta)K_{t-1}$ at real prices (in terms of consumption goods) Q_t. Then they augment the various stocks using distributed goods and facing adjustment costs. The augmented stocks are sold back to entrepreneurs at the end of the period at the same prices. The decision problem of capital stock producers is given by

$$\max_{\{K_t, I_t\}} \mathbb{E}_t \sum_{k=0}^{\infty} \Xi_{t,t+k} \left\{ Q_{t+k}(K_{t+k} - (1-\delta)K_{t+k-1} \diagup \gamma) - I_{t+k} \right\}$$

subject to the constraints

$$K_t = (1-\delta)K_{t-1} \diagup \gamma + \left[1 - S \left(\gamma \frac{I_t \varepsilon_t^I}{I_{t-1}} \right) \right] I_t$$

S is a non-negative adjustment cost function formulated in terms of the gross rate of change in investment and ε_t^I is an efficiency shock to the technology of fixed capital accumulation. The functional form adopted is $S\left(x\right) = \phi/2 \ \left(x - \gamma\right)^2$.

2.6 Final and intermediate goods producers

Final producers are perfectly competitive firms producing an aggregate final good Y_t, expressed in effective terms, that may be used for consumption and investment. This production is obtained using a continuum of differentiated intermediate goods $Y_t(z)$, $z \in [0,1]$ (also expressed in effective terms) with the Kimball (1995) technology. Here again, the Kimball aggregator is defined by

$$\int_0^1 G \left(\frac{Y_t(z)}{Y_t}; \theta_p, \psi \right) \mathrm{d}z = 1$$

with

$$G\left(\frac{Y_t(z)}{Y_t}\right) = \frac{\theta_p}{(\theta_p(1+\psi)-1)}\left[(1+\psi)\frac{Y_t(z)}{Y_t}-\psi\right]^{\frac{\theta_p(1+\psi)-1}{\theta_p(1+\psi)}}$$
$$-\left[\frac{\theta_p}{(\theta_p(1+\psi)-1)}-1\right].$$

The representative final goods producer maximises profits $P_t Y_t - \int_0^1 P_t(z) Y_t(z) dz$ subject to the production function, taking as given the final goods price P_t and the prices of all intermediate goods. In the intermediate goods sector, firms $z \in [0,1]$ are monopolistic competitors and produce differentiated products by using a common Cobb-Douglas technology:

$$Y_t(z) = \varepsilon_t^a \left(u_t K_{t-1}(z)/\gamma\right)^\alpha \left[N^D(z)\right]^{1-\alpha} - \Omega \tag{24}$$

where ε_t^a is an exogenous productivity shock, $\Omega > 0$ is a fixed cost and γ is the trend technological growth rate. A firm z hires its capital, $\widetilde{K}_t(z) = u_t K_{t-1}(z)$, and labour, $N_t^D(z)$, on a competitive market by minimising its production cost. Due to our assumptions on the labour market and the rental rate of capital, the real marginal cost is identical across producers. We introduce a time-varying tax on a firm's revenue that is affected by an i.i.d shock defined by $1 - \tau_{p,t} = \left(1 - \tau_p^\star\right)\varepsilon_t^p$. In each period, a firm z faces a constant (across time and firms) probability $1 - \alpha_p$ of being able to reoptimise its nominal price, say $P_t^*(z)$. If a firm cannot reoptimise its price, the nominal price evolves according to the rule $P_t(z) = \pi_{t-1}^{\xi_p}\left[\pi^\star\right]^{(1-\xi_p)}P_{t-1}(z)$, ie the nominal price is indexed on past inflation and steady state inflation. In our model, all firms that can reoptimise their price at time t choose the same level, denoted p_t^* in real terms.

2.7 Government sector and monetary policy instruments

Public expenditures $G^\star$ in steady state, expressed in effective terms, are subject to random shocks ε_t^g. The government finances public spending with labour tax, product tax and lump-sum transfers, so that the government debt $Q_{B,t} B_G$, expressed in real effective terms, accumulates according to

$$Q_{B,t} B_{G,t} = \frac{R_{G,t}}{\pi_t} Q_{B,t-1} B_{G,t-1}/\gamma + G^\star \varepsilon_t^g - \tau_{w,t} w_t L_t - \tau_{p,t} Y_t - T_t. \tag{25}$$

In the empirical analysis, we neglect the dynamics of public debt and assume that lump-sum taxes T_t are adjusted to ensure that

$$\forall t > 0, \quad B_{G,t} = \overline{B_G}.$$

In order to introduce long-term sovereign debt, we assume that government securities are perpetuities which pay geometrically-decaying coupons (c_g the first period, $(1-\tau_g)c_g$ the second one, $(1-\tau_g)^2 c_g$ the third one, etc...). The nominal return on sovereign bond holding from period t to period $t+1$ is therefore

$$R_{G,t+1} = \epsilon_{t+1}^{R_G}\frac{c_g + (1-\tau_g)Q_{B,t+1}}{Q_{B,t}}. \tag{26}$$

For the purpose of the empirical analysis, we introduced an *ad hoc* government bond valuation shock, ϵ_t^{RG}. This "reduced-form" shock is meant to capture time-variation in the excess bond return not captured by our bank-centric formulation of the term premium. In particular, the rise in sovereign risk pricing during the euro-area financial crisis is not accounted for within the micro-foundation of the model. Note that the estimation period stops before the start of the ECB's asset purchase programme, so that the introduction of the government bond valuation shock does not partially substitute for an unconventional monetary policy shock in the estimation.

Within the government sector, the monetary authority controls the deposit interest rate $R_{D,t}$. Similar to Smets and Wouters (2007), the monetary authority follows an interest rate feedback rule which incorporates terms on lagged inflation, lagged output gap and its first difference. The output gap is defined as the log-difference between actual and flexible-price output. The reaction function also incorporates a non-systematic component ε_t^r.

Written in deviation from the steady state, the interest rule used in the estimation has the form:

$$\hat{R}_{D,t} = \rho \hat{R}_{D,t-1} + (1-\rho)\left[r_\pi \hat{\pi}_{t-1} + r_y \hat{y}_{t-1}\right] + r_{\Delta y}\Delta \hat{y}_t + \log\left(\varepsilon_t^r\right) \tag{27}$$

where a hat over a variable denotes log-deviation of that variable from its deterministic steady-state level.

Finally, we assume as in Gertler and Karadi (2013) that the monetary authority can manage a bond portfolio $B_{CB,t}$. The functional form, we will discuss later.

2.8 Market clearing

In the private credit market, due to nominal rigidity in the setting of interest rates by retail banking branches, the following condition holds

$$L_{BE,t} = \Delta_{E,t}^R L_{E,t} \tag{28}$$

where $\Delta_{E,t}^R = \int_0^1 \left(\frac{R_{E,t}(j)}{R_{E,t}}\right)^{-\frac{\mu_E^R}{\mu_E^R-1}} \mathrm{d}j$ is the dispersion index among retail bank interest rates. Moreover, in equilibrium the lump-sum transfer to bankers per unit of net worth from retail lending and loan officer profits and losses is given by

$$\Pi_{B,t+1}^R = \left(\widetilde{R}_{LE,t+1} - R_{BLE,t}\right)\kappa_{B,t}^l. \tag{29}$$

We can now rewrite the recursive formulation of the bankers' value function $\mathcal{V}_{B,t}$ from equation (11) using bankers' incentive constraint (9) and first-order conditions (12)-(13). This gives a relationship between bank leverage and intermediation spreads:

$$\lambda_b \widetilde{\kappa}_{B,t}/\zeta_b = \mathbb{E}_t\left[\Xi_{t,t+1}\left(\frac{R_{BLE,t} - R_{D,t}}{\pi_{t+1}}\widetilde{\kappa}_{B,t} + \frac{\widetilde{R}_{LE,t+1} - R_{BLE,t}}{\pi_{t+1}}\kappa_{B,t}^l + R_{D,t}\right)(\lambda_b \widetilde{\kappa}_{B,t+1} + (1-\zeta_b))\right] \tag{30}$$

where we denoted $\widetilde{\kappa}_{B,t} \equiv \kappa_{B,t}^l + \delta_b \kappa_{B,t}^g$.

Finally, on the government bond market, the fixed supply is distributed across holdings

by households, bankers and the central bank:

$$B_{H,t} + B_{B,t} + B_{CB,t} = \overline{B_G}.$$

The market clearing condition on goods market is given by:

$$Y_t = C_t + I_t + G^\star \varepsilon_t^g + \Psi(u_t) K_{t-1}/\gamma + \mu_e \int_0^{\overline{\omega}} \omega \mathrm{d}F_e(\omega) K_{t-1}/\gamma \qquad (31)$$

$$\Delta_{pk,t} Y_t = \varepsilon_t^a (u_t K_{t-1}/\gamma)^\alpha (N_t^D)^{1-\alpha} - \Omega \qquad (32)$$

with $\Delta_{pk,t}$ is a price dispersion index. Equilibrium in the labour market implies that

$$\Delta_{wk,t} N_t^D = N_t^S \qquad (33)$$

with $N_t^D = \int_0^1 N_t^D(z)dz$ and $N_t^S = \int_0^1 N_t^S(h)dh$ and $\Delta_{wk,t}$ is the wage dispersion index.

3 Estimation

In this section, we present the estimation of the DSGE model which is similar to that in Smets and Wouters (2007). The model is estimated on euro-area data using Bayesian likelihood methods. We consider 10 key macroeconomic quarterly time series from 1995q1 to 2014q2: output, consumption, fixed investment, hours worked, real wages, the GDP deflator inflation rate, the three-month short-term interest rate, bank loans, bank lending spreads and the (GDP-weighted) 10-year euro-area sovereign spread. The data are not filtered before estimation, with the exception of loans which are linearly detrended. We limit the number of shocks to be equal to the number of observed variables. As in Smets and Wouters (2007), we introduce a correlation between the government spending shock and the productivity shock, $\rho_{a,g}$.

The exogenous shocks can be divided in three categories[1]:

1. Efficient shocks: AR(1) shocks on technology ϵ_t^a, investment ϵ_t^I, public expenditures ϵ_t^g and consumption preferences ϵ_t^b.

2. Inefficient shocks: ARMA(1,1) shocks on price markups ϵ_t^p, and AR(1) on wage markups ϵ_t^w.

3. Financial shocks: AR(1) shock on entrepreneurs' idiosyncratic risk $\epsilon_t^{\sigma_e}$, on entrepreneurs' net worth accumulation $\epsilon_t^{\zeta_e}$, as well as on government bond valuation $\epsilon_t^{R_G}$ in equation 26.

4. Policy shocks: AR(1) shock on the Taylor-rule residual ϵ_t^r.

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[1] All the AR(1) processes are written as: $\log(\varepsilon_t^x) = \rho_x \log(\varepsilon_{t-1}^x) + \epsilon_t^x$ where $\epsilon_t^x \sim \mathcal{N}(0, \sigma_{\varepsilon^x})$. ARMA(1,1) are of the form $\log(\varepsilon_t^x) = \rho_x \log(\varepsilon_{t-1}^x) - \eta_x \epsilon_{t-1}^x + \epsilon_t^x$. All shock processes ε_t^x are equal to one in the steady state.

3.1 Posterior distributions for the key portfolio rebalancing parameters

Most parameters of the model are left free in the estimation. As most of the data used in the estimation are not filtered, some of the deep parameters, notably on the financial side, capture both steady state and cyclical properties of the model. In the Appendix, we document at length the calibration strategy and the choice of prior distributions for the financial block. Regarding the other structural parameters, the prior distributions are similar to Smets and Wouters (2007).

The posterior distribution of estimated parameters, characterised by the mean and the 80% density intervals, are reported in Tables 1 and 2. We focus here on the key parameters which drive the transmission of central bank asset purchases: the bankers' relative diversion rate on government bonds, δ_b, households' portfolio frictions, χ_H, and the rigidity parameter on retail lending rates, ξ_E^R. All these parameters are relatively well-identified in the estimations as the posterior distributions are sizeably narrower compared with the prior distributions.

As explained in Gertler and Karadi (2013), δ_b and χ_H are crucial parameters for the transmission of central bank asset purchases. When δ_b goes to 0, bank portfolio constraints on holdings of sovereign bonds weaken, and the macroeconomic impact of asset purchases vanishes. The authors considered the value of $\delta_b = 0.5$ to match the fact that the level of sovereign spreads in the data is half the intermediation spreads measured with mortgage and corporate bonds. In our model, δ_b creates a link between sovereign spreads and the bankers' loan rate which corresponds to the quarterly return on the bank loan book, net of expected losses and net of the monopolistic margin. Therefore, higher values of δ_b than in Gertler and Karadi (2013) can still be consistent with the sample mean of the sovereign spread and lending rate spread introduced in the estimation, as the latter includes credit risk compensation and a retail margin. Moreover the diversion rate of sovereign holdings introduced in the bankers' incentive compatibility constraint (9) is $\lambda_b \delta_b$ and with calibrated λ_b at around 0.3, δ_b could in principle take values significantly higher than 1. Those considerations explain the choice of a relatively loose prior distribution for δ_b which does not strongly constrain the inference towards low levels.

Turning to χ_H, Gertler and Karadi (2013) set it to 1 in order to broadly match empirical evidence on the impact of the second modification of quantitative easing in the US (QE2) on output and sovereign spreads. But χ_H also affects the distribution of sales of sovereign securities between households and banks in the context of central bank asset purchases. And for values higher than 0.1, the macroeconomic propagation of central bank asset purchases becomes relatively insensitive to χ_H: in particular, households would sell almost no bonds to the central bank, which is at odds with empirical evidence from the US or the UK.[2] Consequently, the support of the prior distribution covers low values for this parameter.

The posterior distribution of the adjustment cost on household portfolio decisions χ_H is low (with mean values below the one of the prior distribution, at less than 0.01). Finally, the bankers' diversion rate for sovereign bond holdings δ_b features a mean posterior distribution around 2. The calibration values of Gertler and Karadi (2013) for χ_H and δ_b

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[2] see for example Carpenter, Demiralp, Ihrig, and Klee (2013) in the case of the Federal Reserve and Joyce, Liu, and Tonks (2013) regarding the Bank of England

Table 1: PARAMETER ESTIMATES 1

Parameters			A priori beliefs			A posteriori beliefs			
		Dist.	Mean	Std.	Mode	Mean	$\mathcal{I}_1$	$\mathcal{I}_2$
σ_c	Intertemp. elasticity of subst.	gamma	1.5	0.2	1.732	1.741	1.453	2.025
η	Habit formation	norm	0.7	0.1	0.739	0.742	0.673	0.813
σ_l	Labour disutility	gamma	2	0.75	0.903	1.203	0.445	1.923
ϕ	Investment adj. cost	norm	4	1.5	4.670	4.968	3.598	6.357
φ	Cap. utilisation adj. cost	beta	0.5	0.15	0.375	0.372	0.229	0.510
α_p	Calvo lottery, price setting	beta	0.5	0.1	0.741	0.737	0.639	0.838
ξ_p	Indexation, price setting	beta	0.5	0.15	0.159	0.192	0.057	0.317
α_w	Calvo lottery, wage setting	beta	0.5	0.1	0.533	0.567	0.427	0.693
ξ_w	Indexation, wage setting	beta	0.5	0.15	0.346	0.362	0.174	0.533
ξ_E^R	Calvo lottery, lending rate	beta	0.5	0.2	0.320	0.323	0.264	0.381
r_μ	Lending rate margin	gamma	0.15	0.05	0.143	0.170	0.083	0.258
δ_b	Diversion rate for sov. bonds	gamma	1	0.5	2.101	2.142	1.540	2.736
χ_H	Portfolio adj. cost	gamma	0.1	0.05	0.007	0.009	0.002	0.017
ζ_b	Bank survival probability	beta	0.95	0.02	0.942	0.946	0.930	0.961
$R_{KK}/R-1$	External finance premium	gamma	0.5	0.1	0.545	0.554	0.386	0.715
μ_e	Monitoring costs	gamma	0.1	0.025	0.027	0.033	0.019	0.047
χ_e	Recovery rate	beta	0.5	0.025	0.477	0.482	0.445	0.517
α	Capital share	norm	0.3	0.05	0.266	0.258	0.217	0.299
μ_p	Price markup	norm	1.25	0.12	1.437	1.473	1.307	1.631
r_β	Time-preference rate	gamma	0.25	0.1	0.093	0.109	0.045	0.171
γ	Trend productivity	gamma	0.3	0.1	0.177	0.177	0.138	0.217
$\overline{L}$	Employment shift	norm	0	5	0.762	0.778	-1.939	3.663
$\overline{\pi}$	SS inflation rate	gamma	0.5	0.05	0.530	0.528	0.449	0.605
ρ	Interest rate smoothing	beta	0.75	0.15	0.890	0.894	0.868	0.919
r_π	Taylor rule coef. on inflation	norm	1.5	0.25	1.608	1.674	1.367	1.964
$r_{\Delta\pi}$	Taylor rule coef. on d(inflation)	norm	0.12	0.05	0.089	0.097	0.065	0.129
$r_{\Delta Y}$	Taylor rule coef. on d(output)	gamma	0.3	0.1	0.076	0.083	0.043	0.120

$[\mathcal{I}_1, \mathcal{I}_2]$ is the shortest interval covering eighty percent of the posterior distribution.

Table 2: PARAMETER ESTIMATES 2

Parameters		*A priori* beliefs			*A posteriori* beliefs			
		Dist.	Mean	Std.	Mode	Mean	$\mathcal{I}_1$	$\mathcal{I}_2$
λ_e	Employment adj. cost	beta	0.5	0.3	0.841	0.835	0.799	0.870
$\rho_{a,g}$	Corr(Tech.,Gov. Spend.)	unif	4.5	3.2	1.155	1.139	0.482	1.769
ρ_a	AR(1) Technology	beta	0.5	0.3	0.907	0.901	0.861	0.943
ρ_b	AR(1) Preference	beta	0.5	0.3	0.072	0.141	0.004	0.266
ρ_g	AR(1) Gov. spending	beta	0.5	0.3	0.995	0.994	0.987	1.000
ρ_I	AR(1) Inv. technology	beta	0.5	0.2	0.559	0.593	0.454	0.729
ρ_p	AR(1) Price markup	beta	0.5	0.2	0.892	0.825	0.670	0.995
η_p	MA(1) Price markup	beta	0.5	0.2	0.769	0.665	0.428	0.888
ρ_w	AR(1) Wage markup	beta	0.5	0.2	0.928	0.916	0.875	0.964
ρ_{σ_e}	AR(1) Entrepr. risk	beta	0.9	0.1	0.712	0.674	0.563	0.782
ρ_{ζ_b}	AR(1) Bankers' net worth	beta	0.5	0.2	0.500	0.422	0.124	0.716
ρ_{R_b}	AR(1) Bond valuation	beta	0.5	0.3	0.991	0.982	0.965	0.999
Standard deviation								
ϵ_t^a	Technology	unif	5	2.9	0.672	0.693	0.457	0.918
ϵ_t^b	Preference	unif	5	2.9	1.719	1.890	1.314	2.471
ϵ_t^g	Gov. spending	unif	5	2.9	1.670	1.740	1.498	1.984
ϵ_t^I	Inv. technology	unif	10	5.8	4.485	4.794	3.405	6.147
ϵ_t^p	Price markup	unif	0.25	0.1	0.146	0.148	0.109	0.187
ϵ_t^w	Price markup	unif	0.25	0.1	0.078	0.080	0.054	0.106
ϵ_t^r	Wage markup	unif	0.25	0.1	0.098	0.103	0.085	0.119
$\epsilon_t^{\sigma_e}$	Entrepreneurs' risk	unif	5	2.9	0.285	0.351	0.194	0.505
$\epsilon_t^{\zeta_b}$	Bankers' net worth	unif	5	2.9	0.872	1.181	0.344	2.056
$\epsilon_t^{R_b}$	Gov. bond valuation	unif	5	2.9	0.008	0.011	0.007	0.014
$P_\lambda(\mathcal{Y})$		-48.716						

$[\mathcal{I}_1, \mathcal{I}_2]$ is the shortest interval covering eighty percent of the posterior distribution.

are far from our posterior mean estimates, and are not even covered by the 80% shortest density interval.

Finally, we introduced to the DSGE a staggered lending rate setting in the retail banking segment, not present in Gertler and Karadi (2013), which significantly affects the pass-through of bankers' required return on loans to the marginal lending rate for entrepreneurs, and therefore the effectiveness of the portfolio rebalancing channel. To control the size of asset purchase output multipliers, lower values of δ_b which would increase the multiplier, everything else being equal, can be compensated for by a higher level of ξ_E^R. The posterior mode for ξ_E^R is around 0.3, which is consistent with a relatively fast lending rate past-through of corporate loans. Obviously, there are differences across retail bank products in terms of the speed and degree with which banks pass through changes in policy rates due to the maturity of the interest rate fixation in the loan contract, the degree of market power of the bank or other indexation scheme on the interest paid during the lifetime of the loan. Darracq-Pariès, Moccero, Krylova, and Marchini (2014), for example, summarise existing time-series evidence showing a more sluggish pass-though of monetary policy rate to mortgages than to corporate lending rates.

4   Asset purchases under *ad hoc* programmes

In this section, we try to shed some light on the general effects of central bank government bond purchase programmes in the presence of the lower bond constraint on the policy rate. However, before we discuss asset purchase programmes by considering the effective lower bound, we compare a standard monetary policy shock with asset purchases and construct realistic crisis scenarios with endogenous periods of binding lower bound constraint on the policy rate. This occasionally binding constraint incorporates some non-linearity into the model and makes the macroeconomic multipliers of central asset purchases sensitive to the underlying crisis scenario. Thereafter, we focus on specific modalities of the *ad hoc* programme. Altogether, our results show that central bank asset purchases are more powerful *i)* in an environment in which the policy rate has reached its effective lower bound, *ii)* the longer the duration of the lower bound period, *iii)* when, at the lower bound, the programme is fully communicated and anticipated, and *iv)* when it is complemented by forward guidance extending the lift-off date for the policy rate beyond agents' expectations.

4.1   Interest rate cuts versus asset purchases

Standard monetary policy accommodation on the one hand, and central bank asset purchases on the other, lead to different credit channels and bank balance sheet conditions. Two sets of impulse-response functions (IRFs) are contrasted in this section regarding their transmission to the broad macroeconomic landscape, together with banks' profitability and capital position. In the first one, the central bank unexpectedly cuts its key interest rates while in the second one, the central bank announces an asset purchase programme. We implement it in the DSGE model like Gertler and Karadi (2013):

$$B_{CB,t} = \epsilon_t^{QE} \overline{B_G} \tag{34}$$

where the asset purchase shock ϵ_t^{QE}, expressed as a percentage of the fixed government bond supply $\overline{B_G}$, follows an AR(2) process,

$$\log(\epsilon_t^{QE}) = \rho_1^{QE} \log(\epsilon_{t-1}^{QE}) + \rho_2^{QE} \log(\epsilon_{t-2}^{QE}) + \nu_t^{QE}, \tag{35}$$

ν_t^{QE} corresponds to unexpected innovations in the purchase strategy of the central bank. For a programme which is announced and completely communicated to the agents ν_t^{QE} is non-zero in one period and has the interpretation that the programme is activated. This can be seen as a rough approximation of the first features of the ECB's asset purchase programme (APP). The next section will investigate in greater detail the design of one-off asset purchase programmes and propose more accurate ways of implementing it within the DSGE model. Nonetheless, for the sake of comparability with the relevant literature, we also present the simulations associated with the AR(2) process.

In normal times, policy rate cuts are favourable for bank profitability both through higher net interest income as well as general equilibrium effects. Figure 1 presents the IRFs for a one-standard-deviation negative shock on the Taylor rule residual (see blue dotted lines). Temporarily lower short-term interest rates shift and steepen the term structure and directly support the profitability of maturity transformation activities of the banking system. In the model, lending rates respond sluggishly to money market rates due to nominal rigidities in the lending rate setting. Besides, the decline in short-term interest rates leads to a higher price of sovereign bonds, which provides some mild holding gains for the banks. Finally, improving economic conditions and increasing asset prices are beneficial to firms' creditworthiness, with receding delinquency rates. Such favourable developments in credit quality allow banks to scale down their credit risk compensation when setting lending rates. Turning to the macroeconomic multipliers of the monetary policy impulse, output increases by 0.3% at its peak, while the rise in the quarterly inflation rate reaches 0.05%. Standard monetary policy interventions entail powerful transmission channels beyond the banking system: on the real side through the intertemporal substitution of spending decisions, and on the financial side, through the discount factor of asset pricing decisions. Therefore, the credit multiplier is relatively low, with real loans increasing by 0.25% while corporate lending rates moderate by more than the policy rate, as the pass-through is almost full over two years in the model and credit risk compensation is lower.

By contrast, the APP entails a strong portfolio rebalancing channel, incentivising banks to ease credit conditions, foregoing profit margins on loans and originating more credit exposures. Figure 1 presents the IRFs of a central bank asset purchase progamme mimicking the ECB's January 2015 APP (see black lines and grey shaded areas). The modelled frictions in bank capital structure decisions embed a constrained portfolio allocation between securities and loans. In this context, the central bank asset purchases do have an impact on government bond yields and compress the excess return on this asset class. The term spread is compressed by around 60 bps (in annual terms). Lower government bond yields encourage banks to shed sovereign bonds and increase loan exposures. Over the course of the programme, bank sales of government bonds account for roughly one-third of the central bank asset purchases. Banks therefore benefit from sizeable holding gains on their securities portfolio. This rebalancing mechanism leads in equilibrium to a narrower "required" excess return on loan books by intermediaries. The pass-through
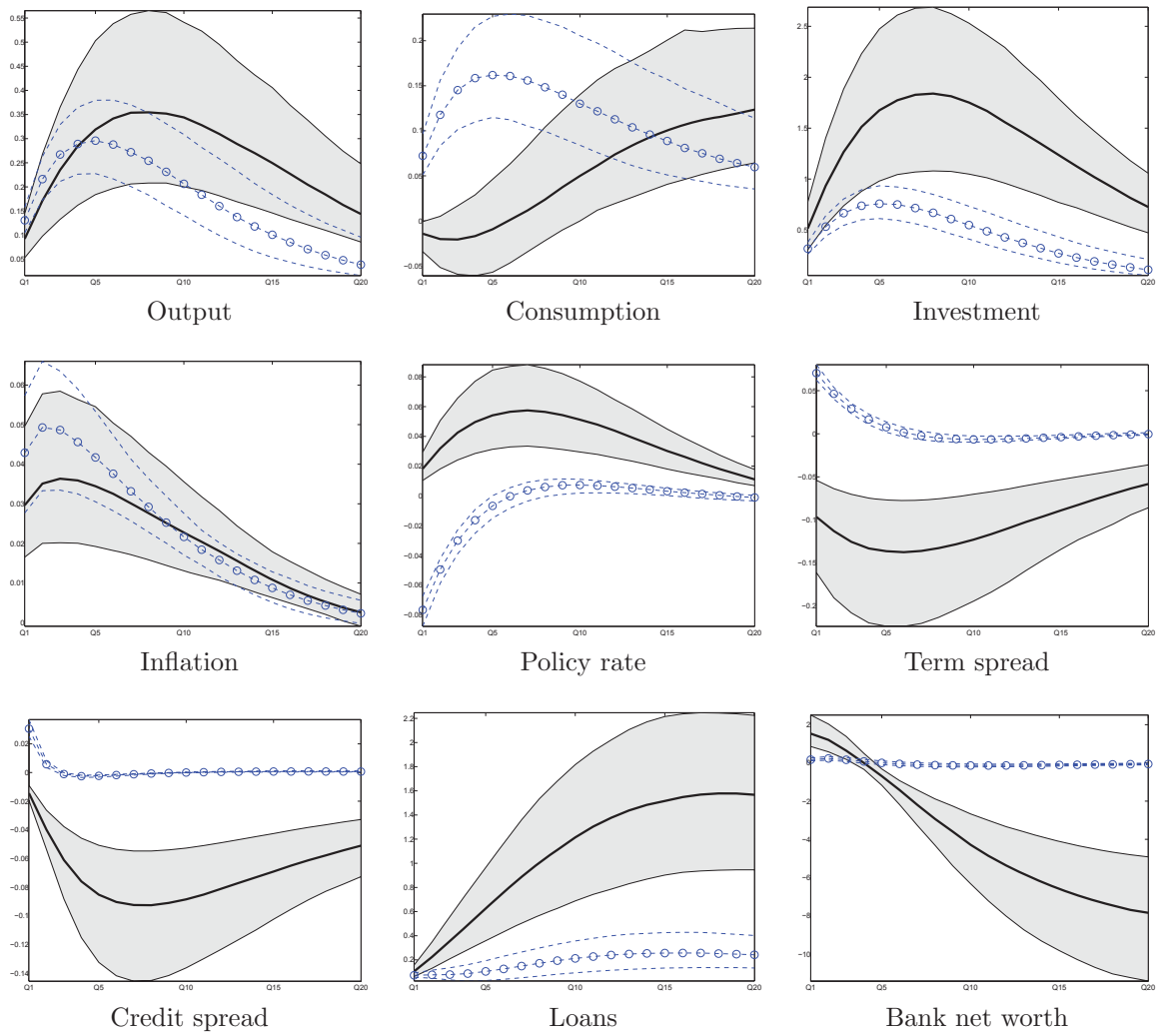
Figure 1: Impulse response functions associated with a shock to ϵ_t^{QE} (black lines for the mode and grey shaded areas for the 90% shortest density interval) and to a shock to ϵ_t^r (blue dashed lines with circles for the mode and blue dashed lines for the 90% shortest density interval).

of sovereign spreads to the required return on loans by the retail lender is around 0.8. Credit expansion through lower borrowing costs is a key propagation mechanism of the central asset purchases in the model, compared with standard monetary policy easing. Net interest income therefore declines over the first two years of the simulation, which depresses banks' net worth in the medium run.[3] As with the standard monetary policy shock, credit quality improves alongside economic activity and asset prices, which contributes to bank profitability. Overall, the easing in financial conditions spurs investment and output, generating inflationary pressures and countercyclical monetary policy adjustment. The output multiplier peaks at 0.35%, and the quarterly inflation rate increases by up to 0.04 pp. Compared with the standard monetary policy shock, the APP transmission features relatively less inflation and more output: indeed, the APP mainly propagates by compressing the overall external finance premium in the economy and thereby entails a stronger cost channel than the standard monetary policy shock. In the simulation, we allowed the monetary policy rate to respond in line with the estimated Taylor rule. The increase in the policy rate partially mitigates the expansionary effects of the central bank asset purchases.

By comparing the transmission of interest rate cuts and asset purchases, we have illustrated the potential strategic complementarities between the two policy instruments. They feature distinctive propagation channels, different macroeconomic stabilisation properties and should not be considered perfect substitutes. Against this background, the rest of the paper aims at exploring the optimal combination of standard and non-standard monetary policy measures. We first start with the specific configuration where the monetary policy rate is constrained at its effective lower bound and the central bank implements a specific asset purchase programme (see Section 4). This exercise enables us to evaluate the macroeconomic multipliers of central bank asset purchases and to perform some sensitivity analysis regarding the implementation design of the programme. In a second stage, we adopt a broader and more normative perspective, looking at the desirability of combining both instruments through the cycle (see Section 5). All in all, we first evaluate the efficiency of programmes similar to the ones introduced by central banks during the crisis before we investigate the optimal policy setting.

4.2   Effective lower bound for the interest rate

In order to evaluate government bond purchases at the effective lower bound for the policy rate, we lean on the parallel literature focusing on fiscal multipliers at the lower bound as we consider central bank asset purchases as an exogenous process (Christiano, Eichenbaum, and Rebelo, 2011b; Woodford, 2011b). The effective lower bound for the interest rate is implemented as follows:

$$
\begin{aligned}
\hat{R}_{D,t} &= max\left(\underline{R}, \hat{R}_{D,t}^{*}\right) & (36)\\
\hat{R}_{D,t}^{*} &= \rho\hat{R}_{D,t-1}^{*} + (1-\rho)\left[r_{\pi}\hat{\pi}_{t-1} + r_{y}\hat{y}_{t-1}\right] + r_{\Delta y}\Delta\hat{y}_{t}
\end{aligned}
$$

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[3]See Kühl (2016) for a discussion of government bond purchases on the financial health of non-financial firms and banks. Kühl (2016) shows that the most important transmission channel runs through affecting the borrowing conditions of non-financial firms.

where $\underline{R}$ is the effective lower bound (see Christiano, Eichenbaum, and Trabandt (2015) for a similar approach). This specification enables us to endogenously determine the length of time for which the lower bound constraint is binding.

When central banks first introduced asset purchase programmes as an additional policy tool, they primarily focused on giving guidance on the path of purchases instead of providing a specific contingency as is usually done when setting the policy rate. Although asset purchase programmes have also been contingent on specific targets, our approach resembles those programmes which have been introduced by some central banks. The Federal Reserve, for instance, announced a purchase path with equally distributed purchases across the months. From this communicated path, it was possible to derive the expected central bank balance sheet. The Eurosystem chose a similar approach when announcing the expanded asset purchase programme in January 2015. Regarding the underlying path for the stock of government bonds held by the central bank, we assume that there is a build-up period until the point is reached when purchases stop, which coincides with the maximum stock of bonds held by the central bank. Following this period in time, the stock is unwound at a specific speed which may be determined by the maturity profile of the market portfolio.

4.3 Constructing realistic lower bound scenario(s)

Asset purchase programmes were usually introduced as an additional policy tool when the short-term interest rate had reached its effective lower bound and thus the scope for further easing the monetary stance through standard measures had been exhausted. To analyse such a policy configuration, we simulate an endogenous lower bound scenario. This requires the selection of shocks which can severely depress economic conditions so that the policy rate reaches its lower bound. Specifying the central bank interest rate policy as in Equation (36) implies that the length of the lower bound period becomes endogenous: as shocks vanish over time, the economy recovers and the policy rate returns to its steady state value. In most of the literature, the lower bound scenario is generated by a single shock - a discount factor shock as done by Eggertsson and Woodford (2003), for example. Since our model has satisfactory data consistency, it allows for well-founded shocks located in the financial sphere which are combined on the basis of real observations.
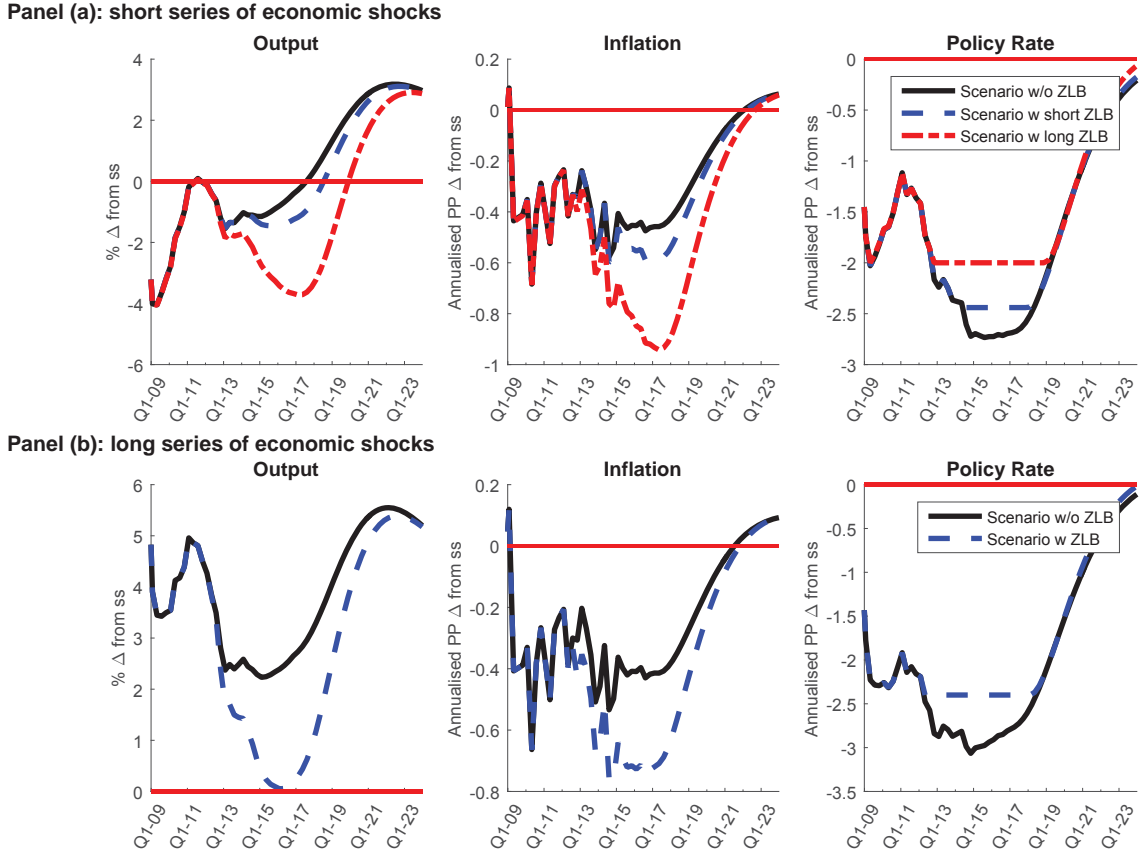
Panel (a): short series of economic shocks

Output — **Inflation** — **Policy Rate**

Legend (Policy Rate):
Scenario w/o ZLB
Scenario w short ZLB
Scenario w long ZLB

Panel (b): long series of economic shocks

Output — **Inflation** — **Policy Rate**

Legend (Policy Rate):
Scenario w/o ZLB
Scenario w ZLB

Figure 2: The zero lower bound scenarios

A more realistic design of the lower bond scenarios is achieved through the lens of the estimated model for the euro area, first constructing shocks which reproduce the salient features of the euro-area crisis, and then injecting those shocks into the model with an occasionally binding constraint on the short-term interest rate. This approach is similar to Christiano et al. (2015), for instance. However, we do not rely on shocks generated outside the model. Instead, we prolong historical data with official forecasts as available by end-2014,[4] and we use the estimated first-order approximation of the model to filter the structural shocks consistent with our set of observable variables, in the absence of the lower bound constraint on the short-term interest rate. We consider all shocks except the monetary policy shock (i.e. the Taylor rule residual) for the scenarios. More importantly, we are interested in showing the sensitivity of macroeconomic multipliers to the underlying lower bound scenarios and the degree of plausibility of the shock selection. Regarding the modelling of the ELB we make use of anticipated shocks and basically follow Laseen and Svensson (2011). In this respect, we allow for an endogenous period in which the effective zero lower is binding, similar to Holden (2016).

Along those lines, we define three different scenarios which differ in terms of the sample used for generating the shocks and the dates for reaching the effective lower bound. For entering the lower bound, we treat two periods (Panel (a) in Figure 2) and follow an approach similar to Christiano et al. (2015), who define a threshold interpreted as the

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[4]This means before the decision of the ECB public securities purchase programme has been made.

zero lower bound. The first one is when the rate for the deposit facility in the euro area reached a level of zero. Although it has been reduced even further, we take this as the first period. The second period starts with the statement by ECB President Mario Draghi that "[...] the key ECB interest rates have reached their lower bound".[5] Given the official projections, the first period is five years long, which we refer to as the long lower bound period. The second period is shorter and comprises roughly three years. In both cases the underlying shocks are the same. We simulate the economy by starting at the end of 2007. To provide further robustness for the results, we also make use of all shocks starting at the beginning of our sample (Panel (b) in Figure 2). Here, we only look at the long lower bound period. By targeting specific dates for hitting the lower bound, we can implicitly control the duration for which the constraint on the policy rate is binding. Both the entry to and exit from the lower bound are endogenous in this exercise and driven by our estimated shocks. The three scenarios are then used for evaluating asset purchases which simultaneously brings large elements of realism and pragmatism to our approach.

4.4 Asset purchases multipliers at the lower bound on nominal interest rates

We now intend to examine in greater detail the distinct features of central bank asset purchases at the lower bound on interest rates, using our estimated DSGE model for the euro area as well as the lower scenarios of the previous section. The benchmark programme in the forthcoming analysis is meant to reproduce the ECB's APP announced in January 2015. Government bonds are purchased gradually over the course of one year. The bond portfolio then shrinks over time as securities are held to maturity.

Conversely to the AR(2) process specification of Gertler and Karadi (2013) examined in Section 4.1, the law of motion for the purchases can be expressed as an AR(1) process with news being used to more precisely match the time profile of the programme: announcements about future purchases can be interpreted as news about future innovations. The formal description, which is close to Kühl (2016), is

$$\log(\epsilon_t^{QE}) = \rho_1^{QE} \log(\epsilon_{t-1}^{QE}) + \sum_{i=0}^{T} \nu_{t-i}^{QE,i}, \tag{37}$$

where the last term on the right-hand side reflects the announcement of the future increase in the stock of government bonds held by the central bank. For $i = 0$ the purchases come as a surprise and for $i > 0$ the agents know at time t the future increase in the stock. Since the entire law of motion is known to the agents, agents can also build expectations about the future path of the stock, and the parameter ρ_1^{QE} controls the maturity profile of the central bank's portfolio.

The simulation of the central bank asset purchase programme is presented as the difference between the lower bound scenario including asset purchases on the one hand, and the lower bound scenario without asset purchases on the other hand. Figure 3 we show the responses of the selected macroeconomic variables under the three different lower bound scenarios: in each case, the profile of asset purchases generated by equation (37) is exactly the same. The black lines correspond to the case from Panel (b) in Figure 2, i.e. a

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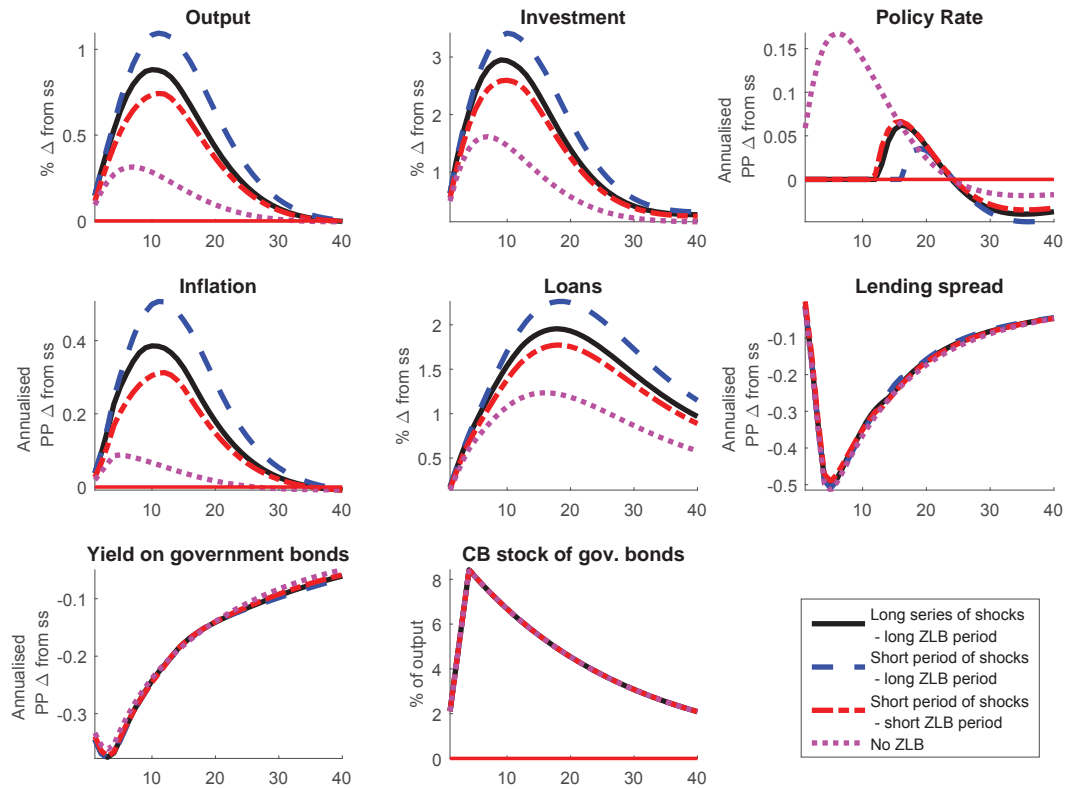[5]Introductory statement to the press conference, 22 January 2015.

Figure 3: Comparison of the benchmark programme at the ZLB for different scenarios

long lower bound period generated by the full series of structural shocks. The dashed blue lines (dashed red lines) illustrate the case where we start the counterfactual simulation after the financial crisis and reflect a long (short) lower bound period. Furthermore, the dotted purple lines represent the case without the lower bound constraints and are broadly similar to the IRFs of Section 4.1.[6]

Qualitatively, the responses are broadly similar across the various lower bound scenarios but display some notable quantitative differences. As is known from the literature on government expenditures, the lower bound environment leads to higher output multipliers (Christiano et al., 2011a; Woodford, 2011b). This property extends to central bank asset purchases, and has also been documented by Gertler and Karadi (2011, 2013) or Chen, C?rdia, and Ferrero (2012). The intuition behind these results is that government bond purchases positively affect output and eventually boost inflation. Since the policy rate is constrained at its lower bound, the countercyclical effects from policy rate increases are missing. Thus, the deterioration of banks' funding costs, through an increase in the policy rate, is absent, which stimulates bank equity and the origination of loans as a consequence. Indeed, the lending rate spread compression is almost 3 times smaller when the policy rate is unconstrained, declining by 20 bps (in annual terms), compared to 50 bps in the other scenarios. The increase in loans is also muted and is twice as weak as in the lower bound simulations. In addition, countercyclical monetary policy reins in the rise of inflation expectations and limit the decline in real interest rates, curtailing the overall output multipliers threefold in comparison with the other scenarios.

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[6]The only difference coming from the stochastic process for the central bank asset purchases.

Across the various lower bound configurations, the quantitative differences in the macroeconomic multipliers seem to be partly related to the duration of the lower bound period. For the scenario in which the shocks are induced not before 2007 and the policy rate remains at its lower bound for longer, government bond purchases have the strongest macroeconomic effects, stimulating output by more than 1% at its peak and annual inflation by around 0.5 pp. At the same time, the responses also differ between the two other scenarios, despite a similar duration of the binding lower bound constraint. Hence, we have illustrated some relevant dimensions of non-linearity: the macroeconomic multipliers of government bond purchases depend on the length of the lower bound period and, more generally, on the underlying crisis shock typology.

4.5   Communication strategy and anticipation effects

We now examine the sensitivity of our previous results with respect to asymmetric information between the central bank and private agents. For this purpose, we consider the same actual path of purchases but assume different announcement strategies. The various simulations are conducted in a lower bound environment as it is the relevant policy configuration for evaluating asset purchases. The underlying lower bound scenario corresponds to the long binding period of Panel (a) in Figure 2.
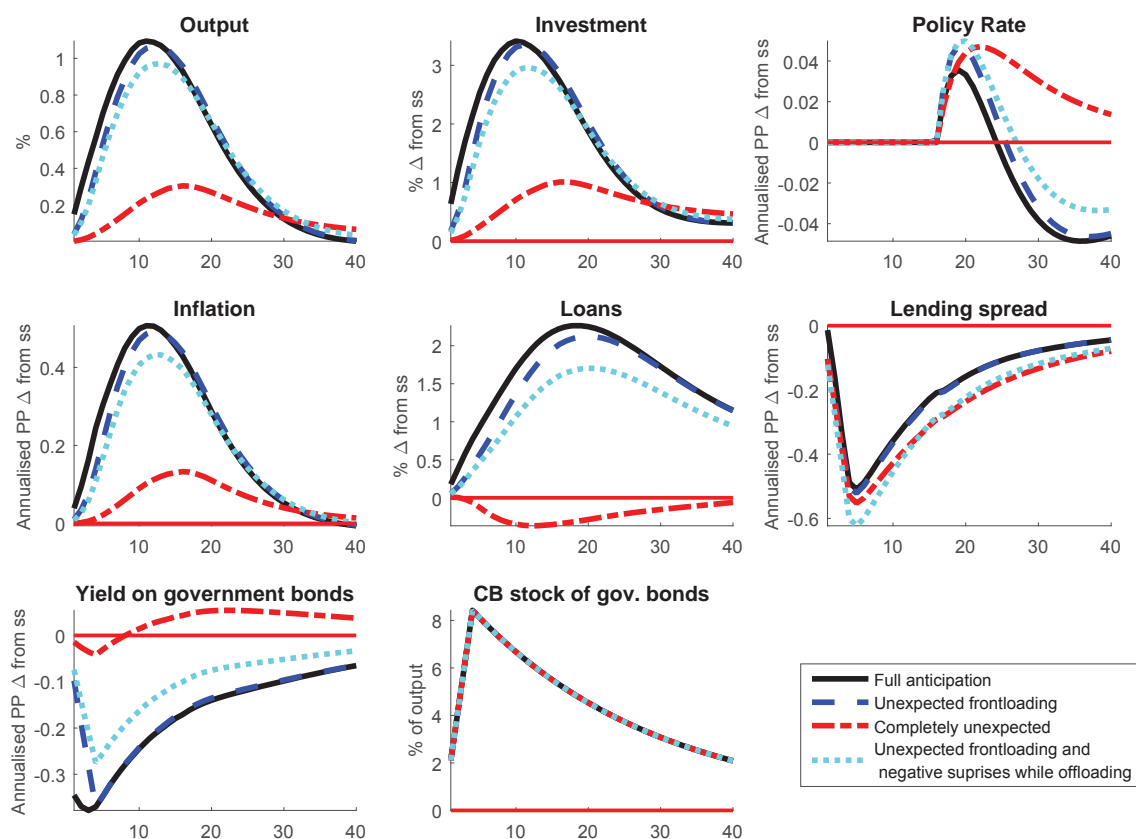

Figure 4: Different information sets regarding ad hoc APP and the zero lower bound - long ZLB period

Figure 4 presents all results. *The benchmark case* (bold black lines) assumes that

agents are fully aware of all the features of the programme and it is the same as the dashed blue lines of Figure 3: the announcement comes as a surprise but the purchase path and the unwinding path are known in their entirety. This comes closest to the ECB's APP of January 2015, as market participants anticipated that purchased assets would be held to maturity. *In the second case*, the unwinding path is known to the agents but the purchases come as a surprise every period (blue dashed lines). *In the third case*, the entire programme comes as a surprise: both the purchases and the unwinding path are unknown and come as a surprise period by period (red dashed lines). *The fourth case* is similar to the second one, with the difference that agents expect unwinding to take place at a slower pace while the actual unwinding is faster (cyan dotted lines). This means that they are faced with positive surprises during the front loading and with negative surprises during the unwinding period. *In the last case*, we combine positive surprises during both the purchase path and the unwinding path (line with stars in magenta).[7] We confine our analysis to the case in which there is a steady surprise (positive or negative), which means that agents cannot learn the actual features of the programme. A more explicit learning formulation would be worth investigating, but that would be beyond the scope of this paper.

Our results show that the correct anticipation of the programme results in the largest macroeconomic responses in the short run (bold black lines). This is especially true for the bond market: only in this case do sovereign yields decline sharply upon announcement of the programme and long before the actual purchases are actually implemented. By contrast, the smallest effects occur when all the features of the programme come as a surprise (red dashed lines). This scenario is an extreme case because agents see the purchases, but they do not know that the purchases will continue in the next period. Nevertheless, this scenario allows us to demonstrate the importance, on the one hand, of the knowledge about the offloading path. On the other hand, the responses can be interpreted as the flow effects of the programme, since agents do not take future stock holdings into account. The closest to the fully announced programme are the ones where agents are "positively" surprised only (see dashed blue lines and lines with stars in magenta). Here, output rises with a short delay. The difference can be traced back to the anticipation effect of the programme. As the programmes include some elements of surprise, the bond market adjusts along with purchases so that portfolio rebalancing takes place with some delay. Smaller surprises during frontloading dampen the output expansion (see cyan dotted lines). This is also true during the unexpectedly faster unwinding of the portfolio.

Overall, the transmission of government bond purchases to the real economy does change when different information sets are assumed, although, based on a visual inspection of the responses in Figure 4, some configurations yield results which are very close to each other.

In order to measure and compare results across specifications, we develop more quantitative indicators of output multipliers. However, we focus on numerical multipliers and do not provide the analytical expression for the multipliers as done by Woodford (2011b), for example. Given the rich structure of our model, an analytical solution would be difficult to obtain and interpret. The output multiplier is therefore defined as the cumulated gains in output relative to the cumulated stock, i.e. the balance sheet profile of the central

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[7]A formal description how we implement these programmes technically is provided in the appendix C.

bank. This present-value multiplier (PVMP) weights the future with the time preference rate. This concept leans on Mountford and Uhlig (2009) for the fiscal multiplier and reflects the idea that output gains today are more valuable for the agents than output gains tomorrow. We compute two multipliers over horizons of one year and ten years.

$$PVMP = \frac{\sum_{i=1}^{T} \beta^{i-1} \Delta Y_{t+i}}{\sum_{i=1}^{T} \beta^{i-1} CBStock_{t+i}} \tag{38}$$

The multipliers for the various information sets are presented in Panel (a) of Table 3, based on different underlying lower bound scenarios (i.e. with either a short or long duration of binding lower bound constraint). The results confirm our previous findings: full anticipation of the programme yields the strongest short- and medium-run improvements in output. This holds qualitatively for both lower bound scenarios, though the effects are stronger for the longer lower bound duration. Conversely, the fully unexpected programme provides the smallest output multiplier over both horizons. Agents cannot build expectations about future purchases and the evolution of the stock. Accordingly, it seems that "stock effects" are key to the effectiveness of the asset purchase programme. Turning to the other information sets, the short-run effects of unexpected frontloading are obviously identical because the programmes differ mainly in terms of the offloading part. As expected, the output gains from the programme with positive surprises are larger than in the "unexpected frontloading" case (without surprises in the offloading strategy), but they are quantitatively very close to each other.

Table 3: Output multipliers from government bond purchases for different programmes calculated over ten years - endogenous lower bound period

	Short ZLB period		Long ZLB period	
	over 1 year	over 10 years	over 1 year	over 10 years
Panel A: Purchases over one year with different information sets				
Full anticipation (benchmark model)	0.0604	0.0798	0.0784	0.1235
Unexpected frontloading	0.0303	0.0746	0.0401	0.1179
Completely unexpected	0.0029	0.027	0.0037	0.0397
Unexpected frontloading and negative surprises while offloading	0.0207	0.0439	0.0282	0.074
Unexpected frontloading and positive surprises while offloading	0.0304	0.0766	0.04	0.1201
Panel B: Purchases over time				
Purchases over one year	0.0604	0.0798	0.0784	0.1235
Purchases over two years	0.0986	0.0796	0.132	0.1252
Purchases over 1 year with 1 year reinvestment	0.065	0.0794	0.085	0.1226
One-off	0.0319	0.0785	0.0409	0.1213

Note: The present value multiplier weights the periods with the discount rate.

4.6 Time profile of asset purchases and reinvestment strategy

While the preceding sensitivity analysis has always assumed the same time profile for the asset purchase programme, we now place more emphasis on the path of the central bank's balance sheet, discussing first the *anticipated* frontloading of purchases and turning afterwards to the offloading strategy.

In the benchmark programme, the purchases are announced and distributed across the first year (recall that the purchases in the first quarter come as a surprise). In Figure 5 we contrast the benchmark programme (black solid lines) with three other programmes. The first one implements quarterly asset purchases equally distributed over two years (blue dashed lines), with the same maximum stock effect as in the benchmark programme (the purchases are therefore smaller in every quarter). Second, we allow for a reinvestment policy which is combined with the initial path of the benchmark model (red dashed lines). Reinvestment means that the maximum stock of government bonds is kept constant over one year before unwinding starts. The reinvestment programme follows the same purchase path as the benchmark model, and has the same unwinding path as the two-year programme. Third, we show one-off purchases with a similar stock effect (cyan dotted lines), noting that the current analysis abstracts from any implementation constraints for the purchase programme that might become binding in this case (like issue or issuer limits, for example).
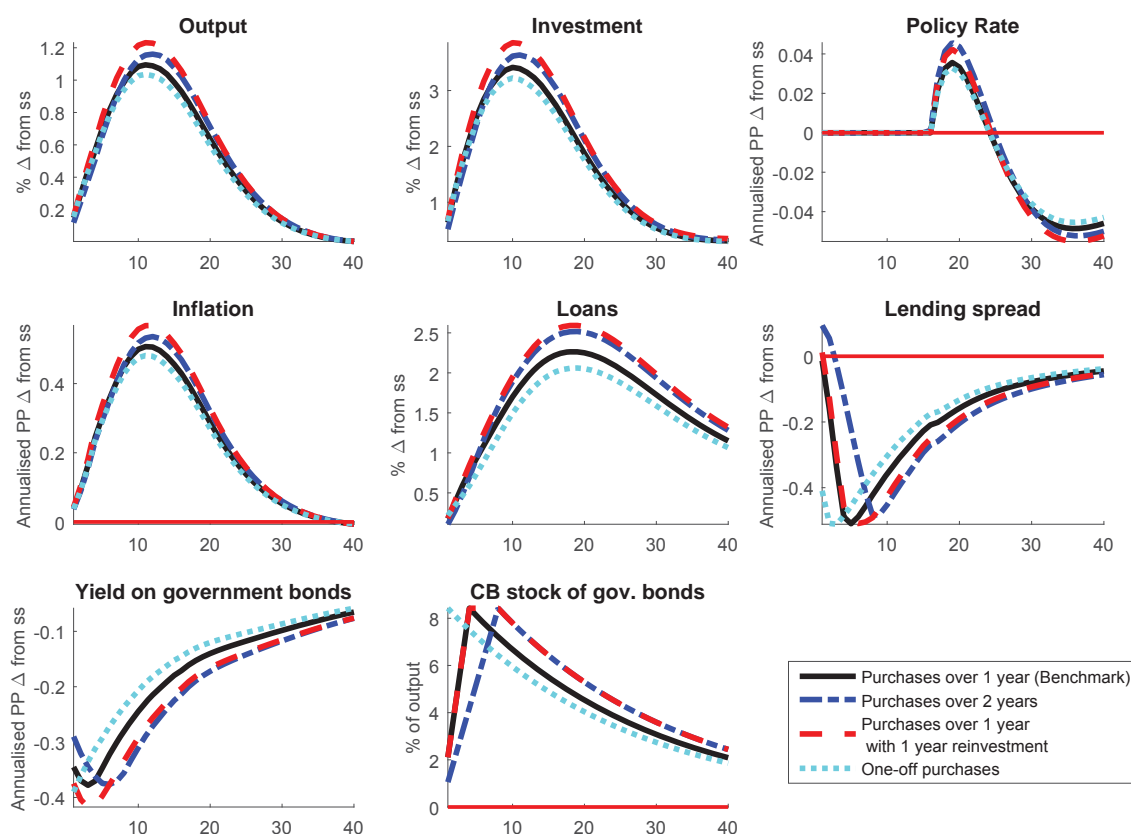

Figure 5: Different distribution of ad hoc APP over time and the zero lower bound - long ZLB period

Among all the cases, the reinvestment programme shows the largest peak effects on

output and inflation, followed by the two-year programme. The one-off purchase programme, however, has a lower average impact on the main variables. The ranking of the different programmes according to their macroeconomic effects might simply be related to a larger or smaller average increase in the central bank's balance sheet.

In order to control for differences across programmes in the average increase in the balance sheet, we look again at the present value output multipliers. They are presented in Panel (b) of Table 3 for the different frontloading paths. Different anticipated purchase paths do little to alter the (relative) effectiveness of government bond purchases on output in the medium run. The output multipliers over a horizon of ten years are widely the same for the benchmark (one-year) programme, the two-year programme, and the "one year with reinvestment" programme. However, the quantitative effects differ more evidently in the short run. Among these three programmes, the two-year programme generates the largest short-term response on output per average unit of purchase. Obviously, anticipation effects about the time profile of the programme affects the output multipliers particularly in the short run. This argument can also be seen by comparing the announced programmes with the one-off programme. In this case, the short-run multipliers fall well below those of the announced programmes, although the long-run effects are broadly similar. This allows us to conclude that the average size of central banks' balance sheets in fully anticipated asset purchase programmes is important for generating output (and inflation) gains. In this respect, for the same peak effect on the central bank balance sheet, a reinvestment policy can provide meaningful macroeconomic amplification. At the same time, output multipliers per unit of purchase are not very sensitive to the time profile of the programmes, even at the lower bound.
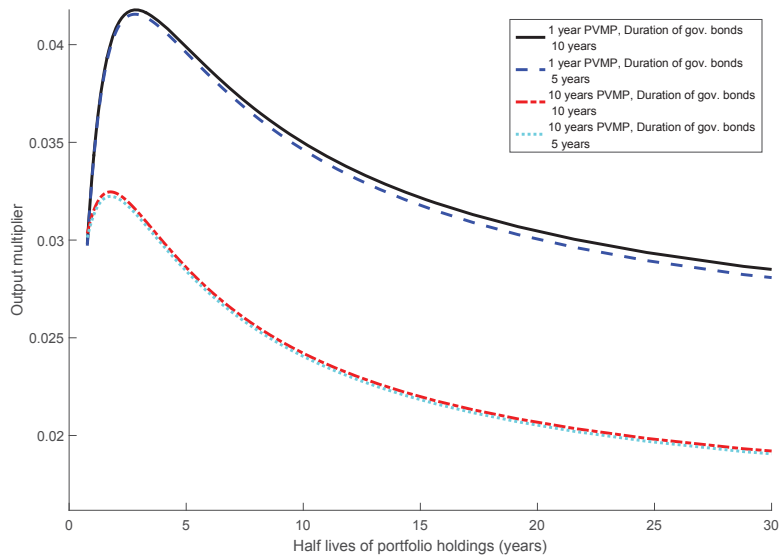

Figure 6: Output multipliers of one-off asset purchases

We now turn to the discussion of the offloading strategy. Figure 6 shows the present-value multipliers on output (y-axis) as a function of the offloading pace of one-off purchases (i.e. the half-life of the portfolio on the unwinding path, which is a function of ρ_1^{QE}

31

from equation (37)).[8] The figure clearly points to non-monotonicity in the relationship. Starting from very short periods of holdings, the output multiplier increases quickly. After reaching a maximum at around a 4-year half-life, the output multiplier falls. For very long periods of holdings, there seems to be a stabilisation in terms of output multipliers. The results are mostly insensitive to the duration of the long-term government bonds in our economy, at least from 5 to 10 years. This result indicates that there is an optimal offloading path in terms of effectiveness per unit of purchases which does not correspond to the duration of the underlying bond. In other words, hold-to-maturity central bank portfolios may not be a dominant unwinding strategy.

Altogether, considering that the "unit" efficiency of a programme is related to the cumulative holdings over its lifetime, this section suggests that holding the assets to maturity might be more efficient for the central bank than earlier offloading strategies which would precisely reduce the cumulative holding. Unwinding the bond portfolio faster would seem to weaken the impact of the programme on asset prices and its pass-through to the broader economy. By contrast, for the same cumulative holding, a very persistent programme would imply much smaller asset purchases in the initial phases of the programme, when the funding constraints are tighter and the lower bound on interest rates is binding, thereby reducing the macroeconomic multipliers.

4.7 Forward guidance on the interest rate

We have studied government bond purchases in a lower bound environment which has been created endogenously. Part of the results obtained are related to the length of the period in which the lower bound is binding, i.e. is expected to hold. Expectations about the duration of the lower bound are related to the structure of the underlying shocks, their impact on the economy, and the policy conduct at the lower bound formulated in Equation (36).

Policymakers can nonetheless influence the economy at the lower bound by using forward guidance on the policy rate: they can commit to keep the policy rate unchanged beyond the lift-off date expected by agents on the basis of the state of the economy and the historical conduct of monetary policy. To investigate the effects of forward guidance on the outcomes of government bond purchases, we depart from the endogenous zero lower bound scenario and assume that the policy rate is being fixed for a given period of time. To disentangle the effects of forward guidance for the interest rate, we consider the benchmark programme from the previous sections and vary the period for which the interest rate is kept fixed. Regarding the modelling of forward guidance, we again lean on Laseen and Svensson (2011).

The responses of the economy for 2 quarters, 4 quarters, and 6 quarters of forward guidance are presented in Figure 7 in deviation from the transmission of the asset purchase programme under the estimated Taylor rule, i.e. we compare the effects under forward guidance with those without forward guidance. The relative perspective allows us to only highlight the effects coming from forward guidance. The corresponding output multipliers are given in Table 4. Government bond purchases become significantly more effective - in terms of resources used - by extending the period of constant interest rates: the

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[8]The half-life of the underlying AR process in years is computed as $half - life = \frac{\log 0.5}{4*\log \rho_1^{QE}}$.

longer the period of forward guidance, the more effective the asset purchase programme. Forward guidance turns out to have exceptionally strong macroeconomic amplification effects on asset purchases. This result is also related to the "forward guidance puzzle" in the DSGE model as explained in Carlstrom, Fuerst, and Paustian (2015), for example. The effectiveness of government bond purchases is much stronger.
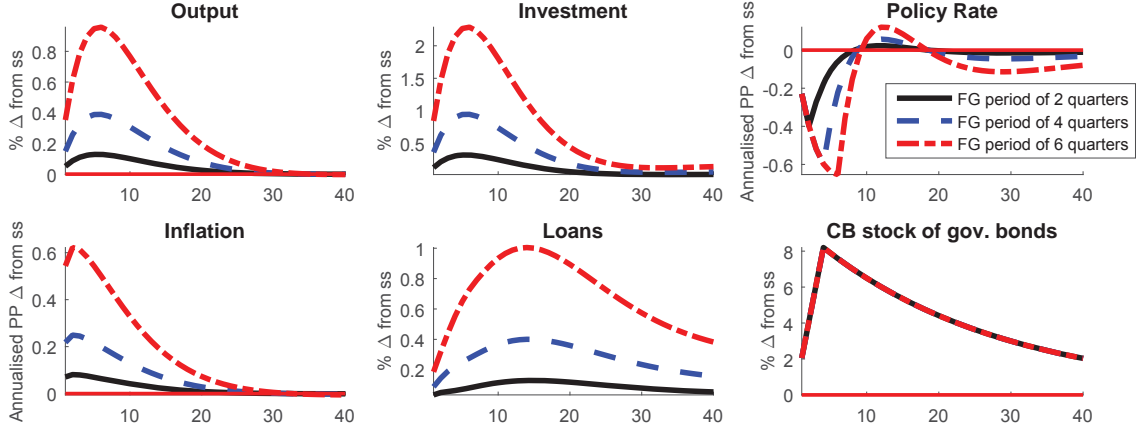

Figure 7: Effects of forward guidance on the interest rate

By comparing Table 4 with Table 3, it turns out that combining government bond purchases with forward guidance on the policy rate beyond a few quarters increases their effectiveness by more than the endogenous lower bound environment. It is worth dwelling on the reasons behind this result. The underlying crisis shocks used to create the endogenous lower bound environment imply first that the standard monetary policy rule is constrained: all agents perceive that the policy rate cannot go down any further and will eventually rise again in line with the Taylor rule as soon as the economic conditions improve tangibly. The binding lower bound constraint can actually be interpreted as "tightening" forward guidance, keeping rates higher than they should. In this context, the asset purchase programme interacts with the underlying shock dynamics at the lower bound and is amplified because they remove some of the "tightening" forward guidance. Only this mitigation of the "tightening" forward guidance should be compared with the previous simulations. Instead, the "easing" forward guidance implemented in this section impacts solely on the asset purchase programme by keeping the interest rate fixed and supporting its effects.

Table 4: Present value output multipliers from government purchases for different periods of forward guidance on interest rates

	1 quarter	2 quarters	3 quarters	4 quarters	6 quarters	8 quarters
Over 1 year	0.0489	0.0608	0.0774	0.1008	0.1851	0.4261
Over 10 years	0.0339	0.0403	0.0496	0.0628	0.1121	0.2569

Note: The present value multiplier weights the periods with the discount rate.

From this point of view, one reading of the results is that the central bank can increase the effectiveness of its asset purchase programme in an endogenous lower bound

environment by combining it with forward guidance on the policy rate. Although coming from a different perspective, this argument resembles the exposition of Woodford (2012) on the strategic complementarities between "quantitative easing" and forward guidance.

5   Asset purchases under optimal policy

After discussing the general properties of *ad hoc* asset purchase programmes, we intend to formulate an optimal path for the stock of government bonds held by the central bank. Similar to the approach followed for standard monetary policy (Woodford (2011a)), we assume that policymakers seek to minimise an intertemporal loss function

$$\min E_0 \left\{ \sum_{t=0}^{\infty} \beta^t Loss_t \right\} \tag{39}$$

subject to the non-negativity constraint

$$R_t \geq \underline{R} \tag{40}$$

with $\underline{R}$ as the effective lower bound on the interest rate and subject to all first-order conditions. We assume that the central bank has the enforcement technology to commit to the optimal policy conduct from a "timeless perspective".[9]   More specifically, the objective function takes the form:

$$Loss_t = \lambda_\pi (\pi_t)^2 + \lambda_Y (\hat{Y}_t - \hat{\bar{Y}}_t)^2 + \lambda_R (\hat{R}_t)^2 + \lambda_{BCB} (\hat{B}_{CB,t})^2, \tag{41}$$

where λ_π is the weight on inflation volatility and λ_Y that on the output gap[10] volatility. We also introduced a penalty for each instrument: λ_R on interest rate volatility and λ_{BCB} on the variability in government debt held by the central bank.[11] There are various theoretical and operational rationales for constraining the fluctuations of the standard and non-standard monetary policy instruments which we will not detail here. We only take for granted that operational monetary policy conduct faces implementation constraints in adjusting its bond portfolio or changing its key interest rates to a very large extent from one period to the next.

In this section, we consider the benchmark optimal policy conduct to be the case where both the short-term rate and the stock of government bonds held by the central bank are set optimally. To evaluate the stabilisation capacity of debt policy, we also consider the case in which only the short-term rate is set optimally. Later in the section, configurations in which the lower bound environment is relevant are contrasted with the unconstrained allocation. We start by reviewing the optimal asset purchase strategy in a specific crisis scenario that brings the economy to the lower bound. This first configuration

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[9]The loss function approach, by contrast to the welfare-based optimal policy, enables to examine different menu of central bank preferences and approximates in some cases the Ramsey allocation.

[10]The output gap is defined as the percentage deviation of output from its flexible price and wage equivalent

[11]Our approach can be seen as a hybrid because we do not explicitly derive the coefficients for λ_Y, λ_R, and λ_{BCB} under optimality. Rather, we derive welfare measures for a grid of possible values reflecting policy makers' possible preferences.

links to the policy evaluation of the previous section and constitutes a relevant preamble to the analysis of optimal instrument combination through the business cycle. Indeed, we investigate the optimal allocation in the absence of lower bound constraint across a range of policymaker preferences before repeating the exercises when the constraint becomes occasionally binding.

Within the confines of the model validity, interest rate policy and asset purchases feature strong strategic complementarities for both normal and crisis times. When constrained by the lower bound on the policy rate, optimal policy displays: *i)* a longer lower bound period and stronger "use of forward guidance" than in the estimated Taylor rule, *ii)* activist asset purchase policy, and *iii)* a clear sequencing of the exit strategy, first stopping (and unwinding) the asset purchases and significantly later raising the policy rate. In terms of macroeconomic performance, optimal asset purchase strategies have the potential to fully offset the costs of lower bound constraint on the policy rate.

5.1 Optimal asset purchase strategy at the lower bound of interest rate

In order to generate a lower bound environment, and as opposed to Section 4, we rely on an underlying scenario which brings the policy rate immediately to its lower bound but in qualitative terms resembles the crisis scenarios of Figure 2. A combination of adverse financial shocks (risk shock and bank-specific shock) and adverse demand-side shocks (investment-specific shock and government expenditures shock) contract the economy. The reason why we depart from the previous crisis scenarios is that, in our characterisation of optimal policy conduct, the central bank does not switch to asset purchases as soon as the policy rate hits its lower bound. Instead, in the optimal allocation, there is always active bond portfolio management, and the probability of reaching the lower bound is strongly mitigated. Since we first want to study the optimal asset purchase strategy at the lower bound, we design a scenario in which the lower bound binds immediately and compare various approaches to conducting policy in this environment. To derive the optimal policy in a lower bound environment, we introduce a non-negativity constraint on interest rates in the intertemporal maximisation programme of the policymaker. In computational terms, we again follow the approach of Holden (2016) to deal with the occasionally binding slackness condition on the interest rate constraint.

Regarding the weights of the loss function in equation (41), we normalise the penalty on inflation, λ_π to 1 and set the penalty for both the output gap, λ_Y, and interest rate volatility, λ_R, to 0.03. Those parameters are chosen such that, in the absence of the lower bound and APP, the volatility of inflation and interest rates under optimal policy is broadly the same as in the Taylor rule allocation. The penalty on the variance of central bank bond portfolio, λ_{BCB}, is set so that in the crisis scenario, the build-up of assets purchased culminates at around 8% of GDP (similar to the initial calibration of the ECB's APP). With the corresponding value for λ_{BCB}, the costs of such asset purchase volatility in terms of the unconditional loss function turns out to be of a similar magnitude to the costs associated with interest rate volatility (using the estimated structural shocks and in the absence of the lower bound constraint).

Note that here, we neglect the inefficiency costs associated with asset purchases, as assumed by Gertler and Karadi (2011, 2013) in a very stylised manner. In principle, the

inefficiency costs would affect the effectiveness of asset purchases, but crucially, we lack micro-foundations for it. Since our aim was to show the general properties of optimal asset purchase strategy, we do not present a sensitivity analysis of our results on inefficiency costs à la Gertler and Karadi (2011, 2013). Instead, the penalty introduced in the policymaker's objective indirectly controls for possible side-effects of excessive reliance on this type of instrument.

In Figure 8 we present the responses of selected variables to the crisis shocks under three different types of monetary policy: *i)* optimal instrument combination (black solid lines), *ii)* optimal interest rate policy only (blue dashed lines), and *iii)* the estimated Taylor rule without asset purchases (dashed red lines with dots). The last case is introduced to show how the economy behaves in the absence of optimal policy conduct based upon our estimation.
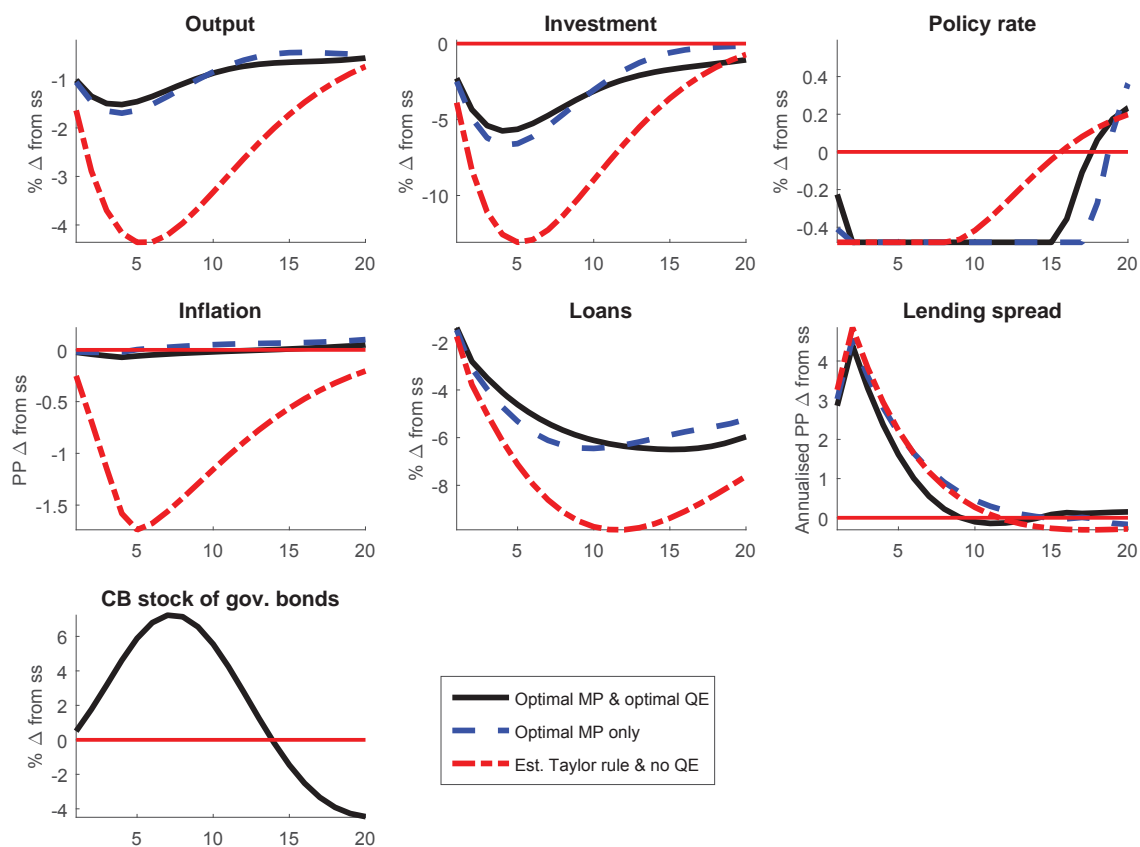
Figure 8: Optimal monetary policy at the effective lower bound on the interest rate

Indeed, under optimal policy conduct, the effects of the crisis scenario on output and inflation are drastically milder than under the estimated Taylor rule. In this latter case, the lower bound constraint is left much earlier. Optimal policy would actually keep interest rates at the lower bound for almost twice as long. This illustrates the inefficiency of a policy rule described by equation (36) at the lower bound. The result is well-known in the literature on optimal monetary policy at the lower bound. In specific modelling frameworks, optimal monetary policy in the absence of the lower bound constraint could take the form of an interest rate feedback rule, to which the estimated Taylor rule used in this paper would not be drastically different (see Giannoni and Woodford (2003a,b)).

However, once the lower bound constraint is introduced, the optimal management of expectations through a feedback rule on the "shadow" interest rate, $\hat{R}_{D,t}^*$, as in equation (36), would not be the same as in the unconstrained case (which is assumed in equation (36)): the larger the constraint imposed by the lower bound in responding to the crisis scenario, the longer the optimal lower bound period and the higher the inflation expectations that the policymaker needs to feed. This intuition about optimal policy conduct at the lower bound is well-framed in the seminal work of Eggertsson and Woodford (2003) and can be interpreted, in a loose sense, as requiring an intensive use of forward guidance.

Comparing the two optimal policy settings, macroeconomic allocation is improved when the central bank also sets its government bonds purchases optimally. The optimal instrument combination dominates even if we introduced a penalty on the volatility of the central bank balance sheet. As opposed to Woodford (2012), the propagation mechanism of asset purchases in our model works through bank portfolio rebalancing and the easing of credit conditions, but it does not entail a specific "signalling channel" through which it could directly support forward guidance on the policy rate. Therefore, whereas the optimal asset purchase strategy appears consistent with a significant degree of forward guidance, it also yields an earlier lift-off date from the lower bound compared to optimal interest policy only.

Figure 9 zooms into the optimal policy allocations. In this figure we reproduce the same cases as in Figure 8 (black lines for the optimal interest rate policy only, and the blue dashed lines labelled *case 1* for the optimal instrument combination). Besides, we add another optimal instrument combination in which the penalty on central bank balance sheet volatility is reduced compared to the benchmark case (see dashed red lines with dots labelled *case 2*). The more activist asset purchase strategy leads to further stabilisation gains and implies an earlier lift-off date for the policy rate, compared to both other cases. This shows that using forward guidance in conjunction with having a more intensive role for debt policy can be a powerful policy mix to stabilise the economy. As a result, the economy would exit the lower bound environment earlier. We can also relate this result to the findings from Section 4.7 on forward guidance on the policy rate. The stabilisation gains from the optimal asset purchase strategy are tangible relative to the optimal interest rate policy. However, these gains appear smaller than the improvements upon the estimated Taylor rule allocation, achieved through the optimal interest rate policy, or in other words, through forward guidance *à la* Eggertsson and Woodford (2003).
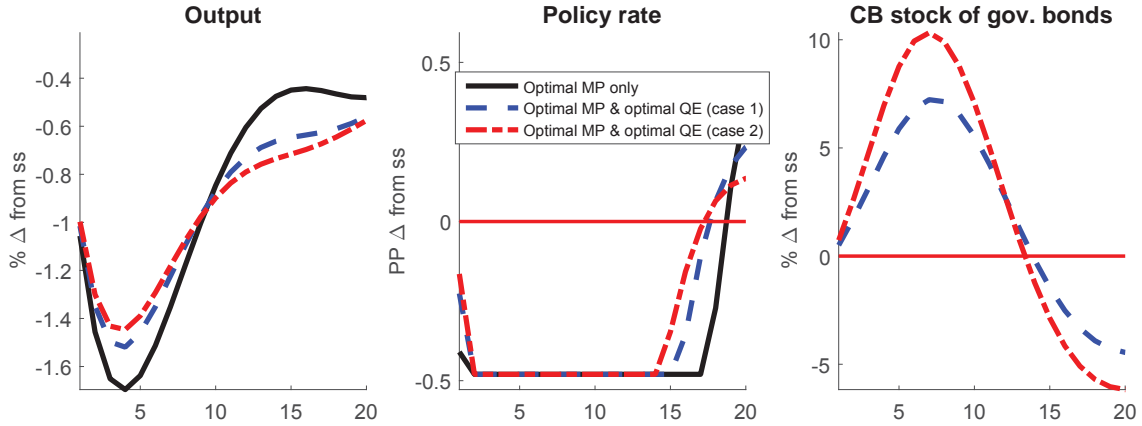
Figure 9: Optimal monetary policy at the effective lower bound on the interest rate and the intensity of government bond purchases

Finally, Figure 9 also illustrates the optimal sequencing of instruments at the lower bound. With the specific calibration of the policymaker loss function, the central bank bond portfolio builds up over 8 quarters before the unwinding of positions starts. The time profile of the programme is not affected much by the more or less activist stance regarding asset purchases. The policy rate is kept at the lower bound for more than 14 quarters, and in any case far beyond the end of the asset purchases. In this precise scenario, the lift-off date is relatively close to the point in time when the bond portfolio returns to its initial size.

5.2 Optimal combination of instruments through the business cycle

Up to now, we have analysed the properties of optimal conventional and unconventional policy in a crisis situation yielding a lower bound environment. In this section, we want to elaborate more on the macroeconomic stabilisation achievable from the class of optimal monetary policy conduct described previously. As a first step, we discard the lower bound constraint on the policy rate and exploit the business cycle regularities captured by the estimated DSGE model. Instead of investigating the optimal response to a specific crisis situation, we activate all structural disturbances in our model and use the estimated shock processes to derive policy efficiency curves. To construct those efficiency curves, we run a grid of weights on inflation in the policymaker loss function, and for every combination of weights, we plot the theoretical second moments for inflation on the y-axis and the model-consistent output gap on the x-axis.
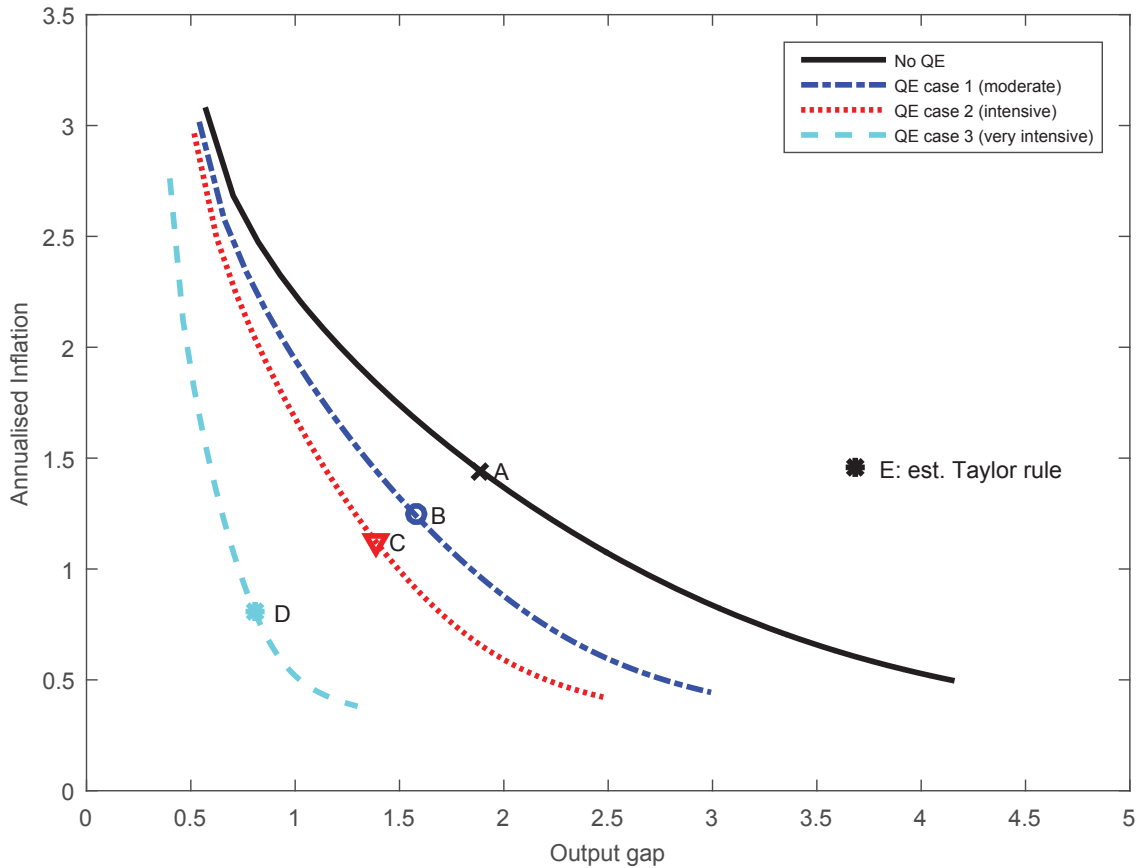
Figure 10: Efficiency frontiers for different penalties on the volatility of asset purchases

In order to quantify the stabilisation benefits provided by the optimal asset purchase strategy (in the absence of the lower bound constraint), we present four cases in Figure 10: *i)* optimal interest rate policy (solid black line), *ii)* optimal instrument combination with moderate asset purchases (dashed blue lines with dots), as in case 1 in Figure 9, *iii)* optimal instrument combination with intensive asset purchases (dotted red lines), as in case 2 in Figure 9, *iv)* optimal instrument combination with more intensive asset purchases (dashed cyan line), for which we further reduced the penalty on government debt in the loss function to obtain case 3 in Figure 9. As can be seen, increasing the activism of the central bank asset purchase strategy shifts the efficiency curves to the south-west quadrant. Combining policy instruments enables the monetary authority to contain the volatility of both inflation and the output gap beyond what can possibly be achieved with interest rate policy only. This result is related to the fact that financial frictions in the banking sector create a wedge between long-term and short-term interest rates. Asset purchases are able to influence this wedge directly, which is also tied to banks' lending decisions.

Moreover, the overall efficiency curve gets steeper when both instruments are combined. This implies that the policy tradeoff between output and inflation stabilisation weakens. With activist asset purchase strategies (case 3), the sacrifice ratio of going from an extremely "dovish" policymaker to an extremely "hawkish" one represents less than 1 pp of output gap standard deviation. The comparable number in the absence of asset purchases (black line) would reach almost 3.5 pp of output gap standard deviation.

In order to see the benefits of combining policy instruments, we pick a point on the efficiency curve in the absence of asset purchases (point **A**) which yields the same inflation (and interest rate) volatility as under the estimated Taylor rule (point **E**). This precise point refers to the same weights in the loss function as the ones used for Figure 9. Now, allowing for asset purchases with the same penalty for central bank balance sheet volatility as in case 1 of Figure 9 brings the allocation to point **B**. It turns out that for the policy preferences at point **A**, the stabilisation gains of asset purchases are relatively even on both inflation and the output gap. The same is true for going from point **B** to point **C**, or from **C** to **D**, as the penalty on central bank balance sheet volatility is reduced and asset purchase strategies are more activist. Therefore, while the overall efficiency curves shift inwards but steepen with increasingly activist asset purchases, the stabilisation improvements for intermediate policy preferences appear to be more homothetic in the inflation/output gap space.

5.3 Sensitivity to portfolio rebalancing frictions

Since our model has a rich banking sector, and given that the effectiveness of government bond purchases depends on the frictions within the banking segments, we now review how these frictions might actually affect the stabilisation performance of optimal asset purchase strategies. The central variable which controls the pricing of government bonds and portfolio rebalancing in the banking sector is δ^{BG}. Hence, we repeat the previous exercise and show efficiency frontiers for different values of δ^{BG}. The results are presented in Figure 11. The solid black line corresponds to the estimated value of the diversion parameter (at around 2). It can be contrasted with two other cases, in which the diversion parameter is lower (dashed blue line with dots) and larger (dotted red line).
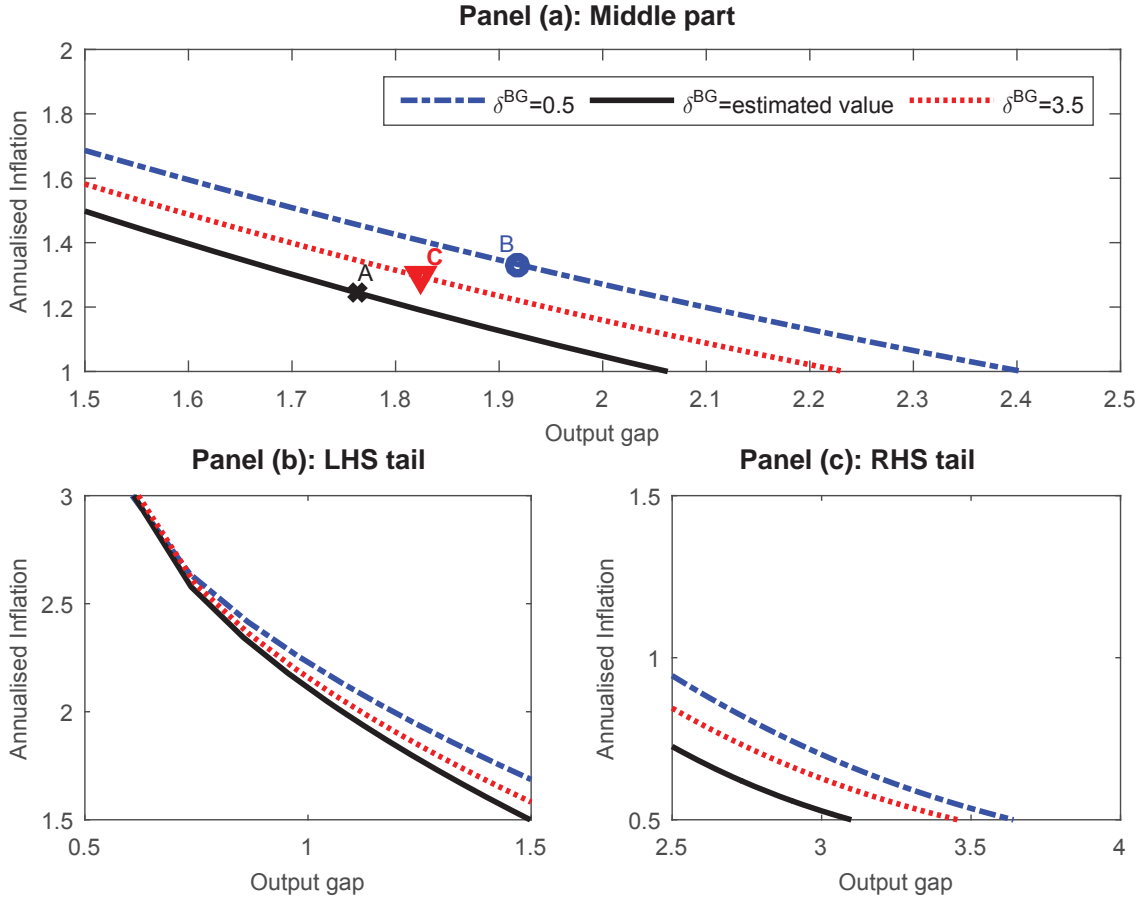
Figure 11: Efficiency frontiers for different degrees of financial frictions in the banking sector

Since the curves are very close to each other at the tails, we split them into three parts: the middle part (Panel (a)), the left-hand side tail (Panel (b)), and the right-hand side tail (Panel (c)). It turns out that the efficiency frontiers for the two alternative values are above the one based on the estimated value of the diversion parameter. Thus, there is evidence of strong non-linearity in the relationship. Furthermore, the high diversion parameter curve is closer to the one based on the estimated value, at the left-hand side tail, while the low diversion parameter curve is closer at the right-end side tail. This means that the shape of the efficiency frontier changes slightly with different diversion parameters.

The non-linearity regarding the size of the diversion parameter is made more explicit in Figure 12, where we present the standard deviations of the output gap and inflation as a function in δ^{BG} and for moderate and intensive asset purchase strategies. As can be seen from the graphs, there seems to be a point at which the variability of both the output gap and inflation is at its lowest. In our specific case, this minimum roughly coincides with the estimated parameter for the diversion rate. For lower values of this parameter, the optimal debt policy becomes less effective, which is why the volatility of the economy is higher. For higher values, the economy becomes more volatile because there is the feedback effect of debt policy on bank equity.
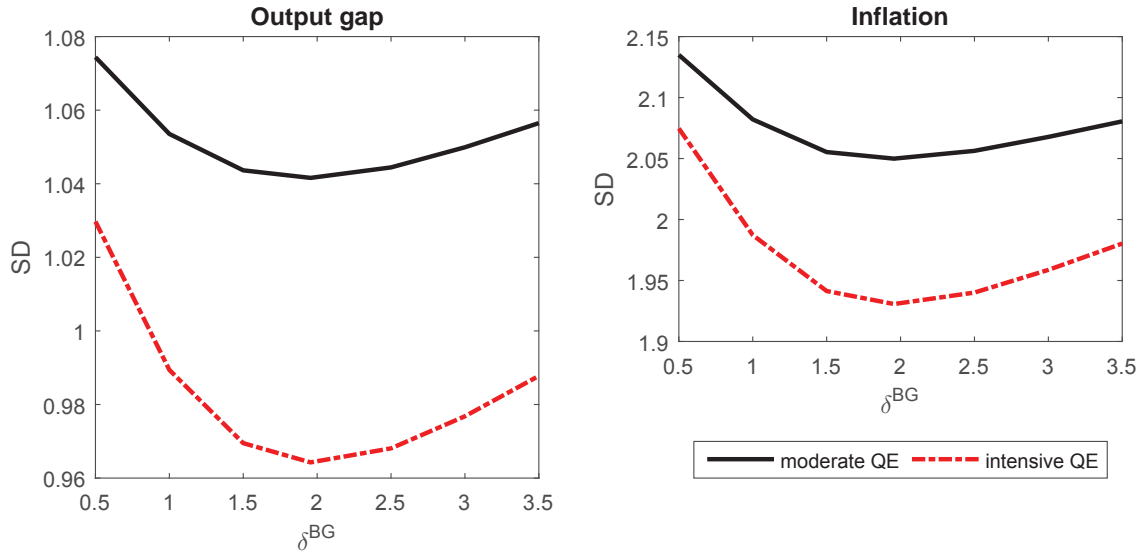
Figure 12: Impact of degrees of financial frictions in the banking sector on macroeconomic volatility

Another important friction which affects the transmission of government bond purchases through the banking sector to the real economy is the imperfect lending rate pass-through in the retail banking segment. If banks are unable to reset their return on assets (due to maturity transformation, indexation schemes or other rigidities in the underwriting of loan contracts), portfolio rebalancing towards credit exposures will be constrained and the associated easing in financing conditions muted. In the DSGE model, the main parameter underlying the staggered lending rate setting and proxying for the retail banking frictions is ξ_E^R. We zoom in again on the sensitivity of the efficiency curve to this parameter. This is shown in Panel (a) of Figure 13, where the black line is based on the estimated parameter value. The case for nearly full pass-through (dashed blue line with dots) corresponds to ξ_E^R close to zero, and is only slightly below the black line. The case for very low pass-through (dotted red line) corresponds to ξ_E^R close to one and is systematically above the black line. The dependency of the efficiency curve on ξ_E^R seems monotonic but non-linear, with marginal stabilisation costs being stronger, the higher the rigidities in the lending rate setting.

Panel (a): Efficiency frontiers

Panel (b): Output gap
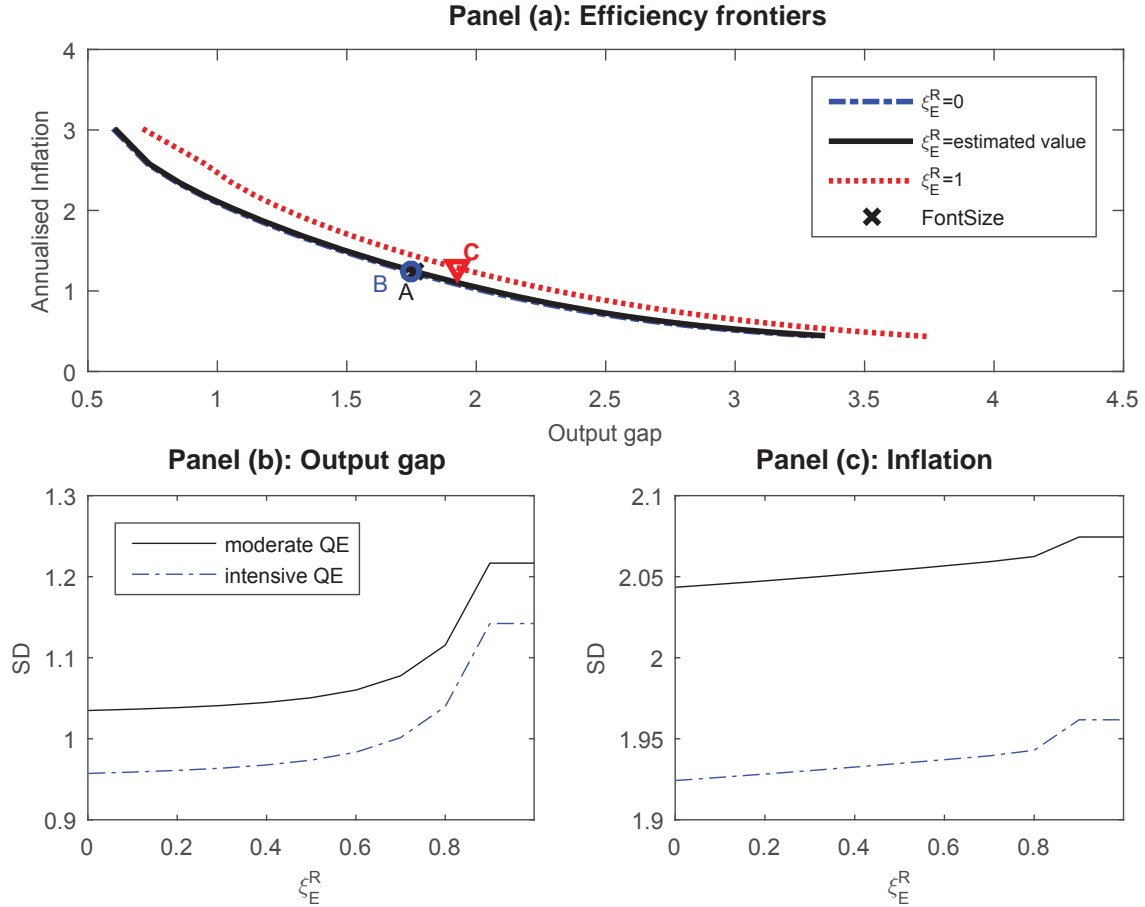
Panel (c): Inflation

Figure 13: Impact of staggered lending rate setting

This is confirmed in Panel (b) and (c) of Figure 13, where we plot, for given policy preferences, the variability of the economy as a function of ξ_E^R. When the lending rate pass-through goes from moderate to high (for values of ξ_E^R between 0.6 and 0), standard deviations are lower as the impulse is transmitted to a large extent into the borrowing conditions of non-financial corporations but the macroeconomic allocation is barely sensitive to the parameter. For lower levels of pass-through (higher values of ξ_E^R), the deterioration in macroeconomic volatilities increases rapidly as the banking system inefficiencies stand in the way of monetary policy actions.

Overall, this sensitivity analysis shows that the effectiveness of optimal policy conduct depends on the transmission of the impulses through the banking sector, which is the reason for our modelling choice.

5.4 Asset purchases and the stabilisation costs of the lower bound on nominal interest rates

In this last subsection, we extend the previous analysis on the efficiency curves to a lower bound environment. We aim at evaluating the stabilisation costs of the lower bound constraint and investigate whether active asset purchase strategies can meaningfully tame such distortions.

For the lower bound case, we need to rely on moments obtained from stochastic simulations compared to the theoretical moments derived in the absence of occasionally binding constraints. More specifically, for each point on the grid of policy preferences, we run simulations producing 500 time series in which we draw for every period in time and for every structural shock from a normal distribution with the corresponding estimated standard deviations. In Figure 14 we present the resulting efficiency curves (from a cubic smoothing approach). Figure 14 is basically the counterpart to Figure 10 except that the former is based upon simulated moments instead of theoretical moments.
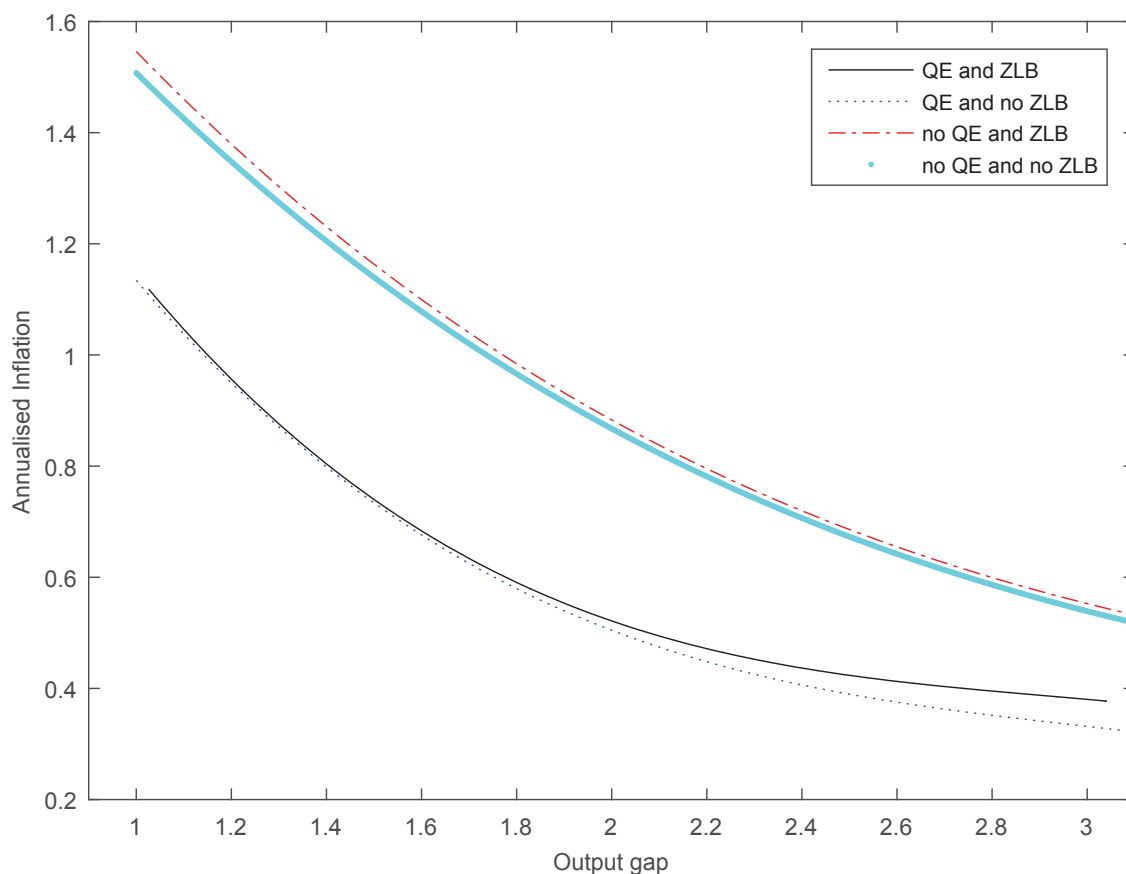

Figure 14: Efficiency frontiers with and without the lower bound constraint

In the absence of asset purchases, the lower bound restriction induces an outward shift in the efficiency curve (moving from the dotted cyan line to the dashed red line with dots): due to the lower bound environment, the achievable set of macroeconomic outcomes deteriorates as optimal monetary policy is not able to fully circumvent the constraint using forward guidance *à la* Eggertsson and Woodford (2003). As before, the activation of optimal asset purchase strategies provides an inward shift in the efficiency frontier. In the absence of the lower bound constraint, the efficiency frontier moves further to the south-west. Depending on the intensity of asset purchases, this gap can actually be fully closed. Therefore, the stabilisation costs of the lower bound environments crucially depend on the operational constraints on activist asset purchase strategies, i.e. on the policy preferences regarding the volatility of the central bank balance sheet.
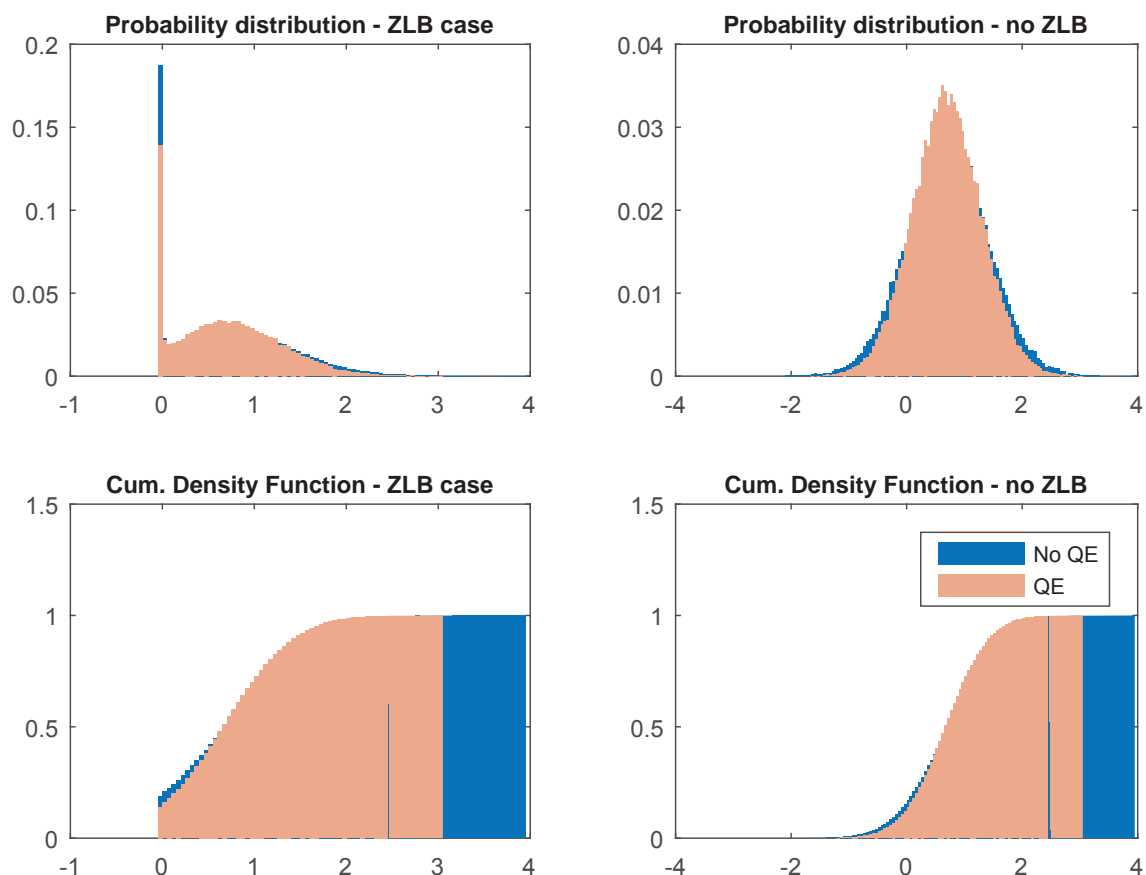
Figure 15: Probability distributions for the policy rate under optimal monetary policy, with or without asset purchases, with or without lower bound constraint

These properties of the optimal asset purchase strategy can also be examined by looking at the distribution of short rates. The top left chart in Figure 15 shows the probability distribution of the short rate for the lower bound case, while the right-hand chart shows the same in the absence of the lower bound constraint. In the second row, the corresponding cumulative density functions are displayed. The red bars correspond to optimal policy with asset purchases, the blue charts to optimal policy without asset purchases. For both cases, with and without considering the effective lower bound constraint, the density distributions have smaller tails. In the absence of the lower bound constraint, this means that large policy rate changes (decreases and increases) occur with a lower probability mass if asset purchases are available as an additional policy measure. For the case of the lower bound constraint, the probability of hitting the lower bound becomes even smaller. Asset purchases obviously reduce the volatility of the policy rate as an additional policy tool is available to stabilise the economy. However, the stabilisation gains crucially depend on the intensity of the purchase strategy.

6   Conclusion

This paper sheds some light on the effectiveness and suitability of central asset purchase programmes. In a lower bound environment, we first evaluate *ad hoc* programmes according to their implementation design and communication strategy. We start by assuming that the central bank wants to impose a specific path for its holdings of government debt. We demonstrate how the effects on the macroeconomy depend on the communication about the programme. Furthermore, we show that it is not only the holding period which matters for the success of the programme. A clearly communicated asset purchase strategy is also conducive to exiting sooner from the lower bound on interest rates as it supports the economic recovery. We try to mimic features of real world programmes in the first part and also investigate central bank asset purchases from an optimal policy perspective. We consider the stabilisation benefits of optimally combining the two instruments, in normal times as well as when the lower bound constraint on interest rates becomes binding.

Our results show that active bond portfolio management by a central bank can efficiently complement interest rate policy. This is particularly true once the lower bound has been reached. However, the macroeconomic stabilisation gains crucially depend on the potential for the asset purchases to influence the term premium. In our model, the term premium mainly results from frictions in the banking sector. Without these frictions, the term premium disappears. In a lower bound environment, asset purchases can be used to substitute the lack of reaction by the short-term policy rate. Otherwise, a policy which is able to reduce frictions in the banking sector might have similar effects to purchases of government bonds in our model. For example, Woodford (2016) investigates the joint conduct of unconventional monetary and macroprudential policy, which certainly represents an inspiring avenue for further research.

In the paper, we consider the euro area as a closed economy. Another interesting avenue for future research would be to investigate optimal government bond purchases in a monetary union consisting of heterogenous countries with different interests regarding their fiscal policy stance.

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A Estimation of the model

A.1 Data

Data for GDP, consumption, investment, employment, wages and the GDP deflator are taken from Fagan, Henry, and Mestre (2001) and Eurostat. Employment numbers replace hours. Consequently, as in Smets and Wouters (2005), hours are linked to the number of people employed e_t^* with the following dynamics:

$$e_t^* = \beta \mathbb{E}_t e_{t+1}^* + \frac{(1 - \beta \lambda_e)(1 - \lambda_e)}{\lambda_e}(l_t^* - e_t^*)$$

The three-month money market rate is the three-month Euribor taken from the ECB website, and we use backdated series for the period prior to 1999 based on national data sources. Data on MFI loans are taken from the ECB website. Data prior to September 1997 have been backdated based on national sources. Meanwhile, data on retail bank loan and deposit rates are based on official ECB statistics from January 2003 onwards, and on ECB internal estimates based on national sources in the period before that. The lending rates refer to new business rates. For the period prior to January 2003, the euro-area aggregate series have been weighted using corresponding loan volumes (outstanding amounts) by country.

For the estimation, the data were transformed as follows. We take the quarterly growth rate of GDP, consumption, investment and loans, all expressed in real terms and divided by working age population. The employment variable is also divided by working age population. Real wages are measured with respect to the consumption deflator. Interest rates and spreads are measured quarterly. With the exception of loan growth and the employment rate for which specific trend developments are not pinned down by the model, transformed data are not demeaned as the model features non-zero steady state values for such variables. A set of parameters are therefore estimated to ensure enough degrees of freedom to account for the mean values of the observed variables. Trend productivity growth γ captures the common mean of GDP, consumption, investment and real wage growth; $\overline{L}$ is a level shift that we allow between the observed detrended employment rate and the model-consistent one; $\overline{\pi}$ is the steady state inflation rate which controls for the mean of the inflation rate; and we also estimate the preference rate $r_\beta = 100(1/\beta - 1)$ which, combined with $\overline{\pi}$ and γ, pins down the mean of the nominal interest rate. Regarding spreads, the bank lending spread mean is related to the monopolistic markup $r_\mu = 100\left(\frac{R_{LE} - R_{BLE}}{\pi}\right)$, while the sovereign spread mean depends on the bankers' intermediation margin $\frac{R_{BLE} - R_D}{\pi}$ and the diversion rate δ_b. We choose loans and lending rates for the non-financial corporate sector. In principle, our model does not formally distinguish housing loans from non-housing loans, or business investment from residential investment. Nonetheless, we adopt a restrictive view on our credit frictions and interpret entrepreneurs in a strict sense as the non-financial corporate sector.

A.2 Calibrated parameters and prior distributions

Some parameters are treated as fixed in the estimation. The depreciation rate of the capital stock δ is set at 0.025 and the share of government spending in output at 18%.

The steady state labour market markup is fixed at 1.5, and we chose curvature parameters of the Kimball aggregators of 10. In the steady state calibration we fix the ratio of banks' holdings of government bonds to their loan book, $\alpha_B = \frac{\kappa_B^g}{\kappa_B^l}$, at 12%, in line with aggregate BSI statistics from the ECB. The total outstanding amount of sovereign debt in the steady state is assumed at 60% of annual GDP. In order to calibrate and choose the prior distribution for the parameters in the financial block of the model, the steady state level of lending rate spreads $\frac{R_{LLE}-R_D}{\pi}$ can be decomposed into three financial wedges.

- First, the *credit risk compensation* corresponds to the spread between the lending rate applied by loan officers and the return on the overall loan portfolio for the retail bankers: $r_{risk} = 100\frac{R_{LLE}-R_{LE}}{\pi}$.

- Second, the lending rate *competitive margin* is related to the retail banking monopolistic segment which applies a markup on the financing rate provided by the bankers: $r_\mu = 100\frac{R_{LE}-R_{BLE}}{\pi}$.

- Finally, the *bank capital channel* spread results from the decision problems of bankers and requires in equilibrium a higher return on private sector intermediation than on deposits, $r_B = 100\frac{R_{BLE}-R_D}{\pi}$.

Starting with the entrepreneurs, we calibrate the entrepreneurs' survival probability, ζ_e, and set the prior distributions of the monitoring costs, μ_e, the standard deviation of the idiosyncratic shock, σ_e, and the limited seizability parameter, χ_e, in order to match four macroeconomic stylised facts: the default frequency for firms of 0.7%; the credit risk compensation on the lending rate of 50 bps (in annual terms), which broadly corresponds to one-third of the sample mean of the lending spreads; the overall external finance premium $100\left(\frac{R_{KK}}{R_{LE}}-1\right)$ of 200 bps (in annual terms); and the average corporate loan to GDP ratio over our sample.

Then, the competitive margin r_μ is a free parameter in the estimation, and its prior distribution has a mean of 40 bps (in annual terms). We also estimate the Calvo lottery parameter related to the retail lending rate setting, ξ_E^R, for which we choose a relatively uninformative prior distribution. Let us now consider the banker's parameter space. In the steady state, equation (30), which links bank leverage to intermediation spreads, is given by

$$\lambda_b\widetilde{\kappa}_B/\zeta_b = \beta\gamma^{-\sigma_C}\left(\frac{r_B}{100}\widetilde{\kappa}_B + \frac{r_\mu}{100}\kappa_B^l + R_D\right)(\lambda_b\widetilde{\kappa}_B + (1-\zeta_b)).$$

Assuming a fixed ratio of government bonds to loans in bank balance sheet, α_B, then this relation can be re-written as

$$\beta\gamma^{-\sigma_C}\left(\frac{r_B}{100} + \frac{r_\mu}{100(1+\delta_b\alpha_B)}\right) = \frac{\lambda_b\widetilde{\kappa}_B - \zeta_b}{\widetilde{\kappa}_B\left(\zeta_b + \lambda_b\widetilde{\kappa}_B\frac{\zeta_b}{(1-\zeta_b)}\right)}. \tag{42}$$

For given values of λ_b and ζ_b, intermediation spreads are a non-monotonic function of bank leverage, $f_{\lambda_b,\zeta_b}(\widetilde{\kappa}_B)$. Moreover, steady state levels for the intermediation spreads and bank leverage can be consistent with multiple combinations of λ_b and ζ_b. Therefore, in order to reduce the parameter space in the estimation and to bring back monotonicity

51

in this steady state relationship, we restrict the steady state allocations for values of $\widetilde{\kappa}_B^*(\lambda_b, \zeta_b)$ which maximise $f_{\lambda_b, \zeta_b}(\widetilde{\kappa}_B)$. This is the case for

$$\lambda_b \widetilde{\kappa}_B^* = \zeta_b + \sqrt{\zeta_b} \tag{43}$$

implying intermediation spreads of

$$\beta \gamma^{-\sigma_C} \left(\frac{r_B}{100} + \frac{r_\mu}{100(1 + \delta_b \alpha_B)} \right)^* = \frac{(1 - \zeta_b)}{\widetilde{\kappa}_B \left(\zeta_b + \sqrt{\zeta_b} \right)}. \tag{44}$$

Under such constraints, the intermediation spread $\frac{r_B}{100} + \frac{r_\mu}{100(1+\delta_b\alpha_B)}$ is a decreasing function of bank leverage $\widetilde{\kappa}_B$ which depends only on ζ_b. Moreover, bank leverage and the survival probability of bankers determine uniquely the diversion rate parameter λ_b. Then, in our calibration strategy, we first set $\widetilde{\kappa}_B$ at 8 (i.e "weighted" capital ratio of 12.5%). After that, we estimate ζ_b, choosing a prior mean which implies a bank capital channel spread r_B of around 50 bps (in annual terms). This is consistent with a prior value for λ_b of around 0.3. Finally, the steady state value of initial transfers to new bankers, Ψ_B, is endogenously set so that the bank net worth accumulation holds (see equation (14)).

From the first-order conditions of the bankers' decision problem (12) and (13), we see that the steady state level of sovereign spread is linked to r_B by

$$\delta_b \frac{(R_G - R_D)}{\pi} = \frac{r_B}{100}. \tag{45}$$

We estimate δ_b using a prior distribution of mean 2. We set the geometric decay of the perpetual coupons on sovereign bond τ_g so that the duration of the securities is 10 years. The initial coupon level is adjusted to ensure that the steady state sovereign bond price Q_B equals 1. Regarding households' portfolio decisions, the adjustment cost parameter on the holding of sovereign securities, χ_H, is left free in the estimation, choosing a prior distribution of a mean 0.1. For the household first-order condition on sovereign bond holdings to be consistent with the steady state sovereign spread and the share of bank holding of sovereign bonds, we let $\overline{B}_H$ clear the steady state relationship associated with equation (1).

B Optimal stabilisation without asset purchases and in the absence of the lower bound on nominal interest rates

This section complements Section 5.2 in the main text and presents in greater detail in the derivation policy efficiency curves without asset purchases and neglecting the incidence of the lower bound on interest rates. We run a grid of weights on inflation in the policymaker loss function, and we plot in Figure 16, for every combination of weights, the theoretical second moments for inflation on the y-axis and the model-consistent output gap on the x-axis. We then repeat the same grid with three different values for the penalty on interest rate volatility. The resulting curves represent efficiency frontiers reflecting the optimal tradeoff between inflation and output gap stabilisation. Furthermore, point **E** shows the

corresponding macroeconomic standard deviations under the estimated Taylor rule.

As expected, the estimated Taylor rule delivers a macroeconomic performance that is sizeably inferior to the optimal policy allocation: point **E** in Figure 16 lies well within the convex area delimited by the efficiency curves. The optimal policy implementing the same inflation (output gap) volatility as the estimated rule would lead to a standard deviation for the output gap (inflation) that is almost twice (three times) lower. Figure 16 also shows that increasing the penalty on interest rate volatility worsens the performance of optimal policy, but the stabilisation costs in terms of output and inflation are limited across a wide range of policy preferences. Indeed, for very "hawkish" policymakers (at the extreme right of the efficiency curve), the highest interest rate penalty (see red dotted curve labelled Case 2) increases output and inflation standard deviations by less then 0.1 pp and 0.05 pp, respectively, compared to the benchmark case (see black curve). For "dovish" policymakers however (towards the extreme left of the efficiency curve), the stabilisation costs become significant, and particularly so for output gap volatility.
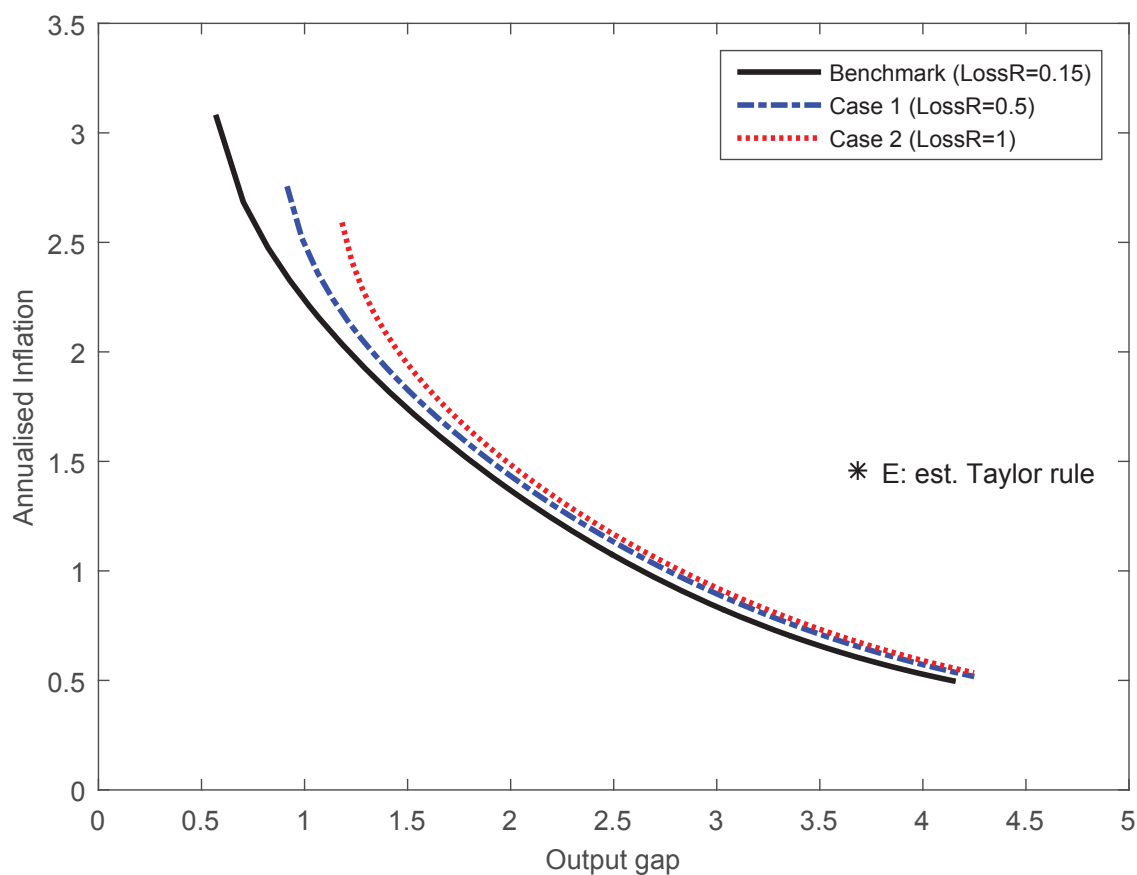

Figure 16: Efficiency frontiers for different penalty on the interest rate volatility

C   Technical implementation for different information sets regarding ad hoc programmes

In this section we present the technicalities behind the investigation of different information sets for the agents, i.e. the communication strategy of the central bank, as done in

Section 4.5.

The full communication of the programme we have already outlined in Equation 37. For the sake of completeness, we start from this equation

$$\log(\epsilon_t^{QE}) = \rho_1^{QE} \log(\epsilon_{t-1}^{QE}) + \sum_{i=0}^{T} \nu_{t-i}^{QE,i}. \tag{46}$$

As is evident from looking at the last part of the equation, the sum represents announced purchases which will occur at a later point in time. In order to investigate the communication strategy of the central bank, we impose the true path for the stock of government bonds purchased by the central bank. However, we assume that agents believe in a different policy rule. This means, for the unexpected frontloading, that they know the offloading path, i.e. the AR1 coefficient, but only see the current purchases. Consequently, we impose the policy rule

$$\log(\epsilon_t^{QE}) = \rho_1^{QE} \log(\epsilon_{t-1}^{QE}) + \tilde{\nu}_t^{QE}, \tag{47}$$

where $\tilde{\nu}_t^{QE}$ represents the hypothetical shocks which produce the same path as under the fully communicated programme. Hence, private agents see only news every period. The completely unexpected programme is implemented accordingly. In this case the AR term only drops out

$$\log(\epsilon_t^{QE}) = \tilde{\nu}_t^{QE} \tag{48}$$

and $\tilde{\nu}_t^{QE}$ are the perceived shocks. In the same vein, the two programmes with unexpected frontloading and unexpected offloading are implemented. In order to investigate the unexpected frontloading, we start from Equation 47 and impose AR coefficients which deviate from those in Equation 37. For the positive (negative) surprises during offloading, the AR1 coefficient that the agents see is smaller (larger) than the actual one.