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The effects of government bond purchases on leverage constraints of banks and non-financial firms

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Non-technical summary

Research Question

Asset purchase programmes have been introduced by several central banks in the past years as additional policy measures designed to support real economic activity by pushing down medium- and long-term interest rates. In the literature there is evidence that the slope of the yield curve is positively related to bank profitability. From this point of view, government bond purchases have the potential to undermine profits in the banking sector. However, a lowering of long-term rates is also transmitted to the borrowing conditions of non-financial firms, supporting their financial health. The aim of this paper is discuss how government bond purchases affect the financial health of non-financial firms and banks in a general equilibrium setting.

Contribution

This paper investigates the effects of government bond purchases in a general equilibrium environment in which both non-financial firms and banks are leverage-constrained. Previous theoretical studies on quantitative easing have focused on affecting consumption and savings decisions, or funding conditions of capital production where funds are intermediated by a banking sector. For the latter, the balance sheet channel is often seen as an important channel through which bank equity is built up. In the present model, loans priced at par dominate in banks' balance sheets, which is why a balance sheet channel is of minor importance. Furthermore, the introduction of a leverage-constrained non-financial sector allows an investigation of the importance of this constraint for the effectiveness of government bond purchases.

Results

My results show that the distinction between two leverage constraints is important, as the financial health of non-financial firms is positively affected by government bond purchases through a reduction in borrowing conditions, while that of banks deteriorates in the medium run after a short-lived improvement. The latter happens because banks' profit margins drop as a result of lower returns on assets. However, the net effect of government bond purchases on real economic activity is positive. This is particularly true if the non-financial sector is faced with large financial frictions.

Nichttechnische Zusammenfassung

Fragestellung

Wertpapierankaufprogramme wurden in jüngster Zeit von diversen Zentralbanken weltweit zur Unterstützung der realwirtschaftlichen Aktivität über die Reduzierung langfristiger Zinsen eingeführt. In der Literatur gibt es Evidenz dafür, dass die Steigung der Zinsstrukturkurve in einem positiven Zusammenhang zur Profitabilität von Banken steht. Folglich können Staatsanleihenkäufe das Potential entfalten, die Gewinne im Bankensektor zu reduzieren. Ein Rückgang der langfristigen Zinsen soll jedoch auch einen positiven Einfluss auf die Finanzierungsbedingungen von nicht-finanziellen Unternehmen ausüben. Das Ziel dieses Papiers ist es, in einem allgemeinen Gleichgewichtsmodell zu untersuchen, welche Effekte Staatsanleihenkäufe auf nicht-finanzielle Unternehmen und Banken haben, wenn beide Kreditbeschränkungen unterliegen.

Beitrag

Bisherige theoretische Studien über Kaufprogramme konzentrieren sich zumeist entweder auf die Beeinflussung von Konsum- und Sparentscheidungen oder betrachten die Finanzierungsbedingungen von Investitionsprojekten, wobei die finanziellen Mittel bei letzterem durch einen Bankensektor bereitgestellt werden. Als ein wichtiger Transmissionskanal im Bankensektor wird hierbei der Bilanzkanal gesehen, bei dem über den Anstieg der Vermögenspreise auch das Eigenkapital der Banken gestützt wird. Im vorliegenden Modell dominieren hingegen Buchkredite in der Bilanz der Banken, wodurch dieser Kanal eine untergeordnete Rolle spielt. Die Betrachtung einer zusätzlichen Kreditbeschränkung im nicht-finanziellen Sektor erlaubt darüber hinaus einen weiteren wichtigen Aspekt der Transmission von Staatsanleihekaufprogrammen zu untersuchen.

Ergebnisse

Die Ergebnisse zeigen, dass die Berücksichtigung von zwei Kreditbeschränkungen zur Beurteilung von Staatsanleihekäufen relevant ist. Während sich die Finanzsituation von nicht-finanziellen Unternehmen verbessert, steigt der Verschuldungsgrad nach einem kurzfristigen Rückgang im Bankensektor mittelfristig an. Die Ursachen von Letzterem sind in einem Rückgang der Gewinne im Bankensektor begründet, der durch den Rückgang der Renditen auf Vermögensobjekte verursacht wird. Staatsanleihekäufe bewirken im Ergebnis einen positiven Nettoeffekt auf die Realwirtschaft. Dies gilt insbesondere bei erheblichen Kreditbeschränkungen im nicht-finanziellen Sektor.

Bundesbank Discussion Paper No 38/2016

The Effects of Government Bond Purchases on Leverage Constraints of Banks and Non-Financial Firms*

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Abstract

This paper investigates how government bond purchases affect leverage-constrained banks and non-financial firms by utilising a stochastic general equilibrium model. My results indicate that government bond purchases not only reduce non-financial firms' borrowing costs, amplified through a reduction in expected defaults, but also lower banks' profit margins. In an economy in which loans priced at par dominate in banks' balance sheets - as a reflection of the euro area's structure - the leverage constraint of non-financial firms is relaxed while that of banks tightens. I show that the leverage constraint in the non-financial sector plays an essential role in transmitting the impulses of government bond purchases to the real economy.

Keywords: DSGE Model, Financial Frictions, Banking Sector, Portfolio Rebalancing Channel, Government Bond Purchases

JEL classification: E44, E58, E61.

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1 Introduction

Asset purchase programmes have been introduced by several central banks in the past years as additional policy measures designed to support real economic activity. The basic idea of government bond purchases as a tool for central bank's balance sheet expansion is to reduce medium- and long-term interest rates by lowering the yields on government bonds. This objective becomes even more striking when the policy rate reaches its effective lower bound and the expectation component of long-term rates is difficult to influence through conventional monetary policy (D'Amico, English, López-Salido, and Nelson, 2012; Krishnamurthy and Vissing-Jorgensen, 2011). The slope of the yield curve, however, is evidently positively related to bank profitability (Aksoy and Basso, 2014; Borio, Gambacorta, and Hofmann, 2015; English, Van den Heuvel, and Zakrajsek, 2012). From this point of view, government bond purchases also have the potential to undermine profitability in the banking sector. This could countervail improvements in the market value of bank equity by stimulating the value of banks' assets and even overcompensate for the improvement in the soundness of non-financial firms which could result from a reduction in borrowing conditions accompanied by a drop in credit risk (Gilchrist and Zakrajsek, 2013).

In this paper, I investigate the effects of government bond purchases on the equity of non-financial firms and banks when both sectors are leverage-constrained, by drawing on a structure for the economy which is similar to that of the euro area. In the banking sector, two main channels through which asset purchases are transmitted are at work which produce opposing effects on banks' soundness. On the one hand, there is a portfolio rebalancing channel and, on the other, there is a balance sheet channel. For the latter, rising asset prices tend to stabilise bank equity, which alleviates financial intermediation. In contrast to the balance sheet channel, the portfolio rebalancing channel works by lowering borrowing rates. Since borrowing rates, in turn, determine bank profits, there could be countervailling effects on bank equity, particularly if changes in asset prices have less impact on banks' balance sheets. The net effect on the economy, however, will also depend on borrowing constraints in the non-financial sector. Since lower borrowing rates for the non-financial corporate sector reduce the cost of debt-financed investment projects, asset purchases lead to an interaction between a borrowing-constrained non-financial corporate sector and a borrowing-constrained banking sector.

The main contribution of this paper is to show that two leverage constraints interact and government bond purchases affect both constraints differently. I will demonstrate that the success of government bond purchases as a policy tool for stimulating output and inflation depends on how they affect financial health in the non-financial and financial sectors, whereas leverage constraints in the non-financial sector tend to be relaxed while they become more binding in the banking sector. An essential source of this effect stems from pricing loans at par while having significant financial frictions in the non-financial sector. Most of the changes in relative returns result from portfolio rebalancing effects. In this regard I am able to investigate portfolio rebalancing effects in the banking sector, in the household sector, and between the banking and the household sector at the same time in an environment where both the real and the financial sectors are faced with endogenous borrowing constraints.

I draw on a fully specified New Keynesian general equilibrium model in which non-financial firms predominantly take out non-market-based loans from banks. Banks are faced with constraints resulting from agency problems with their creditors. Moreover, agency problems also exist between banks and non-financial firms. Thus, I allow for financial frictions on both sides of banks' balance sheets, i.e. I allow for two-sided financial contracting. In the model, defaults by firms in the real sector are relevant, and expected defaults raise non-financial firms' borrowing costs, while unexpected bankruptcy costs are transmitted to the banking sector. Consequently, I assume a costly state verification problem between the financial sector and non-financial firms, while funders of banks are confronted with a limited enforcement problem.[1] In doing so, I additionally introduce two distinct debt instruments for non-financial firms: non-market-based debt (loans) and market-based debt (corporate bonds). Since households cannot provide their savings to capital-producing firms directly, banks intermediate funds to non-financial firms by granting loans and buying corporate bonds. Furthermore, I allow the banks to hold government bonds.

One essential feature of the banking sector is that banks' balance sheets are dominated by loans priced at par value. Purchases of government bonds can increase the price of assets, which stimulates banks' net worth and increases demand for private securities (Gertler and Karadi, 2013). This is particularly true if these securities are marked-to-market, and the resulting increase in private securities' prices is sufficient to support the build-up of bank net worth. I utilise a framework in which banks predominantly hold loans at constant prices in addition to corporate bonds and government bonds at variable prices. Thus, for a large share of bank assets, a balance sheet effect is not relevant because loans are priced at par. This setting more closely reflects the situation in the euro area, for instance.

Estimating the model using euro-area data helps to find parameter values for the elements controlling the strength of portfolio rebalancing. It turns out that the portfolio rebalancing channel in the banking sector following outright purchases of government bonds tends to weaken bank net worth. Purchases of government bonds reduce their returns, which is why banks begin to demand more corporate bonds and supply more loans. At the same time, portfolio rebalancing in the household sector contributes to changes in government bond prices. As a consequence, corporate bond returns and loan rates also fall, which means that borrowing conditions of non-financial firms improve. Lower borrowing conditions for firms alleviate their leverage constraint by raising net worth, for which reason their financial health improves. Consequently, non-financial firms produce more capital with positive effects on output.

Lower returns on bonds and a reduction in loan rates automatically squeeze banks' profit margins, therefore putting banks' leverage under upward pressure. The resulting need to delever countervails the stimulating effects of relaxed borrowing constraints in the non-financial corporate sector. A comparison with a case in which bank equity is kept constant reveals that the balance sheet channel relaxes bank leverage constraints first and contributes to the expansion in loans. Nevertheless, the portfolio rebalancing channel

[1] Since I want to build on the rich environment of two-sided financial contracting but do not want to emphasise banks' default risk, which would be related to bank runs or deposit insurance, I combine the approaches of Bernanke, Gertler, and Gilchrist (1999) and Gertler and Kiyotaki (2010). The advantages of such a setting are also discussed by Rannenberg (2016). Defaults in the banking sector are considered in Clerc, Derviz, Mendicino, Moyen, Nikolov, Stracca, Suarez, and Vardoulakis (2015), for example.

starts to dominate quickly and dampens the expansion in loans, which is predominantly driven by credit demand in the medium run rather than credit supply. A reduction in firms' leverage would not materialise without an improvement in non-financial firms' net worth.

Government bond purchases ultimately stimulate the real economy against the backdrop of improving the financial health of non-financial firms while depressing the financial health of banks in the medium run. Although anticipation effects support the improvement in banks' soundness first, an increase in banks' leverage follows when the programme completely unfolds. The more binding the leverage constraint in the non-financial sector, the greater the positive net effect on real economic activity because in this case, the drop in borrowing conditions has greater effects through the financial accelerator.

As is known from the literature, higher financial frictions in the banking sector make government bond purchases more effective because these frictions mostly prevent full arbitrage, which is needed to obtain non-trivial effects from portfolio rebalancing. However, I argue that larger financial frictions in the non-financial corporate sector also raise the efficiency of bond purchases because reductions in borrowing costs have stronger effects. If there are no financial frictions in the non-financial corporate sector, the deleveraging effect in the banking sector has a larger negative impact on the loan supply. This credit risk channel has a quantitative equivalent to the balance sheet channel, which can be seen by increasing the share of assets in banks' balance sheet priced mark to market. The balance sheet channel contributes to a stronger improvement in output given a specific level of frictions in the other sector. If the frictions are low, the same output gains can be obtained with increasing shares of market finance, i.e. through the balance sheet channel. The paper as a whole shows that government bond purchases can undermine the soundness of the banking sector. Their success in affecting real economic activity, however, largely depends on the financial frictions in the non-finanical sector. Against the backdrop of the estimated model for the euro area, there is an indication that real economic activity might improve, although the financial soundness of the banking sector could suffer in the medium run.

Related literature. There are several contributions in the literature which deal with government bond purchases in general equilibrium. One strand focuses on affecting predominantly savings (and consumption) decisions by altering the return on households' assets (Chen, Cúrdia, and Ferrero, 2012; Ellison and Tischbirek, 2014; Jones and Kulish, 2013), in which case the channel is similar to standard monetary policy. Compared to these approaches, I place a stronger emphasis on the banking sector and the impact on capital production. In this regard, my approach is closely related to Gertler and Karadi (2013) and Carlstrom, Fuerst, and Paustian (2016). While the banking sector shares elements of Gertler and Karadi (2013), I introduce a second leverage constraint on behalf of the non-financial firms and additionally allow for both loans priced at par and corporate bonds in banks' balance sheet. Carlstrom et al. (2016) also allow for two constraints affecting investment decisions. Regarding the banking sector, their and my approach also start from an incentive problem for bankers. However, my second constraint is related to firms' net worth, i.e. a leverage constraint arises like it does in the banking sector, while theirs is a "loan-in-advance" constraint which binds the market value of funds to the market value of investment opportunities. A combination of a leverage-constrained banking sector with a leverage-constrained non-financial sector can also be found in Hirakata, Sudo, and Ueda

(2011), Rannenberg (2016), Sandri and Valencia (2013), or Zeng (2013).[2] Compared to these papers, my focus is on government bond purchases, and my model introduces three assets to banks' balance sheet. Curdia and Woodford (2011) investigate asset purchases and treat several inefficiencies in financial intermedation which are imposed exogenously. In the present model, similar inefficiencies evolve endogenously by giving leverage a role. The purchases in my model are initially financed by issuing agency debt, wherease this modelling devise can also be found in Gertler and Karadi (2011). Since it is assumed that the agency has full credibility, it pays the risk-free rate on its debt. However, there is a feedback to taxpayers in the present model. If profits (or losses) are realised, they are redistributed to the fiscal authority, which has to adjust (lump-sum) taxes to keep the government solvent. Hence, I additionally introduce the possibility of feedback effects on taxes which takes elements discussed by Christiano and Ikeda (2013) into account.

The remainder of the paper is structured as follows. Section 2 provides a description and derivation of the model. Section 3 contains the empirical analysis. Dynamics from simulations of the model are presented in Section 4. Section 5 concludes.

2 Model

A standard DNK model following Smets and Wouters (2003) or Christiano, Eichenbaum, and Evans (2005) is extended to include a financial sector with borrowing constraints between all borrowers and lenders in the private sector. In this respect, I assume that there are borrowing constraints between the real sector and the financial market and between the bank and its creditors. While, in the first case, lenders are confronted with a costly state verification problem, as outlined in Bernanke et al. (1999), the agency problem between the bank and its lenders is modelled as proposed by Gertler and Kiyotaki (2010). This allows us to abstract from default problems in the banking sector.[3] The model consists of households, two types of entrepreneur, intermediate goods firms, final goods firms, mutual funds, banks and a public sector as active agents.

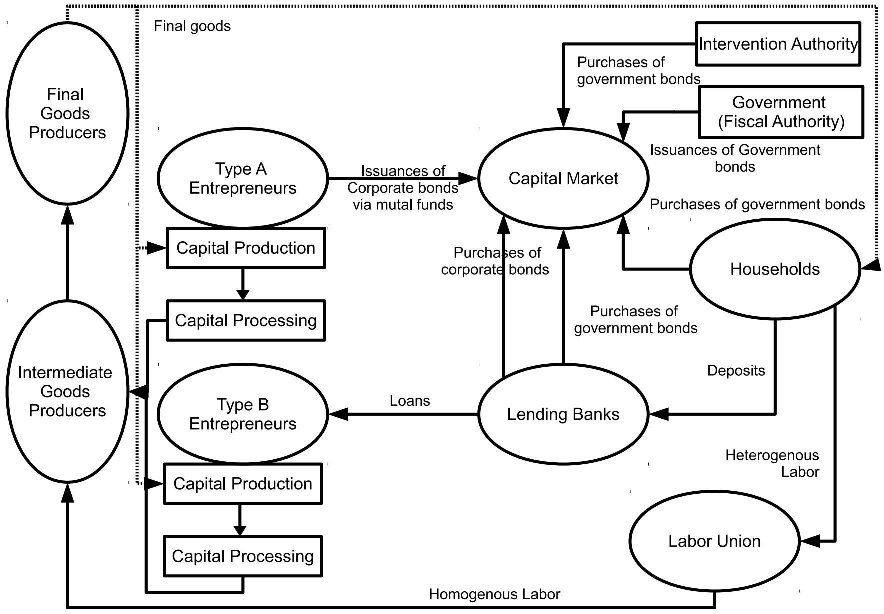
A continuum of households saves, consumes and supplies labour to the intermediate goods firms. Households receive income from labour and from financial assets to consume a bundle of final goods, purchased from final goods firms. The financial wealth of households arises from holdings of government bonds and bank deposits.

I start from Bernanke et al. (1999) but distinguish between two types of entrepreneur (type A and type B entrepreneurs) in order to introduce a role for different debt instruments. Both entrepreneurs process newly produced physical capital, which is exposed to the individual skills of each entrepreneur and is then rented out to intermediate goods firms. Related to the entrepreneurs, I allow for two different stocks of capital. The two types of entrepreneur own different types of capital which are both used complementarily in the production of intermediate goods. Both types of entrepreneur can finance their projects by raising external funds in excess of their net worth. The goods producing sector is similar to Smets and Wouters (2003). The two types of physical capital are rented out to intermediate goods firms, which combine physical capital with rented labour to

[2]In this repect, the present model is closely related to Rannenberg (2016), who combines a costly state verification problem with an incentive-compatibility constraint in the banking sector.

[3]Such a framework is discussed by Rannenberg (2016) and also utilised in Kühl (2014b).

Figure 1: Overview of central relationships in the model



produce differentiated intermediate goods. The intermediate goods firms sell their goods in a market of monopolistic competition to final goods producers. Finally, the final goods firms bundle the differentiated goods into a homogeneous final good. The final good can be used for consumption, in capital utilization, as investment goods, or as government expenditures. Banks receive funds from households (short-term debt) and invest in loans, corporate bonds, and government bonds. A rough sketch of the model can be found in Figure (1).

2.1 Households

The economy is populated by a continuum of households which are indexed by h with $h \in (0,1)$. Each h-th household decides on the supply of labour, how much to consume and to save, and on the allocation of its wealth. Households' utility function is given in Equation (1)

$$E_0^j \sum_{j=0}^{\infty} \beta^j \left[\ln \left(C_{h,t+j} - h^C C_{h,t-1+j} \right) - \kappa \nu_{t+j}^N \frac{\left(N_{h,t+j}\right)^{1+\varphi}}{1+\varphi} \right] \tag{1}$$

with discount factor β and a shock on labour supply ν_t^N, which follows a stationary AR(1) process. The term h^C reflects the internal habits in consumption with $h^C \in (0,1)$.

The households supply differentiated labour services $(N_{h,t})$ to the intermediate goods sector. Because of a monopolistically competitive labour market in which labour services are imperfect substitutes, each household has market power to set its nominal wage (W_t). Following Erceg, Henderson, and Levin (2000), I assume, in analogy to Calvo pricing, that the household is not able to renegotiate its nominal wage in each period. Instead, it can only reoptimise with a specific probability $(1-\gamma^w)$. In periods in which the household cannot renegotiate, it follows an indexation rule $\tilde{W}_t = \tilde{\pi}_{w,t} W_{t-1}$, with

$$\tilde{\pi}_{w,t} = (\pi_{t-1})^{\xi_w} (\pi)^{1-\xi_w} (z_t)^{\xi_z} (z_s)^{1-\xi_z} ,$$

where ξ_w is the weighting parameter for the past rate of inflation and ξ_z the weighting parameter for the shock on the growth rate of technology z_t. Relatedly to this, z_s is the steady-state growth rate of a non-stationary productivity process. A labour agency is introduced that buys differentiated labour from households and pays the individual wage in order to produce a representative labour aggregate as output

$$N_t = \left[\int_0^1 N_{h,t}{}^{\frac{1}{\lambda_w}} dh \right]^{\lambda_w} , \tag{2}$$

where λ_w represents the degree of substitution and is the mark-up of the wage over the household's marginal rate of substitution. By minimizing the costs of producing this aggregator, the labour agency takes the wage rates of each differentiated labour input as given. From this optimisation problem follows the demand for labour of household h for use in goods production

$$N_{h,t} = N_t \left(\frac{W_{h,t}}{W_t} \right)^{\frac{\lambda_w}{1-\lambda_w}} . \tag{3}$$

By combining Equations (2) and (3), one obtains the aggregate wage index

$$W_t = \left[\int_0^1 W_{h,t}^{\frac{1}{1-\lambda_w}} dh \right]^{1-\lambda_w} . \tag{4}$$

With the knowledge of demand for its labour, the household can proceed with determining the optimal wage rate $(W^*_{h,t})$ and the optimal labour supply $(N^*_{h,t})$. Thus, it maximises

$$\max_{\{W_{h,t}\}} E_t \sum_{s=0}^{\infty} (\beta\gamma_w)^s \left[-\kappa \frac{\left(N^*_{h,t+s}\right)^{1+\varphi}}{1+\varphi} + \lambda_{h,t+s} \frac{\Psi^w_{t+s} (1-\tau^w) W^*_{h,t}}{P_{t+s}} N^*_{h,t+s} \right] \tag{5}$$

by making use of Equation (3). The term φ reflects the inverse Frisch elasticity and households pay taxes on their labour income with the tax rate τ^w. Marginal utility of consumption is denoted by $\lambda_{h,t}$. Changes in rates of inflation until date s, which are important for indexation, are summarised in Ψ^w_{t+s} in Equation (5). Before utility maximisation is carried out, the optimal nominal wage emerges from a sub-problem in which the household minimises its disutility of labour by choosing its nominal wage given the labour demand of firms.[4]

It is assumed that some household members leave the household sector for a random time. A specific group of them becomes bank managers, who operate banks, while another group becomes entrepreneurs who conduct investment projects in the real sector.[5] The remaining household members place deposits (D_t) with banks, buy risk-free bonds which

[4]The derivation is presented in the appendix.

[5]A more detailed description can be found in the technical appendix.

consist of short-term $\left(B_t^{short,gov}\right)$ and long-term government bonds (B_t^{gov}) and (short-term) bonds issued by a public agency $\left(B_t^{IA}\right)$.[6] On holdings of short-term government bonds and agency's bonds, summarised as short-term public sector debt B_t^{PS}, they receive the risk-free rate i_t, while they obtain the risk-free return $r_t^{B,gov}$ on long-term government bonds which are traded at price $Q_t^{B,gov}$.[7]

In order to allow for longer-term bonds, I follow Woodford (2001) and assume that only a fraction of the government bonds $(1-\rho^{B,gov})$ issued during the last period are repaid in this period.[8] Regarding the definition of the bond rate $(r_t^{B,gov})$ I obtain

$$r_t^{B,gov} = \pi_t \left(\frac{\rho^{B,gov} Q_t^{B,gov} + 1}{Q_{t-1}^{B,gov}} \right) - 1. \tag{6}$$

In addition, holdings of government bonds are related to costs

$$\Theta_t^{gov,H} = \frac{\upsilon^{B,gov}}{2} \left(B_{h,t+j}^{gov} - B_h^{gov}\right)^2 Q_{h,t+j}^{B,gov} + \tau^{B,gov} Q_{h,t+j}^{B,gov} B_{h,t+j}^{gov} \tag{7}$$

with $\upsilon^{B,gov}$ and $\tau^{B,gov}$ as a scaling parameters and B_h^{gov} as the steady-state holdings of government bonds. The cost function in Equation (7) captures two ideas. The first part on the right-hand side takes into account arguments from the "preferred habitat" theory of the term structure (as argued by Gertler and Karadi (2013)), while the last part borrows slightly from the literature on trading costs (see Harris and Piwowar (2006), for example).[9]

Households receive income from dividend payments $(Div_{h,t})$ provided by intermediate goods firms and capital producers, from their supply of labour, and from investments in financial assets. Following Erceg et al. (2000) households are assumed to buy state-contingent securities with a lump-sum transfer to equalise income differences among the continuum of households. Households' expenditures are allotted to consumption, to lump-sum taxes, to transfers including payments to capital producers, entrepreneurs and bank managers, $\Xi_{h,t}$, and to the purchases of financial assets, i.e. public sector bonds, corporate bonds, and deposits.

[6] I introduce long-term bonds mainly because I want to allow government bonds to have a time-varying market price, similar to Gertler and Karadi (2013).

[7] Below, I mean "default-free" when I talk about risk-free rates and spreads.

[8] In Woodford (2001) $\rho^{B,gov}$ is interpreted as exponentially decaying coupons. This statement is economically equivalent to the representation here as the coupon payment is one. Chen et al. (2012) also allow for a maturity structure. While they use the yield-to-maturity in their model, I draw on the period return.

[9] Harris and Piwowar (2006) propose different functions capturing trading costs which are related to transactions. For municipal bonds in the US, transaction costs fall with larger transactions and vanish with large trades. However, fixed costs also play a role by looking at Equations (1) and (6) in their model. I translate their arguments into the functional form above.

The budget constraint in real terms becomes

$$
\begin{aligned}
& \left(1+i_{t-1+j}\right)\frac{B^{n,PS}_{h,t-1+j}}{P_{t+j}}+\left(1+r^{B,gov}_{t+j}\right)\frac{Q^{B,gov}_{t-1+j}B^{n,gov}_{h,t-1+j}}{P_{t+j}} \\
+ \quad & \left(1+r^{D}_{t-1+j}\right)\frac{D^{n}_{h,t-1+j}}{P_{t+j}}+\left(1-\tau^{w}\right)\frac{W_{h,t+j}}{P_{t+j}}N_{h,t+j}+\frac{Div_{h,t+j}}{P_{t+j}}+\Xi_{h,t+j} \\
\geq \quad & \left(1+\tau^{C}\right)C_{h,t+j}+T_{t+j}+\frac{D^{n}_{h,t+j}}{P_{t+j}}+\frac{B^{n,PS}_{h,t+j}}{P_{t+j}}+\Theta^{gov,H}_{t},
\end{aligned}
$$

where the superscript n denotes nominal terms. Households pay taxes on their labour income and on their consumption expenditures, $\tau^{w}$and τ^{C}, respectively.

From the no-arbitrage conditions it follows that each household holds the same amount of assets, which is why I can aggregate easily. All first-order conditions can be found in the technical appendix.

2.2 Final goods firms

The final good (Y_t) is a composite of the continuum of differentiated intermediate goods purchased from all i monopolistic competitive firms in the intermediate goods market which is populated by perfectly competitive final goods producers

$$Y_t=\left[\int_0^1 Y_{i,t}{}^{\frac{1}{\lambda_{p,t}}}\,di\right]^{\frac{\lambda_{p,t}}{1-\lambda_{p,t}}}, \tag{8}$$

where $\lambda_{p,t}$ represents the mark-up of prices over marginal costs. It follows a stationary stochastic AR(1) process in logs.

By taking the prices of the intermediate goods as well as the price of the final good as given, the final goods firm maximises its profits by choosing the amount of intermediate goods and the amount of output of final goods. From the optimisation problem there follows the demand function for intermediate goods

$$Y_{i,t}=Y_t\left(\frac{P_{i,t}}{P_t}\right)^{\frac{\lambda_{p,t}}{1-\lambda_{p,t}}} \tag{9}$$

where P_{it} is the price of the i-th intermediate good and P_t the price of the final good.

2.3 Intermediate goods firms

A continuum of the intermediate goods firms with mass one plan to rent capital ($\widetilde{K}_{i,t}$) from the entrepreneurs and homogeneous labour ($\widetilde{N}_{i,t}$) from the households for use in production. Intermediate goods arise following a standard production function of the Cobb-Douglas type with constant returns to scale and fixed costs (Ω)

$$Y_{i,t}=A_t\left(\widetilde{K}_{i,t}\right)^{\alpha}\left(Z_t\widetilde{N}_{i,t}\right)^{1-\alpha}-Z_t\Omega_i, \tag{10}$$

where the term α is the share of capital in production. The production technology is affected by a (stationary) shock on total factor productivity A_t which follows an AR(1) process in logs and a non-stationary technology shock Z_t, whereas its growth rate follows an AR(1) process in logs, i.e. $\log(z_t) \equiv \log(Z_t/Z_{t-1}) = (1-\rho_z)\log(z_s) + \rho_z \log(Z_{t-1}/Z_{t-2}) + \epsilon_{z,t}$, with ρ_z as the autoregressive parameter and $\epsilon_{z,t}$ as the iid innovation. I allow for different types of capital in the production process such that the stock of capital is a composite index. I modify the production technology because I later introduce two different debt instruments. For this reason, I will attribute the production of one capital good to one specific debt instrument.[10]

Firms minimise their real costs by choosing inputs given their production technology. Thus,

$$\min_{\{\widetilde{K}^A_{i,t},\widetilde{K}^B_{i,t},\widetilde{N}_{i,t}\}} r^{k,A}_t \widetilde{K}^A_{i,t} + r^{k,B}_t \widetilde{K}^B_{i,t} + w_t \widetilde{N}_{i,t}$$

$$s.t. \quad Y_{i,t} = A_t \left(\widetilde{K}_{i,t}\right)^{\alpha} \left(Z_t \widetilde{N}_{i,t}\right)^{1-\alpha} - Z_t \Omega_i \tag{11}$$

$$\text{and} \quad \widetilde{K}_{i,t} = \left((\zeta^K)^{\frac{1}{\gamma^K}} \left(\widetilde{K}^A_{i,t}\right)^{\frac{\gamma^K-1}{\gamma^K}} + (1-\zeta^K)^{\frac{1}{\gamma^K}} \left(\widetilde{K}^B_{i,t}\right)^{\frac{\gamma^K-1}{\gamma^K}} \right)^{\frac{\gamma^K}{\gamma^K-1}}. \tag{12}$$

The terms $r^{k,A}_t$ and $r^{k,B}_t$ are the costs of capital and w_t is the real wage. The terms A and B refer to type A and B entrepreneurs and ζ^K is the share of utilised type A entrepreneurs' capital in utilised total capital with γ^K as the elasticity of substitution.[11] The first-order conditions for the minimisation problem of each intermediate goods firm are presented in the technical appendix. With their help it can be shown that the ratio of type B entrepreneurs' capital to type A entrepreneurs' capital as well as the capital to labour ratios are the same across all firms.

Following on Calvo (1983), optimal pricing is only possible with a probability of $1-\gamma$, whereas the remaining fraction of firms that cannot optimise their price set the price equal to its value last period multiplied by the past rate of inflation (π_{t-1}) which is weighted by the steady-state rate of inflation (π). Consequently, the optimisation problem for adjusting firms becomes

$$\max_{\{P^*_{i,t}\}} E_t \sum_{j=0}^{\infty} \beta^j \lambda_{t+j} \gamma^j \left[Y_{i,t} \left(P^*_{i,t} - mc_{i,t+j} P_{t+j} \right) \right]$$

[10] Bernanke et al. (1999) and Fisher (1999) also introduce heterogeneous financially constrained firms and proceed similarly. In contrast to them, I do not introduce two complete goods-producing sectors with fixed-input shares and a bundling technology to produce the final good. Instead, I split up the physical stock of capital. The approach allows in a sense to endogenise the financing decision in terms of the intermediate goods by varying the capital input. Carlstrom, Fuerst, and Paustian (2010) distinguish between two different types of labour, whereas one of both is subject to credit constraints. The model here is basically very similar to their approach, except that I introduce a third input for production and relate credit constraints to two of the inputs. The idea of having two different types of entrepreneurs is akin to Aksoy and Basso (2014) who introduce two type of entrepreneurs to have a segmentation for short and long-term debt.

[11] This approach is akin to the one in Krusell, Ohanian, Ríos-Rull, and Violante (2000), where unskilled and skilled labour are combined. However, I favor the properties of the CES function to have constant elasticities of substitution between the inputs.

subject to Equation (9). The optimal price of the intermediate good is denoted by $P_{i,t}^{*}$ and $mc_{i,t}$ represents the marginal costs. The first-order conditions can be found in the technical appendix.

2.4 Capital producers

The economy is populated by capital producers that are owned by households and work in a market of perfect competition. By doing so, they combine undepreciated physical capital with investment goods of class e ($e \in [A, B]$) to produce new physical capital of the same class.

$$K_t^e = K_{t-1}^e \left(1 - \delta^e\right) + I_t^e \left[1 - \Psi\left(\frac{I_t^e}{I_{t-1}^e}\right)\right] \mu_{I,t} \tag{13}$$

Equation (13) presents the law of motion of capital, where K_t^e is the capital stock, δ^e the rate of depreciation, I_t^e the amount of investment goods, and $\mu_{I,t}$ an investment-specific technology shock which follows a stationary AR(1) process in logs and hits both sectors simultaneously. Adjustment costs for investment are denoted by $\Psi\left(\frac{I_t^e}{I_{t-1}^e}\right)$ and follow

$$\Psi\left(\frac{I_t^e}{I_{t-1}^e}\right) = \frac{1}{2}\left[\exp\left[\sqrt{\Psi''}\left(\frac{I_t^e}{I_{t-1}^e} - 1\right)\right] + \exp\left[-\sqrt{\Psi''}\left(\frac{I_t^e}{I_{t-1}^e} - 1\right)\right] - 2\right] \tag{14}$$

where $\Psi(1) = \Psi'(1) = 0$ and $\Psi''(1) > 0$. Capital producers maximise their profits distributed to households, Div_t^I, by determining the amount of newly produced investment goods

$$\max_{\left\{I_t^A, I_t^B\right\}} E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} Div_{t+j}^I$$

subject to the laws of motion for capital, by taking the prices for capital Q_t^e into account, and to the flow of funds constraint $I_t^A + I_t^B = F_t^I$, where F_t^I denotes the funds received from households. The variable $\Lambda_{t,t+j}$ represents the discount factor which is households' pricing kernel $\beta\frac{\lambda_{t+j}}{\lambda_t}$. For convenience, investment goods have the same price as physical capital.

2.5 Entrepreneurs

I follow Bernanke et al. (1999) and assume that the economy is populated by a continuum of entrepreneurs that buy capital from capital producers, transform the capital into new capital exposed to an idiosyncratic processing risk, and rent the capital to intermediate goods producers after observing the shock.

Furthermore, I borrow from Bernanke et al. (1999) and Fisher (1999) and introduce two borrowing-constrained firms to allow for different debt instruments as in De Fiore and Uhlig (2015). I split the continuum of entrepreneurs, with $m \in [0, 1]$, into two groups $e \in [A, B]$ with $A : m \in [0, \varrho)$ and $B : m \in [\varrho, 1]$. The grouping of new entrepreneurs into the two groups is exposed to a random process with fixed probabilities, whereas the population of each entrepreneurial group remains constant. In line with De Fiore and Uhlig (2011), I assume that type B entrepreneurs will solely rely on bank finance, while type B entrepreneurs issue bonds in the capital market.

Financial intermediaries are faced with a costly state verification problem.[12] The productivity shock on type e entrepreneurs' skills ω_t^e can only be observed by the intermediaries if they pay a fixed fraction μ^e of the amount that can be recovered in the case of a default while entrepreneurs always have knowledge about their productivity. As a result, entrepreneurs finance their investment projects by external funds (debt) and internal funds (net worth).

For capital processing, the entrepreneurs' individual skills are of importance and the entrepreneurs decide on the capital utilization $(u_{m,t}^e)$. The skills of both type A and B entrepreneurs are subject to idiosyncratic shocks which affect the physical properties of capital. These shocks $\omega_{m,t}^e$ are drawn from a lognormal distribution with unit mean and are independent over time and across entrepreneurs. For the m-th entrepreneur, I obtain the amount of processed capital $\hat{K}_{m,t}^e$

$$\hat{K}_{m,t}^e = \omega_{m,t}^e K_{m,t}^e. \tag{15}$$

Both types of entrepreneur finance the capital purchases with their own net worth $(NW_{m,t}^e)$ and external funds $\left(L_{m,t}^e\right)$

$$Q_t^e K_{m,t}^e = NW_{m,t}^e + L_{m,t}^e \tag{16}$$

where Q_t^e is the real price of entrepreneurs' capital. For type A entrepreneurs, this means that they borrow from the capital market by issuing bonds $B_{m,t}$ at real price $Q_t^{B,corp}$, i.e. $L_{m,t}^A = Q_t^{B,corp} B_{m,t}$. Type B entrepreneurs obtain loans $\left(L_{m,t}^B = L_{m,t}\right)$ from banks.

For the case where the value of the project is exactly equal to the debt service, I can define $\overline{\omega}_{m,t}^e$ as a productivity threshold for which the borrower is just able to satisfy the debt contract. I assume that the contract is signed before the shocks materialise.[13] Since the contract is negotiated based upon the expected capital return, I have to distinguish between ex ante and ex post thresholds. After the shock has occurred, the realised (gross) capital return emerges as

$$\begin{aligned} 1 + R_{m,t}^{k,e,\omega} &= \pi_t \frac{\left(1-\tau^K\right)\left(r_{m,t}^{k,e} u_t^e - \Gamma(u_{m,t}^e)\right) + Q_t^e(1-\delta^e) + \tau^K \delta^e Q_t^e}{Q_{t-1}^e} \omega_{m,t}^e \\ &= (1 + R_{m,t}^{k,e})\omega_{m,t}^e. \end{aligned} \tag{17}$$

The term τ^K is the tax rate on capital income which is identical to both sectors. If the realised idiosyncratic shock is greater than (or equal to) the ex post threshold, the entrepreneur will be able to repay his debt as contractually agreed and keep the difference as net earnings. A realization of the shock that is below the ex post threshold level results in a default, and the entrepreneur has to liquidate the remaining amount in order to satisfy its lenders.

Similar to Carlstrom, Fuerst, and Paustian (2015), entrepreneurs have a long-run perspective and maximise the expected utility $V_{m,t}^{E,e}$ of continuing entrepreneurs at the

[12] I discuss the optimality of the contract in Section D in the technical appendix.

[13] Thus, I follow slightly Benes and Kumhof (2015) and replace the realised capital return by the expected capital return. This timing convention proxies reality more closely, particularly for bank financing, and allows for unexpected defaults in the period of the shocks.

end of period t, i.e. the franchise value,

$$V_{m,t}^{E,e} = \max_{\left\{K_{m,t}^{e},\overline{\omega}_{m,t+1}^{e}\right\}} E_t \left[\sum_{i=1}^{\infty} \Lambda_{t,t+i} \left(1 - p_t^{E,e}\right) \left(p_t^{E,e}\right)^{i-1} \Pi_{m,t+i}^{E,e} \right],$$

whereas $p_t^{E,e}$ is the probability that an entrepreneur stays in business and is exposed to an iid shock, $\Pi_{t+i}^{E,e}$ are the terminal funds available for exiting entrepreneurs at $t+i$ and transferred to households. Terminal funds are simply their net worth at that period in time, i.e. $\Pi_{m,t}^{E,e} = NW_{m,t}^{E,e}$. Net worth, in turn, results from the net payoffs entrepreneurs receive from their projects after taking the profitability of their projects into account

$$NW_{m,t+1}^{E,e} = E_t \left(1 - \Theta(\overline{\omega}_{m,t+1}^{e}; \sigma_t^e)\right) \left(\frac{1 + R_{t+1}^{k,e}}{\pi_{t+1}} \right) Q_t^e K_{m,t}^e, \tag{18}$$

where $\Theta(\overline{\omega}_{m,t+1}^{e}; \sigma_t^e)$ reflects the payments to the creditors given the outcome of the project. The standard deviation σ_t^e of the distribution can be time-varying, i.e. deviating from its steady-state value σ^e, and obeys a stationary AR(1) process in logs. A time-varying standard deviation gives rise to the possibility of a "financial risk shock", which increases the range of realizations of the shocks (see, for instance, Christiano, Motto, and Rostagno (2014)).

The franchise value of their firm $V_{m,t}^e$ can be expressed recursively and the maximisation problem of each e-type entrepreneur can be written as

$$\max_{\left\{K_{m,t}^{e},\overline{\omega}_{m,t+1}^{e}\right\}} \left(1 - p_t^{E,e}\right) NW_{m,t}^{E,e} + E_t \Lambda_{t,t+1} p_{t+1}^{E,e} V_{m,t+1}^{E,e}$$

$$s.t. \quad \begin{aligned} & E_t \left[\Theta(\overline{\omega}_{m,t+1}^{e}; \sigma_t^e)) - \mu^e G(\overline{\omega}_{m,t+1}^{e}; \sigma_t^e))\right] \left(1 + R_{t+1}^{k,e}\right) Q_t^e K_{m,t}^e \\ & = (1 + r_t^e) \left(Q_t^e K_{m,t}^e - NW_{m,t}^{E,e}\right), \end{aligned} \tag{19}$$

by taking Equation (18) into account. In the participation constraint of the intermediaries (Equation (19)) I have for the two sectors: $r_t^A = E_t \left(r_{t+1}^{B,corp}\right)$ and $r_t^B = r_t^L$. The function $G(\overline{\omega}_{m,t+1}^{e}; \sigma_t^e))$ reflects the expected payoffs for the financial intermediaries given defaults of the entrepreneurs and μ^e denotes the share of assets lost for monitoring purposes.[14] The derivation of the model and the related first-order conditions are presented in the technical appendix.

While the current risk-free loan rate r_t^L enters the participation constraint for the type B entrepreneur, the expected (risk-free) bond return $E_t \left(r_{t+1}^{B,corp}\right)$, which is defined as

$$r_t^{B,corp} = \pi_t \left(\frac{\rho^{B,corp} Q_t^{B,corp} + 1 - \frac{\Upsilon_t^{B,e}}{Q_{t-1}^{B,corp} B_{h,t-1}^{corp}}}{Q_{t-1}^{B,corp}} \right) - 1, \tag{20}$$

[14]The expression $1 - \Theta(\overline{\omega}_{m,t+1}^{e}; \sigma_t^e)$ is the share of entrepreneurial earnings of non-defaulting entrepreneurs, while $\Theta(\overline{\omega}_{m,t+1}^{e}; \sigma_t^e)) - \mu^e G(\overline{\omega}_{m,t+1}^{e}; \sigma_t^e))$ represents earnings of financial intermediaries by taking default cases into account.

becomes relevant in intermediaries' participation constraint for the type A entrepreneurs. As with government bonds, I allow for long-term bonds with a specific maturity structure, which is modelled according to Woodford (2001) with maturity parameter $\rho^{B,corp}$.[15] Similar to government bonds, $Q_t^{B,corp}$ is the price of the corporate bond. Regarding the loan contract of type B entrepreneurs, the loan rate is not state-contingent. As long as the default is not unexpected, its costs are taken into account in bond pricing. Since the contract is written before the productivity shock on ω_t^A is realised, i.e. before the capital return is known, the contract has a state contingent nature. Deviations of the realised capital return from its expected value matter for the debt servicing capacity, i.e. bond holders face ex post losses $\left(\Upsilon_t^{B,e}\right)$ while all ex ante costs of defaults can be completely diversified.

In each period, entrepreneurs leave the market with a given probability of $1 - p_t^{E,e}$ and are exactly replaced by new entrepreneurs just endowed with households' transfers $(NW_t^{E,e,new} = w_m^e)$ to keep the population of entrepreneurs stable. The aggregate law of motion for aggregate entrepreneurial net worth $(NW_t^{E,e})$ becomes

$$NW_t^{E,e} = p_t^{E,e} NW_t^{E,e,old} + NW_t^{E,e,new} \tag{21}$$

with $NW_t^{E,e,old} = \left(1 - \Theta(\overline{\omega}_t^e; \sigma_{t-1}^e)\right) \left(\frac{1+R_t^{k,e}}{\pi_t}\right) Q_{t-1}^e K_{t-1}^e$.

After processing the capital with the help of individual skills, the entrepreneurs decide on capital utilization, which entails costs in the form of

$$\Gamma(u_{m,t}^e) = \frac{r^{k,e}}{\psi^{k,e}} \left(\exp\left[\psi^{k,e} \left(u_{m,t}^e - 1\right)\right] - 1\right). \tag{22}$$

The aggregate amount of physical capital distributed to the intermediate goods sector, after the second stage is accomplished, is obtained by aggregating over the distribution of the productivity shock and over the continuum of entrepreneurs.

$$\hat{K}_{t+1} = \int_0^{\varrho} \int_0^{\infty} u_{m,t} \omega K_{m,t} dF(\omega) f(m) dm + \int_{\varrho}^{0} \int_0^{\infty} u_{m,t} \omega K_{m,t} dF(\omega) f(m) dm = u_t K_t. \tag{23}$$

Equation (23) shows that shocks to entrepreneurs' skills do not matter for the economy as a whole, because the idiosyncratic risk can be diversified perfectly and the utilization rate is identical across all entrepreneurs.

2.6 Financial intermediaries

2.6.1 Mutual funds

Mutual funds are introduced to proxy the capital market. This idea follows Bernanke et al. (1999) and Christiano et al. (2014). The mutual funds serve as intermediaries that channel funds from banks, rent them out by buying bonds from type A entrepreneurs and operate on zero profits. The main objective is to model the linkage between the issuance of bonds and the financing by banks.

[15] See Kühl (2014a) for the implications of introducing bonds with a maturity into the BGG approach.

2.6.2 Banking sector

Since households cannot provide funds to the entrepreneurial sector directly, I introduce a banking sector, which basically follows Gertler and Kiyotaki (2010) and Gertler and Karadi (2011). The economy is populated by a continuum of lending banks n with $n \in [0,1]$. In addition to loans which the lending banks can grant to type B entrepreneurs directly, they also buy (corporate) bonds issued by type A entrepreneurs.[16] In addition, each lending bank buys government bonds. Hence, each n-th lending bank holds three assets which together constitute bank's total assets $A^B_{n,t}$. Funds are raised by issuing debt $D_{n,t}$ combined with net worth $E^I_{n,t}$ which is built up by retaining earnings.[17] Thus, the balance sheet constraint becomes

$$A^B_{n,t} = L_{n,t} + Q^{B,corp}_t B^{corp}_{n,t} + Q^{B,gov}_t B^{gov}_{n,t} = E^I_{n,t} + D_{n,t}. \tag{24}$$

Since the loan rate r^L_t is negotiated before the shocks are realised such that it becomes non-state contingent, ex post defaults can occur, which must be borne by the lending bank. In the corporate bond market, unexpected losses materialise in the ex post period return $r^{B,corp}_t$ as a result of unexpected changes in the price of the corporate bonds (see again Equation (20)). Government bonds have a return of $r^{B,gov}_t$.

Looking at funding, the bank borrows from households at the rate r^D_t.[18] The law of motion for net worth is written in real terms, while financial assets are denominated in nominal terms

$$\begin{aligned} E^I_{n,t} \;=\; & \left(1+r^L_{t-1}\right) L_{n,t-1}\frac{1}{\pi_t} + \left(1+r^{B,corp}_t\right) Q^{B,corp}_{t-1} B^{corp}_{n,t-1}\frac{1}{\pi_t} + \left(1+r^{B,gov}_t\right) Q^{B,gov}_{t-1} B^{gov}_{n,t-1}\frac{1}{\pi_t} \\ & - \left(1+r^D_{t-1}\right)\frac{1}{\pi_t} D_{n,t-1}\frac{1}{\pi_t} - \Upsilon^L_{n,t} + \mu_{EI,t}. \end{aligned} \tag{25}$$

The term $\Upsilon^L_{n,t}$ in Equation (25) comprises losses from the loan portfolio, while $\mu_{EI,t}$ represents an exogenous shock an bank equity. Lending banks maximise the terminal consumption which is equivalent to maximizing the present value of their net worth, i.e. the value of the bank $V^B_{n,t}$. In this case, the bank managers would choose $L_{n,t}$, $B^{corp}_{n,t}$, $B^{gov}_{n,t}$, and $D_{n,t}$ optimally.[19]

$$V^B_{n,t} = \max_{\left\{L_{n,t},B^{corp}_{n,t},B^{gov}_{n,t},D_{n,t}\right\}} E_t \sum_{i=1}^{\infty} \Lambda_{t,t+i}\left(1-p^B\right)\left(p^B\right)^{i-1}\Pi^B_{n,t+i} \tag{26}$$

The term $\left(1-p^B\right)$ in Equation (26) reflects the exit from the banking business and $\Pi^B_{n,t+i}$ are the terminal funds available for exiting bankers at time $t+i$ which is simply the volume of their equity at that period in time, i.e. $\Pi^B_{n,t} = E^I_{n,t}$.

[16]Note that they do not buy bonds directly from type A entrepreneurs. More precisely, they buy bonds from mutual funds, which, in turn, hold bonds issued by type A entrepreneurs. For the sake of simplicity, I argue that lending banks buy bonds from entrepreneurs, but, technically, the funds are intermediated by mutual funds.

[17]Bank net worth can also be interpreted as inside equity.

[18]Because of the formulation of the bank, these loans comprise both deposits and bank bonds. From this point of view, the interest rate is an "average" rate on bank's debt.

[19]I discuss the optimality of the contract in Section E in the technical appendix.

An agency problem in the banking sector arises because bank managers can divert a fraction θ^{IC} of the bank's resources which cannot be recovered because of high enforcement costs. Furthermore, different assets can be diverted to different degrees. Thus, the bank's incentive constraint becomes

$$V_{n,t}^{B} \geq \theta^{IC} \left(L_{n,t} + \Delta^{B,corp} Q_t^{B,corp} B_{n,t}^{corp} + \Delta^{B,gov} Q_t^{B,gov} B_{n,t}^{gov} \right), \tag{27}$$

where $\Delta^{B,corp}$ and $\Delta^{B,gov}$ denote the specific relative shares which can be diverted related to corporate and government bonds, respectively.[20] Hence, bankers maximise Equation (26) subject to Equation (27).

Since a fraction of bank managers resign, bank managers continue to operate a lending bank with probability p^B. While the exiting bank managers' net worth is no longer available, the remaining net worth is a fraction of aggregate net worth

$$E_t^{I,old} = p^B \left(\begin{array}{c} \left(R_{t-1}^L - R_{t-1}^D\right) L_{t-1} \frac{1}{\pi_t} + \left(R_t^{B,corp} - R_{t-1}^D\right) Q_{t-1}^{B,corp} B_{t-1}^{corp} \frac{1}{\pi_t} \\ + \left(R_t^{B,gov} - R_{t-1}^D\right) Q_{t-1}^{B,gov} B_{t-1}^{gov} \frac{1}{\pi_t} + R_{t-1}^D E_{t-1}^I \frac{1}{\pi_t} - \Upsilon_t^L + \mu_{EI,t} \end{array} \right), \tag{28}$$

where I made use of the balance sheet constraint and using gross interest rates, i.e. $R_t^L = 1 + r_t^L$, $R_t^{B,corp} = 1 + r_t^{B,corp}$, $R_t^{B,gov} = 1 + r_t^{B,gov}$, and $R_t^D = 1 + r_t^D$. New bank managers fill the gap created by the exit of old bank managers and enter the market in order to start operating a lending bank. From their households they obtain an endowment with which net worth is built up

$$E_t^{I,new} = \gamma^B \left(\begin{array}{c} \left(R_{t-1}^L - R_{t-1}^D\right) L_{t-1} \frac{1}{\pi_t} + \left(R_t^{B,corp} - R_{t-1}^D\right) Q_{t-1}^{B,corp} B_{t-1}^{corp} \frac{1}{\pi_t} \\ + \left(R_t^{B,gov} - R_{t-1}^D\right) Q_{t-1}^{B,gov} B_{t-1}^{gov} \frac{1}{\pi_t} + R_{t-1}^D E_{t-1}^I \frac{1}{\pi_t} \end{array} \right)$$

that is a fraction γ^B of assets. Consequently, aggregate net worth is the sum of both components

$$E_t^I = E_t^{I,old} + E_t^{I,new}.$$

2.7 Public sector

2.7.1 Fiscal authority

To finance government expenditures G_t, the fiscal authority uses internal funds, i.e. from tax revenues (T_t) and profits received from an intervention authority $\left(\mathcal{P}_t^{IA}\right)$, and external funds, i.e. from the issuance of short-term $B_t^{short,gov}$ and long-term government bonds B_t^{gov} in the capital market traded at price $Q_t^{B,gov}$. Short-term government bonds are in zero net supply. The budget constraint of the fiscal agent is given in Equation (29).

$$G_t + \left(1 + r_t^{B,gov}\right) Q_{t-1}^{B,gov} B_{t-1}^{gov} = \mathcal{P}_t^{IA} + T_t + Q_t^{B,gov} B_t^{gov} \tag{29}$$

[20] Like Rannenberg (2016), I combine a BGG-type problem with a GK-type problem. An advantage of this approach is that I am able to investigate the different frictions separately by abstracting from a risky bank environment. The treatment of risky banks would require either an insurance mechanism or the need to deal with bank runs.

Government expenditures follow a stationary AR(1) process in logs around its steady-state value. Tax revenues stem from labour income, capital returns and consumption taxes. The fiscal agent adjusts the tax rate on labour income in order to stabilise the level of real government debt, where the term ξ^{BG} is a positive number which reflects the fact that governments' insolvency is ruled out by conducting a passive fiscal policy (see, for example, Leeper, 1991). The tax rule is presented in Equation (30).

$$\begin{aligned} T_t &= T\exp\left(\mu_{T,t}\right) + \tau^C C_t + \tau^K r_t^{k,A} K_t^A + \tau^K r_t^{k,B} K_t^B + \tau^w w_t N_t \\ &\quad + \xi^{BG}\left(Q_{t-1}^{B,gov} B_{t-1}^{gov} - Q_s^{B,gov} B_s^{gov}\right), \end{aligned} \tag{30}$$

whereas T denote lump-sum taxes and $\mu_{T,t}$ is a shock on lump-sum taxes which follows an AR(1) process.

2.7.2 Central bank

The central bank conducts monetary policy by controlling the policy rate $i_t^{PR}\left(=i_t=r_t^D\right)$. For this purpose, it obeys a Taylor rule, the objective of which is to set the policy rate according to

$$\left(1+i_t^{PR}\right) = \left(1+i_{t-1}^{PR}\right)^{\rho^{smooth}} (1+i)^{\left(1-\rho^{smooth}\right)} \left(\frac{\pi_t}{\pi}\right)^{\phi^\pi\left(1-\rho^{smooth}\right)} \left(\frac{Y_t}{Y_{t-1}}\right)^{\phi^y\left(1-\rho^{smooth}\right)} \epsilon_{mp,t}, \tag{31}$$

with smoothing parameter ρ^{smooth}. The term ϕ^π is the weight given to inflation and ϕ^y to output growth. Furthermore, the term $\epsilon_{mp,t}$ represents an unexpected monetary policy shock.

2.7.3 Intervention authority

In order to investigate government bond purchases, I introduce an intervention authority that is assigned to the public sector. It has full credibility and is able to issue riskless short-term debt. The reason why I introduce an intervention authority is that I want to sever the direct link to taxes, on the one hand, and do not want to assign the policies solely to the central bank, on the other.[21] In the end, the balance sheet of the intervention authority ($Q_t^{B,gov} B_t^{gov,IA} = B_t^{IA}$) feeds into the public sector's balance sheet. The profits arise as the difference in the returns and the costs $\mathcal{P}_t^{IA} = r_t^{B,gov} Q_{t-1}^{B,gov} B_{t-1}^{gov,IA} \frac{1}{\pi_t} - i_{t-1} B_{t-1}^{IA} \frac{1}{\pi_t}$. Government bond purchases are induced into the model as shocks $\epsilon_{IA,t}$ on the stock of government bonds which are held by the intervention authority

$$B_t^{gov,IA} = \rho_{IA} B_{t-1}^{gov,IA} + \sum_{i=0}^{N} \epsilon_{IA,t-i}, \tag{32}$$

where ρ_{IA} controls how long the intervention authority holds the stock of government debt. The last term on the right-hand side is able to reflect announced purchases for $i>0$. The stock of government bonds in the steady state is zero.

[21] Profits can be redistributed to the fiscal authority or, in appropriate circumstances, losses are balanced by the fiscal authority.

2.8 Market clearing

In the following equation, I present the market clearing condition for the economy

$$
\begin{aligned}
Y_t &= I_t^A + I_t^B + C_t + G_t + K_{t-1}^A \, \Gamma_t^A + \Gamma_t^B \, K_{t-1}^B \\
&+ K_{t-1}^A \, Q_{t-1}^A \, \frac{\left(1 + R_t^{k,A}\right) \, G(\omega_t^A) \, \mu^{f,A}}{\pi_t} + K_{t-1}^B \, Q_{t-1}^B \, \frac{\left(1 + R_t^{k,B}\right) \, G(\omega_t^B) \, \mu^{f,B}}{\pi_t} .
\end{aligned} \tag{33}
$$

Investment spending by type A and type B entrepreneurs constitutes aggregate investment $I_t^A + I_t^B$. Costs resulting from changes in the utilization rates in both sectors are expressed as $K_{t-1}^A \, \Gamma_t^A + \Gamma_t^B \, K_{t-1}^B$. In addition, monitoring type A and B entrepreneurs by the financial intermediaries absorbs resources, which is embodied in the second line of Equation (33). The market for physical capital clears by equating capital supply and capital demand $\widehat{K}_t^e = \widetilde{K}_t^e$.

In terms of asset holdings, a continuum of households meets a continuum of lending banks. The market for corporate bonds clears by introducing mutual funds in the intermediation process, which hold the market portfolio, $\int_0^1 B_{n,t}^{corp} dn = B_t^{corp,B} = \int_0^{\varrho} B_{m,t}^{corp} dm$, where $B_t^{corp,B}$ denotes aggregate holdings of banks. In the market for loans, it is also assumed that each lending bank holds the market portfolio of loans.[22] The market clearing condition results as $\int_0^1 L_{n,t} dn = \int_{\varrho}^1 L_{m,t} dm$. Regarding the asset market for government bonds, the demand for assets resulting from the continuum of households and banks equals the supply of government bonds, $\int_0^1 B_{h,t}^{gov} dh + \int_0^1 B_{n,t}^{gov} dn + B_t^{gov,IA} = B_t^{gov,H} + B_t^{gov,B} + B_t^{gov,IA} = B_t^{gov}$. Accordingly, the market for the intervention authority's bonds clears, $\int_0^1 B_{h,t}^{IA} dh = B_t^{IA}$. The deposit rate r_t^D is linked to the policy rate.

2.9 The source of effects from government bond purchases

In general, a necessary condition for public asset purchases to be effective is that the Wallace Irrelevance proposition of full arbitrage does not hold, i.e. limits to arbitrage exist (Chen et al., 2012; Christiano and Ikeda, 2013; Eggertsson and Woodford, 2003). Hence, the main channel through which asset purchases work is by influencing the relative price of assets which are imperfect substitutes (Andrés, Lopez-Salido, and Nelson, 2004). As a reflection of the bond purchases, the quantity available to investors changes and the returns of the target asset diminish with the consequence that other assets seem to be preferable in terms of returns. Since limits to arbitrage render the assets imperfect substitutes, the related adjustment processes also reduce the returns of the other assets under consideration. This argument can also be applied to term premia as long as market fragmentation across maturities causes deviations from the expectation hypothesis (Vayanos and Vila, 2009). In the present model, portfolio costs in the household sector and financial frictions in the banking sector are the source of market segmentation.

Regarding government bonds, limits to arbitrage arise from two distinct domains. As discussed in Section 2.1, households are faced with portfolio costs and have to bear costs if their portfolio holdings of government bonds deviate from the desired level (see Equation

[22]For technical reasons, as for corporate bonds, I need an aggregator that guarantees the same payoff per unit of loans.

(7)). Both frictions make government bonds imperfect substitutes for short-term assets and prevent full arbitrage, i.e. cause spreads between the yield on government bonds and interest rates of short-term assets.

$$E_t\beta\frac{\lambda_{t+1}b_{t+1}}{\lambda_t b_t}\left(\frac{r_{t+1}^{B,gov}-r_t^D}{\pi_{t+1}}\right)=\upsilon^{B,gov}\left(B_t^{gov,H}-B_s^{gov,H}\right)+\tau^{B,gov} \tag{34}$$

In addition to the leverage constraint, different diversion shares related to the assets held by banks also prevents full arbitrage between the (expected) returns on corporate bonds, government bonds and loans, and the interest rate on short-term assets.[23]

$$\theta^{IC}\left(L_t+\Delta^{B,corp}Q_t^{B,corp}B_t^{corp}+\Delta^{B,gov}Q_t^{B,gov}B_t^{gov}\right) \tag{35}$$
$$=E_t\Lambda_{t+1}\Omega_{t+1}\frac{1}{\pi_{t+1}}\left(R_t^L L_t+R_{t+1}^{B,corp}Q_t^{B,corp}B_t^{corp}+R_{t+1}^{B,gov}Q_t^{B,gov}B_t^{gov}-\left(R_t^D E_t^I-\Upsilon_t^L\right)\right)$$

Different diversion shares, however, also drive wedges between the returns on assets held by the banks. By combining the first-order conditions from the optimisation problem in the banking sector, it turns out that the spreads between (expected) returns on corporate and government bonds and between the loan rate and the (expected) return on government bonds differ by a ratio comprising the diversion shares directed to each asset.

$$E_t\Lambda_{t+1}\Omega_{t+1}\left(\frac{r_{t+1}^{B,corp}-r_{t+1}^{B,gov}}{\pi_{t+1}}\right)=\frac{\left(\Delta^{B,corp}-\Delta^{B,gov}\right)}{\left(1-\Delta^{B,gov}\right)}E_t\Lambda_{t+1}\Omega_{t+1}\left(\frac{r_t^L-r_{t+1}^{B,gov}}{\pi_{t+1}}\right) \tag{36}$$

As can be seen in Equations (34), (35) and (36), the frictions in both sectors drive wedges between the returns on short-term assets and on government bonds and between the returns on loans, corporate bonds, and government bonds. In order to assess the relative strength of the portfolio rebalancing and the balance sheet channel it is important to have knowledge about the parameters which drive these channel.

3 Empirical Analysis

3.1 Data

For the estimation I use quarterly data for the euro area. Limited by data availability the period of observation starts in the fourth quarter of 1997 and ends in the third quarter of 2013. Since the aim of this paper is to investigate the effects of government bonds purchases in the euro area, I decide to stop with the period of observation at the end of 2013. During 2014 there was speculation about a broad quantitative easing in the EMU which could have affected asset prices. To reduce the impact of these effects on the estimates, I cut the period of observation in 2013.

For the estimation I make use of 17 variables. I can split the variables used for the estimation into two groups. The first group consists of seven standard macroeconomic

[23] This is the source of the effects from portfolio rebalancing.

time series: GDP, consumption, investment, the rate of inflation, the real wage, total employment, and the policy rate. GDP, consumption and investment are in real terms deflated by their own price deflators and are expressed in per capita terms. The rate of inflation is computed as the quarterly growth in the GDP deflator. The latter is also used to deflate the nominal wage. GDP, consumption and investment are collected from the European Central Bank and originally stem from Eurostat. The nominal wage (per head), total employment, and the policy rate are taken from the 14th update of the Area-wide Model (AWM) database. As proposed by Smets and Wouters (2003), I proxy hours worked by total employement.[24]

The remaining ten variables belonging to the second group are specific to the model. As the model reflects portfolios of government and corporate bonds, I draw on the redemption yields from indices as provided by Merrill Lynch comprising all maturities. In order to remove the impact of the European debt crisis on government bond yields, I choose the redemption yield of German government bonds. For corporate bonds, I take the redemption yield of bonds from non-financial corporations with a BBB credit rating which is Z_t^A in the model. Loans rates Z_t^B stem from ECB's MFI Interest Rate Statistic combined with the ECB's Retail Interest Rate Statistics.[25] Besides the interest rates, I also address time series to the quantities. Regarding loans, corporate and government bonds, I make use of the ECB's balance sheet items covering data from banks in the EMU. Since government bonds are held by banks and households in the model, I try to identifiy the effects outside the banking sector by drawing on the ECB's securities statistics for the entire amount outstanding. Bank equity is reflected by capital and reserves as provided by the ECB's balance sheet items. Regarding net worth of entrepreneurs, I follow Christiano et al. (2014) and proxy net worth by stock price indices. However, I link stock price indices to the present value of the entrepreneurs instead of linking them to net worth. In the model I have two distinct entrepreneurs. For the aggregate net present value, as the sum of the net present values in both sectors, I take the broad EuroStoxx index. By assuming that market-based debt plays a more important role for blue chips, I proxy the net present value of entrepreneurs in the A sector by the EuroStoxx50 index. Stock prices are taken from the ECB's financial markets statistics. The quantities including the stock price indices are deflated by the GDP deflator and expressed in per capita terms. Except for the interest rates, the rate of inflation, and total employment, I compute the logarithmic first difference for all variables. Finally, I remove the sample mean from all time series.

[24]I consequently make use of the transformation in linearised form

$$\hat{E}_t = \frac{\beta}{1+\beta}\hat{E}_{t+1} + \frac{1}{1+\beta}\hat{E}_{t-1} + \frac{\left(1-\beta\gamma^E\right)\left(1-\gamma^E\right)}{\left(1+\beta\right)\gamma^E}\left(\hat{N}_t - \hat{E}_t\right),$$

whereas $\hat{E}_t$ denotes total employment and $\hat{N}_t$ hours worked with hats as log deviations from steady state.

[25]While the ECB's Retail Interest Rate Statistic stops in 2003, the MFI Interest Rate Statistic starts in 2003. The latter replaced in some sense the former. I choose the annualised agreed rate for loans to non-financial corporations (new business coverage) inlcuding all maturities.

3.2 Priors and calibrated parameters

The parameters of the models are estimated with the help of Bayesian techniques as described in An and Schorfheide (2007). Before I discuss the choice of the prior distributions, I present the parameters which I calibrate in the model as given in Table 1. Hours worked in the steady state, N_s, are normalised to be unity. Following results from Smets and Wouters (2003) I set the inverse Frisch elasticity, φ, to 2.5. The depreciation rates in both sectors, δ^e, at 0.025, take the same value that is usually applied in the literature. Following Christiano, Motto, and Rostagno (2010), I choose 0.999 as a value for the time preference rate, β. In analogy to the same reference, I calibrate the tax rates on capital, τ^K, on consumption, τ^C, and on labour, τ^N, to 0.28, 0.2, and 0.45, respectively. The steady-state rate of inflation, π_s, I fix at 1.8 per cent annualised and the steady technology growth, z_s, is fixed at 1.5 per cent annualised, which corresponds roughly to historical averages. Regarding the business failure rates $F(\overline{\omega}^e)$, I take an average value of bankruptcy rates in the euro area of 0.008.[26] The standard deviations of the idiosyncrativ productivity shock in the entrepreneurial sector, σ^e, are also calibrated to be identical in both sectors. The value of 0.26 is close to the number as used in Christiano et al. (2010). The same reference is taken to calibrate the survival rates of entrepreneurs, $p^{E,e}$, which is assumed to be identical in both sectors. Although it seems to be straightforward, the leverage ratio in the banking sector, ϕ^{IE}, is trickier to calibrate. Historical averages from the ECB's balance sheet items indicate a value close to 16. However, the balance sheets of the banks in my model feature just three assets. I experiment with different values and it turns out that, as an outcome of the estimation, data would prefer a number of eight, which I consequently choose. In almost the same manner, it is not straightforward to calibrate the share of government bonds held by banks. Measured against total assets, the share of government bonds (general government) in banks' balance sheets is 0.06, while its share relative to the sum of loans, corporate bonds, and government bonds (total assets in the model) is 0.24. I decide to set the share at 0.13, which is in the middle between both values. Regarding the fiscal sector, I need to calibrate the ratios of government expenditures and of government bonds to output. Based upon empirical averages, I set the former to 0.2, while the latter takes the value of 0.8.

The prior distributions for the model parameters can be found in the left-hand side columns in Table 2 while those of the shock processes, i.e. the autoregressive parameters and the standard deviations of the shocks, are shown in Table 3. Regarding the monetary policy rule, I assign a value of 0.8 to the mean and 0.15 to the standard deviation of the Beta distribution for the interest rate smoothing parameter ρ_π. The weight on inflation, ϕ_π is given a mean of 1.7 and a standard deviation of 0.1, while the prior mean for the weight on output growth ϕ_y is set to 0.1 with a standard deviation of 0.05. These values are largely in line with Smets and Wouters (2003). The means for price and wage stickiness, γ and γ_w respectively, with values of 0.7 are a bit smaller compared to values from the literature. Since the sample starts at the end of the 1990s, this choice reflects the fact that price and wage stickiness might have changed over the past decades. The standard deviation under the Beta distribution is 0.05 in each case. In addition to the stickiness parameters I also estimate the price and wage mark-up, λ_p and λ_w respectively. In this respect, I start from values for the mean which are close to calibrations in Smets

[26] The data derive from various publications by Creditreform.

Table 1: Calibration of parameters

Description	Symbol	Value
Discount factor	β	0.999
Steady-state labour input in goods' production	N_s	1
Inverse Frisch elasticity	ϕ	2.5
Depreciation rate - type A and type B entrepreneurs	$\delta^A = \delta^B$	0.025
Steady-state rate of inflation, annualised	π_s	1.8 %
Steady-state growth rate, annualised	z_s	1.5 %
Share of government expenditures on steady-state output	G/Y	0.2
Steady-state indebtedness of government relative to steady-state output	$Q^{B,gov}B^{gov}/Y$	0.8
Business failure rates in steady state	$F(\omega_s^A)$,$F(\omega_s^B)$	0.008
Standard deviation of idiosyncratic productivity parameters	σ_s^A,σ_s^B	0.26
Survival rates of entrepreneurs	$p^{E,A}$, $p^{E,B}$	0.978
Leverage ratio	ϕ^{IE}	8
Tax rate on capital	τ^K	0.28
Tax rate on consumption	τ^C	0.2
Tax rate on labour	τ^N	0.45

and Wouters (2003) and Christiano et al. (2010). I choose values of 1.2 for the means of a Beta distribution which is bounded to be within the range of 1 and 2 with a standard deviation of 0.1. To the weights on past inflation in price and wage indexation, ξ and ξ_w respectively, I assign means of 0.15 with a standard deviations of 0.15 under the Beta distribution, which is in line with Christiano et al. (2010). The same is true of the weight on technology growth ξ_z in the indexation rule for wages. The prior mean for habit persistence in consumption h^C is set to be 0.7 with a standard deviation of 0.15 which is close to the literature.

Analogously, the means for investment adjustment costs Ψ'' and costs related to varying the capital utilization are set. The former takes the value of 4 with a standard deviation of 1.5, while the latter is 5 with a standard deviation of 2. I also estimate the power on capital in the production function α and choose the conventional value of 0.3 as a mean with related standard deviation of 0.15. Since I modified the production function, I am interested in estimating the two new parameters. These are the parameters which control the degree of substitution between the two types of capital, γ^K, and the weight ζ^K in the capital bundler. The latter is easier to choose as it also controls the share of loans relative to corporate bonds. For this reason I take a value of 0.1 as a mean which reflects that fact that loans dominate in the euro area. The standard deviation of 0.05 for the normal distribution allows a relatively high variation around. Since the mean for the degree of substitution controls how much capital financed by loans can be replaced by capital financed by corporate bonds, I take a conservative value of 3 and a standard deviation of 1, which states that the degree of substitution is neither complete nor absolutely imperfect. As a prior distribution I choose the Gamma distribution.

Although it is not completely new, the inclusion of a tax rule is new to the model setting. Regarding the parameters controlling the response of taxes on changes in government debt, ξ^{BG}, I take a rather small value of 0.1 as a mean, which just guarantees that fiscal policy is passive. For the Calvo employment parameter γ_E to match total employment with hours worked I follow Smets and Wouters (2003) and work with a mean of 0.5 and a standard deviation of 0.15. Fairly standard priors are used for monitoring costs in both sectors μ^e. The means become 0.2 with standard deviations of 0.05.

Table 2: Model priors and estimated posteriors (parameters)

		Prior Distribution				Posterior Distribution			
		Prior Density	Domain	Mean	SD	Mode	Mean	5 percent	95 percent
ρ_π	Coeff. on lagged interest rate	beta	[0, 1)	0.8	0.15	0.781	0.776	0.739	0.814
ϕ_π	Weight on inflation in Taylor rule	gamma	$\mathbb{R}^+$	1.7	0.1	1.680	1.691	1.550	1.824
ϕ_y	Weight on output growth in Taylor rule	gamma	$\mathbb{R}^+$	0.1	0.05	0.190	0.186	0.083	0.278
γ	Calvo prices	beta	[0, 1)	0.7	0.05	0.776	0.779	0.726	0.827
γ_w	Calvo wages	beta	[0, 1)	0.7	0.05	0.701	0.706	0.637	0.776
λ_p	Steady-state mark-up, prices	beta	(1, 2)	1.2	0.1	1.632	1.634	1.519	1.751
λ_w	Mark-up, wages	beta	(1, 2)	1.2	0.1	1.163	1.195	1.063	1.316
ξ	Weight on past inflation, prices	beta	[0, 1)	0.5	0.15	0.087	0.101	0.033	0.171
ξ_w	Weight on past inflation, wages	beta	[0, 1)	0.5	0.15	0.116	0.142	0.048	0.232
ξ_z	Weight on technology growth, wages	beta	[0, 1)	0.5	0.15	0.654	0.653	0.498	0.805
h^C	Habit persistence parameter	beta	[0, 1)	0.7	0.15	0.652	0.649	0.570	0.723
Ψ''	Investment adjustment costs	gamma	$\mathbb{R}^+$	4	1.5	4.220	4.367	2.756	5.958
ψ_n	Capital utilization costs	gamma	$\mathbb{R}^+$	5	2	3.143	4.087	1.379	6.778
α	Power on capital in production function	beta	[0, 1)	0.3	0.15	0.346	0.345	0.293	0.396
γ^K	Parameter of substitution of capital	gamma	$\mathbb{R}^+$	3	1	3.570	4.023	2.078	5.827
ζ^K	Share of capital sector A	normal	$\mathbb{R}$	0.1	0.05	0.026	0.029	0.006	0.050
ξ^{BG}	Response on debt	gamma	$\mathbb{R}^+$	0.1	0.05	0.243	0.286	0.143	0.415
γ_E	Persistency in labour	beta	[0, 1)	0.5	0.15	0.617	0.625	0.570	0.682
μ^A	Monitoring costs, sector A	beta	[0, 1)	0.2	0.05	0.065	0.065	0.040	0.091
μ^B	Monitoring costs, sector B	beta	[0, 1)	0.2	0.05	0.082	0.081	0.053	0.108
$\upsilon^{B,gov}$	Portfolio costs for long-term gov. bonds	normal	$\mathbb{R}$	0.5	0.25	0.162	0.207	0.062	0.358
p^B	Survival rate of bankers	beta	[0, 1)	0.9	0.05	0.972	0.955	0.927	0.983
ρ^B	Maturity parameter in corporate bonds	beta	[0, 1)	0.95	0.025	0.927	0.919	0.871	0.968
$\rho^{B,gov}$	Maturity parameter in government bonds	beta	[0, 1)	0.95	0.025	0.884	0.878	0.838	0.920
$r^{B,gov}$	Steady-state government bond rate	beta	[0, 1)	0.01	0.0025	0.013	0.014	0.012	0.016
r^B	Steady-state corporate bond rate	beta	[0, 1)	0.01	0.0025	0.010	0.010	0.008	0.012

Table 3: Model priors and estimated posteriors (shocks)

		Prior Distribution				Posterior Distribution			
		Prior Density	Domain	Mean	SD	Mode	Mean	5 percent	95 percent
Autoregressive parameters of shocks									
ρ_A	Transitory technology shock	beta	[0, 1)	0.75	0.15	0.946	0.930	0.891	0.971
ρ_N	Labour supply shock	beta	[0, 1)	0.75	0.15	0.991	0.988	0.978	0.998
ρ_G	Gov. spending shock	beta	[0, 1)	0.75	0.15	0.898	0.890	0.816	0.963
ρ_z	Persistent technology shock	beta	[0, 1)	0.75	0.15	0.234	0.247	0.111	0.377
$\rho_{\sigma,B}$	Riskiness shock, sector B	beta	[0, 1)	0.75	0.15	0.908	0.904	0.878	0.932
$\rho_{\sigma,A}$	Riskiness shock, sector B	beta	[0, 1)	0.75	0.15	0.769	0.754	0.650	0.863
ρ_λ	Price mark-up shock	beta	[0, 1)	0.75	0.15	0.136	0.146	0.062	0.227
ρ_I	Marginal effic. of invest. shock	beta	[0, 1)	0.75	0.15	0.358	0.374	0.228	0.525
Standard deviations of shocks									
ϵ_M	Monetary policy shock	invg	$\mathbb{R}^+$	0.002	0.01	0.001	0.001	0.0010	0.0014
ϵ_G	Gov. expenditures shock	invg	$\mathbb{R}^+$	0.005	0.01	0.016	0.016	0.014	0.019
ϵ_A	Transitory technology shock	invg	$\mathbb{R}^+$	0.005	0.01	0.003	0.004	0.003	0.004
ϵ_z	Persistent technology shock	invg	$\mathbb{R}^+$	0.005	0.01	0.004	0.004	0.003	0.005
ϵ_θ	Price mark-up shock	invg	$\mathbb{R}^+$	0.002	0.01	0.009	0.009	0.007	0.012
ϵ_N	Labour supply shock	invg	$\mathbb{R}^+$	0.01	0.01	0.024	0.026	0.020	0.032
ϵ_{σ^A}	Risk shock, sector A	invg	$\mathbb{R}^+$	0.005	0.01	0.125	0.132	0.104	0.158
ϵ_{σ^B}	Risk shock, sector B	invg	$\mathbb{R}^+$	0.005	0.01	0.076	0.079	0.064	0.094
ϵ_{EI}	Bank equity shock	invg	$\mathbb{R}^+$	0.005	0.01	0.012	0.013	0.007	0.018
ϵ_{γ^A}	Wealth shock, sector A	invg	$\mathbb{R}^+$	0.005	0.01	0.046	0.048	0.041	0.054
ϵ_{γ^B}	Wealth shock, sector B	invg	$\mathbb{R}^+$	0.005	0.01	0.006	0.006	0.005	0.007
$\epsilon_{\mu,I}$	Marginal effic. of invest. shock	invg	$\mathbb{R}^+$	0.005	0.01	0.011	0.012	0.010	0.014
ϵ_τ	Shock on taxes	invg	$\mathbb{R}^+$	0.005	0.01	0.043	0.045	0.036	0.053
Standard deviations of measurement errors									
	Net worth, aggregate	uniform	(0, 10)	5	2.8868	0.125	0.127	0.108	0.143
	Net worth, sector A	uniform	(0, 10)	5	2.8868	0.134	0.137	0.117	0.157
	Yields on long-term gov. bonds	uniform	(0, 10)	5	2.8868	1.009	1.043	0.875	1.200
	Growth of long-term bond in banks' balance sheet	uniform	(0, 10)	5	2.8868	0.031	0.033	0.028	0.038

The remaining parameters are specific to the model. For the costs of deviating from the desired level of government bonds holdings in the government bond sector, $\upsilon^{B,gov}$, I choose a value of 0.5 as a mean with a standard deviation of 0.25 under the Gamma distribution. In the banking sector, I decide to estimate the survival rate of bankers p^B in contrast to the strategy applied to the entrepreneurs.[27] Given that I expect a rather high parameter for the survival rate, I nevertheless set the mean to 0.9 with a standard deviation of 0.05, so that I allow for a broader range of values but do not deviate too much from values in the literature used for the calibration. Although I know the empirical average maturities (durations) of the yields on government and corporate bonds, I nevertheless decide to estimate them in the model. The reason for this is that the model assumption about the implementation of maturities does not exactly match real world features. For this reason, I let the data speak to identify them. I opt for priors with means of 0.95 and standard deviations of 0.025 under the Beta distribution. Furthermore, I estimate the steady-state rates for government bonds and the default-free rate for corporate bonds, $r^{B,gov}$ and r^B, respectively. The reason for this is that the default-free corporate bond rate also controls frictions in the type A sector and the banking sector. In the latter case, it helps to pin down the diversion share related to corporate bonds $\Delta^{B,corp}$. The same argument is applied to the steady-state rate for government bonds, as it is related to portfolio costs in the household sector τ^{gov} and to the diversion share related to corporate bonds $\Delta^{B,gov}$ in the banking sector. For both cases, I choose a mean of 0.01 implying annualised interest rates of 4%. The standard deviations become 0.0025.

For the autoregressive parameters of the shock processes, I opt for conventional priors of 0.75 as means and 0.15 as standard deviations. For the standard deviations of the shocks, I make use of the Inverse Gamma distribution with means of 0.005 with the exceptions of the monetary policy shock, the price mark-up shock, and the consumer preference shock, with 0.002 for each, as well as for the labour supply shock with a mean of 0.01. I furthermore introduce four measurement errors. Two are related to entrepreneurial net worth and are inspired by Christiano et al. (2014). In addition, I include measurement errors for the yields on government bonds and the growth of the stock for long-term government bonds in banks' balance sheets. The justification for the latter is related to the fact that specific agents might hold government bonds in reality which behavior cannot be captured in the model.

3.3 Posteriors

The mode and the mean together with the 5% and the 95% highest probability density intervals for the model parameters can be found on the right-hand side in Table 2 and for the shock processes in Table 3. Most of the estimates for the standard parameters are in line with earlier findings in the literature. As known for the euro area, price stickiness exceeds wage stickiness, i.e. γ is 0.776 at its mode while it is 0.701 for γ_w. Accordingly, the mark-up for prices is with a mode of 1.632 larger than the mark-up for wages 1.163. Furthermore, the largest weight in both indexation rules, for prices and wages, is assigned to steady-state inflation. It is important to note that wages are more tied to current technology growth than to its steady-state rate. Compared to the literature, investment

[27] In the entrepreneurial sector it is difficult to identify all of the related parameters, which is why I do not estimate the survival rates in this sector.

adjustment costs are rather low in the model (a mode of 4.22). The reason for this might be related to the introduction of the banking sector, which introduces frictions into capital production on its own.[28] While the share of capital in production is broadly consistent with the well-known figure in the literature (a mode of 0.346), I am more interested in the estimates for the parameters introduced through the modification of the production functions. Compared with my prior for the degree of substitution, the mode and the mean are slightly above this number (3.57 and 4.023, respectively), which indicates that loans and corporate bonds are not completely imperfect substitutes. However, the share of capital financed by corporate bonds is quite small with a figure of 0.026 at the mode. The rather small share of corporate bonds can be explained by the fact that the loan sector is related to book value accounting, while the corporate bond sector refers to mark-to-market accounting in banks balance sheets. Since ζ^K controls the share from loans to corporate bonds in banks' balance sheets, I see the clear dominance of loans priced at par. Regarding lending to the non-financial sector, the balance sheet channel does not seem to play an important role in the euro area. The mode for the survival rate of bankers, at a figure of 0.972, is close to values used in the literature for calibration.

The monitoring costs in both non-financial sectors are very close to each other. Consistent with De Fiore and Uhlig (2011), for instance, monitoring of corporate bonds seems to be less costly than monitoring of bank loans by looking at the modes. The estimates for monitoring costs in the loan sector are with a value of around 0.065 closer to the calibrated values in Bernanke et al. (1999) than to more recent estimates for the US (see Christiano et al., 2014). The reason for this might be that the European economies are more bank-financed with strong customer relationships, which might reduce monitoring costs.

Regarding the effectiveness of government bond purchases, the degree of frictions in the banking sector and the household sector is of importance. Through the estimate for the steady-state government bond yield the trading cost parameter in the household sector τ^{gov} 0.0034. The estimate for the steady-state default-free corporate bond rate is 3.92% annualised, which translates into an asset-specific diversion share of $1.0477 \times 0.0938 = 0.0983$ ($\Delta^{B,corp} \times \theta^{IC}$). Following from the mode for the steady-state government bond yield of 5.08% annualised, the asset-specifc diversion share related to government bonds becomes $6.8477 \times 0.0938 = 0.6422$ ($\Delta^{B,gov} \times \theta^{IC}$) which shows that frictions related to government bonds are larger than to loans or corporate bonds. Furthermore, the estimate for the portfolio deviation costs in the household sector $\upsilon^{B,gov}$ of 0.162 at the mode shows that limits to arbitrage also exist in the household sector regarding the pricing of government bonds. The parameters representing the maturity structure are 0.884 (a duration of roughly two years) for government bonds and 0.927 (a duration of roughly three years) for corporate bonds.[29] In the next section, I inspect the dynamics resulting from the parameter estimates.

[28]Since it is beyond the scope of this paper to discuss this relationship I take the results as granted.

[29]A reason why these values fall short of the average duration from the market portfolio might be related to the fact that the estimates could capture further features not reflected in the model.

4 Results

In this section, I will discuss the effects of government bond purchases conducted by a public agency which I have called the "intervention authority" to investigate how the purchases affect leverage-constrained financial and non-financial sectors along with the consequences for the real economy.

4.1 Effects of government bond purchases on macroeconomy and financial health

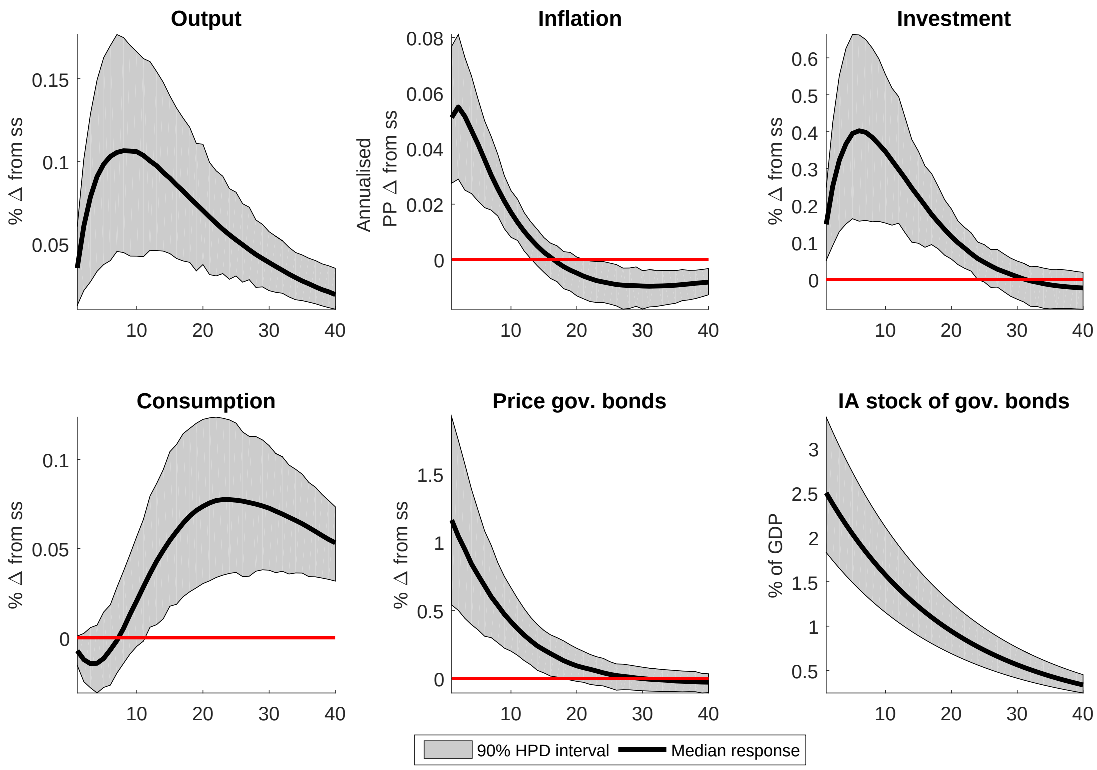
4.1.1 Baseline effects from one-off programme

First, I present the effects of purchases conducted in one period, where the stock of bonds held by the intervention authority dissipates over time. In Figures 2 and 3, the solid black lines represent the effects of government bond purchases amounting to 2.5% of GDP. The responses are the medians and are surrounded by their 90% highest posterior density intervals (grey areas). Figure 2 shows the responses of output, inflation, investment, consumption, the price of government bonds, and the trajectory for the stock of public intermediated government bonds. Figure 3 presents the responses of the net worth of banks, the (aggregate) net worth of entrepreneurs together with the corresponding leverage ratios and spreads; it also depicts lending to entrepreneurs.

The model is able to distinguish between different spreads. The (external) finance premium for the non-financial sector, defined as $\left(1 + E_t\left(R^{k,e}_{t+1}\right)\right) / \left(1 + r^e_t\right)$, measures the costs related to the indebtedness of entrepreneurs. It captures the difference between the return on capital and the costs of external finance, i.e. for issuing corporate bonds in the capital market in the case of entrepreneur A and bank borrowing for entrepreneur B. This spread is related to expected defaults in the non-financial sector. Furthermore, the bank profit margin reflects financial conditions related to bank leverage by taking the asset-specific conditions into account. It is defined as $\left(1 + r^e_t\right) / \left(1 + r^D_t\right)$, and is the respective spread between the returns on assets and banks' costs for external funds. This spread mainly reflects the demand-supply schedule of credit. From this point of view, it resembles the "excess bond premium" from Gilchrist and Zakrajsek (2012). I label this spread "profit margin" because it simply dominates the profit situation in the banking sector from investing in assets. Both spreads together, the finance premium and the profit margin, constitute what I call the "credit spread", which reflects the overall costs of financial intermediation as a result of the two frictions, $\left(1 + E_t\left(R^{k,e}_{t+1}\right)\right) / \left(1 + r^D_t\right)$. Accordingly, the credit spread comprises financing conditions of entrepreneurs and of banks. The aggregate credit spread is weighted by sector-specific capital. The splitting of the spreads is a clear advantage of this model setup, as it shows how government bond purchases affect financial frictions in the real and the banking sectors differently.

Outright purchases of government bonds increase their price, which stimulates banks' net worth (balance sheet channel). Since bank leverage falls as a consequence, financial frictions in the banking sector are alleviated, as households are willing to provide more funds, and banks raise their credit supply. An increase in credit supply has two consequences: banks demand more corporate bonds, causing the price of corporate bonds to increase on impact and reinforcing the initial stimulus to banks' net worth. This is

Figure 2: Effects of government bond purchases (1)

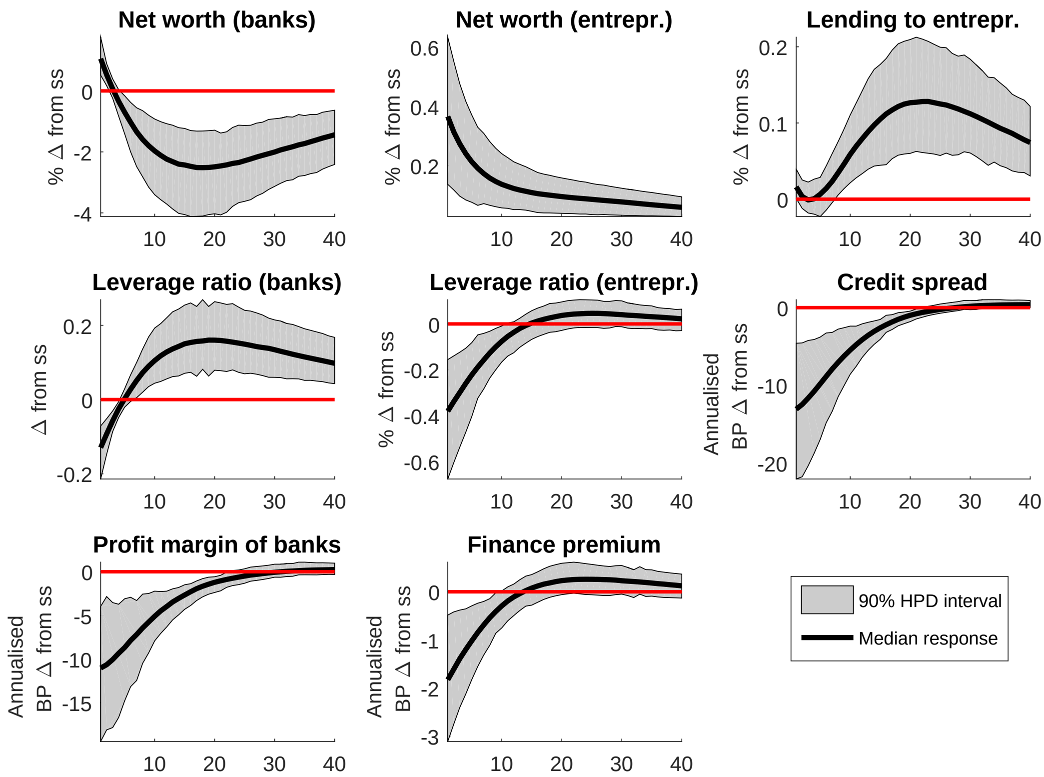


Notes: The figure presents the effects of government bond purchases which are induced as "purchase shocks" as presented in Equation (32) with $N = 0$. The black solid lines represent the median from a purchase shock amounting to 2.5% of gross domestic product (GDP). The grey areas show the 90% highest posterior density intervals.

very similar to the effects described in Gertler and Karadi (2013). The other outcome is that banks increase their supply of loans. As a result, banks' profit margins in both sectors fall, making investment in capital more attractive. The higher demand for capital increases its price, which reduces entrepreneurs' leverage ratio by boosting the value of entrepreneurs' total assets. The finance premiums of entrepreneurs fall as a result, which is the channel discussed by Gilchrist and Zakrajsek (2013). Thus, aggregate investment is boosted on impact, which results in an increase in output and consumption. Government bond purchases eventually improve the financial health of the non-financial sector.

Regarding the financial sector, lower loan rates and bond returns also affect banks' balance sheets. As a consequence of introducing loans priced at par, the balance sheet channel of boosting bank equity by raising asset prices plays a minor role. Hence, lower profit margins in the banking sector translate into lower bank profits, which contracts banks' net worth. Thus, the drop in bank equity resulting from the reduction in lending rates drives banks' leverage ratio upwards in the medium run. Consequently, borrowing constraints are intensified in the banking sector while lending activity generally expands. Asset purchases only stabilise banks' balance sheets in the initial periods of policy implementation. As public intermediation becomes weaker, bank deleveraging even drives bond prices below their steady-state values. By having leverage-constrained non-financial

Figure 3: Effects of government bond purchases (2)



Notes: The figure presents the effects of government bond purchases which are induced as "purchase shocks" as presented in Equation (32) with $N = 0$. The black solid lines represent the median from a purchase shock amounting to 2.5% of gross domestic product (GDP). The grey areas show the 90% highest posterior density intervals.

firms and banks in conjunction with pricing loans at par in banks' balance sheets, government bond purchases affect the financial health of both sectors in opposit directions. They alleviate borrowing constraints for non-financial firms, but falling profit margins, as a reflection of portfolio rebalancing effects, undermine the health of the banking sector in the medium run.[30] Nevertheless, the net effect is positive in this model based upon the estimated parameters for the euro area.[31]

4.1.2 The effects of a pre-announced programme

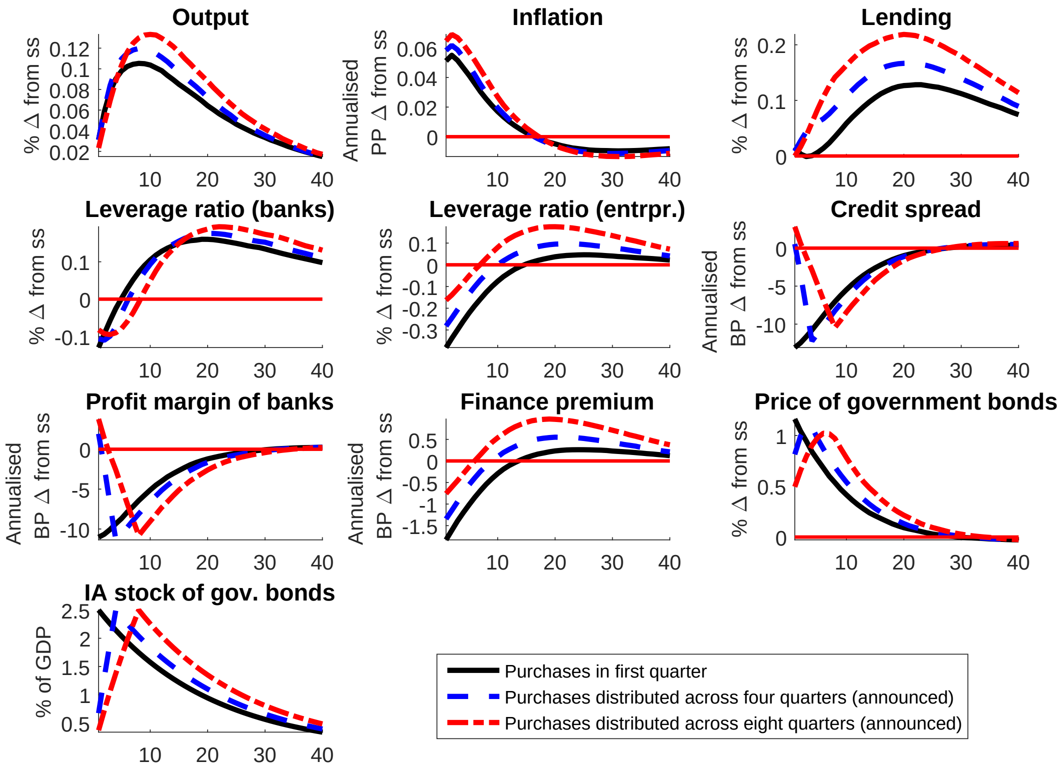
In the previous section, I started to investigate government bond purchases by assuming that the purchases come as a policy surprise (one-off programme) in order to highlight the essential channels. In reality however, central banks announced government bond

[30]Following from portfolio rebalancing, both households and banks reduce their holdings of government bonds; see Figure F in the appendix.

[31]The results are discussed by abstaining from the zero lower bound environment, although government bond purchases are usually introduced when the policy rate reaches its lower bound. Section H compares the zero lower bound scenario with the benchmark scenario. As can be seen, the response of consumption mainy drives the differences between the two cases. Since the lower bound has negligible effects on the main channels stressed in the main text, I have delegated the lower bound case to the technical appendix.

purchases. Exploring how the announcement of a programme affects the responses of an economy with two leverage constraints is the aim of this section. Consequently, I assume that the intervention agency announces the purchases taking place for a specified period. I present three different scenarios in Figure 4. The benchmark scenario - denoted by solid black lines - is the previous case, in which purchases are conducted as a surprise. In addition, I present results for purchases which are announced four (blue dashed lines) and eight quarters (red dashed lines with dots) in advance and are distributed equally over the respective period. All programmes reach the same maximum stock of 2.5% of output.

Figure 4: Comparison of responses to a one-period government bond purchase programme (black solid lines), and previously announced programmes distributed over one (blue dashed lines) and two years (red dashed lines with dots)



Notes: The figure presents the effects of government bond purchases which are induced as "purchase shocks" as presented in Equation (32). The purchases are scaled to achieve a maximal stock of 2.5% of GDP in each case. The responses are median responses from the estimated model.

Similarly to the one-off programme the government bond purchases here ultimately improve borrowing conditions of non-financial firms, which boosts output via investment. Hence, the macroeconomy does not show qualitatively different effects in the face of these programmes compared to the one-off programme. However, anticipation effects do have impact on the financial side of the economy. With the prospect of lower borrowing rates, agents raise their demand for external funds following the announcement of the measures. As a result, the excess demand for credit slightly increases the borrowing rates of the real sector. However, higher profit margins for banks improve their profits, which stimulates the build-up of bank net worth. As opposed to the surprise programme, the build-up of bank net worth accelerates until the period in which the purchases are conducted. The surprise programme raises net worth on impact as a result of the balance sheet channel before net worth starts to shrink again. The improvement in bank net worth peakes at a slightly higher level for the announced programmes. Lending activity to the non-financial sector is bolstered in the announcement cases which underscores that a higher credit demand drives the effects.

Regarding the quantitative responses of the economy, output rises by more for the announced programmes than for the one-off programme. However, the balance sheet of the intervention authority has different time profiles in all cases. For the announced programmes, the purchased stock is obviously held for longer. In order to allow for a fair comparison, I look at output multipliers for evaluating the effectiveness of the government bond purchases. Table 4 reports the (discounted) gain in output as the discounted sum of output deviations from steady state relative to the discounted sum of the stock of government bonds held by the agency. The present value gains are calculated over two different horizons: one year and ten years.[32]

A comparison of the present value gains in Table 4 clearly show that there is a announcement effect. For a horizon of ten years, present value gains of output amount to roughly 1.46% of the (discounted) stock of government bonds held by the intervention authority over the same period in the case where purchases are announced eight quarters in advance. For the surprise case, the respective output gain is 1.36%. It turns out that in the short (a horizon of 1 year) and medium run (10 years) the announcement of a programme produces stronger output effects.

Table 4: Present value gains in output following one-off and announced programmes

in %	1 year	10 years
Purchases in current quarter	0.69	1.36
Distributed across four quarters, announced	1.07	1.42
Distributed across eight quarters, announced	1.62	1.46

Notes: The table shows the present value gains in output over a specified period for government bond purchases. The gains in output are expressed in percentage deviations from steady state and are weighted with the time-preference rate. The present value gain is defined as: $gain = \sum_{k=1}^{K} \beta^k \left(X_{t+k-1} - X_s\right) / \sum_{k=1}^{K} \beta^k \left(Z_{t+k-1}\right) \cdot 100$, with X as output and Z the stock of government bonds held by the central bank.

The main difference between the pre-announced programmes and the surprise programme can be seen in expectational effects. Against the backdrop of the results discussed above, the anticipation effect causes outright purchases to be more successful compared to the one-off programme. Compared to the latter, the financial sector shows a different behaviour as a reflection of these anticipation effects. The financial health of the banking sector first improves and then deteriorates in an environment in which the stabilising effect

[32]This is measured as $gain = \sum_{k=1}^{K} \beta^k \left(X_{t+k-1} - X_s\right) / \sum_{k=1}^{K} \beta^k \left(Z_{t+k-1}\right) \cdot 100$, with X as output and Z the stock of government bond purchases held by the central bank with β as the time-preference rate. This measure is based on the present value multiplier, as it is used to assess the effectiveness of fiscal policies (see, for instance, Mountford and Uhlig, 2009).

of prices, as argued by Gertler and Karadi (2013), is of minor importance.[33] The response of the non-financial sector's financial health mainly depends on the length of the purchases which is announced in advance. For very strong anticipation effects, non-financial firms' leverage can even increase.[34]

4.2 What drives the effects of government bond purchases?

Having discussed the effects of government bond purchases based upon an estimated model of the euro area, I shed more light on the driving forces in this section. I take the estimated model as a point of departure and run several counterfactual experiments by simulating the model at its mode.

4.2.1 Role for evolution of sector-specific net worth

In the presence of two leverage constraints, government bond purchases conducted by the intervention authority tend to relax the borrowing constraint in the corporate sector and make the corresponding constraint in the banking sector more binding. This is predominantly a reflection of changes in net worth. To identify the importance of changes in equity in both sectors I consider cases in which net worth in one sector remains unchanged. The results are presented in Figure 5. The black solid lines reflect the benchmark case while the blue dashed lines show the responses of the economy when bank net worth is kept constant and the red dashed lines with dots when firms' net worth remains unchanged.[35]

For the case without constraints on the path of net worth, banks' leverage ratio first drops and then recovers before overshooting its steady-state level. Since bank net worth is the main driver, banks' leverage ratio is obviously prevented from exceeding its steady-state value if bank net worth does not change. Without an effect on bank net worth, output rises by less on impact compared to the unconstrained case because the improvement in banks' financial health is less elaborated. However, the missing strong rise in banks' leverage in the medium run causes output to remain above the baseline for longer, i.e. the initiated boom lasts for longer, which is also translated into a more persistent rise in the rate of inflation. Although credit spreads fall by less, which is related to an upward shift in the trajectory of firms' leverage ratio, lending activity is stronger than in the benchmark case. The weaker response of output in the medium run for the unconstrained case is related to the fact that banks need to bring their leverage ratio down.

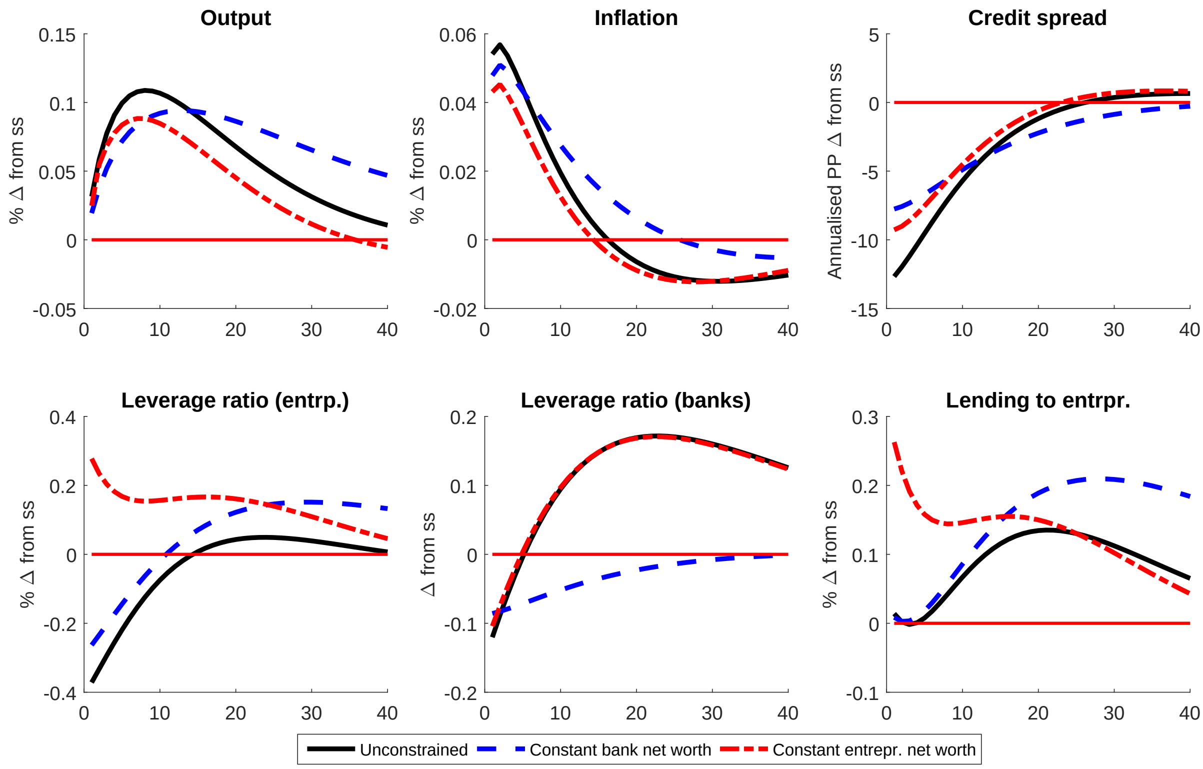
The case where the net worth of entrepreneurs is kept constant has nearly no effect on the leverage of banks. In contrast, the response of firms' leverage ratio is reversed. Firms' leverage rises on impact, which is reflected in a less elaborated drop in the credit spread. Output improves by less and is therefore on a lower trajectory, which also means that it falls below the baseline in the medium run, causing a drop in the trajectory for the rate of inflation. Lending activity accelerates because firms need to finance a larger fraction

[33]Section G in the appendix tries to disentangle stock from flow effects regarding the anticipation of the programme.

[34]It should be noted that full information and rational expectations are important for the finding that the anticipation effects for a long duration of the programme are very strong. The relaxation of these assumptions goes beyond the scope of the paper.

[35]News shocks to net worth are introduced to keep net worth constant in the respective case.

Figure 5: Effects of government bond purchases where bank net worth and entrepreneurial net worth are kept constant



Notes: The figure presents the effects of government bond purchases which are induced as "purchase shocks" as presented in Equation (32) by keeping bank net worth (blue dashed lines) and by keeping entrepreneurial net worth (red dashed lines with dots) constant. These cases are contrasted with the unconstrained benchmark case (black solid lines). The purchases are scaled to achieve a maximal stock of 2.5% of GDP in every case. The responses base upon the simulation of the model at its mode.

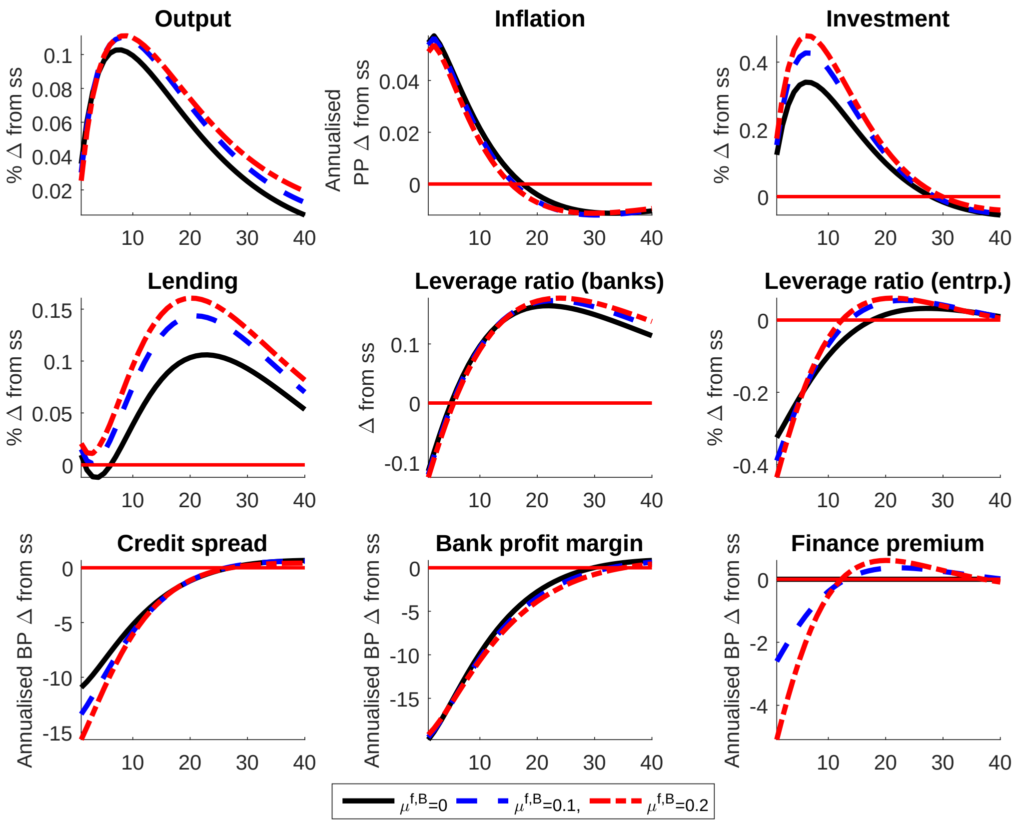
of their investment projects with external funds as the improvement in their net worth is missing. This is eventually the reason why firms' leverage rises.

These results show that the response of bank leverage mainly controls the shape of the output response, while firms' leverage affects the level of the improvements in output. Thus, my results underline that the financial health of both the banking and the non-financial sector is important for government bond purchases to be effective.

4.2.2 The role of financial frictions in the non-financial sector

While it is well-known that the degree of limits to arbitrage controls the effectiveness of government bond purchases, I provided evidence in the previous section that the financial health of the non-financial sector also plays an important role in this regard. This section has the objective of taking a closer look at the role played by the severity of the leverage constraint in the non-financial sector. In the model, financial frictions in the non-financial sector are driven by monitoring costs, as they determine the costs of expected defaults. The notion of credit risk is related to expected defaults. Consequently, costs of expected defaults rise if more resources are lost given the probability of default. Hence, higher monitoring costs make defaults more costly, with the result that the finance premium will react more sensitively to changes in the financial health of non-financial firms. Accordingly, a larger role for a leverage constraint arises if monitoring costs are large. For this reason, I treat changes in monitoring costs as a reflection of different degrees of financial frictions in the non-financial sector.

Figure 6: Comparison of responses to pure bond purchase shocks for different degrees of financial friction in the entrepreneurial sector B

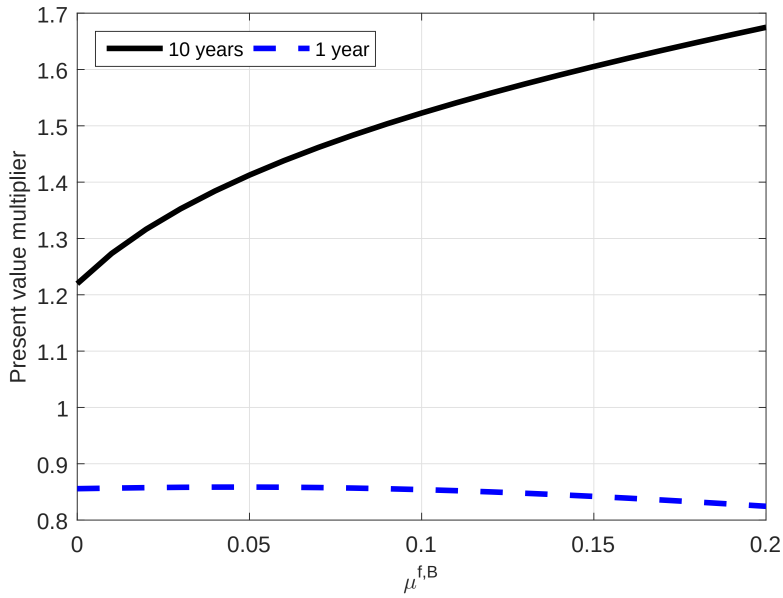


Note: The figure presents the effects of government bond purchases which are induced as "purchase shocks" as presented in Equation (32) with $N = 0$ (black solid lines) for different values of the monitoring costs in the entrepreneurial sector B (μ^B in equation (19)). The responses are based upon the simulation of the model at its mode.

In Figure 6, I present the responses of output, inflation, investment, lending, bank leverage, firm leverage, the (aggregate) credit spread, the overall bank profit margin (covering returns on loans and bonds), and the (aggregate) finance premium on a purchase programme conducted in one period for three cases. In the first case (black solid lines) the monitoring costs in the loan sector are set to zero, which deactivates financial frictions in this sector. For the two other cases, financial frictions in the loan sector (sector B) are activated and set at two different levels ($\mu^{f,B} = 0.1$ blue dashed lines, $\mu^{f,B} = 0.2$ red dashed lines with dots). It turns out that the higher the financial constraints for non-financial firms, the more effective government bond purchases are in supporting output.

As can be seen in Figure 6, the finance premium reacts more sensitively than banks' profit margin to changes in monitoring costs in relative terms. Higher frictions in the credit market for entrepreneurial loans make the finance premium fall more sharply given a drop in entrepreneurial leverage, because the costs of expected defaults decline more strongly.

Figure 7: Impact of financial frictions in the loan sector



Notes: The figure shows the present value gains in output following government bond purchases, which are conducted entirely in one period, by varying monitoring costs for the loan sector. The gains in ouput are expressed in percentage deviations from the steady state and are weighted with the time-preference rate. The present value is defined as: $gain = \sum_{k=1}^{K} \beta^k \left(X_{t+k-1} - X_s\right) / \sum_{k=1}^{K} \beta^k \left(Z_{t+k-1}\right) \cdot 100$, with X as output and Z the stock of government bonds held by the central bank.

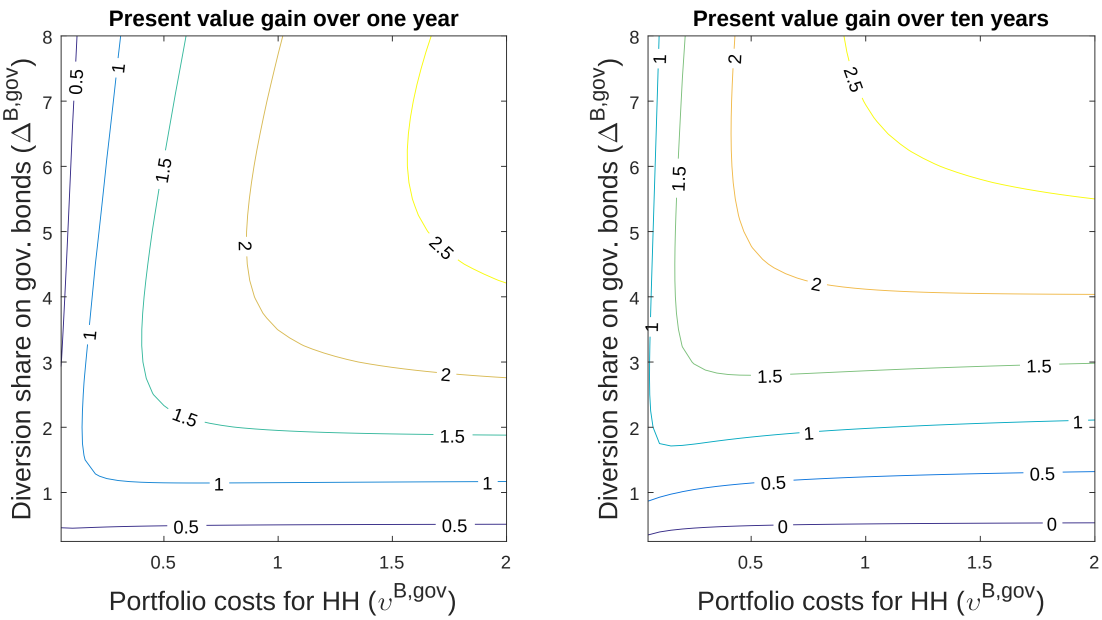
This makes asset purchases more effective if financial frictions in the non-financial sector are quite large. For the "no frictions" case, the accelerating mechanism is not present, with the result that the decrease in banks' profit margin dominates. The stimulus to output is mainly driven by the fall in the loan rate, which is completely unrelated to credit risk. Output, inflation, and investment show different trajectories, because the path of the finance premium is lower.

Figure 7 summarises the impact of frictions (x-axis) on output by looking at the present value multipliers (y-axis) for a one-year and 10-year horizon, as blue dashed and black solid lines, respectively. While the short-run effect on output is rather independent from monitoring costs, the output multiplier clearly increases as monitoring costs rise. Thus, government bond purchases stimulate the economy particularly if the non-financial sector is exposed to strong binding leverage constraints.

4.2.3 The role of market segmentation in conjunction with financial frictions

As is known, from Andrés et al. (2004), for example, limits to arbitrage are crucial for obtaining non-trivial effects of outright purchases. Portfolio adjustment costs for households and the diversion share related to government bonds in the banking sector, $\upsilon^{B,gov}$ and $\Delta^{B,gov}$, respectively, produce limits to arbitrage in the present model. I provided evidence in the previous section that the severity of financial frictions in the non-financial sector are also highly relevant in achieving non-trivial effects on output following government bond purchases. This section aims to shed light on the interplay between the three main parameters which control the impact of government bond purchases.

Figure 8: Impact of limits to arbitrage from frictions in the banking sector and the household sector on the present value gain in output over 1 year (lhs) and 10 years (rhs) resulting from government bond purchases



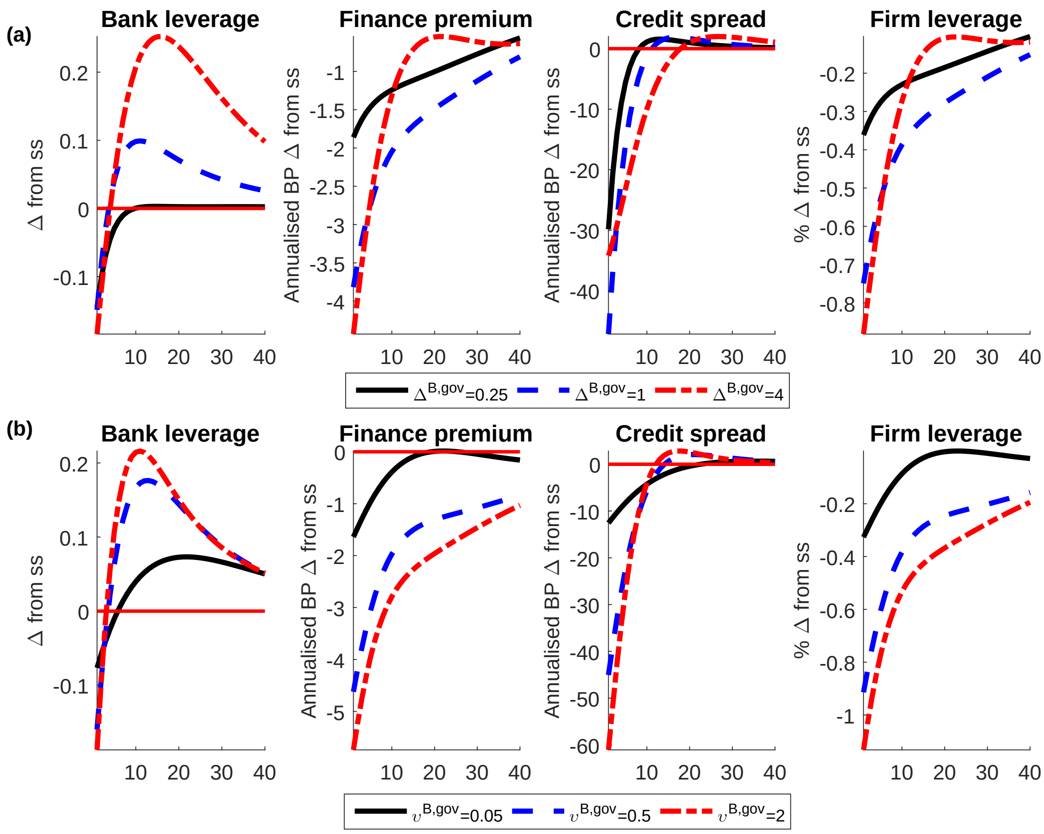
Notes: The figure shows the present value gains in output following government bond purchases, which are conducted entirely in one period, by varying the diversion share related to government bonds in the banking sector (y-axis) and portfolio costs in the household sector (x-axis). The gains in output are expressed in percentage deviations from steady state and are weighted with the time-preference rate. The present value is defined as: $gain = \sum_{k=1}^{K} \beta^k \left(X_{t+k-1} - X_s\right) / \sum_{k=1}^{K} \beta^k \left(Z_{t+k-1}\right) \cdot 100$, with X as output and Z the stock of government bonds held by the central bank.

In Figure 8 I depict the present value gains over one year (left-hand side) and ten years (right-hand side) in output by varying the diversion share in the banking sector related to government bonds on the y-axis and the market segmentation in the household sector on the x-axis. Consistent with earlier findings, the present value gains rise with higher frictions in both sectors.[36] For the shorter horizon, the present value gains in output grow quickly for large frictions in both sectors. With a low level of frictions in one sector, the present value gains are nearly independent from the level of frictions in the other sector. For a longer horizon, the present value gain starts to rise very quickly with larger frictions in the banking sector. Except for the case of nearly no frictions in the household sector (small values for $\upsilon^{B,gov}$), frictions in the banking sector dominate the long-run effects of government bond purchases. This means that it is predominantly the banking sector which affects the pricing of government bonds and therefore controls the medium-run effects on output.

In Figure 9, I show the driving forces. The responses of bank leverage, the finance premium, the credit spread, and the bank profit margin are given for different values of the diversion parameter $\Delta^{B,gov}$ (panel (a)) and for different values for the parameter driving the market fragmentation in the household sector $\upsilon^{B,gov}$ (panel (b)). In both cases,

[36] I do not report results for the case in which the frictions in one sector are deactivated since government bond purchases are ineffective in this case. I refer for this case to Gertler and Karadi (2013) or Christiano and Ikeda (2013).

Figure 9: Effects of different degrees of friction in the banking sector (panel (a)) and in the household sector (panel (b))

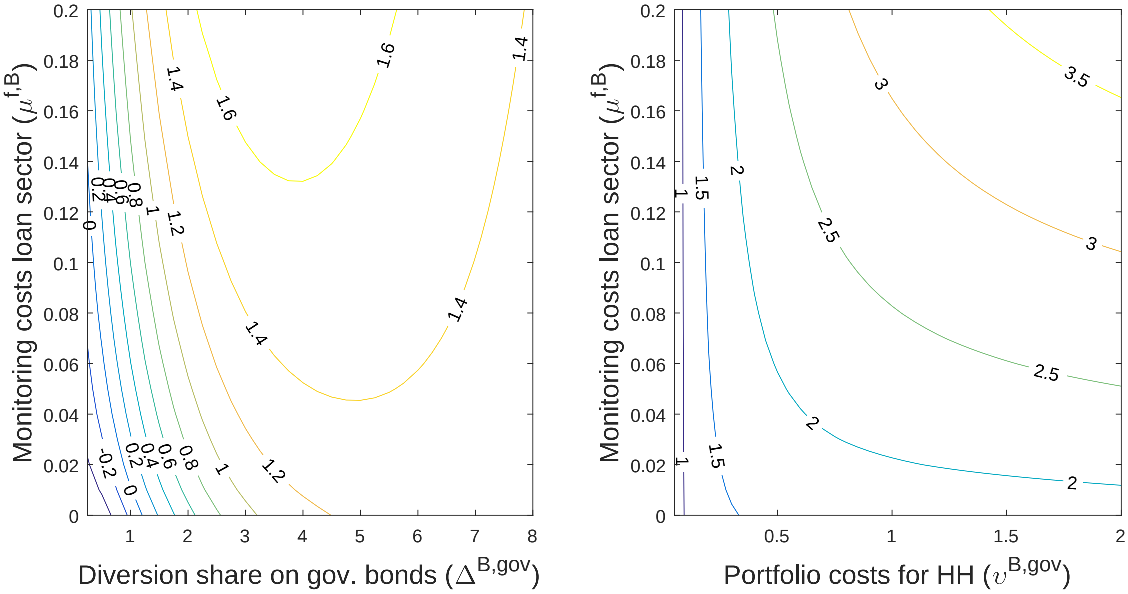


Notes: The figure presents the effects of government bond purchases which are induced as "purchase shocks" as presented in Equation (32) with $N = 0$ (black solid lines) for different values of the monitoring costs in the entrepreneurial sector B (μ^B in equation (19)). The responses are based upon the simulation of the model at its mode.

the qualitative changes of the transmission resulting from larger market segmentation are qualitatively very similar. For larger values of the diversion or the portfolio costs parameter, limits to arbitrage are stronger and the arbitrage between the returns on the assets and the short-term rate is weaker. For low values of the diversion share or the portfolio costs parameter, the degree of imperfect substitution is lower, with the result that the portfolio rebalancing effect becomes weaker. Consequently, lending rates to the non-financial sector fall following government bond purchases, but only for a shorter period before they start to rise. One qualitative difference between the impact of both frictions is that a low level of the diversion share directed to government bonds in the banking sector does not show a strong overshooting of the leverage ratio.

Hence, the government bond purchases become more efficient in boosting output if frictions prevail in both sectors. This result is consistent with earlier findings but also holds for the case of leverage-constrained banks and non-financial firms. Changes in the factors which drive limits to arbitrage with respect to government bonds only control the size of effects on firm leverage, but do not affect its trajectory much. These results are derived by keeping the monitoring costs in the loan sector at its mode from the estimation.

Figure 10: Impact of financial frictions in the loan sector



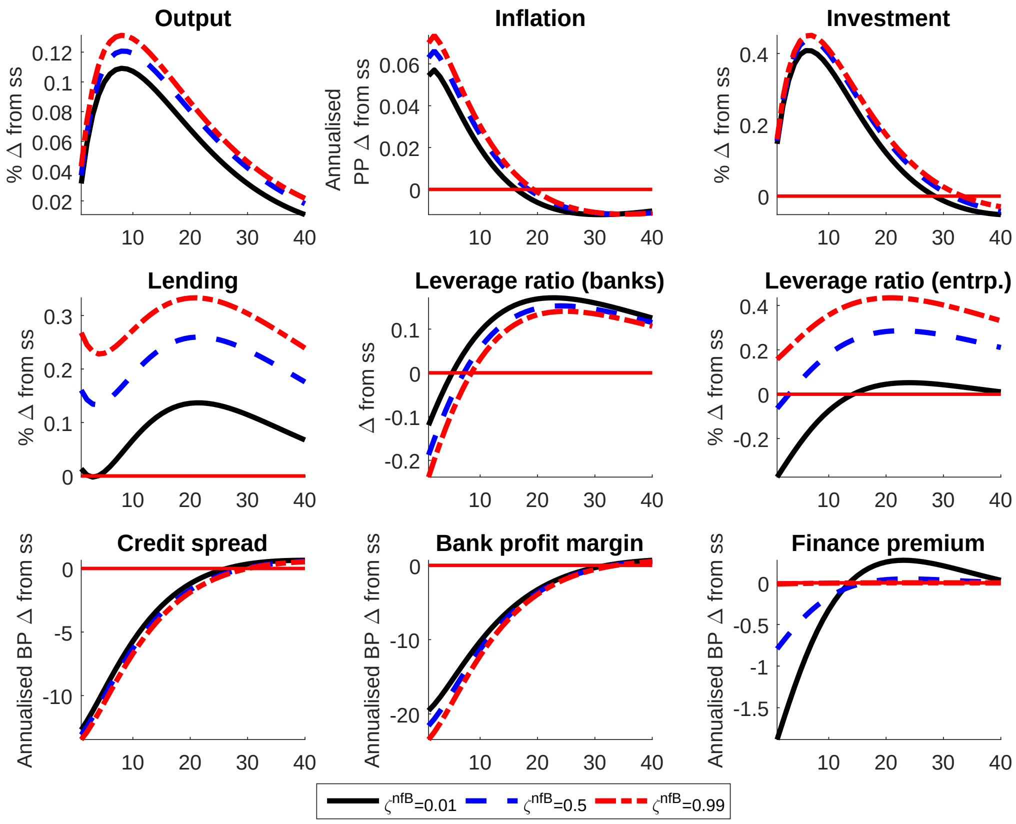
Notes: The figure shows the present value gains in output following government bond purchases, which are conducted entirely in one period, in relation to variations in different frictions. The gains in ouput are expressed in percentage deviations from steady state and are weighted with the time-preference rate. The present value is defined as: $gain = \sum_{k=1}^{K} \beta^k (X_{t+k-1} - X_s) / \sum_{k=1}^{K} \beta^k (Z_{t+k-1}) \cdot 100$, with X as output and Z the stock of government bonds held by the central bank.

Regarding the effectiveness of government bond purchases, the interplay between financial frictions in the non-financial sector and the factors determining the pricing of government bonds plays an essential role. Figure 10 depicts the present value gains in output for an horizon of ten years in relation to the monitoring costs in the loan sector (always on the y-axis). On the left-hand side, the diversion share on government bonds is altered (x-axis) while it is the portfolio costs parameter for the households on the right-hand side (x-axis). It turns out that both monitoring costs and the diversion share control the effectiveness of government bond purchases the most. As long as households contribute to limits to arbitrage in a significant way, the effectiveness of government bond purchases to have sizeable effects on output is driven by leverage constraints in the banking sector but also to a large extent by leverage constraints in the non-financial sector. While the first result is known from the literature, the second one can be derived from my model in which loans priced at par dominate in banks' balance sheets. For low monitoring costs in the loan sector, government bond purchases only have sizeable effects on output in the medium run if there are strong forces in the banking sector which create limits to arbitrage.

4.2.4 Bank-based vs. market-based economy

An essential feature of the model, which is backed by the estimation, is that loans priced at par dominate in banks' balance sheets. In this case, financial frictions in the loan sector have an important impact on the effectiveness of government bond purchases in achieving output gains. To show how the results change if corporate bonds start to dominate in banks' balance sheets, I increase the share of capital financed by corporate bonds, ζ^K as given in Equation (11). In Figure 11, I present the variables' responses to the purchase shock for three different cases. The first case (solid black lines) reflects an economy with nearly no meaning for the balance sheet effect. Given the same maturity structure and monitoring costs, cases two (blue dashed lines) and three (red dashed lines with dots) comprise situations with a higher share of corporate bonds in banks' balance sheets. For the third case, loans play nearly no role. In all cases, the corporate bond is a consol and monitoring costs are set to zero in the corporate bond sector.[37] The share of government bonds in banks' balance sheet remains unchanged.

Figure 11: Dependence of effects of government bond purchases on the share of corporate bonds in banks' balance sheets



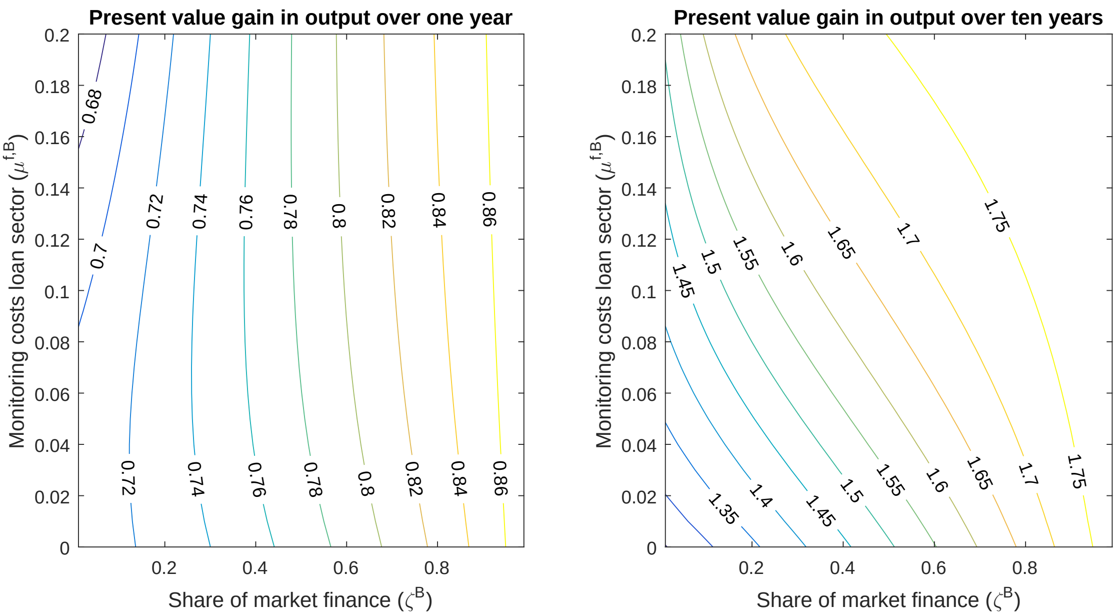
Notes: The figure shows the impulse-response function following purchases of government bonds. For the first case (black solid lines), the parameter γ^K controlling the share of corporate bonds is set to 0.01, for the second case (blue dashed lines) it is 0.5 and for the third case (red dashed lines with dots) 0.99. For all cases, the monitoring costs in the A sector are set to 0 and the parameter controlling the maturity of the corporate bond portfolio ρ^B is set to 1. The responses are based upon the simulation of the model at its mode.

If corporate bonds start to dominate in banks' balance sheets, the balance sheet channel plays an dominating role. Similar to Gertler and Karadi (2013), purchases of government bonds stimulate banks' net worth, amplified through an increase in the price of corporate bonds, which together lower the leverage ratio and relax financial frictions in the banking sector. Output increases by more with higher shares of corporate bonds

[37] The last case is very similar to the benchmark in Gertler and Karadi (2013).

in banks' balance sheets following government bond purchases. In the third case, the response of the finance premium does not react as a reflection of no monitoring costs and a dominance of corporate bonds in banks' balance sheet. Here, the positive impact on bank equity is the strongest. Lending activity is more pronounced compared to the case in which loans dominate in banks' balance sheet as a reflection of the missing effect on the finance premium.

Figure 12: Present value gains in output by varying the share of corporate bonds and the financial frictions in the entrepreneurial sector B for different horizons



Note: The figure shows present value gains in output for government bond purchases which are induced as "purchase shocks" as presented in Equation (32) with $N = 0$ for combinations of the parameter which controls the share of corporate bonds γ^K (x-axis) and monitoring costs in the entrepreneurial sector B (y-axis). The gains in output are measured as deviations weighted with the time-preference rate and expressed in units of purchases in period one per units of steady-state output. The present value gain is defined as: $gain = \sum_{k=1}^{K} \beta^k \left(X_{t+k-1} - X_s\right) / \sum_{k=1}^{K} \beta^k \left(Z_{t+k-1}\right) \cdot 100$, with X as output and Z the stock of government bonds held by the central bank.

Financial frictions in the loan sector and the balance sheet effects of asset price changes interact. In Figure 12, I present the present value gain in output over one year (left-hand side) and ten years (left-hand side) by varying monitoring costs in the loan sector (vertical axis) and the share of market finance (horizontal axis). For longer horizons, the present value output gain is clearly rising with higher monitoring costs and changes in the market structure starting from low levels of monitoring costs and a low share of market finance. However, the present value gains in output become less sensitive to changes in monitoring costs for shorter horizons as the balance sheet channel clearly dominates the short-run effects. For higher levels of monitoring costs and lower shares of market finance the effects of government bond purchases on output are equivalent to larger shares of corporate bonds in banks' balance sheet and low monitoring costs. The largest output gains occur for large shares of market finance and high levels of monitoring costs.

This again shows that the alleviation of financial frictions in the non-financial sector is an important channel for realising output effects compared to stabilising asset prices. In the short run, the balance sheet channel plays a more important role with rising shares of market finance independent from whether frictions in the loan market exist. An increase in the present value multipliers for large shares of market finance and for larger monitoring costs shows that the credit channel can be a substitute for the balance sheet channel in terms of output gains following government bond purchases. Government bond purchases in an economy with a low level of market-finance while having large frictions in the loan sector achieves similar effects as in an economy with a large share of market-finance and a low level of frictions in the loan sector. However, output gains react more sensitively to the increase in market finance compared to an intensification of frictions in the loan sector.

As can be seen, the success of government bond purchases in stimulating output depends heavily on the severity of the leverage constraint in the non-financial sector. This credit risk channel can even be so strong that it is able to compensate for a missing balance sheet channel.

5 Conclusion

In response to a low inflation environment and a slow economic recovery, central banks around the globe started with asset purchases as an additional policy tool to boost economic activity and rates of inflation. In these programmes, purchases of government bonds play the most important role. Reducing the interest rates of medium and long-term maturities is the main objective of this policy. This paper investigates the effects of a reduction in returns on long-term government bond on the soundness of non-financial firms and banks in a New Keynesian DSGE model which is estimated with euro-area data. Both sectors are leverage-constrained, and government bond purchases improve the financial health of the non-financial sector, while this is only true in the short run for the banking sector as a result from the balance sheet channel. In the medium run, banks' profitability deteriorates and undermines the financial health of the banking sector. Nevertheless, positive effects on output and the rate of inflation remain, predominantly as a result of the reduction in non-financial firms' borrowing conditions and are amplified by a related reduction in firms' credit risk. Regarding the latter, this paper is able to highlight the credit risk channel as discussed by Gilchrist and Zakrajsek (2013). I can show that this channel becomes more important, the larger financial frictions in the non-financial sector are. My results provide evidence for the fears that the soundness of the banking sector is affected negatively by government bond purchases. However, these effects do not dominate as long as the non-financial sector is sufficiently balance-sheet constrained despite the fact that assets priced at par dominate in banks' balance sheets.

In the model, I induce the purchases of government bonds as shocks but do not formulate a specific policy rule as in Jones and Kulish (2013). The aim was to shed light on the transmission of government bond purchases through the financial sector against the backdrop of two-sided financial frictions and in an environment where loans priced at par dominate in banks' balance sheets. It is conceivable that some factors might alleviate the negative effects on banks' net worth as banks in reality could start investing in other asset classes not reflected in the model. Moreover, the paper is unable to discuss the circumstances under which the introduction of government bond purchases is useful, nor is it able to discuss further potential side-effects. Hence, the paper does not conduct a welfare analysis directed at government bond purchases as a policy measure. All this goes beyond the scope of the paper.

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A Derivation of the Model

A.1 Households

The household consists of three different groups: bankers, entrepreneurs, and the remaining household members. Similar to Gertler and Kiyotaki (2010) or Gertler and Karadi (2011), the share of household members in the banking sector in each period is s^B. In order to keep the shares constant over time, I assume that exactly the same number of workers become bankers as bankers return to the goods producing sector. The probability of staying a banker p^B is exogenously fixed and does not change over time. The profits each bank manager potentially earns are not transferred to the household before the bank manager leaves the bank, which happens with a probability of $\left(1 - p^B\right)$.[38] In addition, a specific share s^e of households becomes entrepreneurs. Like bank managers, entrepreneurs survive with a probability of $p^{E,e}$. During the time they are entrepreneurs, household members accumulate net wealth, which is transferred back to the household when they leave the entrepreneurial sector.

A.1.1 Utility maximisation

The economy is populated by a continuum of households which are indexed by h with $h \in (0,1)$. Each h-th household decides on the supply of labour, how much to consume and to save, and on the allocation of its wealth. Households' utility function is given in Equation (37)

$$U_0 = E_0^j \sum_{j=0}^{\infty} \beta^j \left[\ln\left(C_{h,t+j} - h^C C_{h,t-1+j}\right) - \kappa \frac{\left(N_{h,t+j}\right)^{1+\varphi}}{1+\varphi} \right] \tag{37}$$

with discount factor β. The term h^C reflects the internal habits in consumption with $h^C \in (0,1)$. The budget constraint in real terms becomes

$$\begin{aligned} & (1+i_{t-1+j})\frac{B^{n,PS}_{h,t-1+j}}{P_{t+j}} + \left(1+r^{B,gov}_{t+j}\right)\frac{Q^{B,gov}_{t-1+j}B^{n,gov}_{h,t-1+j}}{P_{t+j}} \\ + & \left(1+r^D_{t-1+j}\right)\frac{D^n_{h,t-1+j}}{P_{t+j}} + (1-\tau^w)\frac{W_{h,t+j}}{P_{t+j}}N_{h,t+j} + \frac{Div_{h,t+j}}{P_{t+j}} + \Xi_{h,t+j} \\ \geq & \left(1+\tau^C\right)C_{h,t+j} + T_{t+j} + \frac{D^n_{h,t+j}}{P_{t+j}} + \frac{B^{n,PS}_{h,t+j}}{P_{t+j}} + \Theta^{gov,H}_t, \end{aligned}$$

where the superscript n denotes nominal terms. Households pay taxes on their labour income and on their consumption expenditures, $\tau^w$and τ^C, respectively.

Resulting from utility maximisation, I obtain the marginal utility of consumption

$$\frac{\partial U_0}{\partial C_{h,t}} : \left(1+\tau^C\right) b_t \lambda_{h,t} = \left(C_{h,t} - h^C C_{h,t-1}\right)^{-1} - \beta h^C \left(C_{h,t+1} - h^C C_{h,t}\right)^{-1}, \tag{38}$$

[38] The reason why bankers exit lending banks is to guarantee that the lending banks do not accumulate equity indefinitely (see Gertler and Karadi (2011) or Gertler and Kiyotaki (2010)).

the Euler Equation for short-term bonds of the public sector

$$\frac{\partial U_0}{\partial B^{PS}_{h,t}} : 1 = E_t \beta \frac{\lambda_{h,t+1}}{\lambda_{h,t}} \frac{(1+i_t)}{\pi_{t+1}}, \tag{39}$$

the Euler Equation for deposits

$$\frac{\partial U_0}{\partial D_{h,t}} : 1 = E_t \beta \frac{\lambda_{h,t+1}}{\lambda_{h,t}} \frac{(1+r^D_t)}{\pi_{t+1}}, \tag{40}$$

and the Euler Equation for long-term government bonds

$$\frac{\partial U_0}{\partial B^{gov}_{h,t}} : 1 + \upsilon^{B,gov} \left(B^{gov,H}_{h,t} - B^{gov,H}_{h,s} \right) + \tau^{B,gov} = E_t \beta \frac{\lambda_{h,t+1}}{\lambda_{h,t}} \frac{\left(1 + r^{B,gov}_{t+1}\right)}{\pi_{t+1}}. \tag{41}$$

A.1.2 Wage setting

The households supply differentiated labour services ($N_{h,t}$) to the intermediate goods sector. Because of a monopolistically competitive labour market in which labour services are imperfect substitutes, each household has market power to set its nominal wage (W_t). Following Erceg et al. (2000), I assume, in analogy to Calvo pricing, that the household is not able to renegotiate its nominal wage each period. Instead, it can only reoptimise with a specific probability $(1 - \gamma_w)$. In periods in which the household cannot renegotiate, it follows an indexation rule $\tilde{W}_t = \tilde{\pi}_{w,t} W_{t-1}$, with

$$\tilde{\pi}_{w,t} = (\pi_{t-1})^{\xi_w} (\pi)^{1-\xi_w} (z_t)^{\xi_z} (z_s)^{1-\xi_z},$$

where ξ_w is the weighting parameter for the past rate of inflation and ξ_z the weighting parameter for the shock on the growth rate of technology z_t. Relatedly, z_s is the steady-state growth rate of a non-stationary productivity shock. Furthermore, the (stationary) real wage is defined as

$$\tilde{w}_t \equiv \frac{W_t}{Z_t P_t}$$

and the growth rate of nominal wages by taking technology growth into account becomes

$$\pi_{w,t+1} \equiv \pi_{t+1} z_{t+1} \frac{\tilde{w}_{t+1}}{\tilde{w}_t}.$$

A labour agency is introduced that buys differentiated labour from households and pays the individual wage in order to produce a representative labour aggregate as output

$$N_t = \left[\int_0^1 N_{h,t}^{\frac{1}{\lambda_w}} dh \right]^{\lambda_w}, \tag{42}$$

where λ_w is the degree of substitution and represents the mark-up of the wage over the household's marginal rate of substitution. By minimizing the costs of producing this aggregator, the labour agency takes the wage rates of each differentiated labour input as given. From this optimisation problem follows the demand for labour of household h for use in goods production

$$N_{h,t} = N_t \left(\frac{W_{h,t}}{W_t} \right)^{\frac{\lambda_w}{1-\lambda_w}}. \tag{43}$$

By combining Equations (42) and (43), one obtains the aggregate wage index

$$W_t = \left[\int_0^1 W_{h,t}^{\frac{1}{1-\lambda_w}} dh \right]^{1-\lambda_w}. \tag{44}$$

With the knowledge of demand for its labour, the household can proceed with determining the optimal wage rate ($W_{h,t}^*$) and the optimal labour supply ($N_{h,t}^*$). Thus, it maximises

$$\max_{\{W_{h,t}\}} E_t \sum_{s=0}^{\infty} (\beta\gamma_w)^s \left[-\kappa \frac{\left(N_{h,t+s}^*\right)^{1+\varphi}}{1+\varphi} + \lambda_{h,t+s} \frac{\Psi_{t+s}^w (1-\tau^w) W_{h,t}^*}{P_{t+s}} N_{h,t+s}^* \right] \tag{45}$$

by making use of Equation (43). The term φ reflects the inverse Frisch elasticity. The term Ψ_{t+s}^w in Equation (45) corrects the nominal wage for inflation and technology growth, i.e. $\Psi_{t+s}^w = \left(\prod_{s=0}^{\infty} \pi_{t-1+s}\right)^{\xi^w} (\pi^s)^{(1-\xi^w)} \left(\prod_{s=0}^{\infty} \frac{Z_{t-1+s}}{Z_{t-2+s}}\right) = \prod_{s=0}^{\infty} \tilde{\pi}_{w,t+s}$. Before utility maximisation is carried out, the optimal nominal wage emerges from a sub-problem in which the household minimises its disutility of labour by choosing its nominal wage given the labour demand of firms. With definitions $\bar{\Psi}_{t+s}^w \equiv \frac{\Psi_{t+s}^w}{P_{t+s} Z_{t+s}}$ and $\tilde{\lambda}_{h,t} \equiv \lambda_{h,t} Z_t$ one obtains for the optimal (stationary) real wage ($\tilde{w}_{h,t}^*$)

$$\partial / \partial W_{h,t} \quad :$$

$$\left(\tilde{w}_{h,t}^*\right)^{\frac{1-\lambda_w(1+\varphi)}{1-\lambda_w}} = \frac{E_t \sum_{s=0}^{\infty} (\beta\gamma^w)^s \left[\kappa \left(\bar{\Psi}_{t+s}^w \frac{\tilde{w}_t}{\tilde{w}_{t+s}} \right)^{\frac{\lambda_w}{1-\lambda_w}(1+\varphi)} N_{t+s}^{1+\varphi} \right]}{E_t \sum_{s=0}^{\infty} (\beta\gamma^w)^s \left[\tilde{\lambda}_{h,t+s} \bar{\Psi}_{t+s}^w (1-\tau^N) \tilde{w}_t \left(\bar{\Psi}_{t+s}^w \frac{\tilde{w}_t}{\tilde{w}_{t+s}} \right)^{\frac{\lambda_w}{1-\lambda_w}} N_{t+s} \left(\frac{1}{\lambda_w} \right) \right]},$$

where the term λ_w is the wage mark-up and φ reflects the inverse Frisch elasticity. The optimal real wage can be expressed as

$$\tilde{w}_t^\star = \left(\lambda_w \kappa \frac{NW_{l,t}}{\tilde{w}_t DW_{l,t}} \right)^{\frac{1-\lambda_w}{1-\lambda_w(1+\varphi)}}, \tag{46}$$

with

$$NW_t = N_t^{1+\varphi} + (\beta\gamma_w) \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}(1+\varphi)} NW_{t+1}$$

and

$$DW_t = \tilde{\lambda}_t \left(1-\tau^N\right) N_t + \gamma_w \beta \left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t} \right) \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}} \right)^{1+\frac{\lambda_w}{1-\lambda_w}} DW_{t+1}$$

The law of motion for the real wage is

$$w_t^{\frac{1}{1-\lambda_w}} = (1-\gamma_w)\ w_t^{*\,\frac{1}{1-\lambda_w}} + \gamma_w \left(\frac{\tilde{w}_t}{\pi_t}\right)^{\frac{1}{1-\lambda_w}}.$$

A.2 Intermediate goods firms

Before intermediate goods firms maximise profits they solve the sub-problem and minimise costs

$$\min_{\{\widetilde{K}^A_{i,t},\widetilde{K}^B_{i,t},\widetilde{N}_{i,t}\}} r_t^{k,A}\widetilde{K}^A_{i,t} + r_t^{k,B}\widetilde{K}^B_{i,t} + w_t\widetilde{N}_{i,t}$$

subject to the production function

$$Y_{i,t}, = A_t\left(\widetilde{K}_{i,t}\right)^{\alpha}\left(Z_t\widetilde{N}_{i,t}\right)^{1-\alpha} - Z_t\Omega_i, \tag{47}$$

and

$$\widetilde{K}_{i,t} = \left(\left(\zeta^K\right)^{\frac{1}{\gamma^K}}\left(\widetilde{K}^A_{i,t}\right)^{\frac{\gamma^K-1}{\gamma^K}} + \left(1-\zeta^K\right)^{\frac{1}{\gamma^K}}\left(\widetilde{K}^B_{i,t}\right)^{\frac{\gamma^K-1}{\gamma^K}}\right)^{\frac{\gamma^K}{\gamma^K-1}}.$$

The term represents a stationary technology shock, i.e. a shock on total factor productivity which is an AR(1) process

$$\log A_t = \rho_A \log A_{t-1} + \epsilon_{A,t},\ \epsilon_{A,t} \sim N\left(0,\sigma_A^2\right)$$

while Z_t is a labour-augmenting technology process with a stationary growth rate $z_t \equiv \frac{Z_t}{Z_{t-1}}$

$$\log\left(z_t\right) = \left(1-\rho_z\right)\log\ z_s + \rho_z\log\left(z_{t-1}\right) + \epsilon_{z,t},\ \epsilon_{z,t} \sim N\left(0,\sigma_z^2\right).$$

The first-order conditions are

$$\frac{\partial}{\partial\tilde{K}^A_{i,t}}:\ r_t^{k,A} - \varrho_{i,t}A_t\alpha\left(\left(\zeta^K\right)^{\frac{1}{\gamma^K}}\left(\tilde{K}^A_{i,t}\right)^{\frac{\gamma^K-1}{\gamma^K}} + \left(1-\zeta^K\right)^{\frac{1}{\gamma^K}}\left(\tilde{K}^B_{i,t}\right)^{\frac{\gamma^K-1}{\gamma^K}}\right)^{\frac{\alpha\gamma^K}{\gamma^K-1}-1} \tag{48}$$

$$\times\left(\zeta^K\right)^{\frac{1}{\gamma^K}}\left(\tilde{K}^A_{i,t}\right)^{\frac{-1}{\gamma^K}}\left(Z_tN_{i,t}\right)^{1-\alpha} \stackrel{!}{=} 0, \tag{49}$$

$$\frac{\partial}{\partial\tilde{K}^B_{i,t}}:\ r_t^{k,B} - \varrho_{i,t}A_t\alpha\left(\left(\zeta^K\right)^{\frac{1}{\gamma^K}}\left(\tilde{K}^A_{i,t}\right)^{\frac{\gamma^K-1}{\gamma^K}} + \left(1-\zeta^K\right)^{\frac{1}{\gamma^K}}\left(\tilde{K}^B_{i,t}\right)^{\frac{\gamma^K-1}{\gamma^K}}\right)^{\frac{\alpha\gamma^K}{\gamma^K-1}-1} \tag{50}$$

$$\times\left(1-\zeta^K\right)^{\frac{1}{\gamma^K}}\left(\tilde{K}^B_{i,t}\right)^{\frac{-1}{\gamma^K}}\left(Z_tN_{i,t}\right)^{1-\alpha} \stackrel{!}{=} 0, \tag{51}$$

$$\frac{\partial}{\partial N_{i,t}}: \quad w_t - \varrho_{i,t} A_t \left(\left(\zeta^K\right)^{\frac{1}{\gamma^K}} \left(\tilde{K}^A_{i,t}\right)^{\frac{\gamma^K-1}{\gamma^K}} + \left(1-\zeta^K\right)^{\frac{1}{\gamma^K}} \left(\tilde{K}^B_{i,t}\right)^{\frac{\gamma^K-1}{\gamma^K}} \right)^{\frac{\alpha\gamma^K}{\gamma^K-1}} \tag{52}$$

$$\times \left(1-\alpha\right) \left(Z_t\right)^{1-\alpha} \left(N_{i,t}\right)^{-\alpha} \stackrel{!}{=} 0, \tag{53}$$

and

$$\frac{\partial}{\partial \varrho_{i,t}} = A_t \left[\left(\zeta^K\right)^{\frac{1}{\gamma^K}} \left(\tilde{K}^A_{i,t}\right)^{\frac{\gamma^K-1}{\gamma^K}} + \left(1-\zeta^K\right)^{\frac{1}{\gamma^K}} \left(\tilde{K}^B_{i,t}\right)^{\frac{\gamma^K-1}{\gamma^K}} \right]^{\frac{\alpha\gamma^K}{\gamma^K-1}} \left(Z_t N_{i,t}\right)^{1-\alpha} - Z_t \Omega - Y_{i,t} \stackrel{!}{=} 0. \tag{54}$$

In Eqs. (48), (50), (52), and (54) $\varrho_{i,t}$ is the Lagrange multiplier related to the production function. By combining the derived conditions I obtain

$$\frac{\tilde{K}^A_{i,t}}{N_{i,t}} = \frac{\alpha}{(1-\alpha)} \left(\left(\zeta^K\right)^{\frac{1}{\gamma^K}} + \left(1-\zeta^K\right)^{\frac{1}{\gamma^K}} \left(\frac{\left(1-\zeta^K\right)}{\left(\zeta^K\right)} \right)^{\frac{\gamma^K-1}{\gamma^K}} \left(\frac{r^{k,A}_t}{r^{k,B}_t} \right)^{\gamma^K-1} \right)^{-1} \left(\zeta^K\right)^{\frac{1}{\gamma^K}} \frac{w_t}{r^{k,A}_t} \tag{55}$$

and

$$\frac{\tilde{K}^B_{i,t}}{N_{i,t}} = \frac{\alpha}{(1-\alpha)} \left(\left(\zeta^K\right)^{\frac{1}{\gamma^K}} \left(\frac{\left(\zeta^K\right)}{\left(1-\zeta^K\right)} \left(\frac{r^{k,B}_t}{r^{k,A}_t} \right)^{\gamma^K} \right)^{\frac{\gamma^K-1}{\gamma^K}} + \left(1-\zeta^K\right)^{\frac{1}{\gamma^K}} \right)^{-1} \left(1-\zeta^K\right)^{\frac{1}{\gamma^K}} \frac{w_t}{r^{k,B}_t}. \tag{56}$$

The capital-to-capital ratio is

$$\frac{\widetilde{K}^B_{it}}{\widetilde{K}^A_{it}} = \frac{\left(1-\zeta^K\right)}{\zeta^K} \left(\frac{r^{k,A}_{it}}{r^{k,B}_{it}} \right)^{\gamma^K}. \tag{57}$$

By integration over all i individuals it is easy to see that all indicies can be dropped. After expressing Eqs. (55) and (56) in terms of the other respective variables (K^A_t, K^B_t, and N_t), including the resulting expressions into the production function, and solving for the other variables which are included in the cost function, an expression for the marginal costs can be derived

$$
\begin{aligned}
mc_t &= \left(\begin{array}{c}
\left(r_t^{k,A}\right)^{\alpha} \left(\frac{\alpha}{1-\alpha}\right) \left(\left(\zeta^K\right)^{\frac{1}{\gamma^K}} + \left(1-\zeta^K\right)^{\frac{1}{\gamma^K}} \left(\frac{\left(1-\zeta^K\right)}{\left(\zeta^K\right)}\right)^{\frac{\gamma^K-1}{\gamma^K}} \left(\frac{r_t^{k,A}}{r_t^{k,B}}\right)^{\gamma^K-1} \right)^{-1} \left(\zeta^K\right)^{\frac{1-\alpha}{\gamma^K}} \\
+\left(r_t^{k,B}\right)^{\alpha} \left(\frac{\alpha}{1-\alpha}\right) \left(\left(\zeta^K\right)^{\frac{1}{\gamma^K}} \left(\frac{\left(\zeta^K\right)}{\left(1-\zeta^K\right)}\right)^{\frac{\gamma^K-1}{\gamma^K}} \left(\frac{r_t^{k,B}}{r_t^{k,A}}\right)^{\gamma^K-1} + \left(1-\zeta^K\right)^{\frac{1}{\gamma^K}} \right)^{-1} \left(1-\zeta^K\right)^{\frac{1-\alpha}{\gamma^K}} \\
+ \left(\begin{array}{c}
\left(\zeta^K\right)^{\frac{1}{\gamma^K}} \left(\left(\left(\zeta^K\right)^{\frac{1}{\gamma^K}} + \left(1-\zeta^K\right)^{\frac{1}{\gamma^K}} \left(\frac{\left(1-\zeta^K\right)}{\left(\zeta^K\right)}\right)^{\frac{\gamma^K-1}{\gamma^K}} \left(\frac{r_t^{k,A}}{r_t^{k,B}}\right)^{\gamma^K-1} \right)^{-1} \left(\zeta^K\right)^{\frac{1}{\gamma^K}} \frac{1}{r_t^{k,A}} \right)^{\frac{\gamma^K-1}{\gamma^K}} \\
+\left(1-\zeta^K\right)^{\frac{1}{\gamma^K}} \left(\left(\left(\zeta^K\right)^{\frac{1}{\gamma^K}} \left(\frac{\left(\zeta^K\right)}{\left(1-\zeta^K\right)} \left(\frac{r_t^{k,B}}{r_t^{k,A}}\right)^{\gamma^K}\right)^{\frac{\gamma^K-1}{\gamma^K}} + \left(1-\zeta^K\right)^{\frac{1}{\gamma^K}} \right)^{-1} \left(1-\zeta^K\right)^{\frac{1}{\gamma^K}} \frac{1}{r_t^{k,B}} \right)^{\frac{\gamma^K-1}{\gamma^K}}
\end{array} \right)^{-\frac{\alpha\gamma^K}{\gamma^K-1}}
\end{array} \right) \quad (58) \\
&\times \frac{\left(\frac{\alpha}{(1-\alpha)}\right)^{-\alpha} \left(w_t\right)^{1-\alpha}}{A_t}. \quad (59)
\end{aligned}
$$

Intermediate goods firms i maximise their profits

$$\max_{\{P_{i,t}^*\}} E_t \sum_{j=0}^{\infty} \beta^j \gamma^j \left[Y_{i,t}\left(P_{i,t}^* - mc_{i,t+j} P_{t+j}\right)\right] \tag{60}$$

subject to

$$Y_{i,t} = Y_t \left(\frac{P_{i,t}}{P_t}\right)^{\frac{\lambda_{p,t}}{1-\lambda_{p,t}}}.$$

From profit maximisation one obtains

$$\partial/\partial P_{i,t}^* : \; P_{i,t}^* = \lambda_{p,t} \frac{E_t \sum_{k=0}^{\infty} \beta^k \gamma^k \frac{P_{t+k}}{\psi_{t+k} P_t} Y_{i,t+k} \; mc_{i,t+k}}{E_t \sum_{k=0}^{\infty} \beta^k \gamma^k Y_{i,t+k}}, \tag{61}$$

where ψ_{t+k} captures the price indexation, i.e. $\left(\pi_{t-1}\right)^{\xi} (\pi)^{1-\xi}$ with indexation parameter ξ, and mc_t the marginal costs. Eq. (61) can be rewritten with $\pi_{i,t}^* \equiv \frac{P_{i,t}^*}{P_t}$

$$\pi_{i,t}^* = \lambda_{p,t} \frac{NP_{i,t}}{DP_{i,t}}, \tag{62}$$

whereas

$$NP_{i,t} = mc_{i,t} \lambda_t Y_t + \beta\gamma E_t \left[\left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}}\right)^{\frac{\lambda_{p,t+1}}{1-\lambda_{p,t+1}}} NP_{i,t+1} \right], \tag{63}$$

$$DP_{i,t} = \lambda_t Y_t + \beta\gamma E_t \left[\left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}}\right)^{\frac{\lambda_{p,t+1}}{1-\lambda_{p,t+1}}} DP_{i,t+1} \right], \tag{64}$$

$$\tilde{\pi}_t = \pi_{t-1}^{\xi} \, \pi_s^{1-\xi}, \tag{65}$$

and

$$1 = \gamma \left(\frac{\tilde{\pi}_t}{\pi_t}\right)^{\frac{1}{1-\lambda_{p,t}}} + (1-\gamma) \, \pi_t^{* \frac{1}{1-\lambda_{p,t}}}, \tag{66}$$

whereas $P^*_{i,t} = P^*_t$. Since all firms that can adjust the price optimally have the same optimum, I can drop the indexes.

A.3 Final goods firms

Final goods producers maximise profits

$$\max_{\{Y_{i,t}\}} P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} di \tag{67}$$

with

$$Y_t = \left[\int_0^1 Y_{i,t}{}^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}} \tag{68}$$

and the demand for good i results as

$$Y_{i,t} = Y_t \left(\frac{P_{i,t}}{P_t}\right)^{-\theta} \tag{69}$$

with which help the price aggregator can be derived:

$$P_t = \left[\int_0^1 P_{i,t}{}^{1-\theta} di\right]^{\frac{1}{1-\theta}}. \tag{70}$$

A.4 Capital goods producers

Capital producers maximise their profits

$$\max_{\{I_t^A, I_t^B\}} E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} Div^I_{t+j}.$$

with

$$\Psi\left(\frac{I_t^e}{I_{t-1}^e}\right) = \frac{1}{2}\left[\exp\left[\sqrt{\Psi''}\left(\frac{I_t^e}{I_{t-1}^e} - 1\right)\right] + \exp\left[-\sqrt{\Psi''}\left(\frac{I_t^e}{I_{t-1}^e} - 1\right)\right] - 2\right] \tag{71}$$

whereas $\Psi(1) = \Psi'(1) = 0$ and $\Psi'' > 0$ are satisfied. This results in

$$Q_t^{K,e} = \frac{1 - E_t\left(\frac{b_{t+1}\lambda_{t+1}\beta Q_{t+1}^{K,e}}{b_t\lambda_t}\right) \upsilon \left(\frac{E_t\left(I_{t+1}^e\right)}{I_t^e}\right)^2 \left(\frac{E_t\left(I_{t+1}^e\right)}{I_t} - 1\right)}{\left(1 - \frac{\upsilon}{2}\left(\frac{I_t^e}{I_{t-1}^e} - 1\right)^2 - \left(\frac{I_t^e}{I_{t-1}^e} - 1\right)\frac{\upsilon I_t^e}{I_{t-1}^e}\right)}. \tag{72}$$

The law of motion for capital is

$$K_t^e = K_{t-1}^e\left(1 - \delta^e\right) + I_t^e\left[1 - \Psi\left(\frac{I_t^e}{I_{t-1}^e}\right)\right]\mu_{I,t} \tag{73}$$

where $\mu_{I,t}$ is a shock to the marginal efficiency of investment and follows and AR(1) process

$$\log \mu_{I,t} = \rho_I \log \mu_{I,t-1} + \epsilon_{I,t} \, \epsilon_{I,t} \sim N\left(0, \sigma_I^2\right).$$

The shock hits to both sectors at the same time.

A.5 Entrepreneurs

Entrepreneurs borrow from financial intermediaries and combine the external funds with internal funds (net worth). They conduct capital processing which means that they buy capital and transform it with their own individual skill into new units of capital. The skills are randomly and independently distributed over time and across entrepreneurs. As a consequence, the shock also affects the realised return on capital, as denoted by $R^{k,e}_{m,t}$ and given in

$$\begin{aligned} 1 + R^{k,e,\omega}_{m,t} &= \pi_t \frac{\left(1-\tau^K\right)\left(r^{k,e}_{m,t} u^e_t - \Gamma(u^e_{m,t})\right) + Q^e_t(1-\delta^e) + \tau^K \delta^e Q^e_t}{Q^e_{t-1}} \omega^e_{m,t} \qquad (74) \\ &= (1 + R^{k,e}_{m,t}) \omega^e_{m,t}. \end{aligned}$$

Since the shock controls the repayment capacity of debt, there is a value for $\omega^e_{m,t}$ which shall be denoted with $\overline{\omega}^e_{m,t+1}$, below which defaults occur. With the contractual rate Z^e_t this threshold becomes

$$\overline{\omega}^e_{m,t} = \frac{Z^e_{t-1} L^e_{m,t-1}}{\left(1 + R^{k,e}_{m,t}\right) Q^e_{t-1} K^e_{m,t-1}}. \qquad (75)$$

The expected earnings of the m-th entrepreneur ($\mathcal{E}^e_{m,t}$) can be calculated based upon the expected capital return and the ex ante productivity threshold as

$$\mathcal{E}^e_{m,t} = E_t \left\{ \left(\begin{array}{c} \int^{\infty}_{E_t(\overline{\omega}^e_{m,t+1})} \omega^e dF\left(\omega^e\right) \\ - \left[1 - F\left(\overline{\omega}^e_{m,t+1}\right)\right] \overline{\omega}^e_{m,t+1} \end{array} \right) \left(1 + R^{k,e}_{t+1}\right) Q^e_t K^e_{m,t+1} \right\}.$$

The first term on the right-hand side characterises the expected earnings from the project by taking all realizations for $\omega^e_{m,t+1} \geq \overline{\omega}^e_{m,t+1}$ into account, and the second term on the right-hand side reflects the payments to satisfy the debt contract. For $\omega^e_{m,t+1} < \overline{\omega}^e_{m,t+1}$, the entrepreneur would be left with no earnings. The function $F\left(\omega^e_{m,t+1}\right)$ in Equation (76) is the cumulative density function for realization of ω^e_m which means that its value for $\overline{\omega}^e_{m,t+1}$ is the related ex ante default probability.

$$E_t\left(\overline{\omega}^e_{m,t+1}\right)\left(1 + E_t\left(R^{k,e}_{m,t+1}\right)\right) Q^e_t K^e_{m,t} = Z^e_t L^e_{m,t}, \qquad (76)$$

Given the expected gross return of the project and its value as well as the borrowed amount, this threshold is linked to the default-free risky bond rate with Z^e_t as the gross contract rate. The term $\overline{\omega}^e_{m,t}$ denotes the realised threshold value while $E_t\left(\overline{\omega}^e_{m,t+1}\right)$ is its expected value.[39]

[39] An important difference between the original BGG setting and my setting is that the intermediaries expect zero profits in the model, while in the BGG model zero profits hold every period (see, for instance, Benes and Kumhof, 2015).

One can make use of the following definitions. The expected profits of the financial intermediary related to the realization of the productivity shock can be expressed as

$$\Theta(\overline{\omega}) = \overline{\omega}\int_{\overline{\omega}}^{\infty} f(\varpi)d\varpi + \int_{0}^{\overline{\omega}} \varpi f(\varpi)d\varpi.$$

The first term on the right-hand side is the return stemming from all non-default cases, from which contractual payments result, while the expected pay-off in the case of defaults is captured by

$$G(\overline{\omega}) = \int_{0}^{\overline{\omega}} \varpi f(\varpi)d\varpi$$

which is the last term on the right-hand side. Consequently, expected monitoring costs of the financial intermediary are

$$\mu G(\overline{\omega}) = \mu\int_{0}^{\overline{\omega}} \varpi f(\varpi)d\varpi.$$

The expected expected net profits of financial intermediaries after paying for monitoring become $\Theta(\overline{\omega}) - \mu G(\overline{\omega})$. The expected profits of the entrepreneurs from all no-default cases are $1 - \Theta(\overline{\omega})$. Furthermore, I have the definitions

$$\begin{aligned} \Theta_{\varpi}(\overline{\omega}) &= 1 - F\left(\overline{\omega}\right) \\ G_{\varpi}(\overline{\omega}) &= \overline{\omega} f\left(\overline{\omega}\right) \\ f\left(\overline{\omega}\right) &= F_{\varpi}\left(\overline{\omega}\right) \end{aligned}$$

whereas X_Y is the first derivative of X with respect to Y.

The debt contract for the lenders reveals that the lenders want to earn as much as they receive by investing in a risk-free asset. That is the reason why lenders' opportunity costs must be equal to the risk-free rate. Where the idiosyncratic shock exceeds the cut-off value, the lender receives the contractual interest payments. The converse probability of the default probability at the cut-off value of the shocks yields the probability of contractual payments. For the range of realizations of the shocks that are below the ex ante cut-off value $\overline{\omega}^{e}_{m,t}$, the assets of the borrower are expected to be liquidated in order to partly redeem the debt contract. Before collecting the remaining assets, the lender has to observe the state of the borrower. Information is asymmetrically distributed, however. While the entrepreneur can always assess its situation, the financial intermediary cannot observe the state of the entrepreneur at no charge. As a consequence, the creditor has to pay transaction costs, which lower its repayments in the case of a default. It is assumed that the transaction costs are proportional to the realizable assets. These considerations are summarised in Equation (77)

$$\begin{aligned} & \left[1 - F\left(E_t\left(\overline{\omega}^{e}_{m,t+1}\right)\right)\right] Z^{e}_{m,t} L^{e}_{m,t} \\ & +(1-\mu^{e})\int_{0}^{E_t\left(\overline{\omega}^{e}_{m,t+1}\right)} \omega^{e}\left(1 + E_t\left(R^{k,e}_{t+1}\right)\right) Q^{e}_t K^{e}_{m,t} dF\left(\omega^{e}\right) \\ = \; & \left(1 + r^{e}_t\right) L^{e}_{m,t+1} \end{aligned} \tag{77}$$

or

$$\begin{aligned} & \left[1 - F\left(E_t\left(\overline{\omega}^e_{m,t+1}\right)\right)\right] E_t\left(\overline{\omega}^e_{m,t+1}\right)\left(1 + E_t\left(R^{k,e}_{t+1}\right)\right) Q^e_t K^e_{m,t} \\ & +(1-\mu^e)\int_0^{E_t\left(\overline{\omega}^e_{m,t+1}\right)} \omega^e\left(1 + E_t\left(R^{k,e}_{t+1}\right)\right) Q^e_t K^e_{m,t} dF\left(\omega^e\right) \\ = \; & \left(1 + r^e_t\right) L^e_{m,t} \end{aligned} \tag{78}$$

with $r^A_t = E_t\left(r^{B,corp}_{t+1}\right)$ and $r^B_t = r^L_t$.

Entrepreneurs have a long-run perspective and maximise the expected utility of continuing their business by choosing the quantity of capital $K^e_{m,t}$ and selecting the expected productivity threshold below which they default $E_t\left(\overline{\omega}^e_{m,t+1}\right)$.

$$V^{E,e}_{m,t} = \max_{\left\{K^e_{m,t}, \overline{\omega}^e_{m,t+1}\right\}} E_t\left[\sum_{i=1}^{\infty} \Lambda_{t,t+i}\left(1 - p^{E,e}_t\right)\left(p^{E,e}_t\right)^{i-1} \Pi^{E,e}_{m,t+i}\right],$$

whereas $p^{E,e}_t$ is the probability that an entrepreneur stays in business, $\Pi^{E,e}_{t+i}$ the terminal funds available for exiting entrepreneurs at $t+i$ which is simply their net worth at that period in time, i.e. $\Pi^{E,e}_{m,t} = NW^{E,e}_{m,t}$. The terminal funds are redistributed to the households. The variable $\Lambda_{t,t+j}$ represents the discount factor which is households' pricing kernel $\beta\frac{\lambda_{t+j}}{\lambda_t}$. The (expected) profits of entrepreneur (in nominal terms) is

$$\left(1 - \Theta\left(E_t\left(\omega^e_{m,t+1}\right)\right)\right) E_t\left(\bar{R}^{k,e}_{t+1}\right) Q^e_{m,t} K^e_{m,t}$$

while the (expected) profits for intermediaries (in nominal terms) is

$$\left(\Theta\left(E_t\left(\omega^e_{m,t+1}\right)\right) - \mu^e G\left(E_t\left(\omega^e_{m,t+1}\right)\right)\right) E_t\left(\bar{R}^{k,e}_{t+1}\right) Q^e_{m,t} K^e_{m,t}$$

where I used the gross return on capital $E_t\bar{R}^{k,e}_{t+1} = E_t\left(1 + R^{k,e}_{t+1}\right)$. The participation constraint for intermediaries in real terms becomes

$$\left(\Theta\left(E_t\left(\omega^e_{m,t+1}\right)\right) - \mu^e G\left(E_t\left(\omega^e_{m,t+1}\right)\right)\right) \frac{\bar{R}^{k,e}_{t+1}}{\pi_{t+1}} Q^e_t K^e_{m,t} \geq \frac{R^e_{t+1}}{\pi_{t+1}} L^e_{m,t}.$$

The value function in recursive form becomes

$$V^{E,e}_{m,t} = \left(1 - p^{E,e}_t\right) NW^{E,e}_{m,t} + \Lambda_{t,t+1} p^{E,e}_{t+1} V^{E,e}_{m,t+1}. \tag{79}$$

Next, I solve the optimisation problem and follow Carlstrom et al. (2015). First, I guess that the individual value function is a linear combination of an aggregate value function multiplied with individual net worth

$$V^{E,e}_{m,t} = V^{E,e}_t NW^{E,e}_{m,t}.$$

With its help Eq. (79) can be rewritten to obtain

$$V_t^{E,e} NW_{m,t}^{E,e} \;=\; \left(1 - p_t^{E,e}\right) NW_{m,t}^{E,e} + \Lambda_{t,t+1} p_t^{E,e} E_t V_{t+1}^{E,e} NW_{m,t+1}^{E,e}.$$

The maximisation problem becomes

$$\begin{aligned}
\max_{\left\{\kappa_{m,t}, \bar{\omega}_{m,t+1}^e\right\}} V_t^{E,e} \;&=\; E_t \left[\left(1 - p_t^{E,e}\right) + \Lambda_{t,t+1} p_t^{E,e} \left(1 - \Gamma\left(\bar{\omega}_{m,t+1}^e\right)\right) \frac{\bar{R}_{t+1}^{k,e}}{\pi_{t+1}} \kappa_{m,t}^e V_{t+1}^{E,e}\right] \\
s.t. \qquad & E_t \left[\left(\Theta\left(\bar{\omega}_{m,t+1}^e\right) - \mu^e G\left(\bar{\omega}_{m,t+1}^e\right)\right) \frac{\bar{R}_{t+1}^{k,e}}{\pi_{t+1}} \kappa_{m,t}^e NW_{m,t}^{E,e} \geq \frac{R_{t+1}^e}{\pi_{t+1}} \left(\kappa_{m,t}^e - 1\right) NW_{m,t}^{E,e}\right]
\end{aligned}$$

where $\kappa_{m,t}^e = \frac{Q_t^e K_{m,t}^e}{NW_{m,t}^e}$ is the leverage ratio.

The first-order conditions with $\phi_{m,t}$ as a Lagrangian multiplier are

$$\begin{aligned}
\partial/\partial\omega_{m,t}^e : \quad & -E_t \Bigg[\Lambda_{t,t+1} p_t^{E,e} \Theta_\omega\left(\omega_{m,t+1}^e\right) \tfrac{\bar{R}_{t+1}^{k,e}}{\pi_{t+1}} \kappa_{m,t}^e V_{t+1}^{E,e} \\
& + \phi_{m,t}^e \left(\left(\Theta_\omega\left(\bar{\omega}_{m,t+1}^e\right) - \mu^e G_\omega\left(\bar{\omega}_{m,t+1}^e\right)\right) \tfrac{\bar{R}_{t+1}^{k,e}}{\pi_{t+1}} \kappa_{m,t}^e\right)\Bigg] = 0 \qquad (80)
\end{aligned}$$

$$\begin{aligned}
\partial/\partial\kappa_{m,t:}^e \quad & E_t \Bigg[\Lambda_{t,t+1} p_t^{E,e} \left(1 - \Theta\left(\bar{\omega}_{m,t+1}^e\right)\right) \tfrac{\bar{R}_{t+1}^{k}}{\pi_{t+1}} V_{t+1}^{E,e} \\
& + \phi_{m,t}^e \left(\left(\Theta\left(\bar{\omega}_{m,t+1}^e\right) - \mu^e G\left(\bar{\omega}_{m,t+1}^e\right)\right) \tfrac{\bar{R}_{t+1}^{k,e}}{\pi_{t+1}} - \tfrac{R_{t+1}^e}{\pi_{t+1}}\right) \Lambda_{t,t+1}\Bigg] = 0 \qquad (81)
\end{aligned}$$

$$\partial/\partial\phi_{m,t}^e : \quad E_t \left[\left(\Theta\left(\bar{\omega}_{m,t+1}^e\right) - \mu^e G\left(\bar{\omega}_{m,t+1}^e\right)\right) \tfrac{\bar{R}_{t+1}^{k,e}}{\pi_{t+1}} \kappa_{m,t}^e - \tfrac{R_{t+1}^e}{\pi_{t+1}} \left(\kappa_{m,t}^e - 1\right)\right] = 0 \qquad (82)$$

The equations to include in the model are

$$E_t \left[\left(\Theta\left(\bar{\omega}_{m,t+1}^e\right) - \mu^e G\left(\bar{\omega}_{m,t+1}^e\right)\right) \frac{\bar{R}_{t+1}^{k,e}}{R_{t+1}^e}\right] = \frac{\kappa_t^e - 1}{\kappa_t^e} \qquad (83)$$

$$\begin{aligned}
0 \;=\; E_t \Bigg[& \Lambda_{t,t+1} p_t^{E,e} \left(1 - \Theta\left(\bar{\omega}_{m,t+1}^e\right)\right) \frac{\bar{R}_{t+1}^{k,e}}{R_{t+1}^e} V_{t+1}^{E,e} \\
& + \frac{\Lambda_{t,t+1} p_t^{E,e} \Theta_\omega\left(\bar{\omega}_{m,t+1}^e\right) V_{t+1}^{E,e}}{\left(\Theta_\omega\left(\bar{\omega}_{m,t+1}^e\right) - \mu^e G_\omega\left(\bar{\omega}_{m,t+1}^e\right)\right)} \left(\left(\Theta\left(\bar{\omega}_{m,t+1}^e\right) - \mu^e G\left(\bar{\omega}_{m,t+1}^e\right)\right) \frac{\bar{R}_{t+1}^{k}}{R_{t+1}^e} - 1\right)\Bigg] \qquad (84)
\end{aligned}$$

$$NW_t^{E,e} = \left(1 - \Theta\left(\bar{\omega}_{m,t}^e\right)\right) \frac{\bar{R}_t^{k,e}}{\pi_t} Q_{t-1}^e K_{t-1}^e = \left(1 - \Theta\left(\bar{\omega}_{m,t}^e\right)\right) \frac{\bar{R}_t^{k,e}}{\pi_t} \kappa_{t-1}^e NW_{t-1}^{E,e} \qquad (85)$$

$$V_t^{E,e} = \Lambda_{t,t+1}\left(1 - p_t^e\right) + E_t \Lambda_{t,t+1} p_t^{E,e} \left(1 - \Theta\left(\bar{\omega}_{m,t+1}^e\right)\right) \frac{\bar{R}_{t+1}^{k,e}}{\pi_{t+1}} V_{t+1}^{E,e} \kappa_t^e \qquad (86)$$

The external finance premium (or credit spread) can be defined as

$$FP_t^e = \frac{E_t\left(\bar{R}_{t+1}^{k,e}\right)}{E_t\left(R_{t+1}^e\right)},$$

with $FP_t^A = \frac{\left(1+E_t\left(R_{t+1}^{k,A}\right)\right)}{\left(1+E_t\left(r_{t+1}^{B,corp}\right)\right)}$ for type A and $FP_t^B = \frac{\left(1+E_t\left(R_{t+1}^{k,B}\right)\right)}{\left(1+r_t^L\right)}$ for type B entrepreneurs. Ex post losses can occur if the realization of the shock leaves the realised capital return below its expected value, so that the risky contract rate is not sufficient to compensate the intermediaries for all defaults.

$$\Upsilon_t^{B,e} = \begin{array}{c} \left[F(\bar{\omega}_t^e) - F\left(E_{t-1}\left(\bar{\omega}_t^e\right)\right)\right] Z_{t-1}^e L_{t-1}^e \\ +\left(1-\mu^{f,e}\right) \left(\begin{array}{c} K_{t-1}^e Q_{t-1}^e G\left(E_{t-1}\left(\bar{\omega}_t^e\right)\right)\left(1+E_{t-1}(R_t^{k,e})\right) \\ -K_{t-1}^e Q_{t-1}^e \left(1+R_t^{k,e}\right) G(\bar{\omega}_t^e) \end{array} \right) \end{array}. \tag{87}$$

The losses can be split up into two parts: the additional losses, because the realised default rate $F(\overline{\omega}_t^e)$ is above its ex ante value $F\left(E_{t-1}\left(\omega_t^e\right)\right)$ (upper line on the right-hand side), and the reduced amount of realizable assets through defaults (lower line on the right-hand side). To complete the description of the financial contracting between the entrepreneurs and the financial intermediaries, I introduce a time-varying standard deviation σ_t^e regarding the distribution of the productivity parameter in each sector by assuming an AR(1) process for the standard deviation in every sector

$$\log \sigma_t^e = \left(1-\rho^{\sigma,e}\right) \log \sigma_{t-1}^e + \epsilon_{\sigma,t}^e \, \epsilon_{\sigma,t}^e \sim N\left(0, \sigma_{\sigma,e}^2\right), \, e \in (A,B)$$

Thus, I have $F(\bar{\omega}_t^e;\sigma_{t-1}^e)$, $\Theta\left(\bar{\omega}_{m,t}^e;\sigma_{t-1}^e\right)$, and $G\left(\bar{\omega}_{m,t}^e;\sigma_{t-1}^e\right)$ which is also applicable to the first derivatives.

After choosing the amount of capital and the (expected) cut-off point given the corresponding intermediaries' expected zero-profit conditions, entrepreneurs decide on the capital utilization. Based on the assembled capital, they supply to intermediate goods producers and they decide on the utilization of capital at the second state

$$\widehat{K}_{m,t+1}^e = u_{m,t}^e \widetilde{K}_{m,t+1}^e. \tag{88}$$

They maximise the profits

$$\max_{\left\{u_{m,t}^e\right\}} \left[r_t^{k,e} u_{m,t}^e - \Gamma(u_{m,t}^e)\right] K_{m,t+1+j}^e$$

given the costs

$$\Gamma(u_{m,t}^e) = \frac{r^{k,e}}{\psi^k}\left(\exp\left[\psi^k\left(u_{m,t}^e - 1\right)\right] - 1\right). \tag{89}$$

In the optimum, the real cost of capital $(r_t^{k,e})$ is related to the adjustment costs on capital utilization (u_t^e)

$$\partial/\partial r_t^{k,e} : \, r_t^{k,e} = r^{k,e} \exp\left[\psi^{k,e}\left(u_t^e - 1\right)\right] \tag{90}$$

and is free of individual characteristics, i.e. every entrepreneur chooses the same utilization rate.

A.6 Mutual funds

In a decentralised market structure, different banks could earn different ex post returns (see Equation (76)) because some banks hold bonds issued by entrepreneurs that do not default, while some have purchased bonds from entrepreneurs that do. By introducing mutual funds, I easily circumvent this problem because I assume that mutual funds manage the market portfolio. Losses are redistributed to the bond holders via a reduced payoff.[40]

A.7 Banks

The balance sheet constraint of the bank is

$$A_{n,t}^{B} = L_{n,t} + Q_t^{B,corp} B_{n,t}^{corp} + Q_t^{B,gov} B_{n,t}^{gov} = E_{n,t}^{I} + D_{n,t} \tag{91}$$

and the law of motion for bank equity is

$$\begin{aligned} E_{n,t}^{I} &= \left(1 + r_{t-1}^{L}\right) L_{n,t-1} \frac{1}{\pi_t} + \left(1 + r_t^{B,corp}\right) Q_{t-1}^{B,corp} B_{n,t-1}^{corp} \frac{1}{\pi_t} + \left(1 + r_t^{B,gov}\right) Q_{t-1}^{B,gov} B_{n,t-1}^{gov} \frac{1}{\pi_t} \\ &\quad - \left(1 + r_{t-1}^{D}\right) \frac{1}{\pi_t} D_{n,t-1} \frac{1}{\pi_t} - \Upsilon_{n,t}^{L} + \mu_{EI,t}. \end{aligned} \tag{92}$$

which can be rewritten as

$$E_{n,t}^{I} = R_t^{A} A_{n,t-1}^{B} \frac{1}{\pi_t} - R_{t-1}^{D} D_{n,t-1} \frac{1}{\pi_t} - \Upsilon_{n,t}^{L} + \mu_{EI,t} \tag{93}$$

by using with R_t^A and R_t^D the gross interest rates and defining $R_t^A = 1 + r_t^A$ as the average (gross) return on assets held by banks

$$1 + r_t^{A} = \left(1 + r_{t-1}^{L}\right) \varsigma_{n,t}^{L} + \left(1 + r_t^{B,corp}\right) \varsigma_{n,t}^{B,corp} + \left(1 + r_t^{B,gov}\right) \varsigma_{n,t}^{B,gov}, \tag{94}$$

with $\varsigma_{n,t}^{L} = \frac{L_{n,t-1}}{A_{n,t-1}^{B}}$, $\varsigma_{n,t}^{B,corp} = \frac{Q_{t-1}^{B,corp} B_{n,t-1}^{corp}}{A_{n,t-1}^{B}}$, $\varsigma_{n,t}^{B,gov} = \frac{Q_{t-1}^{B,gov} B_{n,t-1}^{gov}}{A_{n,t-1}^{B}}$, and $\varsigma_{n,t}^{B,gov} = 1 - \varsigma_{n,t}^{L} - \varsigma_{n,t}^{B,corp}$. The term $\mu_{EI,t}$ in Equation (93) is a shock to bank equity and is assumed to follow the exogenous process

$$\mu_{EI,t} = \epsilon_{EI,t},\ \epsilon_{EI,t} \sim N\left(0, \sigma_{EI}^{2}\right).$$

Bank managers maximise the franchise value of the bank, which is equivalent to their terminal available funds $\Pi_{n,t}^{B} = E_{n,t}^{I}$, by choosing the portfolio compositon of their assets and the external funds

$$V_{n,t}^{B} = \max_{\left\{L_{n,t}, B_{n,t}^{corp}, B_{n,t}^{gov}, D_{n,t}\right\}} E_t \sum_{i=1}^{\infty} \Lambda_{t,t+i} \left(1 - p^{B}\right) \left(p^{B}\right)^{i-1} E_{n,t+i}^{I}. \tag{95}$$

[40] This setting is discussed in greater depth in Kühl (2014a).

Bank managers like to divert a specific fraction of assets, from which the incentive constraint can be derived

$$V^B_{n,t} \geq \theta^{IC} \left(L_{n,t} + \Delta^{B,corp} Q^{B,corp}_t B^{corp}_{n,t} + \Delta^{B,gov} Q^{B,gov}_t B^{gov}_{n,t} \right) . \tag{96}$$

Similar to Gertler and Karadi (2013), the assets can be diverted to different degrees; $\Delta^{B,corp}$ and $\Delta^{B,gov}$ denote the specific relative shares which can be diverted related to corporate and government bonds, respectively. The franchise value of the banks can be expressed in a linear form, which is

$$V^B_{n,t} = \nu^L_t L_{n,t} + \nu^{B,corp}_t Q^{B,corp}_t B^{corp}_{n,t} + \nu^{B,gov}_t Q^{B,gov}_t B^{gov}_{n,t} - \nu^D_t D_{n,t} - \eta^A_{n,t} E^I_{n,t} \ , \tag{97}$$

or

$$V^B_{n,t} = \upsilon^L_t L_{n,t} + \upsilon^{B,corp}_t Q^{B,corp}_t B^{corp}_{n,t} + \upsilon^{B,gov}_t Q^{B,gov}_t B^{gov}_{n,t} + \eta_t E^I_{n,t} \ ,$$

where $\bar{\eta}^A_{n,t}$ catches the losses from the loan portfolio and is expressed in terms of bank's equity. Thus, bank managers optimisation problem is

$$\max_{\left\{L_{n,t}, B^{corp}_{n,t}, B^{gov}_{n,t}\right\}} \upsilon^L_t L_{n,t} + \upsilon^{B,corp}_t Q^{B,corp}_t B^{corp}_{n,t} + \upsilon^{B,gov}_t Q^{B,gov}_t B^{gov}_{n,t} + \eta_t E^I_{n,t}$$

$$s.t. \quad V^B_{n,t} \geq \theta^{IC} \left(L_{n,t} + \Delta^{B,corp} Q^{B,corp}_t B^{corp}_{n,t} + \Delta^{B,gov} Q^{B,gov}_t B^{gov}_{n,t} \right) A^B_{n,t}.$$

The first-order conditions become

$$\partial / \partial L_{n,t} : \qquad \upsilon^L_t = \frac{\lambda^{IC}_{n,t}}{\left(1 + \lambda^{IC}_{n,t}\right)} \theta,$$

$$\partial / \partial B^{corp}_{n,t} : \ \upsilon^{B,corp}_t = \frac{\lambda^{IC}_{n,t}}{\left(1 + \lambda^{IC}_{n,t}\right)} \theta^{IC} \Delta^{B,corp},$$

$$\partial / \partial B^{gov}_{n,t} : \ \upsilon^{B,gov}_t = \frac{\lambda^{IC}_{n,t}}{\left(1 + \lambda^{IC}_{n,t}\right)} \theta^{IC} \Delta^{B,gov},$$

and

$\partial / \partial \lambda^{IC}_{n,t}$:

$$\frac{A^B_{n,t}}{E^I_{n,t}} = \frac{\eta_t}{\theta^{IC} \left(\varsigma^L_{n,t} + \Delta^{B,corp} \varsigma^{B,corp}_{n,t} + \Delta^{B,gov} \varsigma^{B,gov}_{n,t} \right) - \left(\upsilon^L_t \varsigma^L_{n,t} + \upsilon^{B,corp}_t \varsigma^{B,corp}_{n,t} + \upsilon^{B,gov}_t \varsigma^{B,gov}_{n,t} \right)}.$$

With the help of the method of undetermined coefficients, I get

$$\begin{aligned}
\upsilon_t^L &= E_t \Omega_{t,t+1} \Lambda_{t,t+1} \left(R_t^L - R_t^D\right) \frac{1}{\pi_{t+1}}, \\
\upsilon_t^{B,corp} &= E_t \Omega_{t,t+1} \Lambda_{t,t+1} \left(R_{t+1}^{B,corp} - R_t^D\right) \frac{1}{\pi_{t+1}}, \\
\upsilon_t^{B,gov} &= E_t \Omega_{t,t+1} \Lambda_{t,t+1} \left(R_{t+1}^{B,gov} - R_t^D\right) \frac{1}{\pi_{t+1}}, \\
\eta_t &= E_t \Omega_{t,t+1} \Lambda_{t,t+1} \left(R_t^D - \frac{\Upsilon_{n,t}^L}{E_{n,t}^I}\right) \frac{1}{\pi_{t+1}}, \\
\nu_t^L &= E_t \Omega_{t,t+1} \Lambda_{t,t+1} R_t^L \frac{1}{\pi_{t+1}} \\
\nu_t^{B,corp} &= E_t \Omega_{t,t+1} \Lambda_{t,t+1} R_{t+1}^{B,corp} \frac{1}{\pi_{t+1}}, \\
\nu_t^{B,gov} &= E_t \Omega_{t,t+1} \Lambda_{t,t+1} R_{t+1}^{B,gov} \frac{1}{\pi_{t+1}} \\
\nu_t^D &= E_t \Omega_{t,t+1} \Lambda_{t,t+1} \left(R_t^D\right) \frac{1}{\pi_{t+1}}, \\
\eta_{n,t}^A &= E_t \Omega_{t,t+1} \Lambda_{t,t+1} \left(\frac{\Upsilon_{n,t}^L}{E_{n,t}^I}\right) \frac{1}{\pi_{t+1}},
\end{aligned}$$

and

$$\Omega_{t,t+1} = \left(1 - p^B\right) + p^B \left[\left(\upsilon_{t+1}^L \varsigma_{t+1}^L + \upsilon_{t+1}^{B,corp} \varsigma_{t+1}^{B,corp} + \upsilon_{t+1}^{B,gov} \varsigma_{n,t+1}^{B,gov}\right) \phi_{t+1}^B + \eta_{t+1}\right].$$

Aggregation By combining all first-order conditions of the banking sector it can be shown that they are free from individual characteristics as long as $\frac{\Upsilon_{n,t}^L}{E_{n,t}^I}$ is identical to all lending banks. One necessary condition for this to hold is that the Lagrangian multiplier for the enforcement constraint is identical across all individuals. It is easy to show by forward iteration and the validity of the transversality condition that the term is (nearly) identical to all individuals in the neighbourhood of the steady state, i.e. as long as the sum of assets does not vary to much.[41] Thus, I can drop all indexes.

Knowing that the leverage ratio $\frac{A_t^B}{E_t^I}$ is identical to all lending banks, I see from the first-order conditions resulting from portfolio managers' maximisation problem that the portfolio shares related to the asset classes depend solely on the respective spreads between the (expected) lending rate and the expected return on government bonds which is consequently identical to all individuals. Hence, the portfolio composition is the same across all lending banks. Thus, aggregation of quantities across the individuals can simply be conducted by integration.

For the sum of assets I get $A_t^B = \int_0^1 A_{n,t}^B dn = \widetilde{\phi}_t^{IE} \int_0^1 E_{n,t}^I dn = \widetilde{\phi}_t^{IE} E_t^I$ and for each asset class $L_t = \int_0^1 L_{n,t} dn = \zeta_t^L \widetilde{\phi}_t^{IE} E_t^I$, $Q_t^{B,corp} B_t^{corp} = \int_0^1 Q_t^{B,corp} B_{n,t}^{corp} dn = \zeta_t^{B,corp} \widetilde{\phi}_t^{IE} E_t^I$,

[41] This is a conventional assumption, particularly in models that work with collateral constraints, see Iacoviello (2005), for instance.

and $Q_t^{B,gov} B_t^{gov} = \int_0^1 Q_t^{B,gov} B_{n,t}^{gov} dn = \zeta_t^{B,gov} \widetilde{\phi}_t^{IE} E_t^I$ respectively. The aggregation of liabilities works similarly and the aggregate amount of external finance evolves as $D_t = \left(\widetilde{\phi}_t^{IE} - 1\right) \int_0^1 E_{n,t}^I dn$.

A.8 Exogenous shocks

- Price mark-up shock. It follows an exogenous stochstic process

$$\log \lambda_{p,t} = \left(1 - \rho_{\lambda_p}\right) \log \lambda_p + \rho_{\lambda_p} \log \lambda_{p,t-1} + \epsilon_{p,t},$$

  with θ_p as the steady-state value and $\epsilon_{p,t} \sim i.i.d.N\left(0, \sigma_p^2\right)$.

- Bank equity shock

$$\mu_{EI,t} = \epsilon_{EI,t}.$$

- Risk shock for type A entrepreneurs with innovations $\epsilon_{\sigma,t}^A$, where the variance of log ω is σ_t^2

$$\log\left(\sigma_t^A\right) = \left(1 - \rho_\sigma^A\right) \log\left(\sigma^A\right) + \rho_\sigma^A \log\left(\sigma_{t-1}^A\right) + \epsilon_{\sigma,t}^A.$$

- Risk shock for type B entrepreneurs with innovations $\epsilon_{\sigma,t}^B$, where the variance of log ω is σ_t^2

$$\log\left(\sigma_t^B\right) = \left(1 - \rho_\sigma^B\right) \log\left(\sigma^B\right) + \rho_\sigma^B \log\left(\sigma_{t-1}^B\right) + \epsilon_{\sigma,t}^B.$$

- Net worth shock for type A entrepreneurs with innovations $\epsilon_{\gamma,t}^A$

$$\log\left(\gamma_t^{E,A}\right) = \log\left(\gamma_s^{E,A}\right) + \epsilon_{\gamma,t}^A.$$

- Net worth shock for type B entrepreneurs with innovations $\epsilon_{\gamma,t}^B$

$$\log\left(\gamma_t^{E,B}\right) = \log\left(\gamma_s^{E,B}\right) + \epsilon_{\gamma,t}^B.$$

- Government expenditures

$$\log G_t = (1 - \rho_G) \log G_{ss} + \rho_G \log G_{t-1} + \epsilon_{G,t}.$$

- Investment-specific technology shock

$$\log \mu_{I,t} = \rho_I \log \mu_{I,t-1} + \epsilon_{I,t}.$$

- Monetary policy shock $\epsilon_{mp,t}$.
- Stationary shock on total factor productivity

$$\log A_t = \rho_A \log A_{t-1} + \epsilon_{A,t}$$

- Non-stationary shock on technology Z_t with stationary growth rate

$$\log(z_t) \equiv \log(Z_t/Z_{t-1}) = (1-\rho_z)\log(z_s) + \rho_z \log(Z_{t-1}/Z_{t-2}) + \epsilon_{z,t}$$

,

- Shock on labour supply

$$\log \nu_t^N = \rho_N \log \nu_{t-1}^N + \epsilon_{N,t}.$$

- Shock on lump sum taxes

$$\mu_{T,t} = \rho_T \mu_{T,t-1} + \epsilon_{T,t}.$$

- Measurement errors on growth of entrepreneurs' franchise value, $\epsilon^{measurement}_{V^{E,A}t}$ and $\epsilon^{measurement}_{V^E t}$, the yield on government bonds $\epsilon^{measurement}_{B^{gov,B}t}$, and the volume of government bonds held by banks $\epsilon^{measurement}_{r^{B,gov}t}$.

B Observables

- GDP growth

$$dGDP_{obs} = \log(I_t + C_t + G_t) - \log(I_{t-1} + C_{t-1} + G_{t-1}) + \log z_t - \log z_s$$

- Investment growth

$$dI_{obs} = \log I_t - \log I_{t-1} + \log z_t - \log z_s$$

- Consumption growth

$$dC_{obs} == \log C_t - \log C_{t-1} + \log z_t - \log z_s$$

- Rate of inflation

$$\pi_{obs} = (\pi_t - \pi_s) \times 400$$

- Corporate bond rate

$$Z^A_{obs} = \left(Z^A_t - Z^A_s\right) \times 400$$

- Loans rate

$$Z^B_{obs} = \left(Z^B_t - Z^B_s\right) \times 400$$

- Policy rate

$$i_{obs} = (i_t - i_s) \times 400$$

- Growth in real wages

$$dw_{obs} = \log w_t - \log w_{t-1} + \log z_t - \log z_s$$

- Growth in bank loans

$$dL_{obs} = \log L_t - \log L_{t-1} + \log z_t - \log z_s$$

- Total employment

$$E_{obs} = \log E_t - \log E_s$$

- Growth in franchise value of entrepreneurs in sector A

$$dV_{obs}^{E,A} = \log V_t^{E,A} - \log V_t^{E,A} + \log z_t - \log z_s + \epsilon_{V^{E,A}t}^{measurement}$$

- Growth in franchise value of entrepreneurs

$$dV_{obs}^{E} = \log V_t^{E} - \log V_t^{E} + \log z_t - \log z_s + \epsilon_{V^{E}t}^{measurement},$$

  with $V_t^E = V_t^{E,A} + V_t^{E,B}$.

- Growth in bank holdings of corporate bonds

$$d\left(Q^{B,corp}B^{corp}\right)_{obs} = \log Q_t^{B,corp}B_t^{corp} - \log Q_{t-1}^{B,corp}B_{t-1}^{corp} + \log z_t - \log z_s$$

- Growth in bank holdings of government bonds

$$d\left(Q^{B,gov}B^{gov,B}\right)_{obs} = \log Q_t^{B,gov}B_t^{gov,B} - \log Q_{t-1}^{B,gov}B_{t-1}^{gov,B} + \log z_t - \log z_s + \epsilon_{B^{gov,B}t}^{measurement}$$

- Growth in stock of government bonds outstanding

$$d\left(Q^{B,gov}B^{gov}\right)_{obs} = \log Q_t^{B,gov}B_t^{gov} - \log Q_{t-1}^{B,gov}B_{t-1}^{gov} + \log z_t - \log z_s$$

- Yield on government bonds

$$r_{obs}^{B,gov} = \left(r_{ytm,t}^{B,gov} - r_{ytm,s}^{B,gov}\right) \times 400 + \epsilon_{r^{B,gov}t}^{measurement},$$

  with $r_{ytm,t}^{B,gov} = \left(1 + r_t^{B,gov}\right)\frac{Q_{t-1}^{B,gov}}{Q_t^{B,gov}} - 1$.

- Growth in bank equity

$$dE_{obs}^{I} = \log E_t^{I} - \log E_t^{I} + \log z_t - \log z_s.$$

C Data

- **Consumption (real):** Individual consumption expenditure - Euro area 18 (fixed composition) - World (all entities, including reference area, including IO), Households and non profit institutions serving households (NPISH), Euro, Chain linked volume (rebased), Non transformed data, Calendar and seasonally adjusted data. Source: European Central Bank,
  MNA.Q.Y.I7.W0.S1M.S1.D.P31._Z._Z._T.EUR.LR.N.

- **Investment (real):** Gross fixed capital formation - Euro area 18 (fixed composition) - World (all entities, including reference area, including IO), Total economy, Fixed assets by type of asset (gross), Euro, Chain linked volume (rebased), Non transformed data, Calendar and seasonally adjusted data.Source: European Central Bank,
MNA.Q.Y.I7.W0.S1.S1.D.P51G.N11G._T._Z.EUR.LR.N.

- **Gross domestic product:** Gross domestic product at market prices - Euro area 18 (fixed composition) - Domestic (home or reference area), Total economy, Euro, Chain linked volume (rebased), Non transformed data, Calendar and seasonally adjusted data.Source: European Central Bank,
MNA.Q.Y.I7.W2.S1.S1.B.B1GQ._Z._Z._Z.EUR.LR.N.

- **GDP Deflator:** Gross domestic product at market prices - Euro area 18 (fixed composition) - Domestic (home or reference area), Total economy, Index, Deflator (index), Non transformed data, Calendar and seasonally adjusted data. Source: European Central Bank,
MNA.Q.Y.I7.W2.S1.S1.B.B1GQ._Z._Z._Z.IX.D.N.

- **Bank equity:** Euro area (changing composition), Outstanding amounts at the end of the period (stocks), MFIs excluding ESCB reporting sector - Capital and reserves, All currencies combined - World not allocated (geographically) counterpart, Unspecified counterpart sector sector, denominated in Euro, data Neither seasonally nor working day adjusted. Source: European Central Bank,
BSI.M.U2.N.A.L60.X.1.Z5.0000.Z01.E.

- **Loans:** Euro area (changing composition), Outstanding amounts at the end of the period (stocks), MFIs excluding ESCB reporting sector - Loans, Total maturity, All currencies combined - Euro area (changing composition) counterpart, Non-Financial corporations (S.11) sector, denominated in Euro, data Neither seasonally nor working day adjusted. Source: European Central Bank,
BSI.M.U2.N.A.A20.A.1.U2.2240.Z01.E.

- **Corporate bond holdings in banking sector:** Euro area (changing composition), Outstanding amounts at the end of the period (stocks), MFIs excluding ESCB reporting sector - Debt securities held, Total maturity, All currencies combined - Euro area (changing composition) counterpart, Non-MFIs excluding general government sector, denominated in Euro, data Neither seasonally nor working day adjusted. Source: European Central Bank,
BSI.M.U2.N.A.A30.A.1.U2.2200.Z01.E.

- **Government bond holdings in banking sector:** Euro area (changing composition), Outstanding amounts at the end of the period (stocks), MFIs excluding ESCB reporting sector - Debt securities held, Total maturity, All currencies combined - Euro area (changing composition) counterpart, General Government sector, denominated in Euro, data Neither seasonally nor working day adjusted Source: European Central Bank,
BSI.M.U2.N.A.A30.A.1.U2.2100.Z01.E.

- **Outstanding amount of government bonds:** Outstanding amounts of euro-denominated long-term debt securities issued by general government in Euro area (changing composition) Outstanding amounts at the end of the period (stocks), Long-term securities other than shares, Nominal value, General government issuing sector, Euro, denominated in Euro,Euro area (changing composition) Source: European Central Bank,
SEC.M.U2.1300.F33200.N.1.EUR.E.Z.

- **Yields on government bonds:** BOFA ML GERMAN FED GVT ALL MATS(E) - RED. YIELD. Source: Datastream, MLBDAME(RY).

- **Yields on non-fin. corp. bonds:** BOFA ML EUR NON-FIN. BBB (E) - RED. YIELD. Source: Datastream, MLNF3BE(RY).

- **Loan Rates:** Euro area (changing composition), Annualised agreed rate (AAR) / Narrowly defined effective rate (NDER), Credit and other institutions (MFI except MMFs and central banks) reporting sector - Loans, Total original maturity, Outstanding amount business coverage, Non-Financial corporations (S.11) sector, denominated in Euro
MIR.M.U2.B.A20.A.R.A.2240.EUR.O
Monetary Union (MU), Credit institutions and other MFIs reporting sector - Interest rate (Unspecified rate type) on Loans, Over 1 year maturity, New business coverage, denominated in All currencies combined - Non-Financial corporations (S.11) counterpart sector
RIR.M.U2.A.A20.K.2240.Z01.N.Z.R
Monetary Union (MU), Credit institutions and other MFIs reporting sector - Interest rate (Unspecified rate type) on Loans, Up to 1 year maturity, Unspecified business coverage, denominated in All currencies combined - Non-Financial corporations (S.11) counterpart sector. Source: European Central Bank,
RIR.M.U2.A.A20.F.2240.Z01.Z.Z.R .

- **Entrepreneurial net worth (aggregate):** Equity/index; Dow Jones Euro Stoxx Price Index, Average of observations through period (A). Source: European Central Bank,
FM.M.U2.EUR.DS.EI.DJEURST.HSTA.

- **Entrepreneurial net worth (sector A):** Equity/index; Dow Jones Euro Stoxx 50 Price Index, Average of observations through period (A). Source: European Central Bank,
FM.M.U2.EUR.DS.EI.DJES50I.HSTA.

D The optimality of the debt contract between entrepreneurs and banks

For the contract described in Bernanke et al. (1999), it is assumed that financial intermediaries are risk-neutral and earn zero profits. The participation constraint of the financial intermediaries states in this case that they need to earn as much as they have to pay for their debt. Given the participation constraint and the verification requirement for defaults statements which are related to monitoring costs, entrepreneurs have no incentive to deviate from the conditions of the contract. From this point of view, the contract is globally optimal. Although banks are risk-neutral in my case, they do not earn zero profits as a result of financial frictions between the banking sector and the households. I argue that the contract is locally optimal from the banks' point of view. Regarding the loan and the corporate bond portfolio, bank managers can earn a rate which is ex ante free from default risk as a result from perfect portfolio diversification. As a consequence of the bank-specific financial frictions related to different shares of diversion, the default-free loan rates differ, whereas the relationship is governed by the first-order conditions. In order to take the portfolio diversification issues and the constraints regarding their diverting behaviour into account, bank managers demand different loan rates to participate in the contract with the entrepreneurs. For this reason, bank managers try to maximise the pay-off resulting from each individual contract. The value of each loan contract between the n-th bank and the m-th entrepreneur becomes

$$V_{n,m,t}^{B,L} = E_t \sum_{j=1}^{\infty} \Lambda_{t,t+1} \left(\tilde{R}_{m,t-1+j}^{L} - R_{t-1+j}^{L} \right) L_{n,m,t-1+j}$$

and for purchases of corporate bonds

$$V_{n,m,t}^{B,Bcorp} = E_t \sum_{j=1}^{\infty} \Lambda_{t,t+1} \left(\tilde{R}_{m,t+j}^{B,corp} - R_{t+j}^{B,corp} \right) Q_{t-1+j}^{B,corp} B_{n,m,t-1+j}^{corp}.$$

The gross returns with tildes are the rates the bank managers require to participate. Maximisation yields

$$\frac{\partial V_{n,m,t}^{B,L}}{\partial L_{n,m,t}} = E_t \Lambda_{t,t+1} \left(\tilde{R}_{m,t}^{L} - R_t^{L} \right) = 0$$

and

$$\frac{\partial V_{n,m,t}^{B,L}}{\partial B_{n,m,t}^{corp}} = E_t \Lambda_{t,t+1} \left(\tilde{R}_{m,t+1}^{B,corp} - R_{t+1}^{B,corp} \right) Q_t^{B,corp} = 0.$$

Up to first order this means that $\tilde{R}_{m,t}^{L} = R_t^{L}$ and $E_t \left(\tilde{R}_{m,t+1}^{B,corp} \right) = E_t \left(R_{t+1}^{B,corp} \right)$. Hence, bank managers need to participate in the contract to earn the group-specific default-free rate, which means that they participate in the contract only if R_t^L or $E_t \left(R_{t+1}^{B,corp} \right)$, depending on the group, can be earned as a minimum.

E The optimality of the debt contract between banks and households

I formulate a two-sided agency problem between non-financial firms and banks and between banks and households. The way in which I solve the contracting problem between banks and households and the agency problem between the non-financial sector and banks does not seem to be a major factor. However, the two problems could be interrelated because the outcome of the contract in the non-financial sector might affect the bankers' decision to run. If the incentive constraint holds, bankers will not run. In this section I show that the combination of the two agency problems does not alter the general conclusion.

The value of the bank under the no-default case is $V_t^{B,no-run}$ which is equivalent to the value that appears for continuing business. The value under running is

$$V_t^{B,run} = E_t^I - \left(1 - \theta^{IC}\right) \left(L_{n,t} + \Delta^{B,corp} Q_t^{B,corp} B_t^{corp} + \Delta^{B,gov} Q_t^{B,gov} B_t^{gov}\right) + D_t,$$

where $\Delta^{B,corp}$ and $\Delta^{B,gov}$ denote asset-specific diversion shares. With the help of banks' balance sheet I can write

$$V_t^{B,run} = \theta^{IC} \left(L_t + \Delta^{B,corp} Q_t^{B,corp} B_t^{corp} + \Delta^{B,gov} Q_t^{B,gov} B_t^{gov}\right).$$

Thus, there is no incentive for the bankers to run if

$$V_t^{B,no-run} \geq \theta^{IC} \left(L_t + \Delta^{B,corp} Q_t^{B,corp} B_t^{corp} + \Delta^{B,gov} Q_t^{B,gov} B_t^{gov}\right)$$

holds.

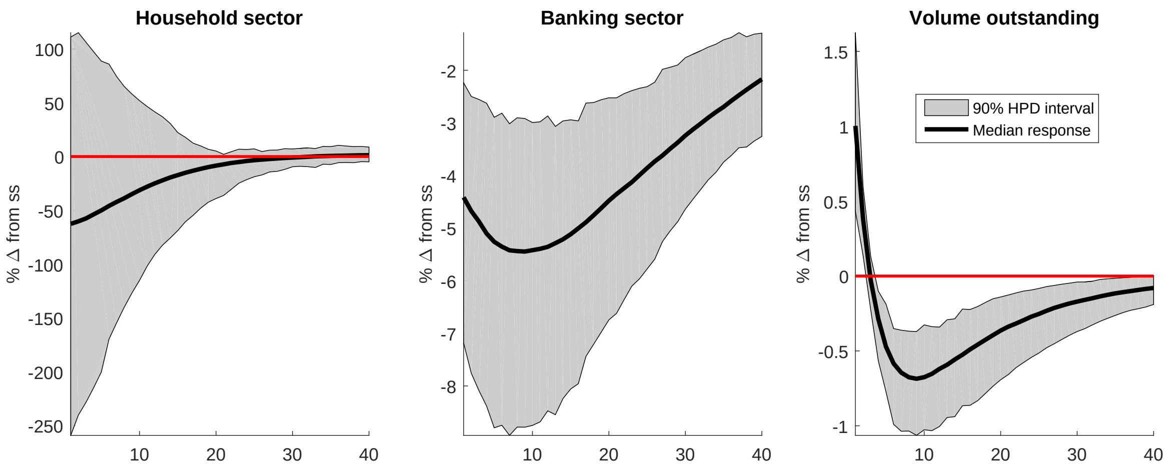
As described in the previous section, entrepreneurs from the non-financial sector sign a contract without the incentive to deviate from its conditions. For this reason, potential misbehaviour by entrepreneurs can be ruled out and does not affect the decision of bankers to run. However, ex post losses from the loan portfolio or from the bond holdings can have an impact on bankers' decision to run as they reduce the realised value of the bank. Following the arguments of Iacoviello (2005) raised for collateral constraints, the incentive constraint is binding in the neighbourhood of the steady state, i.e. as long as the shocks are not too large. This is true in my case because unexpected losses play a minor role compared to expected losses which are completely captured through the external finance premium.

F Portfolio rebalancing effects from government bond purchases

In Figure (13) I show how the purchases of government bonds by the intervention authority affect the portfolio holdings of government bonds in the household sector and the banking sector and how they influence the supply of government bonds. The purchases of government bonds by the intervention authority lead to a redistribution of government bond holdings from the private sector to the public sector. The lower returns of government bonds as a consequence of the purchases causes households and banks to reduce their holdings, whereas the majority of the intervention authority's purchases stem from the banking sector.[42] At the same time government bond supply is lower for two reasons. On the one hand, the fiscal agent collects more taxes as consumption, capital, and labour increase. On the other hand, the fiscal authority also benefits from lower borrowing costs.

[42]It should be noted that there is a large degree of uncertainty around the median responses of the household sector, reflected by the HPD intervals covering positive and negative values.

Figure 13: Effects of government bond purchases on portfolio holdings

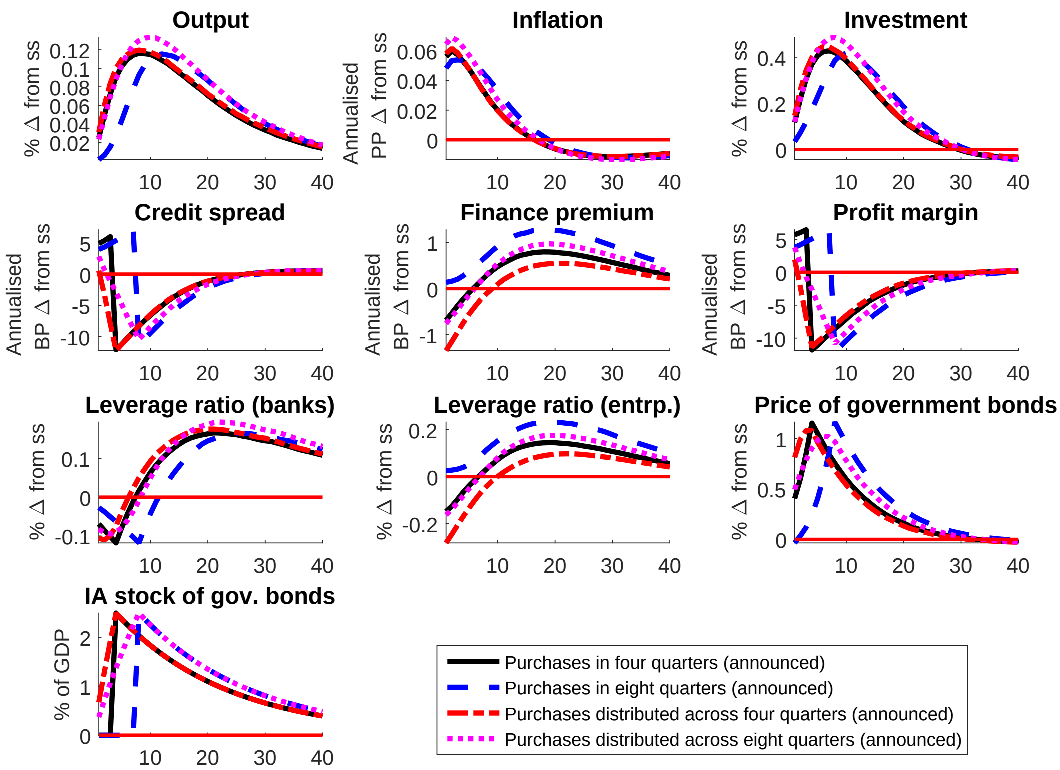


Notes: The figure presents the effects of government bond purchases which are induced as “purchase shocks” as presented in Equation (32) with $N = 0$. The black solid lines represent the median from a purchase shock amounting to 2. 5% of gross domestic product (GDP). The grey areas show the 90% highest posterior density intervals.

G The stock and flow effects in announced and anticipated programmes

Next, I compare the cases of distributing the purchases across a specific period in time. In Figure 14 I treat four cases, whereas two of them are stemming from Figure 4. The black solid lines and the blue dashed lines refer to the cases in which purchases are announced in advance to occur in four and eight quarters, respectively. After the purchases have stopped, the stock dissipates over time. In addition I present results for cases treated before, i.e. cases in which the purchases are distributed across four (red dashed lines with dots) and eight quarters (magenta dotted lines), respectively. The maximum stock of government bonds held by the intervention authority is in all cases the same but the time profile of holdings is different. In the new two cases the purchases are not distributed across periods which means that the purchases in every period are obviously larger than for the two other programmes. By investigating an announced one-off programme, I am basically able to treat an anticipation effect related to the expected stock of government bonds held by the authority. With the two other programmes there is also an anticipation effect but it is additionally related to the expected stock and to the expected path of purchases. By comparing the two different programmes for each of the two cases, I am able to identify flow effects of purchases. As can be seen in Figure 14, the qualitative responses of the real economy do not change. Nevertheless, the maximum effects on output and investment are slightly stronger which can solely be related to the anticipation effects. Thus, the stronger boost in investment results mainly from the expectational effect of agents. Since agents anticipate the drop in borrowing conditions, the real sector starts to produce more capital. Hence, output is driven by more investment and the wealth effect stimulates consumption. However, there are remarkable changes in the financial sector. A stock effect again pushes bank profits upwards until the period in which the purchases end. Obviously, flow effects through the purchases contract bank equity more quickly.

Figure 14: Comparison of responses to a one-period government bond purchase programme (black solid lines), and previously announced programmes distributed over one (blue dashed lines) and three years (red dashed lines with dots)



Notes: The figure presents the effects of government bond purchases which are induced as "purchase shocks" as presented in Equation (32). The purchases are scaled to achieve a maximal stock of 2.5% of GDP in every case. The responses are median responses from the estimated model.

For the announcement cases including the purchases' distribution over time, a small amount of purchases is already being conducted in the first period; however, the lion's share of the volume is reached in later periods. With a longer duration of purchases, the anticipation effect more than offsets again the portfolio rebalancing effect, which depresses banks' profit margins on impact and weakens bank net worth. Only once the entire volume is realised does the portfolio rebalancing channel start to dominate, weakening the balance sheet of the banks. It seems as distributing the purchases over a time while reaching the same maximum stock at the same time, produces larger stabilisation gains in the real economy for a longer duration of the programme. However, the evaluation of the quantitative effects on output in Figure 14 can be misleading because the time profile of intervention authority's balance sheet is different. For taking a size effect into account I refer again to present value gains which are shown Table 4 and also draw on the present value gains given in Table 5 for the other cases. In the case where purchases are distributed across the periods, the long-run gain in output is not stronger compared to the announced purchases in one period. An output gain of 1.479% in terms of a cumulated balance sheet can be realised in ten years for the distribution of purchaes over eight quarters while a output gain of 1.483% is achieved for purchases conducted in eight periods. For the two accounced ad hoc programmes the long-run effects are quantitatively very similar. It turns out that the present value gain in output is larger for the announced one-off programmes compared to the distribution of the purchases over time. This indicates that a flow effect is responsible for the differences among the cases which materialises in a quicker drop of bank equity in the case of distributing the purchases over time.

Table 5: Present value gains in output following from announced one-off programmes

in %	1 year	10 years
Purchases in four quarters, announced	2.38	1.479
Purchases in eight quarters, announced		1.483

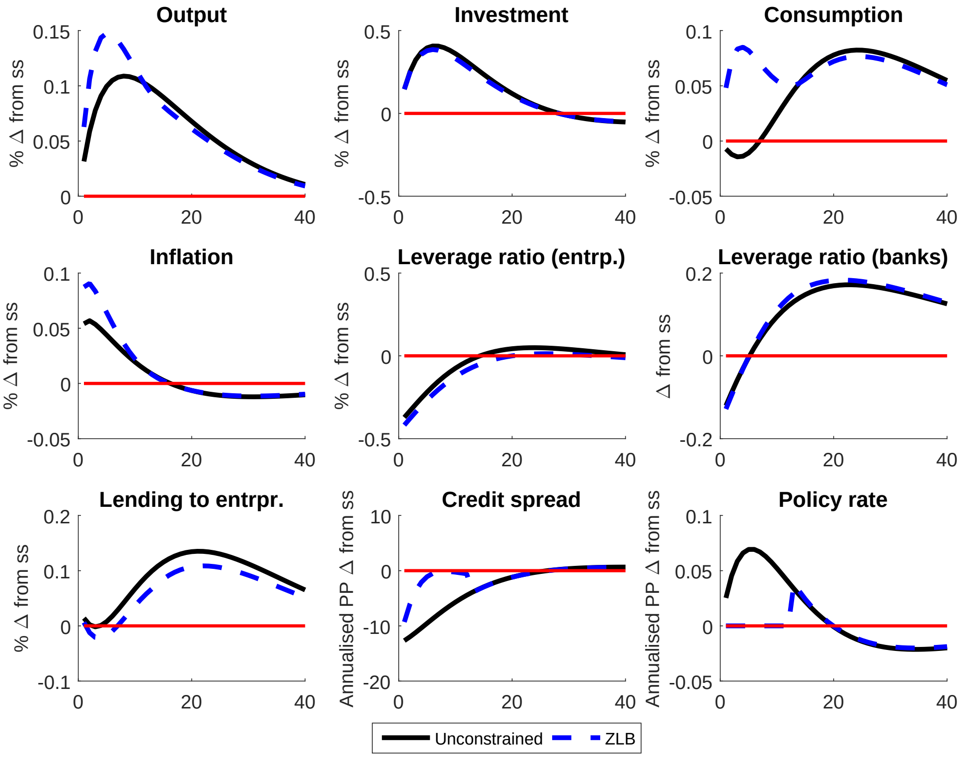
Notes: The table shows the present value gains in output over a specified period for government bond purchases. The gains in output are expressed in percentage deviations from steady state and are weighted with the time-preference rate. The present value gain is defined as: $gain = \sum_{k=1}^{K} \beta^k \left(X_{t+k-1} - X_s\right) / \sum_{k=1}^{K} \beta^k \left(Z_{t+k-1}\right) \cdot 100$, with X as output and Z the stock of government bonds held by the central bank. Since the balance sheet is zero for announced purchases taking place in the eighth quarter, the present value gains for the first year are not computed.

H Government bond purchases and a lower bound on the interest rate

The results in the main text are derived by letting the policy rate adjust endogenously according to the Taylor rule. Since government bond purchases eventually raise the rate of inflation, the policy rate rises as a consequence. However, government bond purchases have been introduced by central banks in an environment in which the policy rate reached its effective lower bound. In order to investigate the effects of government bond purchases for an lower bound scenario, I approximate this environment by keeping the policy rate constant for a specific time period.[43] In Figure 15 I compare the benchmark case from the main text (surprise programme) with the generic effective lower bound case (denoted by ZLB). The results from the main text do not change qualitatively by introducing a period of a constant policy rate. As a result from preventing the policy rate to rise, consumption improves on impact in the lower bound case which boosts output by more compared to the benchmark case. This is also the reason why inflation rises by more. Investment behaves similarly in both cases. Lending to the entrepreneurs is on a lower trajectory in the lower bound case because entrepreneurs' leverage ratio drops by more showing that investment is financed to a greater extent through internal funds, i.e. net worth. The build-up of net worth is stronger because the rise of the policy rate in the benchmark case slightly offset the improvement in borrowing conditions.

[43]Technically, this is imposed by anticipated shocks to the policy rate. Agents consequently know the exact time profile for the policy rate for buidling expectations accordingly.

Figure 15: Effects of government bond purchases and a lower bound on the policy rate



Notes: The figure presents the effects of government bond purchases which are induced as "purchase shocks" as presented in Equation (32) by keeping the policy rate (blue dashed lines) for three years constant. These cases are contrasted with the unconstrained benchmark case (black solid lines). The purchases are scaled to achieve a maximal stock of 2.5% of GDP in every case. The responses base upon the simulation of the model at its mode.