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Chow-Lin X N: how adding a panel dimension can improve accuracy

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Non-technical summary

Research Question

Practitioners and academics are often faced with the problem that some time series of interest are not available at high frequencies. When related series exist, the Chow and Lin (1971) methodology can be used to disaggregate a low frequency series into its high frequency counterpart using the available related series. The aim of this paper is to increase accuracy of the Chow and Lin (1971) methodology by exploiting information from the cross-sectional dimension.

Contribution

We suggest jointly estimating multiple Chow and Lin (1971) equations, one for each cross-sectional unit, restricting the coefficients to be the same across units in order to interpolate unit-specific data. In contrast to the estimation of single equations, unobservable time-varying characteristics (e.g. structural breaks) that are common across units can be taken into account. The proposed approach is straightforward to implement and can readily be applied to various settings.

Results

We provide empirical evidence that the panel-based approach can improve accuracy compared to single equation models, in particular when the time dimension of available data is short. Furthermore, the results suggest that controlling for unobservable time-varying characteristics can improve accuracy of the resulting interpolated series.

Nichttechnische Zusammenfassung

Fragestellung

Forscher werden häufig mit dem Problem konfrontiert, dass betrachtete Zeitreihen nicht in hoher Frequenz verfügbar sind. Liegen jedoch verwandte, hochfrequente Zeitreihen vor, kann eine niederfrequente Zeitreihe nach Chow und Lin (1971) disaggregiert werden. Ziel des vorliegenden Papiers ist es, die Genauigkeit des Chow und Lin (1971)-Ansatzes durch Einbeziehung von Querschnittsinformationen zu verbessern.

Beitrag

Mehrere Chow und Lin (1971)-Gleichungen werden (bei Annahme identischer Regressionskoeffizienten für die einzelnen Querschnittseinheiten) simultan geschätzt. Auf Basis dieser Schätzungen können einheitenspezifische Zeitreihen interpoliert werden. Im Gegensatz zu einer Schätzung von Einzelgleichungen können unbeobachtbare, zeitvariable Charakteristika (z.B. Strukturbrüche), die allen Querschnittseinheiten gemeinsam sind, berkücksichtigt werden. Der vorgeschlagene Ansatz ist einfach zu implementieren und kann leicht an verschiedene Anwendungen angepasst werden.

Ergebnisse

Die empirische Evidenz zeigt, dass der Panel-basierte Ansatz inbesondere dann zu genaueren interpolierten Zeitreihen führen kann, wenn die verfügbaren Daten eine geringe Zeitdimension aufweisen. Darüber hinaus deuten die Ergebnisse darauf hin, dass die Berücksichtigung unbeobachtbarer, zeitvariabler Charakteristika die Genauigkeit der interpolierten Reihen verbessern kann.

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Abstract

Single equation models are well established among academics and practitioners to perform temporal disaggregation of low frequency time series using available related series. In this paper, we propose an extension that exploits information from the cross-sectional dimension. More specifically, we suggest jointly estimating multiple Chow and Lin (1971) equations, one for each cross-sectional unit (e.g. country), restricting the coefficients to be the same across units in order to interpolate unit-specific data. Using actual data on real GDP and industrial production for euro area countries we provide evidence that this approach can result in more accurate interpolated time series for individual countries. The results suggest that the inclusion of time fixed effects, which is not feasible in standard single equation models, can be helpful in increasing accuracy of the resulting series.

Keywords: temporal disaggregation, interpolation, panel data.

JEL classification: C23, C53.

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1 Introduction

This paper proposes a panel-based variant of Chow and Lin (1971) to disaggregate time series using related series. Practitioners and academics are often faced with the problem that some time series of interest are not available at high frequencies. When related series exist, however, the Chow and Lin (1971) methodology can be used to disaggregate a low frequency series into its high frequency counterpart using the available related series. A common example is that of GDP and industrial production. While the highest available frequency of the former is quarterly, the latter is also available at a monthly frequency. Chow and Lin (1971) suggest to disaggregate the low frequency series of interest by using information provided by the related high frequency series. In order to do so, the low frequency series is first regressed on the high frequency series that has been transformed in a way such that it matches the frequency of the low frequency series. The obtained slope coefficient can then be used to disaggregate the low frequency series. This approach has proven to generate relatively accurate high frequency estimates of the series considered.

As mentioned by Chow and Lin (1971), a GLS estimator may perform better than OLS by allowing for autocorrelated residuals. Further improvements can be obtained, for instance, by assuming that residuals follow a random walk (Fernández, 1981). Apart from changing assumptions on the underlying autocorrelation structure of the residuals, previous research improved estimates by adapting the Chow and Lin (1971) methodology to various settings: Santos Silva and Cardoso (2001) to dynamic models and Proietti (2006) to state space models, among others.

We contribute to the latter strand of the literature by adding a panel dimension to different estimation strategies. More specifically, we suggest jointly estimating multiple Chow and Lin (1971) equations, one for each cross-sectional unit (e.g. country), restricting the coefficients to be the same across units in order to interpolate unit-specific data. In an empirical exercise for euro area countries[1], we provide evidence that the panel-based variants can indeed improve accuracy compared to single equation models. The approach is straightforward to implement and can readily be applied to various settings.

Section (2) briefly reviews the standard Chow and Lin (1971) methodology and outlines how a panel dimension can be added in a straightforward way. Section (3) challenges the proposed panel-based approach using actual data for euro area countries as a benchmark against which to evaluate the interpolated time series. The final section concludes.

2 Econometric procedure

2.1 Standard approach

Without loss of generality, assume that an unobservable quarterly series y_q with dimension $4T \times 1$ shall be interpolated based on an available annual series y_a with dimension $T \times 1$. In line with Chow and Lin (1971), the series to be constructed can be expressed as a function of observable quarterly series x_q with dimension $4T \times K$. Note that x_q may also contain an intercept and/or multiple indicator series. In the case with intercept and one indicator series, $K = 2$. Hence,

[1] Codes are available upon request.

$$y_q = x_q\beta + \epsilon, \tag{1}$$

where β denotes a $K \times 1$ estimable parameter vector. The residual vector ϵ has a $4T \times 4T$ covariance matrix $\sigma^2\Omega$, where σ is a constant scalar. In order to aggregate the quarterly series y_q into its annual counterpart, assume that $y_a = Cy_q$, where $C = [I \otimes c']$ is a suitably defined $T \times 4T$ aggregation matrix.[2] y_a can then be expressed as

$$y_a = x_a\beta + C\epsilon, \tag{2}$$

where $x_a = Cx_q$. Note that the covariance matrix of $C\epsilon$ is given by $\sigma^2 C\Omega C'$. β can then be estimated by GLS:

$$\hat{\beta} = [x_a'(C\Omega C')^{-1}x_a]^{-1}x_a'(C\Omega C')^{-1}y_a. \tag{3}$$

An estimate of the interpolated quarterly series is given by

$$\hat{y}_q = x_q\hat{\beta} + \Omega C'(C\Omega C')^{-1}(y_a - x_a\hat{\beta}), \tag{4}$$

where $x_q\hat{\beta}$ is the conditional expectation of y_q given x_q and $\Omega C'(C\Omega C')^{-1}(y_a - x_a\hat{\beta})$ are predicted values of the disturbances. Weak data availability, however, may either render the estimation of single Chow and Lin (1971) equations based on (3) infeasible or yield rather inaccurate disaggregated times series. In what follows, we therefore add a panel dimension to this set-up.

2.2 Panel-based approach

Again, we assume that an unobservable quarterly series y_q with dimension $4T \times 1$ shall be interpolated based on an available annual series y_a with dimension $T \times 1$. Contrary to before, those series are available for a cross-section of $i = 1, ..., N$ units.[3] Stacked vectors are defined as follows: $\bar{y}_j \equiv [y'_{1,j}, ..., y'_{N,j}]'$ and $\bar{x}_j \equiv [x'_{1,j}, ..., x'_{N,j}]'$ for $j \in \{a, q\}$, where the matrices/vectors of the quarterly series, x_q and y_q, are of dimension $4TN \times K$ and $4TN \times 1$. The annual counterparts are of dimension $TN \times K$ and $TN \times 1$, respectively. Hence,

$$\bar{y}_q = \bar{x}_q\beta + \epsilon, \tag{5}$$

$$\hat{\beta} = [\bar{x}'_a(C\Omega C')^{-1}\bar{x}_a]^{-1}\bar{x}'_a(C\Omega C')^{-1}\bar{y}_a, \tag{6}$$

$$\hat{\bar{y}}_q = \bar{x}_q\hat{\beta} + \Omega C'(C\Omega C')^{-1}(\bar{y}_a - \bar{x}_a\hat{\beta}), \tag{7}$$

where the dimensions of Ω and C change to $4TN \times 4TN$ and $TN \times 4TN$, respectively.[4] Similar to the standard approach, the dimension K depends on the number of indicator

[2] The form of C depends upon the specific problem considered. For flow data, for instance, $c' = [1111]$.

[3] We here consider a balanced panel.

[4] Under standard assumptions it can be shown that the GLS estimator is consistent and efficient (see, for instance, Newey and McFadden, 1994). In practice, however, Ω is not known and has to be replaced by an estimator $\hat{\Omega}$. For brevity, we do not give a detailed overview on the choice of $\hat{\Omega}$ and refer, for instance, to Santos Silva and Cardoso (2001).

series as well as the intercept term. In the empirical exercise below, we consider the case of a common intercept term across units together with one indicator (labelled "pooled") as well as unit-specific intercepts with one indicator series (labelled "within-group"). We assume that the slope coefficient is homogeneous across the N units. This assumption allows us to estimate β in (5) using stacked panel data which can yield efficiency gains whenever the number of cross-sectional units N is sufficiently large.[5]

Note that the proposed panel-based approach has two additional appealing features compared to single equation models: First, temporal disaggregation using related series is possible even if only one low frequency data point is available for each cross-sectional unit. Second, the panel-based approach allows for the inclusion of time fixed effects which may increase the accuracy of the resulting interpolated series by controlling for unobservable time-varying characteristics (e.g. structural breaks) that are common across units.

3 Empirical evidence

3.1 Set-up

We use the standard single equation as well as the proposed panel-based variants of the Chow and Lin (1971) methodology to disaggregate annual real GDP for 12 euro area countries into the quarterly counterpart using industrial production as the related series.[6] Since quarterly real GDP is available, it can then be used as a natural benchmark against which to evaluate the interpolated high frequency GDP series without resorting, for instance, to Monte-Carlo simulation exercises.

Two strategies are considered in order to evaluate the performance of the proposed panel-based variants: First, we keep the beginning of the sample (i.e. 2001) fixed and increase the sample length recursively from 2001 to 2015 (labelled "forward extension"). This strategy yields samples characterized by a relatively high degree of cross-sectional homogeneity up until the Great Recession. Second, we consider an alternative loop across the sample in which we keep the final observation (i.e. 2015) fixed and decrease the starting point of the sample recursively from 2015 to 2001 (labelled "backward extension"). This set-up yields samples with a higher degree of cross-sectional heterogeneity in particular in smaller samples owing to the incidence of the Great Recession.[7]

Practitioners are usually interested in interpolating one particular unit-specific time series. A way for us to assess the accuracy of the proposed panel-based approach would then be to focus on just one country series and to compare the high frequency estimates with the actual counterpart. Focusing on just one country, however, provides little in-

[5]The realization of efficiency gains obviously crucially depends on the validity of this homogeneity assumption.

[6]Data for the period 2000 to 2015 are obtained from Deutsche Bundesbank sources. We transform the data into first differences of the log-levels and disaggregate the year-on-year growth rate of real GDP by Q4 using the standard single equation model as well as the two panel-based variants: pooled and within-group. The choice of the aggregation matrix C ensures that the sum of four quarter-on-quarter growth rates equals the corresponding year-on-year rate. Countries included: Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal, and Spain.

[7]Country-specific averages of quarter-on-quarter real GDP growth rates, for instance, have a standard deviation of 0.28 percentage points up until 2008Q3 (the collapse of Lehman Brothers) and 0.42 afterwards.

formation to what extent the panel-based approach is *ex ante* superior to estimating country-specific Chow and Lin (1971) equations. We therefore focus on all countries considered and present the mean absolute errors[8] from comparing the actual quarterly real GDP series with the respective high frequency estimates for the different estimators and data samples considered. Concerning the choice of $\hat{\Omega}$, we focus on Chow and Lin (1971) with the assumption of uncorrelated as well as autocorrelated residuals. Moreover, we follow Fernández (1981) who assumes that errors follow a random-walk.

3.2 Results

Results for the forward extension can be found in Table (1). In the absence of time fixed effects, we observe that single equation models tend to yield more accurate results as the data sample increases. This is, for instance, the case after 9 years for the Chow and Lin (1971) with AR(1)-distributed residuals[9] and also after 9 years for the Fernández (1981) version. When time fixed effects are included, however, the panel-based approach outperforms the single equation counterpart in all cases (recall that including time fixed effects is not possible in the single equation models). Overall, the results therefore suggest that adding a panel dimension to disaggregate low frequency times series can improve accuracy of the resulting interpolated series in particular when time fixed effects are controlled for.[10] This finding is invariant to the choice of the specific panel-based variant as indicated by underlined entries in Table (1). The choice between the within-group and the pooled estimator, however, is not straightforward, in particular for $\hat{\Omega} = I$ and the alternative of Chow and Lin (1971) with AR(1)-distributed residuals.

Table (2) shows the results for the backward extension. Similar to before, the statistics show that both panel-based variants including time fixed effects yield smaller errors compared to the single equation models in almost all of the samples considered. In this case, however, the within-group estimator seems to outperform the pooled counterpart. This finding may be driven by the Great Recession which caused strong and quite heterogeneous declines in GDP and industrial production across the euro area countries. Statistically, these declines can be interpreted as structural breaks in the series considered and cause the samples generated by the backward extension to be quite heterogeneous. Panel regressions with unit-specific intercepts are able to account for this heterogeneity to some extent. Hence, the within-group estimator tends to yield lower errors and should thus be preferred. As expected, the more heterogeneous the samples become, the more

[8] Average (unweighted) errors across countries. Root mean squared errors yield similar results and statistics are available upon request.

[9] As this paper is not about choosing the autocorrelation coefficient optimally, we exemplarily set $\rho = 0.9$. Results are, however, robust to the specific choice of ρ.

[10] One might argue that results are driven by some omitted variable bias: The standard single equation models ("Single" in Tables (1) and (2)) by construction only capture the country-specific industry. In the proposed panel-based variants, however, a European dimension is added. Thus, in addition to the country-specific and thus idiosyncratic component, the panel-based variants include information on aggregate euro area business cycle fluctuations which is absent in the single equation models. To address this issue, we construct a measure of average industrial production for the euro area countries (excluding country i) and add this measure as a separate regressor in the single equations models (in addition to industrial production of country i). Again, mean absolute errors are computed and compared to those resulting from the different panel-based variants. Results are robust suggesting that some omitted variable bias as argued before is not driving the results. Statistics are available upon request.

beneficial is the within-group estimator relative to the pooled counterpart *et vice versa*.

4 Conclusion

This paper proposes an alternative approach to disaggregate low frequency time series using available related series. We show how a panel dimension can be added to Chow and Lin (1971) type of methodologies and demonstrate in an empirical exercise using actual data for euro area countries that this approach can yield more accurate interpolated time series. More specifically, the results suggest that the inclusion of time fixed effects in the panel-based variants – a feature which cannot be implemented in standard single equation models – can increase the accuracy of the resulting series. Hence, even if academics and practitioners are only interested in interpolating data for one particular unit (e.g. country), information from the cross-sectional dimension may be used if corresponding panel data are available. The proposed approach is straightforward to implement and can readily be applied to various settings. It may thus serve as an additional tool to generate more accurate estimates of unobservable high frequency series.

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Table 1: Model evaluation: Forward extension

	Single	Pooled	Within	Single	Pooled	Within
Year Fixed Effects	No	No	No	No	Yes	Yes
From 2001 to ...		$\hat{\Omega} = I$			$\hat{\Omega} = I$	
2001	—	**0.4581**	—	—	—	—
2002	1.1079	0.4499	**0.4342**	1.1079	**0.4998**	0.5422
2003	0.6391	0.5028	**0.4717**	0.6391	0.5238	**0.4695**
2004	0.5261	0.4838	**0.4759**	0.5261	0.4883	**0.4779**
2005	0.6671	0.4774	**0.4612**	0.6671	0.4826	**0.4615**
2006	0.6182	0.4847	**0.4839**	0.6182	0.4702	**0.4647**
2007	0.5527	0.5064	**0.4985**	0.5527	0.4940	**0.4828**
2008	0.5766	**0.5630**	0.5845	0.5766	**0.4936**	0.4971
2009	**0.6202**	0.6244	0.6441	0.6202	**0.5285**	0.5329
2010	0.6084	**0.6067**	0.6247	0.6084	**0.5509**	0.5724
2011	0.6225	**0.6120**	0.6243	0.6225	**0.5673**	0.5810
2012	0.6180	**0.6177**	0.6278	0.6180	**0.5595**	0.5639
2013	**0.6169**	0.6175	0.6242	0.6169	0.5620	**0.5607**
2014	**0.6087**	0.6090	0.6102	0.6087	0.5533	**0.5425**
2015	0.6050	0.6028	**0.6016**	0.6050	0.5521	**0.5391**
From 2001 to ...		$\hat{\Omega} = AR(1), \rho = 0.9$			$\hat{\Omega} = AR(1), \rho = 0.9$	
2001	—	**0.4571**	—	—	—	—
2002	1.1079	0.4476	**0.4438**	1.1079	**0.5283**	0.5391
2003	0.7433	**0.4835**	0.4878	0.7433	**0.4898**	0.5031
2004	0.5845	0.4789	**0.4786**	0.5845	0.4816	**0.4815**
2005	0.6127	0.4665	**0.4647**	0.6127	0.4713	**0.4693**
2006	0.5201	**0.4785**	0.4805	0.5201	**0.4756**	0.4777
2007	0.5275	0.5034	**0.5029**	0.5275	0.5033	**0.5027**
2008	0.5818	**0.5716**	0.5750	0.5818	**0.5131**	0.5150
2009	0.6087	**0.6008**	0.6009	0.6087	0.5575	**0.5571**
2010	**0.5843**	0.5965	0.5977	0.5843	**0.5677**	0.5686
2011	**0.5673**	0.5798	0.5792	0.5673	0.5538	**0.5526**
2012	**0.5606**	0.5745	0.5747	0.5606	**0.5510**	0.5511
2013	**0.5678**	0.5758	0.5759	0.5678	0.5500	**0.5499**
2014	**0.5601**	0.5667	0.5663	0.5601	0.5403	**0.5395**
2015	**0.5603**	0.5721	0.5717	0.5603	0.5430	**0.5420**
From 2001 to ...		$\hat{\Omega}$ = Fernández (1981)			$\hat{\Omega}$ = Fernández (1981)	
2001	—	**0.4905**	—	—	—	—
2002	1.1079	0.4464	**0.4442**	1.1079	**0.4577**	0.5382
2003	0.7959	**0.4959**	0.4969	0.7959	**0.5051**	0.5167
2004	0.7027	0.4822	**0.4778**	0.7027	0.4839	**0.4799**
2005	0.5982	0.4675	**0.4634**	0.5982	0.4712	**0.4682**
2006	0.5339	0.4790	**0.4766**	0.5339	0.4788	**0.4773**
2007	0.5236	0.5051	**0.5023**	0.5236	0.5056	**0.5034**
2008	0.5771	**0.5629**	0.5631	0.5771	0.5141	**0.5123**
2009	0.6125	0.5966	**0.5963**	0.6125	0.5587	**0.5566**
2010	**0.5862**	0.5989	0.5980	0.5862	0.5657	**0.5651**
2011	**0.5684**	0.5765	0.5752	0.5684	0.5513	**0.5506**
2012	**0.5586**	0.5733	0.5722	0.5586	0.5509	**0.5506**
2013	**0.5669**	0.5751	0.5740	0.5669	0.5501	**0.5493**
2014	**0.5593**	0.5665	0.5655	0.5593	0.5408	**0.5397**
2015	**0.5589**	0.5710	0.5703	0.5589	0.5416	**0.5409**

Notes: The table presents the mean absolute errors from comparing the interpolated quarterly real GDP series (using industrial production as related series) with actual quarterly real GDP. For each sample, the average (unweighted) error across countries is calculated and rescaled by the factor 100 for better visualization. "Single" refers to the error resulting from standard country-wise interpolation of the quarterly series, "Pooled" to an interpolation based on a panel regression with a common intercept, and "Within" to an interpolation based on a panel regression with unit-specific intercepts. Note that a country-wise interpolation ("Single") based on just one yearly observation is not feasible. Similarly, adding year fixed effects is not possible for the single equation models. Bold entries highlight the lowest errors for each sample and assumption concerning $\hat{\Omega}$. Underlined entries indicate that both panel-based variants yield lower errors compared to the single equation counterpart.

Table 2: Model evaluation: Backward extension

	Single	Pooled	Within	Single	Pooled	Within
Year Fixed Effects	No	No	No	No	Yes	Yes
From ... to 2015		$\hat{\Omega} = I$			$\hat{\Omega} = I$	
2015	—	**0.5861**	—	—	—	—
2014	0.7900	**0.5370**	0.7369	0.7900	**0.5402**	1.0029
2013	0.5661	0.5567	**0.4982**	0.5661	0.5515	**0.4845**
2012	0.6220	0.5811	**0.5702**	0.6220	0.5295	**0.4908**
2011	0.6255	0.5918	**0.5626**	0.6255	0.5528	**0.4999**
2010	0.6212	0.5988	**0.5644**	0.6212	0.5908	**0.5350**
2009	0.6327	0.6415	**0.6124**	0.6327	0.6299	**0.5790**
2008	0.6114	0.6136	**0.6113**	0.6114	0.5989	**0.5895**
2007	0.6301	**0.6291**	0.6323	0.6301	0.5944	**0.5895**
2006	0.6397	**0.6383**	0.6418	0.6397	0.5826	**0.5751**
2005	0.6344	**0.6314**	0.6315	0.6344	0.5735	**0.5598**
2004	0.6228	0.6169	**0.6164**	0.6228	0.5585	**0.5459**
2003	0.6312	**0.6183**	0.6224	0.6312	0.5616	**0.5541**
2002	0.6187	0.6134	**0.6126**	0.6187	0.5573	**0.5434**
2001	0.6050	0.6028	**0.6016**	0.6050	0.5521	**0.5391**
From ... to 2015		$\hat{\Omega} = AR(1), \rho = 0.9$			$\hat{\Omega} = AR(1), \rho = 0.9$	
2015	—	**0.5855**	—	—	—	—
2014	0.7900	**0.5853**	0.7422	0.7900	**0.6083**	1.0057
2013	0.6345	0.5093	**0.4909**	0.6345	0.5059	**0.4873**
2012	0.5256	0.5184	**0.5154**	0.5256	0.4763	**0.4690**
2011	0.5507	0.5323	**0.5167**	0.5507	0.5002	**0.4831**
2010	0.5096	0.5123	**0.5048**	0.5096	0.4938	**0.4834**
2009	**0.5626**	0.5809	0.5783	0.5626	0.5585	**0.5529**
2008	**0.5619**	0.5729	0.5752	0.5619	**0.5586**	0.5614
2007	**0.6197**	0.6231	0.6233	0.6197	0.5810	**0.5794**
2006	**0.5993**	0.6138	0.6143	0.5993	0.5742	**0.5739**
2005	**0.5891**	0.6004	0.5996	0.5891	0.5615	**0.5594**
2004	**0.5833**	0.5918	0.5919	0.5833	0.5554	**0.5547**
2003	**0.5839**	0.5918	0.5920	0.5839	0.5608	**0.5600**
2002	**0.5658**	0.5800	0.5787	0.5658	0.5471	**0.5450**
2001	**0.5603**	0.5721	0.5717	0.5603	0.5430	**0.5420**
From ... to 2015		$\hat{\Omega}$ = Fernández (1981)			$\hat{\Omega}$ = Fernández (1981)	
2015	—	**0.5819**	—	—	—	—
2014	0.7900	**0.5828**	0.7427	0.7900	**0.5942**	1.0056
2013	0.6678	**0.4796**	0.4831	0.6678	**0.4820**	0.4839
2012	0.4959	**0.4770**	0.4864	0.4959	**0.4578**	0.4579
2011	0.5416	0.5932	**0.5018**	0.5416	0.5661	**0.4742**
2010	0.4983	0.5267	**0.4936**	0.4983	0.5148	**0.4743**
2009	**0.5555**	0.5845	0.5684	0.5555	0.5756	**0.5444**
2008	**0.5544**	0.5588	0.5681	0.5544	**0.5499**	0.5558
2007	**0.6158**	0.6185	0.6189	0.6158	0.5811	**0.5777**
2006	**0.5988**	0.6039	0.6120	0.5988	**0.5659**	0.5733
2005	**0.5882**	0.5994	0.5975	0.5882	0.5620	**0.5585**
2004	**0.5830**	0.5834	0.5909	0.5830	**0.5492**	0.5544
2003	**0.5820**	0.5916	0.5896	0.5820	0.5633	**0.5586**
2002	**0.5660**	0.5749	0.5777	0.5660	**0.5437**	0.5453
2001	**0.5589**	0.5710	0.5703	0.5589	0.5416	**0.5409**

Notes: The table presents the mean absolute errors from comparing the interpolated quarterly real GDP series (using industrial production as related series) with actual quarterly real GDP. For each sample, the average (unweighted) error across countries is calculated and rescaled by the factor 100 for better visualization. “Single” refers to the error resulting from standard country-wise interpolation of the quarterly series, “Pooled” to an interpolation based on a panel regression with a common intercept, and “Within” to an interpolation based on a panel regression with unit-specific intercepts. Note that a country-wise interpolation (“Single”) based on just one yearly observation is not feasible. Similarly, adding year fixed effects is not possible for the single equation models. Bold entries highlight the lowest errors for each sample and assumption concerning $\hat{\Omega}$. Underlined entries indicate that both panel-based variants yield lower errors compared to the single equation counterpart.