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Optimal fiscal substitutes for the exchange rate in a monetary union

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Non-technical summary

Research Question

Tax rate adjustments can change relative prices between countries in a similar fashion as the exchange rate. It has therefore been debated whether member states of the European Monetary Union (EMU) could consider "fiscal" instead of nominal devaluations in response to macroeconomic turmoil. This paper asks to what extent optimal, that is welfare-maximizing, fiscal policy should substitute for the role of the exchange rate in the EMU.

Contribution

This question is answered using a New Keynesian 2-region model that is calibrated to the EMU. A common central bank controls monetary policy at the level of the union, while governments in each country levy a value added tax and issue debt in order to finance a given amount of public spending. The paper adds to the literature on the conduct of monetary and fiscal policy in monetary unions by analysing if fiscal devaluations should be part of optimally-designed policy.

Results

Simulating the EMU in the model shows that optimal fiscal policy can reduce the welfare costs from giving up exchange rate flexibility by up to 86%. Fiscal devaluations can be observed as an optimal policy response to macroeconomic shocks. As a policy recommendation, the model suggests that whenever a nominal devaluation of a region were optimal in the monetary union, it is optimal to raise its value added tax relative to other regions. The reason is that this policy cheapens domestic exports relative to imports, since value added taxes apply only to goods sold within a country.

Nichttechnische Zusammenfassung

Fragestellung

Steuersatzveränderungen können Relativpreisverhältnisse zwischen Ländern auf ähnliche Art und Weise verändern wie der Wechselkurs. Es wird daher diskutiert, ob Mitgliedsstaaten der Europäischen Währungsunion (EWU) als Antwort auf makroökonomische Turbulenzen „fiskalische“ anstelle von nominalen Abwertungen vornehmen könnten. Dieses Papier behandelt die Frage in welchem Maße optimale, im Sinne von wohlfahrtsmaximierender, Fiskalpolitik genutzt werden sollte, um für die Funktion des Wechselkurses innerhalb der EWU zu substituieren.

Beitrag

Diese Frage wird mithilfe eines Neu-Keynesianischen Zwei-Regionen Modells beantwortet, das anhand der EWU kalibriert wurde. Die Geldpolitik wird von einer gemeinsamen Zentralbank auf Unionsebene bestimmt, während die Regierung jedes Mitgliedslandes eine Mehrwertsteuer erhebt und Schulden aufnehmen kann, um Staatsausgaben in gegebener Höhe zu finanzieren. Das Papier trägt zur Literatur über die Durchführung von Geld- und Fiskalpolitik in Währungsunionen bei, indem es analysiert, ob fiskalische Abwertungen Teil einer optimal ausgestalteten Politik sein sollten.

Ergebnisse

Simulationen der EWU anhand des Modells ergeben, dass optimale Fiskalpolitik in der Lage ist, die Wohlfahrtskosten, die sich aus dem Verlust der Wechselkursflexibilität ergeben, um bis zu 86% zu reduzieren. Fiskalische Abwertungen können als optimale Antwort auf makroökonomische Schocks beobachtet werden. Das Modell empfiehlt als Politikmaßnahme für Situationen, in denen die nominale Abwertung einer Region innerhalb der Währungsunion optimal wäre, stattdessen den Mehrwertsteuersatz der Region relativ zu dem anderer Regionen zu erhöhen. Der Grund dafür ist, dass diese Politik heimische Exporte relativ zu Importen verbilligt, da Mehrwertsteuern nur auf Güter anfallen, die innerhalb eines Landes verkauft werden.

Optimal Fiscal Substitutes for the Exchange Rate in a Monetary Union*

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Abstract

This paper studies Ramsey-optimal monetary and fiscal policy in a New Keynesian 2-country open economy framework, which is used to assess how far fiscal policy can substitute for the role of nominal exchange rates within a monetary union. Giving up exchange rate flexibility leads to welfare costs that depend significantly on whether the law of one price holds internationally or whether firms can engage in pricing-to-market. Calibrated to the euro area, the welfare costs can be reduced by 86% in the former and by 69% in the latter case by using only one tax instrument per country. Fiscal devaluations can be observed as an optimal policy in a monetary union: if a nominal devaluation of the domestic currency were optimal under flexible exchange rates, optimal fiscal policy in a monetary union is an increase of the domestic relative to the foreign value added tax.

Keywords: Monetary union; Optimal monetary and fiscal policy; Exchange rate

JEL classification: F41; F45; E63.

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1 Introduction

Free floating exchange rates are generally regarded as an important shock absorber for countries facing macroeconomic turmoil. Giving up this device by joining a monetary union (MU) or committing to a peg clearly reduces the abilities of business cycle stabilization policy in reacting to country-specific shocks, as an independent monetary policy is no longer feasible anymore. The fixed exchange rate regime of the European Monetary Union is also blamed for the slack or even missing recovery of some southern European countries in the aftermath of the global financial crisis.¹

Within a monetary union, fiscal policy can take up the role of the exchange rate, since taxes can in principle affect international relative prices—the terms of trade and the real exchange rate—in a similar fashion as the exchange rate does. Policies of this type are referred to as fiscal devaluations. Setting a theoretical benchmark, [Farhi, Gopinath, and Itskhoki \(2014\)](#) show that the effects of the nominal exchange rate on the allocation of an economy can be replicated entirely using a sufficient number of tax instruments. Following a related approach, [Adao, Correia, and Teles \(2009\)](#) conclude that the exchange rate regime can be completely irrelevant for stabilization policy.

In this paper, I show that even under a minimum set of fiscal instruments being in a monetary union does not have to be unduly painful. In a common New Keynesian 2-country open economy framework, optimal use of only one tax instrument per country reduces the welfare costs of giving up exchange rate flexibility in a MU already significantly. Fiscal devaluation policies are not only viable, but can be observed as the optimal policy response to country-specific shocks.

The 2-country model features complete international capital markets and staggered price setting à la [Calvo \(1983\)](#). I differentiate between the case where prices are sticky in the country of the producer only such that the law of one price (LOOP) holds internationally, and the case where firms are capable of pricing-to-market (PTM), implying an additional sticky price friction for imported goods.² This is important as the welfare costs of fixed exchange rates as well as the capabilities of fiscal policy to reduce these costs depend decisively on the pricing scheme. The model allows for home bias and asymmetries between the countries along several dimensions, such as country size, the degree of competition, and the size of the public sector. Each country has a fiscal authority, whose objective is to finance a given amount of public spending by collecting distortionary taxes and issuance of debt. Only one tax instrument is available for each authority—a value added tax (VAT) payable by firms, which is levied on all goods sold within a country. Optimal policy is characterized using a Ramsey approach. This procedure involves to find sequences for the policy instruments that support the welfare-maximizing competitive equilibrium.

Calibrating the model to characteristics of the euro area, I find that optimal fiscal policy reduces the welfare costs of pegged exchange rates by 86% in case the law of one price holds and by 69% in case of pricing-to-market. The order of magnitude of these results is highly robust to changes in the parametrization and also if payroll taxes are

¹See, for instance, [Mankiw \(2015\)](#) with a particular focus on Greece.

²In case of a flexible exchange rate regime, these two pricing schemes are also referred to as producer currency pricing and local currency pricing. Regarding the high empirical relevance of both schemes and a recent overview of the literature on international price setting, see [Burstein and Gopinath \(2014\)](#).

used instead of the VAT.

Besides analysing the welfare effects, I describe the conduct of optimal stabilization policy depending on the exchange rate regime and the way prices are set. In general, under flexible exchange rates, taxes aim to finance public expenditures in the least distortionary way, while at the same time they can be used for stabilizing marginal costs of firms in response to shocks. This trade-off involves a further dimension in case of a monetary union, where taxes can additionally substitute for the role of the nominal exchange rate, e.g. in inducing expenditure switching effects. In this way, optimal fiscal policy can compensate at least partially for the loss of country-specific monetary policy as a stabilization instrument, thereby bringing the economy closer to the efficient allocation.

The intuition for the simplest form of a fiscal devaluation policy is that, say, an increase of the domestic relative to the foreign VAT rate induces firms to charge higher prices for goods sold at home, resulting in higher prices of domestic imports relative to exports, for the latter are subject to the relatively reduced foreign VAT. Comparable to a nominal devaluation, this fiscal devaluation policy leads to a deterioration of the terms of trade. As shown by [Farhi et al. \(2014\)](#), reproducing the depreciation of the real exchange rate that would emerge under a nominal devaluation and stabilizing internal prices of domestically produced goods that are distorted by the change in the VAT requires additional instruments, though.

In a monetary union, I find that optimal fiscal policy is indeed actively concerned with replicating the flexible exchange rate allocation. Optimal policy favours replicating the behaviour of the terms of trade under a free float over reproducing the response of the real exchange rate, in line with the intuition given above. In situations where a nominal devaluation of a region were optimal, optimal fiscal policy in a MU is a relative increase of the VAT of that region, i.e. to conduct a fiscal devaluation. Although the transmission of fiscal policy is different under LOOP and PTM due to the limited pass-through of tax changes on prices in the latter case, this finding is independent of the pricing scheme. Simulating the economy under both exchange rate regimes yields correlations between the hypothetical optimal exchange rate response and the ratio of VAT rates in the MU of 81% when the LOOP holds and of 59% under PTM. The reaction of the level of tax rates depends on the specific types of shocks, though. In case of shocks for which an efficient response could be attainable under flexible exchange rates (I consider productivity, government spending, and demand preference shocks), replicating the effects of the exchange rate does not conflict with marginal cost stabilization—an instance of "divine coincidence" for fiscal policy under fixed exchange rates. This manifests in correlations between tax rate increases in the MU and the counterfactual nominal devaluations of about 90%. Translated into a general policy recommendation, this implies to increase the VAT of a MU member whenever its exchange rate should be devaluated and vice-versa. In case of mark-up shocks, optimal policy needs to trade-off the objective of stabilizing firms' marginal costs with the incentive to replicate the effect of the exchange rate. Correlations between the hypothetical exchange rate and taxes also depend on the origin of the shock in this instance.

This paper contributes to the literature on optimal stabilization policy for monetary unions in a New Keynesian framework.³ [Benigno \(2004\)](#) offers a description of optimal

³Noteworthy, a monetary union always makes the economy worse off in this literature, as its sole focus lies on the cost-side of giving up flexible exchange rates. For an overview of other (beneficial) aspects of

monetary policy in a 2-country setting. He finds that inflation should be stabilized at the level of the union, with a higher weight attached to the country with more rigid prices. The efficient response is generally not achievable, though. [Lombardo \(2006\)](#) builds on the model of Benigno, focusing in particular on the role of different degrees of imperfect competition for monetary policy. [Beetsma and Jensen \(2005\)](#) add fiscal policy to the model in the form of lump-sum financed government spending that enters households' utility. In this setting, optimal monetary policy is still used to stabilize aggregate inflation, while fiscal policy aims at affecting cross-country inflation differentials. Using a similar fiscal setting, [Galí and Monacelli \(2008\)](#) study optimal policy in a monetary union consisting of a continuum of small open economies.

The closest antecedent to my article is by [Ferrero \(2009\)](#). In his model, fiscal policy also needs to finance an exogenous stream of government spending by distortionary taxes and debt. Optimal policy is described by targeting rules using a linear-quadratic approach. The focus of the paper lies on the question how far simple policy rules can approximate optimal policy in a monetary union. My article assesses how far optimal fiscal policy can reduce the welfare costs of a fixed exchange rate regime. I further show that fiscal devaluation policies can be an optimal policy response to idiosyncratic shocks. To this end, I generalize the modelling framework of [Ferrero \(2009\)](#) by adding PTM, by allowing for asymmetries between countries, and by comparing policy scenarios that differ in terms of the exchange rate regime and the availability of fiscal policy as a stabilization device.

This paper further contributes to the literature on fiscal devaluations. Besides the work of [Farhi et al. \(2014\)](#), this entails, amongst others, [Lipinska and von Thadden \(2012\)](#), and [Engler, Ganelli, Tervala, and Voigts \(2014\)](#), who study the quantitative effects of tax swaps from direct (payroll taxes) to indirect taxation (VATs). In general, this literature studies the economic effects of given fiscal policies, but it does not provide a normative analysis. I show that in fixed exchange rate regimes it is not only viable, but indeed optimal to use fiscal devaluations as a substitute for the exchange rate.

The rest of the paper is structured as follows. Section 2 presents the open economy model. The setup of the Ramsey policy problem is described in Section 3, while Section 4 features a description of the calibration of the model to the euro area. All results are provided in Section 5, with a description of the steady state in Section 5.1, the analysis of welfare costs of giving up exchange rate flexibility in 5.2, and results on optimal policy conduct in 5.3. A conclusion including a discussion of the results is given in Section 6.

2 The Model

The model economy consists of two countries or regions i , labelled as the core ($i = H$) and the periphery ($i = F$), that can form a monetary union. The world population of households (indexed by h) and firms (indexed by k) each sums up to one, of which a fraction $n \in (0, 1)$ of households and firms lives in the core and a fraction $1 - n$ in the periphery. In each region, households choose consumption of domestic and foreign goods, supply labour, which is mobile only within the region, and trade assets internationally. Firms demand labour to produce tradable goods under monopolistic competition. Price setting is subject to a Calvo-type friction. International prices are either set according

monetary unions, see [Beetsma and Giuliodori \(2010\)](#), and [Santos Silva and Tenreyro \(2010\)](#).

to the law of one price or taking into account local market conditions. Fiscal authorities levy distortionary taxes and issue debt to finance an exogenously given amount of public spending. Depending on whether the countries form a monetary union, there are two separate or one single central bank, whose policy instrument is the nominal interest rate. The economy operates at the cashless limit. Periphery variables are denoted by an asterisk (*). The following exposition focuses on the core region; the periphery economy is modelled symmetrically.

2.1 Households

A representative household h living in region H derives utility from consumption and disutility from work efforts. The consumption bundle $C_t(h)$ consists of tradable goods only and is defined as a composite index over domestic- and foreign-produced consumption goods,

$$C_t(h) = \left[\gamma_H^{\frac{1}{\xi}} C_{Ht}(h)^{\frac{\xi-1}{\xi}} + \gamma_F^{\frac{1}{\xi}} C_{Ft}(h)^{\frac{\xi-1}{\xi}} \right]^{\frac{\xi}{\xi-1}}, \quad (1)$$

with $\xi > 0$ being the Armington elasticity of substitution between core and periphery goods, and $\gamma_H = 1 - \gamma_F \in (0, 1)$ the share of domestic goods in the consumption bundle. If $\gamma_H > n$, a home bias in preferences exists. Consumption of domestic and imported goods by household h itself is given via Dixit-Stiglitz aggregators over imperfectly substitutable individual varieties k ,

$$C_{Ht}(h) = \left[\left(\frac{1}{n} \right)^{\frac{1}{\rho}} \int_0^n C_{Ht}(k, h)^{\frac{\rho-1}{\rho}} dk \right]^{\frac{\rho}{\rho-1}}, \quad (2)$$

$$C_{Ft}(h) = \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\rho^*}} \int_n^1 C_{Ft}(k, h)^{\frac{\rho^*-1}{\rho^*}} dk \right]^{\frac{\rho^*}{\rho^*-1}}, \quad (3)$$

where $\rho, \rho^* > 1$ are the elasticities of substitution between the varieties in each country. To express specialization of countries in production, the elasticity of substitution between varieties within a country is assumed to be greater than between goods of different origin, i.e. $\rho > \xi$.

The corresponding price indices can be shown to equal:

$$P_t = \left[\gamma_H P_{Ht}^{1-\xi} + \gamma_F P_{Ft}^{1-\xi} \right]^{\frac{1}{1-\xi}}, \quad (4)$$

$$P_{Ht} = \left[\left(\frac{1}{n} \right) \int_0^n P_{Ht}(k)^{1-\rho} dk \right]^{\frac{1}{1-\rho}}, \quad (5)$$

$$P_{Ft} = \left[\left(\frac{1}{1-n} \right) \int_n^1 P_{Ft}(k)^{1-\rho^*} dk \right]^{\frac{1}{1-\rho^*}}. \quad (6)$$

P_t denotes the core's consumer price index (CPI), P_{Ht} the producer price index (PPI) of

core goods, and P_{Ft} the price index of imported goods. Given the definitions of the price indices, it is easy to show that consumer expenditures are given by $P_t C_t(h) = P_{Ht} C_{Ht}(h) + P_{Ft} C_{Ft}(h)$ with $P_{Ht} C_{Ht}(h) = \int_0^n P_{Ht}(k) C_{Ht}(k, h) dk$ and $P_{Ft} C_{Ft}(h) = \int_n^1 P_{Ft}(k) C_{Ft}(k, h) dk$. Consumption demand functions are characterized by:

$$C_{Ht}(h) = \gamma_H \left(\frac{P_{Ht}}{P_t} \right)^{-\xi} C_t(h), \quad C_{Ft}(h) = \gamma_F \left(\frac{P_{Ft}}{P_t} \right)^{-\xi} C_t(h), \quad (7)$$

$$C_{Ht}(k, h) = \frac{1}{n} \left(\frac{P_{Ht}(k)}{P_{Ht}} \right)^{-\rho} C_{Ht}(h), \quad C_{Ft}(k, h) = \frac{1}{1-n} \left(\frac{P_{Ft}(k)}{P_{Ft}} \right)^{-\rho^*} C_{Ft}(h). \quad (8)$$

Each household h maximizes the utility function

$$\mathbb{U}_0(h) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\zeta_t^c \frac{C_t(h)^{1-\sigma}}{1-\sigma} - \frac{N_t(h)^{1+\eta}}{1+\eta} \right], \quad (9)$$

subject to the flow budget constraint

$$P_t C_t(h) + \mathbb{E}_t \{ Q_{t,t+1} [D_{t+1}(h) + B_{t+1}(h)] \} \leq W_t N_t(h) + D_t(h) + B_t(h) + \Pi_t(h), \quad (10)$$

where ζ_t^c denotes a demand preference shock, $N_t(h)$ labour supply, W_t the wage rate, and $\Pi_t(h)$ the profit share of a well-diversified portfolio of firms in possession of household h . Asset markets are complete within and across countries. $Q_{t,t+1}$ is the period t price of one unit of domestic currency in a particular state of period $t+1$, normalized by the probability of occurrence of that state, i.e. the stochastic discount factor. Accordingly, $\mathbb{E}_t Q_{t,t+1}$ is the price of an asset portfolio that pays off one unit of domestic currency in every state of period $t+1$ and, therefore, equals the inverse of the risk-free gross nominal interest rate, $R_t = 1/\mathbb{E}_t Q_{t,t+1}$. $D_{t+1}(h)$ is the quantity of an internationally-traded state-contingent private asset portfolio denominated in domestic currency, while $B_{t+1}(h)$ denotes holdings of government debt. It is assumed without loss of generality that sovereign debt of country i can be held only by agents of that country. Besides its budget, the household has to regard the transversality conditions

$$\lim_{s \rightarrow \infty} \mathbb{E}_t [Q_{t,s} D_{t+s}(h)] = 0 \quad \text{and} \quad \lim_{s \rightarrow \infty} \mathbb{E}_t [Q_{t,s} B_{t+s}(h)] = 0, \quad (11)$$

where $Q_{t,s} = \prod_{z=t}^s Q_{t,z}$ denotes the stochastic discount factor from period s to period t .

The first-order conditions of the household's problem imply the Euler equation,

$$Q_{t,t+1} = \beta \frac{\zeta_{t+1}^c}{\zeta_t^c} \left(\frac{C_{t+1}(h)}{C_t(h)} \right)^{-\sigma} \frac{P_t}{P_{t+1}}, \quad (12)$$

as well as an intratemporal consumption-leisure trade-off, given by

$$\frac{N_t(h)^\eta}{\zeta_t^c C_t(h)^{-\sigma}} = \frac{W_t}{P_t}. \quad (13)$$

Foreign households behave analogously and in particular hold a quantity $D_{t+1}^*(h)$ of the internationally-traded asset portfolio. From the periphery's perspective, the stochastic

discount factor is priced as

$$Q_{t,t+1} = \beta \frac{\zeta_{t+1}^{c*}}{\zeta_t^{c*}} \left(\frac{C_{t+1}^*(h)}{C_t^*(h)} \right)^{-\sigma} \frac{P_t^*}{P_{t+1}^*} \frac{E_t}{E_{t+1}}, \quad (14)$$

where P_t^* is the CPI of the periphery, and E_t is the nominal exchange rate, which is defined as the price of one unit of periphery currency in terms of core currency ($E_t = [H]/[F]$). An increase in E_t accordingly implies a nominal devaluation of the core region. In case the countries form a monetary union, the exchange rate is fixed at unity ($\bar{E} = 1$). Combining (12) and (14) yields the well-known condition of international risk sharing that links consumption of the two countries and determines their (real) exchange rate:

$$q_t = \frac{\zeta_t^{c*}}{\zeta_t^c} \left(\frac{C_t^*(h)}{C_t^c(h)} \right)^{-\sigma} \kappa. \quad (15)$$

The real exchange rate is defined as the nominal exchange rate weighted ratio of the CPIs, $q_t = (E_t P_t^*)/P_t$, while $\kappa = q_0(C_0/C_0^*)^{-\sigma}$ is a positive constant that depends on preferences and the initial asset distribution. As pointed out, amongst others, by [Faia and Monacelli \(2004\)](#), $\kappa = 1$ if markets are complete, the initial net foreign indebtedness is zero ($D_{t+1}(h) = D_{t+1}^*(h) = 0 \forall h$), and preferences are symmetric across countries.

2.2 Firms and Price Setting Assumptions

In the core a continuum of firms $k \in [0, n]$ operates under monopolistic competition. Each firm produces a variety k according to the production plan

$$Y_t(k) = A_t N_t^\alpha(k), \quad (16)$$

where $Y_t(k)$ is total supply of variety k , A_t a country-specific stochastic productivity level, and $N_t(k)$ the firm's labour demand. Labour is the sole input of production, and α is the input elasticity of production. Labour supply by households is perfectly mobile across firms within the country, but immobile between countries. Total demand for the good produced by firm k is given by the demand of domestic ($C_{Ht}(k)$) and foreign ($C_{Ht}^*(k)$) households as well as public demand by the domestic government ($G_t(k)$):

$$Y_t(k) = \int_0^n C_{Ht}(k, h) dh + \int_n^1 C_{Ht}^*(k, h) dh + G_t(k). \quad (17)$$

The period t profit function of firm k reads

$$\begin{aligned} \Pi_t(k) &= (1 - \tau_t^v) P_{Ht}(k) \left[\int_0^n C_{Ht}(k, h) dh + G_t(k) \right] \\ &\quad + (1 - \tau_t^{v*}) E_t P_{Ht}^*(k) \int_n^1 C_{Ht}^*(k, h) dh - W_t N_t(k), \end{aligned} \quad (18)$$

where $P_{Ht}^*(k)$ is the price of core good k abroad. τ_t^v and τ_t^{v*} are country-specific value-added taxes (VAT) in region H and F respectively. As common in existing tax systems, τ_t^v is levied on all goods sold within the Home country, but not on exports. The latter

are taxed at the border with the foreign VAT rate τ_t^{v*} .

Price setting of firms is impaired by Calvo-type price stickiness. Each period t , a firm can adjust prices with probability $1 - \theta$, independent of the date of previous price changes. With probability θ the firm has to maintain last period's prices. Optimal prices are set as to maximize the net present value of future profits

$$\sum_{s=t}^{\infty} \theta^{s-t} \mathbb{E}_t [Q_{t,s} \Pi_s(k)] \quad (19)$$

subject to the production technology and demand. Prices always include taxes. The price of domestic goods sold within the core, $P_{Ht}(h)$, is always set in domestic currency. The setting of export prices for the periphery, $P_{Ht}^*(k)$, is conducted according to the assumption of either the law of one price or pricing-to-market.

2.2.1 Law of One Price (LOOP)

Under this pricing scheme, firms set a price for their good in domestic currency, while the price in the other region satisfies the law of one price, adjusted for tax rates:

$$\begin{aligned} (1 - \tau_t^{v*}) E_t P_{Ht}^*(k) &\stackrel{!}{=} (1 - \tau_t^v) P_{Ht}(k) \\ \Leftrightarrow P_{Ht}^*(k) &= \frac{(1 - \tau_t^v)}{(1 - \tau_t^{v*})} \frac{1}{E_t} P_{Ht}(k). \end{aligned} \quad (20)$$

Following [Farhi et al. \(2014\)](#), this expression is derived from the assumption that one unit of sales should yield the same revenue to the firm, independent of the origin of the buyer. (20) implies complete and immediate pass-through of both exchange rates and taxes on international prices. A relative increase of the core's VAT rate has the same effect on prices abroad as a nominal devaluation.

The optimality condition for the price set in period t , $\bar{P}_{Ht}(k)$, is derived in [Appendix A.1](#) and reads

$$\mathbb{E}_t \sum_{s=t}^{\infty} \theta^{s-t} Q_{t,s} \left(\frac{\bar{P}_{Ht}(k)}{P_{Hs}} \right)^{-1-\rho} \frac{Y_s}{P_{Hs}} \left[\frac{\rho}{\rho - 1} \mu_s MC_{Hs}(k) - (1 - \tau_s^v) \bar{P}_{Ht}(k) \right] = 0, \quad (21)$$

where $MC_{Ht}(k) = W_t / [\alpha A_t N_t^{\alpha-1}(k)]$ denotes marginal costs, and μ_t a stochastic mark-up shock. The equation shows the standard result that the optimal price is set equal to a mark-up over a weighted average of current and future marginal costs.

2.2.2 Pricing-to-Market (PTM)

Under the alternative assumption of PTM, firms set separate prices at home, $\bar{P}_{Ht}(k)$, and abroad, $\bar{P}_{Ht}^*(k)$, each of them subject to a Calvo friction. As a result, there is only limited direct pass-through of exchange rates and taxes on international prices, and the law of one price can be violated. Optimal price setting is now described by two conditions, also derived in [Appendix A.1](#):

$$\mathbb{E}_t \sum_{s=t}^{\infty} \theta^{s-t} Q_{t,s} \left(\frac{\bar{P}_{Ht}(k)}{P_{Hs}} \right)^{-1} \frac{(n C_{Hs} + G_s)}{P_{Hs}} \left[\frac{\rho}{\rho-1} \mu_s M C_{Hs}(k) - (1 - \tau_s^v) \bar{P}_{Ht}(k) \right] = 0, \quad (22)$$

$$\mathbb{E}_t \sum_{s=t}^{\infty} \theta^{s-t} Q_{t,s} \left(\frac{\bar{P}_{Ht}^*(k)}{P_{Hs}^*} \right)^{-1} \frac{C_{Hs}^*}{P_{Hs}^*} \left[\frac{\rho}{\rho-1} \mu_s M C_{Hs}(k) - (1 - \tau_s^{v*}) E_s \bar{P}_{Ht}^*(k) \right] = 0. \quad (23)$$

A devaluation of the domestic currency has the same effect on the firm's pricing decision for exports as a reduction in marginal costs, since every unit sold abroad leads to higher revenues than selling on the domestic market. Note that reducing the periphery's VAT rate τ^{v*} induces, ceteris paribus, the same effect on import prices in the periphery as a rise in E_t .

2.2.3 Foreign Firms

Foreign firms are modelled symmetrically. Under the LOOP, they set a price $\bar{P}_{Ft}^*(k)$ at which periphery goods are sold in F . The price at which goods are sold internationally is again determined by the law of one price, adjusted for taxes:

$$P_{Ft}(k) = \frac{(1 - \tau^{v*})}{(1 - \tau^v)} E_t P_{Ft}^*(k). \quad (24)$$

In case of PTM, firms in the periphery can also set separate prices for their domestic and the international market. Optimal prices are implicitly given by:

$$\mathbb{E}_t \sum_{s=t}^{\infty} \theta^{*s-t} Q_{t,s}^* \left(\frac{\bar{P}_{Ft}^*(k)}{P_{Fs}^*} \right)^{-1-\rho^*} \frac{((1-n) C_{Fs}^* + G_s^*)}{P_{Fs}^*} \cdot \left[\frac{\rho^*}{\rho^*-1} \mu_s^* M C_{Fs}^*(k) - (1 - \tau_s^{v*}) \bar{P}_{Ft}^*(k) \right] = 0, \quad (25)$$

$$\mathbb{E}_t \sum_{s=t}^{\infty} \theta^{*s-t} Q_{t,s}^* \left(\frac{\bar{P}_{Ft}^*(k)}{P_{Fs}^*} \right)^{-1-\rho^*} \frac{C_{Fs}^*}{P_{Fs}^*} \cdot \left[\frac{\rho^*}{\rho^*-1} \mu_s^* M C_{Fs}^*(k) - \frac{(1 - \tau_s^v)}{E_s} \bar{P}_{Ft}^*(k) \right] = 0, \quad (26)$$

where $M C_{Ft}^*(k) = W_t^* / [\alpha A_t^* N_t^{*\alpha-1}(k)]$.

2.3 Monetary and Fiscal Authorities

The public sector consists of separate fiscal authorities and central banks at the country level. The policy instruments of the central banks are their nominal interest rates, R_t and R_t^* . If the regions share the same currency, only one central bank for the union as a whole exists, whose policy instrument is denoted by R_t^{MU} .

The task of the fiscal authorities is to finance an exogenously given stochastic amount of public spending G_t . In each country, government spending consists of an index of

locally produced goods only,

$$G_t = \left[\left(\frac{1}{n} \right) \int_0^n G_t(k)^{\frac{\rho-1}{\rho}} dk \right]^{\frac{\rho}{\rho-1}}, \quad (27)$$

with corresponding demand functions for each variety k , given by

$$G_t(k) = \frac{1}{n} \left(\frac{P_{Ht}(k)}{P_{Ht}} \right)^{-\rho} G_t. \quad (28)$$

These expenditures are financed by distortionary value-added taxes and state-contingent public debt. The budget constraint of the domestic government reads

$$\begin{aligned} P_{Ht}G_t + B_t &\leq \mathbb{E}_t Q_{t,t+1} B_{t+1} + \tau_t^v \int_0^n P_{Ht}(k) \left(\int_0^n C_{Ht}(k, h) dh + G_t(k) \right) dk \\ &\quad + \tau_t^v \int_n^1 \int_0^n P_{Ft}(k) C_{Ft}(k, h) dh dk. \end{aligned} \quad (29)$$

Note that the VAT is not only levied on domestically produced goods, but also on imports C_{Ft} .

2.4 Aggregation and Equilibrium

Due to symmetry among agents within a country, households and firms, respectively, will in each situation come to the same decisions. In the process of aggregation, one can, therefore, drop indices h and k .

By the law of large numbers, today's PPIs consist of the prices set today and last period's price index, weighted with the probabilities of adjustment and non-adjustment, respectively. As shown in Appendix A.2, the law of motion for P_{Ht} can be expressed as

$$\tilde{p}_{Ht} = \frac{\bar{P}_{Ht}}{P_{Ht}} = \left(\frac{1 - \theta \pi_{Ht}^{\rho-1}}{1 - \theta} \right)^{\frac{1}{1-\rho}}, \quad (30)$$

where $\pi_{Ht} = P_{Ht}/P_{Ht-1}$ denotes the PPI inflation rate of domestically produced goods in H .

(21) gives an expression for the Philips curve of core goods inflation under the LOOP. In order to solve the model, it is required to rewrite the Philips curve in a recursive way, which avoids the use of infinite sums. To do so, I follow [Schmitt-Grohé and Uribe \(2006\)](#) and restate (21) by defining two auxiliary variables, $X1_{Ht}$ and $X2_{Ht}$ (for a derivation, see Appendix A.3), such that

$$\frac{\rho}{\rho-1} \mu_t X1_{Ht} = X2_{Ht}, \quad (31)$$

where

$$X1_{Ht} = \tilde{p}_{Ht}^{-1-\rho} Y_t m c_{Ht} + \theta \beta \mathbb{E}_t \frac{\zeta_{t+1}^c}{\zeta_t^c} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{\pi_{Ht+1}^{1+\rho}}{\pi_{t+1}} \left(\frac{\tilde{p}_{Ht}}{\tilde{p}_{Ht+1}} \right)^{-1-\rho} X1_{Ht+1}, \quad (32)$$

$$X2_{Ht} = \tilde{p}_{Ht}^{-\rho} Y_t (1 - \tau_t^v) + \theta \beta \mathbb{E}_t \frac{\zeta_{t+1}^c}{\zeta_t^c} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{\pi_{Ht+1}^\rho}{\pi_{t+1}} \left(\frac{\tilde{p}_{Ht}}{\tilde{p}_{Ht+1}} \right)^{-\rho} X2_{Ht+1}, \quad (33)$$

with $\pi_t = P_t/P_{t-1}$ being the CPI inflation rate of the core, and $m c_{Ht} = MC_{Ht}/P_{Ht}$ being real marginal costs.

The resource constraint of the economy can be obtained by integrating the production function (16) over firms. The result differs depending on whether pricing follows the law of one price or firms can engage in pricing-to-market:⁴

$$A_t n N_t^\alpha = \Delta_{Ht} (n C_{Ht} + (1 - n) C_{Ht}^* + G_t) \quad (34)$$

$$A_t n N_t^\alpha = \Delta_{Ht} (n C_{Ht} + G_t) + \Delta_{Ht}^* (1 - n) C_{Ht}^* \quad (35)$$

The first equation holds under the LOOP, the latter one under PTM. Δ_{Ht} and Δ_{Ht}^* are indices of price dispersion that render inflation costly in efficiency terms. They are defined as

$$\Delta_{Ht} = \frac{1}{n} \int_0^n \left(\frac{P_{Ht}(k)}{P_{Ht}} \right)^{-\rho} dk, \quad (36)$$

$$\Delta_{Ht}^* = \frac{1}{n} \int_0^n \left(\frac{P_{Ht}^*(k)}{P_{Ht}^*} \right)^{-\rho} dk. \quad (37)$$

Their laws of motion are given by

$$\Delta_{Ht} = (1 - \theta) \tilde{p}_{Ht}^{-\rho} + \theta \pi_{Ht}^\rho \Delta_{Ht-1}, \quad (38)$$

$$\Delta_{Ht}^* = (1 - \theta) \tilde{p}_{Ht}^{*-\rho} + \theta \pi_{Ht}^{*\rho} \Delta_{Ht-1}^*. \quad (39)$$

As clarified by [Schmitt-Grohé and Uribe \(2006\)](#), price dispersion is irrelevant for the allocation if the non-stochastic (steady state) level of inflation is zero and only a first-order approximation to the equilibrium conditions is used.

An equilibrium in this economy is characterized by prices and quantities that fulfil the optimality conditions of households and firms in both countries such that all markets clear, given stochastic processes for all shocks, and sequences for the policy instruments. Goods markets under LOOP and markets for private assets clear at the international level; goods markets under PTM, government bond, and labour markets clear at national levels. A complete list of all equilibrium conditions under both LOOP and PTM is given in [Appendix B](#).

The following definition of the terms of trade will be useful for the rest of the analysis. The terms of trade indicate how much of exports the economy has to give for one unit of

⁴For derivations, see [Appendix A.4](#).

imports,

$$z_t = \frac{P_{Ft}}{P_{Ht}} \stackrel{(LOOP)}{=} \underbrace{\frac{(1 - \tau_t^{v*})}{(1 - \tau_t^v)}}_{FD_t} E_t \frac{P_{Ft}^*}{P_{Ht}}, \quad (40)$$

where the second equality sign holds under the law of one price. In this case of complete pass-through only, exchange rate and tax adjustments translate directly into changes of the terms of trade. The formula shows that under LOOP an increase of H 's VAT relative to F 's has the same effect on z_t as a nominal devaluation. The term $FD_t = (1 - \tau_t^{v*})/(1 - \tau_t^v)$ will, therefore, also be referred to as the fiscal devaluation factor.

In case of pricing-to-market, the pass-through of the exchange rate and taxes on the terms of trade is limited by their effect on P_{Ft} and P_{Ht} . The price setting conditions (23) and (26) make clear that the tax rates can have the same effect on import prices as the nominal exchange rate. The speed of pass-through depends on the degree of price stickiness, with the law of one price and, so, the second part of (40) only holding in the long-run. The short-run efficacy of fiscal devaluation policies to affect the terms of trade will, therefore, be higher under LOOP than under PTM.

3 The Ramsey Problem

Optimal monetary and fiscal policy is determined using a Ramsey approach. This procedure involves to find the sequences of the available policy instruments that support the welfare-maximizing competitive equilibrium. All policy authorities can credibly commit to their announced policies, and I assume full cooperation between all entities. The objective of the Ramsey planner is a utilitarian world welfare function that weights utility of core and periphery households according to their population size:

$$W_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ n \left(\zeta_t^c \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta} \right) + (1-n) \left(\zeta_t^{c*} \frac{C_t^{*1-\sigma}}{1-\sigma} - \frac{N_t^{*1+\eta}}{1+\eta} \right) \right\}. \quad (41)$$

If prices were flexible, the optimal policy problem could be described by maximizing (41) subject to one implementability and one resource constraint for each country only. Using this so-called primal approach to the Ramsey problem, proposed by Lucas and Stokey (1983) also in the context of optimal stabilization policy, the planner directly chooses an equilibrium allocation, from which prices and instruments can be backed out afterwards. In presence of a sticky price friction, this reduction of the problem to just two constraints per country is generally not possible anymore, as the Philips curves now effectively constrain the evolution of prices.⁵ The dual approach to the Ramsey problem, which involves choosing prices and instruments directly, has to be used instead.

In the following analysis, I compare optimal policy under various scenarios to assess the consequences of being in a monetary union. The scenarios differ by the type of price setting (LOOP vs. PTM) and by the availability of different policy instruments:

⁵The work of, for instance, Schmitt-Grohé and Uribe (2004), and Faia and Monacelli (2004) is also subject to this issue.

flexible exchange rates vs. monetary union, and monetary and fiscal policy vs. monetary policy only. In all of these scenarios, the dual solution to the policy problem is found by maximizing (41) subject to the relevant equilibrium conditions, described in Appendix B. If fiscal policy is an instrument to the Ramsey planner, the time path of the VAT rates, $\{\tau_t^v, \tau_t^{v*}\}_{t=0}^{\infty}$, has to ensure solvency of the fiscal authorities in both countries. To this end, the problem is augmented with the intertemporal fiscal budget constraints of both countries. As an example, I describe the solution to the Ramsey problem by means of its first-order conditions for the case of a monetary union, where the law of one price holds, with fiscal policy in detail in Appendix C.⁶

4 Calibration

I calibrate the model to characteristics of the euro area using quarterly data between 2001:1 and 2014:4 from Eurostat. In the calibration, the core (region H) comprises Austria, Belgium, Finland, France, Germany, the Netherlands, and Slovakia. The periphery (region F) consists of Greece, Ireland, Italy, Portugal, and Spain. This leads to a population share of the core of 60%; hence, $n = 0.6$. In total, these 12 countries cover 98% of euro area GDP in 2014.

Table 1: PARAMETER VALUES

Parameter	Core	Periphery
Size of region	$n = 0.6$	$(1 - n) = 0.4$
Discount factor	$\beta = 0.99$	
Risk aversion	$\sigma = 2$	
Inverse Frisch elasticity	$\eta = 2$	
Home bias	$\gamma_H = 0.72$	$\gamma_F^* = 0.48$
Armington elasticity (Home-Foreign goods)	$\xi = 1.2$	
Elasticity of substitution between varieties	$\rho = 6$	$\rho^* = 4$
Labor input elasticity of production	$\alpha = 1$	
1 – Probability of price adjustment	$\theta = 0.75$	$\theta^* = 0.75$
Gov. spending ratio to GDP in steady state	$G/Y = 0.21$	$G^*/Y^* = 0.19$
Annual gov. debt to GDP ratio in steady state	$B/Y = 0.78$	$B^*/Y^* = 1.08$

The discount factor β is set to 0.99, which is the standard value in the business-cycle literature for quarterly data, implying an annual real interest rate of about 4% in steady state. Risk aversion and the inverse Frisch elasticity are both set equal to 2, also following conventions of the literature. A mild home bias in demand preferences of 20% exists in both countries, yielding $\gamma_H = 1.2n = 0.72$ and $\gamma_F^* = 1.2(1 - n) = 0.48$. Following estimates by Feenstra, Luck, Obstfeld, and Russ (2014), I set the Armington elasticity between goods of different origin to $\xi = 1.2$. Initial international private debt in steady state is set to match the average trade balance surplus relative to GDP of the core of 2% between 2001 and 2014.

The elasticities of substitution between individual goods varieties, ρ and ρ^* , are set to match aggregate mark-ups. Høj, Jimenez, Maher, Nicoletti, and Wise (2007) provide

⁶Solutions to all other scenarios are available on request.

estimates for several OECD countries that suggest a mark-up of 1.2 in the core and 1.3 in the periphery, which implies $\rho = 6$ and $\rho^* = 4$. The labour input elasticity of production is set to one, which implies that the production technology is linear in labour. Following empirical evidence by ECB (2005), the probability of price stickiness is set to $\theta, \theta^* = 0.75$ so that price contracts last on average 4 quarters.⁷ Cross-country evidence by Druant, Fabiani, Kezdi, Lamo, Martins, and Sabbatini (2012) confirms that the frequency of price adjustments is similar across core and periphery countries.

The ratio of government spending to GDP in steady state (G/Y) is set to the average values between 2001 and 2014, which are 21% for the core and 19% for the periphery. The government debt to GDP ratio in annualized steady state (B/Y) matches the 2010-2014 average debt-to-GDP-ratios of the core (78%) and the periphery (108%). This calibration requires a steady state primary surplus relative to quarterly GDP of 3.1% in the core and of 4.3% in the periphery. Balanced public budgets imply steady state VAT rates of 24.6% and 22.6%, respectively. Table 1 summarizes all parameter values.

Table 2: SHOCK PROCESSES

Parameter	Core	Periphery
Persistence of productivity shocks (φ_A, φ_{A^*})	0.9301	0.9434
Persistence of demand preference shocks (φ_C, φ_{C^*})	0.8135	0.8990
Persistence of government spending shocks (φ_G, φ_{G^*})	0.7731	0.6439
Std. dev. of productivity shocks (σ_A, σ_{A^*})	0.0034	0.0032
Std. dev. of demand preference shocks (σ_C, σ_{C^*})	0.0139	0.0209
Std. dev. of government spending shocks (σ_G, σ_{G^*})	0.0071	0.0194
Std. dev. of mark-up shocks ($\sigma_\mu, \sigma_{\mu^*}$)	0.0057	0.0140

Note: Parameters calibrated to match autocorrelations and standard deviations of GDP, government spending, private consumption, and wage data between 2001:1 and 2014:4.

The evolution of the economy outside steady state is driven by region-specific stochastic processes for productivity A_t and government spending G_t , the demand preference shocks ζ_t^c , and the mark-up shocks μ_t in both countries. All but the mark-up shocks are modelled as AR(1)-processes, while the latter are assumed to be white noise.⁸ Persistence and Variance of the shocks are calibrated to match autocorrelations and standard deviations of seasonally adjusted and quadratically detrended data on GDP, government spending, private consumption, and average wage rates of the core and periphery between 2001:1 and 2014:4. The resulting parameters are given in Table 2. Details on the used data, including the target moments, are shown in Appendix D.

⁷The average time until a firm gets a chance to adjust its price is given by $1/(1-\theta)$, as Calvo-type price stickiness implies a Poisson process, where time until next adjustment is an exponentially-distributed random variable.

⁸Allowing the mark-up shock to follow an AR(1)-process as well yields persistence parameters of (μ_t, μ_t^*) close to zero and does not affect the moments of the other processes significantly, which is in line with results of Smets and Wouters (2003).

5 Results

The solution to the Ramsey problem, calibrated to the euro area, is quantitatively assessed in this chapter. Section 5.1 provides a brief description of the steady state. Section 5.2 analyses to what extent optimal fiscal policy reduces the welfare costs of giving up flexible exchange rates within the European Monetary Union. The conduct and mechanisms of optimal policy are subsequently described in Section 5.3.

5.1 The Allocation in Steady State

Gross inflation rates in all sectors, domestic goods and imports, are equal to one in the Ramsey-optimal steady state, since price dispersion that would arise otherwise impairs an efficient bundling of individual goods. Given this result, optimal price setting of domestic firms in steady state when the LOOP holds is described by

$$\frac{\rho}{\rho - 1} \frac{1}{(1 - \tau^v)} MC_H = P_H, \quad (42)$$

while under PTM the following condition for export prices additionally holds:

$$\frac{\rho}{\rho - 1} \frac{1}{(1 - \tau^{v*})} \frac{1}{E} MC_H = P_H^*. \quad (43)$$

Combining (42) and (43) immediately yields the law of one price (20). Hence, there are no long-run deviations from the law of one price, which would distort the composition of consumption between domestic and imported goods.

Also visible from (42) and (43), the distortions that render the long-run allocation different from its first-best level are the reduction in activity due to monopolistic competition and the necessity to use distortionary taxation to finance public expenditures. As taxes have to be positive in steady state, they cannot be used for mark-up elimination. Instead, taxes exacerbate the wedge driven by the mark-up between prices and marginal costs. The steady state is, therefore, in general not efficient.

5.2 The Welfare Costs of Giving up Exchange Rate Flexibility

The welfare comparison of the various policy scenarios is discussed next. The welfare measure used to assess the scenarios is units of steady state consumption that households are willing to give up in order to live in the deterministic steady state economy instead of a stochastic economy—that is a percentage amount of steady state consumption $\omega_I^\mathcal{E}$ satisfying

$$\frac{1}{1 - \beta} \left\{ n \left(\frac{[C(1 + \omega_I^\mathcal{E})]^{1-\sigma}}{1 - \sigma} - \frac{N^{1+\eta}}{1 + \eta} \right) + (1 - n) \left(\frac{[C^*(1 + \omega_I^\mathcal{E})]^{1-\sigma}}{1 - \sigma} - \frac{N^{*1+\eta}}{1 + \eta} \right) \right\} \stackrel{!}{=} \mathbb{W}_0^{\mathcal{E}, I},$$

where $\mathbb{W}_0^{\mathcal{E}, I}$ is the expected net present value of aggregate welfare as defined by (41) for a given exchange rate regime $\mathcal{E} \in \{MU, FLEX\}$ and a given set of policy instruments $I \in \{MP, MFP\}$. To evaluate welfare, the model is solved by a second-order approximation

to the policy functions and simulated for $T = 1000$ periods.⁹ I average the welfare measure over $J = 100$ simulations with different stochastic seeds to obtain ergodic means.

Results are given in Table 3. The evaluated scenarios differ along 3 dimensions. A first distinction is made in terms of the pricing scheme, law of one price or pricing-to-market. Second, the two columns, headed MU and FLEX, indicate whether the exchange rate regime is a monetary union or flexible. Third, rows mark if only monetary policy is available for stabilization purposes (abbreviated by MP) or if both monetary and fiscal policy can be used (MFP). To allow for comparisons between these scenarios, the underlying steady state is calibrated to be identical across all 8 scenarios. This implies for the MP scenarios that VAT rates in steady state have to be set on the optimal values obtained under MFP.

As is well-known, absolute numbers calculated for the welfare costs of business cycles are in general small in representative agent models.¹⁰ The focus of this analysis, yet, lies on the comparison across different scenarios, which yields more expressive outcomes. Results for the benchmark calibration are given in Panel (A) of the table. Under LOOP and exclusive availability of monetary policy, households are willing to give up $\omega_{MP}^{MU} = 5.16 * 10^{-2}\%$ of steady state consumption (hereafter c%) to avoid living in the stochastic economy of a monetary union and $\omega_{MP}^{FLEX} = 4.53 * 10^{-2}\%$ with flexible exchange rates. The difference between these two numbers, $\Delta\omega_{MP} = \omega_{MP}^{MU} - \omega_{MP}^{FLEX} = 0.63 * 10^{-2}\%$, given in the last column, shows the welfare costs of giving up exchange rate flexibility in a monetary union. Allowing for the VAT rates of both countries as a stabilization tool reveals that fiscal policy is almost irrelevant under flexible exchange rates—welfare costs are reduced from $\omega_{MFP}^{MU} = 4.45 * 10^{-2}\%$ to $\omega_{MFP}^{FLEX} = 4.36 * 10^{-2}\%$. By contrast, fiscal policy is an effective instrument in a monetary union: welfare costs of entering a MU are reduced by 85.76% from $\Delta\omega_{MP} = 0.63 * 10^{-2}\%$ to $\Delta\omega_{MFP} = 0.09 * 10^{-2}\%$.

Engel (2011) shows that optimal exchange rate volatility is lower in presence of pricing-to-market since in this case exchange rate movements do not directly translate into changes of international relative prices as they would under the LOOP, but merely distort price mark-ups of firms, thereby making inefficient deviations from the law of one price to occur.¹¹ Welfare costs of fixed exchange rate regimes are, therefore, strictly lower with PTM than under LOOP, a point also raised by Corsetti (2008). Additionally, as shown in Section 2, fiscal policy can potentially be much more effective in manipulating the terms of trade when the LOOP holds due to the assumption of full pass-through than under PTM. To take into account the effect of the pricing scheme on the welfare costs of exchange rate pegs on the one hand, and to avoid an overestimation of the beneficial effect of fiscal policy because of full pass-through on the other hand, the reduction of welfare costs is studied next for the case of PTM. The welfare costs of entering a MU are now about $\Delta\omega_{MP} = 0.099 * 10^{-2}\%$ under monetary policy only, which is about 6.4 times smaller than when the law of one price holds. Adding fiscal policy to the set of instruments also

⁹I use the Dynare toolbox to solve the model. The second-order simulations are obtained using the pruning algorithm proposed by Kim, Kim, Schaumburg, and Sims (2008).

¹⁰An exemption is the analysis of Schmitt-Grohé and Uribe (2016), which relies on downward nominal wage rigidity.

¹¹Under very specific conditions, it can even be optimal to completely stabilize the nominal exchange rate in presence of PTM, as shown by Devereux and Engel (2003). Duarte and Obstfeld (2008) emphasize that this extreme result holds only under a restrictive set of assumptions. Among these are one period in advance price stickiness and the absence of home bias.

Table 3: WELFARE COSTS OF FIXED EXCHANGE RATES

(A) Benchmark				
		MU	FLEX	Difference
LOOP				
Monetary Policy (MP)	$10^{-2} *$	-5.1612	-4.5269	0.6343
Monetary+Fiscal Policy (MFP)	$10^{-2} *$	-4.4485	-4.3582	0.0903
		Reduction of Welfare Costs:		85.76%
PTM				
Monetary Policy	$10^{-2} *$	-5.1593	-5.0605	0.0988
Monetary+Fiscal Policy	$10^{-2} *$	-4.9095	-4.8785	0.0310
		Reduction of Welfare Costs:		68.66%
(B) Productivity, Preference, Gov. Spending Shocks				
LOOP				
Monetary Policy	$10^{-2} *$	-4.1227	-3.6759	0.4468
Monetary+Fiscal Policy	$10^{-2} *$	-3.7696	-3.6983	0.0713
		Reduction of Welfare Costs:		84.03%
PTM				
Monetary Policy	$10^{-2} *$	-4.1212	-4.0689	0.0523
Monetary+Fiscal Policy	$10^{-2} *$	-4.0826	-4.0593	0.0233
		Reduction of Welfare Costs:		55.42%
(C) Mark-up Shocks				
LOOP				
Monetary Policy	$10^{-2} *$	-0.9617	-0.7742	0.1875
Monetary+Fiscal Policy	$10^{-2} *$	-0.6064	-0.5868	0.0194
		Reduction of Welfare Costs:		89.58%
PTM				
Monetary Policy	$10^{-2} *$	-0.9613	-0.9148	0.0465
Monetary+Fiscal Policy	$10^{-2} *$	-0.7505	-0.7428	0.0077
		Reduction of Welfare Costs:		83.43%

Note: Welfare measure: consumption equivalents between deterministic and stochastic world economy. Exchange rate regime either monetary union (MU) or flexible (FLEX). Panel (A): productivity, demand preference, government spending, & mark-up shocks in both countries. Panel (B): all but mark-up shocks. Panel (C): mark-up shocks only. Second-order approximation to policy functions. $T = 1000$, $J = 100$.

helps to reduce welfare costs considerably by 68.66%. Hence, even under PTM, fiscal policy is capable of reducing the welfare costs of fixed exchange rates substantially.

The two bottom panels, (B) and (C), of Table 3 decompose the shocks into those, for which the efficient response is attainable by the use of monetary policy only when the law of one price holds and exchange rates are fully flexible (productivity, demand preferences, and government spending shocks), and the mark-up shocks, which cannot be fully stabilized. While the size of the welfare costs in the various cases naturally depends on the type and number of shocks considered, the percentage reduction of the welfare costs of the fixed exchange rate regime is of comparable magnitude as in the benchmark (Panel A). Under the law of one price, using the tax instruments for stabilization policy purposes reduces the welfare costs of the monetary union by 84% in Panel (B) and by almost 90% in presence of the mark-up shocks. Under pricing-to-market, the reduction

in welfare costs depends to a larger extent on the type of shock. Allowing for active fiscal policy reduces welfare costs by 55.42% in Panel (B) and by 83.43% in Panel (C). The cause for the effective stabilization of mark-up shocks can directly be understood from the firms' first-order conditions (21) to (23). The VAT rates can directly offset the effect of the mark-up shocks μ_t on the firms' price setting.

Various sensitivity checks confirm that the results of Table 3 are very robust to changes in the parametrization of the model. Table 7 in Appendix E provides results, where standard deviations of all shocks are doubled compared to the benchmark calibration. Increasing the shock size naturally raises the shares of steady state consumption that households are willing to give up to avoid living in the stochastic economy. The percentage reduction of the welfare costs of fixed exchange rates by using fiscal policy, however, remains virtually the same. Optimal exchange rate volatility and the costs of pegs also depend on the structural parameters of the model. For instance, Lombardo and Ravenna (2014) and Faia and Monacelli (2008) emphasize the role of trade openness for the exchange rate, while De Paoli (2009) analyses the impact of the Armington elasticity. Results in Table 8 show that the findings of this section regarding the reduction of welfare costs are qualitatively fully maintained for changes in all structural parameters as well as the amount of government spending and debt.

Instead of VAT rates, policymakers could in principle also use payroll taxes as the fiscal instrument to substitute for the effect of the exchange rate.¹² A change in the labour tax implies that the prices of all goods produced within a country are affected equally, while changes of the domestic VAT alter only prices of goods sold at home, but not of exports. To analyse whether these differences influence the capability of fiscal policy to reduce the welfare costs of a fixed exchange rate regime, I repeat the welfare analysis of Table 3 with a payroll tax in each country levied on firms instead of a VAT.

Table 9 in Appendix E presents the results of that exercise. To ensure comparability, I use the same calibration as before. Most importantly, the reduction of welfare costs by the additional use of fiscal policy remains to be high with payroll taxes. Under the benchmark calibration, welfare costs of a peg can be reduced by about 60% under the LOOP and by 80% under PTM. The relatively smaller reduction of welfare costs under the LOOP is driven by the low reduction for productivity, demand preference, and government spending shocks in Panel (B) of 41% only compared to 84% with VATs. The main reason for these different results is that, in opposition to VATs, payroll tax changes do not directly pass through on the terms of trade via the law of one price (recap equations 20 and 40). In case of mark-up shocks, on the other hand, the welfare costs of fixed exchange rates can be avoided almost completely—by 99% under LOOP and 97% under PTM. A change in the domestic payroll tax suffices to neutralize the effect of a domestic mark-up shock, while with VATs the rates of both countries would have to adjust for stabilization of the shock along all relevant margins.

In sum, these results suggest that optimal use of only one fiscal instrument per country could substantially reduce welfare costs in the euro area that arise from the fixed exchange rate regime.

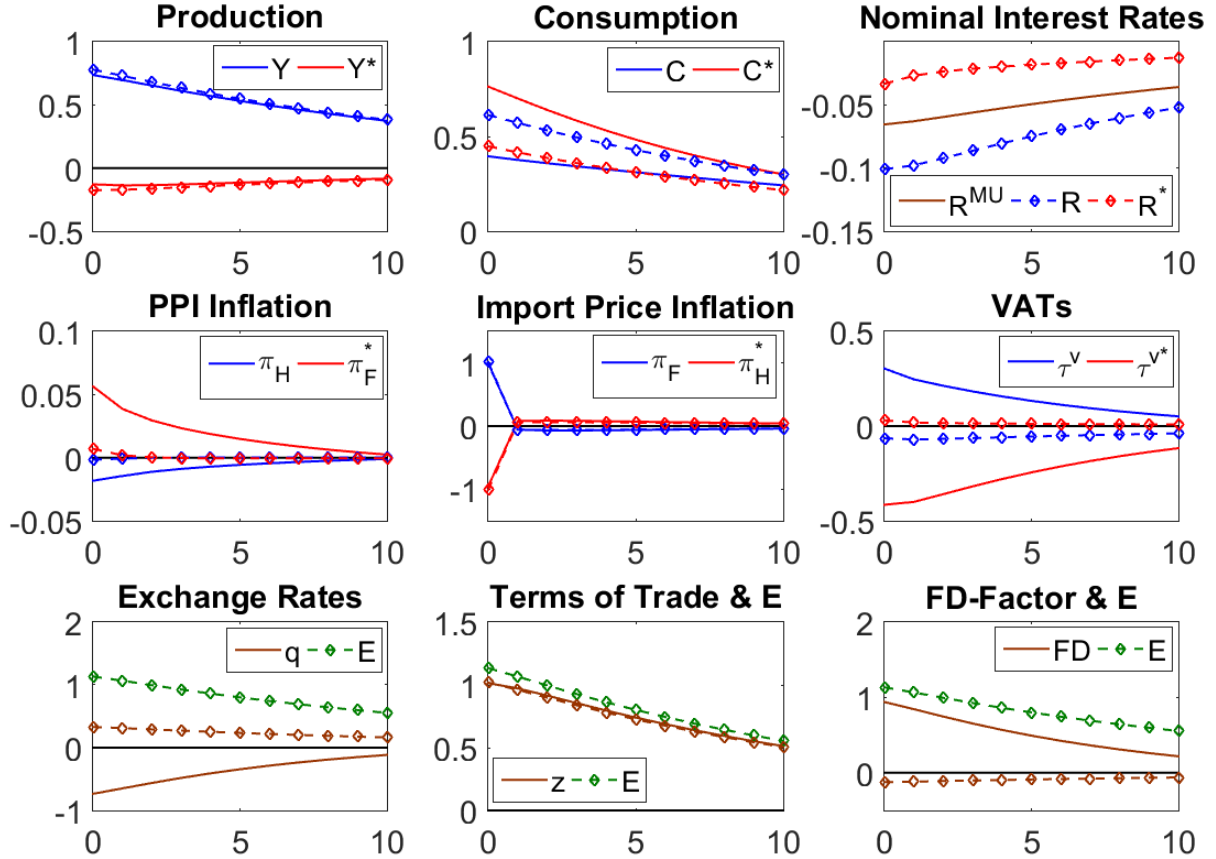
¹²If a payroll tax τ_t^n is levied on the employers, profits of firm k become

$$\Pi_t(k) = (1 - \tau_t^v) P_{Ht}(k) [nC_{Ht}(k, h) + G_t(k)] + (1 - \tau_t^{v*}) E_t P_{Ht}^*(k) C_{Ht}^*(k, h) - (1 + \tau_t^n) W_t N_t(k).$$

5.3 Optimal Fiscal Substitutes for the Exchange Rate

This section describes the conduct of optimal policy and shows how taxes should be used to substitute for the nominal exchange rate inside the euro area. Optimal fiscal policy in the monetary union depicts a fiscal devaluation policy: in case it would be optimal to devalue the exchange rate of a region, it is optimal to increase its VAT relative to the other region of the monetary union.

Figure 1: PRODUCTIVITY SHOCK IN CORE, LOOP



Note: Comparison of impulse responses to 1% productivity shock in the core under the law of one price. **Solid lines:** monetary union. **Dashed diamond lines:** flexible exchange rate. **Blue lines:** core. **Red lines:** periphery. Unit of y-axis is % deviation from steady state (p.p. deviation in Panels 3 & 6). X-axis indicates quarters after impulse.

5.3.1 Dynamic Response to a Productivity Shock

To gain intuition for the findings of the welfare analysis, Figure 1 compares the impulse response to a 1% productivity shock in the core under LOOP in the monetary union (solid lines) with the counterfactual response under flexible exchange rates (dashed diamond lines). As shown by Corsetti, Dedola, and Leduc (2010), the latter case constitutes the benchmark of "divine coincidence" in open economies, where stabilizing PPI inflation by monetary policy in both regions is sufficient to obtain the efficient allocation in presence of the shock.

The increase of productivity implies that it is efficient to produce a larger share of world output in the core. Y_t increases strongly, while Y_t^* declines on impact (Panel 1). To induce the required expenditure-switching towards core goods, the terms of trade of the core have to deteriorate (i.e. z_t has to increase). According to (40), this can be achieved by changes of the PPIs (P_{Ht}, P_{Ft}^*), by nominal or by fiscal devaluation. As long as exchange rates are flexible (dashed diamond lines), this shift in the terms of trade is generated by the nominal exchange rate due to its feature of immediate pass-through under LOOP (see Panel 8), while PPI inflation rates are kept constant to avoid welfare-reducing price dispersion among goods (Panel 4). The adjustment of the exchange rate leads to strong effects on the prices of imports (Panel 5). As imports behave as under flexible prices, inflation in that sector does not have to be minimized to avoid welfare losses. The VAT rates are basically unused under flexible E_t (Panels 6 and 9) since the efficient response to the shock can in this case be brought about by monetary policy alone.

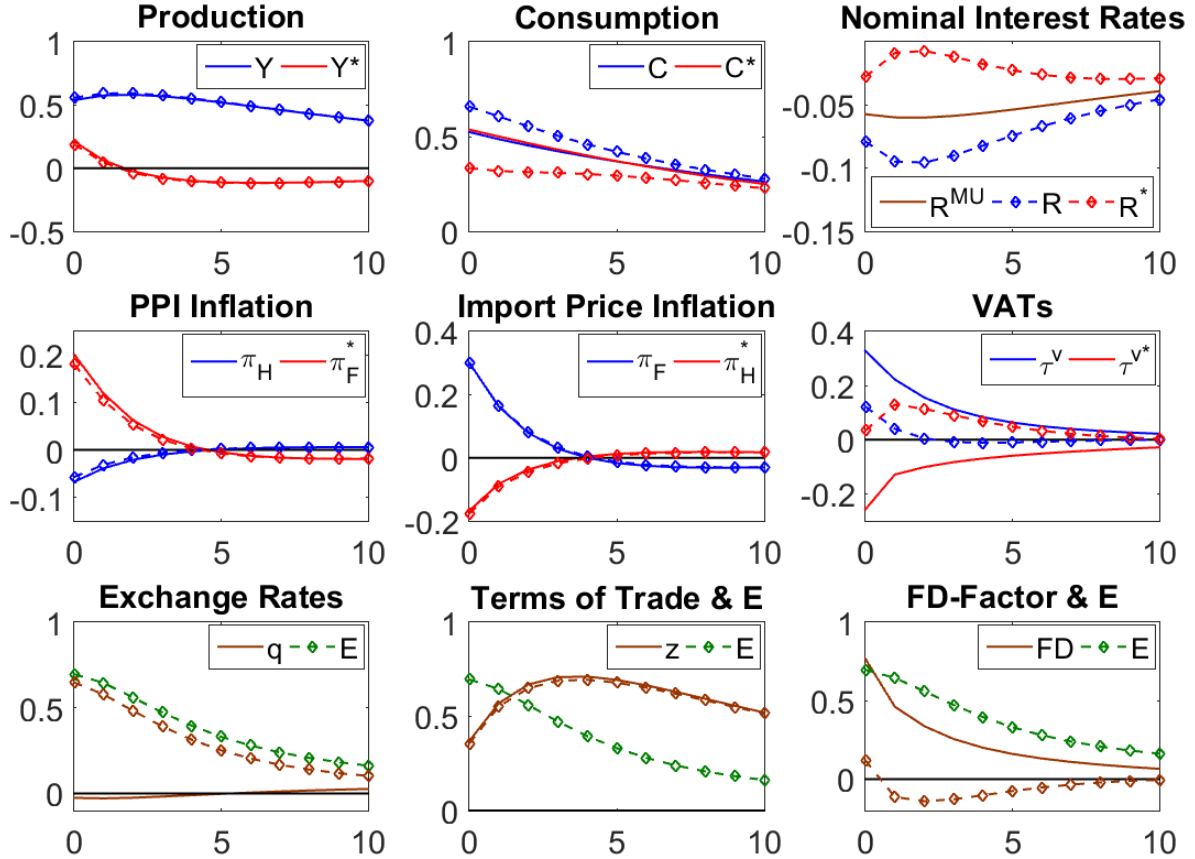
These dynamics change altogether in the monetary union (solid lines). Monetary policy on its own is not able to reach the efficient response any more. The reaction of the nominal interest rate is now in between the responses of core and periphery under flexible exchange rates, which implies a rate too low for the periphery and rate too high for the core (Panel 3). As a consequence, deviations of PPI inflation from steady state are now slightly larger than under exchange rate flexibility. The reaction is somewhat stronger in the periphery due to its relatively lower weight in the welfare function of the Ramsey planner. Even though E_t is now fixed, the efficient response of the terms of trade can again be reached (the brown solid and dashed lines in Panel 8 cover up each other perfectly). The way the reaction of the terms of trade is induced is completely different, though. The VAT rates are now used actively to substitute for the effect of E_t on the terms of trade. Panel 9 shows that the response of the fiscal devaluation factor, FD_t , in the monetary union is very close to the counterfactual flexible exchange rate response. On impact, 93% of the response of z_t in the monetary union are due to a fiscal devaluation policy. Only the remaining 7% are due to changes in PPIs. To implement the fiscal devaluation, the VAT of the core increases, while the VAT of the periphery decreases. Besides its effect on the terms of trade, these tax responses at the same time help to stabilize firm mark-ups. The increase of τ_t^v supports monetary policy in fighting back deflationary pressures in the core that arise from the increased productivity, while the decrease of τ_t^{v*} reduces inflationary pressures in the periphery, which are the result of the relatively loose monetary policy for that region.

Under the free floating regime, the real exchange rate q_t depreciates because the devaluation of the core's currency dominates the relative increase of the core's CPI (Panel 7). In the monetary union, the real exchange rate appreciates instead. The fiscal devaluation also increases the CPI of the core relative to the periphery by making core imports more expensive, but the relative currency value between the regions now remains fixed. As a result, in case of the monetary union consumption in the periphery increases by more than in the core due to international risk sharing (Panel 2).

Taken together, the optimal fiscal devaluation policy focusses on replicating the behaviour of the terms of trade under flexible exchange rates to induce expenditure switching effects, but it does not reproduce the response of the real exchange rate that affects levels of consumption via the international risk sharing condition (15). The policymaker thereby favours production efficiency over an efficient allocation of aggregate consumption in the

monetary union. Addressing the latter would require an additional instrument to affect the real exchange rate. [Farhi et al. \(2014\)](#) show that a consumption subsidy payable to households could succeed to that task. They also prove that a complete replication of the allocation under flexible exchange rates lacks even further instruments. A payroll subsidy to firms would be needed to stabilize internal prices of domestically produced goods, which are distorted by the VAT, while a labour income tax levied household would be required to neutralize distortions by the consumption subsidy on wages.¹³

Figure 2: PRODUCTIVITY SHOCK IN CORE, PTM



Note: Comparison of impulse responses to 1% productivity shock in the core under pricing-to-market. **Solid lines:** monetary union. **Dashed diamond lines:** flexible exchange rate. **Blue lines:** core. **Red lines:** periphery. Unit of y-axis is % deviation from steady state (p.p. deviation in Panels 3 & 6). X-axis indicates quarters after impulse.

Figure 2 compares impulse responses of the monetary union with the flexible exchange rate scenario for the case of pricing-to-market. [Engel \(2011\)](#) shows for the case of flexible exchange rates and PTM that CPI inflation (as the weighted average of PPI and import price inflation) instead of PPI inflation only ought to be stabilized, since the import sector is now also subject to a sticky price friction. However, avoiding inflation and closing output gaps is not sufficient to obtain the efficient allocation, because deviations of the terms of trade from their efficient level and deviations from the law of one price

¹³Depending on the specific model setting, still more instruments may be required. See also [Adao et al. \(2009\)](#) on that point.

can still occur. These wrong price signals translate into inefficient shifts in the level and composition of consumption between the regions. Accordingly, in opposition to the LOOP case, the response to the productivity shock under flexible exchange rates and PTM does not reach the efficient allocation.

Beginning the description with the case of flexible exchange rates again (dashed diamond lines), PPI inflation is more pronounced under PTM, but import inflation is decisively weaker, leading to a terms of trade deterioration which falls short of its efficient response. Under PTM, z_t rises by 0.36% in the first quarter, while the efficient response under the LOOP renders 1%. Expenditure switching from periphery to core is, therefore, not sufficient. The reason for the dampened reaction of the terms of trade is that exchange rate pass-through on international prices is now limited by the sticky price friction in the import sector, visible in the low co-movement between z_t and E_t (Panel 8). Policymakers generate a weaker devaluation of the nominal exchange rate under PTM (0.7% on impact) than under LOOP (1.1% on impact), for they now have to trade off the costs of additional import price dispersion against the benefits of deteriorated terms of trade due to higher import prices P_{Ft} . Taxes have a comparable effect on prices as monetary policy (confer equations 22 and 26 for the perspective of the core). Increasing the domestic VAT, τ_t^v , dampens the deflationary pressure on the core's PPI, but incentivizes higher import prices also. As long as exchange rate flexibility is given, taxes are used only to a limited extent for stabilization purposes.

In the monetary union (solid lines), the response of the terms of trade under flexible exchange rates can again be replicated entirely by fiscal policy. The VAT rates are used to induce the same price setting behaviour as the flexible exchange rate would. Relative VAT rates, i.e. the fiscal devaluation factor, are highly correlated with the counterfactual exchange rate (Panel 9), in order to shift relative prices and to reduce deviations from the law of one price. On impact, FD_t even overshoots the response of E_t by 7%. The pass-through of these tax changes on prices and the terms of trade remains, however, limited again.

Fiscal policy in the monetary union is successful in replicating the path of the terms of trade under flexible exchange rates, but again the fiscal devaluation policy does not keep track of the respective real exchange rate path. Since CPIs are implicitly stabilized under PTM, q_t barely moves in the monetary union (Panel 7), leading to almost perfectly correlated reactions of consumption in the core and periphery because of international risk sharing. Under flexible exchange rates instead, the real exchange rate follows the depreciation of E_t closely. Consumption in the core, hence, increases by more than in the periphery in this case.

5.3.2 Business-Cycle Properties of the Ramsey Allocations

In this section, I show to what extent the findings and intuitions obtained under the productivity shock generalize to the other shocks as well. To do so, I analyse second moments of key variables, generated from simulated business cycle data.

Table 4 presents correlations between the counterfactual flexible exchange rate and various tax measures in the monetary union for both types of price setting and different types of shocks. Correlations are calculated with the fiscal devaluation factor, FD_t , and with the tax rates in levels, τ_t^v and τ_t^{v*} .

Table 4: CORRELATIONS BETWEEN EXCHANGE RATES AND TAXES

LOOP	Benchmark	No Mark-up Shocks	Mark-up, Core	Mark-up, Periphery
$Corr(E_t, FD_t)$	0.81	0.89	0.52	0.52
$Corr(E_t, \tau_t^v)$	0.03	0.81	0.43	-0.49
$Corr(E_t, \tau_t^{v*})$	-0.41	-0.91	0.40	-0.50
PTM				
$Corr(E_t, FD_t)$	0.59	0.89	0.86	0.65
$Corr(E_t, \tau_t^v)$	0.11	0.88	0.90	-0.73
$Corr(E_t, \tau_t^{v*})$	-0.31	-0.86	0.93	-0.69

Note: Correlations between tax measures obtained in monetary union scenario and counterfactual flexible exchange rate. Columns indicate shock processes used for simulation: 'Benchmark' includes productivity, demand preference, government spending, & mark-up shocks in both countries. 'No Mark-up' includes all but mark-up shocks. Last two columns include mark-up shocks in the respective region only. Second-order approximation to policy functions. $T = 1000$, $J = 100$.

The correlation between FD_t and E_t is generally found to be high. In the benchmark scenario with all shocks, it reads 81% when the LOOP holds and 59% under PTM. It is even higher at 89% for both pricing schemes when looking at the productivity, demand preference, and government spending shocks, and it ranges between 52% and 86% for the mark-up shocks. These results indicate that the policymaker actively uses fiscal policy to replicate the path of the terms of trade in absence of a flexible exchange rate.

The results regarding tax rates in levels do not allow for general conclusions in the benchmark scenario, both under the LOOP and PTM.¹⁴ A more detailed inspection reveals that the correlations—and, hence, the exact conduct of tax policy—depend decisively on the type of shocks. With the productivity, demand preference, and government spending shocks, the correlations with taxes under the LOOP (PTM) read 81% (88%) and -91% (-86%), respectively. Whenever it were optimal to devalue the exchange rate of a region in the monetary union, its VAT ought to be increased, while the tax of the other (re-valuing) region should decrease. Fiscal devaluation policies as outlined in the introduction can accordingly be observed, independent of the type of price setting.

Under mark-up shocks, the tax responses additionally depend on the origin of the shock. The VAT rates of both regions are now positively correlated with the exchange rate of that region which experiences a mark-up shock.¹⁵ In response to a positive mark-up shock, e.g., in the core, it is efficient to shift production to the periphery, which requires an appreciated exchange rate (i.e. a decline of E_t) for the core. The optimal response in a monetary union is to decrease taxes in both regions. Under the LOOP, this policy attenuates the higher mark-up in the core and fosters the expenditure-switch by reducing prices for periphery goods, while still taking heed of solvency of the fiscal authority. In order to achieve a decline of FD_t nevertheless, the VAT of the periphery should decline by less than its core counterpart. Under PTM, it is clear from (23) that a rise in τ_t^{v*} aimed at replicating the decline in E_t , would even exacerbate the mark-up distortion for

¹⁴The asymmetry between the correlations of τ_t^v and τ_t^{v*} is mainly driven by the different shock sizes in core and periphery.

¹⁵Note that E_t denotes the exchange rate from the perspective of the core. Correlations are, therefore, negative for a mark-up shock in the periphery.

the periphery's import goods. τ_t^{v*} , therefore, also declines instead, which explains the positive correlation of τ_t^{v*} with E_t .

Altogether, policymakers in the monetary union always adjust the ratio of tax rates between the regions to induce relative price shifts in a similar fashion as the exchange rate would. The behaviour of tax rates in levels and their correlation with the exchange rate crucially depends on the type of shocks.

Table 5: STANDARD DEVIATIONS OVER THE BUSINESS-CYCLE

(A) Benchmark						
LOOP	q_t	z_t	E_t	FD_t	τ_t^v	τ_t^{v*}
Monetary Union (MU)	0.89	1.69		1.19	1.46	1.73
Flexible Exchange Rate (FLEX)	0.49	1.68	1.84	0.26	1.52	1.53
PTM						
Monetary Union	0.11	1.29		1.67	1.43	1.95
Flexible Exchange Rate	0.69	1.27	0.81	1.33	1.43	1.87
(B) Productivity, Preference, Gov. Spending Shocks						
LOOP						
Monetary Union	0.75	1.44		1.00	0.32	0.46
Flexible Exchange Rate	0.46	1.42	1.58	0.17	0.11	0.09
PTM						
Monetary Union	0.10	1.21		0.56	0.26	0.17
Flexible Exchange Rate	0.63	1.19	0.77	0.15	0.09	0.13
(C) Mark-up Shocks						
LOOP						
Monetary Union	0.47	0.87		0.64	1.43	1.67
Flexible Exchange Rate	0.14	0.88	0.89	0.19	1.52	1.53
PTM						
Monetary Union	0.03	0.39		1.57	1.41	1.94
Flexible Exchange Rate	0.28	0.39	0.39	1.33	1.42	1.86

Note: Standard deviations are measured in percentage points. Exchange rate regime either monetary union (MU) or flexible (FLEX). Panel (A): productivity, demand preference, government spending, & mark-up shocks in both countries. Panel (B): all but mark-up shocks. Panel (C): mark-up shocks only. Second-order approximation to policy functions. $T = 1000$, $J = 100$.

Table 5 compares standard deviations of international relative prices and of taxes in the monetary union and the flexible exchange rate regime, for both types of price setting and different shock compositions. The following observations stand out.

In all scenarios, standard deviations of the terms of trade, z_t , in the monetary union are found to be close to their counterpart under flexible exchange rates, e.g. 1.69% versus 1.68% in the benchmark with LOOP. The volatility of real exchange rates, instead, differs markedly between MU and FLEX. This indicates a generalization of the finding, obtained from the productivity shock, that optimal fiscal devaluation policies focus on replicating the time path of the terms of trade, but not of the real exchange rate.

Confirming Engel’s (2011) result, nominal exchange rate volatility is in all panels found to be lower under PTM than under the LOOP, at least by a factor of two.

Also in line with the results obtained from the analysis of the productivity shock, volatilities of the fiscal devaluation factor are smaller under flexible exchange rates than in the monetary union in all scenarios. In the benchmark (Panel A), the volatility increases from 0.26% to 1.19% when the LOOP holds and from 1.33% to 1.67% under PTM. The volatility of the tax rates itself is found to be of similar size in the MU as well as the FLEX scenario in Panel (A). The decomposition into the different shock types in Panel (B) and (C) reveals that this is primarily driven by the mark-up shocks, for the latter require an active fiscal policy response even under flexible exchange rates. In case of the productivity, demand preference, and government spending shocks, the intuition, obtained from the impulse responses, is restored that taxes are used only mildly under flexible exchange rates, but intensely in the monetary union.

The volatility of tax rates is of the same order of magnitude as the volatility of E_t . In the benchmark of panel (A), this implies that taxes on average do not have to fluctuate more than about 2 percentage points for an optimal policy response to the business cycle—thereby rendering fiscal devaluations as a practically implementable policy option.¹⁶

6 Conclusion

This paper analyses to what extent fiscal policy can compensate for the absent nominal exchange rate in a monetary union in terms of business cycle stabilization. Various Ramsey-optimal policy scenarios are studied in a New Keynesian 2-region model, calibrated to the euro area, that differ regarding the exchange rate regime and the availability of fiscal policy for stabilization purposes. Optimal use of only one tax instrument per country enables policymakers to reduce the welfare costs of giving up flexible exchange rates in a monetary union by up to 86% when the law of one price holds for traded goods, and up to 69% when different prices can be set for the regions. Fiscal devaluations arise as an outcome of optimal fiscal policy. Whenever a nominal exchange rate devaluation were optimal for a region, a relative increase of the region’s VAT is the optimal fiscal policy in the monetary union. In particular in case of mark-up shocks, policymakers face a trade-off between replicating the effects of the nominal exchange rate and stabilizing firms’ costs, however. Optimal fiscal policy in the monetary union is successful in the reproduction of the flexible exchange rate path of the terms of trade, but not of the real exchange rate.

The analysis of optimal policy studies the relevant benchmark of full cooperation between the central bank(s) and the fiscal authorities at the region level. The strategic interactions in form of a dynamic Nash game between the different entities are not considered so far. This constitutes a probably fruitful exercise. Fiscal policy, as considered in the model, requires tax changes at a business cycle frequency, whose implementation

¹⁶Naturally, the standard deviations of both exchange rates and taxes increase with the size of the underlying shocks. The seemingly small volatility of E_t found in the simulations, nevertheless, does not need to be entirely unrealistic. The model provides an optimal policy response that reacts on changes in fundamentals only, compared to actual exchange rate data, which notoriously entails a sizeable amount of unexplainable volatility. Regarding this point, see also the vast literature on the “exchange rate disconnect” puzzle following [Obstfeld and Rogoff \(2001\)](#).

surely poses political economy issues. However, first steps in direction of a unified VAT framework for all member states of the European Union are already taken that will facilitate a higher degree of coordination in fiscal policy in future.¹⁷

The paper focuses on VAT-based fiscal devaluation policies. Further research could also study the optimality of more general fiscal devaluation policies in the sense of tax swaps from direct to indirect taxation (e.g. an increase in the VAT, paired with a reduction of payroll taxes of employers), which can be revenue neutral to public budgets. An analysis of such policies is, however, impeded in the present class of models due to an indeterminacy between consumption and income taxes. The inclusion of another, untaxed, production factor could possibly remedy this issue.

Other interesting augmentations of the model include the introduction of non-tradable goods and downward nominal wage rigidity as an additional friction. [Schmitt-Grohé and Uribe \(2016\)](#) show that the combination of these two components can lead to welfare costs of fixed exchange rates that are more in line with conventional wisdom than those usually obtained in representative agents models.

¹⁷See in particular European Council Directive 2006/112/EC, which lays down a common system of value added tax regulation for the EU. It covers aspects such as the tax base, the allowed number of reduced tax rates besides the standard rate, and also defines which types of goods are eligible for exemptions. It further regulates which country's rate applies to imported goods, and even directs upper and lower bounds for tax rates.

Appendix

A Derivations for Section 2

A.1 Optimal Firm Price Setting

In order to derive the conditions for optimal price setting, one first needs to derive an aggregate demand equation $Y_t(k)$ for firm k . Consider (17), which can be rewritten as

$$\begin{aligned} Y_t(k) &= nC_{Ht}(k) + (1-n)C_{Ht}^*(k) + G_t(k) \\ &= \frac{1}{n} \left(\frac{P_{Ht}(k)}{P_{Ht}} \right)^{-\rho} (nC_{Ht} + G_t) + \frac{(1-n)}{n} \left(\frac{P_{Ht}^*(k)}{P_{Ht}^*} \right)^{-\rho} C_{Ht}^*. \end{aligned} \quad (44)$$

In the first line, I integrated over households using the fact that agents within a country behave identically. In the second line, I applied consumption demand functions (8) of both households and the domestic government.

Law of One Price: Using the law of one price (20) and the fact that the law also holds for price indices under the given structure, (44) reduces to

$$Y_t(k) = \frac{1}{n} \left(\frac{P_{Ht}(k)}{P_{Ht}} \right)^{-\rho} Y_t, \quad (45)$$

where $Y_t = nC_{Ht} + (1-n)C_{Ht}^* + G_t$. By means of the law of one price again and (45), firm profits (18) change to

$$\Pi_t(k) = (1 - \tau_t^v) P_{Ht}(k) \frac{1}{n} \left(\frac{P_{Ht}(k)}{P_{Ht}} \right)^{-\rho} Y_t - W_t N_t(k). \quad (46)$$

The optimal price $\bar{P}_{Ht}(k)$ is then determined by maximizing the expected present discounted value of profits subject to the production technology (16) and demand (45):

$$\begin{aligned} \max_{\bar{P}_{Ht}(k), N_s(k)} \mathbb{L}^{LOOP} &= \mathbb{E}_t \sum_{s=t}^{\infty} \theta^{s-t} Q_{t,s} P_{Hs} \left\{ \left[(1 - \tau_s^v) \frac{1}{n} \left(\frac{\bar{P}_{Ht}(k)}{P_{Hs}} \right)^{1-\rho} Y_s - w_s N_s(k) \right] \right. \\ &\quad \left. + m c_{Hs}(k) \left[A_s N_s(k)^\alpha - \frac{1}{n} \left(\frac{\bar{P}_{Ht}(k)}{P_{Hs}} \right)^{-\rho} Y_s \right] \right\}, \end{aligned}$$

where $w_t = W_t/P_{Ht}$ is the producer real wage. The associated first-order conditions are:

$$\begin{aligned} \frac{\partial \mathbb{L}^{LOOP}}{\partial \bar{P}_{Ht}(k)} &= \mathbb{E}_t \sum_{s=t}^{\infty} \theta^{s-t} Q_{t,s} \left(\frac{\bar{P}_{Ht}(k)}{P_{Hs}} \right)^{-1-\rho} Y_s \left\{ (1 - \tau_s^v) (1 - \rho) \frac{\bar{P}_{Ht}(k)}{P_{Hs}} + m c_{Hs}(k) \rho \right\} = 0, \\ \frac{\partial \mathbb{L}^{LOOP}}{\partial N_s(k)} &= \mathbb{E}_t \theta^{s-t} Q_{t,s} P_{Hs} [-w_s + m c_{Hs}(k) \alpha A_s N_s(k)^{\alpha-1}] = 0. \end{aligned}$$

Combining the two conditions and rearranging the result yields the optimal pricing condition under LOOP (21).

Pricing-to-Market: The price setting problem of the firm under PTM implies maximizing profits (18) subject to (16) and (44):

$$\begin{aligned} \max_{\bar{P}_{Ht}(k), \bar{P}_{Ht}^*(k), N_s(k)} \mathbb{L}^{PTM} &= \mathbb{E}_t \sum_{s=t}^{\infty} \theta^{s-t} Q_{t,s} \left\{ (1 - \tau_s^v) \bar{P}_{Ht}(k) \frac{1}{n} \left(\frac{\bar{P}_{Ht}(k)}{P_{Hs}} \right)^{-\rho} [nC_{Hs} + G_s] \right. \\ &+ (1 - \tau_s^{v*}) E_s \bar{P}_{Ht}^*(k) \frac{1}{n} \left(\frac{\bar{P}_{Ht}^*(k)}{P_{Hs}^*} \right)^{-\rho} (1 - n) C_{Hs}^* - W_s N_s(k) \\ &\left. + MC_{Hs}(k) \left[A_s N_s(k)^\alpha - \frac{1}{n} \left(\frac{\bar{P}_{Ht}(k)}{P_{Hs}} \right)^{-\rho} [nC_{Hs} + G_s] - \frac{(1-n)}{n} \left(\frac{\bar{P}_{Ht}^*(k)}{P_{Hs}^*} \right)^{-\rho} C_{Hs}^* \right] \right\}. \end{aligned}$$

The associated first-order conditions are:

$$\begin{aligned} \frac{\partial \mathbb{L}^{PTM}}{\partial \bar{P}_{Ht}(k)} &= \mathbb{E}_t \sum_{s=t}^{\infty} \theta^{s-t} Q_{t,s} \left(\frac{\bar{P}_{Ht}(k)}{P_{Hs}} \right)^{-1-\rho} [nC_{Hs} + G_s] \\ &\cdot \left\{ (1 - \tau_s^v) \frac{\bar{P}_{Ht}(k)}{P_{Hs}} - \frac{\rho}{\rho - 1} \frac{MC_{Hs}(k)}{P_{Hs}} \right\} = 0, \\ \frac{\partial \mathbb{L}^{PTM}}{\partial \bar{P}_{Ht}^*(k)} &= \mathbb{E}_t \sum_{s=t}^{\infty} \theta^{s-t} Q_{t,s} \left(\frac{\bar{P}_{Ht}^*(k)}{P_{Hs}^*} \right)^{-1-\rho} C_{Hs}^* \\ &\cdot \left\{ (1 - \tau_s^{v*}) E_s \left(\frac{\bar{P}_{Ht}^*(k)}{P_{Hs}^*} \right) - \frac{\rho}{\rho - 1} \frac{MC_{Hs}(k)}{P_{Hs}^*} \right\} = 0, \\ \frac{\partial \mathbb{L}^{PTM}}{\partial N_s(k)} &= \mathbb{E}_t \theta^{s-t} Q_{t,s} [-W_s + MC_{Hs}(k) A_s \alpha N_s(k)^{\alpha-1}] = 0. \end{aligned}$$

Combining the conditions and rearranging the results yields the optimal pricing conditions under PTM, (22) and (23).

A.2 Evolution of Price Indices

Price index (5) can be written as

$$\begin{aligned} nP_{Ht}^{1-\rho} &= \int_0^{n\theta} P_{Ht-1}^{1-\rho}(k) dk + \int_{n\theta}^n \bar{P}_{Ht}^{1-\rho}(k) dk \\ \Leftrightarrow nP_{Ht}^{1-\rho} &= n\theta P_{Ht-1}^{1-\rho} + n(1-\theta) \bar{P}_{Ht}^{1-\rho} \\ \Leftrightarrow 1 &= \theta \pi_{Ht}^{\rho-1} + (1-\theta) \left(\frac{\bar{P}_{Ht}}{P_{Ht}} \right)^{1-\rho} \end{aligned}$$

$$\Leftrightarrow \tilde{p}_{Ht} = \frac{\bar{P}_{Ht}}{P_{Ht}} = \left(\frac{1 - \theta \pi_{Ht}^{\rho-1}}{1 - \theta} \right)^{\frac{1}{1-\rho}},$$

which is (30) in the main text. Similar expressions hold for P_{Ft}^* and, under PTM, also for P_{Ft} and P_{Ht}^* .

A.3 Recursive Philips Curves

The recursive form of a Philips curve is derived here by way of example for the PPI of home goods under the LOOP. The optimal pricing condition (21) can be written as

$$\begin{aligned} \frac{\rho}{\rho-1} \mu_s \mathbb{E}_t \sum_{s=t}^{\infty} \theta^{s-t} Q_{t,s} \left(\frac{\bar{P}_{Ht}}{P_{Ht}} \right)^{-1-\rho} Y_s m_{CHs} &= \mathbb{E}_t \sum_{s=t}^{\infty} \theta^{s-t} Q_{t,s} \left(\frac{\bar{P}_{Ht}}{P_{Ht}} \right)^{-\rho} Y_s (1 - \tau_s^v) \\ \frac{\rho}{\rho-1} \mu_t X1_{Ht} &= X2_{Ht} \end{aligned}$$

with

$$\begin{aligned} X1_{Ht} &= \mathbb{E}_t \sum_{s=t}^{\infty} \theta^{s-t} Q_{t,s} Y_s \left(\frac{\bar{P}_{Ht}}{P_{Hs}} \right)^{-1-\rho} m_{CHs} \\ &= \left(\frac{\bar{P}_{Ht}}{P_{Ht}} \right)^{-1-\rho} Y_t m_{CHt} + \mathbb{E}_t \sum_{s=t+1}^{\infty} \theta^{s-t} Q_{t,s} \left(\frac{\bar{P}_{Ht}}{P_{Hs}} \right)^{-1-\rho} Y_s m_{CHs} \\ &= \left(\frac{\bar{P}_{Ht}}{P_{Ht}} \right)^{-1-\rho} Y_t m_{CHt} \\ &\quad + \theta \mathbb{E}_t Q_{t,t+1} \left(\frac{\bar{P}_{Ht}}{\bar{P}_{Ht+1}} \right)^{-1-\rho} \mathbb{E}_{t+1} \sum_{s=t+1}^{\infty} \theta^{s-t-1} Q_{t+1,s} \left(\frac{\bar{P}_{Ht+1}}{P_{Hs}} \right)^{-1-\rho} Y_s m_{CHs} \\ &= \left(\frac{\bar{P}_{Ht}}{P_{Ht}} \right)^{-1-\rho} Y_t m_{CHt} + \theta \mathbb{E}_t Q_{t,t+1} \left(\frac{\bar{P}_{Ht}}{\bar{P}_{Ht+1}} \right)^{-1-\rho} X1_{Ht+1} \end{aligned}$$

and

$$\begin{aligned} X2_{Ht} &= \mathbb{E}_t \sum_{s=t}^{\infty} \theta^{s-t} Q_{t,s} Y_s \left(\frac{\bar{P}_{Ht}}{P_{Hs}} \right)^{-\rho} (1 - \tau_s^v) \\ &= \left(\frac{\bar{P}_{Ht}}{P_{Ht}} \right)^{-\rho} Y_t (1 - \tau_t^v) + \theta \mathbb{E}_t Q_{t,t+1} \left(\frac{\bar{P}_{Ht}}{\bar{P}_{Ht+1}} \right)^{-\rho} X2_{Ht+1}. \end{aligned}$$

Inserting the definition of the stochastic discount factor (12) and the law of motion of the PPI (30) yields equation (32) and (33) in the text.

Corresponding expressions for PPIs and import price indices under PTM can be derived accordingly from (22), (23), (25), and (26).

A.4 Aggregate Resource Constraint

To derive the aggregate resource constraints, combine production (16) with demand (44), and integrate over firms:

$$\begin{aligned}
A_t N_t^\alpha(k) &= \frac{1}{n} \left(\frac{P_{Ht}(k)}{P_{Ht}} \right)^{-\rho} (nC_{Ht} + G_t) + \frac{1}{n} \left(\frac{P_{Ht}^*(k)}{P_{Ht}^*} \right)^{-\rho} (1-n) C_{Ht}^* \\
A_t \int_0^n N_t^\alpha(k) dk &= \frac{1}{n} \int_0^n \left(\frac{P_{Ht}(k)}{P_{Ht}} \right)^{-\rho} dk (nC_{Ht} + G_t) \\
&\quad + \frac{1}{n} \int_0^n \left(\frac{P_{Ht}^*(k)}{P_{Ht}^*} \right)^{-\rho} dk (1-n) C_{Ht}^*.
\end{aligned}$$

As $(P_{Ht}(k)/P_{Ht}) = (P_{Ht}^*(k)/P_{Ht}^*)$ if the law of one price holds, this reduces to (34) under LOOP, but to (35) under PTM.

The law of motion for price dispersion emerges from (36) as follows:

$$\begin{aligned}
\Delta_{Ht} &= \frac{1}{n} \int_0^n \left(\frac{P_{Ht}(k)}{P_{Ht}} \right)^{-\rho} dk \\
&= \frac{1}{n} \left[n(1-\theta) \left(\frac{\bar{P}_{Ht}}{P_{Ht}} \right)^{-\rho} + n(1-\theta)\theta \left(\frac{\bar{P}_{Ht-1}}{P_{Ht}} \right)^{-\rho} + \dots \right] \\
&= (1-\theta) \sum_{j=0}^{\infty} \theta^j \left(\frac{\bar{P}_{Ht-j}}{P_{Ht}} \right)^{-\rho} \\
&= (1-\theta) \left(\frac{\bar{P}_{Ht}}{P_{Ht}} \right)^{-\rho} + (1-\theta) \sum_{j=1}^{\infty} \theta^j \left(\frac{\bar{P}_{Ht-j}}{P_{Ht}} \right)^{-\rho} \\
&= (1-\theta) \left(\frac{\bar{P}_{Ht}}{P_{Ht}} \right)^{-\rho} + \theta \left(\frac{P_{Ht-1}}{P_{Ht}} \right)^{-\rho} (1-\theta) \sum_{j=1}^{\infty} \theta^{j-1} \left(\frac{\bar{P}_{Ht-j}}{P_{Ht-1}} \right)^{-\rho} \\
&= (1-\theta) \tilde{p}_{Ht}^{-\rho} + \theta \pi_{Ht}^\rho \Delta_{Ht-1}.
\end{aligned}$$

B Competitive Equilibrium

This appendix lists equilibrium conditions for the cases of LOOP and PTM. All prices are expressed in relative terms.

B.1 Law of One Price

Let $p_{Ht} = P_{Ht}/P_t$ and $p_{Ft}^* = P_{Ft}^*/P_t^*$ be the PPI-CPI ratios, and $w_t = W_t/P_{Ht}$ and $w_t^* = W_t^*/P_{Ft}^*$ the producer real wages. A competitive equilibrium under the LOOP and autonomous monetary policy in both countries is a set of sequences $\{C_t, C_{Ht}, C_{Ft}, C_t^*, C_{Ht}^*, C_{Ft}^*, Y_t, Y_t^*, N_t, N_t^*, q_t, p_{Ht}, p_{Ft}^*, w_t, w_t^*, \pi_t, \pi_t^*, \pi_{Ht}, \pi_{Ft}^*, \Delta_{Ht}, \Delta_{Ft}^*, \tilde{p}_{Ht}, \tilde{p}_{Ft}^*, X1_{Ht}, X2_{Ht}, X1_{Ft}^*, X2_{Ft}^*\}_{t=0}^{\infty}$, satisfying

- Demand for Home and Foreign goods:

$$C_{Ht} = \gamma_H p_{Ht}^{-\xi} C_t, \quad C_{Ft} = \gamma_F \left(\frac{(1 - \tau_t^{v*})}{(1 - \tau_t^v)} p_{Ft}^* q_t \right)^{-\xi} C_t$$

$$C_{Ht}^* = \gamma_H^* \left(\frac{(1 - \tau_t^v) p_{Ht}}{(1 - \tau_t^{v*}) q_t} \right)^{-\xi} C_t^*, \quad C_{Ft}^* = \gamma_F^* p_{Ft}^{*-\xi} C_t^*$$

- Euler equations and international risk sharing:

$$\frac{1}{R_t} = \beta \mathbb{E}_t \left[\frac{\zeta_{t+1}^c}{\zeta_t^c} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{\pi_{t+1}} \right]$$

$$\frac{1}{R_t^*} = \beta \mathbb{E}_t \left[\frac{\zeta_{t+1}^{c*}}{\zeta_t^{c*}} \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{1}{\pi_{t+1}^*} \right]$$

$$q_t = \kappa \frac{\zeta_t^{c*}}{\zeta_t^c} \left(\frac{C_t^*}{C_t} \right)^{-\sigma}$$

- Labour supply:

$$N_t^\eta C_t^\sigma = \zeta_t^c w_t p_{Ht}$$

$$N_t^{*\eta} C_t^{*\sigma} = \zeta_t^{c*} w_t^* p_{Ft}^*$$

- Aggregate demand:

$$Y_t = n C_{Ht} + (1 - n) C_{Ht}^* + G_t$$

$$Y_t^* = n C_{Ft} + (1 - n) C_{Ft}^* + G_t^*$$

- Resource constraints:

$$A_t n N_t^\alpha = \Delta_{Ht} Y_t$$

$$A_t^* (1 - n) N_t^{*\alpha} = \Delta_{Ft}^* Y_t^*$$

- Phillips curves:

$$\frac{\rho}{\rho - 1} \mu_t X1_{Ht} = X2_{Ht},$$

$$X1_{Ht} = \tilde{p}_{Ht}^{-1-\rho} Y_t \frac{w_t}{A_t \alpha N_t^{\alpha-1}} + \theta \beta \mathbb{E}_t \frac{\zeta_{t+1}^c}{\zeta_t^c} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{\pi_{Ht+1}^{1+\rho}}{\pi_{t+1}} \left(\frac{\tilde{p}_{Ht}}{\tilde{p}_{Ht+1}} \right)^{-1-\rho} X1_{Ht+1}$$

$$X2_{Ht} = \tilde{p}_{Ht}^{-\rho} Y_t (1 - \tau_t^v) + \theta \beta \mathbb{E}_t \frac{\zeta_{t+1}^c}{\zeta_t^c} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{\pi_{Ht+1}^\rho}{\pi_{t+1}} \left(\frac{\tilde{p}_{Ht}}{\tilde{p}_{Ht+1}} \right)^{-\rho} X2_{Ht+1}$$

$$\frac{\rho^*}{\rho^* - 1} \mu_t^* X1_{Ft}^* = X2_{Ft}^*$$

$$\begin{aligned}
X1_{Ft}^* &= \tilde{p}_{Ft}^{*-1-\rho^*} Y_t^* \frac{w_t^*}{A_t^* \alpha N_t^{*\alpha-1}} + \theta^* \beta \mathbb{E}_t \frac{\zeta_{t+1}^{c^*}}{\zeta_t^{c^*}} \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{\pi_{Ft+1}^{*1+\rho^*}}{\pi_{t+1}^*} \left(\frac{\tilde{p}_{Ft}^*}{\tilde{p}_{Ft+1}^*} \right)^{-1-\rho^*} X1_{Ft+1}^* \\
X2_{Ft}^* &= \tilde{p}_{Ft}^{*- \rho^*} Y_t^* (1 - \tau_t^{v^*}) + \theta^* \beta \mathbb{E}_t \frac{\zeta_{t+1}^{c^*}}{\zeta_t^{c^*}} \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{\pi_{Ft+1}^{*\rho^*}}{\pi_{t+1}^*} \left(\frac{\tilde{p}_{Ft}^*}{\tilde{p}_{Ft+1}^*} \right)^{-\rho^*} X2_{Ft+1}^*
\end{aligned}$$

- Consumer price indices:

$$\begin{aligned}
1 &= \gamma_H p_{Ht}^{1-\xi} + \gamma_F \left(\frac{(1 - \tau_t^{v^*})}{(1 - \tau_t^v)} p_{Ft}^* q_t \right)^{1-\xi} \\
1 &= \gamma_H^* \left(\frac{(1 - \tau_t^v)}{(1 - \tau_t^{v^*})} p_{Ht} \right)^{1-\xi} + \gamma_F^* p_{Ft}^{*1-\xi}
\end{aligned}$$

- Evolution of PPIs:

$$\begin{aligned}
\tilde{p}_{Ht} &= \left(\frac{1 - \theta \pi_{Ht}^{\rho-1}}{1 - \theta} \right)^{\frac{1}{1-\rho}} \\
\tilde{p}_{Ft}^* &= \left(\frac{1 - \theta^* (\pi_{Ft}^*)^{\rho^*-1}}{1 - \theta^*} \right)^{\frac{1}{1-\rho^*}}
\end{aligned}$$

- Evolution of price dispersion:

$$\begin{aligned}
\Delta_{Ht} &= (1 - \theta) \tilde{p}_{Ht}^{-\rho} + \theta \pi_{Ht}^\rho \Delta_{Ht-1} \\
\Delta_{Ft}^* &= (1 - \theta^*) \tilde{p}_{Ft}^{*-\rho^*} + \theta^* (\pi_{Ft}^*)^{\rho^*} \Delta_{Ft-1}^*
\end{aligned}$$

- Evolution of relative prices:

$$\frac{p_{Ht}}{p_{Ht-1}} = \frac{\pi_{Ht}}{\pi_t}, \quad \frac{p_{Ft}^*}{p_{Ft-1}^*} = \frac{\pi_{Ft}^*}{\pi_t^*},$$

given the transversality conditions, sequences of the policy instruments $\{R_t, R_t^*, \tau_t^v, \tau_t^{v^*}\}_{t=0}^\infty$ and of the shocks $\{A_t, A_t^*, \mu_t, \mu_t^*, \zeta_t^c, \zeta_t^{c^*}, G_t, G_t^*\}_{t=0}^\infty$.

If the two countries form a monetary union, the equation defining R_t^* drops out. Instead, an expression that restricts the evolution of the real exchange rate needs to be added:

$$\frac{q_t}{q_{t-1}} = \frac{\pi_t^*}{\pi_t}.$$

B.2 Pricing-to-Market

Let $p_{Ft} = P_{Ft}/P_t$ and $p_{Ht}^* = P_{Ht}^*/P_t^*$ be the import-price-to-CPI ratios. A competitive equilibrium under PTM and autonomous monetary policy in both countries is a set of sequences $\{C_t, C_{Ht}, C_{Ft}, C_t^*, C_{Ht}^*, C_{Ft}^*, N_t, N_t^*, q_t, E_t, p_{Ht}, p_{Ft}, p_{Ht}^*, p_{Ft}^*, w_t, w_t^*, \pi_t, \pi_t^*,$

$\pi_{Ht}, \pi_{Ft}, \pi_{Ht}^*, \pi_{Ft}^*, \Delta_{Ht}, \Delta_{HF}, \Delta_{Ht}^*, \Delta_{Ft}^*, X1_{Ht}, X2_{Ht}, X1_{Ft}, X2_{Ft}, X1_{Ht}^*, X2_{Ht}^*, X1_{Ft}^*, X2_{Ft}^* \}_{t=0}^{\infty}$, satisfying

- Demand for Home and Foreign goods:

$$\begin{aligned} C_{Ht} &= \gamma_H p_{Ht}^{-\xi} C_t, & C_{Ft} &= \gamma_F p_{Ft}^{-\xi} C_t \\ C_{Ht}^* &= \gamma_H^* p_{Ht}^{*\xi} C_t^*, & C_{Ft}^* &= \gamma_F^* p_{Ft}^{*\xi} C_t^* \end{aligned}$$

- Euler equations and international risk sharing:

$$\begin{aligned} \frac{1}{R_t} &= \beta \mathbb{E}_t \left[\frac{\zeta_{t+1}^c}{\zeta_t^c} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{\pi_{t+1}} \right] \\ \frac{1}{R_t^*} &= \beta \mathbb{E}_t \left[\frac{\zeta_{t+1}^{c*}}{\zeta_t^{c*}} \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{1}{\pi_{t+1}^*} \right] \\ q_t &= \kappa \frac{\zeta_t^{c*}}{\zeta_t^c} \left(\frac{C_t^*}{C_t} \right)^{-\sigma} \end{aligned}$$

- Labour supply:

$$\begin{aligned} N_t^\eta C_t^\sigma &= \zeta_t^c w_t p_{Ht} \\ N_t^{*\eta} C_t^{*\sigma} &= \zeta_t^{c*} w_t^* p_{Ft}^* \end{aligned}$$

- Resource constraints:

$$\begin{aligned} A_t n N_t^\alpha &= \Delta_{Ht} (n C_{Ht} + G_t) + \Delta_{Ht}^* (1 - n) C_{Ht}^* \\ A_t^* (1 - n) N_t^{*\alpha} &= \Delta_{Ft}^* ((1 - n) C_{Ft}^* + G_t^*) + \Delta_{Ft} n C_{Ft} \end{aligned}$$

- Philips curves:

$$\begin{aligned} \frac{\rho}{\rho - 1} X1_{Ht} &= X2_{Ht} \\ X1_{Ht} &= \left(\frac{1 - \theta \pi_{Ht}^{\rho-1}}{1 - \theta} \right)^{\frac{\rho+1}{\rho-1}} (n C_{Ht} + G_t) \frac{w_t}{A_t \alpha N_t^{\alpha-1}} \\ &\quad + \theta \beta \mathbb{E}_t \frac{\zeta_{t+1}^c}{\zeta_t^c} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{\pi_{Ht+1}^{1+\rho}}{\pi_{t+1}} \left(\frac{1 - \theta \pi_{Ht}^{\rho-1}}{1 - \theta \pi_{Ht+1}^{\rho-1}} \right)^{\frac{\rho+1}{\rho-1}} X1_{Ht+1} \\ X2_{Ht} &= \left(\frac{1 - \theta \pi_{Ht}^{\rho-1}}{1 - \theta} \right)^{\frac{\rho}{\rho-1}} (n C_{Ht} + G_t) (1 - \tau_t^v) \\ &\quad + \theta \beta \mathbb{E}_t \frac{\zeta_{t+1}^c}{\zeta_t^c} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{\pi_{Ht+1}^\rho}{\pi_{t+1}} \left(\frac{1 - \theta \pi_{Ht}^{\rho-1}}{1 - \theta \pi_{Ht+1}^{\rho-1}} \right)^{\frac{\rho}{\rho-1}} X2_{Ht+1} \\ \frac{\rho}{\rho - 1} X1_{Ht}^* &= X2_{Ht}^* \end{aligned}$$

$$\begin{aligned}
X1_{Ht}^* &= \left(\frac{1 - \theta \pi_{Ht}^{*\rho-1}}{1 - \theta} \right)^{\frac{\rho+1}{\rho-1}} C_{Ht}^* E_t \frac{p_{Ht}}{q_t p_{Ht}^*} \frac{w_t}{A_t \alpha N_t^{\alpha-1}} \\
&\quad + \theta \beta \mathbb{E}_t \frac{\zeta_{t+1}^c}{\zeta_t^c} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{\pi_{Ht+1}^{*1+\rho}}{\pi_{t+1}} \left(\frac{1 - \theta \pi_{Ht}^{*\rho-1}}{1 - \theta \pi_{Ht+1}^{*\rho-1}} \right)^{\frac{\rho+1}{\rho-1}} X1_{Ht+1}^* \\
X2_{Ht}^* &= \left(\frac{1 - \theta \pi_{Ht}^{*\rho-1}}{1 - \theta} \right)^{\frac{\rho}{\rho-1}} C_{Ht}^* E_t (1 - \tau_t^{v*}) \\
&\quad + \theta \beta \mathbb{E}_t \frac{\zeta_{t+1}^c}{\zeta_t^c} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{\pi_{Ht+1}^{*\rho}}{\pi_{t+1}} \left(\frac{1 - \theta \pi_{Ht}^{*\rho-1}}{1 - \theta \pi_{Ht+1}^{*\rho-1}} \right)^{\frac{\rho}{\rho-1}} X2_{Ht+1}^* \\
&\quad \frac{\rho^*}{\rho^* - 1} X1_{Ft}^* = X2_{Ft}^*
\end{aligned}$$

$$\begin{aligned}
X1_{Ft}^* &= \left(\frac{1 - \theta^* \pi_{Ft}^{*\rho^*-1}}{1 - \theta^*} \right)^{\frac{\rho^*+1}{\rho^*-1}} ((1-n) C_{Ft}^* + G_t^*) \frac{w_t^*}{A_t^* \alpha N_t^{*\alpha-1}} \\
&\quad + \theta^* \beta \mathbb{E}_t \frac{\zeta_{t+1}^{c*}}{\zeta_t^{c*}} \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{\pi_{Ft+1}^{*1+\rho^*}}{\pi_{t+1}^*} \left(\frac{1 - \theta^* \pi_{Ft}^{*\rho^*-1}}{1 - \theta^* \pi_{Ft+1}^{*\rho^*-1}} \right)^{\frac{\rho^*+1}{\rho^*-1}} X1_{Ft+1}^* \\
X2_{Ft}^* &= \left(\frac{1 - \theta^* \pi_{Ft}^{*\rho^*-1}}{1 - \theta^*} \right)^{\frac{\rho^*}{\rho^*-1}} ((1-n) C_{Ft}^* + G_t^*) (1 - \tau_t^{v*}) \\
&\quad + \theta^* \beta \mathbb{E}_t \frac{\zeta_{t+1}^{c*}}{\zeta_t^{c*}} \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{\pi_{Ft+1}^{*\rho^*}}{\pi_{t+1}^*} \left(\frac{1 - \theta^* \pi_{Ft}^{*\rho^*-1}}{1 - \theta^* \pi_{Ft+1}^{*\rho^*-1}} \right)^{\frac{\rho^*}{\rho^*-1}} X2_{Ft+1}^* \\
&\quad \frac{\rho^*}{\rho^* - 1} X1_{Ft}^* = X2_{Ft}^*
\end{aligned}$$

$$\begin{aligned}
X1_{Ft} &= \left(\frac{1 - \theta^* \pi_{Ft}^{\rho^*-1}}{1 - \theta^*} \right)^{\frac{\rho^*+1}{\rho^*-1}} \frac{C_{Ft}}{E_t} \frac{q_t p_{Ft}^*}{p_{Ft}} \frac{w_t^*}{A_t^* \alpha N_t^{*\alpha-1}} \\
&\quad + \theta^* \beta \mathbb{E}_t \frac{\zeta_{t+1}^{c*}}{\zeta_t^{c*}} \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{\pi_{Ft+1}^{*1+\rho^*}}{\pi_{t+1}^*} \left(\frac{1 - \theta^* \pi_{Ft}^{\rho^*-1}}{1 - \theta^* \pi_{Ft+1}^{\rho^*-1}} \right)^{\frac{\rho^*+1}{\rho^*-1}} X1_{Ft+1} \\
X2_{Ft} &= \left(\frac{1 - \theta^* \pi_{Ft}^{\rho^*-1}}{1 - \theta^*} \right)^{\frac{\rho^*}{\rho^*-1}} \frac{C_{Ft}}{E_t} (1 - \tau_t^v) \\
&\quad + \theta^* \beta \mathbb{E}_t \frac{\zeta_{t+1}^{c*}}{\zeta_t^{c*}} \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{\pi_{Ft+1}^{\rho^*}}{\pi_{t+1}^*} \left(\frac{1 - \theta^* \pi_{Ft}^{\rho^*-1}}{1 - \theta^* \pi_{Ft+1}^{\rho^*-1}} \right)^{\frac{\rho^*}{\rho^*-1}} X2_{Ft+1}
\end{aligned}$$

- Consumer price indices:

$$\begin{aligned} 1 &= \gamma_H p_{Ht}^{1-\xi} + \gamma_F p_{Ft}^{1-\xi} \\ 1 &= \gamma_H^* p_{Ht}^{*1-\xi} + \gamma_F^* p_{Ft}^{*1-\xi} \end{aligned}$$

- Evolution of price dispersion:

$$\begin{aligned} \Delta_{Ht} &= (1 - \theta) \left(\frac{1 - \theta \pi_{Ht}^{\rho-1}}{1 - \theta} \right)^{\frac{\rho}{\rho-1}} + \theta \pi_{Ht}^\rho \Delta_{Ht-1} \\ \Delta_{Ht}^* &= (1 - \theta) \left(\frac{1 - \theta \pi_{Ht}^{*\rho-1}}{1 - \theta} \right)^{\frac{\rho}{\rho-1}} + \theta \pi_{Ht}^{*\rho} \Delta_{Ht-1}^* \\ \Delta_{Ft}^* &= (1 - \theta^*) \left(\frac{1 - \theta^* (\pi_{Ft}^*)^{\rho^*-1}}{1 - \theta^*} \right)^{\frac{\rho^*}{\rho^*-1}} + \theta^* (\pi_{Ft}^*)^{\rho^*} \Delta_{Ft-1}^* \\ \Delta_{Ft} &= (1 - \theta^*) \left(\frac{1 - \theta^* (\pi_{Ft})^{\rho^*-1}}{1 - \theta^*} \right)^{\frac{\rho^*}{\rho^*-1}} + \theta^* (\pi_{Ft})^{\rho^*} \Delta_{Ft-1} \end{aligned}$$

- Evolution of relative prices:

$$\frac{p_{Ht}}{p_{Ht-1}} = \frac{\pi_{Ht}}{\pi_t}, \quad \frac{p_{Ft}}{p_{Ft-1}} = \frac{\pi_{Ft}}{\pi_t}, \quad \frac{p_{Ht}^*}{p_{Ht-1}^*} = \frac{\pi_{Ht}^*}{\pi_t^*}, \quad \frac{p_{Ft}^*}{p_{Ft-1}^*} = \frac{\pi_{Ft}^*}{\pi_t^*}$$

- Evolution of the real exchange rate

$$\frac{q_t}{q_{t-1}} = \frac{E_t \pi_t^*}{E_{t-1} \pi_t}$$

given the transversality conditions, sequences of the policy instruments $\{R_t, R_t^*, \tau_t^v, \tau_t^{v*}\}_{t=0}^\infty$ and of the shocks $\{A_t, A_t^*, \mu_t, \mu_t^*, \zeta_t^c, \zeta_t^{c*}, G_t, G_t^*\}_{t=0}^\infty$, and an initial $E_{-1} = 1$. Unlike with the LOOP, the nominal exchange rate is itself a relevant argument to the equilibrium.

If the two countries form a monetary union, the equation defining R_t^* drops out, and the nominal exchange rate is fixed, i.e. $E_t = 1 \forall t$.

C The Ramsey Problem

C.1 Derivation of the Intertemporal Fiscal Budget Constraint

Integrating (29) over h and k , and dividing by P_t yields

$$b_t = \mathbb{E}_t Q_{t,t+1} \pi_{t+1} b_{t+1} + s_t, \tag{47}$$

where $b_t = B_t/P_t$ is real debt and the primary surplus reads

$$s_t = \frac{1}{P_t} [\tau_t^v n (P_{Ht} C_{Ht} + P_{Ft} C_{Ft}) - (1 - \tau_t^v) P_{Ht} G_t].$$

Repeatedly iterating on (47) using successive future terms of it, beginning in period $t = 0$, yields the present-value fiscal budget constraint

$$b_0 = \mathbb{E}_0 \sum_{t=0}^T Q_{0,t} \pi_{0,t} s_t + \mathbb{E}_0 Q_{0,T+1} \pi_{0,T+1} b_{T+1},$$

where $\pi_{0,T+1} = P_{T+1}/P_0$ is the product of inflation rates between $t = 0$ and $t = T + 1$. Imposing the transversality condition

$$\lim_{T \rightarrow \infty} \mathbb{E}_0 Q_{0,T+1} \pi_{0,T+1} b_{T+1} = 0$$

and using the definition of $Q_{0,t}$, one ends up with

$$\zeta_0^c C_0^{-\sigma} b_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \zeta_t^c C_t^{-\sigma} s_t.$$

C.2 The Lagrangian of the Ramsey Problem

The scenario under study assumes the law of one price, the availability of fiscal policy as an instrument, and that the countries form a monetary union. The objective of the policy planner is, hence, to find sequences $\{R_t^{MU}, \tau_t^v, \tau_t^{v*}\}_{t=0}^{\infty}$.

$$\begin{aligned} \mathbb{V} = & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ n \left(\zeta_t^c \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta} \right) + (1-n) \left(\zeta_t^{c*} \frac{C_t^{*1-\sigma}}{1-\sigma} - \frac{N_t^{*1+\eta}}{1+\eta} \right) \right. \\ & + \Lambda^H \zeta_t^c C_t^{-\sigma} \left[\tau_t^v n \left(p_{Ht} C_{Ht} + \frac{(1-\tau_t^{v*})}{(1-\tau_t^v)} p_{Ft}^* q_t C_{Ft} \right) - (1-\tau_t^v) p_{Ht} G_t \right] \\ & + \Lambda^F \zeta_t^{c*} C_t^{*-\sigma} \left[\tau_t^{v*} (1-n) \left[p_{Ft}^* C_{Ft}^* + \frac{(1-\tau_t^v)}{(1-\tau_t^{v*})} \frac{p_{Ht}}{q_t} C_{Ht}^* \right] - (1-\tau_t^{v*}) p_{Ft}^* G_t^* \right] \\ & + \lambda_t^1 \left[\gamma_H p_{Ht}^{-\xi} C_t - C_{Ht} \right] + \lambda_t^2 \left[\gamma_F \left(\frac{(1-\tau_t^{v*})}{(1-\tau_t^v)} p_{Ft}^* q_t \right)^{-\xi} C_t - C_{Ft} \right] \\ & + \lambda_t^3 \left[\gamma_H^* \left(\frac{(1-\tau_t^v)}{(1-\tau_t^{v*})} \frac{p_{Ht}}{q_t} \right)^{-\xi} C_t^* - C_{Ht}^* \right] + \lambda_t^4 \left[\gamma_F^* p_{Ft}^{*-\xi} C_t^* - C_{Ft}^* \right] \\ & + \lambda_t^5 [N_t^\eta C_t^\sigma - \zeta_t^c w_t p_{Ht}] + \lambda_t^6 [N_t^{*\eta} C_t^{*\sigma} - \zeta_t^{c*} w_t^* p_{Ft}^*] \\ & + \lambda_t^7 [A_t n N_t^\alpha - \Delta_{Ht} (n C_{Ht} + (1-n) C_{Ht}^* + G_t)] \\ & + \lambda_t^8 [A_t^* (1-n) N_t^{*\alpha} - \Delta_{Ft}^* (n C_{Ft} + (1-n) C_{Ft}^* + G_t^*)] \\ & + \lambda_t^9 \left[\frac{\rho}{\rho-1} \mu_t X_{1Ht} - X_{2Ht} \right] \\ & + \lambda_t^{10} \left[\tilde{p}_{Ht}^{1-\rho} (n C_{Ht} + (1-n) C_{Ht}^* + G_t) \frac{w_t}{\alpha A_t N_t^{\alpha-1}} \right] \end{aligned}$$

$$\begin{aligned}
& +\theta\beta\frac{\zeta_{t+1}^c}{\zeta_t^c}\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{\pi_{Ht+1}^{1+\rho}}{\pi_{t+1}}\left(\frac{\tilde{p}_{Ht}}{\tilde{p}_{Ht+1}}\right)^{-1-\rho}X1_{Ht+1}-X1_{Ht}\Big] \\
& +\lambda_t^{11}\left[\tilde{p}_{Ht}^\rho(nC_{Ht}+(1-n)C_{Ht}^*+G_t)(1-\tau_t^v)\right. \\
& \left. +\theta\beta\frac{\zeta_{t+1}^c}{\zeta_t^c}\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{\pi_{Ht+1}^\rho}{\pi_{t+1}}\left(\frac{\tilde{p}_{Ht}}{\tilde{p}_{Ht+1}}\right)^{-\rho}X2_{Ht+1}-X2_{Ht}\right] \\
& +\lambda_t^{12}\left[\frac{\rho^*}{\rho^*-1}\mu_t^*X1_{Ft}^*-X2_{Ft}^*\right] \\
& +\lambda_t^{13}\left[\tilde{p}_{Ft}^{*-1-\rho^*}(nC_{Ft}+(1-n)C_{Ft}^*+G_t^*)\frac{w_t^*}{\alpha A_t^*N_t^{*\alpha-1}}\right. \\
& \left. +\theta^*\beta\frac{\zeta_{t+1}^{c*}}{\zeta_t^{c*}}\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma}\frac{\pi_{Ft+1}^{*1+\rho^*}}{\pi_{t+1}^*}\left(\frac{\tilde{p}_{Ft}^*}{\tilde{p}_{Ft+1}^*}\right)^{-1-\rho^*}X1_{Ft+1}^*-X1_{Ft}^*\right] \\
& +\lambda_t^{14}\left[\tilde{p}_{Ft}^{*-\rho^*}(nC_{Ft}+(1-n)C_{Ft}^*+G_t^*)(1-\tau_t^{v*})\right. \\
& \left. +\theta^*\beta\frac{\zeta_{t+1}^{c*}}{\zeta_t^{c*}}\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma}\frac{\pi_{Ft+1}^{*\rho^*}}{\pi_{t+1}^{*\rho^*}}\left(\frac{\tilde{p}_{Ft}^*}{\tilde{p}_{Ft+1}^*}\right)^{-\rho^*}X2_{Ft+1}^*-X2_{Ft}^*\right] \\
& +\lambda_t^{15}\left[\gamma_H p_{Ht}^{1-\xi}+\gamma_F\left(\frac{(1-\tau_t^{v*})}{(1-\tau_t^v)}p_{Ft}^*q_t\right)^{1-\xi}-1\right]+\lambda_t^{16}\left[\gamma_H^*\left(\frac{(1-\tau_t^v)}{(1-\tau_t^{v*})}\frac{p_{Ht}}{q_t}\right)^{1-\xi}+\gamma_F^*p_{Ft}^{*1-\xi}-1\right] \\
& +\lambda_t^{17}\left[\left(\frac{1-\theta\pi_{Ht}^{\rho-1}}{1-\theta}\right)^{\frac{1}{1-\rho}}-\tilde{p}_{Ht}\right]+\lambda_t^{18}\left[\left(\frac{1-\theta^*\pi_{Ft}^{*\rho^*-1}}{1-\theta^*}\right)^{\frac{1}{1-\rho^*}}-\tilde{p}_{Ft}^*\right] \\
& +\lambda_t^{19}\left[\kappa\frac{\zeta_t^{c*}}{\zeta_t^c}\left(\frac{C_t^*}{C_t}\right)^{-\sigma}-q_t\right] \\
& +\lambda_t^{20}\left[(1-\theta)\tilde{p}_{Ht}^{-\rho}+\theta\pi_{Ht}^\rho\Delta_{Ht-1}-\Delta_{Ht}\right]+\lambda_t^{21}\left[(1-\theta^*)\tilde{p}_{Ft}^{*-\rho^*}+\theta^*\pi_{Ft}^{*\rho^*}\Delta_{Ft-1}^*-\Delta_{Ft}^*\right] \\
& +\lambda_t^{22}\left[\frac{p_{Ht}}{p_{Ht-1}}-\frac{\pi_{Ht}}{\pi_t}\right]+\lambda_t^{23}\left[\frac{p_{Ft}^*}{p_{Ft-1}^*}-\frac{\pi_{Ft}^*}{\pi_t^*}\right]+\lambda_t^{24}\left[\frac{q_t}{q_{t-1}}-\frac{\pi_t^*}{\pi_t}\right]\Big\} \\
& -\Lambda^H\zeta_0^cC_0^{-\sigma}b_0-\Lambda^F\zeta_0^{c*}C_0^{*-\sigma}b_0^*.
\end{aligned}$$

C.3 First-order Conditions for $t \geq 1$

The solution to the optimal policy problem can be described by the first-order conditions with respect to all Lagrange multipliers and with respect to all endogenous variables of the model:

- W.r.t. C_t :

$$\begin{aligned}
0 & = n_H\zeta_t^cC_t^{-\sigma}-\Lambda^H\zeta_t^c\sigma C_t^{-\sigma-1}s_t+\lambda_t^1\gamma_H p_{Ht}^{-\xi}+\lambda_t^2\gamma_F\left(\frac{(1-\tau_t^{v*})}{(1-\tau_t^v)}p_{Ft}^*q_t\right)^{-\xi}+\lambda_t^5N_t^\eta\sigma C_t^{\sigma-1} \\
& +\frac{\sigma}{C_t}\left[\lambda_t^{10}\theta\beta\frac{\zeta_{t+1}^c}{\zeta_t^c}\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{\pi_{Ht+1}^{1+\rho}}{\pi_{t+1}}\left(\frac{\tilde{p}_{Ht}}{\tilde{p}_{Ht+1}}\right)^{-1-\rho}X1_{Ht+1}\right.
\end{aligned}$$

$$\begin{aligned}
& - \lambda_{t-1}^{10} \theta \frac{\zeta_t^c}{\zeta_{t-1}^c} \left(\frac{C_t}{C_{t-1}} \right)^{-\sigma} \frac{\pi_{Ht}^{1+\rho}}{\pi_t} \left(\frac{\tilde{p}_{Ht-1}}{\tilde{p}_{Ht}} \right)^{-1-\rho} X1_{Ht} \Big] \\
& + \frac{\sigma}{C_t} \left[\lambda_t^{11} \theta \beta \frac{\zeta_{t+1}^c}{\zeta_t^c} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{\pi_{Ht+1}^\rho}{\pi_{t+1}} \left(\frac{\tilde{p}_{Ht}}{\tilde{p}_{Ht+1}} \right)^{-\rho} X2_{Ht+1} \right. \\
& \left. - \lambda_{t-1}^{11} \theta \frac{\zeta_t^c}{\zeta_{t-1}^c} \left(\frac{C_t}{C_{t-1}} \right)^{-\sigma} \frac{\pi_{Ht}^\rho}{\pi_t} \left(\frac{\tilde{p}_{Ht-1}}{\tilde{p}_{Ht}} \right)^{-\rho} X2_{Ht} \right] + \lambda_t^{19} \kappa \frac{\sigma}{C_t} \frac{\zeta_t^{c*}}{\zeta_t^c} \left(\frac{C_t^*}{C_t} \right)^{-\sigma}
\end{aligned}$$

- W.r.t. C_{Ht} :

$$0 = \Lambda^H \zeta_t^c C_t^{-\sigma} \tau_t^v p_{Ht} n_H - \lambda_t^1 - \lambda_t^7 \Delta_{Ht} n_H + \lambda_t^{10} \tilde{p}_{Ht}^{-1-\rho} \frac{n_H w_t}{\alpha A_t N_t^{\alpha-1}} + \lambda_t^{11} \tilde{p}_{Ht}^{-\rho} n_H (1 - \tau_t^v)$$

- W.r.t. C_{Ft} :

$$0 = \Lambda^H \zeta_t^c C_t^{-\sigma} \tau_t^v \frac{(1 - \tau_t^{v*})}{(1 - \tau_t^v)} p_{Ft}^* q_t n_H - \lambda_t^2 - \lambda_t^8 \Delta_{Ft}^* n_H + \lambda_t^{13} \tilde{p}_{Ft}^{*-1-\rho^*} \frac{n_H w_t^*}{\alpha A_t^* N_t^{*\alpha-1}} + \lambda_t^{14} \tilde{p}_{Ft}^{*- \rho^*} n_H (1 - \tau_t^{v*})$$

- W.r.t. C_t^* :

$$\begin{aligned}
0 & = n_F \zeta_t^{c*} C_t^{*\sigma} - \Lambda^F \zeta_t^{c*} \sigma C_t^{*\sigma-1} s_t^* + \lambda_t^3 \gamma_H^* \left(\frac{(1 - \tau_t^v) p_{Ht}}{(1 - \tau_t^{v*}) q_t} \right)^{-\xi} + \lambda_t^4 \gamma_F^* p_{Ft}^{*\xi} + \lambda_t^6 N_t^{*\eta} \sigma C_t^{*\sigma-1} \\
& + \frac{\sigma}{C_t^*} \left[\lambda_t^{13} \theta^* \beta \frac{\zeta_{t+1}^{c*}}{\zeta_t^{c*}} \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{\pi_{Ft+1}^{*1+\rho^*}}{\pi_{t+1}^*} \left(\frac{\tilde{p}_{Ft}^*}{\tilde{p}_{Ft+1}^*} \right)^{-1-\rho^*} X1_{Ft+1}^* \right. \\
& \left. - \lambda_{t-1}^{13} \theta^* \frac{\zeta_t^{c*}}{\zeta_{t-1}^{c*}} \left(\frac{C_t^*}{C_{t-1}^*} \right)^{-\sigma} \frac{\pi_{Ft}^{*1+\rho^*}}{\pi_t^*} \left(\frac{\tilde{p}_{Ft-1}^*}{\tilde{p}_{Ft}^*} \right)^{-1-\rho^*} X1_{Ft}^* \right] \\
& + \frac{\sigma}{C_t^*} \left[\lambda_t^{14} \theta^* \beta \frac{\zeta_{t+1}^{c*}}{\zeta_t^{c*}} \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{\pi_{Ft+1}^{*\rho^*}}{\pi_{t+1}^*} \left(\frac{\tilde{p}_{Ft}^*}{\tilde{p}_{Ft+1}^*} \right)^{-\rho^*} X2_{Ft+1}^* \right. \\
& \left. - \lambda_{t-1}^{14} \theta^* \frac{\zeta_t^{c*}}{\zeta_{t-1}^{c*}} \left(\frac{C_t^*}{C_{t-1}^*} \right)^{-\sigma} \frac{\pi_{Ft}^{*\rho^*}}{\pi_t^*} \left(\frac{\tilde{p}_{Ft-1}^*}{\tilde{p}_{Ft}^*} \right)^{-\rho^*} X2_{Ft}^* \right] - \lambda_t^{19} \kappa \frac{\sigma}{C_t^*} \frac{\zeta_t^{c*}}{\zeta_t^c} \left(\frac{C_t^*}{C_t} \right)^{-\sigma}
\end{aligned}$$

- W.r.t. C_{Ht}^* :

$$0 = \Lambda^F \zeta_t^{c*} C_t^{*\sigma} \tau_t^{v*} \frac{(1 - \tau_t^v) p_{Ht}}{(1 - \tau_t^{v*}) q_t} n_F - \lambda_t^3 - \lambda_t^7 \Delta_{Ht} n_F + \lambda_t^{10} \tilde{p}_{Ht}^{-1-\rho} \frac{n_F w_t}{\alpha A_t N_t^{\alpha-1}} + \lambda_t^{11} \tilde{p}_{Ht}^{-\rho} n_F (1 - \tau_t^v)$$

- W.r.t. C_{Ft}^* :

$$0 = \Lambda^F \zeta_t^{c*} C_t^{*\sigma} \tau_t^{v*} p_{Ft}^* n_F - \lambda_t^4 - \lambda_t^8 \Delta_{Ft}^* n_F + \lambda_t^{13} \tilde{p}_{Ft}^{*-1-\rho^*} \frac{n_F w_t^*}{\alpha A_t^* N_t^{*\alpha-1}} + \lambda_t^{14} \tilde{p}_{Ft}^{*- \rho^*} n_F (1 - \tau_t^{v*})$$

- W.r.t. p_{Ht} :

$$\begin{aligned}
0 &= \Lambda^H \zeta_t^c C_t^{-\sigma} (\tau_t^v n_H C_{Ht} - (1 - \tau_t^v) G_t) + \Lambda^F \zeta_t^{c*} C_t^{*\sigma} \tau_t^{v*} \frac{(1 - \tau_t^v)}{(1 - \tau_t^{v*})} \frac{n_F C_{Ht}^*}{q_t} \\
&\quad - \lambda_t^1 \gamma_H \xi p_{Ht}^{-\xi-1} C_t - \lambda_t^3 \gamma_H^* \left(\frac{(1 - \tau_t^v)}{(1 - \tau_t^{v*})} \frac{p_{Ht}}{q_t} \right)^{-\xi} \frac{\xi C_t^*}{p_{Ht}} - \lambda_t^5 \zeta_t^c w_t \\
&\quad + \lambda_t^{15} \gamma_H (1 - \xi) p_{Ht}^{-\xi} + \lambda_t^{16} \gamma_H^* \left(\frac{(1 - \tau_t^v)}{(1 - \tau_t^{v*})} \frac{p_{Ht}}{q_t} \right)^{1-\xi} \frac{(1 - \xi)}{p_{Ht}} + \frac{\lambda_t^{22}}{p_{Ht-1}} - \lambda_{t+1}^{22} \beta \frac{p_{Ht+1}}{p_{Ht}^2}
\end{aligned}$$

- W.r.t. p_{Ft}^* :

$$\begin{aligned}
0 &= \Lambda^H \zeta_t^c C_t^{-\sigma} \tau_t^v \frac{(1 - \tau_t^{v*})}{(1 - \tau_t^v)} q_t n_H C_{Ft} + \Lambda^F \zeta_t^{c*} C_t^{*\sigma} (\tau_t^{v*} n_F C_{Ft}^* - (1 - \tau_t^{v*}) G_t^*) \\
&\quad - \lambda_t^2 \gamma_F \left(\frac{(1 - \tau_t^{v*})}{(1 - \tau_t^v)} p_{Ft}^* q_t \right)^{-\xi} \frac{\xi C_t}{p_{Ft}^*} - \lambda_t^4 \gamma_F^* \xi p_{Ft}^{*\xi-1} C_t^* - \lambda_t^6 \zeta_t^{c*} w_t^* \\
&\quad + \lambda_t^{15} \gamma_F \left(\frac{(1 - \tau_t^{v*})}{(1 - \tau_t^v)} p_{Ft}^* q_t \right)^{1-\xi} \frac{(1 - \xi)}{p_{Ft}^*} + \lambda_t^{16} \gamma_F^* (1 - \xi) p_{Ft}^{*\xi} + \frac{\lambda_t^{23}}{p_{Ft-1}^*} - \lambda_{t+1}^{23} \beta \frac{p_{Ft+1}^*}{p_{Ft}^{*2}}
\end{aligned}$$

- W.r.t. q_t :

$$\begin{aligned}
0 &= \Lambda^H \zeta_t^c C_t^{-\sigma} \tau_t^v \frac{(1 - \tau_t^{v*})}{(1 - \tau_t^v)} p_{Ft}^* n_H C_{Ft} - \Lambda^F \zeta_t^{c*} C_t^{*\sigma} \tau_t^{v*} \frac{(1 - \tau_t^v)}{(1 - \tau_t^{v*})} \frac{p_{Ht}}{q_t} n_F C_{Ht}^* \\
&\quad - \lambda_t^2 \gamma_F \left(\frac{(1 - \tau_t^{v*})}{(1 - \tau_t^v)} p_{Ft}^* q_t \right)^{-\xi} \frac{\xi C_t}{q_t} + \lambda_t^3 \gamma_H^* \left(\frac{(1 - \tau_t^v)}{(1 - \tau_t^{v*})} \frac{p_{Ht}}{q_t} \right)^{-\xi} \frac{\xi C_t^*}{q_t} \\
&\quad + \lambda_t^{15} \gamma_F \left(\frac{(1 - \tau_t^{v*})}{(1 - \tau_t^v)} p_{Ft}^* q_t \right)^{1-\xi} \frac{(1 - \xi)}{q_t} - \lambda_t^{16} \gamma_H^* \left(\frac{(1 - \tau_t^v)}{(1 - \tau_t^{v*})} \frac{p_{Ht}}{q_t} \right)^{1-\xi} \frac{(1 - \xi)}{q_t} \\
&\quad - \lambda_t^{19} + \frac{\lambda_t^{24}}{q_{t-1}} - \lambda_{t+1}^{24} \beta \frac{q_{t+1}}{q_t^2}
\end{aligned}$$

- W.r.t. π_t :

$$\begin{aligned}
0 &= -\lambda_{t-1}^{10} \theta \frac{\zeta_t^c}{\zeta_{t-1}^c} \left(\frac{C_t}{C_{t-1}} \right)^{-\sigma} \frac{\pi_{Ht}^{1+\rho}}{\pi_t^2} \left(\frac{\tilde{p}_{Ht-1}}{\tilde{p}_{Ht}} \right)^{-1-\rho} X1_{Ht} \\
&\quad - \lambda_{t-1}^{11} \theta \frac{\zeta_t^c}{\zeta_{t-1}^c} \left(\frac{C_t}{C_{t-1}} \right)^{-\sigma} \frac{\pi_{Ht}^\rho}{\pi_t^2} \left(\frac{\tilde{p}_{Ht-1}}{\tilde{p}_{Ht}} \right)^{-\rho} X2_{Ht} + \lambda_t^{22} \frac{\pi_{Ht}}{\pi_t^2} + \lambda_t^{24} \frac{\pi_t^*}{\pi_t^2}
\end{aligned}$$

- W.r.t. π_t^* :

$$\begin{aligned}
0 &= -\lambda_{t-1}^{13} \theta^* \frac{\zeta_t^*}{\zeta_{t-1}^*} \left(\frac{C_t^*}{C_{t-1}^*} \right)^{-\sigma} \frac{\pi_{Ft}^{*1+\rho^*}}{\pi_t^{*2}} \left(\frac{\tilde{p}_{Ft-1}^*}{\tilde{p}_{Ft}^*} \right)^{-1-\rho^*} X1_{Ft}^* \\
&\quad - \lambda_{t-1}^{14} \theta^* \frac{\zeta_t^*}{\zeta_{t-1}^*} \left(\frac{C_t^*}{C_{t-1}^*} \right)^{-\sigma} \frac{\pi_{Ft}^{*\rho^*}}{\pi_t^{*2}} \left(\frac{\tilde{p}_{Ft-1}^*}{\tilde{p}_{Ft}^*} \right)^{-\rho^*} X2_{Ft}^* + \lambda_t^{23} \frac{\pi_{Ft}^*}{\pi_t^{*2}} - \frac{\lambda_t^{24}}{\pi_t}
\end{aligned}$$

- W.r.t. π_{Ht} :

$$\begin{aligned}
0 &= \lambda_{t-1}^{10} \theta \frac{\zeta_t^c}{\zeta_{t-1}^c} \left(\frac{C_t}{C_{t-1}} \right)^{-\sigma} \frac{(1+\rho)\pi_{Ht}^\rho}{\pi_t} \left(\frac{\tilde{p}_{Ht-1}}{\tilde{p}_{Ht}} \right)^{-1-\rho} X1_{Ht} \\
&+ \lambda_{t-1}^{11} \theta \frac{\zeta_t^c}{\zeta_{t-1}^c} \left(\frac{C_t}{C_{t-1}} \right)^{-\sigma} \frac{\rho\pi_{Ht}^{\rho-1}}{\pi_t} \left(\frac{\tilde{p}_{Ht-1}}{\tilde{p}_{Ht}} \right)^{-\rho} X2_{Ht} \\
&+ \lambda_t^{17} \left(\frac{1-\theta\pi_{Ht}^{\rho-1}}{1-\theta} \right)^{\frac{\rho}{1-\rho}} \frac{\theta}{1-\theta} \pi_{Ht}^{\rho-2} + \lambda_t^{20} \theta \rho \pi_{Ht}^{\rho-1} \Delta_{Ht-1} - \frac{\lambda_t^{22}}{\pi_t}
\end{aligned}$$

- W.r.t. π_{Ft}^* :

$$\begin{aligned}
0 &= \lambda_{t-1}^{13} \theta^* \frac{\zeta_t^{c*}}{\zeta_{t-1}^{c*}} \left(\frac{C_t^*}{C_{t-1}^*} \right)^{-\sigma} \frac{(1+\rho^*)\pi_{Ft}^{*\rho^*}}{\pi_t^*} \left(\frac{\tilde{p}_{Ft-1}^*}{\tilde{p}_{Ft}^*} \right)^{-1-\rho^*} X1_{Ft}^* \\
&+ \lambda_{t-1}^{14} \theta^* \frac{\zeta_t^{c*}}{\zeta_{t-1}^{c*}} \left(\frac{C_t^*}{C_{t-1}^*} \right)^{-\sigma} \frac{\rho^*\pi_{Ft}^{*\rho^*-1}}{\pi_t^*} \left(\frac{\tilde{p}_{Ft-1}^*}{\tilde{p}_{Ft}^*} \right)^{-\rho^*} X2_{Ft}^* \\
&+ \lambda_t^{18} \left(\frac{1-\theta^*\pi_{Ft}^{*\rho^*-1}}{1-\theta^*} \right)^{\frac{\rho^*}{1-\rho^*}} \frac{\theta^*}{1-\theta^*} \pi_{Ft}^{*\rho^*-2} + \lambda_t^{21} \theta^* \rho^* \pi_{Ft}^{*\rho^*-1} \Delta_{Ft-1}^* - \frac{\lambda_t^{23}}{\pi_t^*}
\end{aligned}$$

- W.r.t. Δ_{Ht} :

$$0 = -\lambda_t^7 (n_H C_{Ht} + n_F C_{Ht}^* + G_t) - \lambda_t^{20} + \lambda_{t+1}^{20} \beta \theta \pi_{Ht+1}^\rho$$

- W.r.t. Δ_{Ft}^* :

$$0 = -\lambda_t^8 (n_H C_{Ft} + n_F C_{Ft}^* + G_t^*) - \lambda_t^{21} + \lambda_{t+1}^{21} \beta \theta^* \pi_{Ft+1}^{*\rho^*}$$

- W.r.t. \tilde{p}_{Ht} :

$$\begin{aligned}
0 &= -\lambda_t^{10} \frac{1+\rho}{\tilde{p}_{Ht}} \left[\tilde{p}_{Ht}^{-1-\rho} (n_H C_{Ht} + n_F C_{Ht}^* + G_t) \frac{w_t}{\alpha A_t N_t^{\alpha-1}} \right. \\
&+ \left. \theta \beta \frac{\zeta_{t+1}^c}{\zeta_t^c} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{\pi_{Ht+1}^{1+\rho}}{\pi_{t+1}} \left(\frac{\tilde{p}_{Ht}}{\tilde{p}_{Ht+1}} \right)^{-1-\rho} X1_{Ht+1} \right] \\
&+ \lambda_{t-1}^{10} \frac{1+\rho}{\tilde{p}_{Ht}} \theta \frac{\zeta_t^c}{\zeta_{t-1}^c} \left(\frac{C_t}{C_{t-1}} \right)^{-\sigma} \frac{\pi_{Ht}^{1+\rho}}{\pi_t} \left(\frac{\tilde{p}_{Ht-1}}{\tilde{p}_{Ht}} \right)^{-1-\rho} X1_{Ht} \\
&- \lambda_t^{11} \frac{\rho}{\tilde{p}_{Ht}} \left[\tilde{p}_{Ht}^{-\rho} (n_H C_{Ht} + n_F C_{Ht}^* + G_t) (1-\tau_t^v) \right. \\
&+ \left. \theta \beta \frac{\zeta_{t+1}^c}{\zeta_t^c} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{\pi_{Ht+1}^\rho}{\pi_{t+1}} \left(\frac{\tilde{p}_{Ht}}{\tilde{p}_{Ht+1}} \right)^{-\rho} X2_{Ht+1} \right] \\
&+ \lambda_{t-1}^{11} \frac{\rho}{\tilde{p}_{Ht}} \theta \frac{\zeta_t^c}{\zeta_{t-1}^c} \left(\frac{C_t}{C_{t-1}} \right)^{-\sigma} \frac{\pi_{Ht}^\rho}{\pi_t} \left(\frac{\tilde{p}_{Ht-1}}{\tilde{p}_{Ht}} \right)^{-\rho} X2_{Ht} - \lambda_t^{17} - \lambda_t^{20} (1-\theta) \rho \tilde{p}_{Ht}^{-\rho-1}
\end{aligned}$$

- W.r.t. \tilde{p}_{Ft}^* :

$$\begin{aligned}
0 = & -\lambda_t^{13} \frac{1 + \rho^*}{\tilde{p}_{Ft}^*} \left[\tilde{p}_{Ft}^{*-1-\rho^*} (n_H C_{Ft} + n_F C_{Ft}^* + G_t^*) \frac{w_t^*}{\alpha A_t^* N_t^{*\alpha-1}} \right. \\
& \left. + \theta^* \beta \frac{\zeta_{t+1}^{c^*}}{\zeta_t^{c^*}} \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{\pi_{Ft+1}^{*1+\rho^*}}{\pi_{t+1}^*} \left(\frac{\tilde{p}_{Ft}^*}{\tilde{p}_{Ft+1}^*} \right)^{-1-\rho^*} X1_{Ft+1}^* \right] \\
& + \lambda_{t-1}^{13} \frac{1 + \rho^*}{\tilde{p}_{Ft}^*} \theta^* \frac{\zeta_t^{c^*}}{\zeta_{t-1}^{c^*}} \left(\frac{C_t^*}{C_{t-1}^*} \right)^{-\sigma} \frac{\pi_{Ft}^{*1+\rho^*}}{\pi_t^*} \left(\frac{\tilde{p}_{Ft-1}^*}{\tilde{p}_{Ft}^*} \right)^{-1-\rho^*} X1_{Ft}^* \\
& - \lambda_t^{14} \frac{\rho^*}{\tilde{p}_{Ft}^*} \left[\tilde{p}_{Ft}^{*- \rho^*} (n_H C_{Ft} + n_F C_{Ft}^* + G_t^*) (1 - \tau_t^{v^*}) \right. \\
& \left. + \theta^* \beta \frac{\zeta_{t+1}^{c^*}}{\zeta_t^{c^*}} \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{\pi_{Ft+1}^{*\rho^*}}{\pi_{t+1}^*} \left(\frac{\tilde{p}_{Ft}^*}{\tilde{p}_{Ft+1}^*} \right)^{-\rho^*} X2_{Ft+1}^* \right] \\
& + \lambda_{t-1}^{14} \frac{\rho^*}{\tilde{p}_{Ft}^*} \theta^* \frac{\zeta_t^{c^*}}{\zeta_{t-1}^{c^*}} \left(\frac{C_t^*}{C_{t-1}^*} \right)^{-\sigma} \frac{\pi_{Ft}^{*\rho^*}}{\pi_t^*} \left(\frac{\tilde{p}_{Ft-1}^*}{\tilde{p}_{Ft}^*} \right)^{-\rho^*} X2_{Ft}^* - \lambda_t^{18} - \lambda_t^{21} (1 - \theta^*) \rho^* \tilde{p}_{Ft}^{*- \rho^* - 1}
\end{aligned}$$

- W.r.t. $X1_{Ht}$:

$$0 = \lambda_t^9 \frac{\rho}{\rho - 1} \mu_t - \lambda_t^{10} + \lambda_{t-1}^{10} \theta \frac{\zeta_t^c}{\zeta_{t-1}^c} \left(\frac{C_t}{C_{t-1}} \right)^{-\sigma} \frac{\pi_{Ht}^{1+\rho}}{\pi_t} \left(\frac{\tilde{p}_{Ht-1}}{\tilde{p}_{Ht}} \right)^{-1-\rho}$$

- W.r.t. $X2_{Ht}$:

$$0 = -\lambda_t^9 - \lambda_t^{11} + \lambda_{t-1}^{11} \theta \frac{\zeta_t^c}{\zeta_{t-1}^c} \left(\frac{C_t}{C_{t-1}} \right)^{-\sigma} \frac{\pi_{Ht}^\rho}{\pi_t} \left(\frac{\tilde{p}_{Ht-1}}{\tilde{p}_{Ht}} \right)^{-\rho}$$

- W.r.t. $X1_{Ft}^*$:

$$0 = \lambda_t^{12} \frac{\rho^*}{\rho^* - 1} \mu_t^* - \lambda_t^{13} + \lambda_{t-1}^{13} \theta^* \frac{\zeta_t^{c^*}}{\zeta_{t-1}^{c^*}} \left(\frac{C_t^*}{C_{t-1}^*} \right)^{-\sigma} \frac{\pi_{Ft}^{*1+\rho^*}}{\pi_t^*} \left(\frac{\tilde{p}_{Ft-1}^*}{\tilde{p}_{Ft}^*} \right)^{-1-\rho^*}$$

- W.r.t. $X2_{Ft}^*$:

$$0 = -\lambda_t^{12} - \lambda_t^{14} + \lambda_{t-1}^{14} \theta^* \frac{\zeta_t^{c^*}}{\zeta_{t-1}^{c^*}} \left(\frac{C_t^*}{C_{t-1}^*} \right)^{-\sigma} \frac{\pi_{Ft}^{*\rho^*}}{\pi_t^*} \left(\frac{\tilde{p}_{Ft-1}^*}{\tilde{p}_{Ft}^*} \right)^{-\rho^*}$$

- W.r.t. N_t :

$$0 = -n_H N_t^\eta + \lambda_t^5 \eta N_t^{\eta-1} C_t^\sigma + \lambda_t^7 A_t n_H \alpha N_t^{\alpha-1} + \lambda_t^{10} \tilde{p}_{Ht}^{-1-\rho} (n_H C_{Ht} + n_F C_{Ht}^* + G_t) \frac{(1 - \alpha)}{\alpha} \frac{w_t}{A_t N_t^\alpha}$$

- W.r.t. N_t^* :

$$0 = -n_F N_t^{*\eta} + \lambda_t^6 \eta N_t^{*\eta-1} C_t^{*\sigma} + \lambda_t^8 A_t^* n_F \alpha N_t^{*\alpha-1}$$

$$+\lambda_t^{13}\tilde{p}_{Ft}^{*-1-\rho^*} (n_H C_{Ft} + n_F C_{Ft}^* + G_t^*) \frac{(1-\alpha)}{\alpha} \frac{w_t^*}{A_t^* N_t^{*\alpha}}$$

- W.r.t. w_t :

$$0 = -\lambda_t^5 \zeta_t^c p_{Ht} + \lambda_t^{10} \tilde{p}_{Ht}^{*-1-\rho} (n_H C_{Ht} + n_F C_{Ht}^* + G_t) \frac{N_t^{1-\alpha}}{\alpha A_t}$$

- W.r.t. w_t^* :

$$0 = -\lambda_t^6 \zeta_t^{c^*} p_{Ft}^* + \lambda_t^{13} \tilde{p}_{Ft}^{*-1-\rho^*} (n_H C_{Ft} + n_F C_{Ft}^* + G_t^*) \frac{N_t^{*1-\alpha}}{\alpha A_t^*}$$

- W.r.t. τ_t^v :

$$\begin{aligned} 0 = & \Lambda^H \zeta_t^c C_t^{-\sigma} \left(p_{Ht} (n_H C_{Ht} + G_t) + \frac{(1-\tau_t^{v*})}{(1-\tau_t^v)^2} p_{Ft}^* q_t n_H C_{Ft} \right) - \frac{\Lambda^F \zeta_t^{c^*} C_t^{*-\sigma} \tau_t^{v*}}{(1-\tau_t^{v*})} \frac{p_{Ht}}{q_t} n_F C_{Ht}^* \\ & - \lambda_t^2 \gamma_F \left(\frac{(1-\tau_t^{v*})}{(1-\tau_t^v)} p_{Ft}^* q_t \right)^{-\xi} \frac{\xi C_t}{(1-\tau_t^v)} + \lambda_t^3 \gamma_H^* \left(\frac{(1-\tau_t^v)}{(1-\tau_t^{v*})} \frac{p_{Ht}}{q_t} \right)^{-\xi} \frac{\xi C_t^*}{(1-\tau_t^v)} \\ & - \lambda_t^{11} \tilde{p}_{Ht}^{-\rho} (n_H C_{Ht} + n_F C_{Ht}^* + G_t) \\ & + \frac{(1-\xi)}{(1-\tau_t^v)} \left[\lambda_t^{15} \gamma_F \left(\frac{(1-\tau_t^{v*})}{(1-\tau_t^v)} p_{Ft}^* q_t \right)^{1-\xi} - \lambda_t^{16} \gamma_H^* \left(\frac{(1-\tau_t^v)}{(1-\tau_t^{v*})} \frac{p_{Ht}}{q_t} \right)^{1-\xi} \right] \end{aligned}$$

- W.r.t. τ_t^{v*} :

$$\begin{aligned} 0 = & -\Lambda^H \zeta_t^c C_t^{-\sigma} \tau_t^v \frac{p_{Ft}^* q_t n_H C_{Ft}}{(1-\tau_t^v)} + \Lambda^F \zeta_t^{c^*} C_t^{*-\sigma} \left(p_{Ft}^* (n_F C_{Ft}^* + G_t^*) + \frac{(1-\tau_t^v)}{(1-\tau_t^{v*})^2} \frac{p_{Ht}}{q_t} n_F C_{Ht}^* \right) \\ & + \lambda_t^2 \gamma_F \left(\frac{(1-\tau_t^{v*})}{(1-\tau_t^v)} p_{Ft}^* q_t \right)^{-\xi} \frac{\xi C_t}{(1-\tau_t^{v*})} - \lambda_t^3 \gamma_H^* \left(\frac{(1-\tau_t^v)}{(1-\tau_t^{v*})} \frac{p_{Ht}}{q_t} \right)^{-\xi} \frac{\xi C_t^*}{(1-\tau_t^{v*})} \\ & - \lambda_t^{14} \tilde{p}_{Ft}^{*-1-\rho^*} (n_H C_{Ft} + n_F C_{Ft}^* + G_t^*) \\ & - \frac{(1-\xi)}{(1-\tau_t^{v*})} \left[\lambda_t^{15} \gamma_F \left(\frac{(1-\tau_t^{v*})}{(1-\tau_t^v)} p_{Ft}^* q_t \right)^{1-\xi} - \lambda_t^{16} \gamma_H^* \left(\frac{(1-\tau_t^v)}{(1-\tau_t^{v*})} \frac{p_{Ht}}{q_t} \right)^{1-\xi} \right] \end{aligned}$$

D Data Sources and Calibration Targets

All data is taken from Eurostat (<http://ec.europa.eu/eurostat/data/database>). Population shares of the core and periphery are calculated using time averages of total population between 2001-2014 (variable name in source: [demo_pjan]). Government debt over GDP in steady state (G/Y) is calculated as time average of general government consolidated gross debt as percentage of GDP using annual data between 2010-2014 (variable name in source: [gov_10dd_edpt1]). Government spending and trade balance relative to GDP in steady state are constructed analogously as time averages on quarterly data between 2001:1 and 2014:4 (variables in source from category: [namq_10_gdp]).

Table 6: EMPIRICAL AND THEORETICAL SECOND MOMENTS

	GDP	Cons.	Gov.	Wage
Target Moments				
CORE:				
Autocorrelation:	0.87	0.81	0.77	
Std. Dev. (in p.p.):	1.67	0.96	1.07	0.99
PERIPHERY:				
Autocorrelation:	0.82	0.88	0.64	
Std. Dev. (in p.p.):	2.06	1.78	2.42	1.78
Model-Generated Moments				
CORE:				
Autocorrelation:	0.87	0.81	0.77	
Std. Dev. (in p.p.):	0.88	1.17	1.12	1.39
PERIPHERY:				
Autocorrelation:	0.82	0.88	0.64	
Std. Dev. (in p.p.):	1.11	2.11	2.54	1.96

Note: Empirical target moments (upper panel) calculated using quarterly data from Eurostat for the period 2001:1 to 2014:4. All series in logs, seasonally adjusted and quadratically detrended. Theoretical moments (lower panel) from calibrated model (see Tables 1 and 2). Available policy instruments: monetary policy at union level only.

The data series used to construct the calibration targets for GDP, consumption, and government spending also stem from [namq_10_gdp]. The variable names are "Gross domestic product at market prices", "Final consumption expenditure of households", and "Final consumption expenditure of general government". The raw series are not seasonally adjusted and measured in current prices. Data on aggregate wages are proxied by the labour cost index (LCI) for the business economy sector (variable name in source: [lc_lci_r2_q]), which provides observations for all required countries but France. The index is given at a quarterly frequency, not seasonally adjusted, and it takes on a value of 100 in 2012. Before calculating the target moments (autocorrelations, standard deviations) for the calibration, the log of all series is quadratically detrended and seasonally adjusted. An overview of the second moments generated from the data and from the model is given in Table 6.

E Sensitivity Analysis

Table 7: WELFARE COSTS OF FIXED EXCHANGE RATES – INCREASED SHOCK SIZE

(A) Benchmark				
LOOP		MU	FLEX	Difference
Monetary Policy (MP)	$10^{-2} *$	-21.059	-18.530	2.5295
Monetary+Fiscal Policy (MFP)	$10^{-2} *$	-18.219	-17.858	0.3604
		Reduction of Welfare Costs:		85.75%
PTM				
Monetary Policy (MP)	$10^{-2} *$	-21.052	-20.658	0.3938
Monetary+Fiscal Policy (MFP)	$10^{-2} *$	-20.058	-19.934	0.1235
		Reduction of Welfare Costs:		68.64%
(B) Productivity, Preference, Gov. Spending Shocks				
LOOP				
Monetary Policy	$10^{-2} *$	-17.069	-15.286	1.7827
Monetary+Fiscal Policy	$10^{-2} *$	-15.654	-15.371	0.2836
		Reduction of Welfare Costs:		84.09%
PTM				
Monetary Policy	$10^{-2} *$	-17.063	-16.854	0.2086
Monetary+Fiscal Policy	$10^{-2} *$	-16.911	-16.818	0.0930
		Reduction of Welfare Costs:		55.43%
(C) Mark-up Shocks				
LOOP				
Monetary Policy	$10^{-2} *$	-3.8470	-3.0972	0.7498
Monetary+Fiscal Policy	$10^{-2} *$	-2.4248	-2.3467	0.0781
		Reduction of Welfare Costs:		89.58%
PTM				
Monetary Policy	$10^{-2} *$	-3.8454	-3.6595	0.1859
Monetary+Fiscal Policy	$10^{-2} *$	-3.0018	-2.971	0.0308
		Reduction of Welfare Costs:		83.43%

Note: Welfare measure: consumption equivalents between deterministic and stochastic world economy. Exchange rate regime either monetary union (MU) or flexible (FLEX). Panel (A): productivity, demand preference, government spending, & mark-up shocks in both countries. Panel (B): all but mark-up shocks. Panel (C): mark-up shocks only. Shock standard deviations in all scenarios doubled compared to benchmark calibration. Second-order approximation to policy functions. $T = 1000$, $J = 100$.

Table 8: PERCENTAGE REDUCTION OF WELFARE COSTS OF FIXED EXCHANGE RATES – PARAMETER CHANGES

Benchmark	$\sigma = 4$	$\eta = 5$	Home Bias 30%	$\xi = 3$	$\rho, \rho^* = 10$	$\theta, \theta^* = 0.85$	$G/Y, G^*/Y^* = 0.30$	$B/Y, B^*/Y^* = 1.80$	
(A) All Shocks									
LOOP	85.76	93.01	90.10	85.23	89.78	93.17	87.12	87.20	85.66
PTM	68.66	84.12	49.99	65.21	83.26	67.32	63.56	78.93	68.61
(B) Productivity, Preference, Gov. Spending Shocks									
LOOP	84.03	90.62	88.66	81.99	91.15	92.00	86.45	86.64	84.29
PTM	55.42	76.84	42.25	54.22	67.57	53.27	54.68	71.74	55.23
(C) Mark-up Shocks									
LOOP	89.58	97.74	96.04	92.95	79.89	94.34	89.37	88.34	88.63
PTM	83.43	89.90	68.18	84.13	96.05	77.21	79.85	91.53	83.71

Note: Table shows percentage reduction of welfare costs of fixed exchange rates by using optimal fiscal policy. Underlying numbers of the welfare measure for all policy scenarios are available on request. Welfare measure: consumption equivalents between deterministic and stochastic world economy. Column 'Benchmark' repeats results of Table 3. Panel (A): productivity, demand preference, government spending, & mark-up shocks in both countries. Panel (B): all but mark-up shocks. Panel (C): mark-up shocks only. Second-order approximation to policy functions. $T = 1000$, $J = 100$.

Table 9: WELFARE COSTS OF FIXED EXCHANGE RATES – PAYROLL TAXES

(A) Benchmark				
		MU	FLEX	Difference
LOOP				
Monetary Policy (MP)	$10^{-2} *$	-5.0352	-4.4208	0.6144
Monetary+Fiscal Policy (MFP)	$10^{-2} *$	-4.0039	-3.747	0.2569
		Reduction of Welfare Costs:		58.20%
PTM				
Monetary Policy (MP)	$10^{-2} *$	-5.0348	-4.9412	0.0936
Monetary+Fiscal Policy (MFP)	$10^{-2} *$	-4.006	-3.988	0.0180
		Reduction of Welfare Costs:		80.77%
(B) Productivity, Preference, Gov. Spending Shocks				
LOOP				
Monetary Policy	$10^{-2} *$	-4.0539	-3.6155	0.4385
Monetary+Fiscal Policy	$10^{-2} *$	-3.8918	-3.6351	0.2567
		Reduction of Welfare Costs:		41.45%
PTM				
Monetary Policy	$10^{-2} *$	-4.0531	-4.0023	0.0507
Monetary+Fiscal Policy	$10^{-2} *$	-3.8749	-3.8582	0.0168
		Reduction of Welfare Costs:		66.96%
(C) Mark-up Shocks				
LOOP				
Monetary Policy	$10^{-2} *$	-0.9066	-0.7306	0.176
Monetary+Fiscal Policy	$10^{-2} *$	-0.0453	-0.0442	0.0012
		Reduction of Welfare Costs:		99.34%
PTM				
Monetary Policy	$10^{-2} *$	-0.9071	-0.8642	0.0429
Monetary+Fiscal Policy	$10^{-2} *$	-0.0565	-0.0553	0.0012
		Reduction of Welfare Costs:		97.11%

Note: Payroll taxes as fiscal instrument instead of VATs. Welfare measure: consumption equivalents between deterministic and stochastic world economy. Exchange rate regime either monetary union (MU) or flexible (FLEX). Panel (A): productivity, demand preference, government spending, & mark-up shocks in both countries. Panel (B): all but mark-up shocks. Panel (C): mark-up shocks only. Shock standard deviations in all scenarios doubled compared to benchmark calibration. Second-order approximation to policy functions. $T = 1000$, $J = 100$.

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